Lagrangian Statistics of Particles in Geophysical Turbulence

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University of Technology

Where innovation starts

Contents

- Motivation

- Inertial particles in stratified turbulence (ST)

- Summary ST

- Lagrangian statistics in rotating turbulence (RT)

- Summary RT

Motivation

Table-Top Geophysical Turbulence

- Dispersion in Geophysical Turbulence

Motivation

Table-Top Geophysical Turbulence

- Dispersion in Geophysical Turbulence
- Dispersion in Forced Stratified Turbulence
- Rotating Turbulence
- Dispersion in Rotating Turbulence
- Convection and Rotation
- Shallow (Q2D) Flows

Motivation

Table-Top Geophysical Turbulence:

Staff "Vortex Dynamics and Turbulence" group at TU/e working on transport in (geophysical) turbulence:

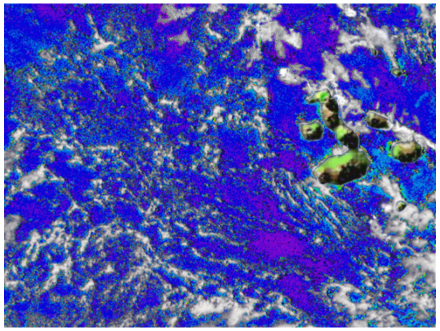
Herman Clercx, GertJan van Heijst & Federico Toschi
Rudie Kunnen & Bas van de Wiel

PhD students (who contributed to this talk):

Marleen van Aartrijk, Laurens van Bokhoven &
Lorenzo Del Castello

Dispersion in geophysical turbulence

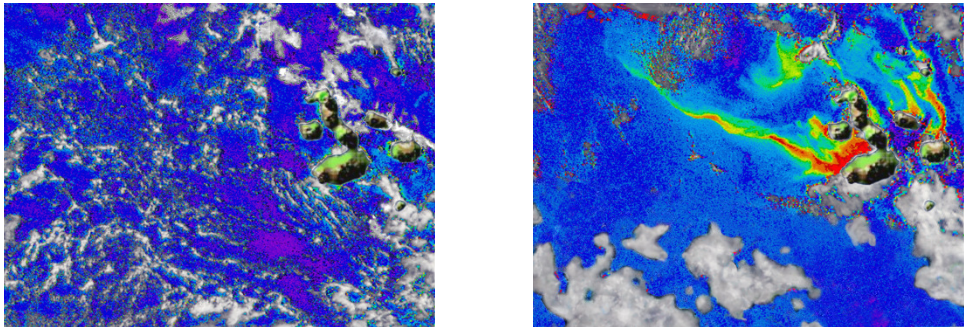
May 9, 1998



Phytoplankton around the Galapagos Islands in May 1998 showing the rapid demise of El Niño.

- El Niño waters are blamed for choking off essential nutrients

Dispersion in geophysical turbulence



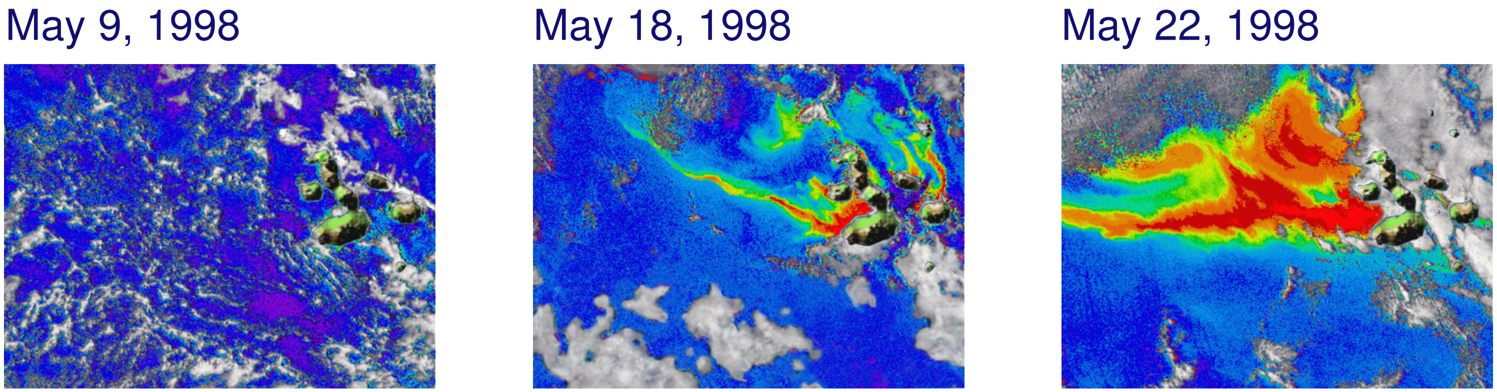
May 9, 1998

May 18, 1998

Phytoplankton around the Galapagos Islands in May 1998 showing the rapid demise of El Niño.

- El Niño waters are blamed for choking off essential nutrients
- Upwelling during replacement of the warm El Niño waters

Dispersion in geophysical turbulence



May 9, 1998

May 18, 1998

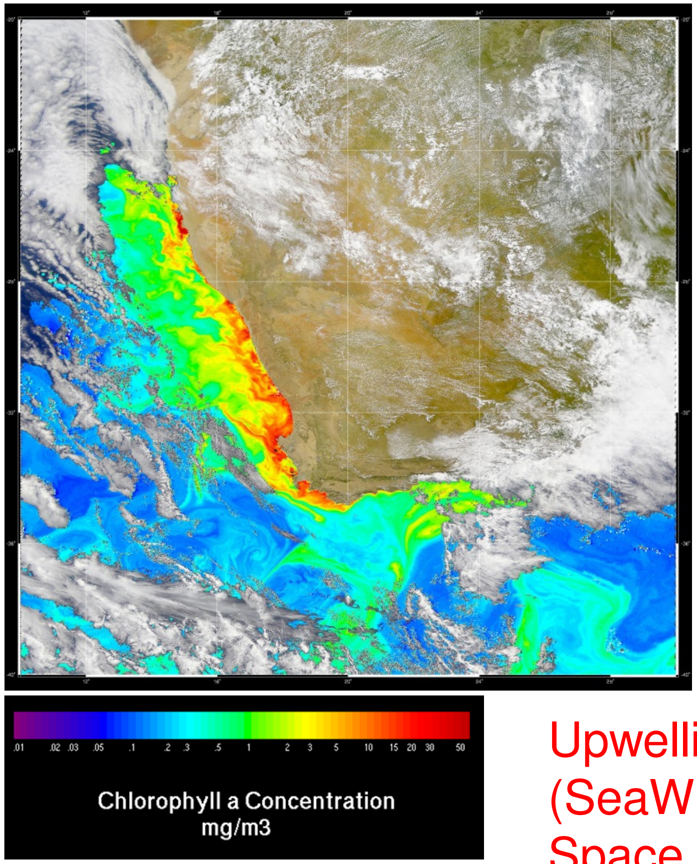
May 22, 1998

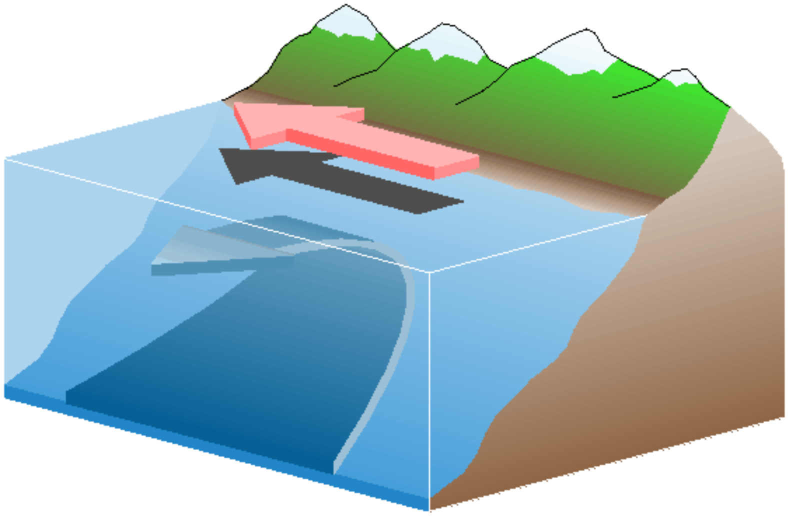
Phytoplankton around the Galapagos Islands in May 1998 showing the rapid demise of El Niño.

- El Niño waters are blamed for choking off essential nutrients
- Upwelling during replacement of the warm El Niño waters
- Cold nutrient-rich waters initiate phytoplankton bloom

Dispersion in geophysical turbulence

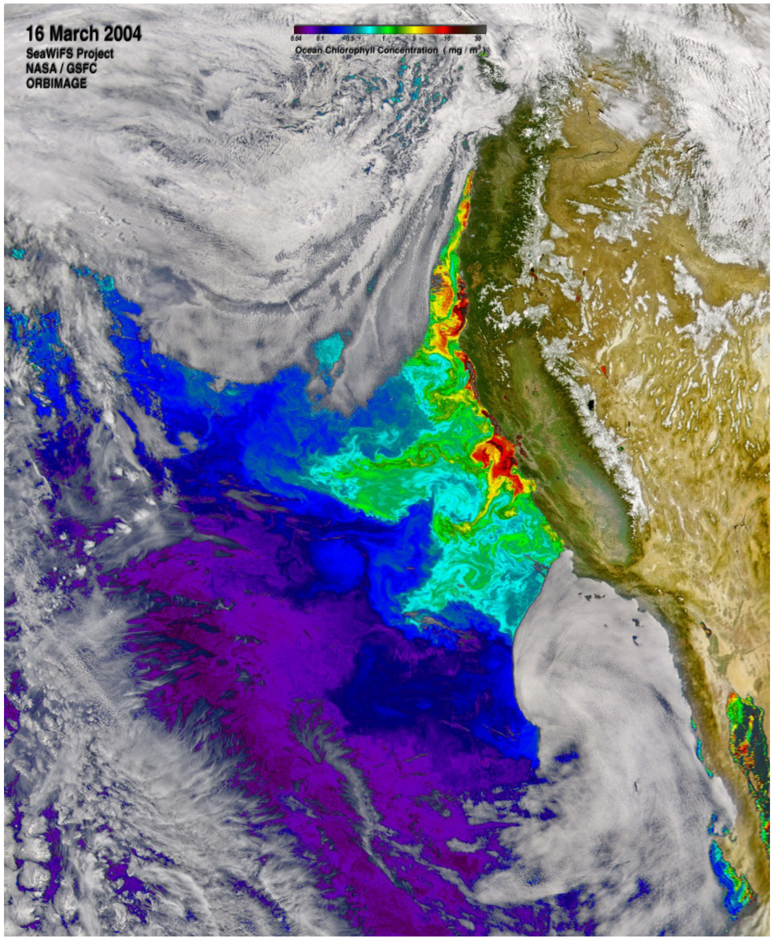
SeaWiFS: Sea-viewing Wide Field-of-view Sensor.





Upwelling off Cape Town, South Africa (SeaWiFS Project, NASA/Goddard Space Flight Center).

Dispersion in geophysical turbulence



Phytoplankton off the coast of California.

Image courtesy the SeaWiFS Project, NASA/Goddard Space Flight Center, and ORBIMAGE.

Strong currents of the coast of California are pulling cold, deep currents up from the sea floor to the surface.

These nutrient-rich waters support populations of microscopic marine plant life.

Rivers and urban areas provide additional nutrients.

Dispersion in geophysical turbulence

- transition zones at land-sea interface
- influenced by riverine input
- nutrient-rich (land)

- strong (vertical) turbulent mixing
- particle-rich and high turbidity
- strong pelagic-benthic interaction

- tide- and wind-induced flows
- interaction with the ocean

Phytoplankton blooms in Shallow Coastal Ecosystems (Chesapeake Bay, USA).

J.E. Cloern, *Phytoplankton bloom dynamics in coastal ecosystems,* Reviews of Geophysics, 34, 127-168 (1996).

Dispersion in geophysical turbulence

What is the relevant range of scales?

1-100 μm: smallest organisms;
0.1-10 cm: Kolmogorov scale;
0.01-10 m: stratification;
10-1000 km: rotation and stratification.

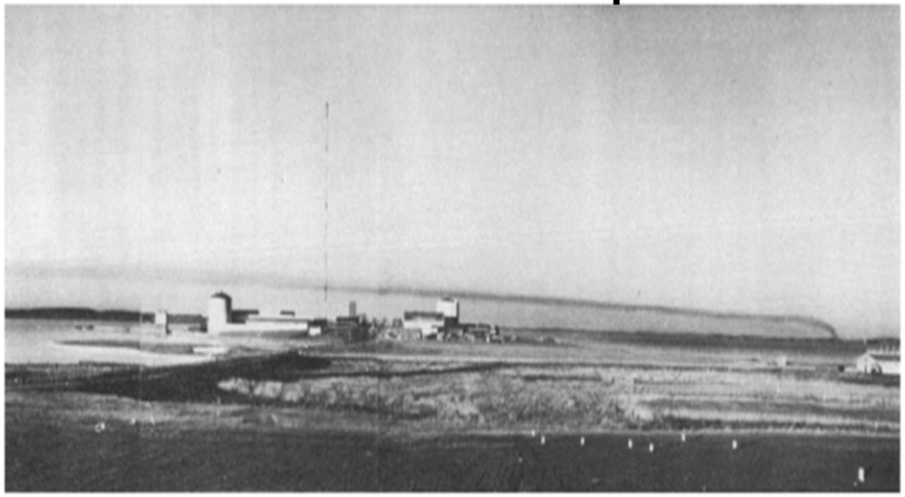
Processes that regulate the total biomass, species composition, and spatial distribution of phytoplankton include:

- turbulent vertical mixing;
- (turbulent) horizontal advection;
- in situ processes of population change.

Inertial particles in stratified turbulence

Algal blooms

Aerosol dispersion



How is particle dispersion affected by the particle's inertial properties?

Does preferential concentration persist for small particle-to-fluid density ratios?

How is preferential concentration affected by stratification?

Inertial particles in stratified turbulence

Direct Numerical Simulation

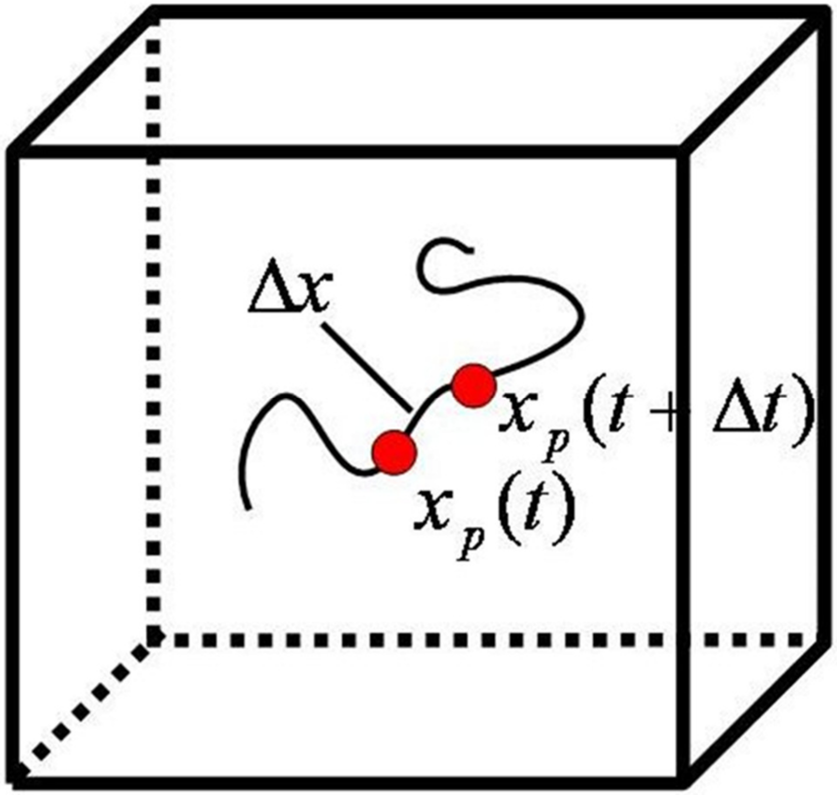
Two steps:

1. Eulerian approach for velocity field

2. Lagrangian approach for particle tracking

Inertial particles in stratified turbulence

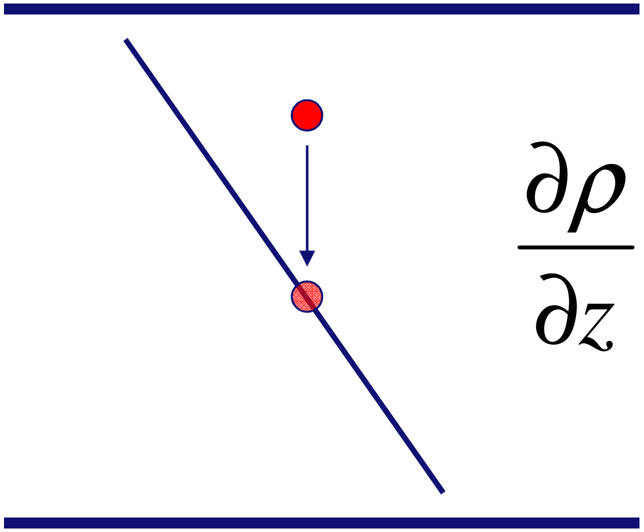
Direct Numerical Simulation



- Boussinesq approx.
- Periodic domain
- 128^3 (256^3)
- Forced DNS
- Parallel

Inertial particles in stratified turbulence

Stable stratification



$$\frac{\partial \rho}{\partial z} < 0$$

Buoyancy frequency N:

$$N = \left(-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \right)^{1/2}$$

