University of
BRISTOL

The effects of wind on the rise of volcanic plumes and the intrusions of volcanic ash

Andrew Hogg

Mark Woodhouse, Jeremy Phillips, Chris Johnson,
Steve Sparks, Herbert Huppert

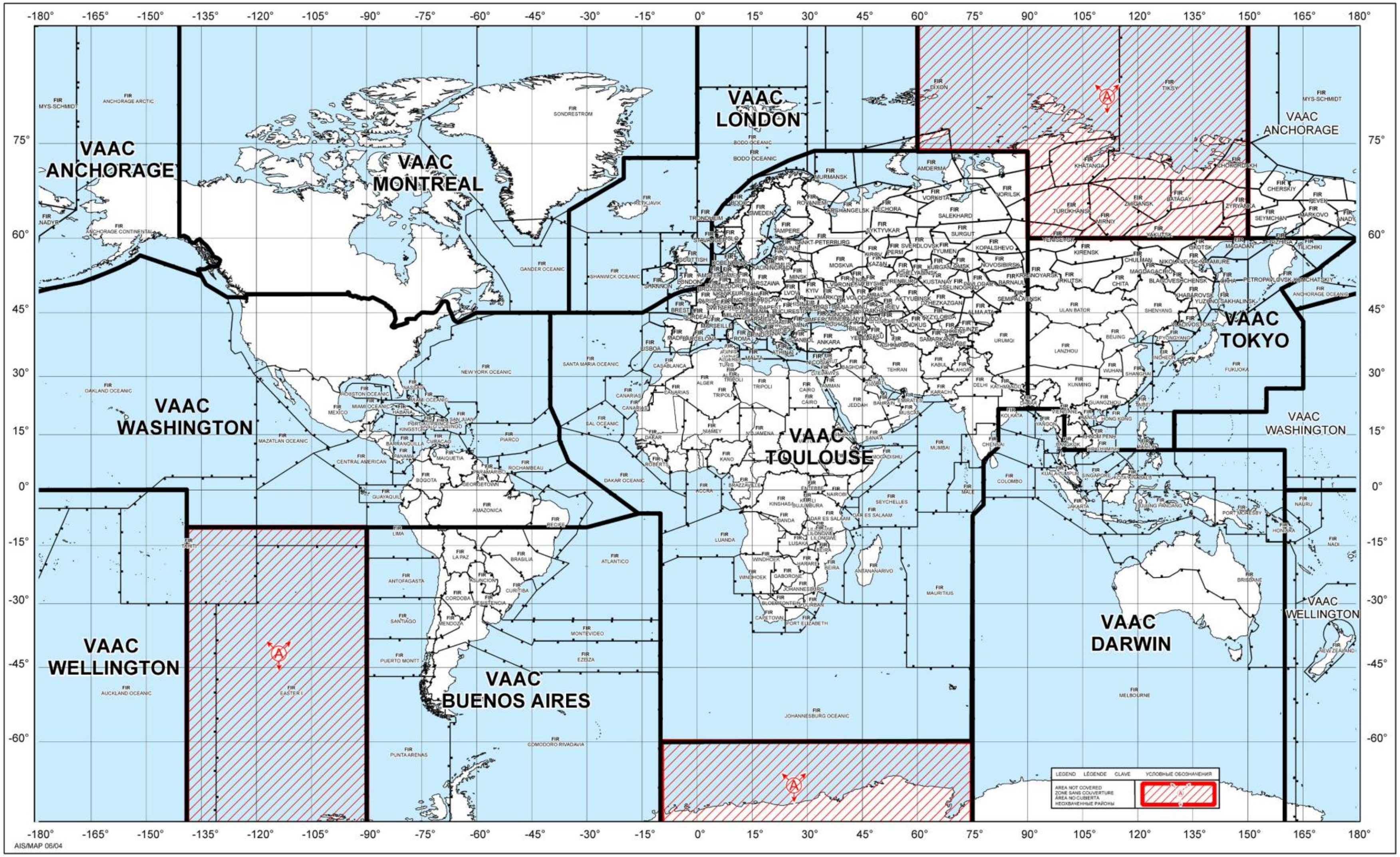
VANAHEIM (NERC Consortium)

Wind effects: *plume rise & ash dispersion*

Volcanic Ash and the risk to aviation

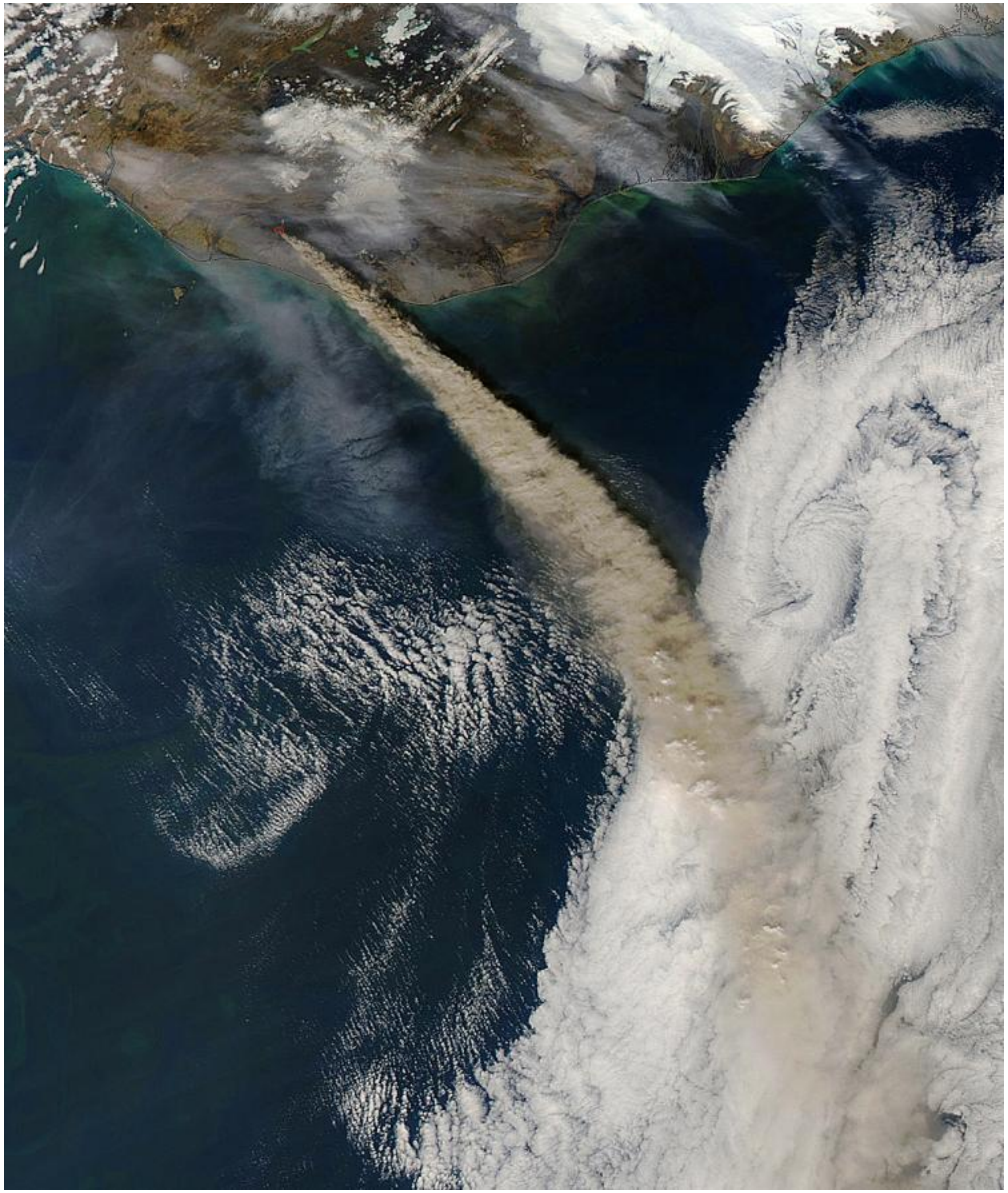
- International airspace is advised by 9 Volcanic Ash Advisory Centres (VAACs) that provide guidance to airlines on the safety of flight paths.

CURRENT STATUS OF ICAO VOLCANIC ASH ADVISORY CENTRES (VAAC) - AREAS OF RESPONSIBILITY
SITUATION ACTUELLE DES CENTRES OACI D'AVIS DE CENDRES VOLCANIQUES (VAAC) - ZONES DE RESPONSABILITÉ
ESTADO ACTUAL DE LOS CENTROS DE AVISOS DE CENIZAS VOLCÁNICAS (VAAC) DE LA OACI - ÁREAS DE RESPONSABILIDAD
СУЩЕСТВУЮЩЕЕ РАСПРЕДЕЛЕНИЕ КОНСУЛЬТАТИВНЫХ ЦЕНТРОВ ИКАО ИО ВУЛКАНИЧЕСКОМУ ПЕПЛУ (VAAC) - РАЙОНЫ ОТВЕТСТВЕННОСТИ



2010 Eruption of Eyjafjallajökull (Iceland)

SOURCE: US Geological Survey



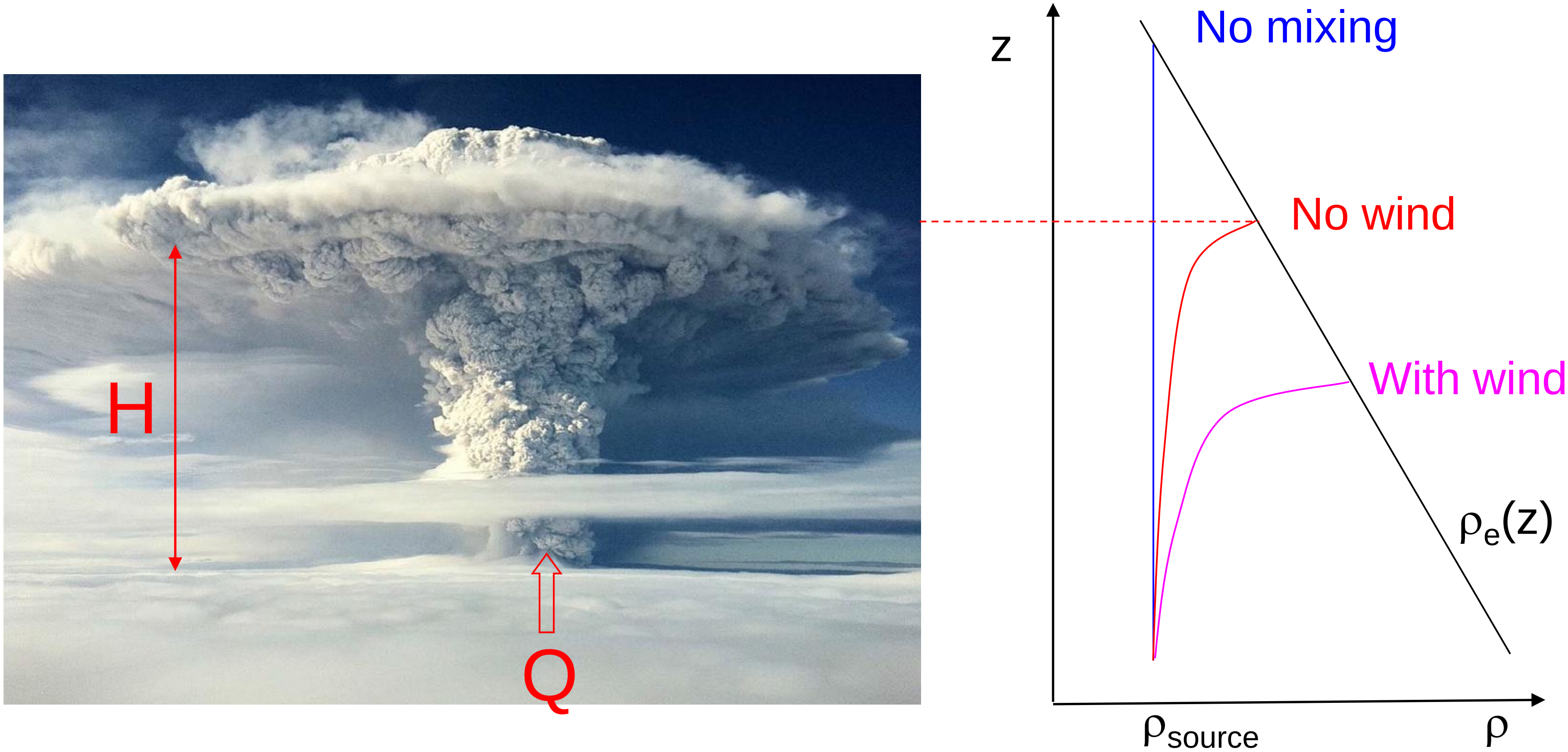
- The Eyja plume was relatively long-lived (April-May 2010)
- Forecasters did not have operational tools to predict its behaviour

- UK government policy changed during the eruption period (2mg/m^3 threshold)

