

# Vortices, Streaks and Coherent Structures: Lessons learned from pipe flow

Bruno Eckhardt



# Content:

- Transition in pipe flow
- Coherent structures
- Particles in pipe flow
- Particles in vortices
- Advection and Taylor's Hypothesis

# **Jim's Open Questions**

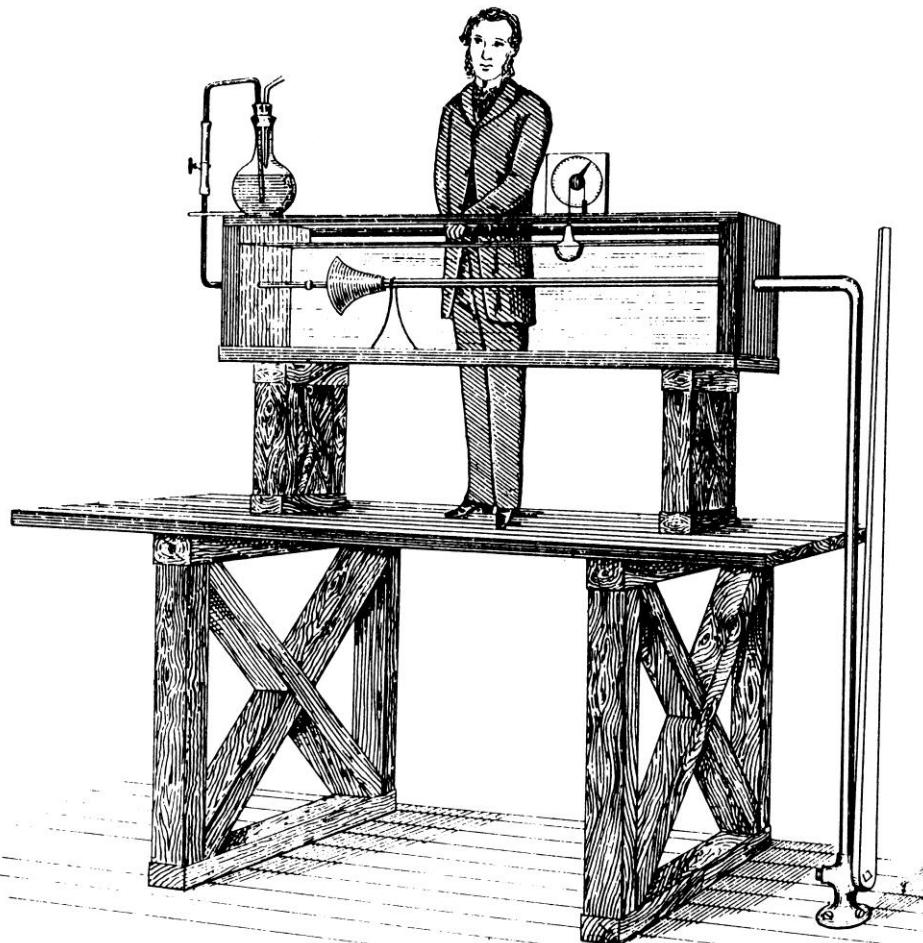
## **A) Particle/turbulence interaction**

A.I) How is a steady, uniform turbulent shear flow modified by particles in suspensions over a range of different particle/fluid density ratios and particle volume fractions?

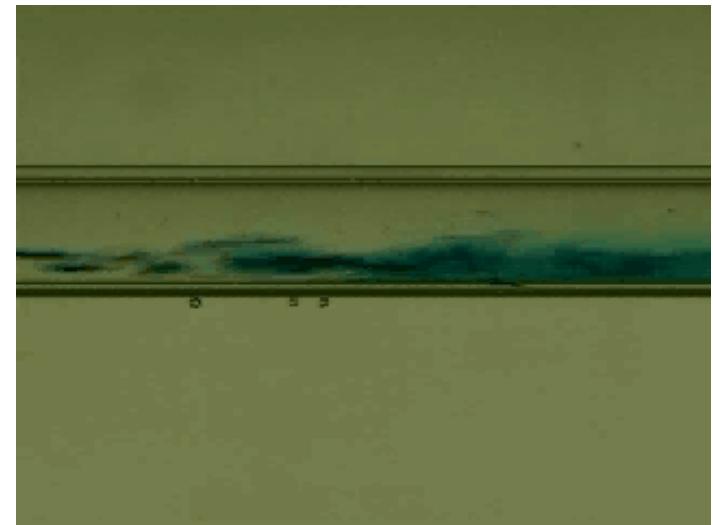
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# Reynolds' experiment (1883)



O. Reynolds, Phil Trans R Soc 1883



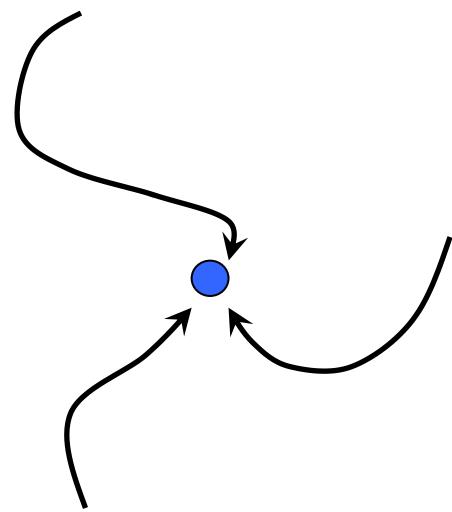
G Homsy et al, Fluid Dynamics CD

# When does pipe flow become turbulent?

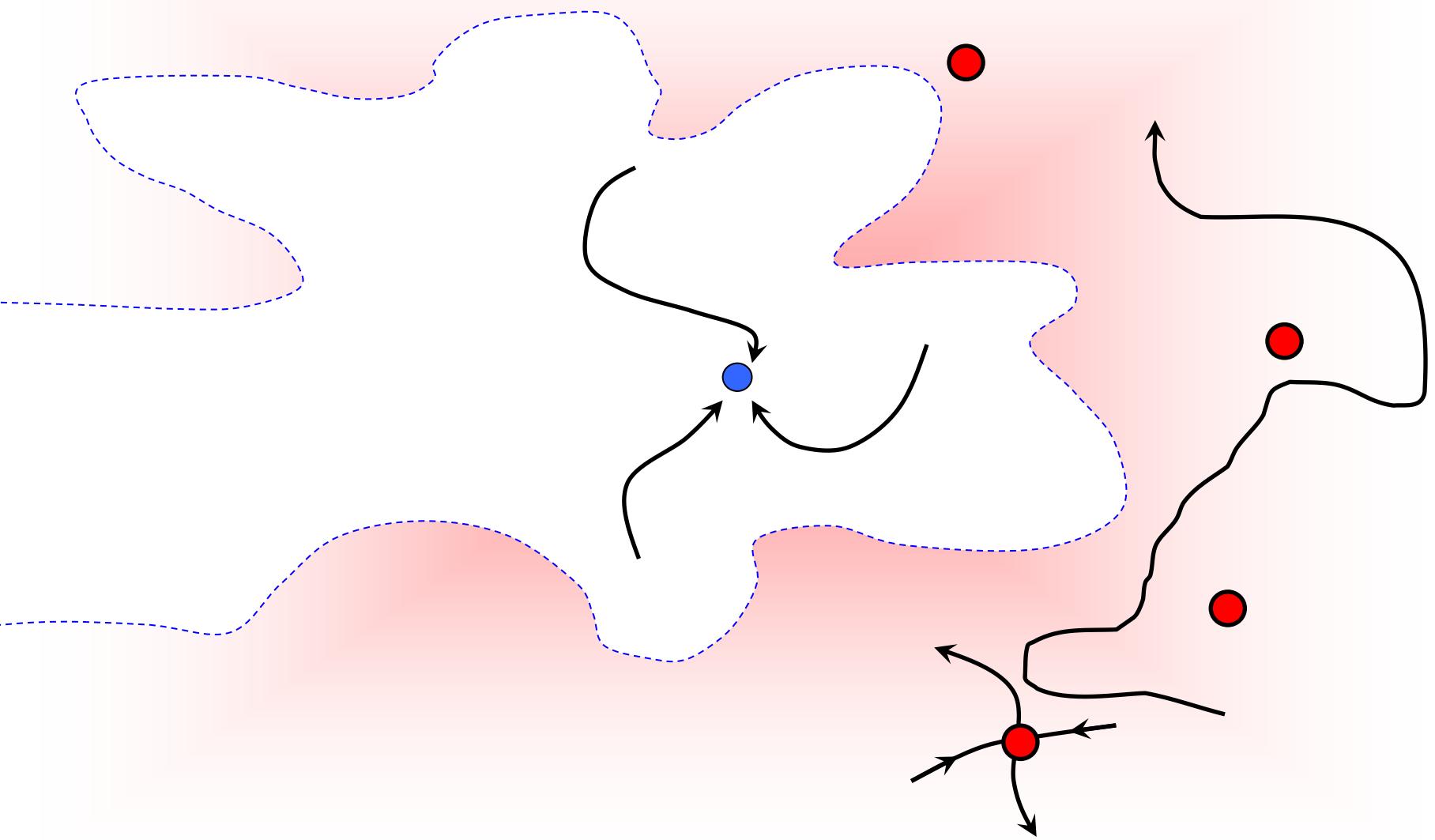
- Reynolds 1800
- Gerthsen (physics textbook) 2000
- Wikipedia 2300
- Fluid mechanics books 2000-3000
- Recent experiments (Mullin) >1650
- Stöcker (Data reference) 1000-2500
- From a lab report:  
Laminar regime  $2100 < \text{Re} < 4000$

???

