Self-Consistent Models of Basal Entrainment in Snow Avalanches

Dieter Issler (NGI, Oslo)
Tómas Jóhannesson (IMO, Reykjavík)
Manuel Pastor (UPM, Madrid)

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NGI

Note

This presentation is largely based on the slides used in the author's talk at KITP on Tuesday, 2013-12-09, but it contains some changes that were made in preparation for a talk at the AGU Fall Meeting in San Francisco on 2013-12-13. Most of the changes aim at making the material easier to understand. There is one important addition to the content, however: An estimate of the entrainment rate under conditions typical of snow avalanches was obtained, and the physical basis for the limitations of the model was more clearly realized. It was felt that it would be a disservice to potential readers not to include them here.

D.I.

What is the problem?

- Most avalanches entrain at least part of the snow cover.
  Mass may grow up to tenfold!

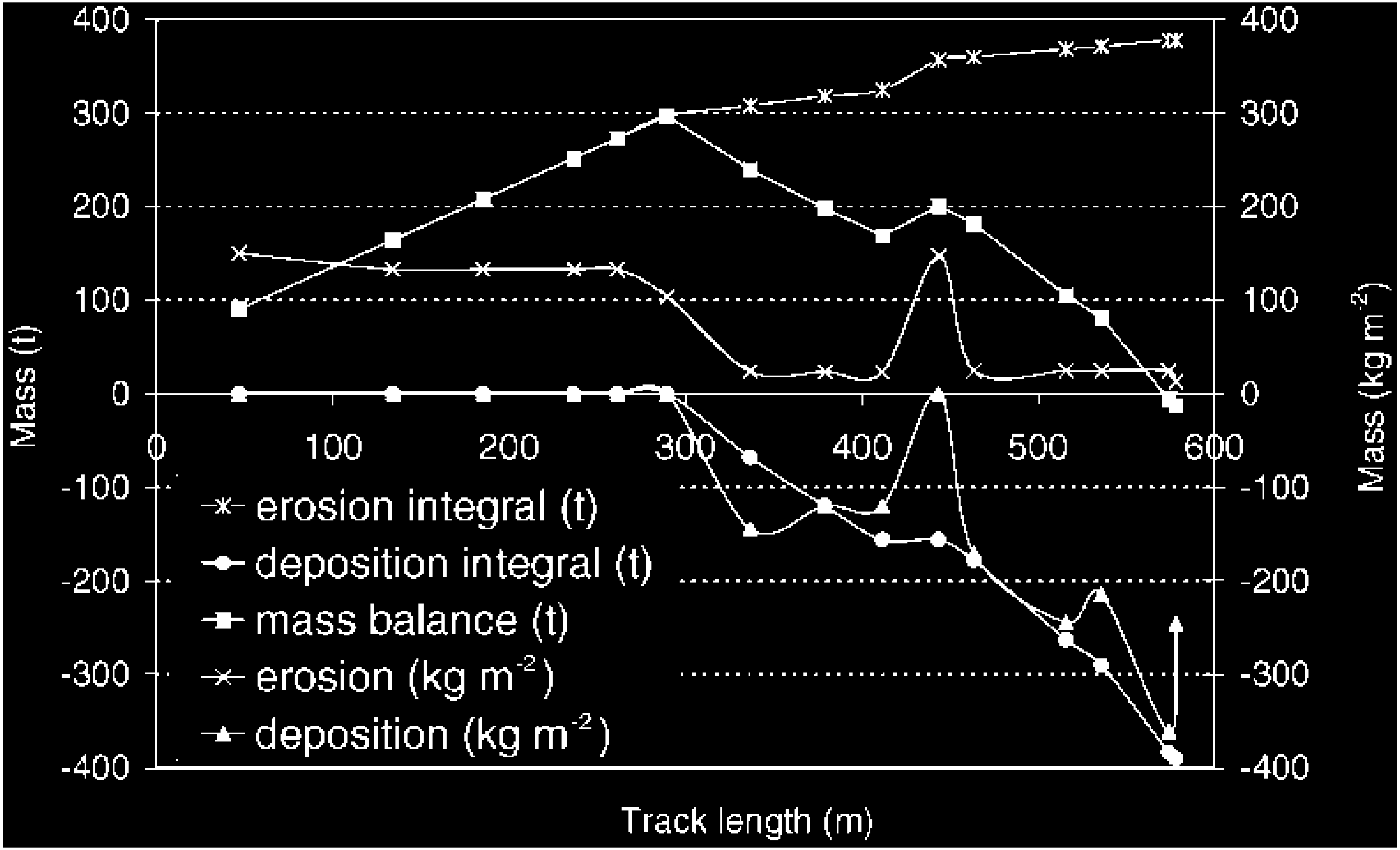
- Entrainment has effect on dynamics, needs to be modeled.

- Most practical models are depth-averaged.
  $\Rightarrow$ Need entrainment rate $q_e = \rho w_e = f(u,h)$.

- Solution must be determined by
  - snow cover properties
  - flow rheology

  without free parameters.

Spatial mass balance in a snow avalanche (measured at Monte Pizzac test site, Italy, in 1998)



From Sovilla et al., Annals Glaciol. 32 (2001), 230–236.

