POLITECNICO DI MILANO

From dry granular flows to collisional sediment transport

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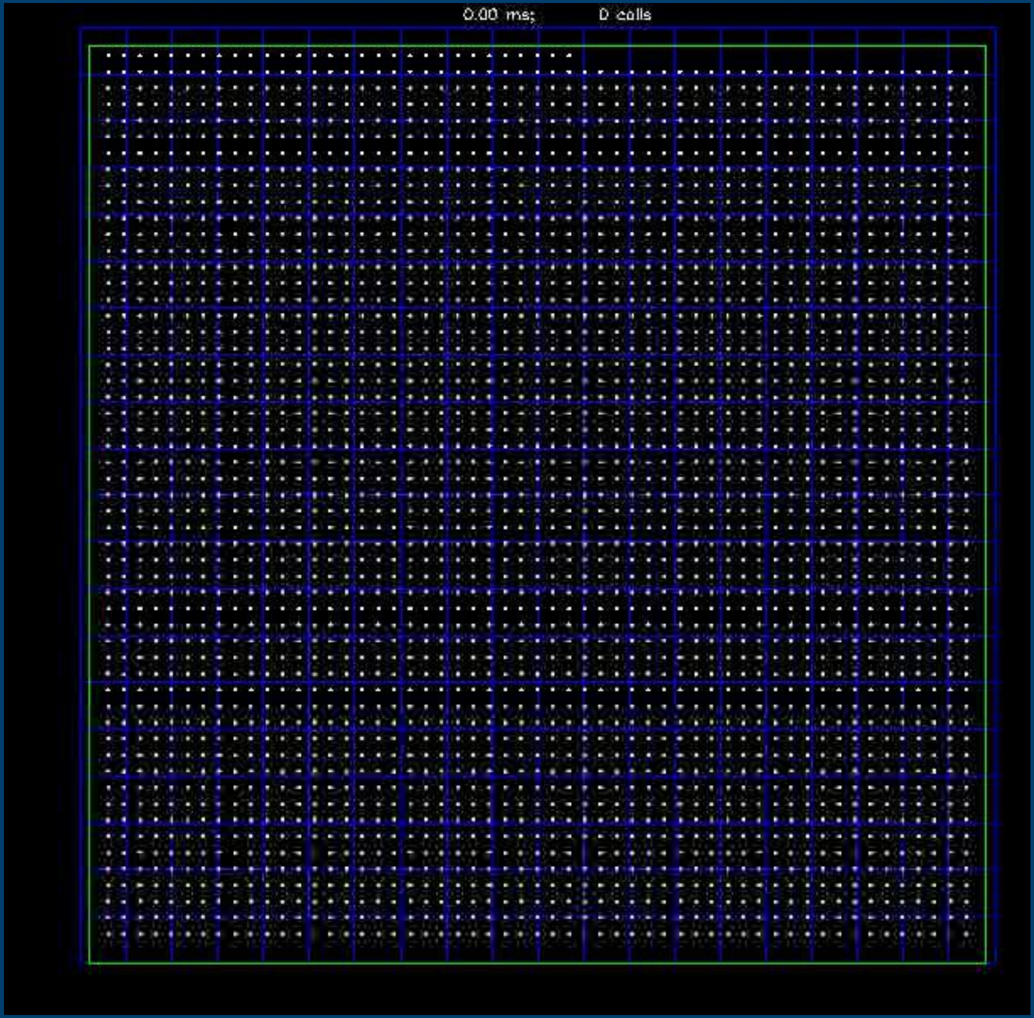
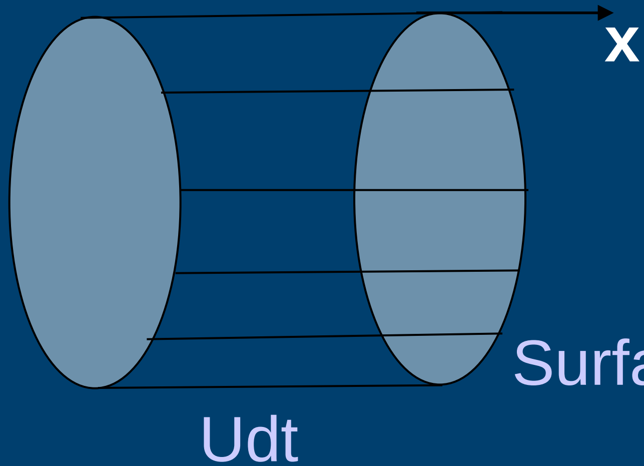
Chaotic motion, instantaneous binary collisions



Mean velocity field equal to zero (equilibrium state)

n number of particles per unit volume in the cylinder UdtdS



$$\overline{m_p U U dt dS} = m_p n \overline{U^2} dt dS$$

Average flux of x-momentum across dS in dt

$$m_p n = \rho_p \nu$$

Normal stresses

$$\rho_p \nu \overline{U^2} = \phi_{xx} \qquad \rho_p \nu \overline{V^2} = \phi_{yy} \qquad \rho_p \nu \overline{W^2} = \phi_{yy}$$

The mean value of the normal stresses is the pressure

$$p = \frac{1}{3}\left(\phi_{xx} + \phi_{yy} + \phi_{zz}\right) = \frac{1}{3}\rho_p \nu \overline{U^2 + V^2 + W^2} = \frac{1}{3}\rho_p \nu \overline{C^2}$$

