Grain Segregation and Levee Formation in Geophysical Mass Flows

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with
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Pete Kokelaar (University of Liverpool),
Dick Iverson (US Geological Survey)

References

Further details of the material in the talk are given in:

C. G. Johnson, B. P. Kokelaar, R. M. Iverson, R. G. LaHusen, M. Logan and J. M. N. T. Gray (2012)
Grain-size segregation and levee formation in geophysical mass flows,
J. Geophys. Res. **117**, F01032, doi:10.1029/2011JF002185

Videos are at `http://onlinelibrary.wiley.com/doi/10.1029/2011JF002185/full`

The model coupling particle size to basal friction, shown in the final slide, is described by:

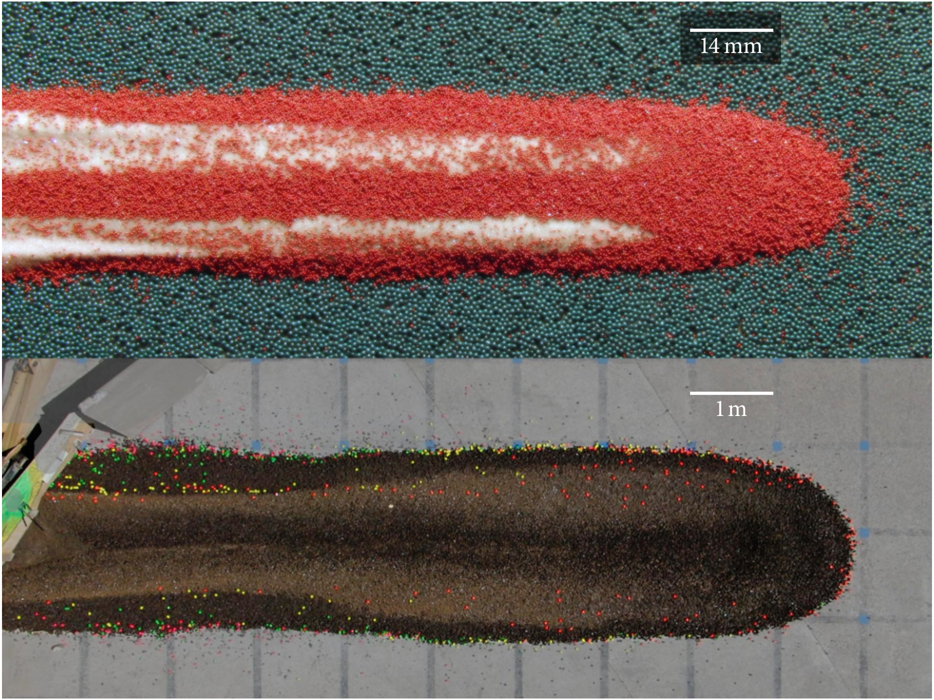
M. J. Woodhouse, A. R. Thornton, C. G. Johnson, B. P. Kokelaar and J. M. N. T. Gray (2012)
Segregation-induced fingering instabilities in granular free-surface flows,
J. Fluid Mech. **709**, p.543–580, doi:10.1017/jfm.2012.348

D. Miller, US Geological Survey

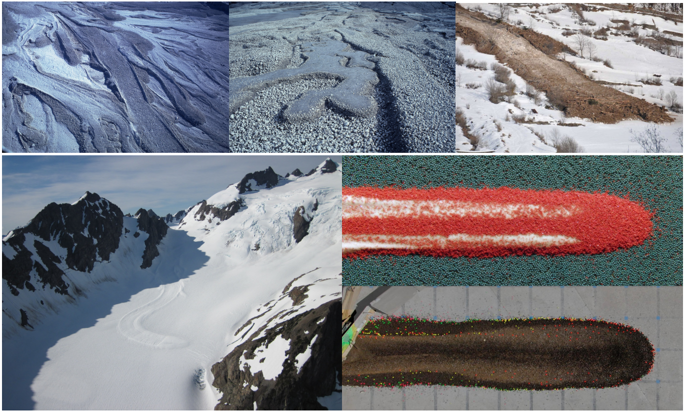
D. Miller, US Geological Survey

P. Bartelt, WSL/SLF, Switzerland

Blue Glacier, Olympic National Park, WA


14 mm
1 m

Leveed deposits



A wide variety of deposits that:

- Are on smooth topography at a shallow incline
- Have an elongated, levee-bounded, (possibly fingered?) morphology
- Have a steep, rounded distal termination

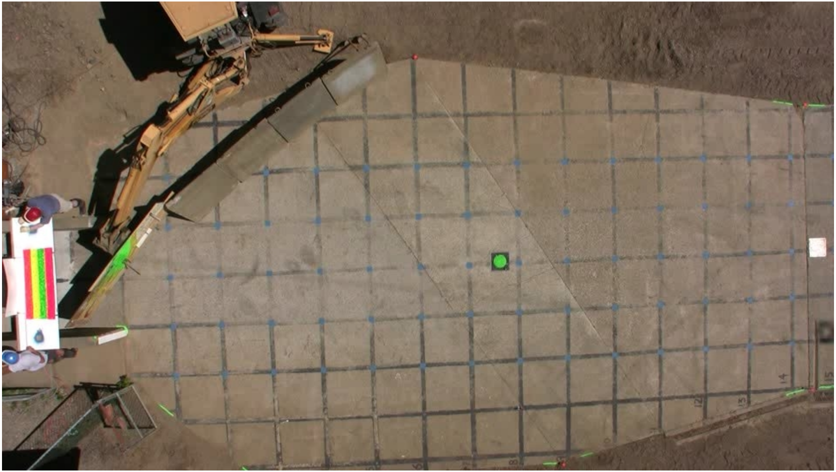
By what mechanism are these deposits emplaced?

USGS Debris Flow Flume

- Flume is 95 m long, 2 m wide
- Inclined at 31°
- Release of 10 m^3 of water-saturated sediment
- Sediment is water-saturated sand and gravel, size-range predominantly 0.0625–32mm
- Flume opens onto flat runout area

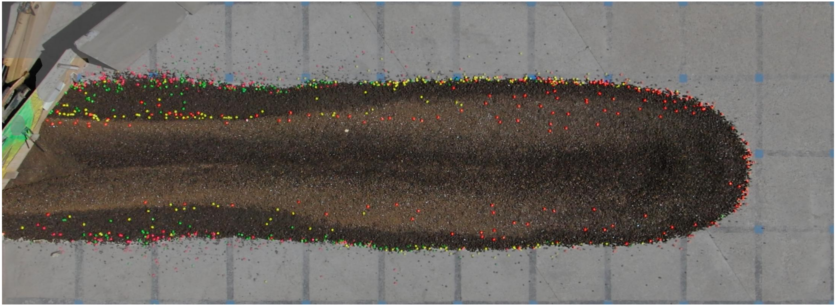
Flow Tracking

Debris Flow Runout



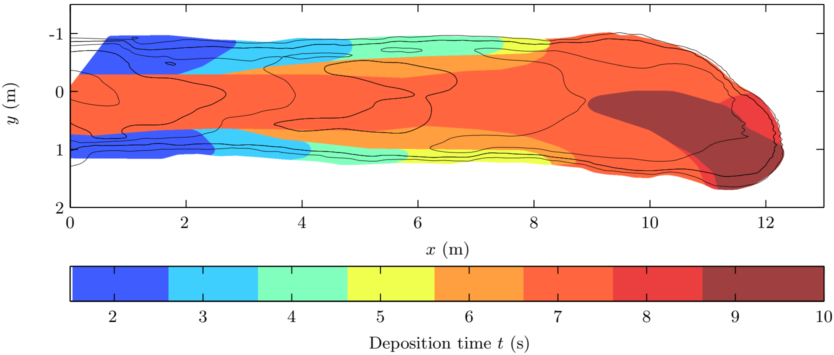
- Flow runs out onto near-horizontal surface (~ 2.4° incline)
- Forms elongated deposit