# Laminar flow in state space



# Turbulent flow in state space

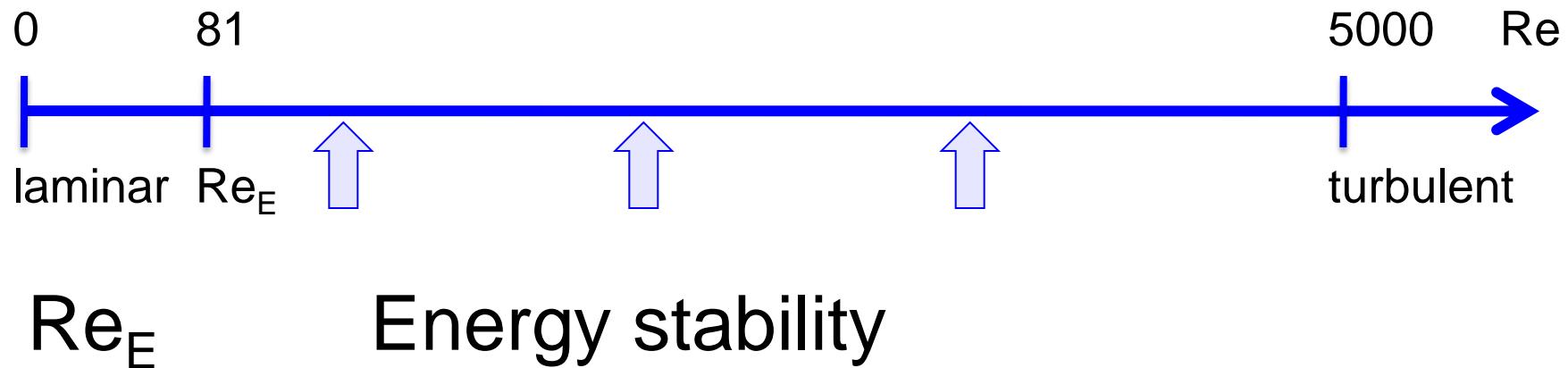


# Reynolds numbers for pipe flow

(not to scale)



# Reynolds numbers for pipe flow



# Reynolds numbers for pipe flow



$Re_E$

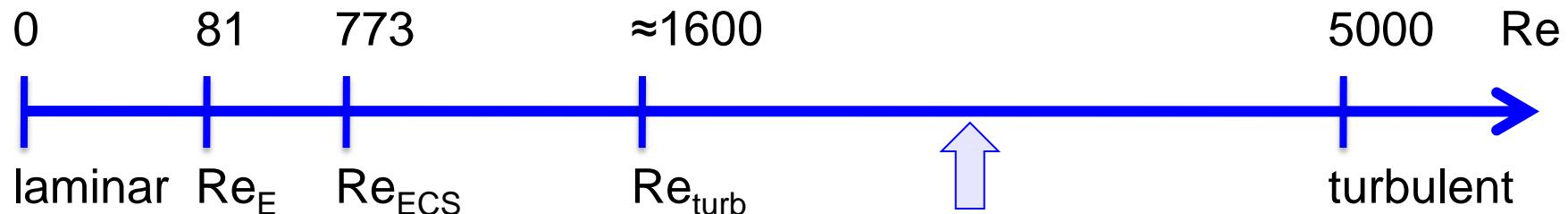
$Re_{ECS}$

Energy stability

Coherent structures appear



# Reynolds numbers for pipe flow



$Re_E$

Energy stability

$Re_{ECS}$

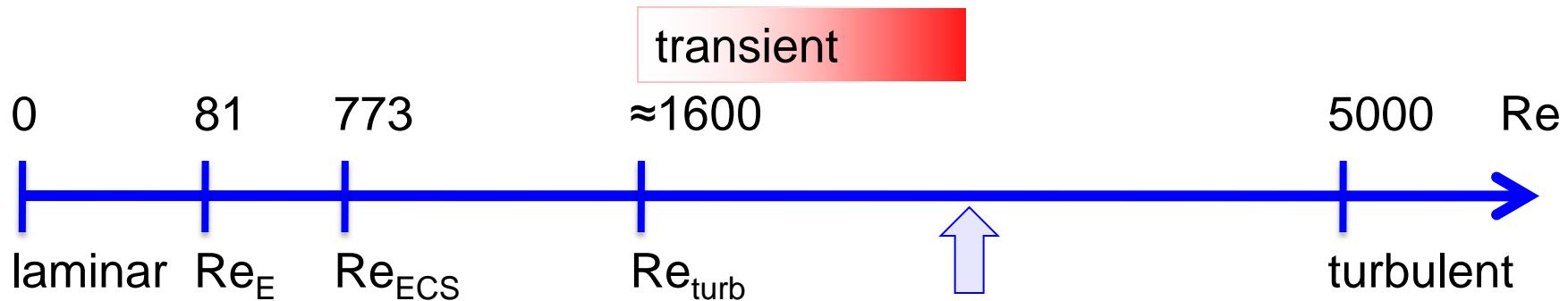
Coherent structures appear

$Re_{turb}$

Turbulence appears in experiments



# Reynolds numbers for pipe flow



$Re_E$

$Re_{ECS}$

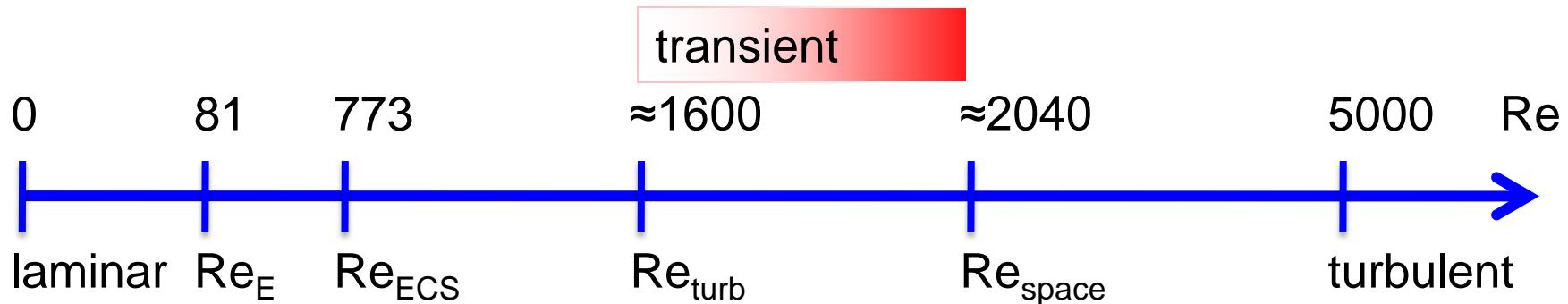
$Re_{turb}$

Energy stability

Coherent structures appear

Turbulence appears in experiments

# Reynolds numbers for pipe flow



$Re_E$

Energy stability

$Re_{ECS}$

Coherent structures appear

$Re_{turb}$

Turbulence appears in experiments

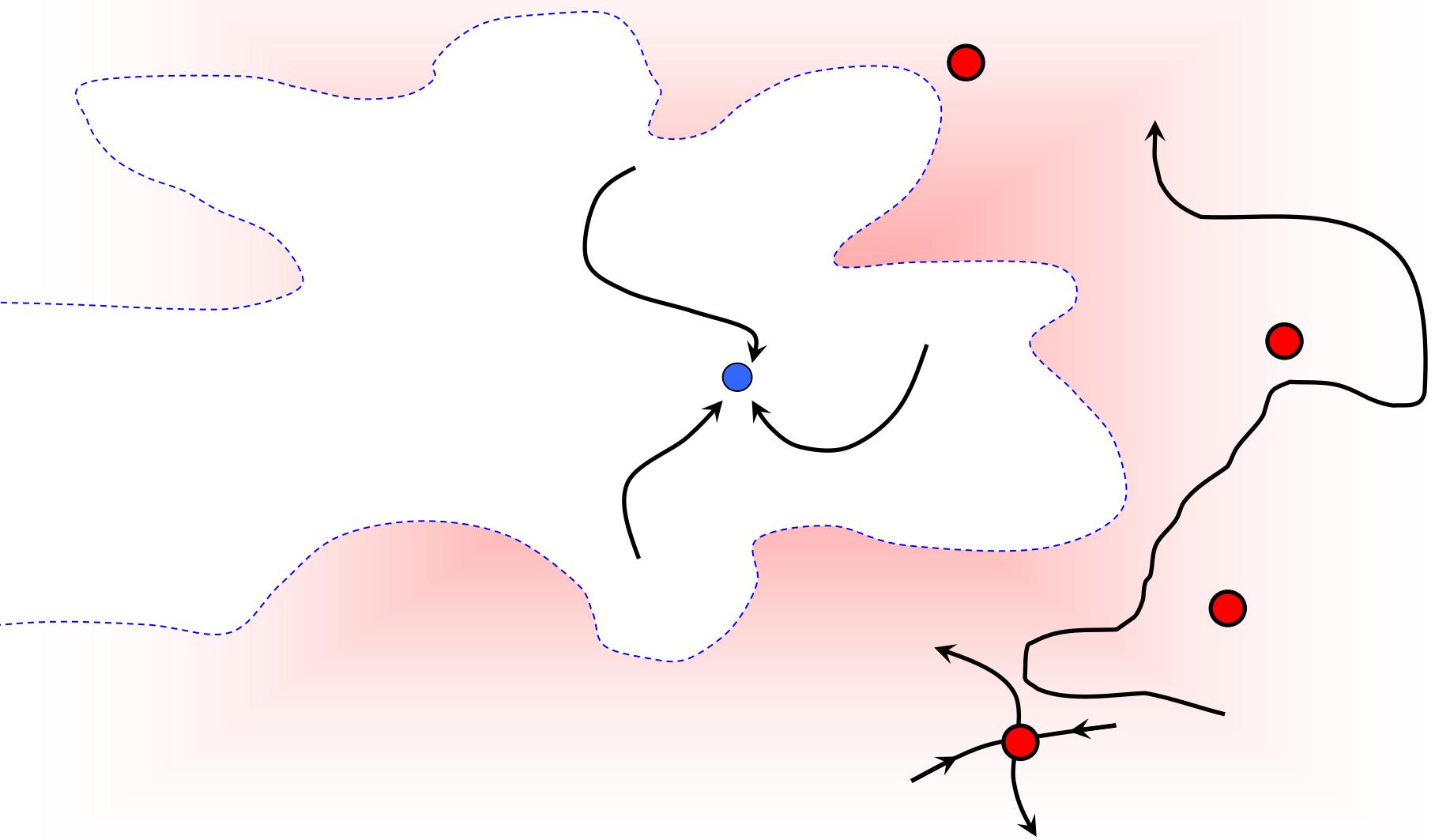
$Re_{space}$

Turbulence becomes spacefilling

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# Turbulent flow in state space