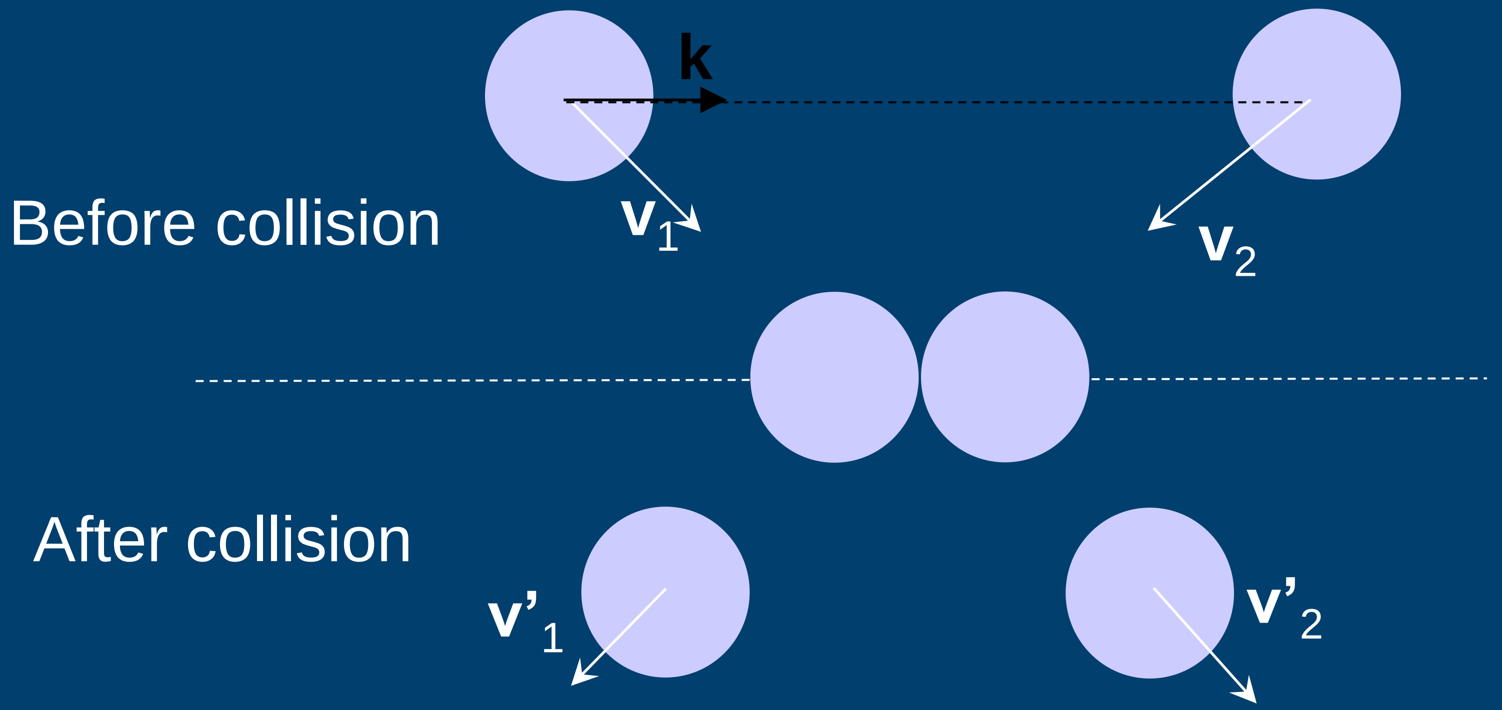
and

$T = \frac{1}{3}\overline{C^2}$ is the granular temperature (measure of the strength of the velocity fluctuations) $\left(T_B = \frac{1}{3}\frac{m_p}{k_B}\overline{C^2}\right)$

⇓

$p = \rho_p \nu T$ Equation of state for a dilute, nearly elastic granular gas $\left(pV = \frac{M}{m_p} k_B T_B\right)$

Chaotic motion of smooth, **inelastic** particles where momentum exchange is dominated by instantaneous, binary collisions



Inelastic collision

$(\mathbf{v}'_1-\mathbf{v}'_2)_k = -e(\mathbf{v}_1-\mathbf{v}_2)_k$

$e<1$ normal restitution coefficient

Functions f_i are given e.g. in Garzo and Dufty, PRE (1999)

Pressure $p = \rho_p f_1(\nu, e) T$

Shear stress $s = \rho_p f_2(\nu, e) d T^{1/2} \dot{\gamma}$

Fluctuating energy balance

$$\frac{3}{2}\rho_p \nu \frac{DT}{Dt} = -\nabla \cdot \mathbf{Q} + s\dot{\gamma} - \Gamma$$

Diffusion ($-\nabla \cdot \mathbf{Q}$) — Production ($s\dot{\gamma}$) — Dissipation (Γ)

Energy flux $\mathbf{Q} = -\rho_p f_4(\nu, e) d\, T^{1/2} \nabla T - \rho_p f_5(\nu, e) d\, T^{3/2} \nabla \nu$

Dissipation rate $\Gamma = \frac{\rho_p}{L} T^{3/2} f_3(\nu)(1 - e^2)$

length scale (L)

Classic kinetic theory

(Jenkins and Savage , JFM1983; Lun et al. JFM 1984; Garzo and Dufty PRE 1999)

Particle velocities are uncorrelated and *L=d*

(dilute to moderate gases,i.e., volume fraction less than 49%)

Extended kinetic theory

(Jenkins, PoF 2006; Jenkins, Granul. Matt. 2007; Jenkins and Berzi, Granul. Matt. 2010 and 2012)

Particle velocities are correlated/presence of clusters of particles and *L>d*

(dense gases)

$$L = f_6(\nu, e)\frac{d\dot{\gamma}}{T^{1/2}}$$

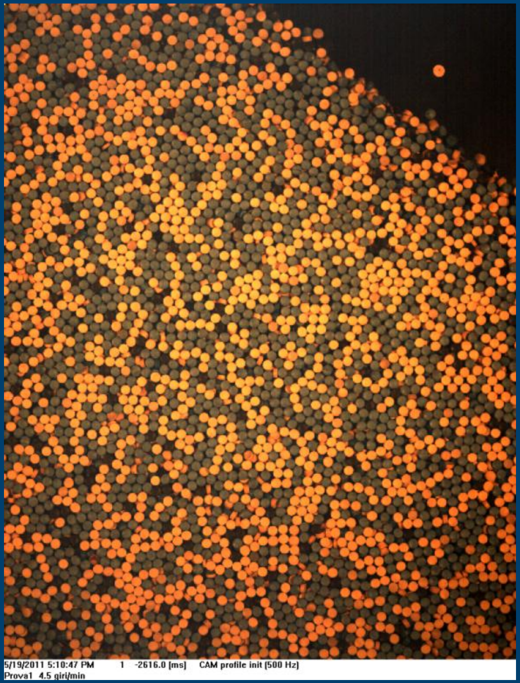
Role of friction in collisions

We need to solve also balances of angular momentum and rotational fluctuating energy for frictional particles: also coefficient of tangential restitution and Coulomb's friction coefficient - **Sticking-Sliding collision** (Walton 1992; Jenkins JAM 1992)

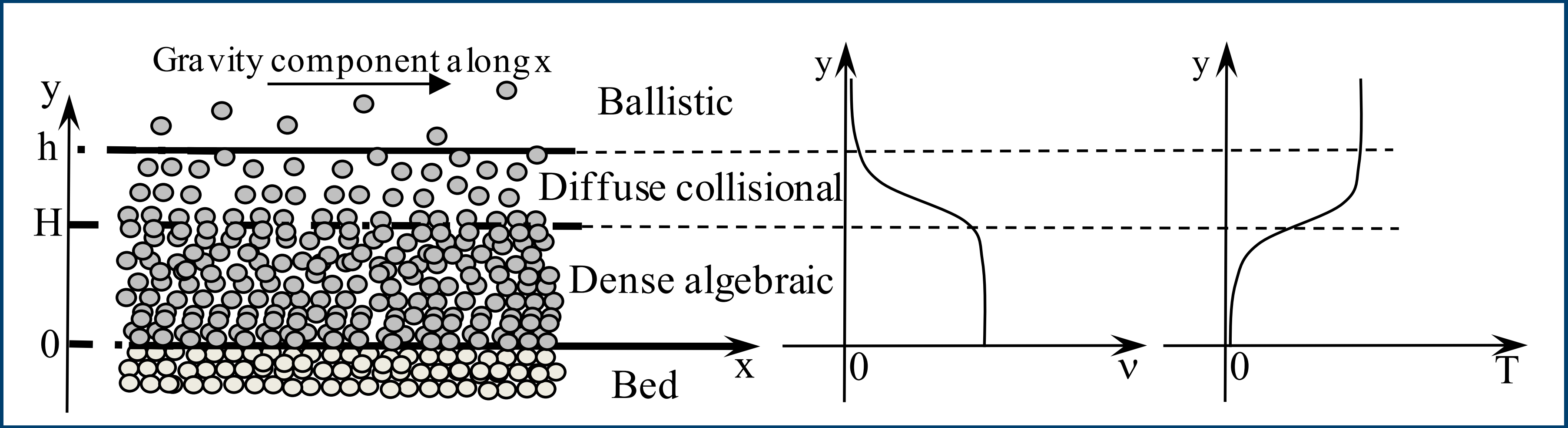
⇓ In the limit of small friction

Effective coefficient of normal restitution which takes into account the additional dissipation of traslational energy due to its transformation into rotational energy (Jenkins and Zhang PoF 2002): we can keep things simple!