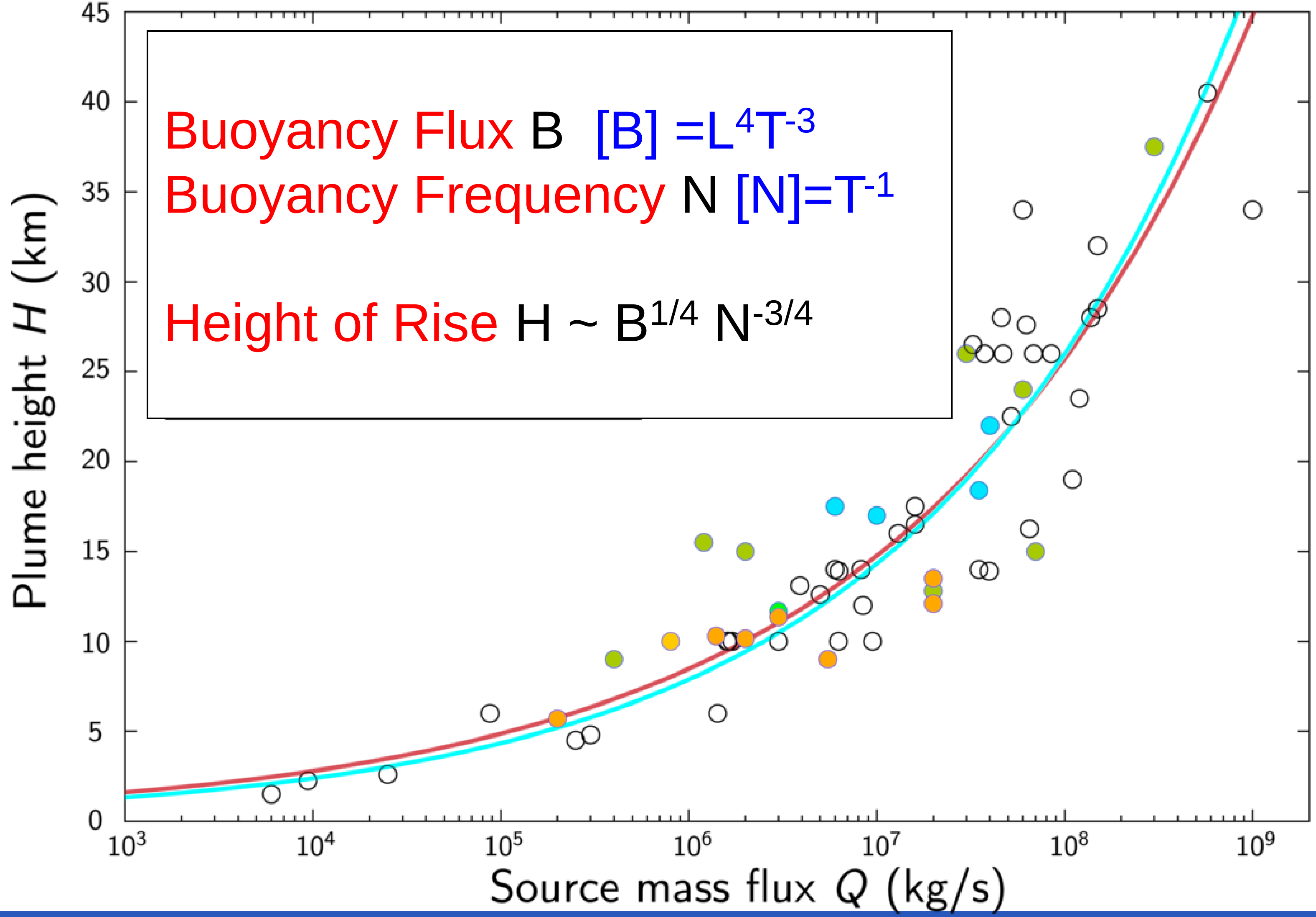
- Estimated cost to European economy £5bn.

Plume rise and entrainment

- How can mass flux of volcanic plume be determined?



Empirical relation: *Sparks & Mastin curves*



Buoyancy Flux B $[B] = L^4T^{-3}$

Buoyancy Frequency N $[N] = T^{-1}$

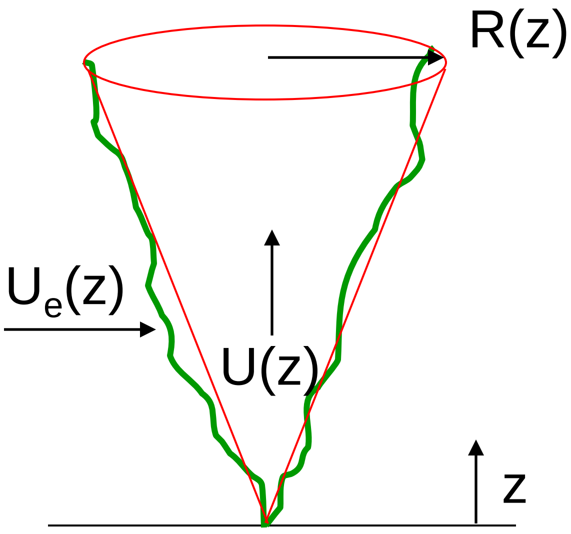
Height of Rise $H \sim B^{1/4} N^{-3/4}$

Plume model *(Morton, Taylor, Turner 1956)*

$\rho(z)$=plume density,

$\rho_e(z)$=atmosphere density

$N^2=-g(d\rho_e/dz)/\rho(0)$=Buoyancy Frequency



Mass $$\frac{\mathrm{d}}{\mathrm{dz}}\left(\pi R^2 U\right)=2\pi R U_e$$

Momentum $$\frac{\mathrm{d}}{\mathrm{dz}}\left(\pi R^2 U^2\right)=\pi R^2 g'$$

Buoyancy $$\frac{\mathrm{d}}{\mathrm{dz}}\left(\pi R^2 g' U\right)=\pi R^2 N^2 U$$

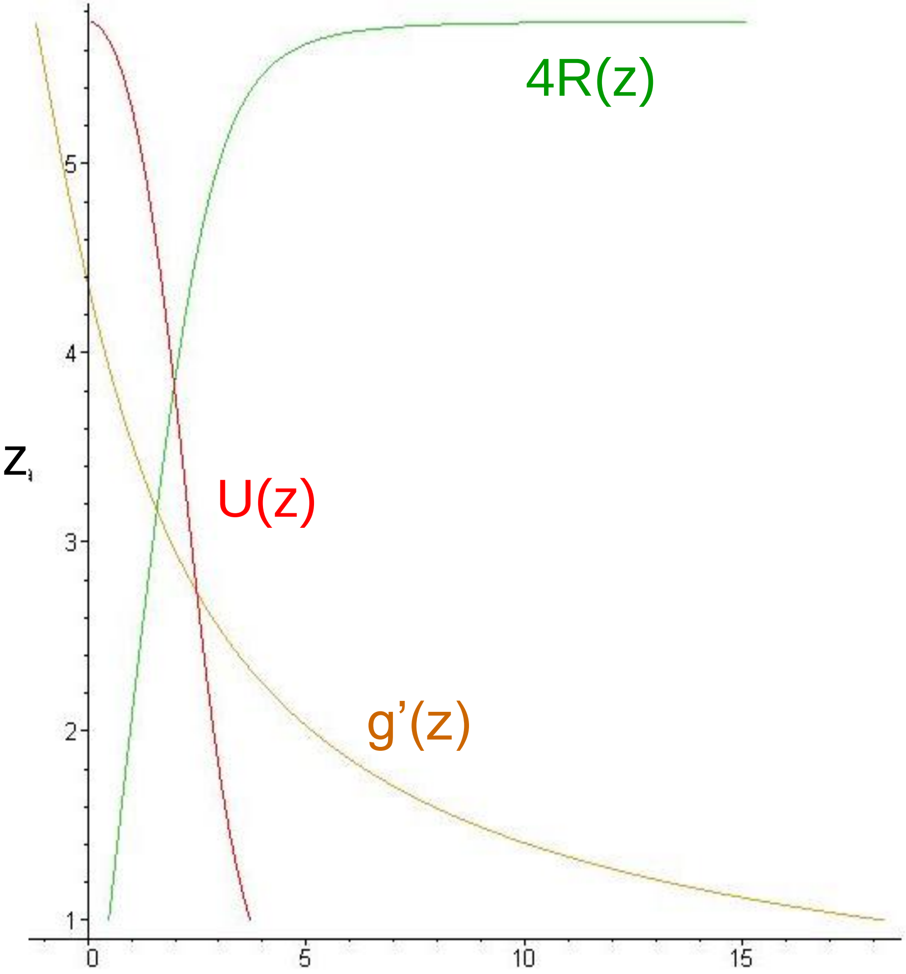
- Source (z=0): Buoyancy flux $R^2 g' U = B$
  Momentum and Mass fluxes

$$R^2 U^2 \to 0$$
$$R^2 U \to 0$$

- Entrainment assumption $U_e = kU$

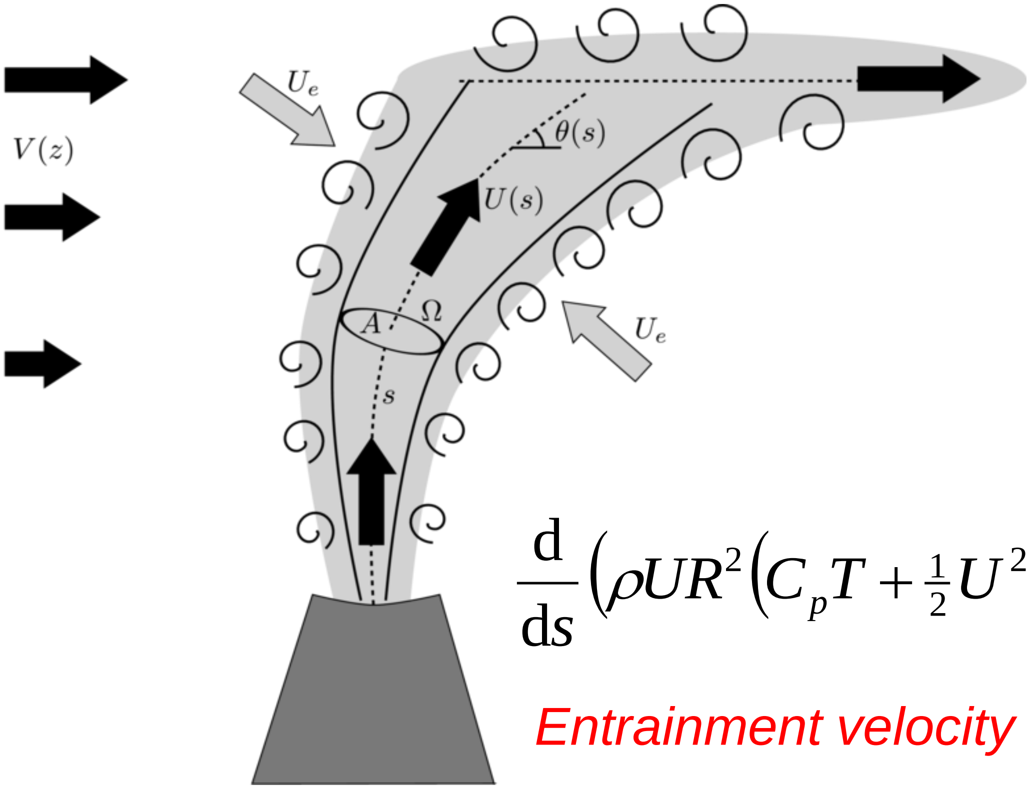
Plumes: *typical results*

- Plume rises to neutral buoyancy level and overshoots due to inertia



Plume model

- Integral model of wind-blown plumes
  - Model evolution of mass, momentum (vertical & horizontal) & energy
  - Mixing with atmosphere plays a key role



$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U R^2\right) = 2\rho_a U_e R$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U^2 R^2 \sin\theta\right) = \left(\rho_a - \rho\right) g R^2$$

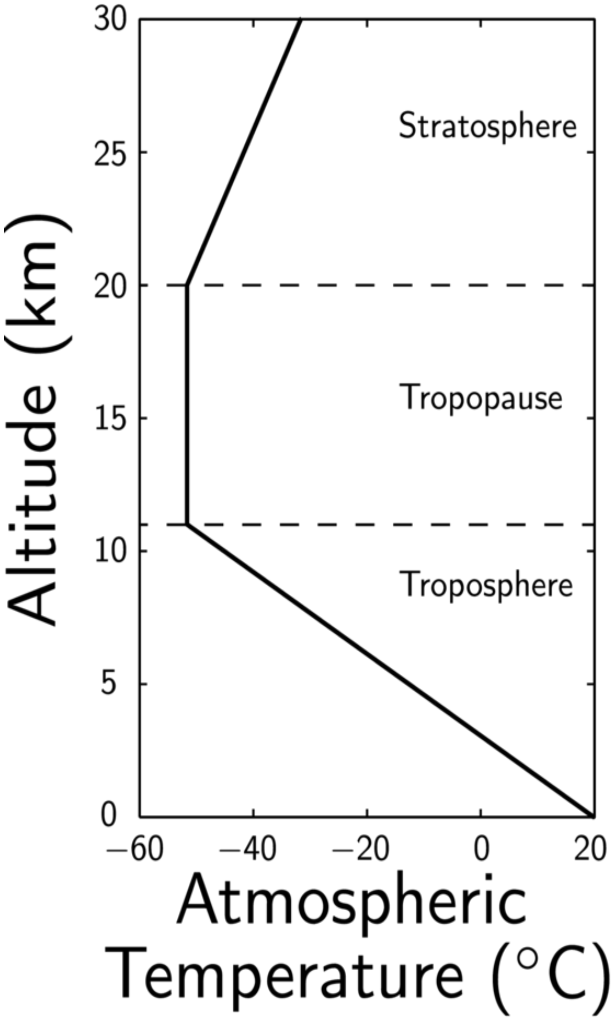
$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U^2 R^2 \cos\theta\right) = 2\rho_a U_e R V$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\rho U R^2\left(C_p T + \tfrac{1}{2}U^2 + gz\right)\right) = 2\rho_a R U_e\left(C_a T_a + \tfrac{1}{2}U_e^2 + gz\right)$$