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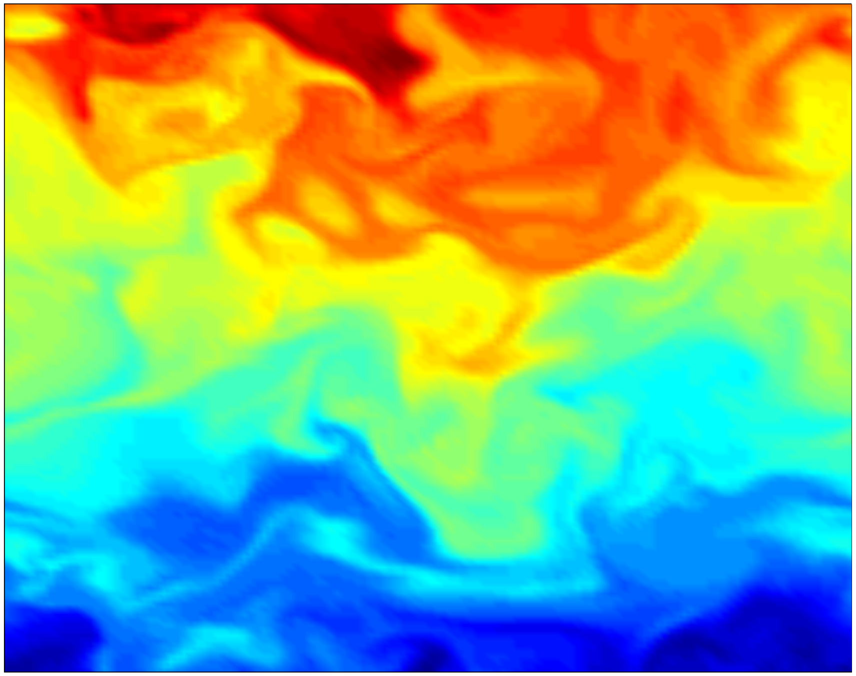
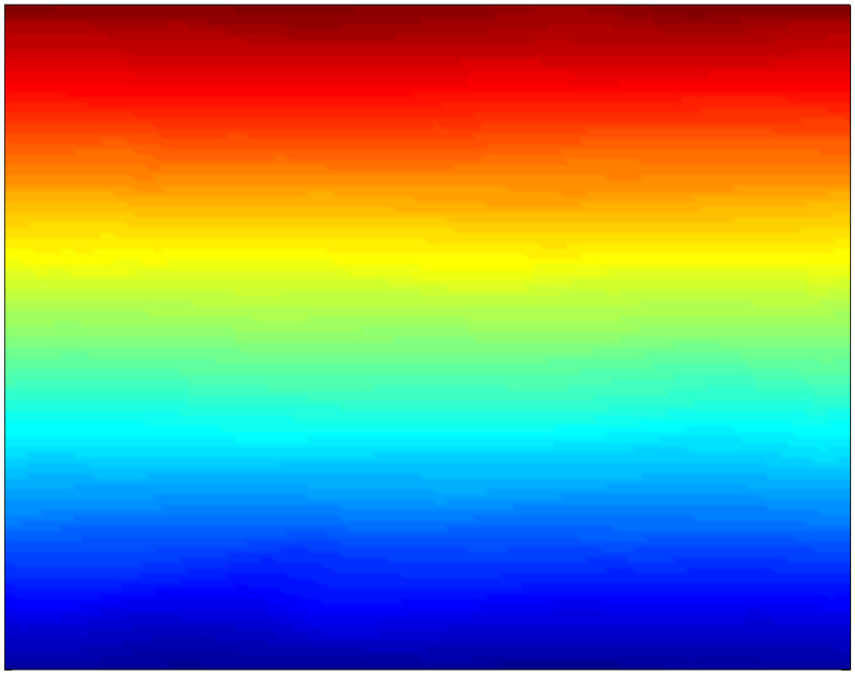
October 2, 2013

Inertial particles in stratified turbulence

Density profiles



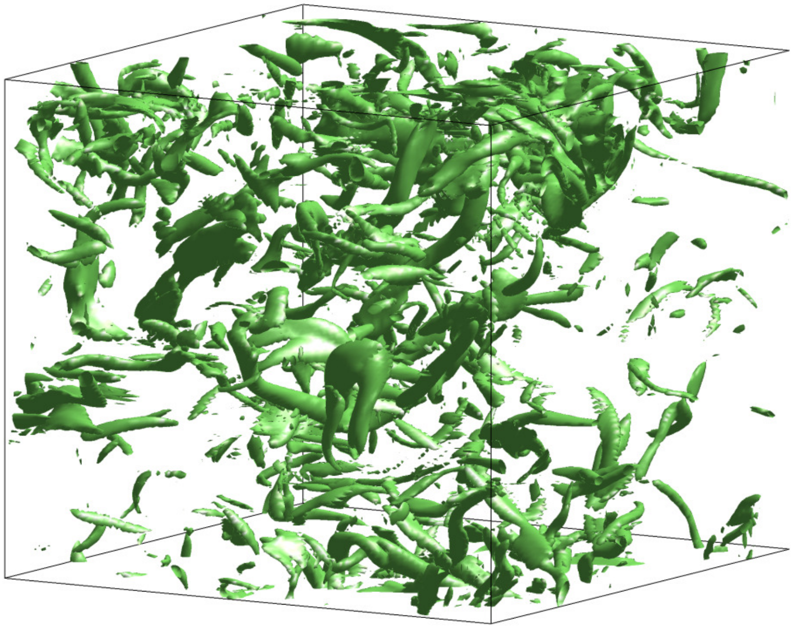
$N \sim 0.1$ (s^{-1})



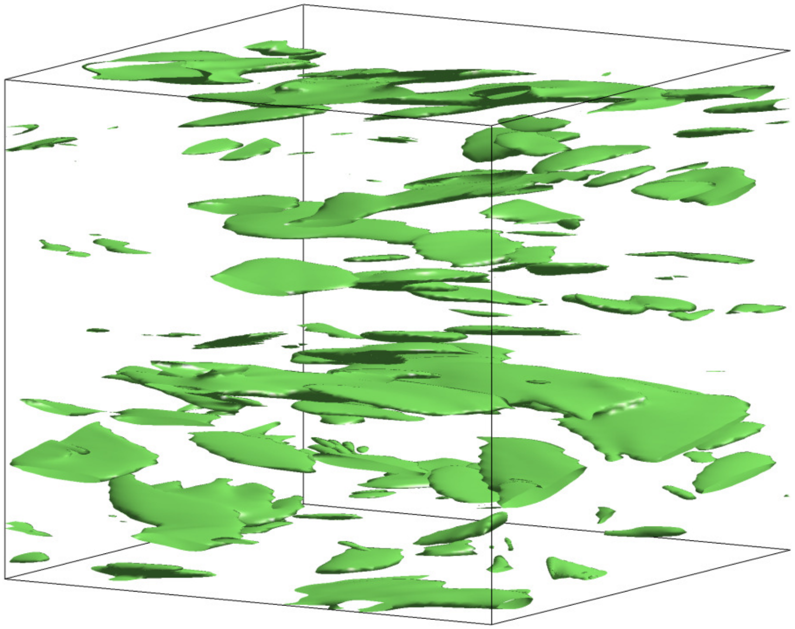
$N \sim 1$ (s^{-1})

Inertial particles in stratified turbulence

Isovorticity



N~0.1 (s^{-1})



N~1 (s^{-1})

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Inertial particles in stratified turbulence

Fluid particle tracking

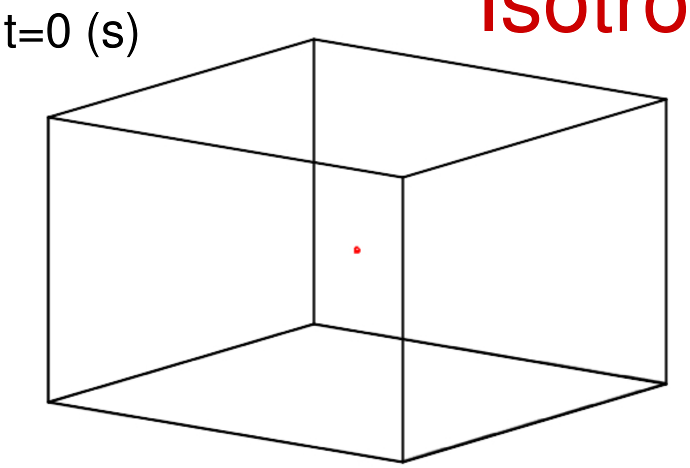
$$\frac{d\vec{x}_p}{dt} = \vec{u}_p$$

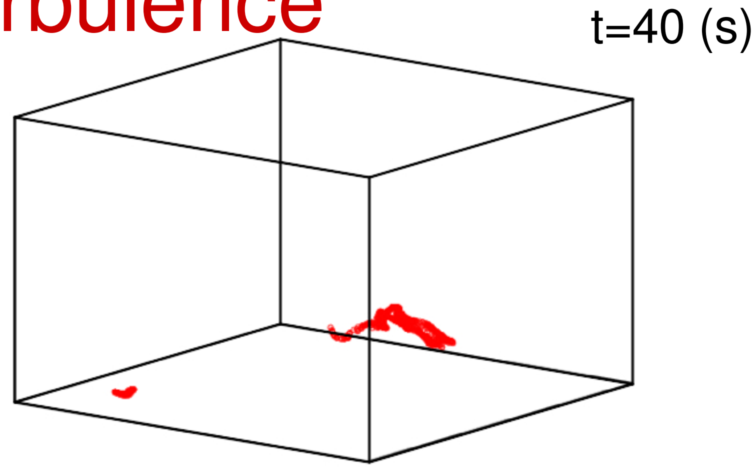
$$\vec{u}_p = \vec{u}_f(\vec{x}_p)$$

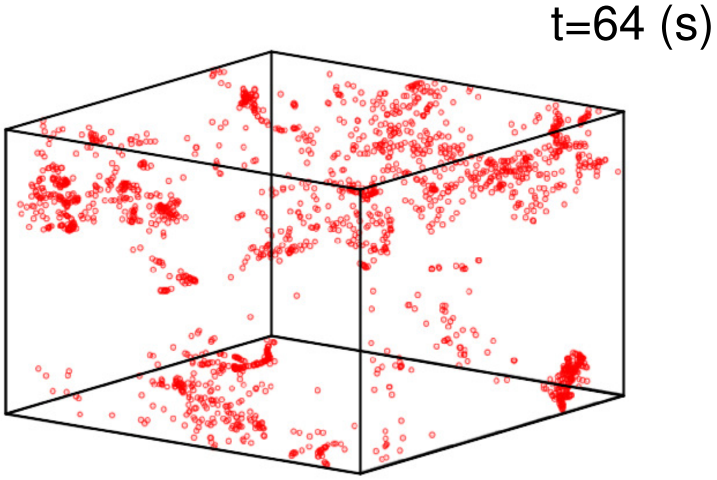
Cubic spline interpolation

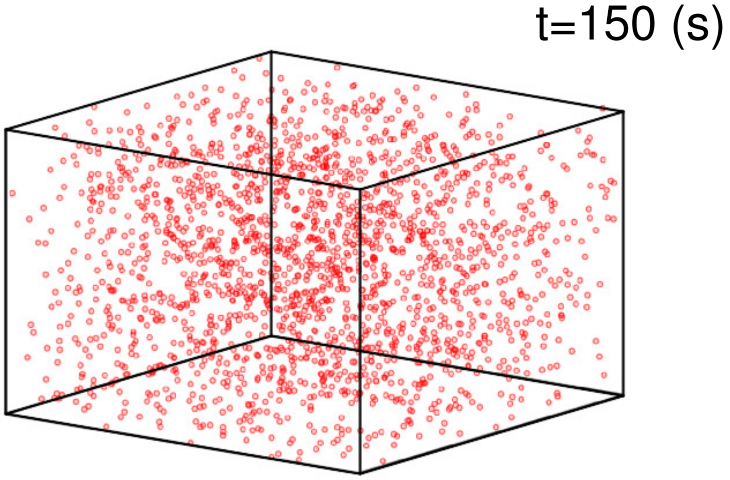
Inertial particles in stratified turbulence

Isotropic turbulence



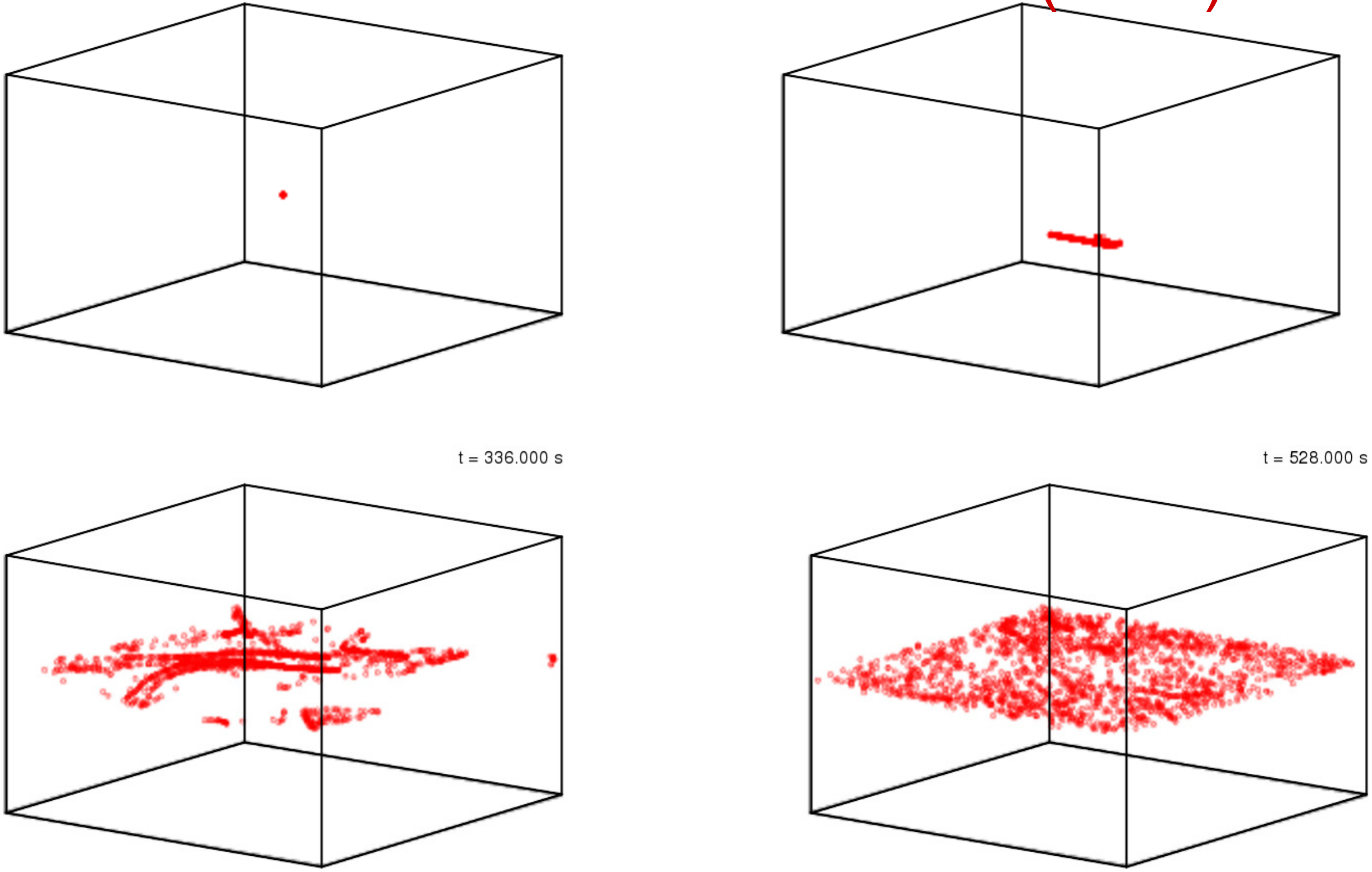






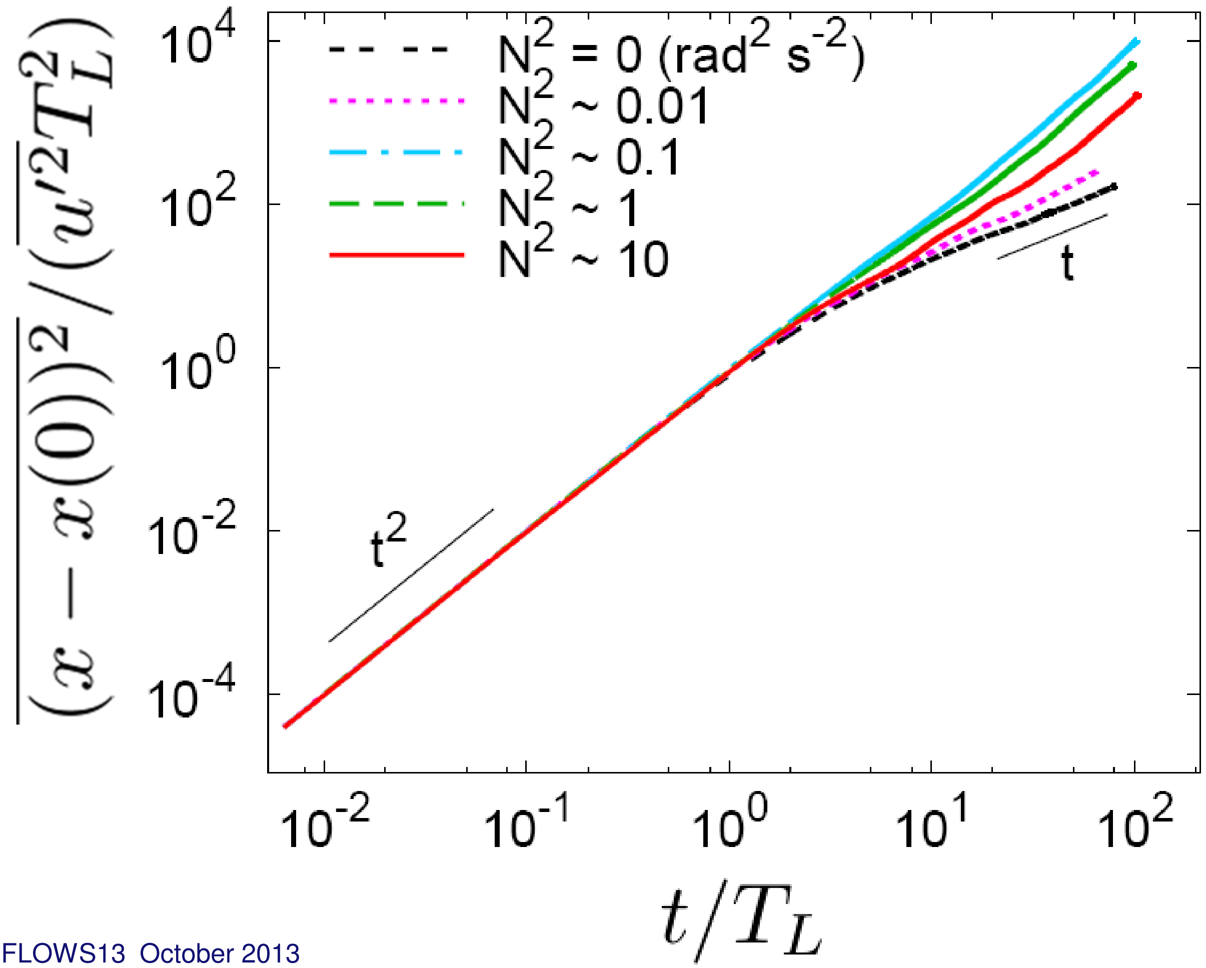
Inertial particles in stratified turbulence

Stratified turbulence (N~1)

