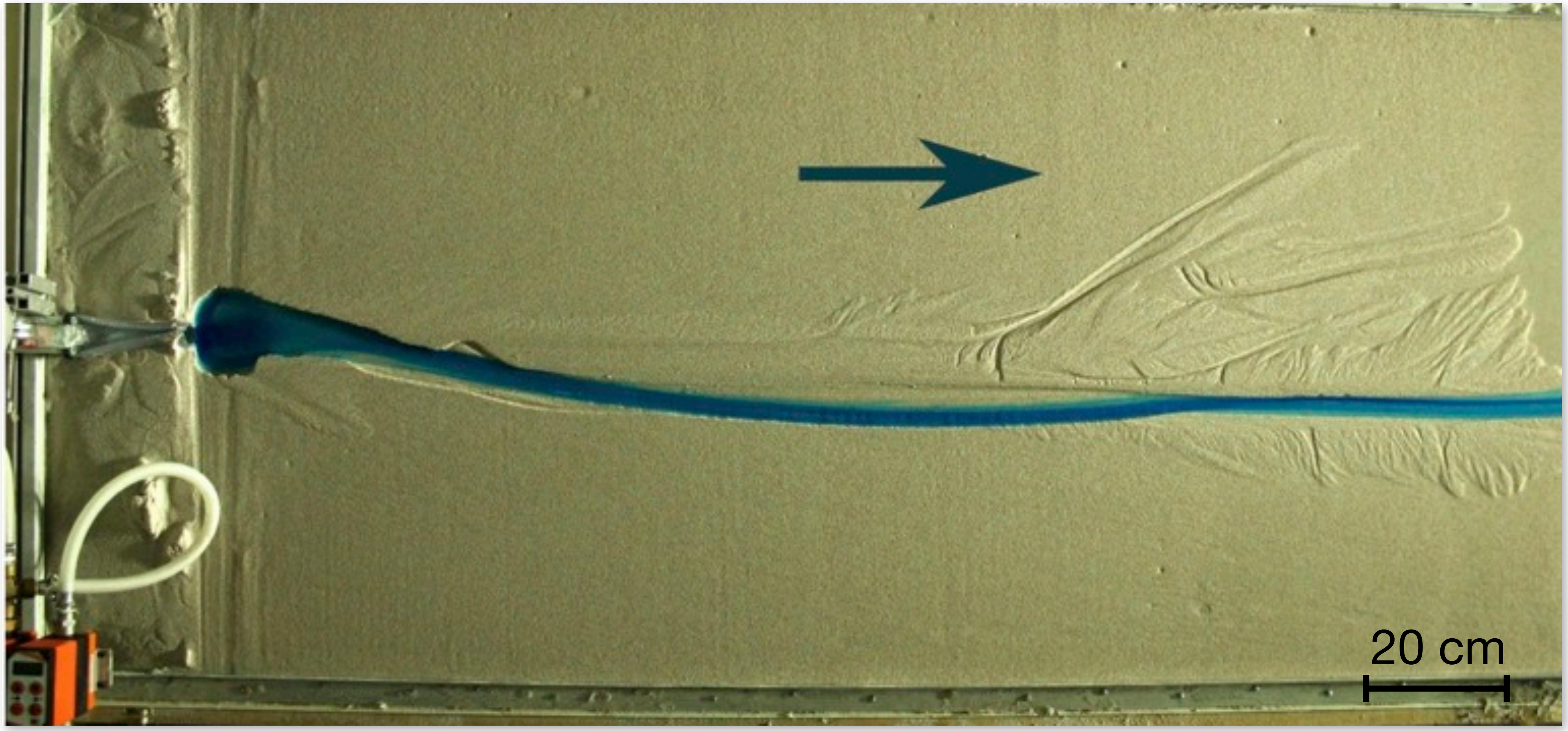
Equilibrium morphology of rivers : Insights from small scale experiments

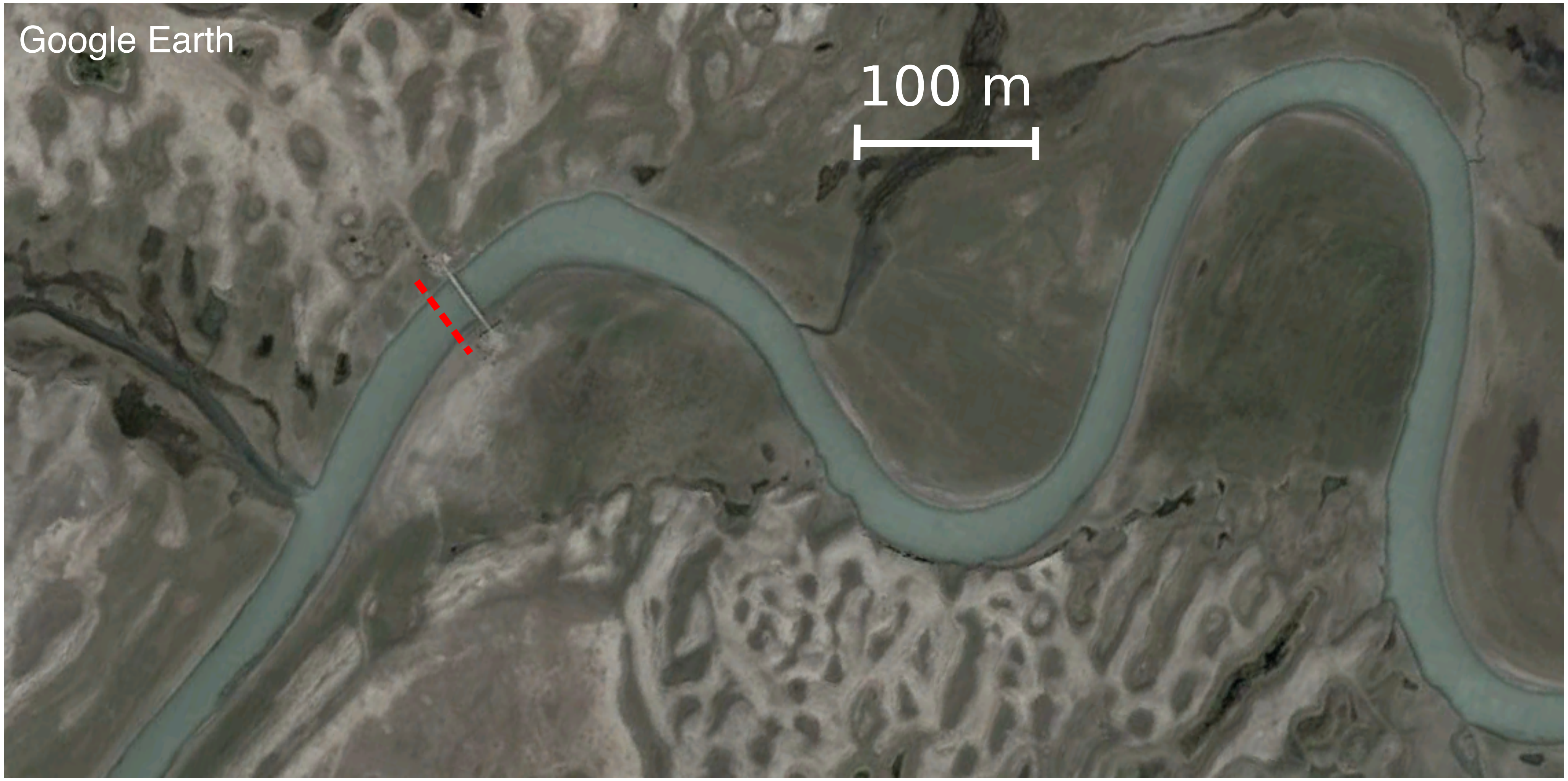
G. Seizilles[1], O. Devauchelle[1], E. Lajeunesse[1], F. Métivier[1] & M. Bak[2]

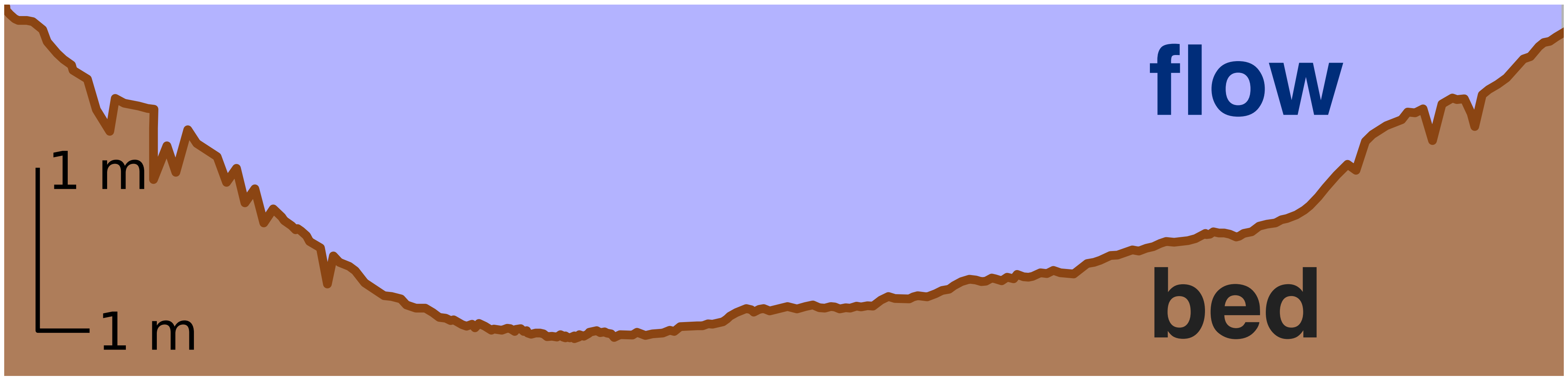
1. Institut de Physique du Globe de Paris
2. Earth and Environmental Science dpt, University of Pennsylvania

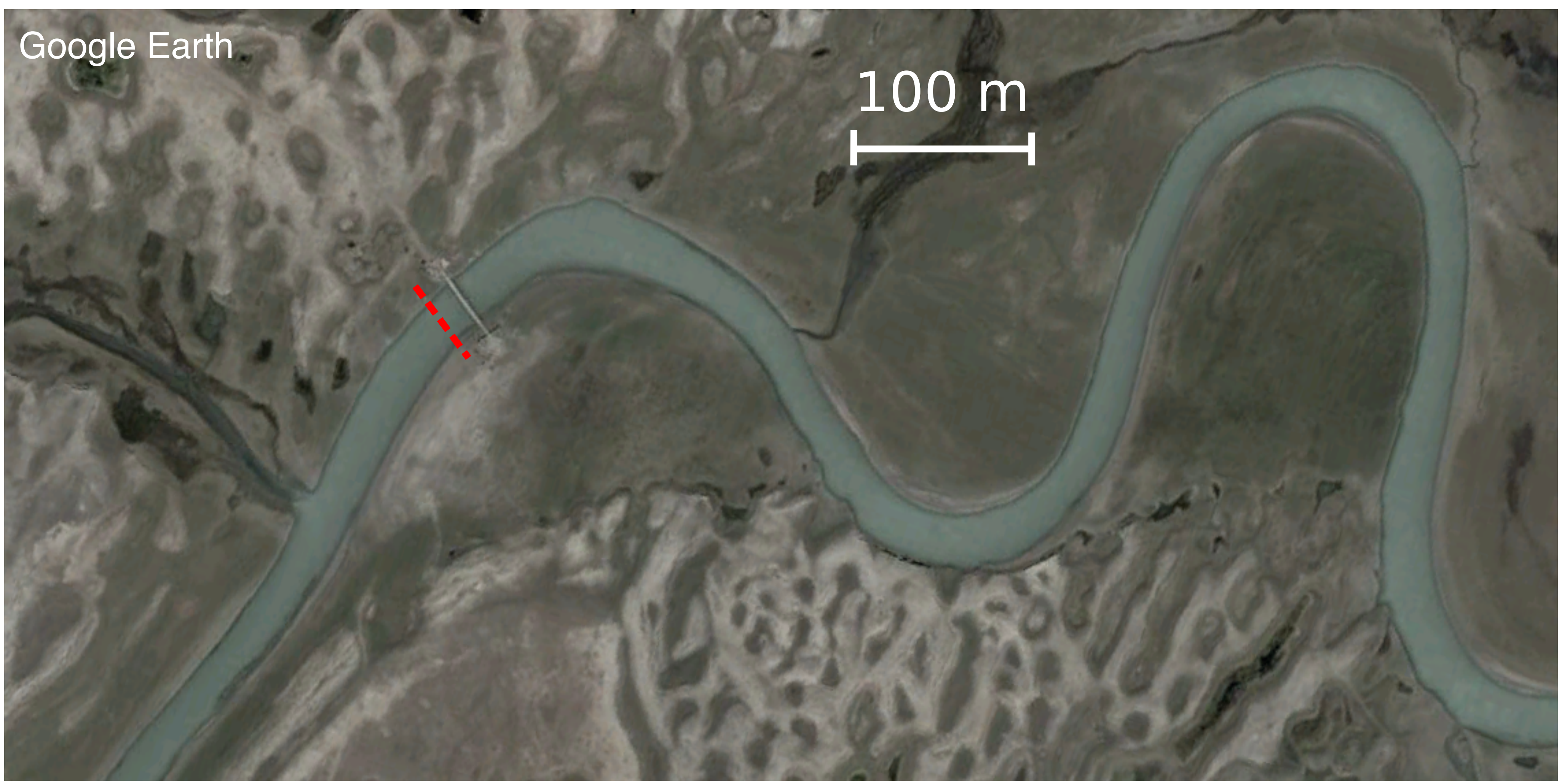


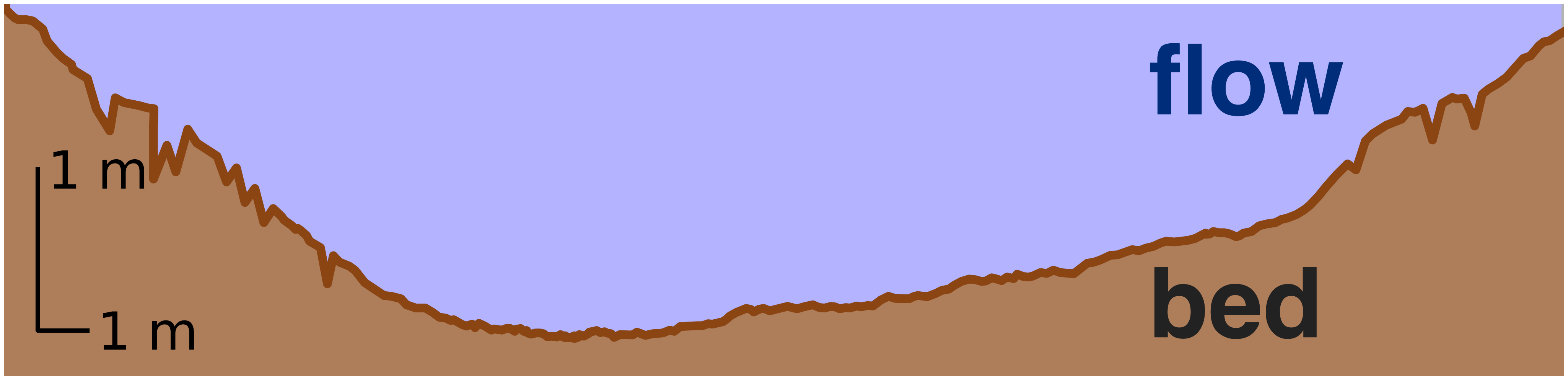

Kaidu river,
chinese Tian-Shan


Kaidu river,
chinese Tian-Shan


Google Earth
100 m

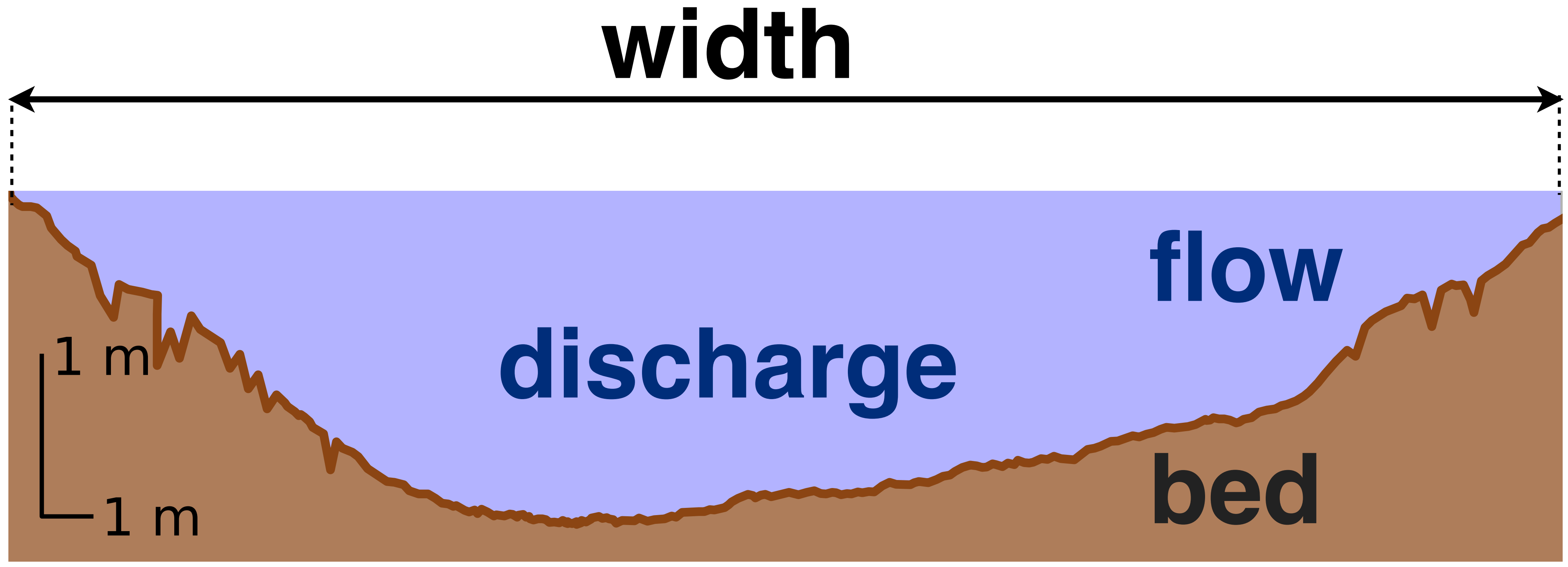

flow
bed
1 m
1 m


Google Earth
100 m



- **What selects the shape of the cross-section ?**
- **What parameters control its size ?**

Parker [1978], Vigilar & Diplas [1997], Cao & Knight [1998], Eaton & Millar [2004], ...



- **What selects the shape of the cross-section ?**
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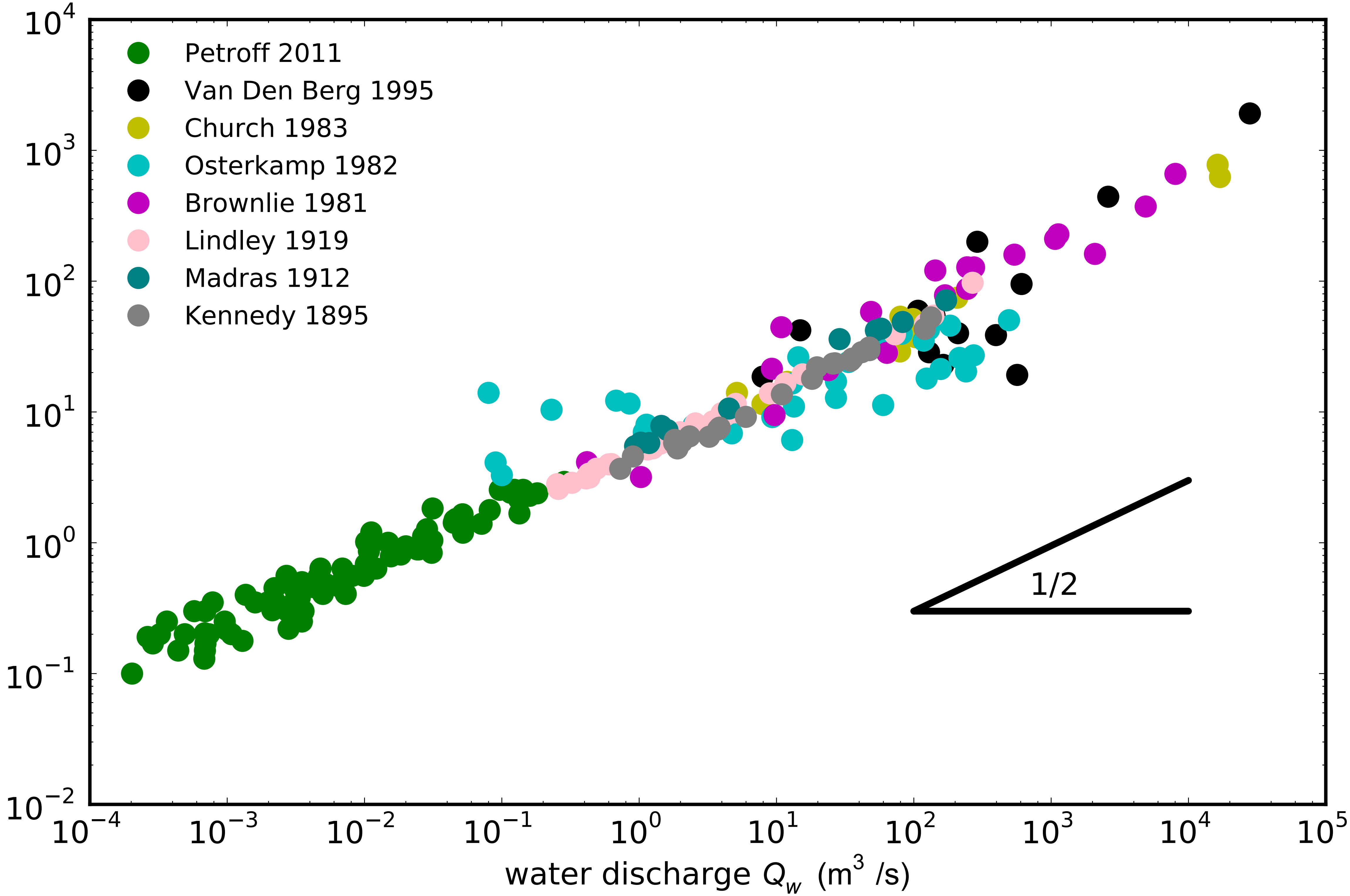
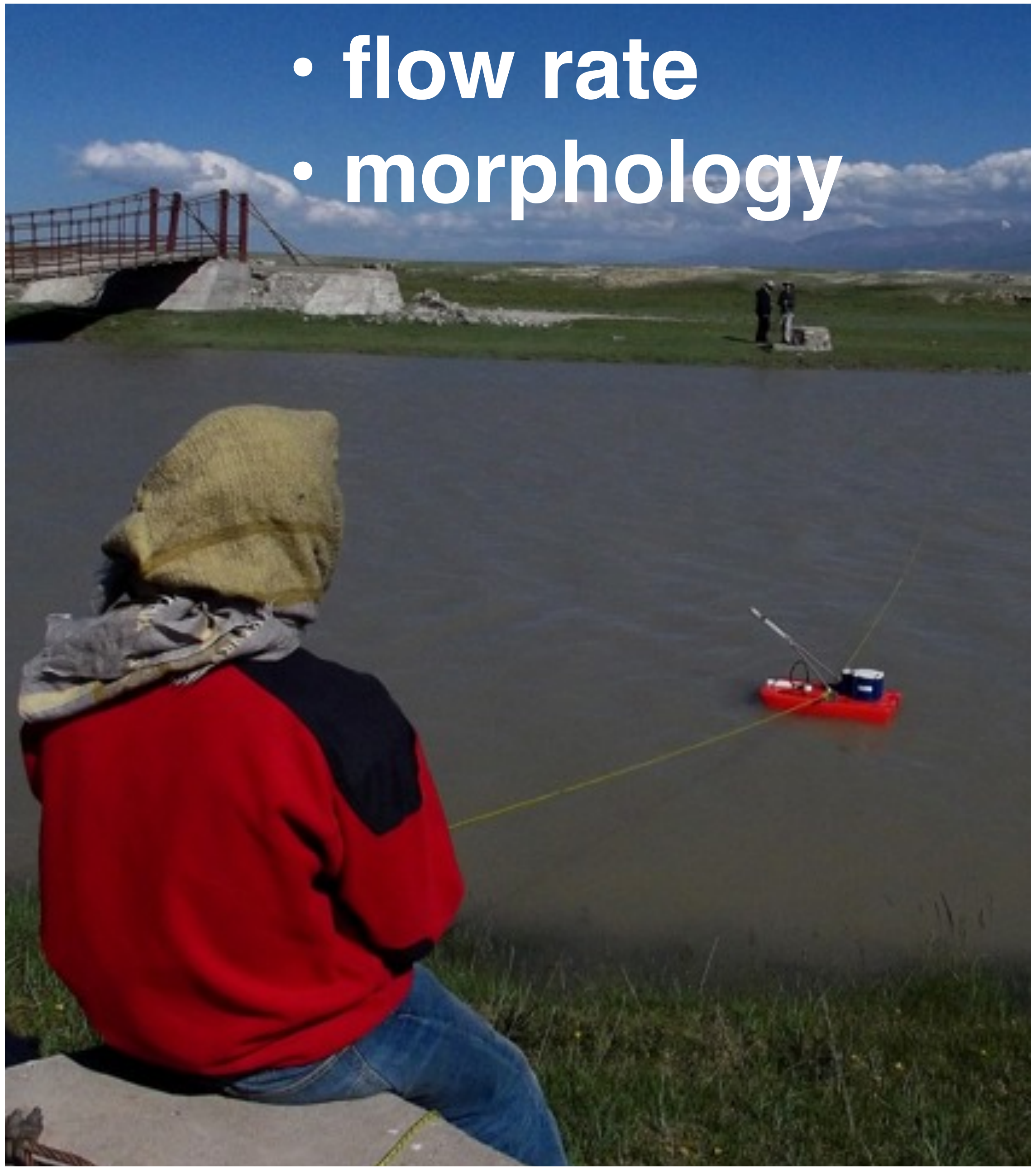
Parker [1978], Vigilar & Diplas [1997], Cao & Knight [1998], Eaton & Millar [2004], ...

Lacey's law - sandy, single thread rivers

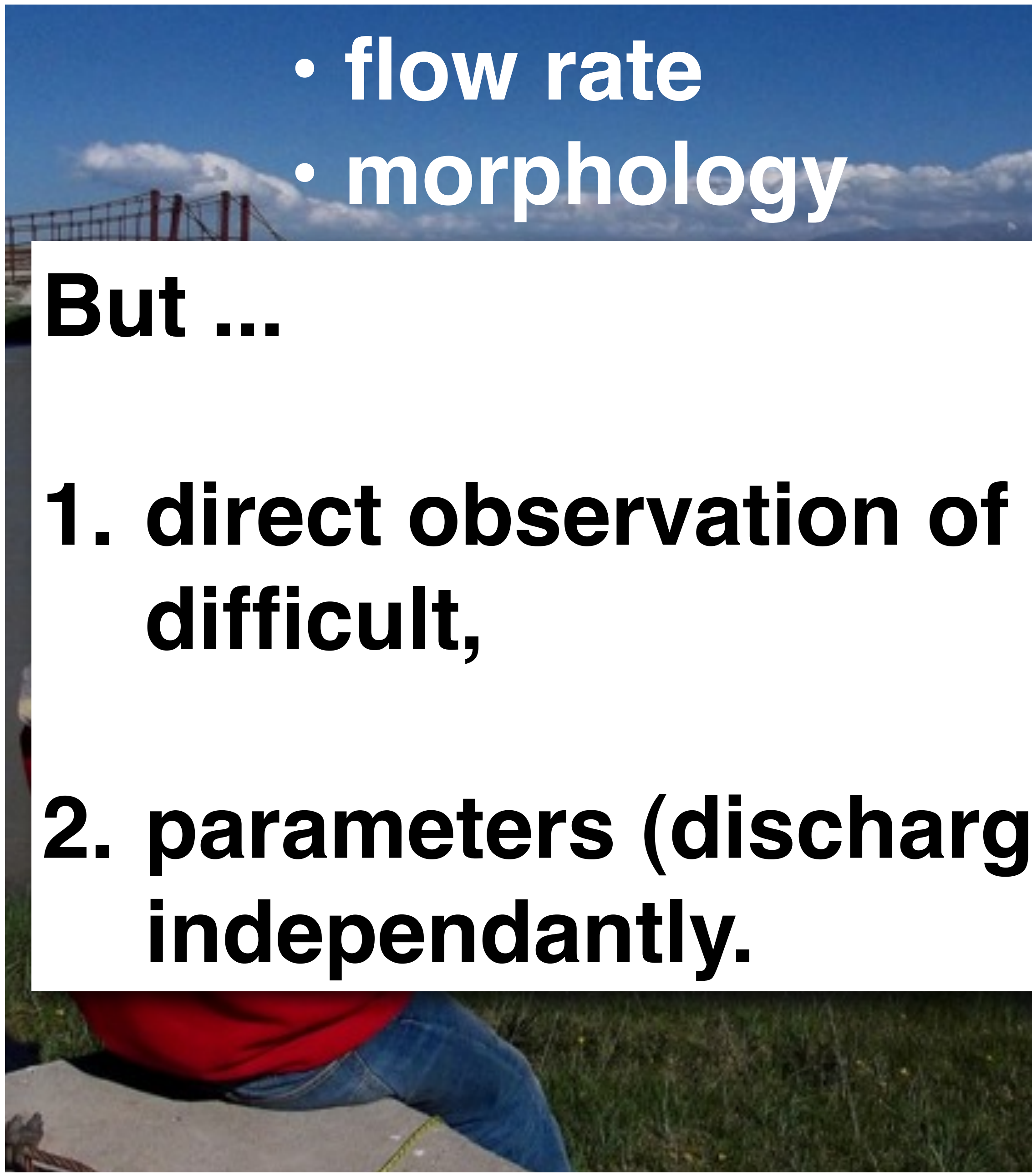
data from Brownlie [1981], Church & Rood [1983], Osterkamp & Hedman [1982], Vand den Berg [1995], Devauchelle et al [2010]

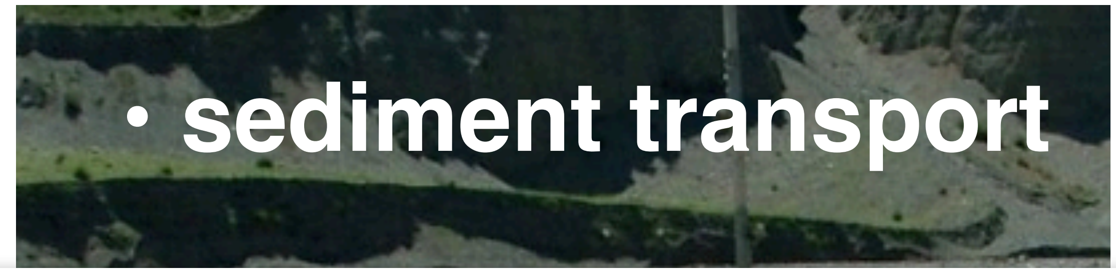


Field measurements



Field measurements





But ...

