Stability of river bed forms cont'd

Marco Colombini
Dipartimento di Ingegneria Civile, Chimica e Ambientale
University of Genova, Italy

Ripples and Dunes

Ripples in the Hunter River, New South Wales, Australia

Image courtesy of M.C. Rygel

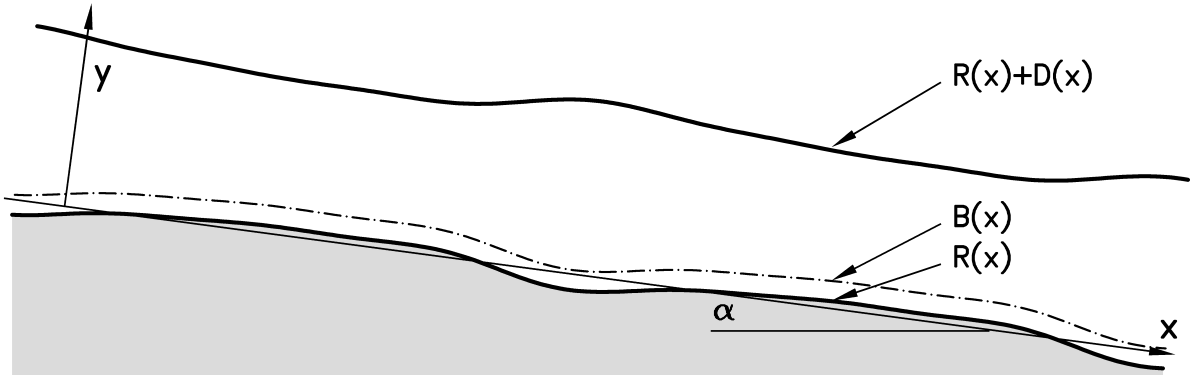
Previous Studies

Linear Stability Analyses

Authors	Flow Regime	Flow model	Results
Richards 1980 *J. Fluid Mech.*	rough	Rotational Effect of the flow depth included	Dunes Ripples
Engelund & Fredsoe, 1982, *Ann. Rev. Fluid Mech.*	Ripples associated to smooth flow regime		
Sumer & Bakioglu, 1984 *J. Fluid Mech.*	smooth	Rotational Boundary layer Infinite flow depth	Ripples
Fourriere et al, 2010 *J. Fluid Mech.*	rough	Rotational Effect of flow depth included	Ripples
Colombini & Stocchino, 2011, *J. Fluid Mech.*	rough transitional smooth	Rotational Effect of flow depth included	Dunes Ripples

All models consider bed load only

Flow model



$$\eta = \frac{y - R}{D}$$

2D REYNOLDS EQUATIONS + CONTINUITY

$$u_{,t} + uu_{,x} + vu_{,y} + p_{,x} - S/Fr^2 - T_{xx,x} - T_{xy,y} = 0$$

$$v_{,t} + uv_{,x} + vv_{,y} + p_{,y} + 1/Fr^2 - T_{xy,x} - T_{yy,y} = 0$$

$$u_{,x} + v_{,y} = 0$$

BOUSSINNESQ's TURBULENCE CLOSURE

$$T_{xx} = 2\nu_T u_{,x} \qquad T_{yy} = 2\nu_T v_{,y} \qquad T_{xy} = \nu_T\left(u_{,y} + v_{,x}\right)$$

EDDY VISCOSITY

$$\nu_T = l^2\sqrt{\left(u_{,x} - v_{,y}\right)^2 + \left(u_{,y} + v_{,x}\right)^2}$$

CONDUCTANCE COEFFICIENT (CHEZY)

$$C = \sqrt{\frac{8}{f}} = \frac{1}{\kappa} \ln\left(\frac{11.09}{2.5d}\right)$$

Rough regime

(ASCE Task Committee, 1963)

$$C = \sqrt{\frac{8}{f}} = \frac{1}{\kappa} \ln\left(\frac{\mathrm{Re}\sqrt{f}}{3.41}\right)$$

Smooth regime

Rough vs. smooth regime

Rough vs. smooth regime

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$$C = \sqrt{\frac{8}{f}} = \frac{1}{\kappa}\ln\left[\left(\frac{11.09}{2.5d}\right)^{1-\beta}\left(\frac{\mathrm{Re}\sqrt{f}}{3.41}\right)^{\beta}\right]$$

Transitional regime

(Chang, 2008)

$$\beta = \exp\left[-0.11\left(\ln \mathrm{Re}_k\right)^{\frac{5}{2}}\right]$$

$$\mathrm{Re}_k = \frac{u_f^* k_S^*}{\nu}$$

Roughness Reynolds number

$$\mathrm{Re} = \frac{4U^* D^*}{\nu}$$

Flow Reynolds number

$$\Rightarrow \quad \mathrm{Re} = \frac{4C}{2.5d}\mathrm{Re}_k$$

Rough vs. smooth regime

MIXING LENGTH – Rough regime

$$l = \kappa D(\eta + \eta_0)(1-\eta)^{1/2}$$

Rule of the thumb: the roughness height is about one thirtieth of the actual nondimensional roughness.
This result can be formally obtained by integrating the log-law along depth, relating the roughness height to the nondimensional Chezy coefficient)

$$1 = \int_0^1 u\,d\eta = \frac{1}{\kappa C}\int_0^1 \ln\left(\frac{\eta+\eta_0}{\eta_0}\right)d\eta \cong -\frac{1}{\kappa C}\left[1+\ln(\eta_0)\right]$$

and then substituting the empirical relationship

$$C = \sqrt{\frac{8}{f}} = \frac{1}{\kappa}\ln\left(\frac{11.09}{2.5d}\right)$$

to obtain

$$\eta_0 = \exp(-\kappa C - 1) = \frac{d}{12} = \frac{2.5d}{30} = \frac{y_R}{30} = R_0$$

Rough vs. smooth regime

MIXING LENGTH – Smooth & Transitional regimes

$$l = \kappa D(\eta + \Delta\eta)\left[1 - \exp\left(-\frac{\eta + \Delta\eta}{A} D \frac{\mathrm{Re}_k}{2.5d}\right)\right](1-\eta)^{1/2}$$

MIXING LENGTH – Smooth & Transitional regimes

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Van Driest (1956) – The exponential correction accounts for the existence of a viscous and buffer layer in the smooth and transitional regimes.

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$$\Delta\eta^{+} = \frac{\mathrm{Re}_k}{30}$$