FMCW radar plot of snow avalanche at Vallée de la Sionne

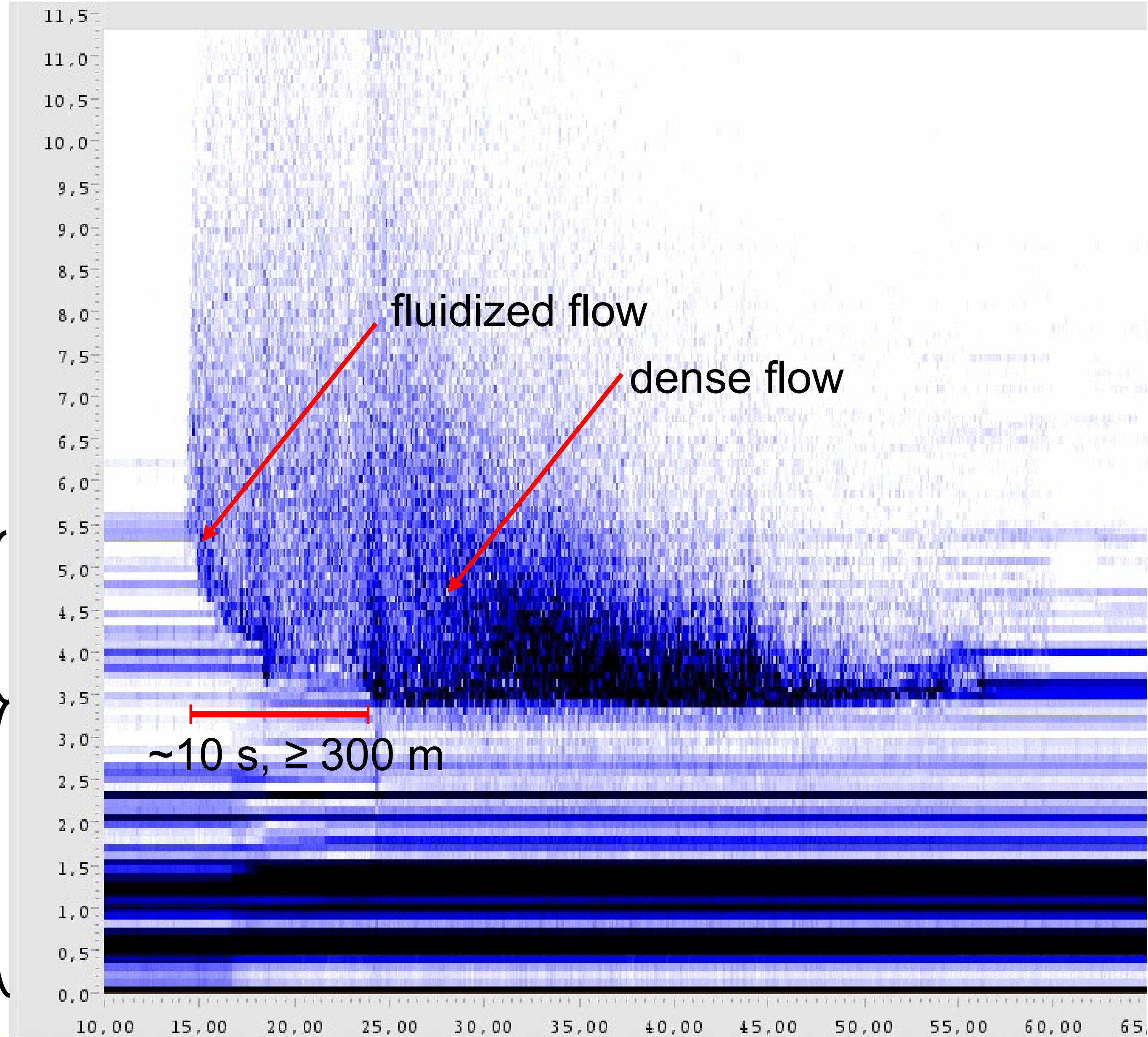
Observed entrainment rate:

10–200 kg m^{-2} s^{-1},

diminishing with time and erosion depth.

