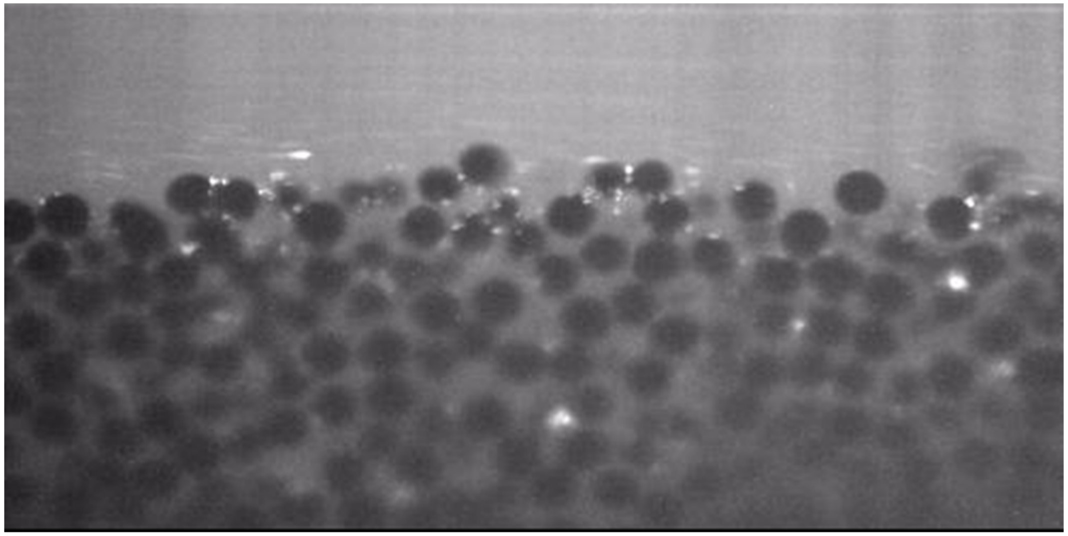
Erosion of a granular bed by fluid flow

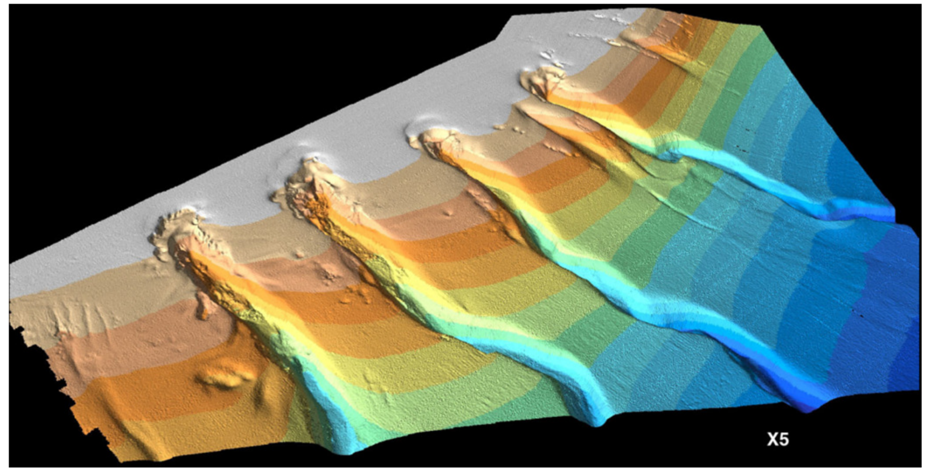


Arshad Kudrolli
Department of Physics, Clark University
Acknowledgements: Ashish Orpe, Ryan Molloy, Anyu Hong, Vikrant Yadav, Alex Lobkovsky, Daniel Rothmam (EAPS, MIT)

Supported by the Department of Energy and the National Science Foundation

Background

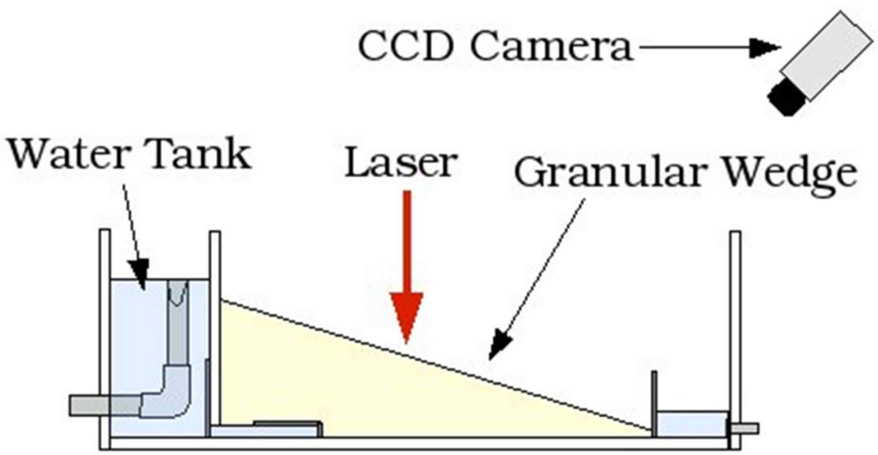
Cape Cod Beach

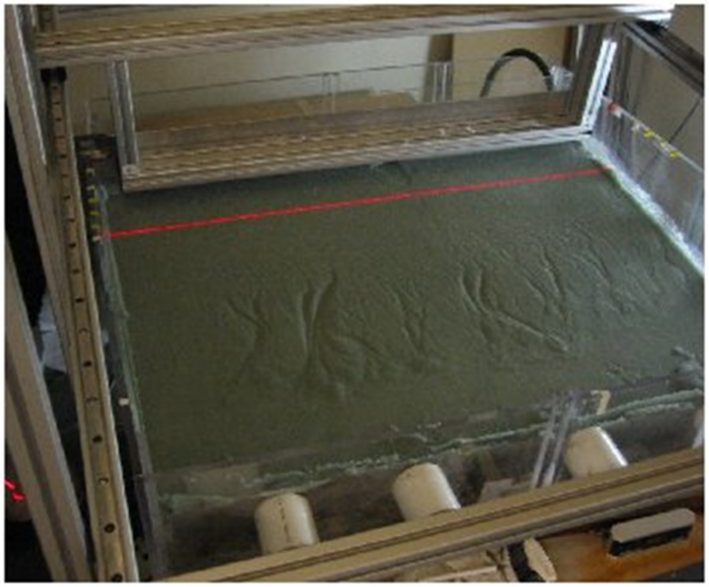


“Headless” Submarine Canyons: Tens of kilometers

Florida Sapping Canyons: Hundreds of meters

Erosion over a Granular Surface Bed

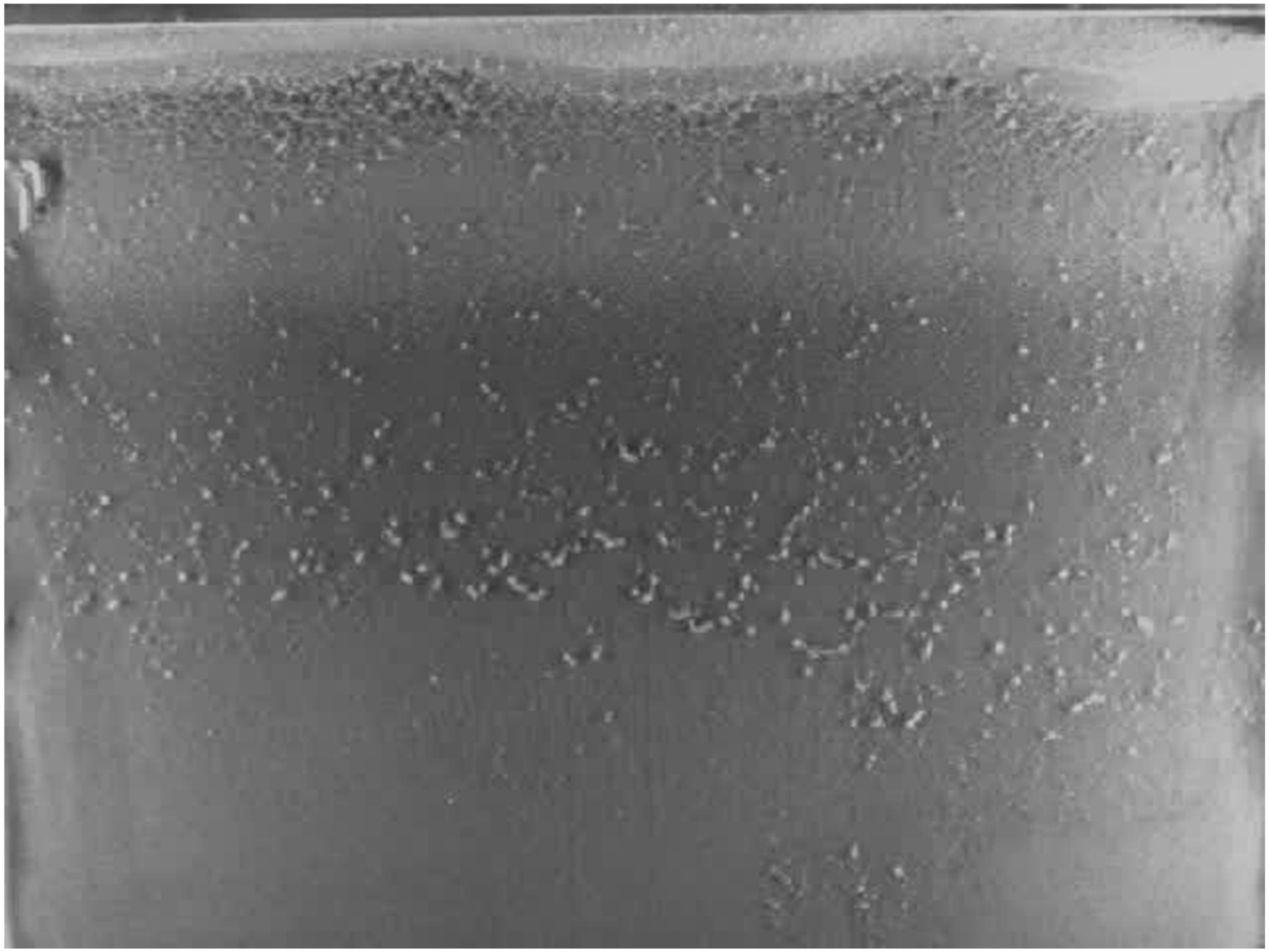




1m x 1m box

Our granular material: Glass beads

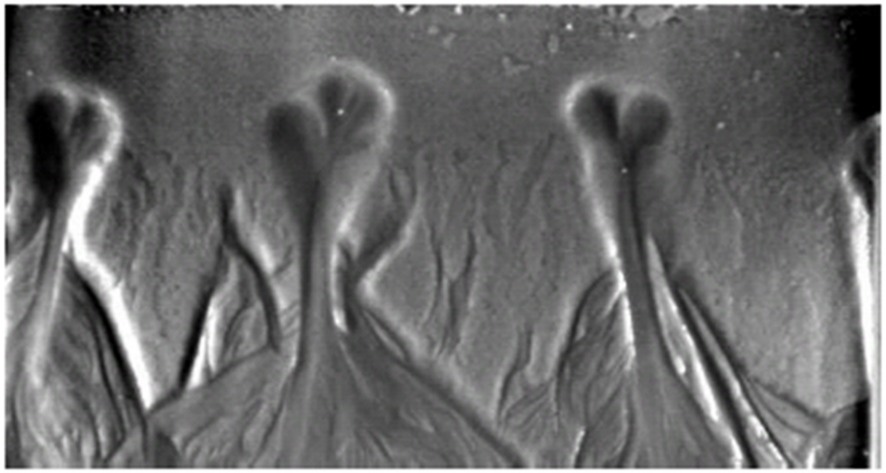
Erosion patterns caused by water sapping



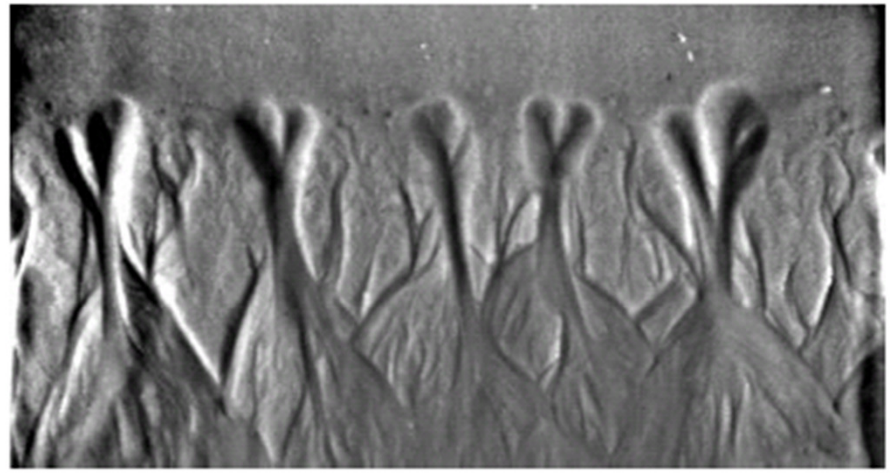
JFM (2004)