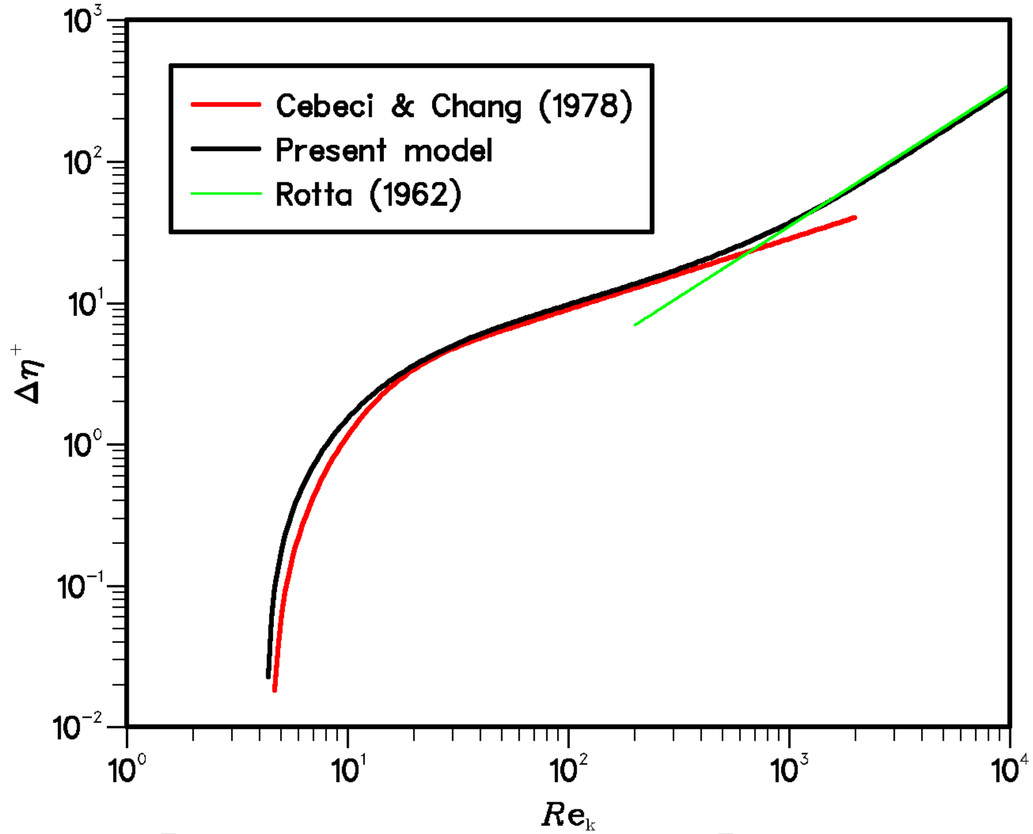
Rotta (1962)
Rough regime

$$\Delta\eta^{+} = 0.9\left[\sqrt{\mathrm{Re}_k} - \mathrm{Re}_k \exp\left(-\frac{\mathrm{Re}_k}{6}\right)\right]$$

Cebeci & Chang (1978)
Smooth & Transitional regime

Rough vs. smooth regime

DISPLACEMENT

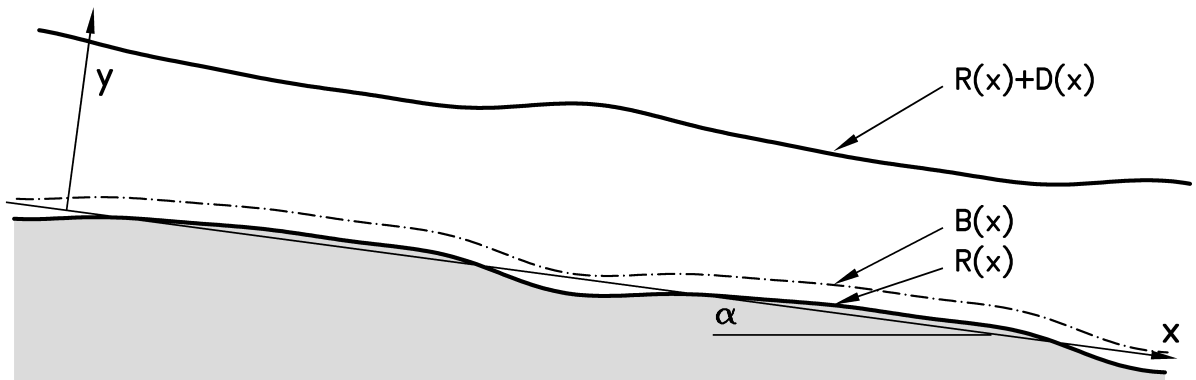


$$C = \frac{1}{\kappa} \ln\left[\left(\frac{11.09}{2.5d} \right)^{1-\beta} \left(\frac{\mathrm{Re}\sqrt{f}}{3.41} \right)^{\beta} \right]$$

$$\beta = \exp\left[-0.11 \left(\ln \mathrm{Re}_k \right)^{\frac{5}{2}} \right]$$

$$\Delta\eta^{+} = f\left(\mathrm{Re}_k\right)$$

Sedime nt transport model



SEDIMENT CONTINUITY EQUATION (EXNER)

$$R_{,t} + Q_S \Phi_{,x} = 0 \qquad Q_S = \frac{1}{F}\frac{\sqrt{(s-1)d^3}}{1-p} << 1$$

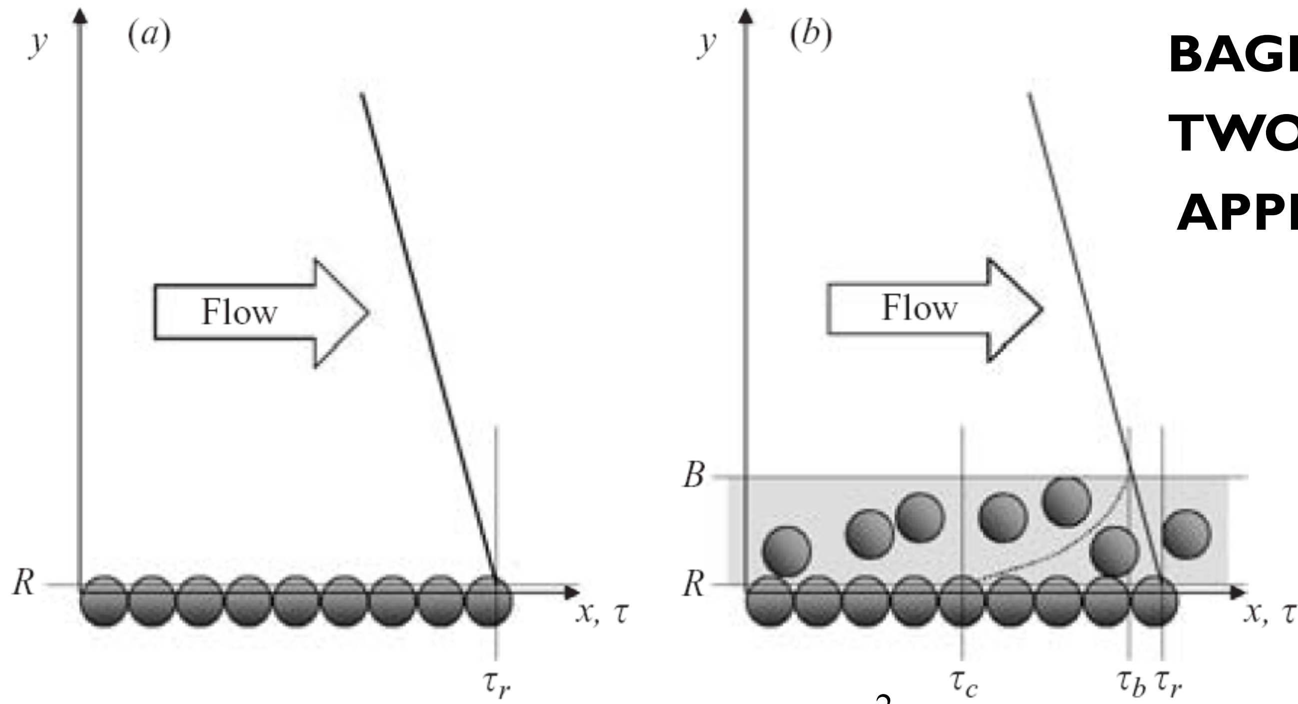
BEDLOAD FUNCTION (MPM after Wong & Parker (2006))

$$\Phi = 3.97\left(\vartheta_B - \vartheta_C\right)^{3/2}$$

BEDLOAD LAYER THICKNESS (scaled with grain size)

$$l_B = 1 + 1.3\left(\frac{\vartheta_0 - \vartheta_{C0}}{\vartheta_{C0}}\right)^{0.55}$$

