KITP and Fluid Mechanics Seminar

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Segregation in dense, inclined flows: binary mixtures of spheres

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Why Bother?

Differences in particle size influence the flow, and particle segregation itself is of interest, but may be undesirable

How to Do It?

Simplest theory for segregation of spheres that differ little in size and mass

Extended kinetic theory for the dense, collisional flow of the mixture

Ingredients

Steady, uniform, dense, inclined flow

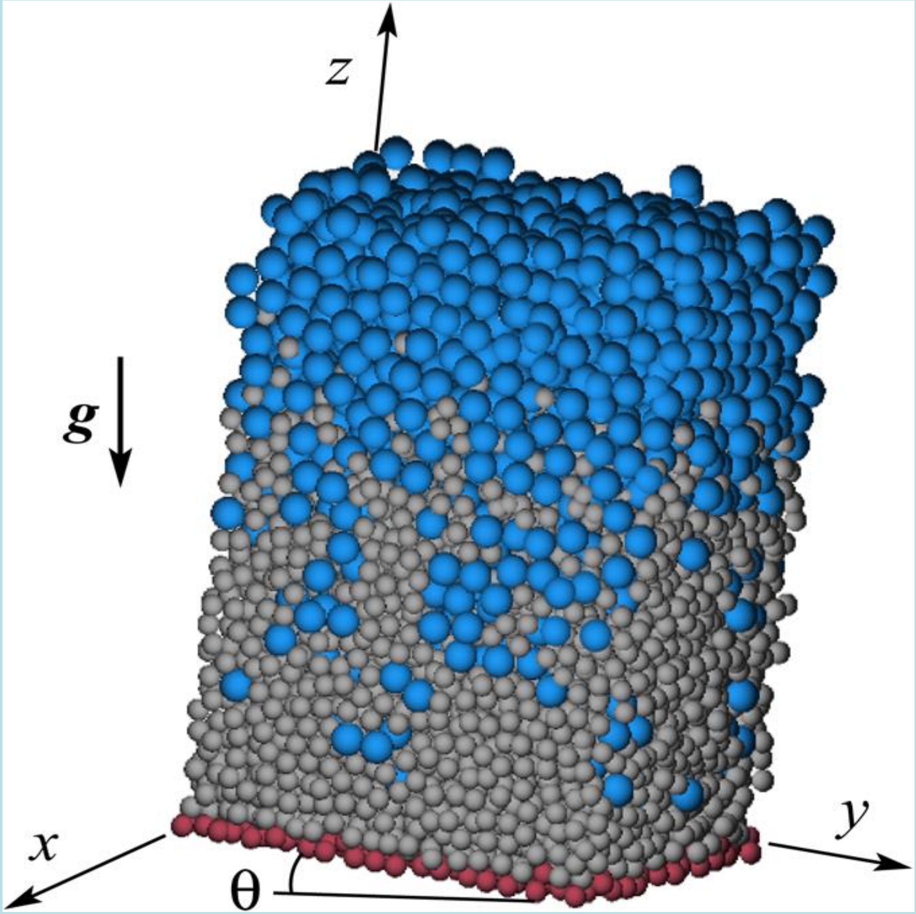
Binary mixture (De Haro, Cohen & Kincaid 1983)

Small differences in diameters and masses (Arnarson & Jenkins 2004)

Extended kinetic theory (Jenkins 2007)

Very dissipative collisions (Garzo & Dufty 1999)

Numerical Simulations



Tripathi and Khakhar, Phys. Fluids 3, 113302 (2011)

Variables

Radii: $r_A, r_B \quad d = r_A + r_B$

Masses: m_A, m_B

$\delta r = r_A / r_B - 1$

$\delta m = (m_A - m_B) / (m_A + m_B)$

Number densities:

$n_A, n_B \quad n = n_A + n_B$

Number fraction, species A:

$f_A = N_A / N$

Mass densities: $\rho_i = m_i \, n_i \qquad \rho = \rho_A + \rho_B$

Volume fractions: $c_i = n_i \, (4/3) \, \pi \, r_i^3 \qquad c = c_A + c_B$

Variables

Mixture velocity: $u = (\rho_A u_A + \rho_B u_B)/\rho$

Granular temperature: $T_i = \dfrac{m_i \left\langle C_i^{\,2} \right\rangle}{3} \quad i = A, B$

