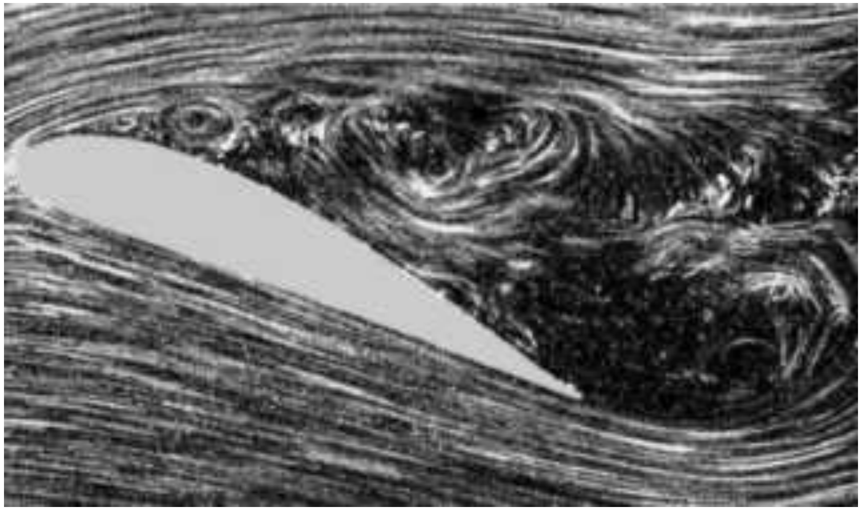
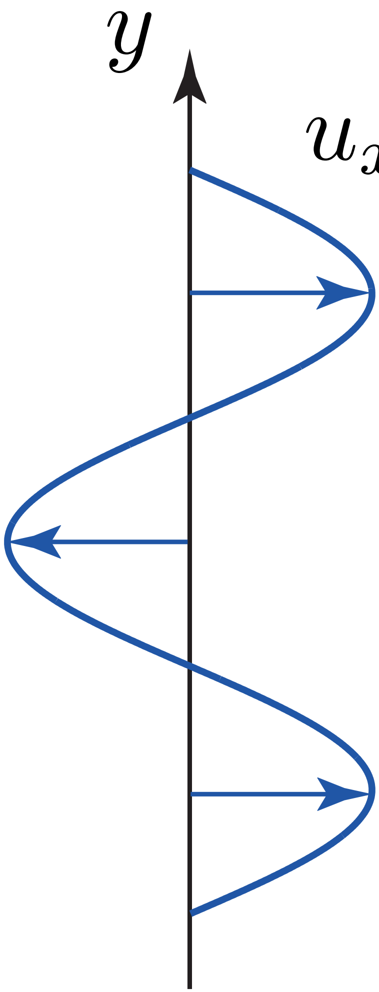
The decay of turbulence
and
particles in turbulence

Gregory P. Bewley
Max Planck Institute for Dynamics and Self-Organization
Göttingen, Germany



http://en.wikipedia.org/wiki/File:Flow_separation.jpg



$$u_x(\mathbf{x}, t_0) = U_0 \sin\left(\frac{y}{\lambda_y}\right)$$

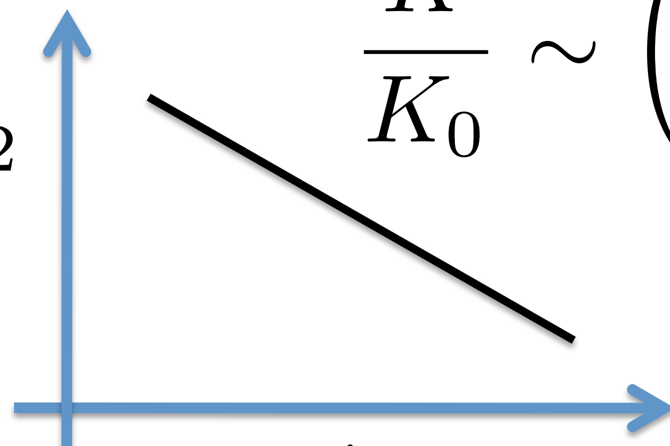
$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u}$$

$$\partial_t u_x = -\frac{\nu}{\lambda_y^2} u_x$$

$$u_x = U_0 \, e^{-\nu t/\lambda_y^2} \sin\left(\frac{y}{\lambda_y}\right)$$

$$\frac{K}{K_0} \sim \left(\frac{tU_0}{\lambda_0}\right)^{-n}$$

$$\log K \equiv \frac{3}{2}u'^2$$



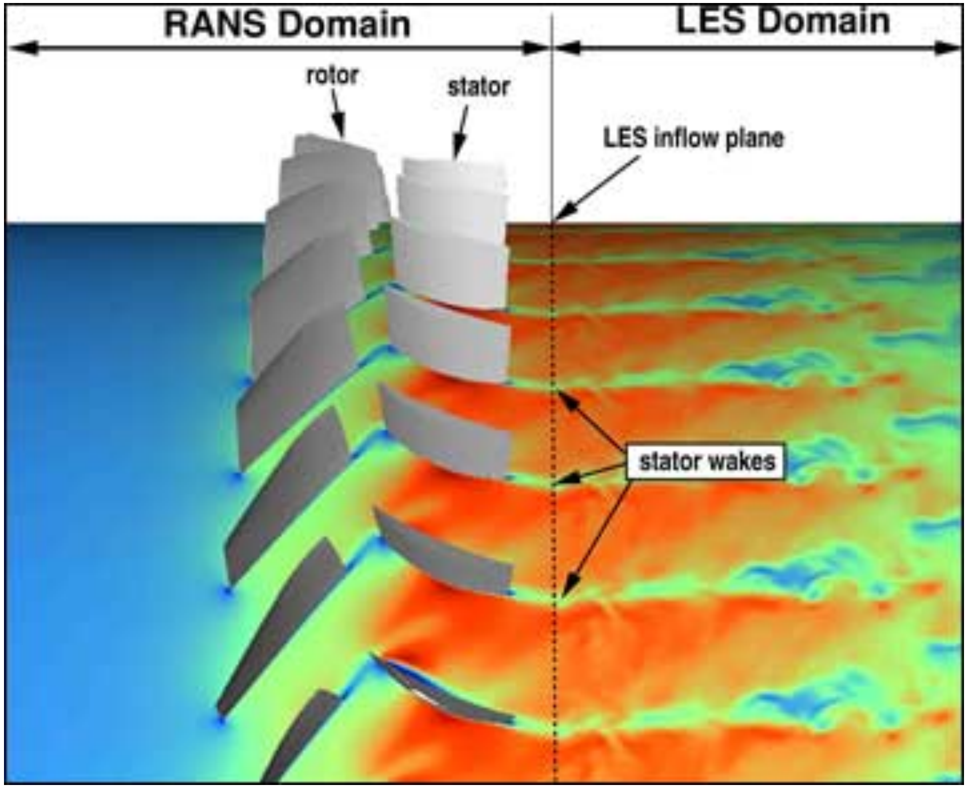
$\log\ t$

Von Kármán and Howarth, Kolmogorov, Dryden, Batchelor, Saffman, etc…

http://captainkimo.com/wp-content/uploads/2012/01/Smoke-Stack-from-Sugar-Factory-in-Belle-Glade-Florida.jpg

http://ict-aeolus.eu/images/horns_rev.jpg

Objectives



http://www.stanford.edu/group/cits/images/integration/stage35wake_bg.jpg

Provide inspiration for models.

Make the next generation of design tools possible.

Statistical analysis
of idealized flows
(e.g. Kolmogorov)

Parameterizations
of non-ideal flows

Dynamical mechanisms

Find useful parameterizations.

The Reynolds number:
its influence on the decay rate
its influence on scaling

Anisotropy:
systematics in the large-scale measures of turbulence

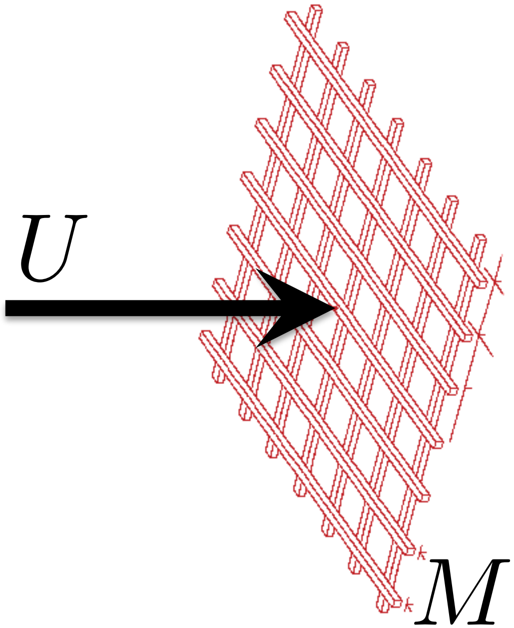
Unsteadiness:
its influence on scaling coefficients

Understand specific mechanisms.

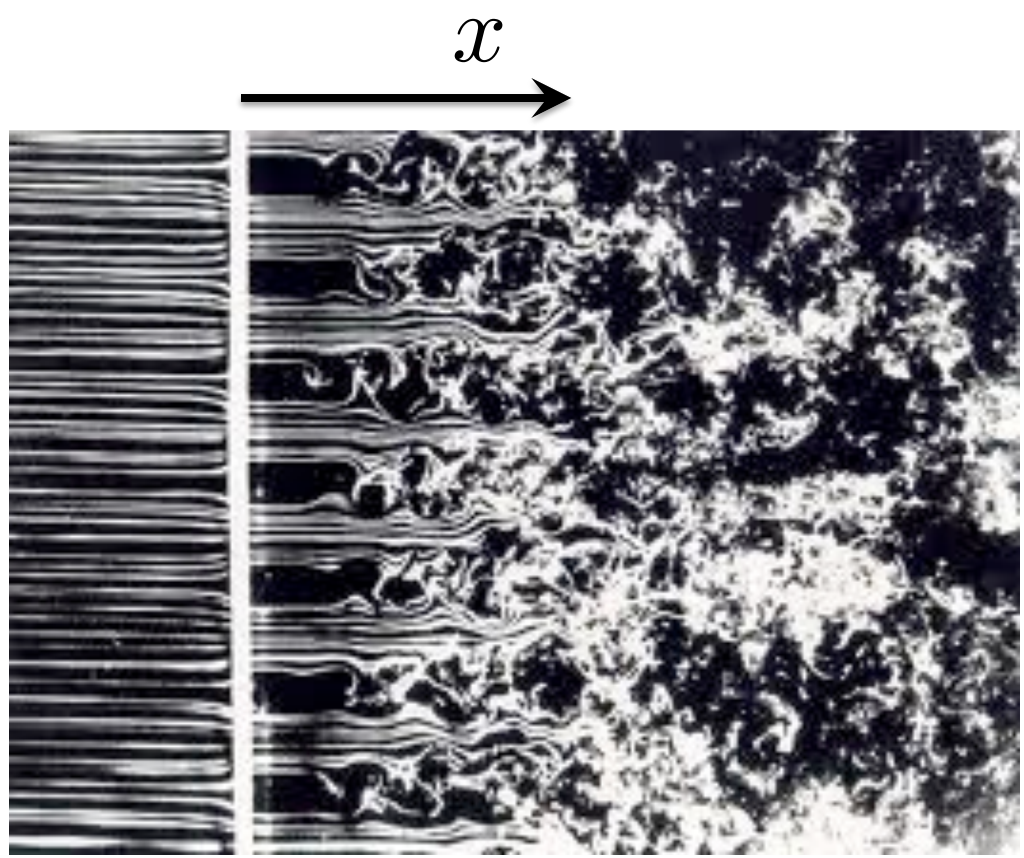
Reconnection:
arises from collisions between vortices and enables dissipation

The sling effect:
produces collisions cloud droplets that enable rain

GRID TURBULENCE



$$Re_M = \frac{UM}{\nu}$$



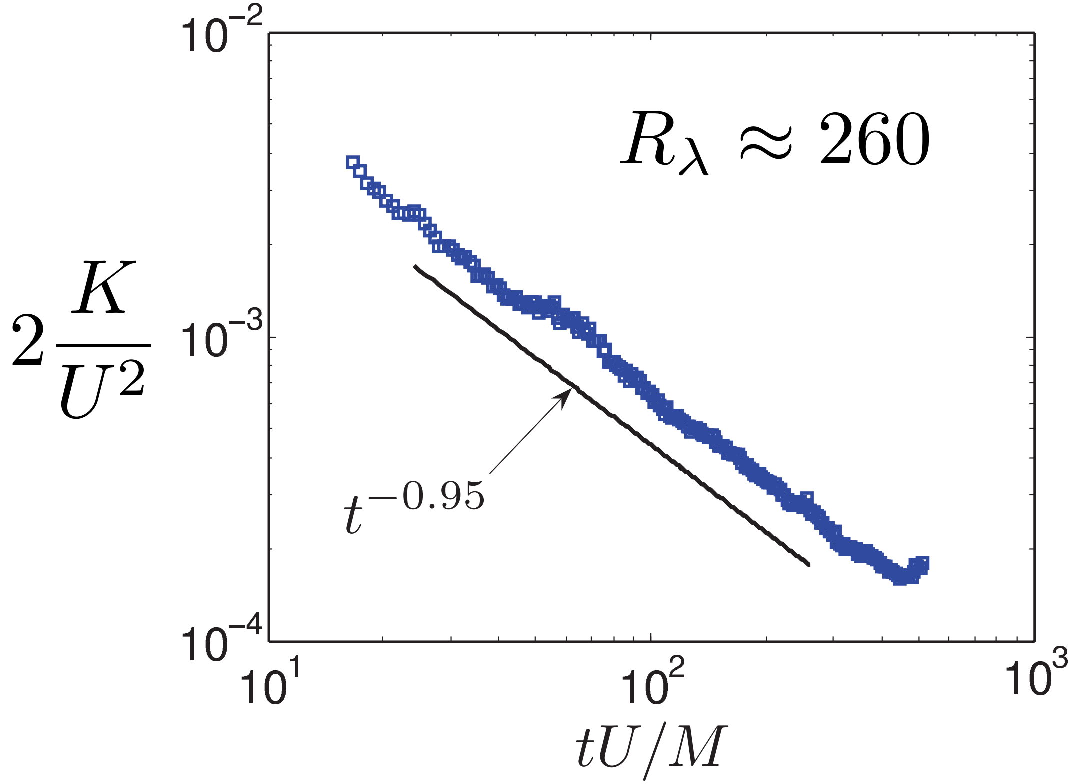
http://fdrc.iit.edu/research/images/GridTurbulenceRe2.jpg

1. $r, t \Rightarrow r/L(t)$

2. $Re = const$

$$K \sim t^{-1}$$

KINETIC ENERGY DECAY
AFTER MIXING LIQUID HELIUM WITH A GRID



Dryden (1941) *Q. Appl. Maths*
Speziale and Bernard (1992) *J. Fluid Mech.*

Bewley et al. (2007) *Phys. Fluids*

$$f(r,t) = \frac{\langle\, u(\vec{x},t)\, u(\vec{x}+\vec{r},t)\,\rangle}{u'^2}$$

$$f(r,t) \sim r^{-2} \quad \Leftrightarrow \quad K \sim t^{-6/5}$$ (Saffman)

$$f(r,t) \sim r^{-6} \quad \Leftrightarrow \quad K \sim t^{-10/7}$$ (Kolmogorov)

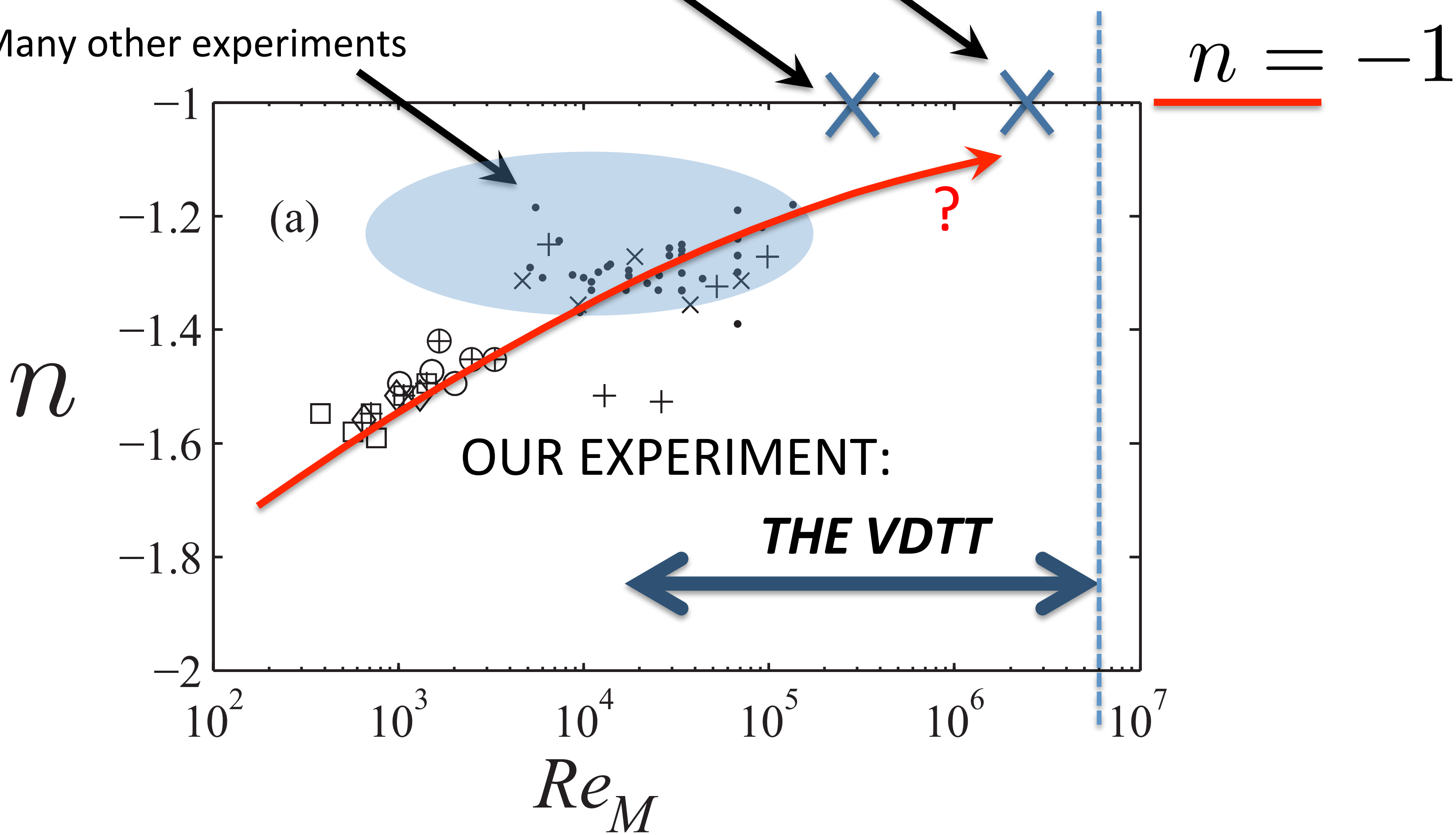
e.g. Davidson (2011) *Phys. Fluids*

Is it possible to imprint desired long-range correlations?