# Non-normal amplification

- Linearized Navier-Stokes equation

$$\partial_t \vec{u} + (\vec{u}_0 \cdot \vec{\nabla}) \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u}_0 = -\vec{\nabla} p + \nu \Delta \vec{u}$$

or:

$$\partial_t \vec{u} = L \vec{u}$$

- $L$  is not self-adjoint and not normal:

$$\boxed{L \neq L^+ \quad LL^+ \neq L^+L}$$

# Non-normal operators

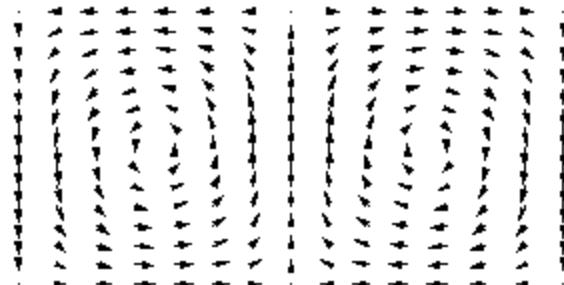
- Eigenvalues need not be real
- Eigenvectors are not orthogonal
- Left and right eigenvectors have to be distinguished
- The representation of small perturbations may need huge components along eigenvectors

# Vortex-Streak interaction

- Downstream vortices create spanwise streaks:

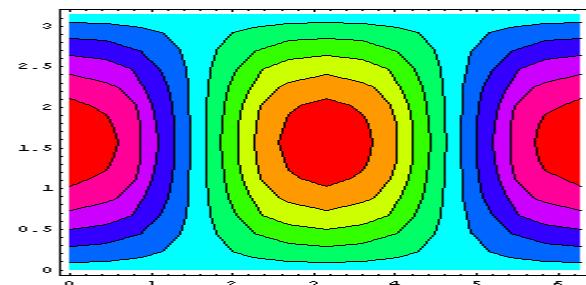
$$\begin{matrix} \ddot{x} & 0 & \ddot{\theta} \\ \ddot{y} & \div & \div \\ \ddot{z} & b \sin(\alpha y) \sin(\beta z) & \div \\ \theta & -a \cos(\alpha y) \cos(\beta z) & \div \end{matrix}$$

Vortex



$$\begin{pmatrix} -\beta \cos(\alpha y) \sin(\beta z) \\ 0 \\ 0 \end{pmatrix}$$

Streak



# Vortex-Streak interaction

Downstream vortex (amplitude  $\omega$ )  
amplifies spanwise streak (amplitude  $s$ )

$$\partial_t \begin{pmatrix} s \\ \omega \end{pmatrix} = \begin{pmatrix} -1/\text{Re} & S \\ 0 & -1/\text{Re} \end{pmatrix} \begin{pmatrix} s \\ \omega \end{pmatrix}$$

$$s(t) = (s_0 + S\omega_0 t)e^{-t/\text{Re}}$$

$$\omega(t) = \omega_0 e^{-t/\text{Re}}$$

transient  
linear growth  
 $\propto \text{Re}$  !

# Waleffes turbulent cycle

- Downstream vortices induce streaks by non-normal amplification
- Streaks undergo shear flow instability forming vortices in normal direction
- Normal vortices are rotated in downstream direction by background flow

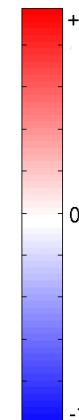
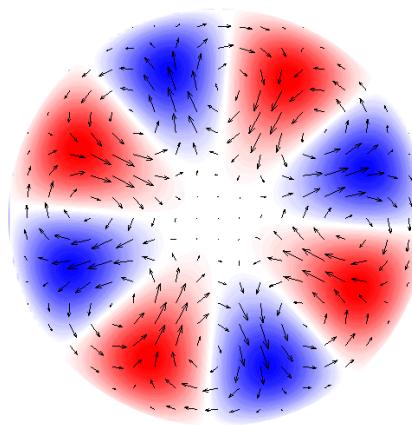
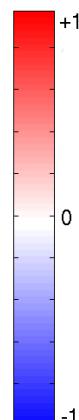
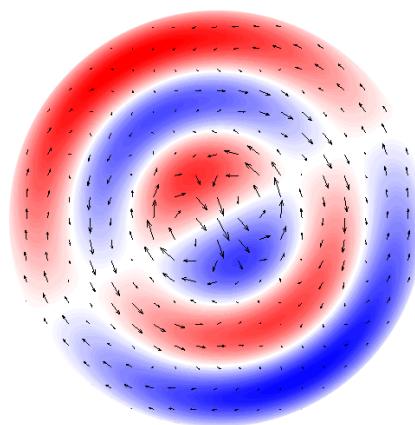
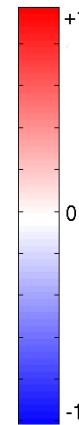
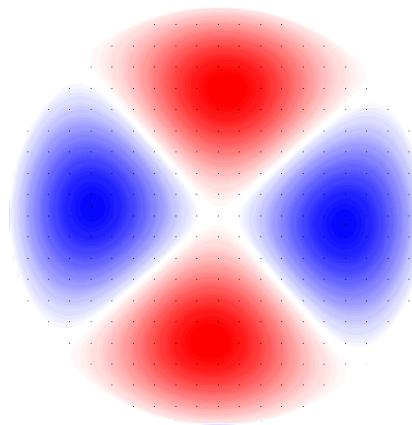
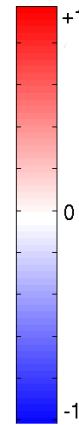
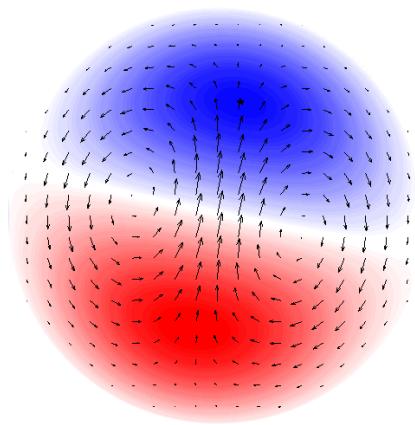
# A variation on Waleffes turbulent cycle:

- Downstream vortices induce streaks by non-normal amplification
- Streaks undergo shear flow instability forming vortices in normal direction
- During this instability, new, symmetry broken states appear that persist

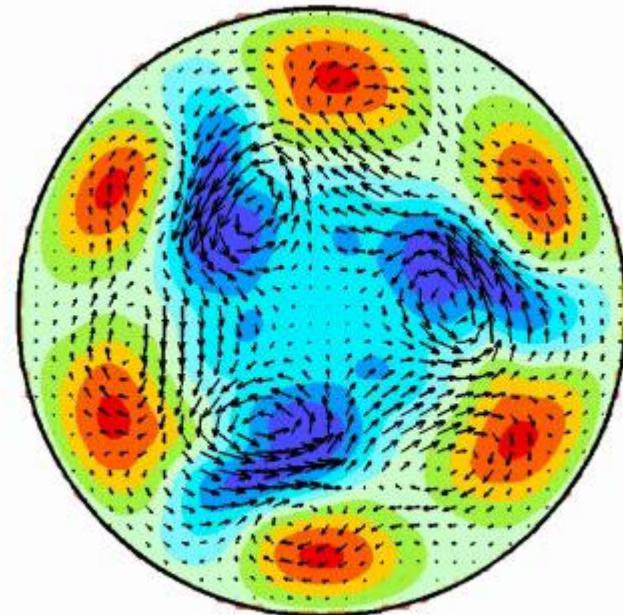
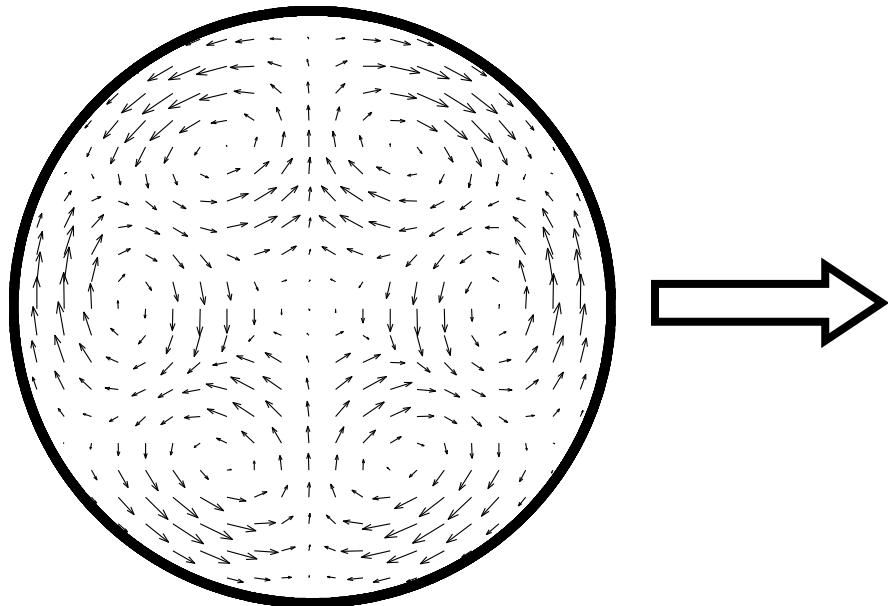
Vortices dominate dynamics