Debris Flow Runout

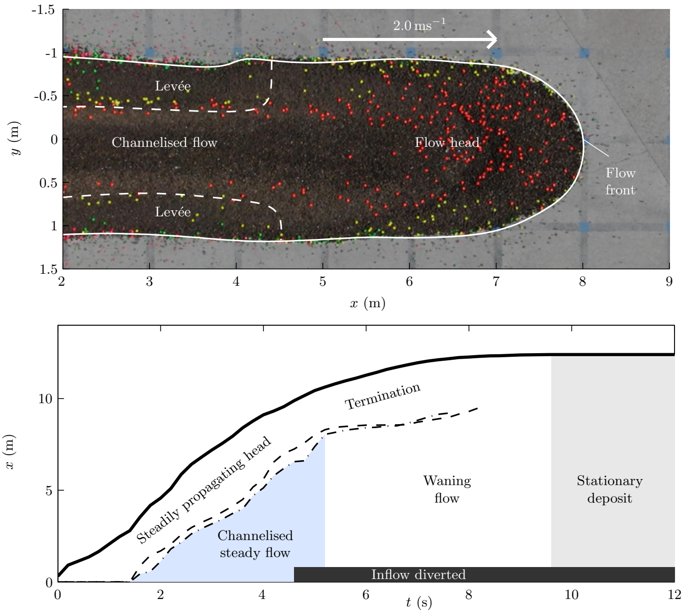


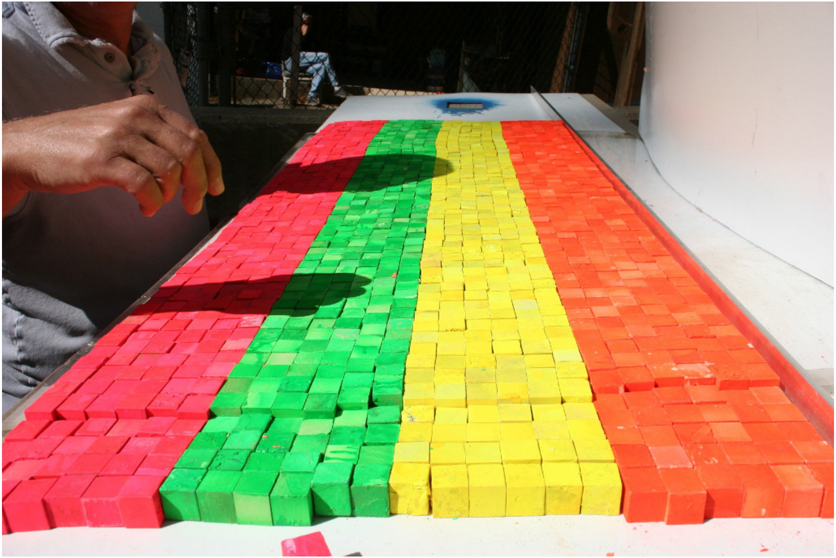
- Flow deposits continuously into stationary levees
- Levees restrict flow width, except close to the flow front

Debris Flow Runout



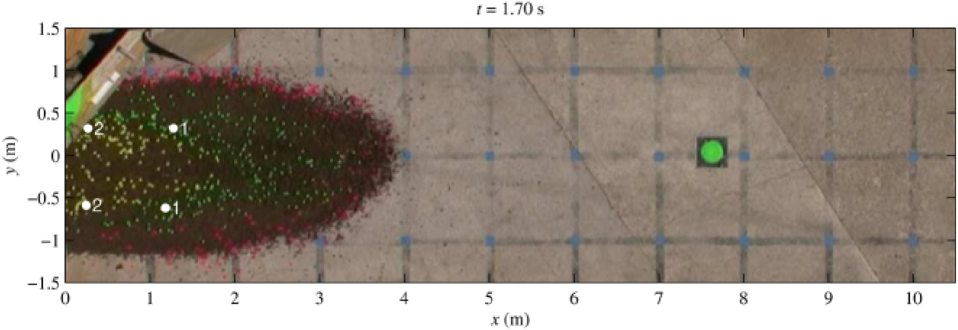
Debris Flow Runout



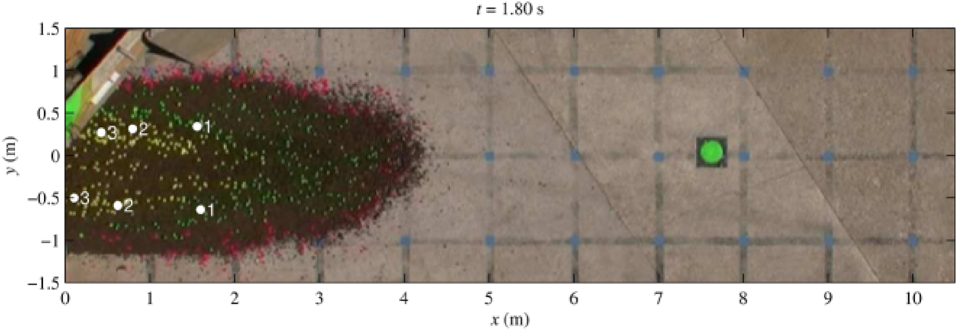


Surface Tracers

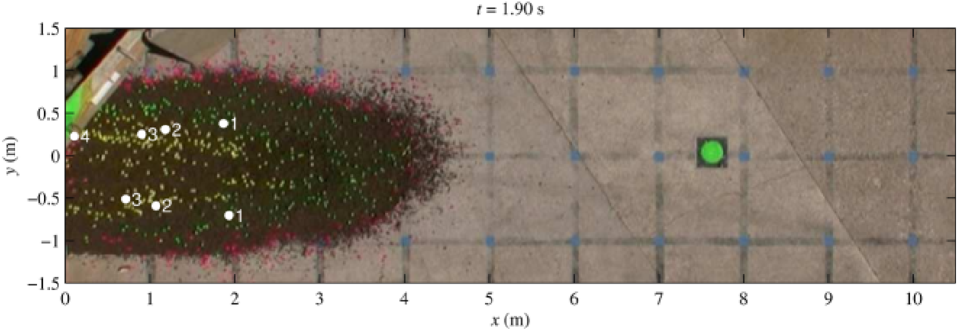
Surface particle paths



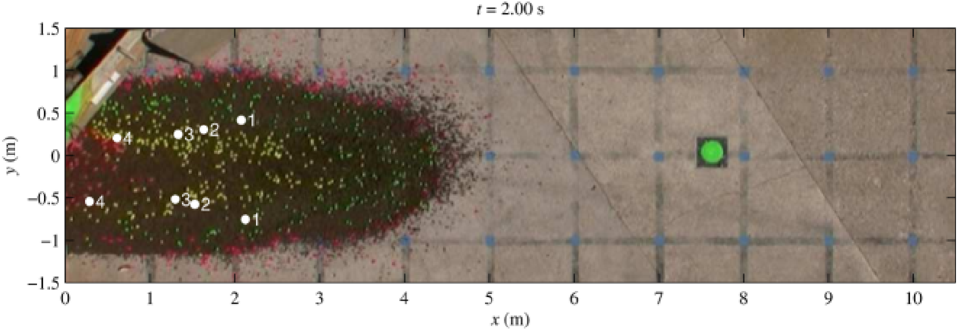
Surface particle paths



Surface particle paths



Surface particle paths



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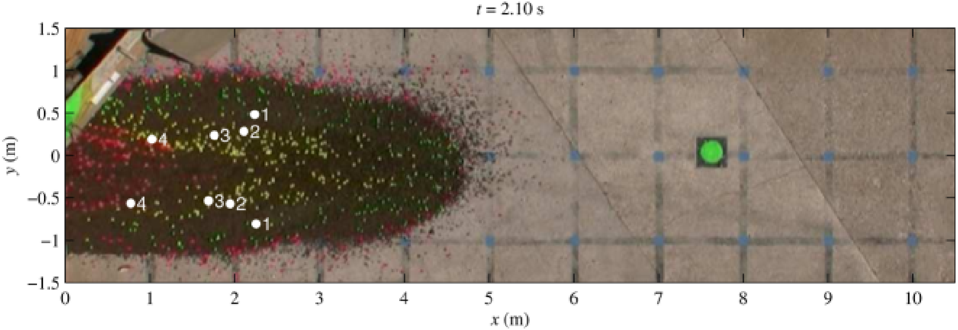
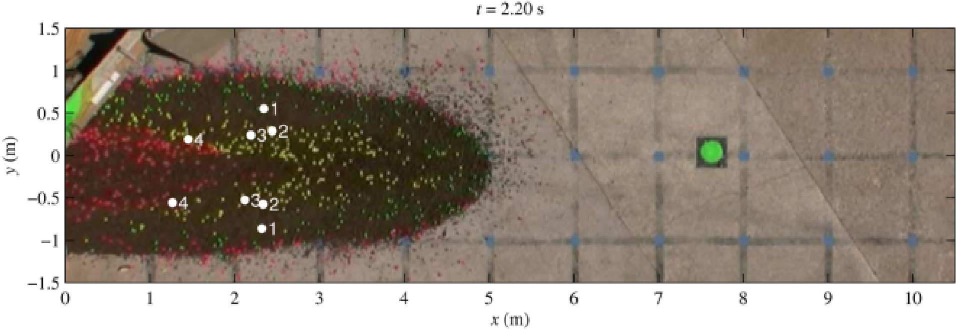
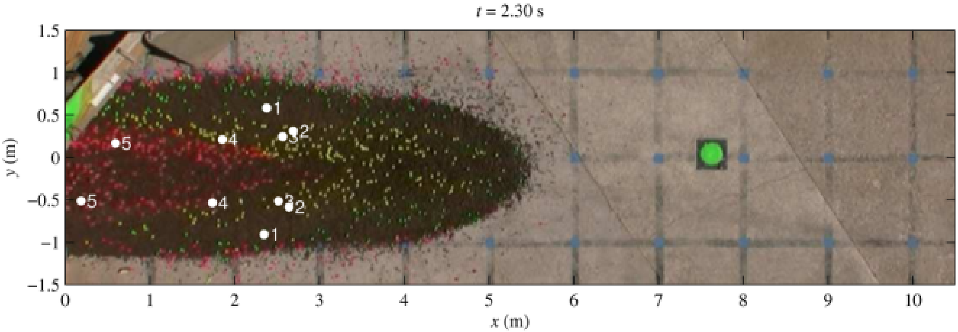