Kistler and Vrebalovich (1966) *J. Fluid Mech.*

Bewley et al. (2007) *Phys. Fluids*

Many other experiments



Kurian and Fransson (2009) *Fluid Dyn. Res.*

e.g. Speziale and Bernard (1992) *J. Fluid Mech.*

THE VARIABLE DENSITY TURBULENCE TUNNEL (VDTT)

Bewley, Nobach, Sinhuber, Xu, Bodenschatz (2013) *in prep.*

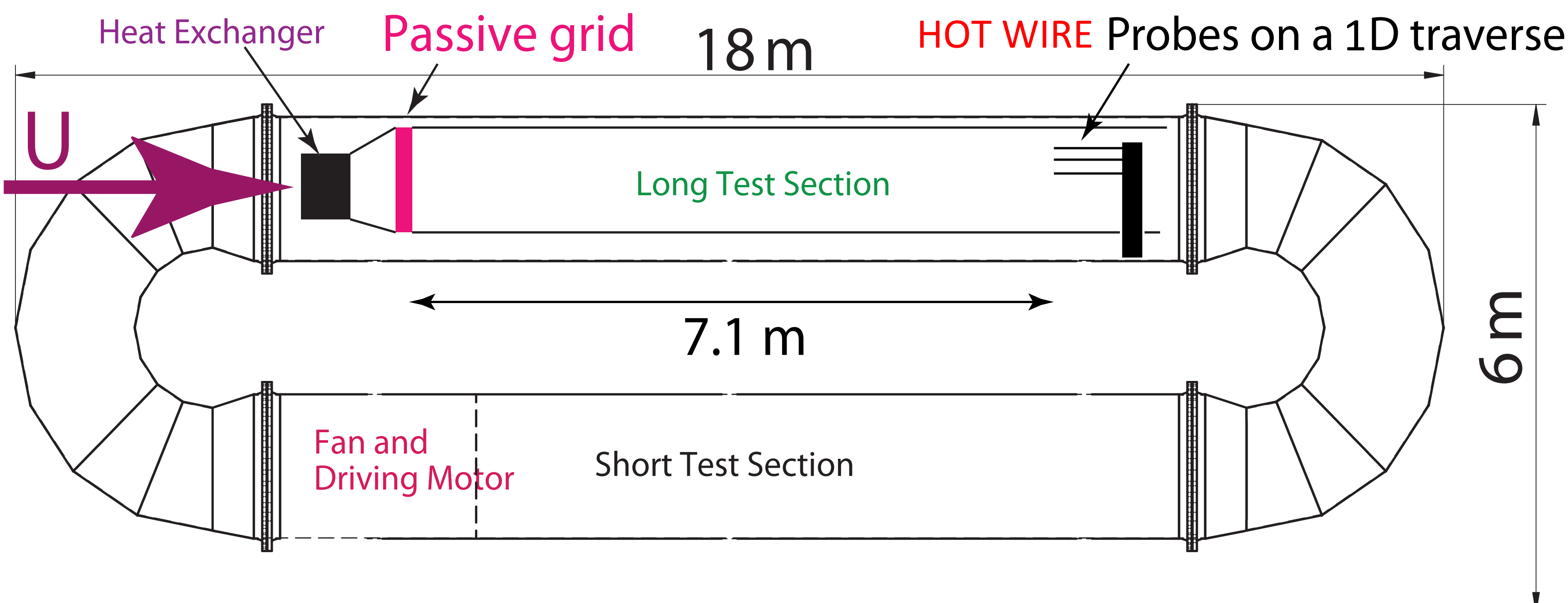
$$Re = \frac{\rho U L}{\mu}$$

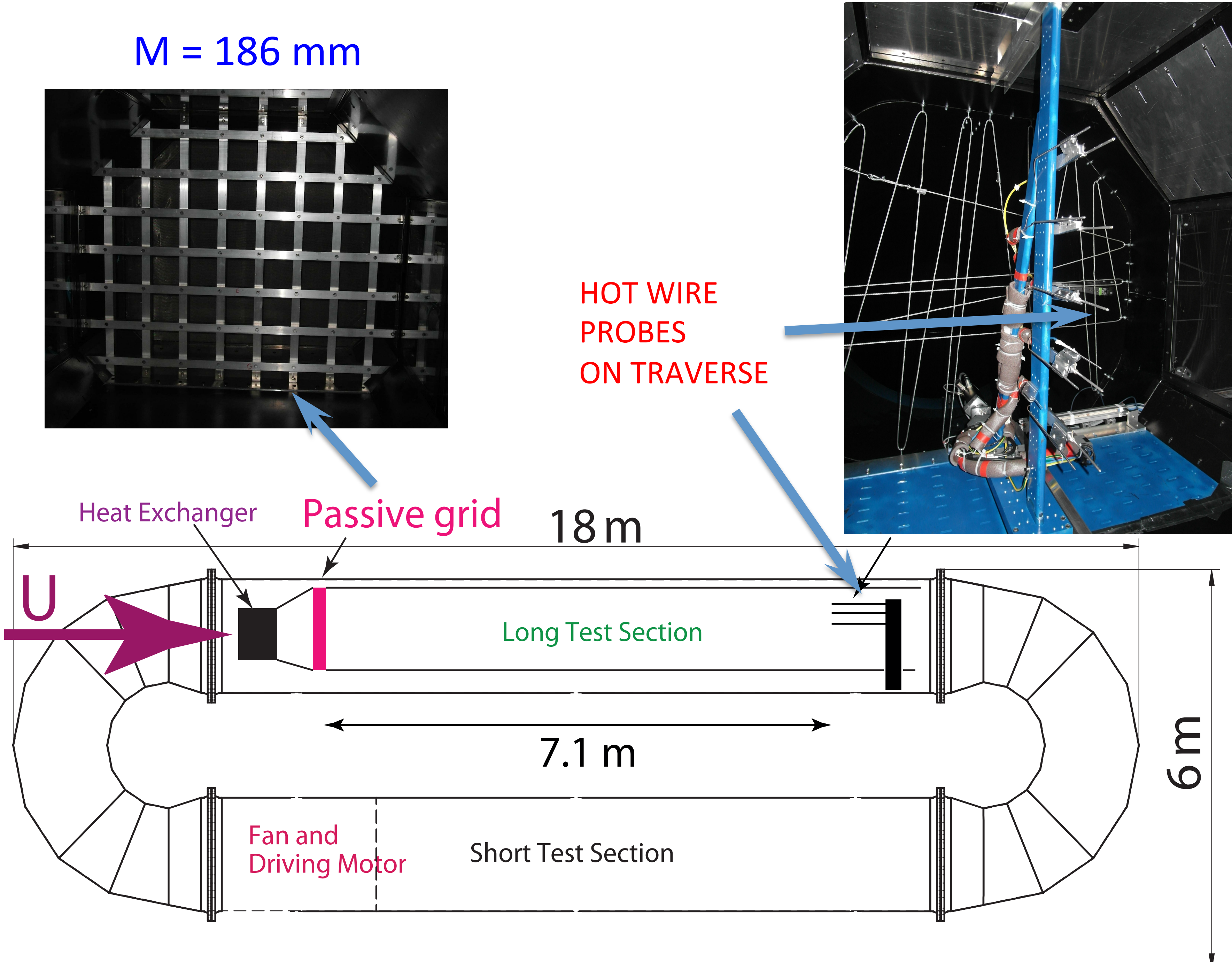
Air and **Sulfur Hexafluoride** gas (SF_6)

$$U \leq 5\, m/s$$

$$\eta \geq 20\, \mu m$$

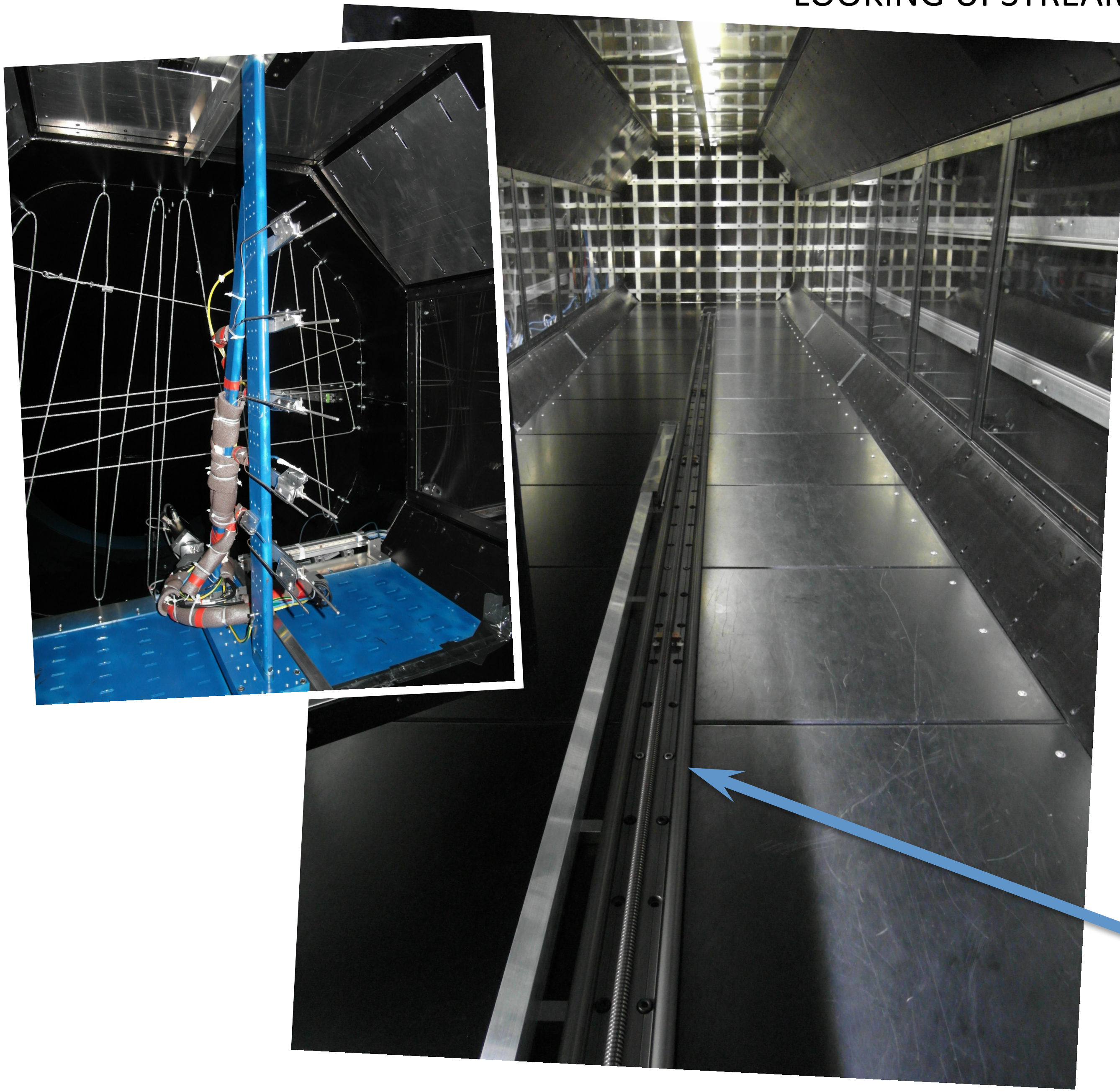
$$\tau_\eta \geq 2\, ms$$





M = 186 mm
HOT WIRE
PROBES
ON TRAVERSE
Heat Exchanger
Passive grid
18 m
U
Long Test Section
7.1 m
6 m
Fan and
Driving Motor
Short Test Section

LOOKING UPSTREAM

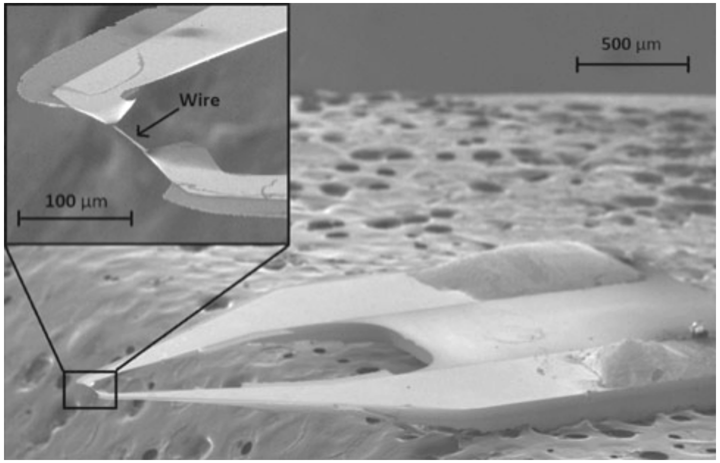


TRAVERSE

THE VDTT

~18 meters

THE NSTAP

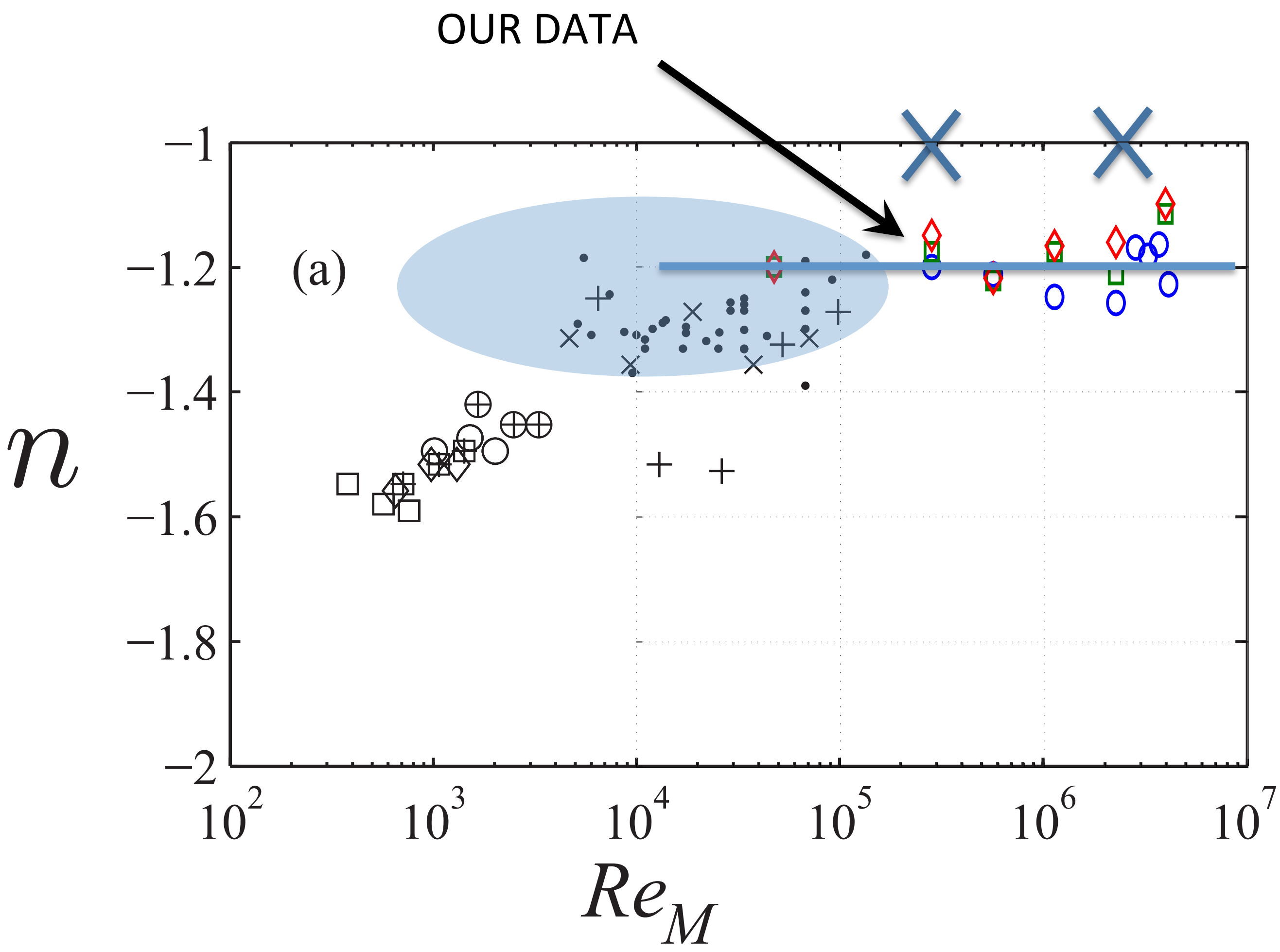


30 – 60 micron

HOT WIRE
PROBES
ON TRAVERSE

Vallikivi et al. (2011)
Expt. Fluids

Princeton University



assuming a low Reynolds number exponent of -1.2

CONTROL OF LARGE-SCALE STRUCTURE

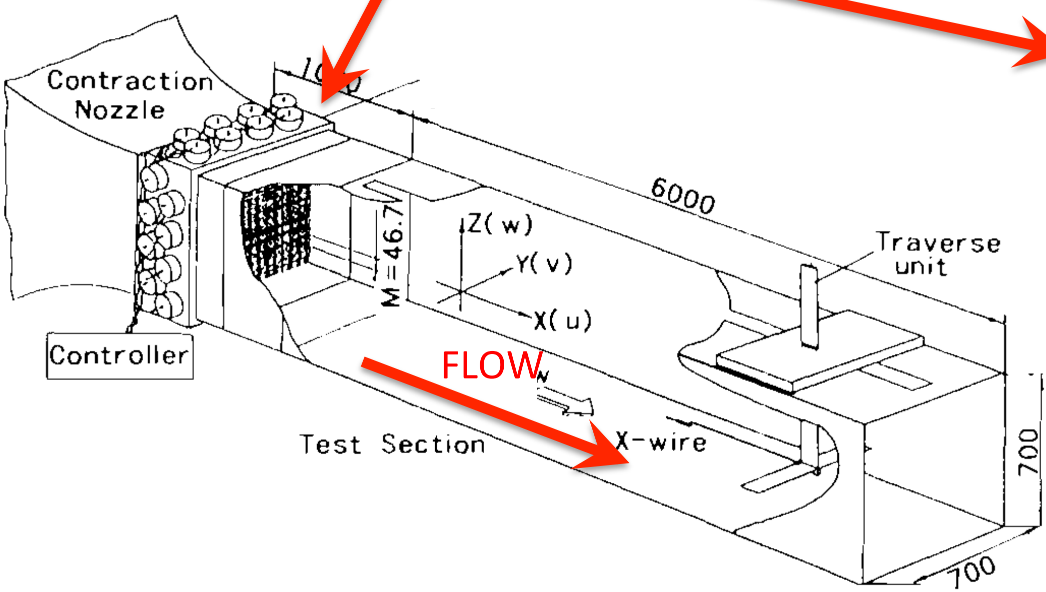
-for high Reynolds numbers

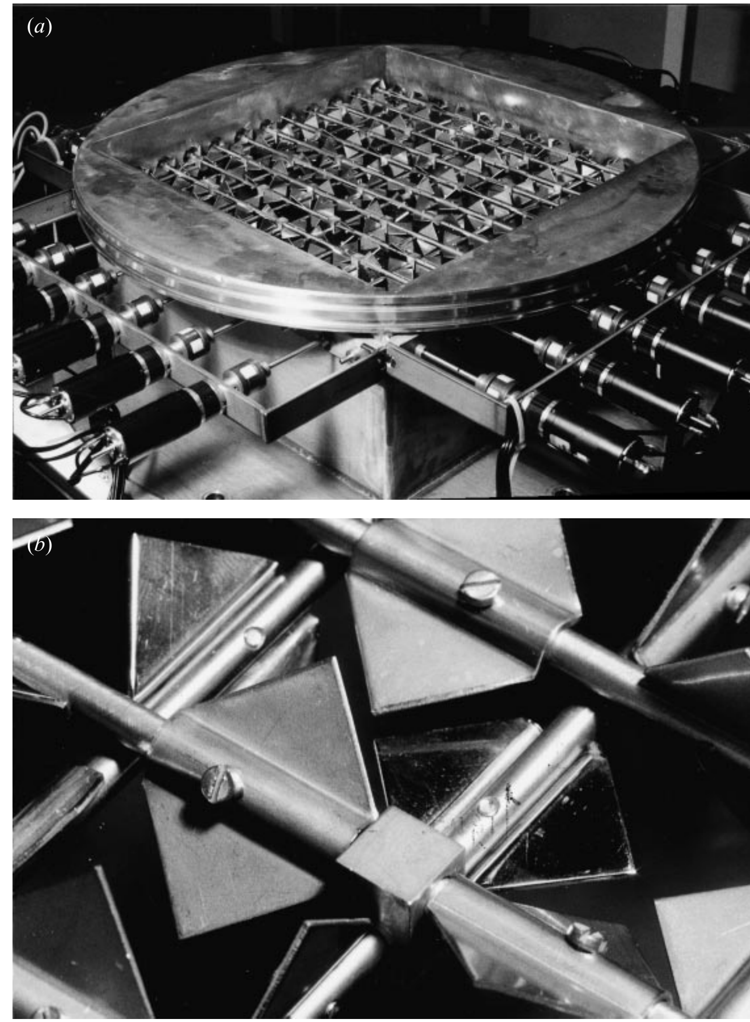
e.g. Makita (1991*) Fluid. Dyn. Res.*

-for control

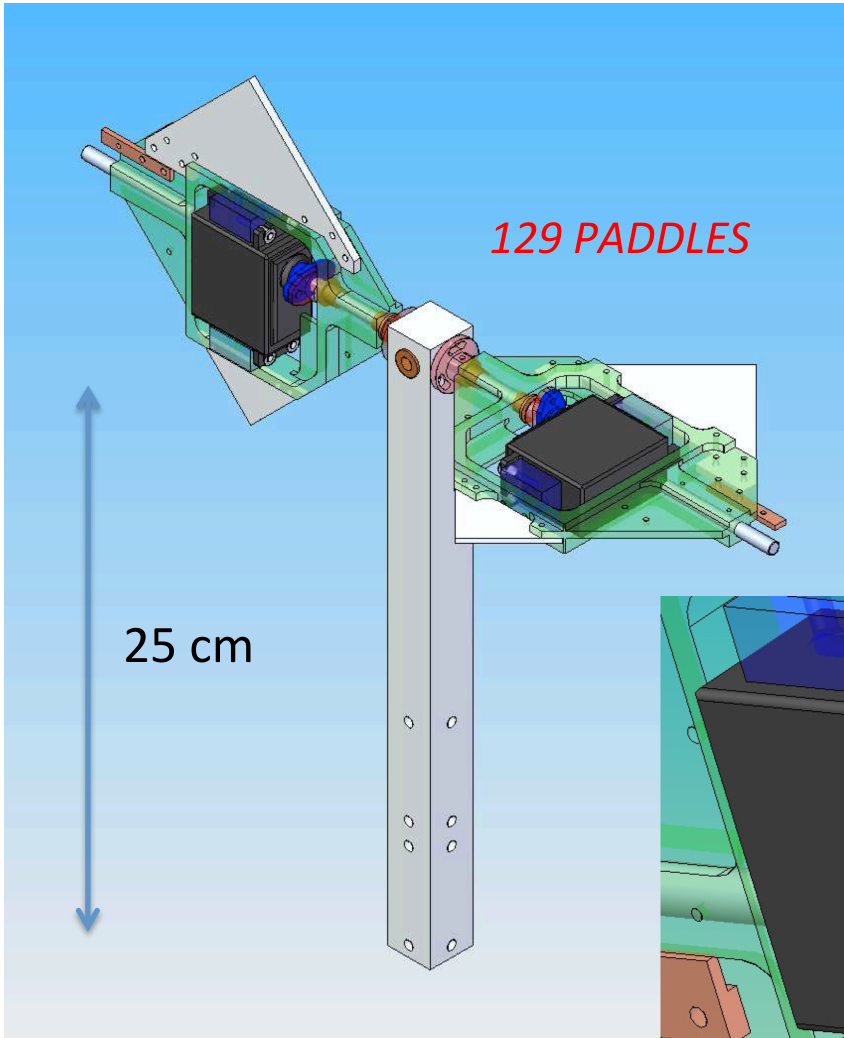
e.g. Poorte and Biesheuvel (2002) *JFM*
Cekli, Tipton and van de Water (2010) *PRL*

