Simulating Saltation

Hans J. Herrmann

Computational Physics
IfB, ETH Zürich, Switzerland
and
Dept. de Física, UFC, Fortaleza, Brasil

Program 'Mediated Particle Transport in Geophysical Flows'
Kavli Inst. for Theoretical Physics
Santa Barbara, Oct.10, 2013

Collaborators:

Murilo Almeida
Jose Soares Andrade
Nuno Araújo
Marcus V. Carneiro
Thomas Pähtz
Eric Parteli
Keld Rasmussen

Particles in Turbulent Fluids

- **Saltation**
- **Aerosols**
- **Fluidized beds**
- **Preferential concentration**
  (e.g. distribution of plankton in oceans)
- **Pneumatic transport**
- **Turbulent mixing**
- **Rheology of suspensions**

Sandstorm

Equation of motion of fluid

$\vec{v}(\vec{x})$ and $p(\vec{x})$ are velocity and pressure field of the fluid, ρ and μ its density and kinematic viscosity.

Incompressible Navier-Stokes equation:

$$\boxed{\frac{\partial \vec{v}}{\partial t} + \vec{v}(\vec{\nabla}\vec{v}) = -\frac{1}{\rho}\vec{\nabla} p + \mu \Delta \vec{v}}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0 \wedge \rho = \mathbf{const} \Rightarrow} \qquad \boxed{\vec{\nabla}\vec{v} = 0}$$

Reynolds number Re

$$\boxed{\mathrm{Re} = \frac{Vh}{\mu}}$$

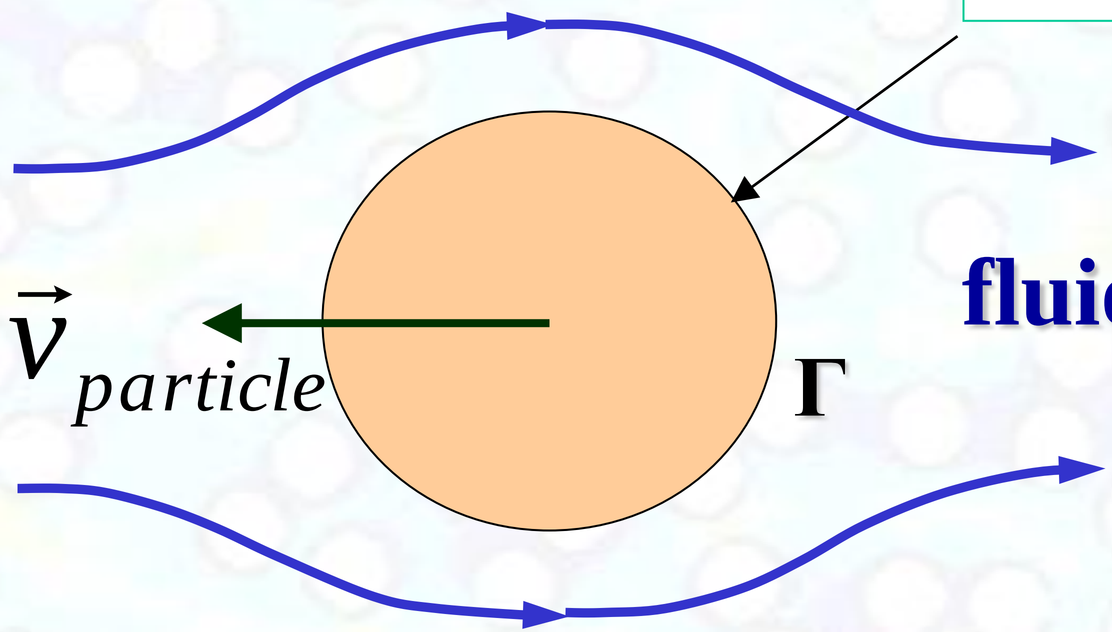
V **is characteristc velocity**
h **is characteristic length**
μ **is kinematic viscosity**

In turbulent regime: Re >> 1

One particle in fluid

e.g. pull sphere through fluid

no-slip condition: $\vec{v}_\Gamma = \vec{v}_{particle}$



moving boundary condition

create shear in fluid : exchange momentum

Drag force

drag force

(Bernoulli's principle)

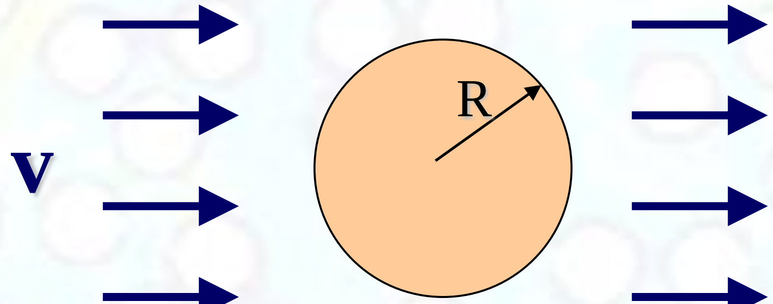
$$\vec{F}_D = \int_{\Gamma} \overleftrightarrow{\Theta} d\vec{A}$$

stress tensor

$$\Theta_{ij} = -p\delta_{ij} + \eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

$\eta = \rho\,\mu$ is dynamic viscosity

Homogeneous flow



Re << 1 Stokes law:

$$F_D = 6\pi\, \eta\, R\, v$$

(exact for Re = 0)

R is particle radius, **v** is relative velocity

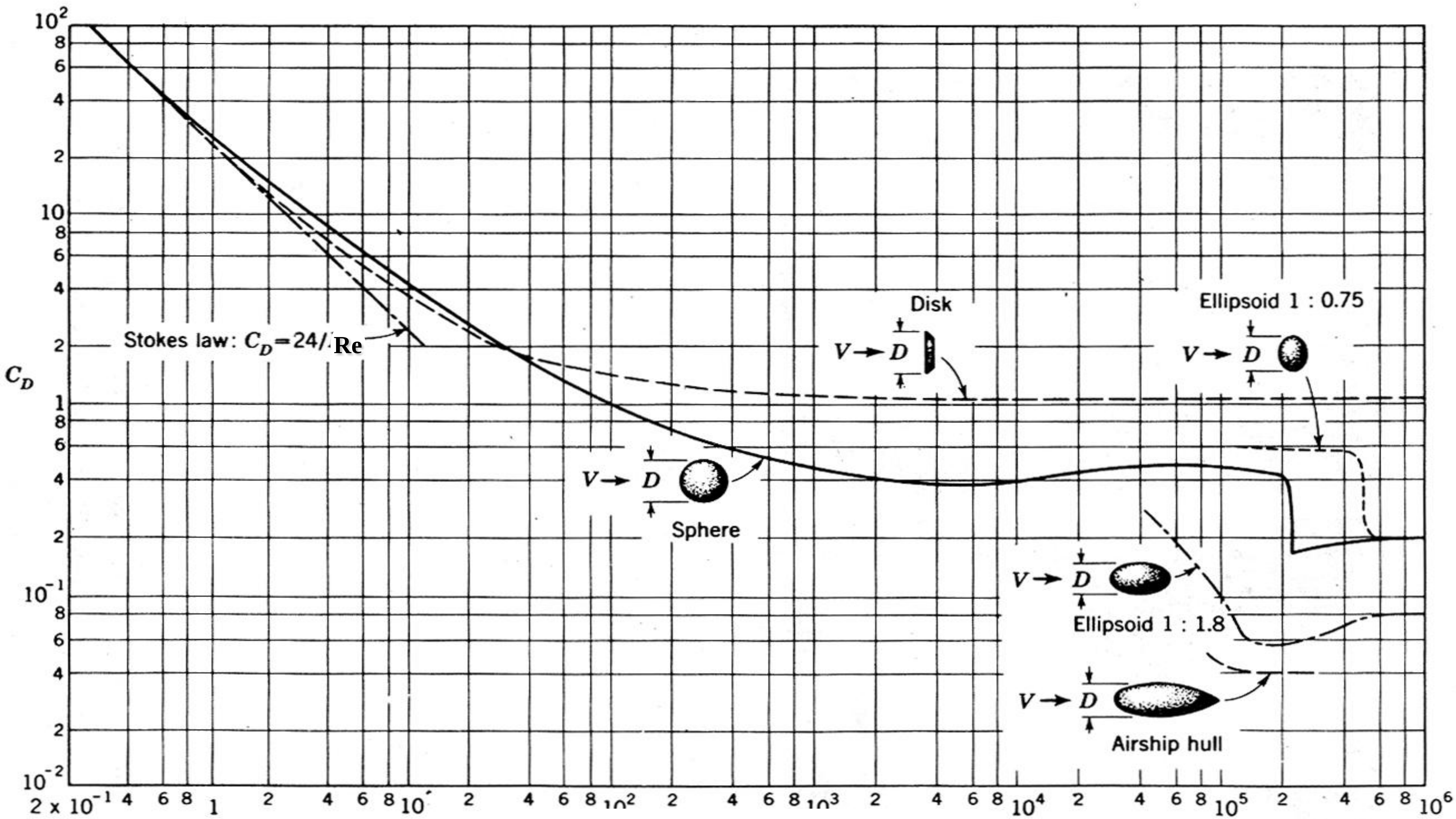
Re >> 1 Newton's law: $F_D = 0.22\pi\, \rho\, R^2 v^2$

general drag law:

$$\boxed{F_D = \frac{\pi}{8}\frac{\eta^2}{\rho} C_D \,\mathrm{Re}^2}$$

C_D is the drag coefficient

Drag coefficient C_D



Reynolds number Re = *D*v/*μ*

Inhomogeneous flow

In velocity or pressure gradients: Lift forces

are perpendicular to the direction of the external flow, important for wings of airplanes.

lift force: $$L = C_L \times \rho \times \frac{V^2}{2} \times A$$

C_L is „lift coefficient“

---

when particle rotates: Magnus effect

important for soccer

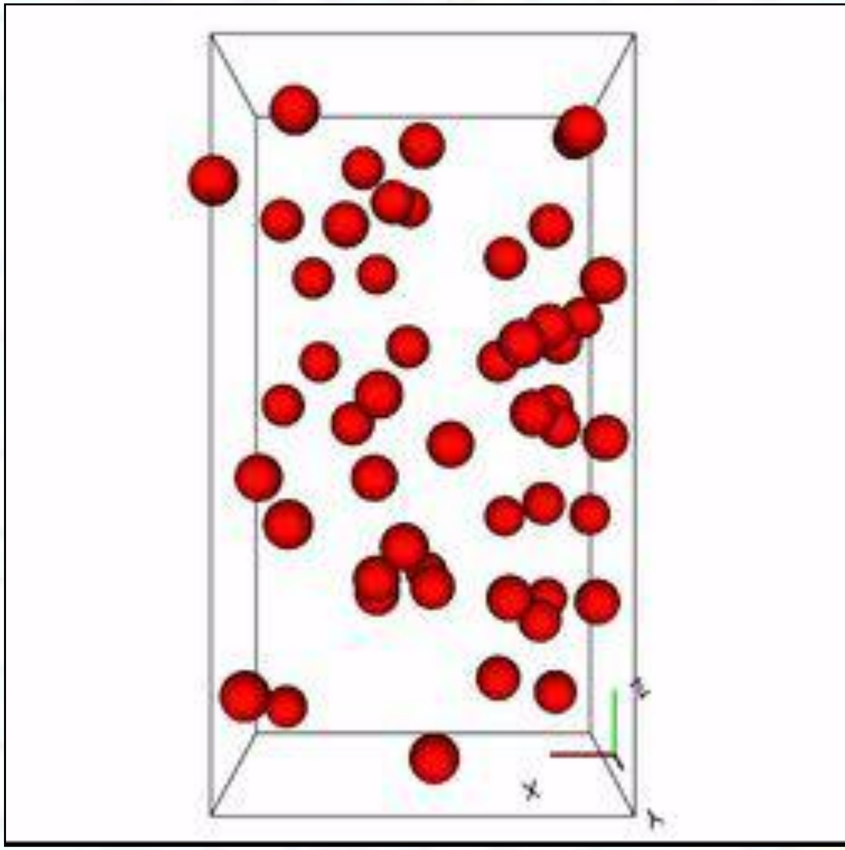
Many particles in fluids

•The fluid velocity field follows the incompressible Navier Stokes equations.

• Many industrial processes involve the transport of solid particles suspended in a fluid. The particles can be sand, colloids, polymers, etc.

•The particles are dragged by the fluid with a force:

$$\boxed{F_D = \frac{\pi}{8}\frac{\eta^2}{\rho} C_D \,\mathrm{Re}^2}$$



simulating particles moving in a sheared fluid

Aeolian Sand transport

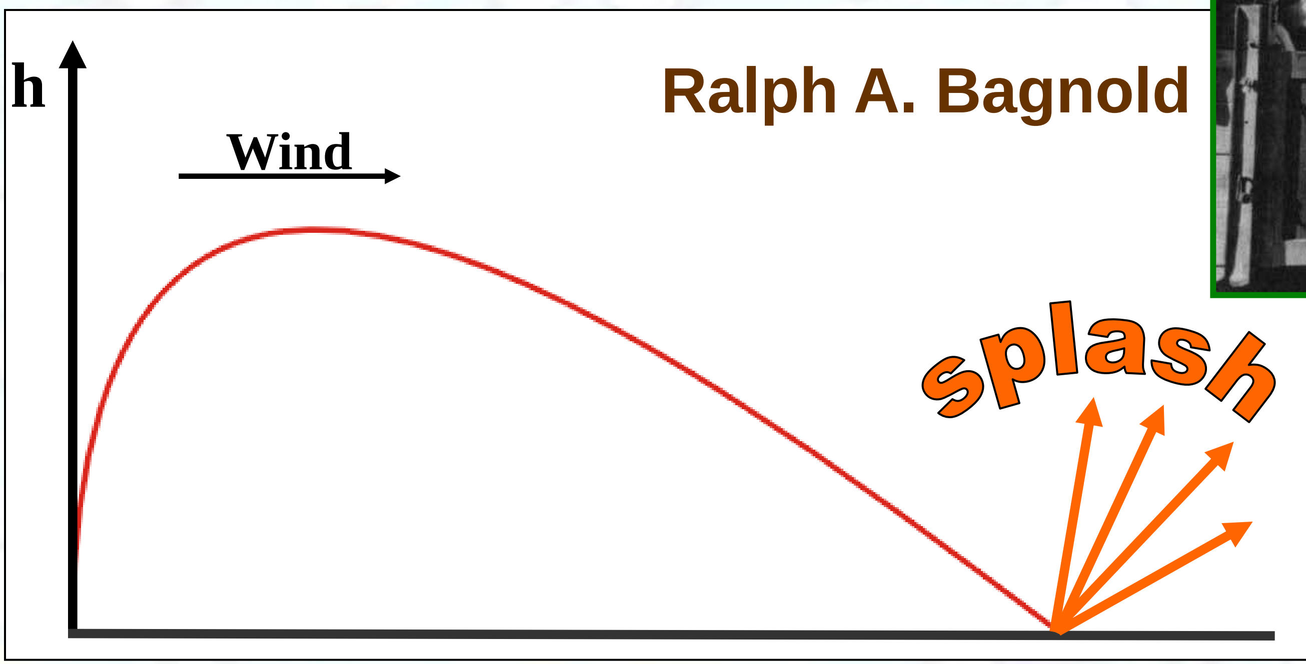
Transport by Saltation

(on coastal dunes)

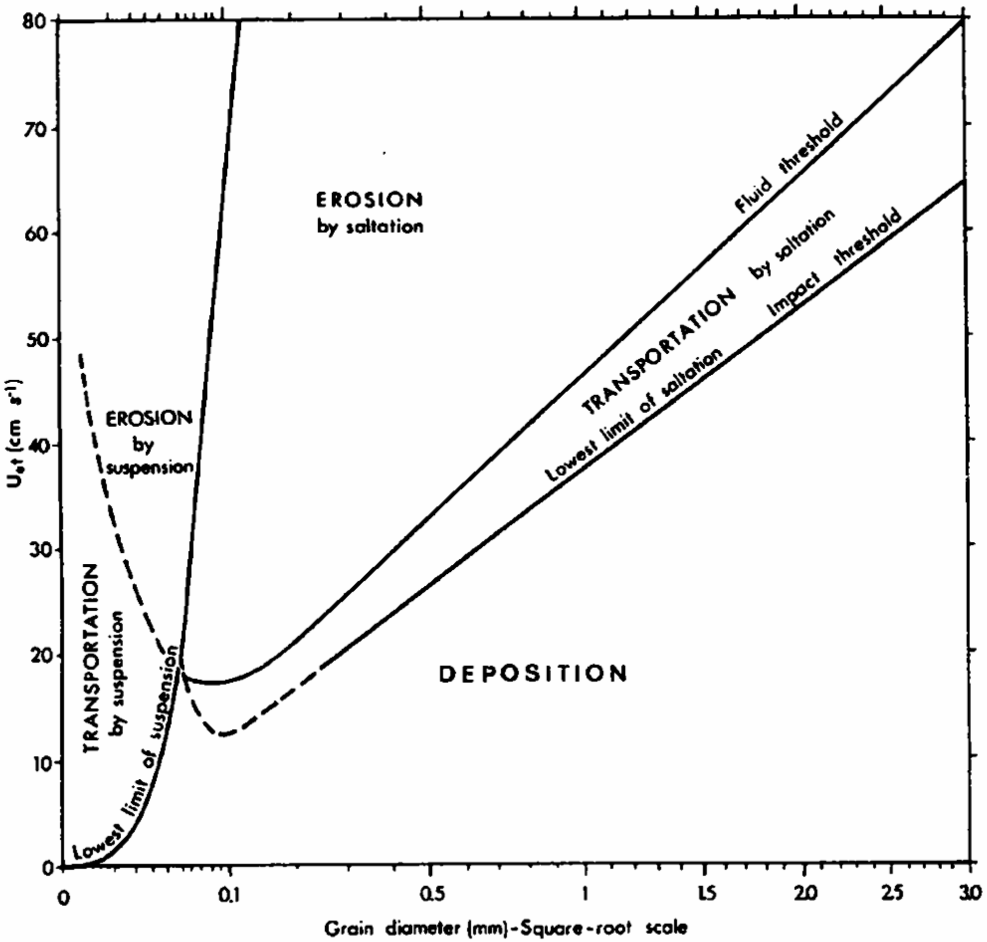
Saltation in the desert

The Mechanism of Saltation

• Grains are drawn from the ground and accelerated by the wind. With more energy they impact again against the surface and eject a splash of new particles. In this way more and more grains saltate until saturation is reached due to momentum conservation.



Dependence on grain size



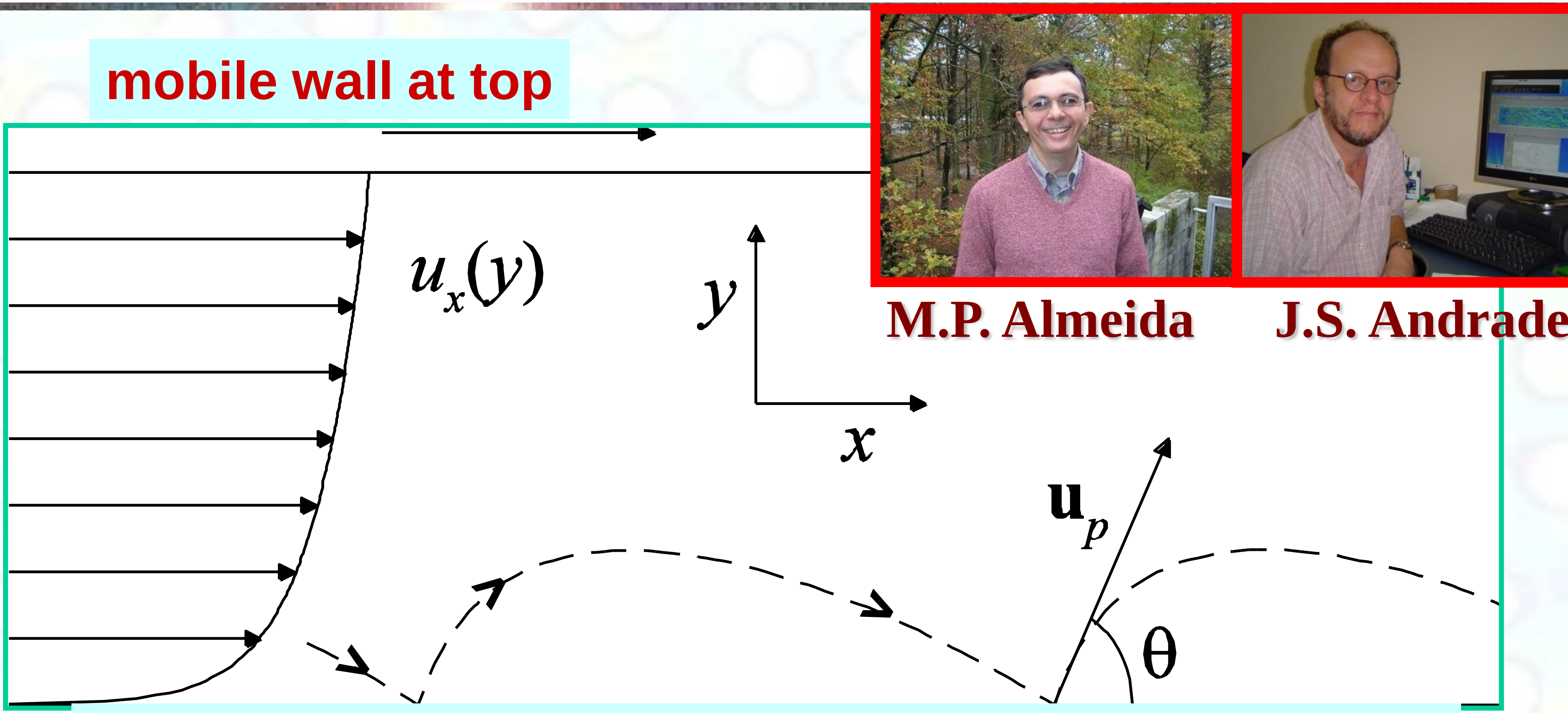
typical grain diameters for saltation on Earth: 100 – 300 μm

wind tunnel measurements

Bagnold and Chepil

Schematic saltating trajectory

mobile wall at top



M.P. Almeida

J.S. Andrade

θ = ejection angle u_p = particle velocity

$u_x(y)$ = wind velocity profile

Physical Review Letters, **96**, 21 (2006)

The turbulent air flow

ETH

logarithmic velocity profile of the horizontal component of the velocity as function of height *y*:

$$u_x(y) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right)$$

$\kappa \approx 0.4$ is the von Kármán constant

z_0 is the roughness length

viscosity of air: $\eta = 1.7895 \cdot 10^{-5}$ kg m^{-1} s^{-1}

density of air: $\rho = 1.225$ kg m^{-3}

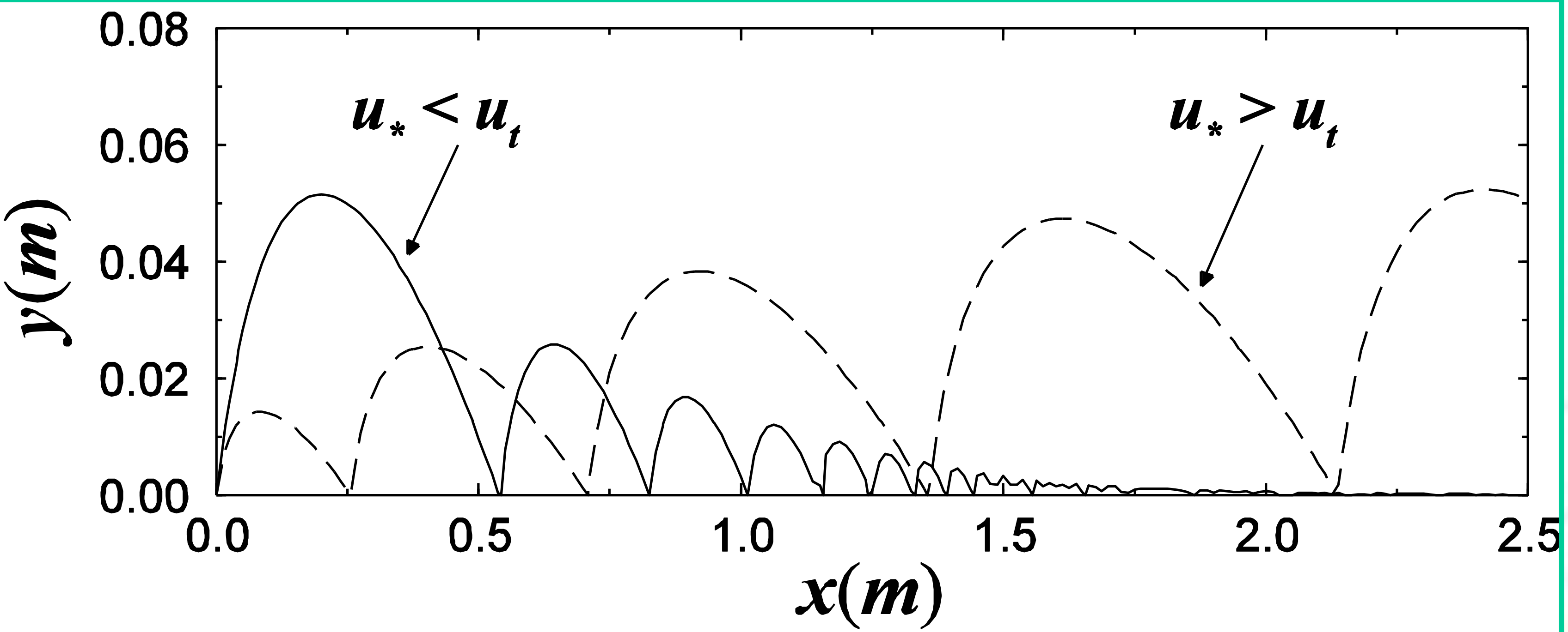
Solve it with k-ε model using Fluent.

a commercial finite volume solver on an adaptive triangulated mesh

Types of transient behaviour

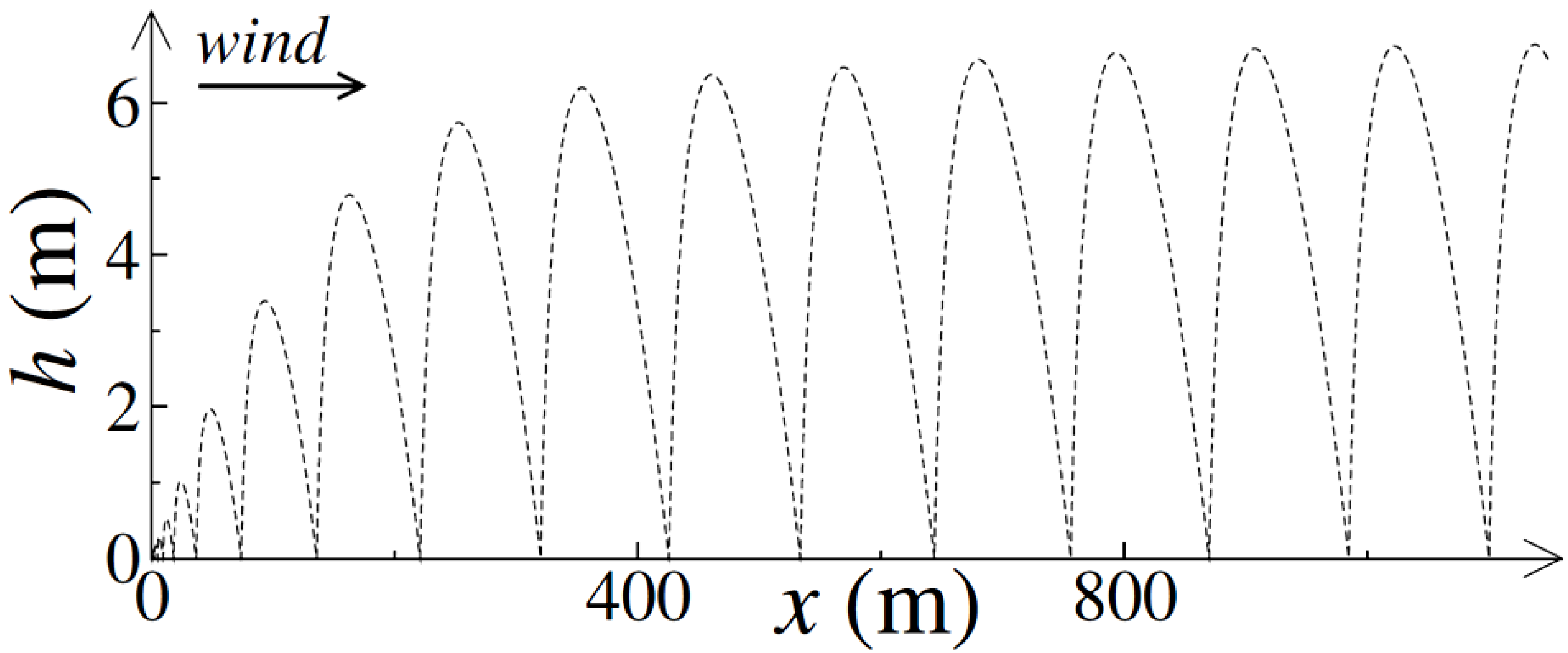
force on particle:

$$\frac{d\vec{u}_p}{dt} = F_D\left(\vec{u} - \vec{u}_p\right) + \frac{1}{\rho}\vec{g}\left(\rho_p - \rho\right)$$



threshold velocity $u_t \approx 0.35$

Steady state



⇒ saturated flux q_s

Saturated flux

Bagnold (1941):