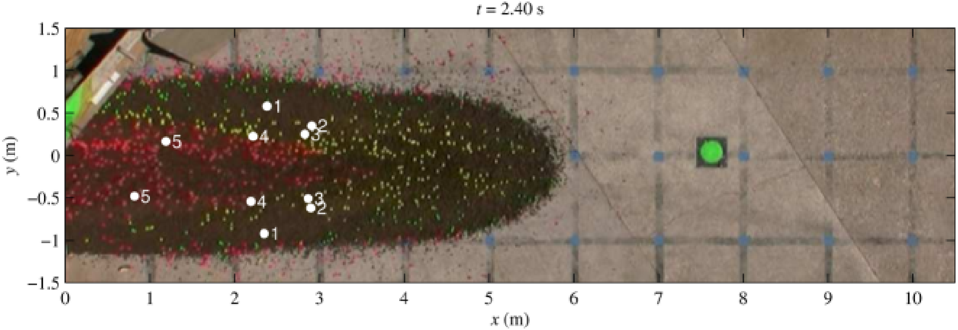
Surface particle paths



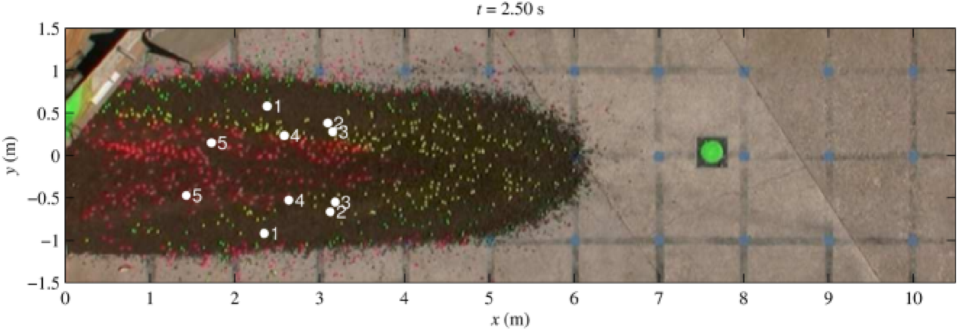
Surface particle paths



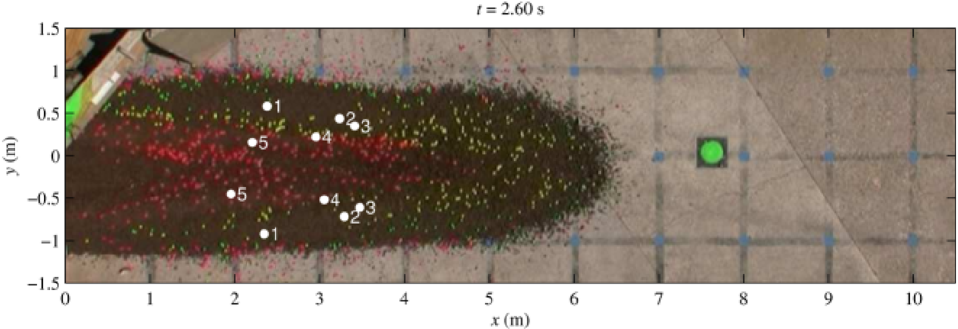
Surface particle paths



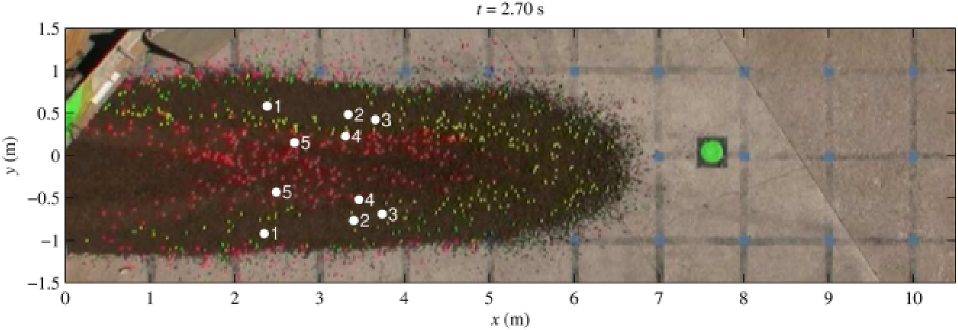
Surface particle paths

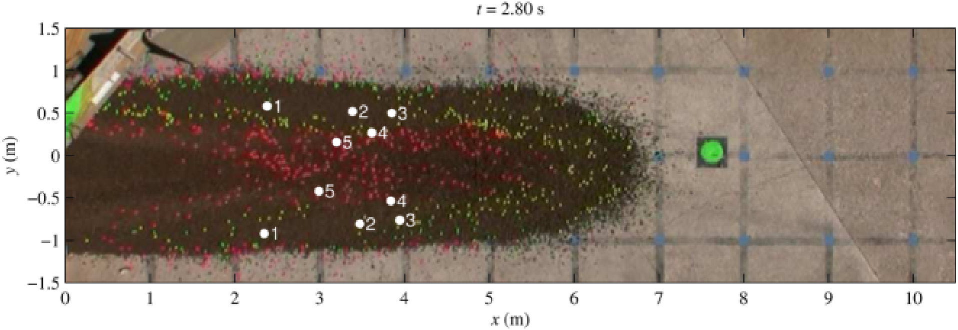


Surface particle paths



Surface particle paths

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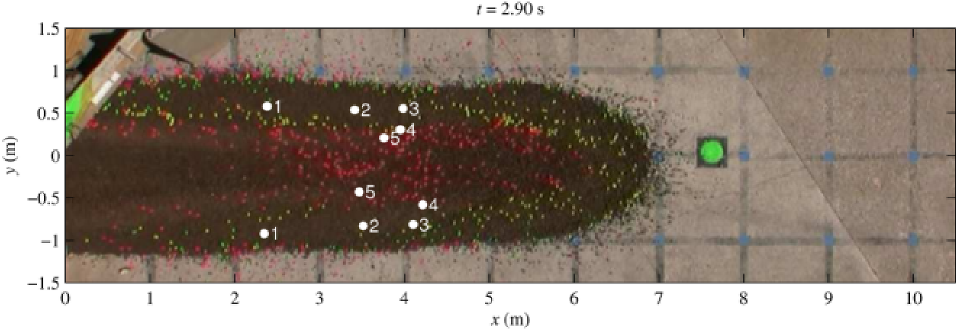
1.5
1
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
5
4
3
2
1

Surface particle paths



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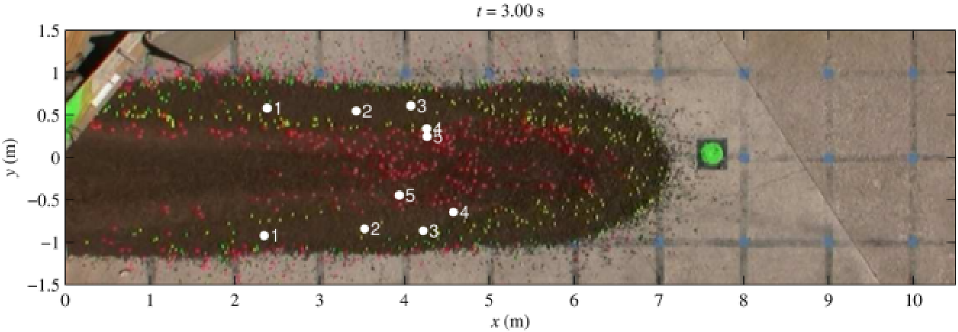
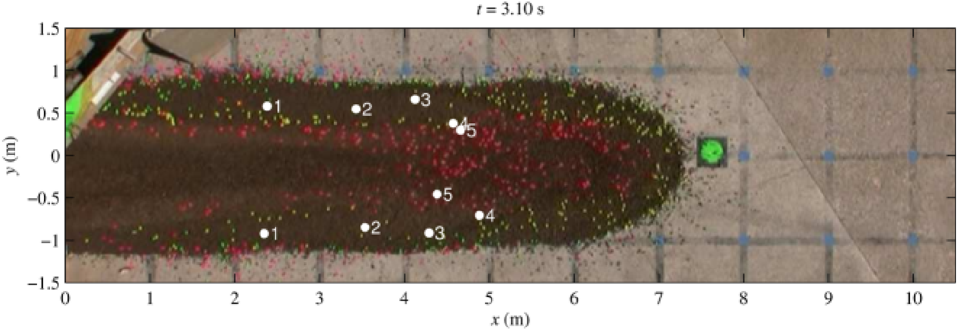
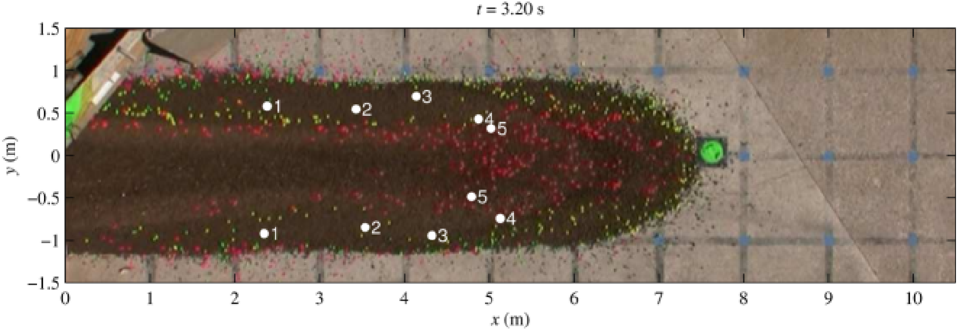