1. **direct observation of the physical processes is difficult,**

2. **parameters (discharge, grain size, ...) cannot be varied independantly.**

Field measurements

Field measurements

Field measurements

Laboratory rivers

water + sugar or glycerol

$r = 1.2\ 10^{3}$ kg m^{-3}

$n = 15\ 10^{-6}$ m s^{-2}

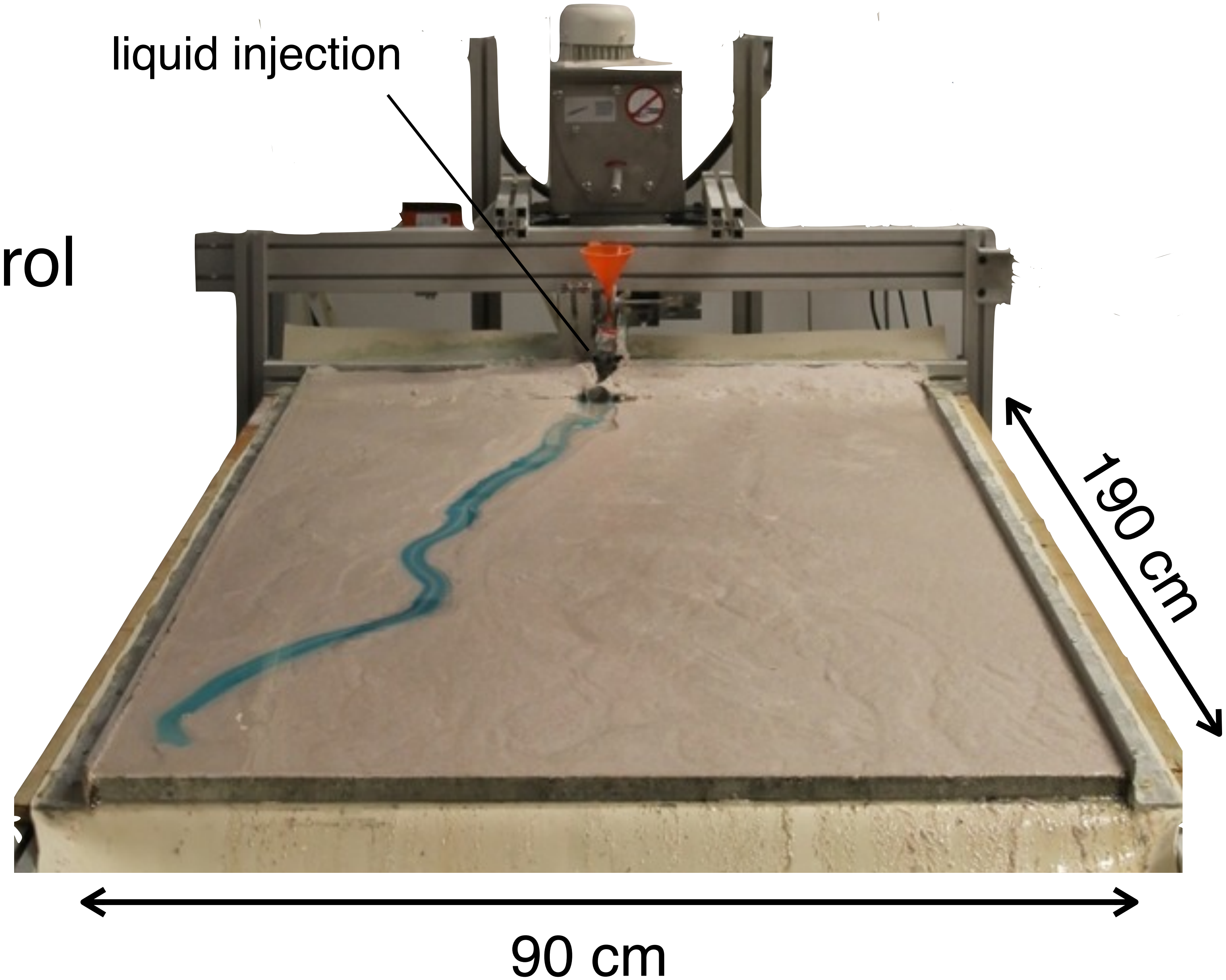
Re $\simeq$10

plastic sand

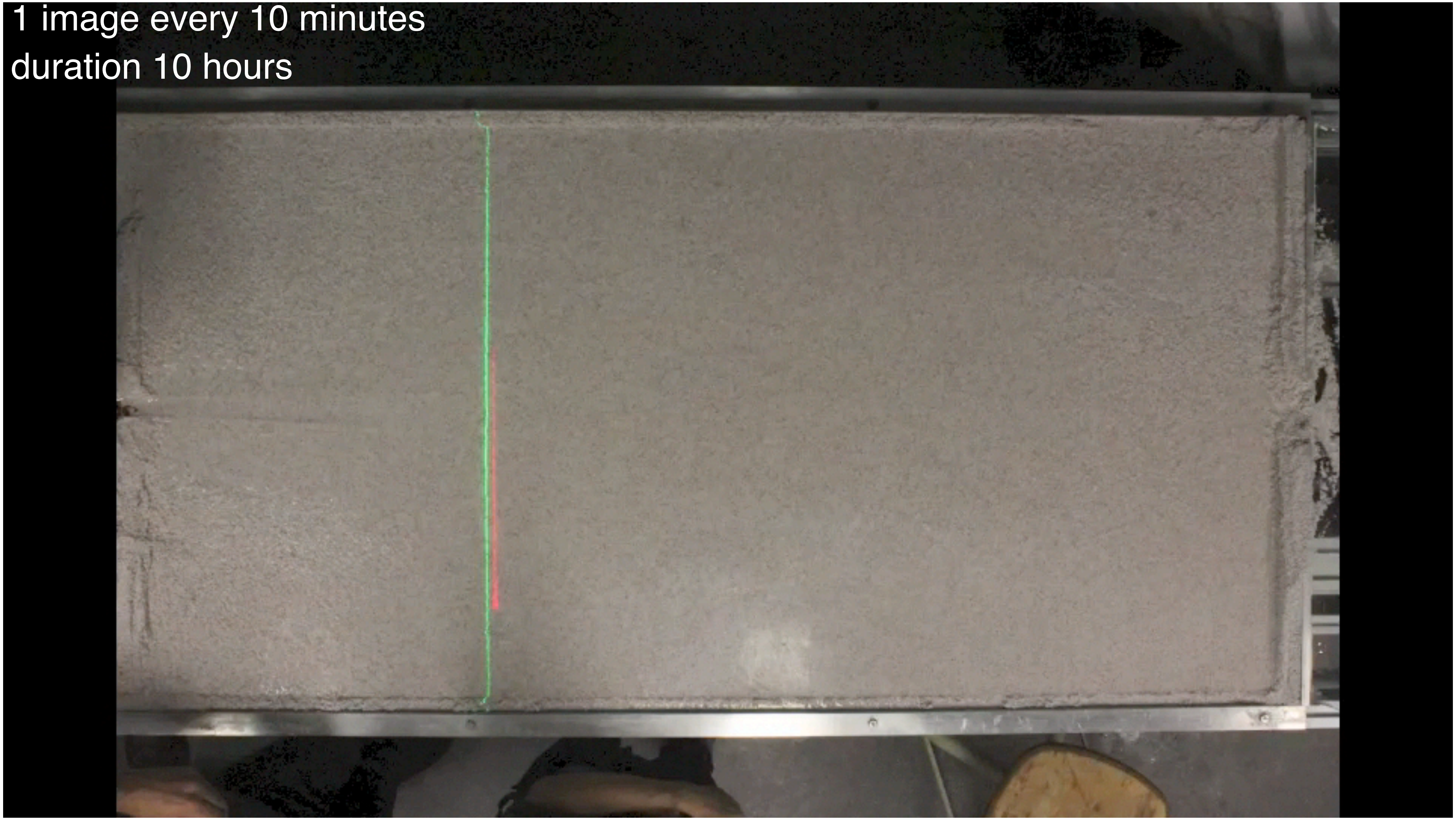
ds = 250 mm

$r_s = 1.5\ 10^{3}$ kg m^{-3}

$q_a = 35°$

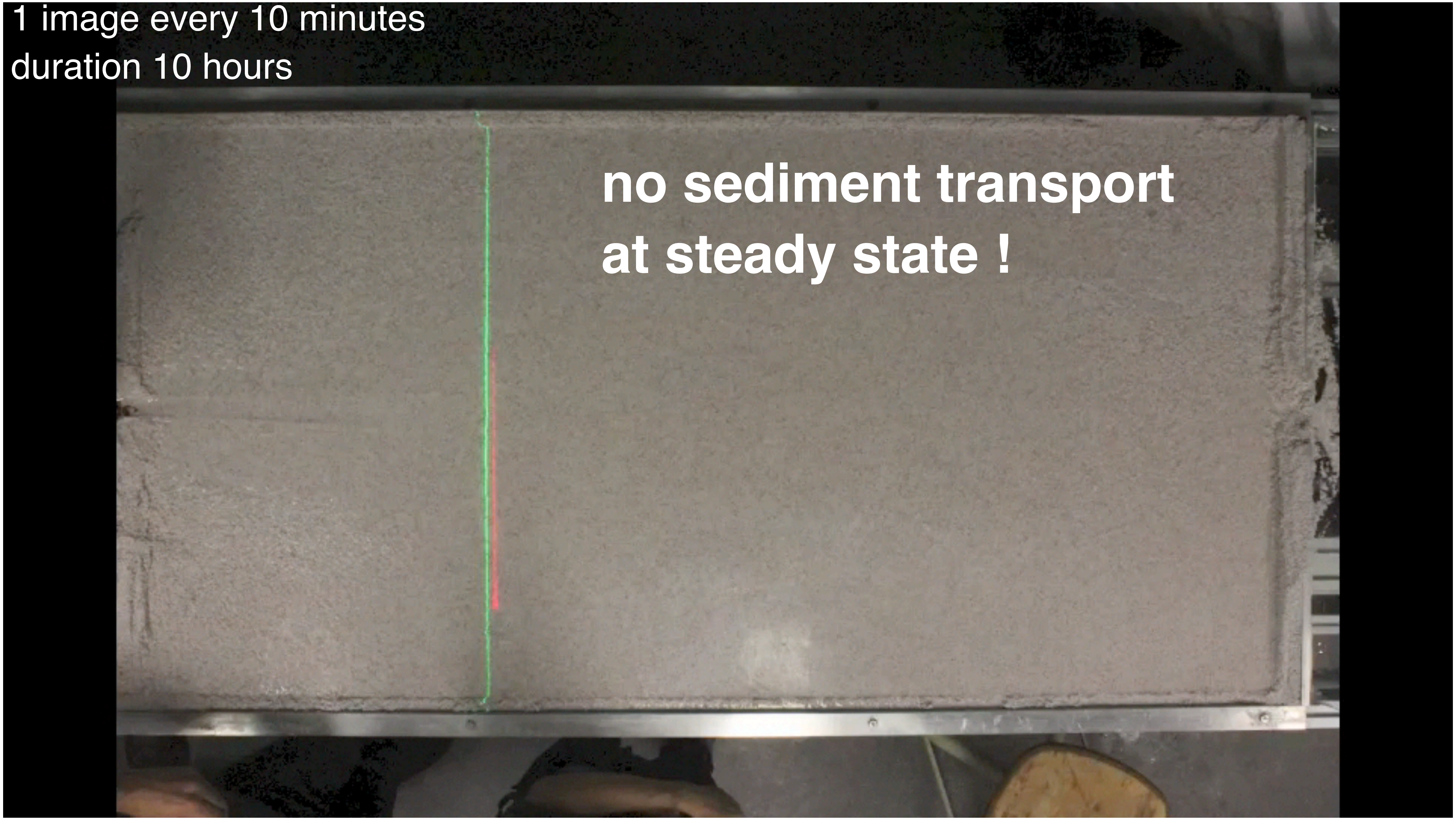


Experimental procedure



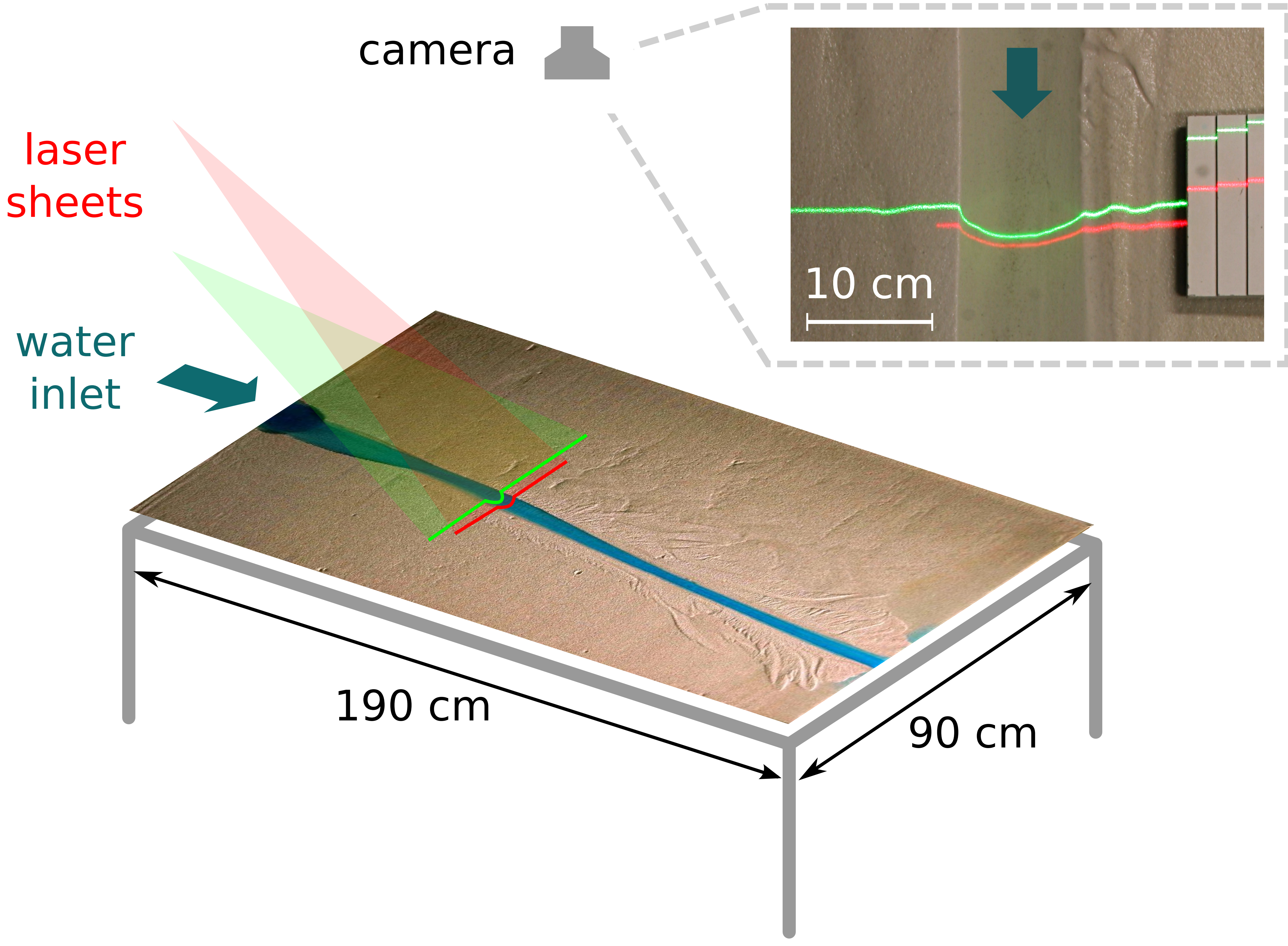
constant discharge
no sediment input

Experimental procedure

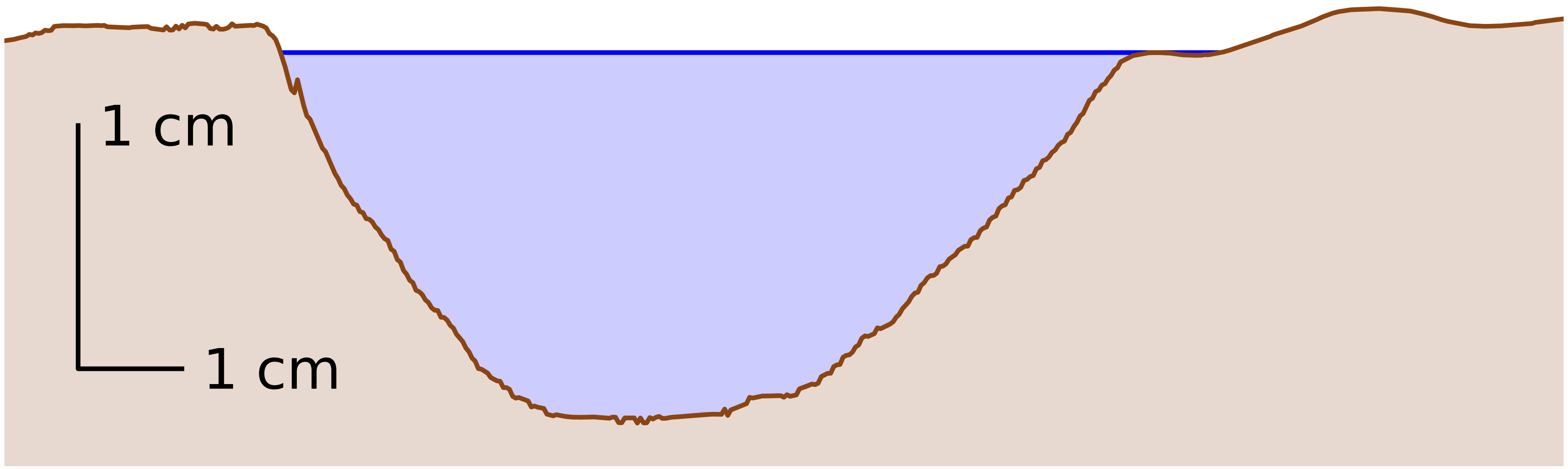


constant discharge
no sediment input

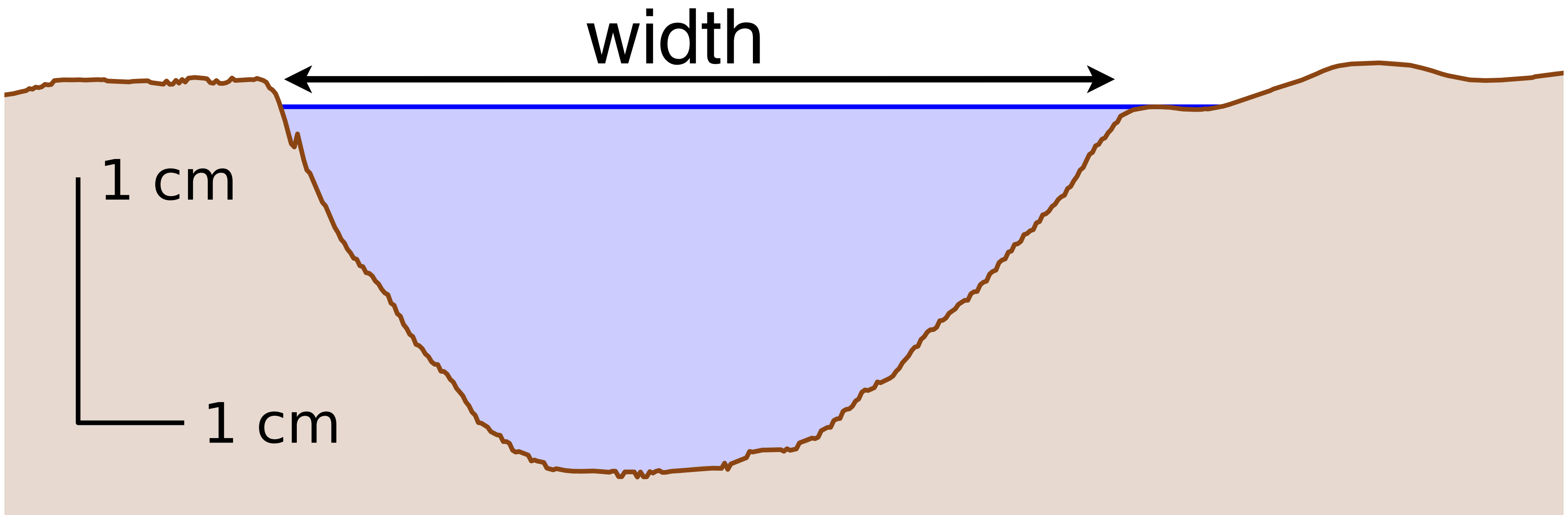
Measurements of flow depth and bed topography

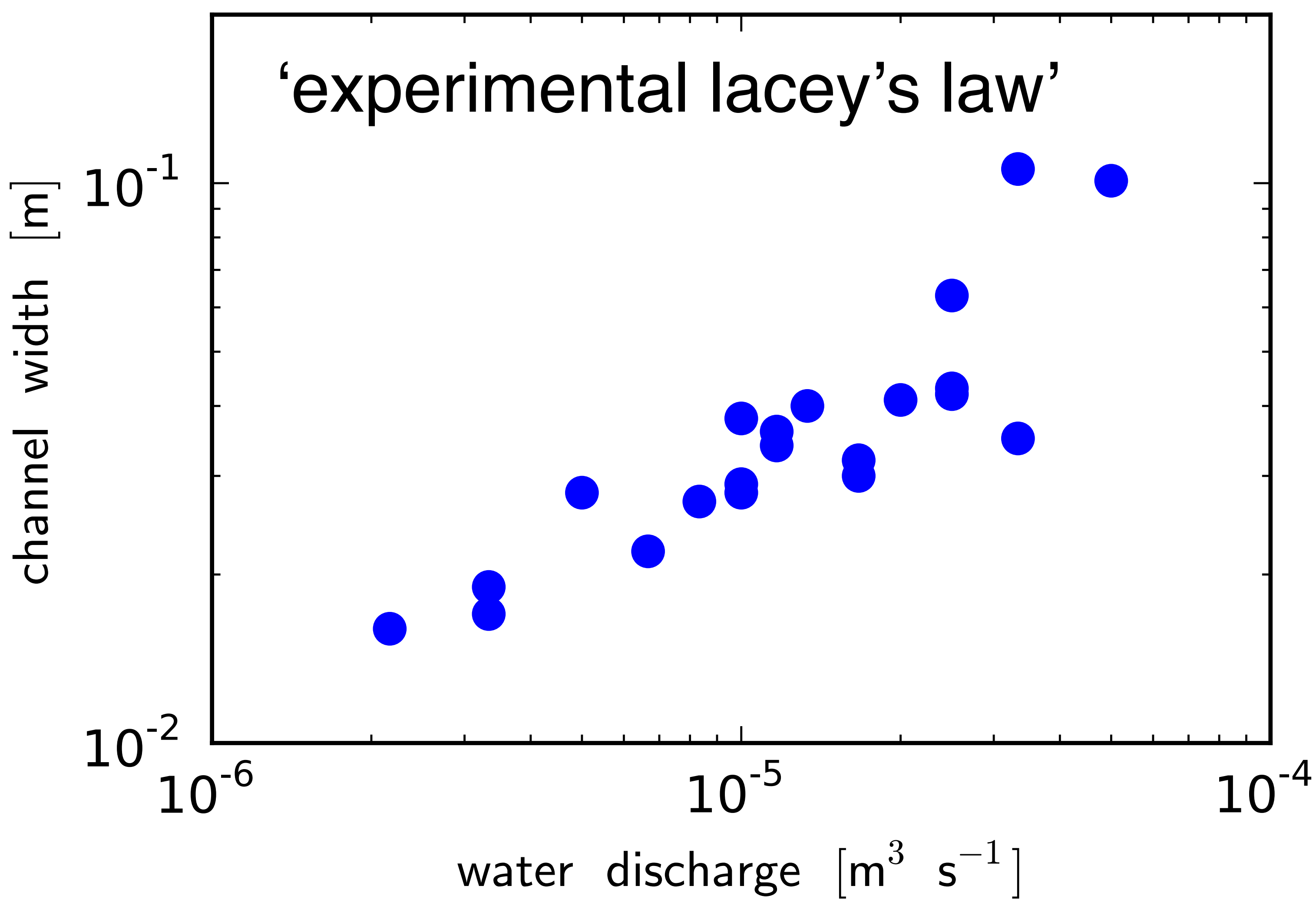


Laboratory rivers

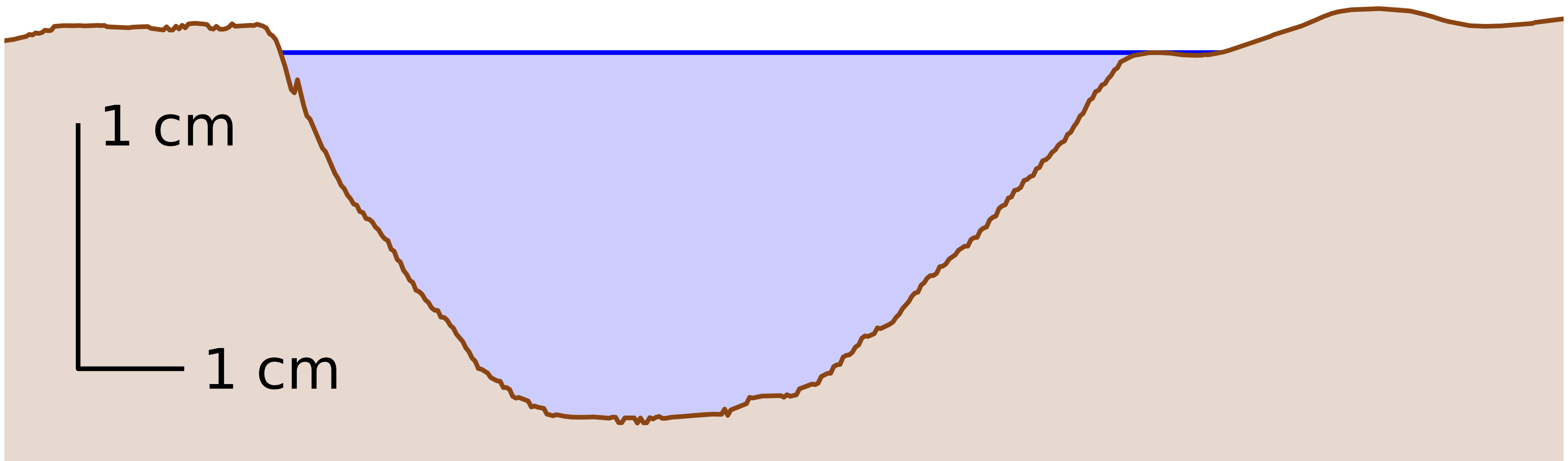


Laboratory rivers





Zero-transport model

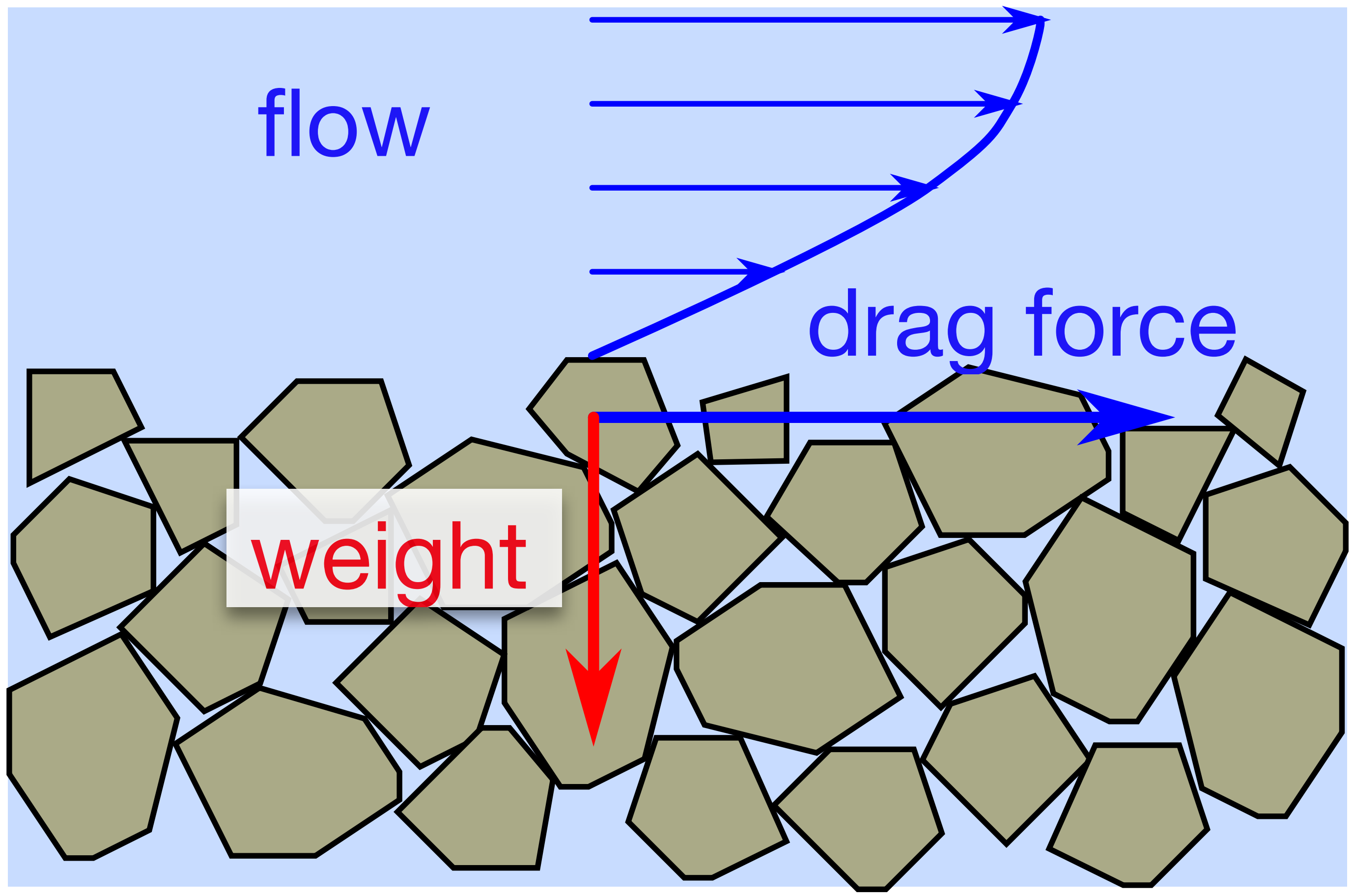


The equilibrium morphology is reached when sediment transport ceases.

At equilibrium, grains must be at the threshold of entrainment everywhere on the bed!