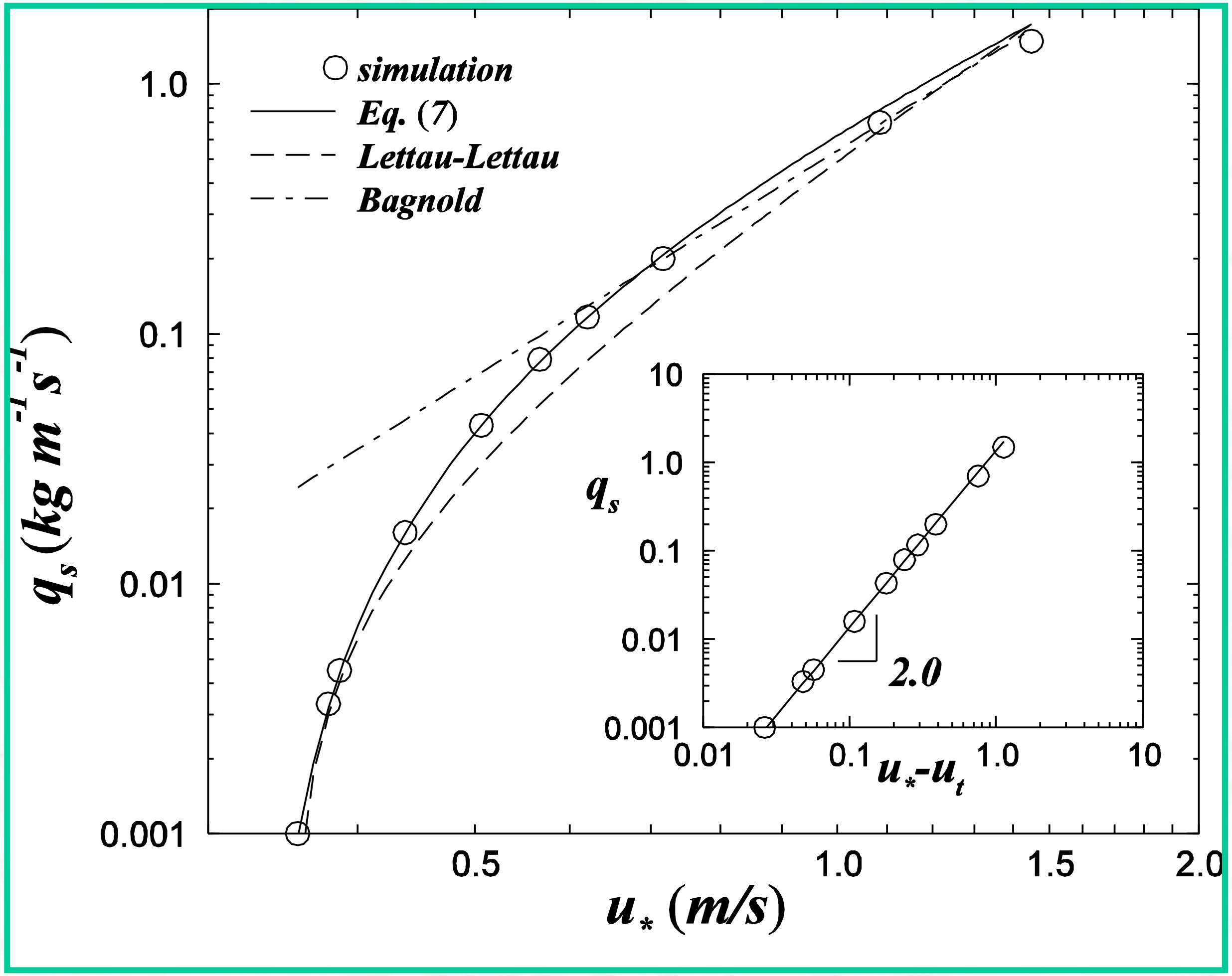
$$q_s \propto u_*^3$$

Lettau and Lettau:
(1978)

$$q_s = C_L \frac{\rho}{g} u_*^2 (u_* - u_t)$$

fit of solid line:

$$q_s = a(u_* - u_t)^2$$



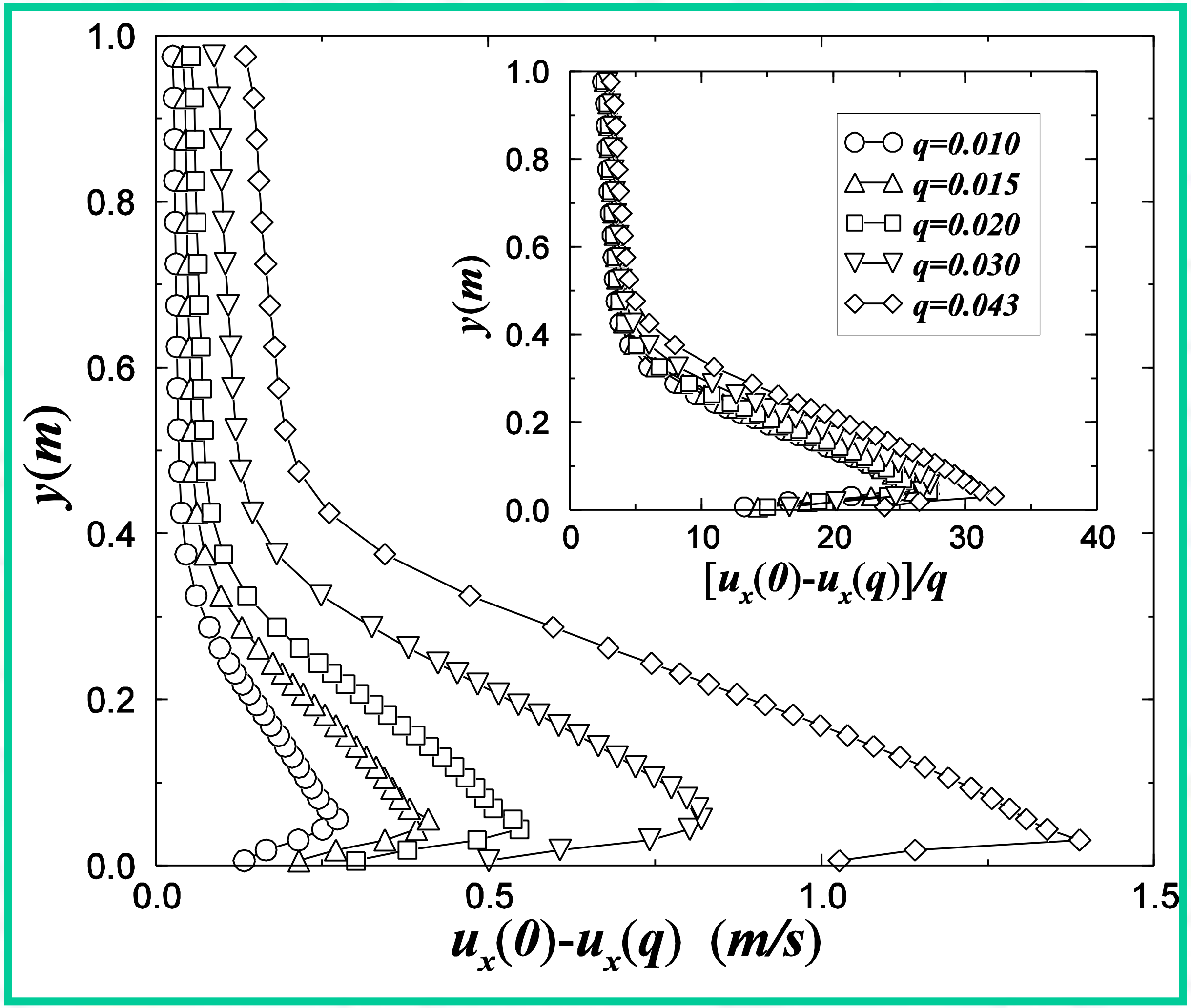
Physical Review Letters, **96**, 21 (2006)

Wind velocity profile

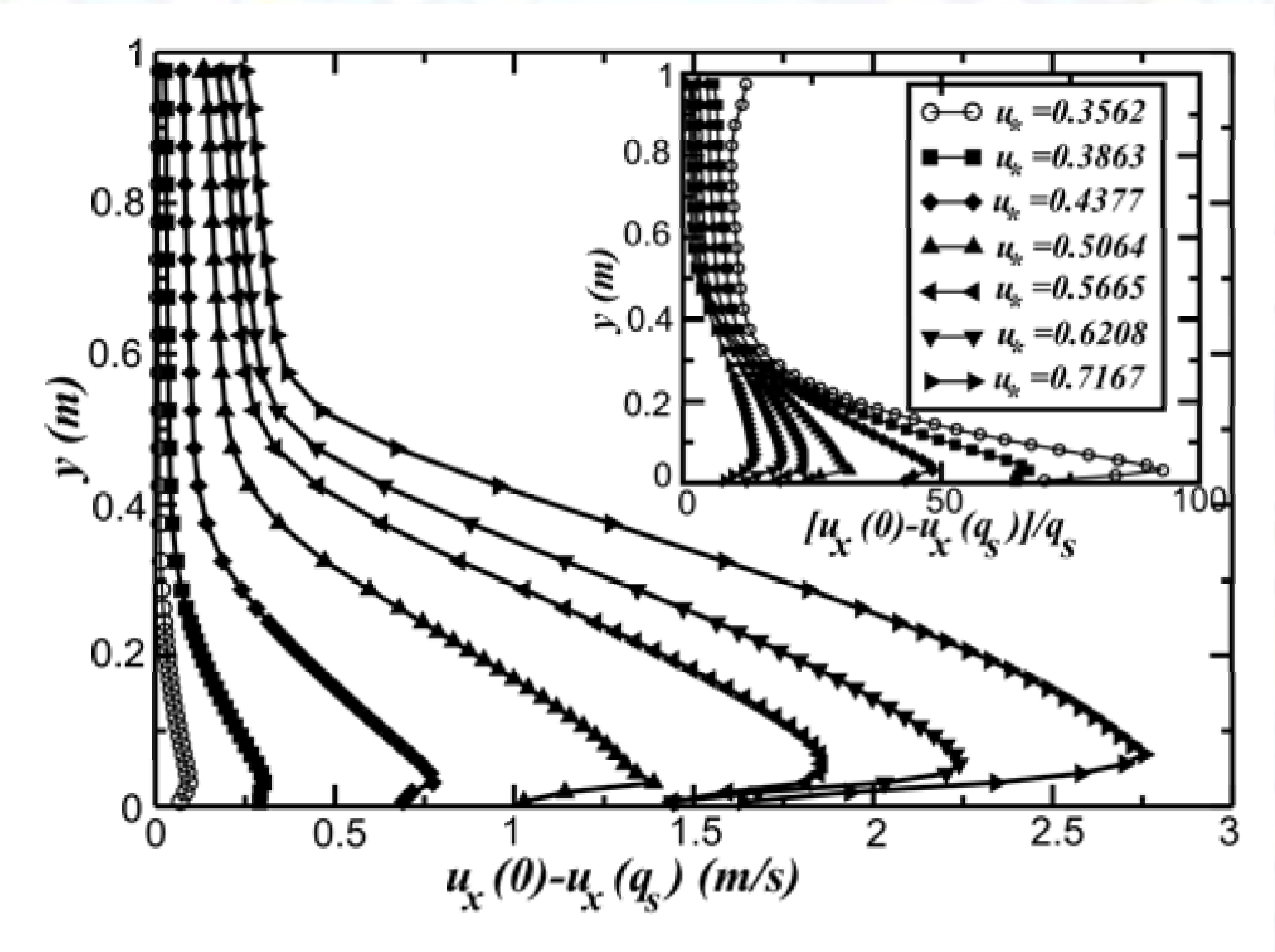
difference between disturbed and undisturbed velocity profile

$u_* = 0.51$

collapse when normalizing with flux q



Wind velocity profile



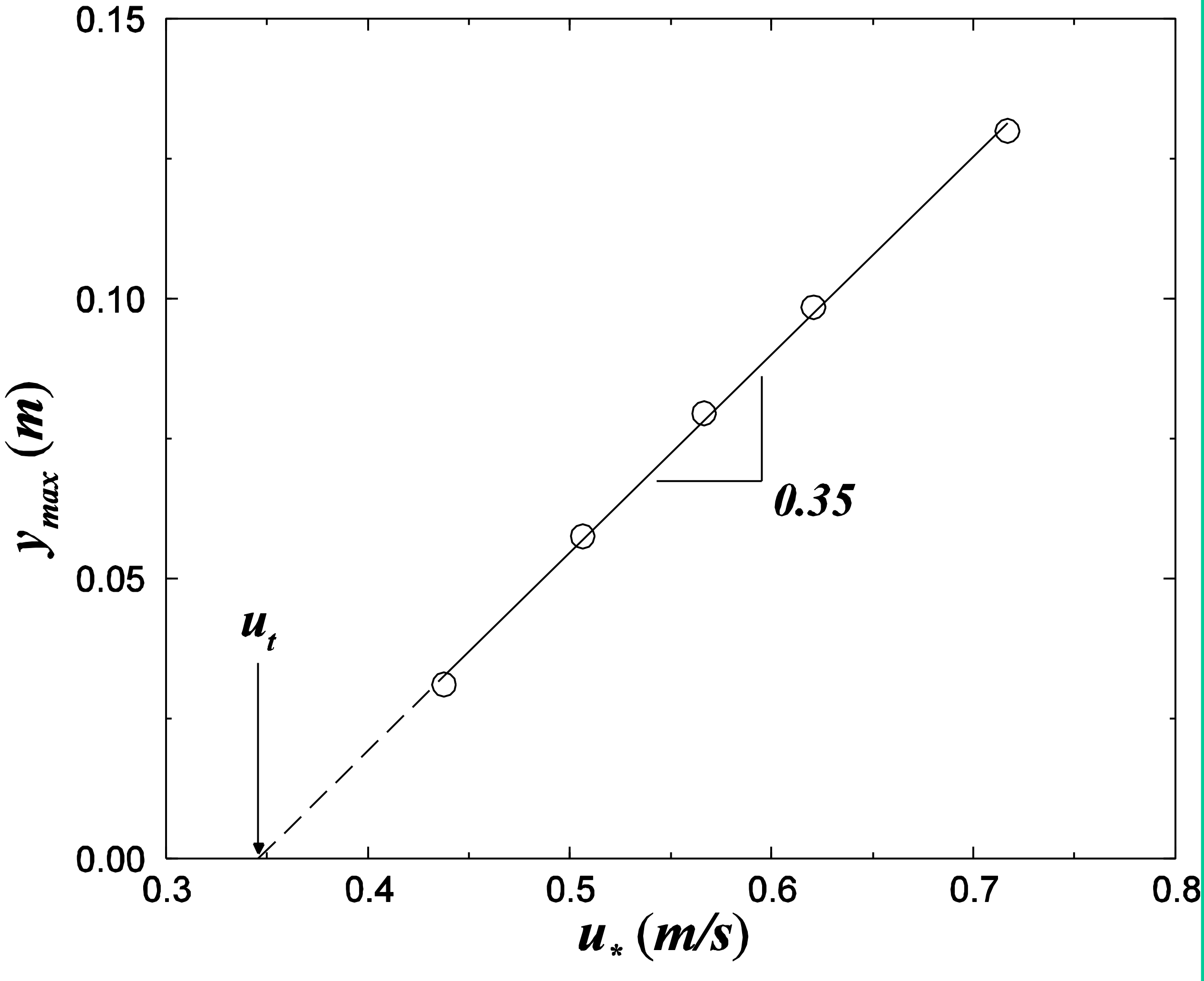
Height of saltation layer

y_{max} height of maximum loss of velocity

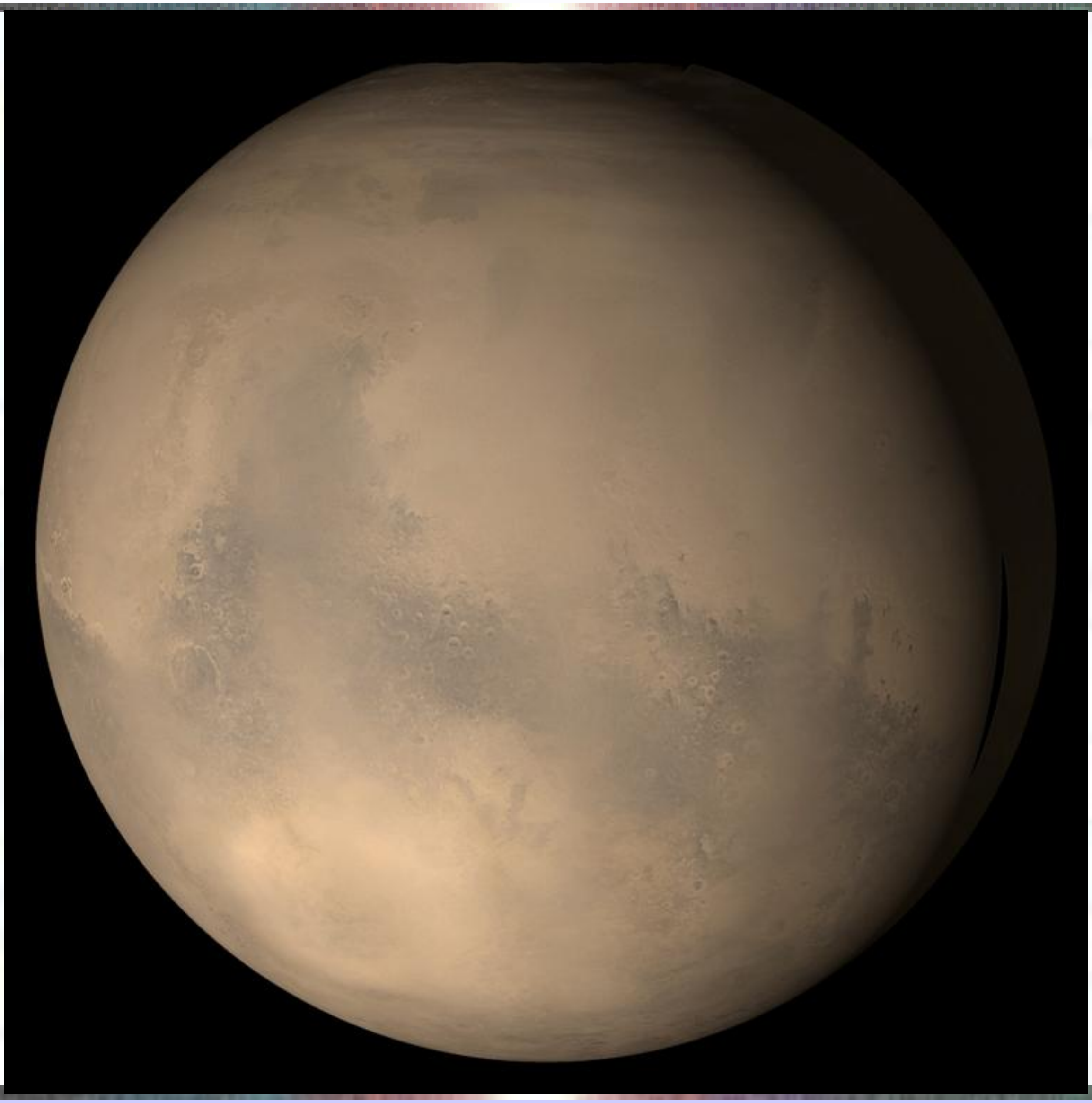
linear increase with u_*

becomes zero at:

u_t = 0.33±0.01



Planet Mars



Eric Parteli

M. Almeida, E. Parteli, J.S. Andrade, HJH
PNAS 105, 6222 (2008)

Parameters on Mars

Earth	Mars
$g = 9.81\,\mathrm{m/s^2}$	$g = 3.71\,\mathrm{m/s^2}$
$\rho_{\mathrm{air}} = 1.225\,\mathrm{kg/m^3}$	$\rho_{\mathrm{air}} = 0.02\,\mathrm{kg/m^3}$
$\rho_{\mathrm{grain}} = 2650\,\mathrm{kg/m^3}$	$\rho_{\mathrm{grain}} = 3200\,\mathrm{kg/m^3}$
$d = 250\,\mu\mathrm{m}$	$d = 600\,\mu\mathrm{m}$
$u_{*\mathrm{t}} \approx 0.2\ \mathrm{m/s}$	$u_{*\mathrm{t}} \approx 2.0\ \mathrm{m/s}$ [*Greeley and Iversen* (1985)]
$\eta \approx 1.8 \times 10^{-6}\ \mathrm{kg/s{\cdot}m}$	$\eta \approx 1.1 \times 10^{-6}\ \mathrm{kg/s{\cdot}m}$ (*Viscosity of* CO_2 at $-100^{\circ}\mathrm{C}$)

- u_* on Earth is 0.4 m/s and on Mars, Pathfinder Mission 1997 found u_* close to threshold. Further, it has been found that the angle of the slip face of martian dunes is the same as of terrestrial dunes.

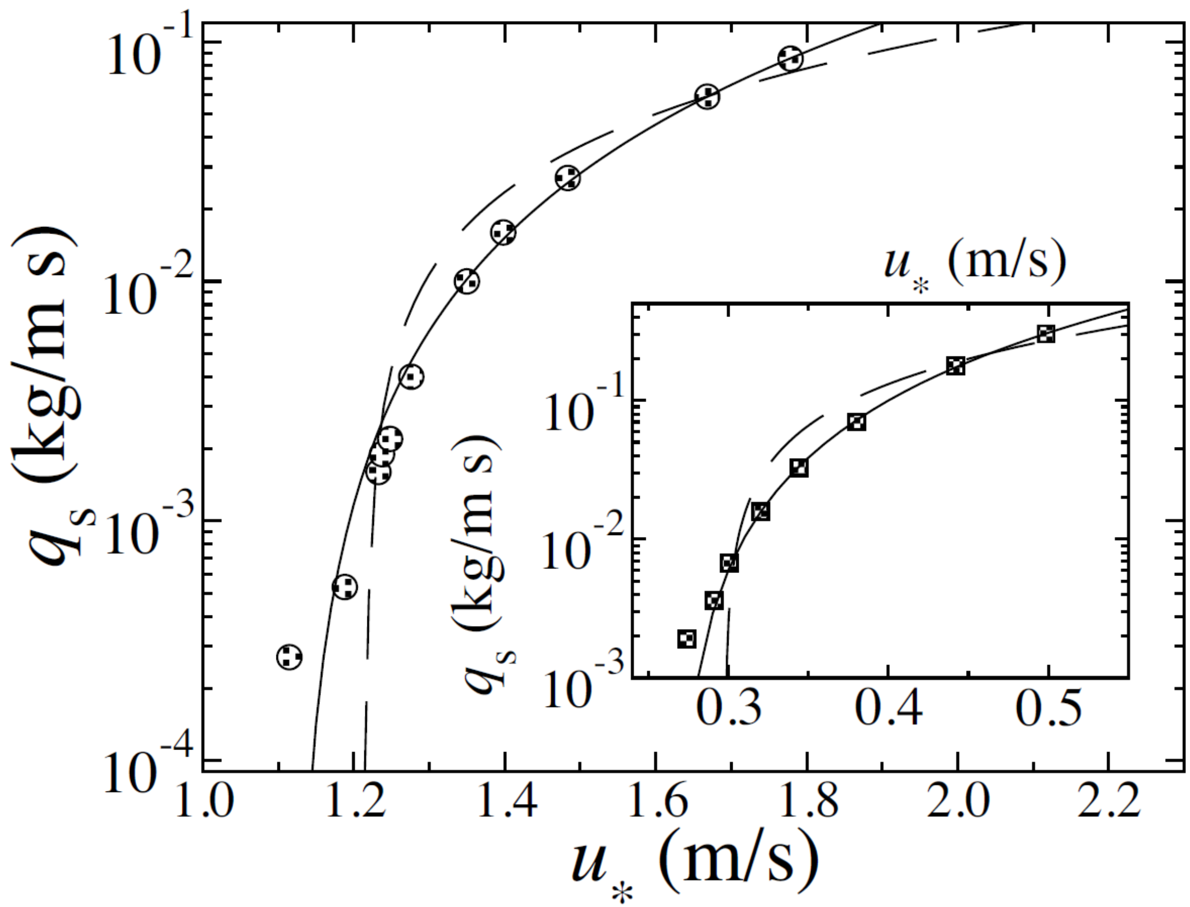
Saltation on Mars

White (1979)
Greenley et al. (1996)

$$q_s = C\frac{\rho_{fl}}{g}u_*^3\left(1-\frac{u_t}{u_*}\right)\left(1+\frac{u_t}{u_*}\right)^2$$

***C* = 18 for Earth**
***C* = 2.9 for Mars**

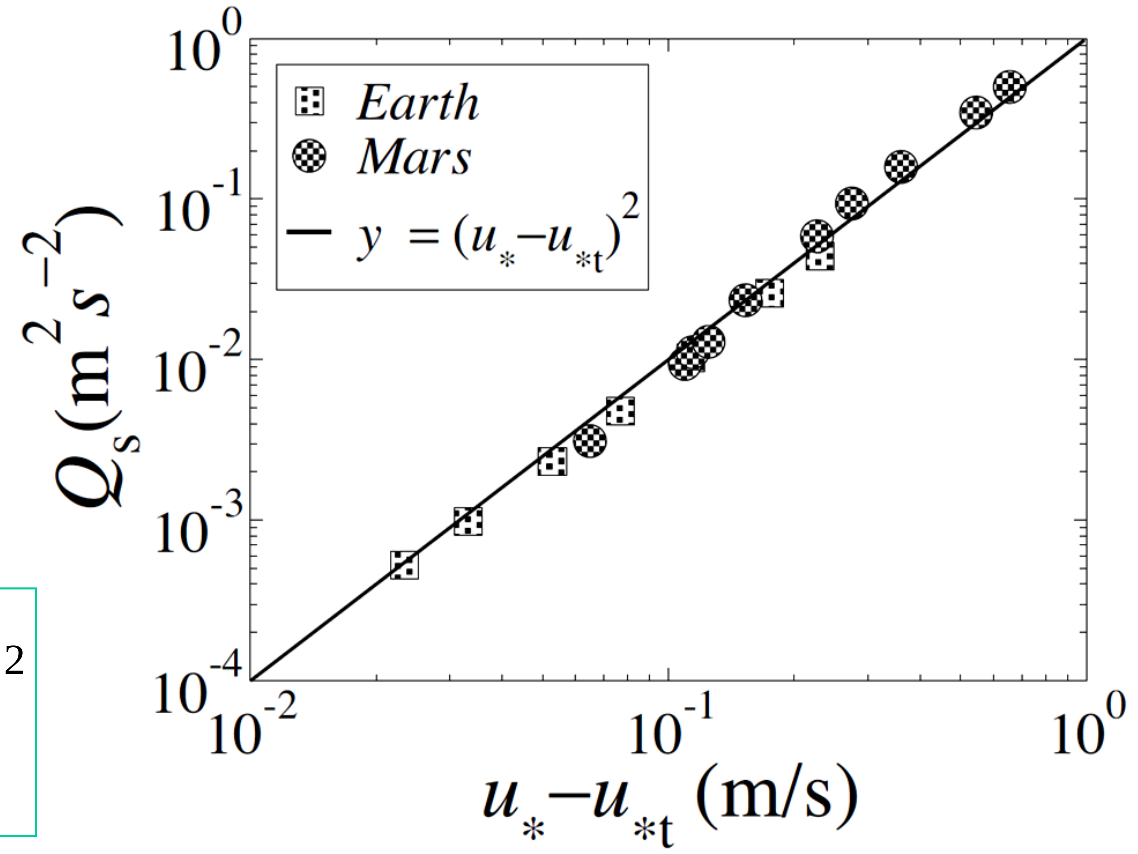
$$C = 19\, d / l_v$$
$$l_v = (\mu^2/g)^{1/3}$$