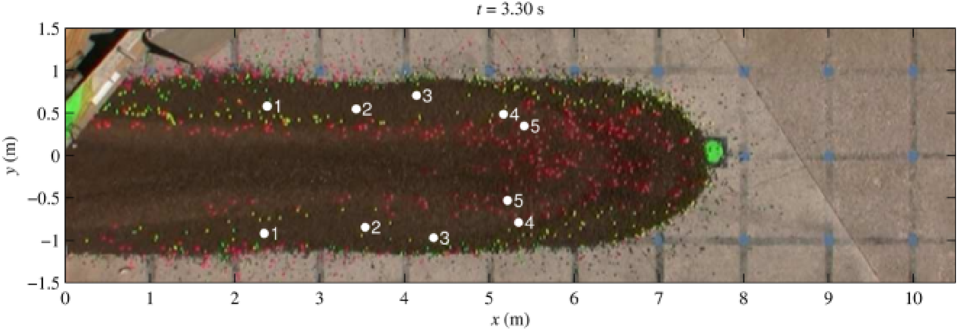
Surface particle paths



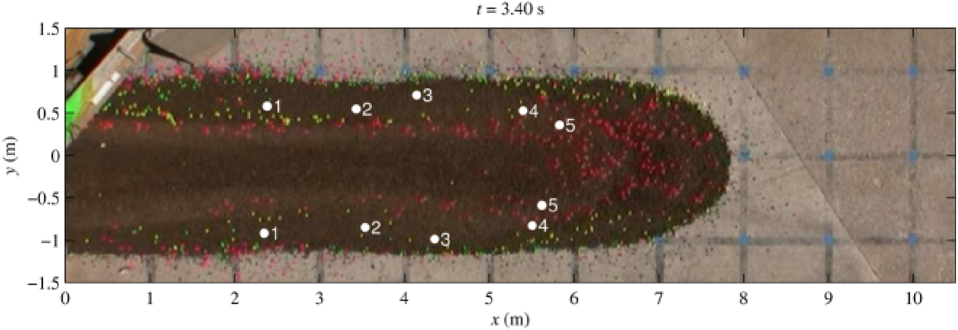
Surface particle paths



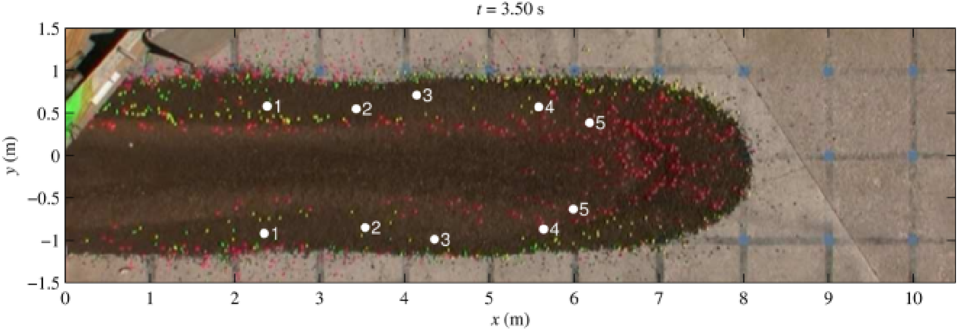
Surface particle paths



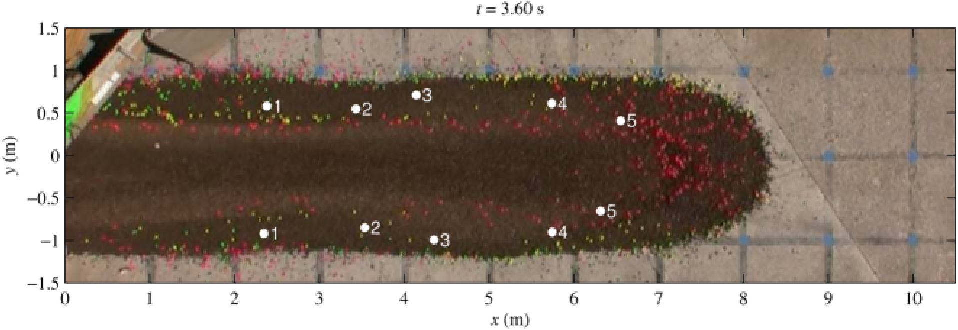
Surface particle paths



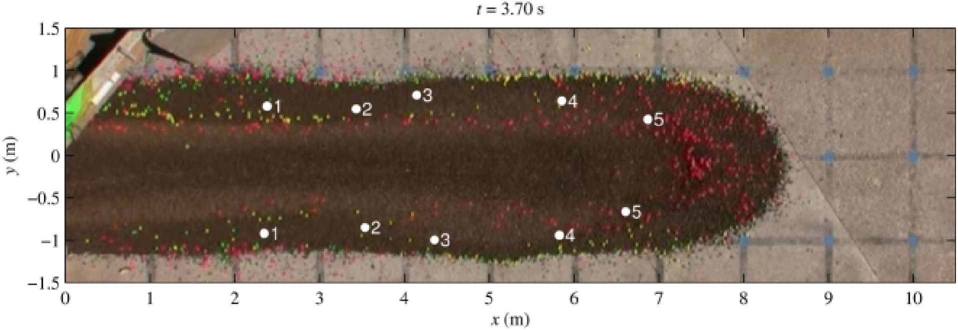
Surface particle paths

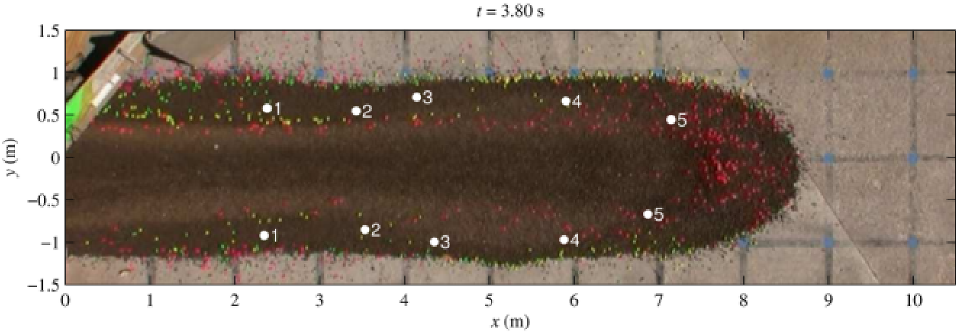


Surface particle paths

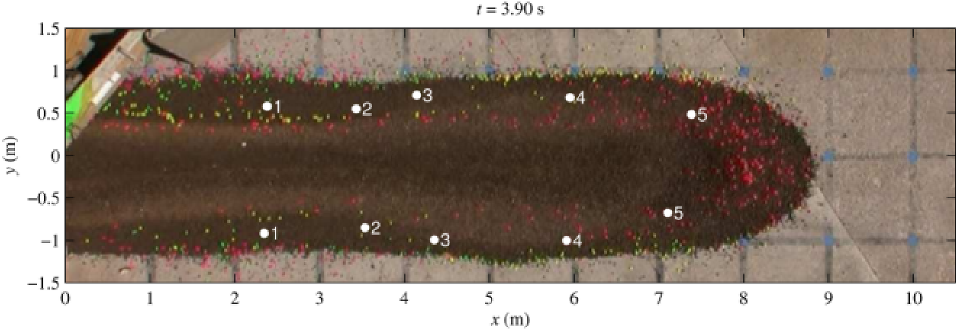


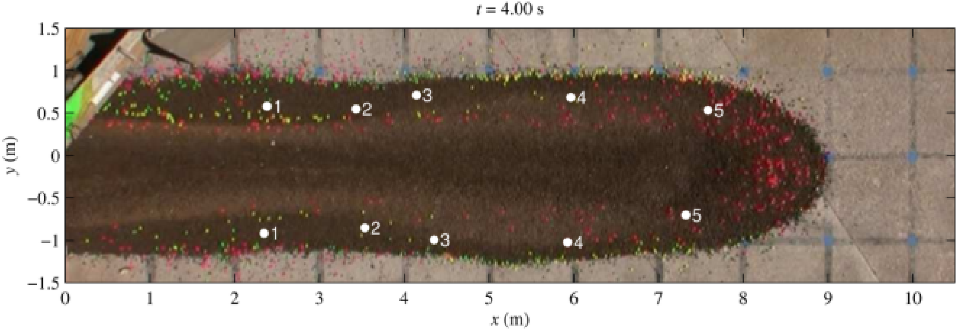
Surface particle paths




t = 3.80 s
1.5
1
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5

Surface particle paths




t = 4.00 s
1.5
1
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
3