Steady flows of plastic beads in a rotating drum
(Fantoli Hydraulic Lab, Politecnico di Milano, 2011)



Steady, inclined (angle of inclination ϕ) granular flows over erodible beds between frictional sidewalls distant W apart (Berzi and Jenkins, PoF 2011)



Everything dimensionless using particle density and diameter and gravitational acceleration

$$p' = -\nu \cos\phi$$

Particle y-momentum balance

$$s' = -\nu \sin\phi + 2\frac{\mu_w}{W} p$$

Particle x-momentum balance

(Coulomb friction with the sidewalls)

$$-Q' + su' - \Gamma = 0$$

Fluctuating energy balance

Key assumptions

- negligible contribution of the collisionless layer
- concentration linearly distributed in the diffuse layer (from 0 to 0.6) and constant in the dense layers ($\approx$0.6)
- velocity approximately linear in the diffuse layer
- negligible diffusion in the dense collisional layer (algebraic balance between production and dissipation)
- yielding at the bed (s/p has a characteristic value α)

Domain decomposition

$y = h$ — $s = p = \nu = 0$

Diffuse layer: classic kinetic theory

$L = 1$

$y = H$

$\nu \approx 0.6$

Dense algebraic layer: extended kinetic theory

$L \geq 1$

$su' = \Gamma$

$y = 0$ — $s / p = \alpha, u = 0$

Erodible bed

Analytical solution

Dense algebraic layer

$$\left(\frac{u'}{T^{1/2}}\right)^3 = \frac{f_3(v,e)}{f_2(v,e)\, f_6(v,e)}$$

from energy balance

$$\frac{s}{p} = \frac{f_2(v,e)T^{1/2}u'}{f_1(v,e)T} = \frac{f_2(v,e)}{f_1(v,e)}\frac{u'}{T^{1/2}} = \mu(v,e)$$

Stress ratio from constitutive relations

$$L = f_6(v,e)\frac{u'}{T^{1/2}} = L(v,e)$$

Correlation length

$$\mu = \mu(L,e)$$

at y=H, L=1

$$k = k(e)$$

$$I = \frac{u'}{p^{1/2}} = \frac{1}{\sqrt{f_1(\nu,e)}}\frac{u'}{T^{1/2}} = I(\nu,e)$$

Inertial parameter (GDR MiDi 2004) from constitutive relation

$\mu(\nu,e)$, $I(\nu,e)$ or equivalently $\mu(I,e)$, $\nu(I,e)$ or equivalently $I(\mu,e)$, $\nu(\mu,e)$

GDR MiDi 2004 rheology in the context of kinetic theory holds only if the energy diffusion is negligible (i.e., far from the boundaries)