Entrainment velocity $U_e = k_s\left|U - V\cos\theta\right| + k_w\left|V\sin\theta\right|$

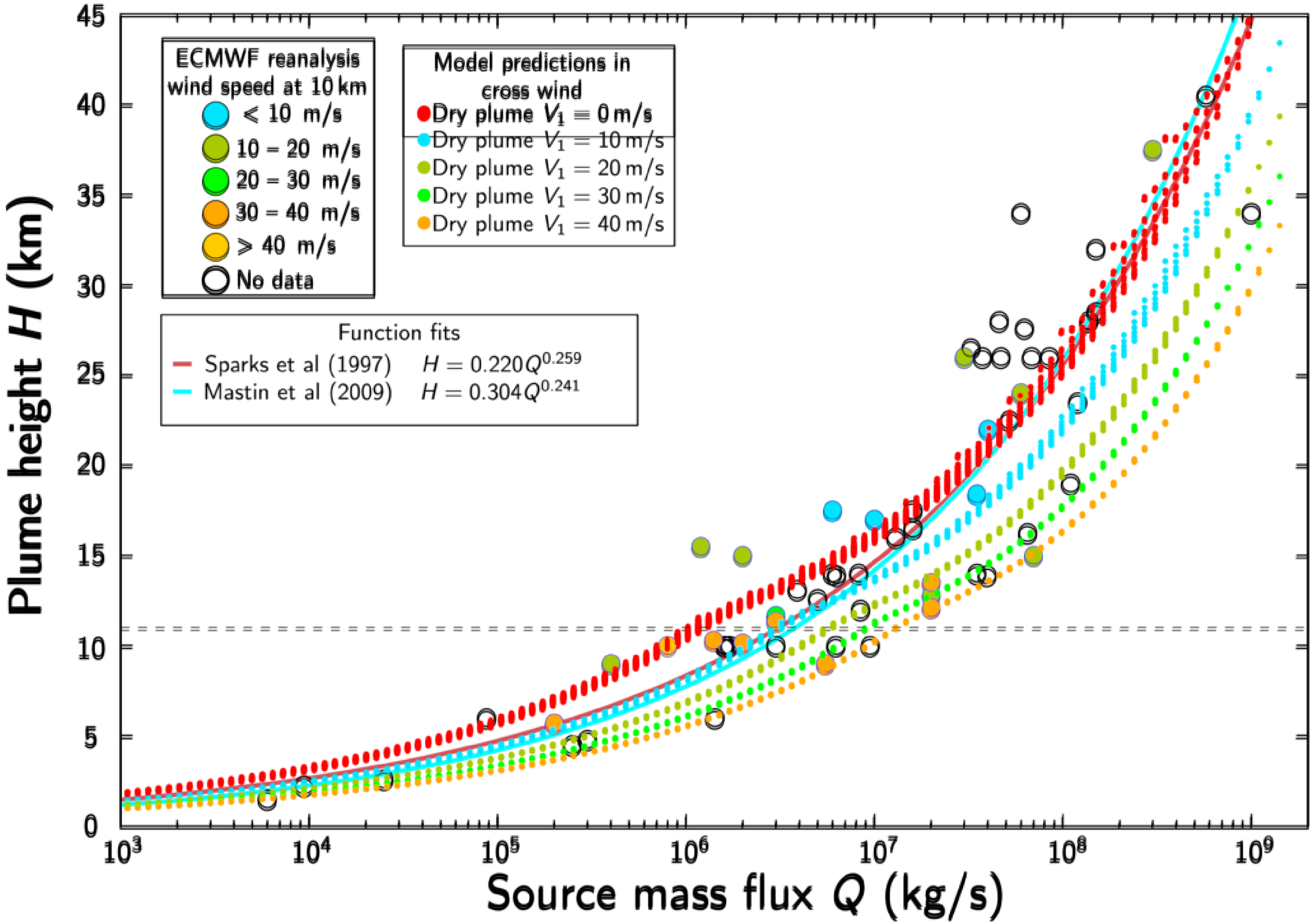
[Can include moisture content and phase changes]

Plume models with Standard Atmosphere

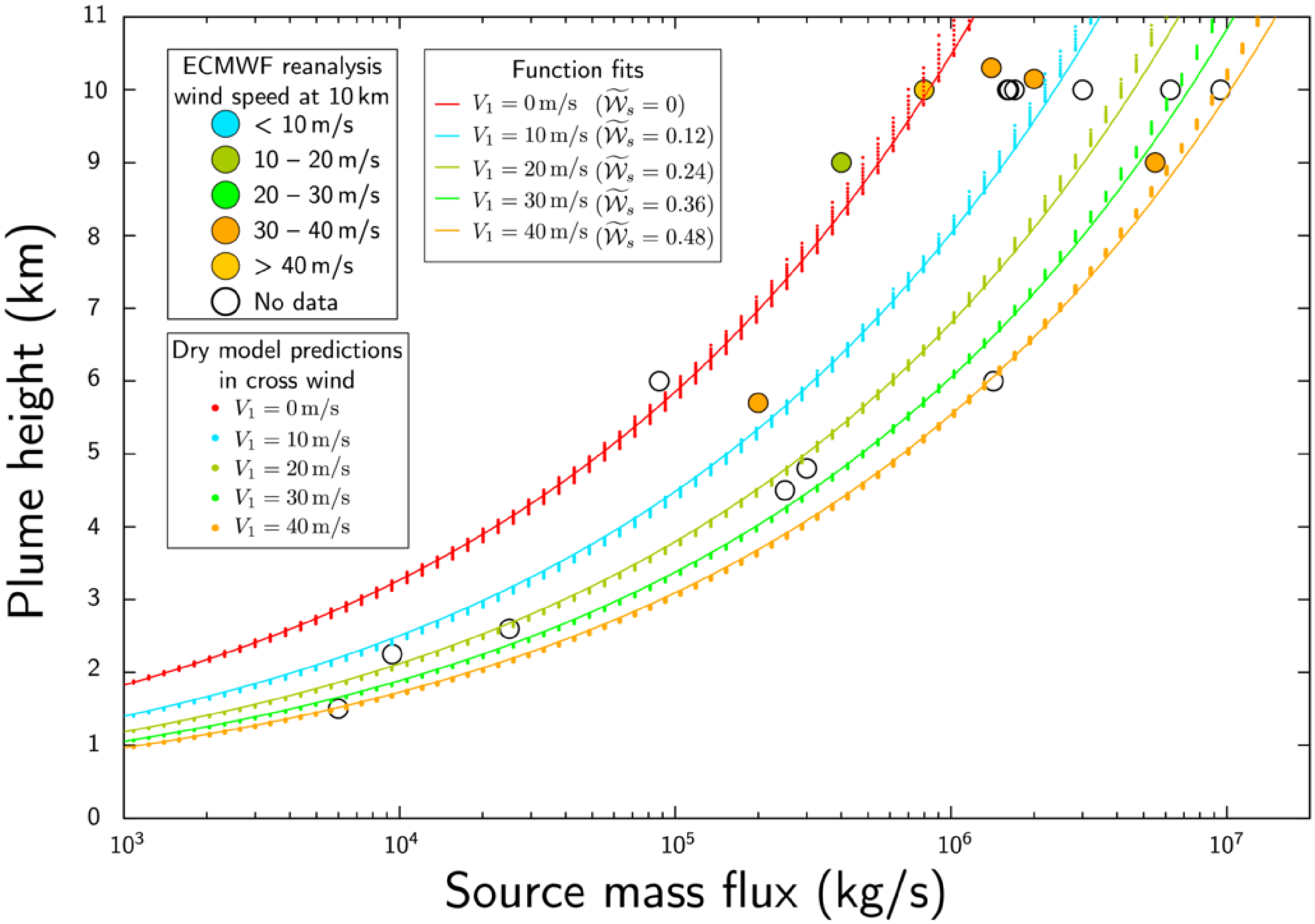


- Plume model integrated in a standard atmosphere for various wind speeds at the tropopause (V_1)