[Glover & Florey, 1951; Henderson, 1961]

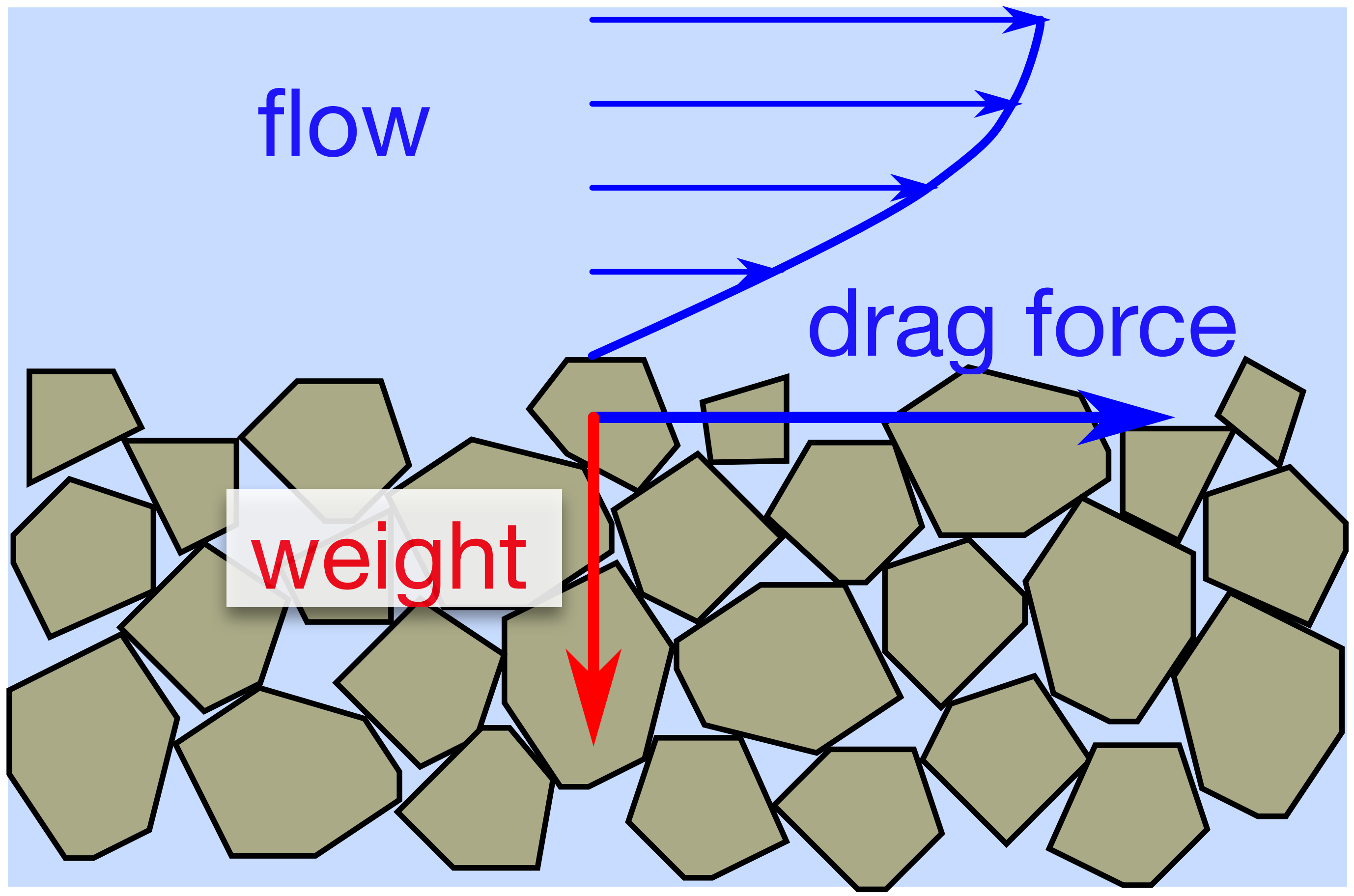
Threshold of motion for a flat sediment bed
Coulomb's law of friction ...



$$\frac{\text{tangeantial force}}{\text{normal force}} = \mu$$

friction coefficient

Threshold of motion for a flat sediment bed
Coulomb's law of friction ...



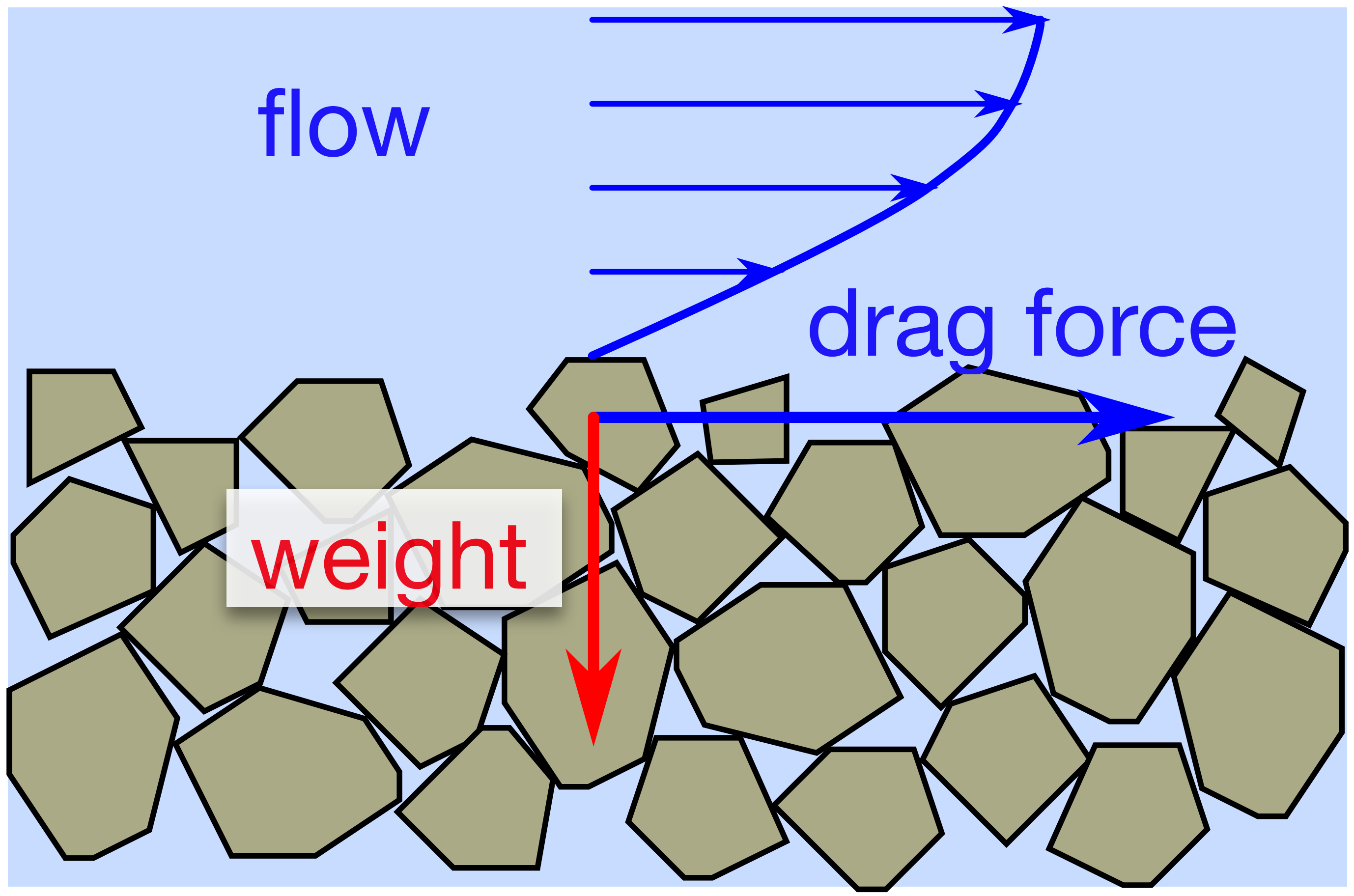
drag force

friction coefficient

$$\frac{\text{tangeantial force}}{\text{normal force}} = \mu$$

weight

Threshold of motion for a flat sediment bed ... and Shields number



$$\frac{\tau}{(\rho_s - \rho_f) g d_s} = \theta_t$$

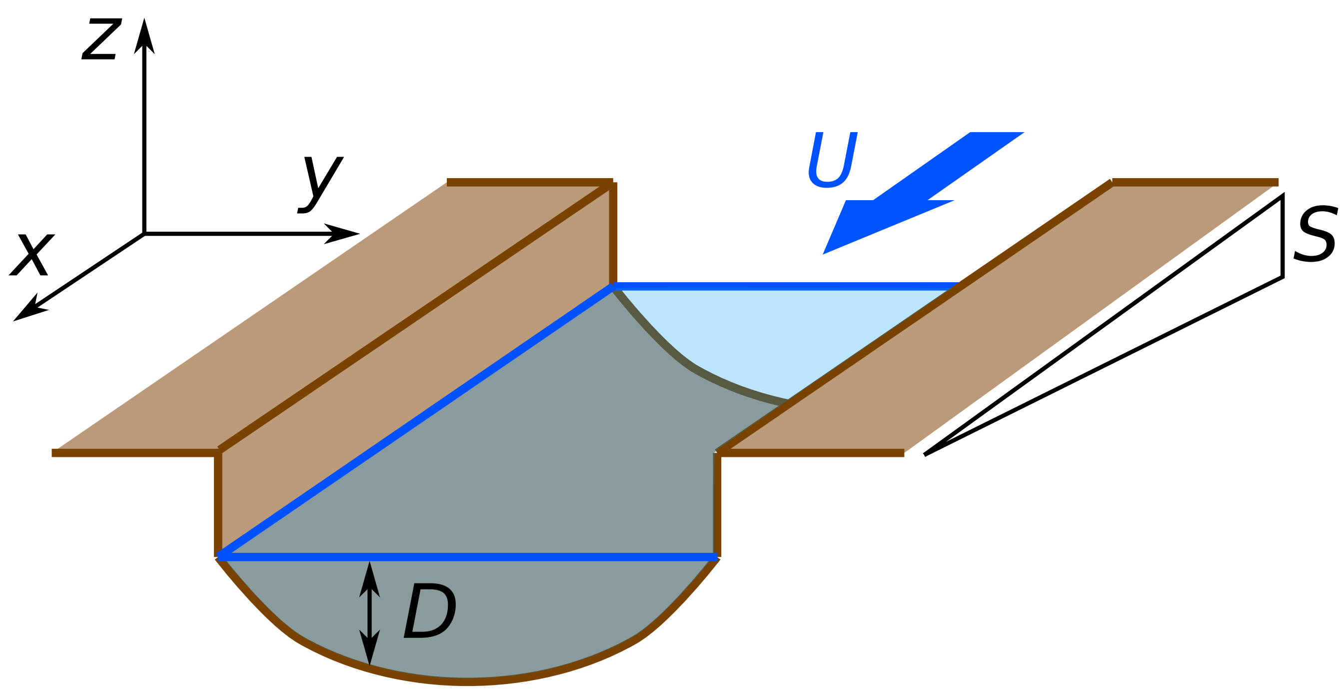
[Shields, 1936]

Shields number

threshold Shields number

t = shear stress
d_s = grain size
r_s, r_f = sediment & fluid densities

Threshold of motion for a channel

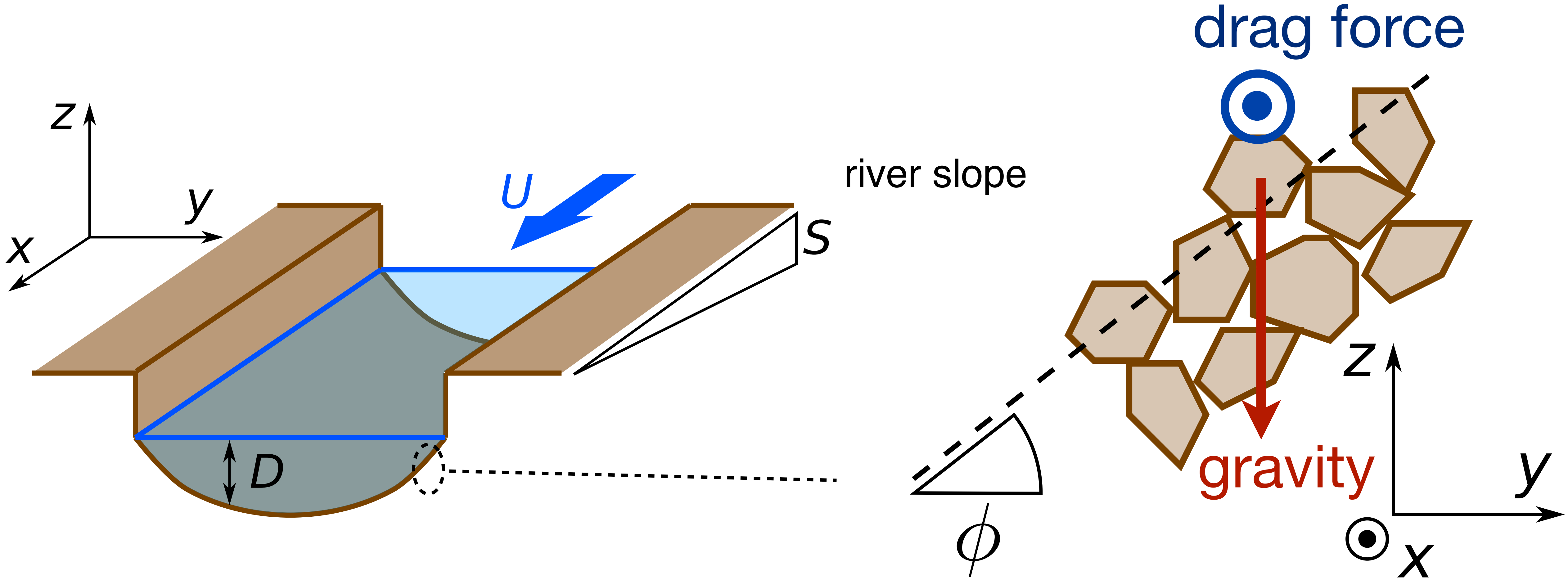


Coulomb's law of friction :

$$\frac{\text{tangeantial force}}{\text{normal force}} = \mu$$

friction coefficient

Threshold of motion for a channel



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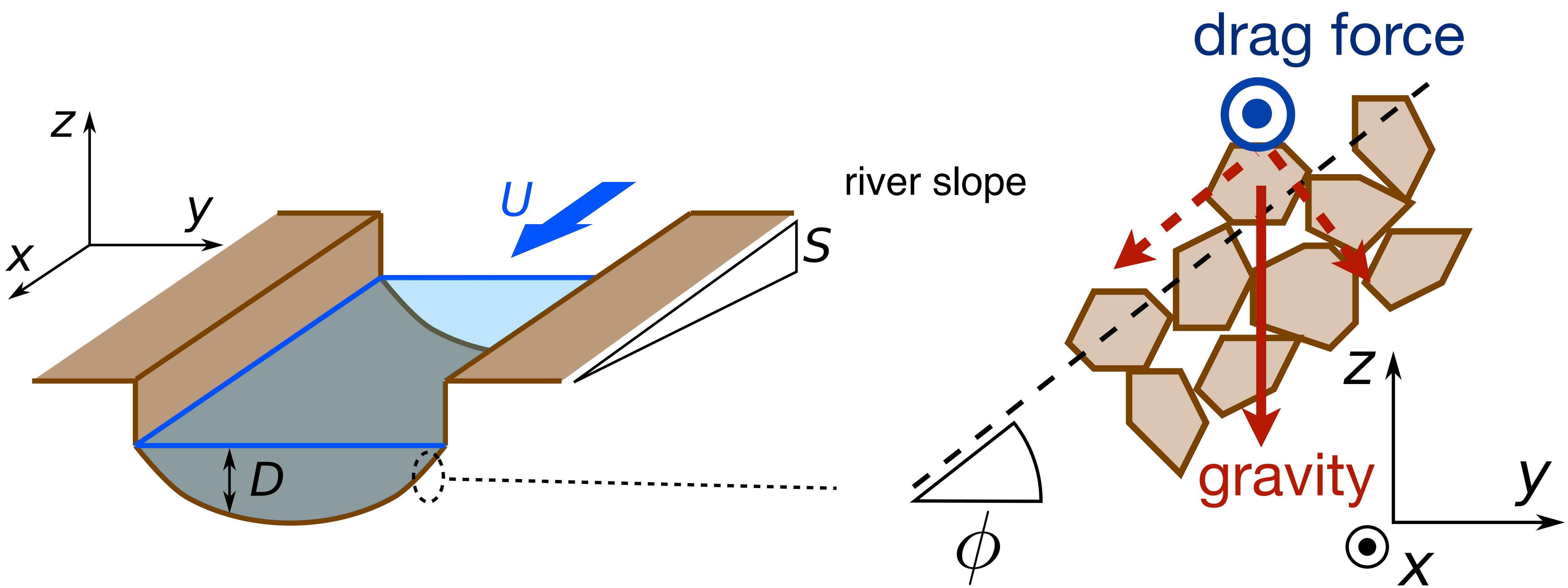
Threshold of motion for a channel



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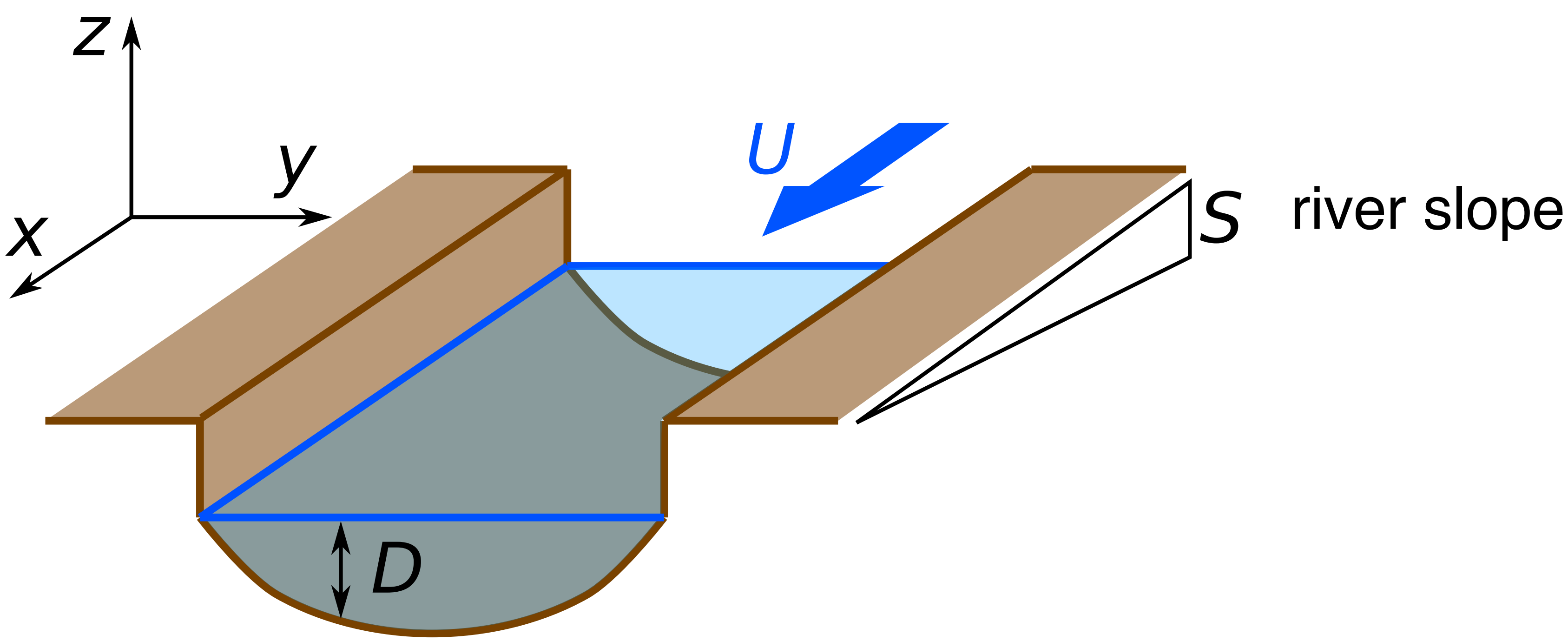


streamwise slope

friction coefficient ≃ 0.7

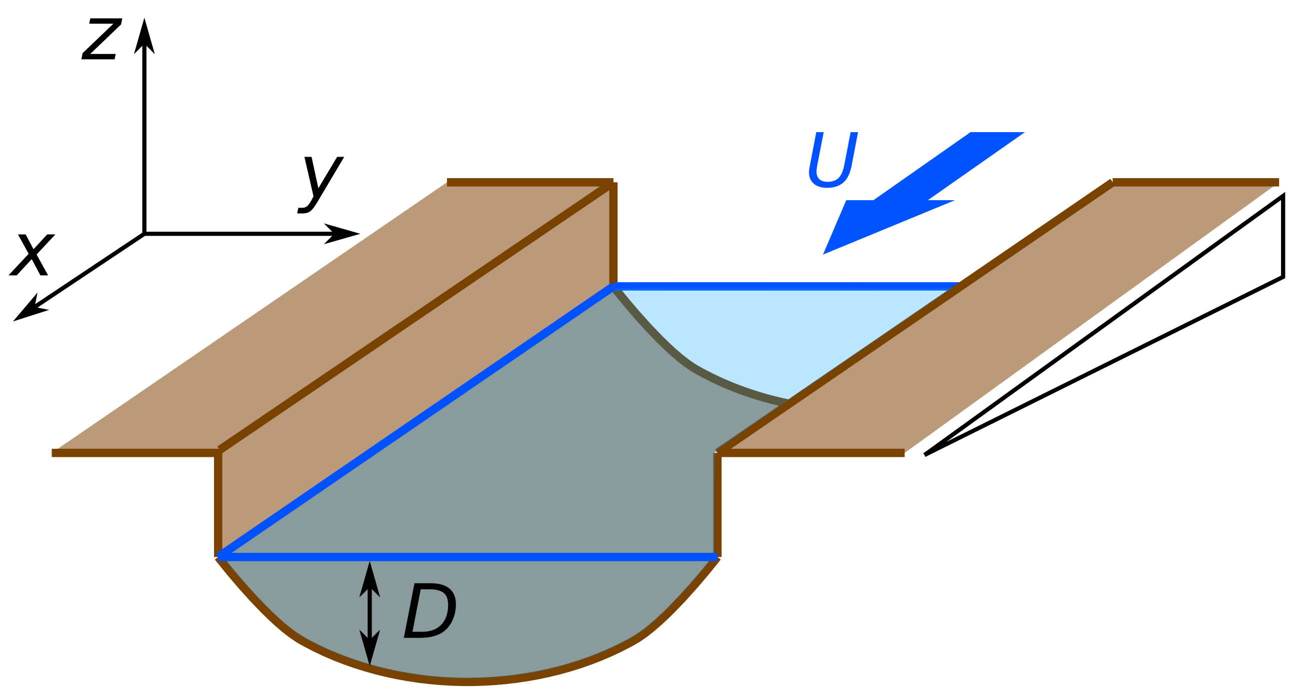
$$\left(\frac{\partial D}{\partial y}\right)^2 + \left(\frac{S\,D}{L}\right)^2 = \mu^2$$

characteristic length ≈ grain size

weight **drag force**

[Seizilles et al., 2013]

Threshold of motion for a channel



S river slope

$$L = \frac{\theta_t \, (\rho_s - \rho_f) \, d_s}{\mu \, \rho_f}$$

- θ_t: threshold Shields stress
- $(\rho_s - \rho_f)$: sediment & fluid density
- d_s: grain size

$$\left(\frac{\partial D}{\partial y}\right)^2 + \left(\frac{S\,D}{L}\right)^2 = \mu^2$$

- S: streamwise slope
- μ: friction coefficient ≃ 0.7
- L: characteristic length ≈ grain size

weight **drag force**

[Seizilles et al., 2013]

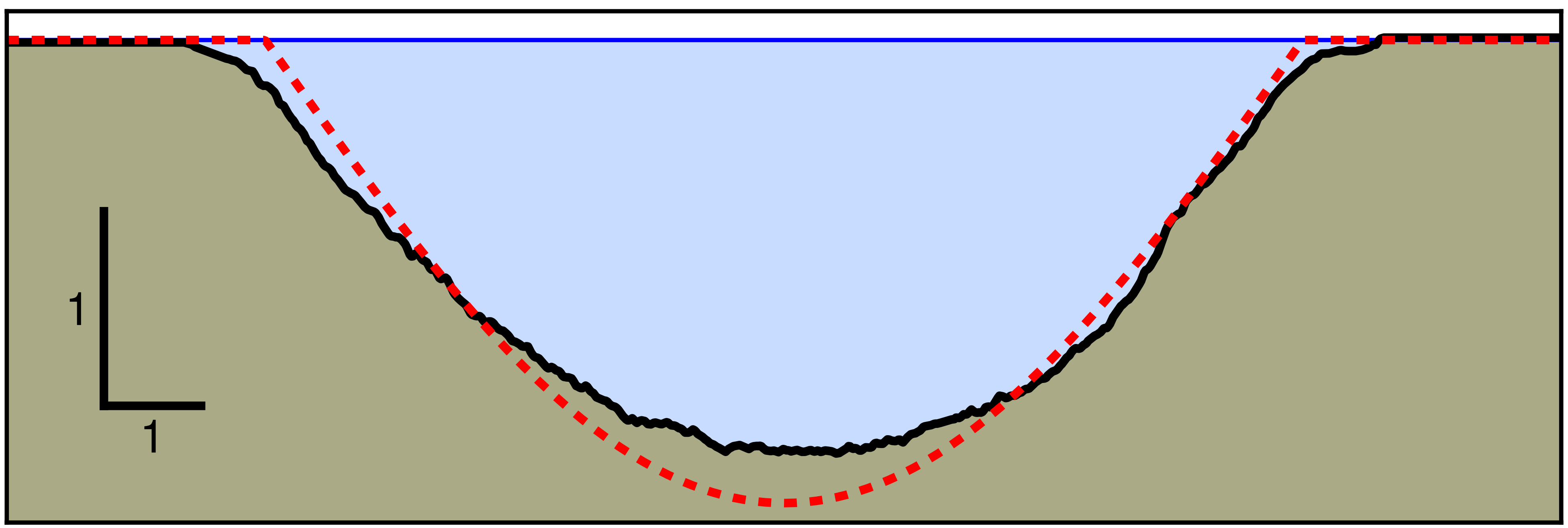
Zero-transport model

friction coefficient

longitudinal slope

$$D = \frac{\mu\, L}{S}\ \cos\left(\frac{S\, y}{L}\right)$$

characteristic length ≈ grain size



no adjustable parameter !

Zero-transport model

friction coefficient

streamwise slope

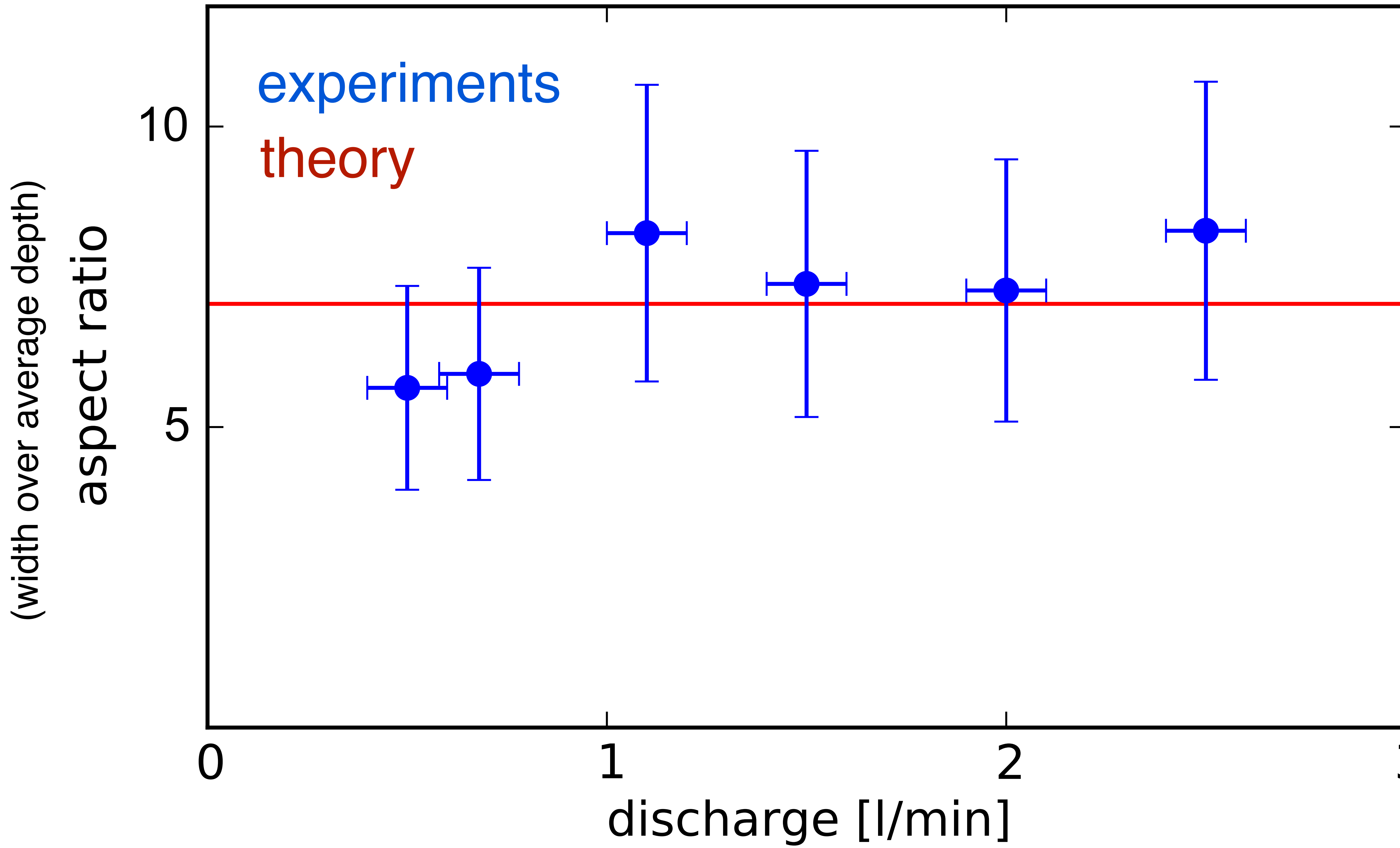
$$D = \frac{\mu\, L}{S}\ \cos\left(\frac{S\, y}{L}\right)$$

characteristic length ≈ grain size

Channel characteristic depth and width scale as L / S !

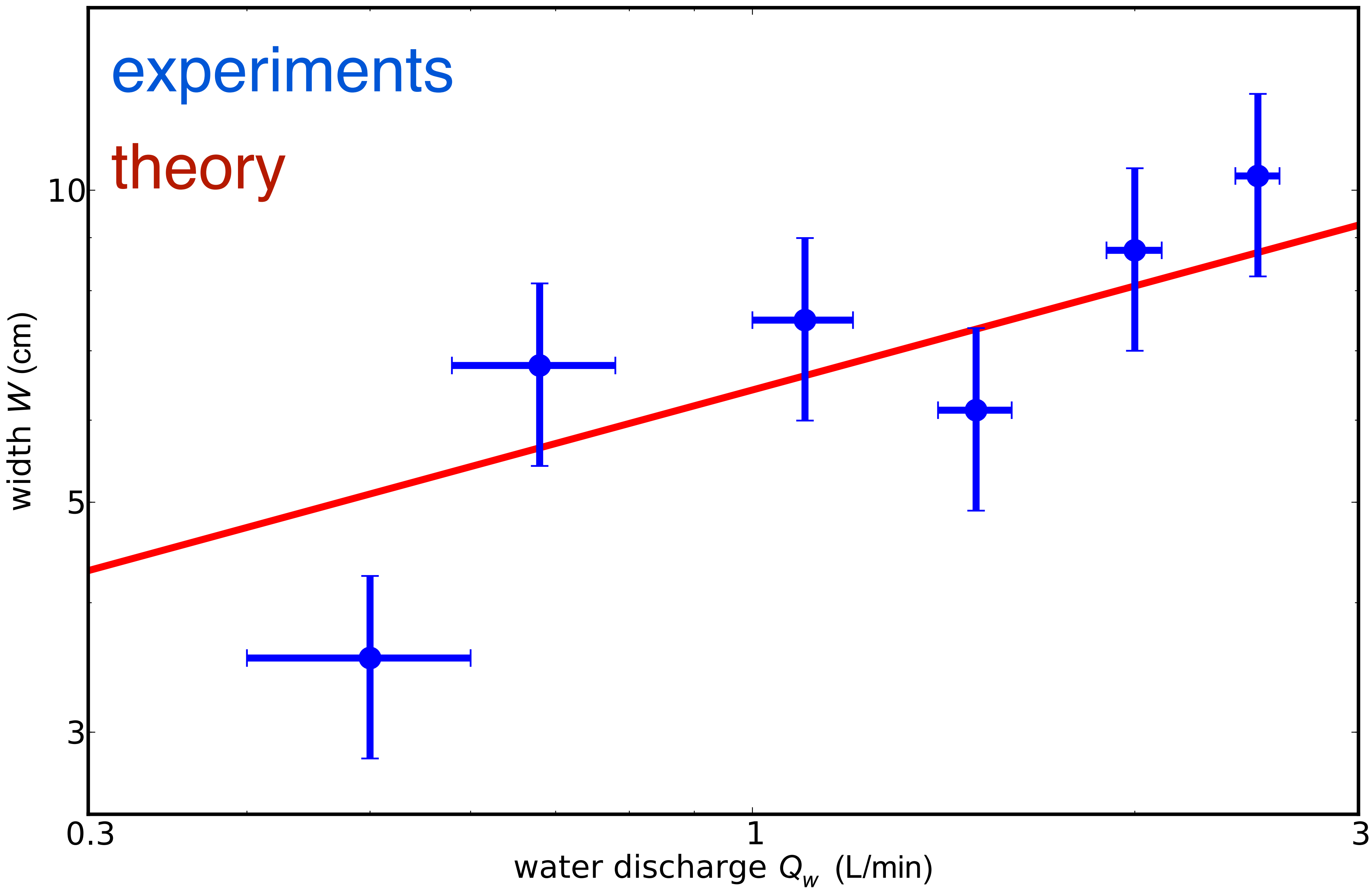
The channel aspect ratio is constant.