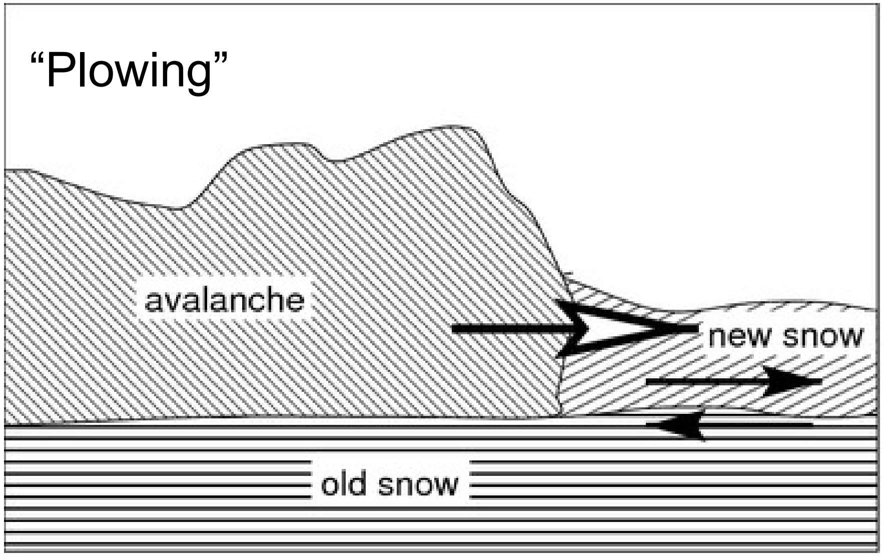
2 m of fresh snow eroded

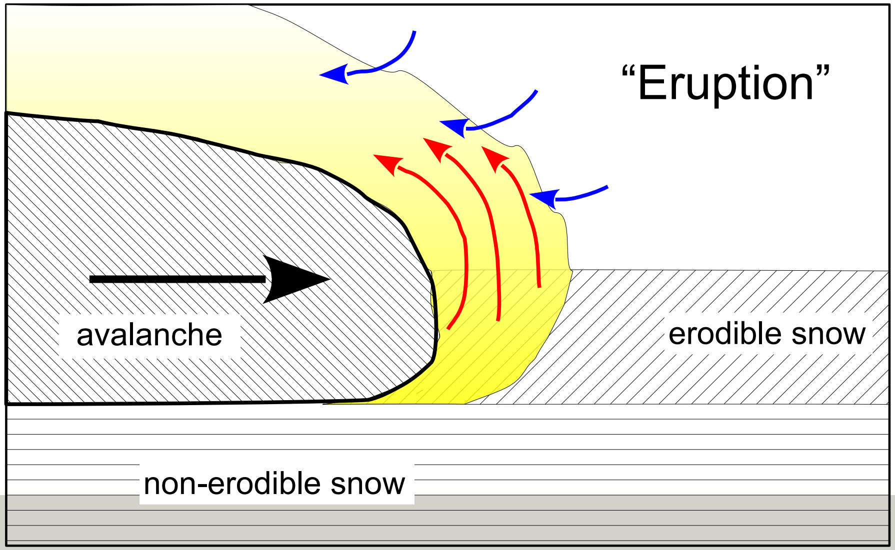
Hard old snow not eroded



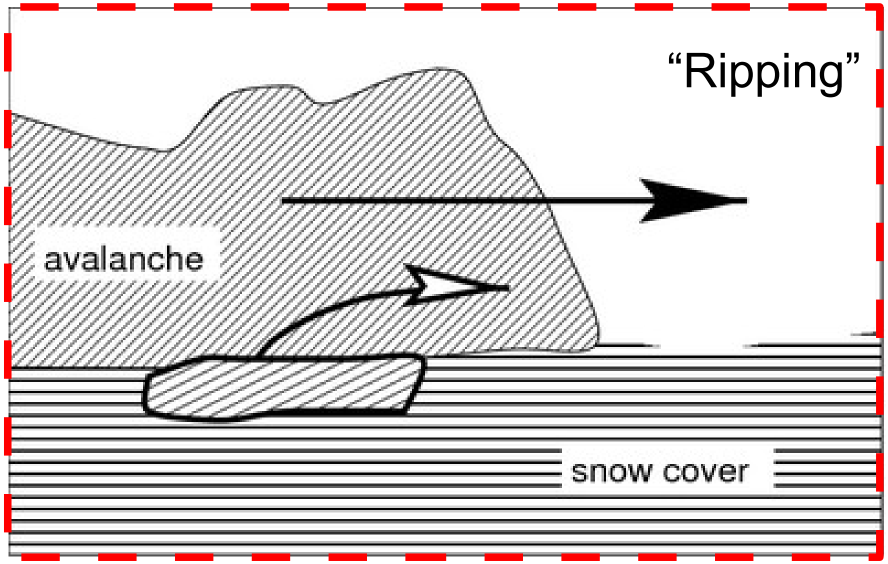
Conjectured erosion mechanisms (Gauer & Issler, 2004)

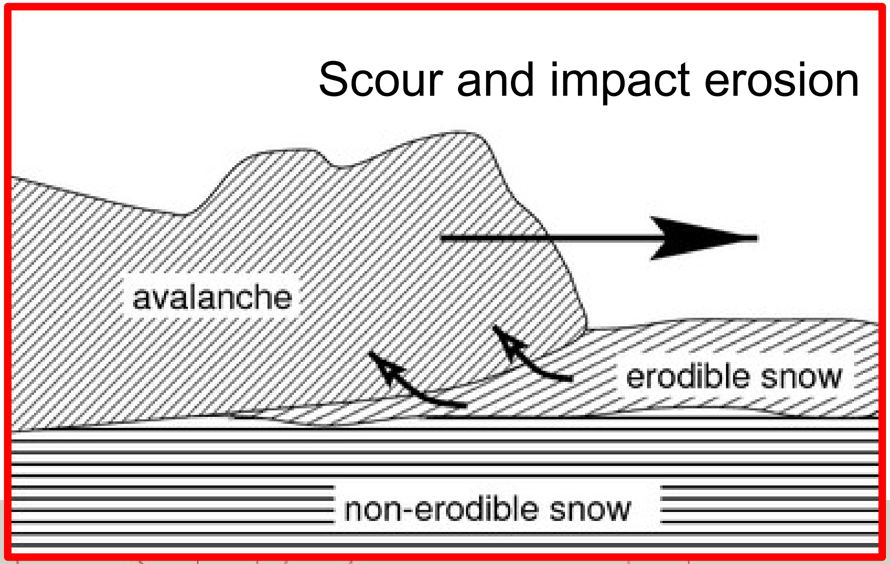
Frontal mechanisms





Mechanisms acting along bottom





Assumptions:

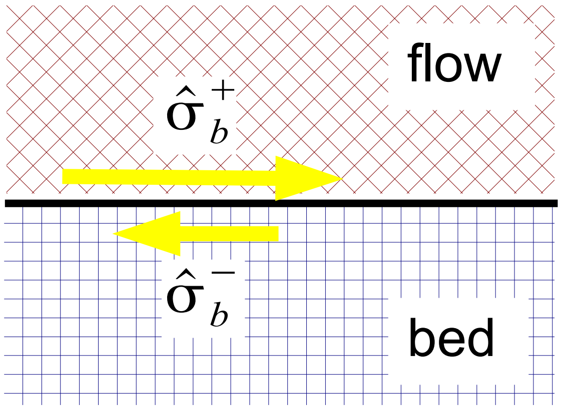
- Concentrate on erosion by scour along flow bottom, neglect frontal entrainment.
- **Bed material is perfectly brittle – breaks at stress τ_c .** Neglect energy required to break the snow cover.
- Interior of snow cover remains stable when surface is eroded. (This is a tricky one and needs to be investigated further!)
- Stress tensor depends on overburden and shear rate $\partial_z u_x \equiv \dot{\gamma}$:

$$\sigma_{ij}(x, z, t) = \sigma_{ij}(z, h, \dot{\gamma}),$$

  where $h = h(x,t)$ and $\dot{\gamma} = \dot{\gamma}(x, z, t)$.

Analytic solution for sliding blocks

- Assume a bed (b) friction law of the form



$$\hat{\sigma}_b^+ \equiv \frac{\sigma_{xz}(z=b^+)}{\rho} = \hat{f}(\bar{u}, h, \ldots)$$

Shear stress at top of bed: $\hat{\sigma}_b^- = \hat{\tau}_c$.

- The block model assumption implies that the velocity profile inside the block is uniform. This implies sliding, i.e. the velocity is discontinuous at the interface.

- At a threshold velocity u_c, the shear stress exerted by the flow becomes larger than the shear strength of the snow cover, leading to a *discontinuity in the shear stress* at the interface.

- With $u > u_c$, the snow cover fails and the difference between actual shear stress at the bottom of the block and the bed shear strength accelerates the eroded bed material to speed u.

- Jump condition for *x*-momentum across bed–flow interface:

$$w_e \cdot \left(u(b^+) - u(b^-) \right) = w_e \bar{u} = \hat{\sigma}_b^+ - \hat{\sigma}_b^- = \hat{f}(\bar{u}, h, \ldots) - \hat{\tau}_c$$

- Now immediately find the entrainment rate:

$$q_e = \rho w_e = \begin{cases} 0 & \text{if } \sigma_b(\bar{u}, h) \leq \tau_c, \\ \dfrac{\sigma(\bar{u}, h) - \tau_c}{\bar{u}} & \text{else.} \end{cases}$$

Note added after talk: Under the stated assumptions, this is the only consistent solution. However, if one considers that the eroded particles have a finite size d, their acceleration takes a finite time during which the relative velocity between the block and the accelerated particles is diminishing. One should then expect that the accelerating force and, consequently, the entrainment rate are reduced.