AN **ACTIVE GRID**



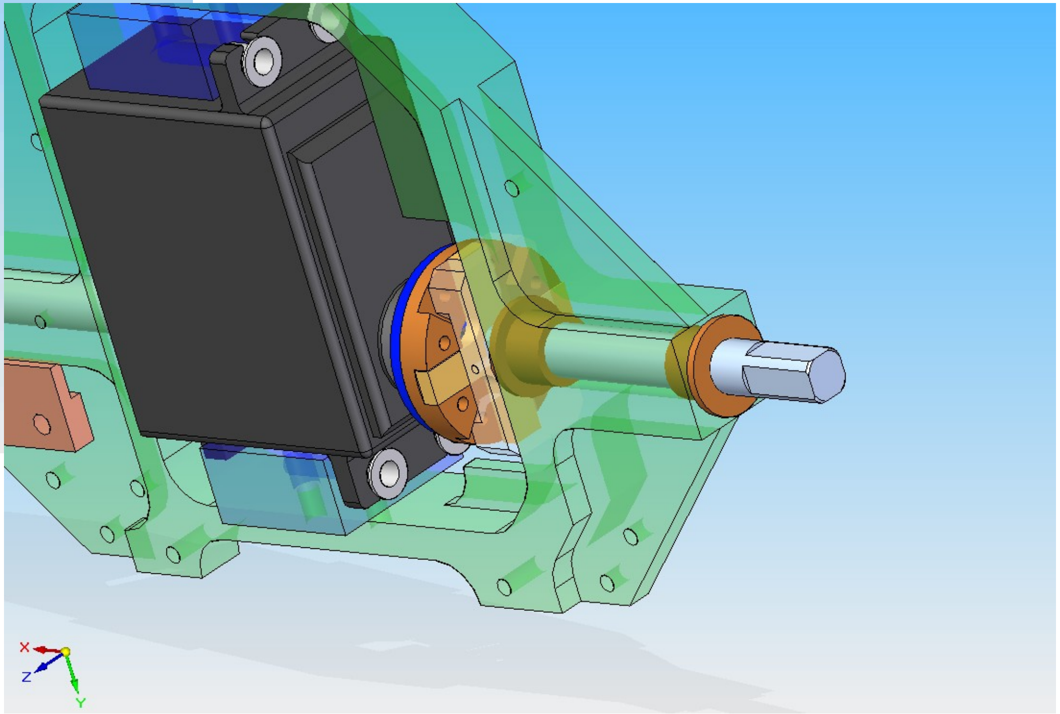


OUR GRID

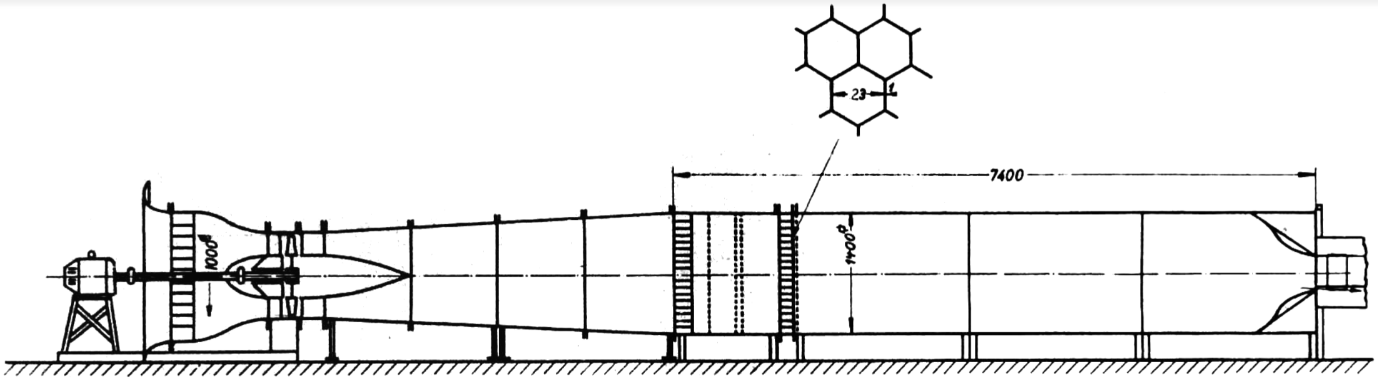


We (*uniquely*) have:

- -independent paddles
- -feedback-control of angle

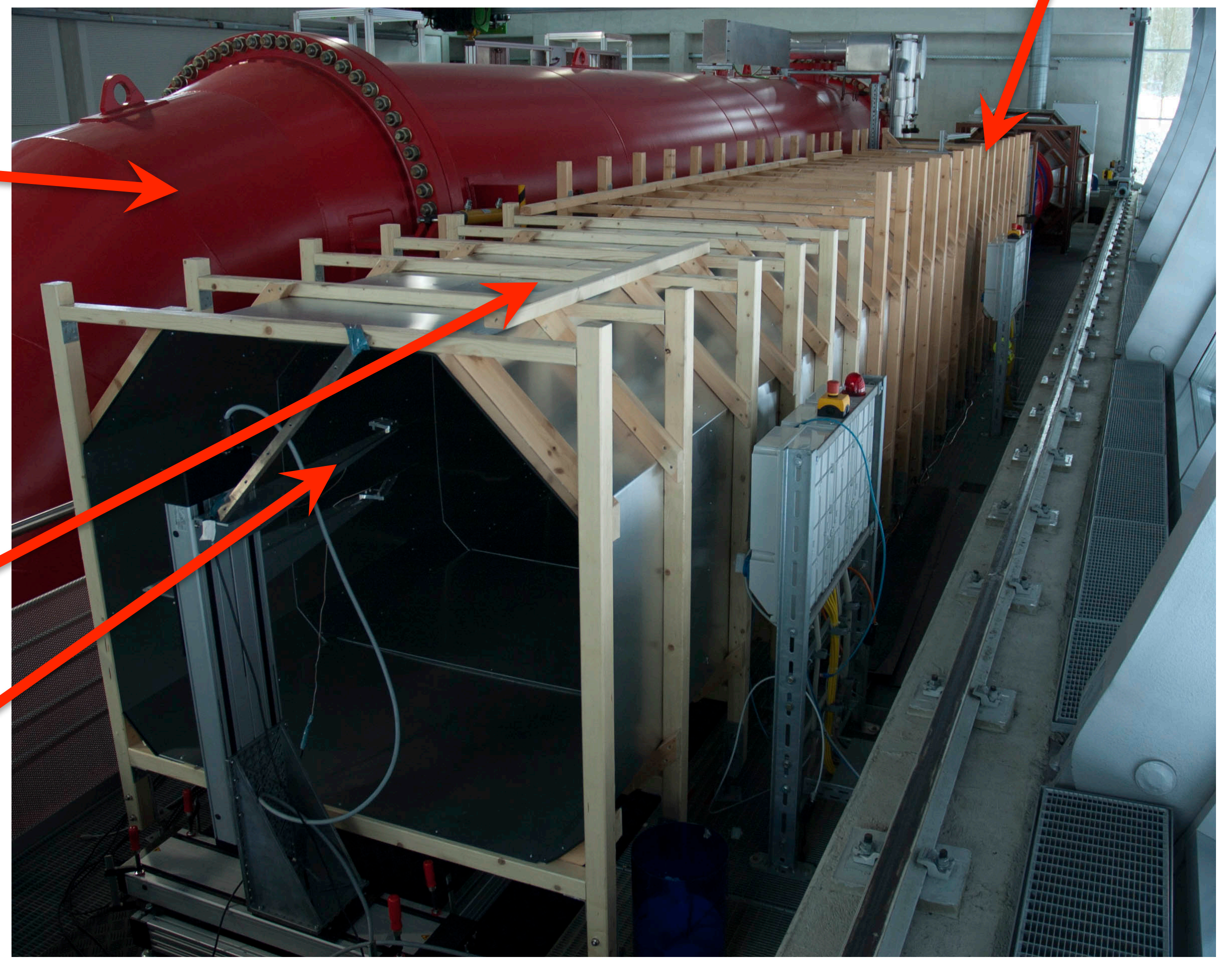


M = 163 mm



THE ACTIVE GRID

THE VDTT

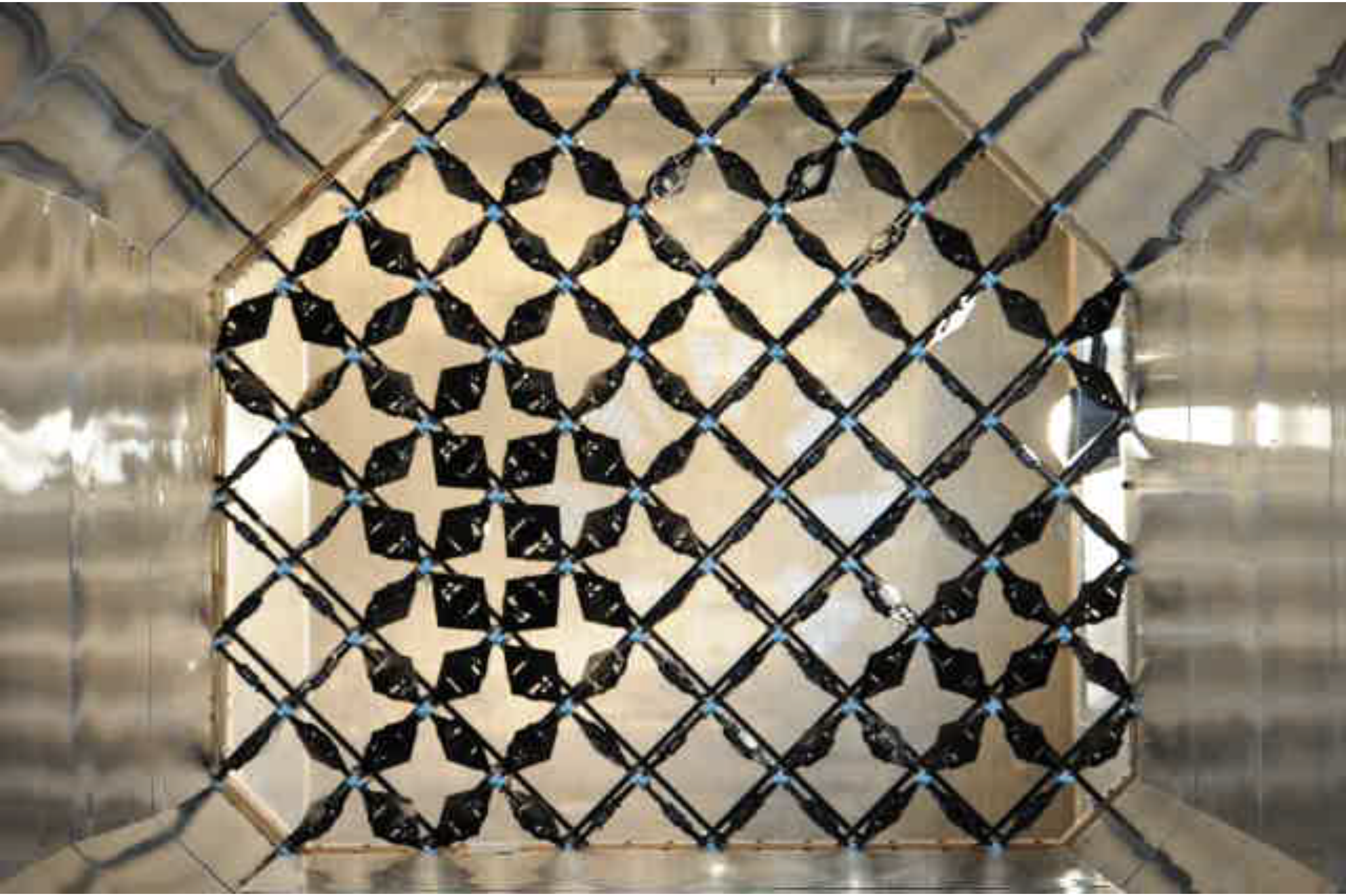


THE PRANDTL TUNNEL

(1938)

HOT WIRE PROBES

50M DOWNSTREAM

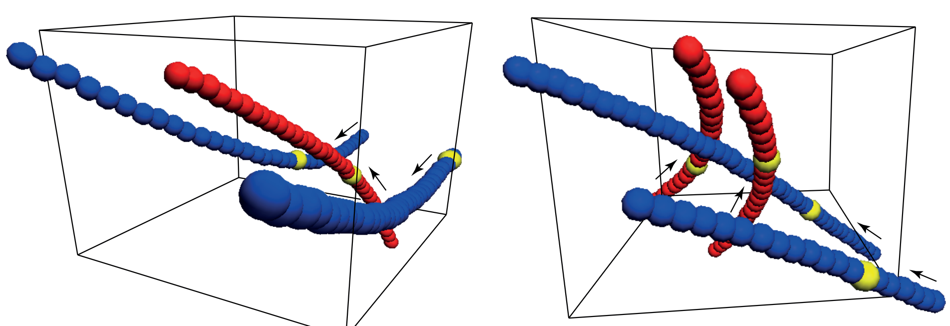


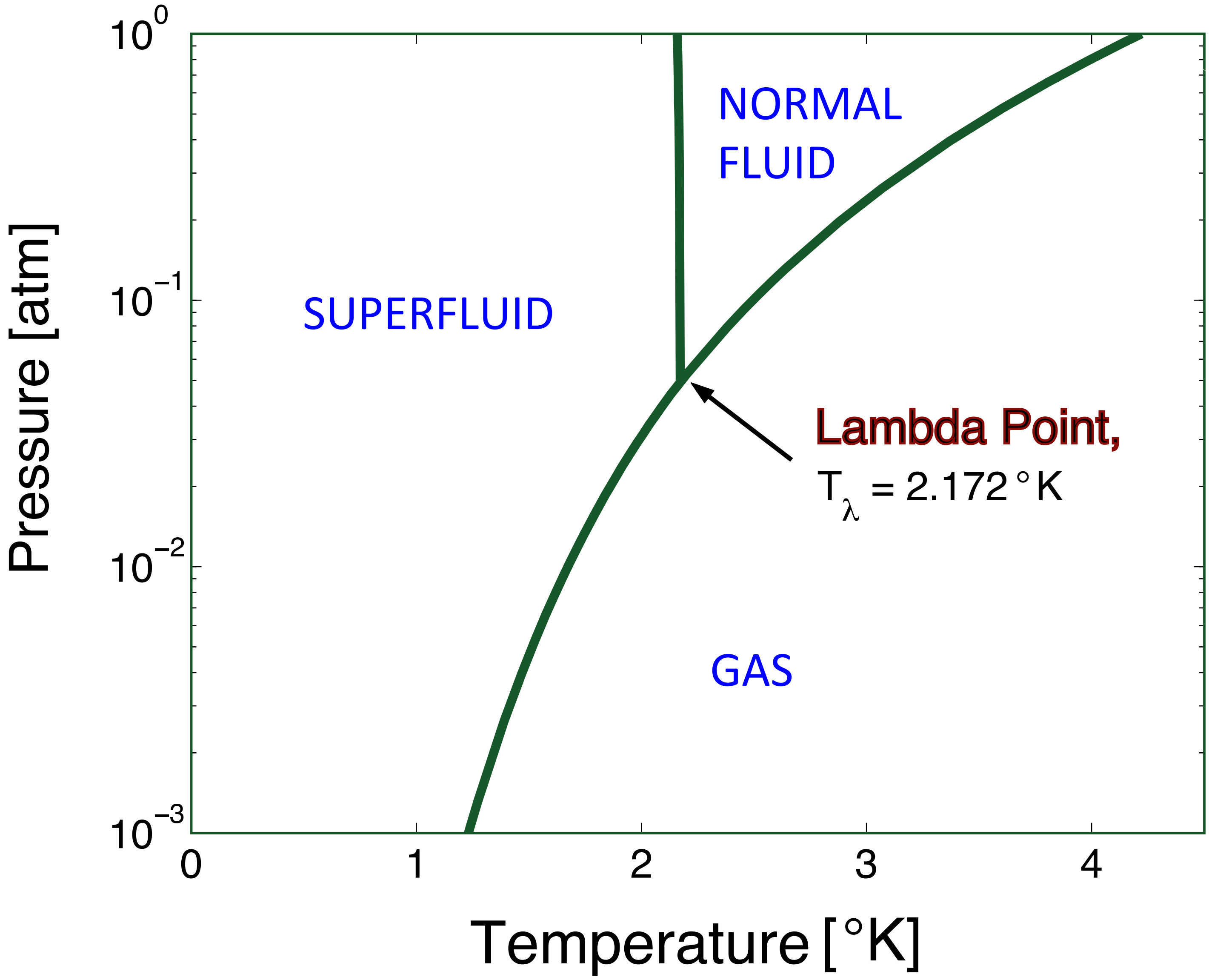
Mechanisms

reconnection

e.g. Kerr (2013) *Phys. Fluids*

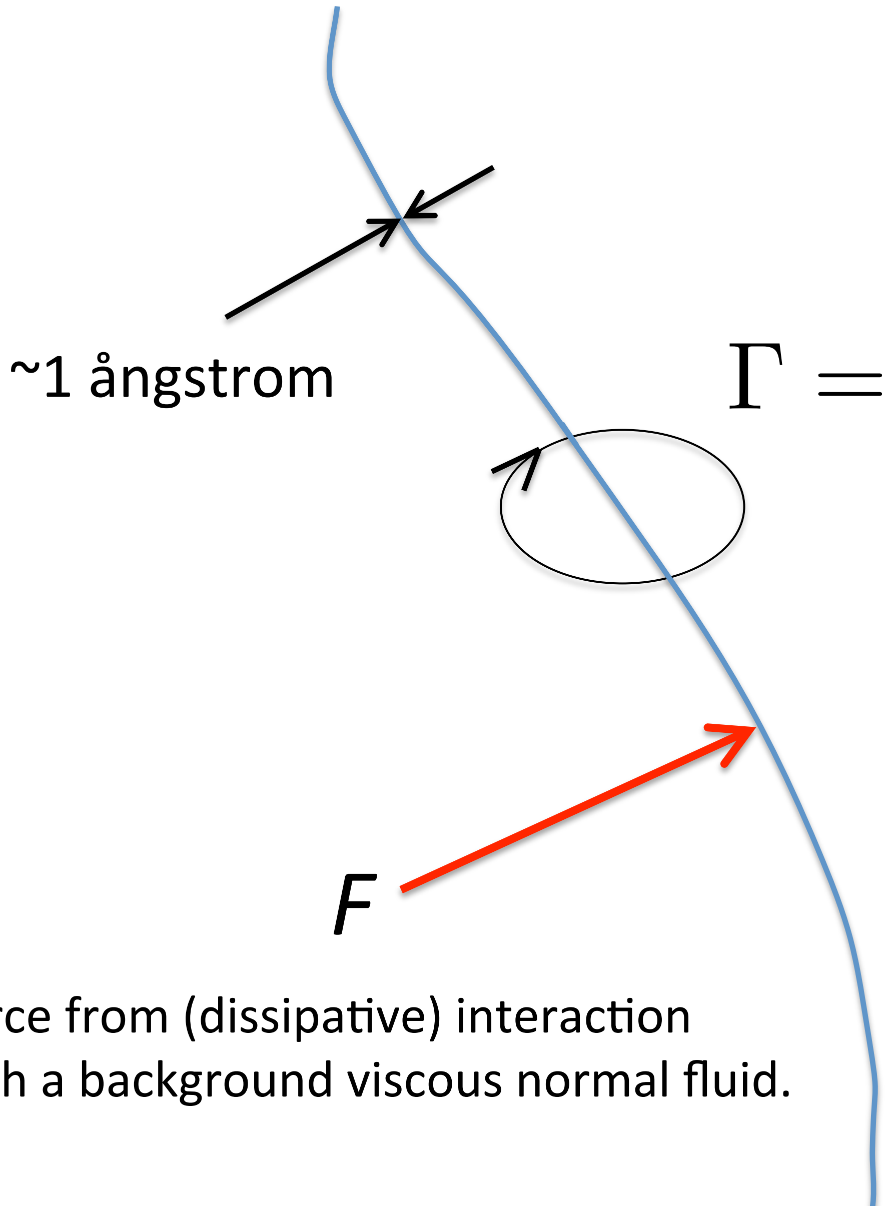
rain formation





NORMAL
FLUID
SUPERFLUID
GAS
Lambda Point,
$T_\lambda = 2.172\,^\circ K$
Pressure [atm]
Temperature [°K]
10^0
10^{-1}
10^{-2}
10^{-3}
0
1
2
3
4