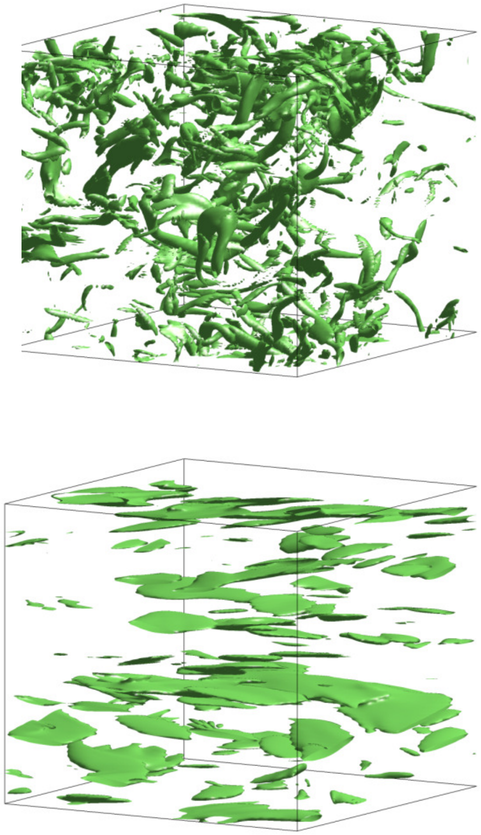


Inertial particles in stratified turbulence

Horizontal dispersion

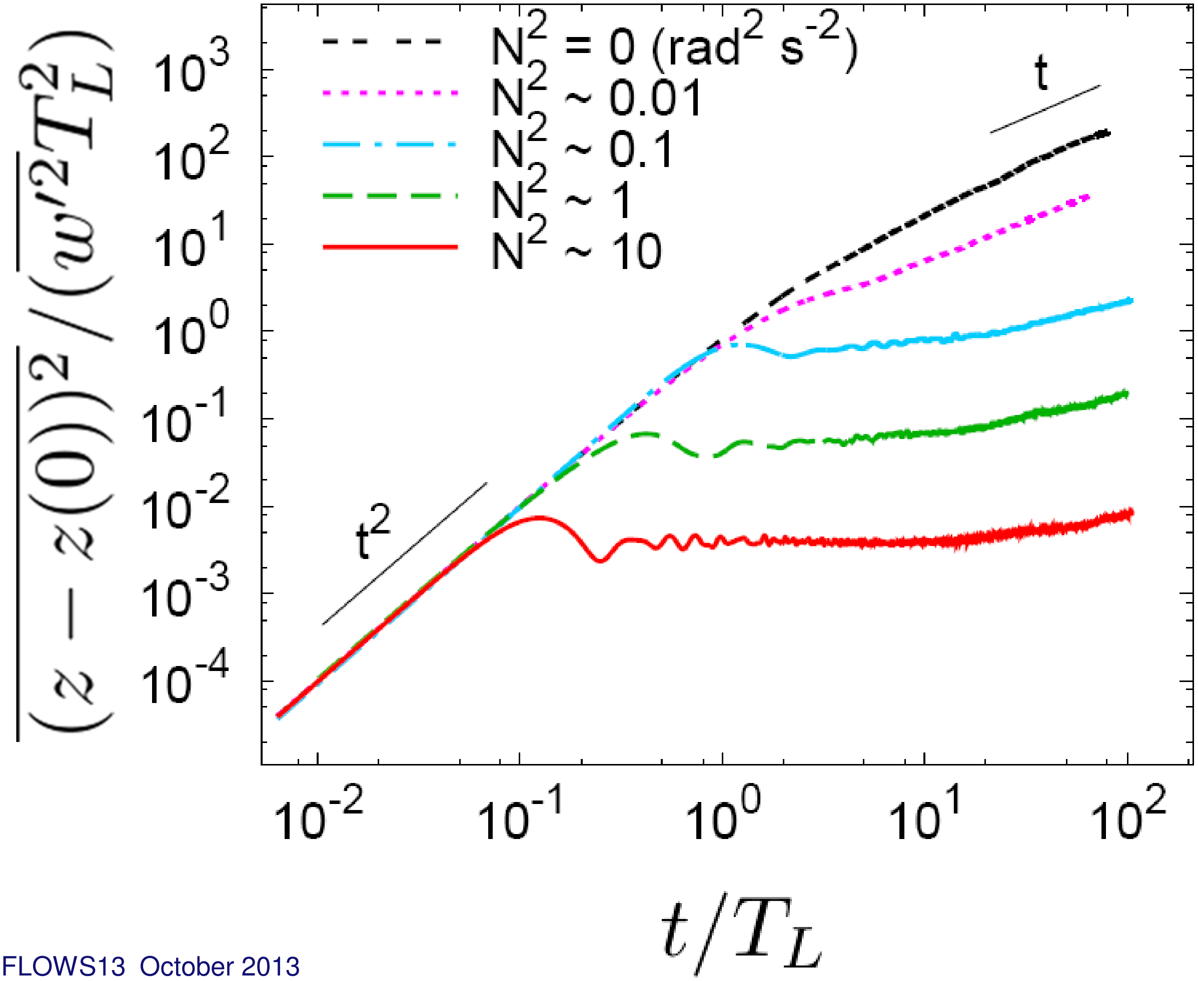


M. van Aartrijk, H.J.H. Clercx & K.B. Winters, *Phys. Fluids* **20**, 025104 (2008).



Inertial particles in stratified turbulence

Vertical dispersion



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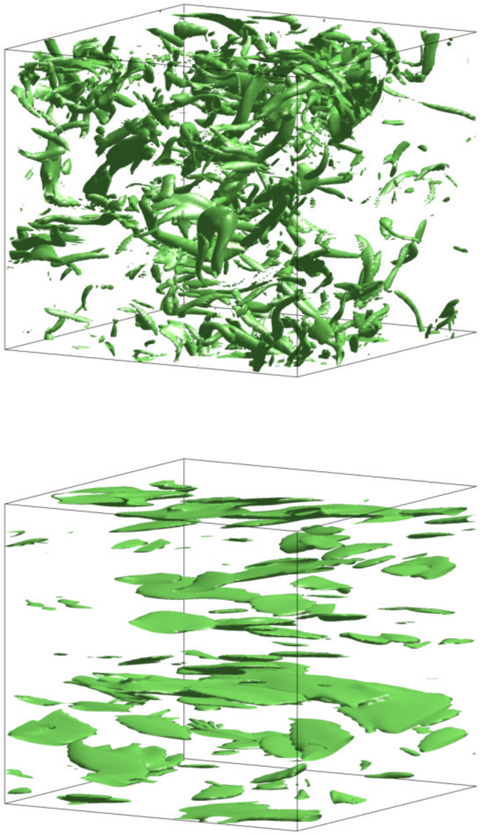

Inertial particles in stratified turbulence

Inertia effect on dispersion

$$\boxed{\overline{x^2} = 2\overline{u'_p{}^2}\int(t-\tau)R(\tau)d\tau}$$

Taylor (1921)

- Increasing inertia → $\overline{u'_p{}^2}\downarrow$ →
  decreasing dispersion
- Increasing inertia → memory, $R(\tau)\uparrow$ →
  increasing dispersion
- Dispersion optimum around $\tau_p = \tau_K$ (iso)

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Inertial particles in stratified turbulence

Inertial particle tracking

$$m_p \frac{d\vec{u}_p}{dt} = 6\pi a\mu\left(\vec{u} - \vec{u}_p + \frac{1}{6}a^2\nabla^2\vec{u}\right) + m_f\frac{D\vec{u}}{Dt}$$

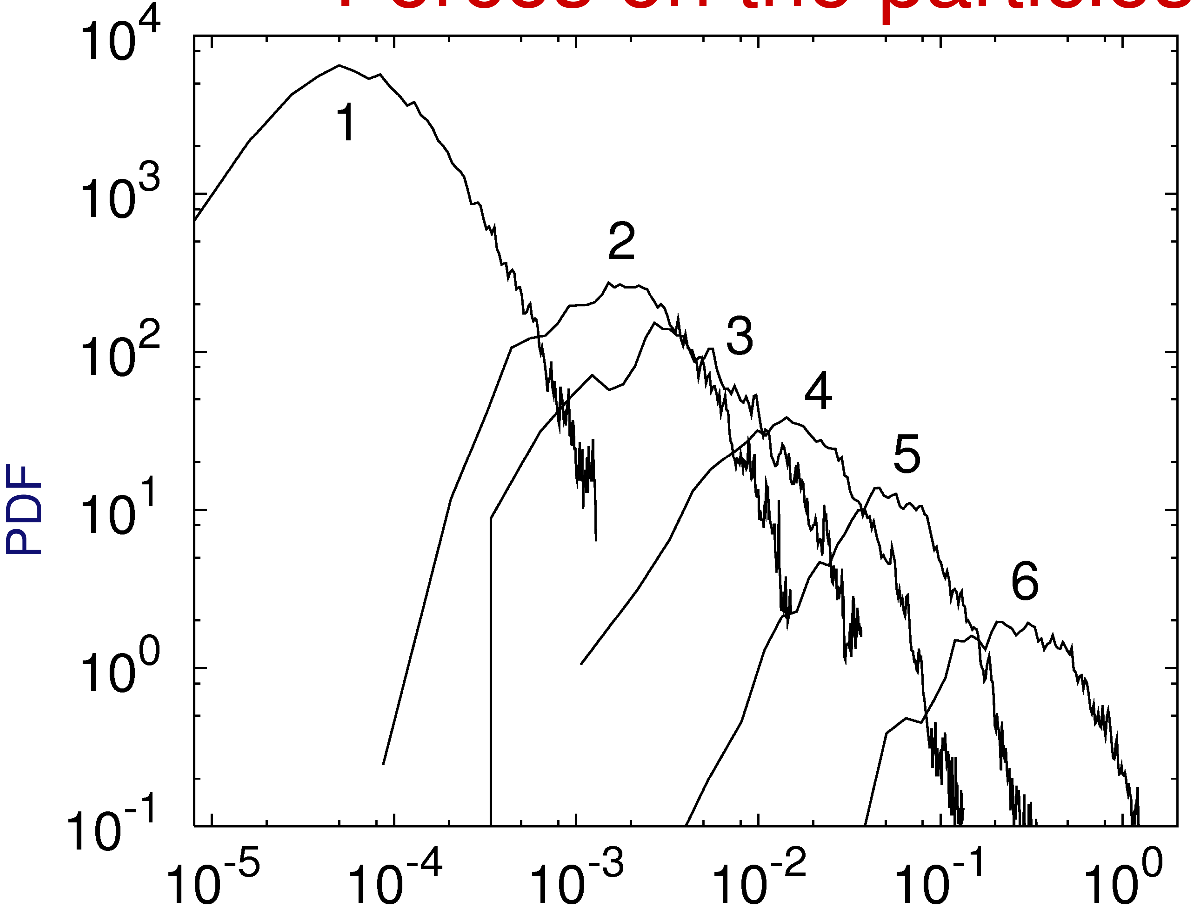
$$-\left(m_p - m_f\right)g\vec{\hat{z}} + \frac{1}{2}m_f\left(\frac{D\vec{u}}{Dt} - \frac{d\vec{u}_p}{dt} + \frac{1}{10}a^2\frac{d}{dt}\nabla^2\vec{u}\right)$$

$$+6\pi a^2\mu\int_0^t d\tau\frac{d\vec{u}/d\tau - d\vec{u}_p/d\tau + \frac{1}{6}a^2 d\nabla^2\vec{u}/d\tau}{\left[\pi\nu(t-\tau)\right]^{1/2}}$$

M.R. Maxey and J.J. Riley, *Phys. Fluids* **26**, 883 (1983).

Inertial particles in stratified turbulence

Forces on the particles (N~0.3)



F_1= added mass Faxèn correction

F_2= Basset force Faxèn correction

F_3= Stokes drag Faxèn correction

F_4= added mass

F_5= pressure gradient

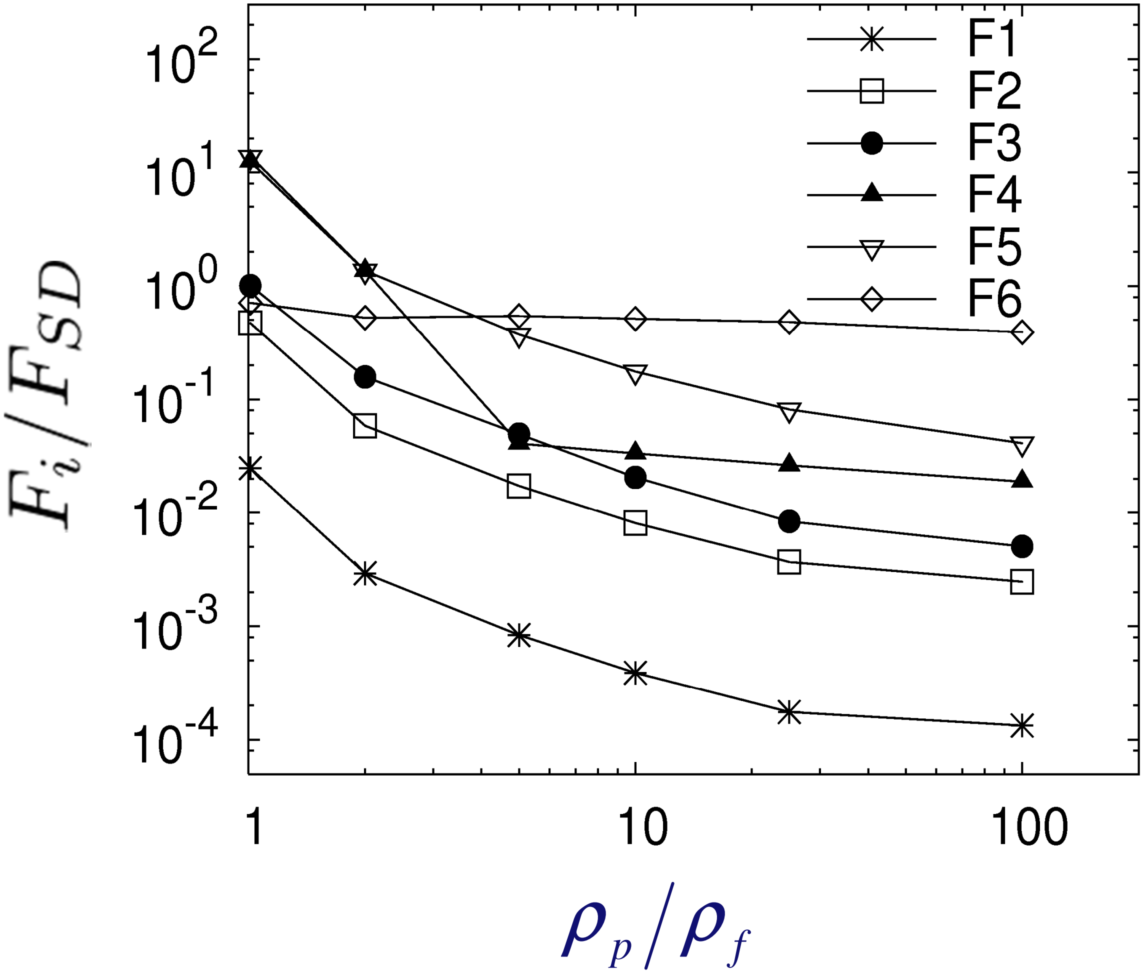
F_6= Basset force

$$\rho_p / \rho_f = 25 \quad St = 1.4$$

Non-stratified, see e.g.: V. Armenio and V. Fiorotto, *Phys. Fluids* **13**, 2437 (2001).

Inertial particles in stratified turbulence

Forces on the particles (N~0.3)



F_1= added mass Faxèn correction

F_2= Basset force Faxèn correction

F_3= Stokes drag Faxèn correction

F_4= added mass

F_5= pressure gradient

F_6= Basset force

Inertial particles in stratified turbulence

Inertial particle tracking

Heavy inertial particles: $\frac{\rho_p}{\rho_f} \gg 1$

$$\frac{du_p}{dt} = \frac{1}{\tau_p}(u - u_p)$$

$$St = \frac{\tau_p}{\tau_k}$$

$$\tau_p = \frac{(\rho_p/\rho_f)\, d_p^2}{18\nu}$$

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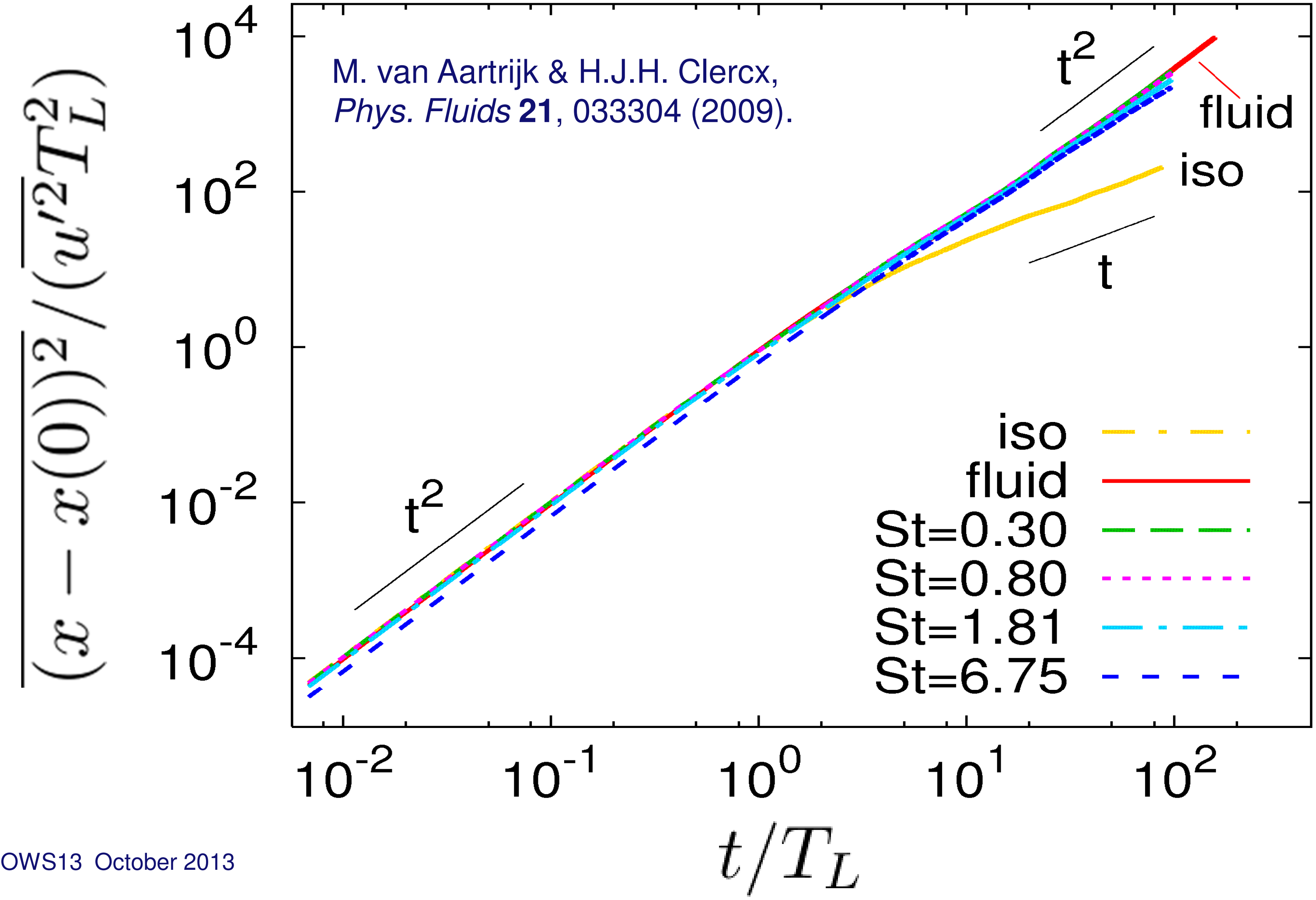
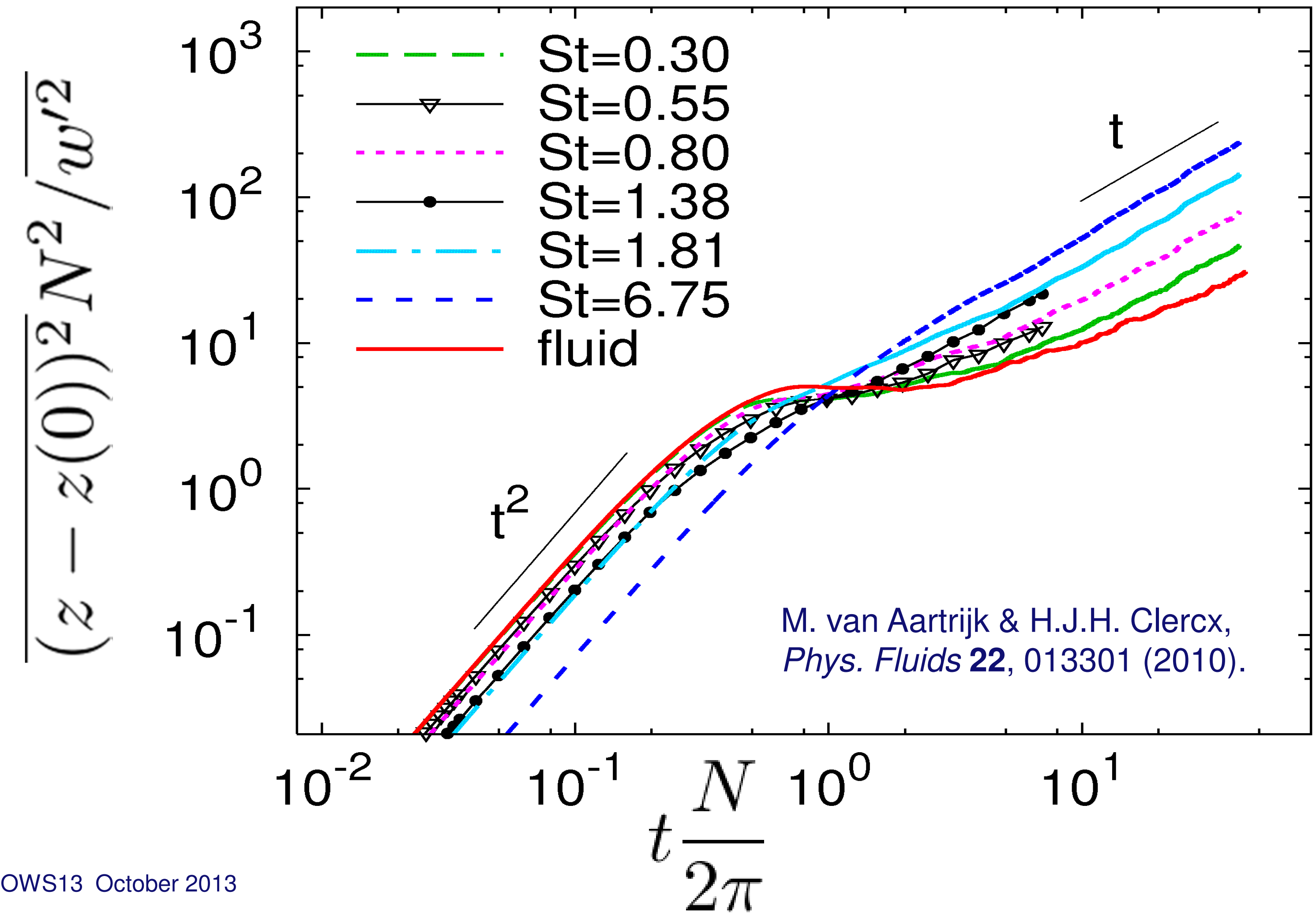
Inertial particles in stratified turbulence