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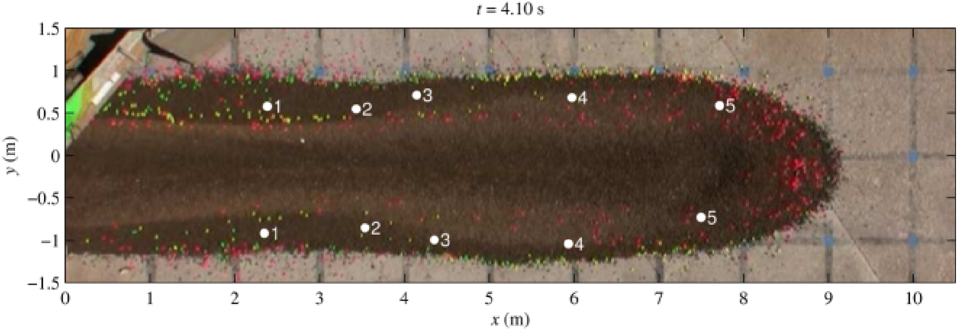
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5



t = 4.10 s
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0.5
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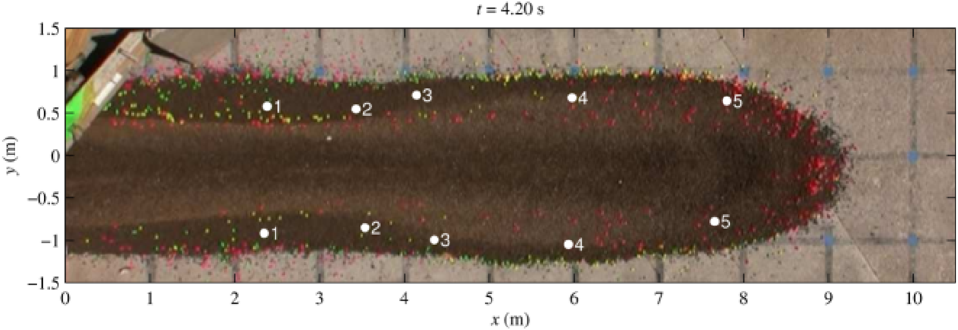
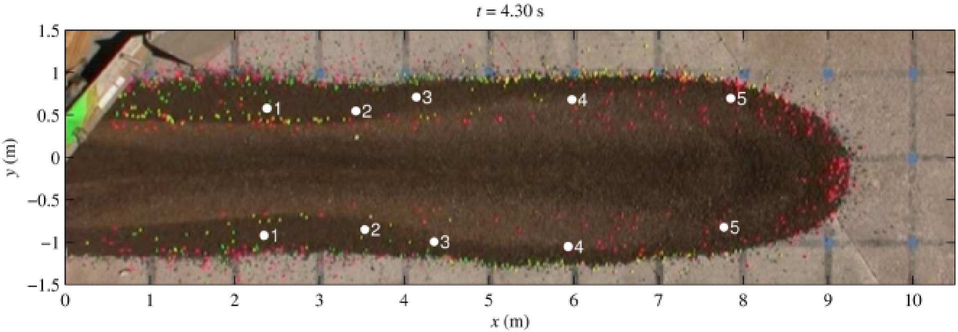
−1
−1.5
y (m)
0
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5

Surface particle paths





t = 4.30 s
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−0.5
−1
−1.5
y (m)
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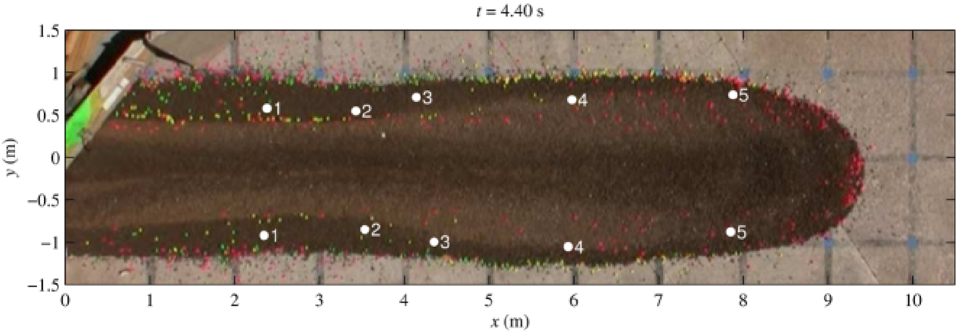
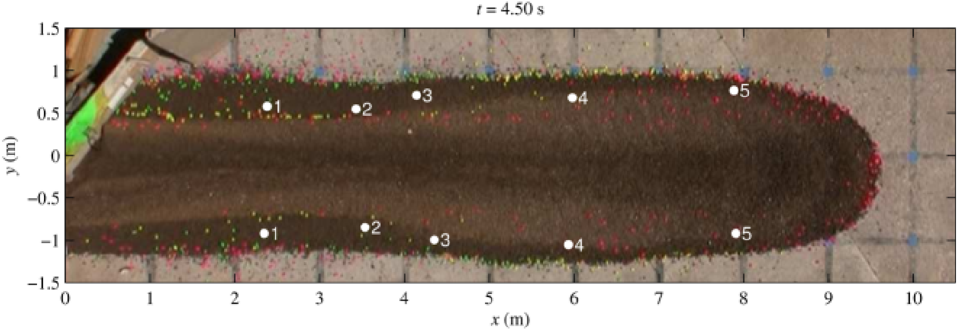
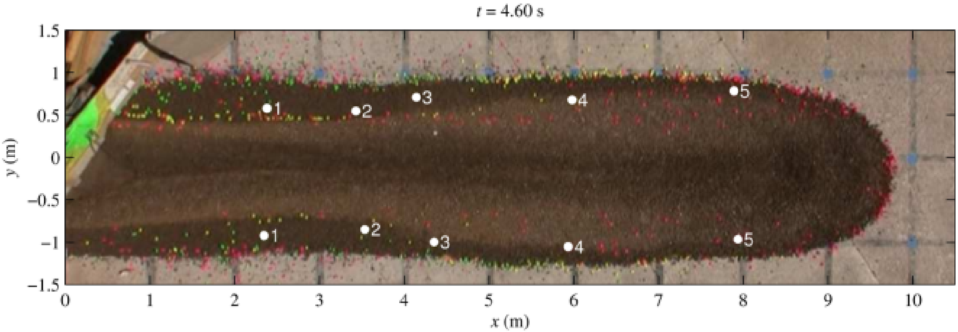
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5