Saltation on Mars

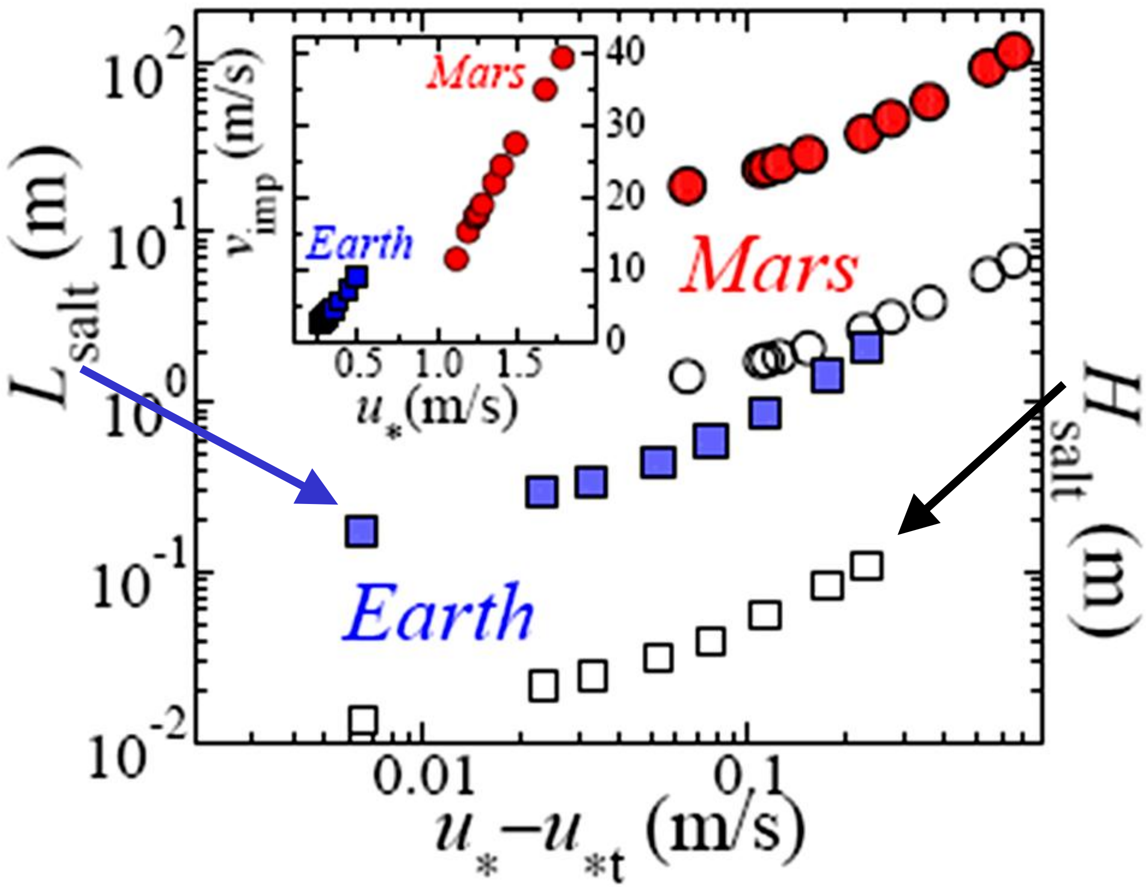
$$Q_s = \frac{q_s}{q_{s0}}$$

$$q_{s0} = \frac{d\rho_{fl}}{(\mu g)^{2/3}} u_t \left(u_* - u_t\right)^2$$



Saltation on Mars

Length L_{salt} and height H_{salt} of saltation trajectory

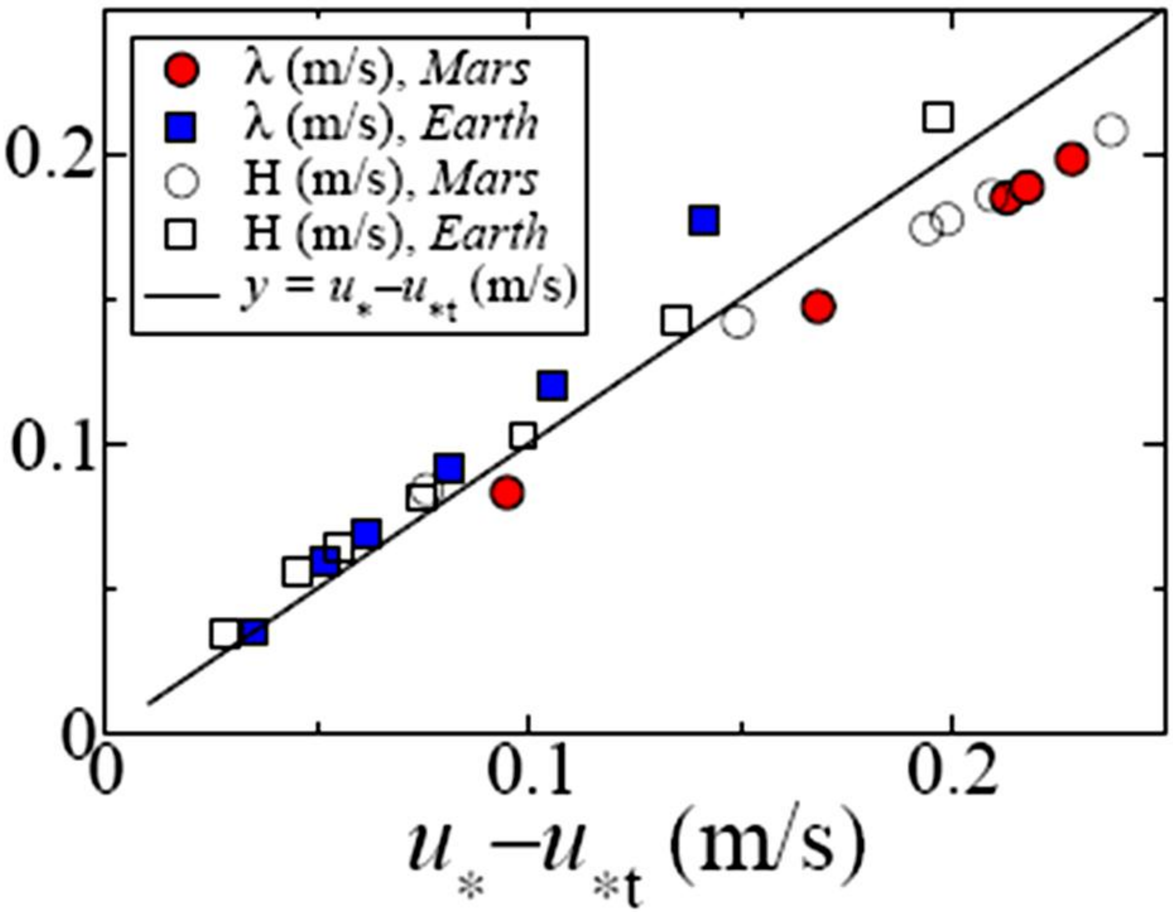


$$L_{salt} \propto t_v \left(u_* - u_t\right)$$

$$H_{salt} \propto t_v \left(u_* - u_t\right)$$

$$t_v = \left(\mu / g^2\right)^{1/3}$$

Saltation on Mars



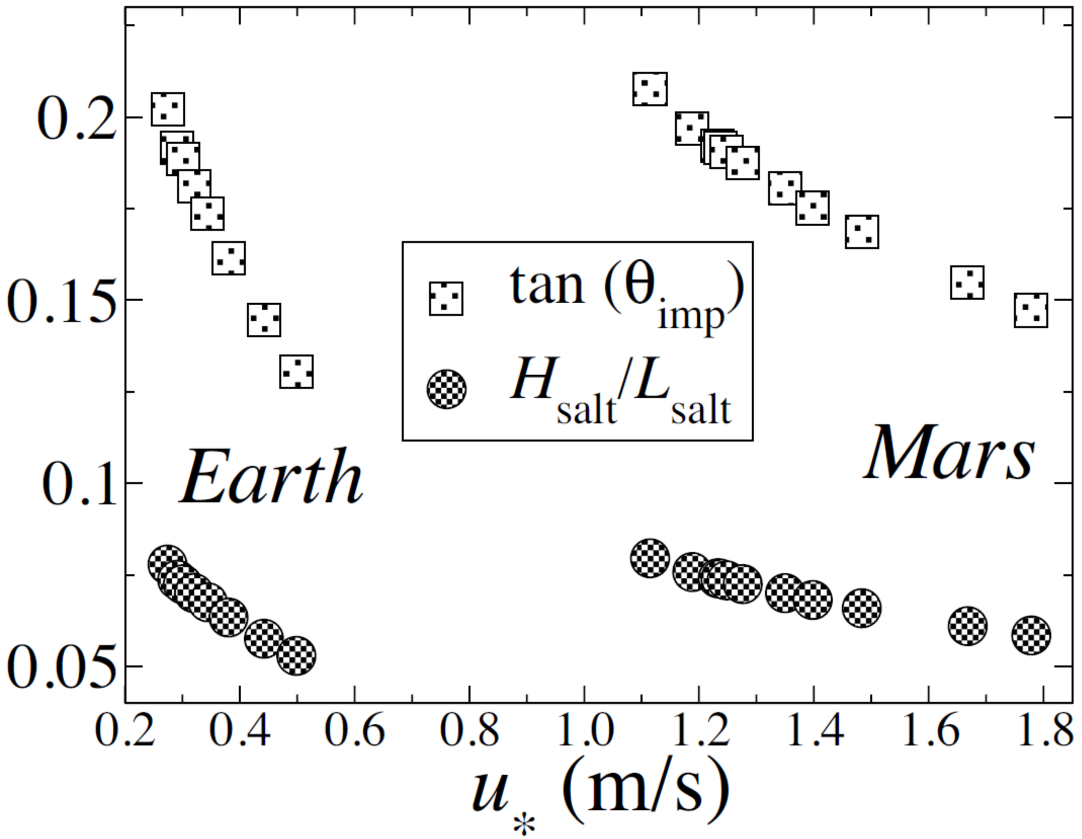
$$\lambda = \frac{L_{salt}}{l_v}$$

$$H = \frac{H_{salt}}{l_v}$$

$$l_v = (\mu^2/g)^{1/3}$$

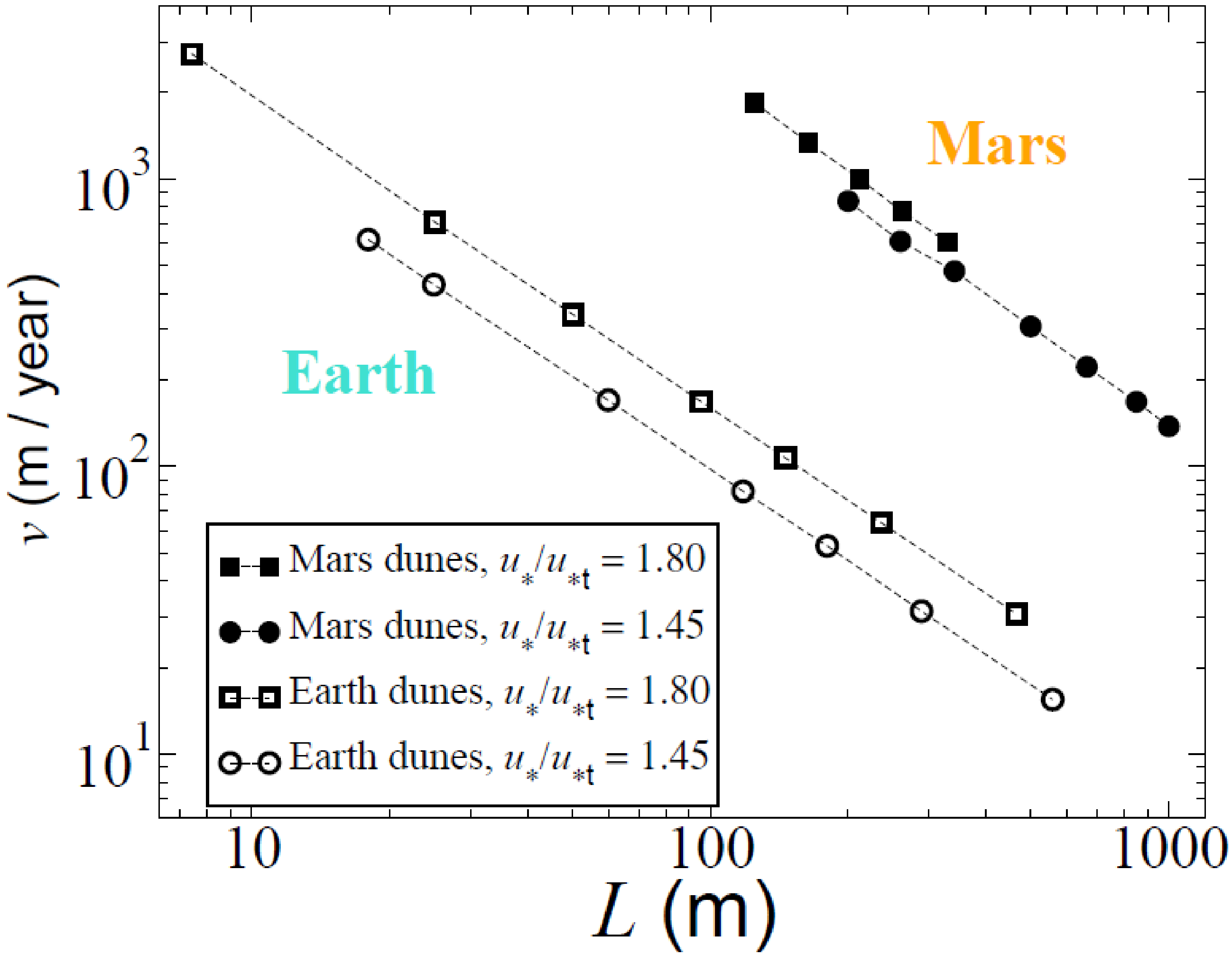
Saltation on Mars

Impact angle and height to length ratio of saltation trajectory as function of u_*



$$\frac{L_{salt}}{H_{salt}} \propto u_*$$

Dune velocities

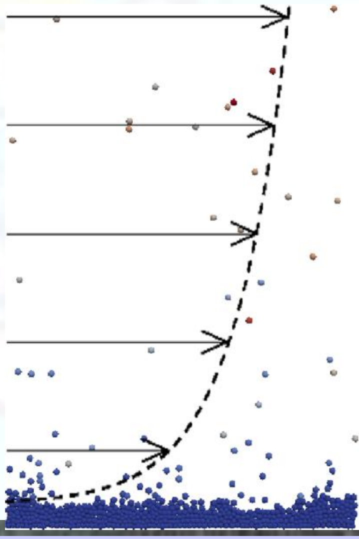


Discrete Element Model for Saltation

- We consider a bed of spheres of similar size under gravity in a channel , with periodic boundary conditions in horizontal direction and reflective conditions at top and bottom walls.
- Dissipation occurs at particle-particle and particle-wall collisions.
- At t = 0, some particles are dropped from random positions triggering the saltation process.

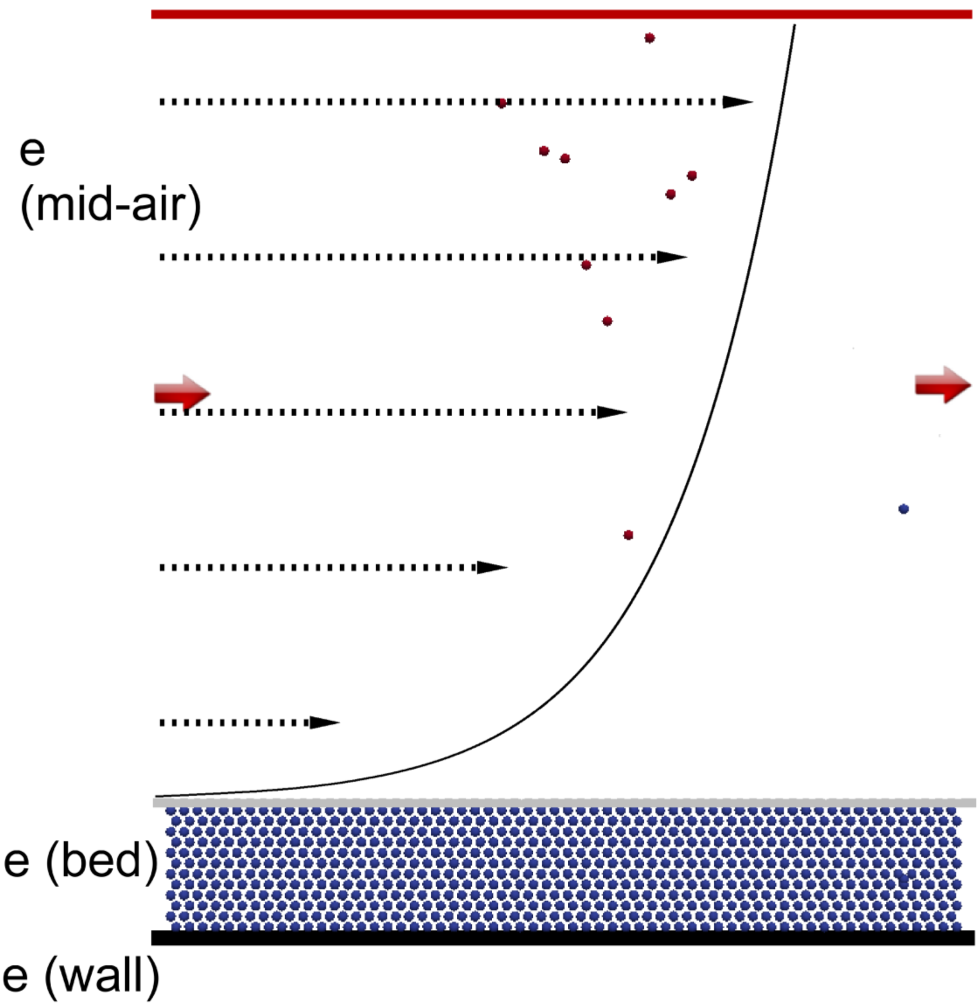
Marcus Vinicius Carneiro

Thomas Pähtz



M.V. Carneiro, T. Pähtz, H.J.H., Phys. Rev. Lett. 107, 098001 (2011), arXiv:1104.2767

Sketch of the system



$$u_x(z) = \frac{u_*}{\kappa} \ln \frac{z + z_0 - h_0}{z_0}$$

$$z_0 = D_{av}/30$$

- Logarithmic wind profile above the sand bed starts at height h_0, which must be determined.

DEM model of bed in moving fluid

$$m_i \ddot{\mathbf{x}}_i = \mathbf{F}_D + \mathbf{F}_{coll} + m_i \mathbf{g}$$

The fluid exerts on the particles the drag force:

$$F_{\boldsymbol{D}} = \frac{\pi \ d_p^2}{8} \rho \ C_{\boldsymbol{D}} \ (u - v_p)(\vec{u} - \vec{v}_p)$$

$$C_D = \left[\left(\frac{32}{\mathrm{Re}} \right)^{2/3} + 1 \right]^{3/2} \quad , \quad \mathrm{Re} = \frac{\rho_f \left| \mathbf{v} - \mathbf{u} \right| D_{av}}{\eta}$$

N. S. Cheng, J. Hydraul. Eng., 123(2), 149 (1997)

Dissipative collisions between particles

$$\boldsymbol{F}_{coll} = \left(k_n \delta_{ij} + c_n \dot{\delta}_{ij} \right) \boldsymbol{n}_{ij}$$

k_n elastic spring constant, c_n damping coefficient

$$\boldsymbol{n}_{ij} = \frac{\boldsymbol{x}_i - \boldsymbol{x}_j}{\left| \boldsymbol{x}_i - \boldsymbol{x}_j \right|}$$

$$\delta_{ij} = \left(R_i + R_j \right) - \left| \boldsymbol{x}_i - \boldsymbol{x}_j \right|$$

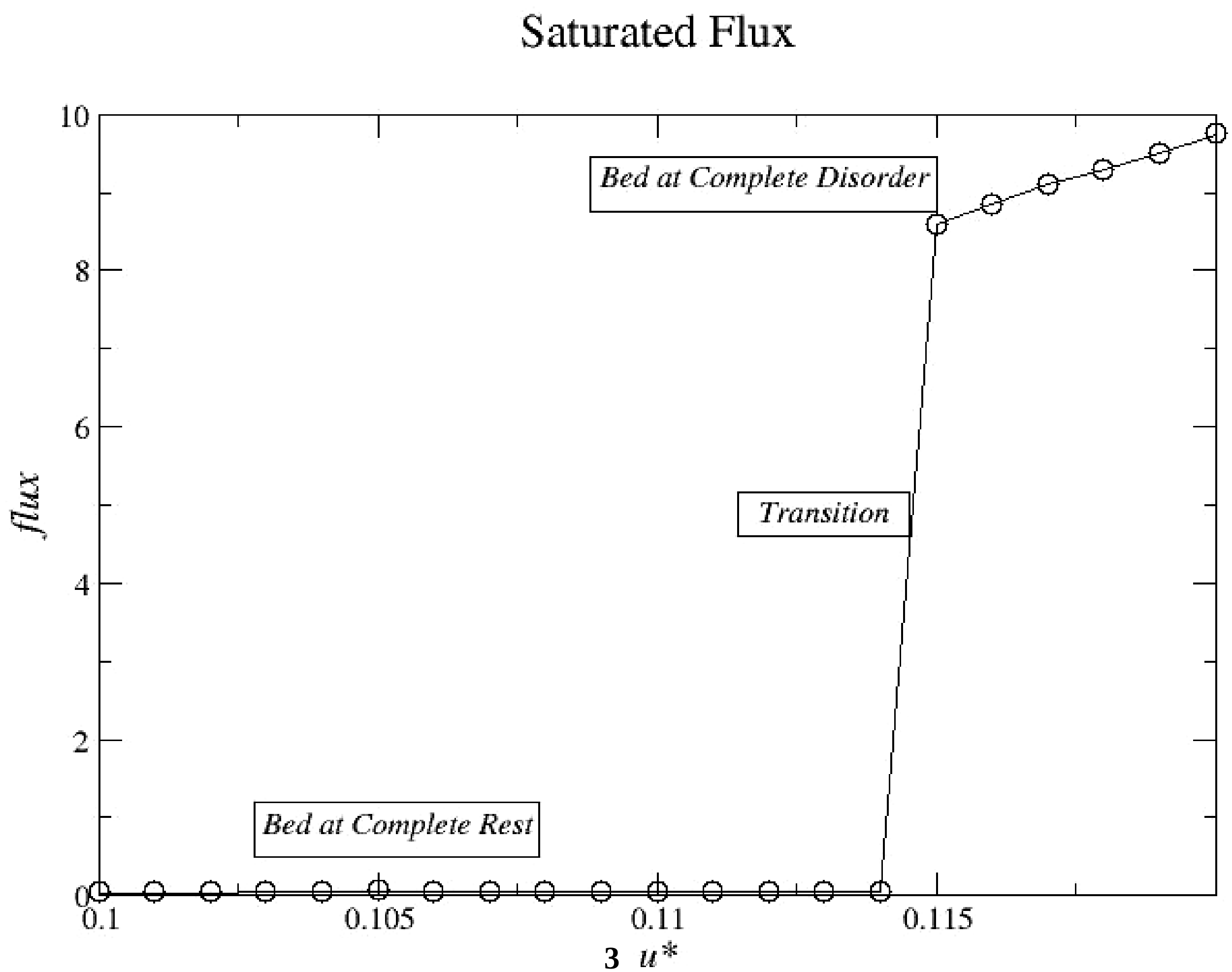
for sand

$\boldsymbol{e_n = 0.7}$

The restitution coefficient e_n is related to c_n through:

$$e_n = \exp\left(-\frac{\pi c_n}{\sqrt{4 m_{eff} k_n - c_n^2}} \right)$$

Results without feedback



2d

DEM model of bed in moving fluid

A feedback procedure corrects the wind profile iteratively before the drag forces are applied on the particles.

Including Feedback

- The reaction-drag forces exerted by the particles on the wind slow down the wind.
- We calculate the grain-stress generated by the accelerated particles through (A = area of sand bed, f = force density)

$$\tau_g(z) = \int_z^{\infty} f(z')dz' \cong \sum_{i:z_i>z} \frac{F_i}{A}$$

- The grain-stress modifies the shear velocity in the saltation layer by

$$u_\tau(z) := u_* \sqrt{1 - \frac{\tau_g(z)}{\rho_a u_*^2}}$$

- With this we determine the wind profile using $\dfrac{du}{dz} = \dfrac{u_\tau(z)}{\kappa z}$

Feedback Algorithm

Wind Profile $v_t^j(z)$

Calculate drag forces $F_i[v_t(z)]$

Calculate grain stress $\tau_g(z) = \sum_{i:z_i>z} \frac{F_i}{L1.3d}$

Integrate $\frac{dv}{dz} = \frac{u_\tau(z)}{\kappa z}$ $u_\tau(z) := u_* \sqrt{1 - \frac{\tau_g(z)}{\rho_a u_*^2}}$