$$u' = I(\mu, e)\, p^{1/2}$$

Shear rate in the dense layer

$$\mu = \tan\phi - \mu_w \frac{p}{\nu W \cos\phi}$$

Stress ratio from momentum balances

$$p_H = 0.3(h-H)\cos\phi$$
$$p_0 = 0.3(h-H)\cos\phi + 0.6H\cos\phi$$

Approximate pressure distribution

$k(e)$ Stress ratio at y=H

α Stress ratio at the bed

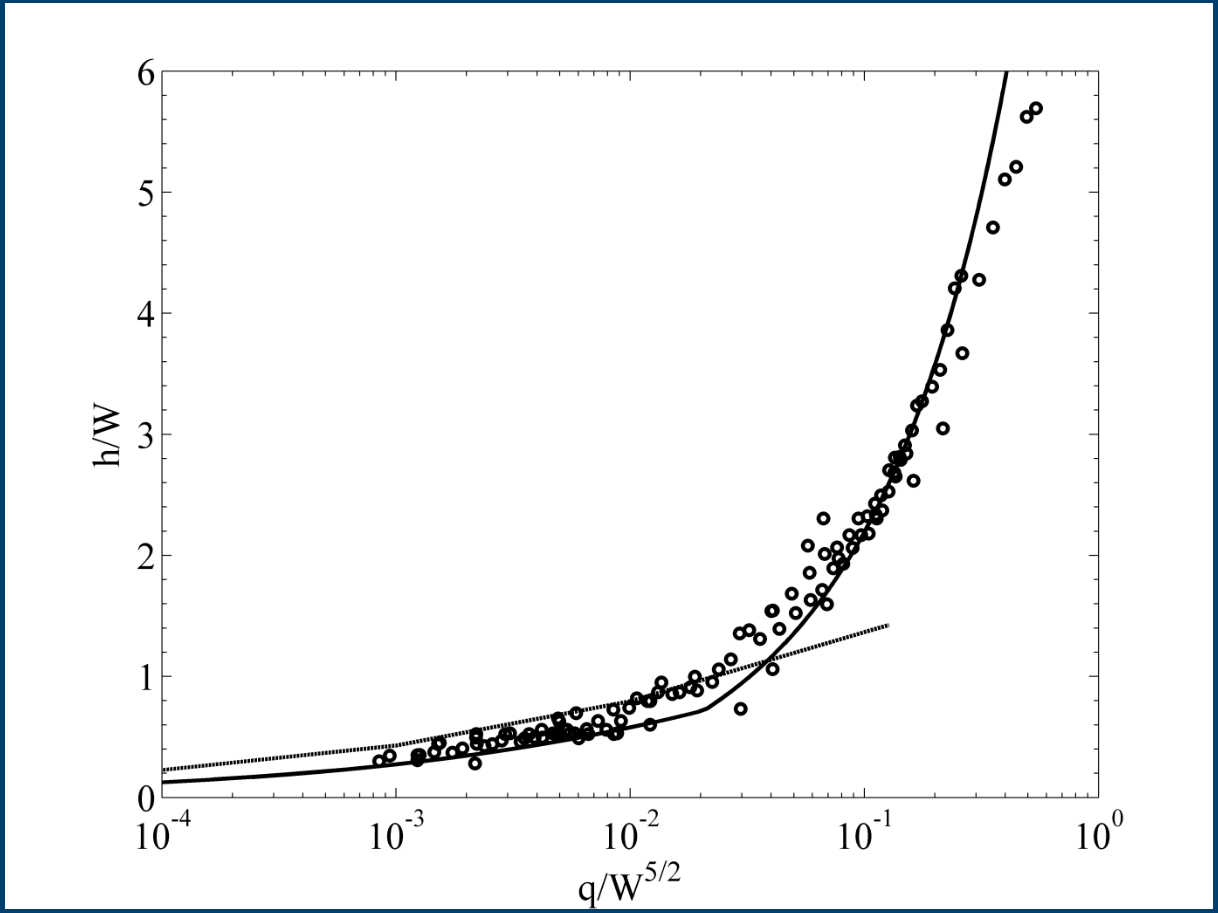
⇓

Depths

Velocity

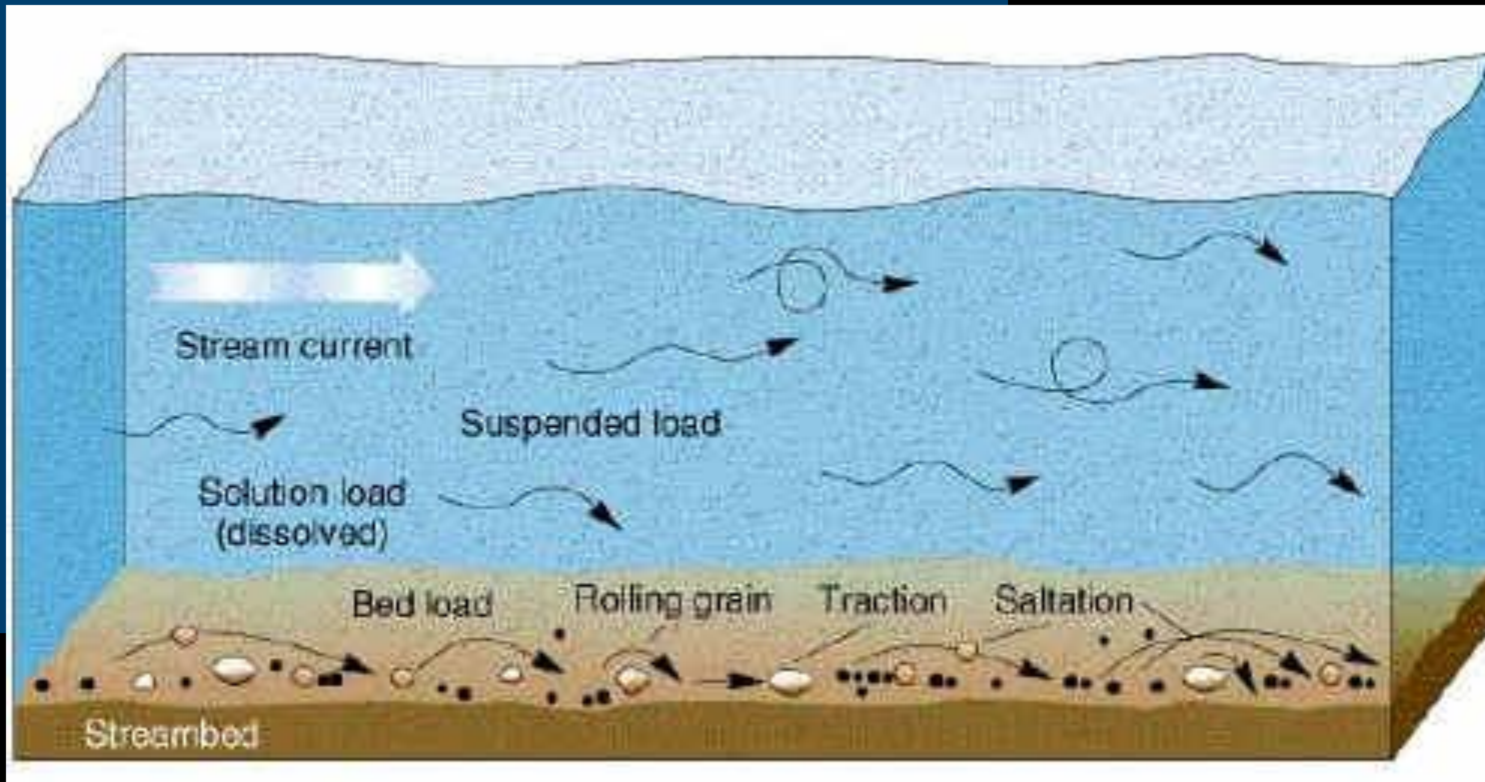
Flow rate

Experiments on glass spheres (heap flow, Jop et al. JFM 2005; rotating drum, Felix et al. EPJE 2007)



Sediment transport

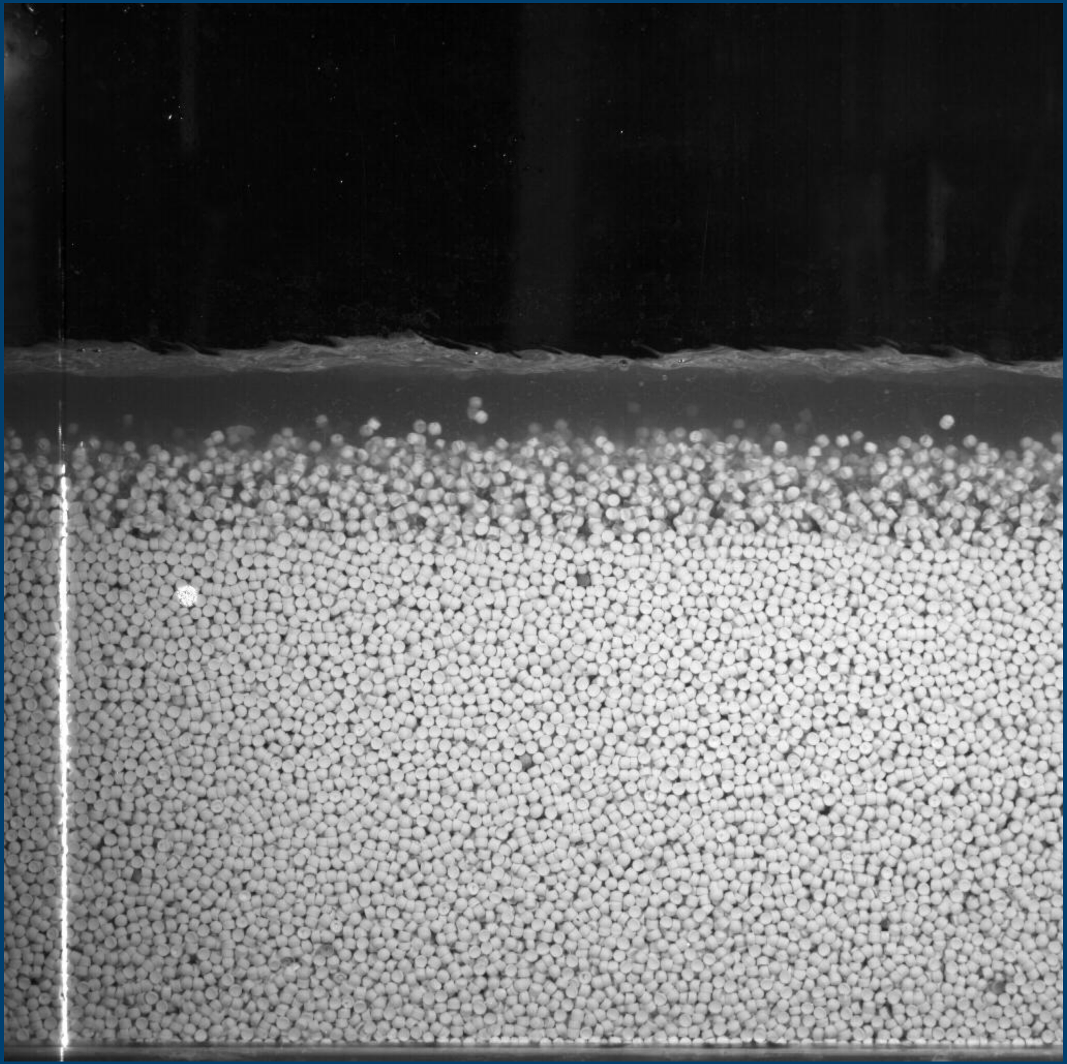
Youtube (by Mark Schmeeckle)



In between?