Comparison with experimental results



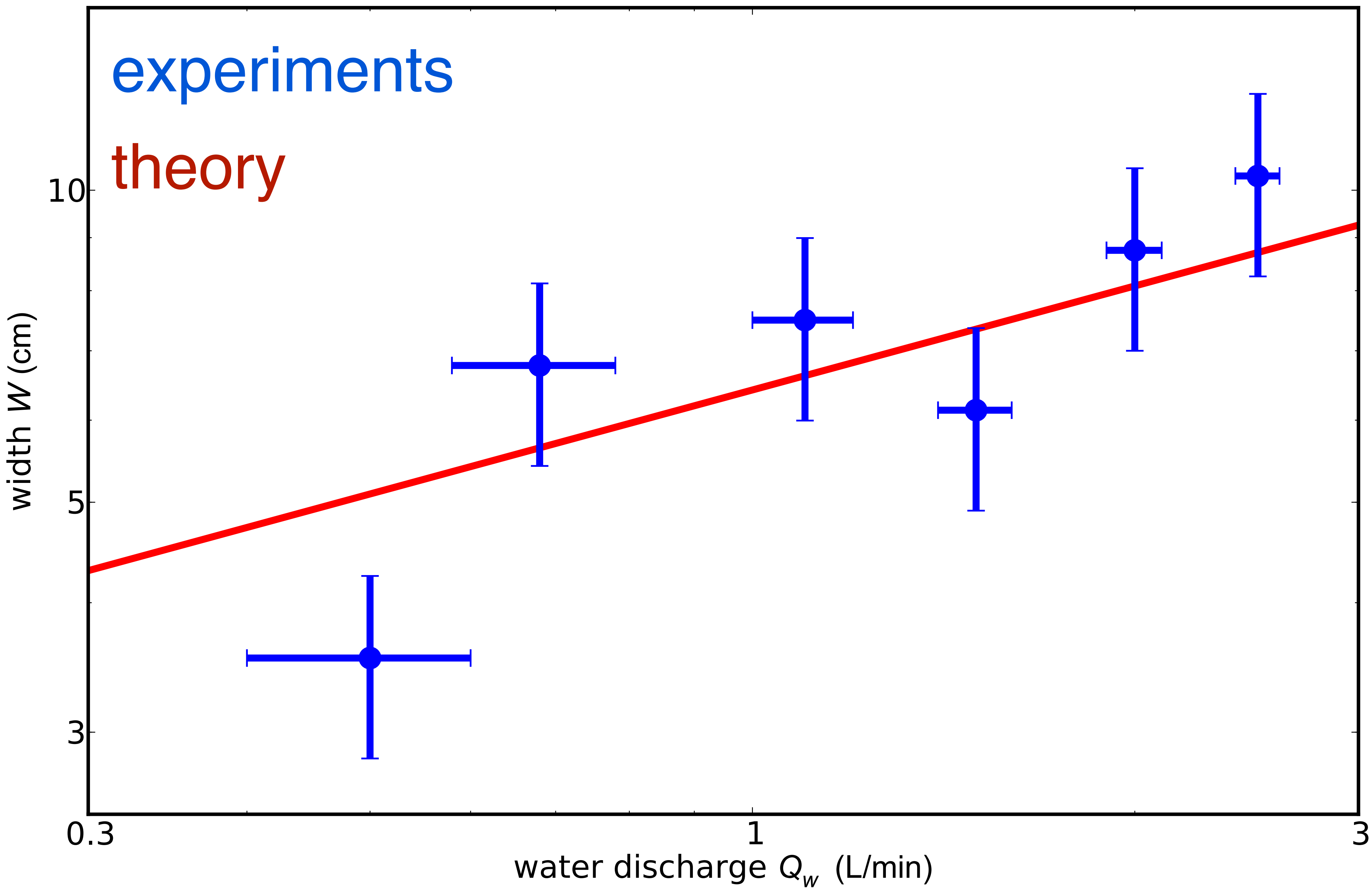
$$\frac{W}{< D >} = \frac{\pi^2}{2\mu} \approx 7$$

Comparison with experimental results



$$\mathbf{width} = \frac{\pi}{\mu^{2/3}} \left(\frac{9\,\nu\,\rho_f}{4\,g\,\theta_t\,(\rho_s - \rho_f)\,d_s} \right)^{1/3} \times \ \mathbf{discharge}^{1/3}$$

Comparison with experimental results



laminar flow

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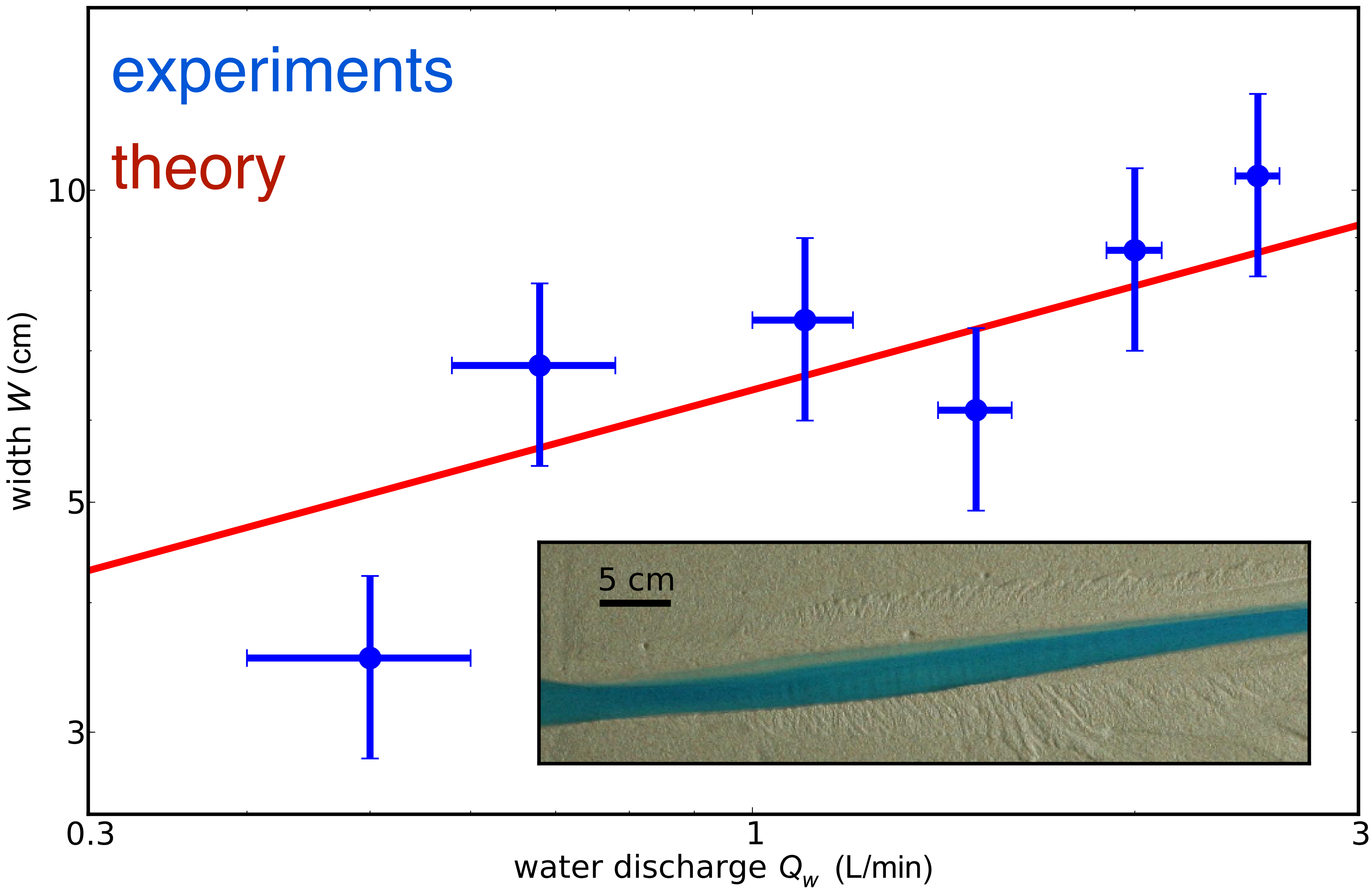

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Comparison with field sandy rivers

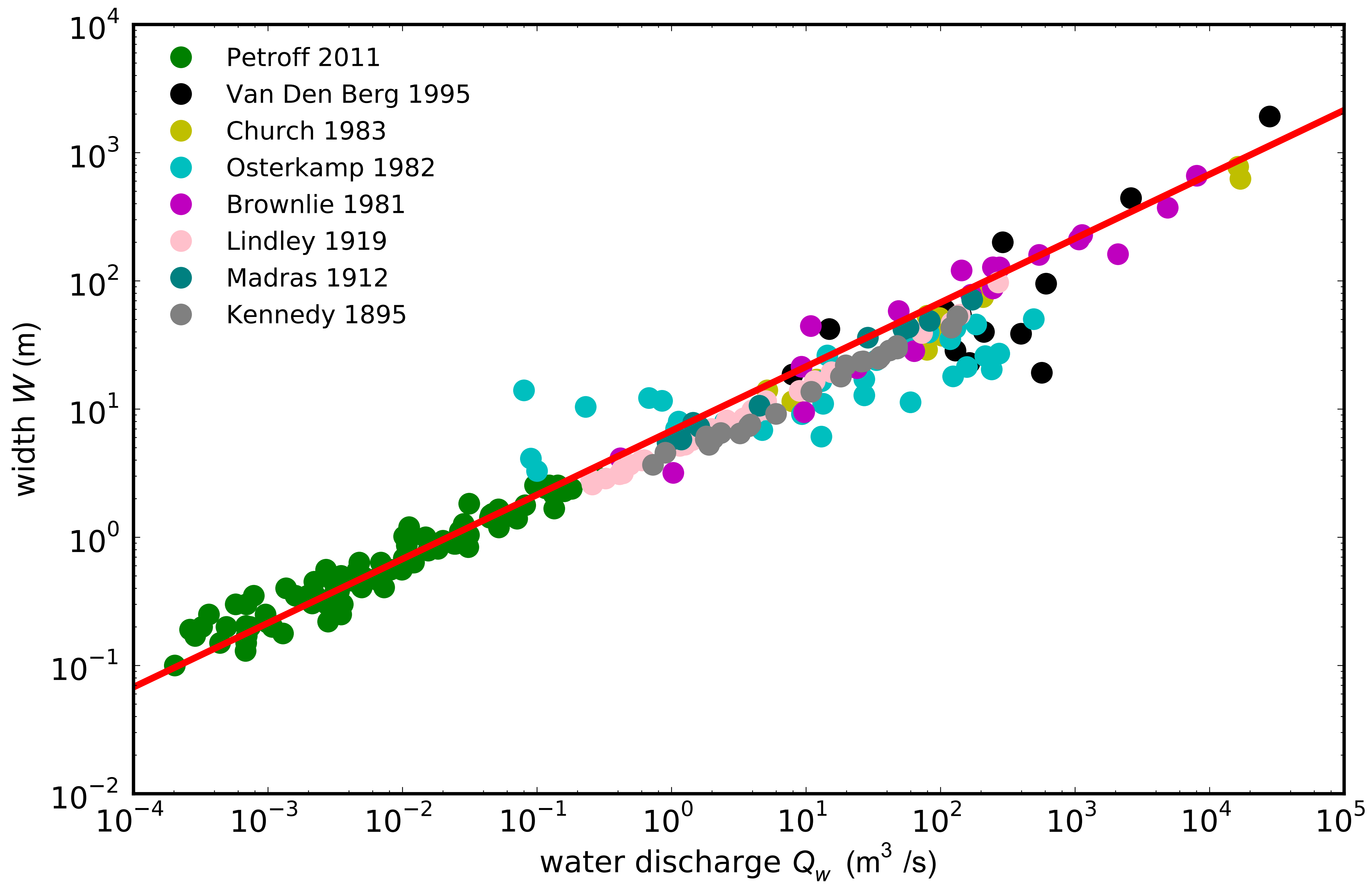
turbulent friction coef.

turbulent flow

$$\text{width} = \frac{\pi}{1.75}\left(\frac{\mu^2 C_f}{gL}\right)^{1/4} \times \text{discharge}^{1/2}$$

[Glover & Florey, 1951]

Comparison with field sandy rivers



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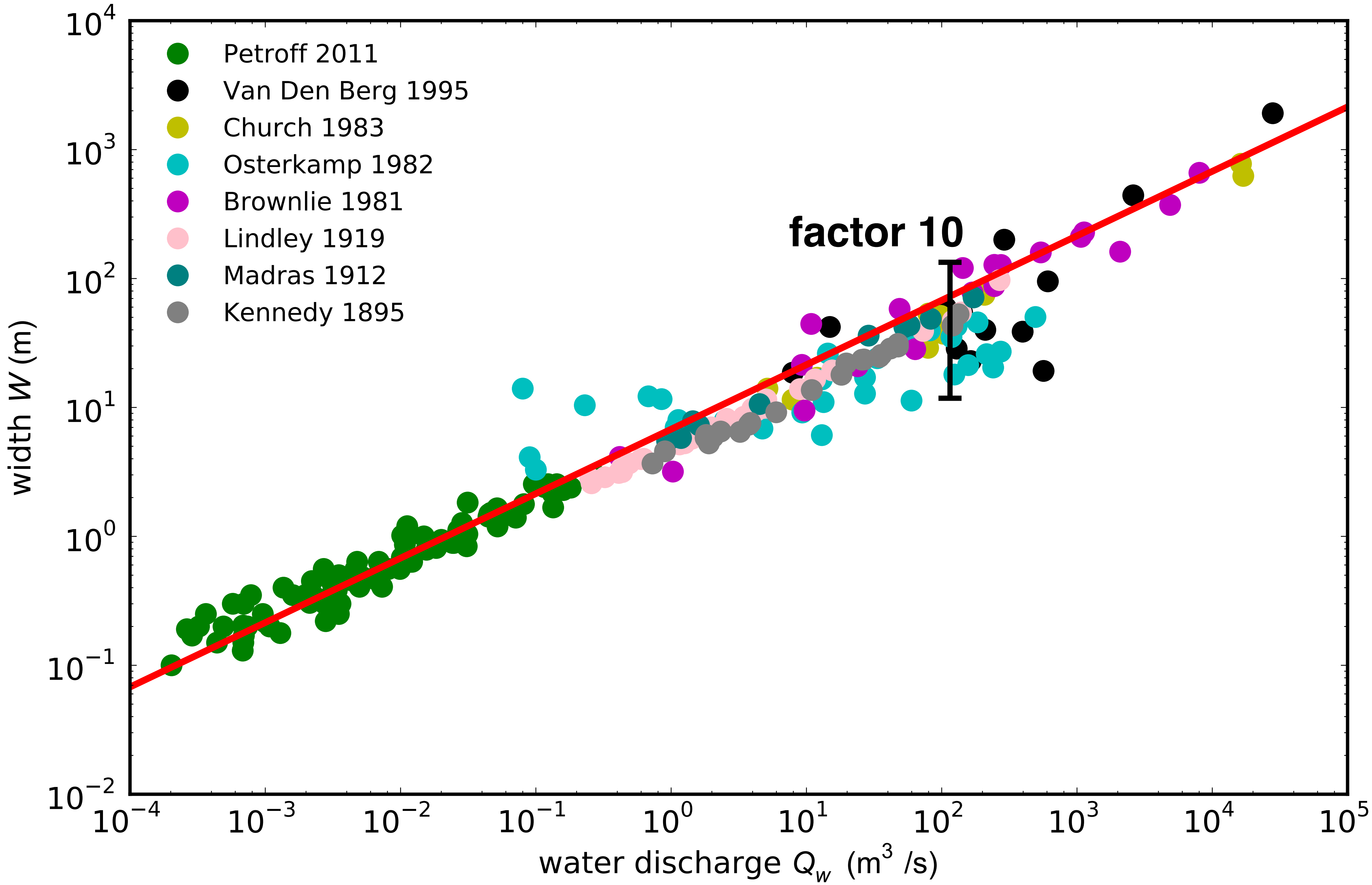
Comparison with field sandy rivers



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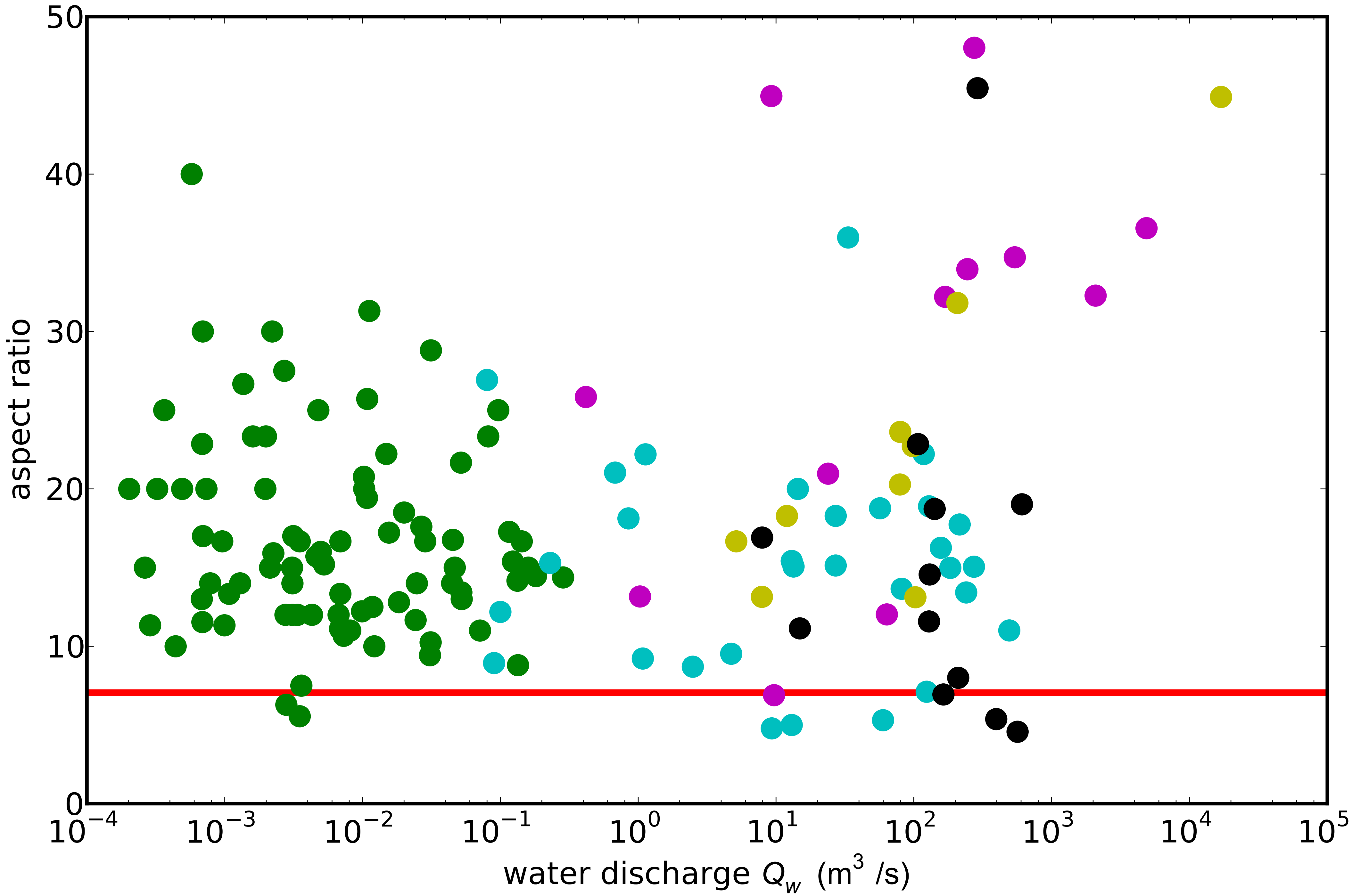
Comparison with field sandy rivers



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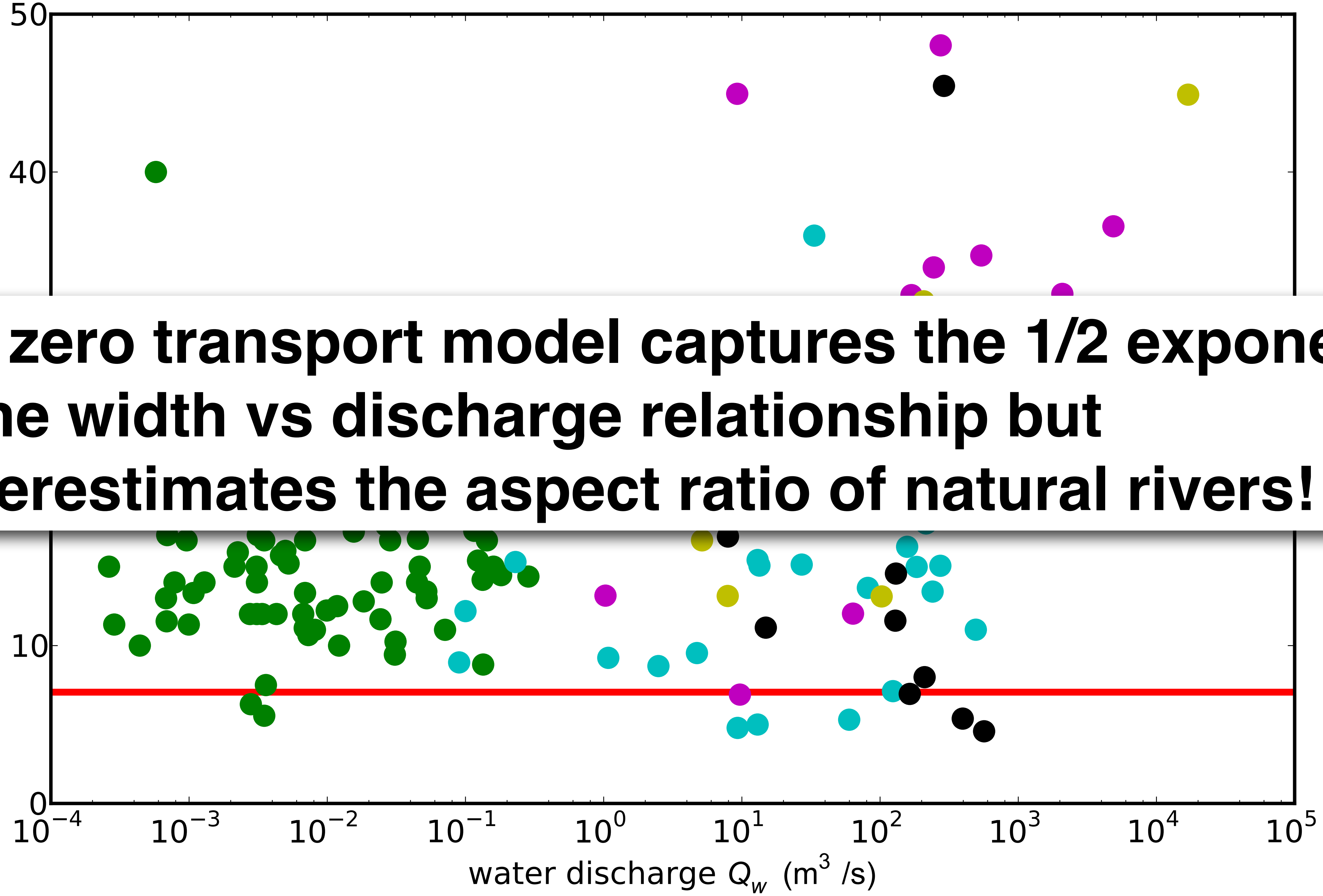
Comparison with field sandy rivers

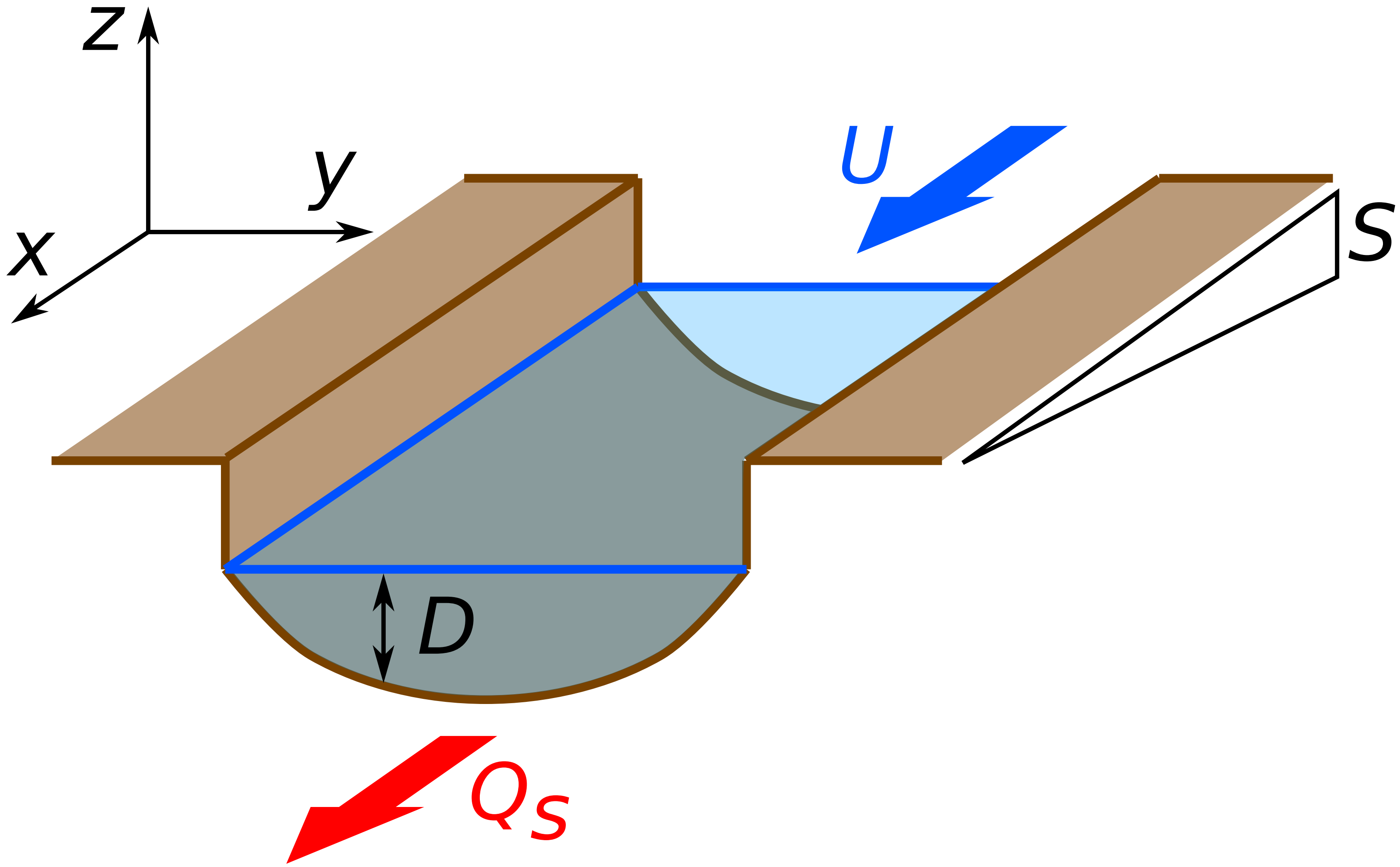


The zero transport model captures the 1/2 exponent of the width vs discharge relationship but underestimates the aspect ratio of natural rivers!