Horizontal dispersion (N~1)



Inertial particles in stratified turbulence

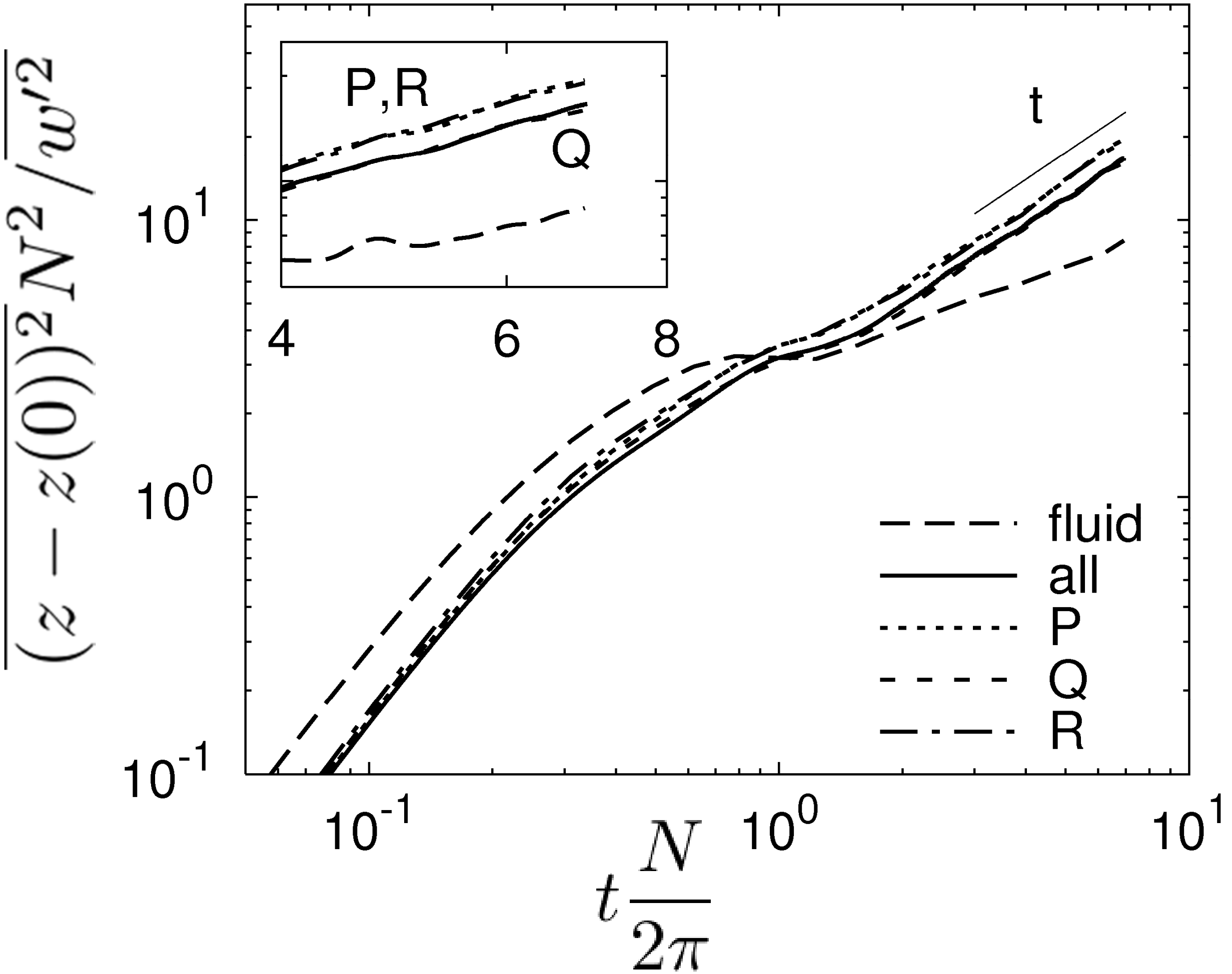
Vertical dispersion (N~1)



M. van Aartrijk & H.J.H. Clercx, *Phys. Fluids* **22**, 013301 (2010).

Inertial particles in stratified turbulence

Vertical dispersion (N~0.3)



P = no Basset

Q = no Faxèn corrections

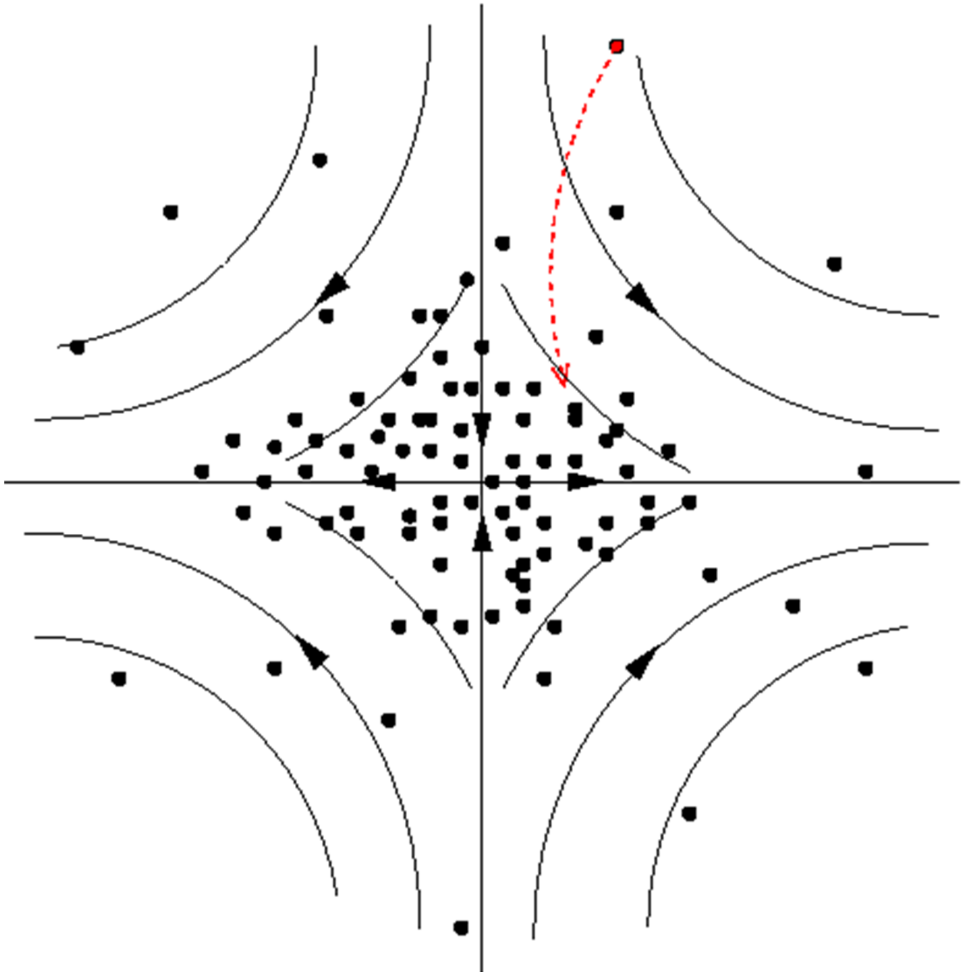
R = only Stokes drag, pressure gradient, added mass,

$$St = 1.4$$

$$\rho_p / \rho_f = 25$$

Inertial particles in stratified turbulence

Preferential concentration

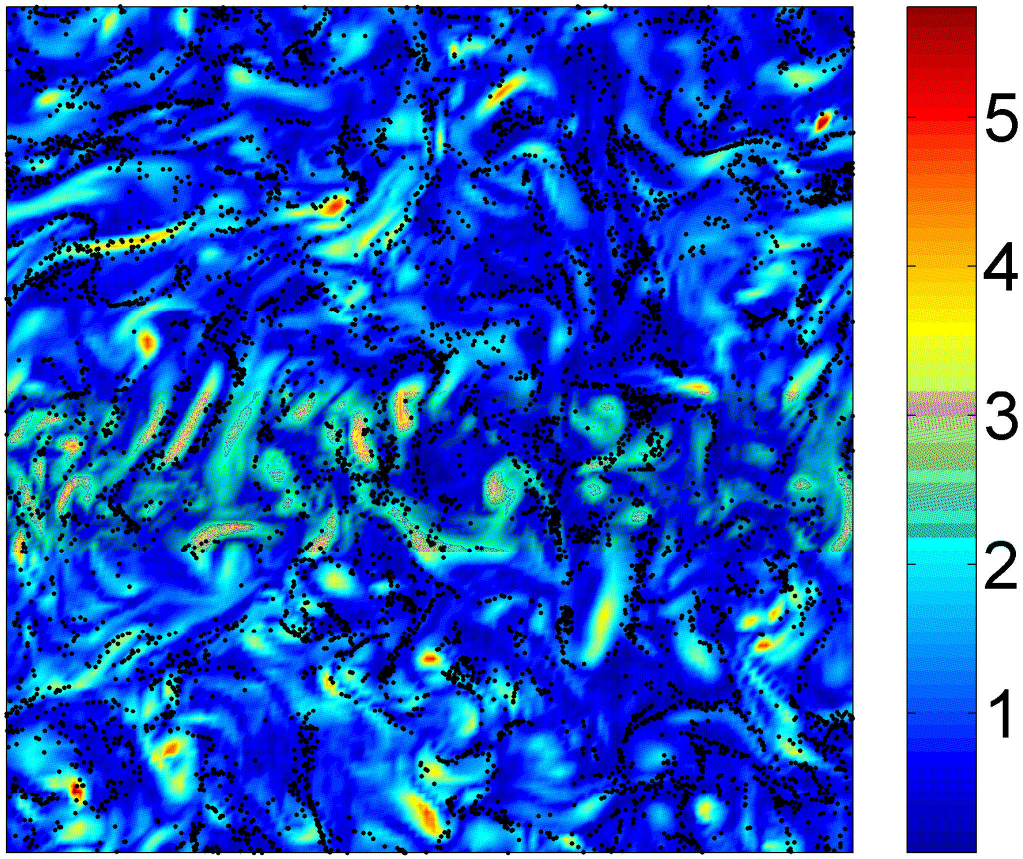


$$\frac{\rho_p}{\rho_f} > 1$$

High strain,
low vorticity

Inertial particles in stratified turbulence

Preferential concentration isotropic turbulence



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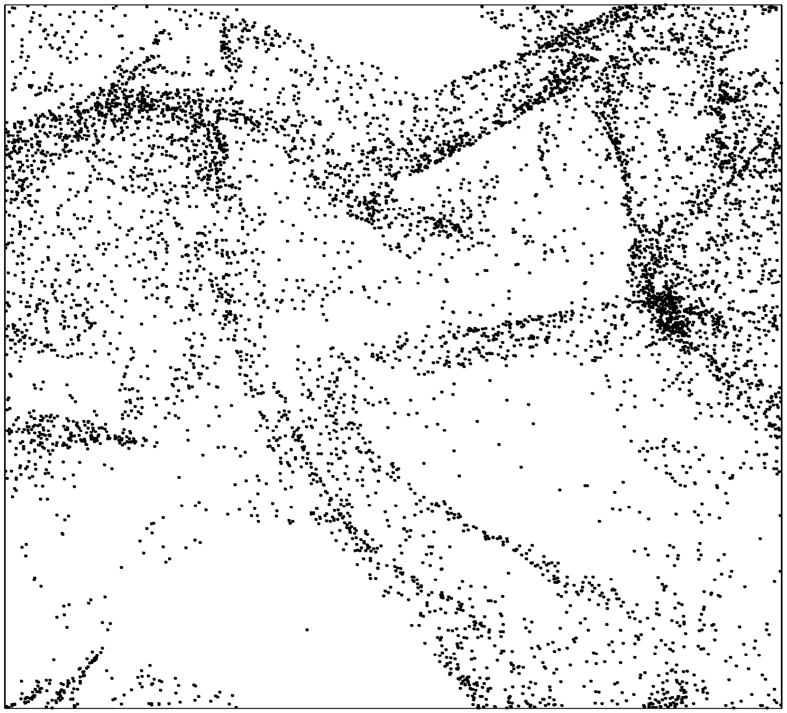
Inertial particles in stratified turbulence