For the Voellmy bed friction law: $\hat{\sigma}_b^+ = \operatorname{sgn}(\bar{u}) \left(\hat{\sigma}_n \tan\delta + k\,\bar{u}^2\right)$

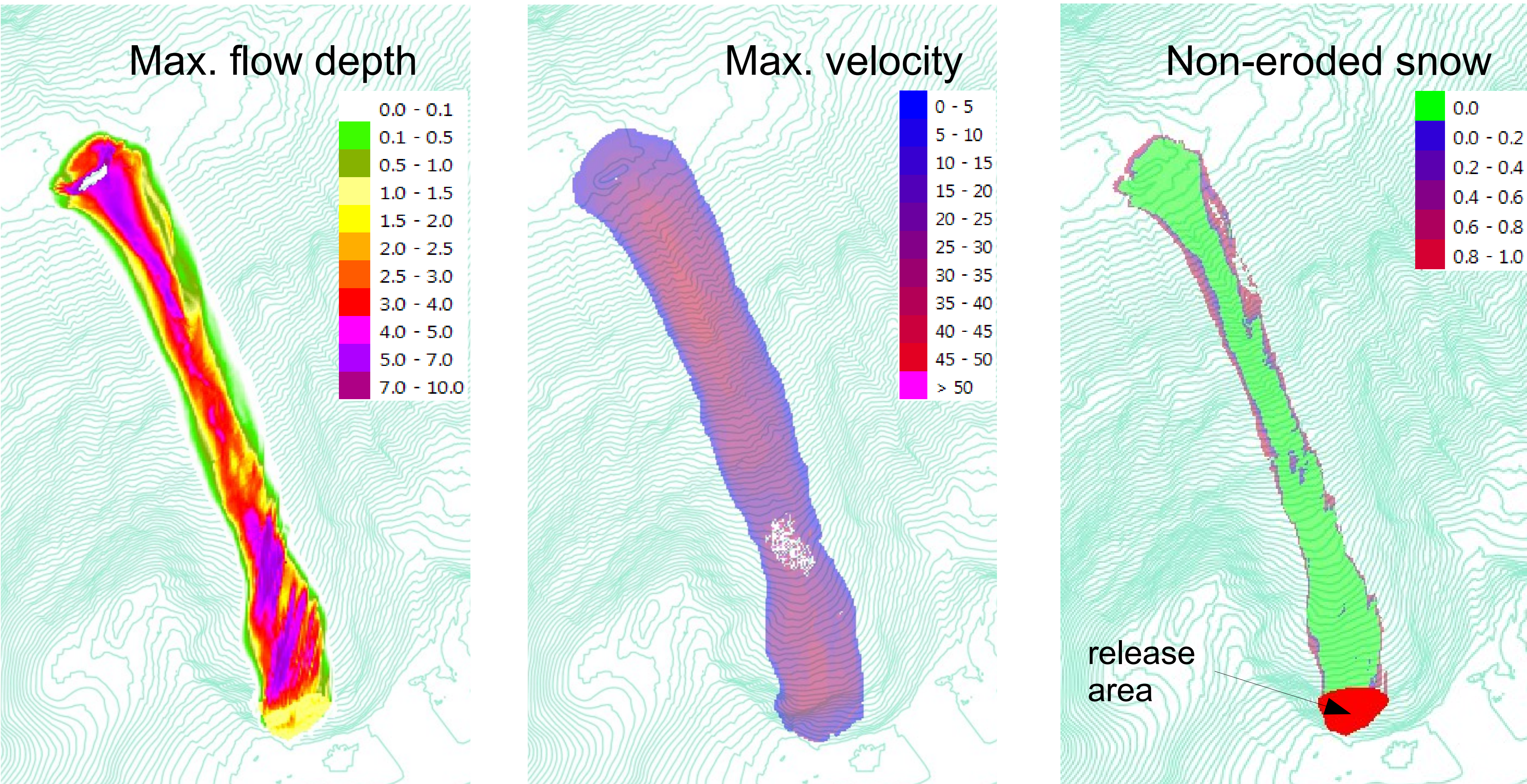
with σ_n = normal stress on bed,
δ = bed friction angle (assume $\delta < \theta$)