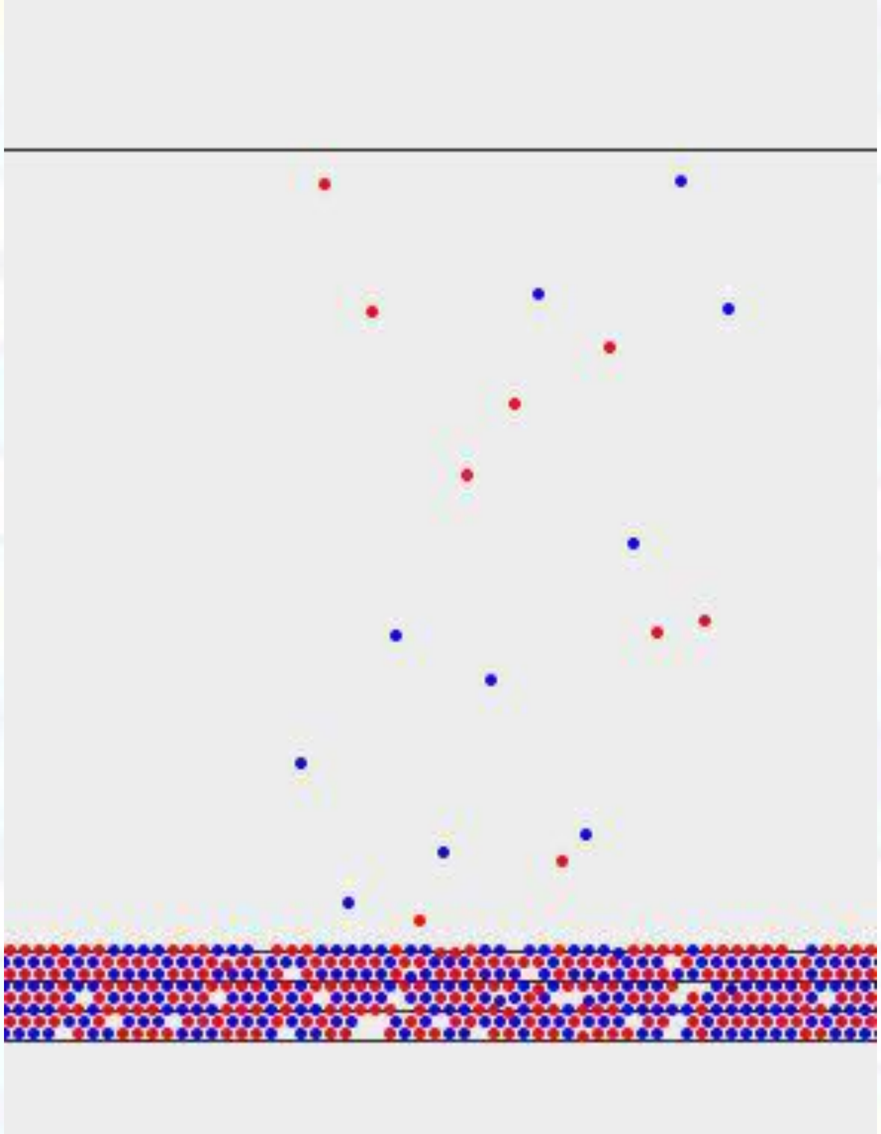
$j := j+1$ $v_t^j(z)$

$v_t^j(z) \cong v_t^{j-1}(z)$

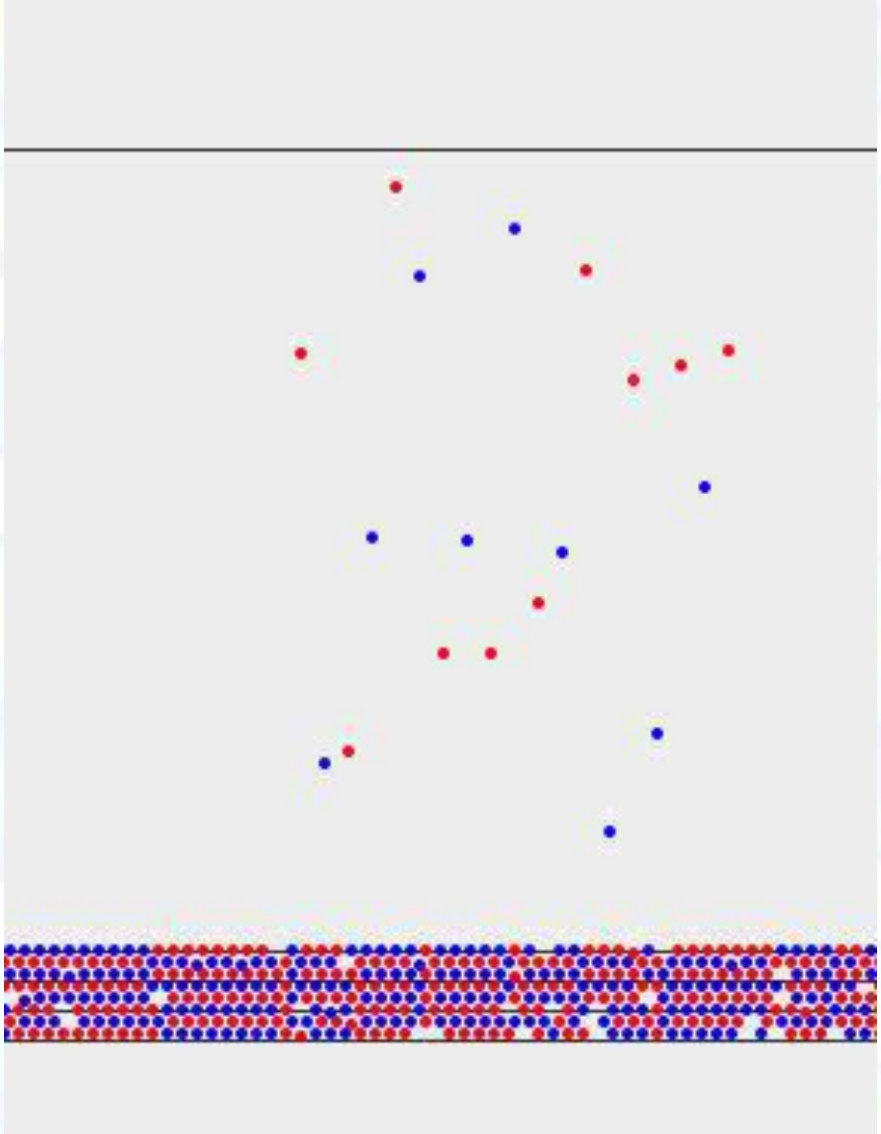
$v_{t+1}^0(z) := v_t^j(z)$

Apply drag forces on particles

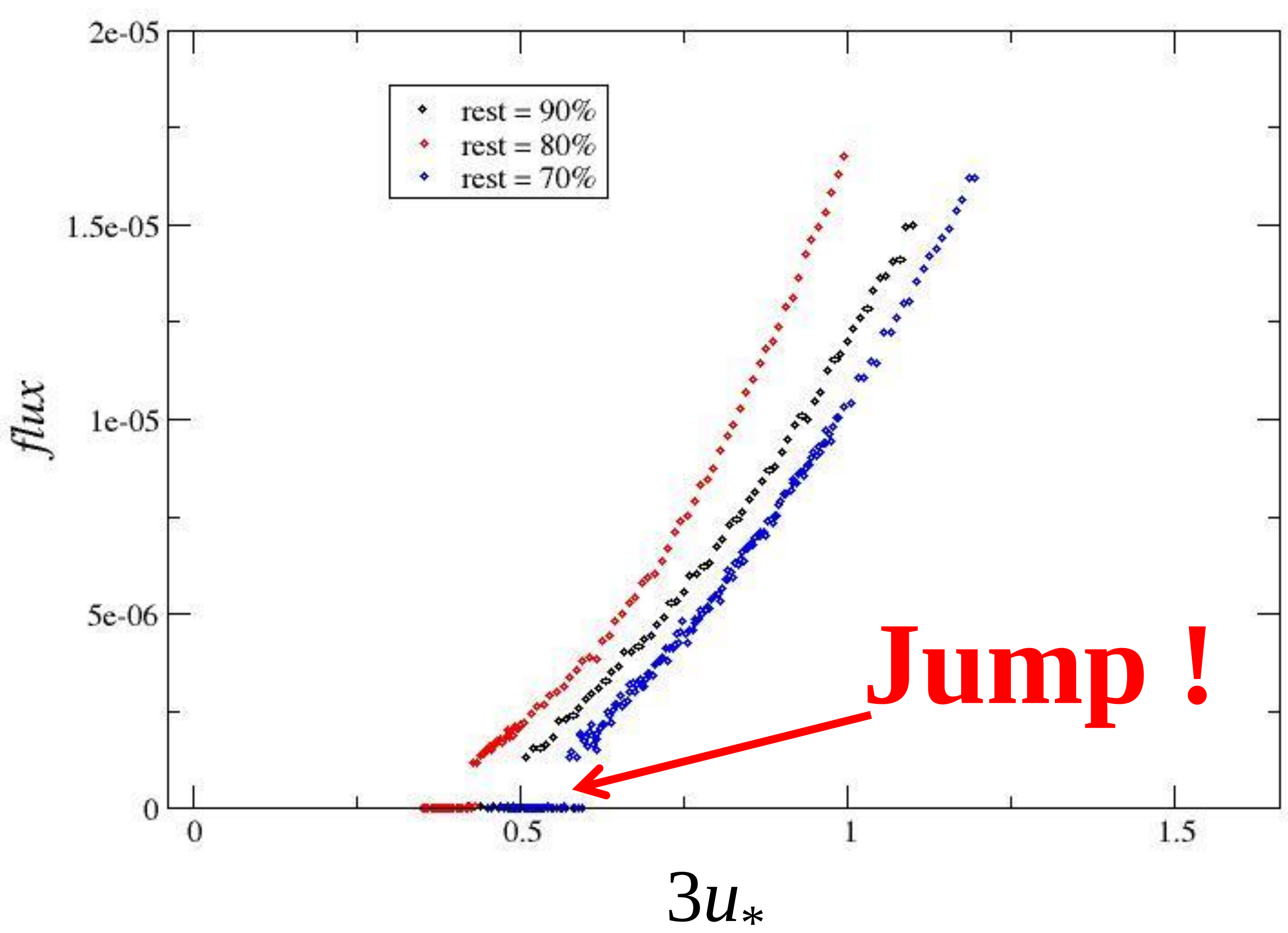
Results for $u_* = 0.52$ m/s in 2d



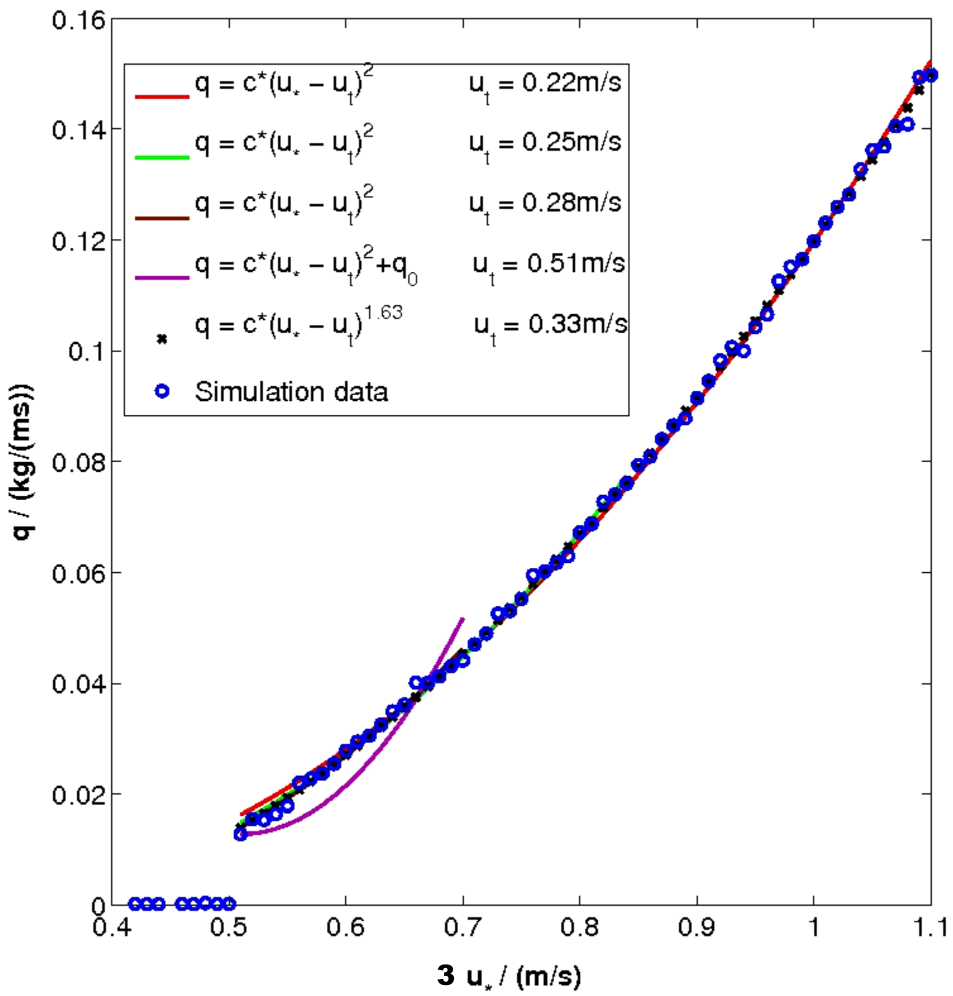
Results for $u_* = 0.82$m/s in 2d



Saturated flux as function of u_* for different restitution coefficients



Comparison with different theories

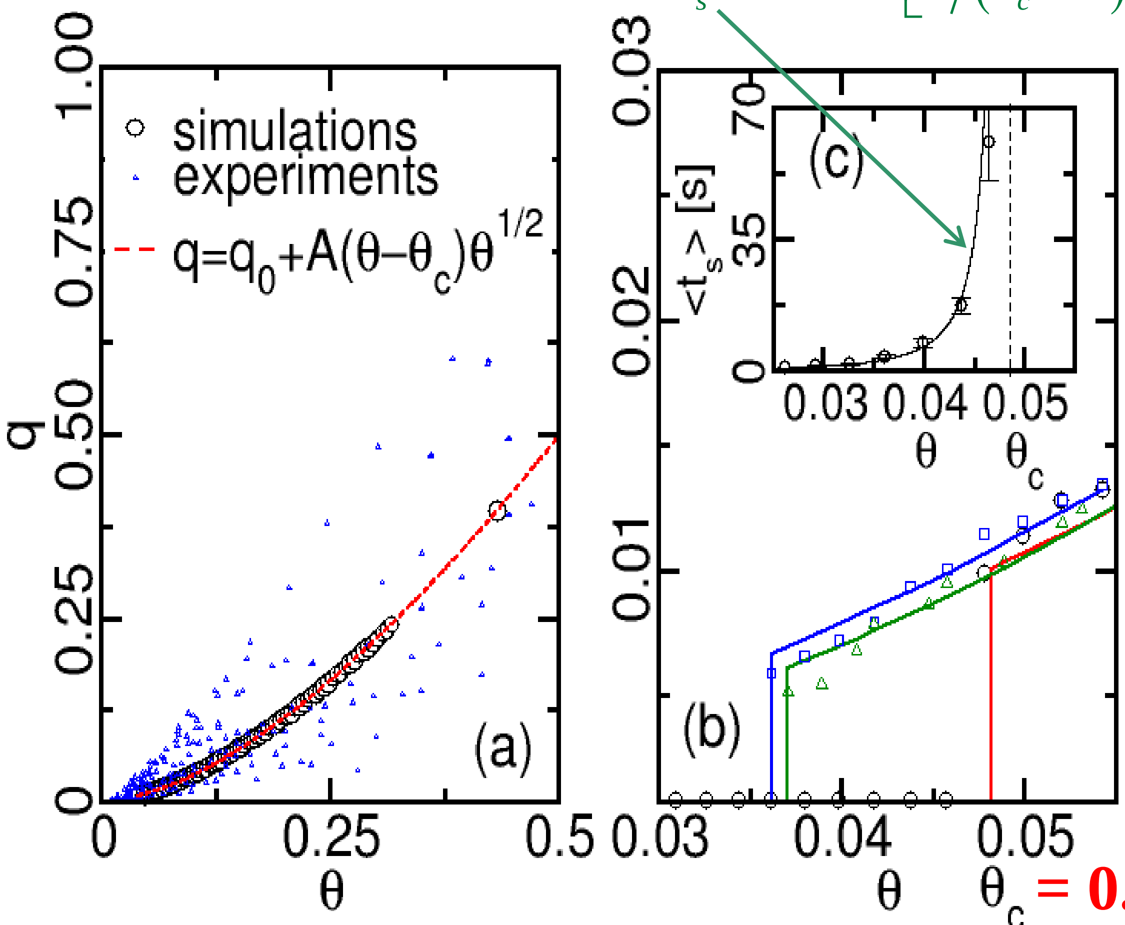


Normalized quantities

ETH

transient time t_s

$t_s = 0.524s\left[\theta/(\theta_c - \theta)\right]^{1.5}$



θ_c **= 0.048**

Iversen and Rasmussen, 1999

Shields number

$$\theta = \frac{u_*^2}{(s-1)\,gD_{av}}$$

with $s = \dfrac{\rho_p}{\rho_f}$

$$\tilde{q} = \frac{q}{\rho_p\sqrt{(s-1)\,gD_{av}^3}}$$

Perturbations

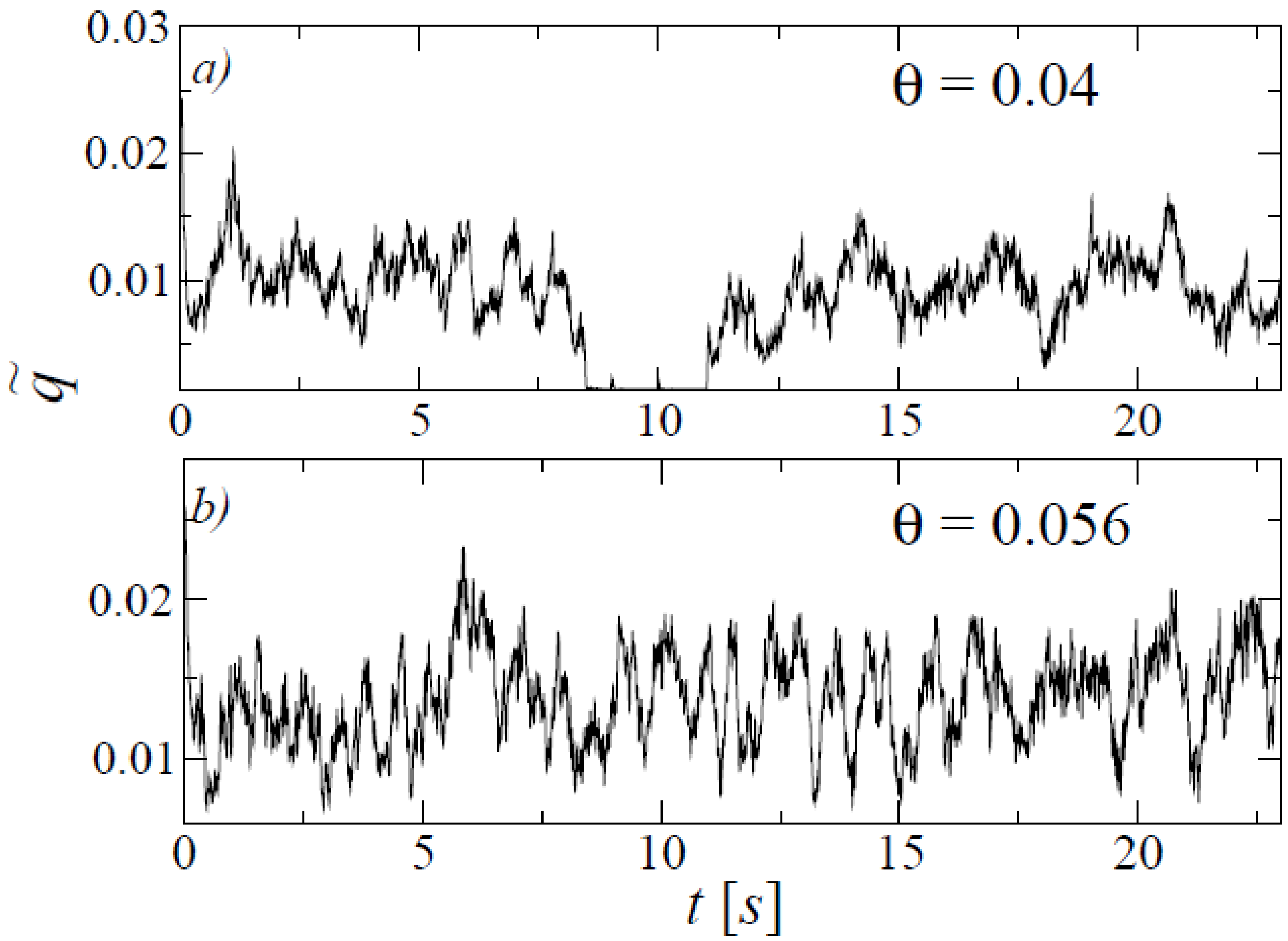
- **lift forces**:
  Apply to the particles close to the surface a force

$$\boxed{F_l = \frac{\pi D_{av}^3}{8} \rho_{fl} C_l \nabla v_{rel}^2} \qquad C_l = 0.425 C_d$$

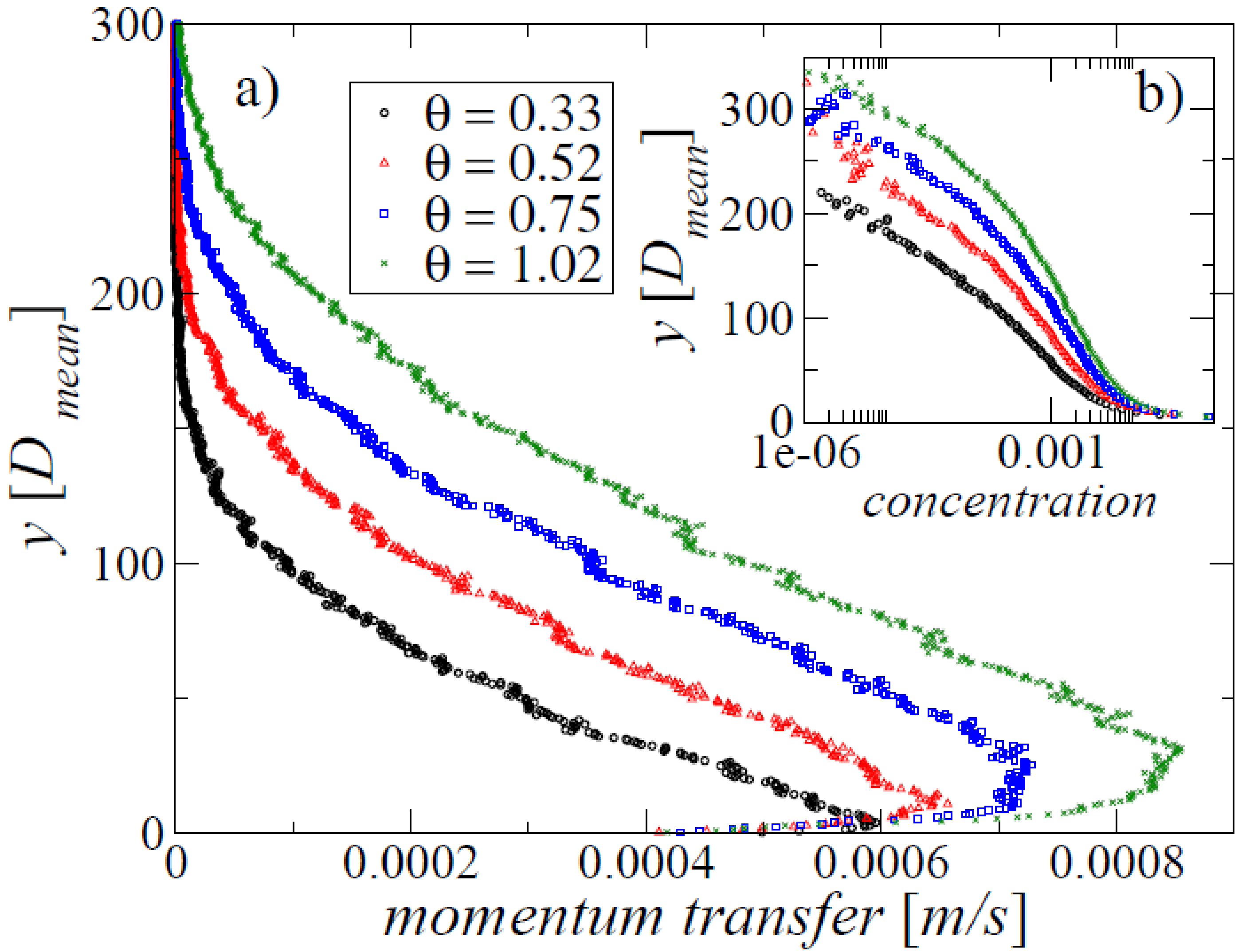
- **random forces**:

Move every second randomly 20%
of the particles at the surface
up by one average diameter D_{av} .

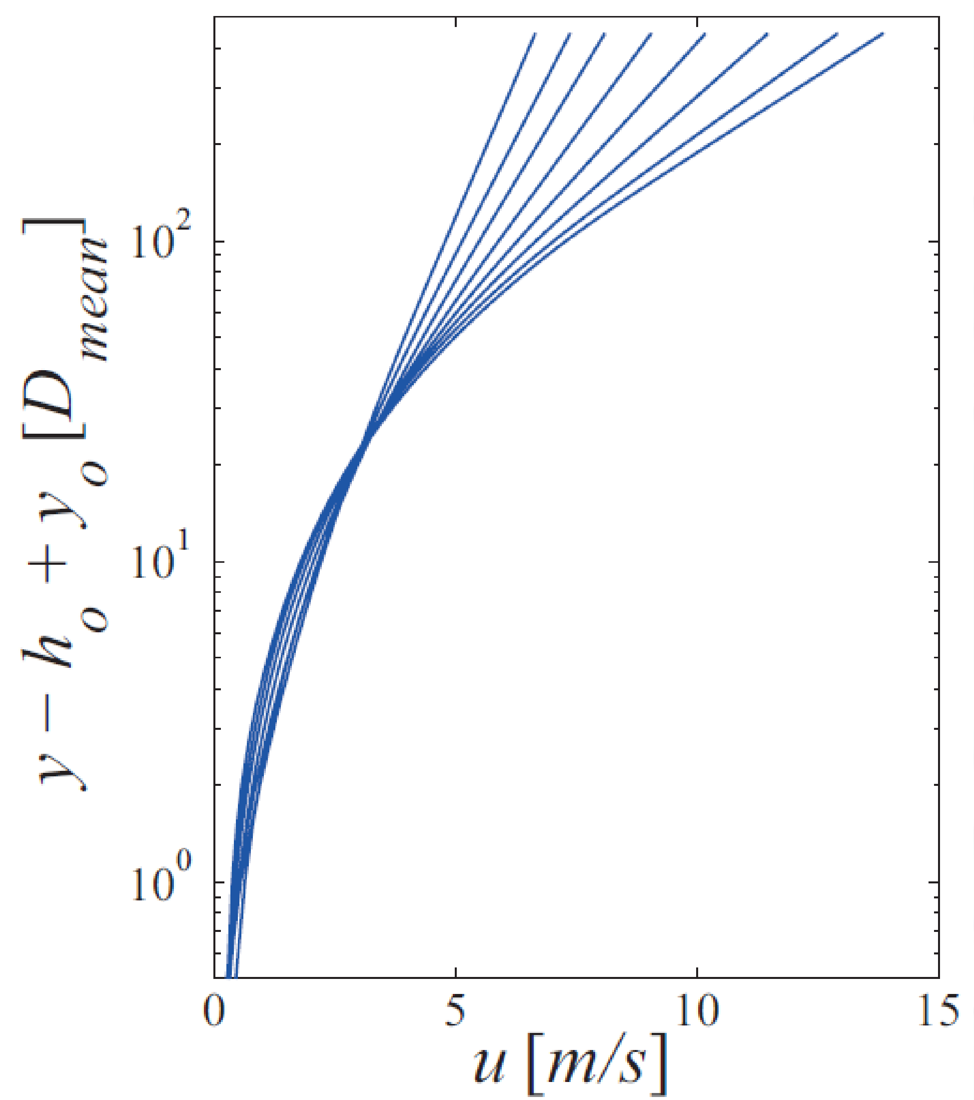
Time series of the flux near threshold



Height dependence of momentum exchange

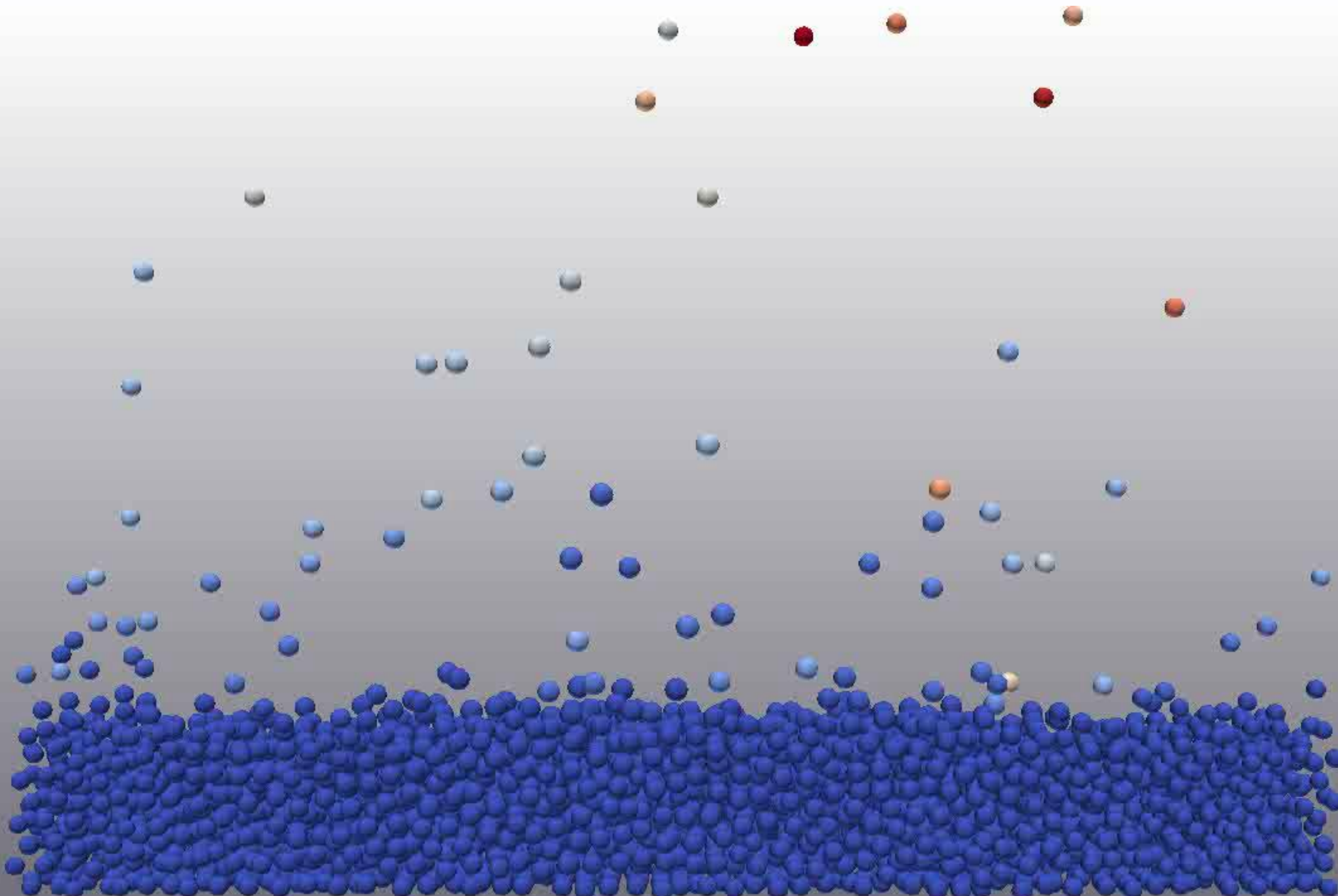


Bagnold Focus



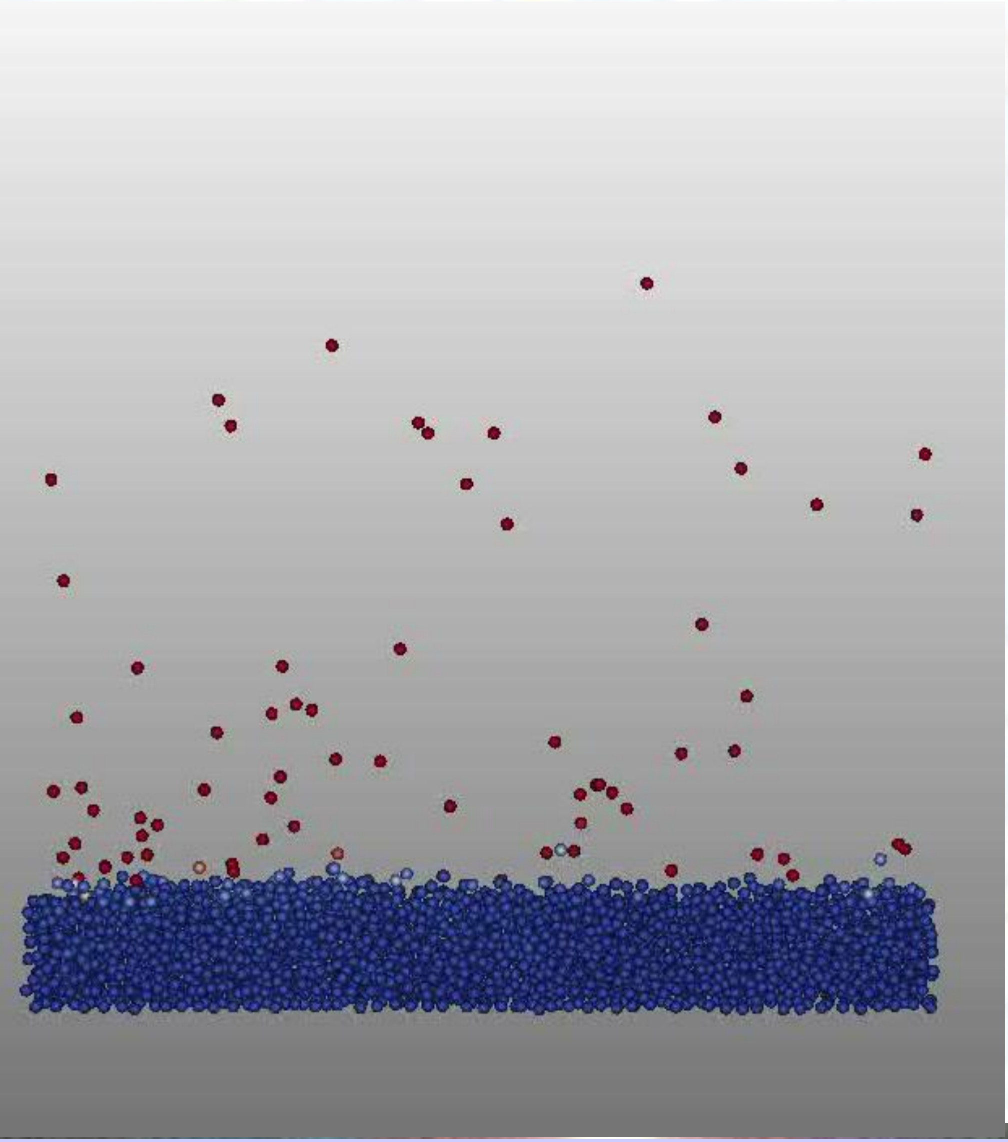
Modified wind profiles intersect for $\theta > \theta_c$