Preferential concentration stratified turbulence



Horizontal

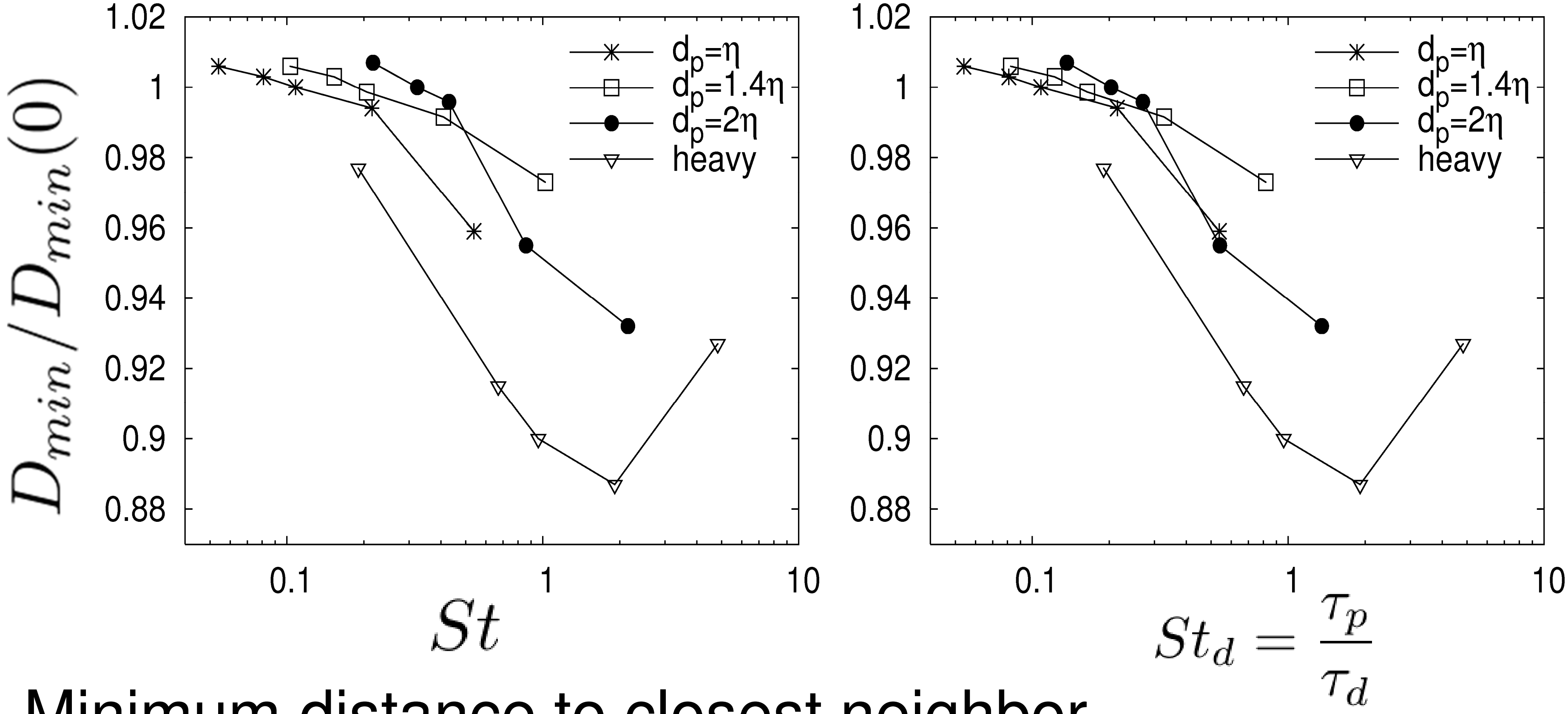
Vertical

M. van Aartrijk & H.J.H. Clercx,
Phys. Rev. Lett. **100**, 254501 (2008).

$N \sim 1\ (s^{-1})$

Inertial particles in stratified turbulence

Pref. conc. isotropic turbulence

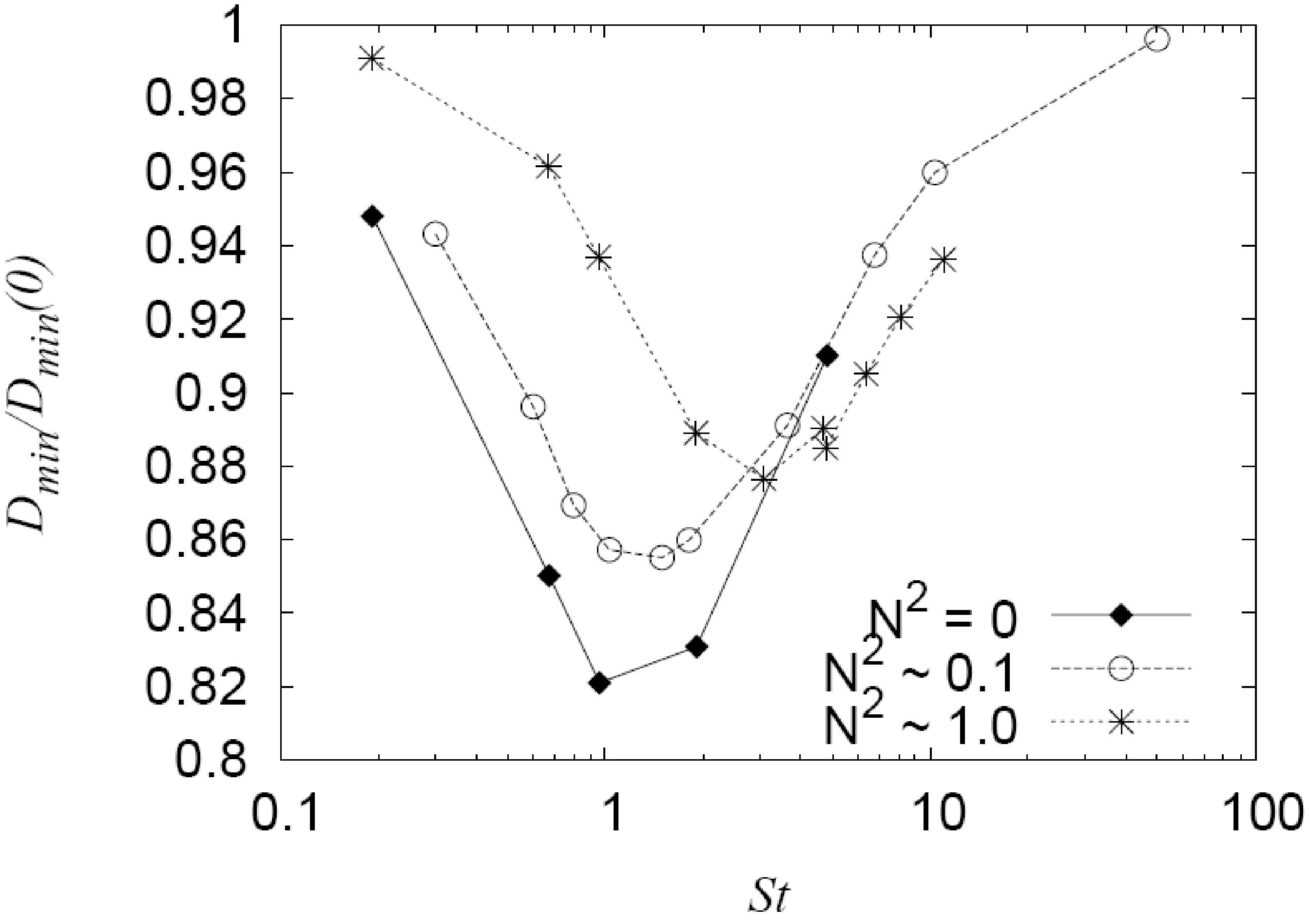


Minimum distance to closest neighbor

$\tau_d = (d^2/\varepsilon)^{1/3}$

Inertial particles in stratified turbulence

Preferential concentration stratified turbulence

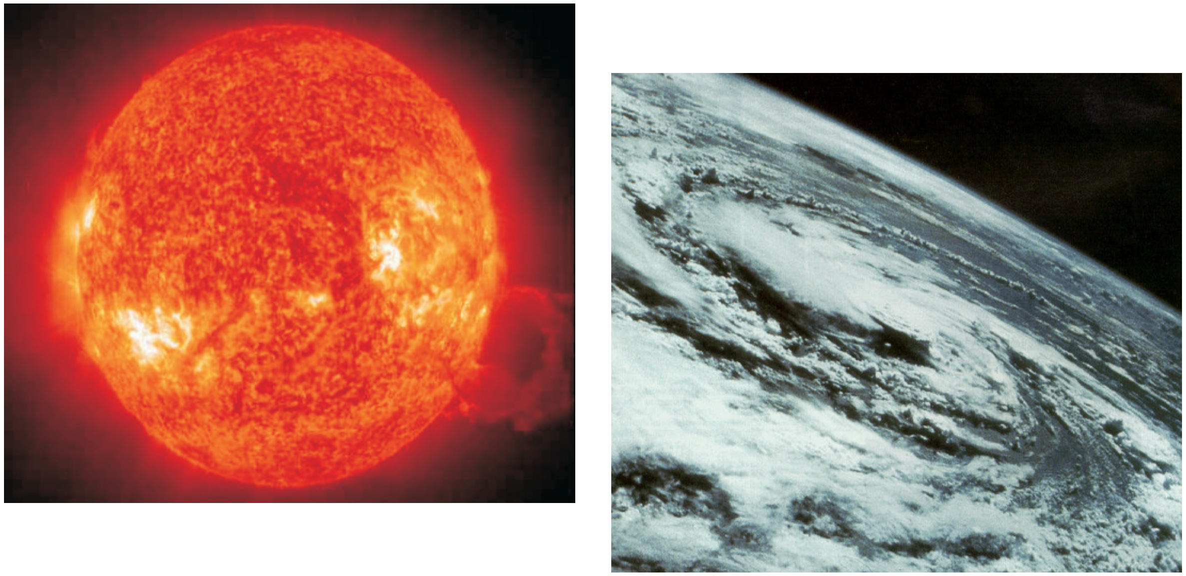


Inertial particles in stratified turbulence

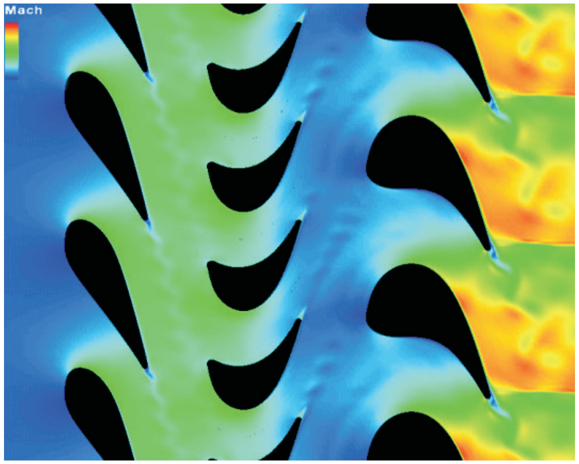
Intermediate Summary

- Stratification enhances horizontal dispersion and reduces vertical dispersion
- Inertia has negligible influence on horizontal dispersion in stratified turbulence
- With increasing inertia ($St \uparrow$), the long-time vertical dispersion in stratified turbulence increases
- Stratification enhances mixing; only within the horizontal plane
- Basset force needs to be taken into account for vertical light particle dispersion in stratified turbulence

Rotating flow



Astrophysical and geophysical applications



Engineering applications

- How does rotation affect velocity and acceleration PDFs?

- How does rotation affect Lagrangian autocorrelations?

- How do they compare with Eulerian autocorrelations?

Rotating flow

Navier-Stokes equations in rotating frame of reference:

$$\rho(\partial \mathbf{u}/\partial t + \mathbf{u}\cdot\nabla\mathbf{u}) + 2\rho\Omega \times \mathbf{u} = -\nabla p + \rho\nu\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0$$

Rossby number: $\dfrac{\text{inertial forces}}{\text{Coriolis force}} \rightarrow \dfrac{|\mathbf{u}\cdot\nabla\mathbf{u}|}{|2\Omega\times\mathbf{u}|} = \dfrac{U}{2\Omega L}$

Ro

Rotating flow

Navier-Stokes equations in rotating frame of reference:

$$\rho(\partial\mathbf{u}/\partial t + \mathbf{u}\cdot\nabla\mathbf{u}) + 2\rho\Omega\times\mathbf{u} = -\nabla p + \rho\nu\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0$$

Ekman number: $$\frac{\text{viscous forces}}{\text{Coriolis force}} \rightarrow \frac{|\nu\nabla^2\mathbf{u}|}{|2\Omega\times\mathbf{u}|} = \frac{\nu}{2\Omega L^2}$$

Ek

Rotating flow

Navier-Stokes equations in rotating frame of reference:

$$\rho(\partial \mathbf{u}/\partial t + \mathbf{u}\cdot\nabla\mathbf{u}) + 2\rho\Omega \times \mathbf{u} = -\nabla p + \nu\rho\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0$$

steady, Ro << 1, viscous effects negligible (Ek << 1):

$$2\Omega \times \mathbf{u} = -(1/\rho)\nabla p$$

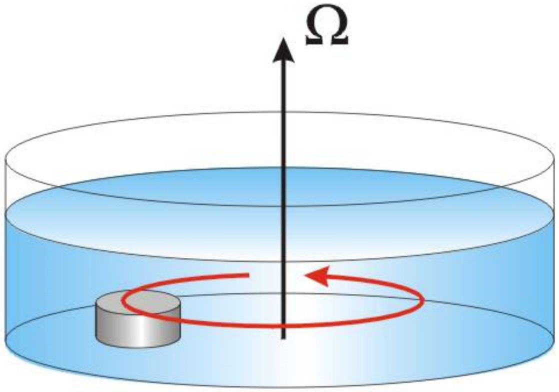
Taylor-Proudman theorem:
under strong rotation

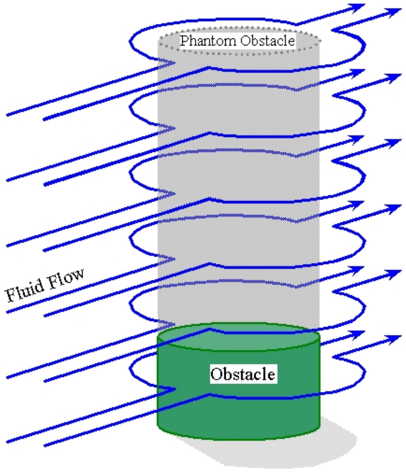
$$\frac{\partial \mathbf{v}}{\partial z} = 0$$

→ columnar flow structuring

Rotating flow

Taylor–Proudman theorem ↔ no vertical variation of velocity under geostrophic conditions

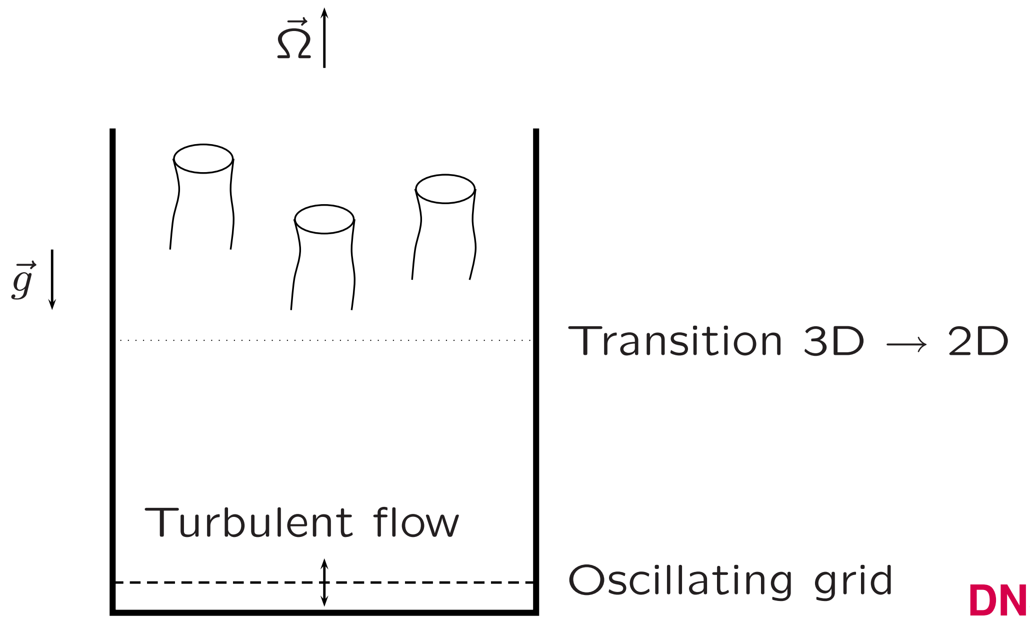




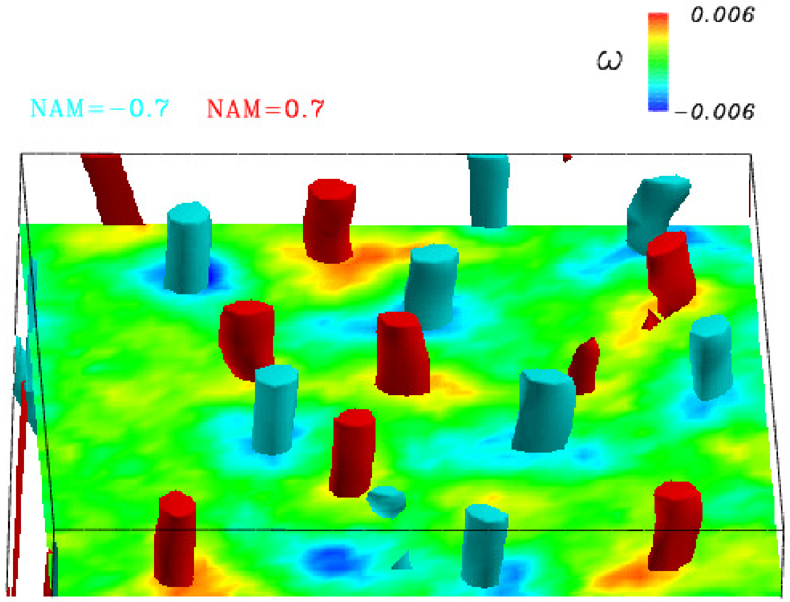
“Taylor column” above object dragged through a rotating fluid

Rotating turbulence

What is the effect of rotation on turbulence?

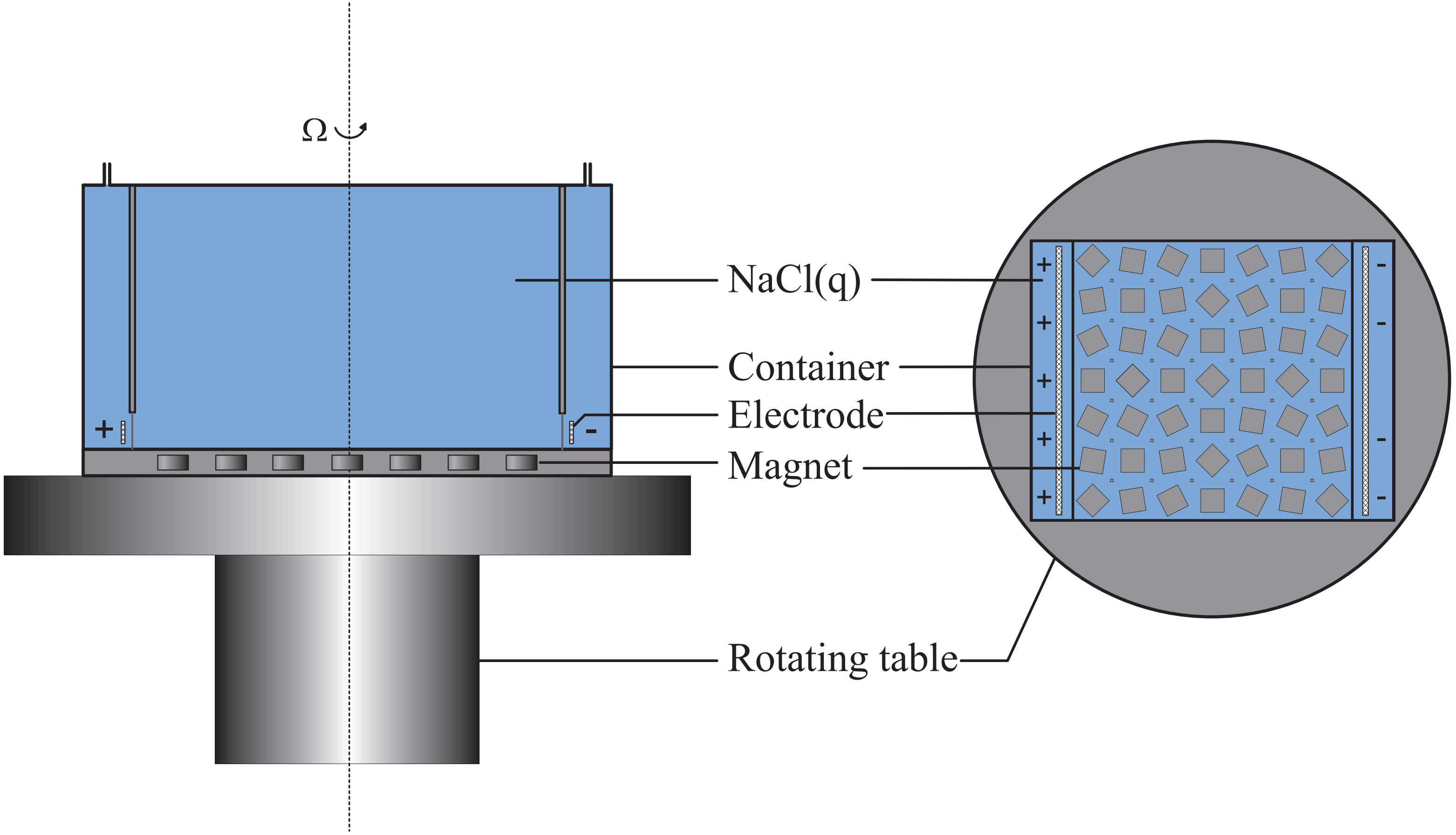


Experiment by Hopfinger, Browand and Gagne, JFM 125 (1982).



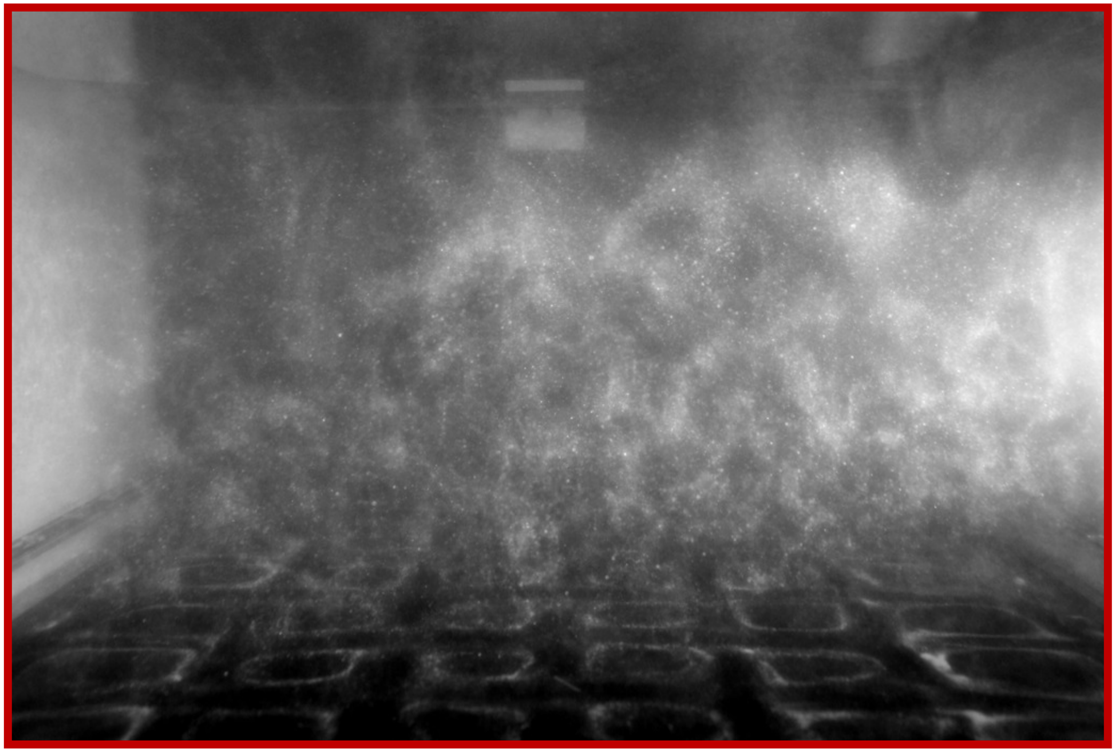
DNS by Godeferd and Lollini, JFM 393 (1999).