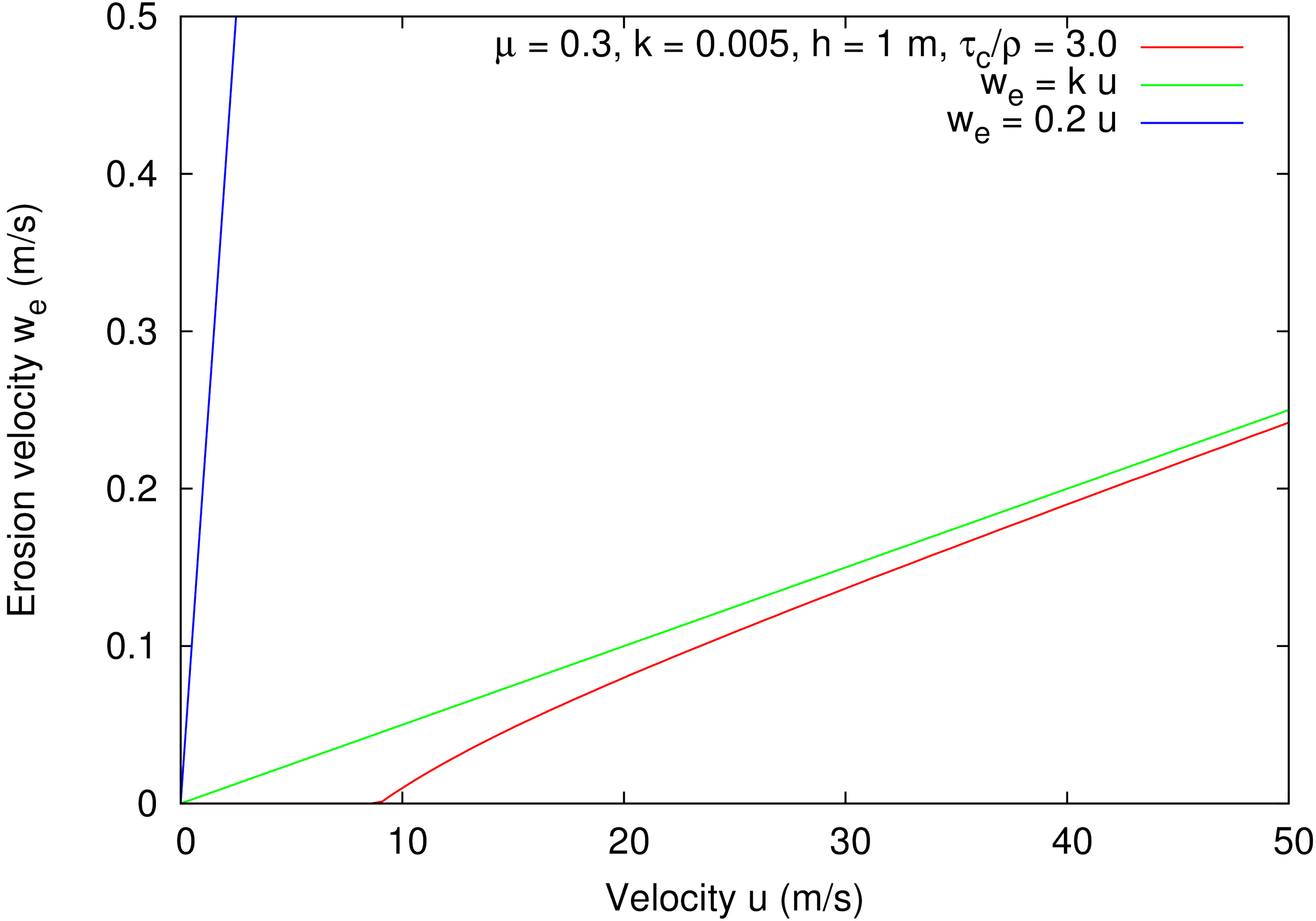
$$w_e = \begin{cases} 0 & \text{if } |\bar{u}| \le \sqrt{\dfrac{\hat{\tau}_c - \hat{\sigma}_n \tan\delta}{k}}, \\[2ex] k\,\bar{u} - \dfrac{\hat{\tau}_c - \hat{\sigma}_n \tan\delta}{\bar{u}} & \text{else.} \end{cases}$$

Compare this with the popular entrainment assumption (e.g. (Eglit, 1984), implementation in the code RAMMS, etc.):

$$w_e = c\,\bar{u} \quad \text{with} \quad c > 0 \ \text{ arbitrary}$$

Avalanche mesured at Ryggfonn test site on 1993-03-27
Simulation with proposed entrainment relation, μ = 0.4, k = 0.001
Snow shear strength varies with altitude from 1 to 1.2 kPa.





μ = 0.3, k = 0.005, h = 1 m, τc/ρ = 3.0
we = k u
we = 0.2 u
Erosion velocity we (m/s)
Velocity u (m/s)
0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
50

Equation for the erosion rate

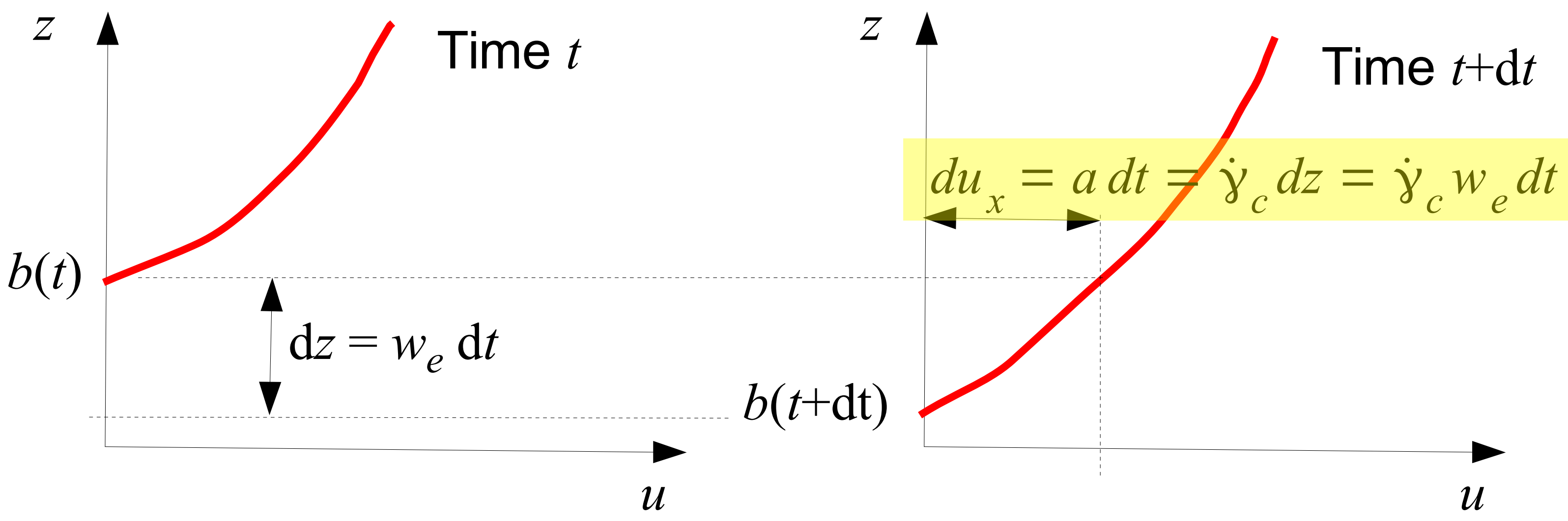
- Equation of motion in the flow (2D for simplicity):

$$\frac{du_x}{dt} = g\sin\theta + \partial_x \hat{\sigma}_{xx} + \partial_z \hat{\sigma}_{xz}$$

- Boundary conditions: $u_x(b,t) = 0, \quad \sigma_{zz}(h,t) = 0, \quad \boldsymbol{\sigma_{xz}(b,t) = \tau_c}.$

  No erosion: $\sigma_{xz}(b,t) < \tau_c.$
  Erosion: $\sigma_{xz}(b,t) = \tau_c.$
  If shear stress exceeds τ_c, erosion rate increases instantaneously and shear stress drops because of acceleration of eroded particles. Self-regulating mechanism similar to Owen's hypothesis for blown sand or snow.)