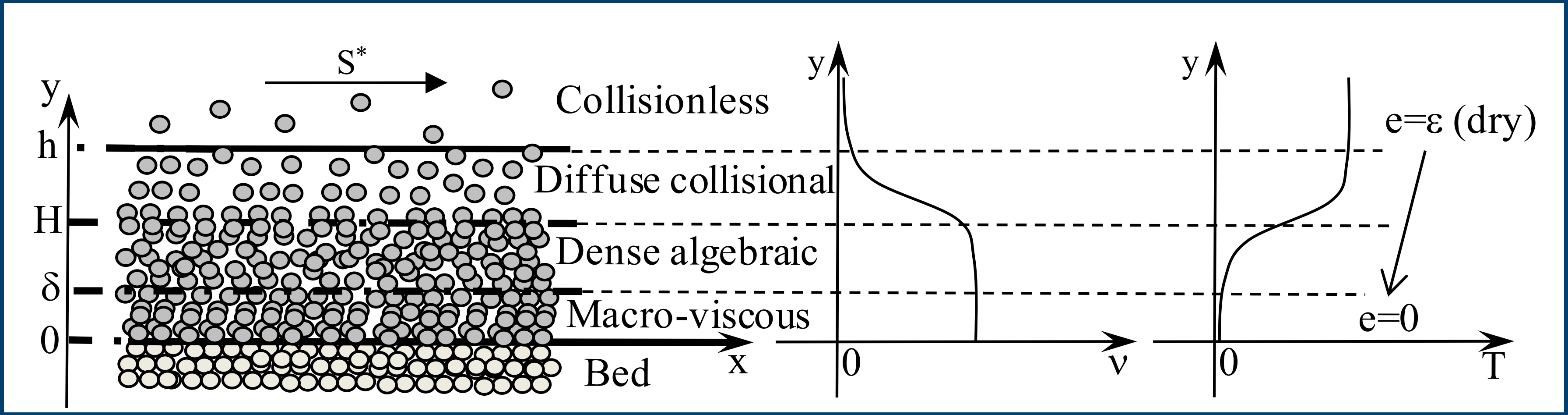
Youtube (by flyer0lines)

Collisional sediment transport



Courtesy of L. Fraccarollo (Università di Trento) and H. Capart (National Taiwan University)

Steady, horizontal sediment transport over an erodible bed (Berzi , JHE 2011, 2013)



S* is the fluid shear stress exerted at the top of the particles; σ particle to fluid density ratio

Everything dimensionless using particle density and diameter and reduced gravity

$$p' = -\nu$$ Particle y-momentum balance

$$s' = -D$$ Particle x-momentum balance

$$S' = D$$ Fluid x-momentum balance

$$-Q' + s\dot{\gamma} - \Gamma + coupling = 0$$ Fluctuating energy balance

If the turbulence is suppressed, the coupling term reduces to a sink term due to the viscous drag. Simpler approach:

$$-Q' + s\dot{\gamma} - \Gamma = 0$$

$$e = \varepsilon - 6.9\frac{1+\varepsilon}{\mathrm{St}}$$

Coefficient of restitution decreases with the Stokes number (Yang and Hunt, PoF 2006)

$$\mathrm{St} \equiv \frac{\rho_\mathrm{p} d T^{1/2}}{\eta_f}$$

Key assumptions

- negligible contribution of the collisionless layer
- s+S≈S* (boundary layer)
- concentration linearly distributed in the diffuse layer (from 0 to 0.6) and constant in the dense layers (≈0.6)
- velocity approximately linear in the diffuse layer
- turbulence suppressed in the dense layers (S≈0)
- negligible diffusion in the dense collisional layer (algebraic balance between production and dissipation)
- yielding at the bed (s/p has a characteristic value α)

Domain decomposition

$$S = S^*$$

$y = h$ — $s = p = \nu = 0$

Diffuse layer: classic kinetic theory

$$L = 1$$

$y = H$

Dense algebraic layer: extended kinetic theory

$$L \geq 1 \qquad \nu \approx 0.6$$

$$su' = \Gamma \qquad s \approx S^*$$

$y = \delta$ — $e = 0$

Macroviscous layer: viscous fluid with concentration-dependent viscosity

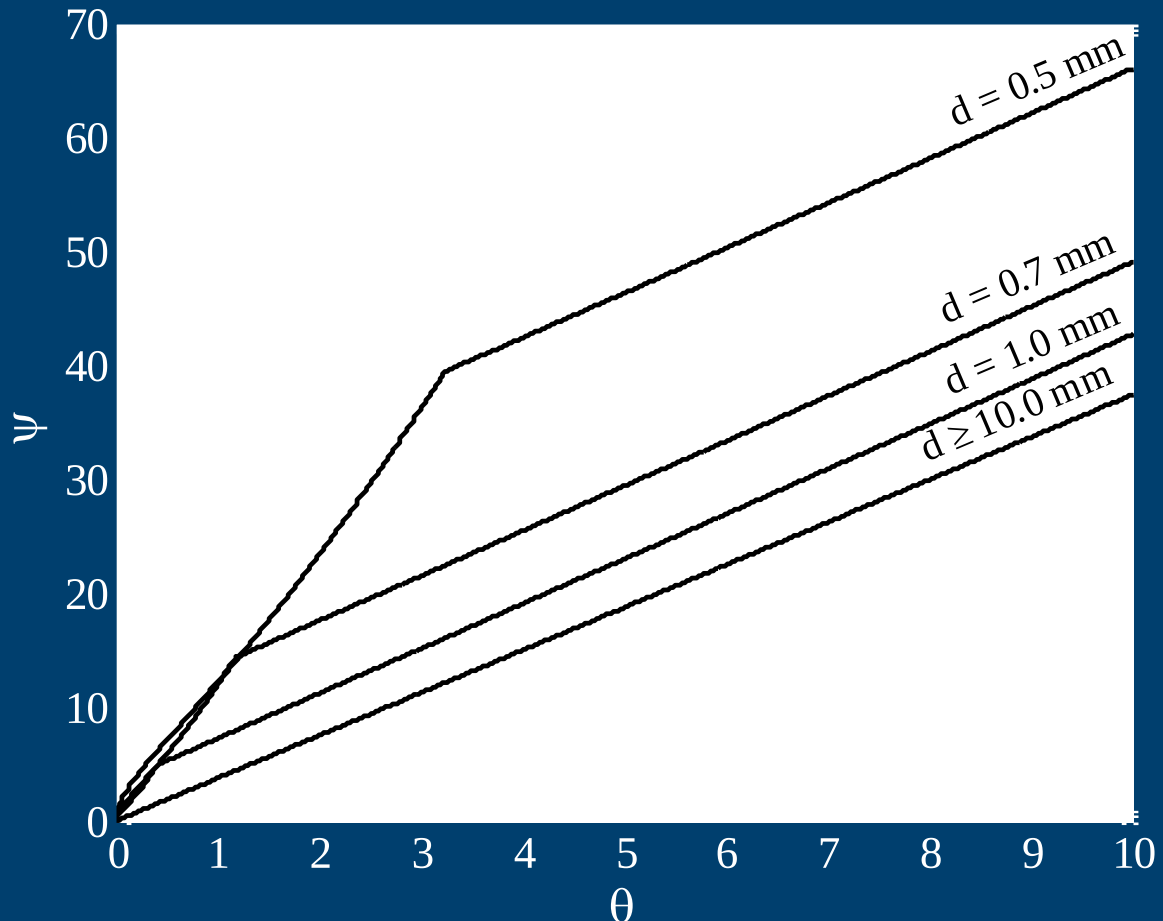
$$\nu \approx 0.6$$

$y = 0$ — $s / p = \alpha, u = 0$

Erodible bed

Analytical solution

$S^* = \theta$ (Shields number) → *Depths* / *Particle velocity* → *Particle flow rate per unit width* Φ