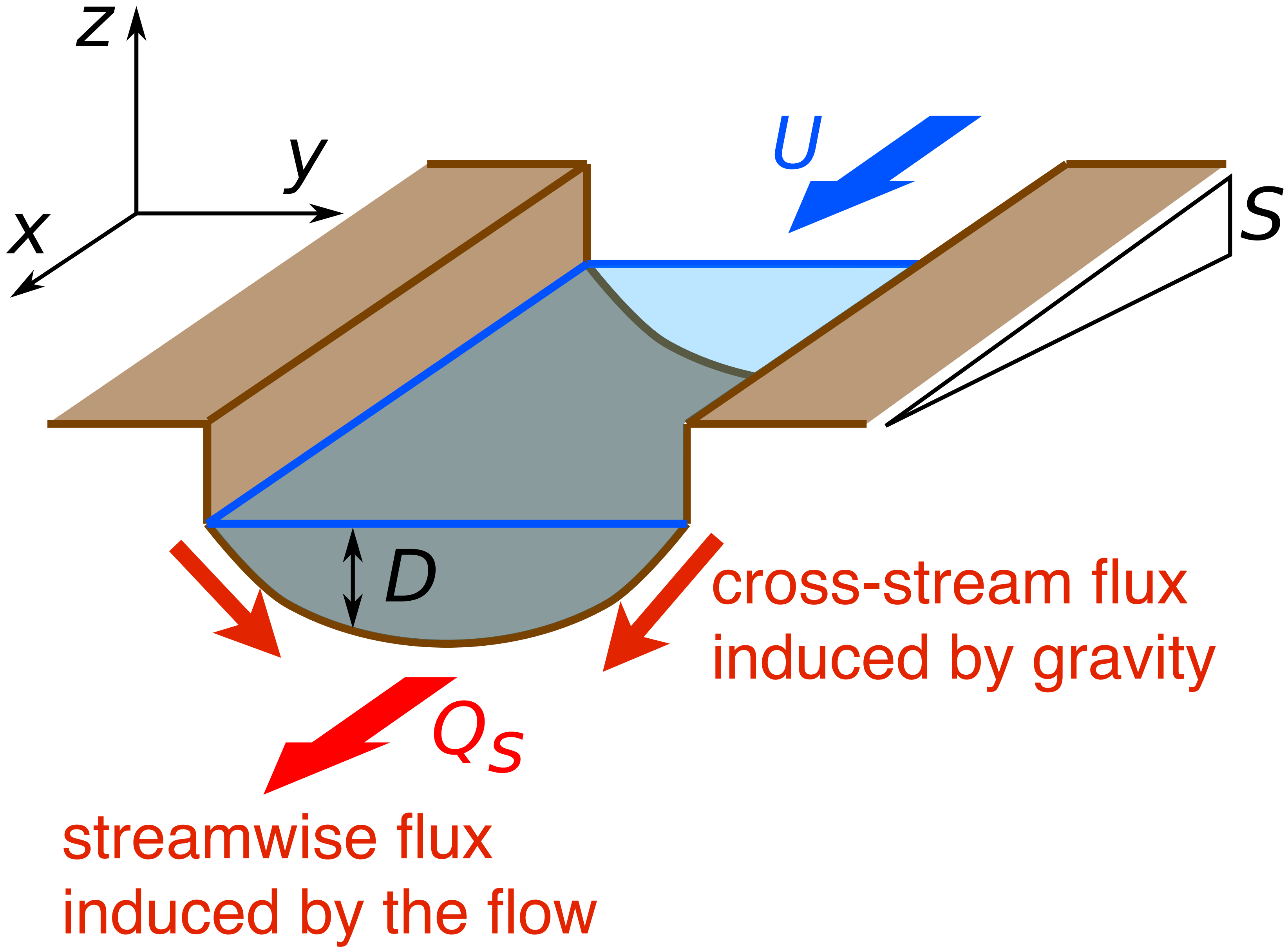
$$\frac{W}{< D >} = \frac{\pi^2}{2\mu} \approx 7$$

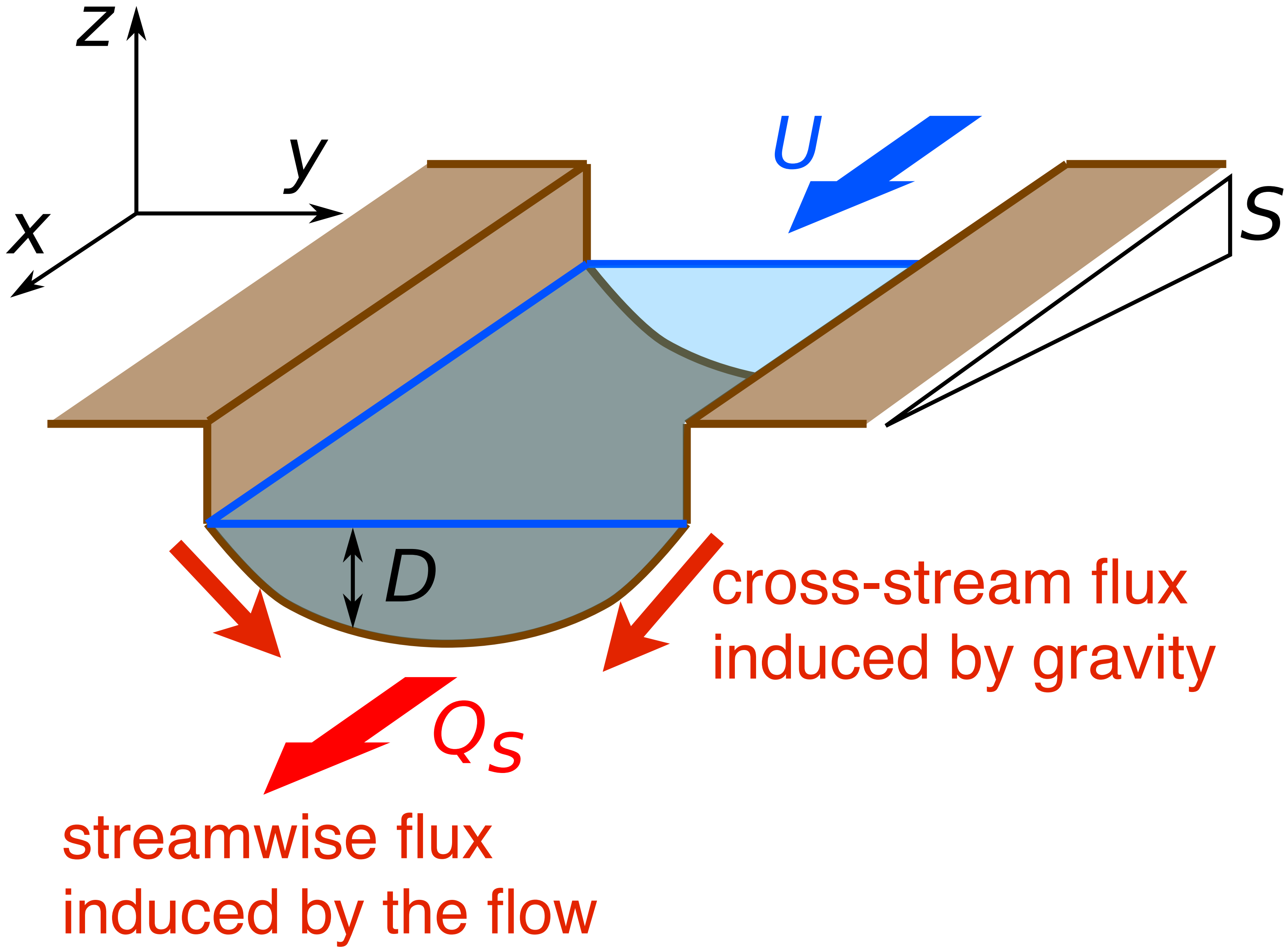
Rivers do transport sediment !

Urümqi He, chinese Tian-Shan



streamwise flux
induced by the flow

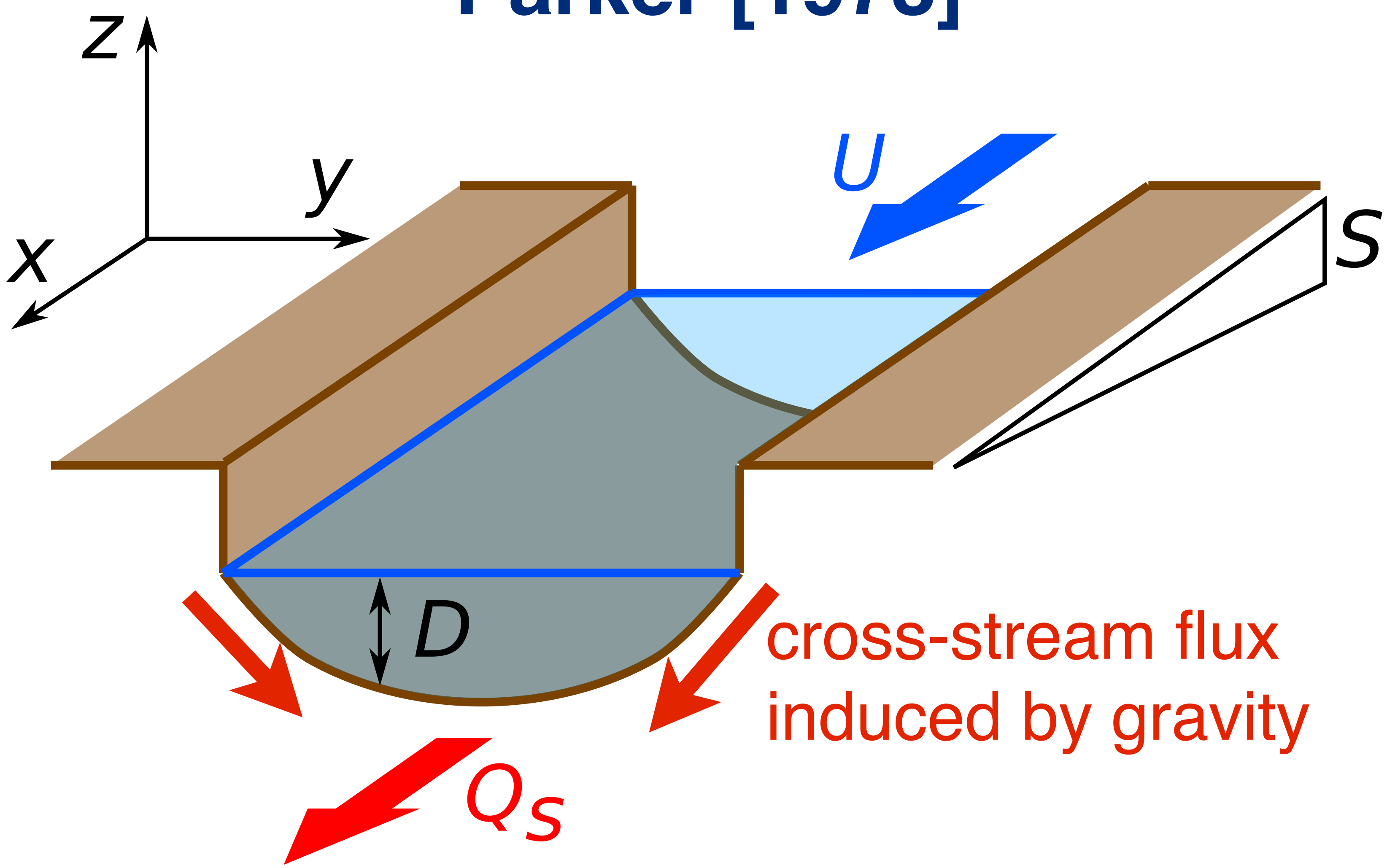

z
y
x
U
S
D
cross-stream flux
induced by gravity
Q_S
streamwise flux
induced by the flow



transverse gravity flux ➔ bank erosion

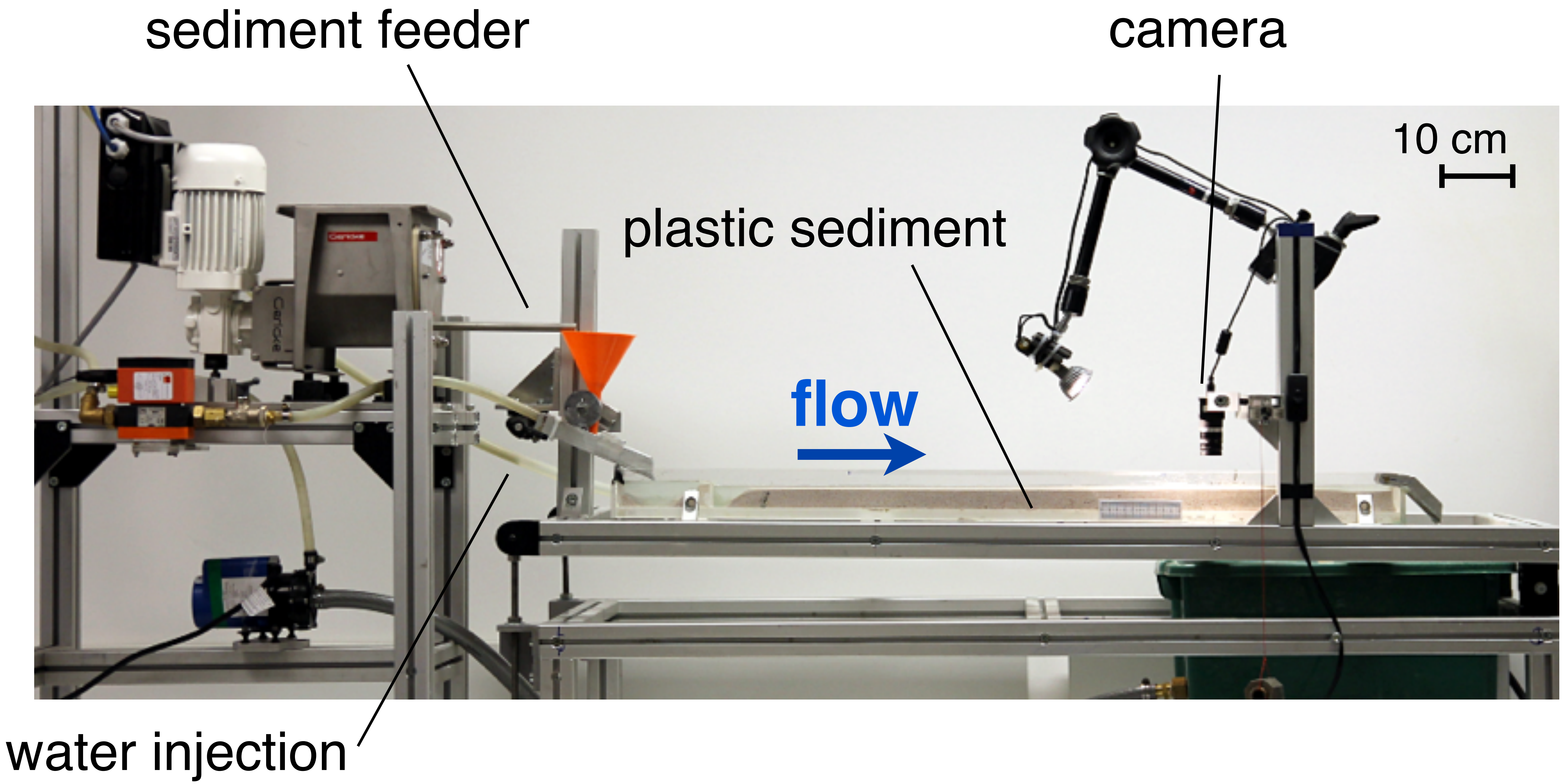
‘stable channel paradox’

Parker [1978]

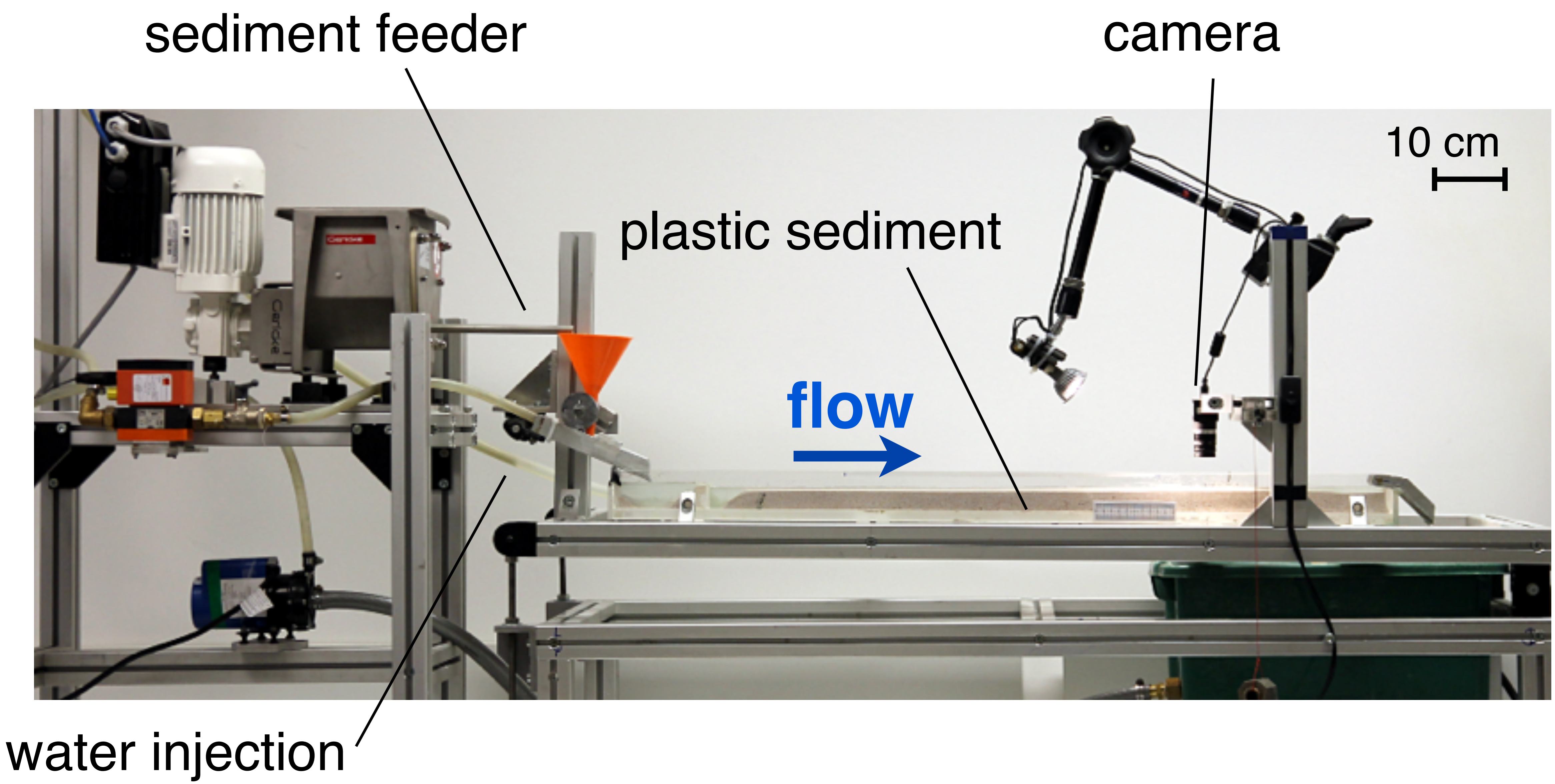


A river transporting sediment cannot be stable unless we find an effect which compensates for the gravity flux.

Experimental channel

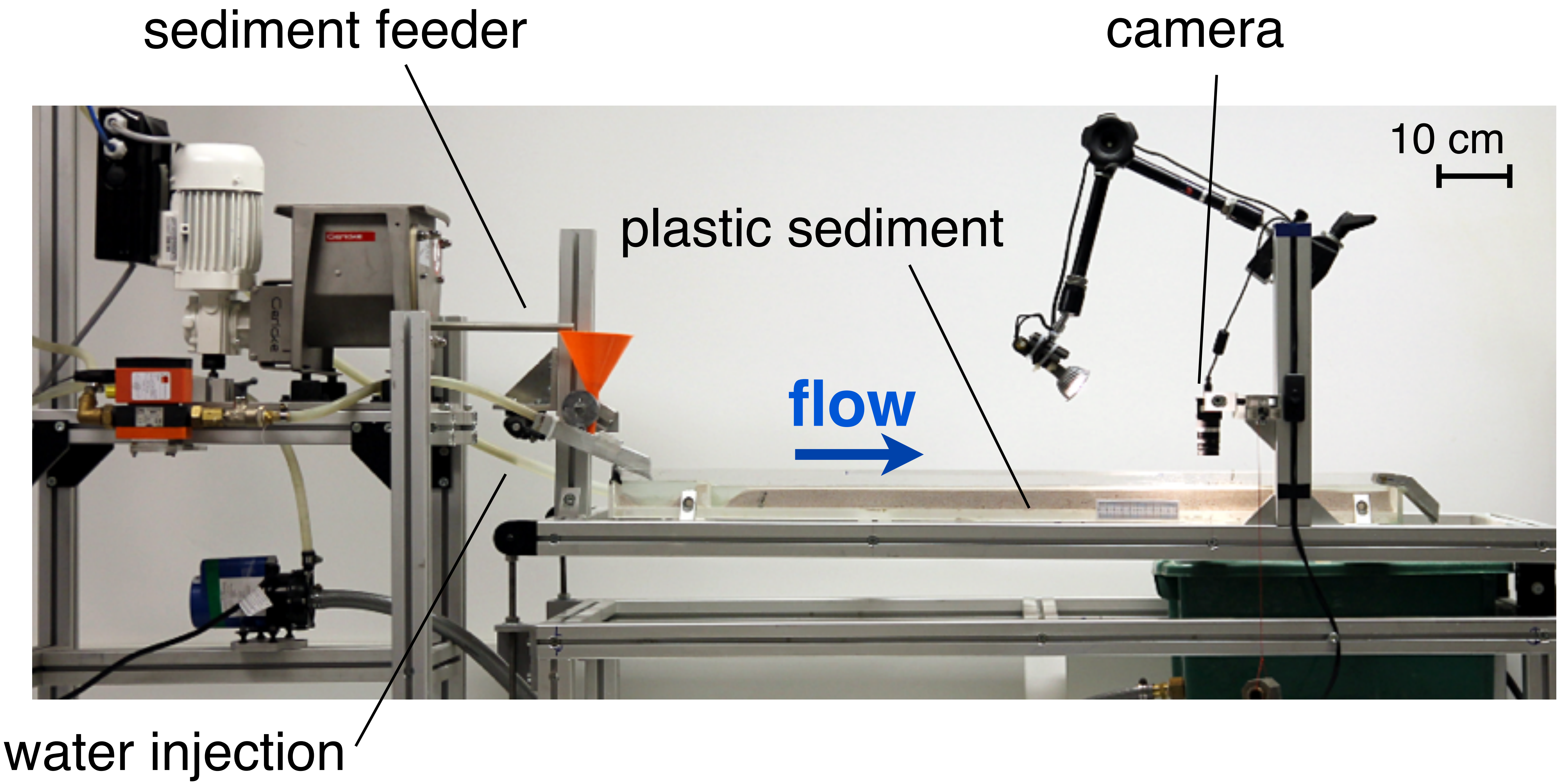


Experimental channel



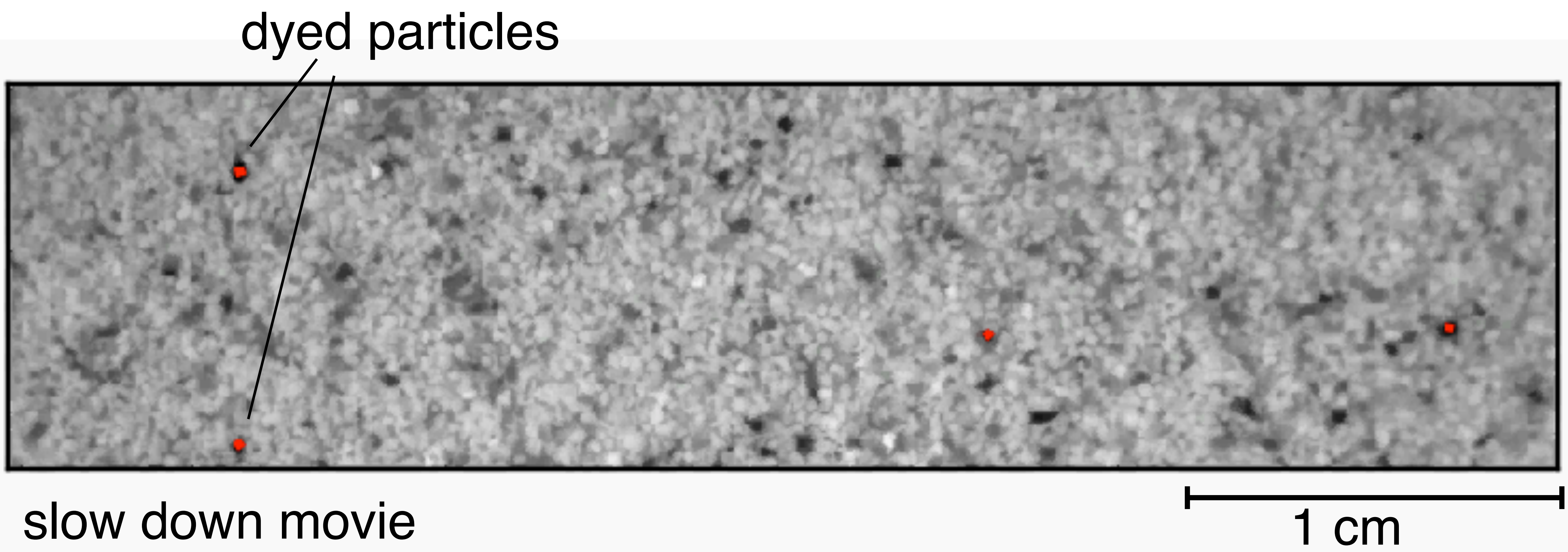
constant flow discharge
constant sediment discharge → steady-state

Experimental channel



- The sediment bed is flat !
- Close to the entrainment threshold !

Particle trajectories

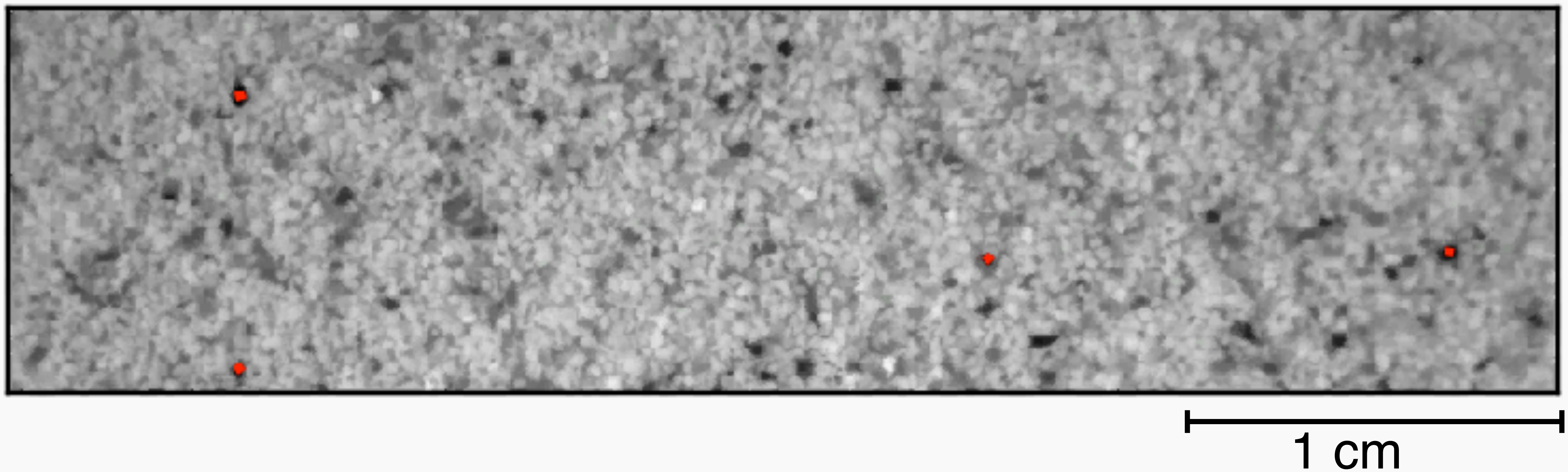


slow down movie
(1second = 0.1 second)

Bedload transport : rolling, jumping & sliding

The motion of an individual particle is stochastic!

Particle trajectories



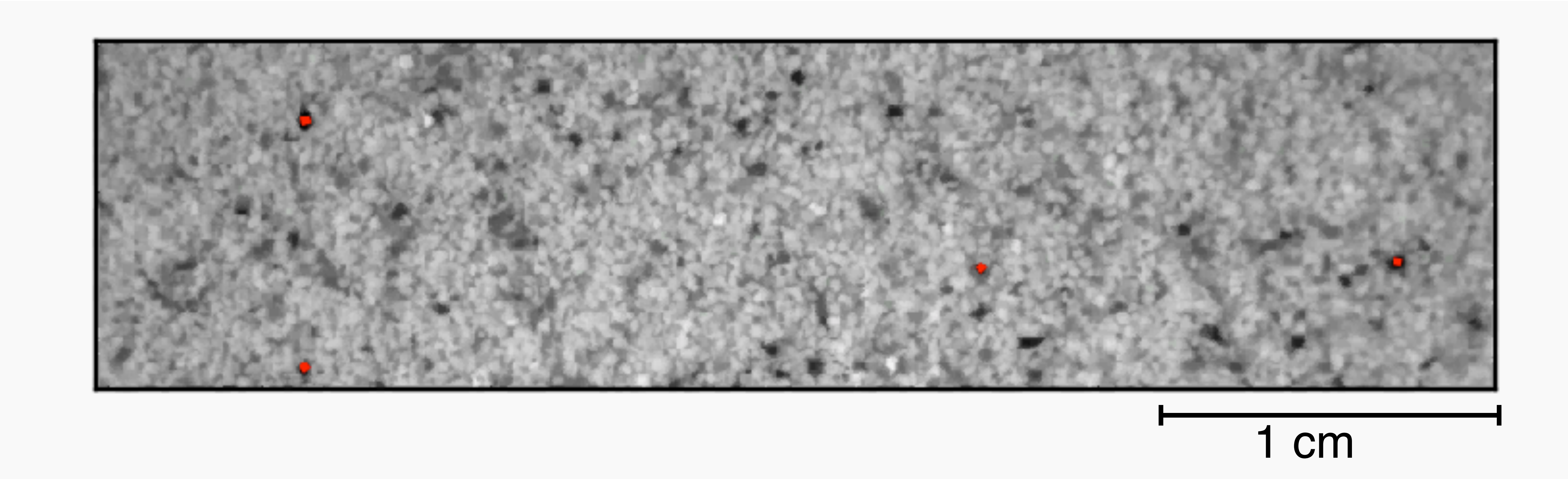
$$q_b = n\,V$$

transport rate

surface concentration of moving particles

average velocity of the particles

Particle trajectories



$$q_b = nV$$

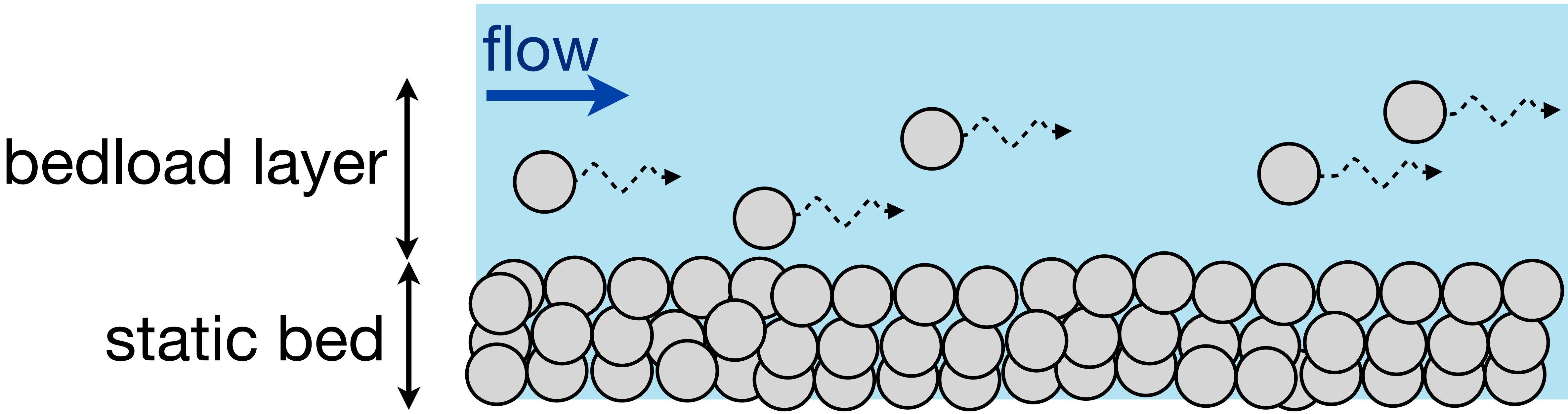
transport rate

surface concentration of moving particles

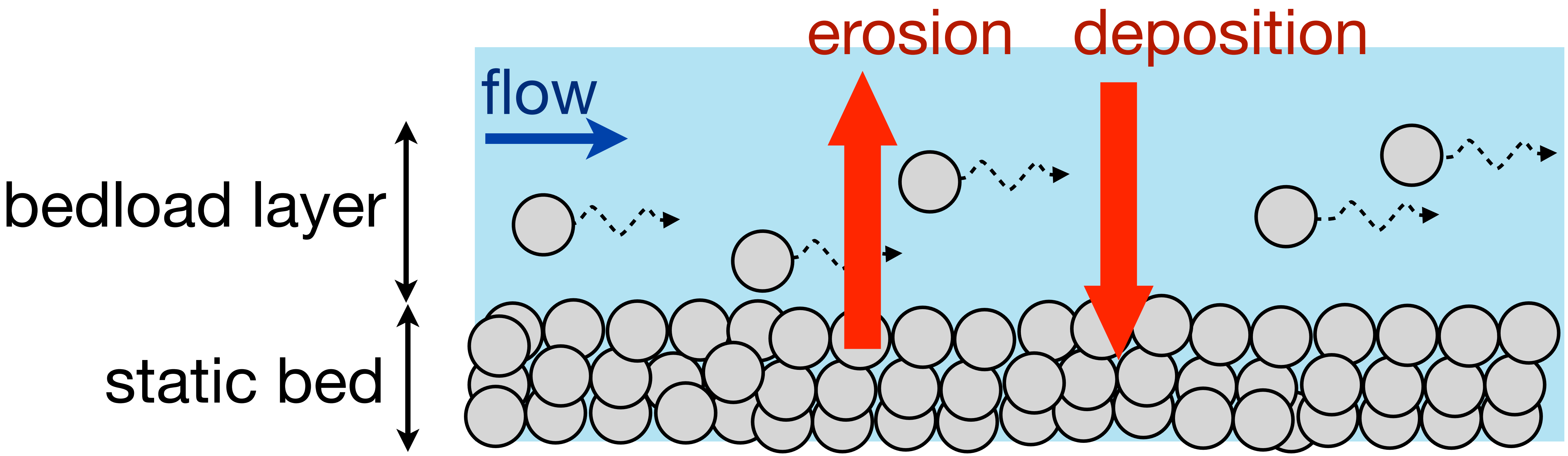
average velocity of the particles

Charru et *al.* [2004]
Lajeunesse et *al.* [2010]
Furbish et al. [2012]
Seizilles et al. [in press]