Surface particle paths




$t = 4.50$ s
1.5
1
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5



$t = 4.60$ s
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1
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
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4
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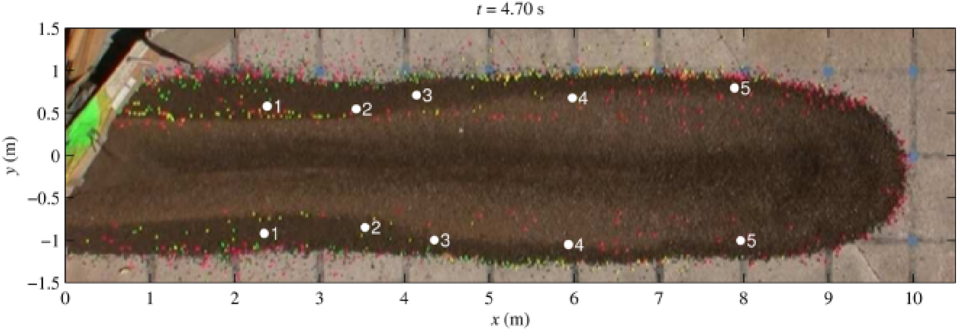
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5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0.5
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−0.5
−1
−1.5
y (m)
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1
2
3
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x (m)
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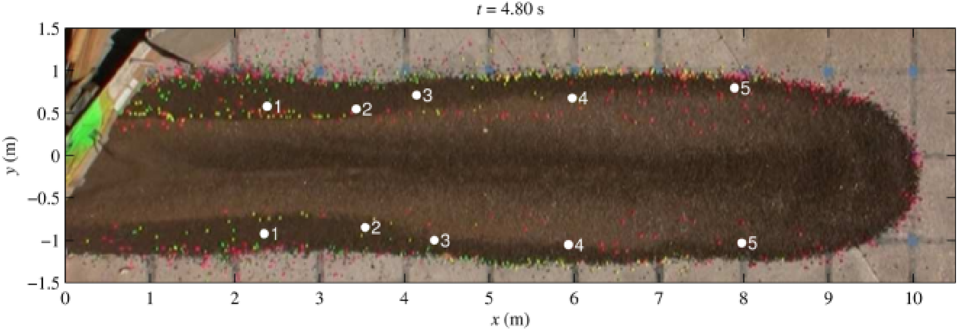
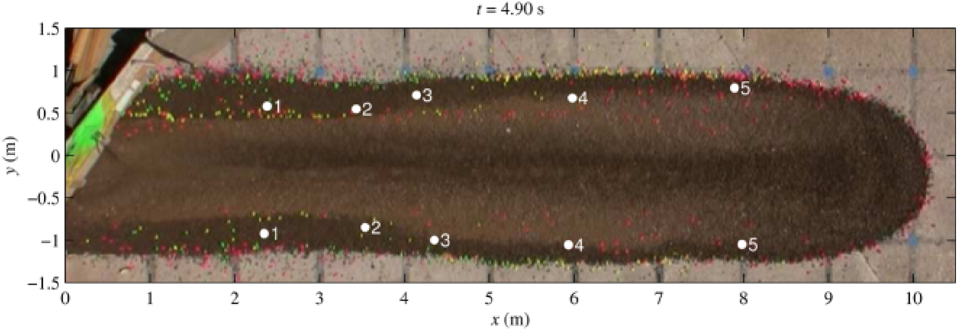
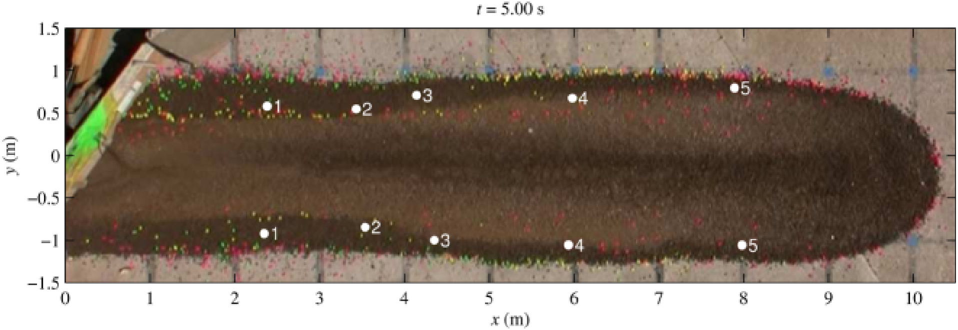
1
2
3
4
5



t = 4.80 s
1.5
1
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5

Surface particle paths

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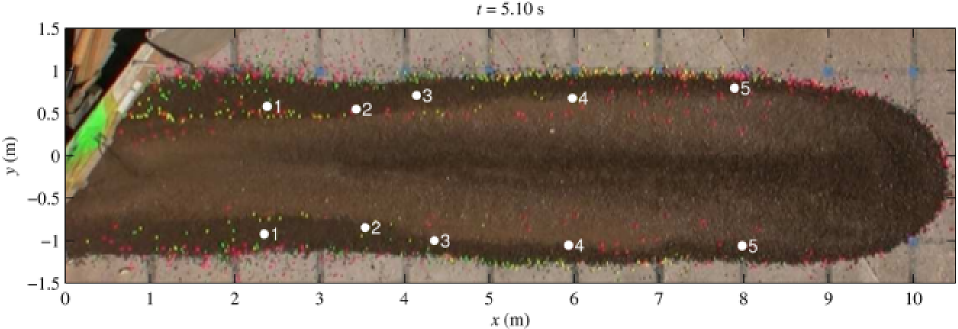
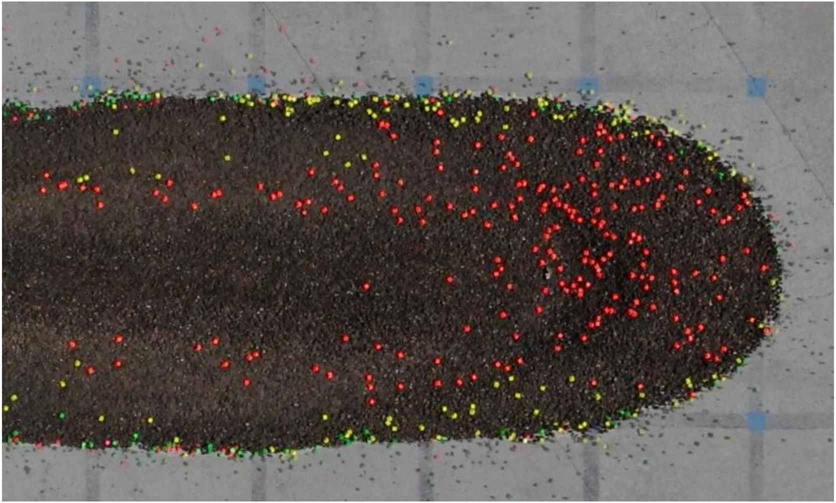
0.5
0
−0.5
−1
−1.5
y (m)
0
1
2
3
4
5
6
7
8
9
10
x (m)
1
2
3
4
5
1
2
3
4
5

Surface particle paths

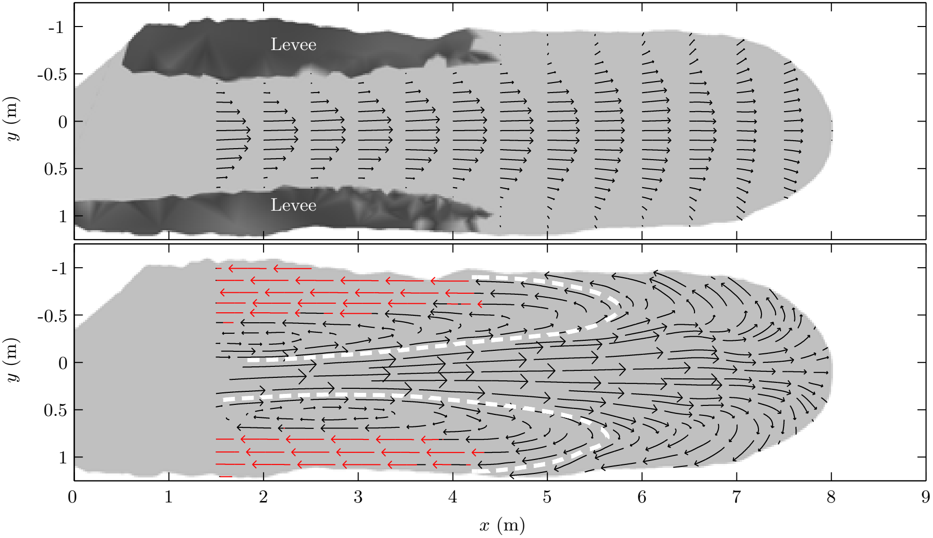


USGS Debris Flow Flume



- Flow steady in a frame moving with the front

USGS Debris Flow Flume



Natural debris flow

Source: Costa & Williams, USGS Open File Report 84-606 (1984)

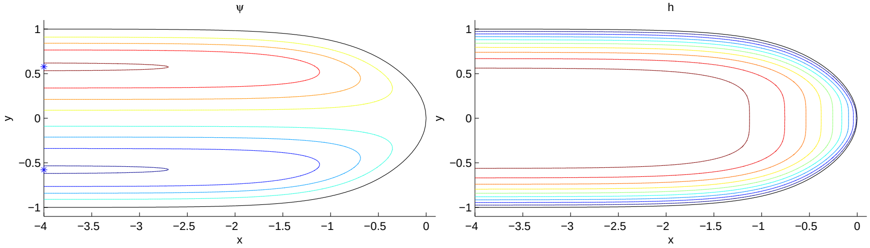
Natural debris flow

Source: Costa & Williams, USGS Open File Report 84-606 (1984)

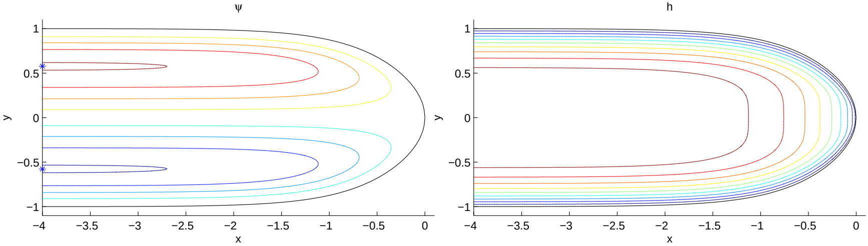
For modelling, construct a flow depth and depth-integrated velocity field that resemble the experimental flows:

$$
\begin{aligned}
h(\xi, y) &= y_0\left(1 - \hat{y}^{2n}\right) \\
h\bar{\mathbf{u}} &= (\psi_y, -\psi_\xi) \qquad \text{with}
\end{aligned}
$$

$$
\begin{aligned}
\psi(\xi, y) &= y_0^3\left(A\hat{y}^{-1} - B\hat{y}^{2n+1} - C\hat{y}^{2m+1} + D\hat{y}^{2n+2m+1}\right) \\
y_0(\xi) &= \sqrt{\tanh(-\xi)}, \qquad \hat{y} = y/y_0.
\end{aligned}
$$