Mixture temperature: $T = \dfrac{n_A T_A + n_B T_B}{n}$

Segregation

Species momentum in normal direction:

$$0 = -\frac{dp_A}{dy} - \rho_A g \cos\phi + \Phi \qquad 0 = -\frac{dp_B}{dy} - \rho_B g \cos\phi - \Phi$$

Sum:

$$0 = -\frac{dp}{dy} - \rho g \cos\phi \qquad p = p_A + p_B \qquad \rho = \rho_A + \rho_B$$

Weighted difference:

$$-\rho_B \frac{dp_A}{dy} + \rho_A \frac{dp_B}{dy} + \rho\Phi = 0$$

Segregation

Partial pressures:

$$p_A = n_A(1 + K_{AA} + K_{AB})T \qquad K_{Aj} = \frac{1}{2}c_j\left(1 + r_A / r_j\right)^3 g_{Aj} \qquad j = A, B$$

Radial distribution functions:

$$g_{ij} = g_{ij}(r_A, r_B, c_A, c_B), \qquad j = A, B$$

Particle interactions:

$$\Phi = K_{AB} n_A T \left\{ \frac{m_B - m_A}{m_B + m_A} \frac{d}{dy}\left(\ln T\right) + \frac{d}{dy}\left[\ln\left(\frac{n_A}{n_B}\right)\right]\right\}$$

Small Differences δr and δm

$$\frac{dx}{dy} = -\frac{1-4x^2}{2}\left[-\frac{1}{T}\frac{dT}{dy}\left(R_1\delta r+\Gamma_1\delta m\right)+\frac{g}{2T}\left(R_2\delta r+\Gamma_2\delta m\right)\cos\phi\right]$$

$$\frac{dI}{dy} = \frac{1}{h}\frac{2\bar{c}x}{\left(1+2x\right)\left(\overline{nc}_A/\bar{n}_A+\overline{nc}_B/\bar{n}_B\right)}$$

$$x \equiv \frac{1}{2}\frac{n_A-n_B}{n}$$

$$I(y) = \frac{1}{h}\int_0^y \frac{2\bar{c}x(x)}{\left[1+2x(x)\right]\left(\bar{n}\bar{c}x_A/\bar{n}_A+\bar{n}\bar{c}_B/\bar{n}_B\right)}dx$$

$$I(0) = 0, \quad I(h) = \frac{N_A}{N} - \frac{1}{2}$$

Coefficients

$$R_1 = \frac{5}{58}\left[2 + \frac{c\left(3-c\right)}{2-c} - \frac{12}{5}G\right] + 2G\left[3 + \frac{c\left(3-c\right)}{2-c}\right] - \frac{12cH\left(1+4G\right)}{1+4G+4cH}$$

$$R_2 = -\frac{12cH}{1+4G+4cH}, \quad \Gamma_1 = \frac{179}{29}G + \frac{105}{116}, \quad \Gamma_2 = 2$$

$$G(c) \equiv c g_0(c), \qquad g_0(c) = 5.69\frac{c_M - 0.49}{c_M - c}, \quad H \equiv \frac{dG}{dc}$$

Dense, Dissipative Mixture

Mixture density:

$$\rho = (1 + 2x\delta m) m_{AB} n / 2 = constant$$

Mixture pressure:

$$p = (1 + e) m_{AB} nGT = \rho g (h - y) \cos\phi$$

Mixture temperature:

$$T = \frac{1}{2(1+e)G} \left(h - y\right) g \cos\phi (1 + 2x\delta m)$$

Dense, Dissipative Mixture

Mixture shear stress:

$$s = \frac{4J}{5\pi^{1/2}} m_{AB} n r_{AB} G T^{1/2} \frac{du}{dy}$$

$$J = \frac{1+e}{2} + \frac{\pi}{4} \frac{(3e-1)(1+e)^2}{24-(1-e)(11-e)}$$

Collisional dissipation:

$$\gamma = \frac{6}{\pi^{1/2}} \frac{m_{AB} n G}{L} (1-e^2) T^{3/2} \left[1 - x(\delta r + \delta m\right]$$