Erosion-deposition model



Erosion-deposition model



steady-state
uniform flow

$$\longrightarrow \quad n \propto \frac{(\theta - \theta_t)}{d_s^2}$$

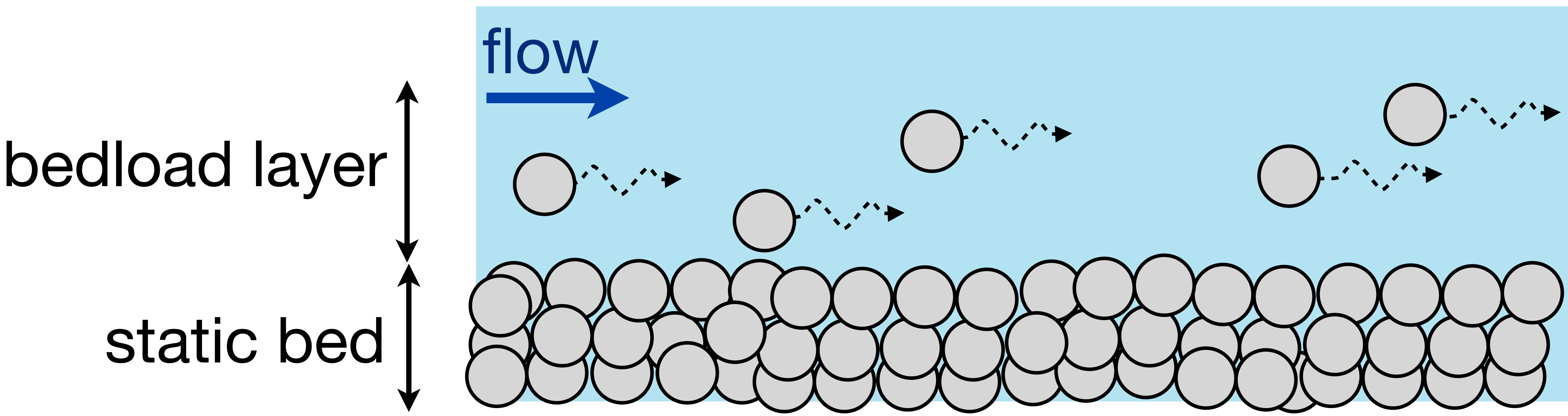
excess Shields stress

concentration
of moving particles

grain size

[Charru et al., 2004;
Lajeunesse et al., 2010]

Erosion-deposition model



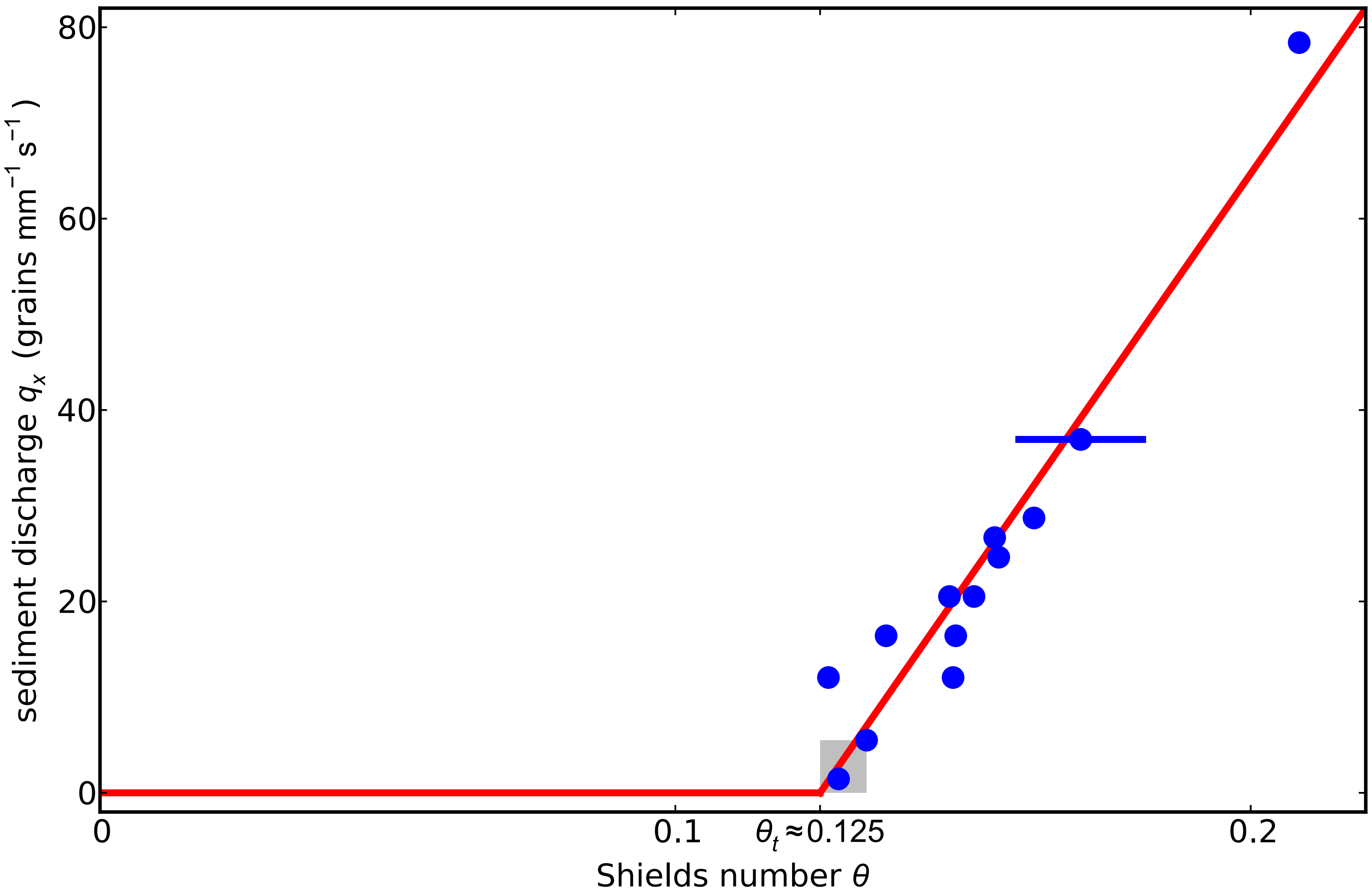
grain size — shear rate — stoke velocity

average particle velocity

$$V_x \sim d_s \frac{\partial u}{\partial z} \sim V_s\,\theta$$

[Charru et al., 2004;
Seizilles et al., in press]

Erosion-deposition model



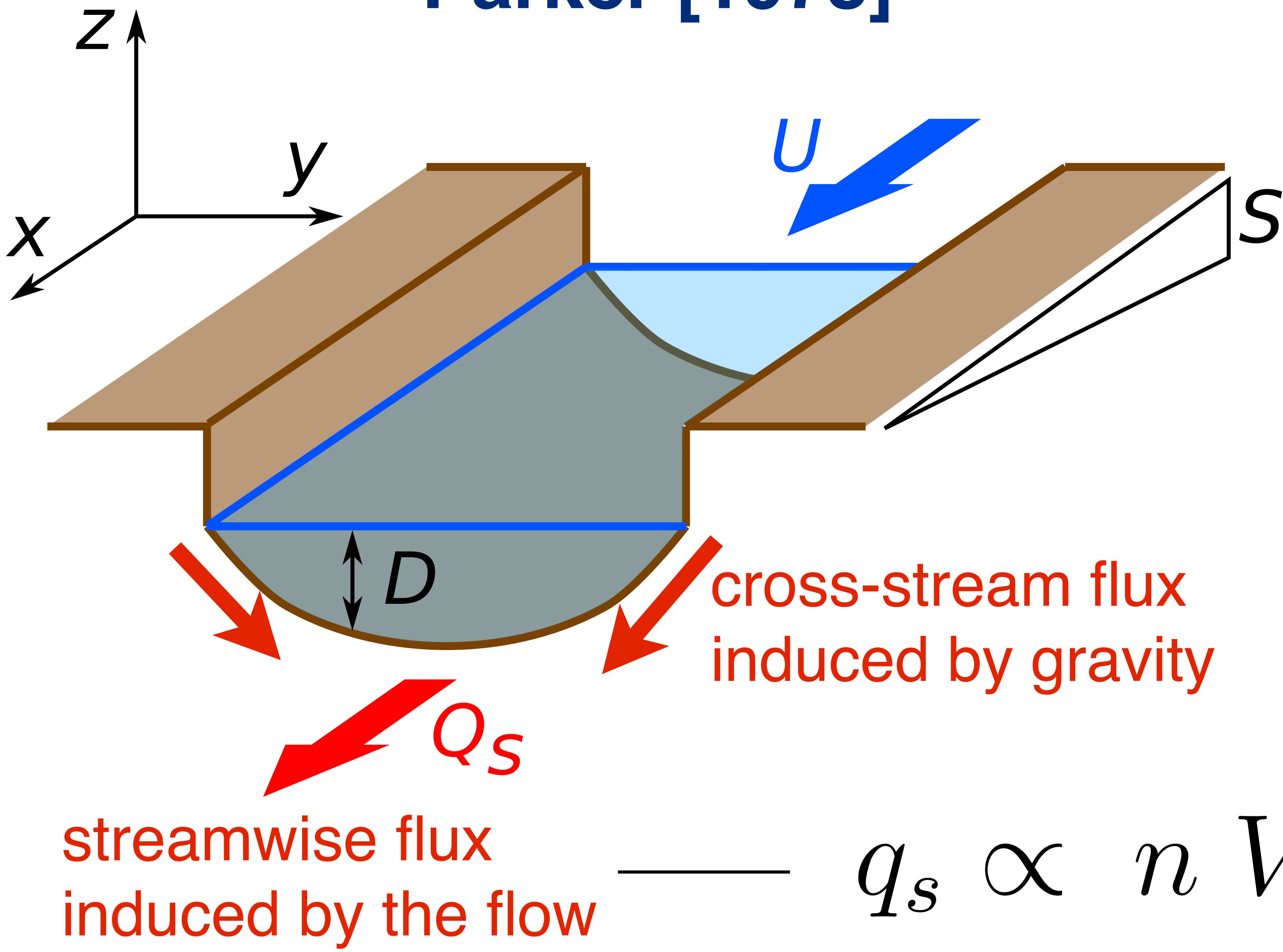
excess Shields stress

At leading order $$q_s \propto n\, V_x \;\propto\; \frac{\theta_t\, V_s}{d_s^2}\,(\theta - \theta_t)$$

[Charru et al., 2004;
Seizilles et al., in press]

'stable channel paradox'

Parker [1978]



$$q_s \propto n V$$

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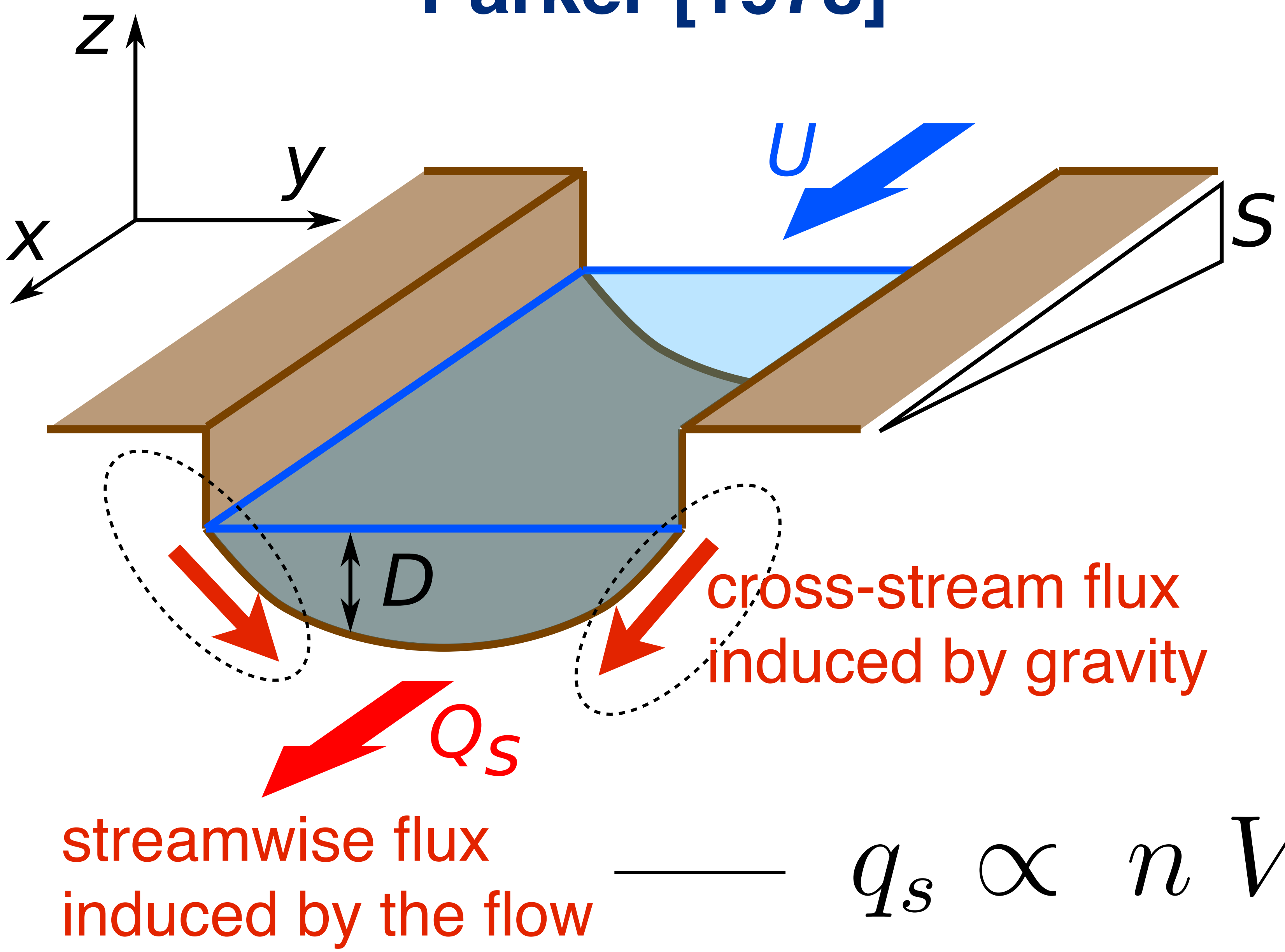
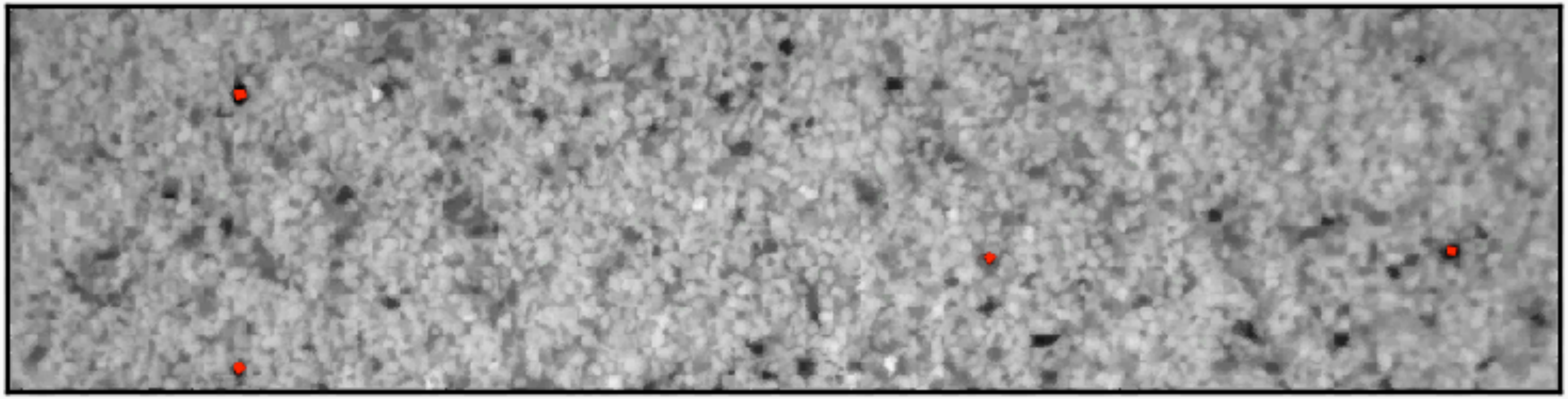


$$q_s \propto n\, V$$

What opposes the cross-stream gravity-induced flux ?

Bedload transport



view from above
slow down movie

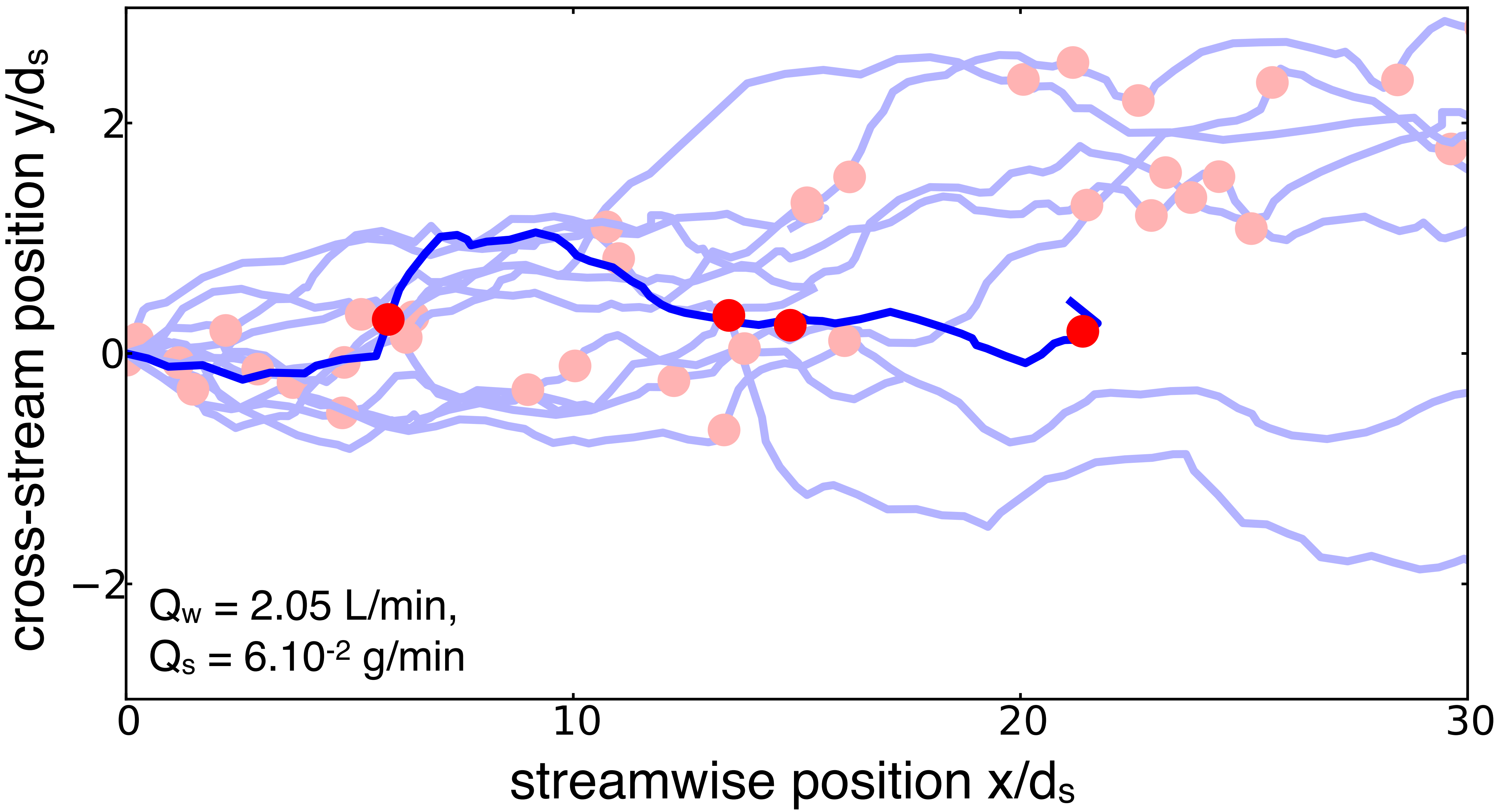
Trajectories are not straight lines!

bed roughness ➔ deviations along the cross stream direction

Samson et al. [1998], Lajeunesse et al. [2010], Roseberry et al. [2012]

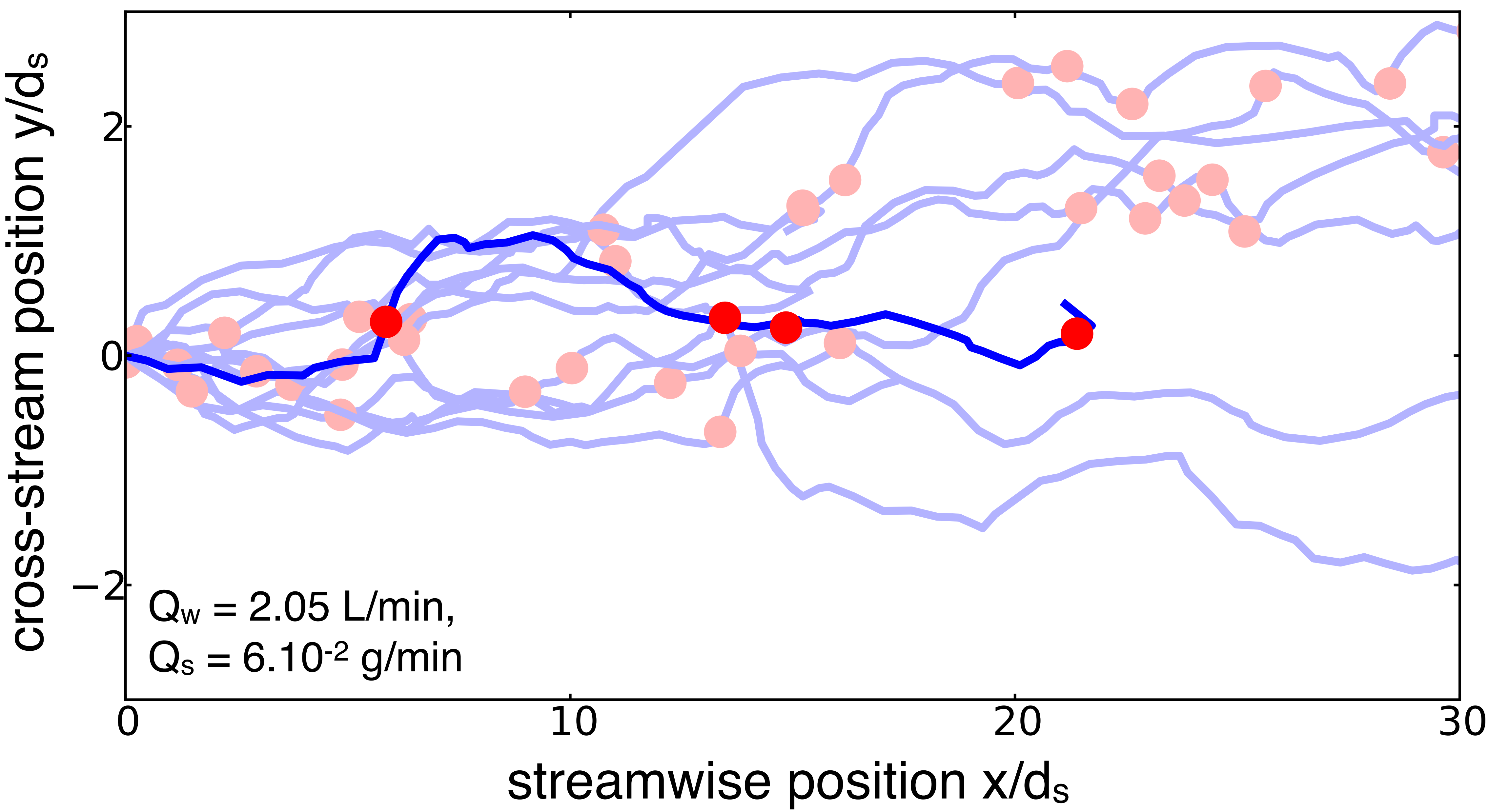
Particle trajectories

translated so that their initial starting point coincide



Particle trajectories

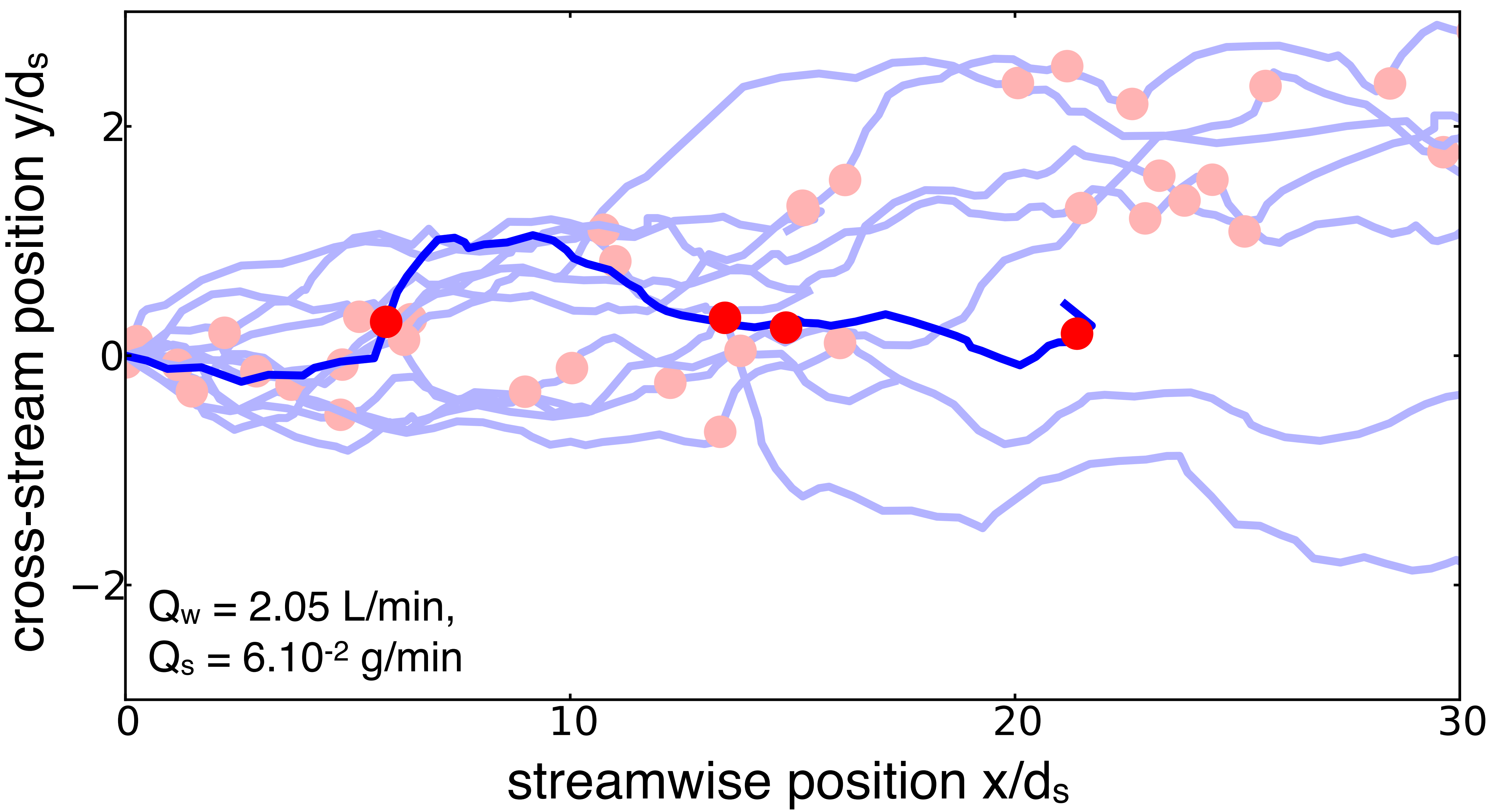
translated so that their initial starting point coincide



spreading along the cross-stream direction

Particle trajectories

translated so that their initial starting point coincide

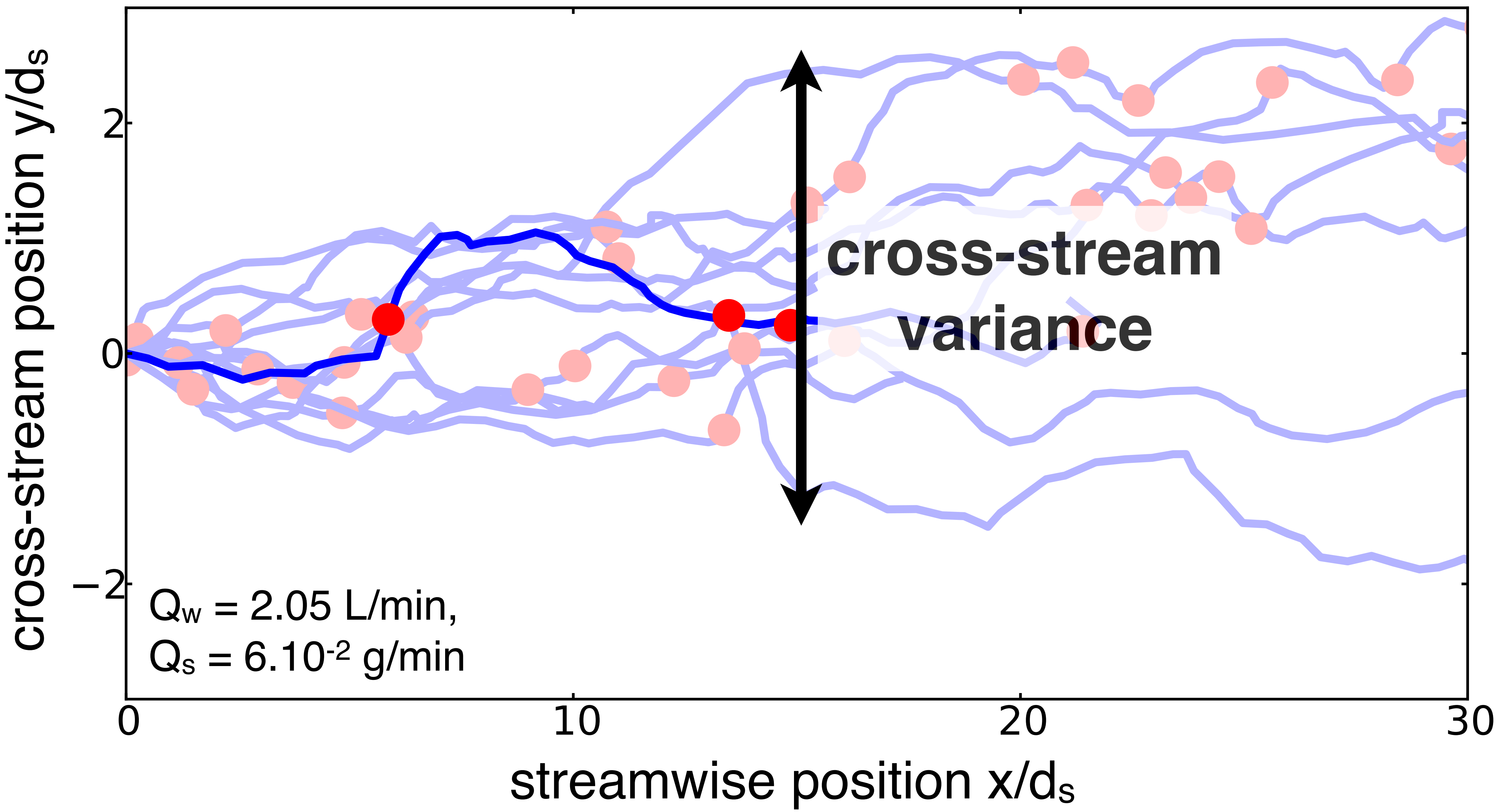


spreading along the cross-stream direction

random walk in the cross-stream direction ?