# Linear stability analysis

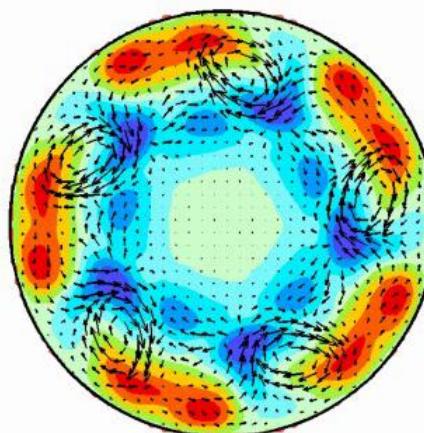
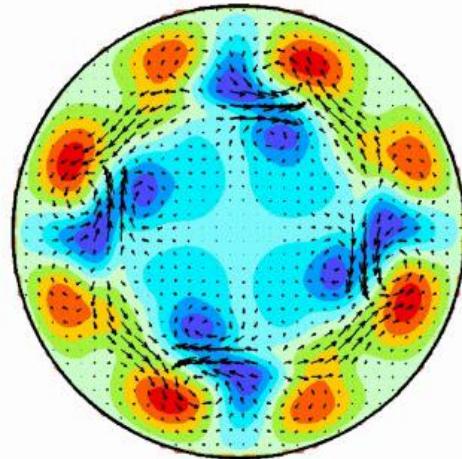
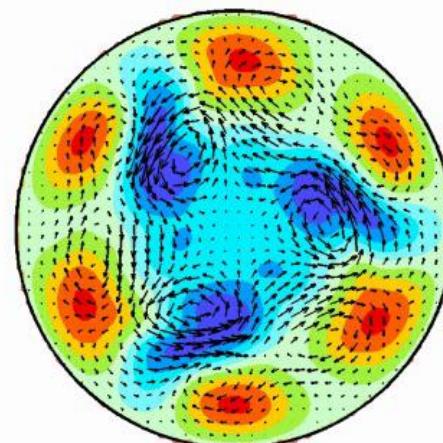
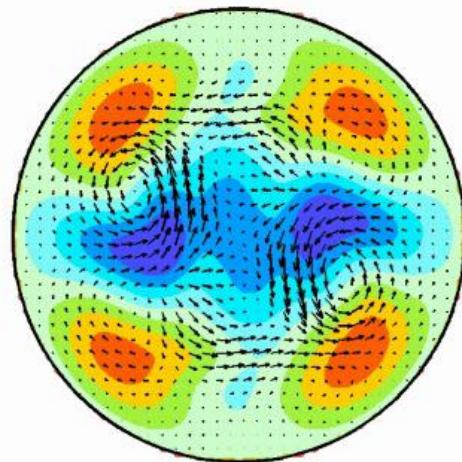
- HP-profile linearly stable for all  $Re$
- eigenmodes show vortex-streak structure



# Travelling wave with threefold symmetry

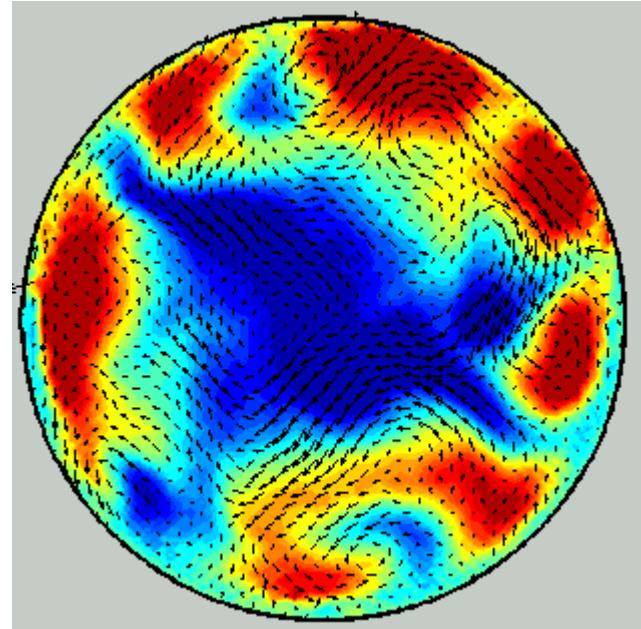
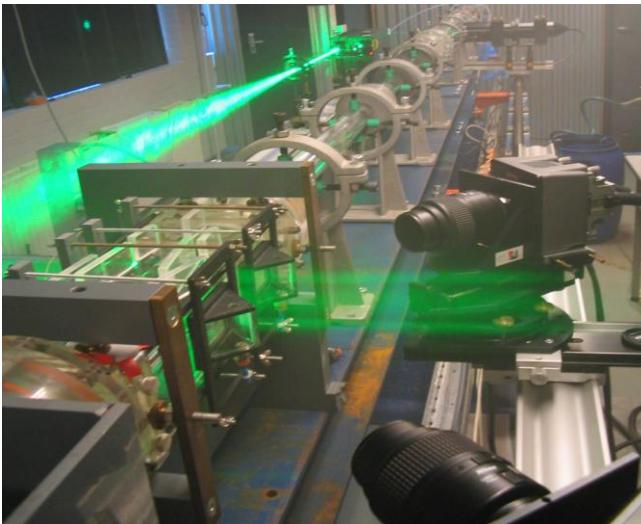
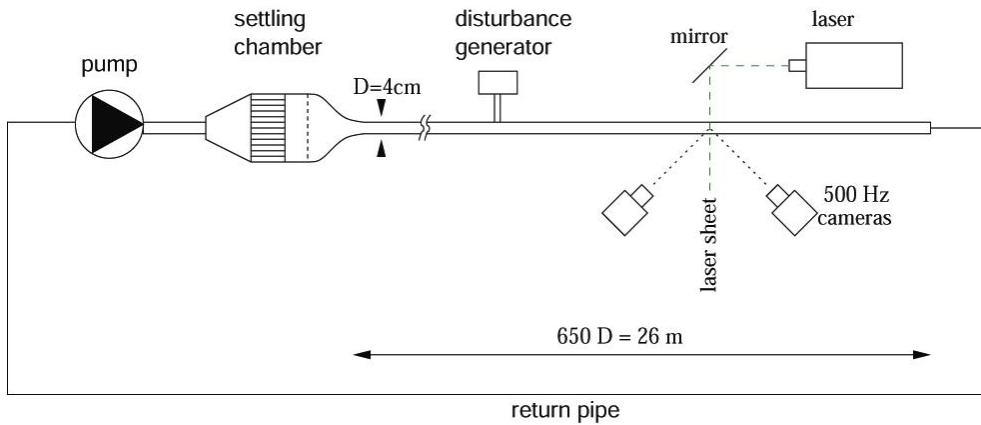


# Travelling waves in pipe flow



PRL 91 (2003)  
224502

# Dynamics in cross sections



Cas van Doorne  
Björn Hof  
(Delft)

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# Particulate pipe flow

VOLUME 90, NUMBER 1

PHYSICAL REVIEW LETTERS

week ending  
10 JANUARY 2003

## Transition to Turbulence in Particulate Pipe Flow

J.-P. Matas,<sup>1</sup> J. F. Morris,<sup>2</sup> and É. Guazzelli<sup>1</sup>

<sup>1</sup>*IUSTI-CNRS UMR 6595, Polytech'Marseille, Technopôle de Château-Gombert, 13453 Marseille Cedex 13, France*

<sup>2</sup>*School of Chemical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332*

(Received 19 December 2001; published 10 January 2003)

We investigate experimentally the influence of suspended particles on the transition to turbulence. The particles are monodisperse and neutrally buoyant with the liquid. The role of the particles on the transition depends upon both the pipe to particle diameter ratios and the concentration. For large pipe-to-particle diameter ratios the transition is delayed while it is lowered for small ratios. A scaling is proposed to collapse the departure from the critical Reynolds number for pure fluid as a function of concentration into a single master curve.