Experimental outcomes

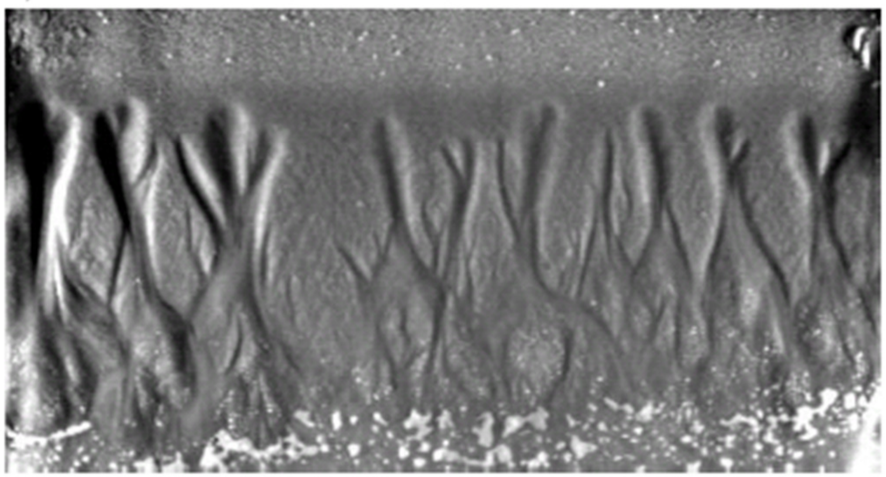
a) h_W=13.8cm, s=11%, t=28 min



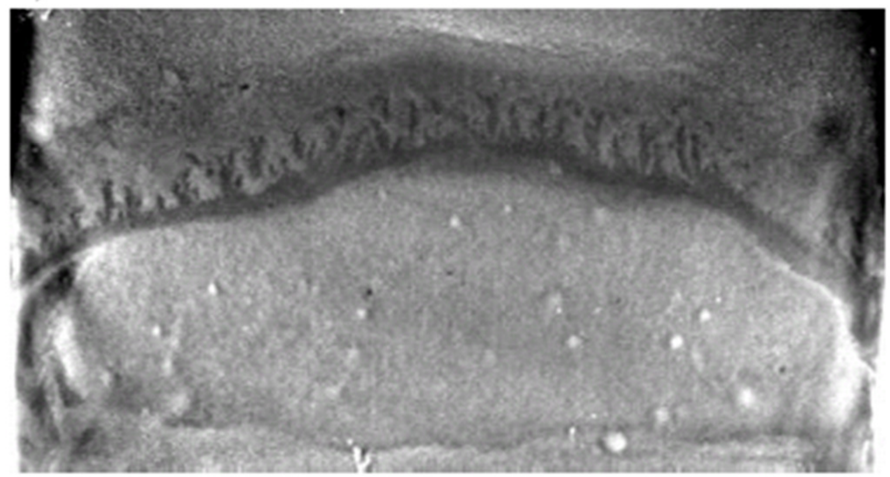
b) h_W=16.2cm, s=11%, t=18 min



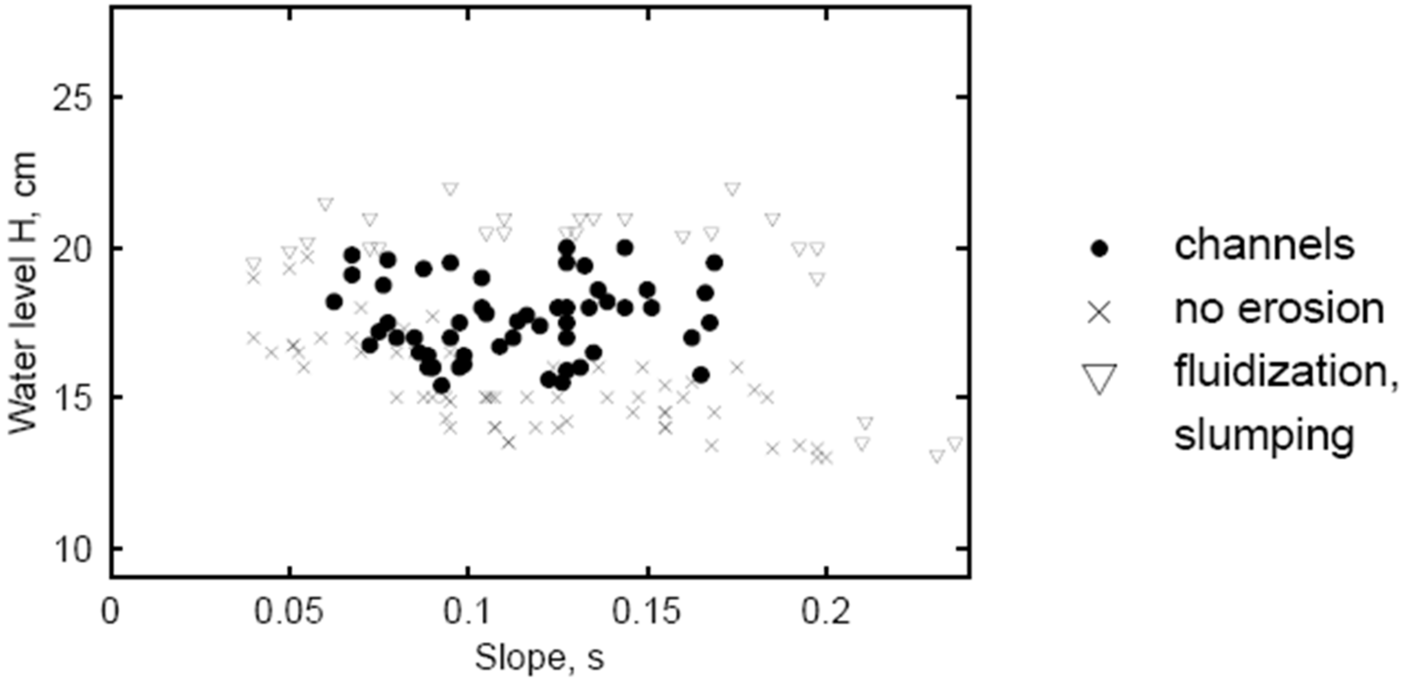
c) h_W=15.8cm, s=10%, t=18 min



d) h_W=13.4cm, s=16%, t=2 min



Phase diagram

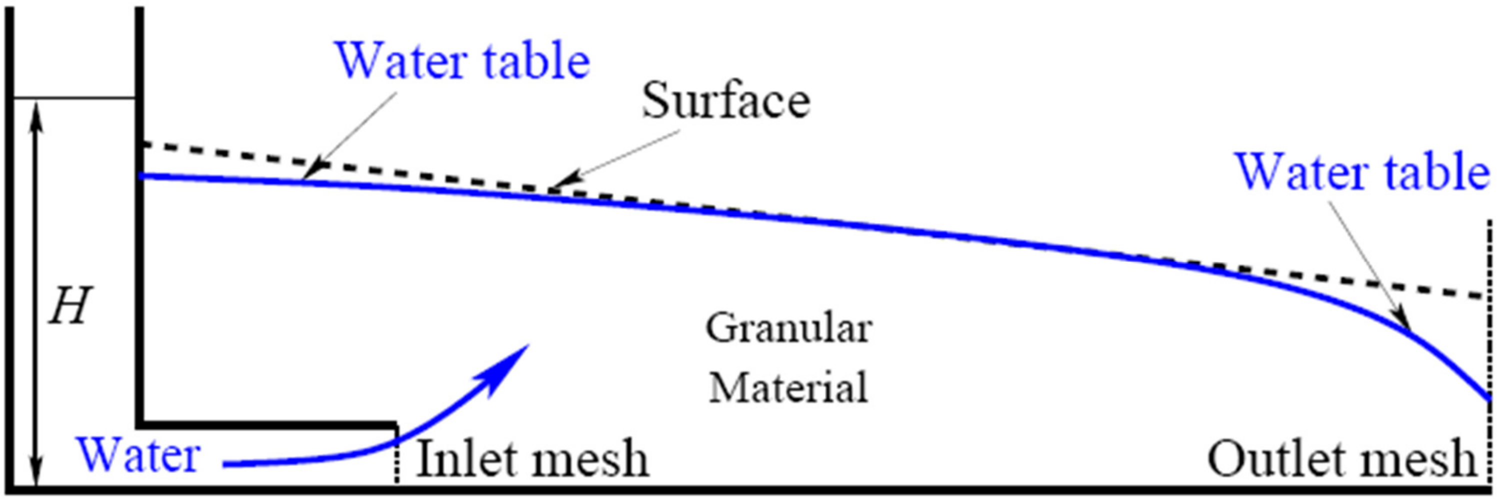


- The onset of channelization occurs when the Shields number

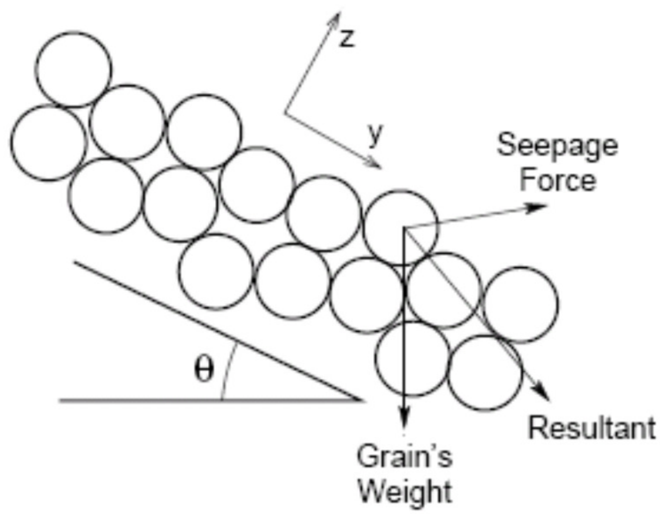
$$\tau = \frac{\text{tangential hydrodynamic forces}}{\text{normal force}} > \tau_c$$

- τ is calculated by numerical solution of $\nabla^2 p = 0$, computation of the seepage flux, and calculation of the shear stress.

Erosion is driven by subsurface flow



- Darcy flow + incompressibility ⇨
- $\nabla^2 p = 0$ in the sand.
- pressure p is hydrostatic at the inlet.
- p vanishes at the free surface (or capillarity is imposed).
- normal velocities vanish at boundaries.



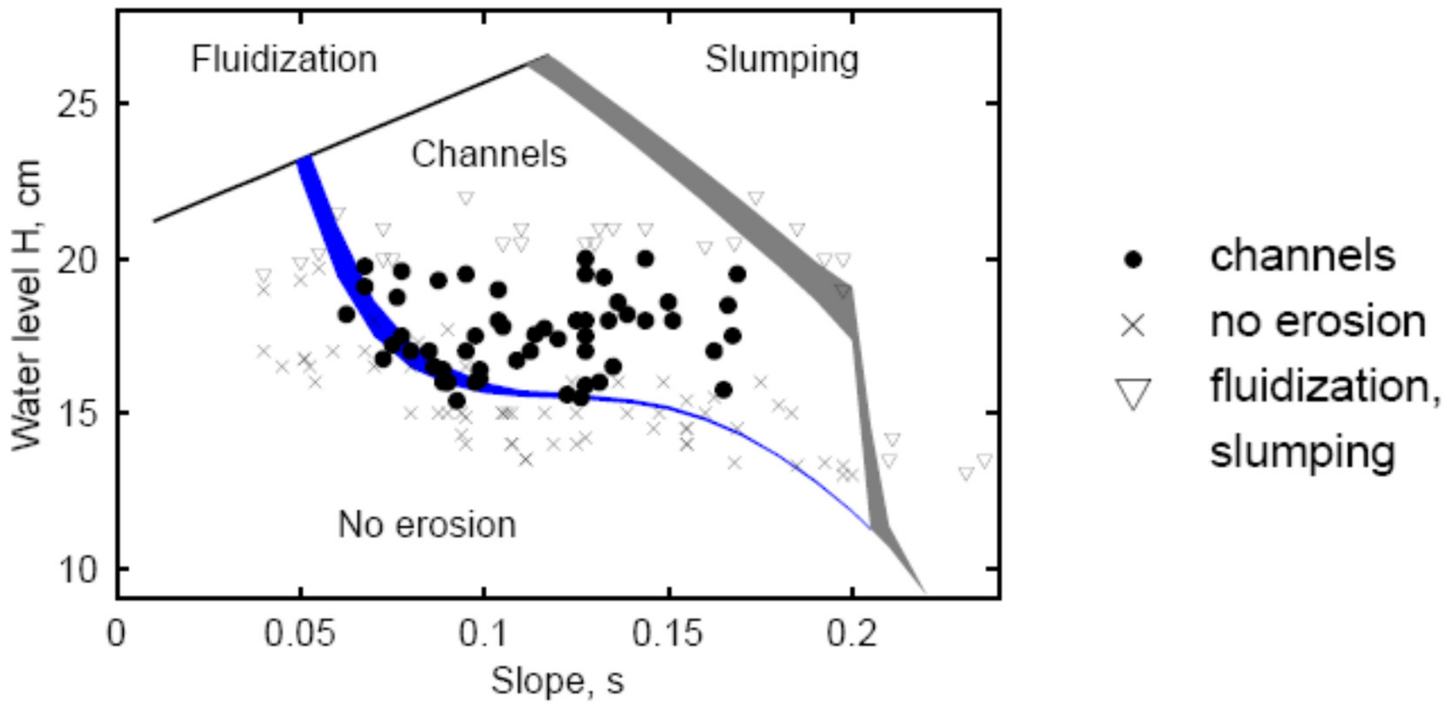
$$\tau^* = \frac{(h/d + a)\sin\theta}{(\rho_s/\rho - 1)\cos\theta + a\,\partial_z\psi}.$$