QUANTIZED VORTEX



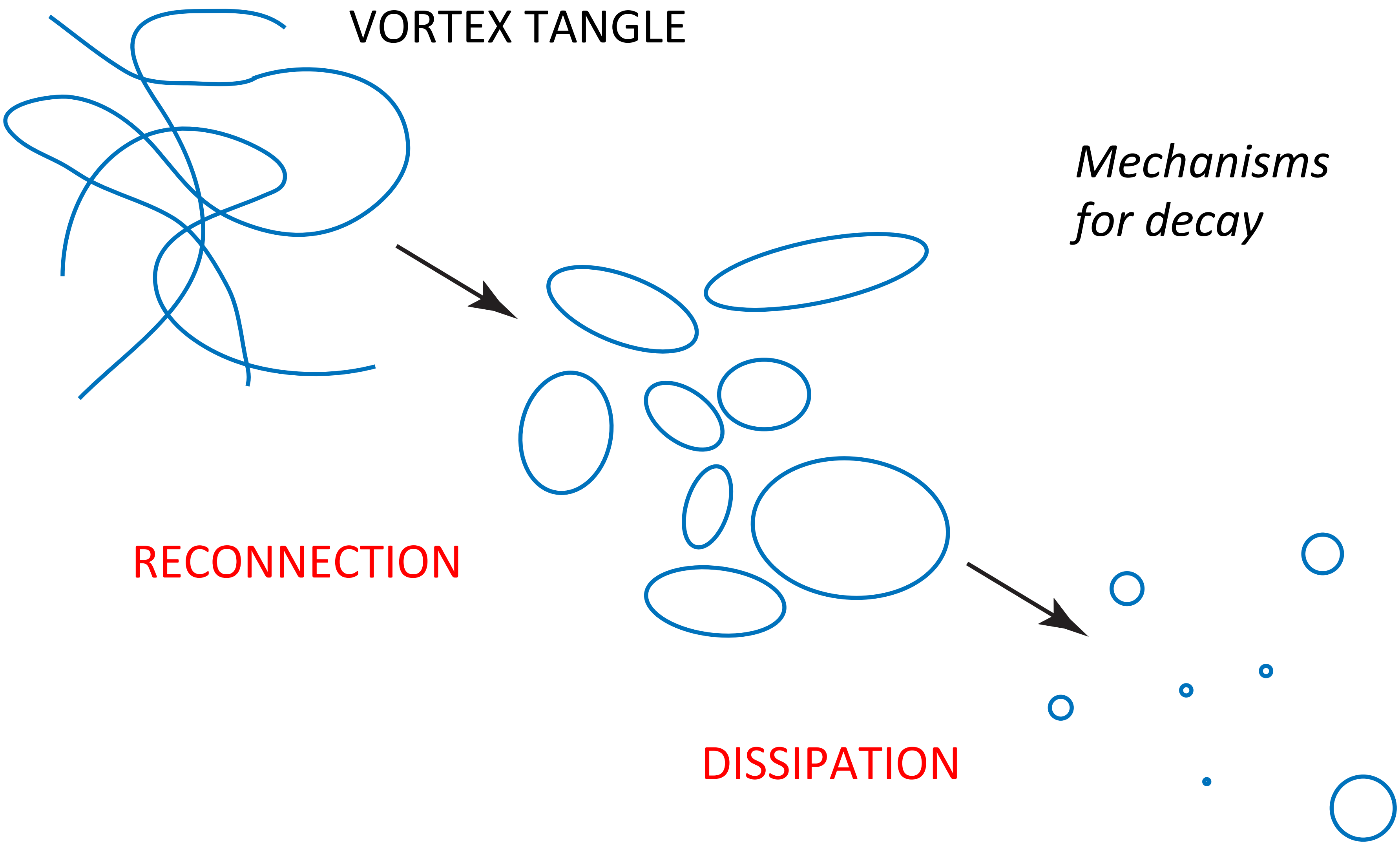
$$\Gamma = \oint v \cdot ds = \kappa$$

$$\kappa = \frac{h}{m} \approx 10^{-3} \, \frac{cm^2}{s}$$

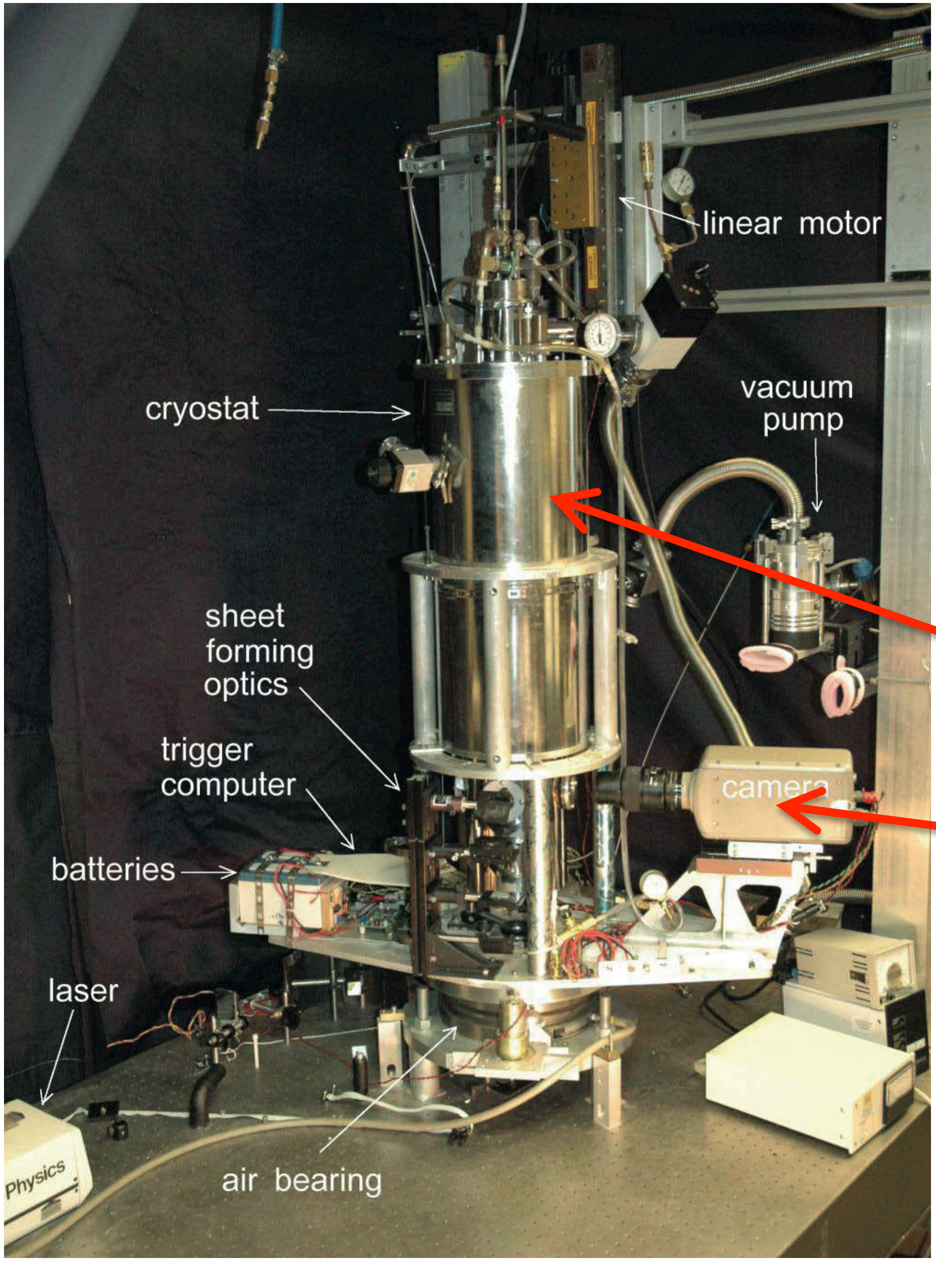
m – mass of a helium atom

Force from (dissipative) interaction
with a background viscous normal fluid.

Onsager, L. *Proc. Intern. Conf. Theor. Phys.*, Kyoto and Tokyo, Science Council of Japan, Tokyo, 877-880 (1953).

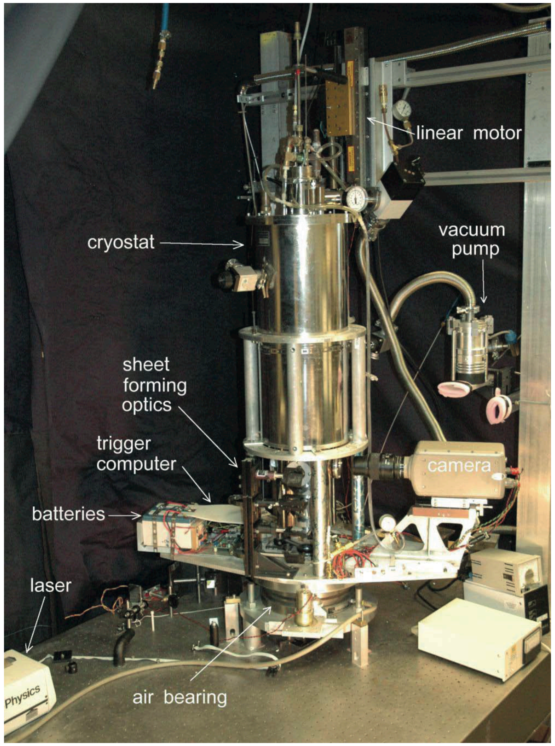


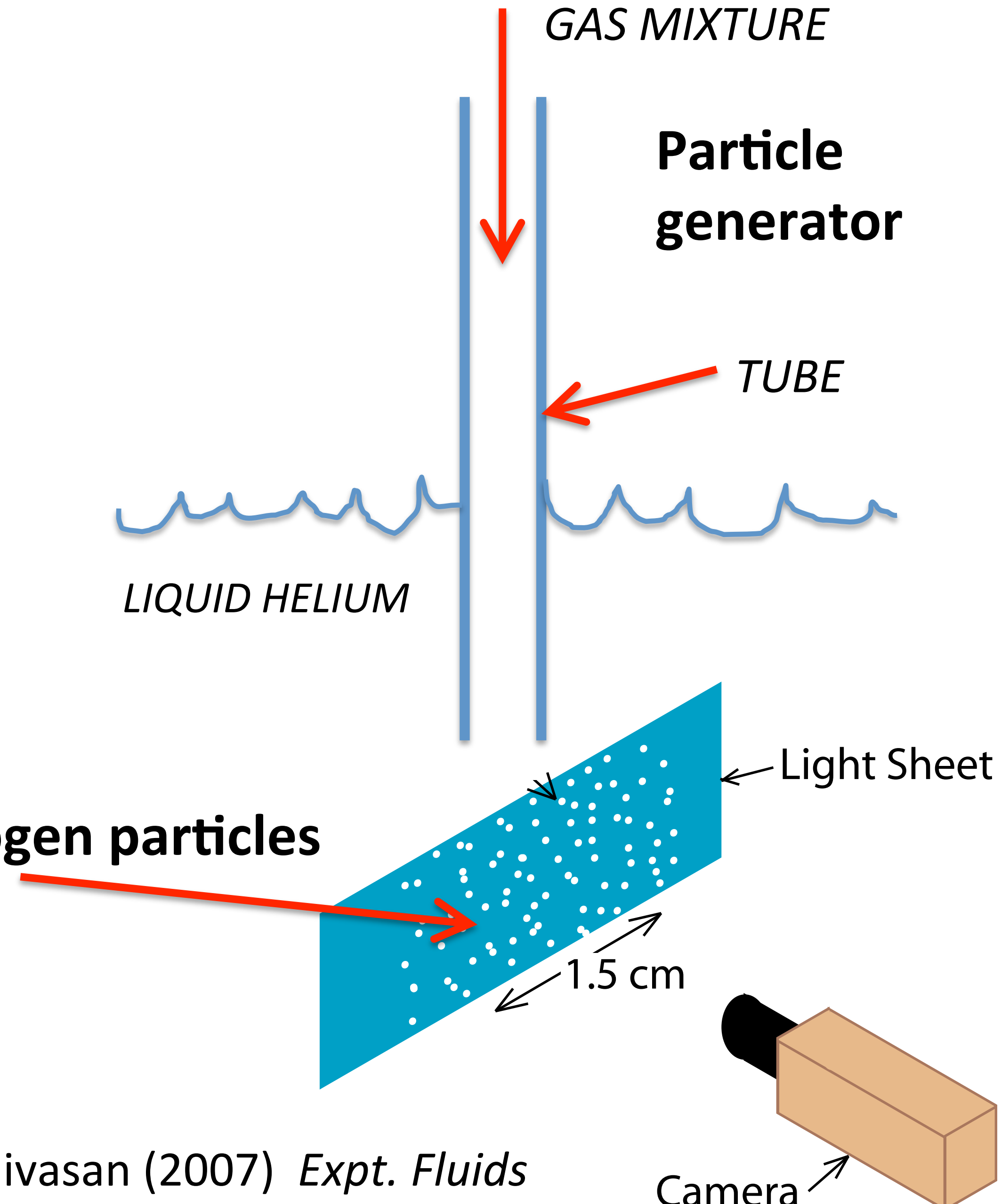
Feynman, R.P. *Progress in Low Temp. Phys. I*, C.J. Gorter, ed.,
North-Holland Publishing Co., 17-53 (1955).



OPTICAL CRYOSTAT

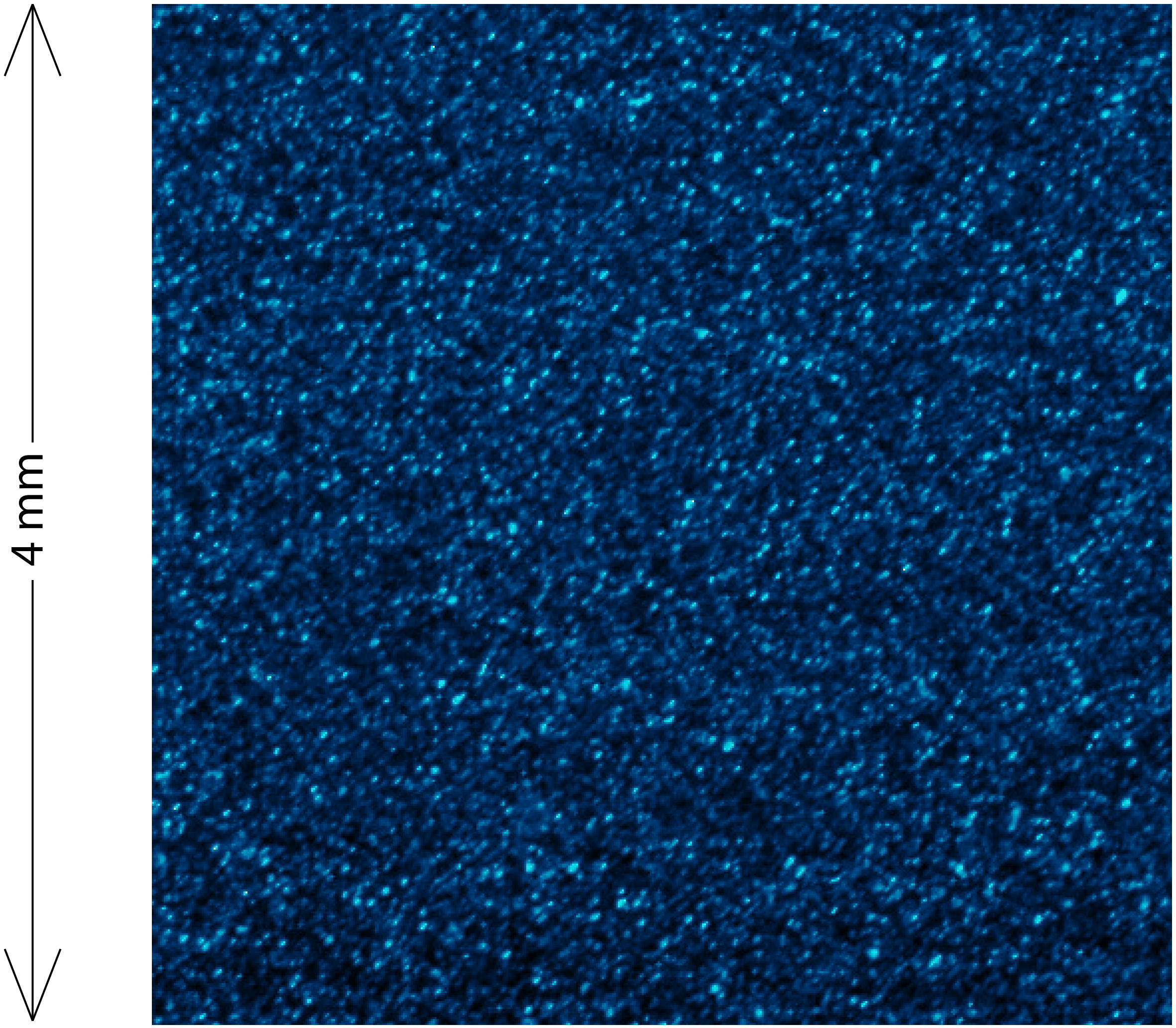
CAMERA



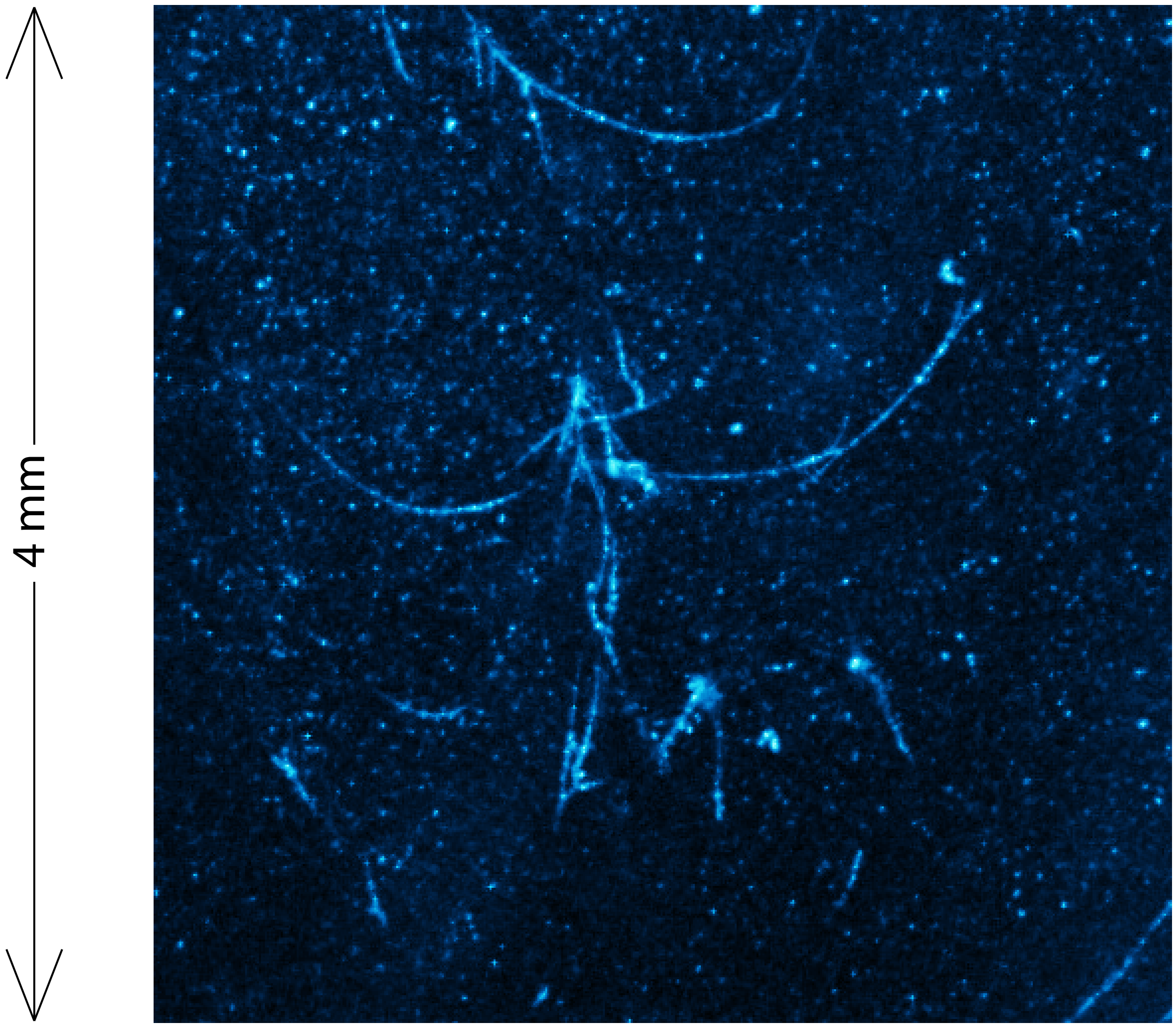


Bewley, Lathrop and Sreenivasan (2007) *Expt. Fluids*
Bewley and Vollmer (2013) *Physica Scripta*

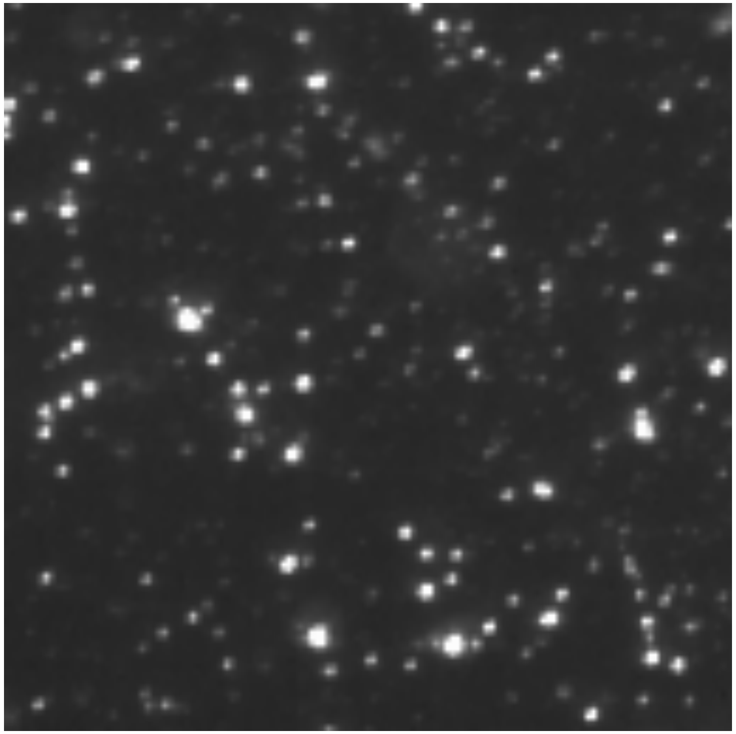
In normal liquid helium, $T > T_\lambda$



In superfluid liquid helium, $T < T_\lambda$



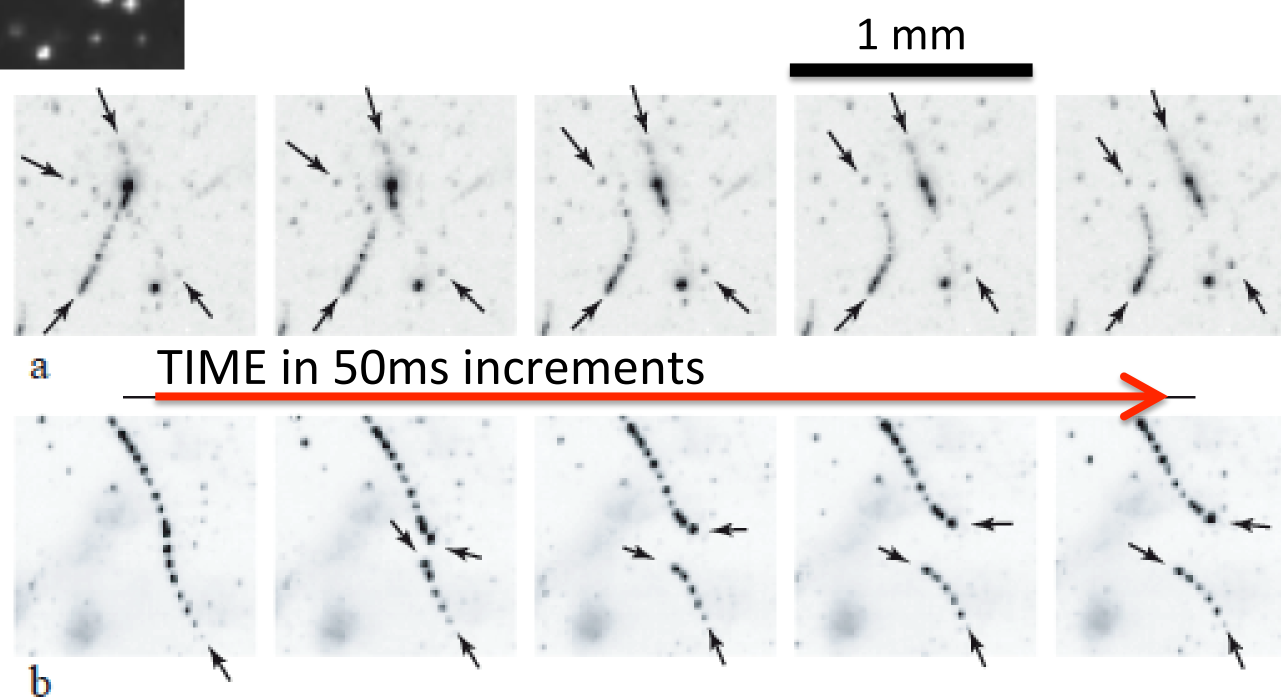
Bewley, Lathrop, Sreenivasan (2006) *Nature*