Universität  
Marburg

Philipps

J.-P. Matas,  
J.F. Morris  
E. Guazelli  
PRL **90** (2003)  
014501

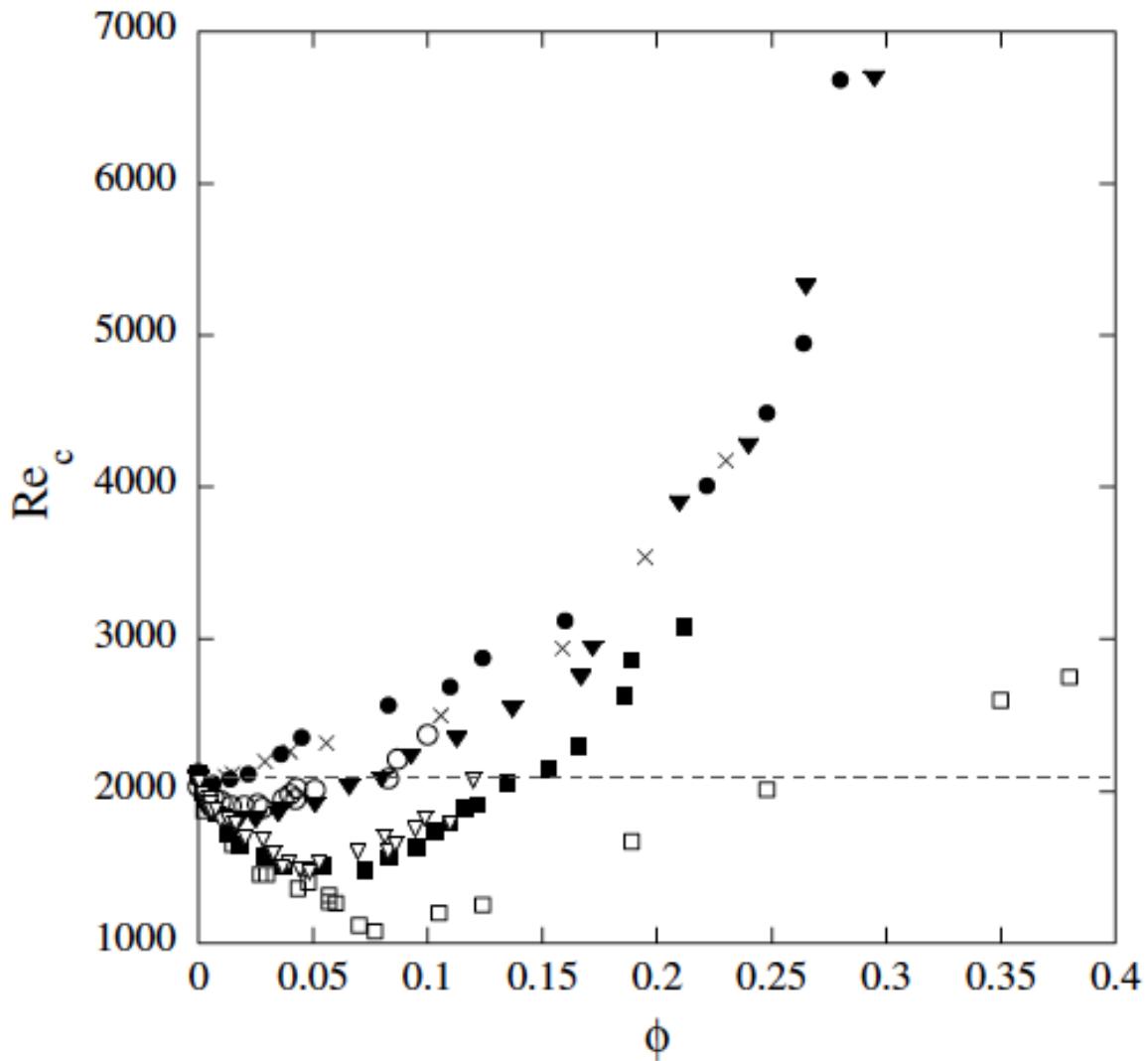


FIG. 2. Critical Reynolds number  $Re_c$  as a function of the volume fraction  $\phi$  of the suspension for different combinations of particles and tubes. Tube  $D_1$  with  $d = 215\text{ }\mu\text{m}$  ( $\circ$ ),  $510\text{ }\mu\text{m}$  ( $\nabla$ ),  $780\text{ }\mu\text{m}$  ( $\square$ ). Tube  $D_2$  with  $d = 40\text{ }\mu\text{m}$  ( $\times$ ),  $215\text{ }\mu\text{m}$  ( $\bullet$ ),  $510\text{ }\mu\text{m}$  ( $\blacktriangledown$ ),  $780\text{ }\mu\text{m}$  ( $\blacksquare$ ).

J.-P. Matas,  
J.F. Morris  
E. Guazelli  
PRL **90** (2003)  
014501

Using:

$$\frac{\mu_e}{\mu} = (1 - \phi/\phi_m)^{-1.82},$$

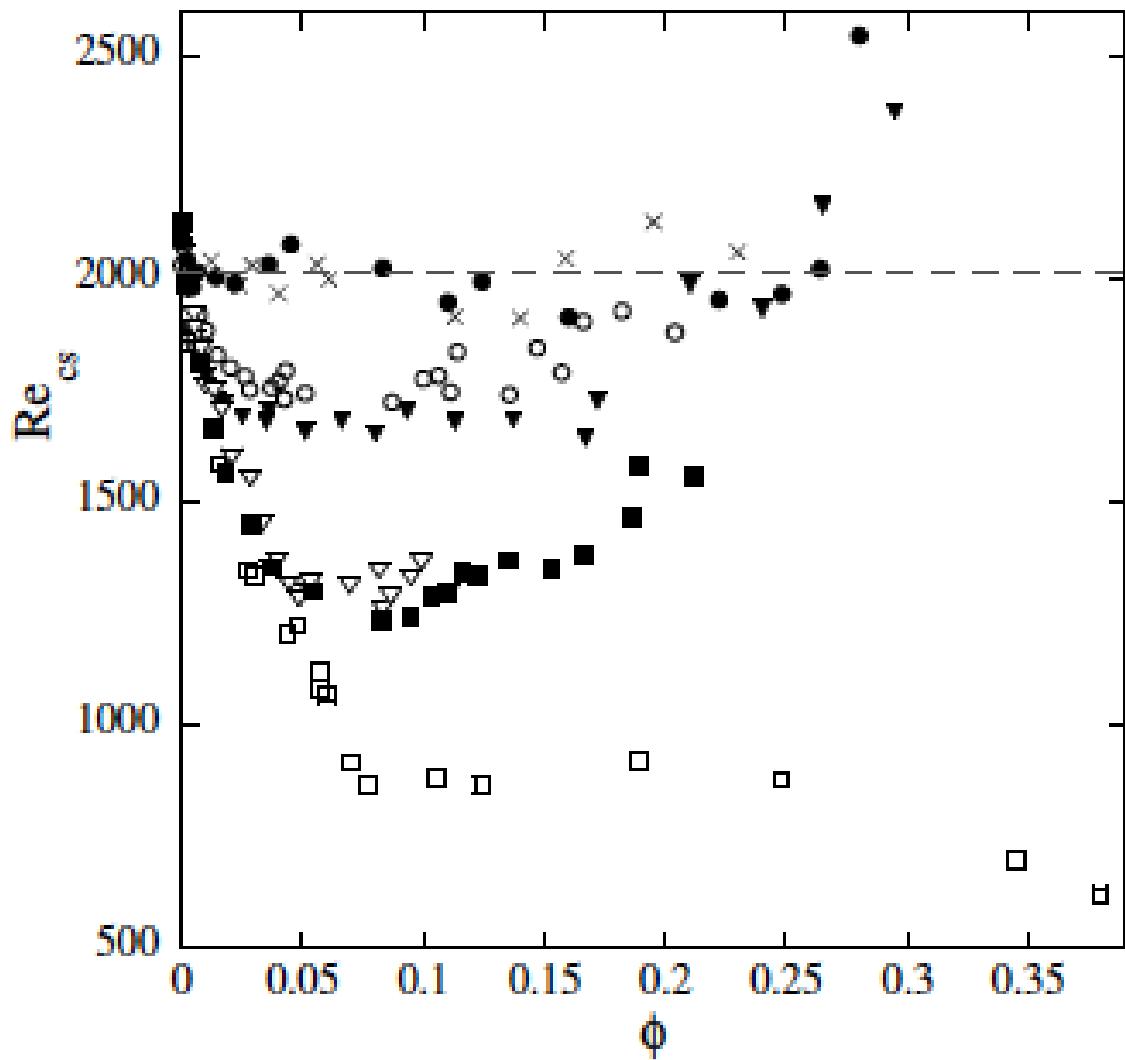


FIG. 3. Critical Reynolds number of the suspension using Krieger's viscosity  $\text{Re}_{ss} = \text{Re}_c \mu_0 / \mu_e$  as a function of particle volume fraction  $\phi$ . Tube  $D_1$  with  $d = 215 \mu\text{m}$  ( $\circ$ ),  $510 \mu\text{m}$  ( $\nabla$ ),  $780 \mu\text{m}$  ( $\square$ ). Tube  $D_2$  with  $d = 40 \mu\text{m}$  ( $\times$ ),  $215 \mu\text{m}$  ( $\bullet$ ),  $510 \mu\text{m}$  ( $\blacktriangledown$ ),  $780 \mu\text{m}$  ( $\blacksquare$ ).

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# Equations of motion (Maxey-Riley)

$$d_t x = u(x(t), t) + W$$

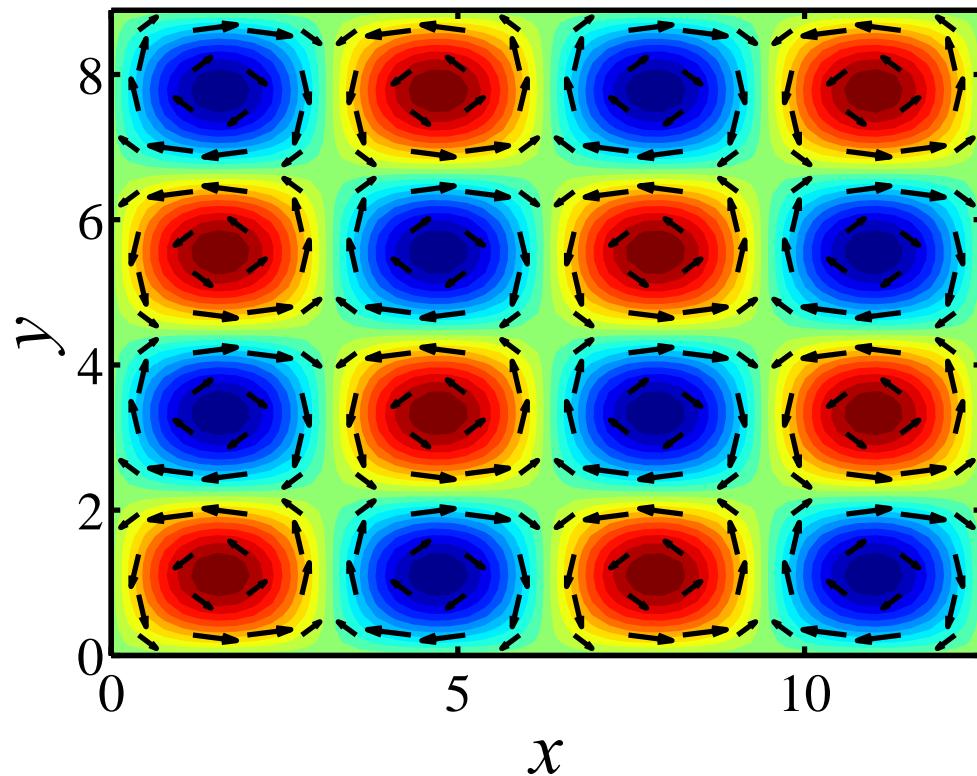
$$(m_p + m_F / 2)d_t W =$$

$$- 6\pi a \mu W - 6\pi a^2 \mu \int_{-\infty}^t \frac{dW/d\tau}{\sqrt{(\nu\pi)(t-\tau)}} d\tau +$$