The modified shields criterion considering the inclination of the pile and the angle of the seepage force

h is the flowing liquid thickness

a ~ 0.5

Phase diagram

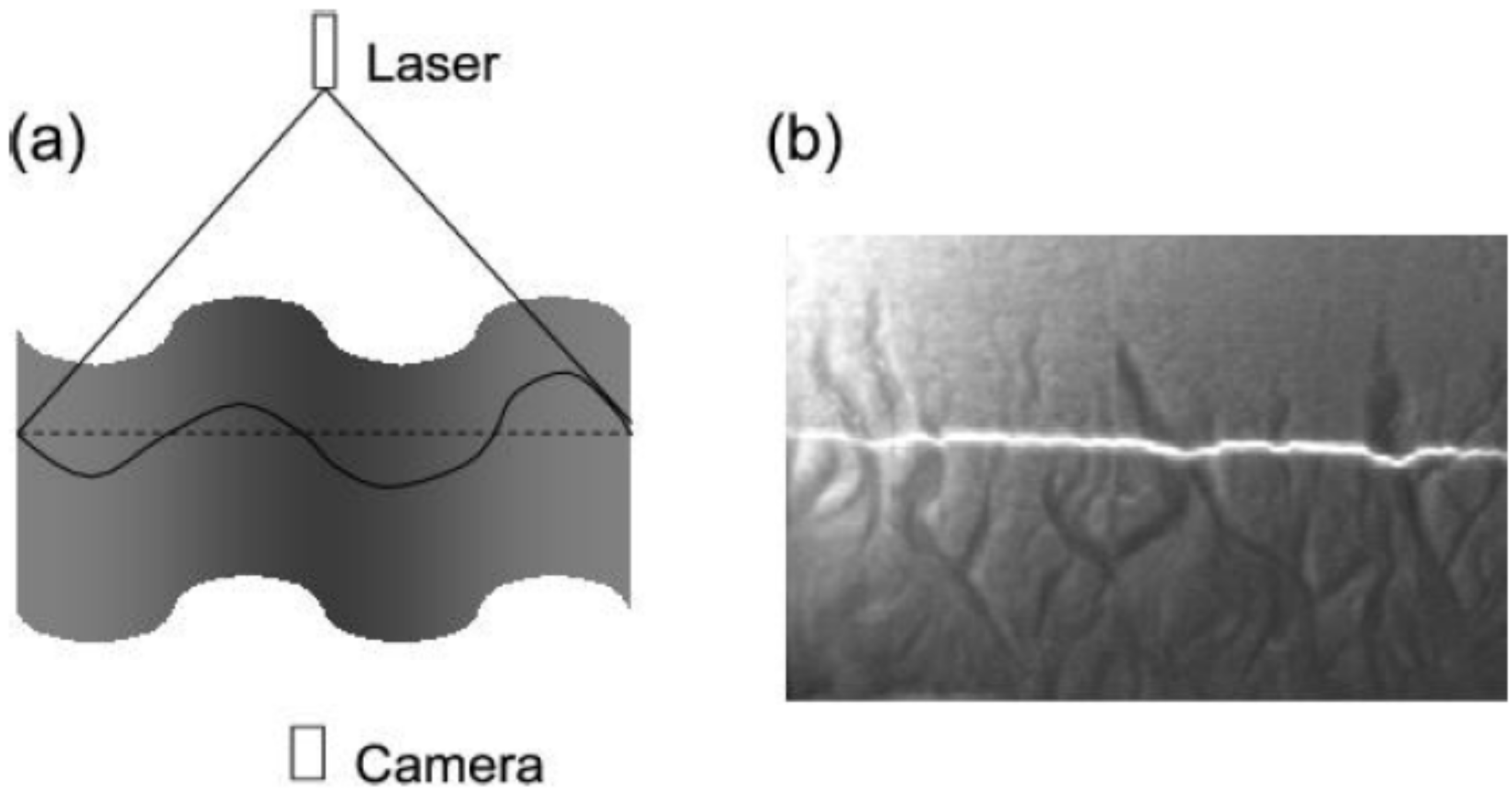


Criteria for fluidization and slumping follow Iverson and Major (1986):

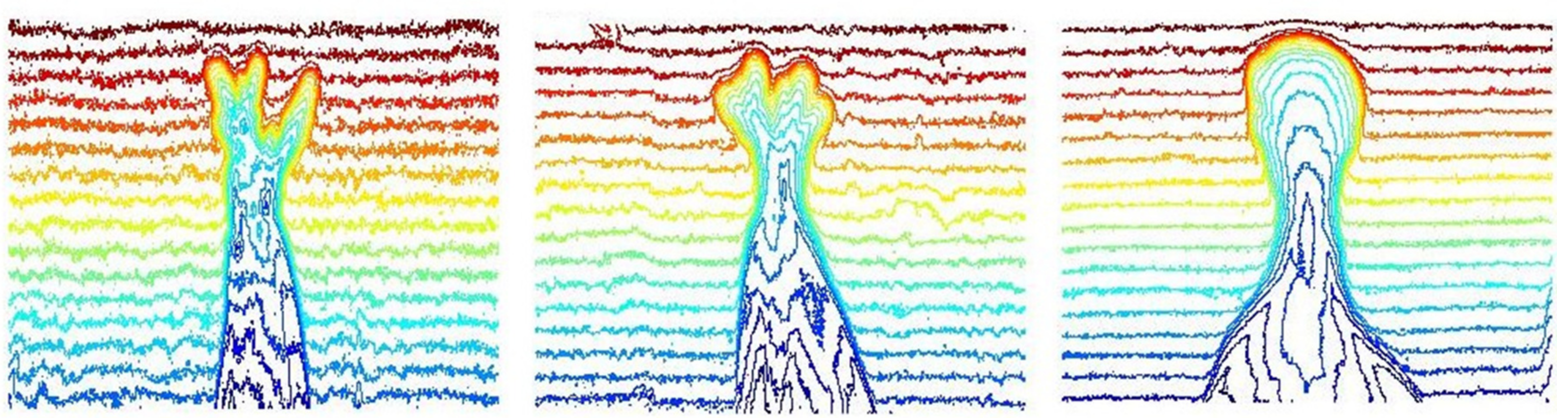
- Fluidization occurs when seepage force exceeds granular weight.
- Slumping occurs when the effective inclination angle—modified by the seepage force—exceeds the maximum angle of stability.

JGR 109, F04010 (2004).

Laser Aided Tomography

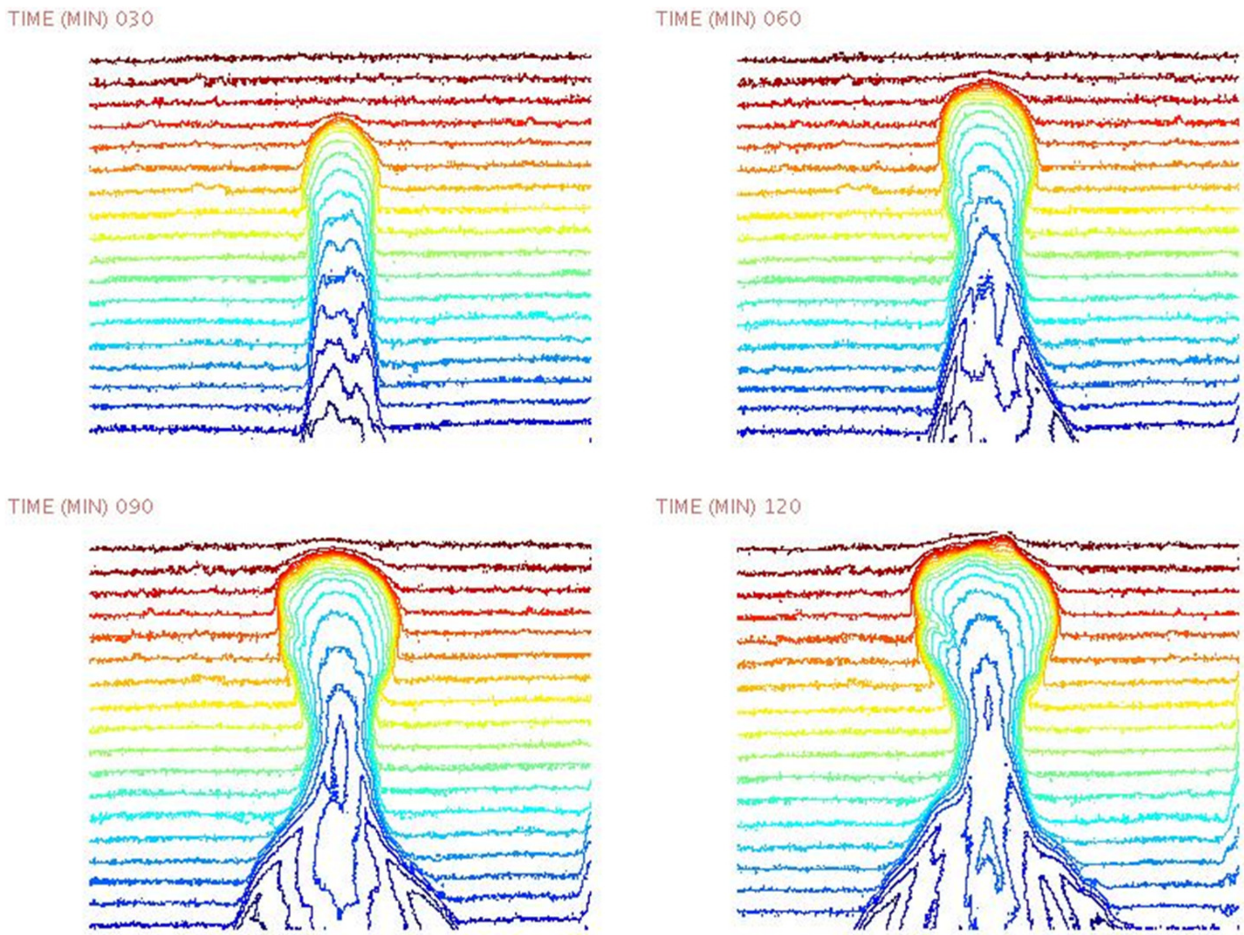


Erosion of a single channel obtained by laser aided topography



Increasing slope

Erosion of a single channel obtained by laser aided topography



Effective equation for surface height $h(x, y, t)$

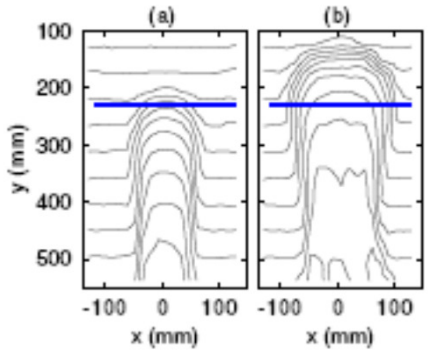
- Erosion rate $\dot{h}$ is a function of local water and sand fluxes
- Granular surface changes slowly $\Rightarrow$ water fluxes slaved to h
- Channel evolves in a roughly self-similar manner $\Rightarrow$ fluxes are functions of **local** topography & global scale factor h_0
- Simplify: evolution of transects

$$\boxed{\dot{h} = \nu h_{xx} + \lambda h_x^2 + \delta |h_x| + \mu \Theta(h/h_0 - f)}$$

- Diffusion constant ν
- Advection speeds λ and δ
- Erosion rate μ and fractional driving depth f

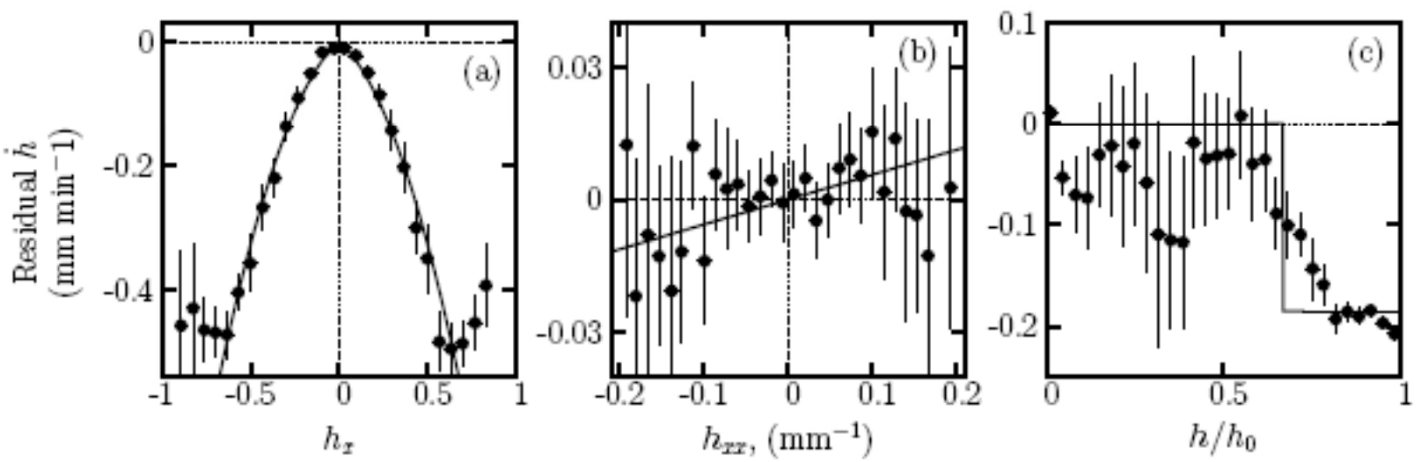
Encode driving as well as granular dynamics