Extended Kinetic Theory

Assume:

$$\frac{L}{r_{AB}} = -\frac{1}{2}\alpha G^{1/3}\frac{u'}{T^{1/2}}$$

Mixture energy balance:

$$su' - \gamma = 0$$

Obtain:

$$\left(\frac{r_{AB}u'}{T^{1/2}}\right)^3 = \frac{15}{J}\frac{\left(1-e^2\right)}{\alpha G^{1/3}}\left[1-2x(\delta r + \delta m)\right]$$

Results

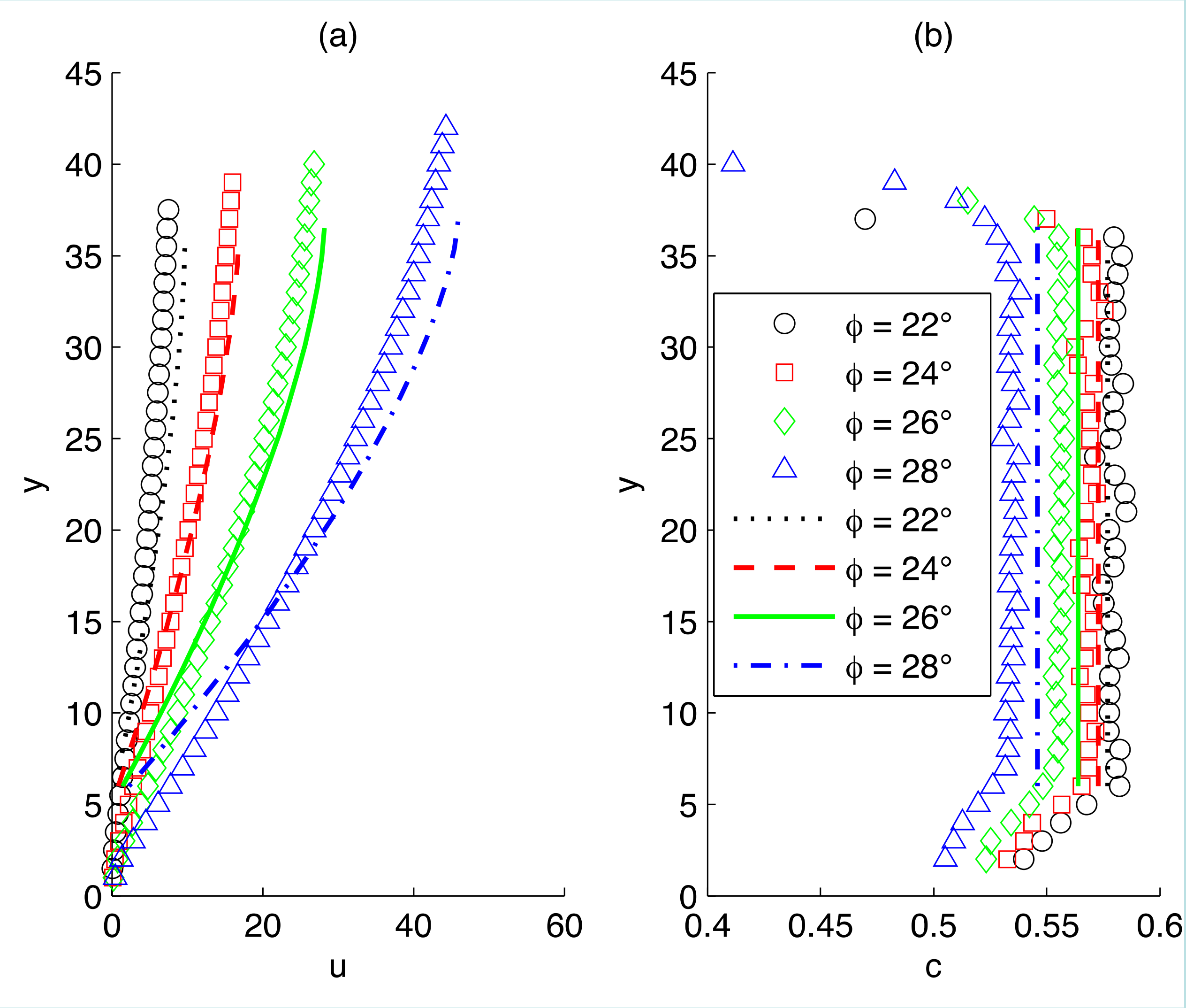
$$G = \left\{ \frac{4J}{5\pi^{1/2}} \frac{1}{1+e} \left[\frac{15}{J} \frac{(1-e^2)}{\alpha} \right]^{1/3} \frac{1}{s/p} \right\}^9$$

$$u = \frac{5\pi^{1/2}}{4Jr_{AB}} \left(\frac{1+e}{2G} g \cos\phi \right)^{1/2} \left[h^{3/2} - (h-y)^{3/2} \right] \frac{s}{p} (1 - x\delta r)$$