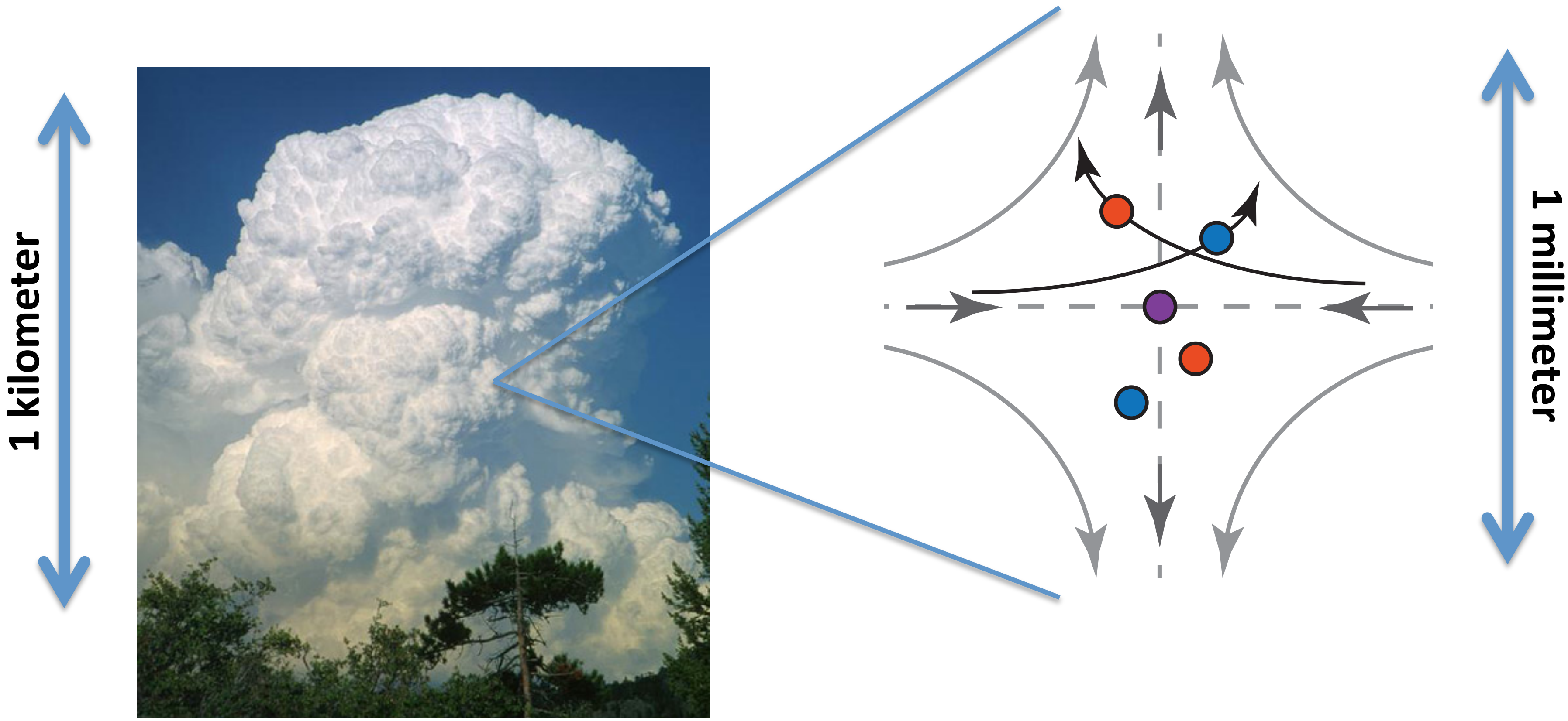
RECONNECTION

Bewley et al. (2007) *PNAS*

Bewley (2009) *Cryogenics*



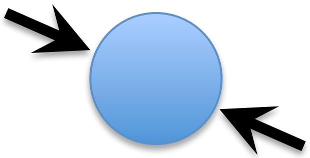
DROPLET DYNAMICS



...what happens when droplet inertia first starts to become important?

particle in a turbulent flow:

$$St = \frac{\tau_p}{\tau_\eta}$$



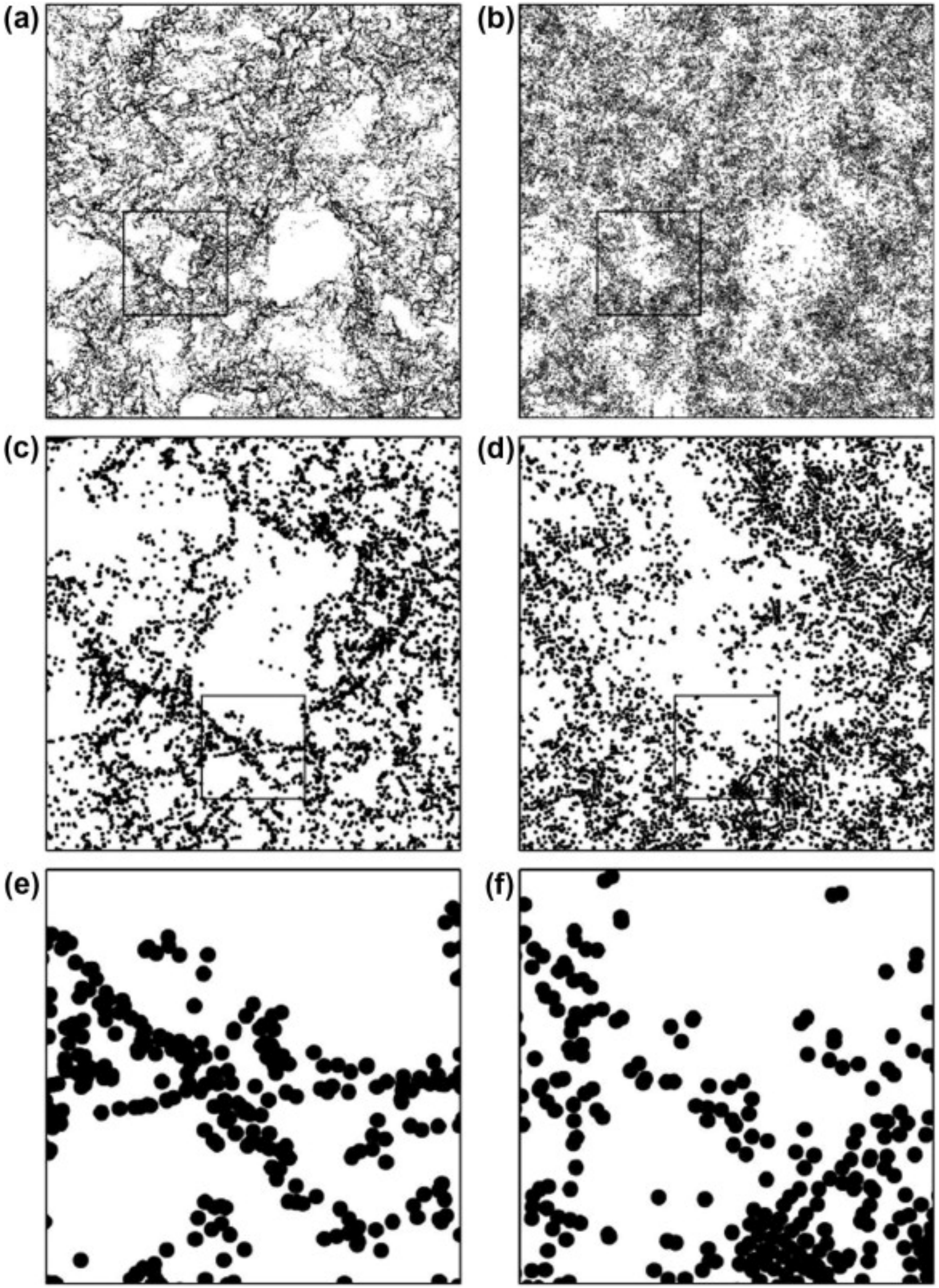
τ_p droplet response time

τ_η turbulence time scale

$St \ll 1$

$St \gg 1$

For intermediate St…



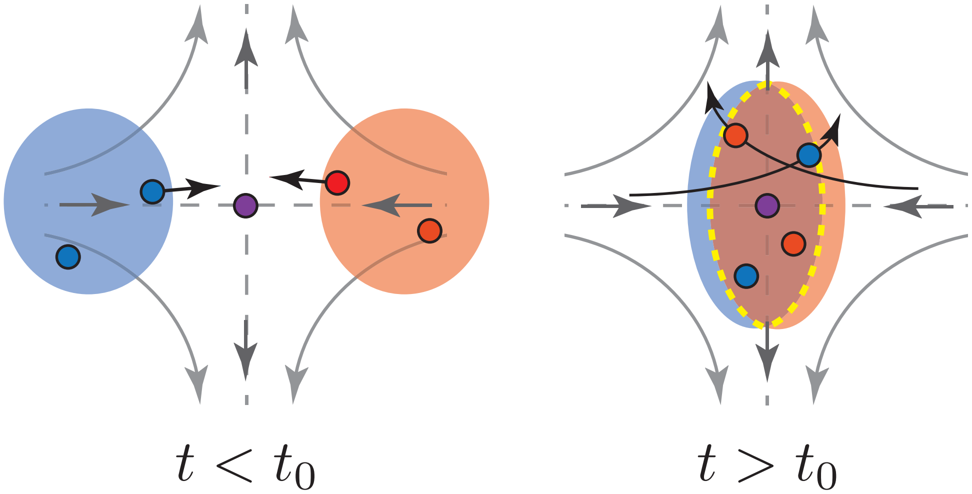
PARTICLE FIELD IS “SOFT”

Maxey (1987) *JFM*

BUT ALSO INTERPENETRATING…

Monchaux, Bourgoin, Cartellier (2012) *Int. J. Multiphase Flow*

PARTICLE FIELD IS INTERPENETRATING

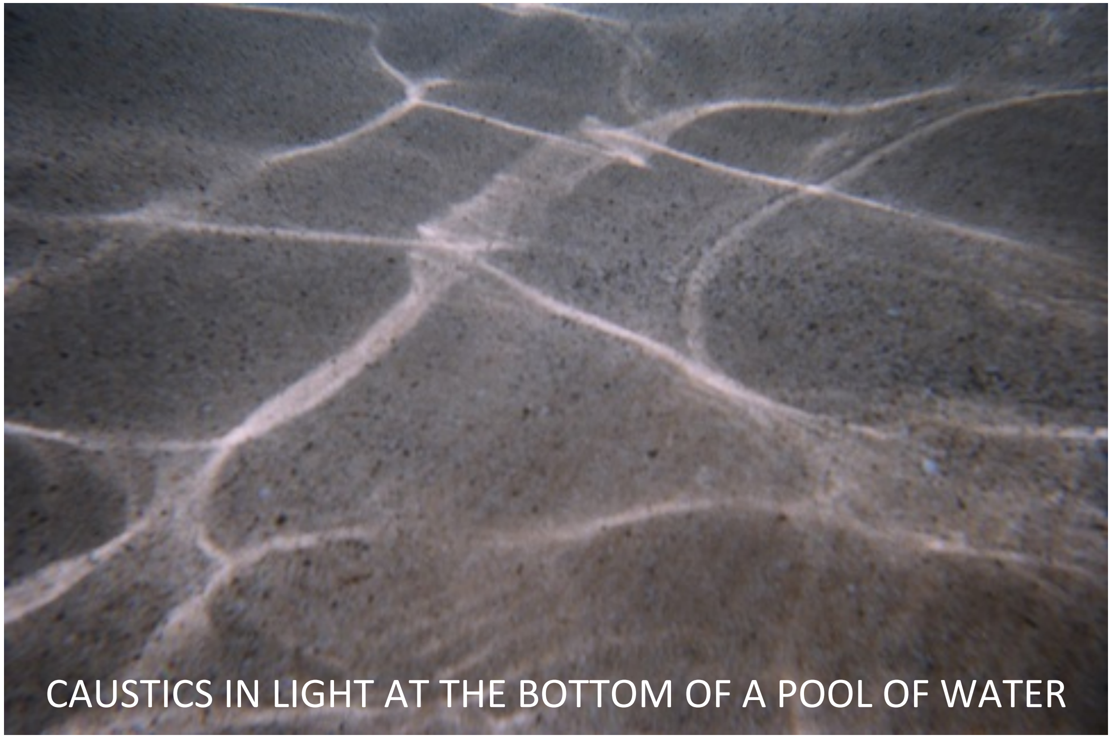


Falkovich, Fouxon and Stepanov (2002) *Nature*

Wilkinson and Mehlig (2005) *EPL*

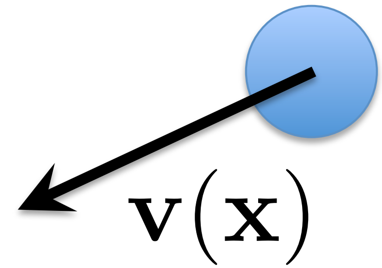
CAUSTICS

Berry (1980) *Les Houches, Session XXXV*



gradient of (simplified) droplet-momentum equation:

$$\nabla\left(\quad \frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u}-\mathbf{v}) + \mathbf{g} \quad \right)$$



$$\sigma = \nabla \mathbf{v}$$

$$s = \nabla \mathbf{u}$$

$$\frac{d\sigma}{dt} = \frac{1}{\tau_p}(s-\sigma) - \sigma^2$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$$

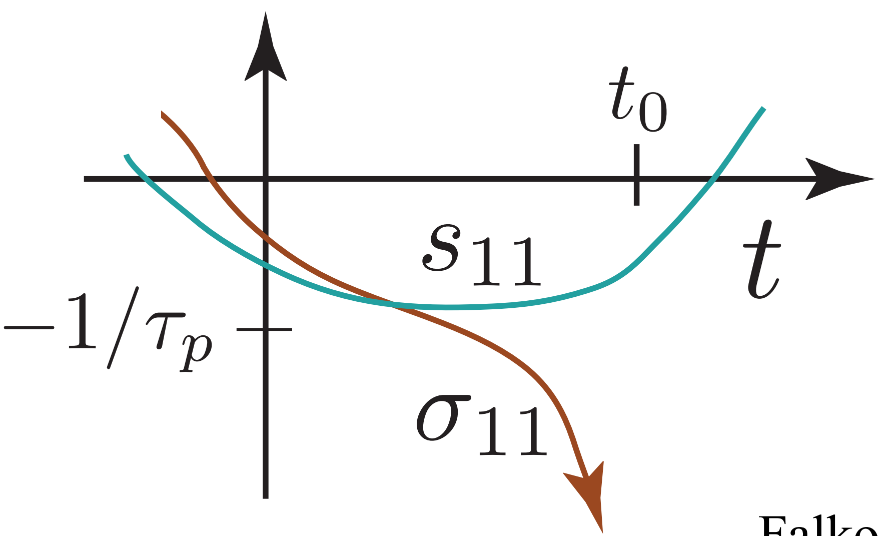
$\mathbf{u}$ fluid velocity

$\mathbf{v}$ droplet velocity

$$\frac{d\sigma}{dt} = \frac{1}{\tau_p}(s - \sigma) - \sigma^2$$

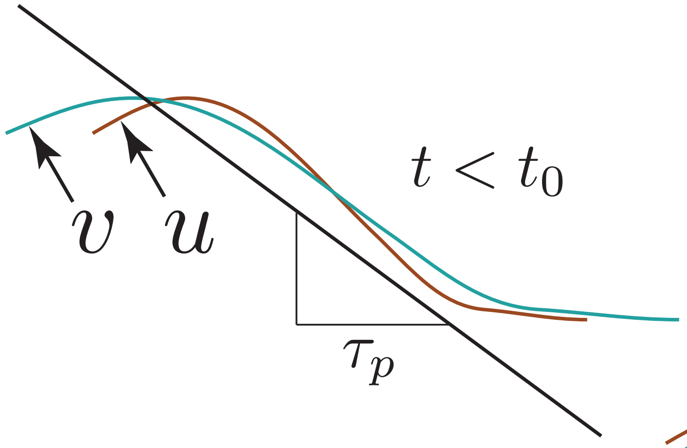
can dominate when $|\sigma| > \frac{1}{\tau_p}$

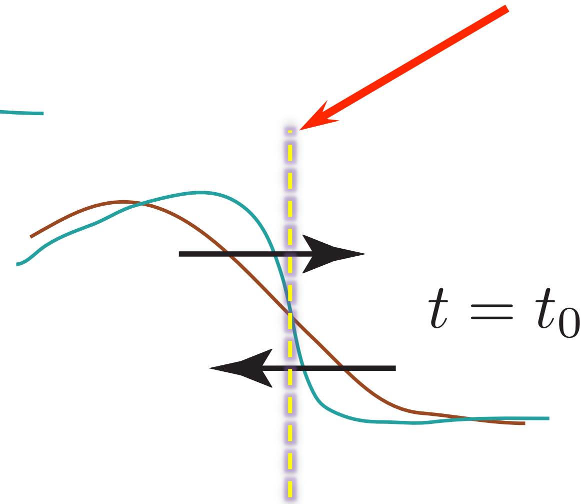
causes unbounded growth when $\sigma < 0$

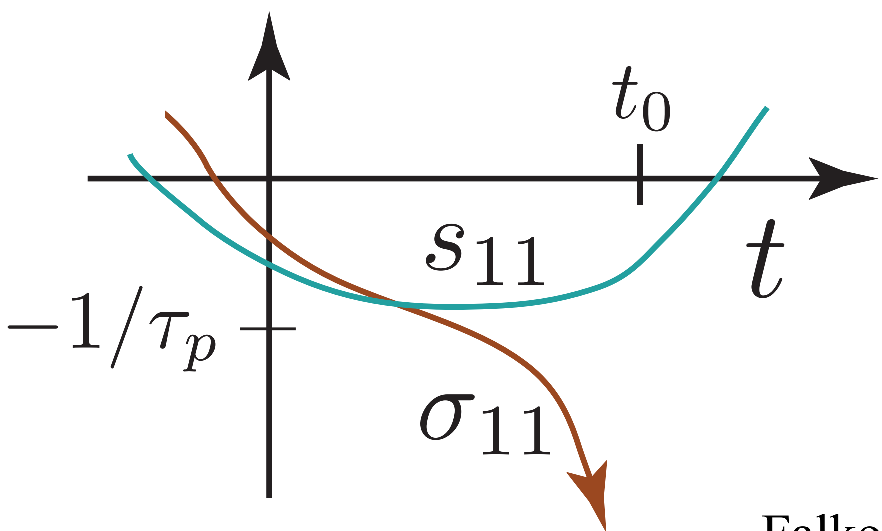


$$\sigma = \nabla \mathbf{v}$$
$$s = \nabla \mathbf{u}$$

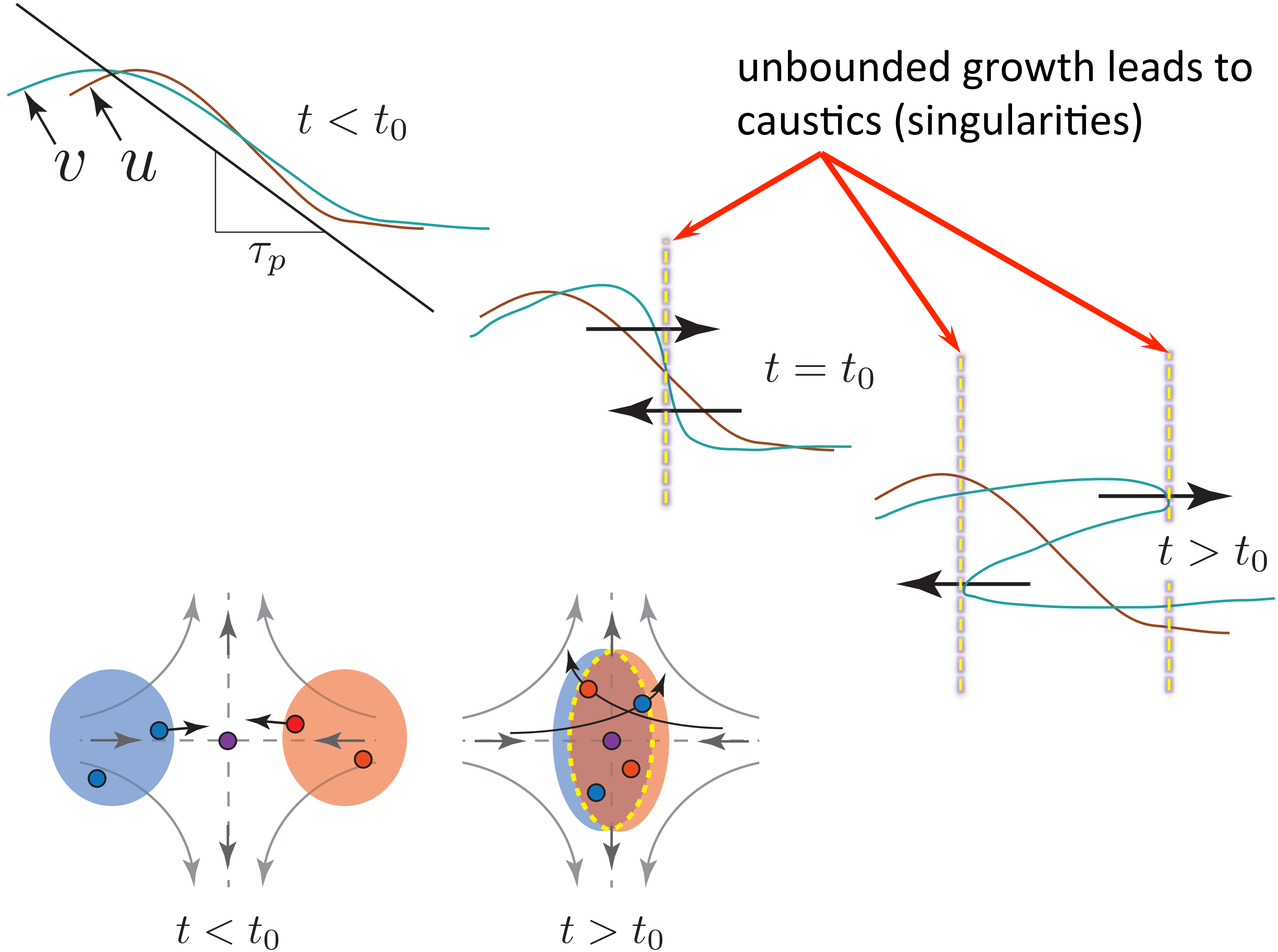
Falkovich, Fouxon and Stepanov (2002) *Nature* unbounded growth leads to caustics (singularities)