$\theta = 0.15$ in 3d

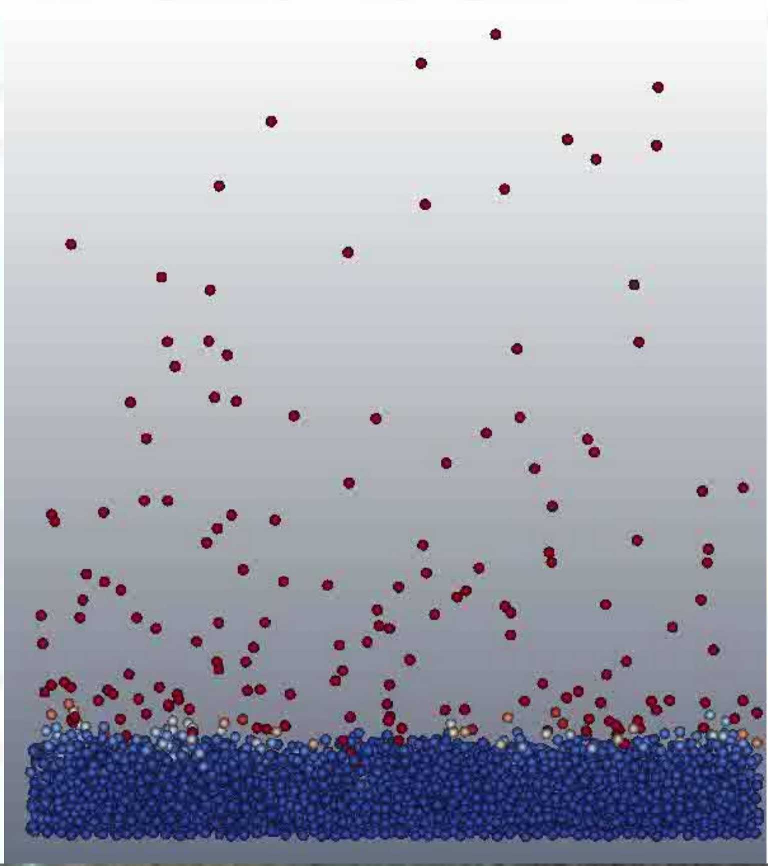


Impact on the ground

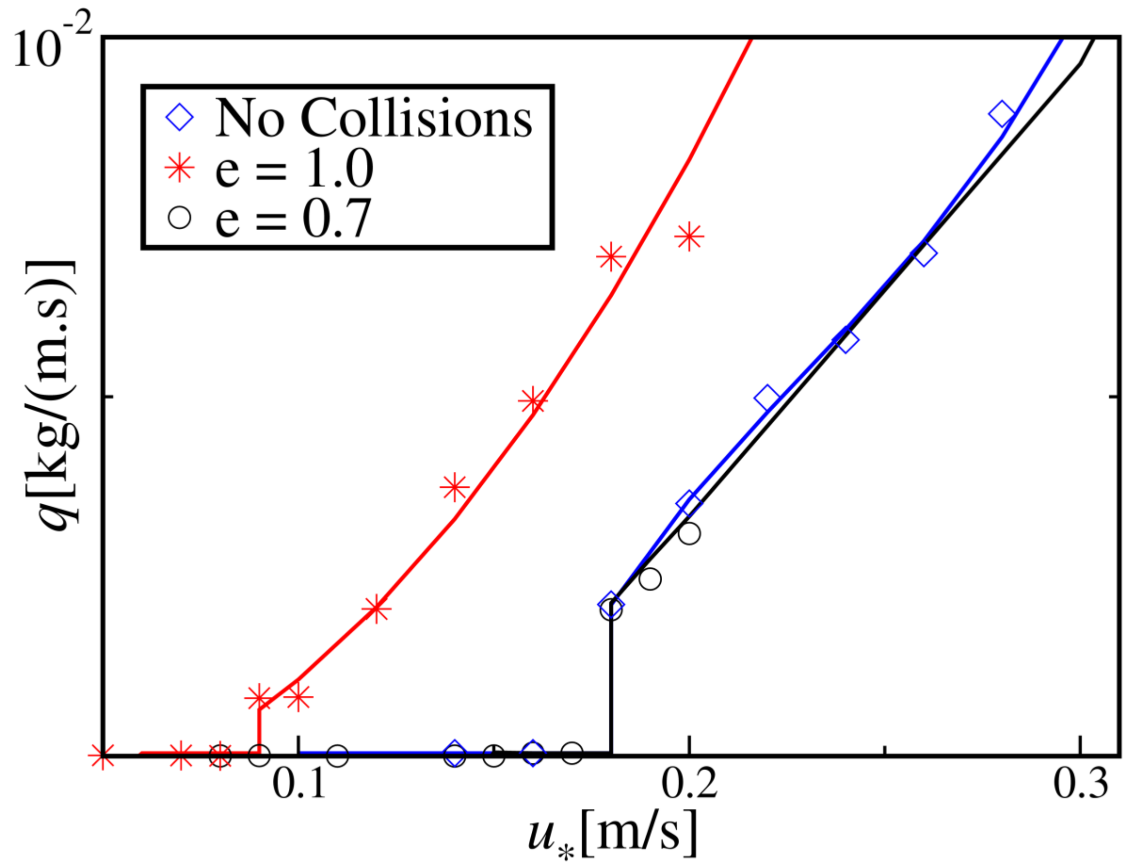
$u_* = 0.8$



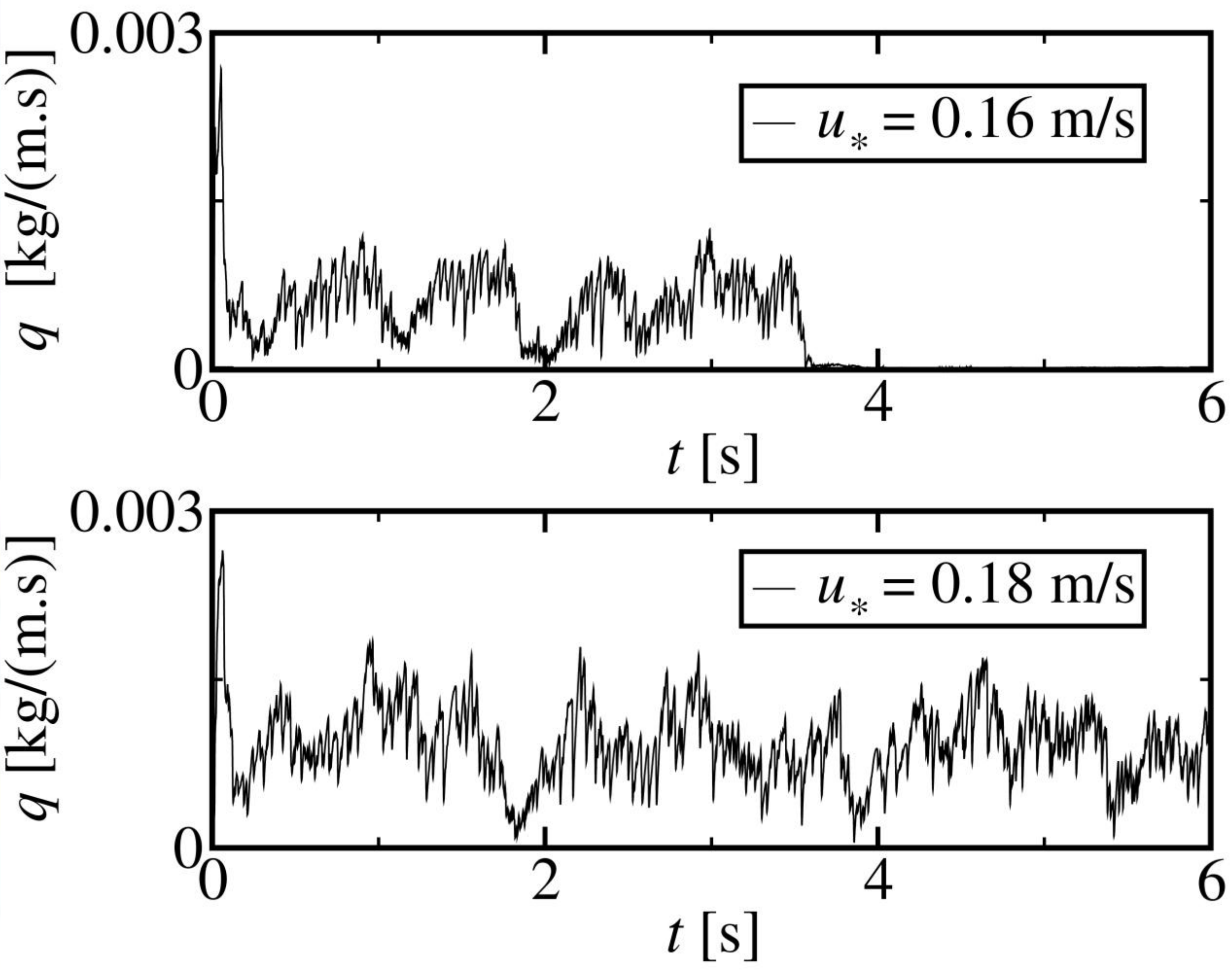
$u_* = 1.2$



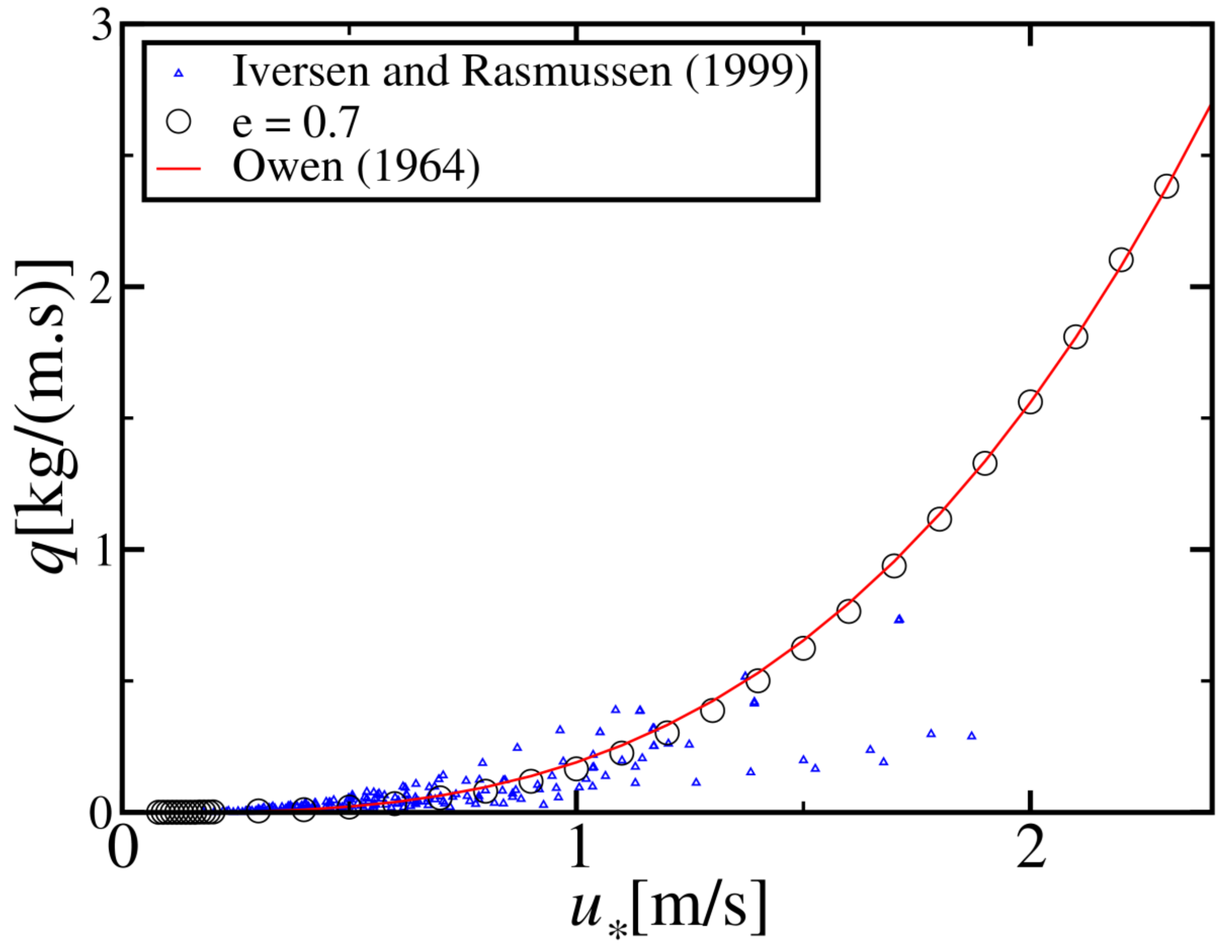
Jump in the saturated flux in 3d



Time series close to jump in 3d

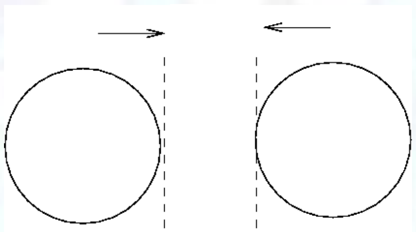


Comparison with experiments and fit



3d

The role of mid-air collisions



Marcus Vinicius Carneiro

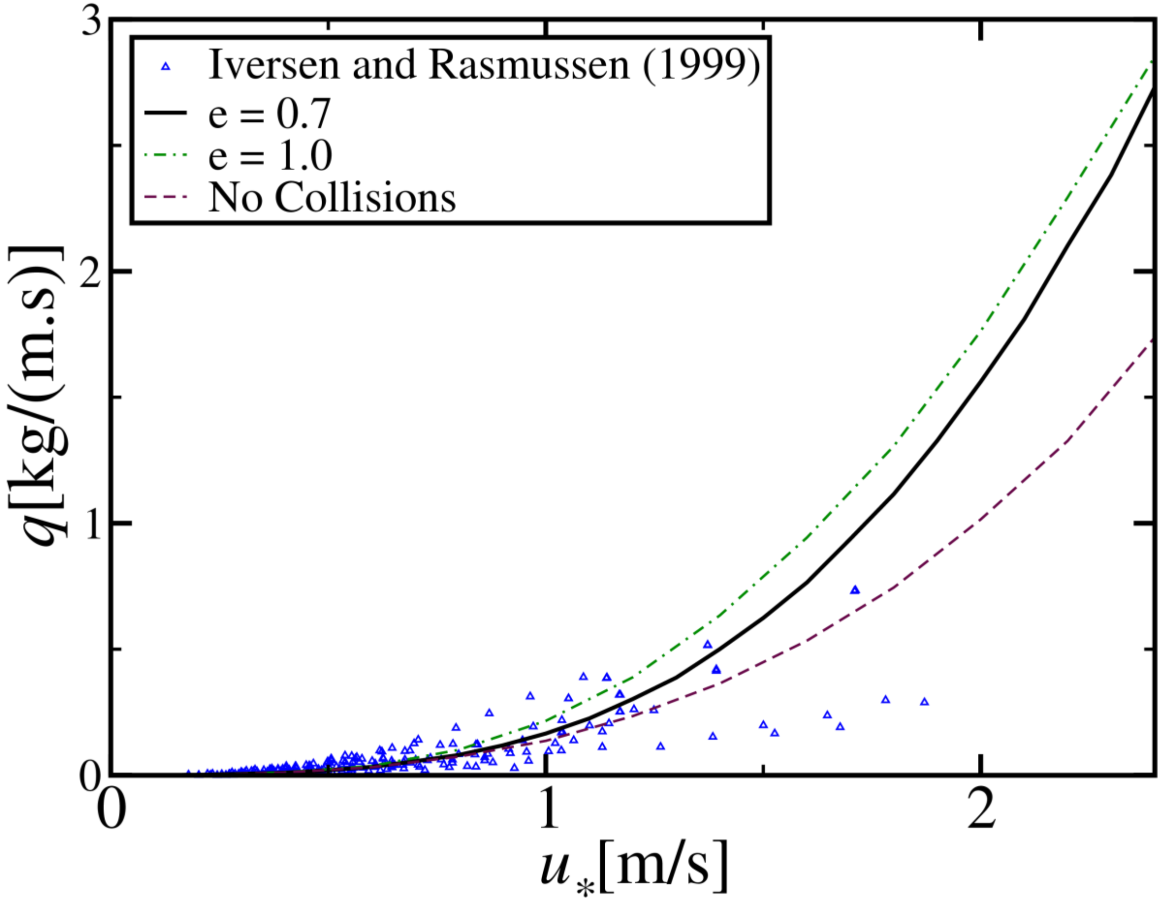
Thomas Pähtz

Nuno Araújo

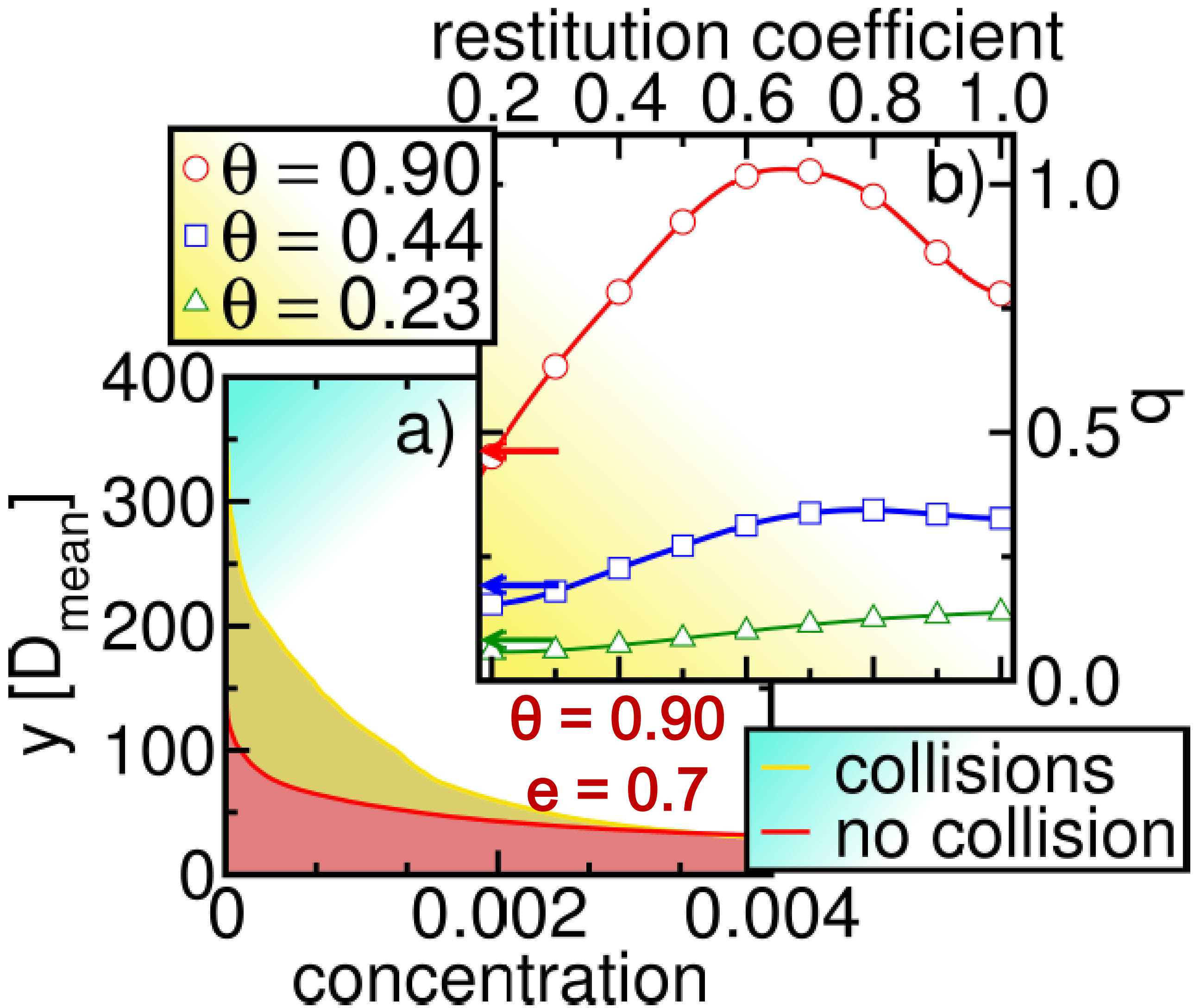
Common belief has generally been that collisions between grains in the air are not relevant.

Effect of collisions in 3d

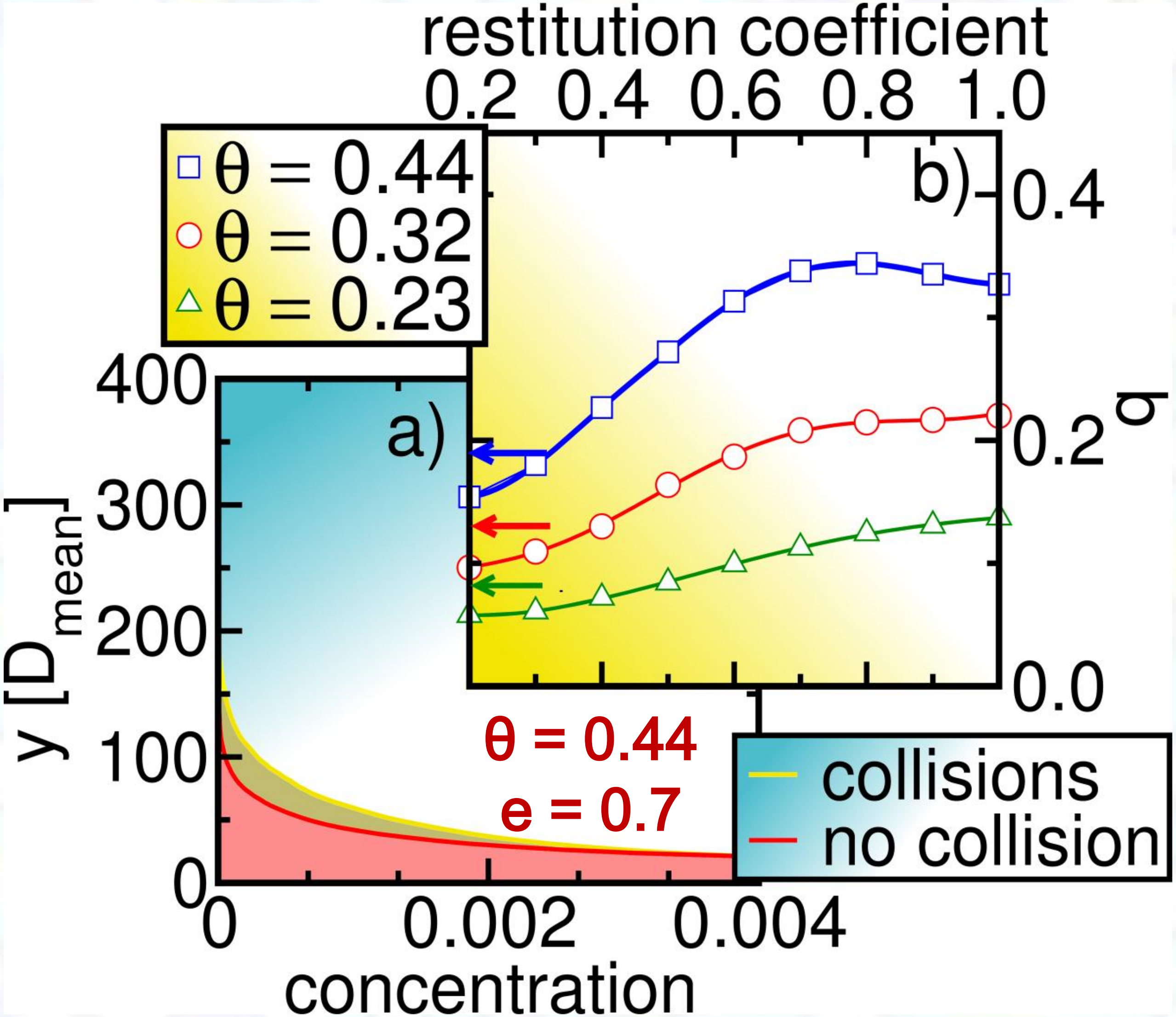
Changing restitution coefficient for mid-air collisions, while for collisions with the ground we keep $e_n = 0.7$.