Particle trajectories

translated so that their initial starting point coincide

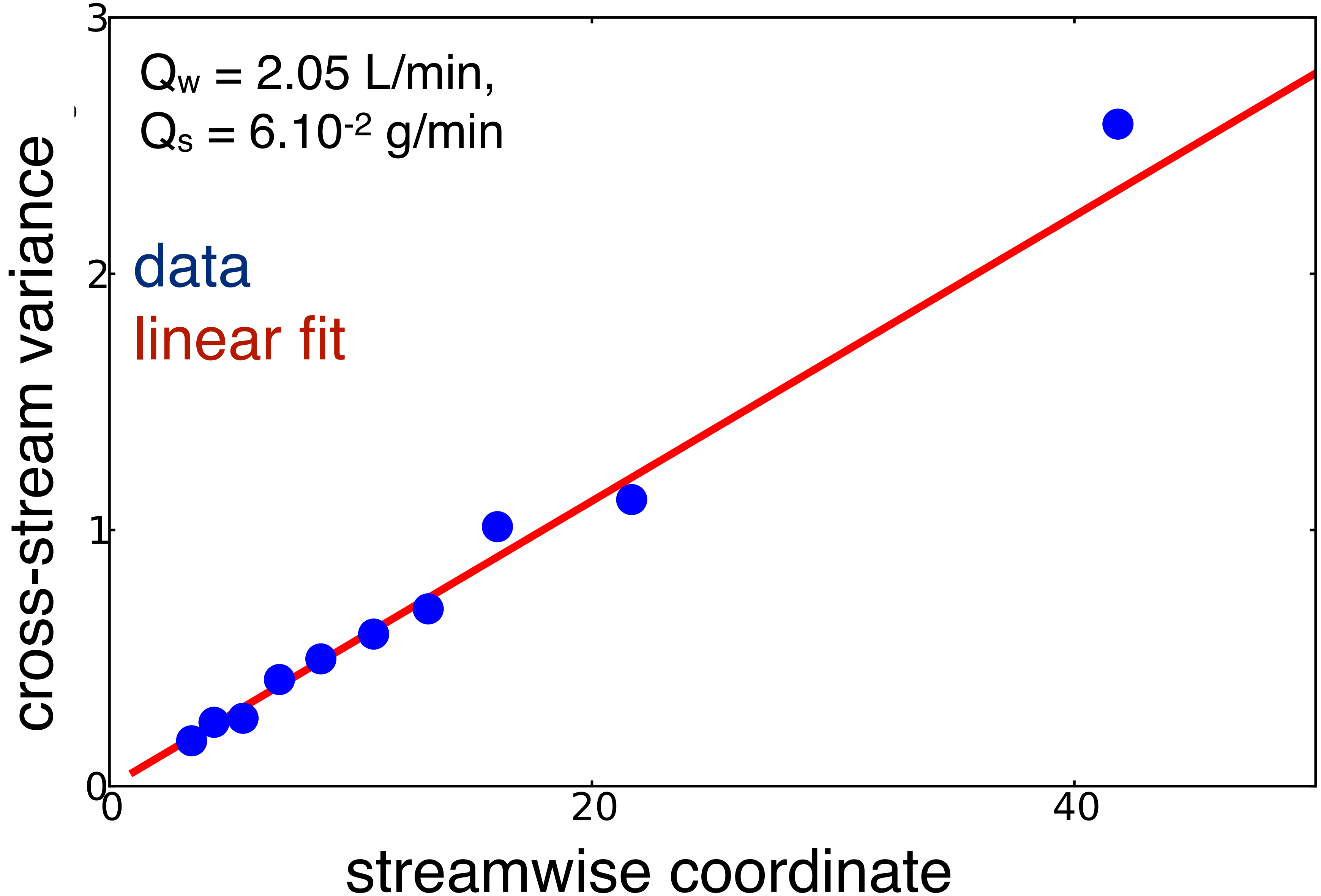


spreading along the cross-stream direction

random walk in the cross-stream direction ?

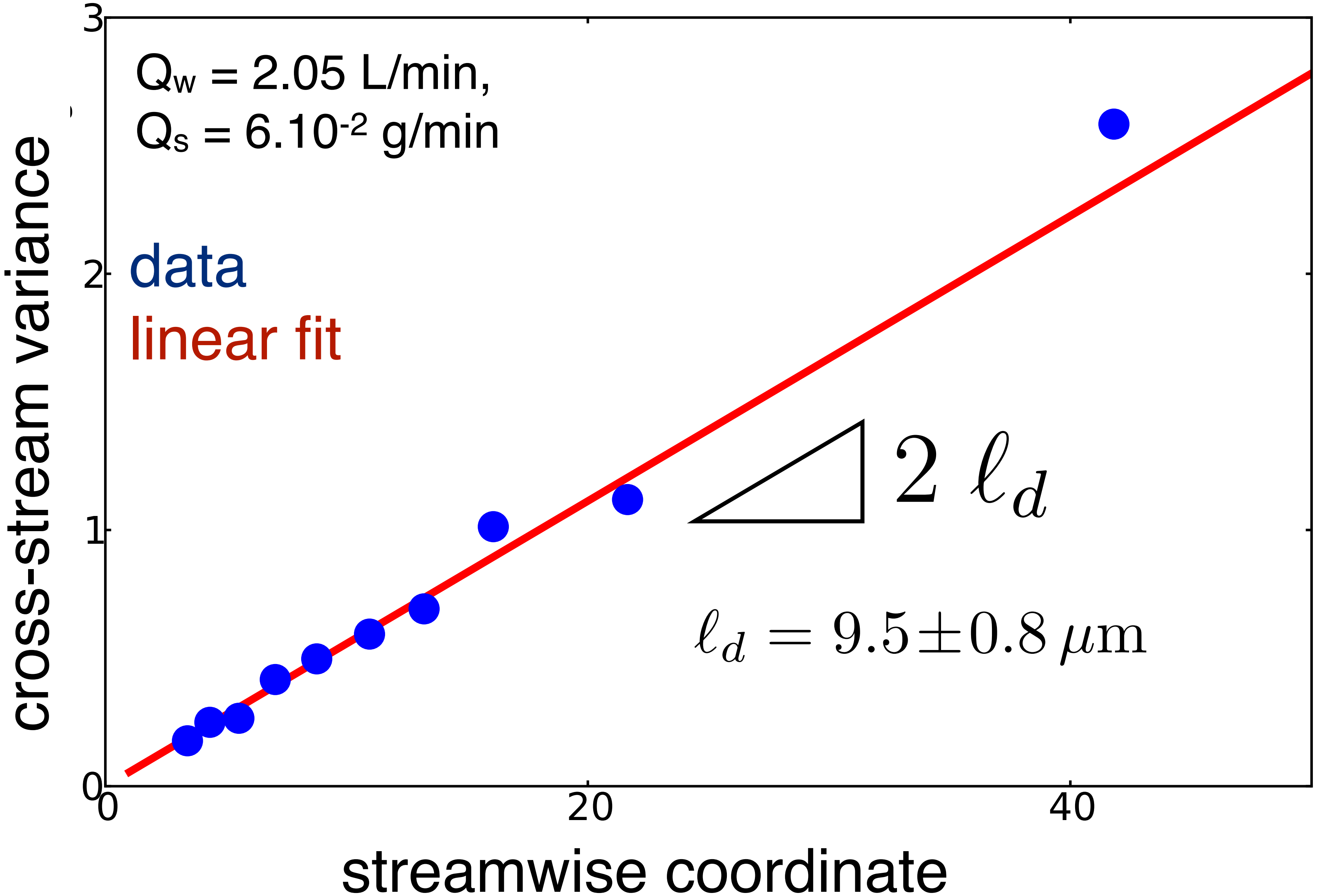
$$\text{variance of y} \propto \text{x} \; ?$$

Random walk



[Seizilles et al., sub.]

Random walk



variance of y

diffusion length

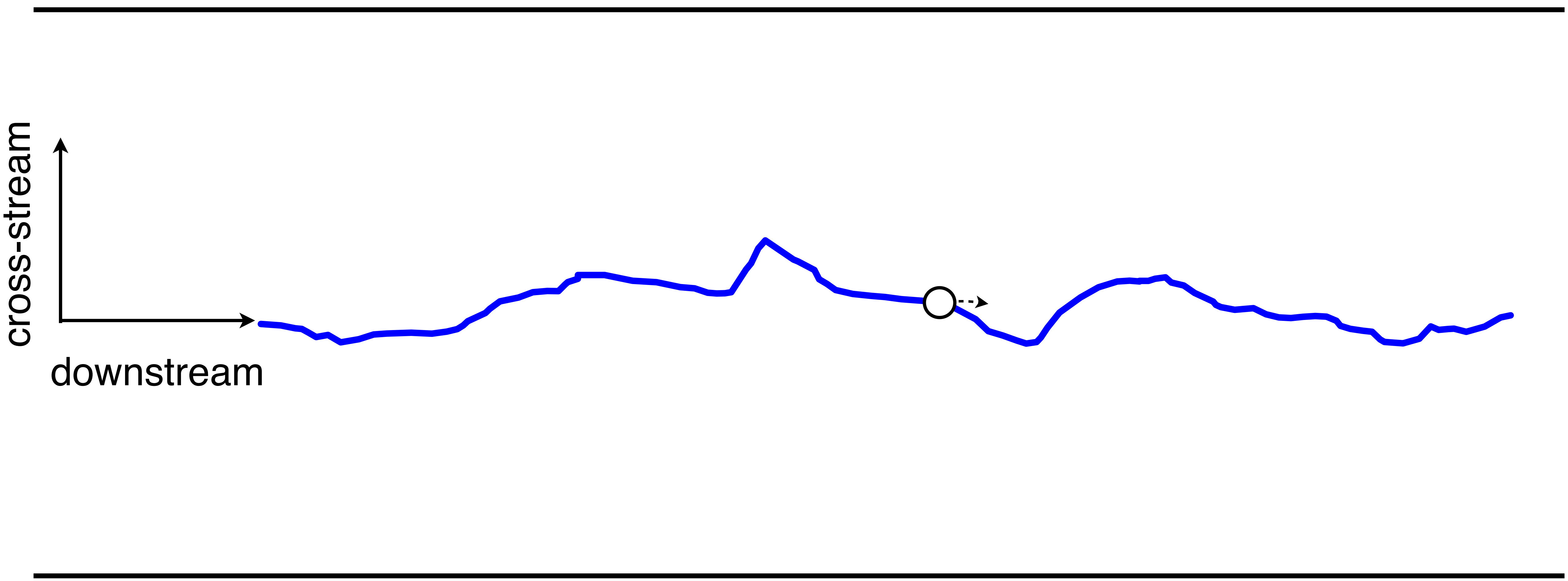
$$< y^2 > = 2\,\ell_d\,x$$

[Seizilles et al., sub.]

From random walk ....

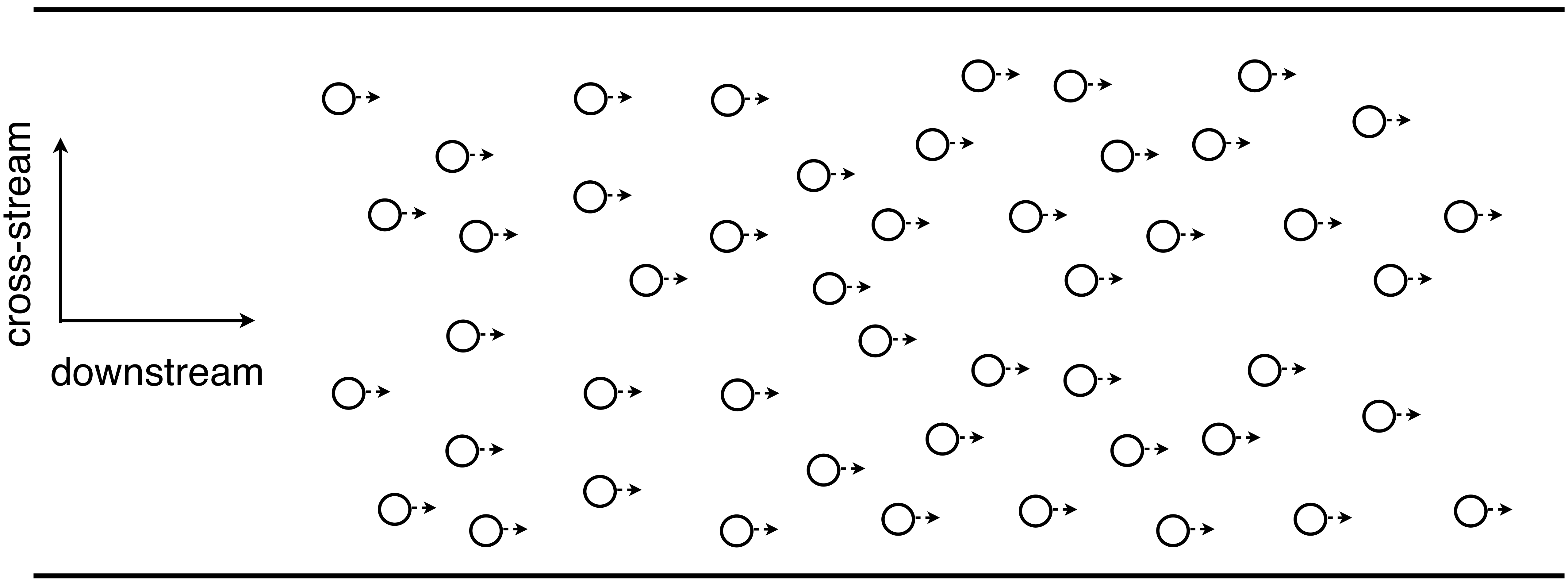
- **single particle = random walker**

Each time the particle takes one step in the streamwise direction, it also takes one random step either to the left or to the right in the cross-stream direction.



... to diffusion

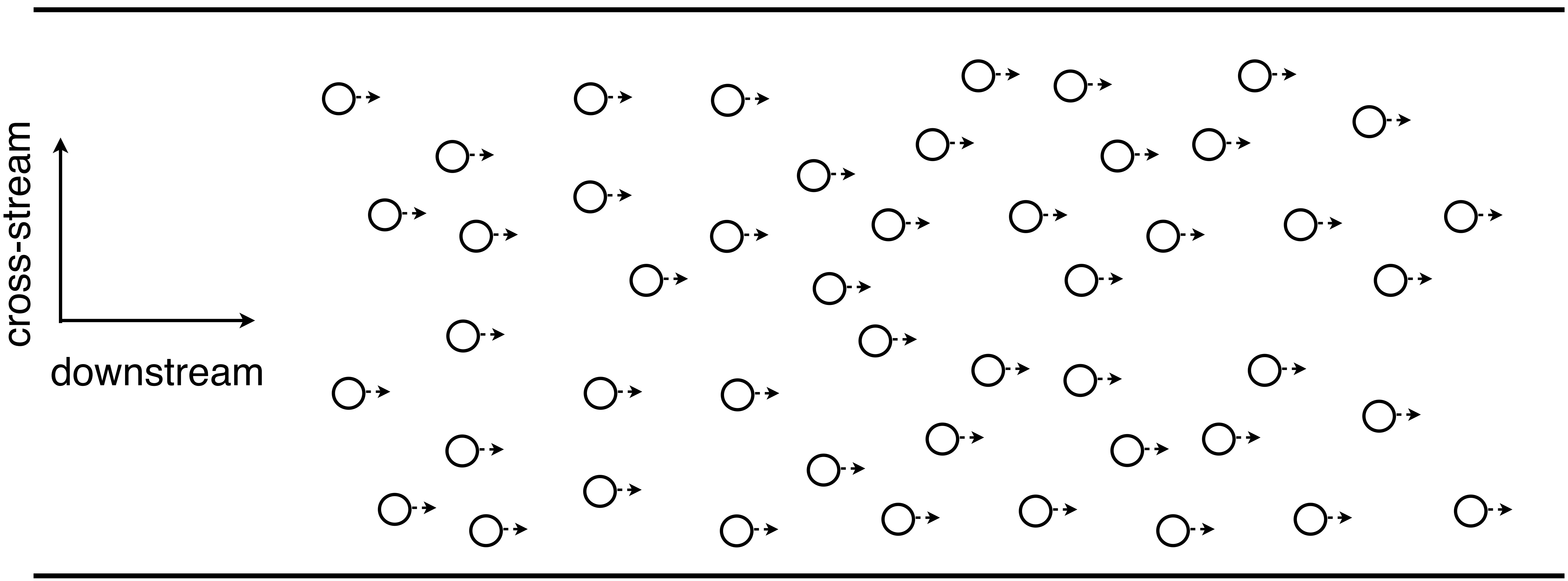
- **many random walkers**



... to diffusion

- **many random walkers**

moving grains homogeneously distributed



... to diffusion

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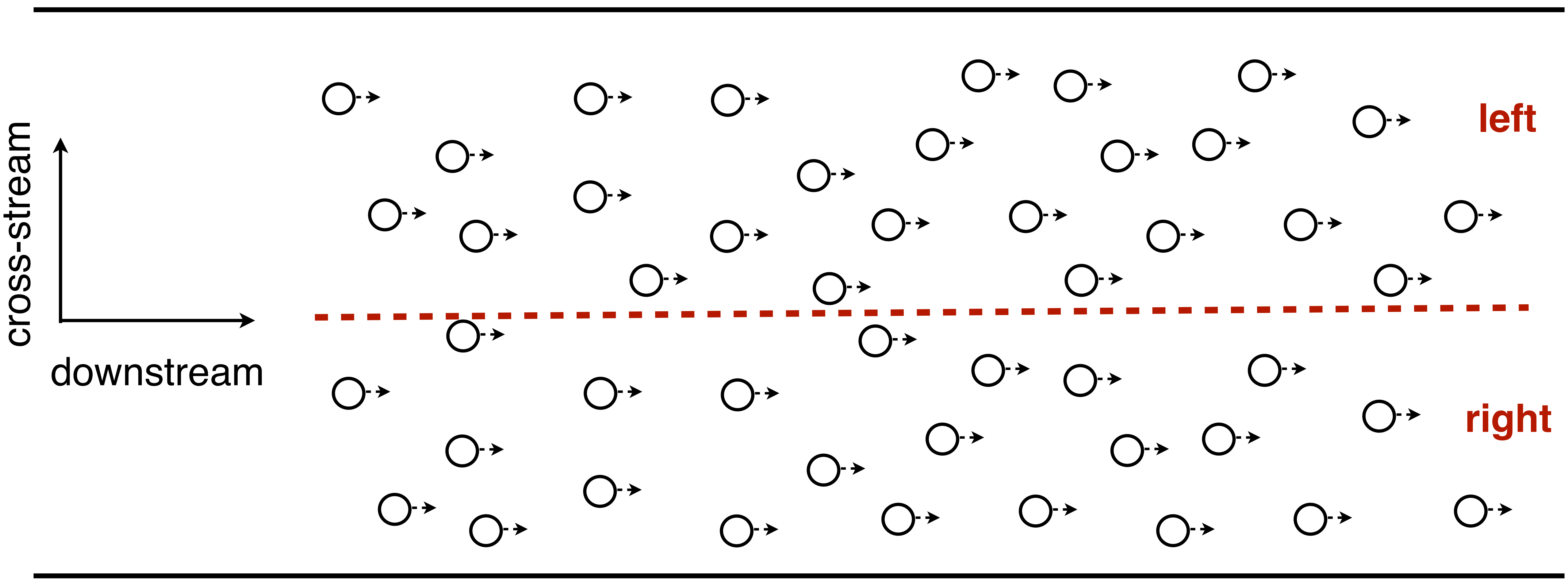
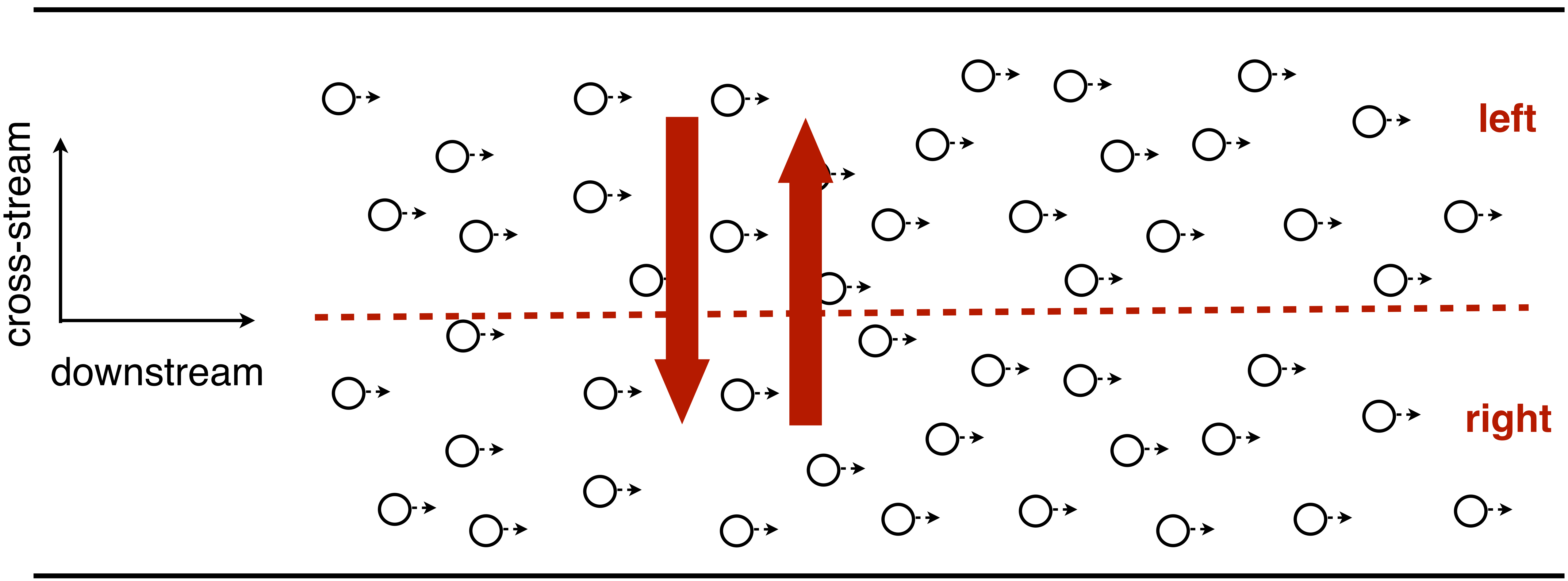