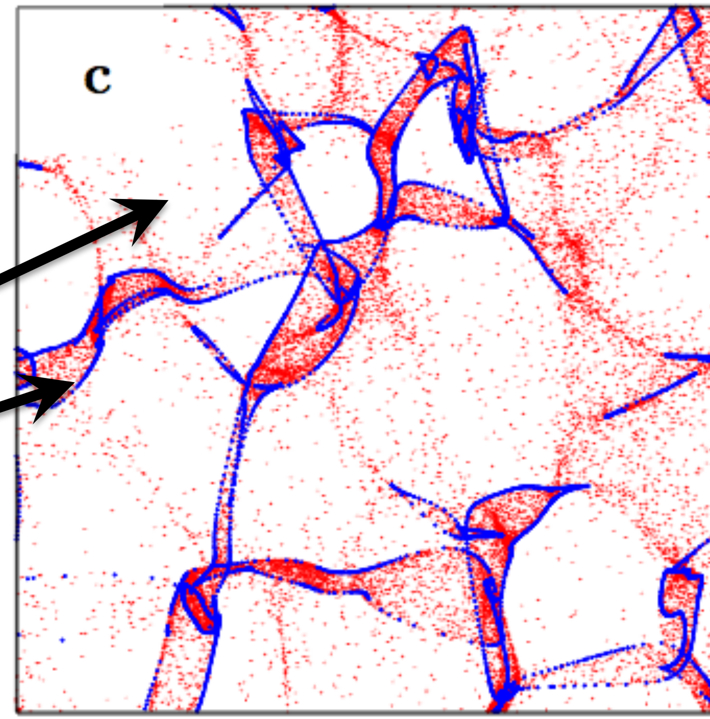


Falkovich, Fouxon and Stepanov (2002) *Nature*


unbounded growth leads to caustics (singularities)
v
u
t < t0
τp
t = t0
t > t0
t < t0
t > t0

CAUSTICS AMONG PARTICLES

NUMERICAL MODEL

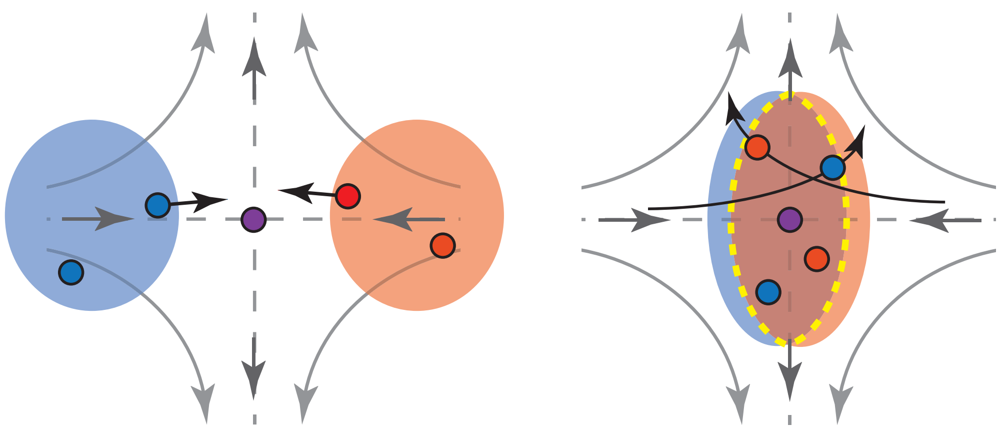


smooth field

singularity

Wilkinson and Mehlig (2005) *EPL*

Salazar and Collins (2012) *JFM*

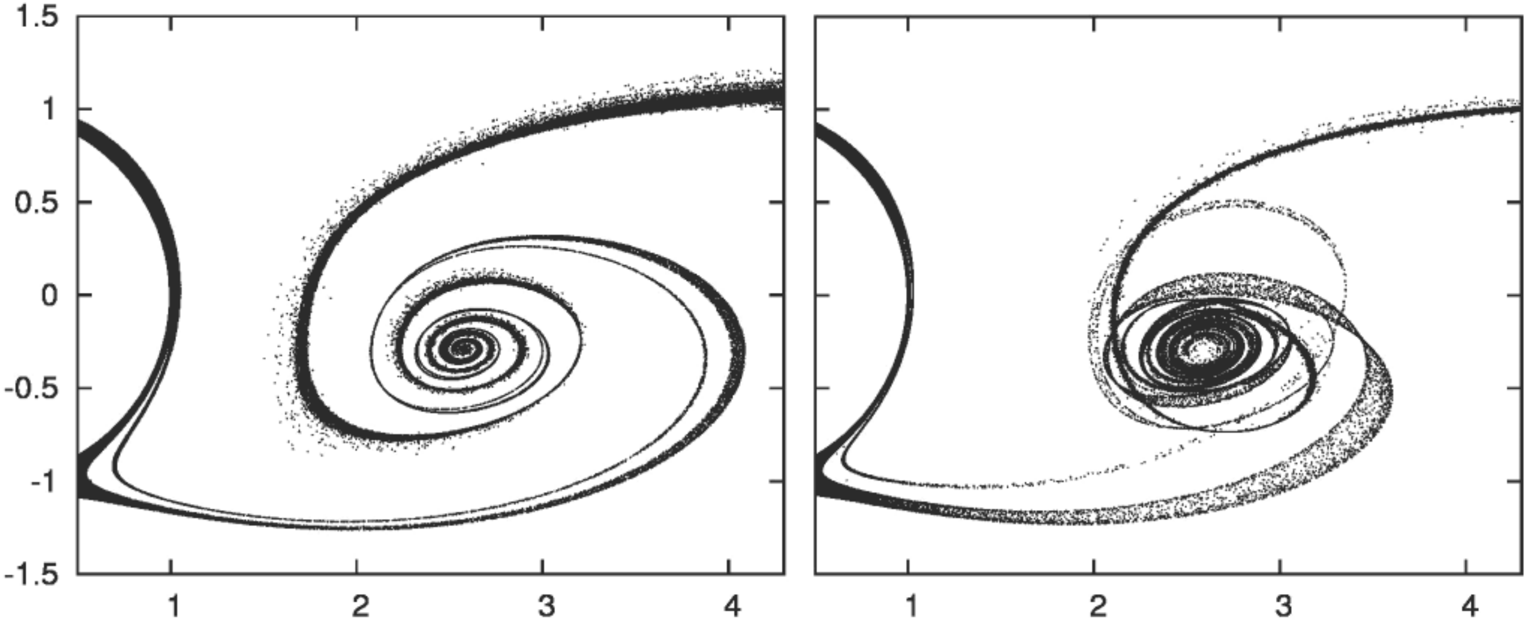


$t < t_0$ $\qquad$ $t > t_0$

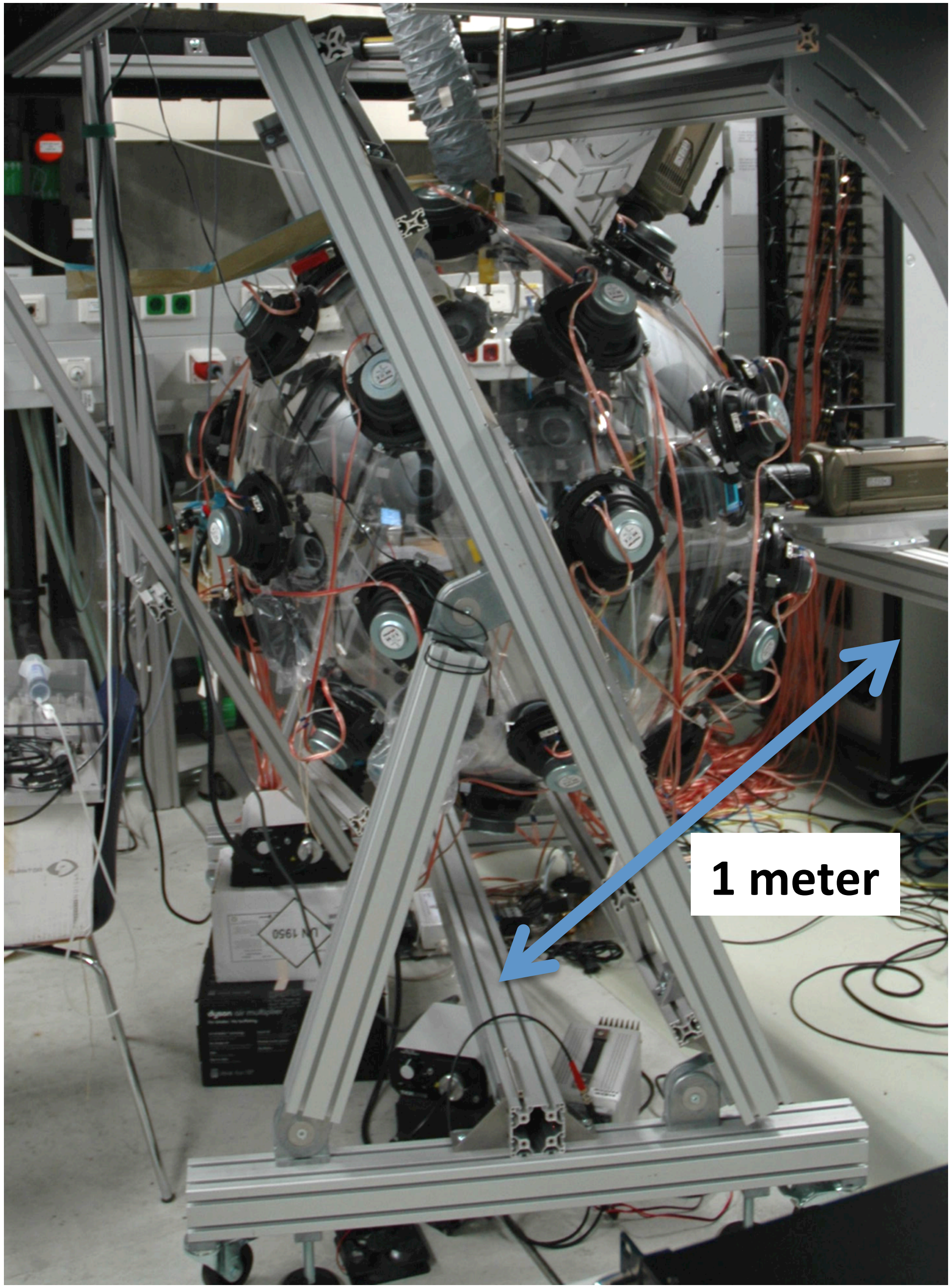
$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u} - \mathbf{v}) + \mathbf{g}$$ + history + Basset + added mass + …

Maxey and Riley (1983) *Phys. Fluids*

History suppresses caustics:



Daitche and Tél (2011) *PRL*



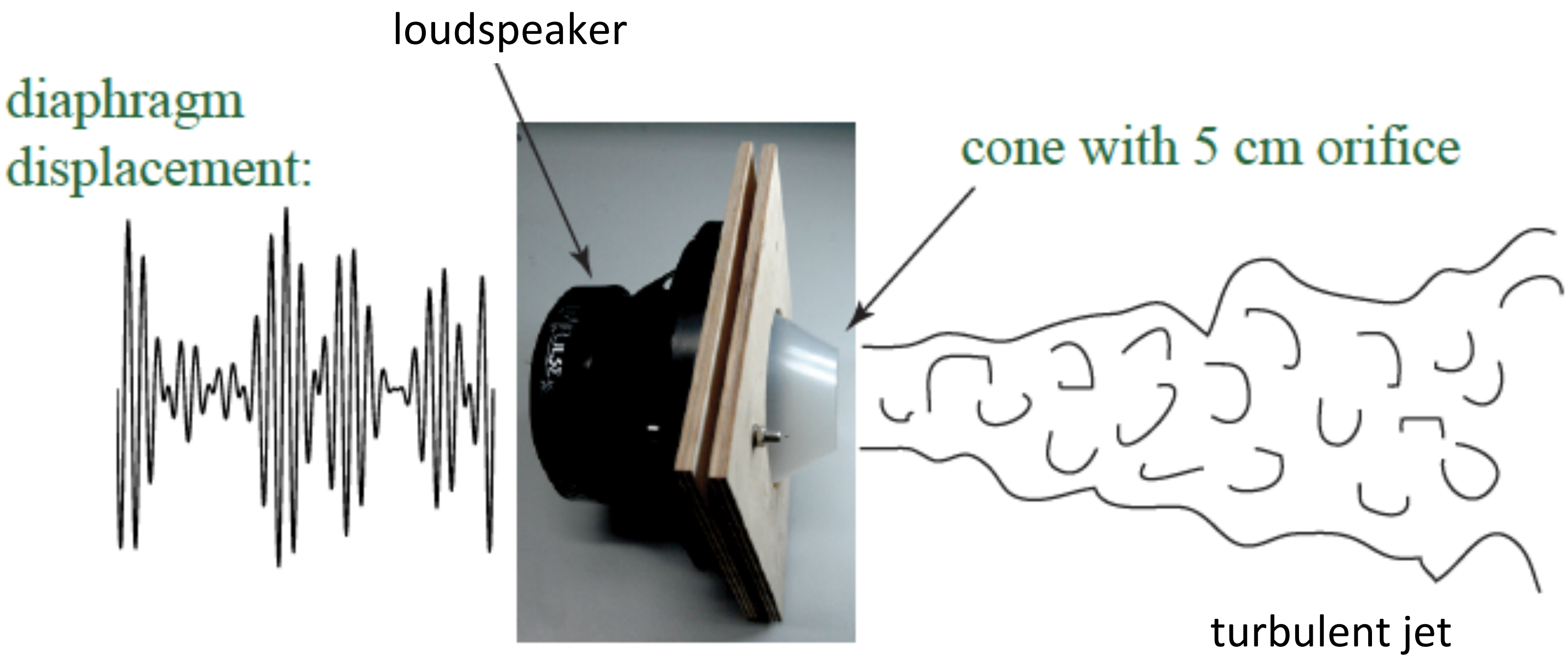
CRYSTAL (SOCCER) BALL

1 m Acrylic ball

32 Independent, random-amplitude jets

$$R_\lambda \approx 200$$

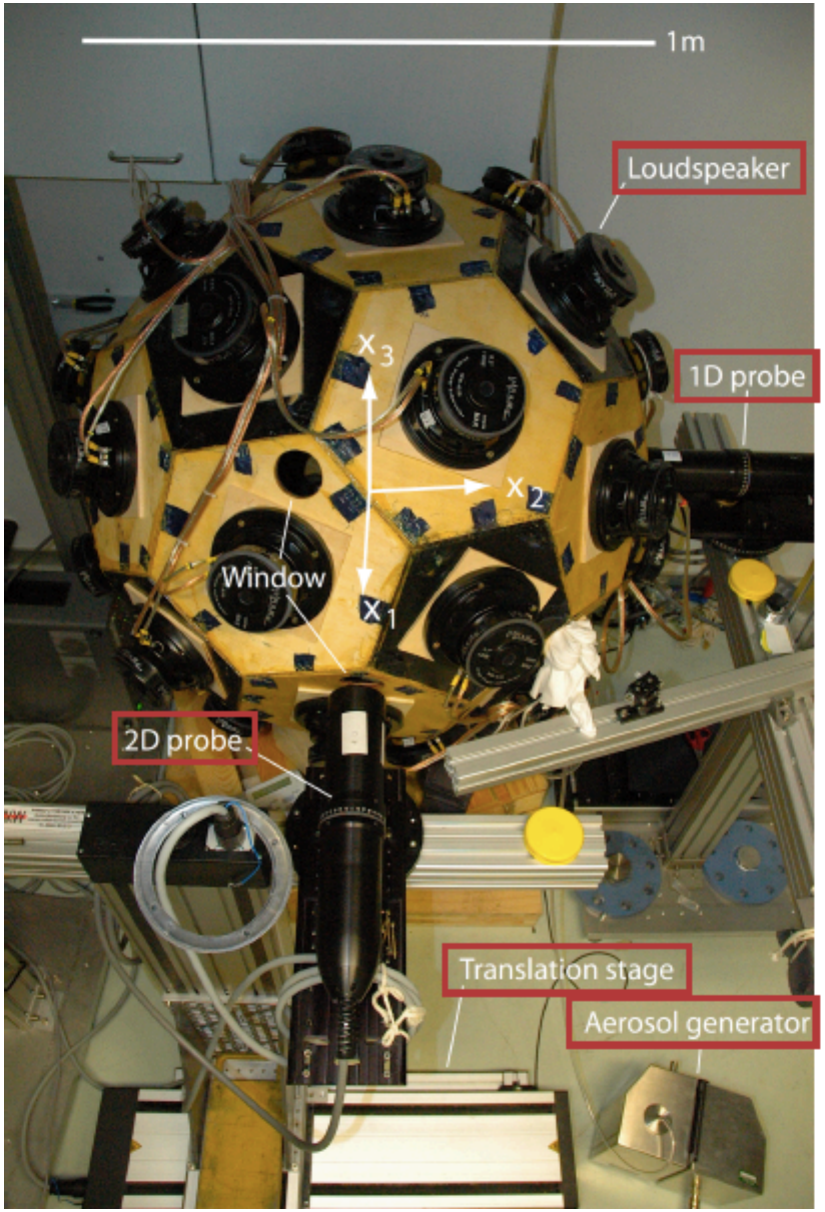
Chang, Bewley and Bodenschatz (2012) *J. Fluid Mech.*



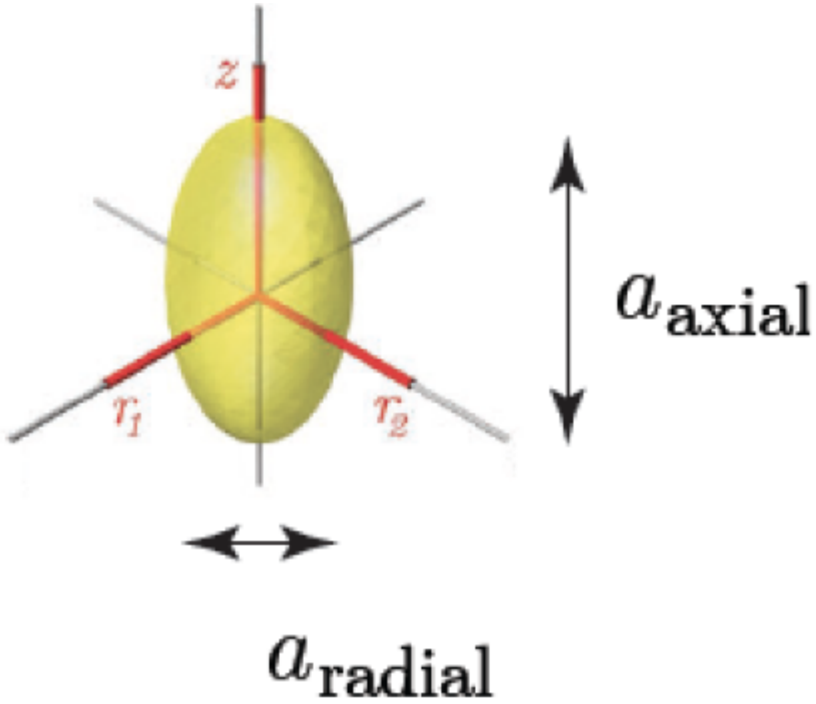
isotropic turbulence

$$R_\lambda \approx 200$$

Hwang and Eaton (2004) *Exp. Fluids*



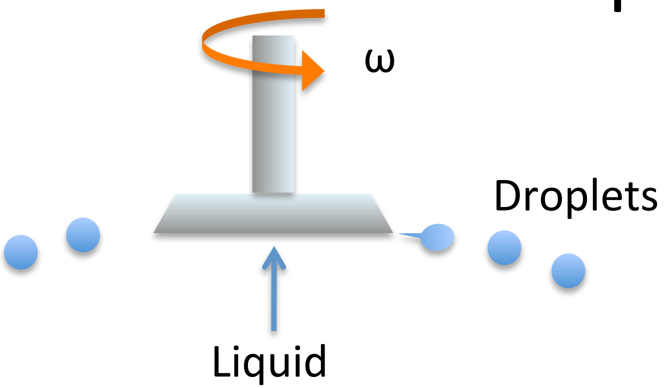
Control of anisotropy



$$\frac{u_z}{u_{r_1}} = \frac{u_z}{u_{r_2}} \equiv \frac{u_z}{u_r}$$

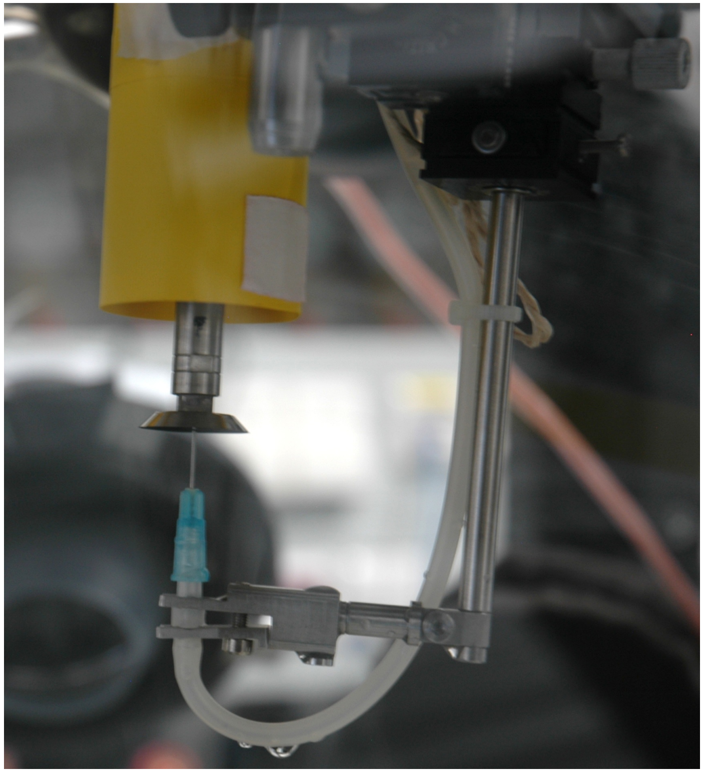
Bewley, Chang and Bodenschatz (2012) *Phys. Fluids*

Spinning Disk Droplet Generator.