Rotating turbulence



Set-up for Rotating Turbulence Studies

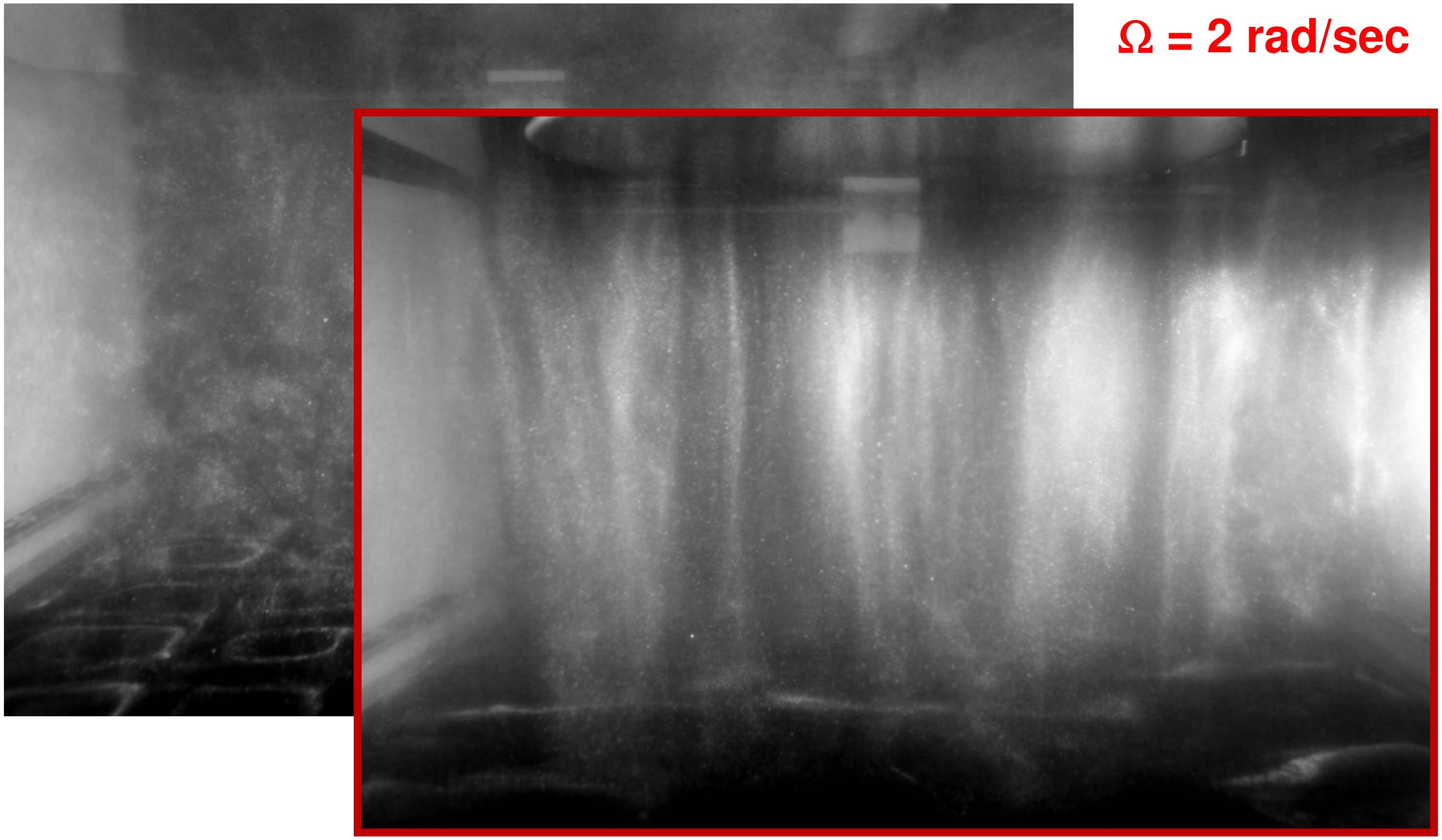
Rotating turbulence



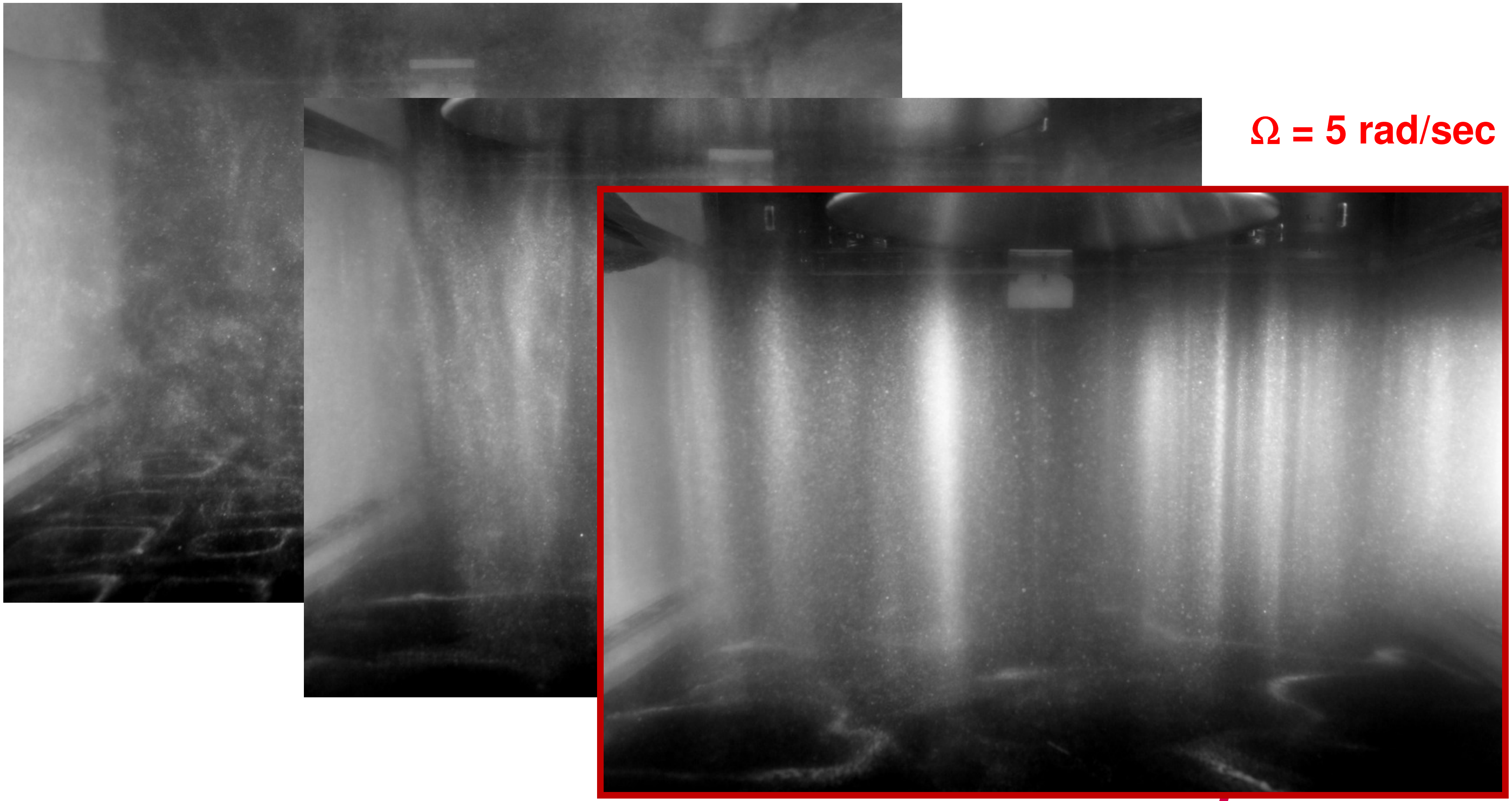
Ω = 0 rad/sec

Rotating turbulence

Ω = 2 rad/sec



Rotating turbulence



$\Omega = 5$ rad/sec

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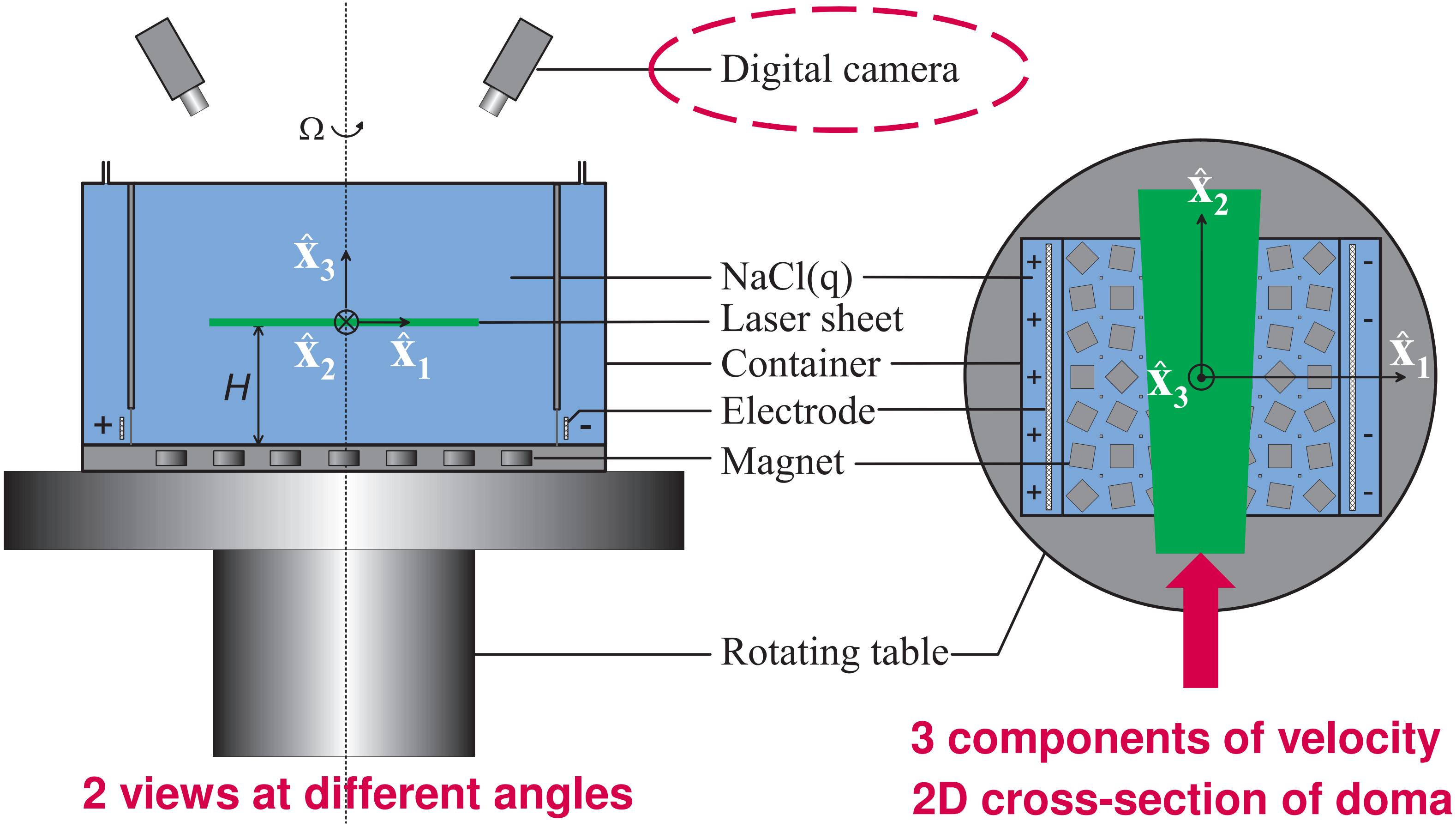
Rotating turbulence



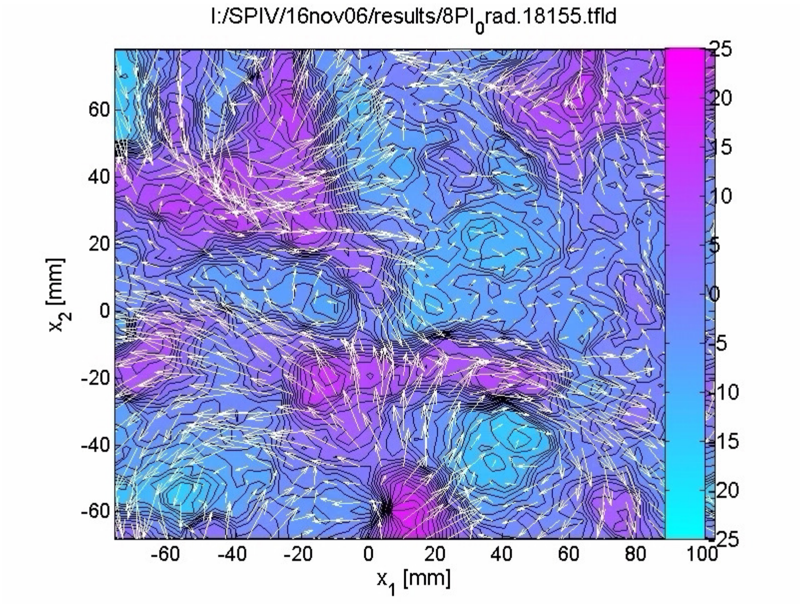
2 views at different angles

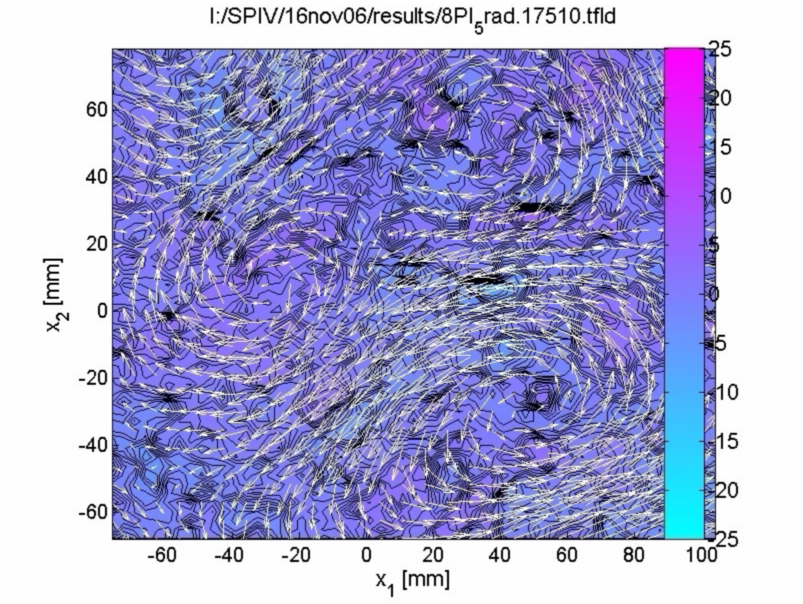
↓

3rd velocity component through geometric reconstruction

3 components of velocity in 2D cross-section of domain

Rotating turbulence





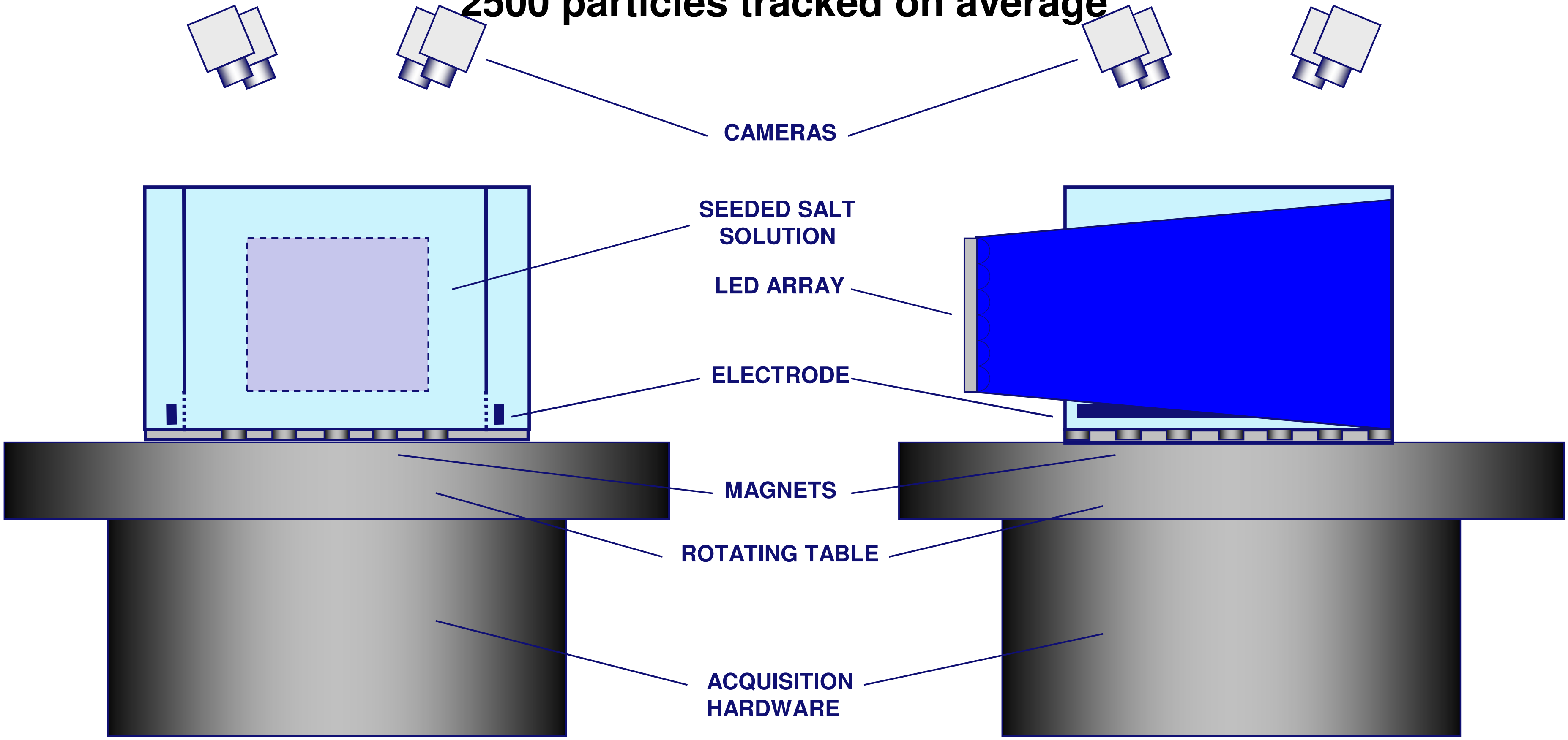
Re_{λ}~150-250 ,   H=5 cm

$\Omega=0$: stationary, reproducible, and $(u')^2 \sim (v')^2 \sim (w')^2$.