Flux as function of restitution coefficient

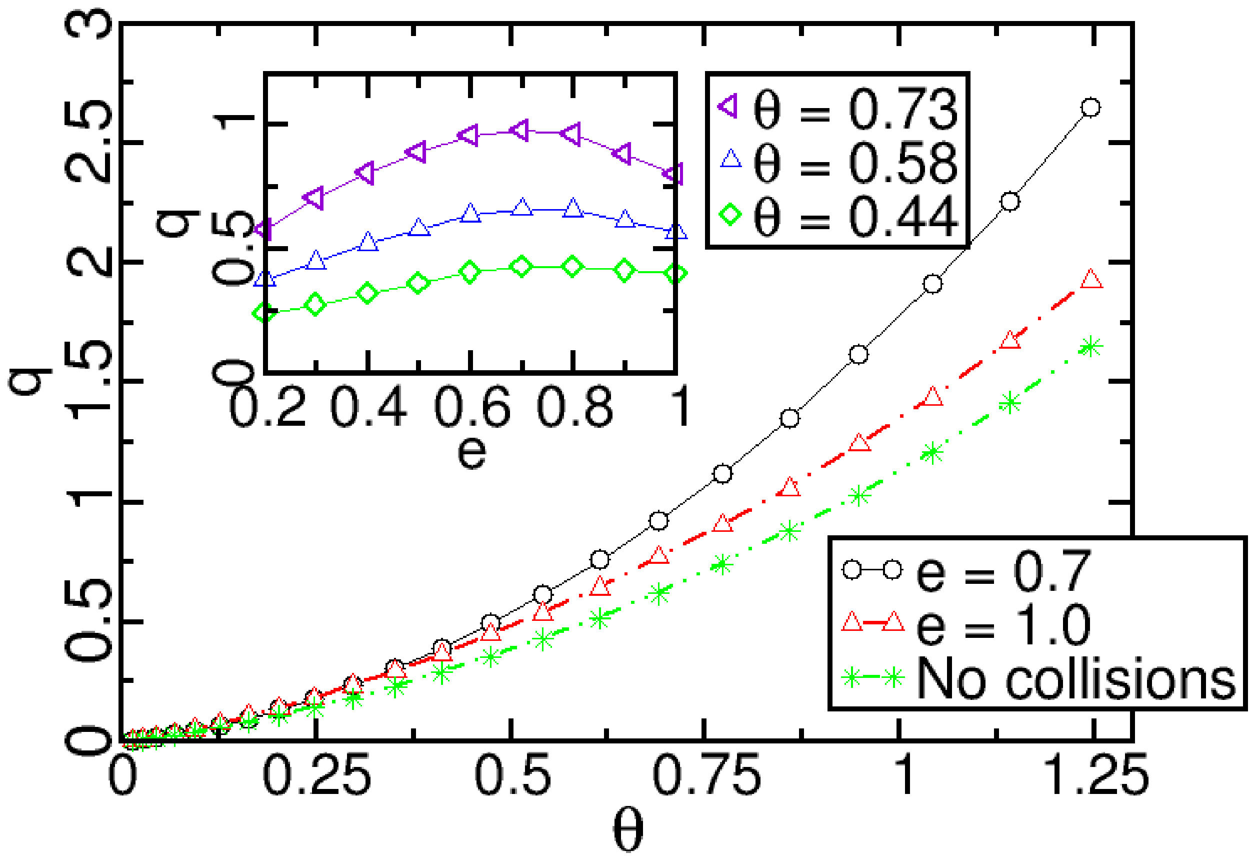


Flux as function of restitution coefficient

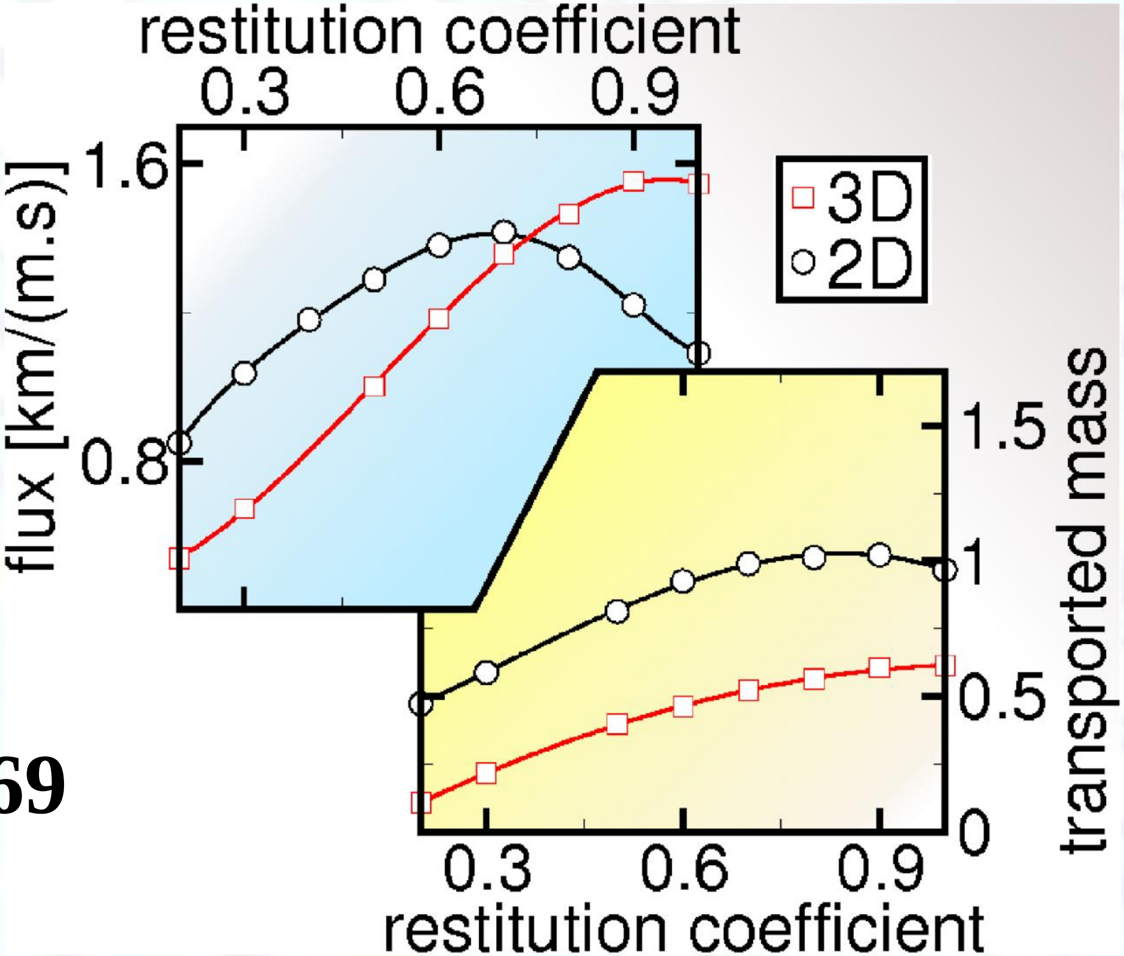


Saturated flux in 2d

Changing restitution coefficient for mid-air collisions, while for collisions with the ground we keep e_n = 0.7.

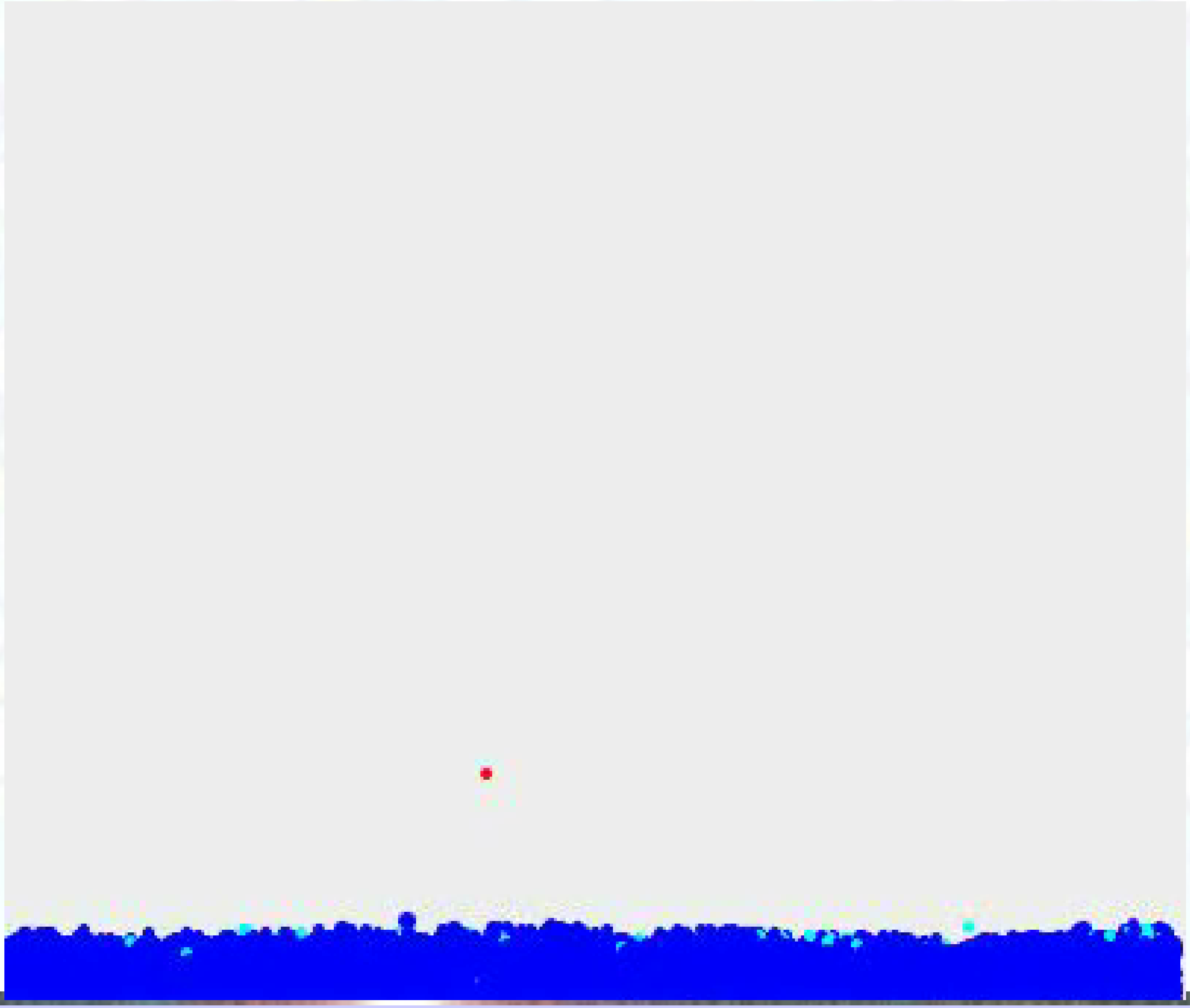


Comparing 2d and 3d



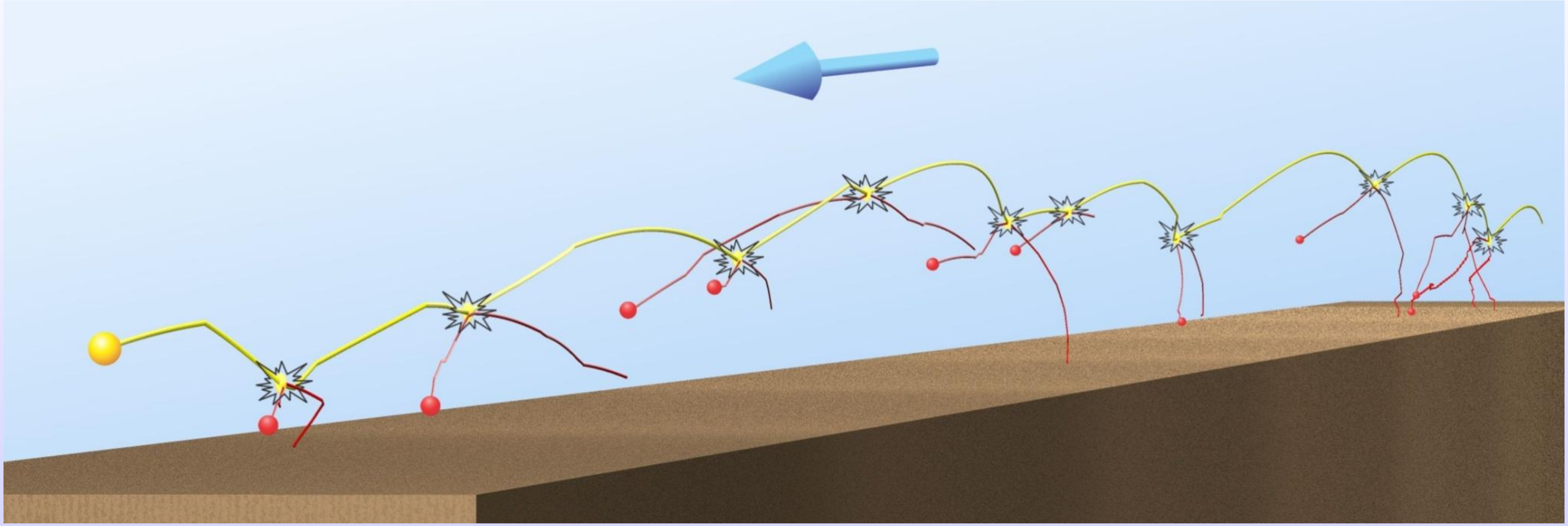
θ = 0.69

Role of collisions: saltons



θ = 0.13
e = 0.7

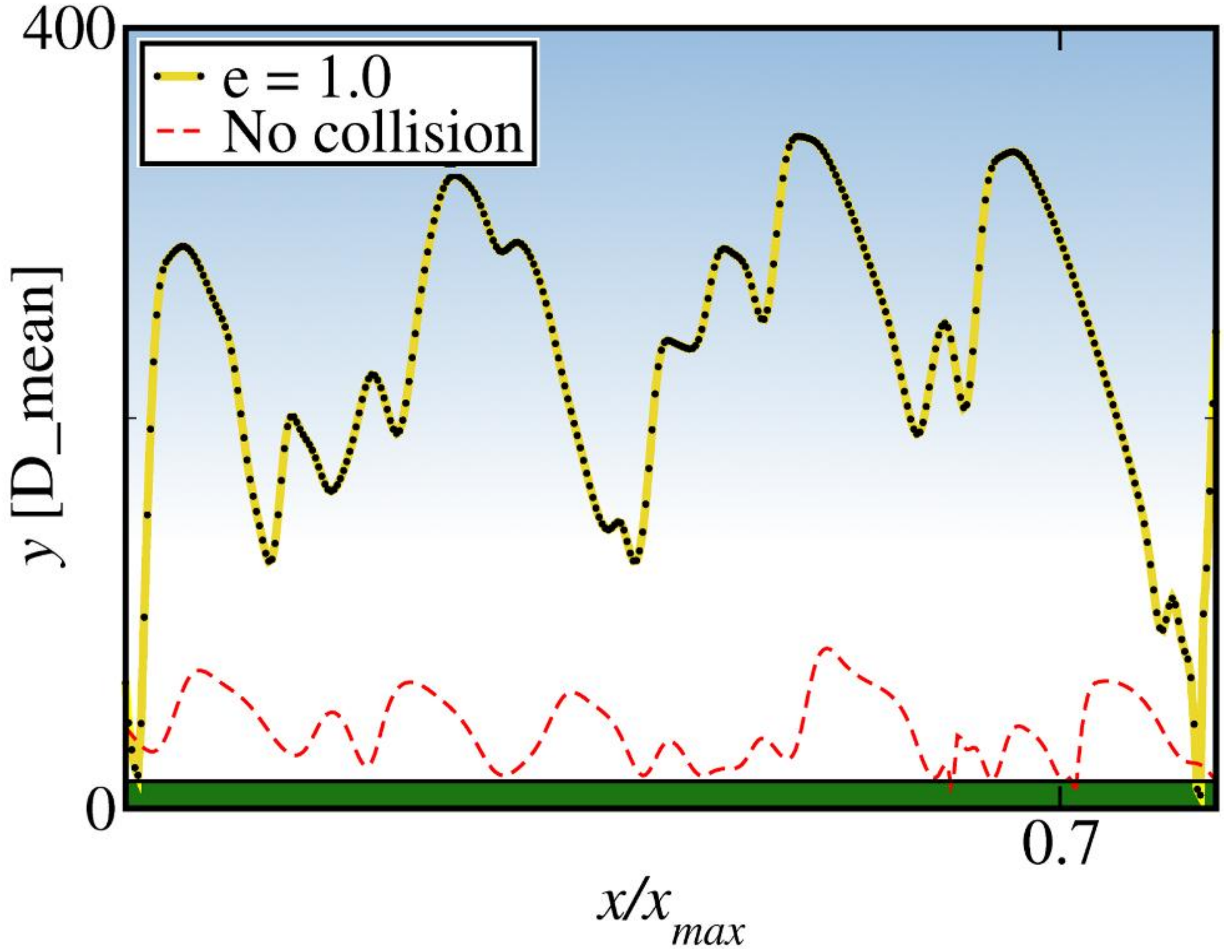
Role of collisions: saltons



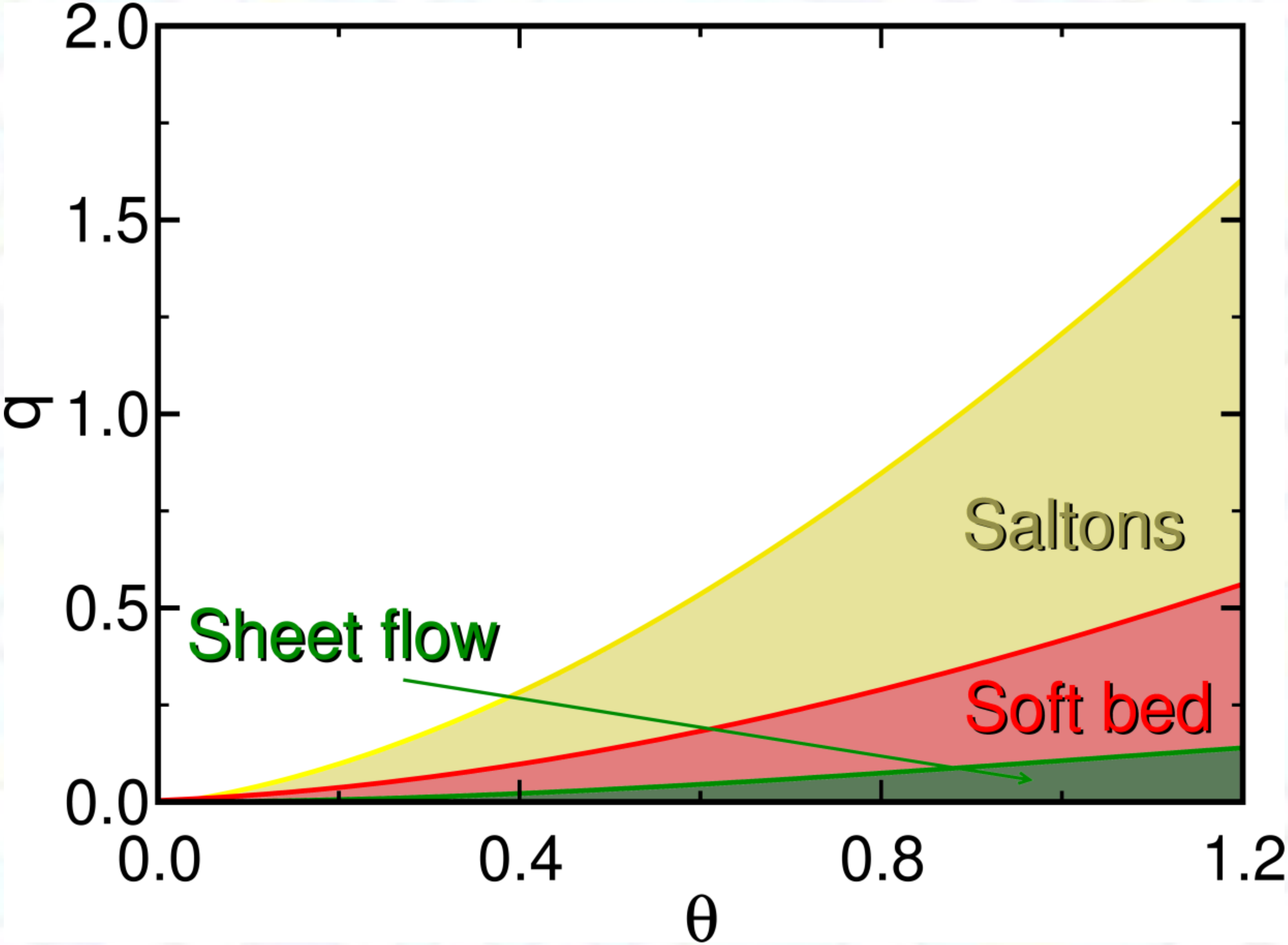
Saltons jumping on the soft bed

$\theta = 0.90$
$e = 0.7$

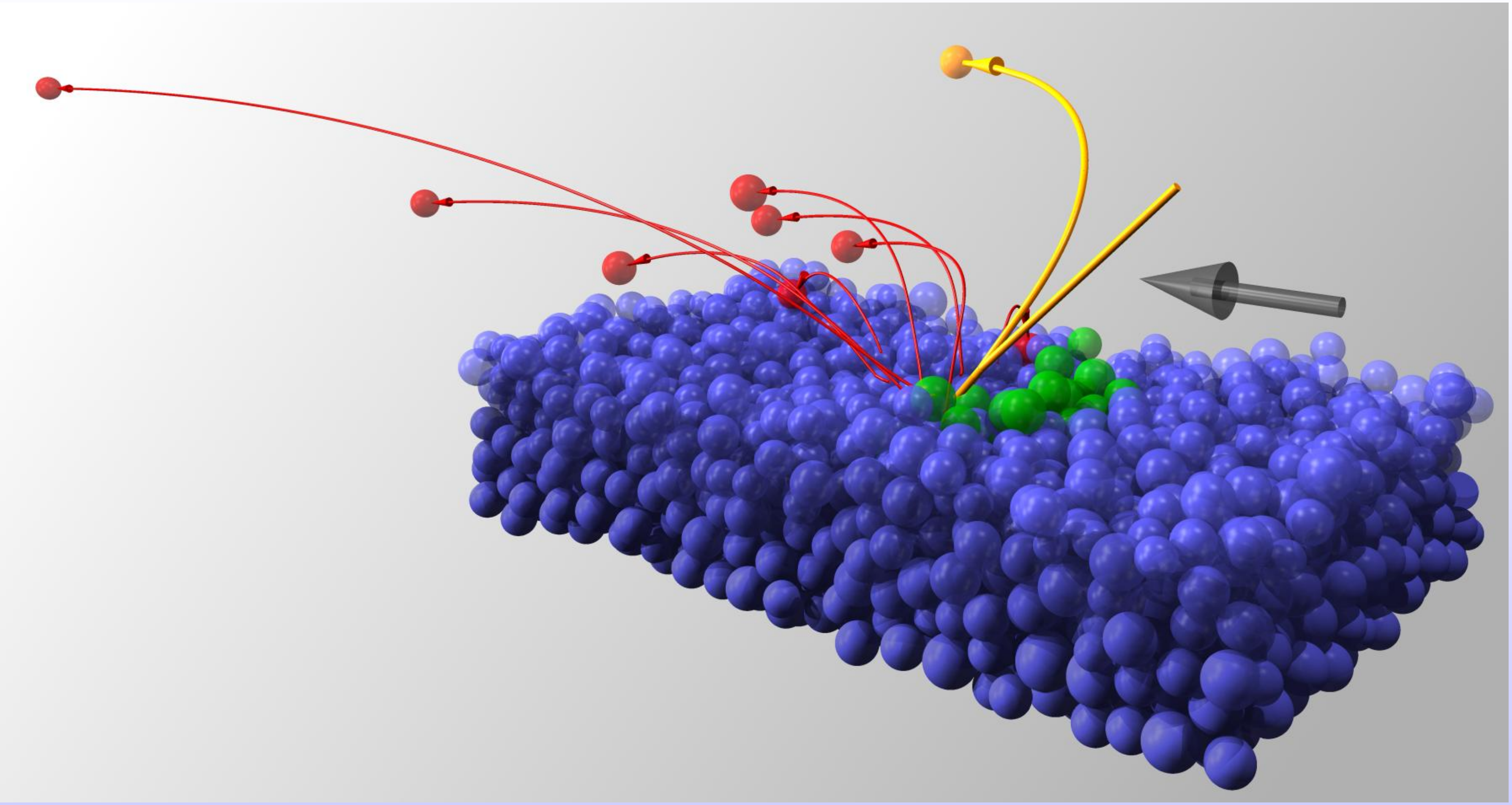
Role of collisions: saltons



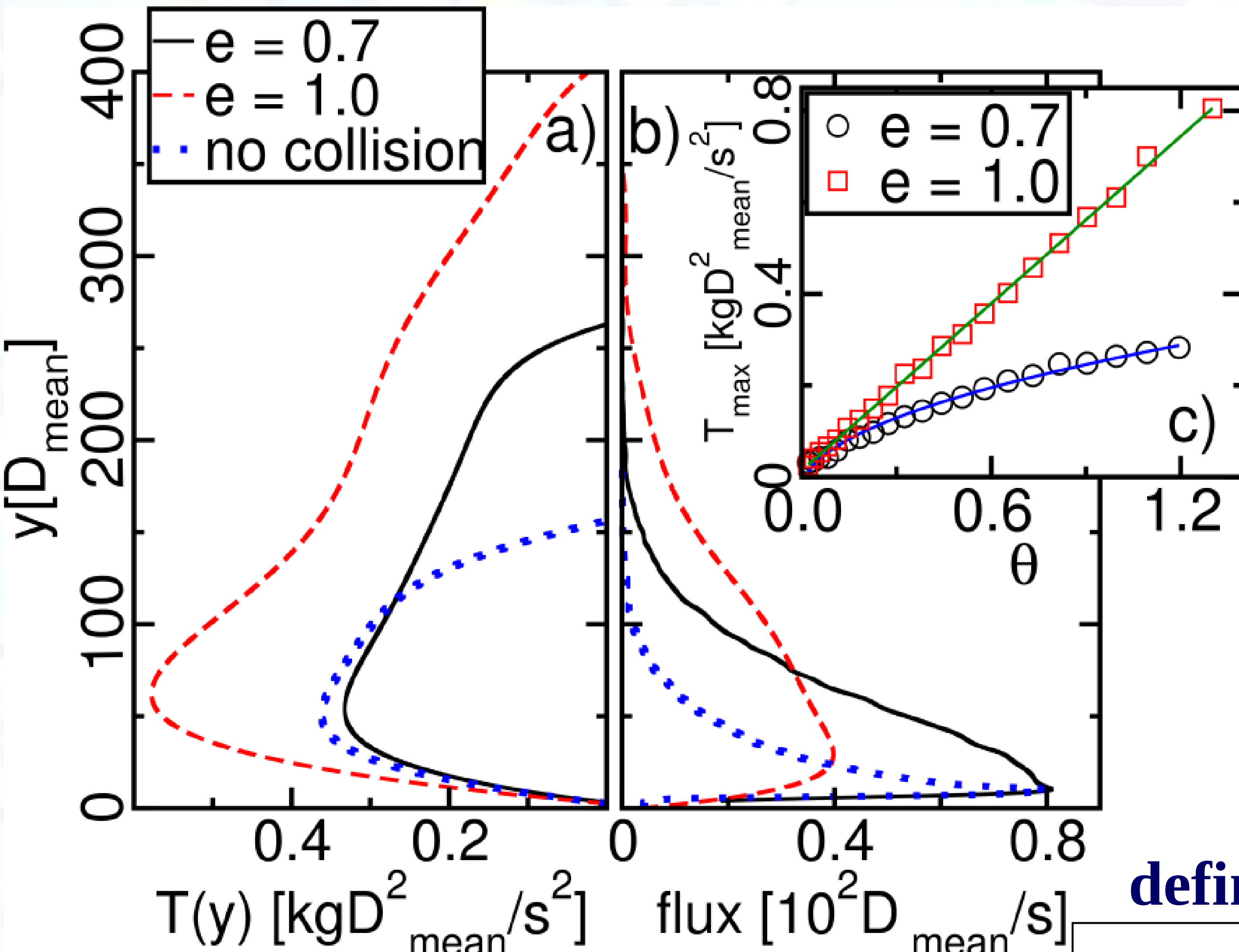
Contribution to the flux



Splash after impact of salton



Temperature and flux profiles



$u_* = 2.0$ m/s

definition of temperature:

$$T(y) = \frac{1}{3N(y)} \sum_{i}^{N(y)} m_i \left(v_i - v_m(y)\right)^2$$

Pasini and Jenkins, 2005

Particle alignment due to dissipation

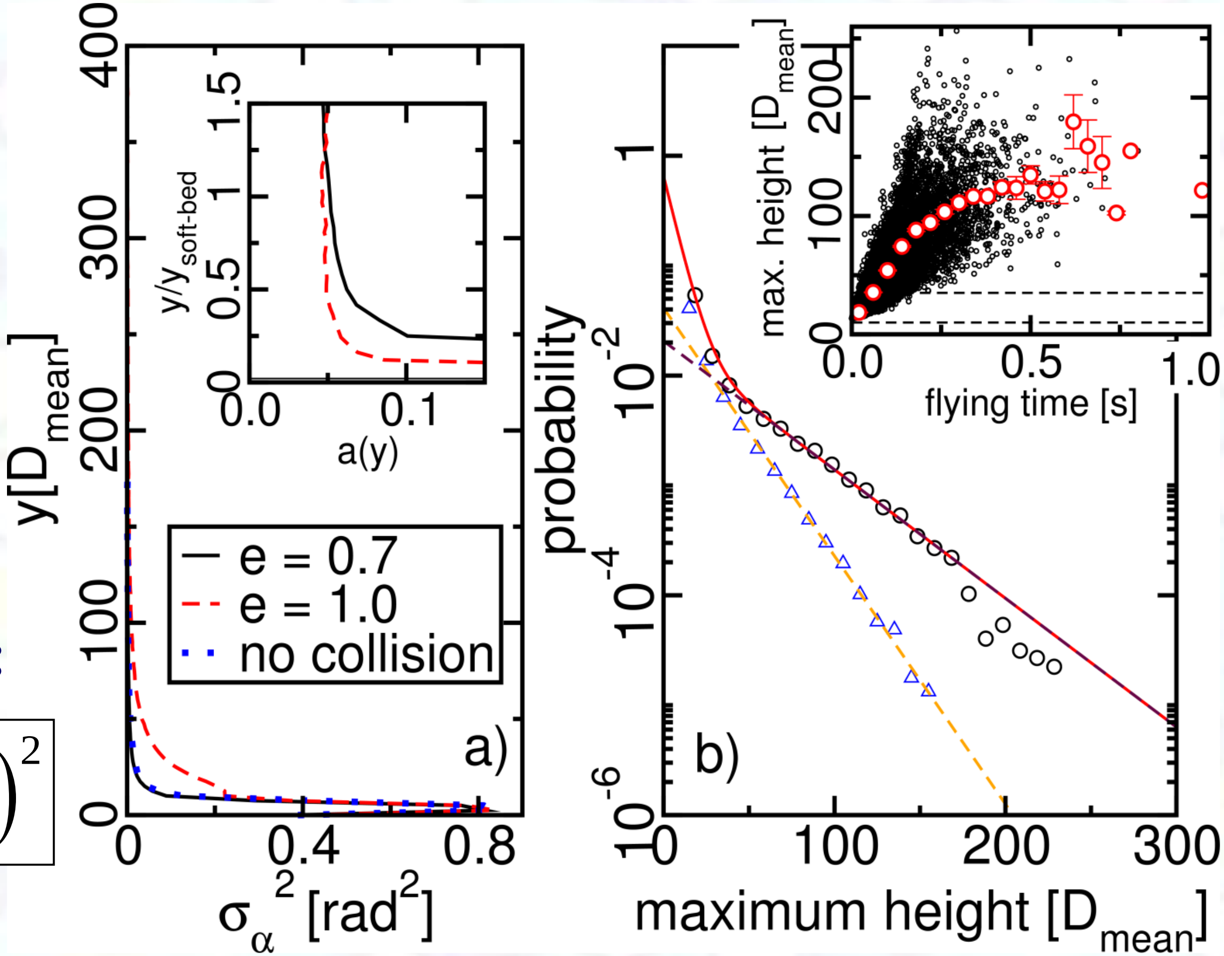
$$\boxed{\sigma_\alpha^2 = \langle \alpha^2 \rangle - \langle \alpha \rangle^2}$$