Validate model with data



Strategy:

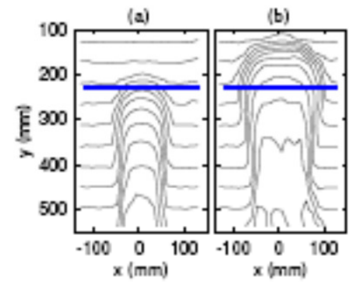
- Measure $h(x, y, t)$
- Compute $\dot{h}$, h_x, etc, in a window
- Fit data cloud to evolution equation
  $\dot{h} = \nu h_{xx} + \lambda h_x^2 + \delta |h_x| + \mu \Theta(h/h_0 - f)$

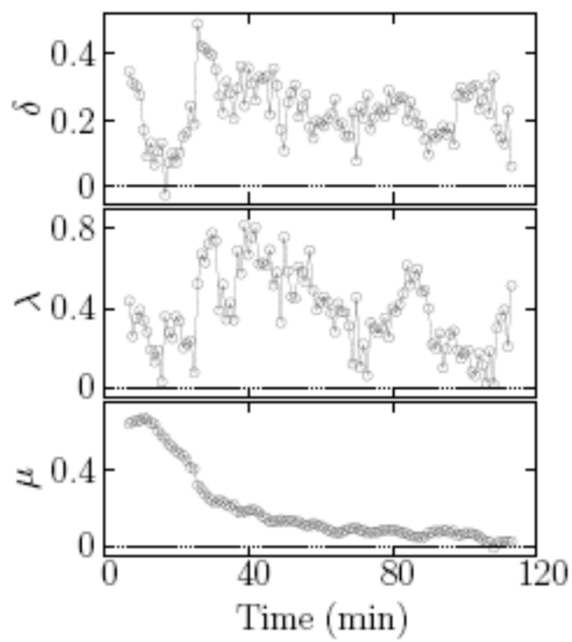


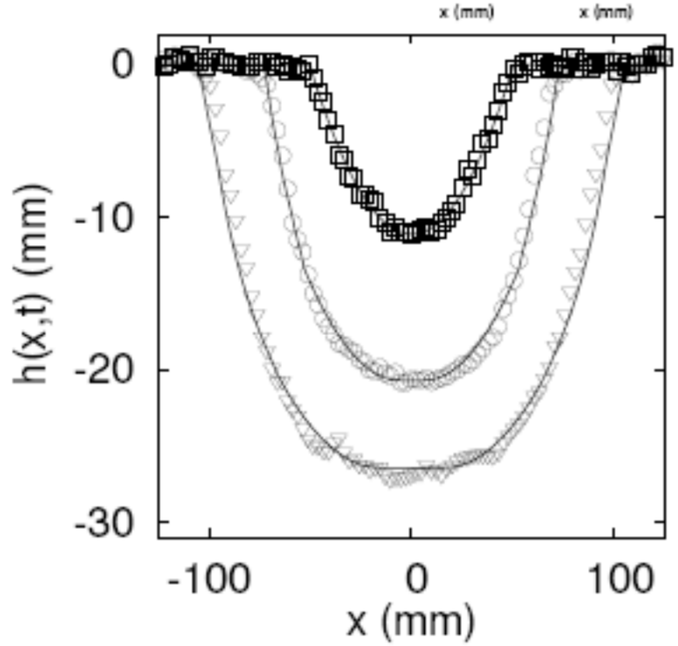
Predicted vs. measured transect shapes

Extracted coefficients vary with time

$$\dot{h} = \nu h_{xx} + \lambda h_x^2 + \delta|h_x| + \mu\Theta(h/h_0 - f)$$







JGR **112**, F03S12 (2007).

Erosion onset and bed-load transport

• Need to understand the relationship between hydrodynamic stress and surfical granular flow from microscopic (grain) level

$$\text{Shields parameter } (\theta) = \frac{\text{Viscous shear stress}}{\text{Normal stress}}$$

($\theta > \theta_c$) for fluid to erode the granular bed

θ_c: Critical shields parameter

• Previous observation and experimental work: Shields (1936), Vanoni (1977), Charru et al (2004), Paphitis & Collins (2005), Loiseleux et al. (2005)

- Large scatter in the values of θ_c
- Lack of controlled experimentation: turbulent vs laminar flows, ambiguous definitions

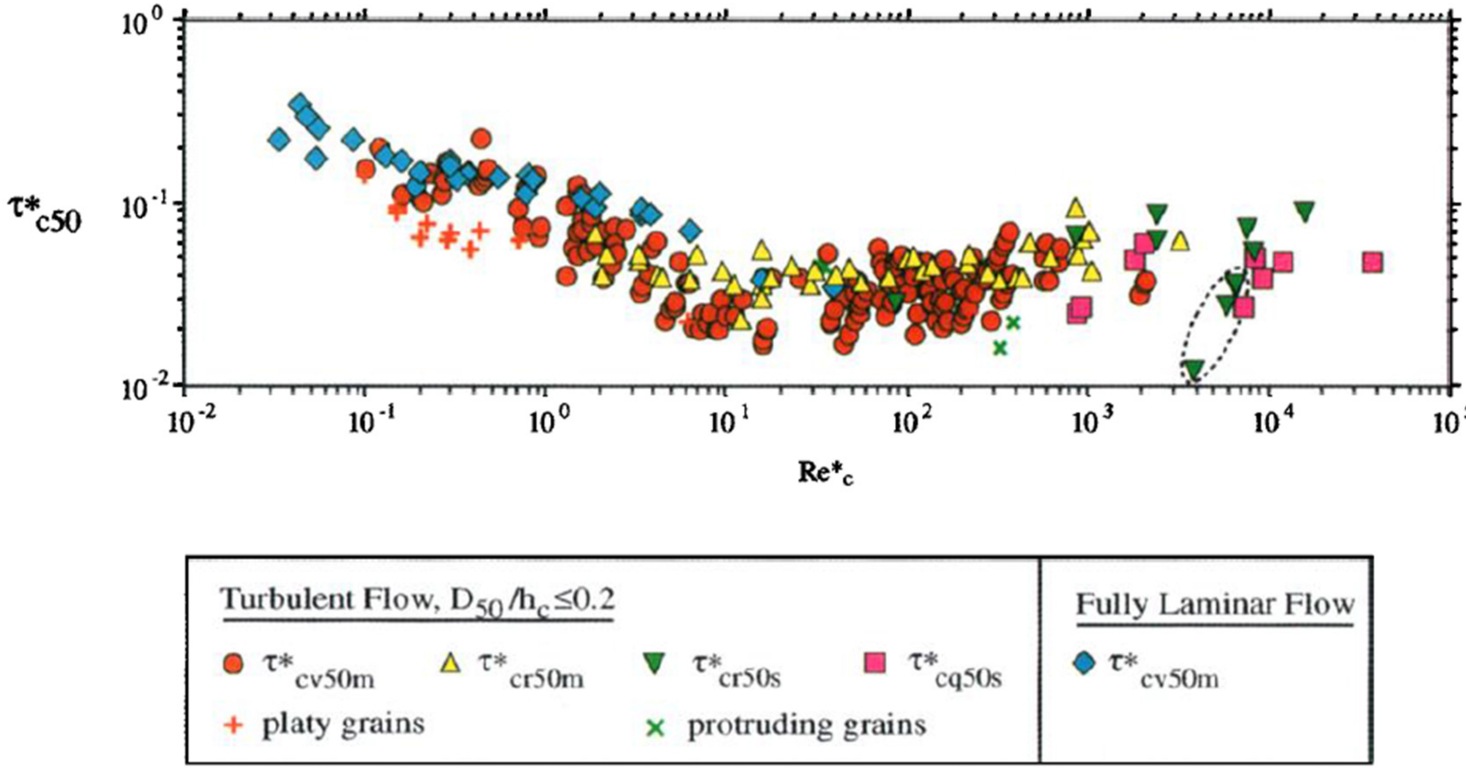


Plate 1. Shields curve for empirical data that represent initial motion of the bed surface material. All mixture-based values have known $\sigma_{gm} \leq 0.5$. Circled triangles are values reported for Oak Creek by *Parker and Klingeman* [1982], *Diplas* [1987], *Wilcock and Southard* [1988], *Parker* [1990], and *Wilcock* [1993]; these values are variations of the same data set (that of *Milhous* [1973]) analyzed using *Parker et al.*'s [1982] definition of incipient motion. The reference-based subcategory of protruding grains indicates significant grain projection and exposure sensu *Kirchner et al.* [1990].

Buffington and Montgomery (1997)

Erosion onset and bed-load transport

Granular flux (q_g) by fluid forcing

$q_g \propto (\theta - \theta_c)^{1.5}$

Empirical relation among others using sampling experiments

--- Meyer-Peter & Muller (1948); with correction by Wong & Parker (2006)

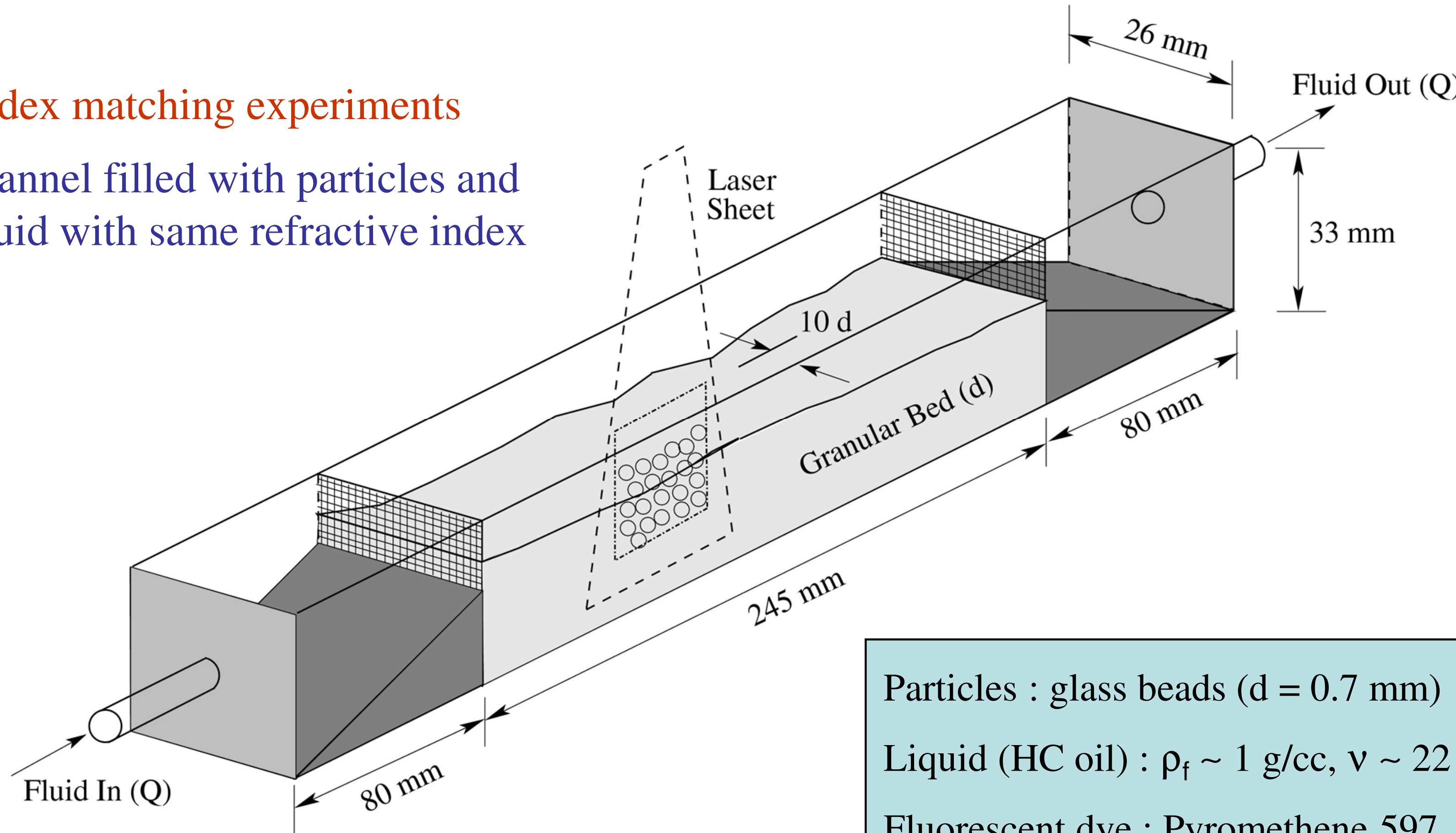
--- Charru, Mouilleron & Eiff (2004, 2009); Aussillous et al (2013) [experiments measuring onset and velocity profiles of moving layer above onset]

--- Derksen (2011) [simulations]

Experimental system

Index matching experiments

Channel filled with particles and liquid with same refractive index



Particles : glass beads (d = 0.7 mm)