BAGNOLD'S TWO-LAYER APPROACH



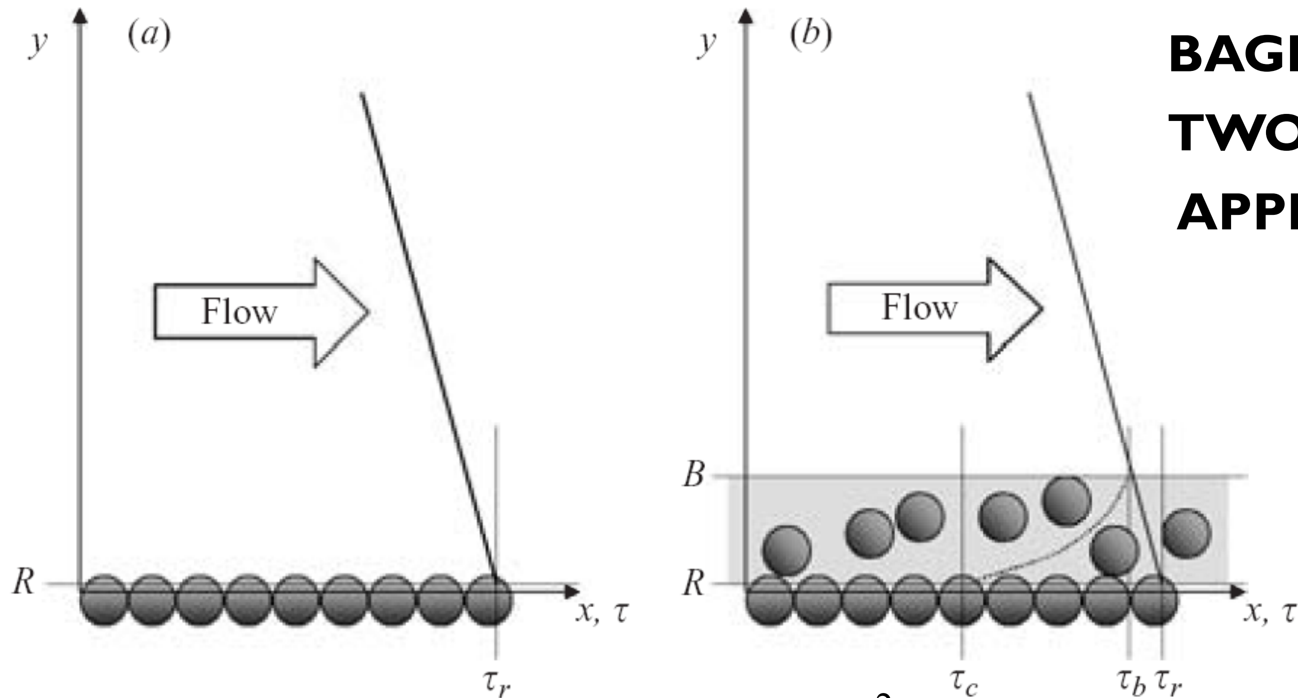
$$\vartheta_B = \vartheta_0 T_B = T_{xy}\Big|_{y=B} \qquad \vartheta_0 = \frac{F^2}{C^2(s-1)d}$$

$$B = R + \eta_B D \qquad \eta_B = l_B d$$

CORRECTION FOR SEDIMENT WEIGHT

$$\vartheta_C = \vartheta_{CH} - \mu_x\left(S - R_{,x}\right) \qquad \mu_x = 0.1$$

Fredsøe (1974)



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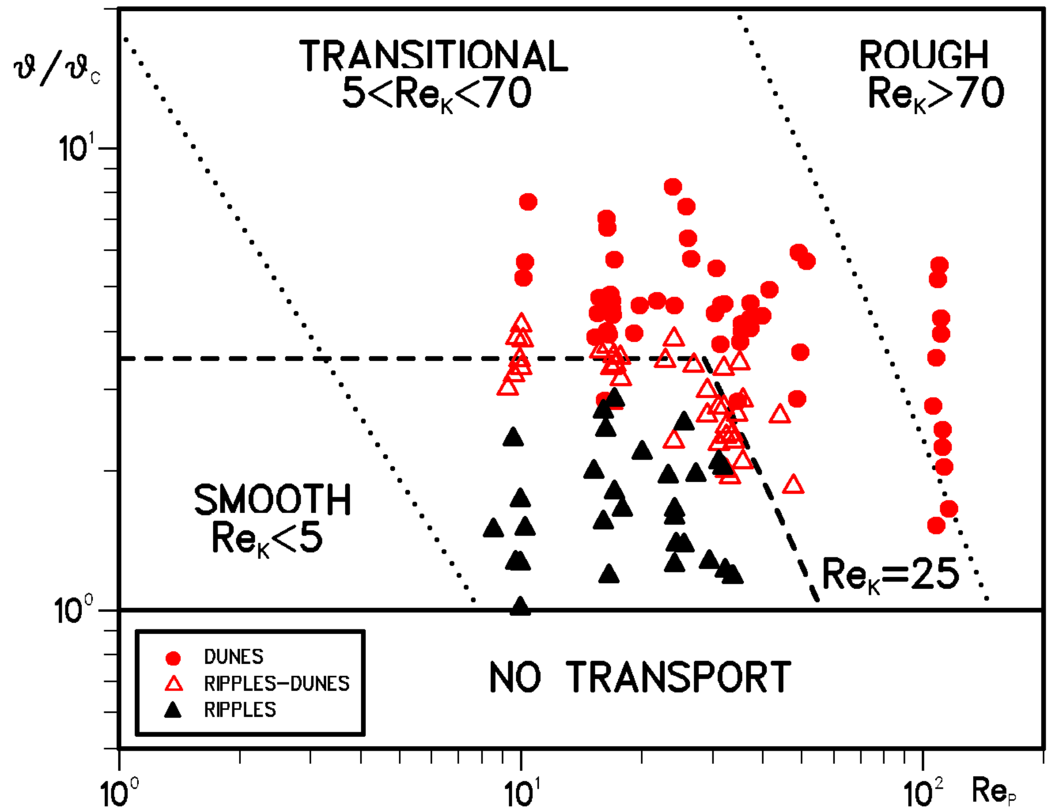
$$\vartheta_{CH} = 0.22\,\mathrm{Re}_P^{-0.6} + 0.06\exp\left(-17.73\,\mathrm{Re}_P^{-0.6}\right)$$ **Brownlie (1981)**

$$\vartheta_{CH} = 0.0495$$ **Wong & Parker (2006)**

Experimental Observations

Experiments by Guy, Simons & Richardson (1966)