- Kinematic relation between erosion rate, velocity profile and particle acceleration at the bed-flow interface:



- Velocity at time $t + dt$ of particles eroded at time t:

$$u(b(t), t+dt) = 0 + (g \sin\theta + \partial_z \hat{\sigma}_{xz})\, dt \ .$$

- Shear rate at erosion front is locked to critical shear rate:

$$\dot{\gamma}(b,t) = \frac{u(b,t+dt) - 0}{dz} = \frac{\left(g \sin\theta + \partial_z \hat{\sigma}_{xz}(b,t)\right) dt}{w_e(t)\, dt} \overset{!}{=} \dot{\gamma}_c$$

Equation for the erosion rate

- Momentum balance equation in the flow (2D for simplicity):

$$\frac{du_x}{dt} = g \sin\theta + \partial_x \hat{\sigma}_{xx} + \partial_z \hat{\sigma}_{xz}$$

- Boundary conditions: $u_x(b,t) = 0$, $\sigma_{zz}(h,t) = 0$, $\boldsymbol{\sigma_{xz}(b,t) = \tau_c}$.

- Kinematic relationship between erosion rate, velocity profile and particle acceleration at the bed-flow interface:

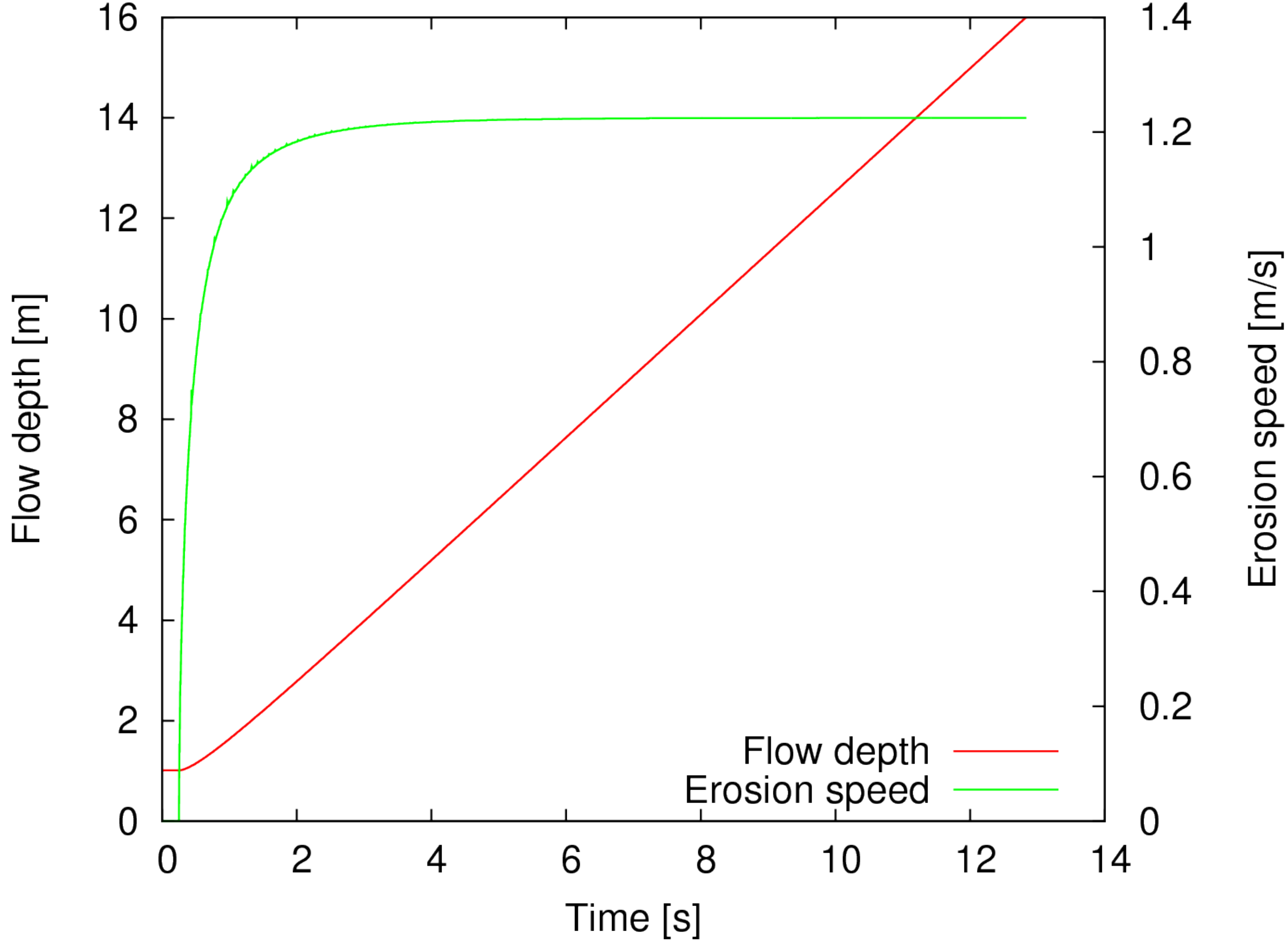
$$a_x dt = \partial_z u_x dz \quad \Rightarrow \quad a_x = \dot{\gamma} w_e$$

- Shear stress at interface equals shear strength → critical shear rate $\dot{\gamma}_c$.