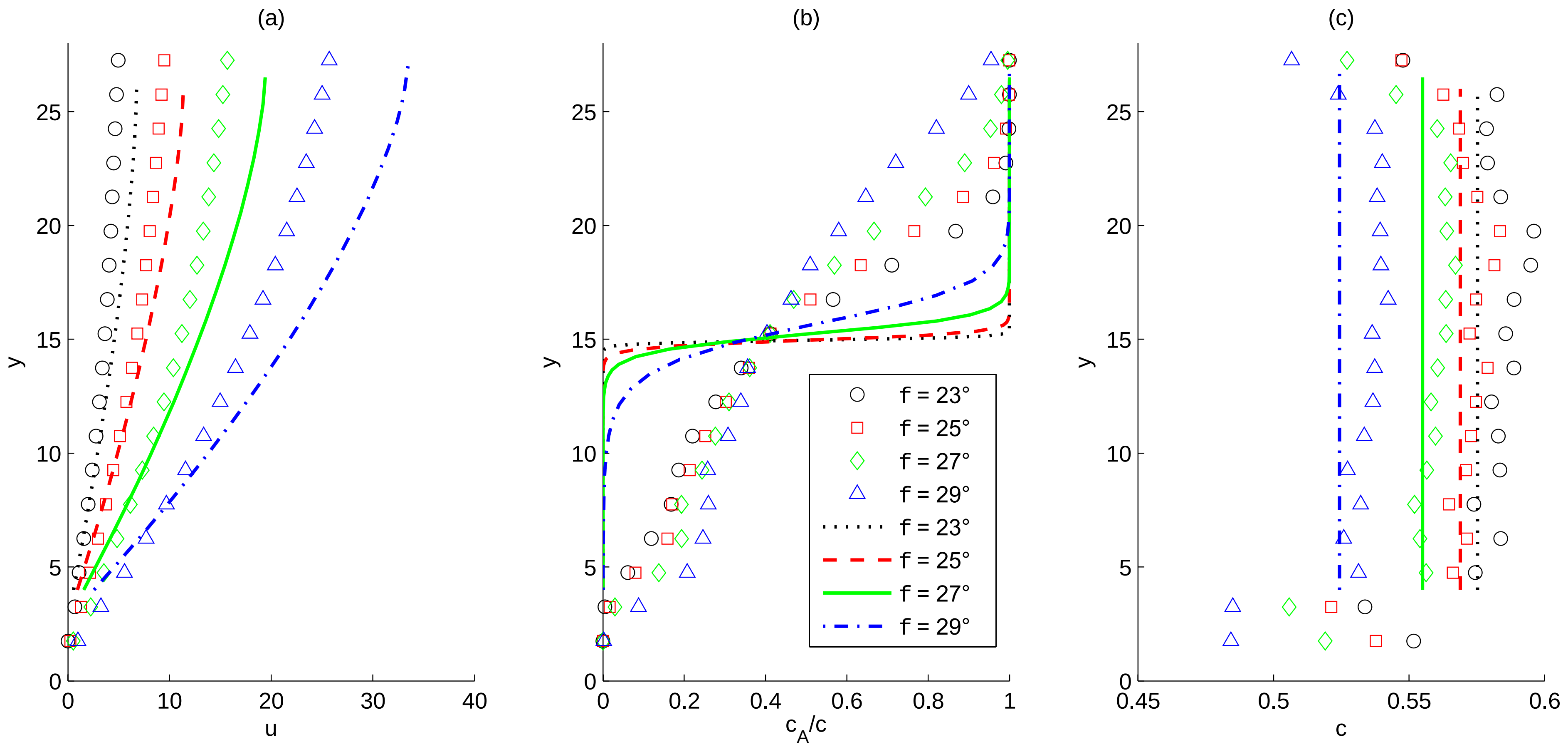
$$\frac{s}{p} = \tan\phi$$

$$c = \frac{c_M G}{G + 5.69(c_M - 0.49)}$$

Dense Flow Volume Fraction

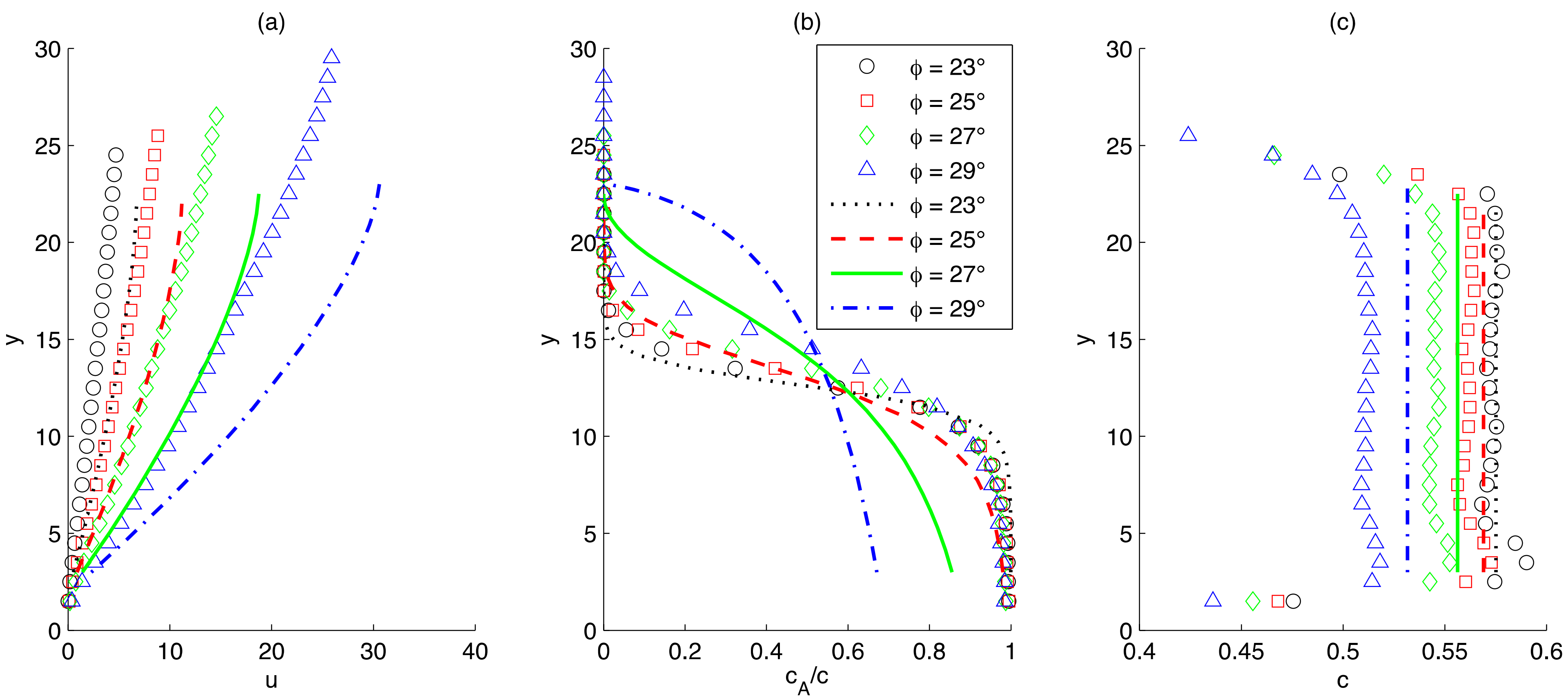


Different Size, Same Material



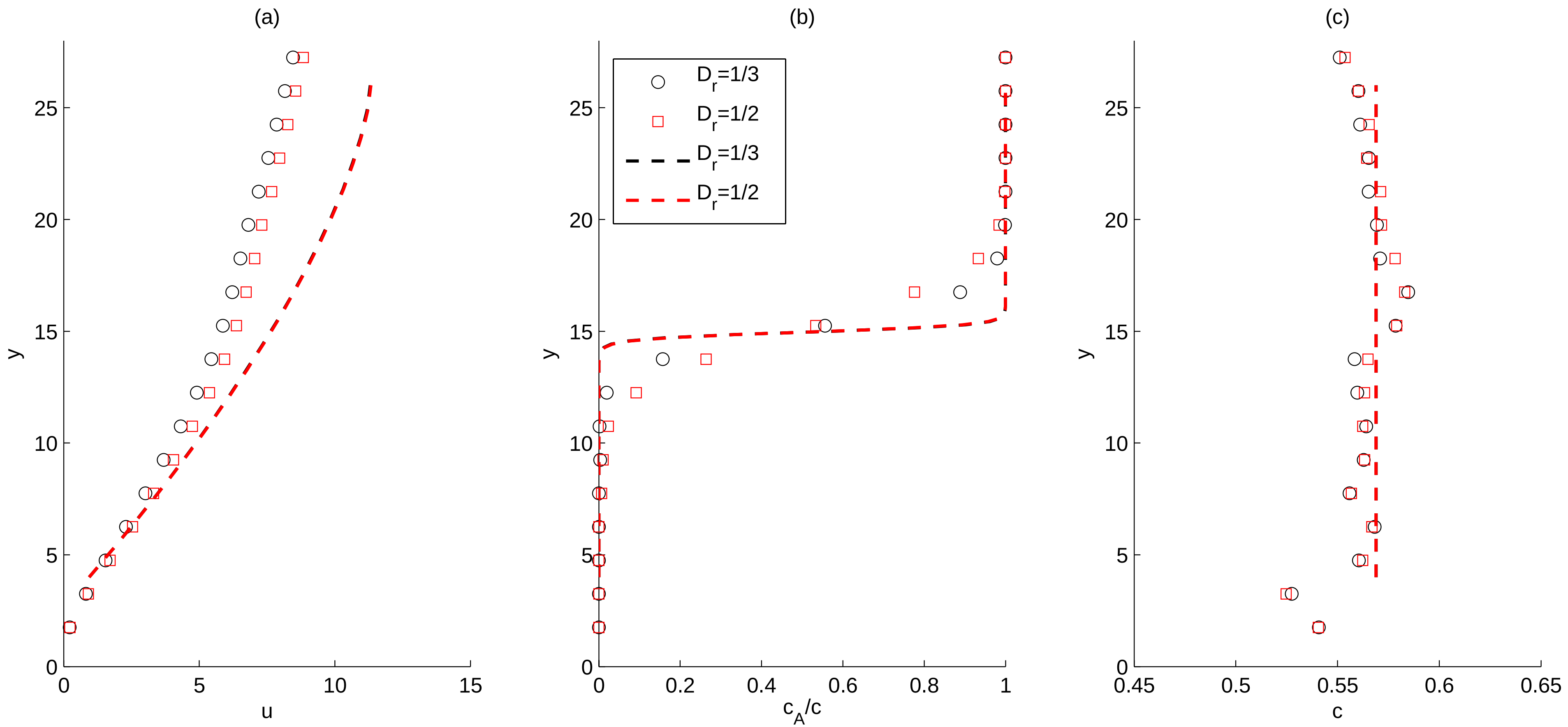
Predicted mixture velocity u, local concentration fraction c_A/c of the larger spheres, and mixture concentration c , for e = 0.65, r_A / r_B = 1.5, m_A / m_B = $(1.5)^3$, α = 0.50 and f_A = 0.23 (V_A/V = 0.5) at four angles of inclination.

Same Size, Different Material



Predicted mixture velocity u, local concentration fraction c_A/c of the more massive spheres and mixture concentration c when e = 0.65, $r_A / r_B = 1$, $m_A / m_B = 3$, $\alpha = 0.50$, $c_M = 0.58$, and $f_A = 0.50$ (equivalent to $V_A/V = 0.5$) for four angles of inclination.