α is angle of particle trajectory with horizontal

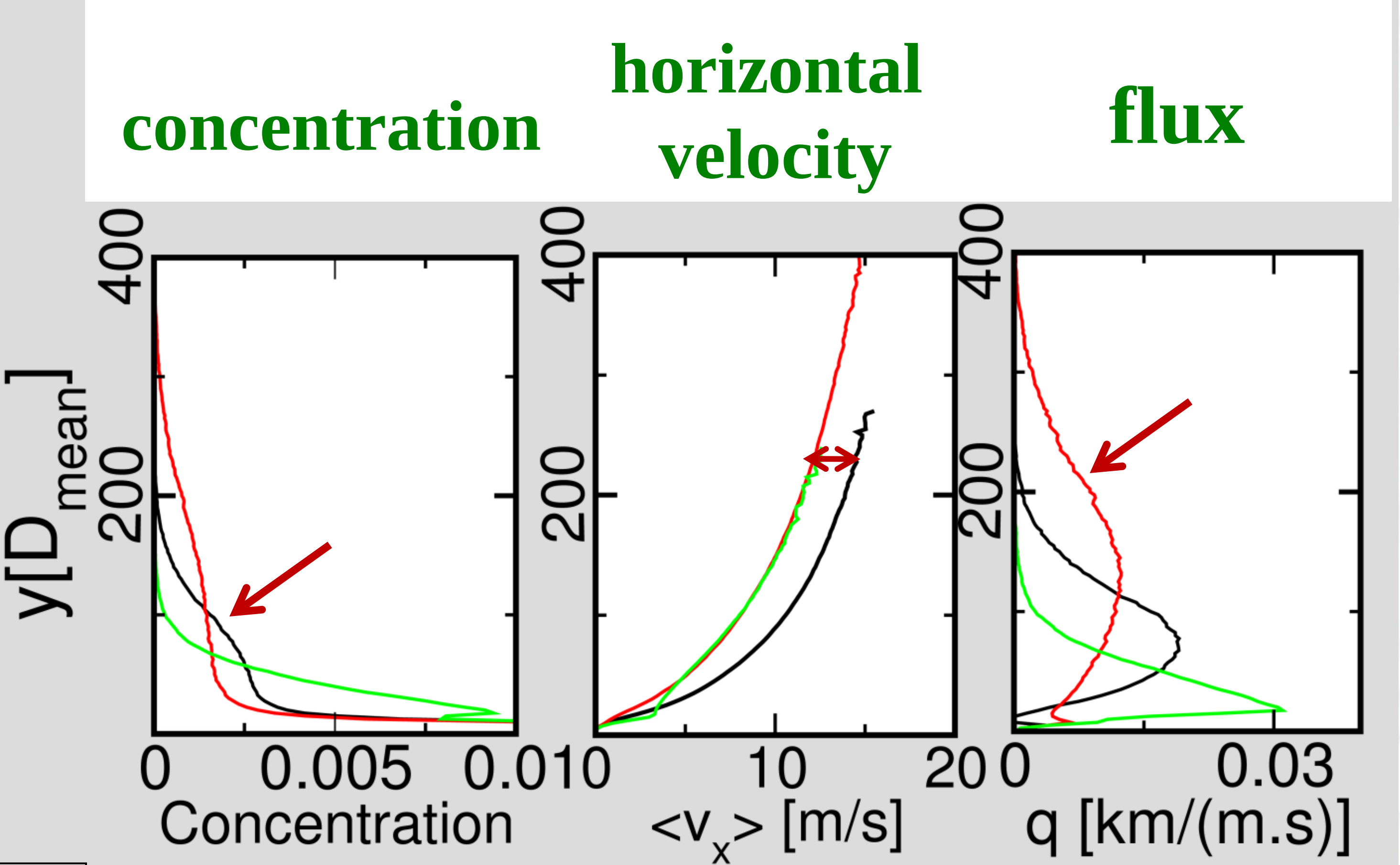
temperature anisotropy:

$$\boxed{a(y) = \langle v_y^2 \rangle / \left(v_x - \langle v_x \rangle\right)^2}$$

θ = 0.69



Height profiles of concentration, horizontal grain velocity and flux in 3d

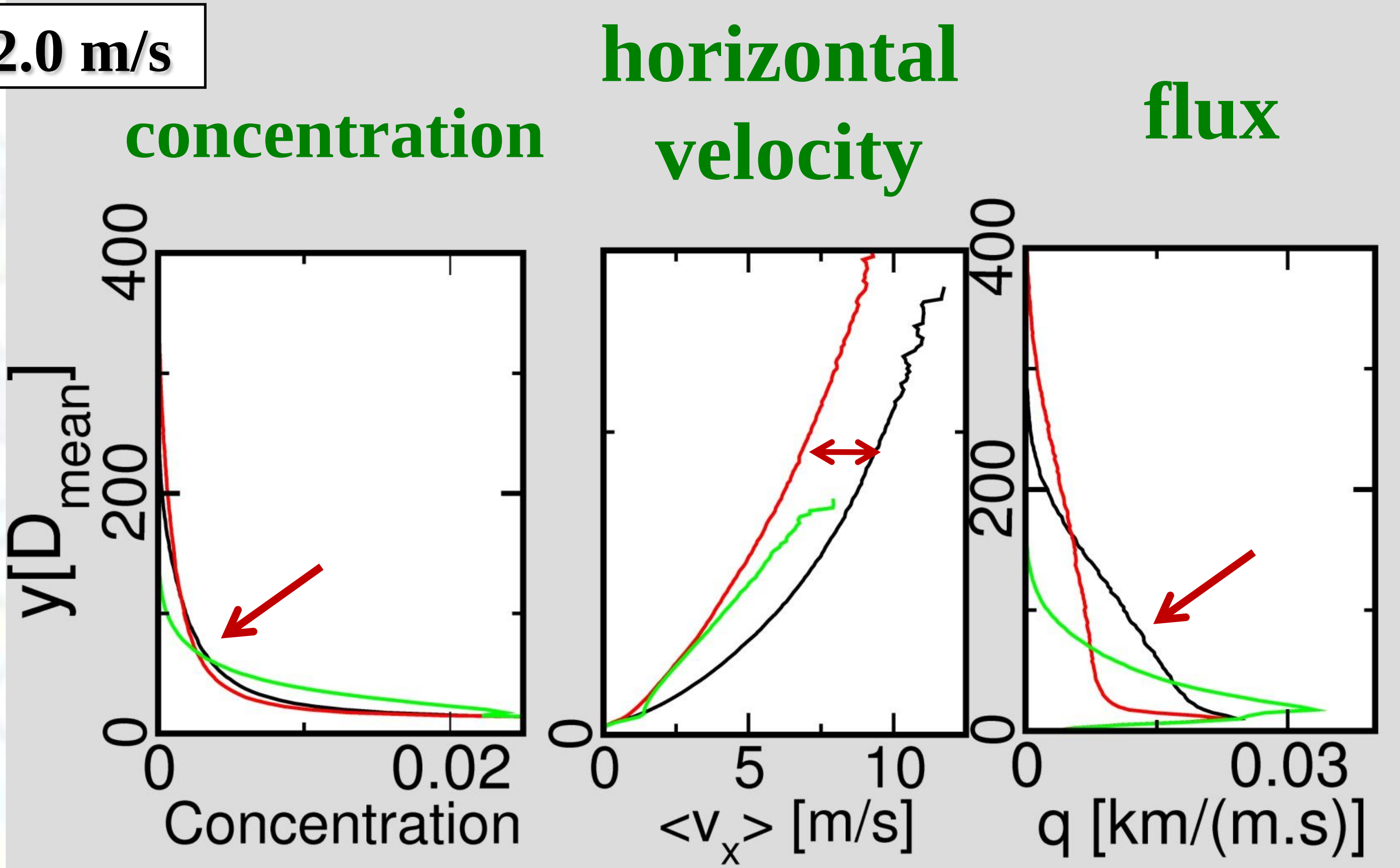


$u_* = 2.0$ m/s

$e_n = 0.7$, $e_n = 1.0$, no collisions

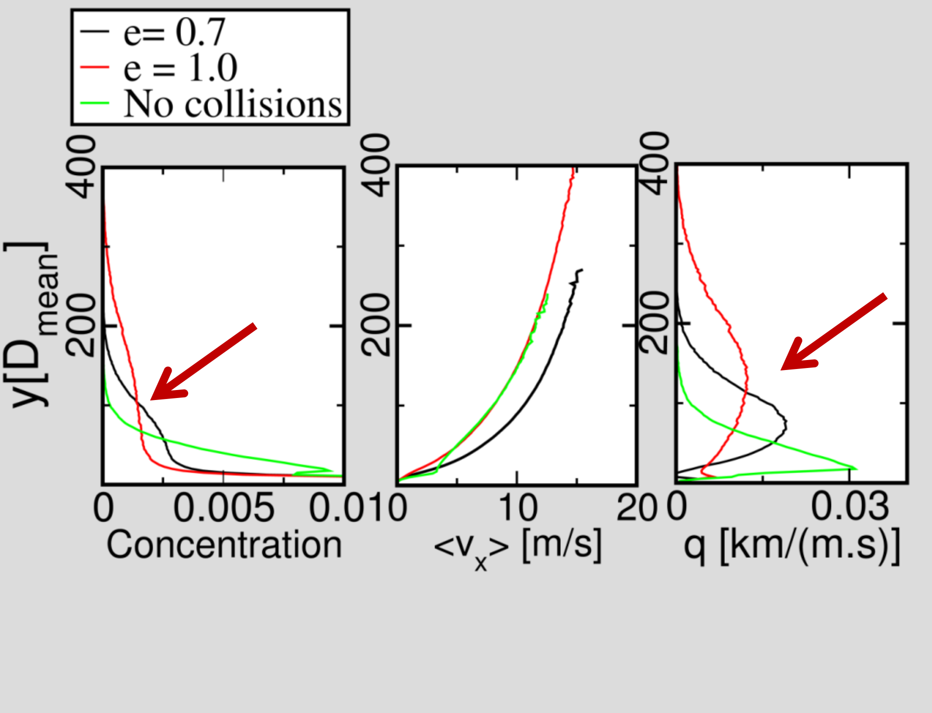
Height profile of concentration, horizontal grain velocity and flux in 2d

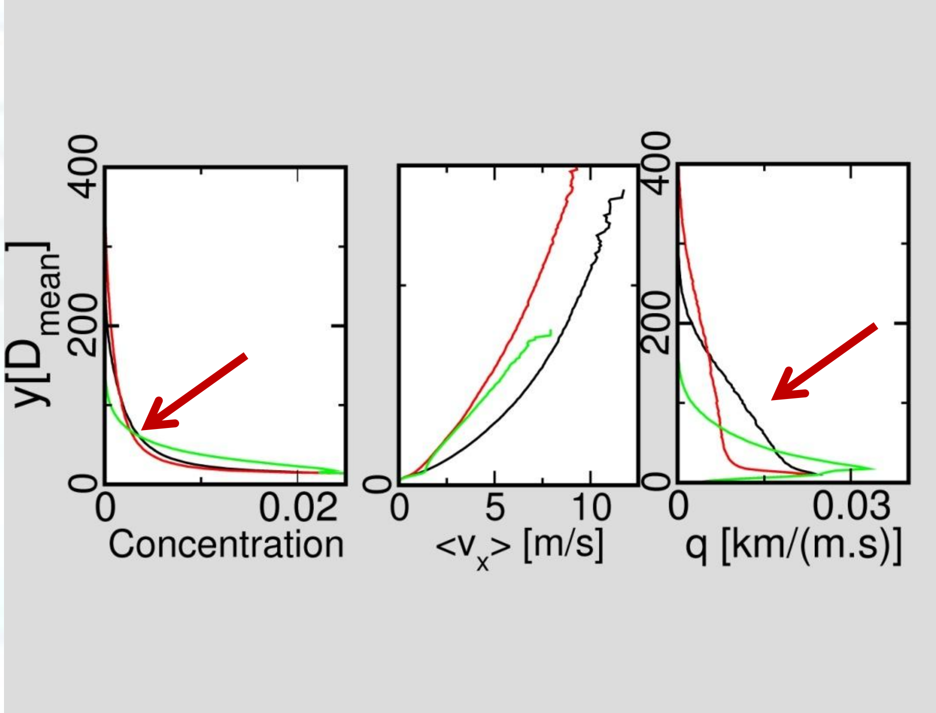
$u_* = 2.0$ m/s



$e_n = 0.7$, $e_n = 1.0$, no collisions

Comparing 2d and 3d





3d

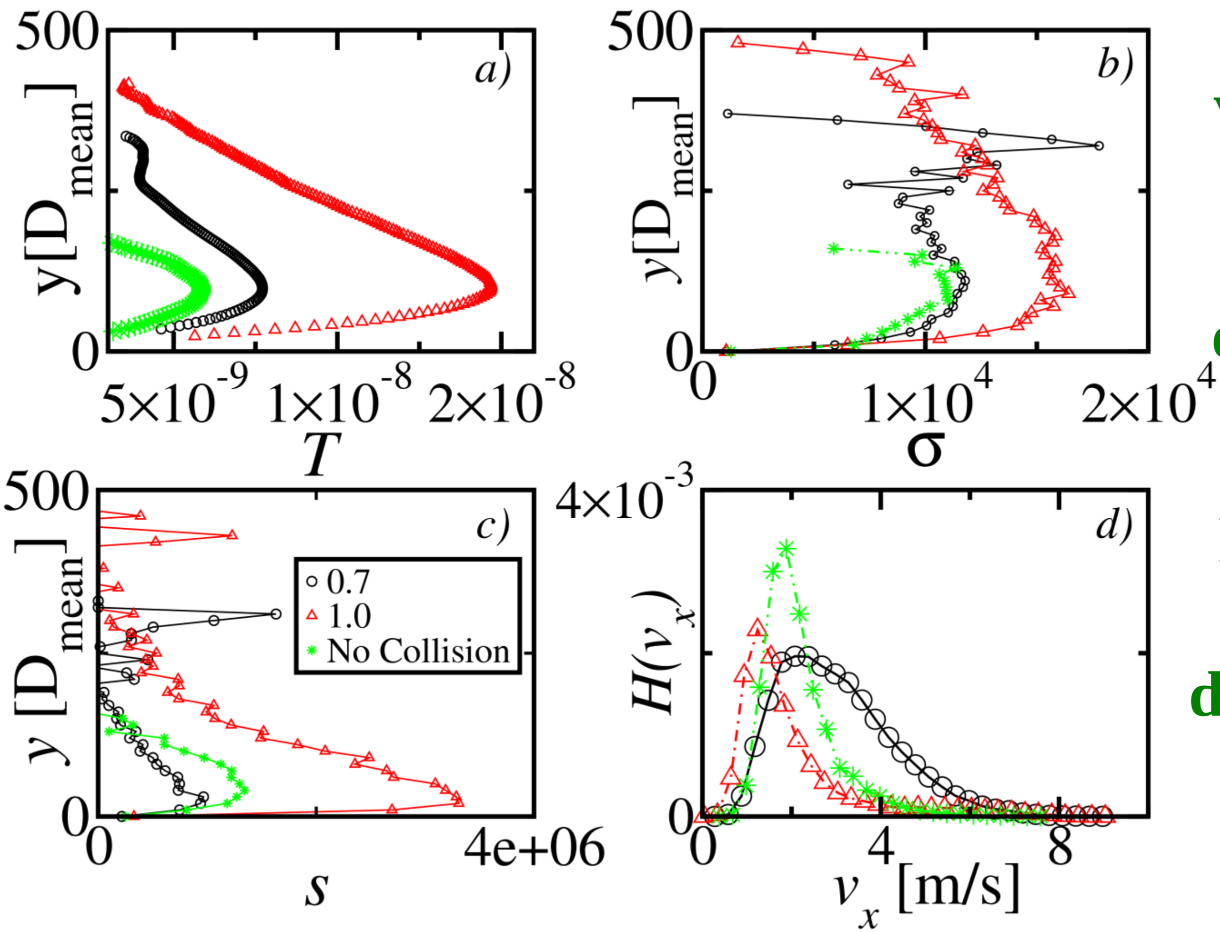
2d

$u_* = 2.0$ m/s

more details in 2d

granular temperature (fluctuation of vertical velocity)

skewness of horizontal velocity distribution



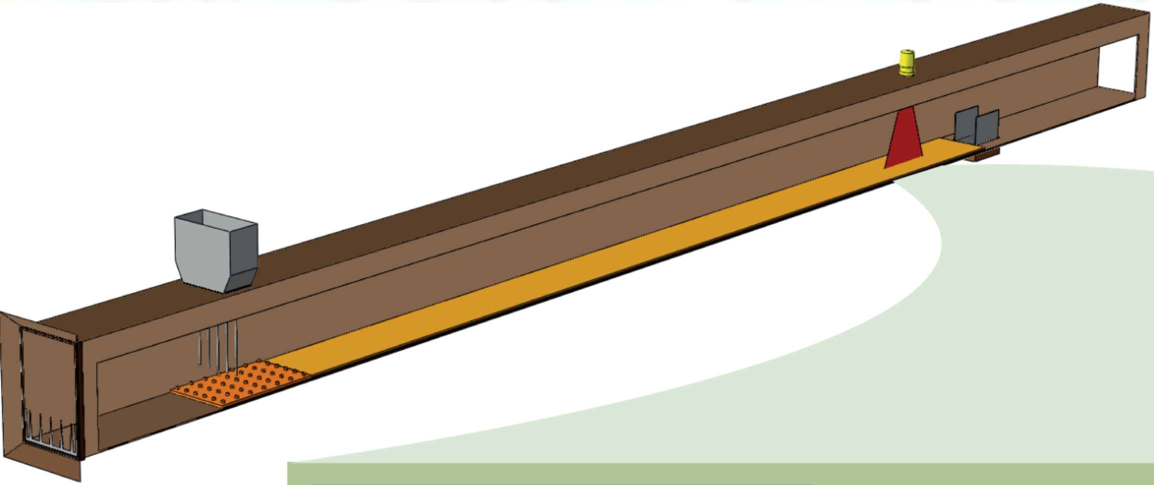
variance of horizontal velocity distribution

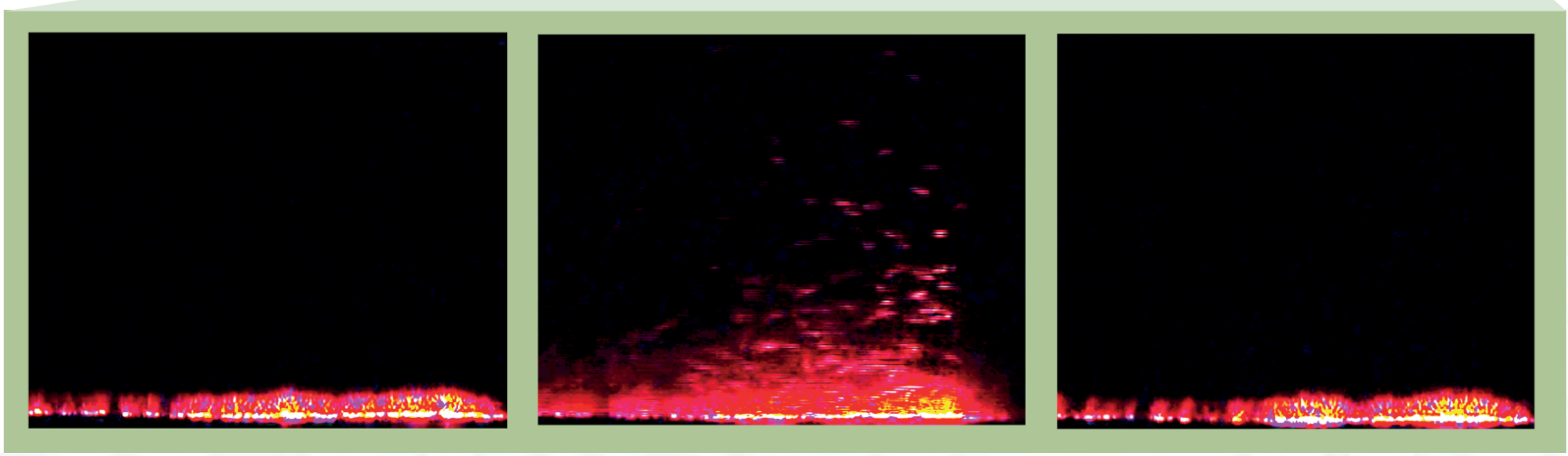
horizontal velocity distribution at y = 35