$$\mathrm{Re}_k = \frac{u_f^* k_S^*}{\nu}$$

Roughness Reynolds number

$$\mathrm{Re}_P = \frac{\sqrt{(s-1)gd^*}\,d^*}{\nu}$$

Particle Reynolds number

$$\mathrm{Re}_k = \sqrt{\vartheta}\,\mathrm{Re}_P 2.5$$

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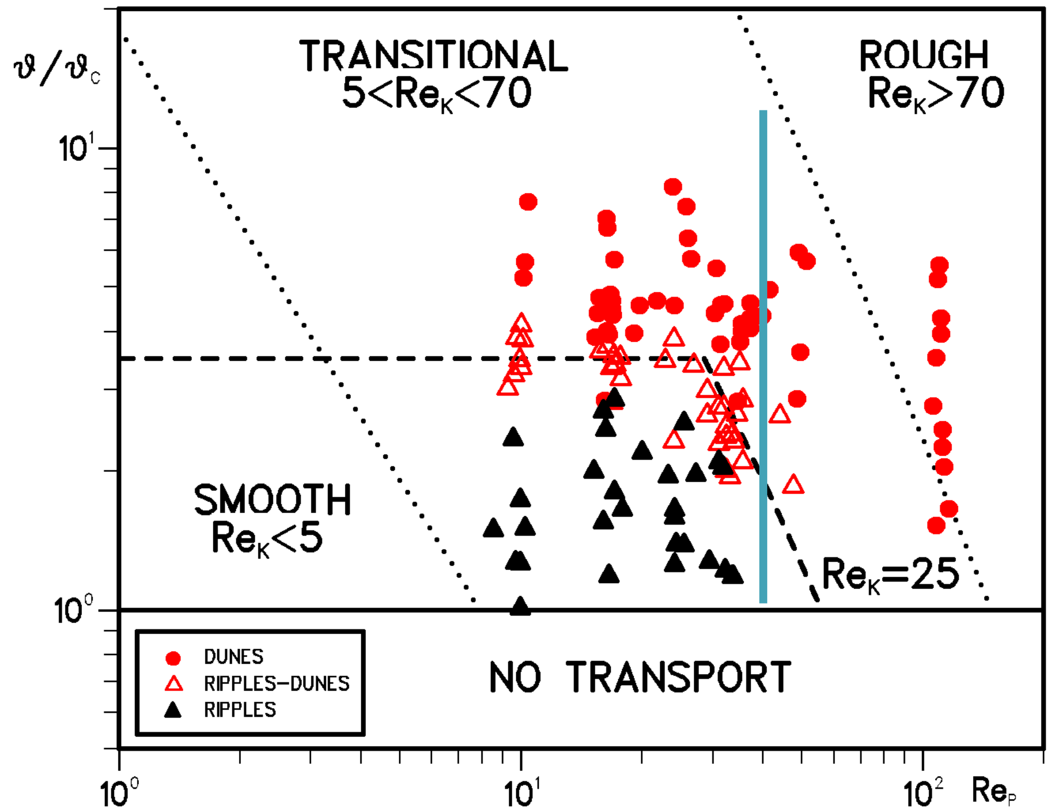

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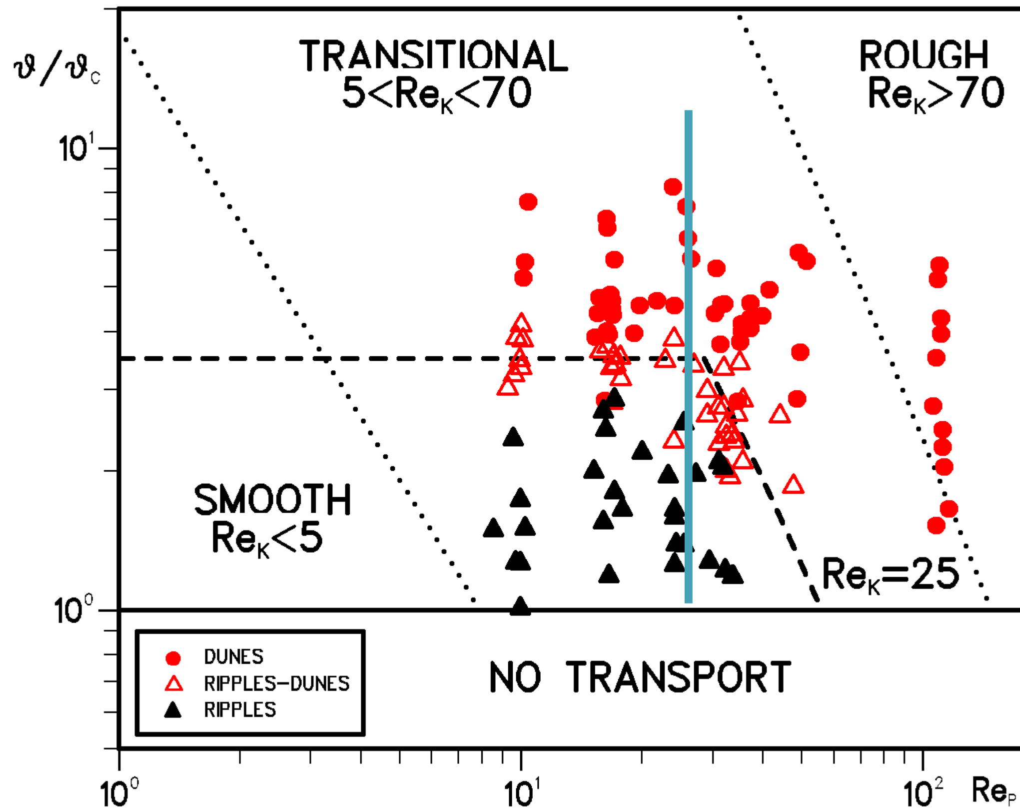

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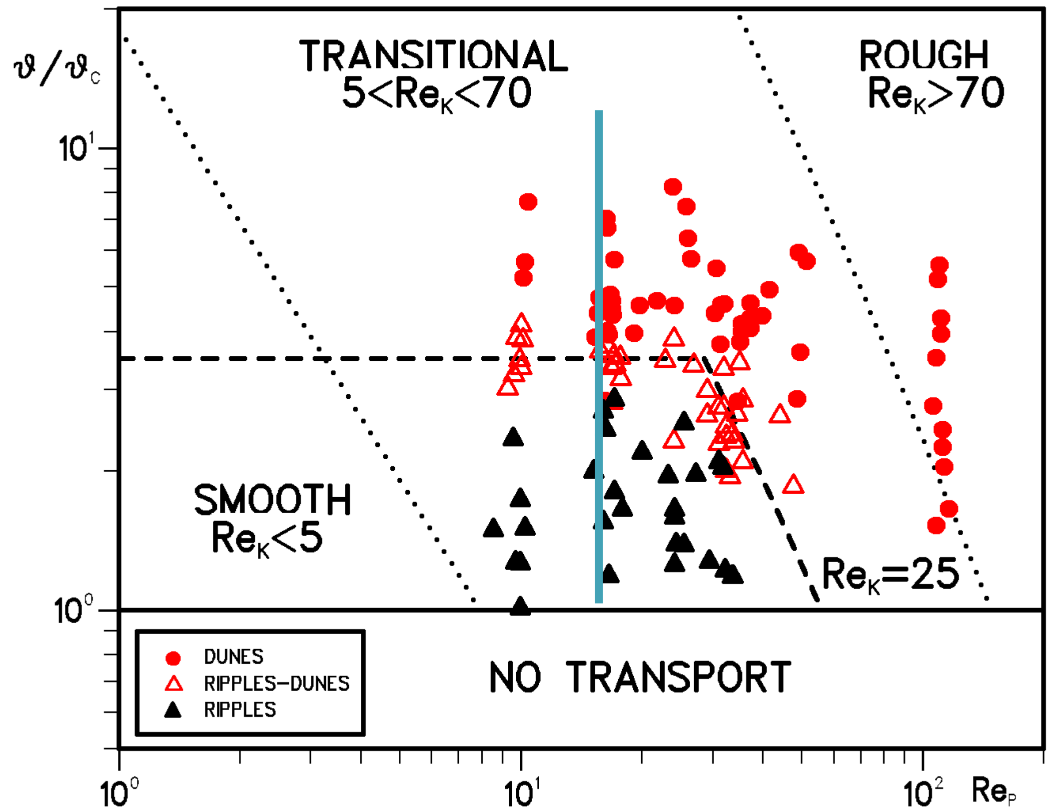