$$- m_p \frac{du}{dt} + m_F \frac{Du}{Dt} + \dots$$

- Stokes 1851, Basset 1888, Oseen 1927,  
Burgers 1938, Maxey and Riley 1983

# Particles in vortical flows



# For particles in a vortical flow

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

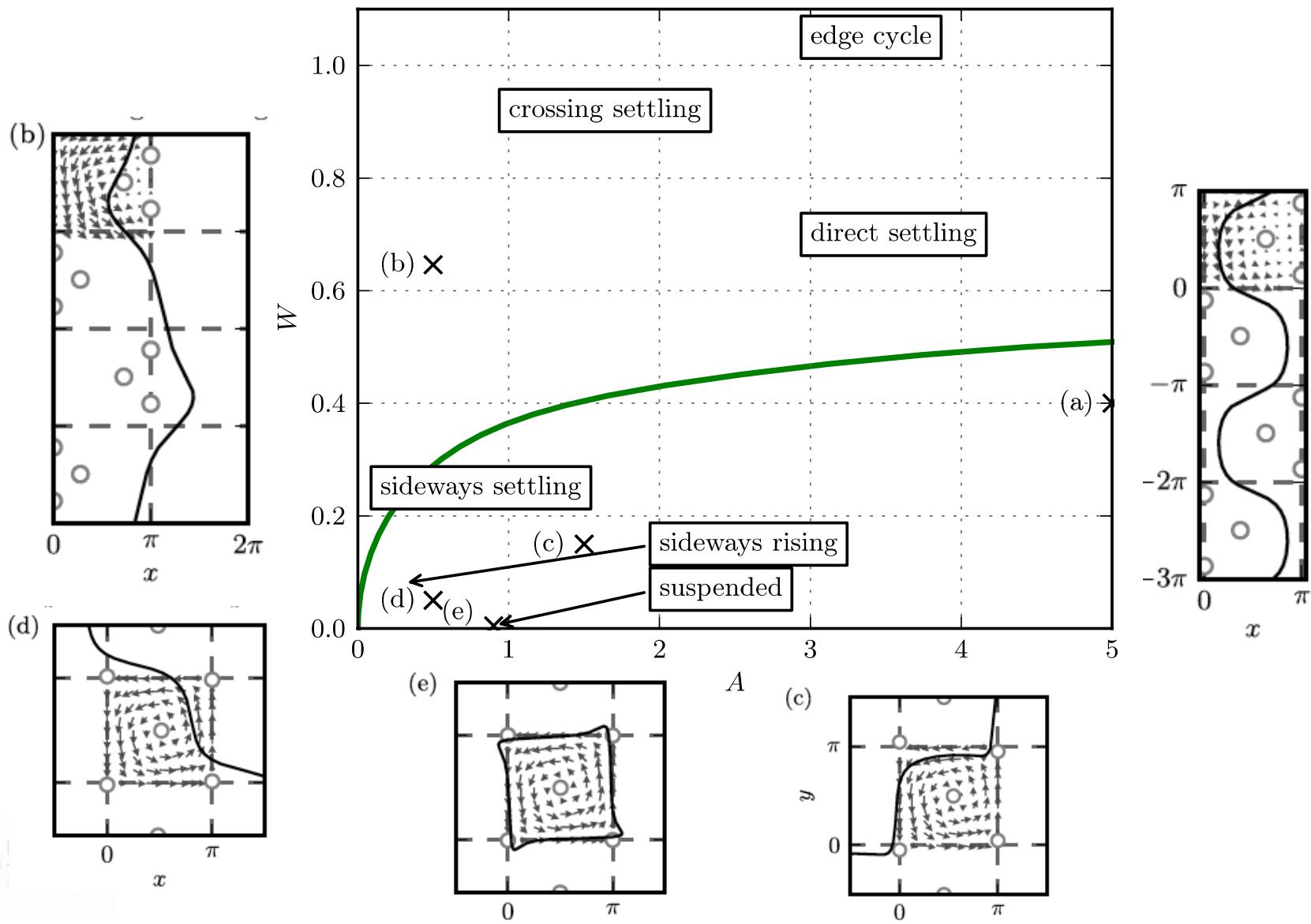
$$\dot{v}_x = A [\sin(x) \cos(y) - v_x]$$

$$+ \frac{1}{2} R [v_x \cos(x) \cos(y) - v_y \sin(x) \sin(y)] + R \sin(x) \cos(x)$$

$$\dot{v}_y = A [-\cos(x) \sin(y) - v_y - W]$$

$$+ \frac{1}{2} R [v_x \sin(x) \sin(y) - v_y \cos(x) \cos(y)] + R \sin(y) \cos(y).$$

# Phase diagram for aerosols ( $R=0$ )



# Content:

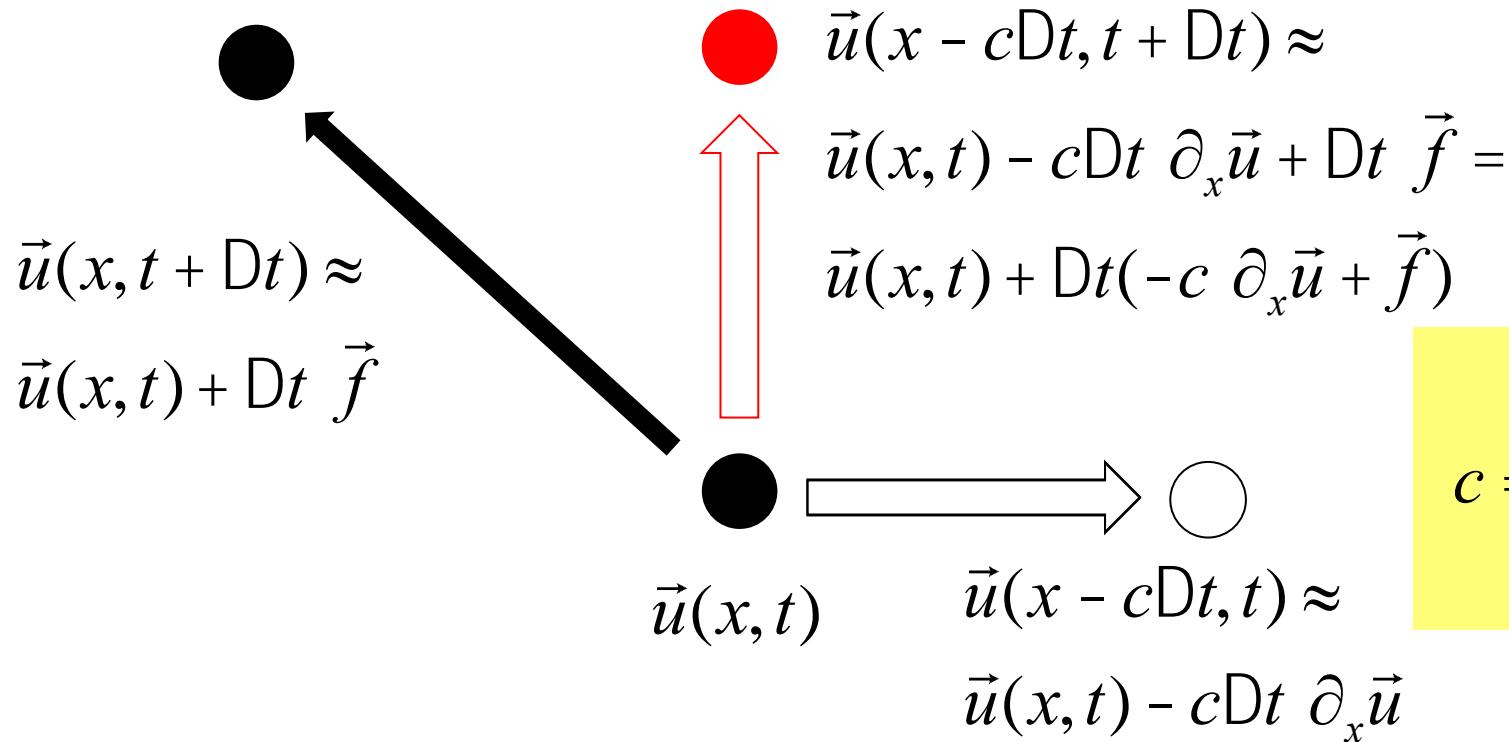
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# Consequences of continuous symmetry (for coherent states):

- Allows for travelling waves:  
states that move downstream without  
changing shape
- Combines changes in the flow field and  
translations thereof

# How to remove the translation?

$$\P_t \vec{u}(x, t) = \vec{f}(\vec{u}(x, t), t)$$

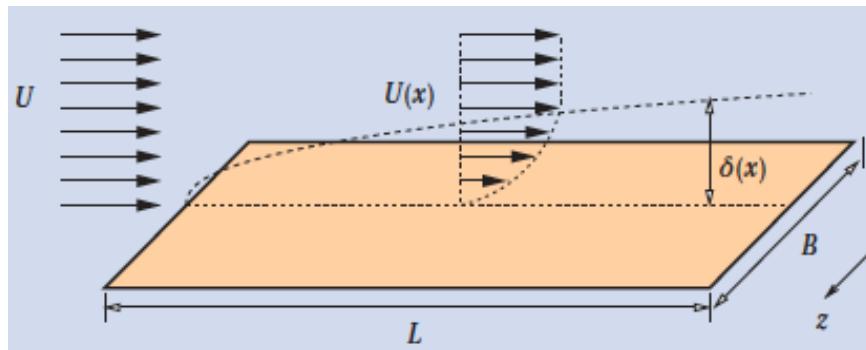


# Example:

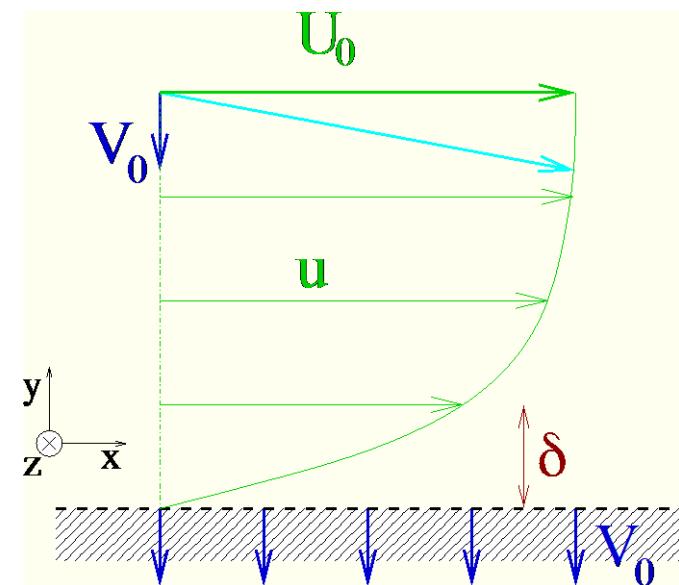
## Coherent structures in the asymptotic suction boundary layer