Characterization of rotating turbulence at several heights in the rotating fluid; $\Omega=0.1, 0.2, 0.5, 1, 2, 5, 10$ rad/s.

Rotating turbulence

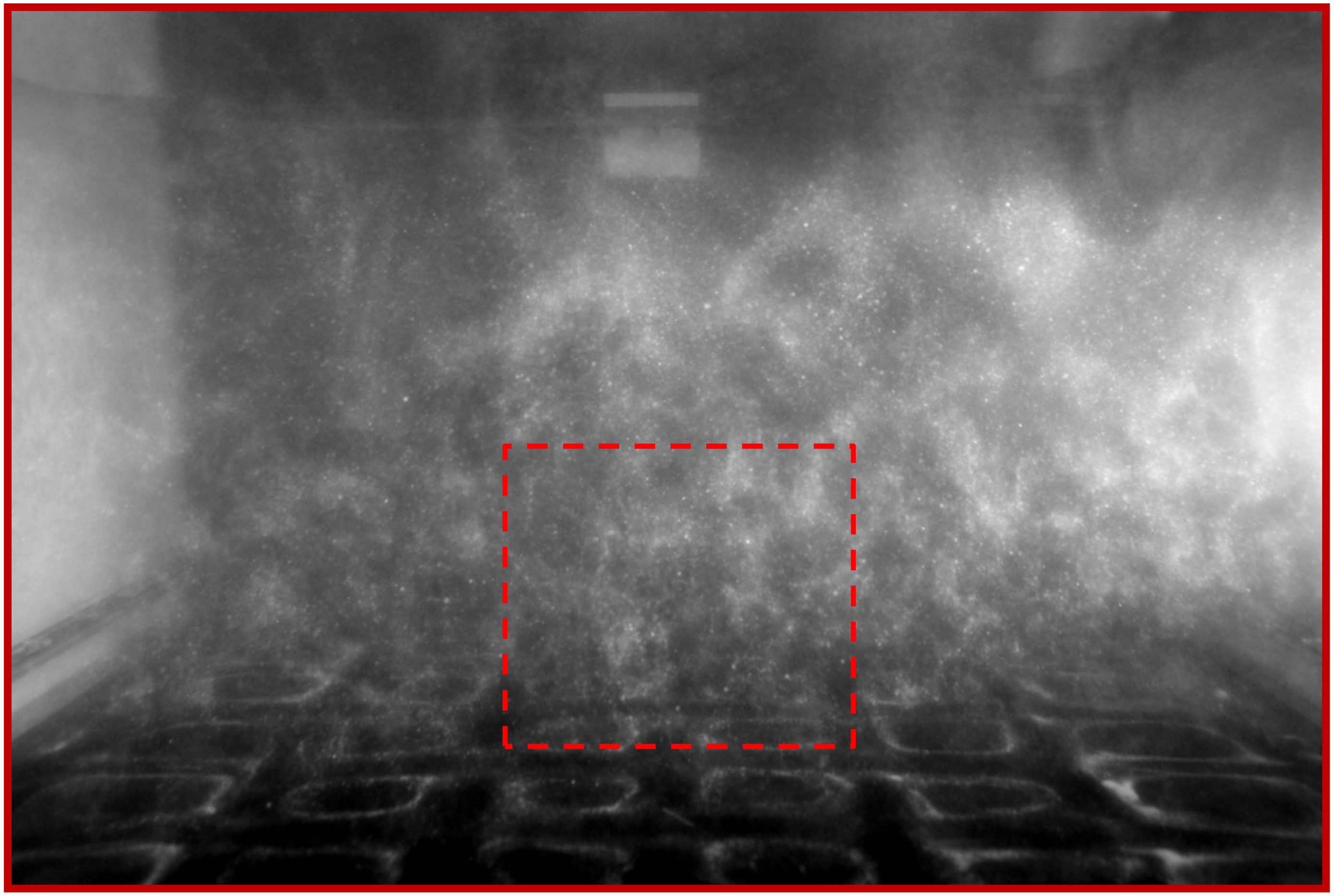
3D Particle Tracking Velocimetry system, four fast cameras, 2500 particles tracked on average



PTV-code from ETH, Zurich

H.G. Maas, A. Gruen, D. Papantoniou, Exp. Fluids 15, 133 (1993).
J. Willneff, A. Gruen. Proc. 9th Int. Symp. Transp. Phen. & Dyn. Rot. Mach. (2002).

Rotating turbulence



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Rotating turbulence

Exp		1	2	3	4	5	6
Ω	rad/sec	0	0.2	0.5	1.0	2.0	5.0
U_{rms}	mm/sec	12.4	11.0	13.5	16.3	22.9	16.9
Ro			0.86	0.30	0.11	0.07	0.04
T_{eddy}	sec	5.66	6.37	5.20	4.31	3.05	4.15
ΔT	sec	160	160	160	160	320	320
$\Delta T/T_{eddy}$		28.3	25.1	30.8	37.1	104.9	77.1

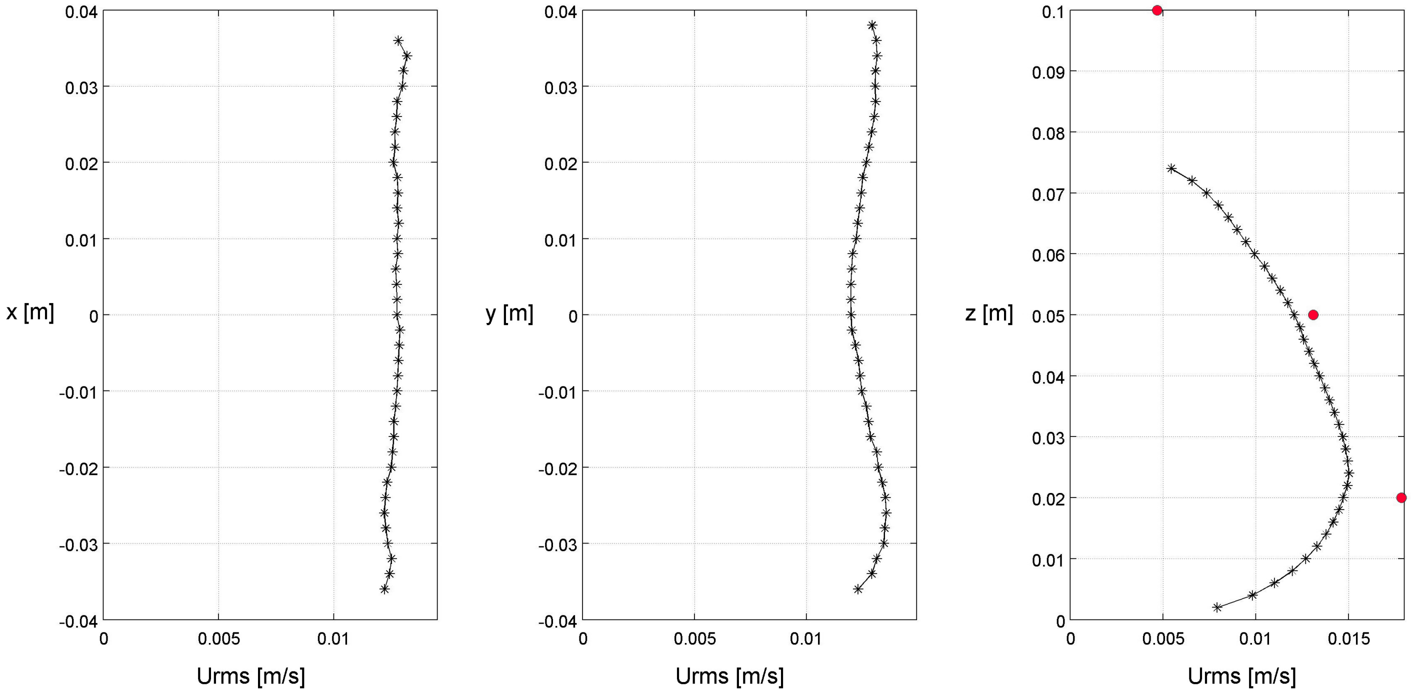
$0.6 < \eta < 0.8$ mm , $0.2 < \tau_\eta < 0.5$ sec

$Ro = U_{rms}/(2\Omega L)$

$50 < Re_\lambda < 150$

$\Delta t_{PTV} = 16.7$ μm for Ω up to 1 rad/sec
and $\Delta t_{PTV} = 33.3$ μm for Ω=2 and 5 rad/sec

Rotating turbulence



- Horizontal homogeneous and isotropic
- Vertical decay of energy (but reduced for rotating turbulence)

For more details, see Van Bokhoven et al., PoF **21**, 096601 (2009)

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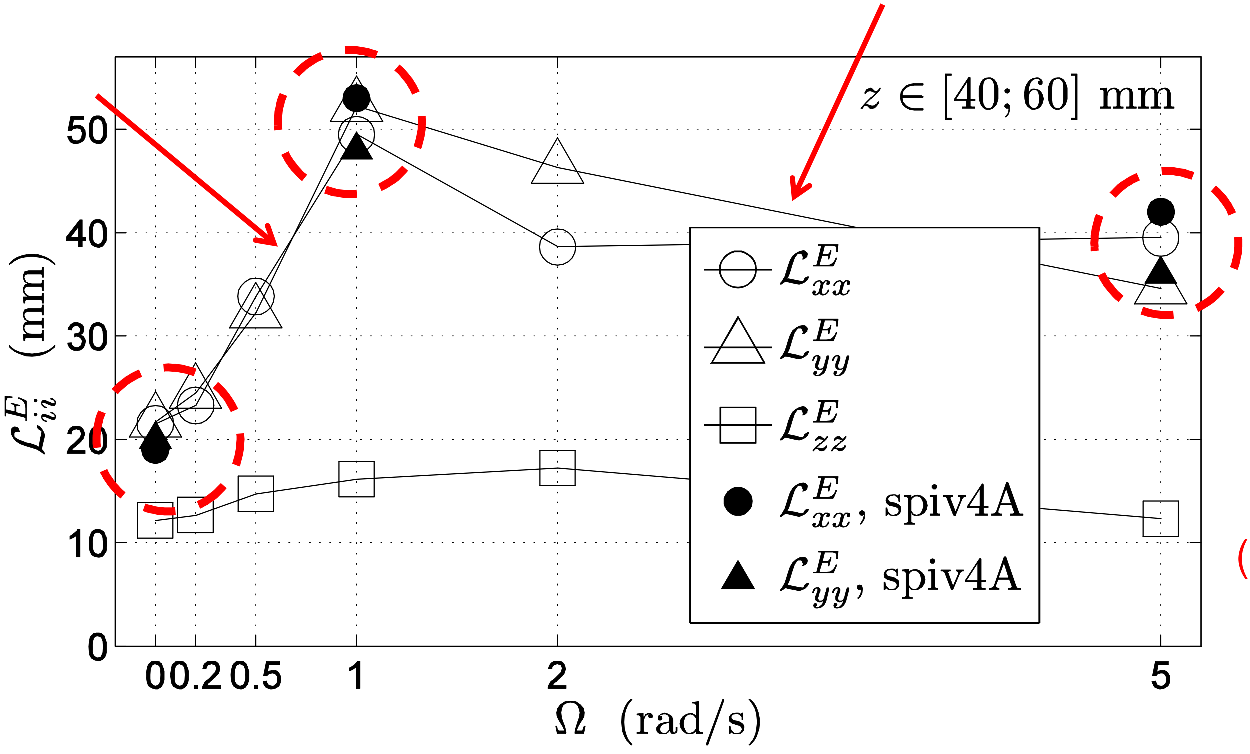
Rotating turbulence

Comparison Eulerian and Lagrangian spatial correlations

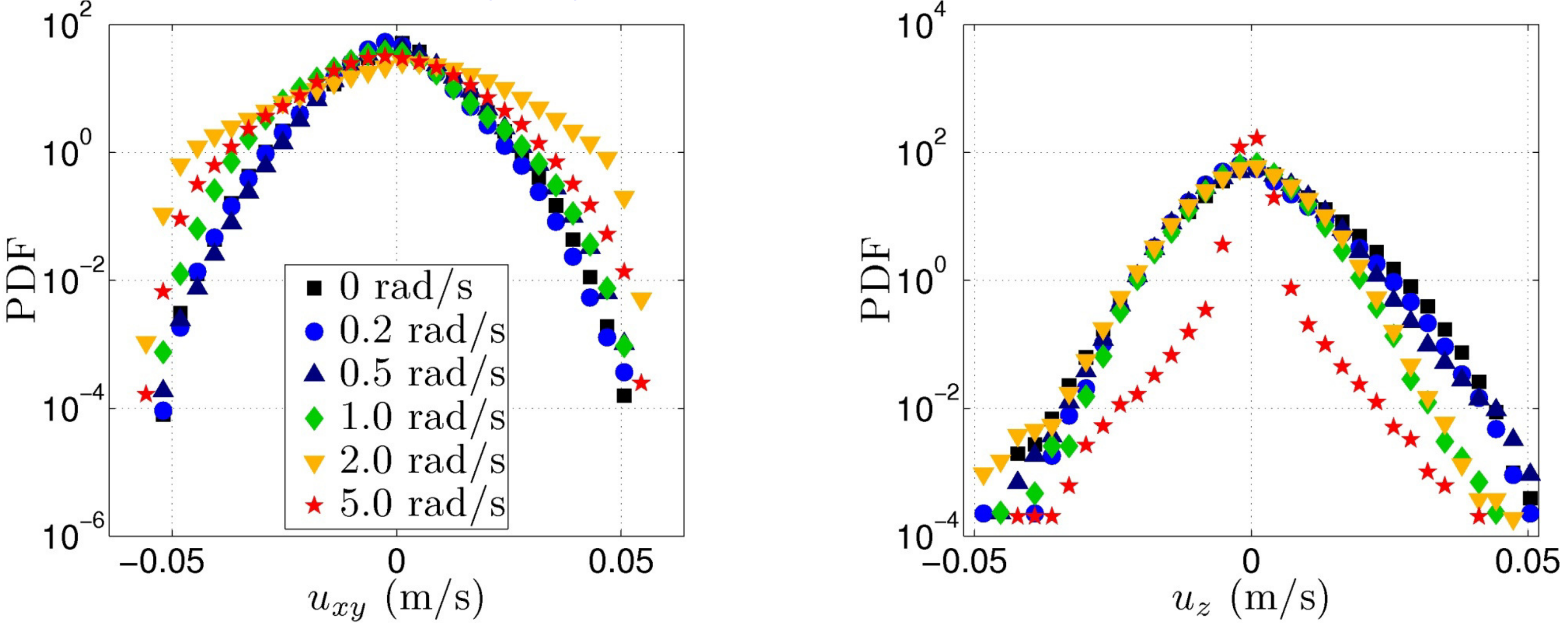


(thickness slabs)

Rotating turbulence

Horizontal and vertical velocity PDFs

Del Castello & Clercx, PRE **83**, 056316 (2011)



8x10^{6} (horizontal velocity) and 4x10^{6} (vertical velocity) data points

Hardly any skewness (except Ω=2 rad/sec)

Kurtosis: 3<K<4 (except Ω=5 rad/sec, u_z)

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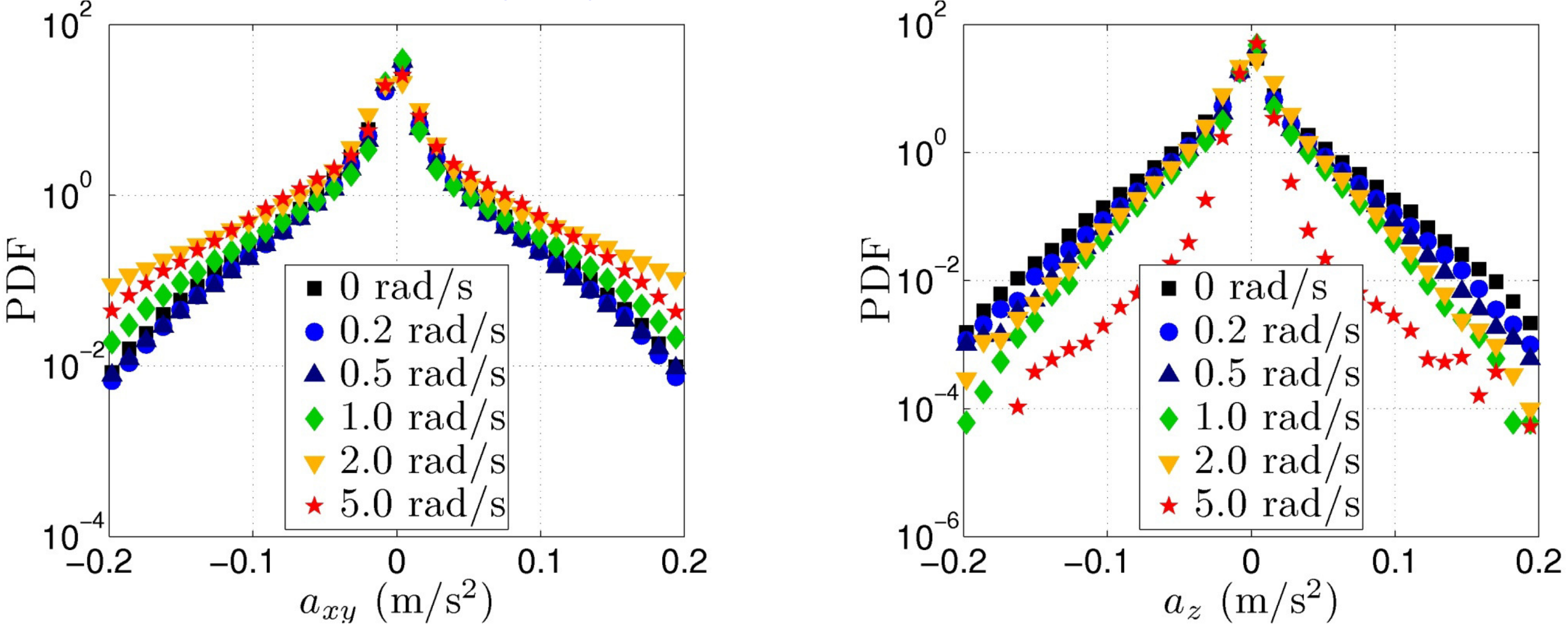
Rotating turbulence

Horizontal and vertical acceleration PDFs

Del Castello & Clercx, PRL **107**, 214502 (2011)



8×10^6 (horizontal velocity) and 4×10^6 (vertical velocity) data points

Hardly any skewness for all Ω; kurtosis: K>10 (except Ω=5 rad/sec)