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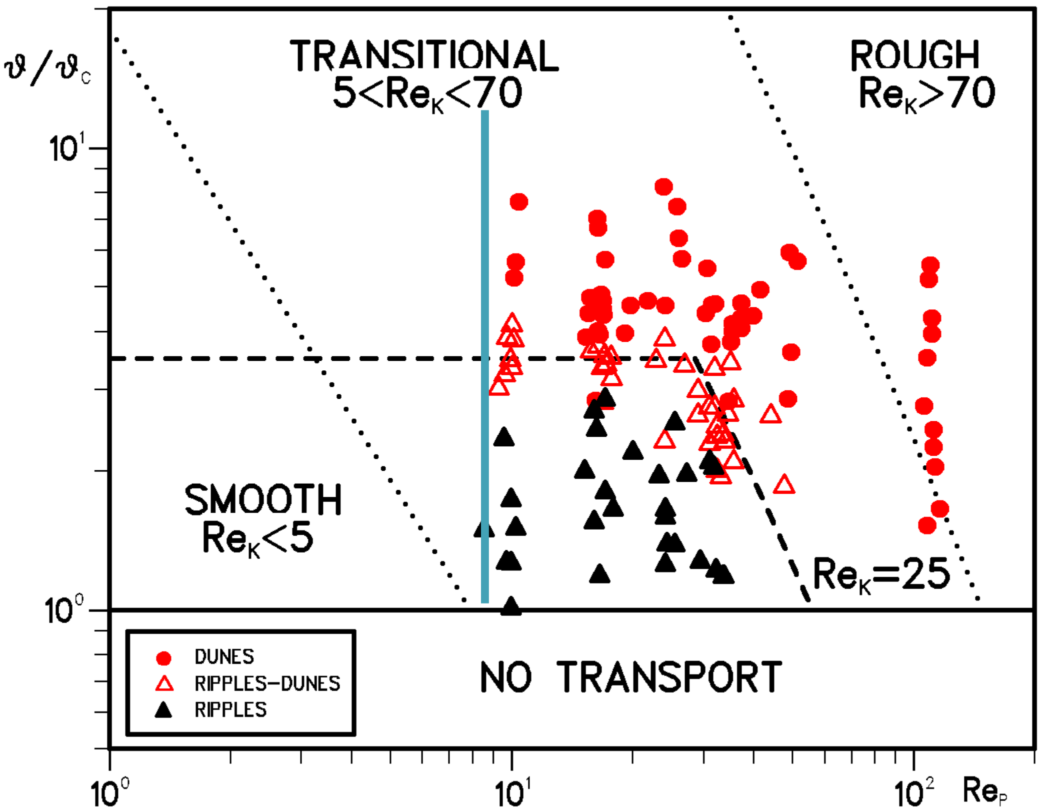

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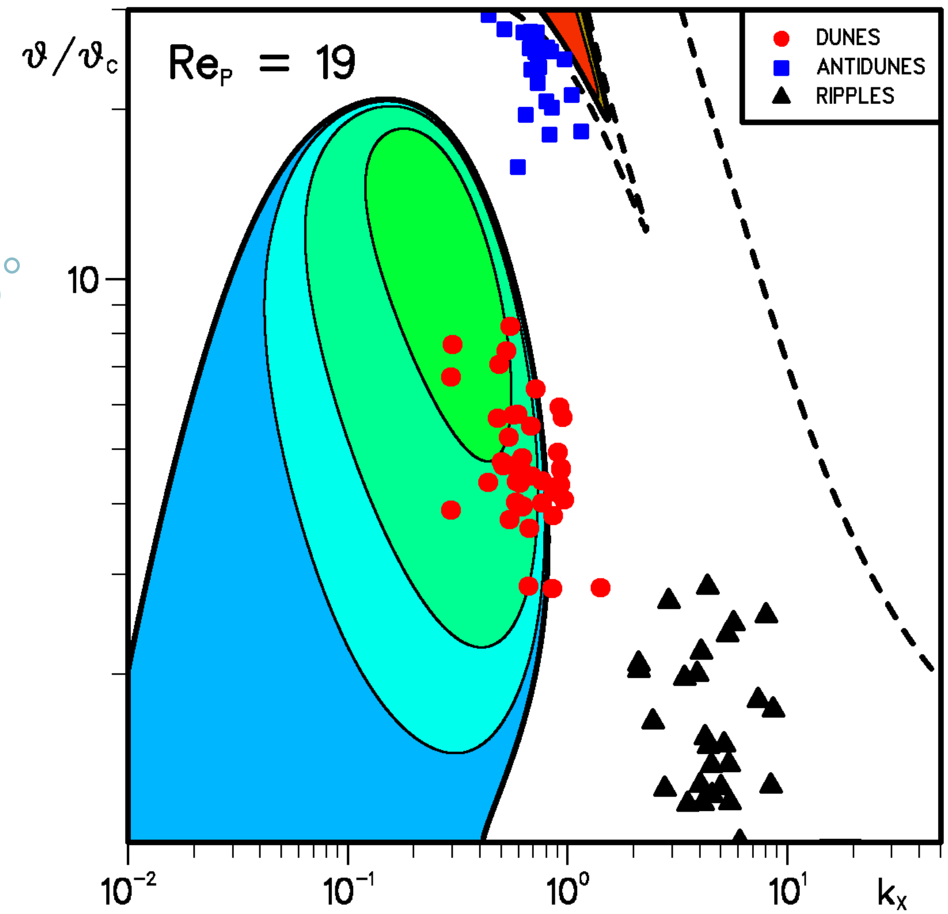
Dune-ripple transtion