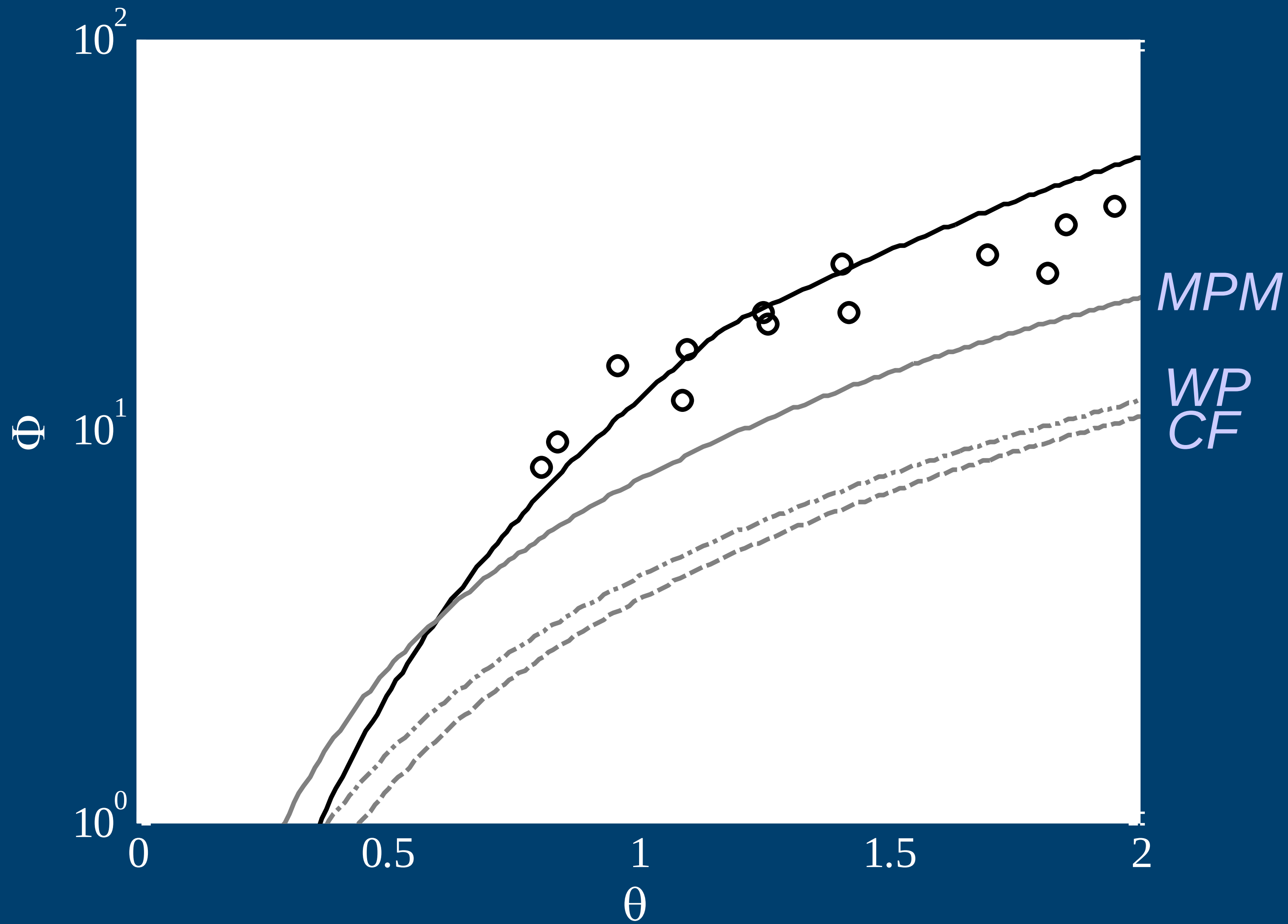
Sand/gravel in water

$$\Phi = \psi \theta^{3/2}$$

Meyer-Peter and Muller (1948) $\psi = 8$

Wong and Parker, JHE (2006); Capart and Fraccarollo, GRL (2012) $\psi \approx 4$

0.7 mm sand in water (Nnadi and Wilson, JHE 1992)



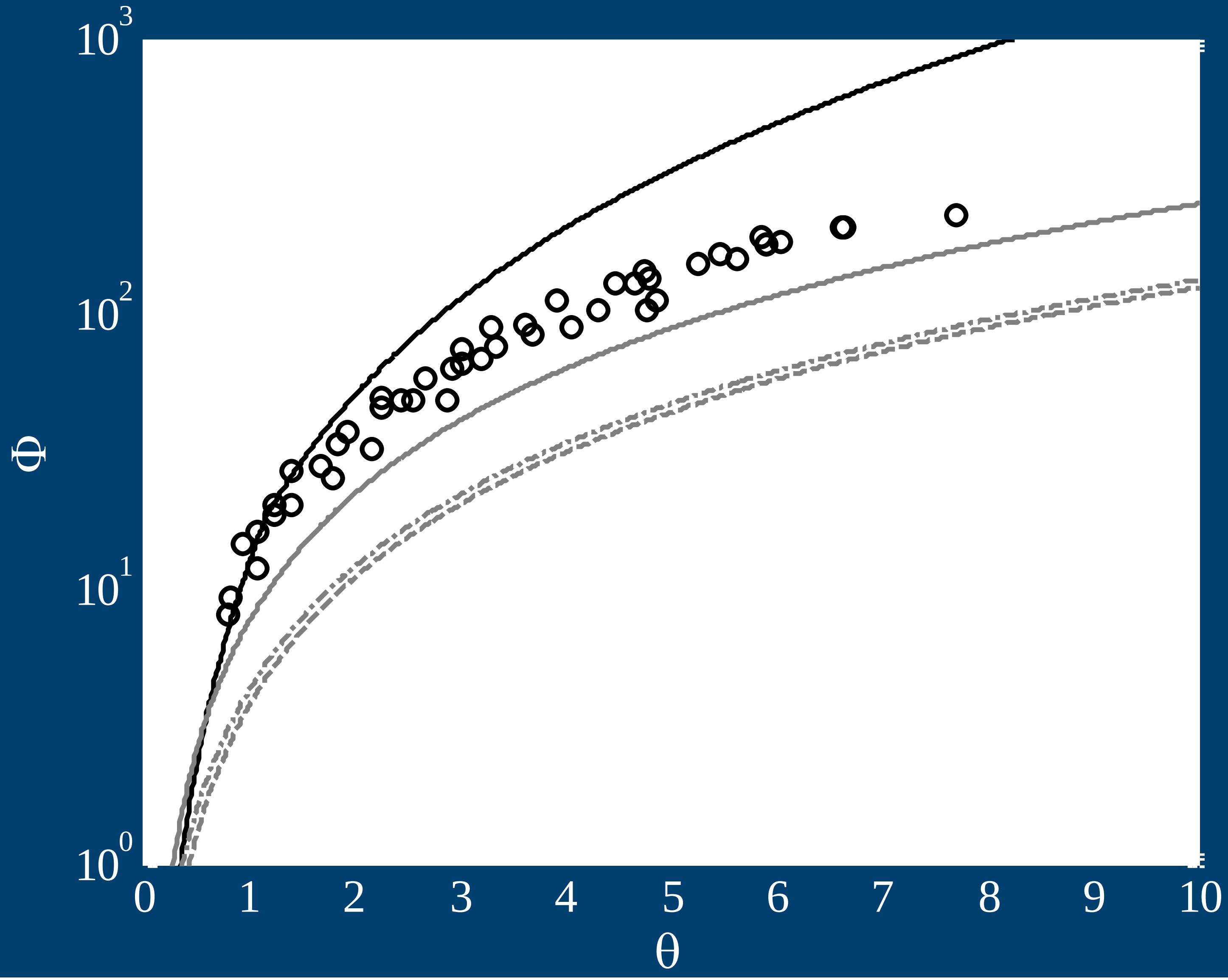
Limits of the theory - 1

Particle depth (h) must be at least 1 diameter (otherwise ordinary bedload) → *Minimum value for the Shields number (around 0.2 for sand, i.e. 4 times the critical Shields number)*

Absence of turbulent suspension, i.e., ratio of fluid shear velocity at the top and single particle settling velocity less than 1 → *Maximum value for the Shields number (around 1.3 for 0.7 mm sand)*