Rise heights in Standard Atmosphere



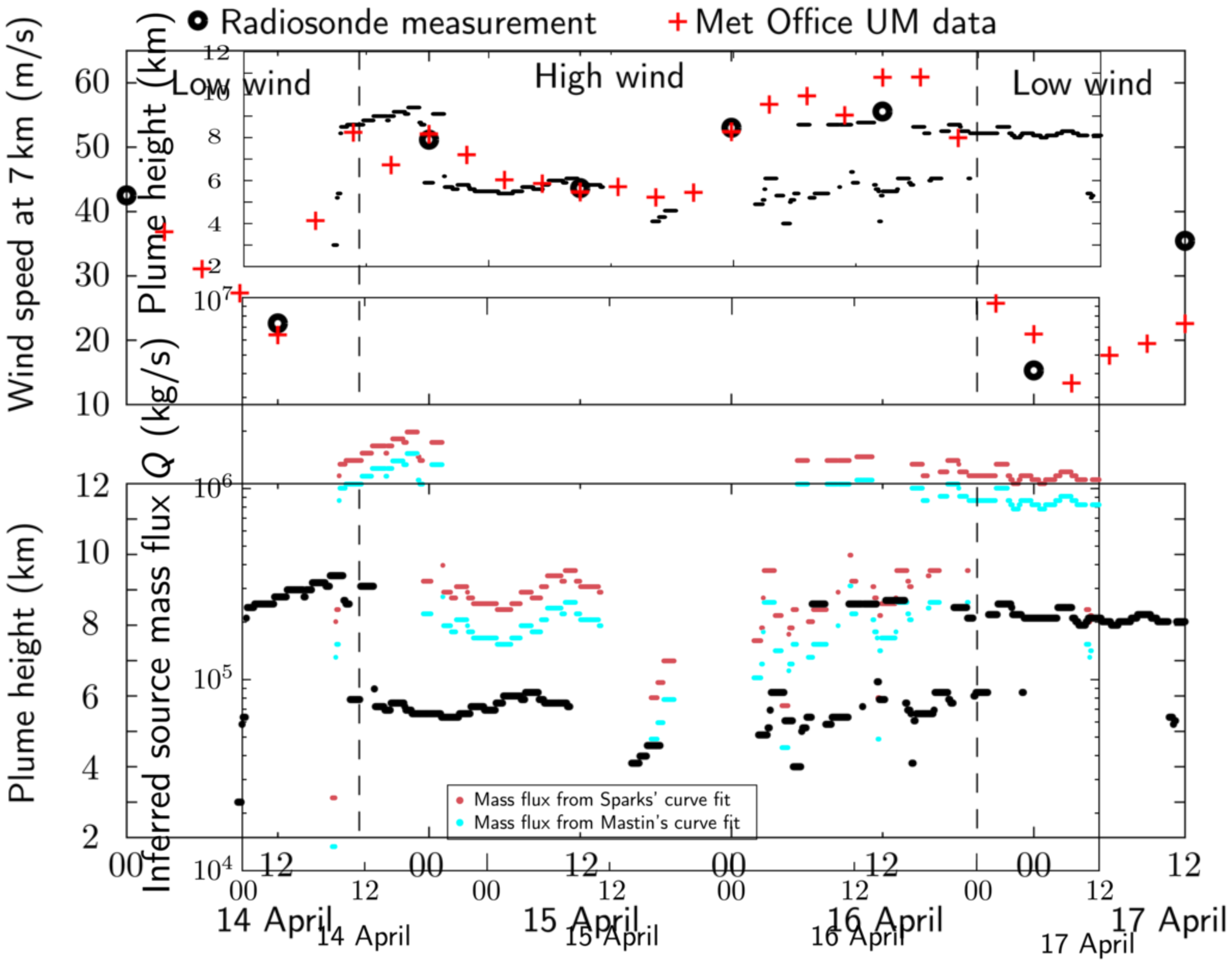
New empirical relationship with wind



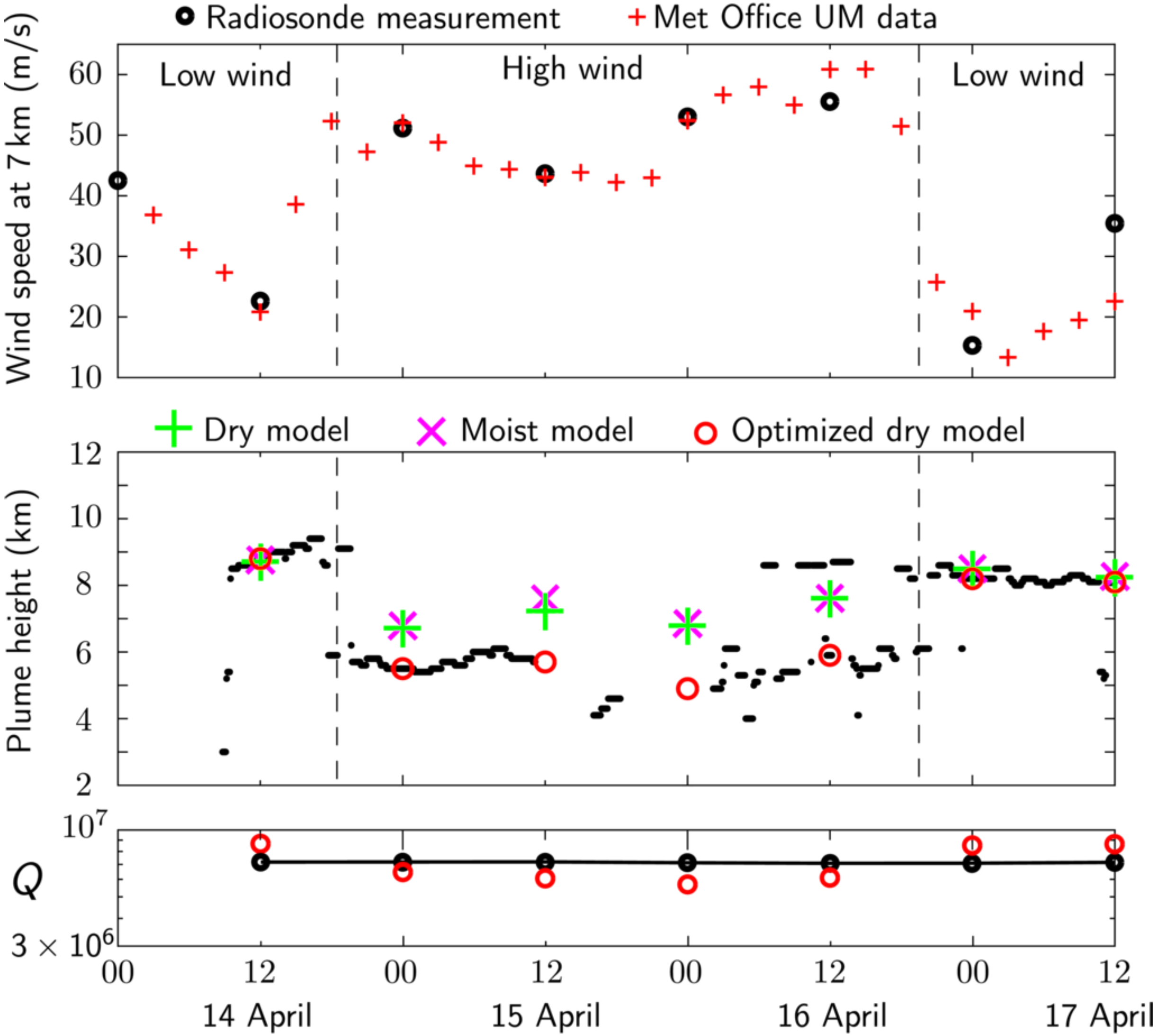
Application to 2010 eruption of Eyjafjallajökull

Observations of
- plume height
- wind
- temperature

Plume height 14-17 April



Source flux: wind effect

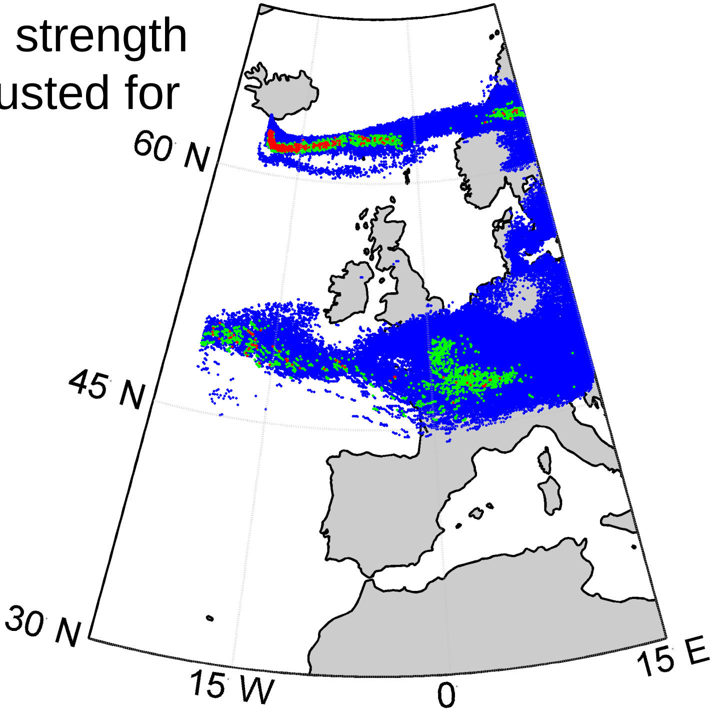


Mass flux upto 100 greater than predicted using wind-free formulae

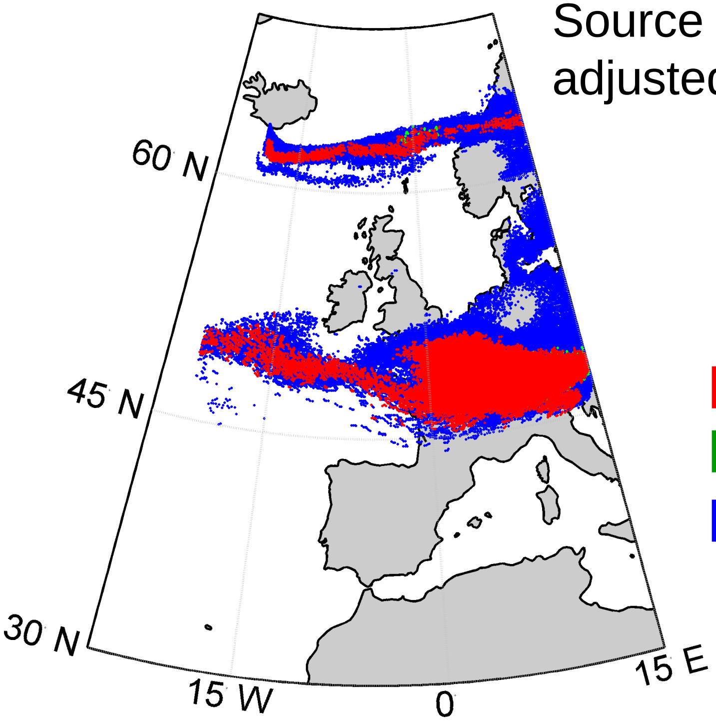
Ash dispersal in atmosphere

- Use NAME (*Numerical Atmospheric dispersion Modeling Environment)* to calculate dispersal
  - 72 hours after eruption; Concentrations at 3km; No proximal adjustment

Source strength not adjusted for wind



Source strength adjusted for wind



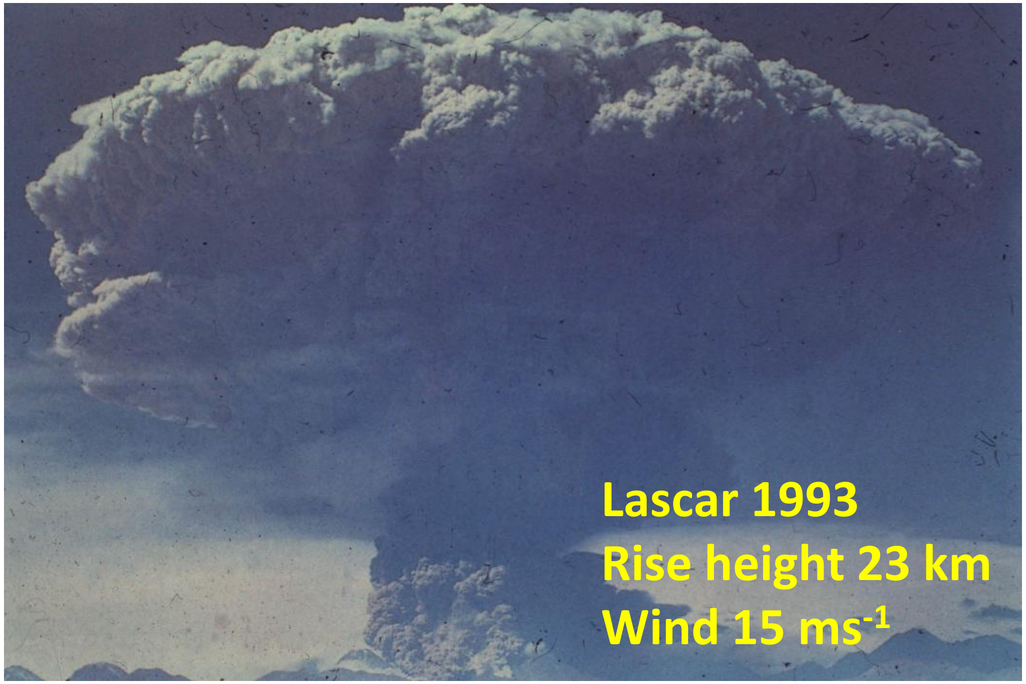
🟥 >4 mgm^{-3}

🟩 2-4 mgm^{-3}

🟦 <2 mgm^{-3}

Wind effects: *plume rise & ash dispersion*

High Intensity eruptions: *weak winds*



Lascar 1993

Rise height 23 km

Wind 15 ms^{-1}

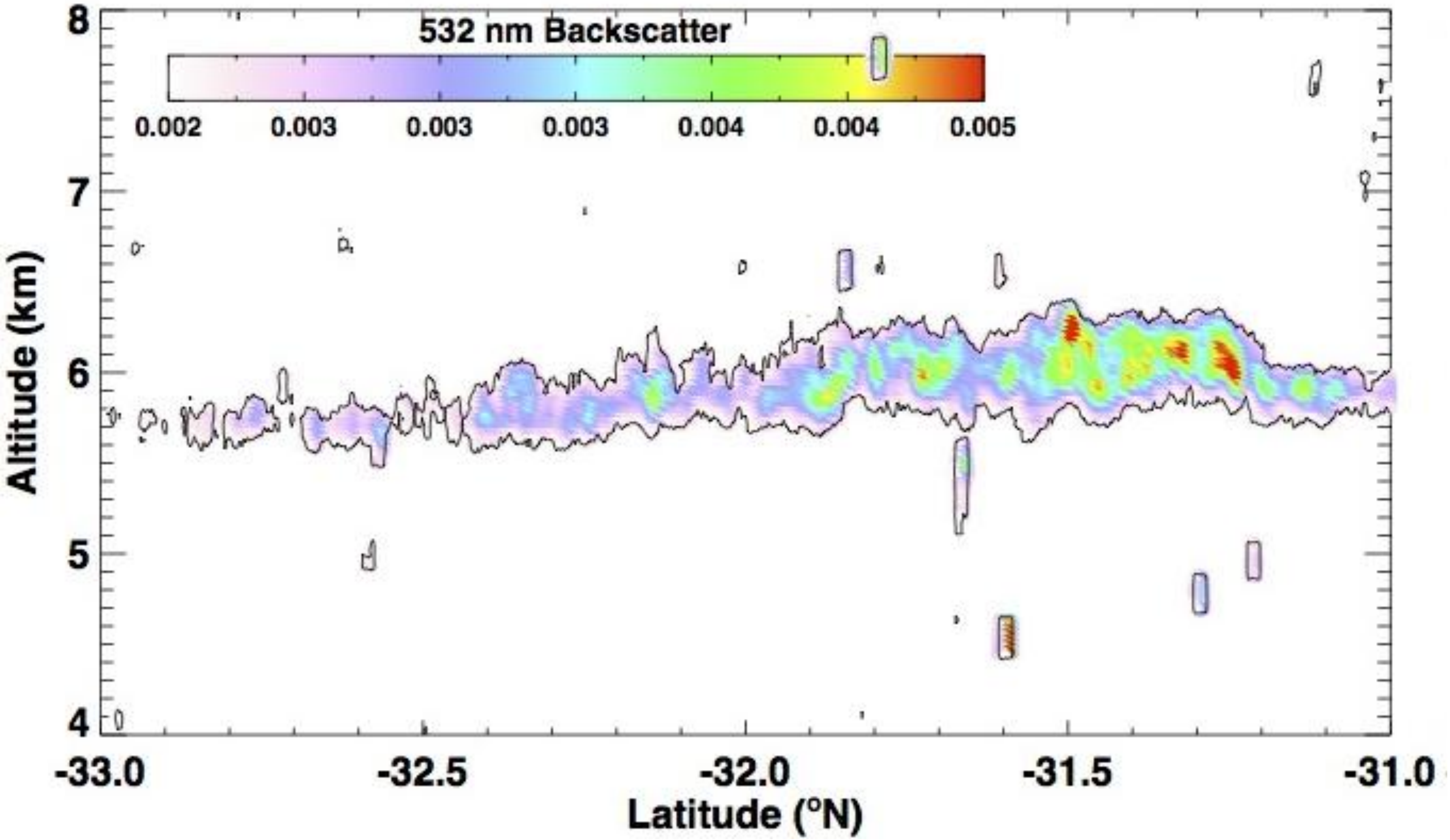
Mount St Helens 1980

Rise height 16 km

Wind 33 ms^{-1}

- For strong eruptions, wind does not significantly influence near source behaviour.
- *Umbrella cloud* expansion