60,000 rpm spinning disk.

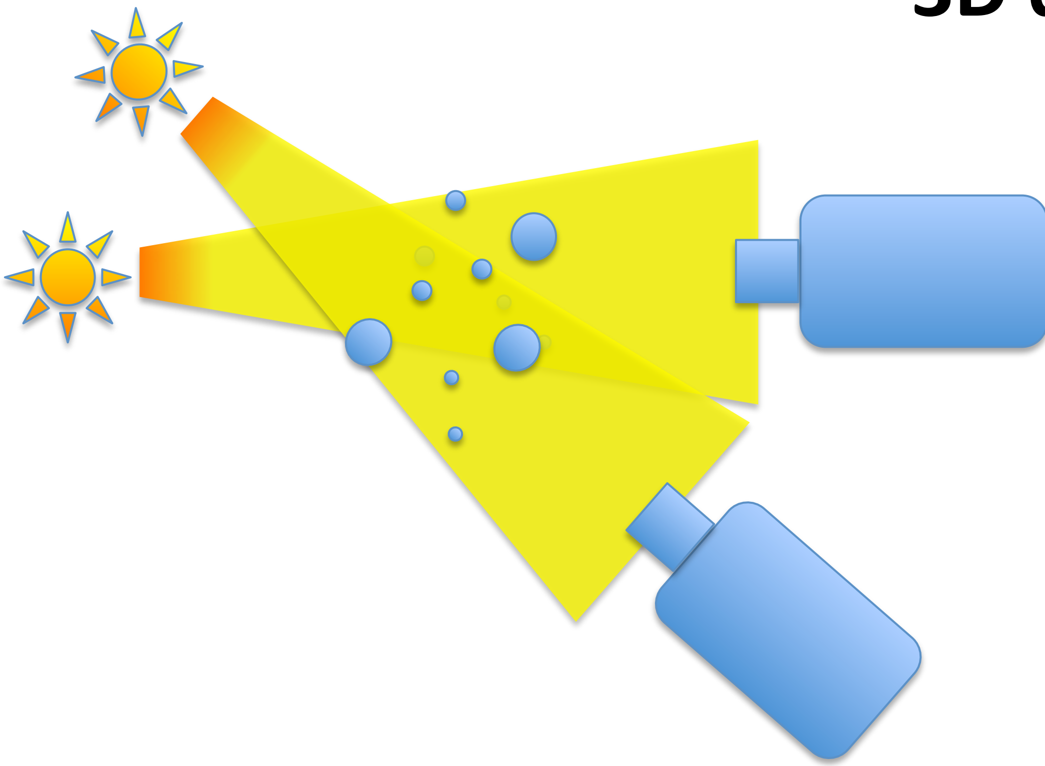
Droplets ejected from disk edge



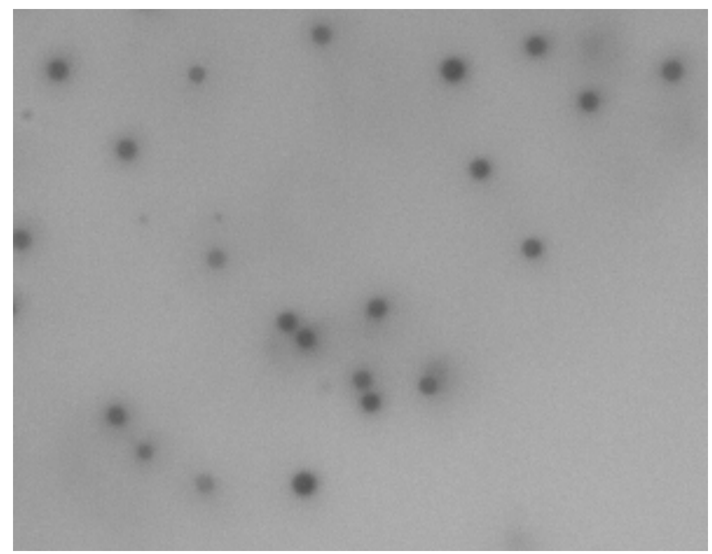
40% ethanol – 60% water

two classes of particles:
- Class 1: 9µm mean diameter
- Class 2: 18µm mean diameter

3D droplet tracking



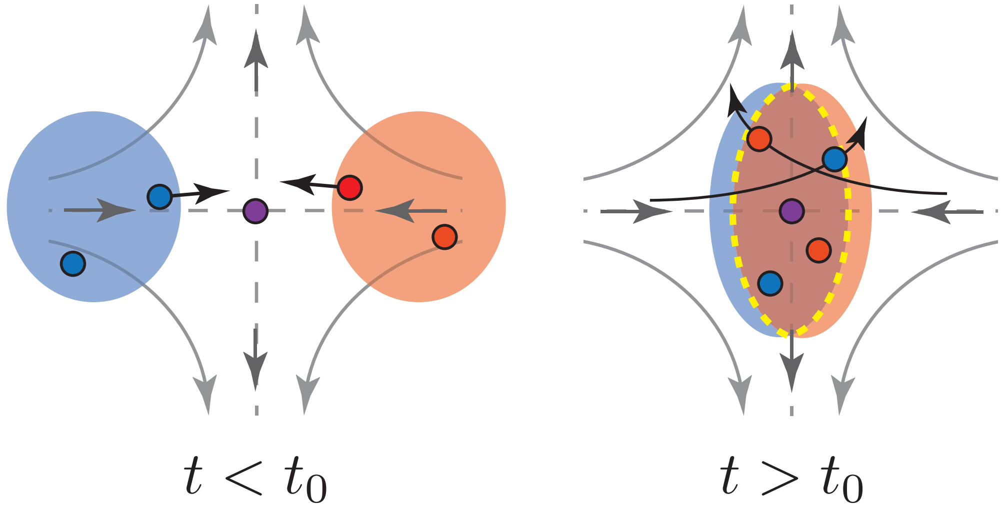
DROPLET IMAGES

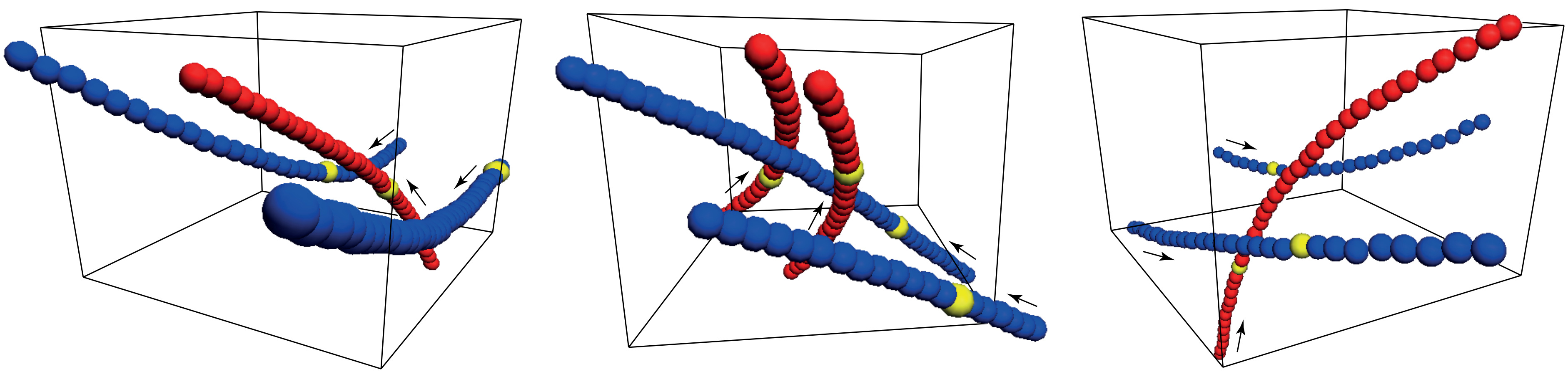


~1 millimeter

3 µm / pixels < η / 50	resolution
2 mm^3	view volume
15 KHz > 30 / τ_η	frame rate

ARE THERE POCKETS?

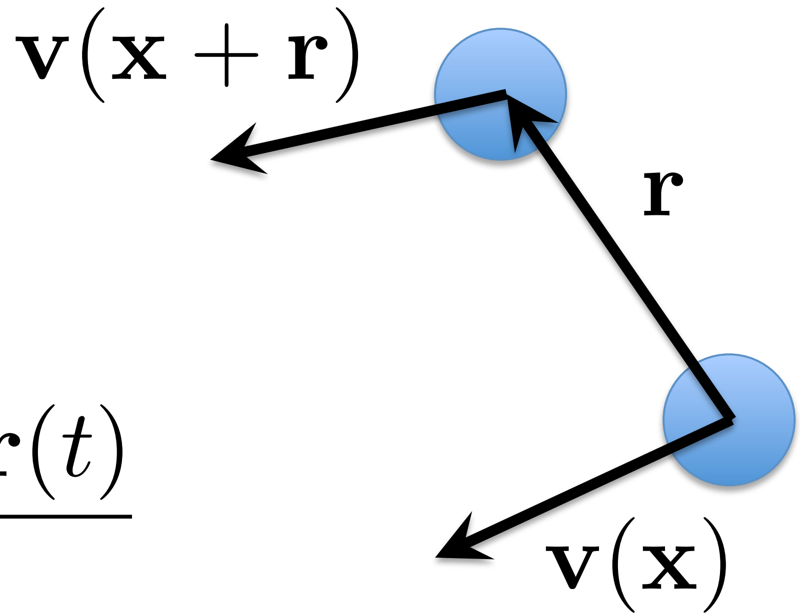




$St = 0.5$

YES

measure two-droplet statistics:



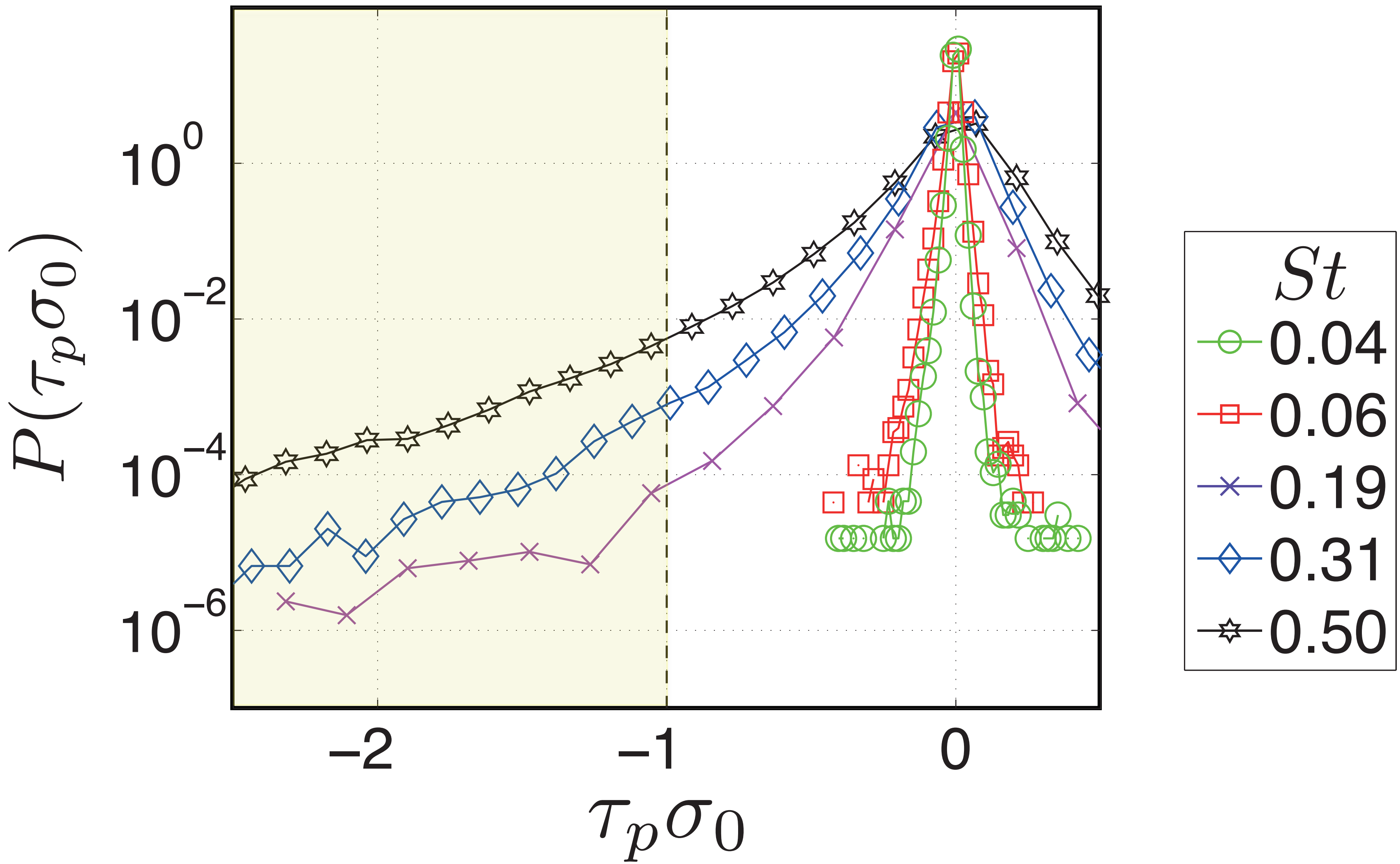
$$\sigma_{11} = \frac{\partial v_1}{\partial x_1} \approx \frac{\delta \mathbf{v}(t) \cdot \mathbf{r}(t)}{r^2}$$

$$\delta \mathbf{v}(\mathbf{r}, t) = \mathbf{v}(\mathbf{x}+\mathbf{r}, t) - \mathbf{v}(\mathbf{x}, t)$$

$$\mathbf{r} = (r, 0, 0)$$

ARE THERE LARGE-ENOUGH GRADIENTS?

$$0 < r/\eta < 3$$



How do the gradients evolve?

$$\boxed{\frac{d\sigma}{dt} = \frac{1}{\tau_p}(s-\sigma) - \sigma^2}$$

measure:

$$Q(t) \equiv \frac{\delta \mathbf{v}(\mathbf{r},t)\cdot\hat{\mathbf{r}}(t_1)}{\mathbf{r}(t)\cdot\hat{\mathbf{r}}(t_1)} \approx \sigma_{11}$$

for small t-t_1, small r

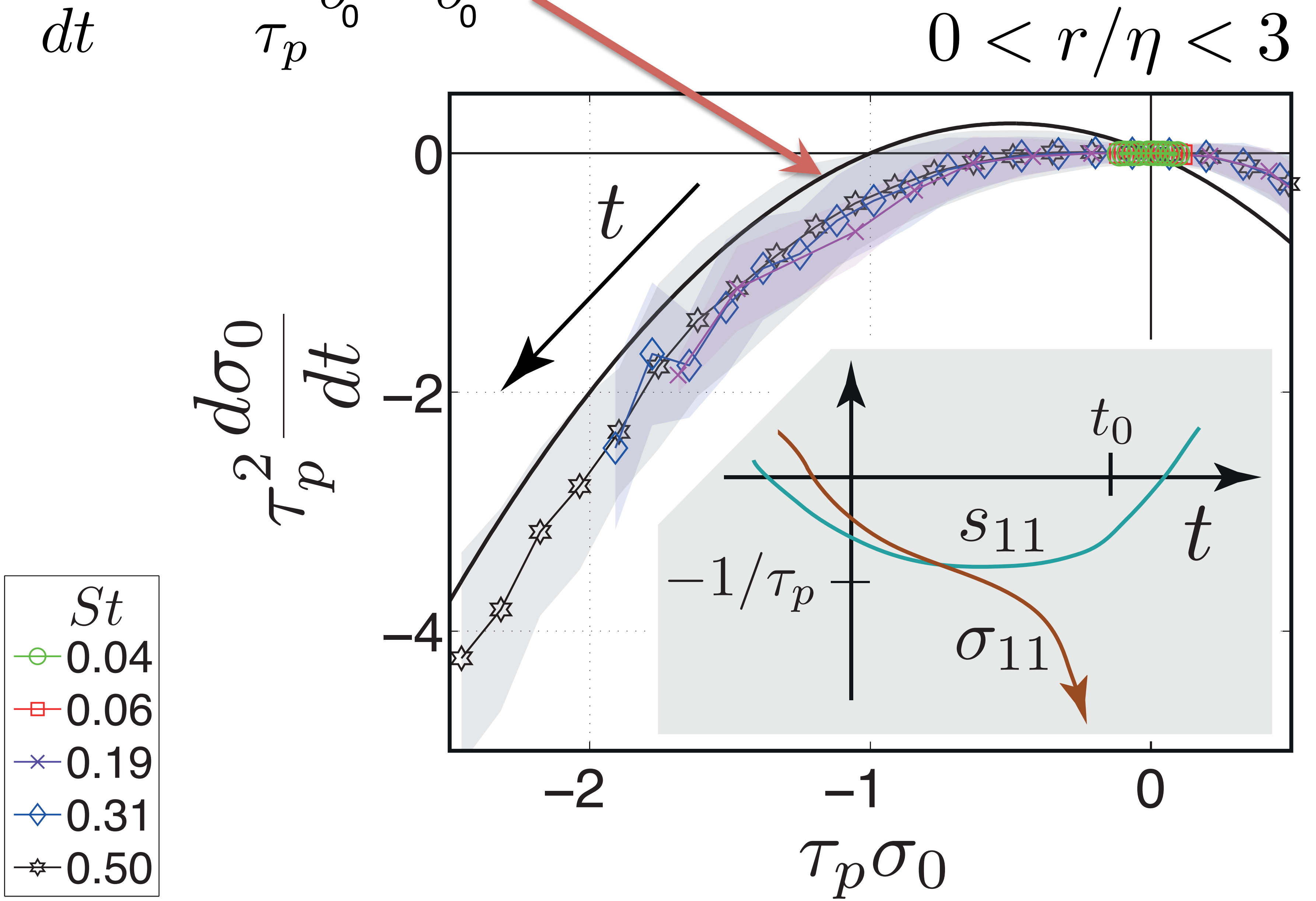
$$\frac{d\sigma_{11}}{dt} = \frac{dQ}{dt} - \sigma_{12}\sigma_{21} - \sigma_{13}\sigma_{31} \quad \approx \quad \frac{dQ}{dt} \approx \frac{Q(t_2)-Q(t_1)}{t_2-t_1}$$

conditional average:

$$\tau_p^2\frac{d\sigma_0}{dt} = \tau_p\langle s_{11}|\sigma_0\rangle - \tau_p\sigma_0 - \tau_p^2\sigma_0^2 - 2\tau_p^2\langle\sigma_{12}\sigma_{21}|\sigma_0\rangle$$

check cross terms later...

$$\frac{d\sigma_0}{dt} = -\frac{1}{\tau_p}\sigma_0 - \sigma_0^2$$



IN THIS FRAMEWORK:

NUMBER OF COLLISIONS WITHIN POCKETS

NUMBER OF COLLISIONS

NUMBER OF POCKETS

$$n_c \;\; [\#/m^3 s] \sim n'_c n_p V_p$$

CHARACTERISTIC VOLUME OF A POCKET

A MORE STANDARD APPROACH:

NUMBER OF COLLISIONS

$$n_c \;\; [\#/m^3 s] \;\sim\; g(d) \int \delta v \; P(\delta v | d) \; d\delta v$$

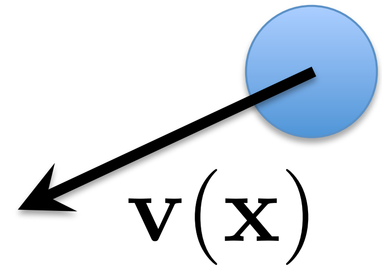
RADIAL DISTRIBUTION FUNCTION

RELATIVE VELOCITY DISTRIBUTION

Sundaram and Collins (1997) *JFM*

(simplified) droplet-momentum equation:

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u} - \mathbf{v}) + \mathbf{g}$$



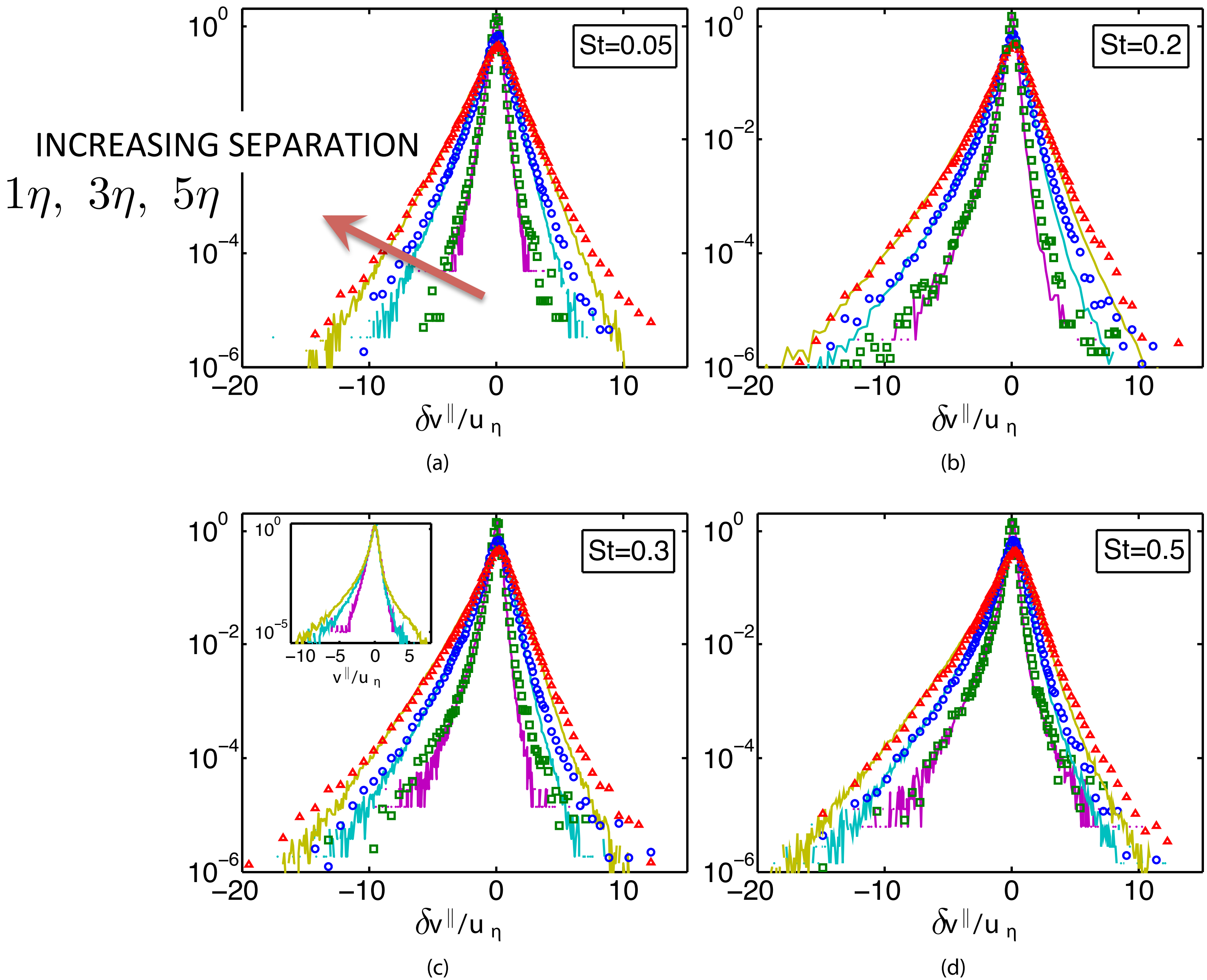
STOKES DRAG

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

$\mathbf{u}$ fluid velocity

$\mathbf{v}$ droplet velocity

IS THIS MODEL ADEQUATE?