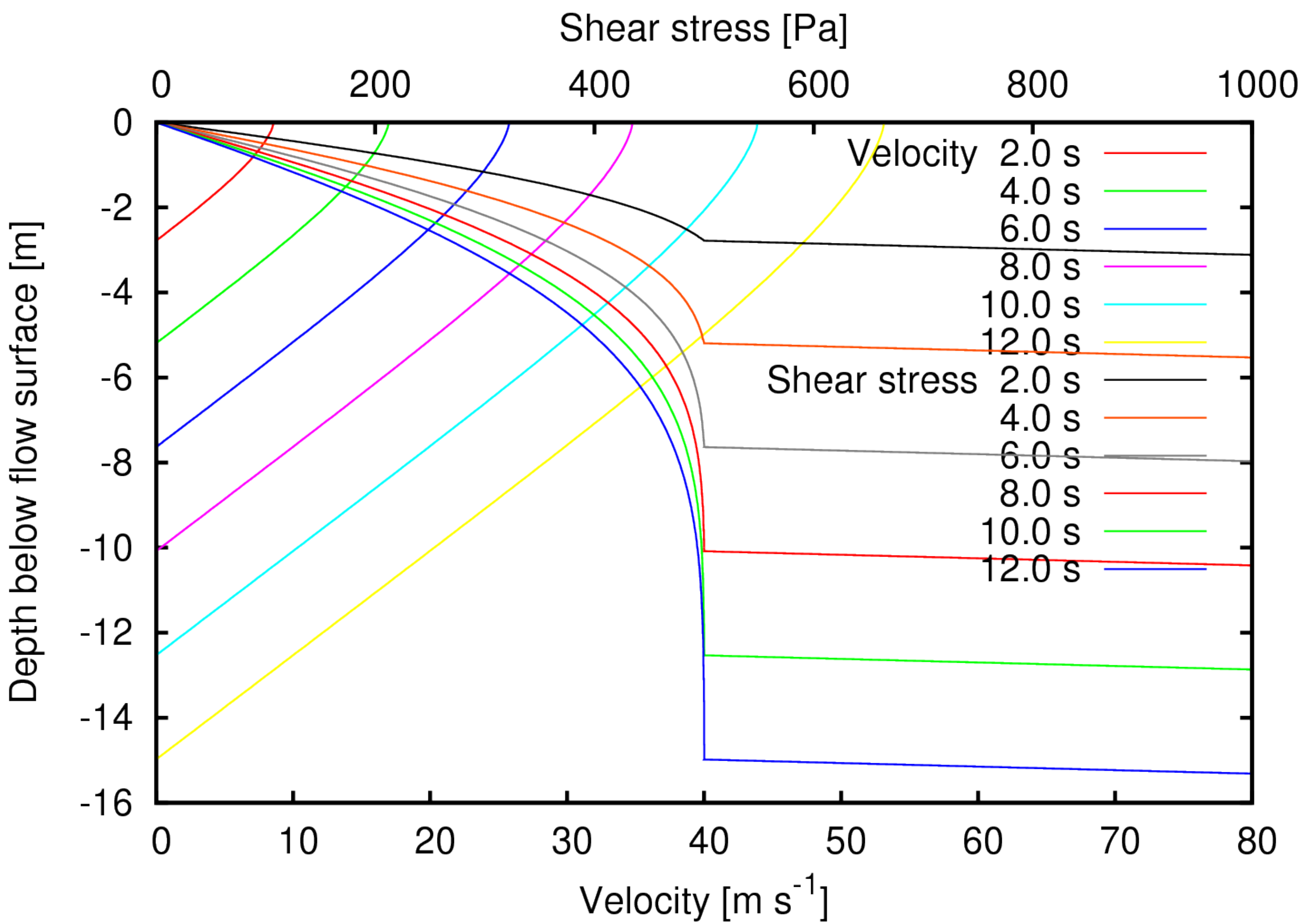
☞

$$w_e(t) = \frac{g \sin\theta + \partial_z \hat{\sigma}_{xz}(\dot{\gamma}_c, h)}{\dot{\gamma}_c}.$$

Numerical solution for a Bagnoldian fluid: Flow depth and erosion rate vs. time



Numerical solution for a Bagnoldian fluid: Evolution of velocity and shear stress profiles



Approximate solution for the NIS model

➔ Aim for *depth-averaged* model with erosion rate as source term.

- Use Norem–Irgens–Schieldrop (NIS) model for rheology:

$$\hat{\sigma}_n = -\hat{p}_e - \nu_0 \dot{\gamma}^2, \qquad \hat{\tau} = \mu \hat{p}_e + \nu_s \dot{\gamma}^2$$

effective pressure
(through skeleton)

collisional contribution
(dispersive stresses)

- Without erosion: Equilibrium profile functions are Bagnoldian,

$$u_x(z) = U \cdot \left[1 - \left(1 - \frac{z}{h} \right)^{3/2} \right].$$

- Critical shear rate for erosion:

$$\mu g h \cos\theta + (\nu_s - \mu \nu_0) \dot{\gamma}_c^{\,2} = \hat{\tau}_c$$

With erosion, approximate the velocity profile by (see next slide):

$$u_x(z) \approx U \cdot \left[1 - \left(1 - \frac{z}{h} \right)^{\alpha(x,t)} \right]$$