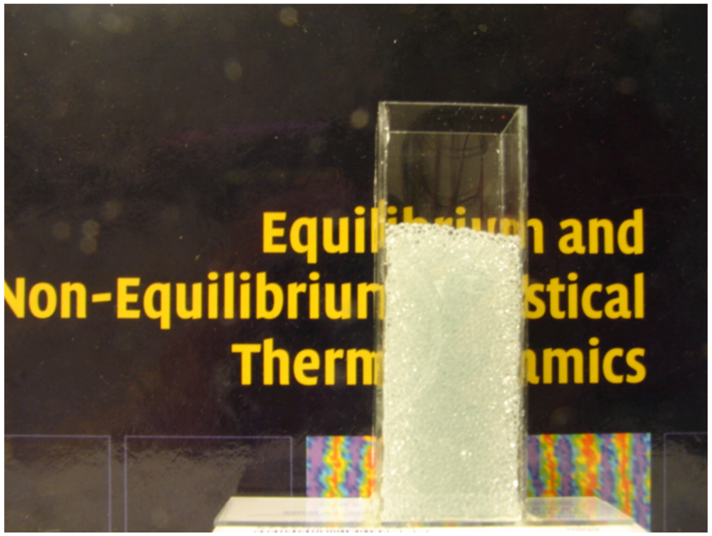
Liquid (HC oil) : $\rho_f \sim 1$ g/cc, $\nu \sim 22$ cS

Fluorescent dye : Pyromethene 597

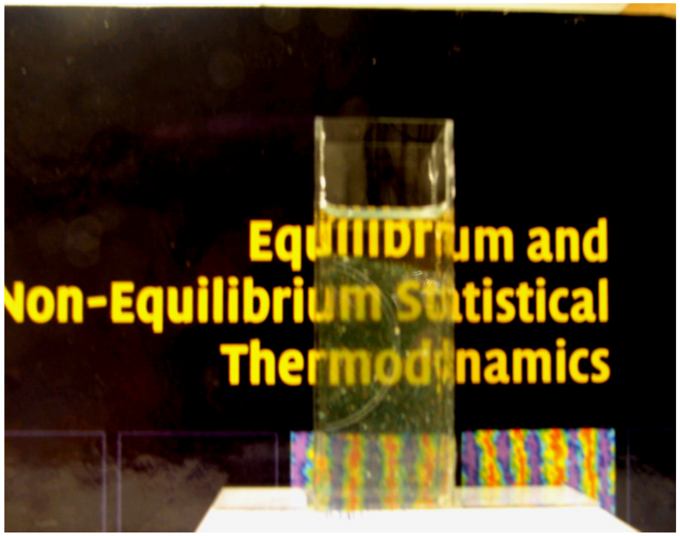
Fluid flow rate (Q) : 60 - 2500 cm^3/min

Experimental technique to investigate properties away from side walls

Index matching technique used commonly to study colloids with confocal microscopy



Container filled with glass beads



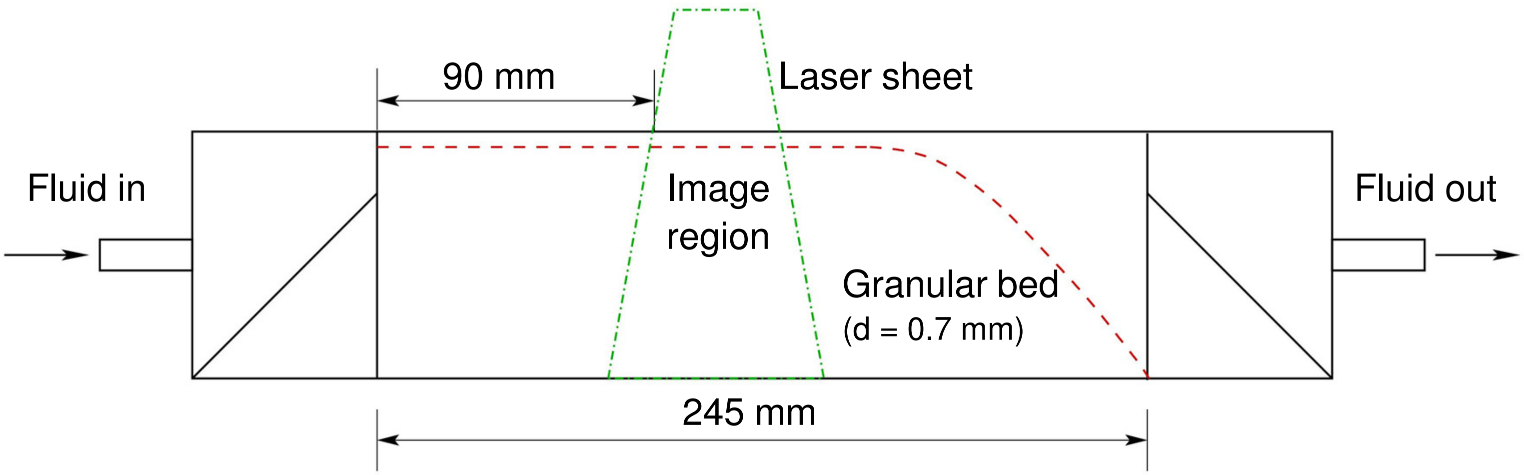
Same container filled with glass beads and a liquid with the same refractive index

Particles : glass beads (d = 0.7mm)
Liquid (HC oil) : RI ~ 1.52, ν ~ 22 cS

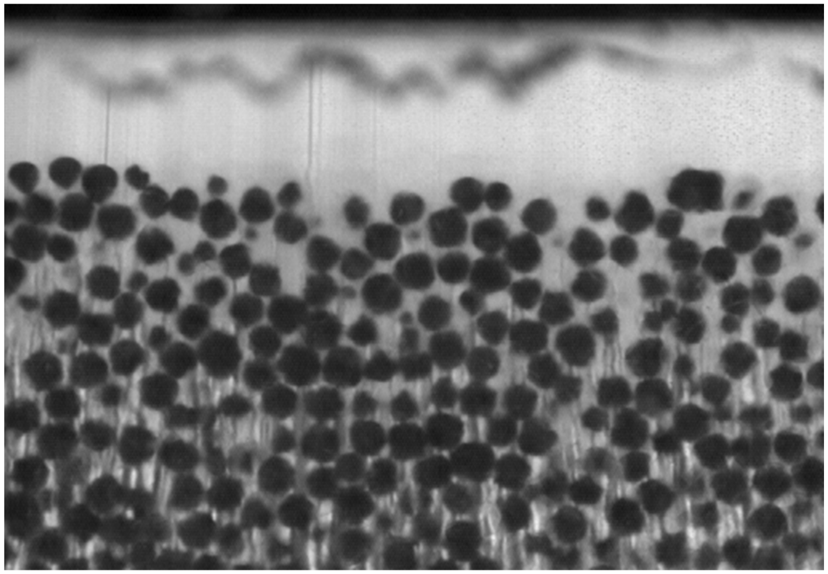
Tsai et al (2003)

Bed height/Fluid gap measurements

Bed initialization



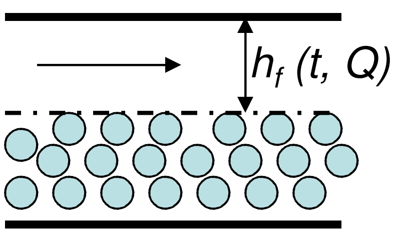
- Images acquired every second interval
- Edge detecting algorithm to obtain flowing layer surface

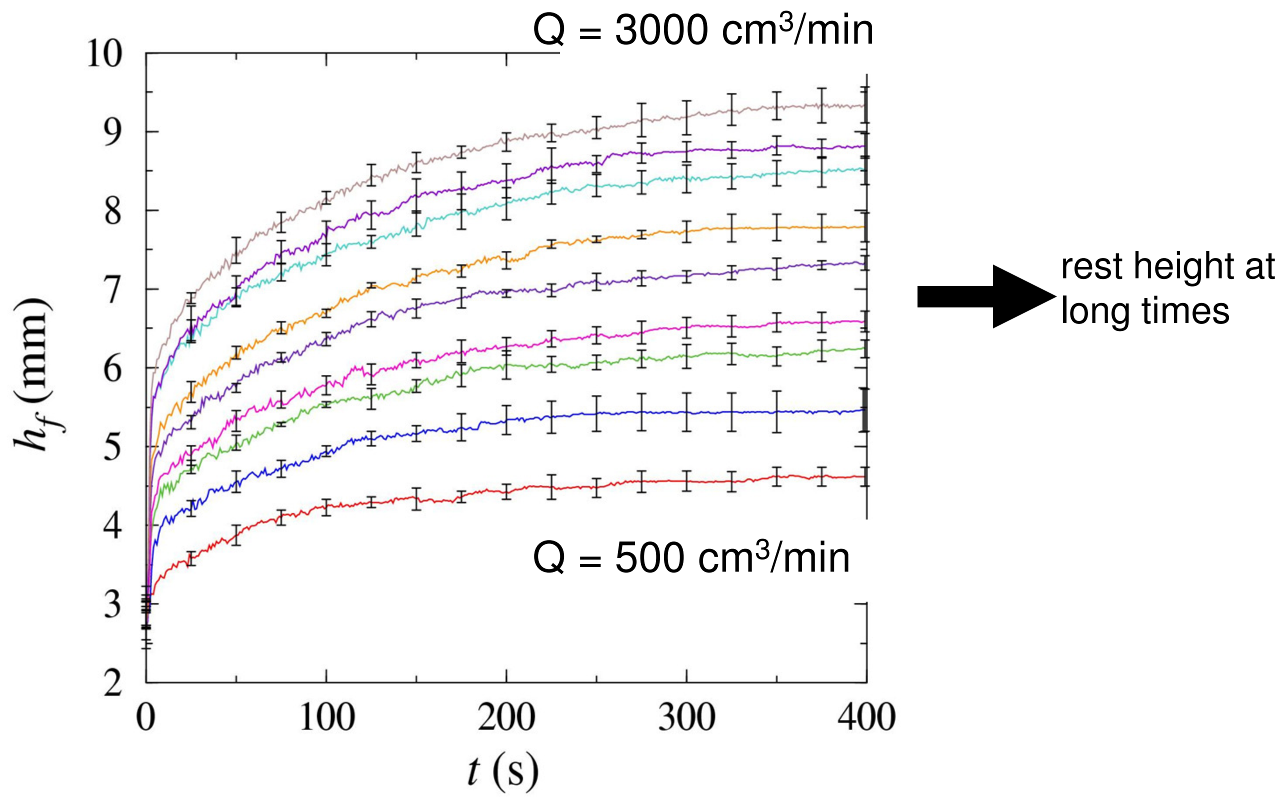


Q = 1000 cm^3/min

Bed evolution

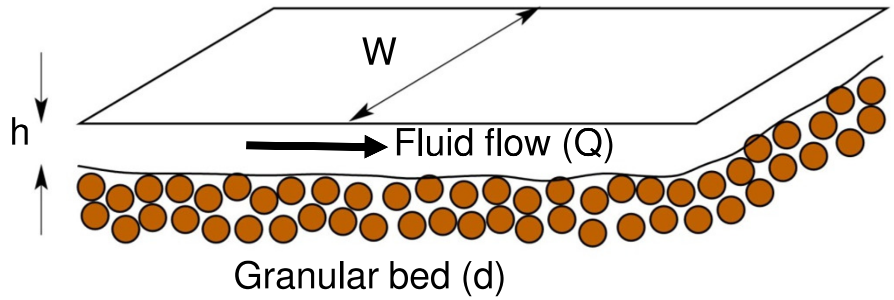
Approach to rest height (h_r)





- Gradual decrease of shear stress in time (approaches threshold from above)
- Final fluid gap (rest height, h_r) increases with flow rate applied

Threshold bed erosion



Shields parameter (θ) = $\dfrac{\text{Viscous shear stress}}{\text{Gravitational normal stress}}$

$$\theta = \frac{6Q\nu}{W\gamma g d h^2}$$

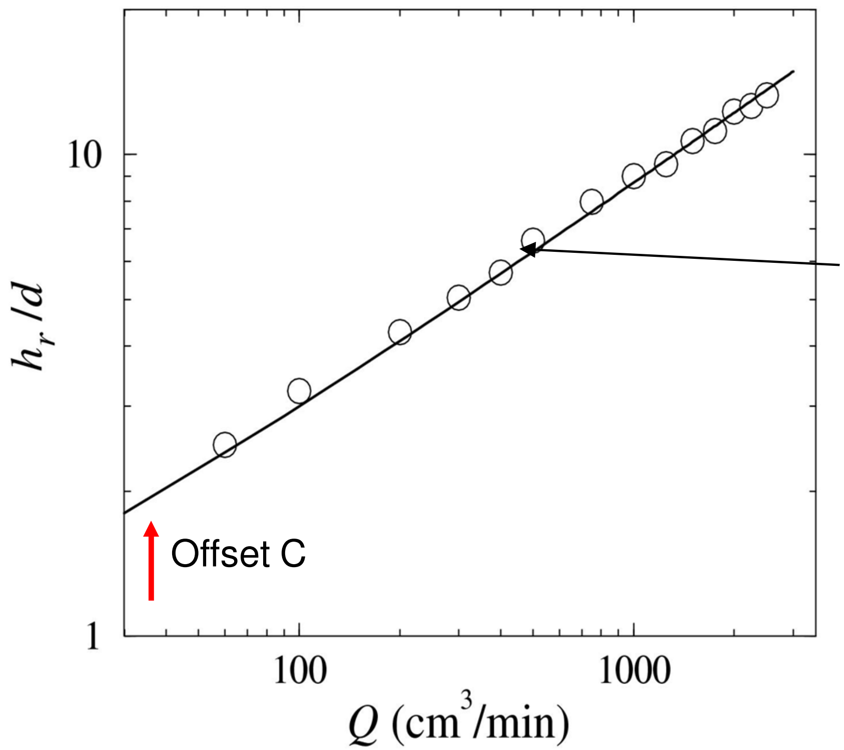
- 2d laminar flow between infinite smooth parallel plates (W >> h)
- Channel Reynolds No : $3Q/W\nu \approx 1$ ($Q = 1000\ cm^3/min$)
- γ (density contrast): $(\rho_g/\rho_f - 1)$, ν : Fluid viscosity

$$h_r = d\sqrt{\frac{Q}{Q_r}}$$

- $\theta = \theta_c$ (Critical Shields parameter)
- $h = h_r$ (Fluid gap after erosion ceases)
- $Q_r = \theta_c \dfrac{W\gamma g d^3}{6\nu}$