Lines:
DNS

Symbols:
Experiments

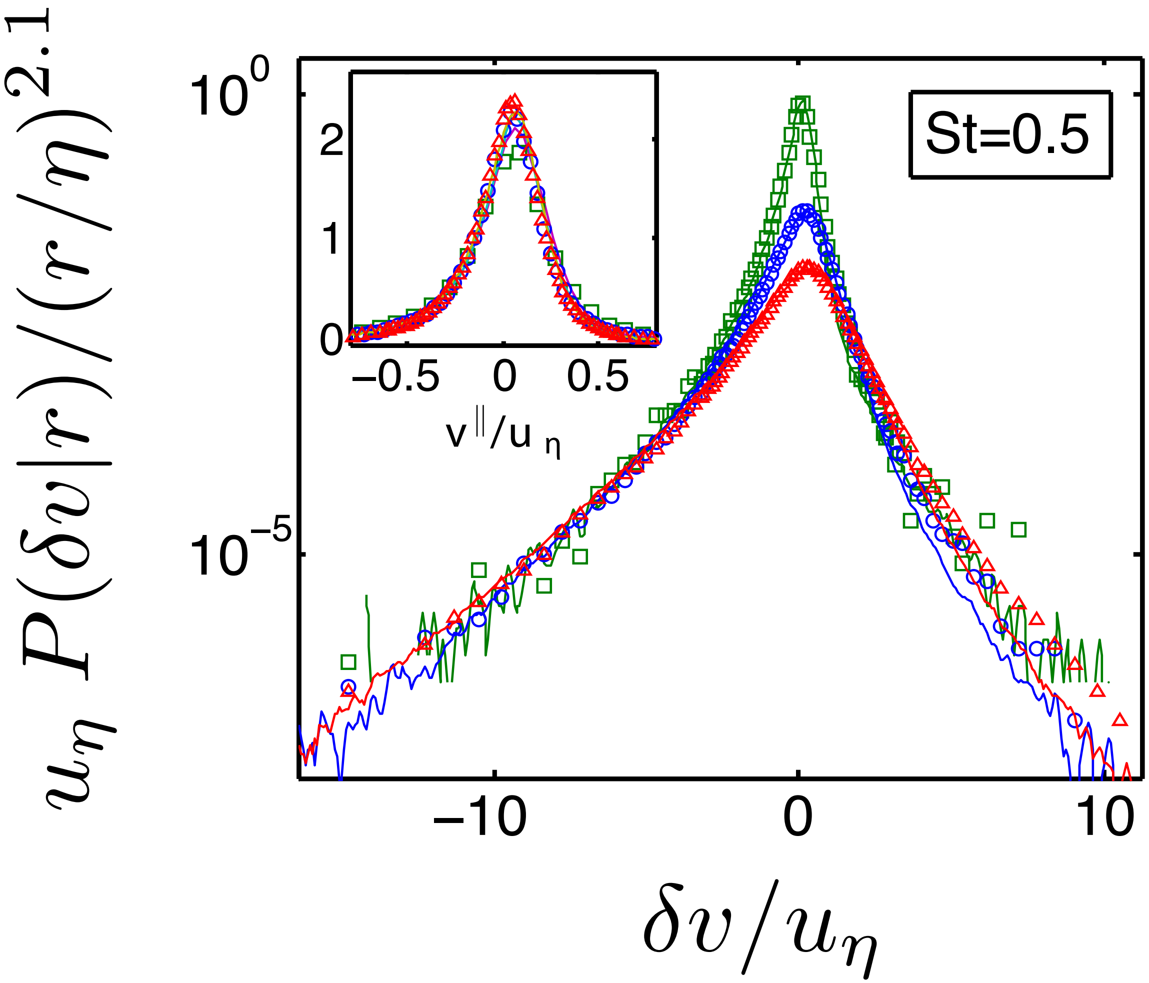
Matched conditions

Saw, Bewley, Bodenschatz, Ray, Homann, Bec *in preparation*

For the extreme events:

$$P(\delta v|r) = r^{\xi_\infty}\phi(\delta v)$$

Gustavsson, Mehlig (2011) *Phys. Rev. E*

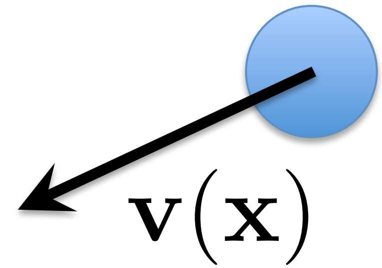


$$\xi_\infty = 2.1$$

Saw, Bewley, Bodenschatz, Ray, Homann, Bec *in preparation*

(simplified) droplet-momentum equation:

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\tau_p}(\mathbf{u} - \mathbf{v}) + \mathbf{g}$$



GRAVITY

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

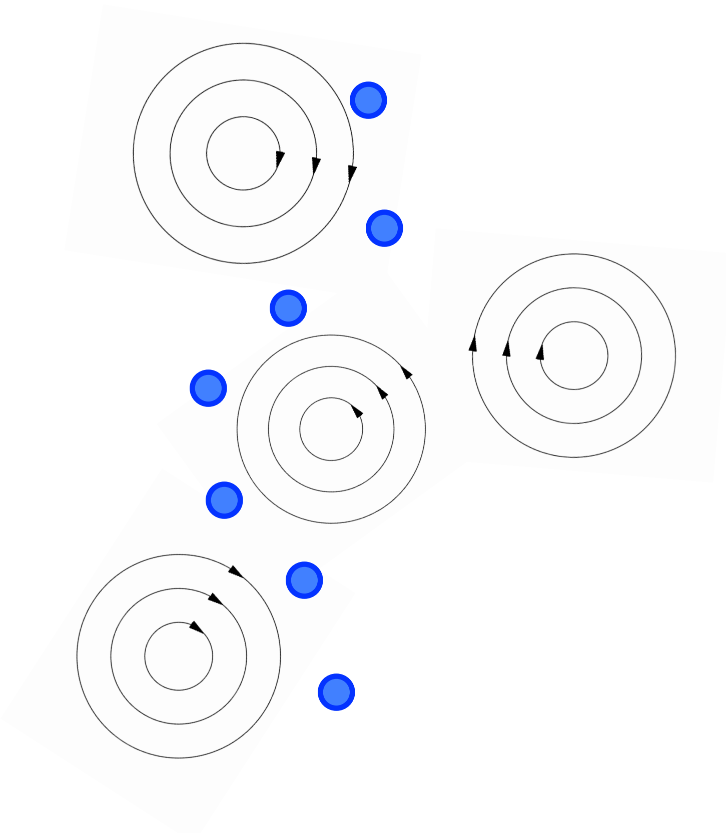
$\mathbf{u}$ fluid velocity

$\mathbf{v}$ droplet velocity

WHAT IS THE ROLE OF GRAVITY?

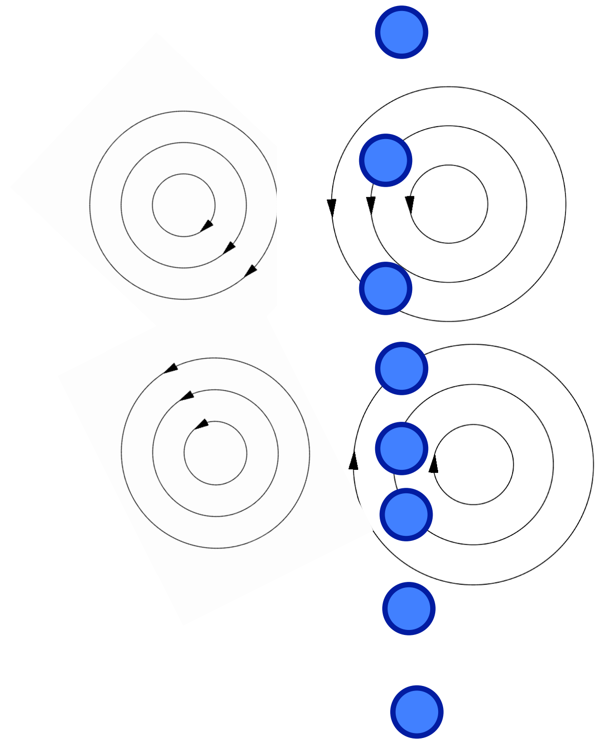
SETTLING VELOCITY MODIFICATION

Biased path: Enhances



Wang, Maxey (1993) *JFM*

Unbiased path: Can reduce



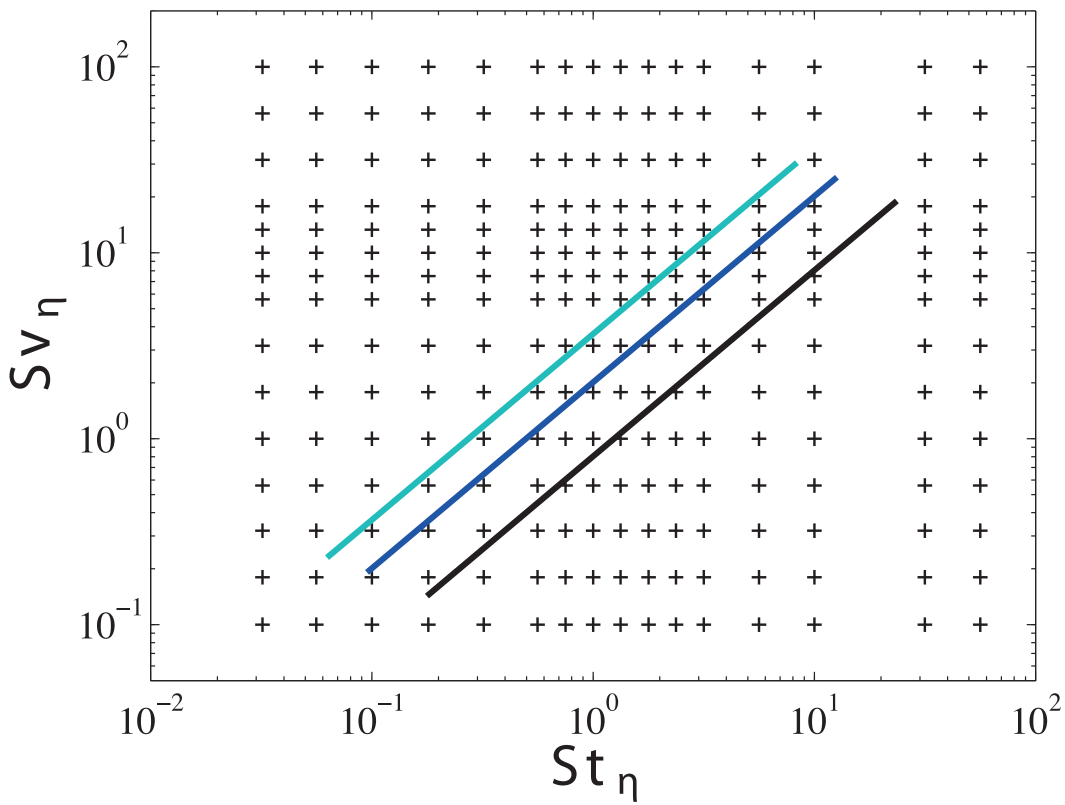
Nielsen (1993) *J. Sediment. Petrol.*

Parameter space

$$St_\eta = \frac{\tau_p}{\tau_\eta} \qquad Sv_\eta = \frac{W_0}{u_\eta}$$

Still air settling velocity:

$$W_0 = \tau_p g$$



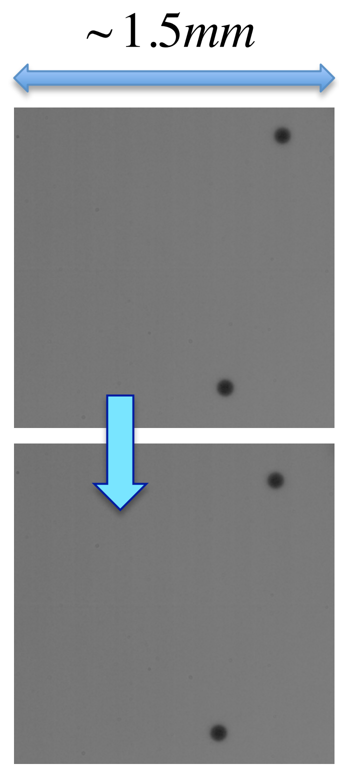
+ DNS

- Experiments

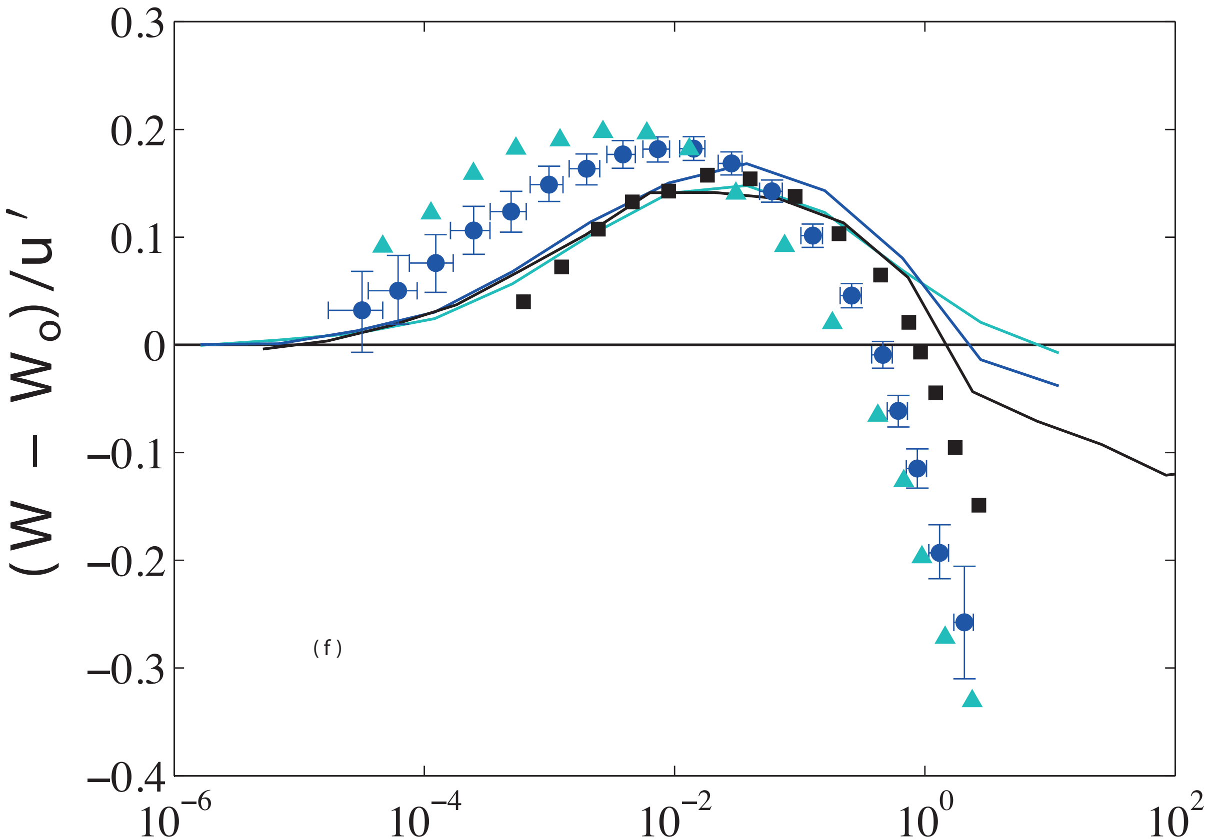
Particles

- $d \approx 15-150\mu m$, sub-Kolmogorov-scale water droplets
- at different Reynolds numbers
- Volume fraction $\phi_V \approx 10^{-6}$

Ultrasonic droplet generator



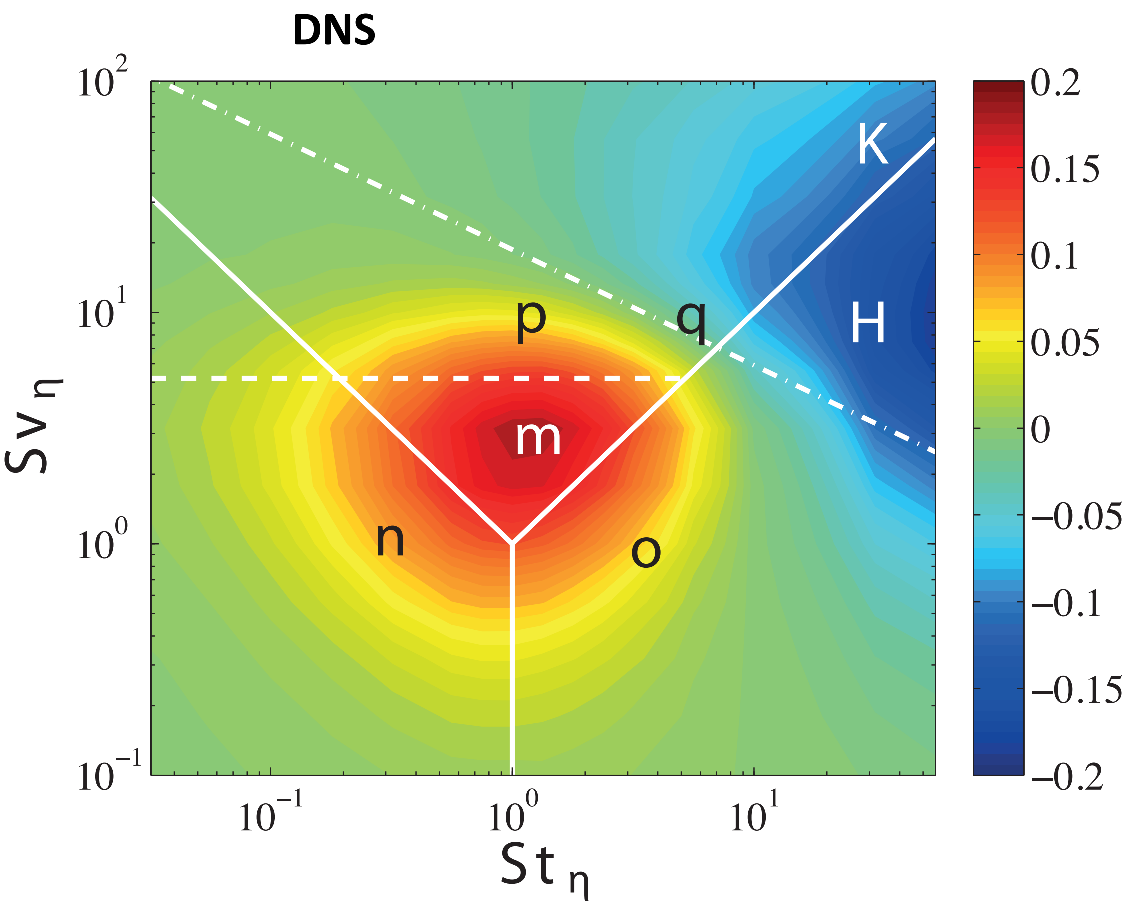
Single-camera 2D Particle tracking



$$F_l \sim Sv_\eta^2 St_\eta$$

Davila, Hunt (2001) *JFM;* Ghosh et al. (2005) *Proc. Roy. Soc. A*

Good, Ireland, Bewley, Bodenschatz, Collins, Warhaft *in preparation*



$$(W - W_0)/u'$$

Good, Ireland, Bewley, Bodenschatz, Collins, Warhaft *in preparation*

REVIEW OF PARTICLE-TURBULENCE WORK

The sling effect happens!

Can we model the onset of rain through the sling effect?

Linear drag alone does not quantitatively predict extreme events.

Turbulence both *enhances* and *retards* gravitational settling!

Acknowledgements

E. Bodenschatz

Active grid

E. Cekli
F. Köhler
J. Kassel
F. Lachaussée
H. Grajewski

Wind tunnel

M. Sinhuber
H. Eckelmann

Funding

Max Planck Gesellschaft
Volkswagen Stiftung

Soccer balls I&II

K. Chang
E.-W. Saw
P.-Y. Lim
G. Good
D. Ivanov

H. Nobach
T. Schneider
J. Vollmer
H. Xu

A. Kopp
A. Kubitzek
O. Kurre
A. Renner
U. Schminke *et al.*