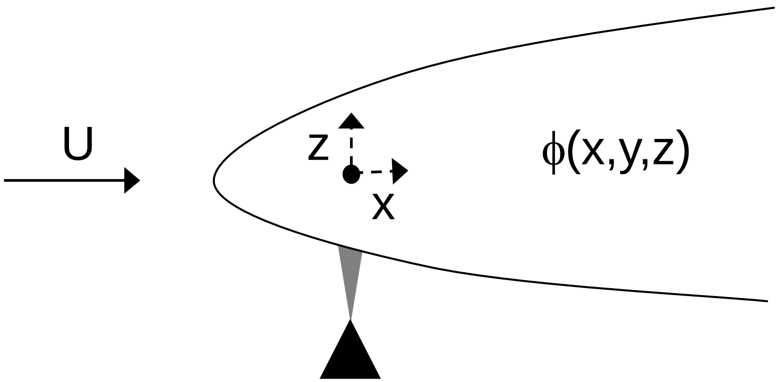
Satellite images of thin ash layers



Puyehue 24-12-2012 (600 km downwind): *Fred Prata*

A diffusion model

- The concentration of volcanic ash is produced by a sustained source (Q); advected by the wind (U); diffused due to action of turbulence (diffusivity: K); and settles under gravity (v_s)

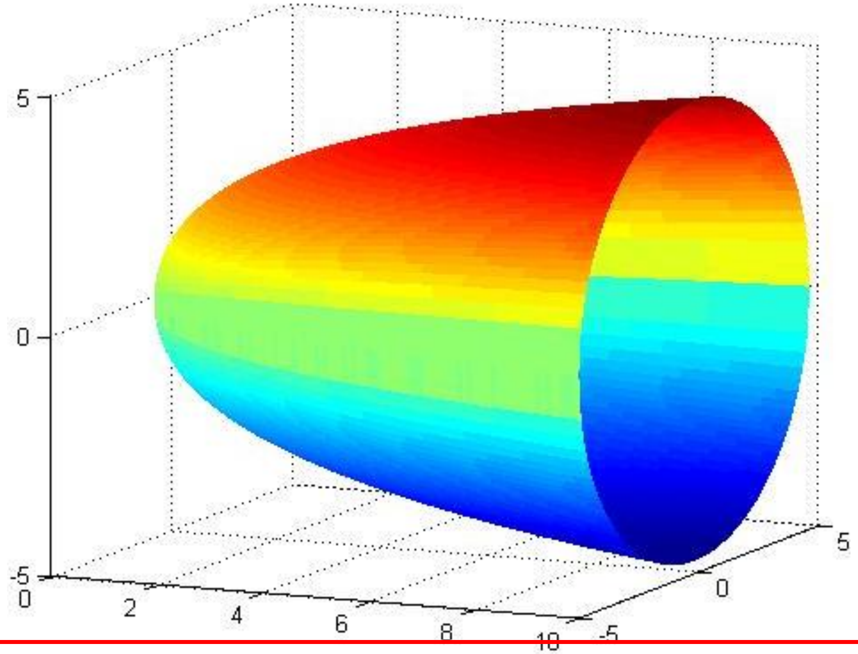


$$U\frac{\partial\phi}{\partial x}-v_s\frac{\partial\phi}{\partial z}=K\nabla^2\phi+Q\delta(\mathbf{x})$$

Wind Settling Diffusion Source

Centreline concentration ~1/x

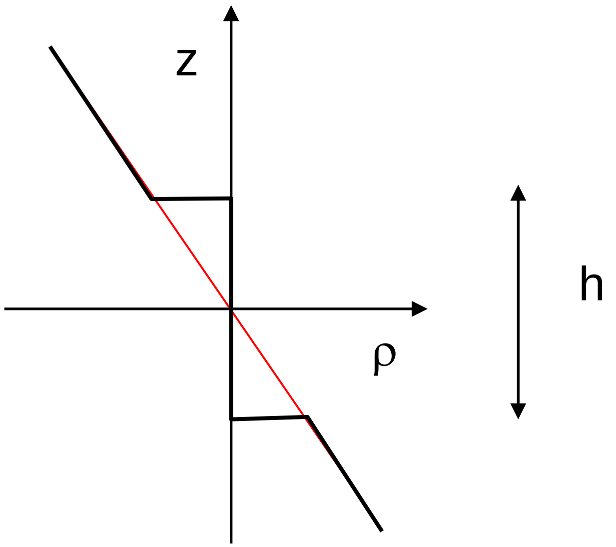
Width of cloud ~$(Kx/U)^{1/2}$



- **But this diffusive model neglects buoyancy-driven motion**

Intrusion dynamics: *buoyancy processes*

Density profile



- Plume delivers fluid at neutral buoyancy height, with uniform density
  - Perturbs atmospheric stratification
- Particles do not add significantly to bulk density of intrusion.
- Thickness of intrusion, h, determines pressure excess above hydrostatic balance.
- Gradient of thickness sets up a horizontal pressure gradient

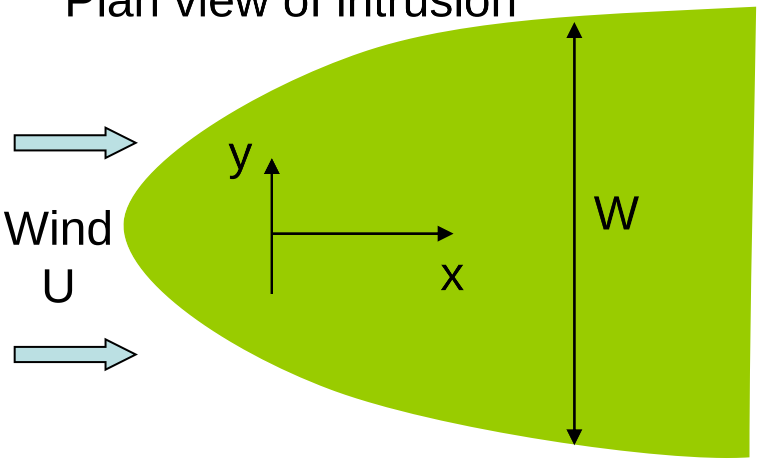
$$\nabla_H P = -\tfrac{1}{4} N^2 h \nabla_H h$$

$$N^2 = -\frac{g}{\rho_0}\frac{\partial \rho_a}{\partial z}$$

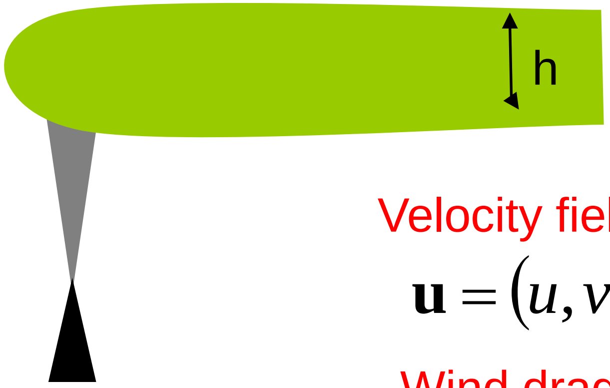
Buoyancy frequency

Shallow layer model of intrusion

Plan view of intrusion



Side view of intrusion



Velocity field

$$\mathbf{u} = (u, v)$$

Wind drag

$$\boldsymbol{\tau} = \rho C_D (\mathbf{u} - U\hat{\mathbf{x}})|\mathbf{u} - U\hat{\mathbf{x}}|$$

Mass

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0$$

Momentum

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) + \frac{\partial}{\partial y}(uvh) + \frac{1}{4}N^2 h^2 \frac{\partial h}{\partial x} = -\tau_x$$

$$\frac{\partial}{\partial t}(vh) + \frac{\partial}{\partial x}(uvh) + \frac{\partial}{\partial y}(v^2 h) + \frac{1}{4}N^2 h^2 \frac{\partial h}{\partial y} = -\tau_y$$

Radial motion: *unsteady, drag-free*

- Close to the source, the intrusion spreads radially

$$\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(ru_r h\right)=0$$

$$\frac{\partial}{\partial t}\left(u_r h\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(ru_r{}^2 h\right)+\frac{1}{4}N^2 h^2\frac{\partial h}{\partial r}=0$$

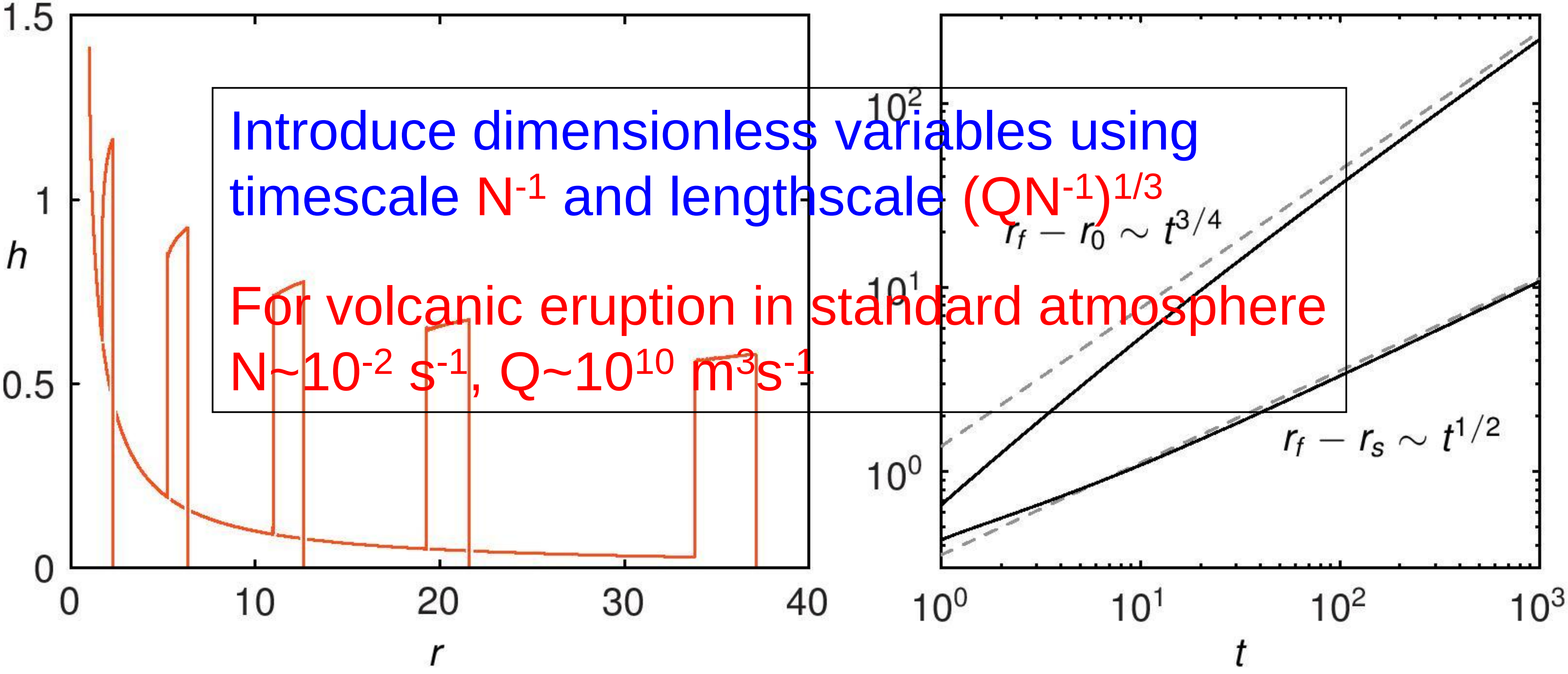
- Source flux $r\ h\ u_r = Q$ at $r=r_0$
- Front condition $u_r = Fr\ Nh$ at $r=r_f(t)$