Different Size, Different Materials



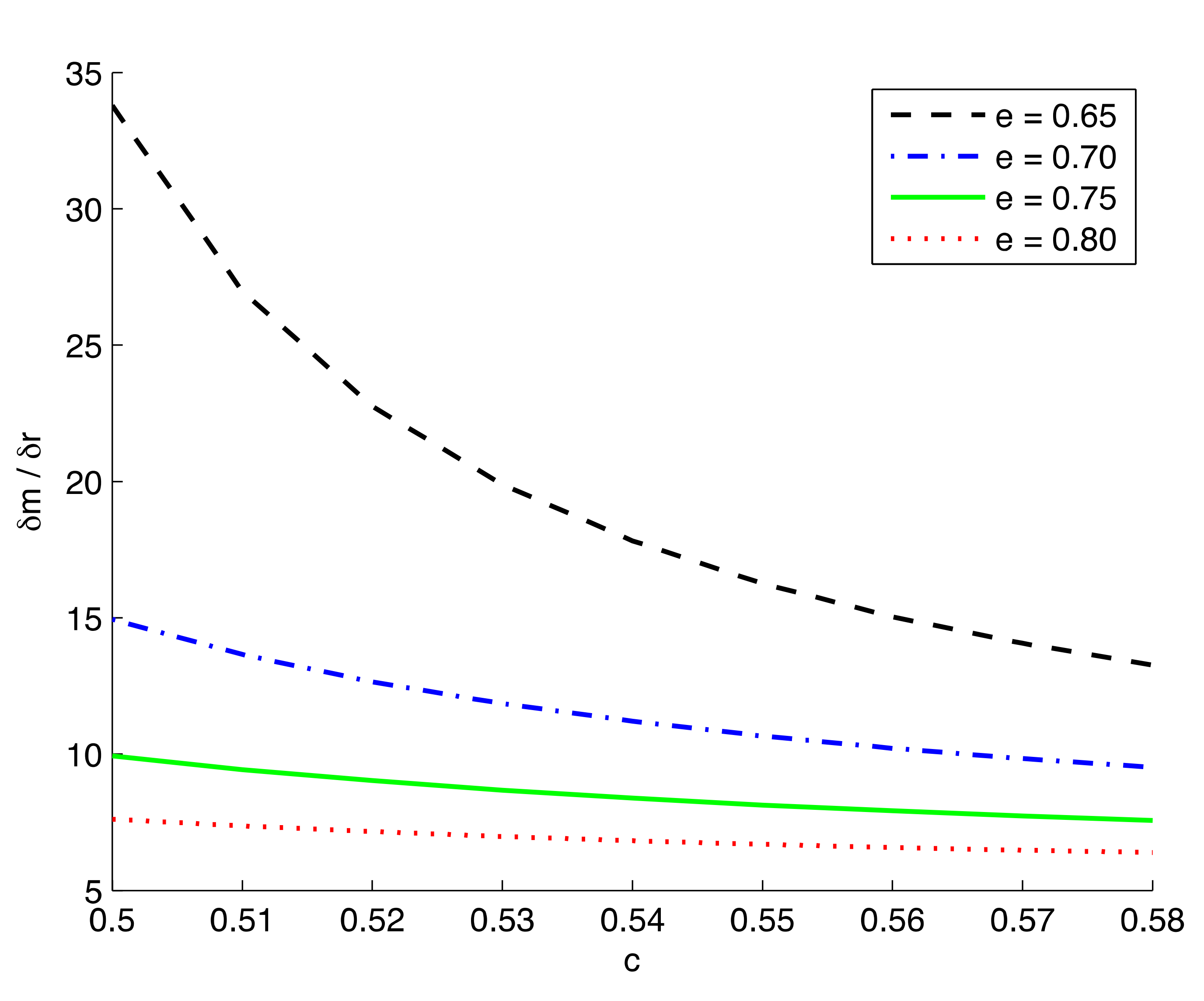
Predicted mixture velocity *u*, local concentration fraction c_A/c of the larger spheres, and mixture concentration *c*, for $r_A / r_B = 1.5$, $e = 0.65$, $\alpha = 0.50$, $\phi = 25^0$ and $V_A/V = 0.5$ ($f_A = 0.23$) at two values of the density ratio.

Special Case: No Segregation

$$\frac{dx}{dy} = \frac{1-4x^2}{4(1-y)} \boxed{\left[\left(R_1\delta r + \Gamma_1 \delta m\right) - 2\left(R_2 \delta r + \Gamma_2 \delta m\right)\left(1+e\right)G\right]}$$

$$\frac{\delta m}{\delta r} = \frac{R_1 - 2R_2(1+e)G}{2\Gamma_2(1+e)G - \Gamma_1}$$

Special Case: No Segregation



Conclusion

A rough amalgamation of two theories: one for segregation, and one for dense, dissipative inclined flow, reproduces qualitative features and provides some quantitative agreement with the results of numerical simulations.

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