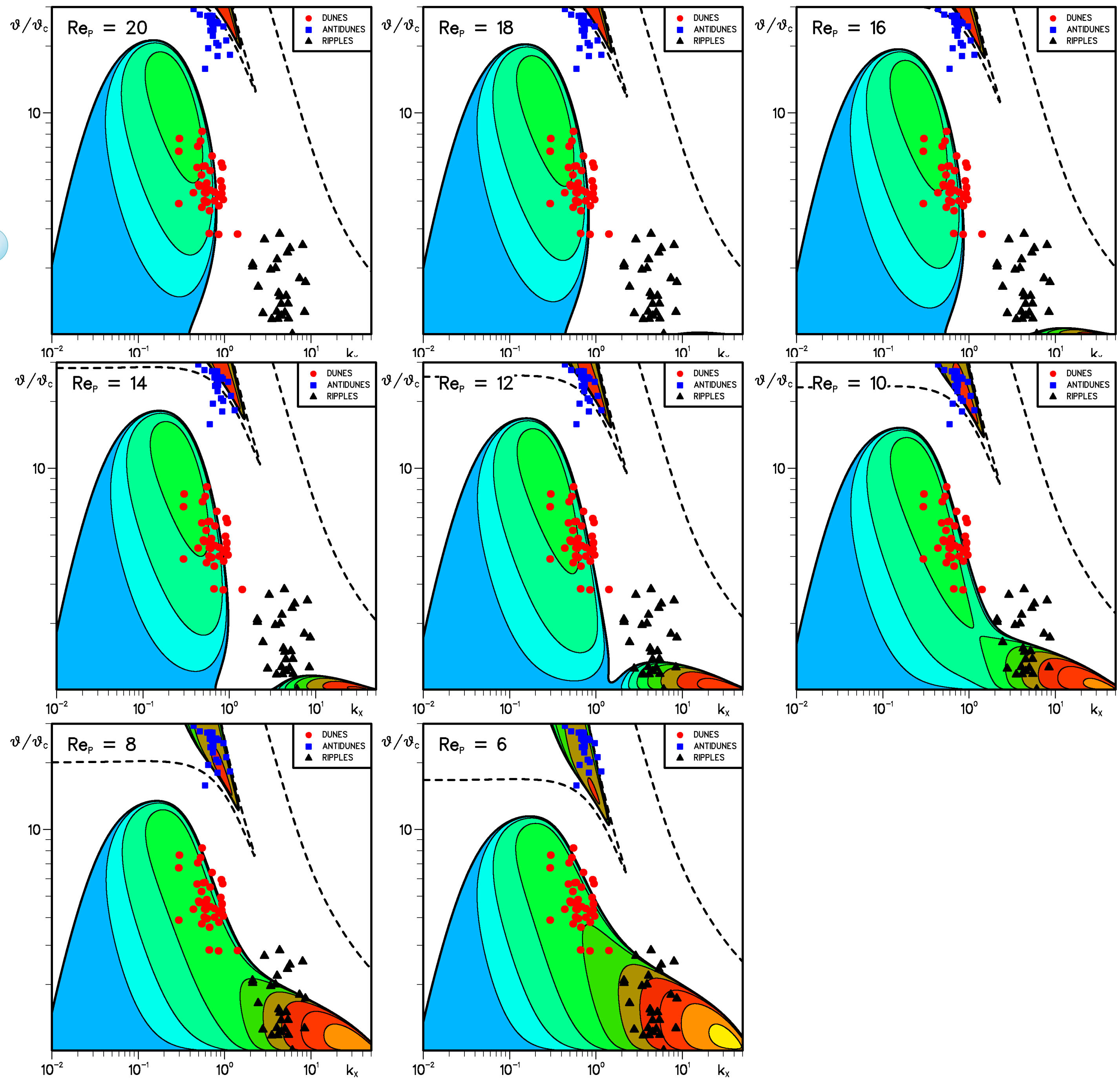
$$Re_p = \frac{\sqrt{(s-1)gd^*}\, d^*}{\nu}$$

Re_p is the parameter that controls the transition between ripples and dunes

Re_p decreasing $\Rightarrow$ d^* decreasing $\Rightarrow$ finer sediment

d constant, d^* decreasing $\Rightarrow$ shallower flow

Dune-ripple transtion



GEOFLOWS 13 - Fluid-Mediated Particle Transport in Geophysical Flows, Sept. 23~Dec 20, 2013, Santa Barbara, California

The neverending story

- Kennedy (1963,1969) - Irrotational D A R
- Engelund (1970), Fredsøe (1974) - Slip velocity D A R
- Parker (1975) - Irrotational D A R

- Richards (1980) - Rotational D A R
- Sumer & Backioglu (1984) - Rotational D A R
- Coleman & Fenton (2000) - Irrotational D A R
- Colombini (2004) - Rotational D A R
- Fourriére, Claudin & Andreotti (2010) - Rotational D A R
- Colombini & Stocchino (2011) - Rotational D A R

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- **Colombini & Stocchino (2011) - Rotational** D A R

- **Charru, Andreotti & Claudin (2013)** - Sand ripples and dunes. Ann. Rev. Fluid Mech. D A R
- **Andreotti & Claudin (2013)** - Aeolian and subaqueous bedforms in shear flows. Phil. Trans. R. Soc. A D A R

Linear Coarsening

Marco Colombini
on behalf of
L. Ridolfi, C. Camporeale, R. Vesipa
Dipartimento di Ingegneria dell'Ambiente, del Territorio e delle Infrastrutture
Politecnico di Torino, Italy

POLITECNICO DI TORINO

The Kavli Institute for Theoretical Physics
University of California, Santa Barbara

Nonmodal analysis and river morphodynamics

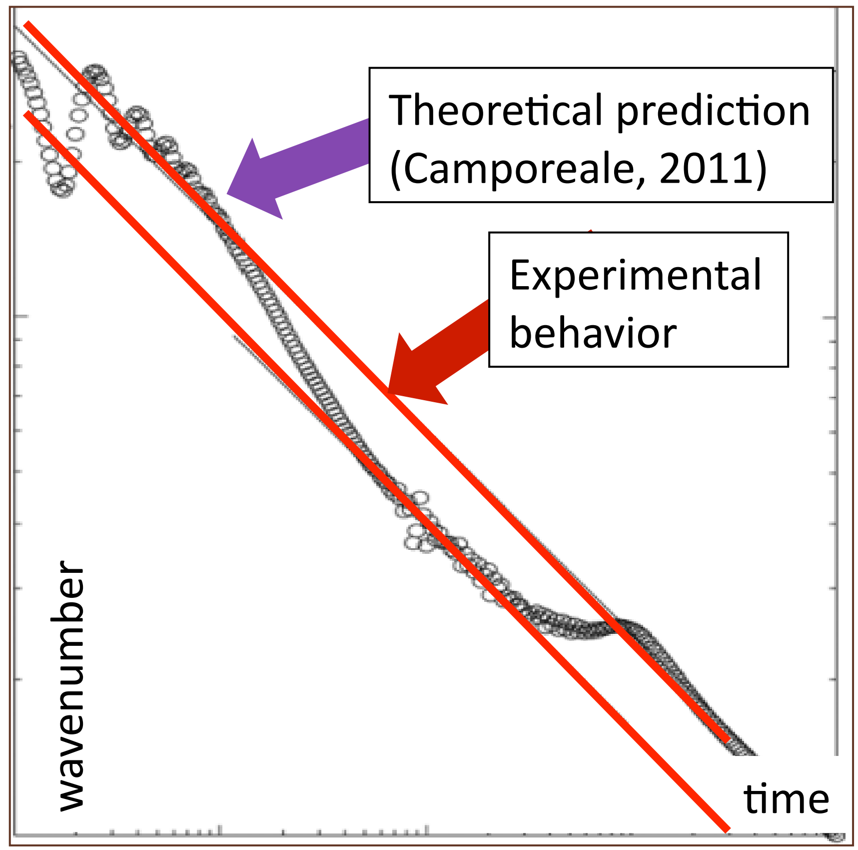
Nonmodal analysis has been proposed for different morhological systems:

- Long sediment waves and bars (Camporeale, 2011; Vesipa 2012)
- Dunes and antidunes (Camporeale, 2011)