- Expectation that $r_f(t) \sim (QNt^2)^{1/3}$

mass: $rhu_r \sim Q$ dynamics: $u_r \sim Nh$ kinematics: $u_r \sim r/t$

$$\boxed{r^3 \sim QNt^2}$$

Numerical solution of shallow layer model

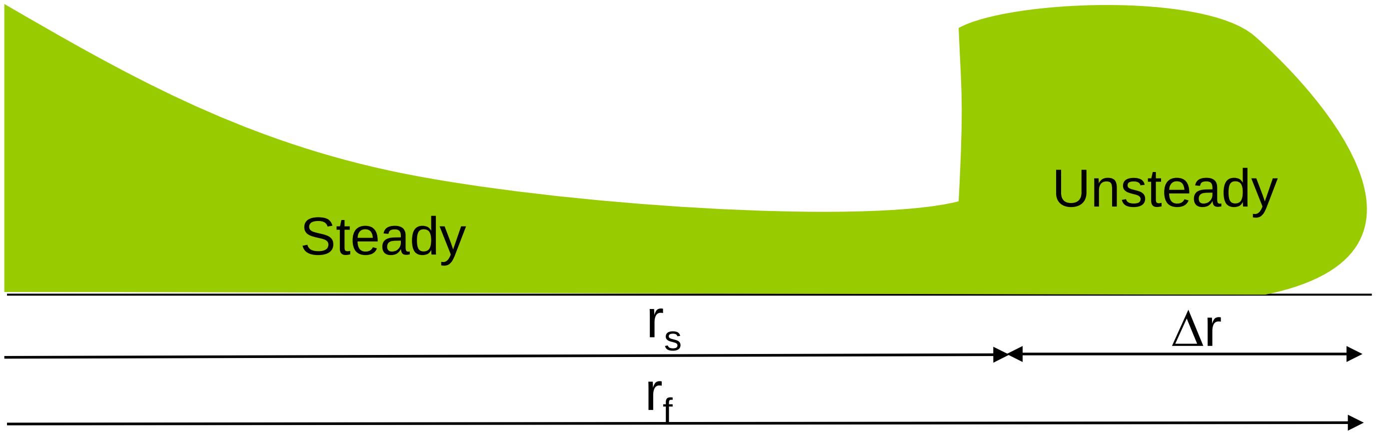


Introduce dimensionless variables using timescale N^{-1} and lengthscale $(QN^{-1})^{1/3}$

For volcanic eruption in standard atmosphere $N \sim 10^{-2}$ s^{-1}, $Q \sim 10^{10}$ $m^3 s^{-1}$

- The front: $r_f \sim t^{3/4}$
- Height & velocity fields are not time-dependent throughout
  - Time dependence in frontal region; steady-state in tail.

Structure of unsteady solution



- Within tail, motion is steady: $r u_r h = 1$ $u_r^2 + h^2/4 = const$ $h \sim 1/r$ $u_r \sim 1$
- Connected to unsteady front via a shock:

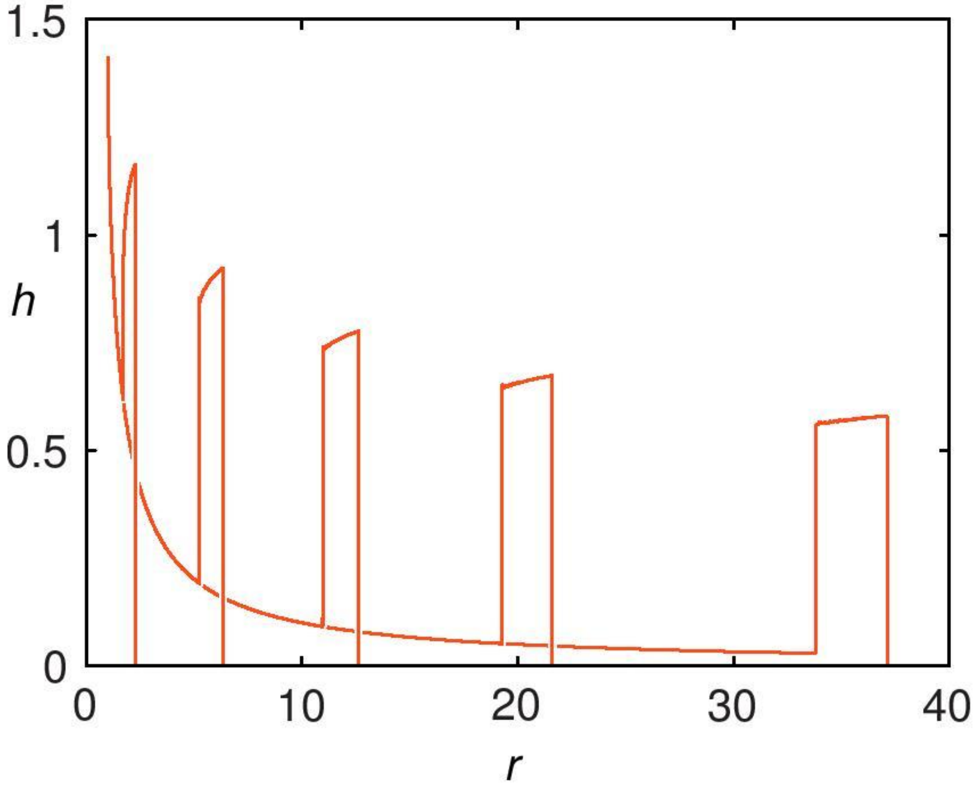
$$\left[(u-c)h\right]_-^+ = 0 \qquad \left[(u-c)^2 h + \tfrac{1}{12}h^3\right]_-^+ = 0$$

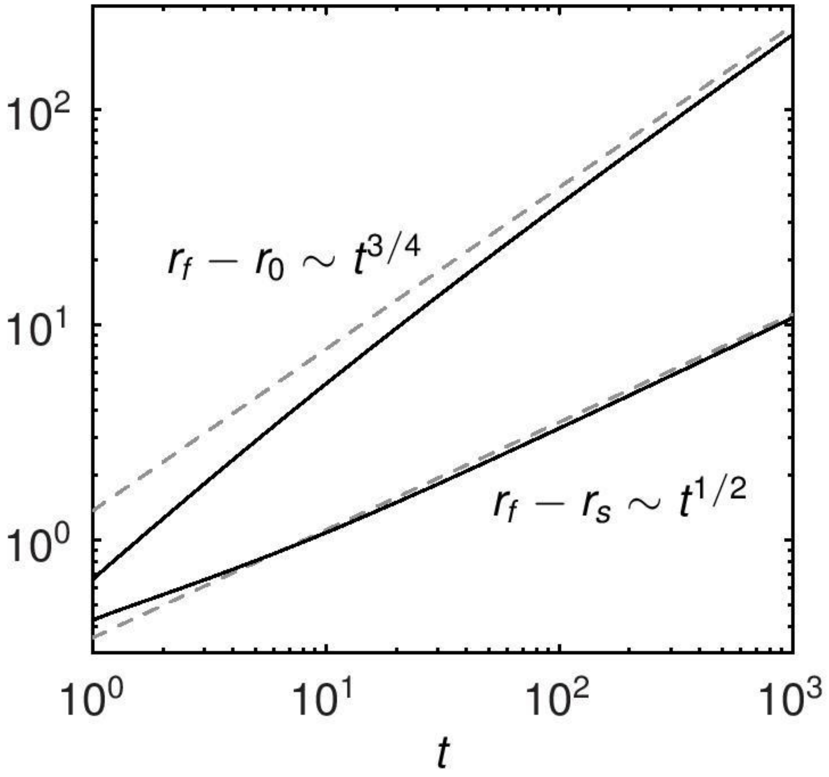
$$\xrightarrow{r>>1} \quad c = u_f \qquad u^2 h\big|_- = \tfrac{1}{12} h_f^3$$

$$h_f^3 \sim \frac{1}{r_f} \qquad u_f \sim h_f \qquad r_f \sim t^{3/4}$$

- Mass conservation $\Delta r\ r_f\ h_f \sim t$ $\Delta r \sim t^{1/2}$

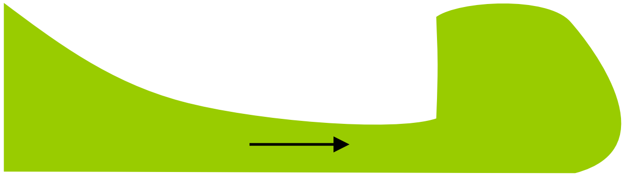
Numerical solution: *radial, drag-free*





Aside: *constant flux, radial gravity currents through uniform environments*

- For currents of excess density ($\Delta\rho$), moving through a uniform environment

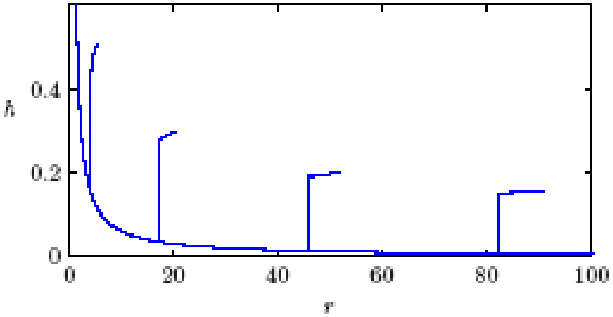


Mass
$$\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(ru_r h\right)=0$$

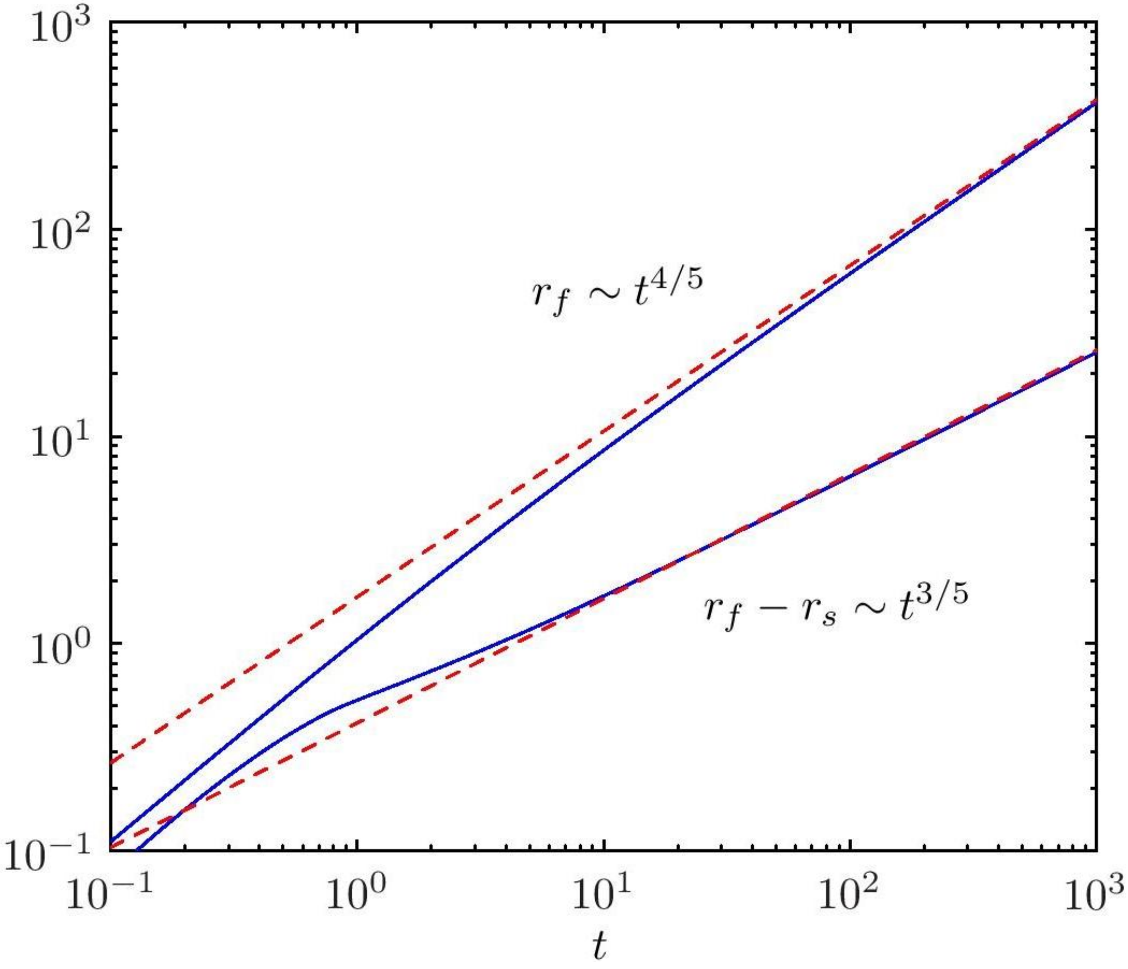
Momentum
$$\frac{\partial}{\partial t}\left(u_r h\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(ru_r{}^2 h\right)+g'h\frac{\partial h}{\partial r}=0$$

Reduced gravity $g'=\Delta\rho g/\rho_0$

- A similarity scaling would indicate that $r_f \sim (g'Qt^3)^{1/4}$
  - But this is not realised from numerical solutions of the governing equations.