Comparisons with experiments 2

0.7 mm sand in water (Nnadi and Wilson, JHE 1992)



McTigue, JHD (1981)

Turbulent suspension due to turbulent lift (correlation between fluctuations of fluid velocity and volume fraction in the drag force in the momentum balance) $\mathbf{D} = C\nu\left(\mathbf{v}_f - \mathbf{v}_p\right)$

$$p' = -\nu + C\left\langle \tilde{v}\tilde{v}_f \right\rangle$$ Particle y-momentum balance

Many-particle settling in still fluid $w = \dfrac{\nu}{C}$

Model for diffusion $\left\langle \tilde{v}\tilde{v}_f \right\rangle \approx -\kappa u^*\left(h - H\right)\nu'$

$$u^* = \left(\sigma S^*\right)^{1/2}$$

Shear velocity

$$p' \approx -\nu\left[1 + \frac{\kappa u^*}{w}\left(h - H\right)\nu'\right] \approx -\nu\left[1 - 0.6\xi\frac{\kappa u^*}{w_0}\right]$$

Single particle settling velocity

Berzi, JHE (2013)

The weight of the particles is partially supported by turbulent suspension

⇓

Collisional pressure decreases (less collisions)

⇓

Only a portion of the actual distant fluid shear stress induces particle motion

⇓

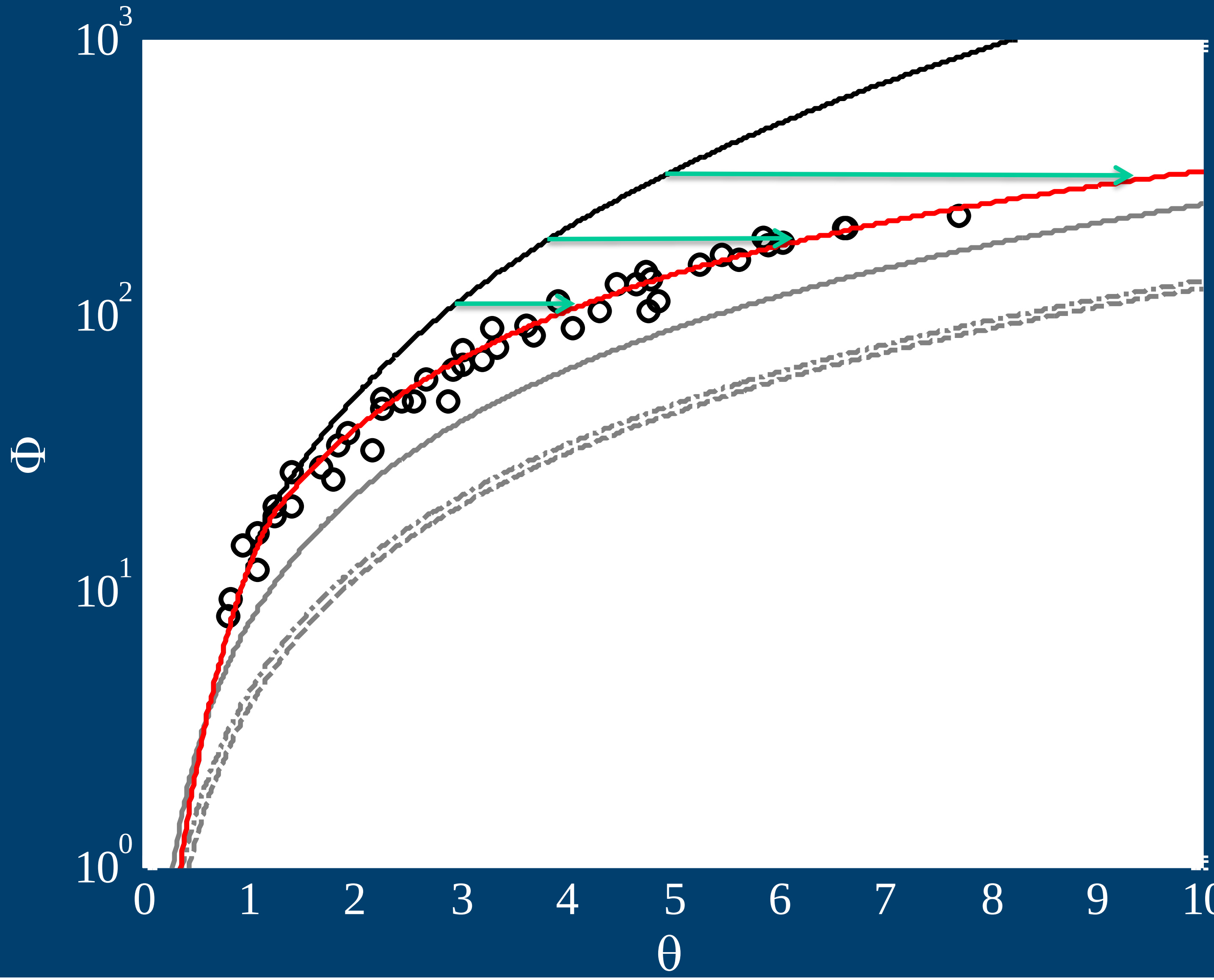
Inverse of the Rouse number

The calculated particle flow rate must be associated with larger values of the Shields number!

$$\frac{\theta_{eff}}{\theta} = 1 - 0.6\xi \frac{\kappa u^*}{w_0}$$

Comparisons with experiments 3

0.7 mm sand in water (Nnadi and Wilson, JHE 1992)



Limits of the theory - 2

h must be at least 1 diameter (otherwise ordinary bedload) ⟹ *Minimum value for the Shields number around 0.2 for sand, i.e. 4 times the critical Shields number*

Weight of the particles not entirely supported by turbulent suspension ⟹ ***Maximum value for the Shields number around 7 for 0.7 mm sand***

Just for fun

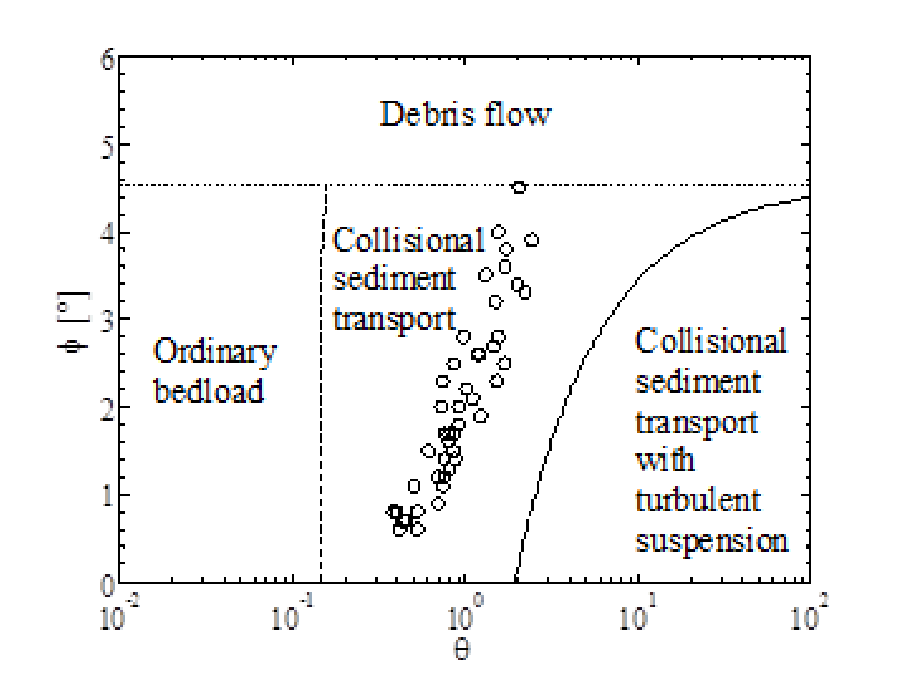
Typical situation of natural particles transported by a water stream (depth or order 1 m) at a certain slope