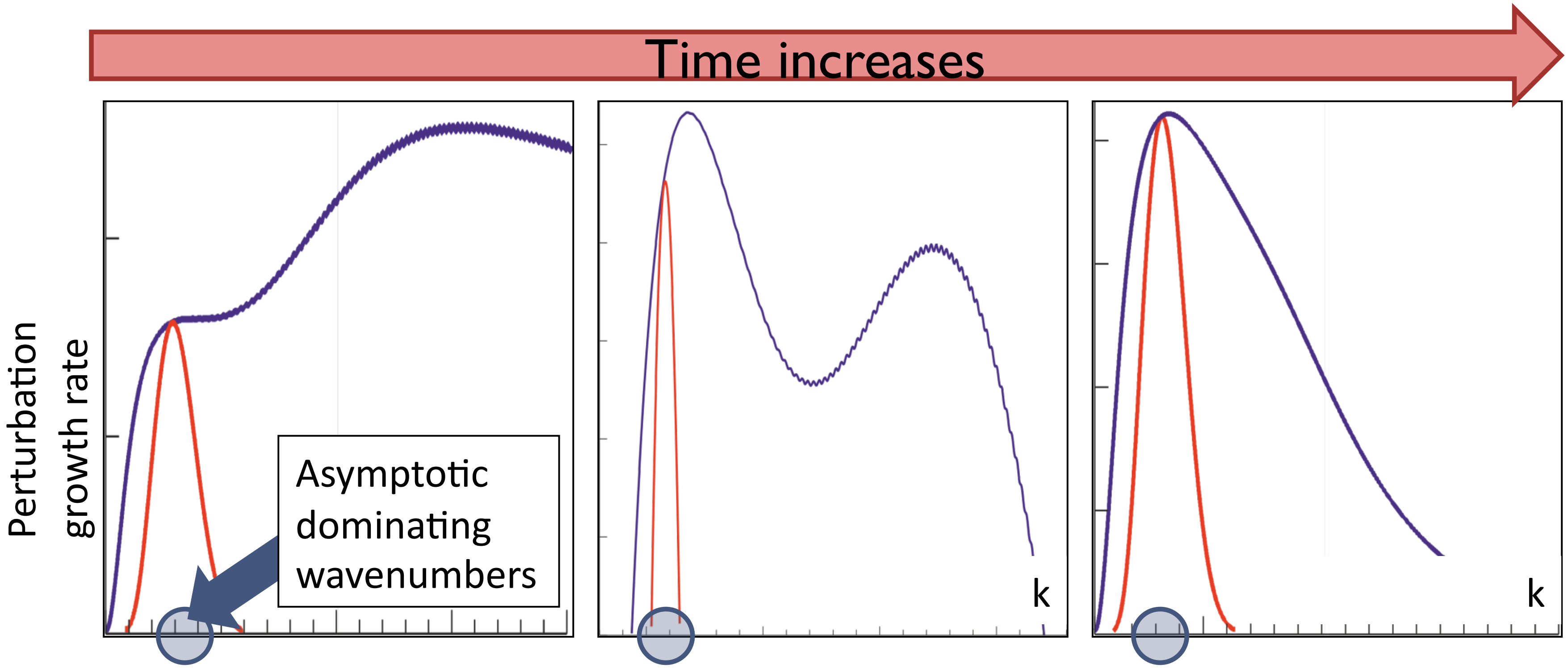
Nonmodal analysis can possibly explain a number of issues:

- Wavelength at initial times are shorter than wavelength at asymptotic times (linear transient excitation of stable modes)

- Sediment wave amalgamation is induced by linear mechanisms (excitation of modes with different phase velocity)



Non modal analysis and bed form wavelengths



At the very beginning

- short waves dominates
- the higher k, the higher the phase velocity

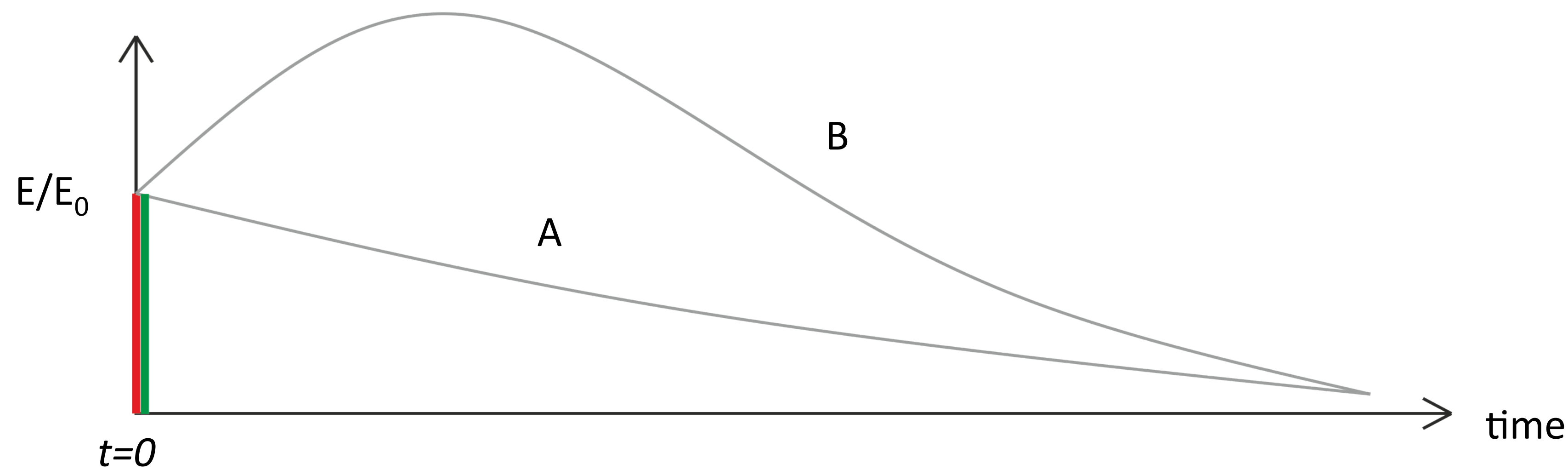
At intermediate times

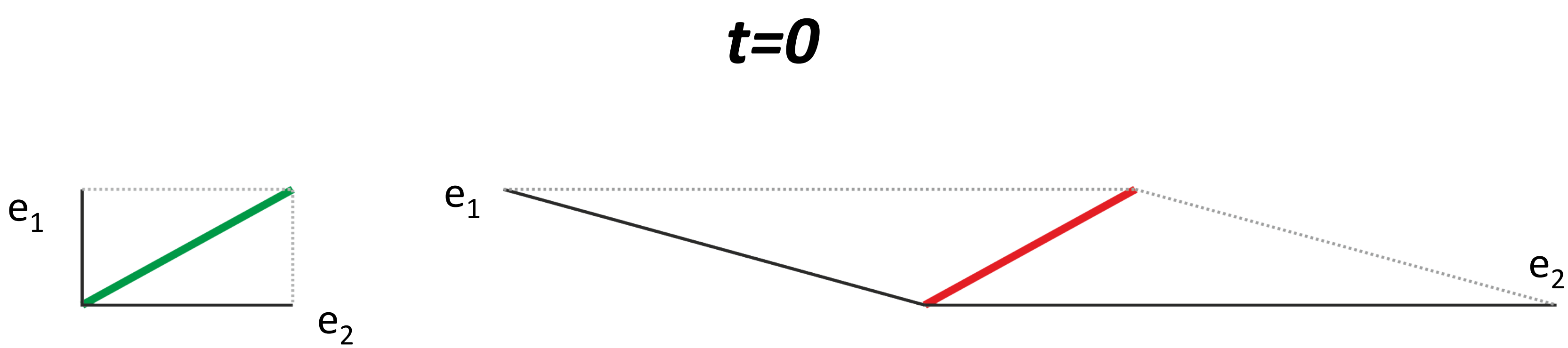
- short and asymptotic waves compete
- waves with different phase velocity can be seen
- amalgamation among waves