e_n = 0.7, e_n = 1.0, no collisions

Wind Channel in Aarhus

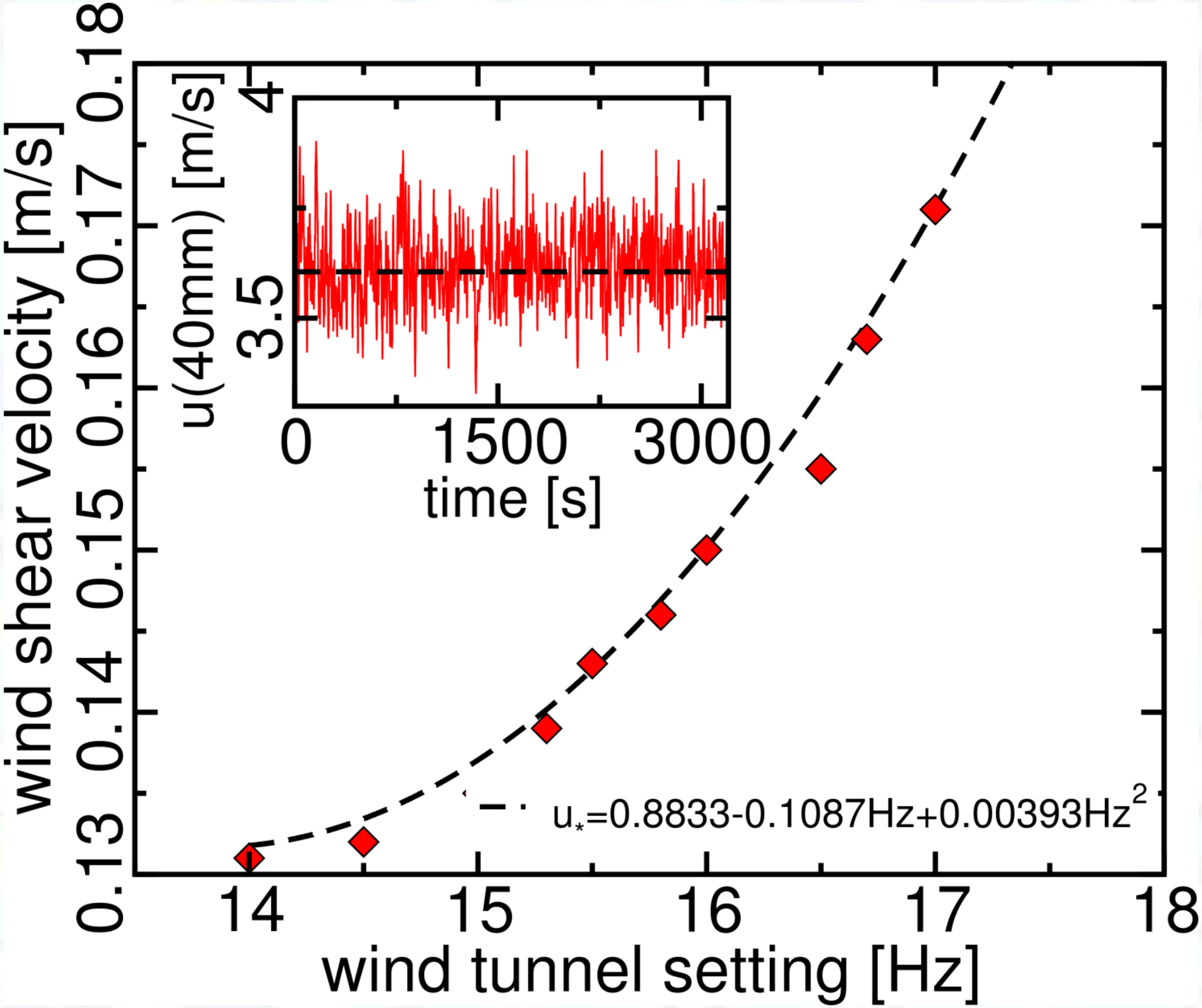
Keld Rasmussen



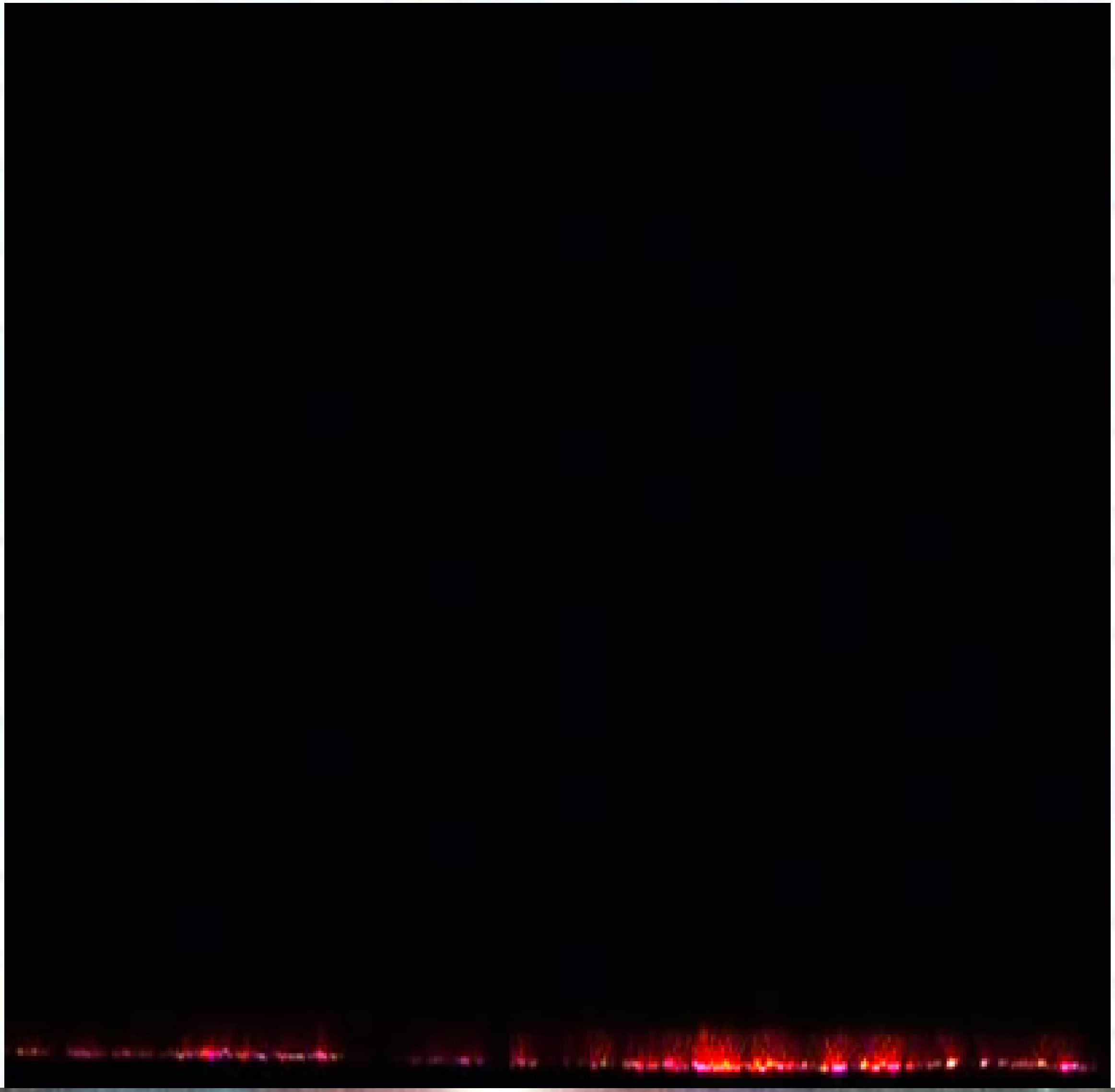


Wind Channel in Aarhus

Measuring the wind velocity in channel

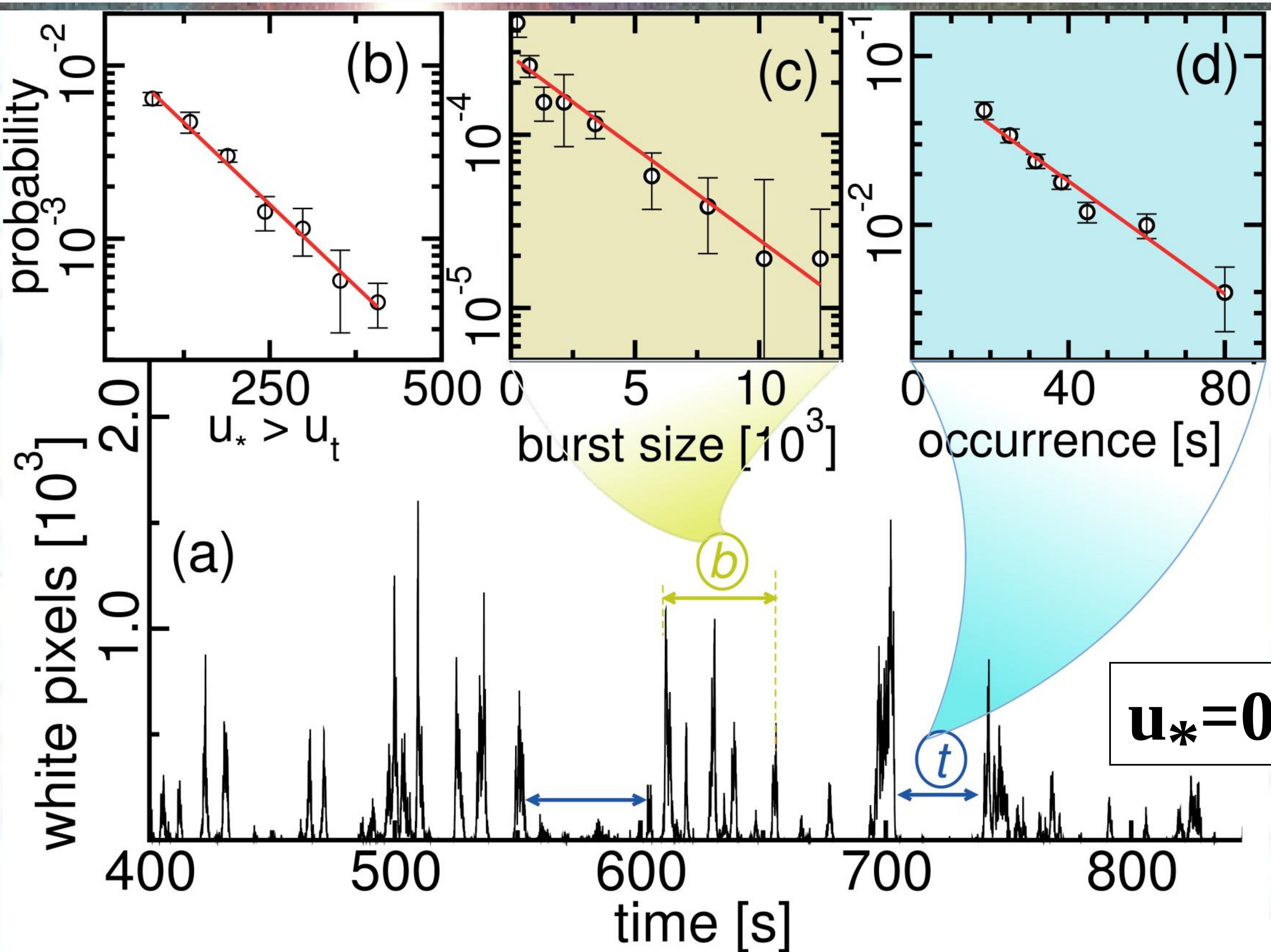


Intermittent Flow



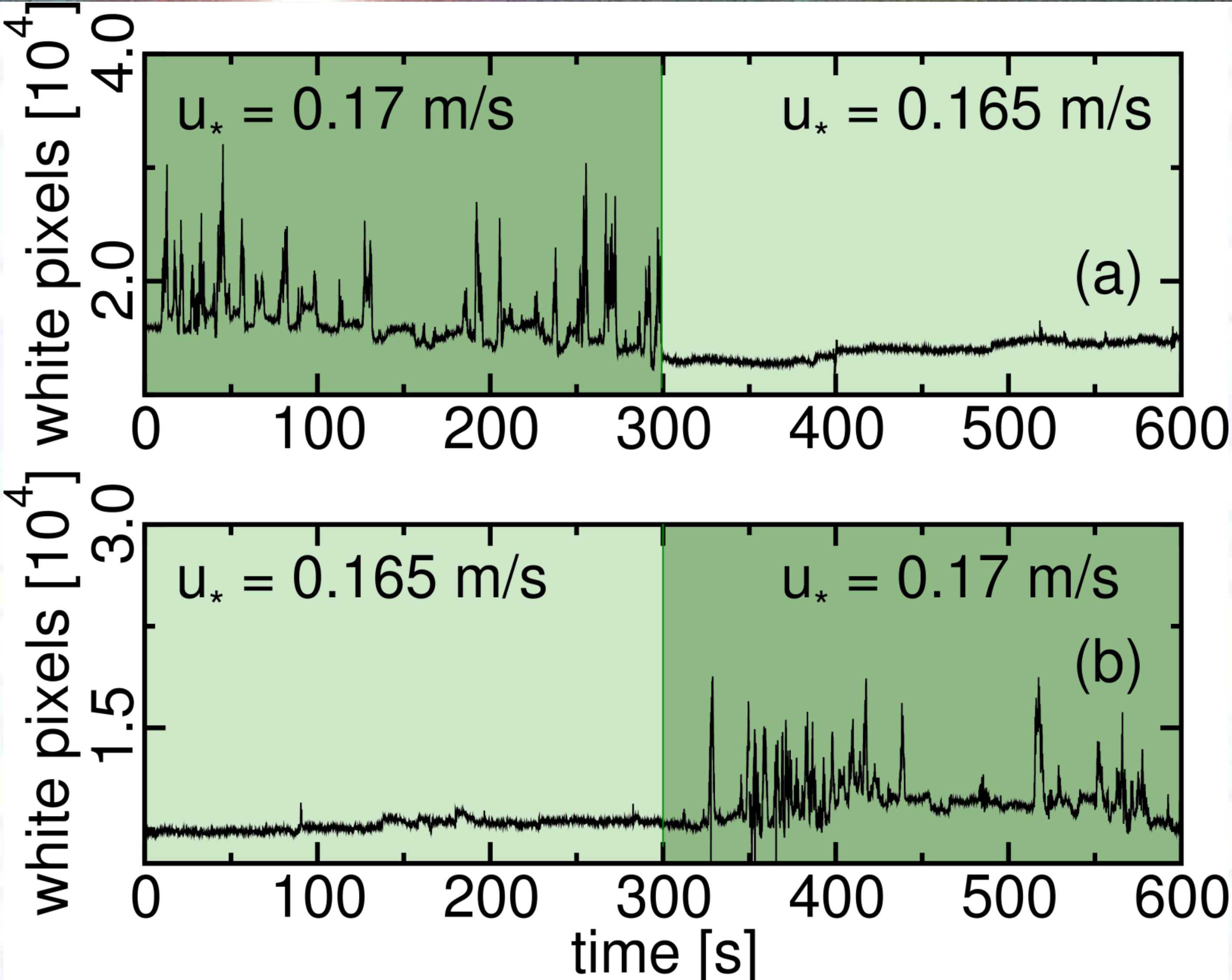
measurement in wind channel in Aarhus

Burst statistics in wind channel



u_*=0.17m/s

Flux in wind channel changing u_*



Intrinsic velocity fluctuations

A.M. Reynolds (2003)

Logarithm of local dissipation rate $\chi = \ln\left(\varepsilon / \langle \varepsilon \rangle\right)$

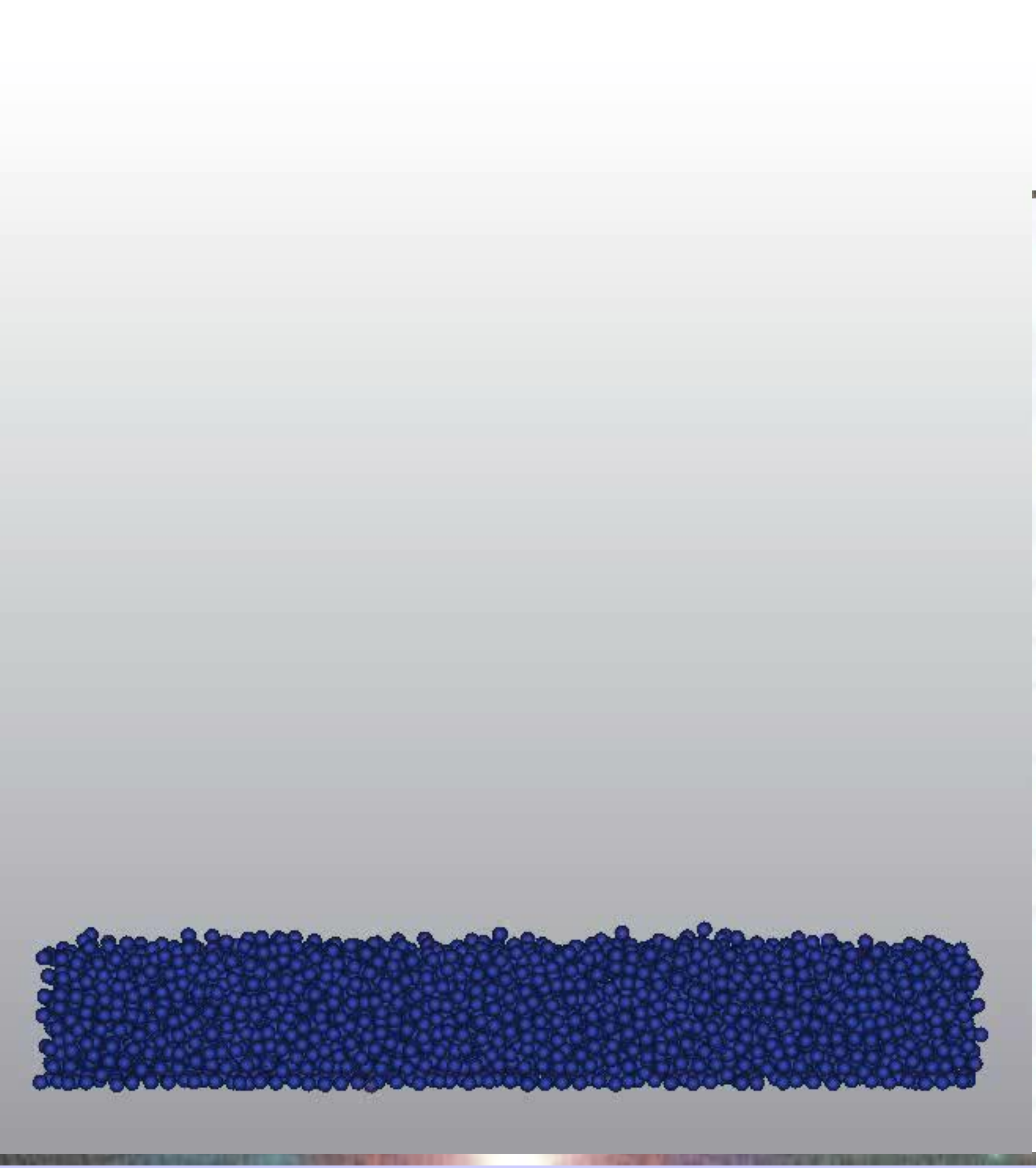
Stochastic differential equations for χ and each component of the acceleration a_t .

$$d\chi = -\left(\chi - \langle \chi \rangle\right) T_\chi^{-1} dt + \sqrt{2\sigma_\chi^2 T_\chi^{-1}}\, d\xi_1$$

$$da_t = -\left(T_\chi^{-1} + t_\eta^{-1} - \sigma_{a_t|\varepsilon}^{-1} \frac{d\sigma_{a_t|\varepsilon}}{dt} \right) a_t dt - T_L^{-1} t_\eta^{-1}$$

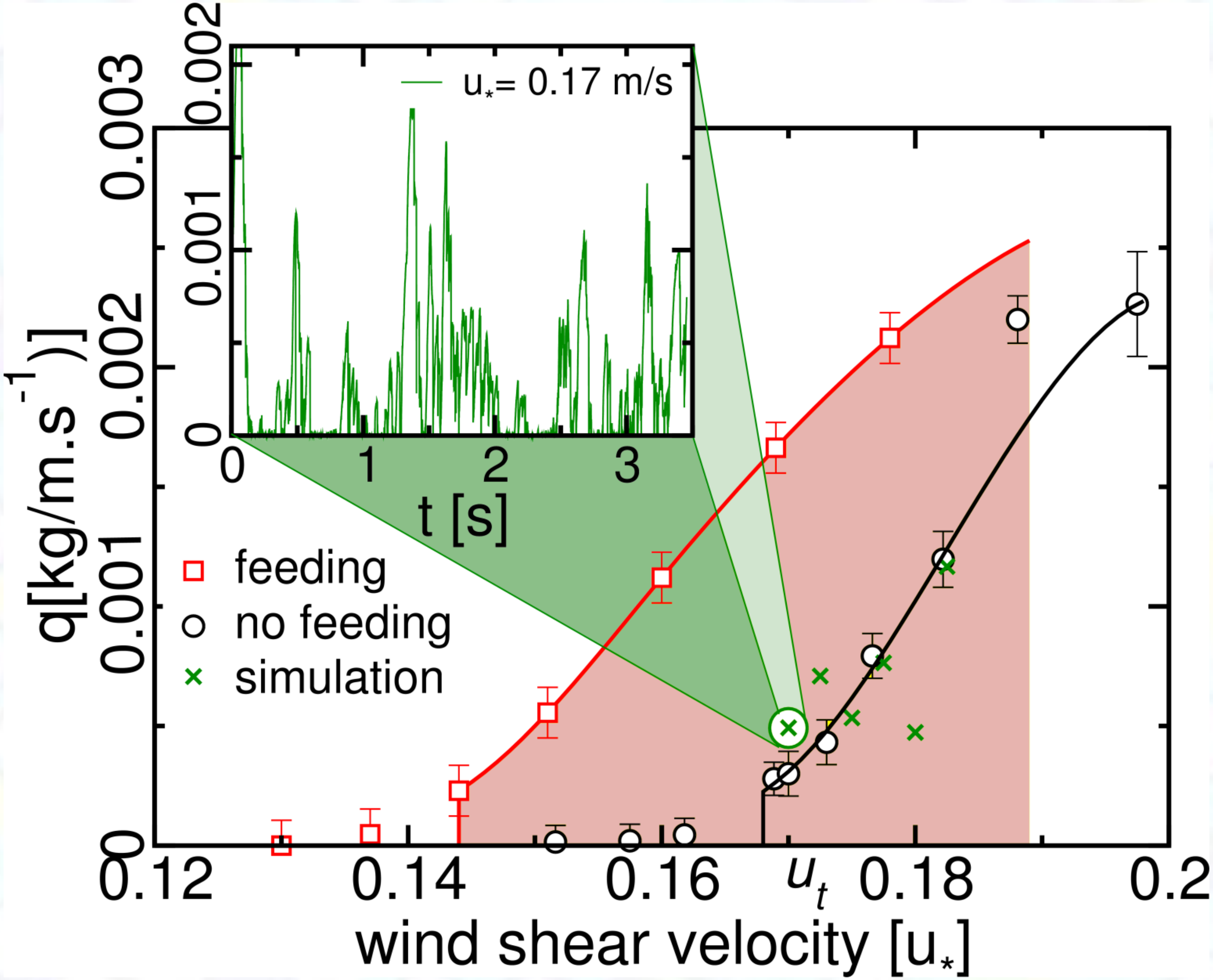
$$+ \sqrt{2\sigma_u^2 \left(T_L^{-1} + t_\eta^{-1}\right) T_L^{-1} t_\eta^{-1}}\, d\xi_2$$

$$du_t = a_t dt \quad , \qquad dx_t = u_t dt$$



3d simulation of a burst

Comparing experiment and simulation

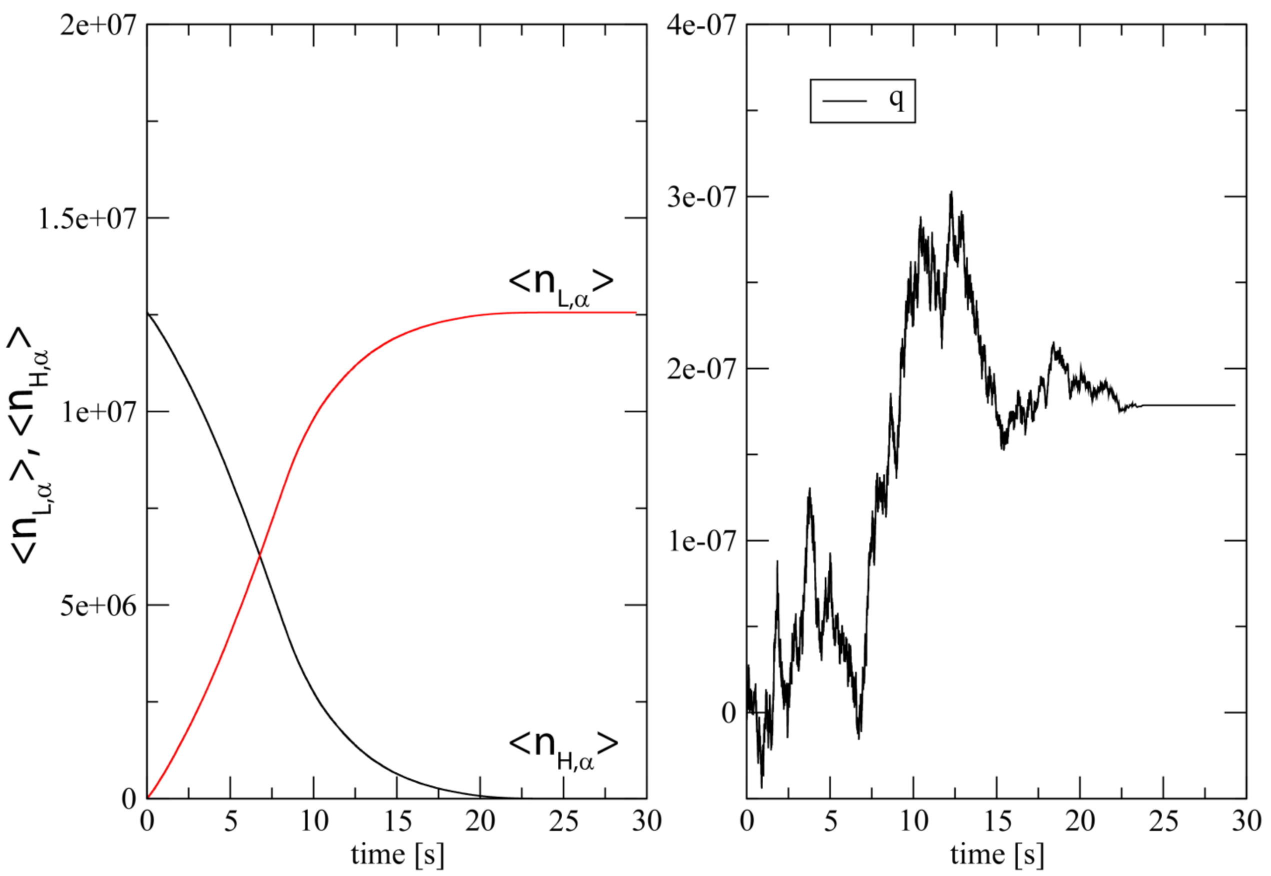


DEM model of bed in moving fluid

- Reproduce splash
- Agreement with experiment and theory
- Jump in saturated flux at onset of saltation
- Mid-air collisions enhance flux.
- We can distinguish saltons, soft-bed and reptons.
- Temporal fluctuations of turbulent produce intermittency.

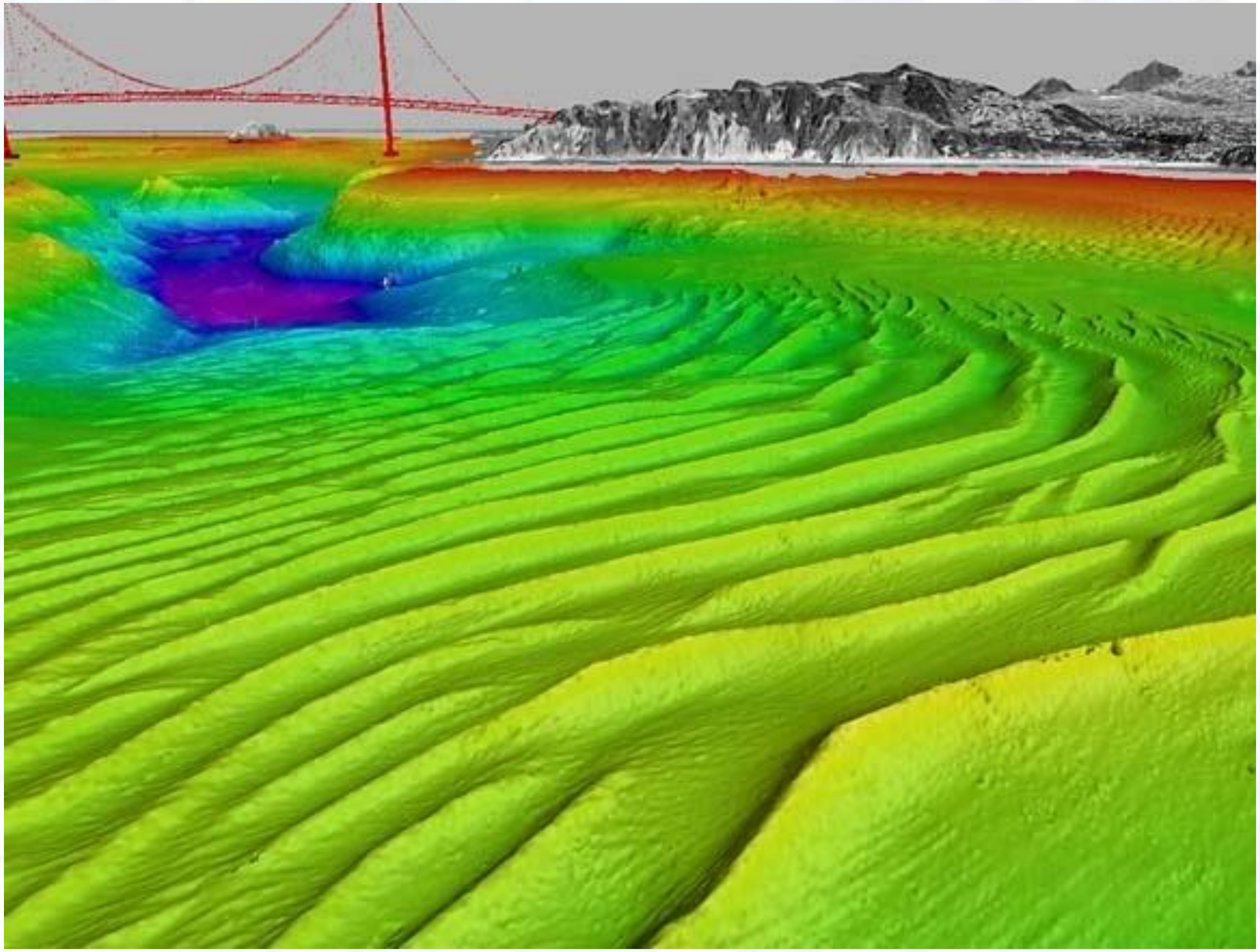
Including electrical charges

N. Duff and D.J. Lacks, J. of Electrostatics 66, 51 (2008)



Electrons trapped locally in high energy states.

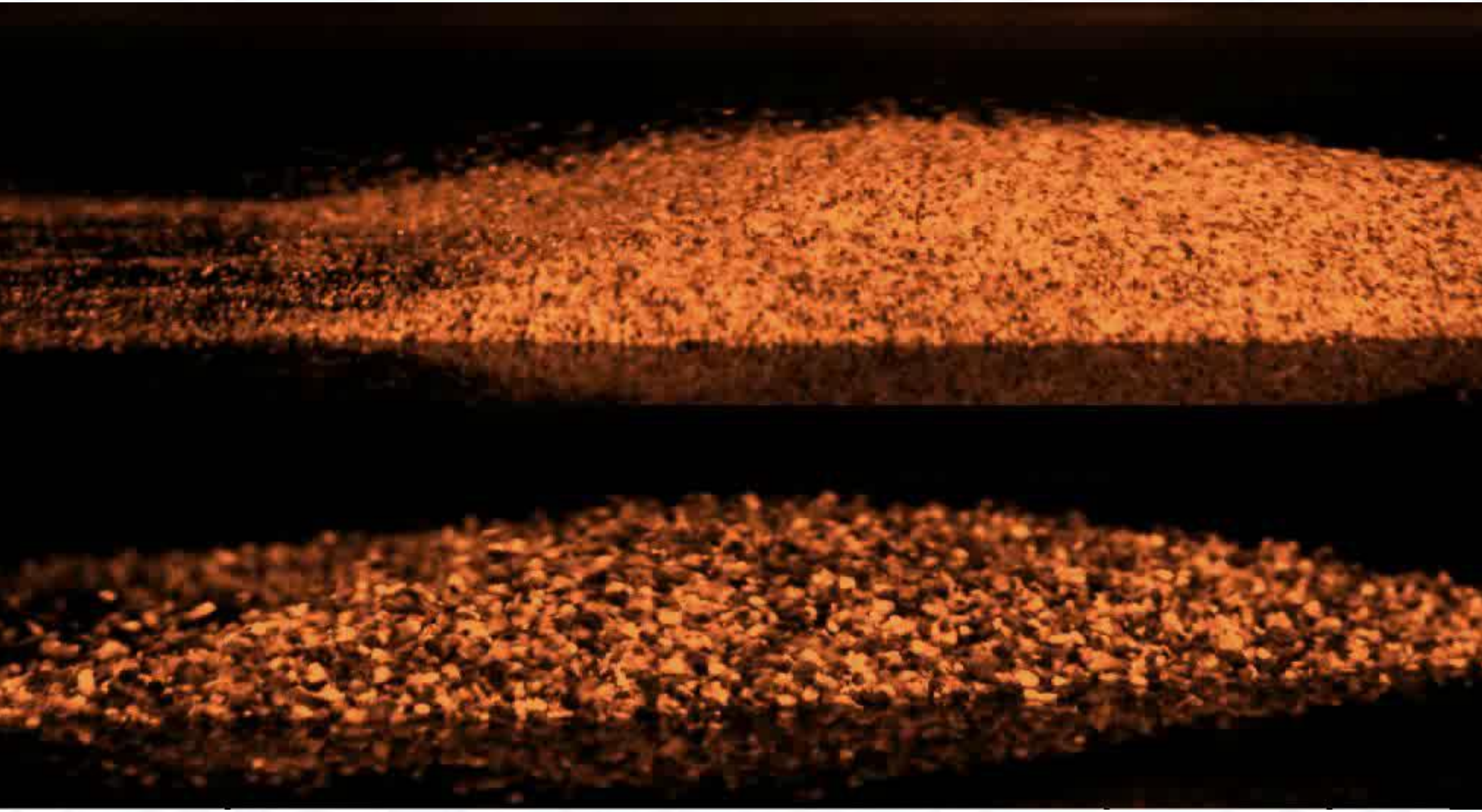
Particle transport by water



under water dunes in front of San Francisco bay

Drag in Water Channel

Drag in Water Channel



Particle transport by water

- **Transport mechanisms on ground:**

- **1. Creep – rolling and sliding of grains on the soil**
- **2. Saltation – hops of grains near the soil**
- **3. Sheet Flow – completely mobile sand bed, grains moving in granular sheets**

Future Perspectives

- Make full 3d turbulent simulation.
- Consider saltation in water.
- Consider electric charges.
- Consider different shapes of grains.
- Consider realistic boundary layers.
- Consider rotations of grains (Magnus forces).
- Consider obstacles in the flow.
- Consider non-Newtonian rheologies.

Thank you !