# Blasius vs. Asymptotic Suction Boundary Layer

## Blasius



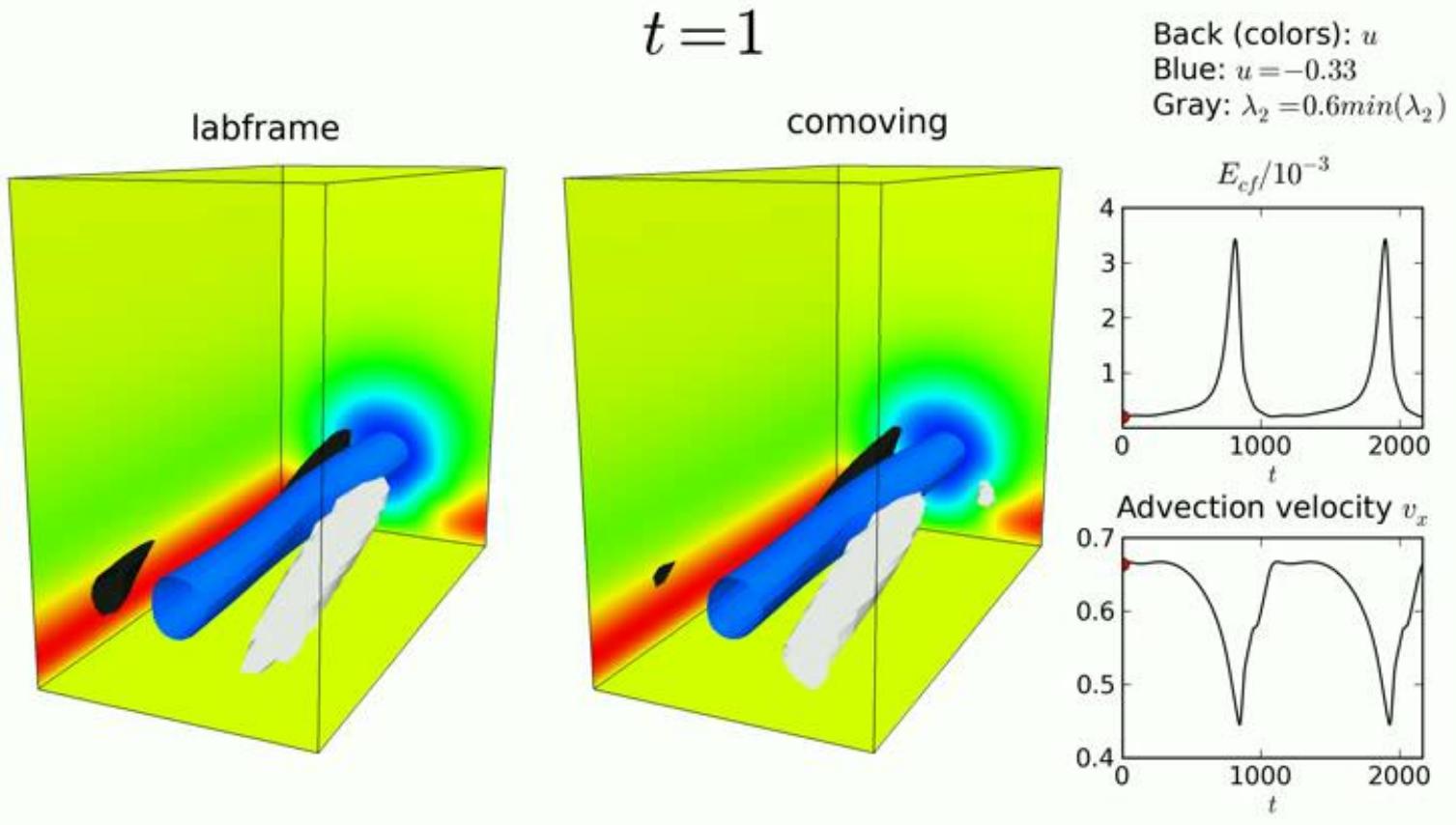
## ASBL



$$\delta(x) = \sqrt{XV / U}$$

$$d = V_0 / U_0$$

# Vortex dynamics



Kreilos and BE, arxiv 1209.0593, JFM **726**, 100 (2013)

# Some Open Questions

## A) Particle/turbulence interaction

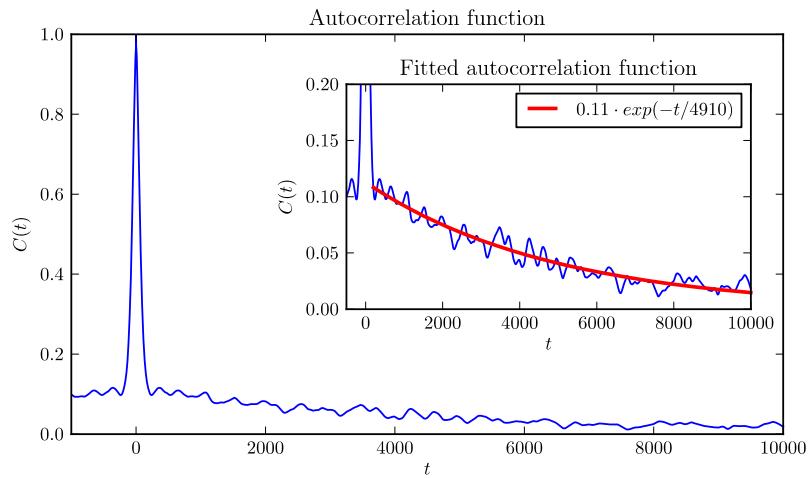
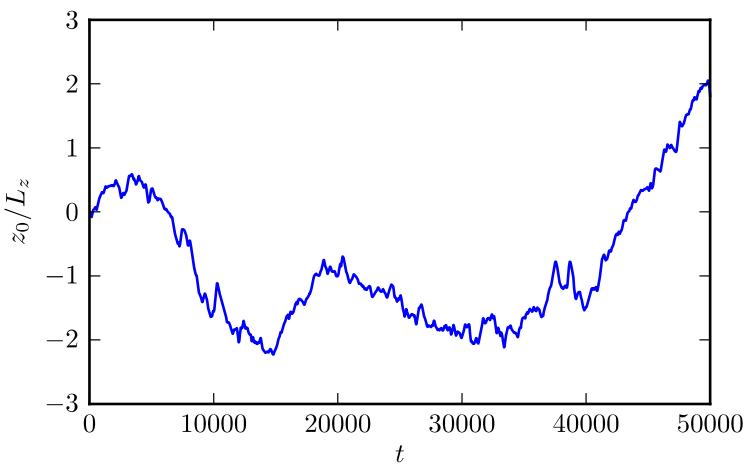
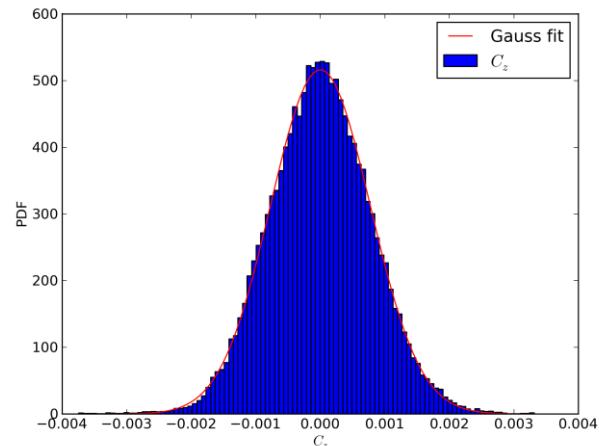
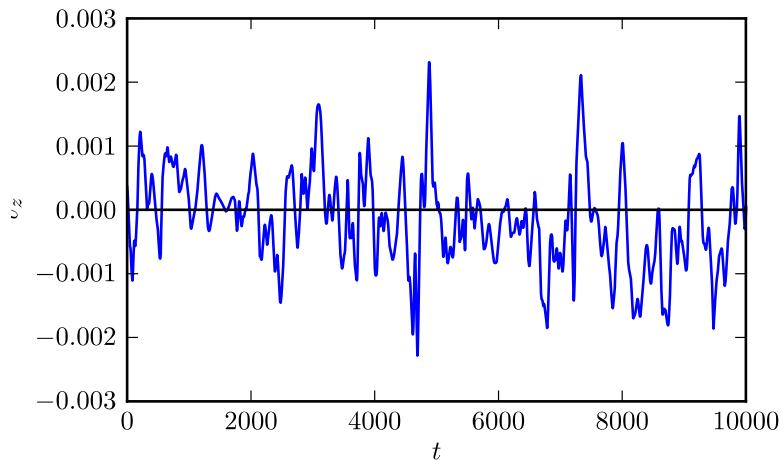
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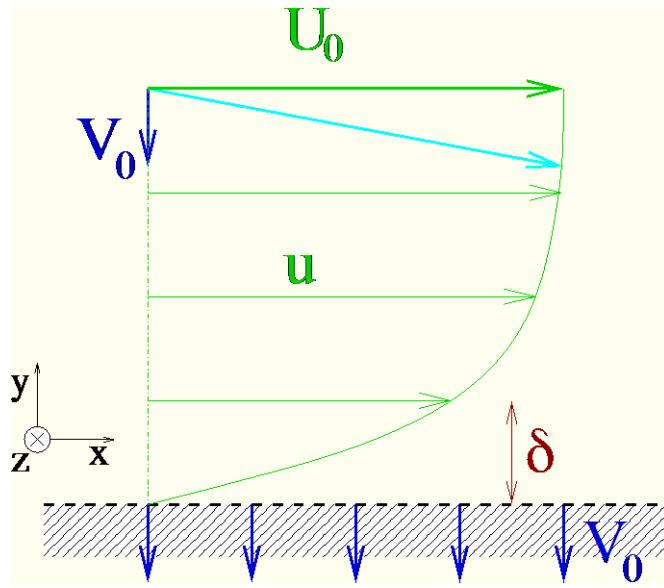
# Example 2:

Transverse motions in the  
asymptotic suction boundary layer

# Spanwise fluctuations

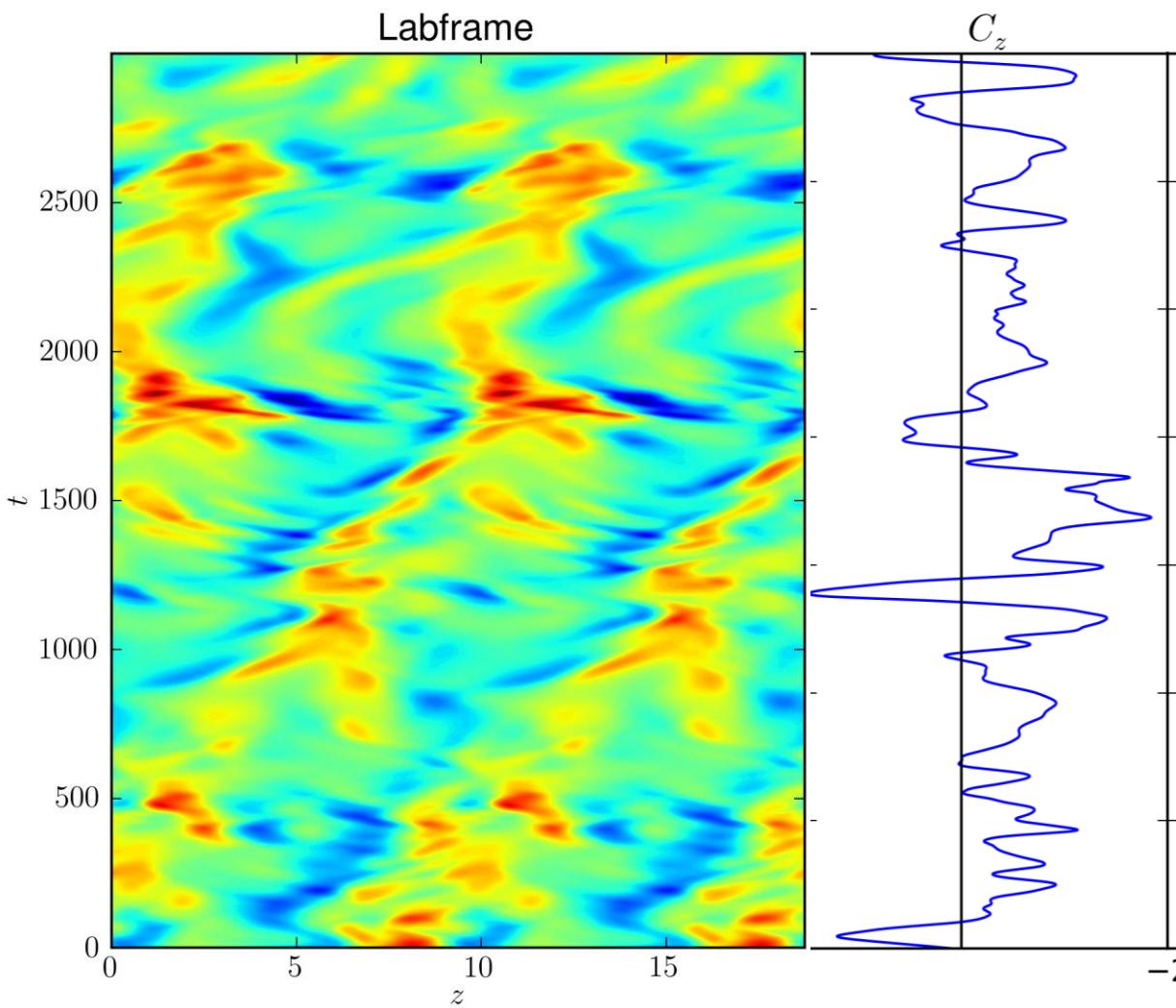


# Visualizing vortices

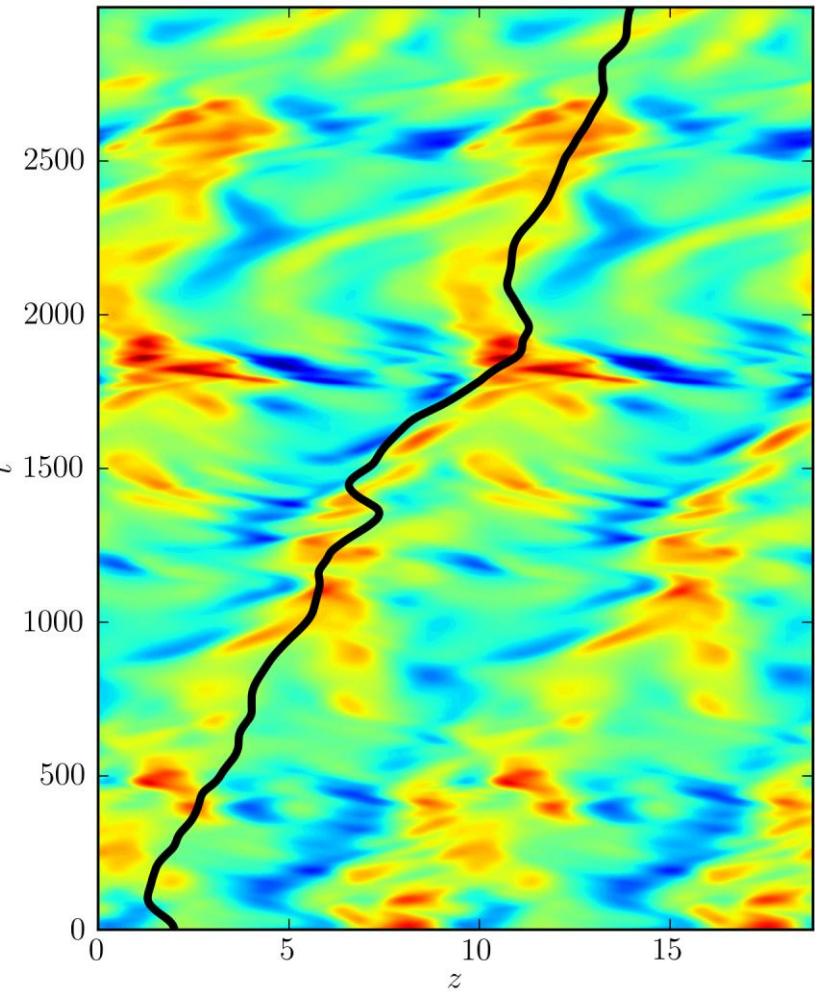


Take  
normal velocity  
as observable:

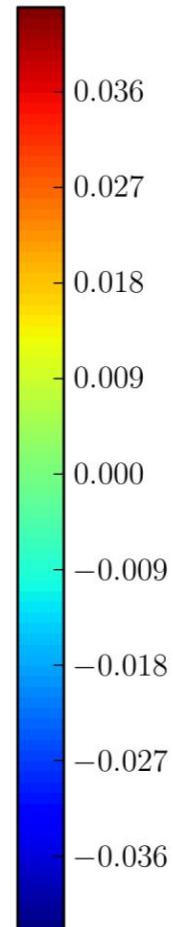
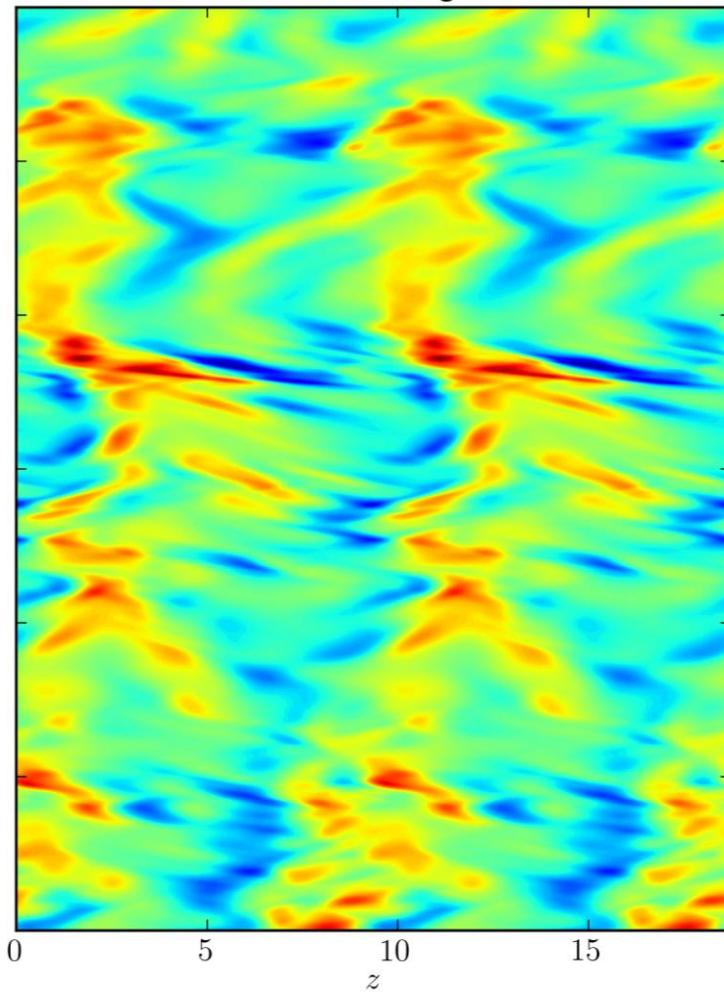
$$A(z, t) = \langle u_y(x, y = d, z, t) \rangle_x$$



Labframe



Comoving



# Summary

- Turbulent time evolution contains motion in neutral directions
- Local phase speed is variable
- Phase speeds usually uncorrelated
- Long time persistent motion in spanwise direction