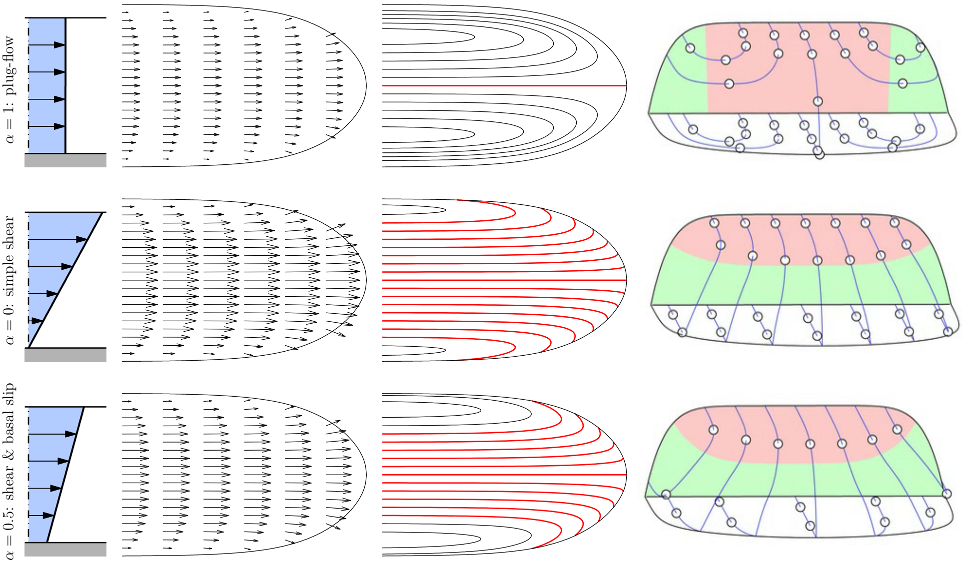


Moving-frame Modelling

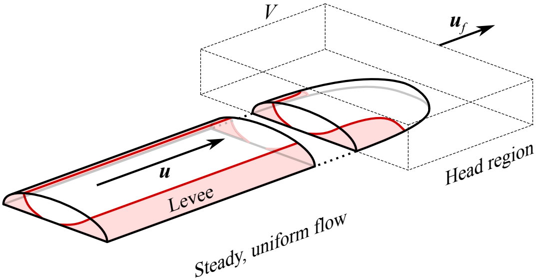


- Assume a constant vertical velocity profile: $(u, v)(\xi, y, z) = \bar{\mathbf{u}}(\xi, y) f(z/h)$
- For example, linear velocity profiles with basal slip: $(u, v) = \bar{\mathbf{u}} \left(\alpha + (1-\alpha) \frac{2z}{h} \right)$
- Vertical velocity w determined by mass conservation: $\nabla \cdot \mathbf{u} = 0$

Moving-frame Modelling



Momentum in the flow head



Momentum conservation: $\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = \nabla\cdot\sigma + \mathbf{f}$

Assume steady in a moving frame, integrate over volume V moving with the front:

$$\underbrace{-\int_{\partial V}\left[\left(\mathbf{u}-\mathbf{u}_f\right)\cdot\mathbf{n}\right]u\,\mathrm{d}A}_{\text{Momentum advected into head}} + \underbrace{\int_{x=u_f t}(\sigma\cdot\mathbf{n})_x\,\mathrm{d}A}_{\text{Pressure force}} + \underbrace{g\sin\theta\int_V \rho\,\mathrm{d}V}_{\text{Gravity}} = \underbrace{-\int_{z=0}(\sigma\cdot\mathbf{n})_x\,\mathrm{d}A}_{\text{Basal friction/ viscous resistance}}$$

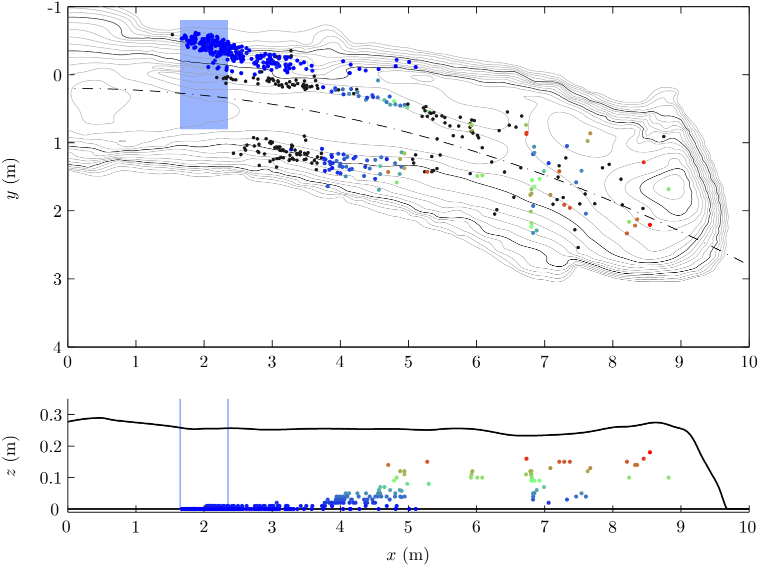
Extra momentum advected into head due to lateral shape profile.

Must be balanced by additional basal friction / viscous resistance in head.

Particle Size Segregation

Particle Size Segregation

Particle Size Segregation



- Infer a typical rise rate of ~ 3.5cm s^{-1}

Segregation Modelling

In flume experiments, continuum particle size distribution forms three classes:

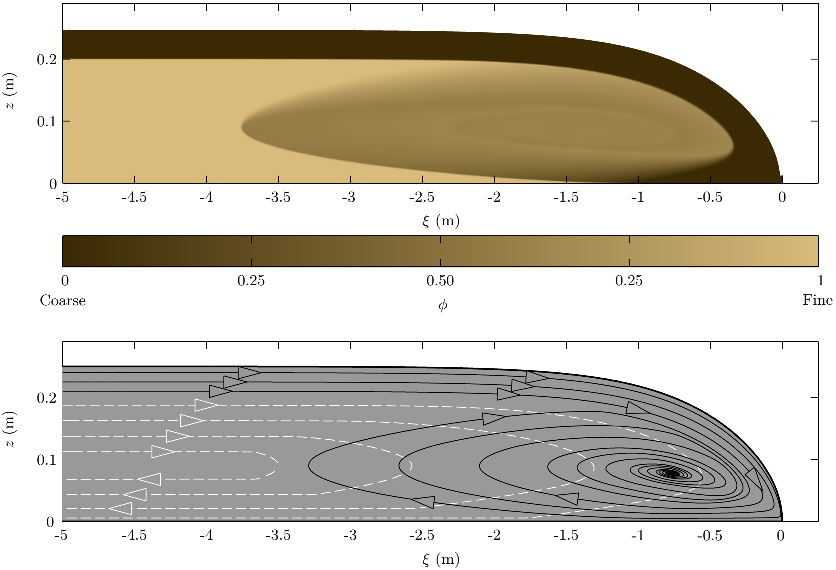
- Gravel (> 8 mm), segregates upwards
- Sand (0.0625–8 mm), segregates downwards
- Mud and fine sand (< 0.0625 mm) (~ 2%), advected with pore fluid

Bidisperse particle segregation model of Gray & Thornton (Proc. R. Soc. **461**)

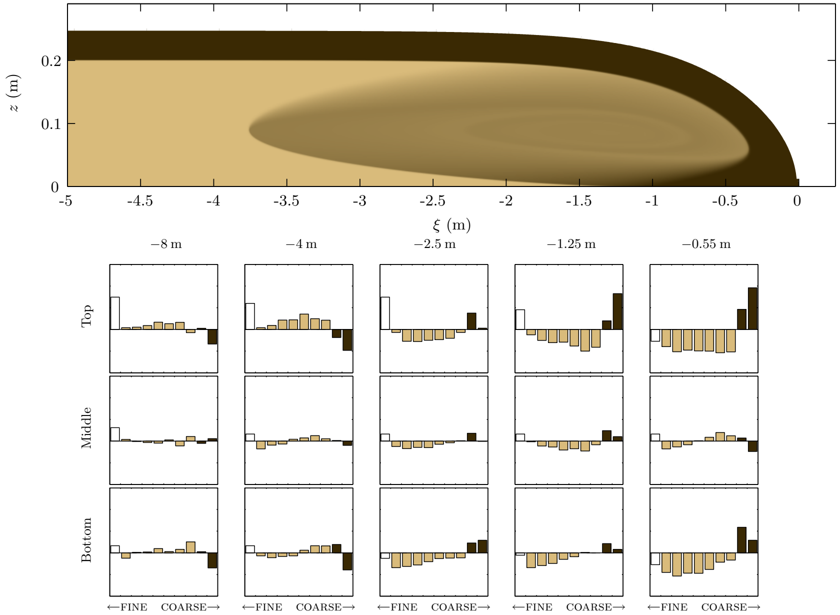
- Large (gravel) and small (sand) particle volume fractions ϕ_l and ϕ_s
- Assume incompressible flow $\phi_l + \phi_s = 1$
- Large particles move with velocity $\mathbf{v}_l = \mathbf{u} + q\phi_s\mathbf{k}$, concentration governed by

$$\frac{\partial \phi_l}{\partial t} + \nabla \cdot (\phi_l \mathbf{v}_l) = 0, \text{ where}$$