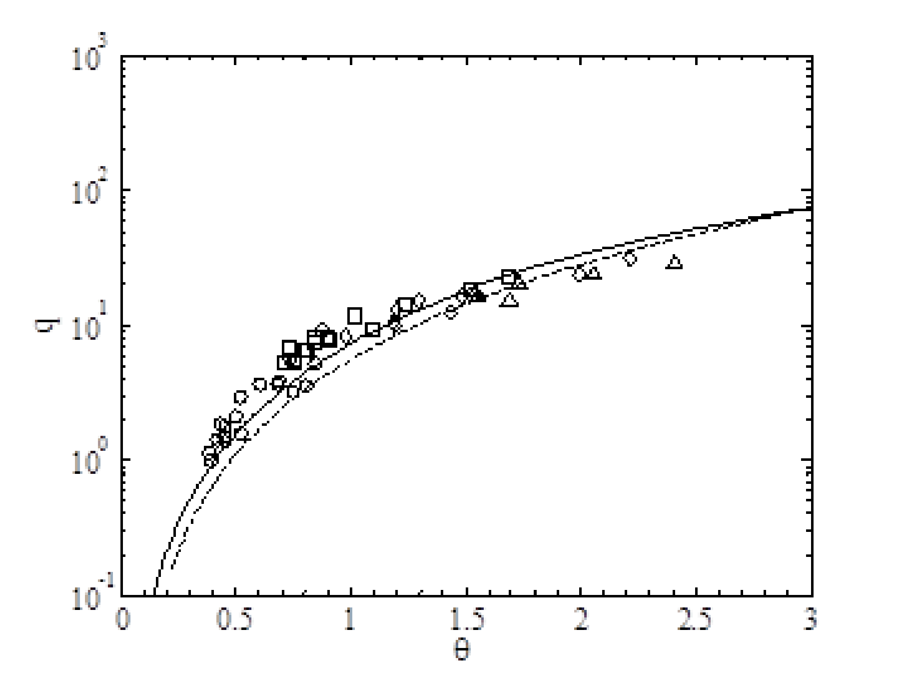
$$\theta = \frac{\rho H_w \cdot slope}{d\left(\rho_p - \rho\right)} \approx \frac{H_w}{d} slope$$

d [m]	θ when slope~0.001	θ when slope~0.01	Regime
10^{-4} (silt)	10	100	Fully suspended
10^{-3} (sand)	1	10	Collisional
10^{-2} (gravel)	0.1	1	Ordinary bedload/ Collisional
10^{-1} (boulders)	0.01	0.1	Ordinary bedload

Inclined sediment transport – plastic beads

Experiments (Capart and Fraccarollo, GRL 2011) on 3.35 mm plastic cylinders and water (specific density 1.5)

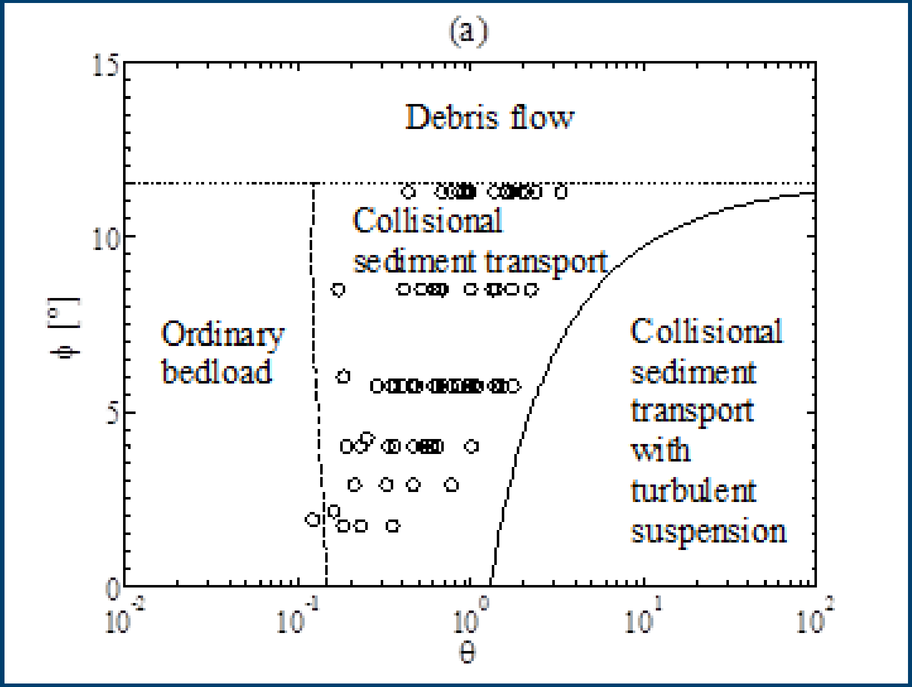


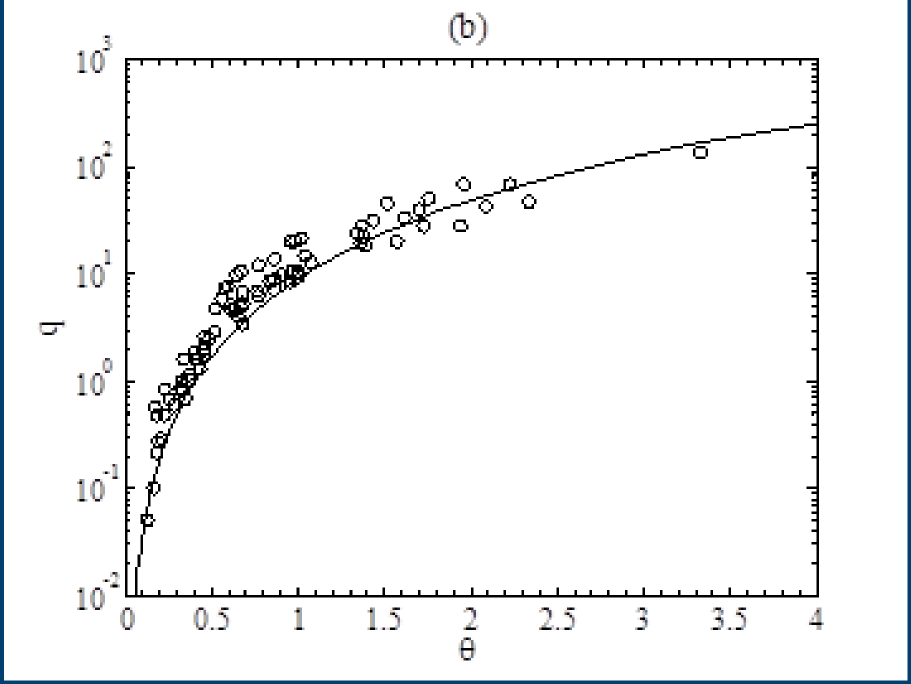


Berzi and Fraccarollo, PoF 2013

Inclined sediment transport – sand/gravel

Experiments (Smart, JHE 1984) on 2-10 mm sand/gravel and water (specific density 2.6)





Berzi and Fraccarollo, PoF 2013

- Collisionless regimes:

  1- fully suspended transport at large Shields numbers (suspended load; turbidity and/or density currents)

  2- aquatic saltation at small Shields numbers

- Polydispersity
- Oscillating flows