Numerical solution



- Within tail, motion is steady: $r u_r h=1$ $u_r^2+h=const$
- Connected to front via a shock:

$$\left[(u-c)h\right]_-^+ = 0$$

$$\left[(u-c)^2 h + \tfrac{1}{2}h^2\right]_-^+ = 0$$

r>>1 →

$$c = u_f$$

$$u^2 h\Big|_- = \tfrac{1}{2} h_f^{\;2}$$

$$h_f^{\;2} \sim \frac{1}{r_f}$$

$$u_f \sim \sqrt{h_f}$$

$$\boxed{r_f \sim t^{4/5}}$$

Radial motion: *unsteady, drag-influenced*

- The motion becomes influenced by atmospheric drag, here modelled $C_D\rho u_r|u_r|$

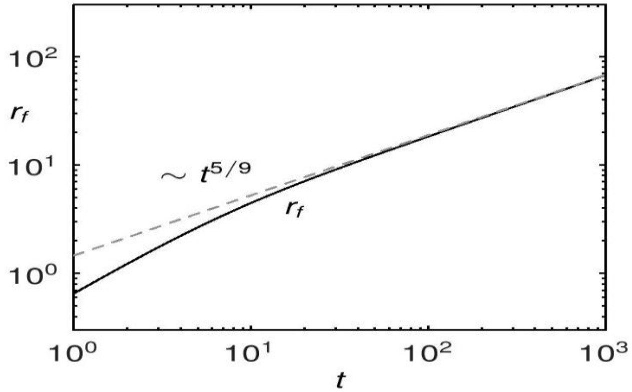
$$\frac{\partial h}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r u_r h\right) = 0$$

$$\frac{\partial}{\partial t}\left(u_r h\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(u_r{}^2 h\right) + \boxed{\frac{1}{4}N^2h^2\frac{\partial h}{\partial r}} = \boxed{-C_D u_r\left|u_r\right|}$$

- At long times: pressure gradient~ drag *[inertia negligible]*

mass: $rhu_r \sim Q$ dynamics: $N^2h^3/r \sim C_Du_r^2$ kinematics: $u_r \sim r/t$

$$\boxed{r^9 \sim Q^3N^2t^5/C_D}$$

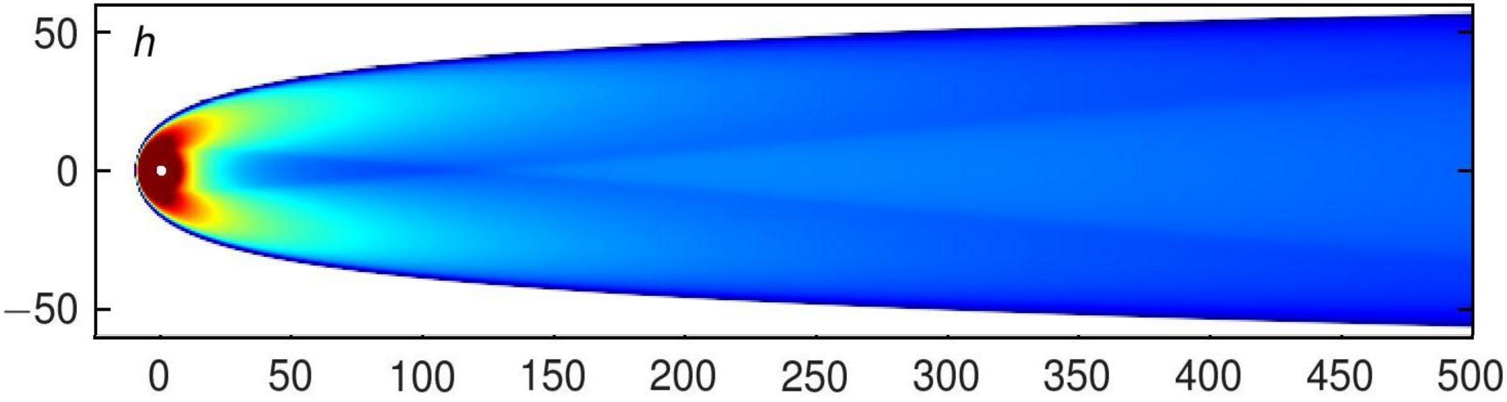


Steady evolution with wind: *Numerical solutions*

$Q=10^{10}$ m^{3}s^{-1}, N = 0.01 s^{-1},
Wind speed 30 ms^{-1}

Dimensionless wind speed
U=0.3

$$\frac{\partial}{\partial x}(uh)+\frac{\partial}{\partial y}(vh)=0$$
$$\frac{\partial}{\partial x}(u^2h)+\frac{\partial}{\partial y}(uvh)+\frac{1}{4}h^2\frac{\partial h}{\partial x}=-\tau_x$$
$$\frac{\partial}{\partial x}(uvh)+\frac{\partial}{\partial y}(v^2h)+\frac{1}{4}h^2\frac{\partial h}{\partial y}=-\tau_y$$



Plan views of the height of an intrusion from a sustained source

Far-field form of intrusion

- When width of intrusion W <<streamwise length L

$$\frac{\partial}{\partial x}\left(u^2h\right)+\frac{\partial}{\partial y}\left(uvh\right)+\frac{1}{4}h^2\frac{\partial h}{\partial x}=\boxed{-\tau_x} \qquad \Rightarrow u=U$$

$$\frac{\partial}{\partial x}\left(uvh\right)+\frac{\partial}{\partial y}\left(v^2h\right)+\boxed{\frac{1}{4}h^2\frac{\partial h}{\partial y}}=\boxed{-\tau_y}=-C_Dv^2$$

- Fluid conservation $\quad \frac{\partial}{\partial x}\left(uh\right)+\frac{\partial}{\partial y}\left(vh\right)=0 \qquad \int_0^W uh=\frac{1}{2}$

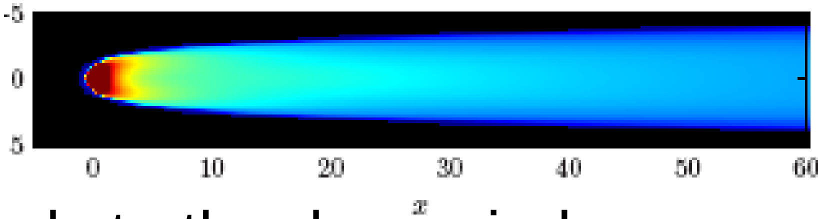
Solution:

$$v=\frac{y}{3x} \qquad h(x,y)=\frac{1}{x^{1/3}}\frac{2^{2/3}C}{3^{2/3}}\left(1-\frac{y^3}{Cx}\right)^{1/3} \qquad W=Cx^{1/3}$$

Constant

$C=0.884$

Far downwind of source



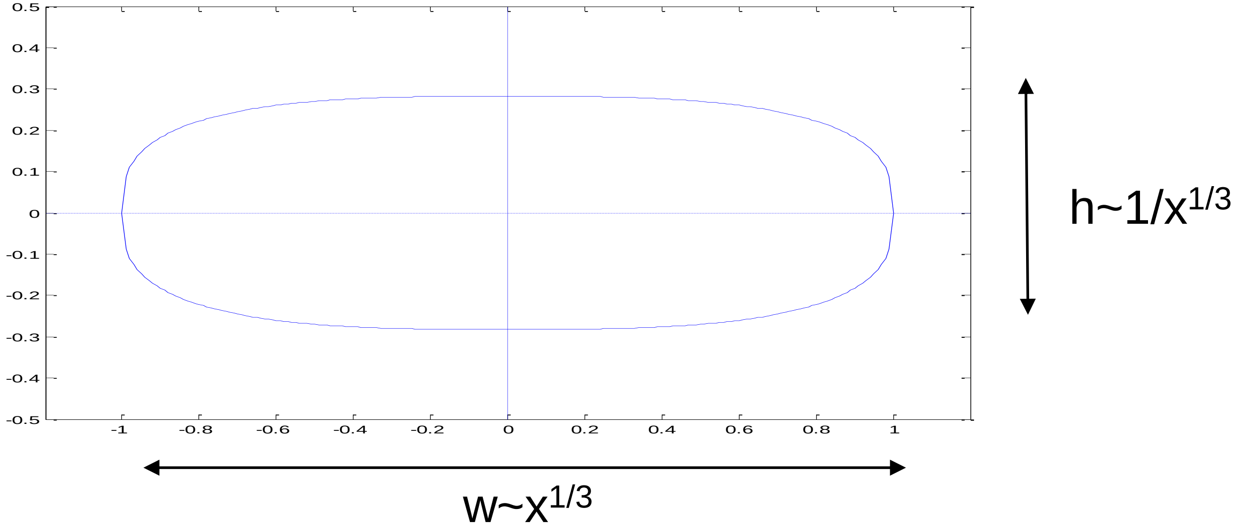
- Far from source the intrusion tends to the downwind velocity (u~U), but continues to spread laterally
- Height of intrusion: *similarity solution far downwind*

Height $$h(x,y)=\left(\frac{Q^3C_D}{N^2U}\right)^{1/6}\frac{1}{x^{1/3}}\frac{2^{2/3}C}{3^{2/3}}\left(1-\frac{y^3}{w^3}\right)^{1/3}$$

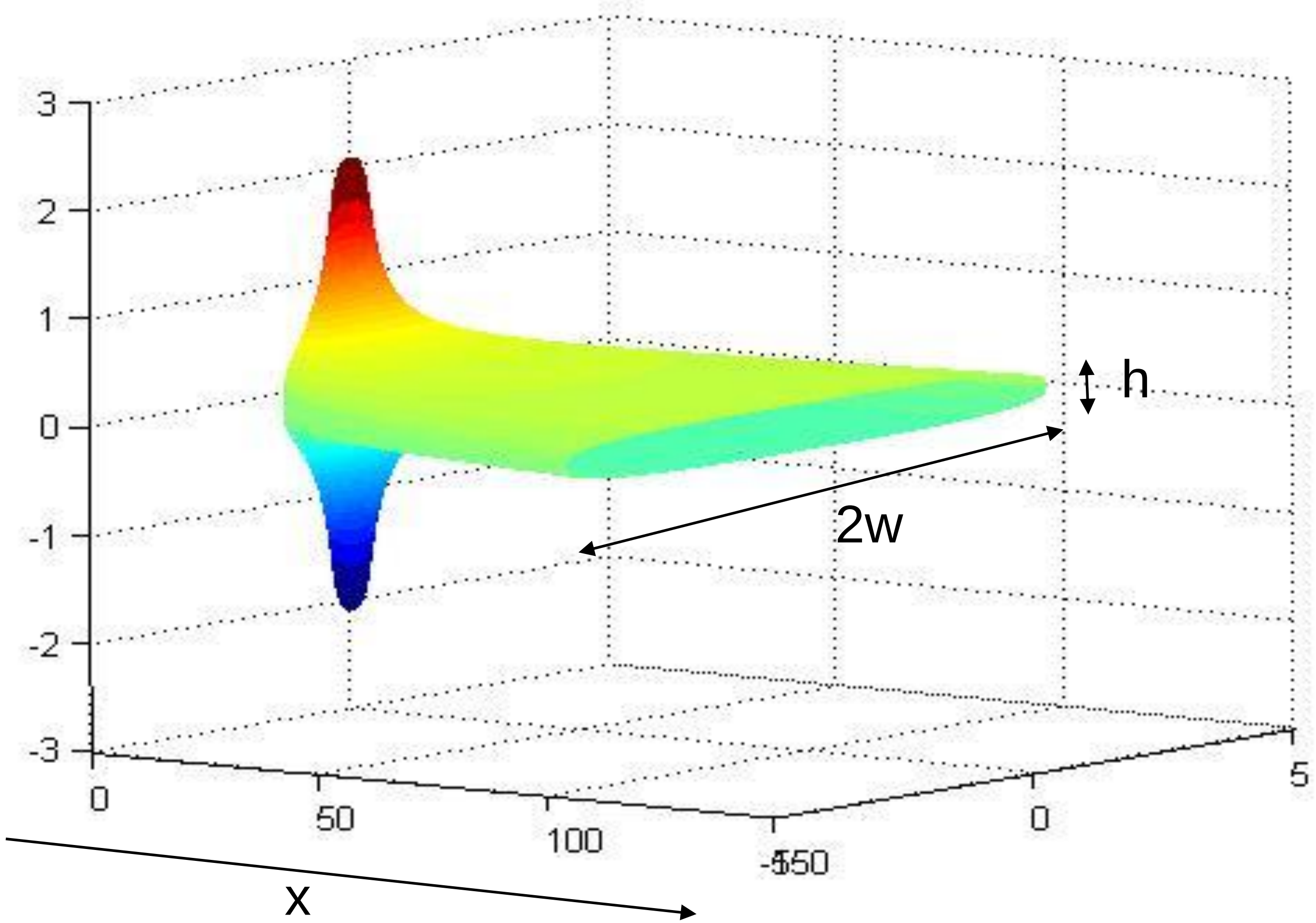
Width $$w=C\left(\frac{Q^3N^2}{U^5C_D}\right)x^{1/3}$$