Rest height

Flow rate dependence



$$\frac{h_r}{d} = \sqrt{\frac{Q}{Q_r}} + C$$

(Differences in the flow near smooth and porous boundary)

C: 0.5d, Q_r: 15.0 cm^3/min

Critical Shields parameter (θ_c) : 0.3

A. Lobkovsky, A. Orpe, R. Molloy, A.Kudrolli, and D. Rothman, JFM (2008).

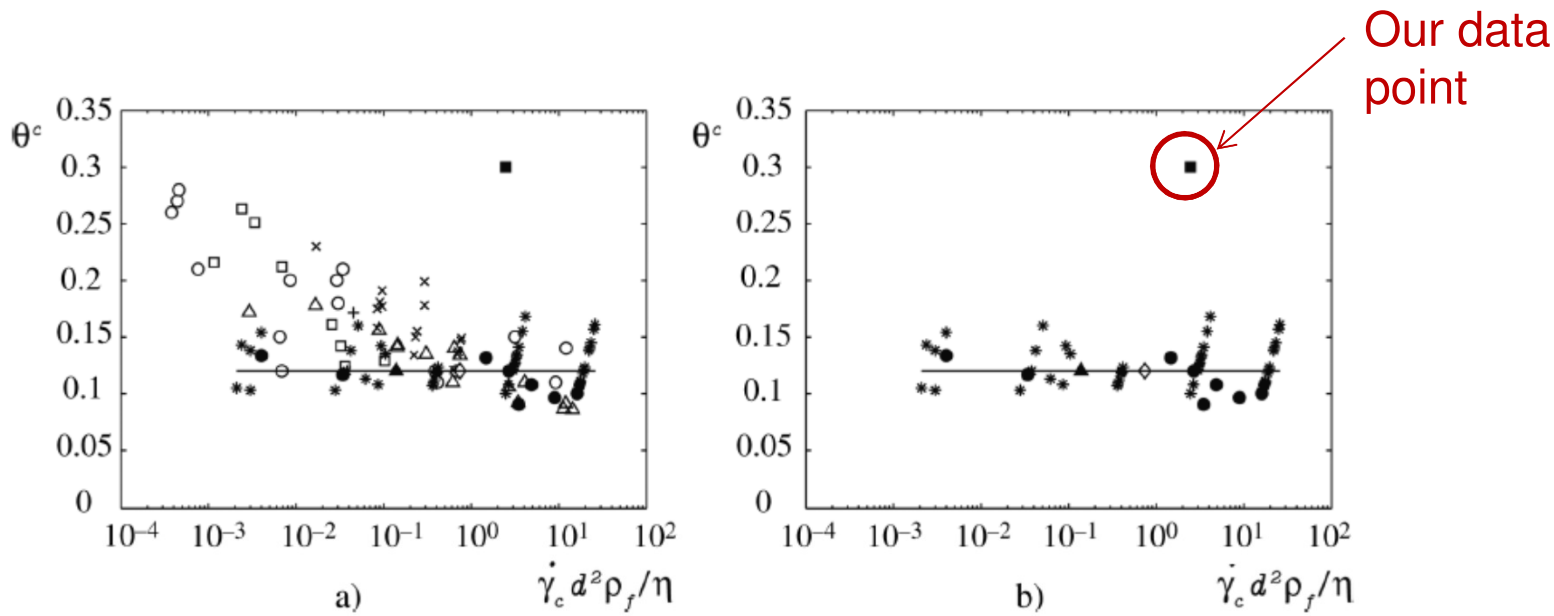


Figure 8.5. *(a) Critical Shields number* θ^c *versus the particle Reynolds number* $\dot{\gamma} d^2 \rho_f / \eta$ *: experimental data of White [WHI 40] quoted by Loiseleux et al. [LOI 05] (+), of White 1970 quoted by Mantz [MAN 77] (□), of Mantz [MAN 77] (×), of Yalin and Karahan [YAL 79] (Δ), of Pilotti and Menduni [PIL 01] (○), of Charru et al. [CHA 04] (▲), of Loiseleux et al. [LOI 05] (●), of Ouriemi et al. [OUR 07] (*), of Lobkovsky et al. [LOB 08] (■) and of Malverti et al. [MAL 08] (◊). The horizontal line represents value* $\theta^c = 0.12$*; (b) same figure but only representing the most recent data where the threshold is clearly defined*

Graph compiled by Aussilious, Guazzelli, Peysson, in *Erosion of Geomaterials* (2012)

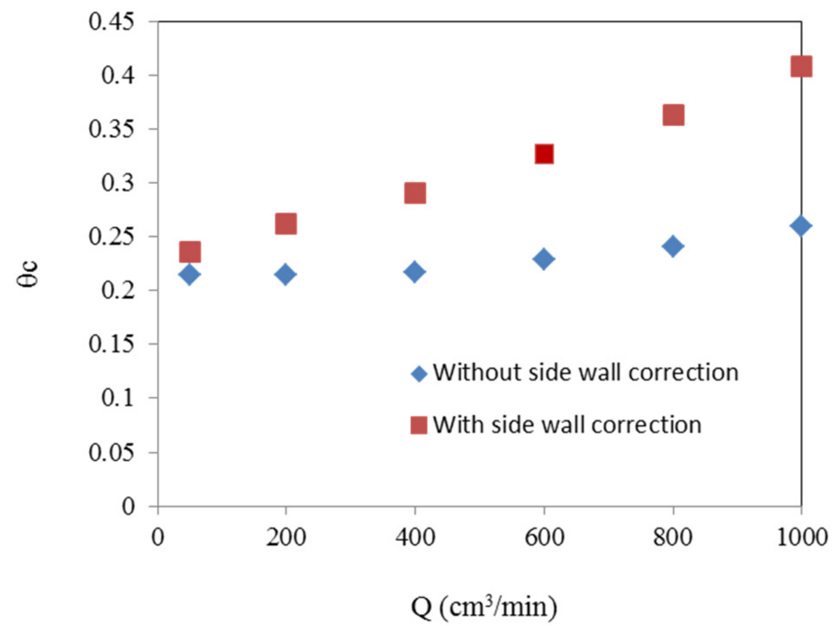
Correcting for side wall effects

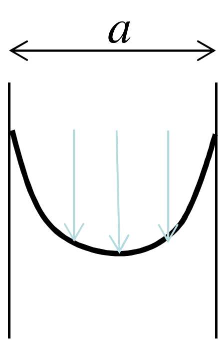
- Anyu Hong

Consider the flow in a rectangular pipe (Cornish, 1928)

$$Q = -\frac{4}{3} \cdot \frac{ab^3}{\mu} \cdot \frac{dp}{dz} \left\{1 - \frac{192}{\pi^5} \cdot \frac{b}{a} \left(\tanh \frac{\pi a}{2b} + \frac{1}{3^5} \cdot \tanh \frac{3\pi a}{2b} + \dots\right)\right\}$$

a = width
b = height

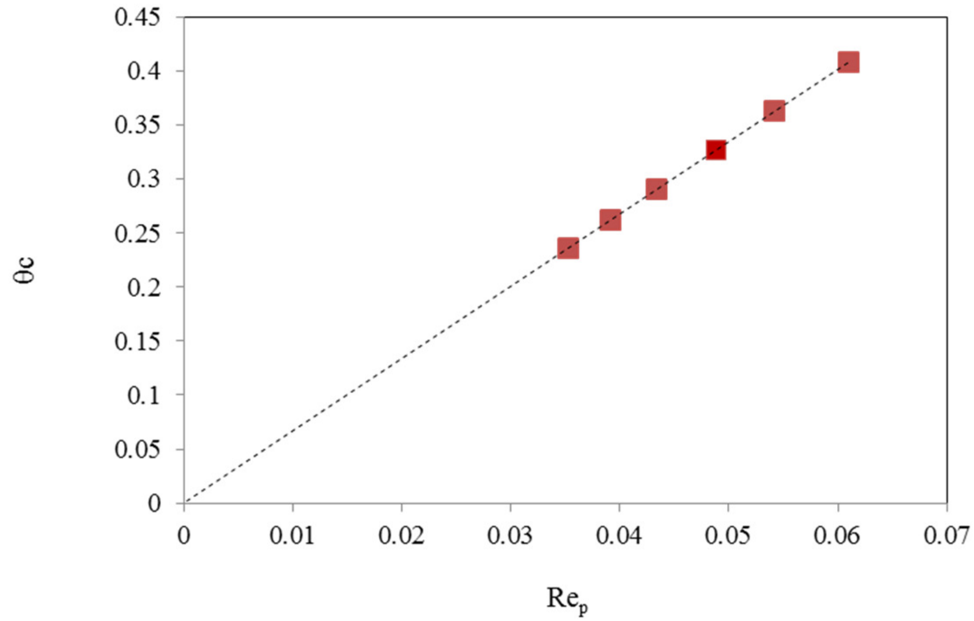




Top view

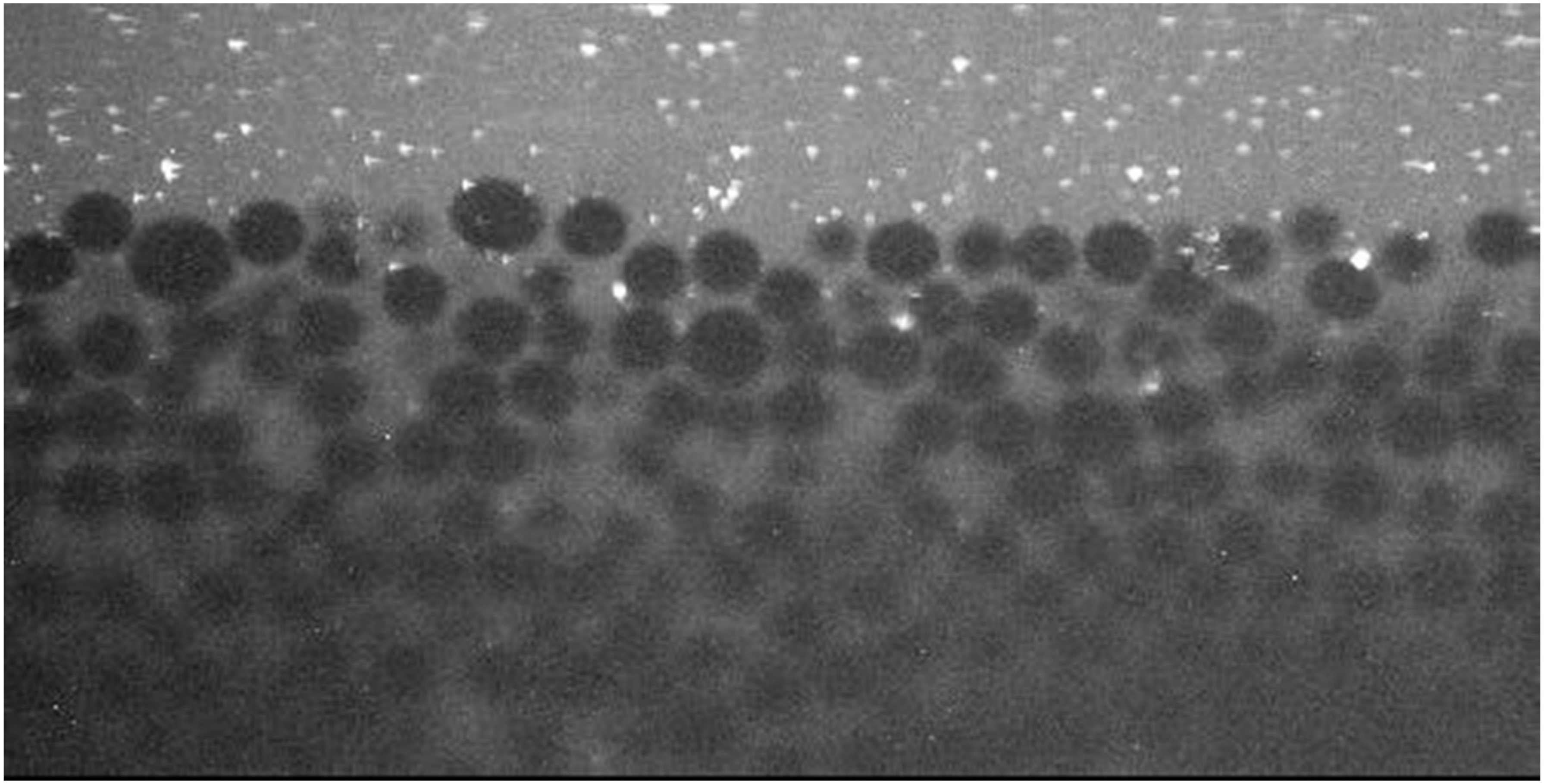
- Experiments were performed over longer times (2 hours) to ensure that erosion had indeed stopped
- Performed with a more viscous fluid to further ensure laminar flow