At asymptotic times

- long waves dominates

Energy exchanges among (stable) modes

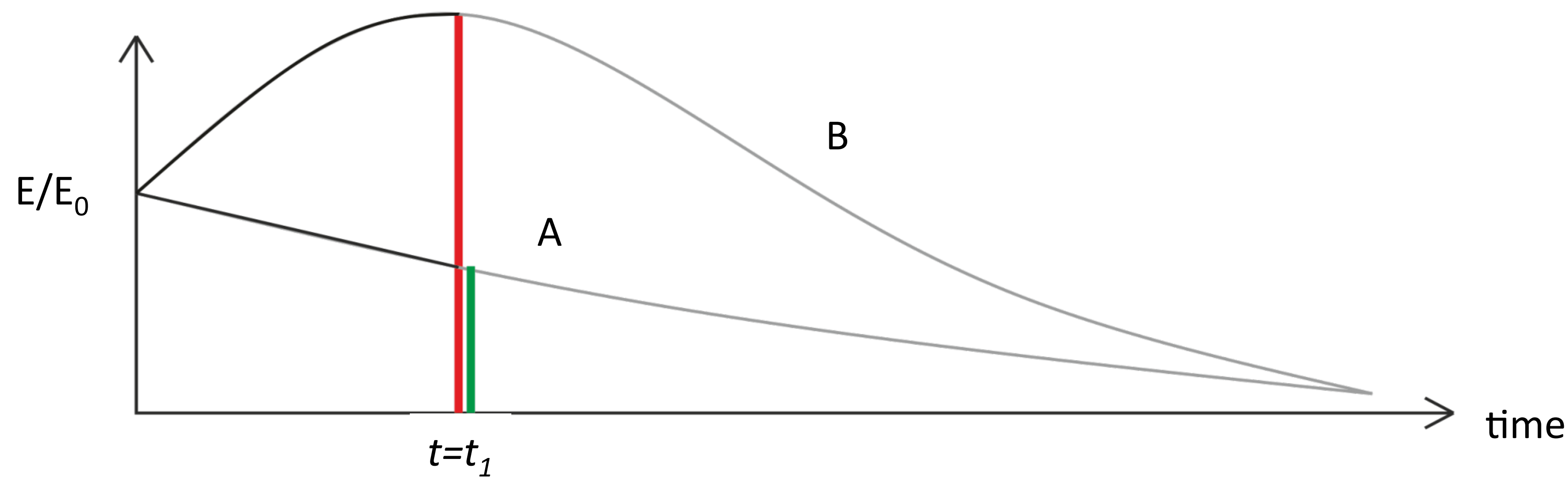




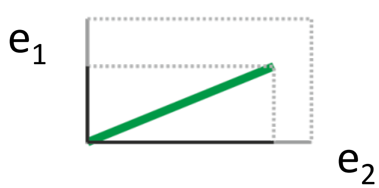
A- Eigenvectors (e_1,e_2) are orthogonal

B- Eigenvectors (e_1,e_2) are not orthogonal

Energy exchanges among (stable) modes



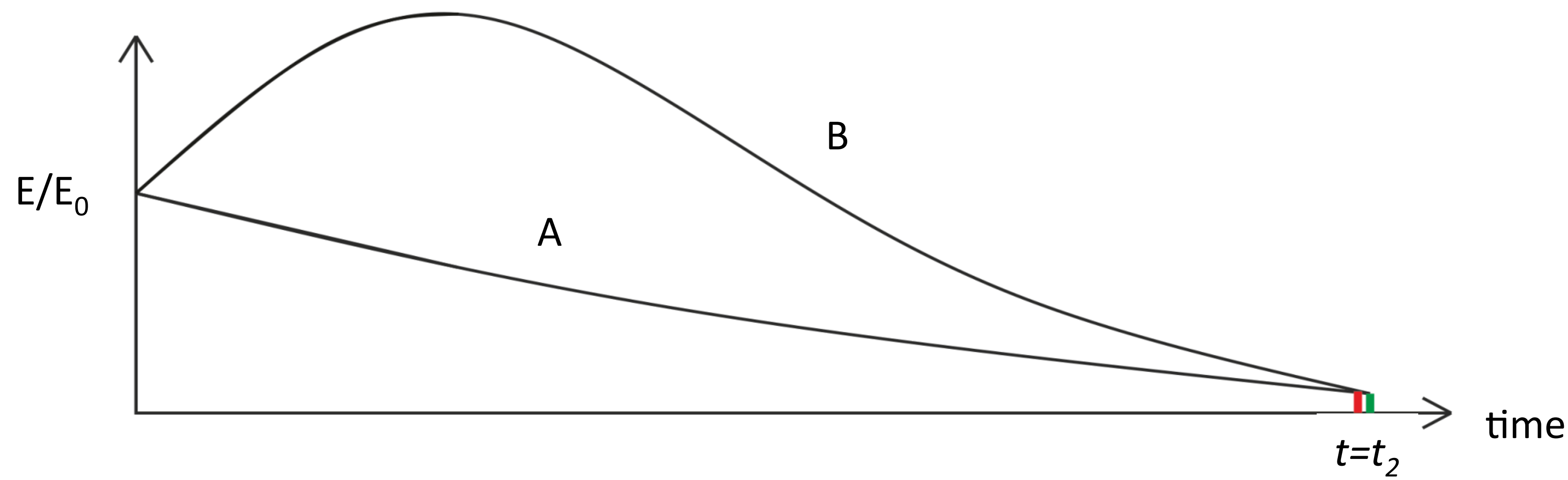
$t=t_1$



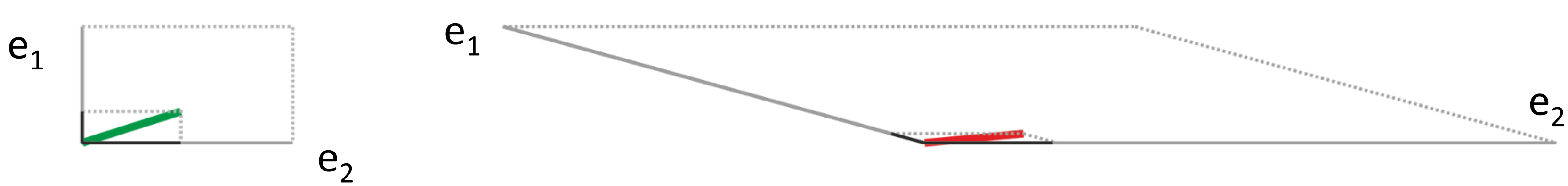
A- Each energetic components of the pert. reduces, the total energy reduces

B- Each energetic components of the pert. reduces, **but the total energy increases**

Energy exchanges among (stable) modes



$t=t_2 \approx \infty$



A- For **asymptotic time** the perturbation energy has decayed

B- After a **transient growth**, unperturbed conditions are resytored

References

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Camporeale, C. and Ridolfi, L. 2011 **Modal versus nonmodal linear stability analysis of river dunes** *Physics of Fluid*s 23(10) 104102

Schmid P.J. and Henningson D.S. 2001 *Stability and Transition in Shear Flows*, Springer

Vesipa, R. and Camporeale, C. and Ridolfi, L. 2012 **Transient growths of stable modes in riverbed dynamics** *EPL* 100(6) 64002