Lateral Velocity $$v=\frac{Uy}{3x}$$

Constant $C=0.884$

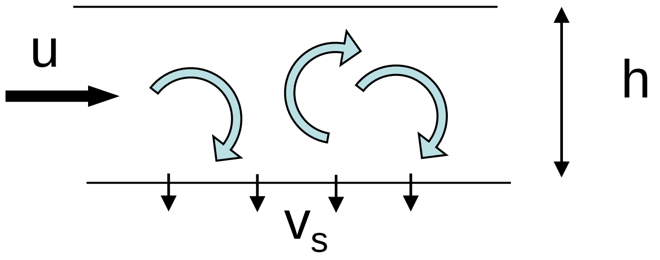


Shape of intrusion



Particle transport

- The concentration of suspended particles, ϕ, within a turbulent layer satisfies: *(Hazen's Law)*



$$\frac{\partial}{\partial t}(h\phi)+\frac{\partial}{\partial x}(hu\phi)+\frac{\partial}{\partial y}(hv\phi)=-v_s\phi$$

Advection with wind

Settling

- Steady, radially symmetric intrusion at neutral buoyancy height (u_r=radial velocity)

Particle transport:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_rh\phi)=-v_s\phi$$

Volume flux:

$$Q=ru_rh$$

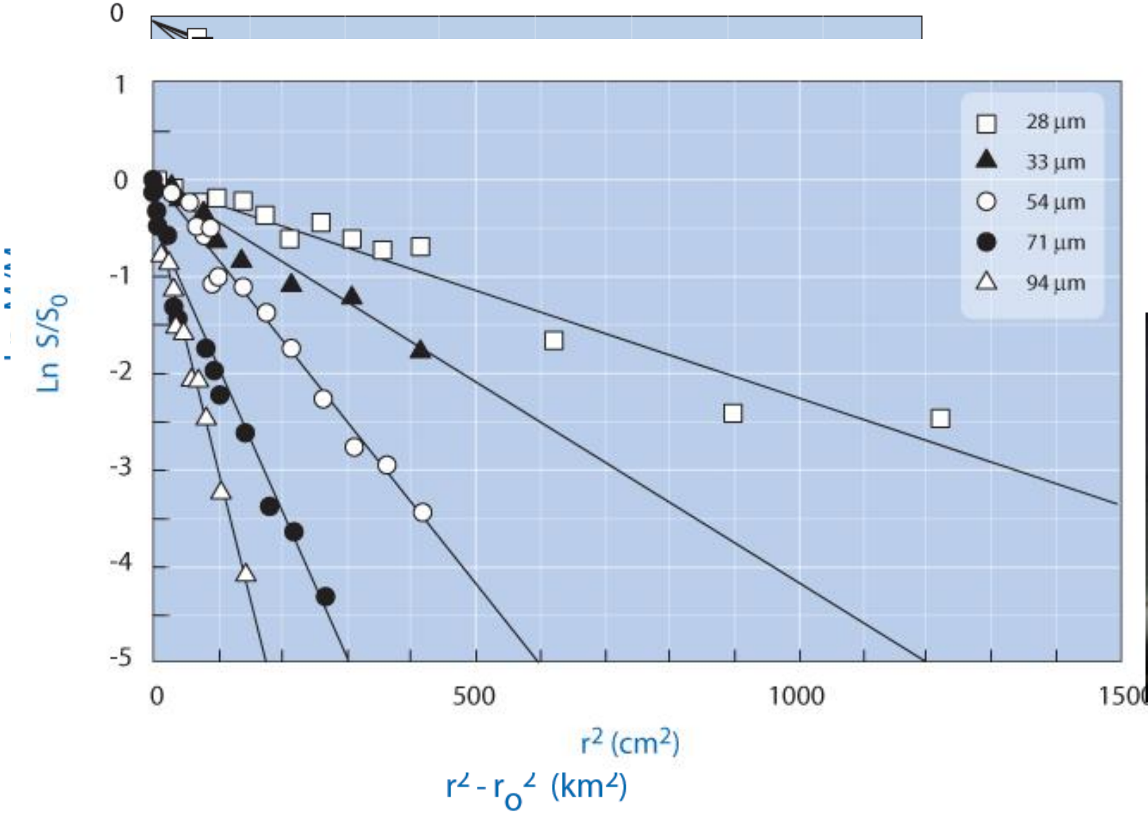
$$\phi=\phi_0\exp\left(-\frac{v_sr^2}{2Q}\right)$$

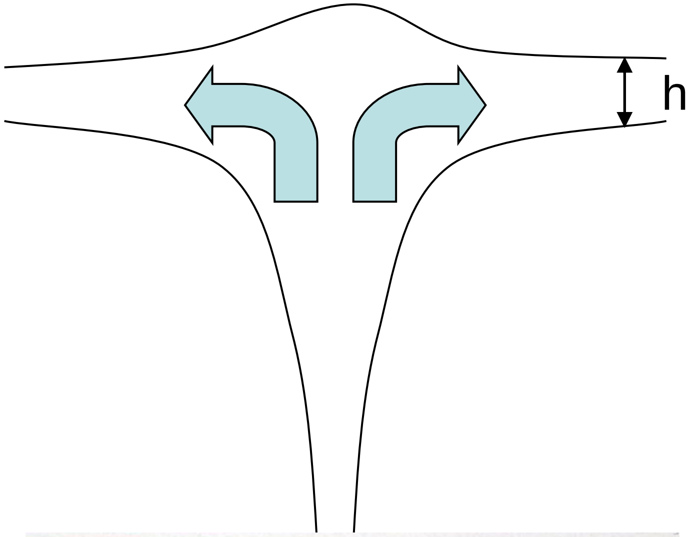
Distribution of deposit: *observations*

- Measure deposit from umbrella clouds

Field data from Fogo Volcano (Azores)

Laboratory experiments

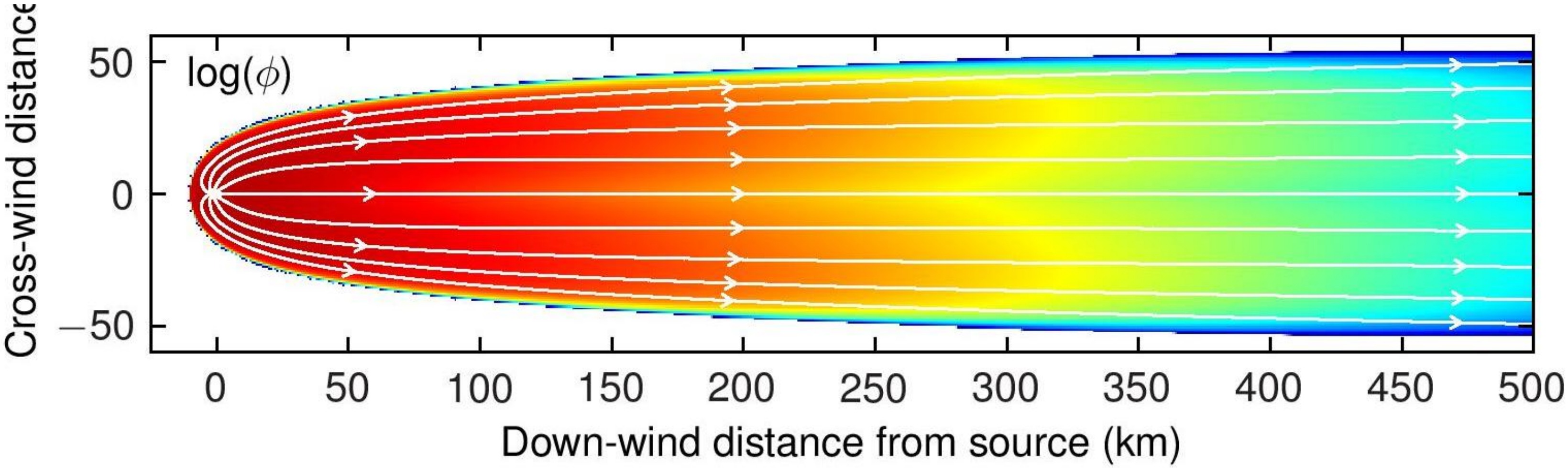




Ash transport in wind

- The concentration of particles satisfies:

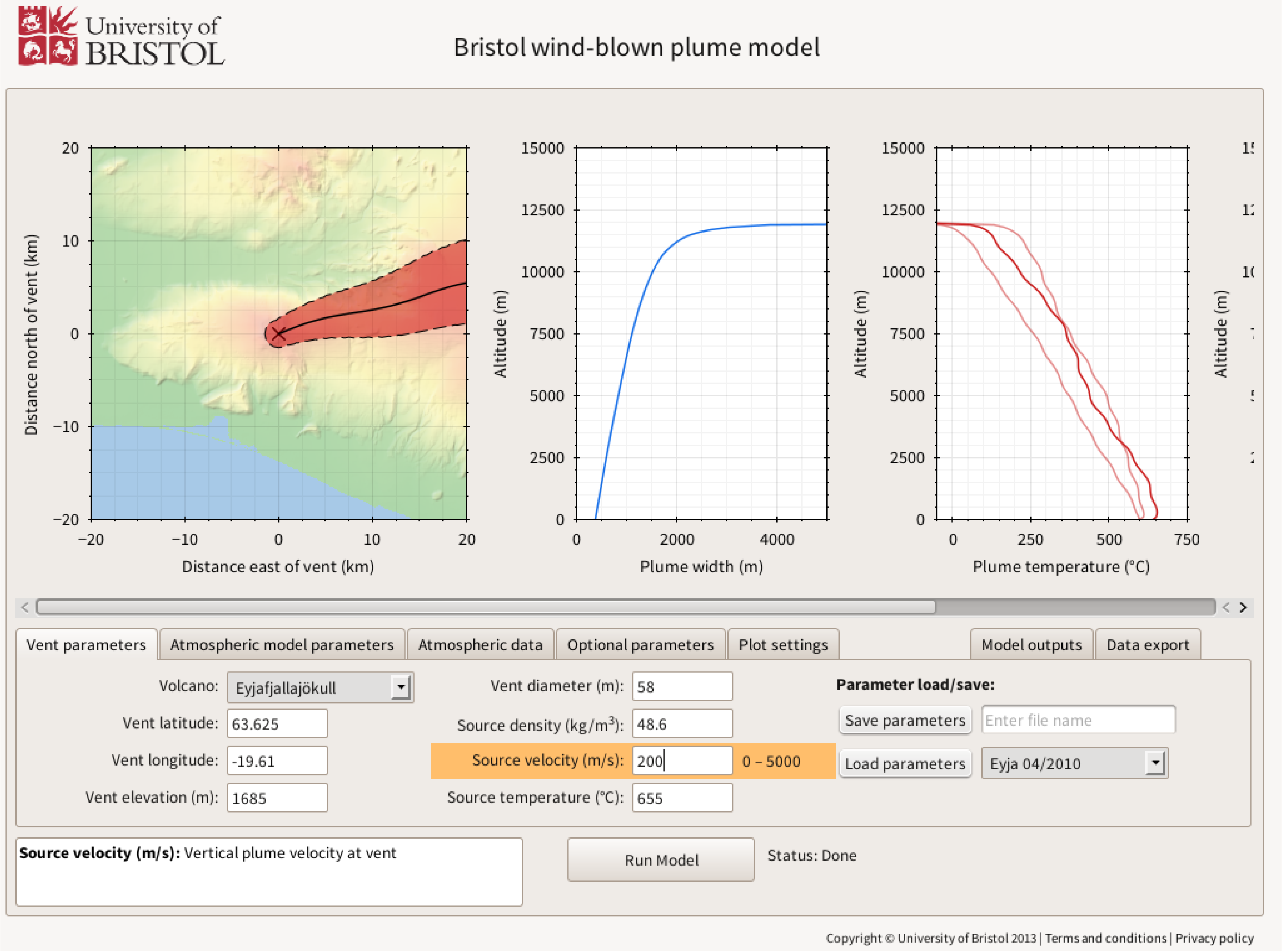
$$\frac{\partial}{\partial x}\left(u\phi\right)+\underset{\substack{\uparrow\\ \text{Advection with wind}}}{\frac{\partial}{\partial y}}\left(v\phi\right)=-\underset{\substack{\uparrow\\ \text{Settling}}}{\frac{v_s\phi}{h}}$$



$$\phi(x,y)=\phi_0\exp\left(-\frac{3^{5/3}}{C2^{8/3}}\left(\frac{v_sN^{1/3}}{U^{5/6}Q^{1/2}}\right)\frac{x^{4/3}}{\left(1-y^3/w^3\right)^{1/3}}\right)$$

Conclusions

- Plume rise heights are reduced by atmospheric winds
- Estimates of source mass flux from height of rise need to be derived from models that account for wind
  - Revised empirical formulae accounting for wind
- Intruding ash clouds are partly driven by buoyancy forces.
- Buoyancy processes lead to progressively thinning layers of ash in the atmosphere *[contrast to diffusive thickening].*
- Down wind the ash cloud moves with the atmospheric wind but continues to spread laterally,
- Ash is suspended in a well-mixed turbulent fluid layer and settles from its base.



Volcano: Eyjafjallajökull

Vent latitude: 63.625

Vent longitude: -19.61

Vent elevation (m): 1685

Vent diameter (m): 58

Source density (kg/m^3): 48.6

Source velocity (m/s): 200 0 – 5000

Source temperature (°C): 655

Parameter load/save:

Enter file name

Eyja 04/2010

Source velocity (m/s): Vertical plume velocity at vent

Status: Done

University of BRISTOL

www.plumerise.bris.ac.uk