Critical Shields Number Versus Particle Reynolds Number

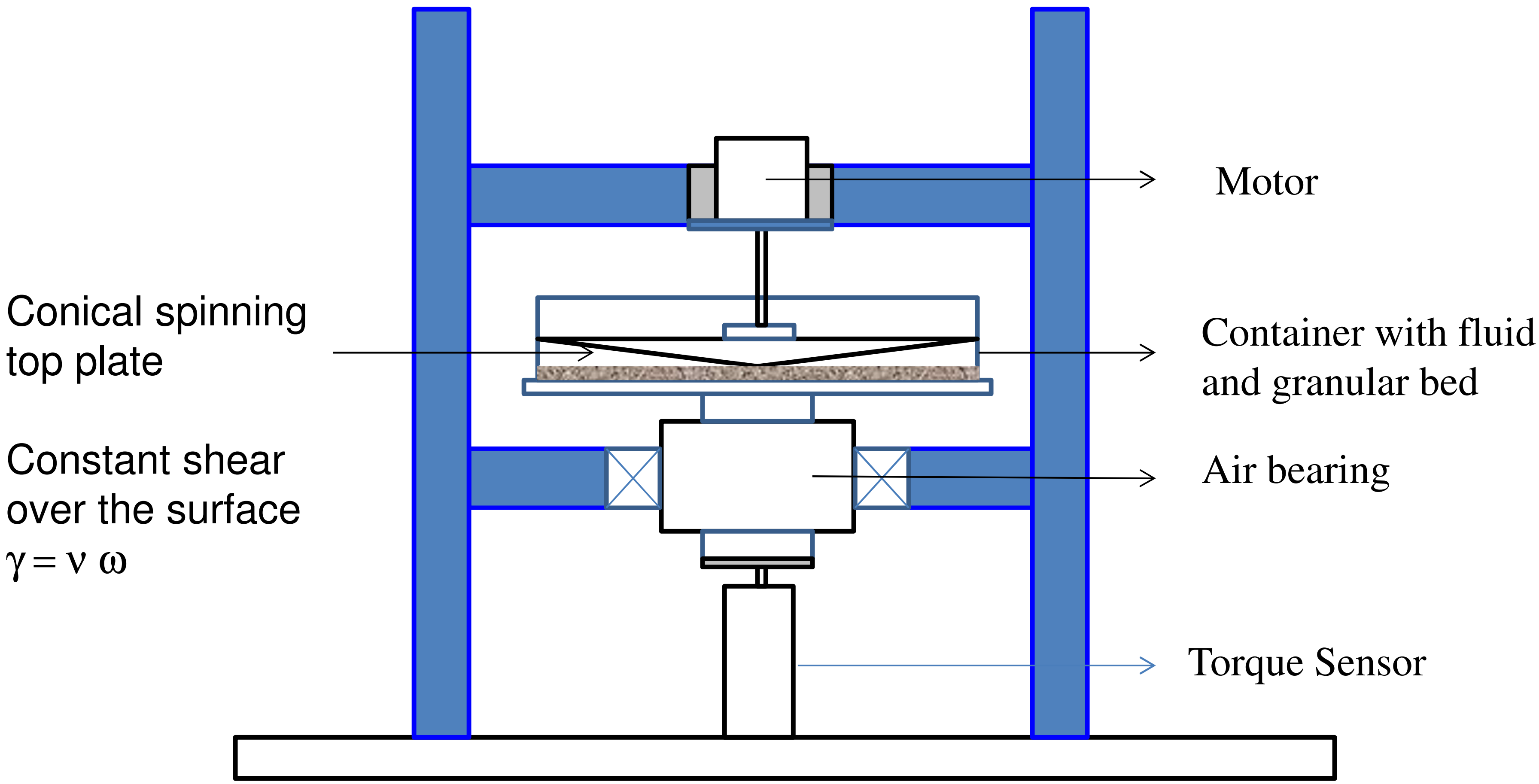


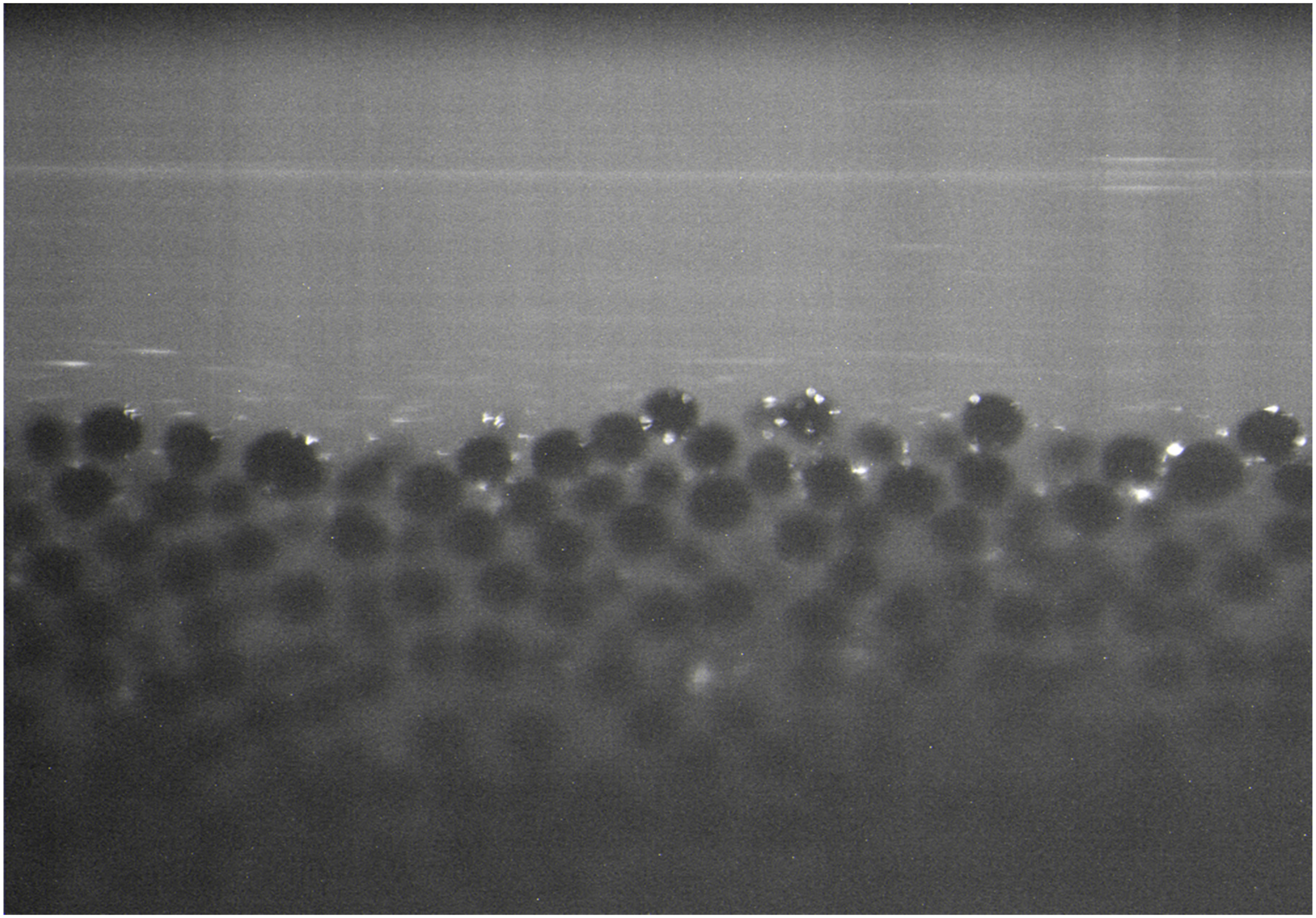
- Critical Shields Number increases with particle Re_p because of the way it is defined

PIV to image fluid motion at interface

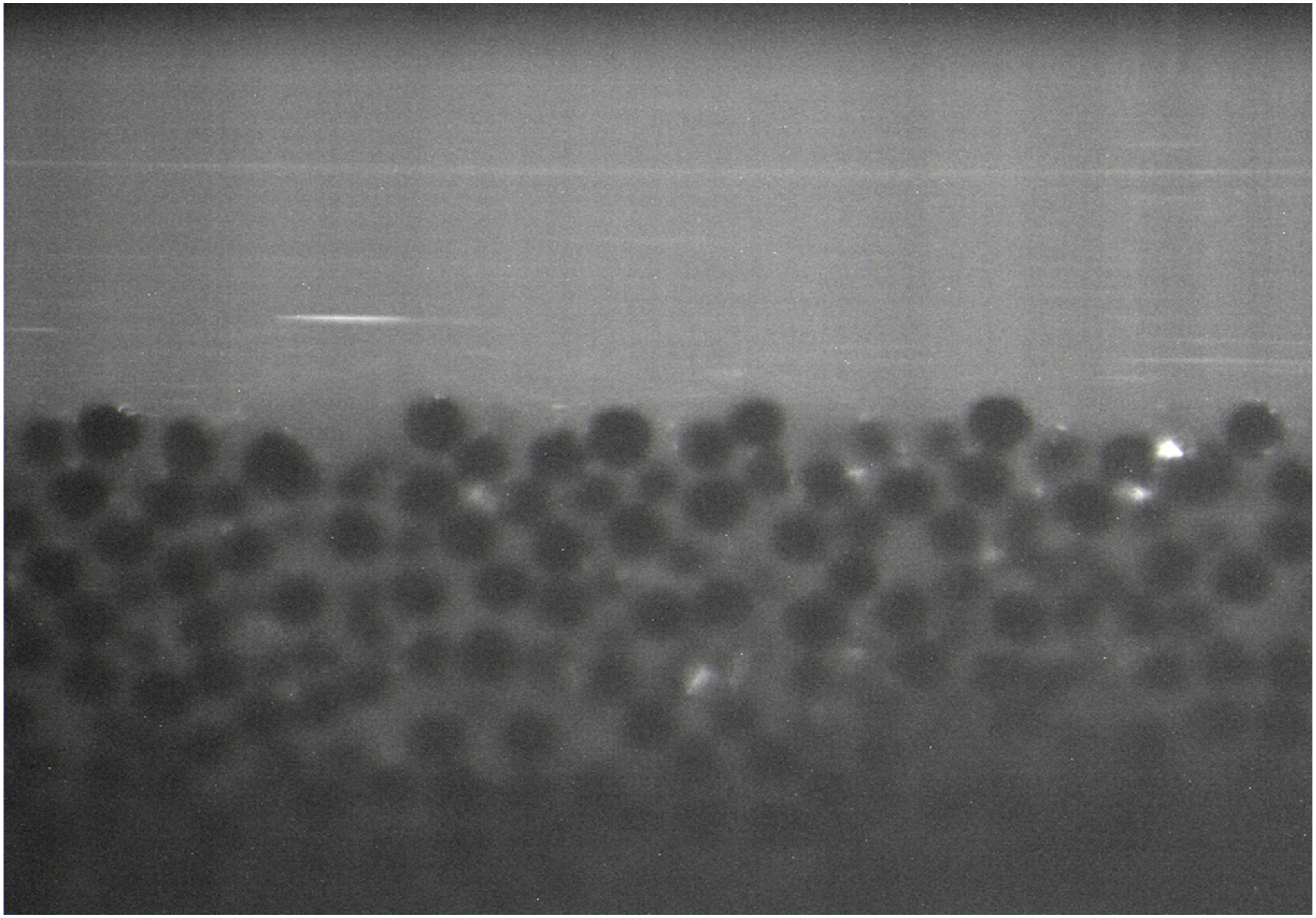


Complementary measurements using a conical "rheometer"