Agreement with literature for small Ω

Note: PDFs converged with $32\Delta t_{PTV} \rightarrow \Delta t_{PTV}$

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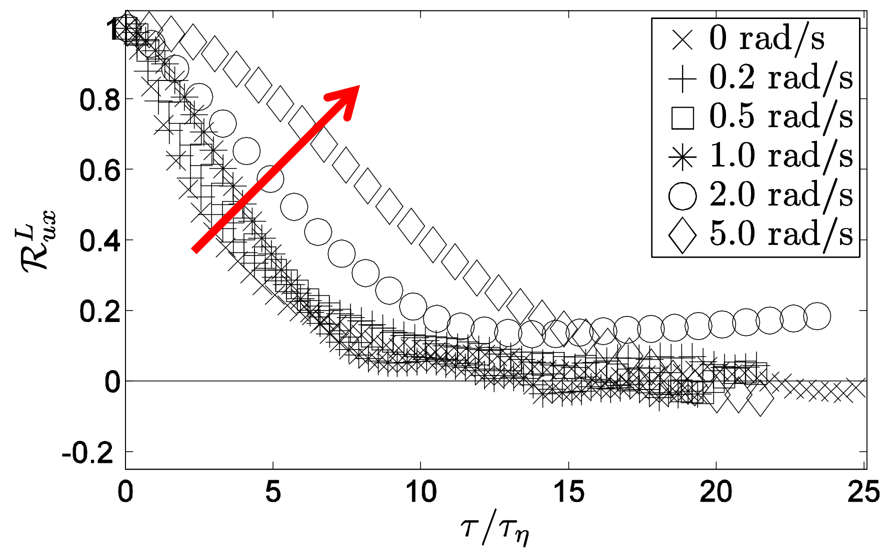
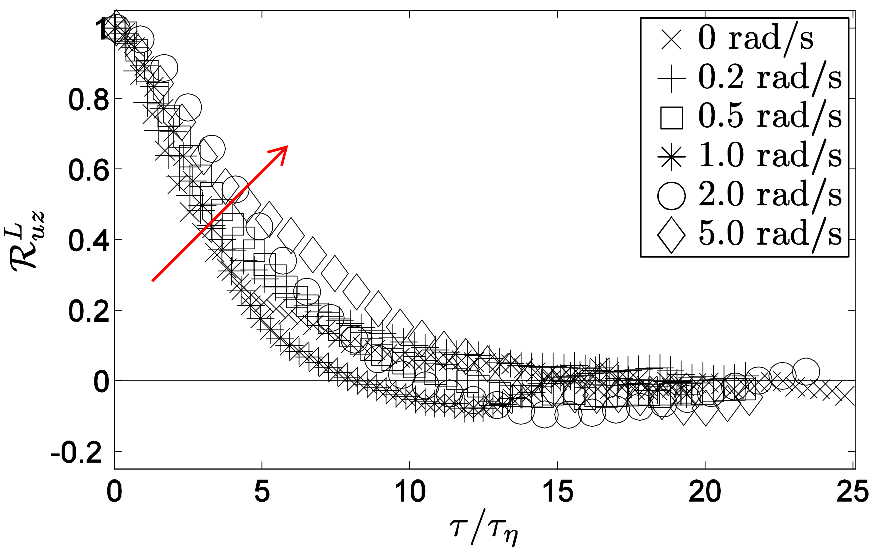
Lagrangian autocorrelations

Lagrangian longitudinal auto-correlation coefficients of Cartesian velocity components





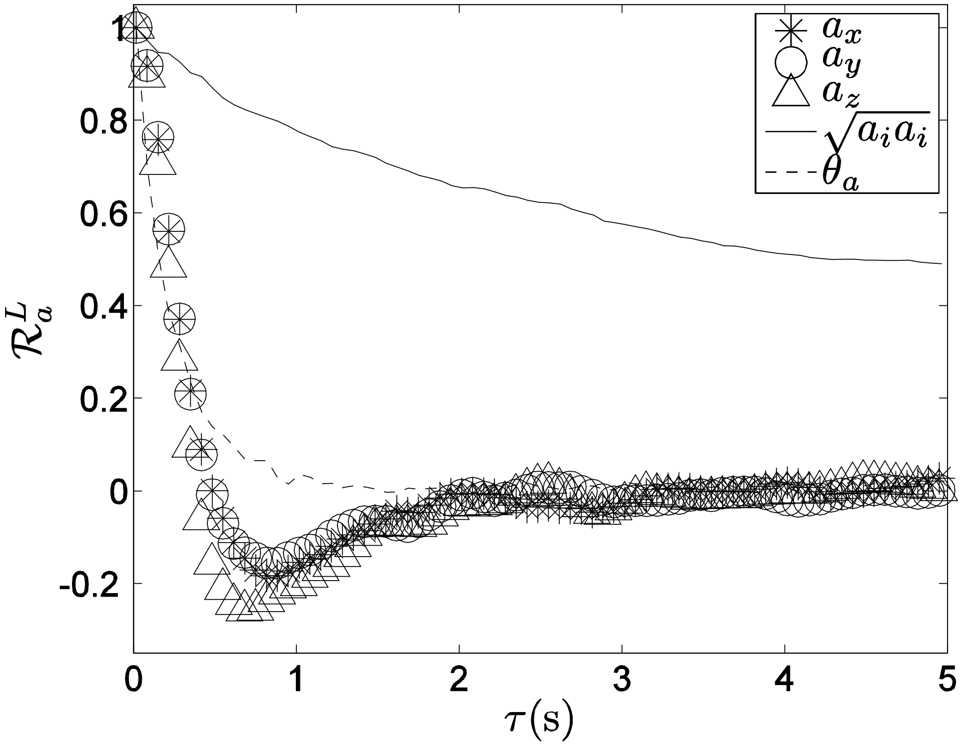
$$R_{L,ui} = \frac{<u_i(t)u_i(t+\tau)>}{<u_i(t)u_i(t)>}$$

But more pronounced Lagrangian vertical correlation compared to Eulerian vertical correlations!

Lagrangian autocorrelations

Lagrangian longitudinal auto-correlation coefficients of Cartesian acceleration components **$\Omega=0$**

$$R_{L,ai} = \frac{\langle a_i(t)a_i(t+\tau)\rangle}{\langle a_i(t)a_i(t)\rangle}$$



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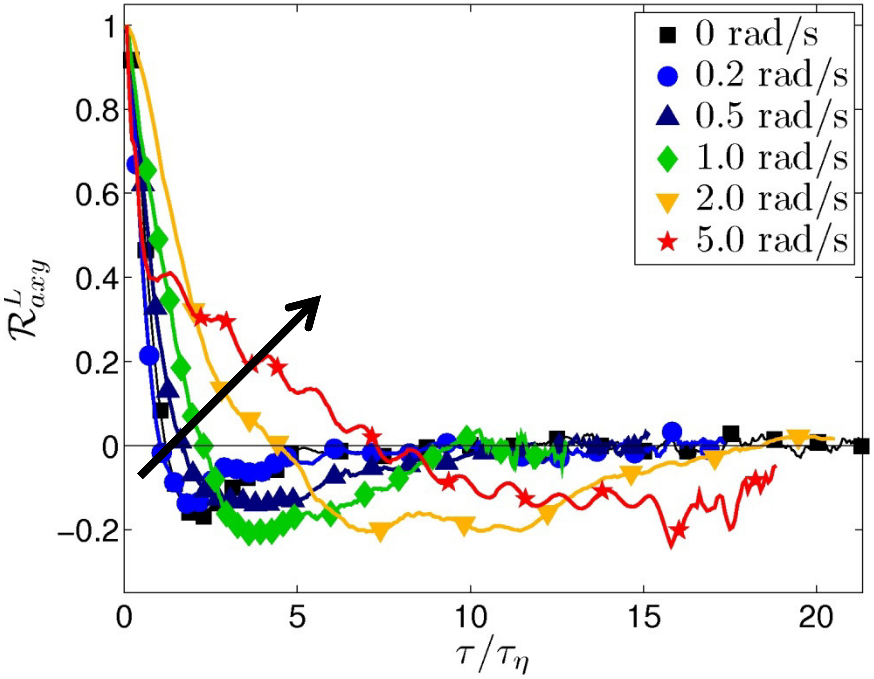
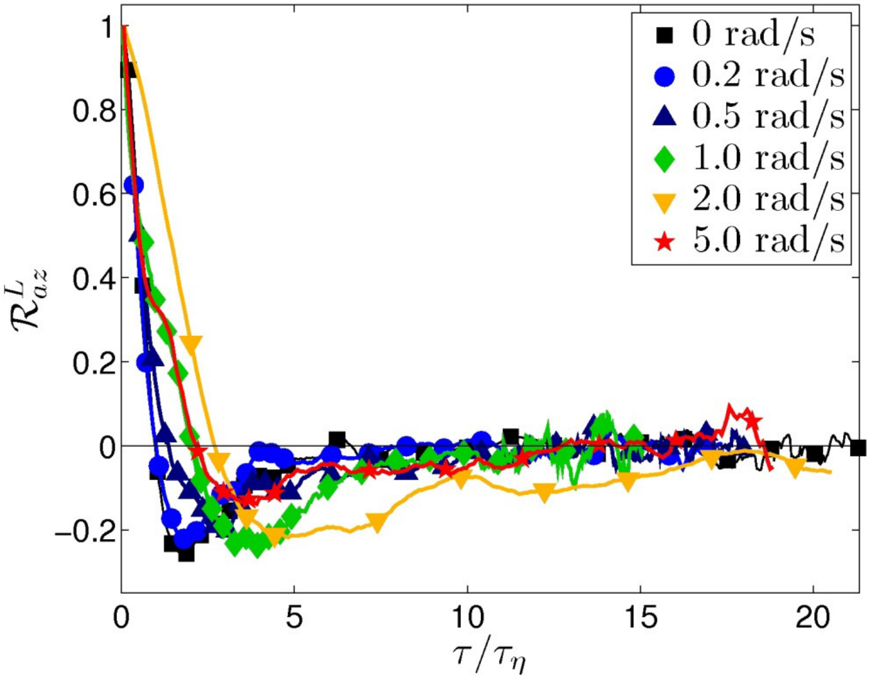
Lagrangian autocorrelations

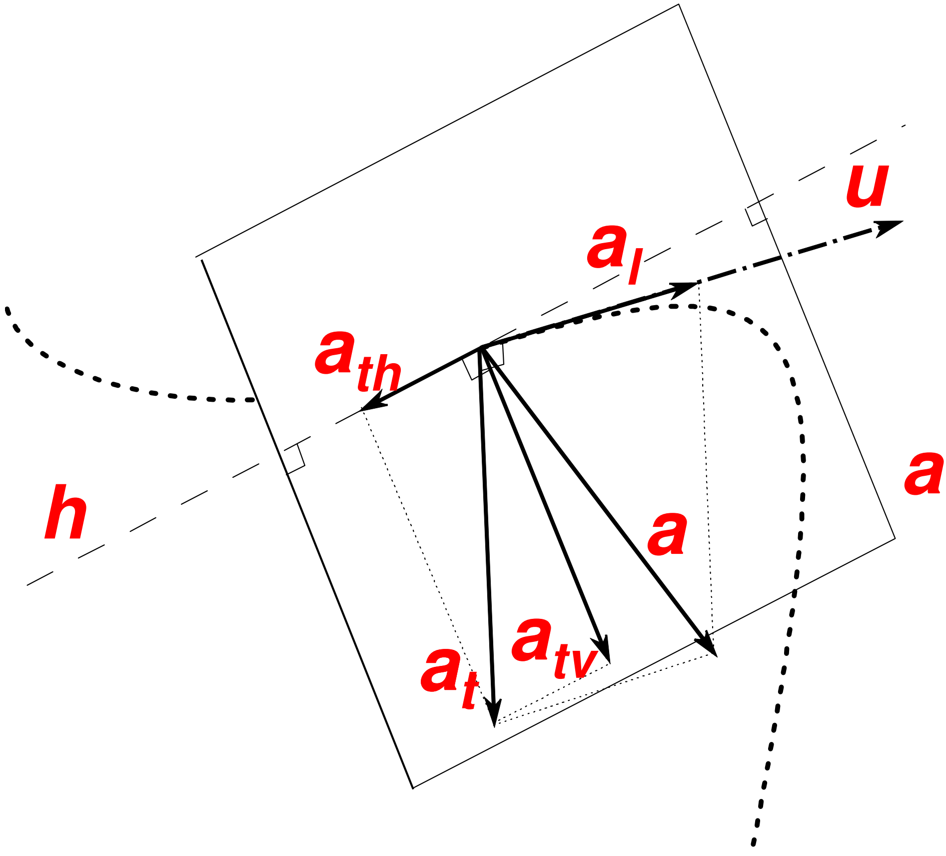
Lagrangian longitudinal auto-correlation coefficients of Cartesian acceleration components **all Ω>0**





Lagrangian autocorrelations

Decomposition of the Lagrangian acceleration vector



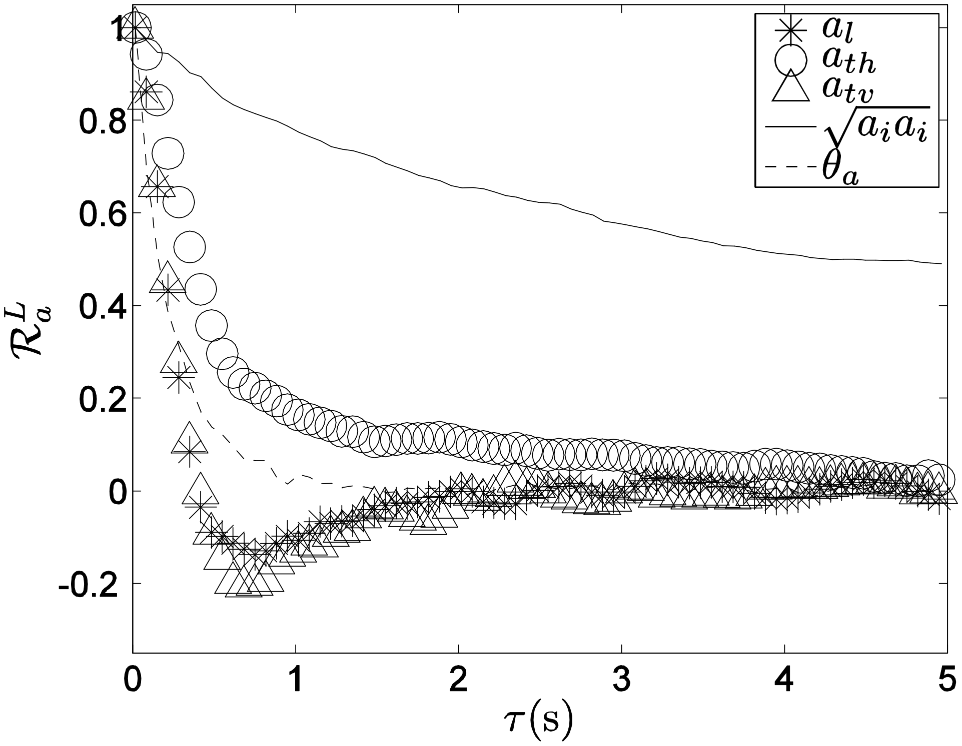
$$a = a_l + a_t$$
$$= a_l + \boxed{a_{th}} + a_{tv}$$

$$a_{coriolis} = (2u \times \Omega) \,||\, a_{th}$$

Lagrangian autocorrelations

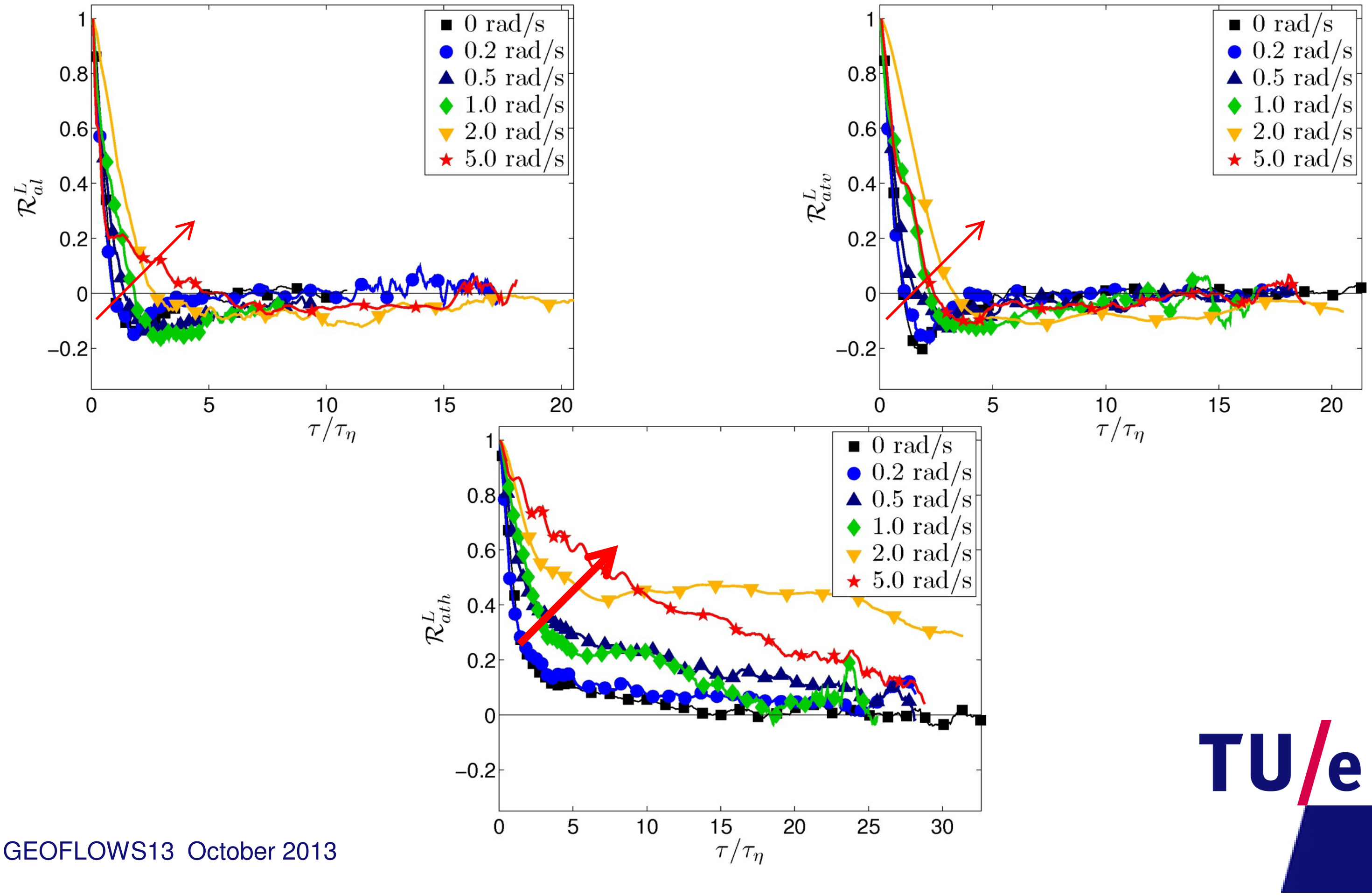
Lagrangian longitudinal auto-correlation coefficients of a_l, a_{tv}, a_{th} acceleration components Ω=0

$$R_{L,ai} = \frac{\langle a_i(t)a_i(t+\tau)\rangle}{\langle a_i(t)a_i(t)\rangle}$$



Lagrangian autocorrelations

Lagrangian longitudinal auto-correlation coefficients of a_l, a_{tv}, a_{th} acceleration components all $\Omega>0$



Lagrangian statistics in rotating turbulence

- Horizontal and verticaly velocity PDFs remain Gaussian except for highest rotation rate (two-dimensionalization). Horizontal acceleration PDFs show enhancement of tails with increasing rotation rate (more extreme events); the vertical acceleration PDFs shows just the opposite effect (two-dimensionalization).

- Amplification of the memory time-scales of the fluid particle velocity and acceleration with rotation.
  Increase of horizontal and vertical length scales and suppression of vertical motion.

- Strong amplification of the transversal horizontal acceleration (a_{th}) correlation, and mild increase of the longitudinal acceleration (a_l) correlation.
  Direct and indirect effects, respectively, of the Coriolis acceleration.

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