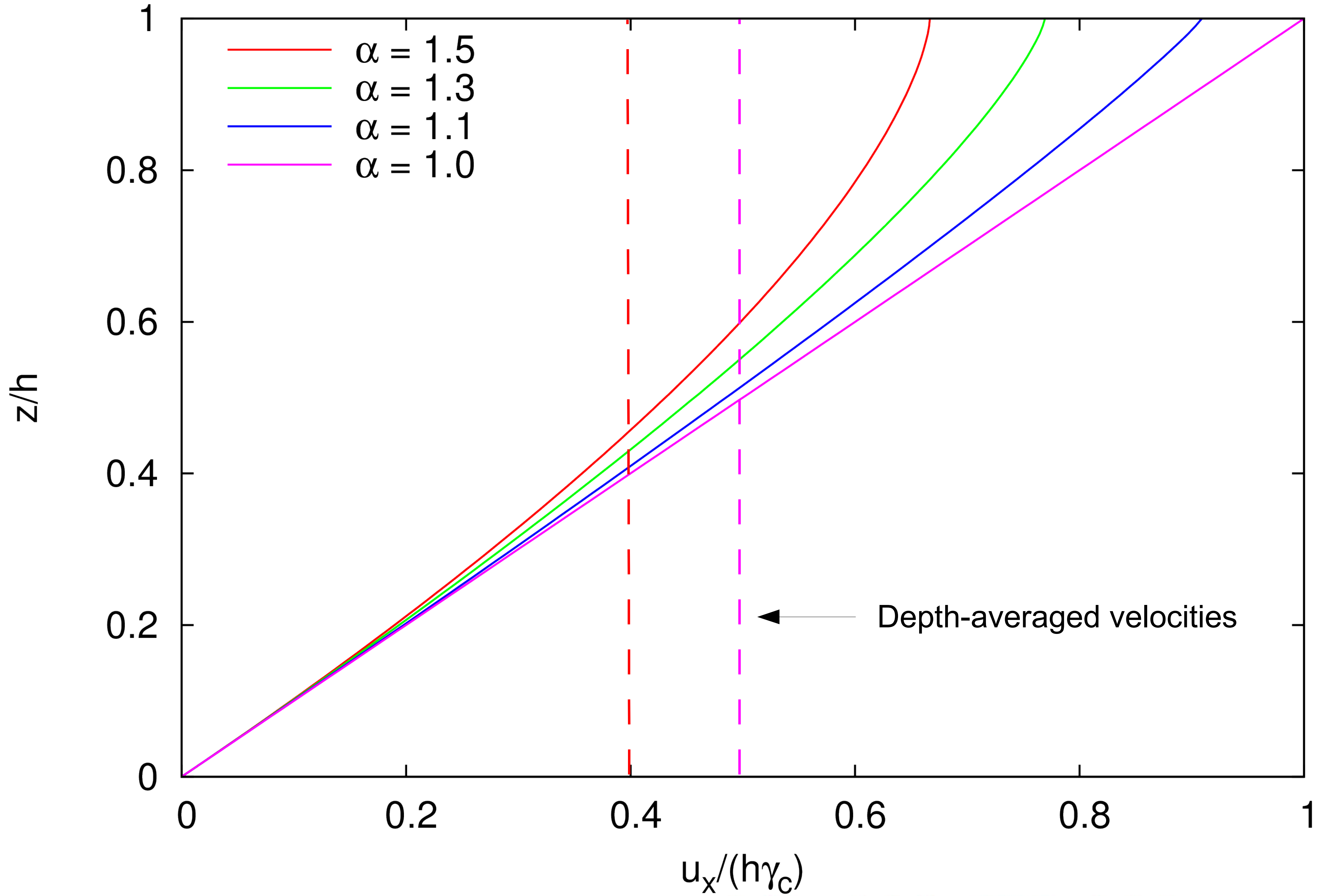
Choose $1 \le \alpha \le 3/2$ *variable* to match velocity $\bar{u}_x$ and shear stress at interface:

$$u_x(0) = 0, \quad \partial_z u_x(h) = 0, \quad \partial_z u_x(0) = \dot{\gamma}_c, \quad \bar{u} = \frac{\alpha}{\alpha + 1} U$$

Fulfilled by construction (first two conditions)

2 equations, 2 unknowns (last two conditions)

All relevant quantities can now be computed.



α = 1.5
α = 1.3
α = 1.1
α = 1.0
Depth-averaged velocities
z/h
u_x/(hγ_c)
0
0.2
0.4
0.6
0.8
1

From velocity profile compute $\partial_z \hat{\sigma}_{xz}(b,t)$ to obtain erosion formula:

$$w_e = \Theta\left(\sigma_{xz}(0) - \tau_c\right)\left[\frac{h\, g \sin\theta - \hat{\tau}_c}{h\,\dot{\gamma}_c} + (\nu_s - \mu\,\nu_0)\left(5\,\frac{\dot{\gamma}_c}{h} - 2\,\frac{\dot{\gamma}_c^2}{\bar{u}_x}\right)\right]$$

- No free parameters in the erosion model!
- Erosion threshold: Shear stress must exceed bed strength.
- For given flow depth, erosion rate grows with velocity, but is limited.

- Model applies only if Coulomb friction is less than bed strength.
- For typical snow avalanche conditions: w_e **= 0.0–0.2 m/s**.

Different approach needed for heavy and/or fast flows!

Notes added after the talk:

1. The failure of the snow pack under an avalanche beyond a critical weight is not specific to this model, but generic if there is Coulomb friction.

2. One might expect that this dilemma can be avoided if one assumes the strength of the snow pack to increase with overburden, for which there is experimental evidence. One may assume, e.g.,

$$\hat{\tau}_c(z) = C + \mu_s \hat{\sigma}_n(z)$$

   with C the cohesion and μ_s the internal friction coefficient of the snowpack (see the conference poster by Jan-Thomas Fischer).
   However, *if* erosion occurs, the shear stress at $z = b$ is equal to τ_c and increases at the rate $g \sin \theta$ down into the snow pack. Hence one finds that $\mu_s > \tan \theta$. This in turn implies that $\tau_c(b) > \rho h g \sin \theta$. In other words, erosion is possible only if the avalanche is sufficiently faster than the equilibrium velocity (without erosion). This can be the case for a short distance after a sufficiently pronounced reduction of the slope angle, but continuous erosion on a smooth slope is not possible.

3. A possible solution to the stability problem of the snow pack is to suppose that the cohesion C also increases with depth, due to the progressive compaction of the snow pack since the snow storm and sintering. In this case, stability of the undisturbed snow pack requires

$$-\partial_z C > g\,(\sin\theta - \mu_s \cos\theta).$$

Then the shear strength at the interface with the avalanche can be less than the gravitational pull on the avalanche, $gh \sin\theta$, yet the interior of the snow pack is stable while erosion occurs at the interface.

4. The encountered bounds on the erosion rate and the velocity should not be interpreted as the model failing, but as indicative of another regime in which some of the assumptions made earlier are relaxed. I conjecture that the assumption $\sigma_{xz}(b) = \tau_c$ cannot be upheld in sufficiently rapid flows: The substrate fails catastrophically at full depth, but is not entrained into the flow at once. The velocity profile has an inflection point, being convex in its lower part and concave in its upper part.
Under such conditions, a description of entrainment as a shock at the avalanche front might be appropriate.

Conclusions / Outlook

1. Erosion and flow rheology are intimately coupled. This makes validation of erosion models difficult.

2. Consistent erosion functions have no adjustable parameters.

3. Stress boundary condition at bed–flow interface is central. Perfectly brittle behavior is an interesting candidate for snow avalanches.

4. Also consider frontal erosion by plowing or eruption.

5. We need more data on the shear-strength profile of new-snow layers and to implement this in numerical flow models.