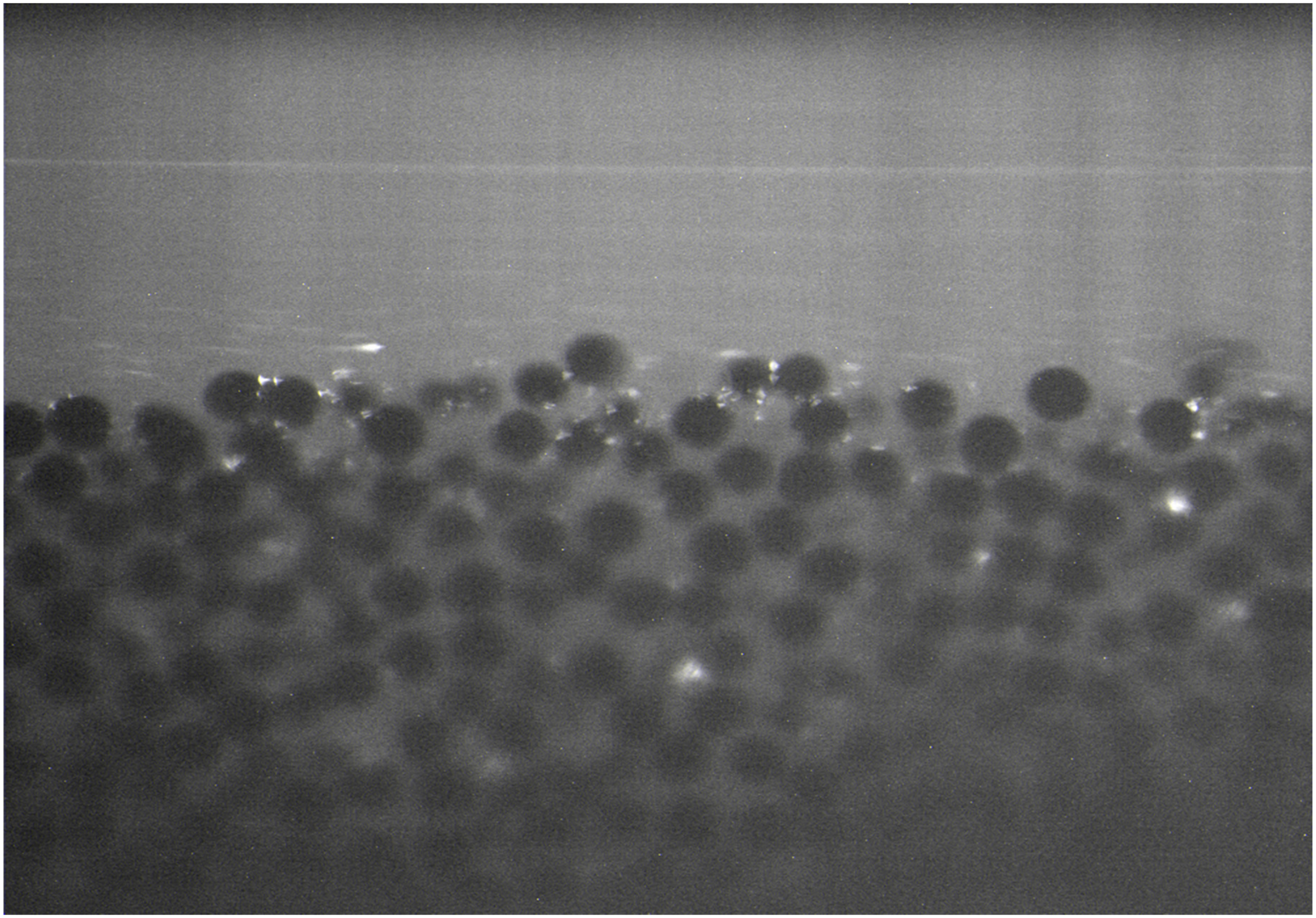




Below onset

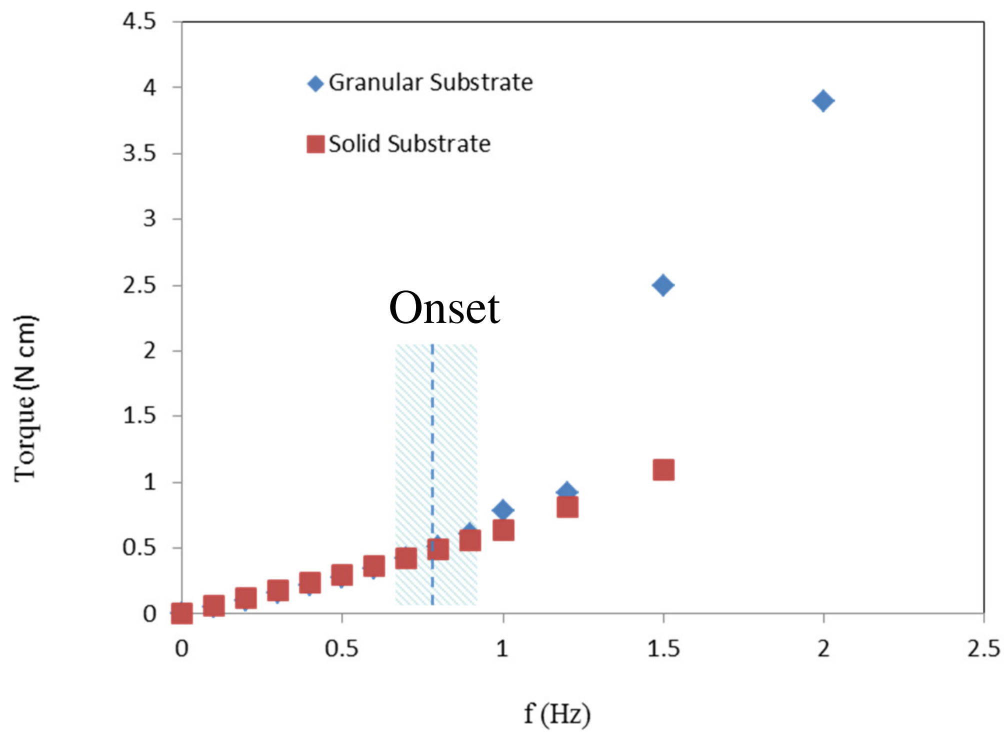


Around onset



Above onset

Measuring onset using a "rheometer"



Glass beads: 0.7 mm

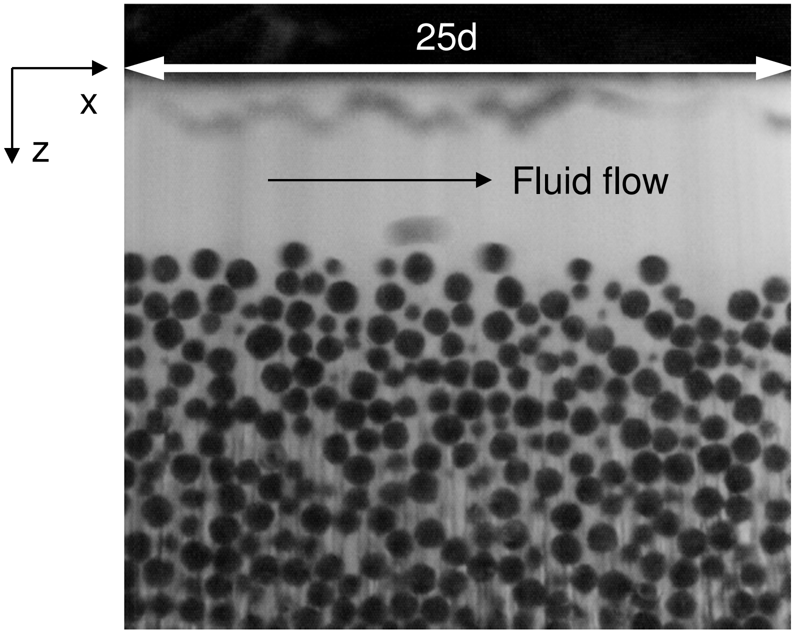
Liquid: Glycerol – Water mixture

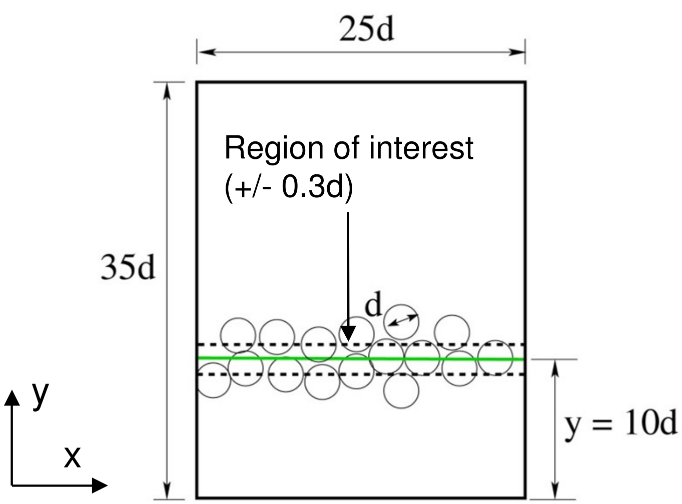
Density: 1207 kg/m^3

Dynamic viscosity: 0.0478 Ns/m^2

$$\theta_c = \frac{3\,T}{2\,\pi\,(\rho\ -\rho_l)g\ d\ R^3} \longrightarrow \theta_c \approx 0.32$$

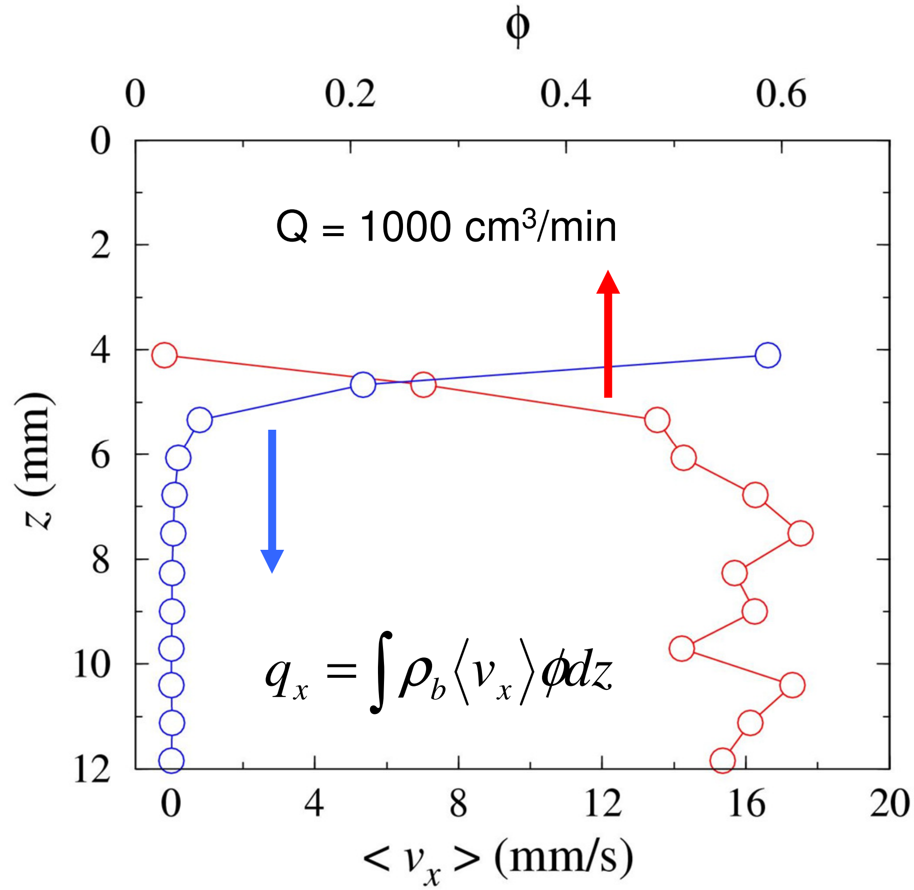
Granular flux measurements





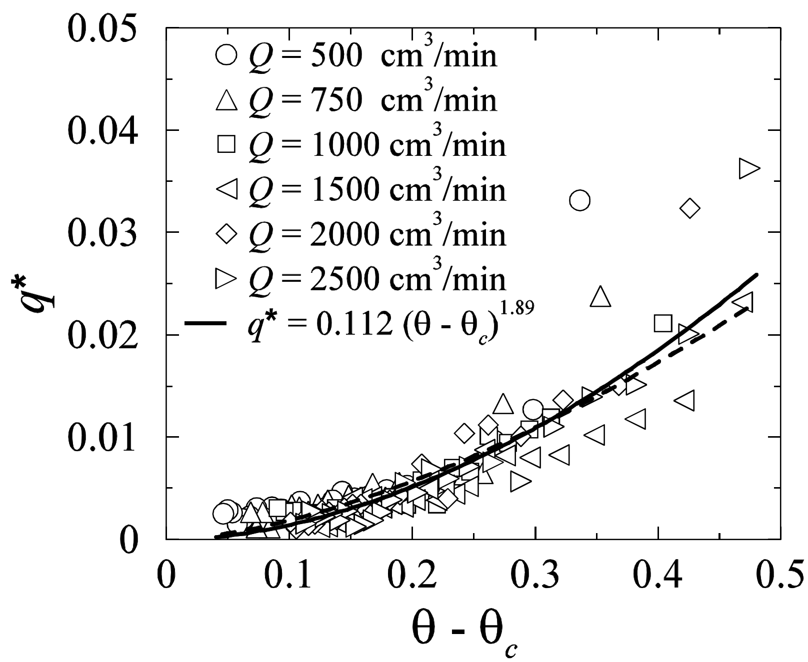
Top view

Digital images acquired at 1-250 Hz



Flow rule

Einstein Number: $q^* = q_g/(\gamma g d^3)^{1/2}$



- $\lambda \sim 1.89$ somewhat higher than accepted value of 1.5

- Strong influence of θ_c on λ

Conclusions

- Critical parameter to have onset of erosion: Shields number ~ 0.3 for laminar flow
- Shields number increases with flow rate

- Possible role of armoring

- Used index matching technique to measure the granular flux as a function of fluid flow

$q_g \propto (\theta - \theta_c)^{\lambda}$, $\lambda = 1.75 \pm 0.25$

http://physics.clarku.edu/~akudrolli