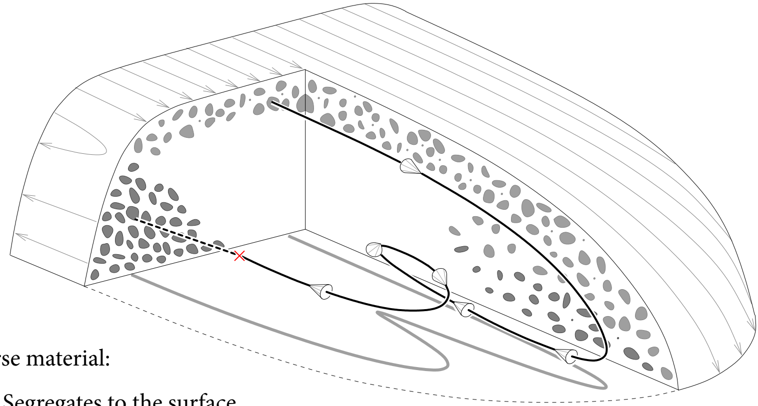
- $\mathbf{u}$ is the prescribed bulk velocity field
- q is a segregation speed ($\approx 3.5\,\mathrm{cm\,s^{-1}}$)


z (m)
0.2
0.1
0
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
ξ (m)
0
0.25
0.50
0.25
1
Coarse
ϕ
Fine
z (m)
0.2
0.1
0
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
ξ (m)

Segregation Modelling



Particle paths in a segregating flow front

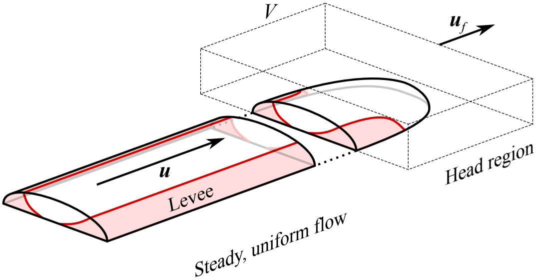


Coarse material:

- Segregates to the surface
- Is transported to the front
- Recirculates with a spiral path within the flow head
- Is transported laterally, into the levees

(Johnson *et al.*, JGR Earth Surface **117** (2012))

Mass balance in the flow head



Large particle conservation: $\dfrac{\partial \phi_l}{\partial t} + \nabla \cdot (\phi_l \mathbf{v}_l) = 0$

Assume steady in a moving frame, integrate over volume V moving with the front:

$$\int_{\partial V} \left[\phi_l \left(\mathbf{u} - \mathbf{u}_f \right) \right] \cdot \mathbf{n} \, \mathrm{d}A = 0$$

Flux of large particles advected into flow head at the surface of the channelised flow must balance flux of large particles leaving head in levees.

Depth-integrated Modelling

Wish to encapsulate vertical structure into a two-dimensional depth-averaged model.

- Integrate three-dimensional equation

$$\frac{\partial \phi_l}{\partial t} + \nabla \cdot \left(\phi_l \left(\mathbf{u} + q\phi_s \mathbf{k} \right) \right) = 0$$

  between $z = 0$ and $z = h$.

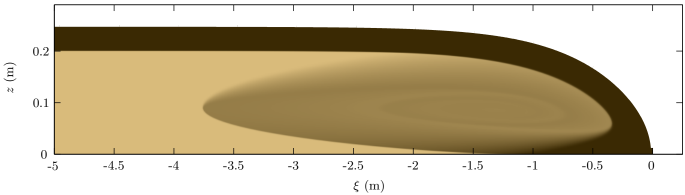
- Project $\phi_l(x, y, z, t)$ onto a 'first moment' function $\phi_l(z) = f(z, \bar{\phi}_l(x, y, t))$.

Assuming instantaneous segregation (Gray & Kokelaar, JFM **652**), obtain

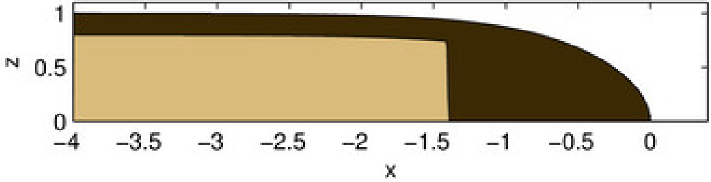
$$\frac{\partial \eta}{\partial t} + \nabla_h \cdot \left[\eta \bar{\mathbf{u}} \left(\alpha + (1 - \alpha) \frac{\eta}{h} \right) \right] = 0$$

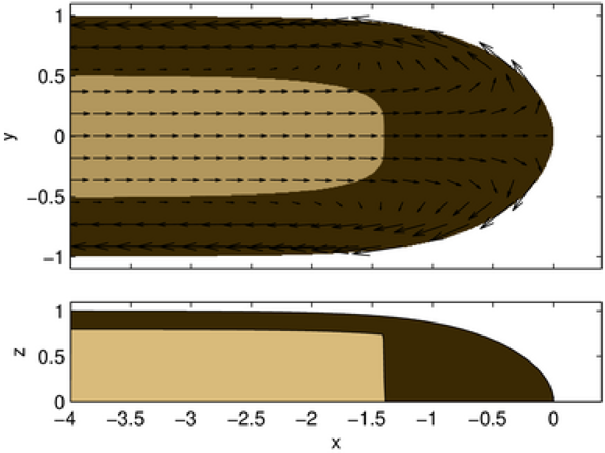
- $\eta(x, y) = h\bar{\phi}_l$ is the height of the interface between segregated small and large particles
- The depth-integrated model captures just the enhanced transport of large particles at the surface

$$\frac{\partial \phi_l}{\partial t} + \nabla \cdot \left(\phi_l \left(\mathbf{u} + q\phi_s \mathbf{k}\right)\right) = 0$$

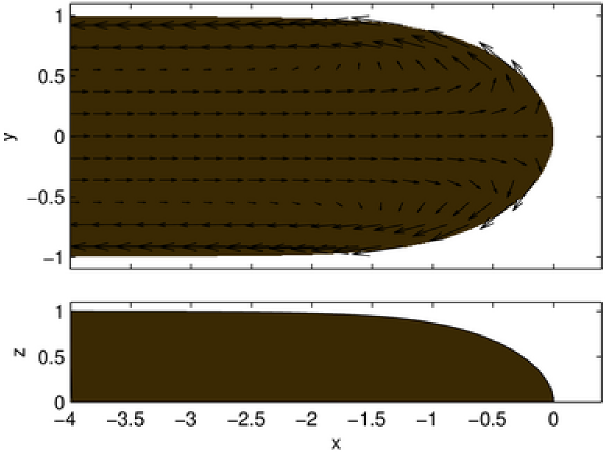


$$\frac{\partial \eta}{\partial t} + \nabla_h \cdot \left[\eta \bar{\mathbf{u}} \left(\alpha + (1-\alpha)\frac{\eta}{h}\right)\right] = 0$$





$\bar{\phi}_l$ = 0.2: Large particles transported away from flow front in levees



$\bar{\phi}_l = 0.3$: Large particles accumulate in flow head

Model depth-integrated:

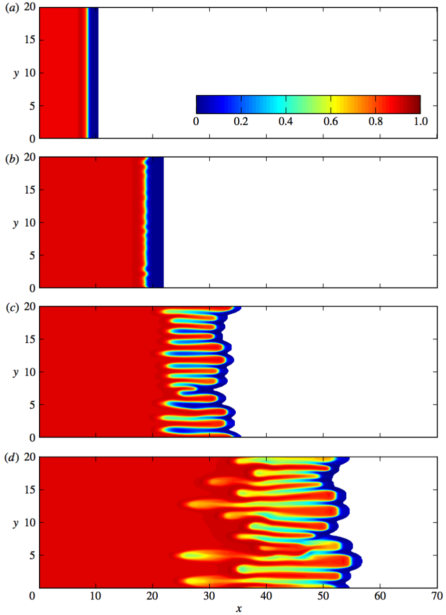
- Mass conservation of each species
- Momentum conservation
- Effect of large particle concentration on friction

$$\frac{\partial h}{\partial t} + \nabla_h \cdot [h\bar{\mathbf{u}}] = 0$$

$$\frac{\partial \eta}{\partial t} + \nabla_h \cdot \left[\eta\bar{\mathbf{u}}\left(\alpha + (1-\alpha)\frac{\eta}{h}\right)\right] = 0$$

$$\frac{\partial}{\partial t}(h\bar{\mathbf{u}}) + \nabla_h \cdot \left[h\bar{\mathbf{u}}\bar{\mathbf{u}} + \frac{gh^2}{2}\right] = \mathbf{i}\sin\theta + \mu\frac{\bar{\mathbf{u}}}{|\bar{\mathbf{u}}|}$$

$$\mu = \mu(h, |\bar{\mathbf{u}}|, \eta/h)$$



(Woodhouse et al., JFM **709** (2012))