... to diffusion

- **many random walkers**

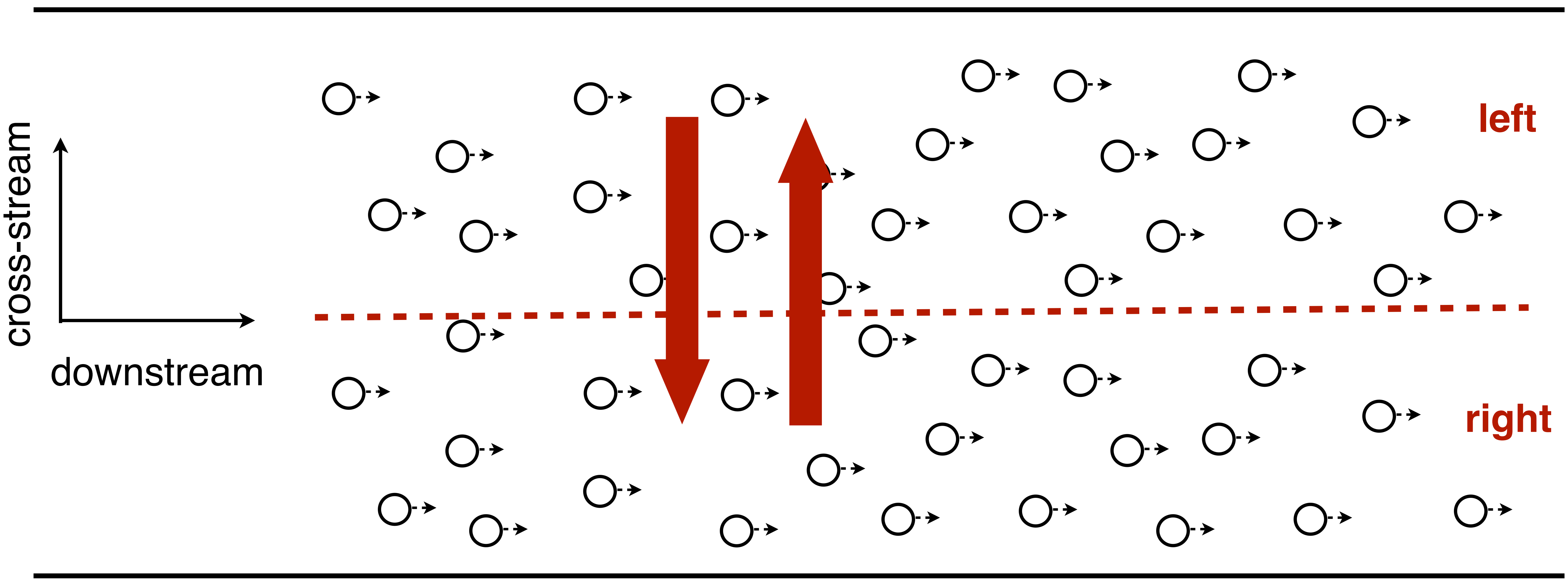
moving grains homogeneously distributed



... to diffusion

- **many random walkers**

moving grains homogeneously distributed

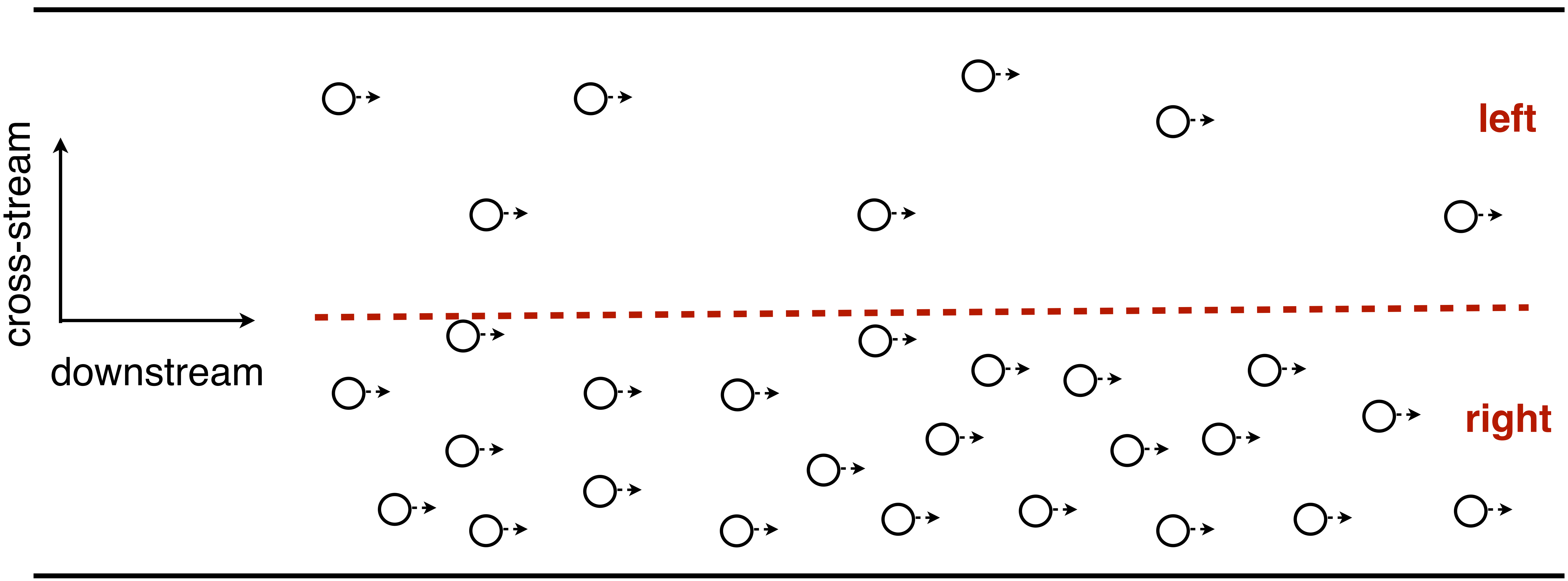


➔ no net flux along the cross-stream direction

... to diffusion

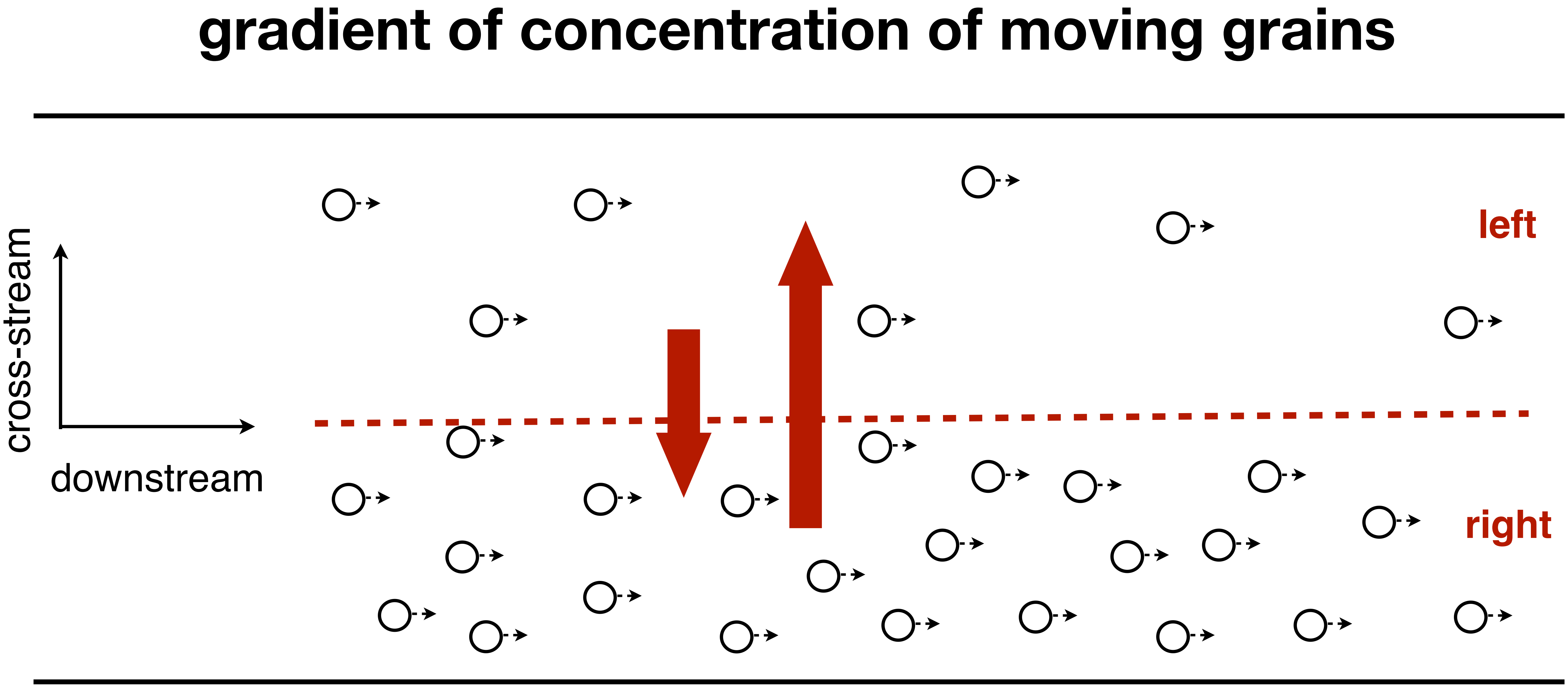
- **many random walkers**

gradient of concentration of moving grains



... to diffusion

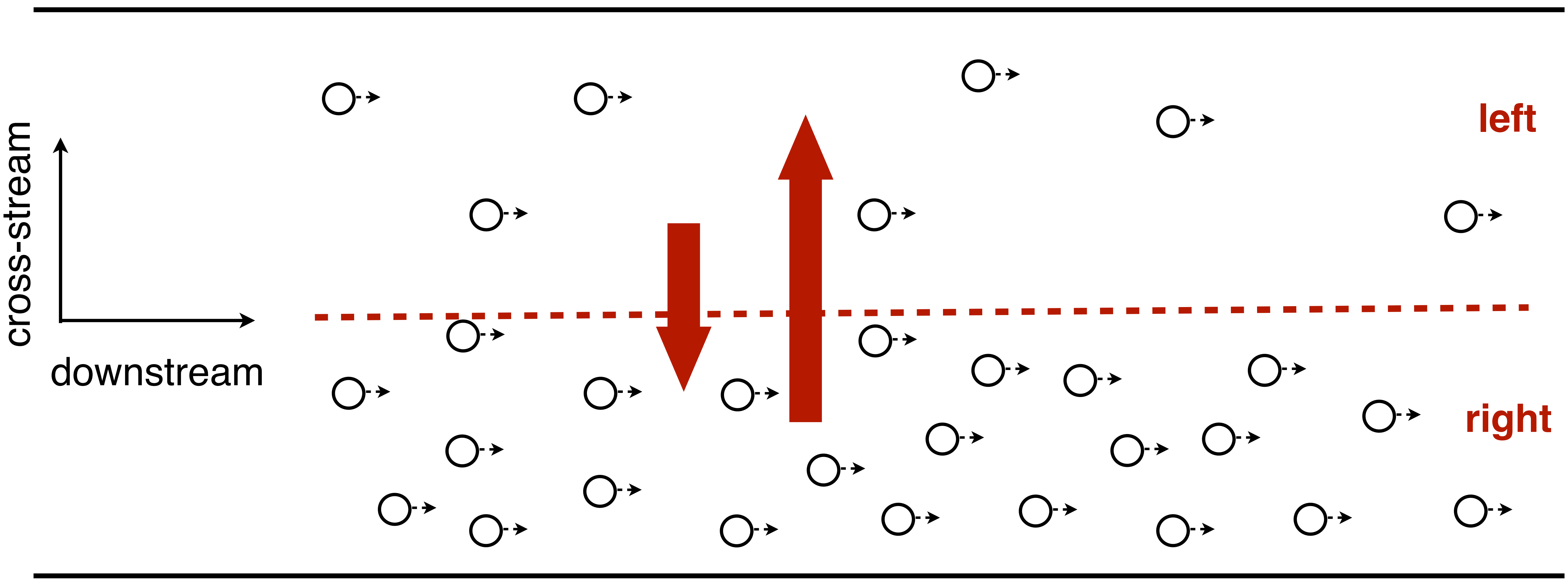
- **many random walkers**



... to diffusion

- **many random walkers**

gradient of concentration of moving grains



➔ net sediment flux

- **directed toward the less populated areas**
- **proportional to the gradient of the number of particles**

Bedload diffusion

diffusive flux — diffusion coef. — concentration of moving particles

$$\vec{q}_{\mathrm{d}} = -\zeta_D \, \vec{\nabla}_{\perp} n$$

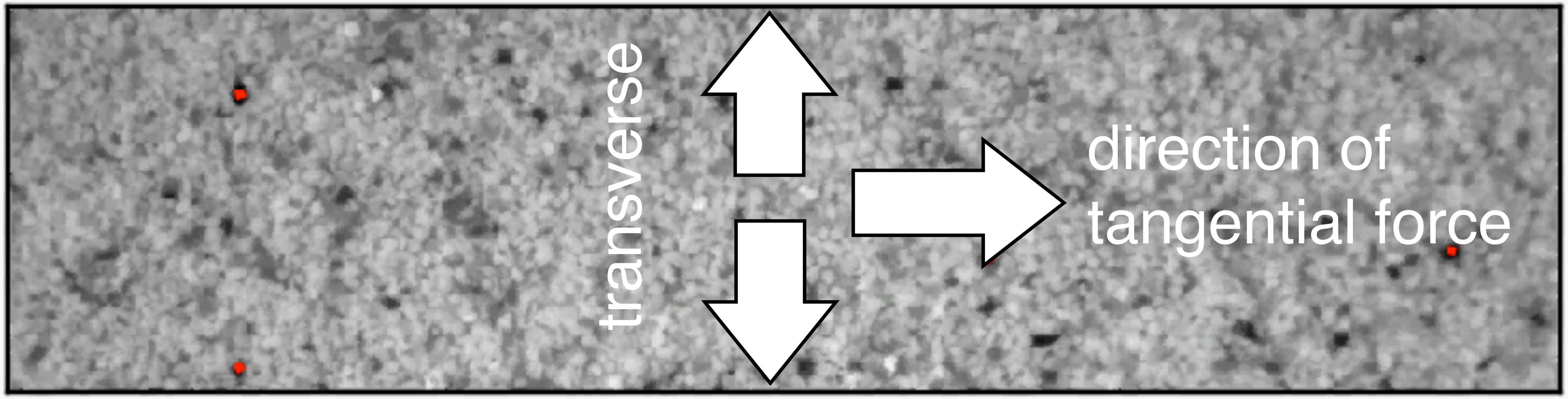
diffusive length — particle velocity

velocity distributions & correlation time ⟶

$$\zeta_D = \ell_D \, V \approx 0.03 \, d_s \, V$$

[Seizilles et al., in press]

Bedload transport : the complete picture !



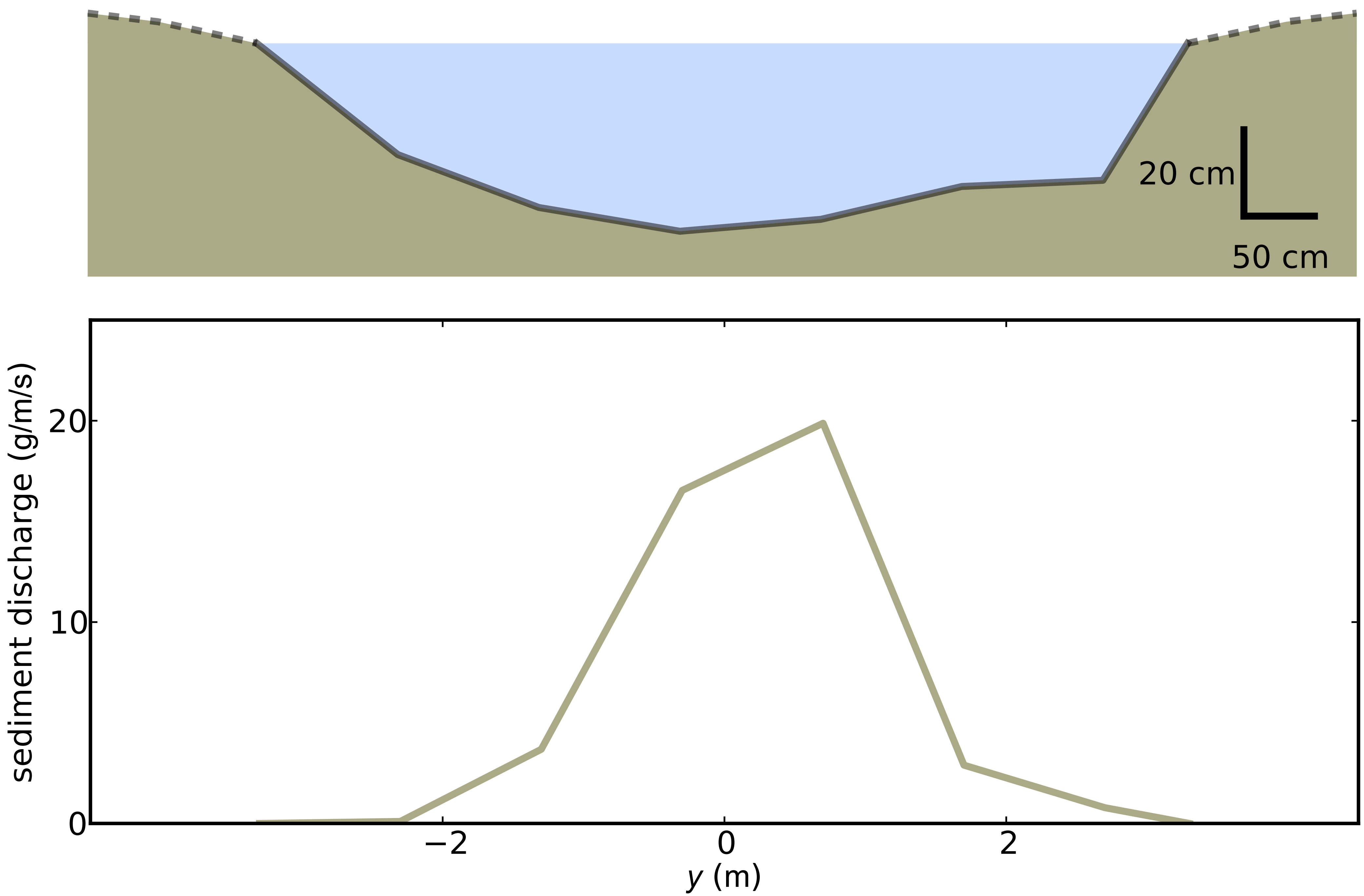
bedload transport, advection $$\vec{q_b} = n\,\vec{V}$$

transverse flux, diffusion $$\vec{q}_{\mathrm{d}} = -\zeta_D\,\vec{\nabla}_{\perp} n$$

Do gradients of bedload particles exist in nature ?

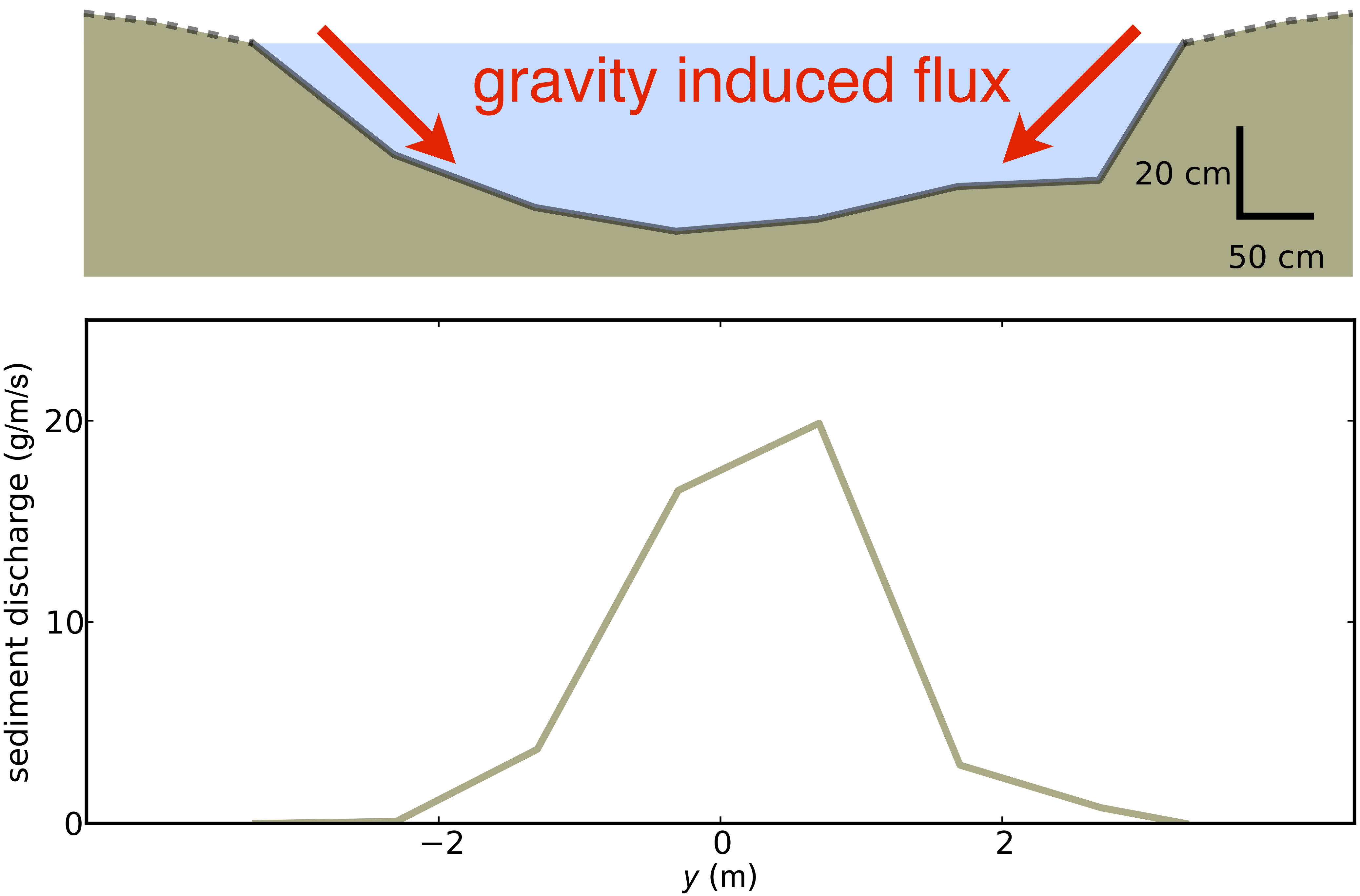
Urumqi He, Tian-Shan, China

Do gradients of bedload particles exist in nature ?



Urumqi He, Tian-Shan, China

Do gradients of bedload particles exist in nature ?



Urumqi He, Tian-Shan, China

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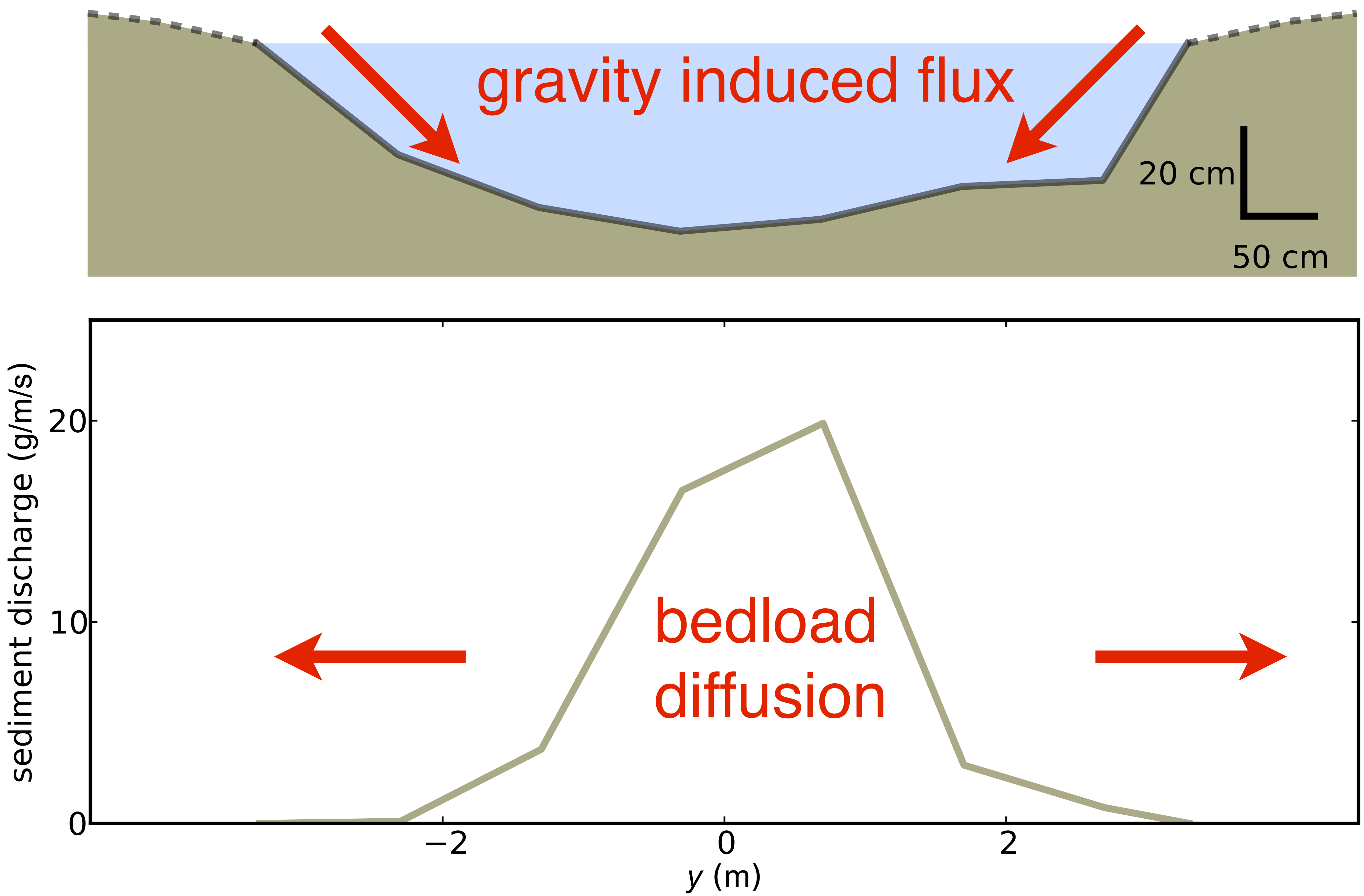
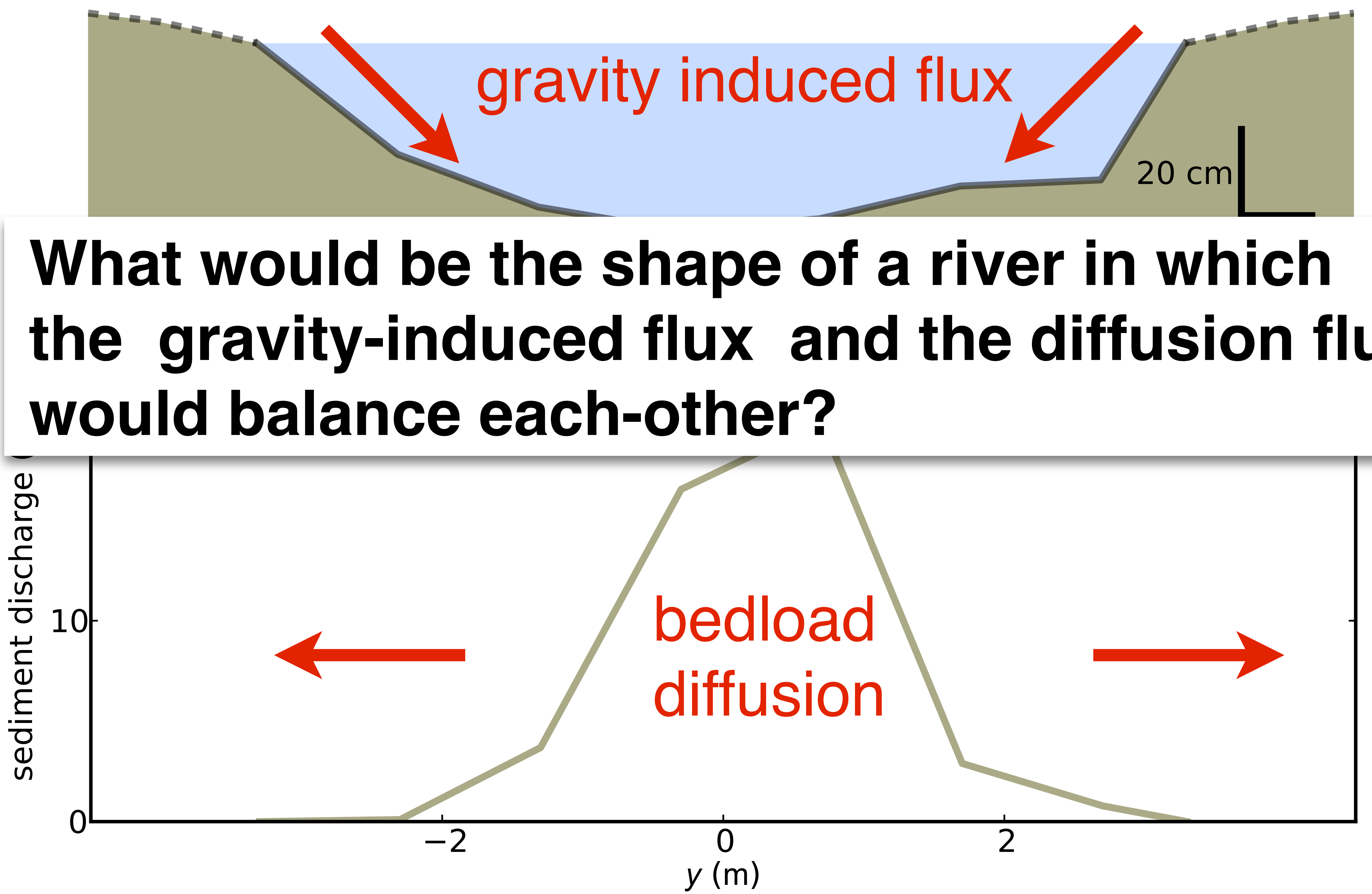


Urumqi He, Tian-Shan, China

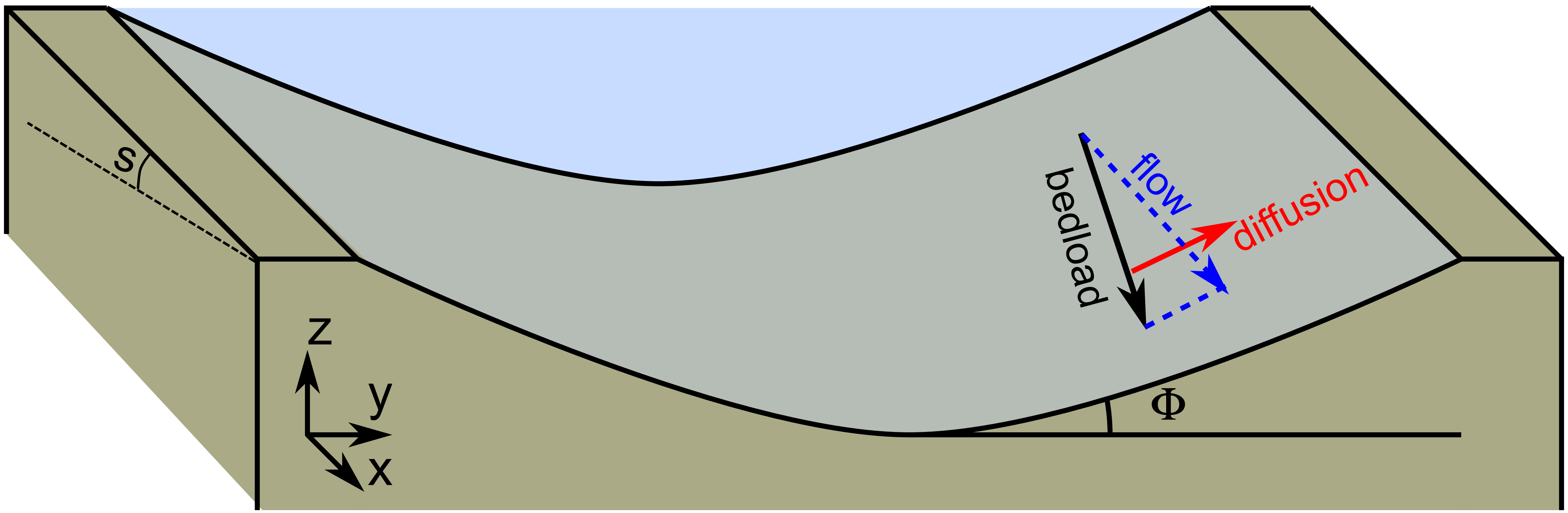
Do gradients of bedload particles exist in nature ?



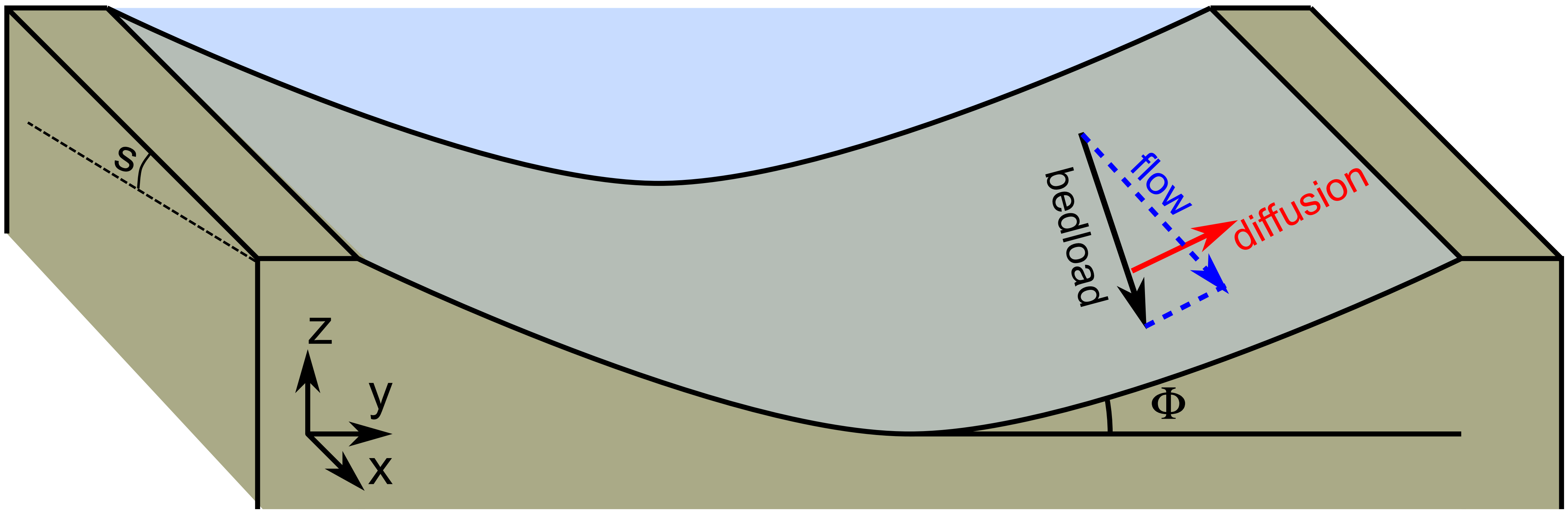
What would be the shape of a river in which the gravity-induced flux and the diffusion flux would balance each-other?

Urumqi He, Tian-Shan, China

Equilibrium with sediment transport



Equilibrium with sediment transport



$$Pe\left(\sqrt{\tilde{D}^2+\tilde{D}'^2}-\mu\right)\left(\tilde{D}^2+\tilde{D}'^2\right)-\tilde{D}^2\left(\tilde{D}+\tilde{D}''\right)=0$$

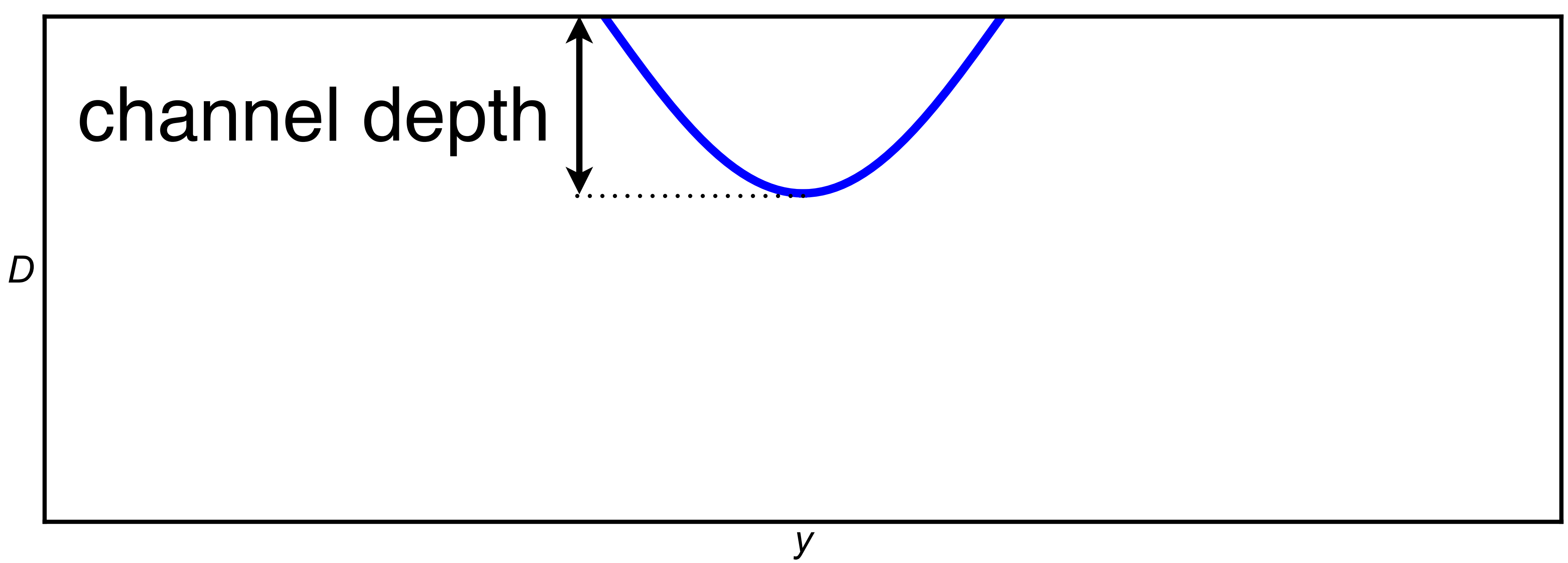
$$\tilde{D}=\frac{S}{L}D$$ dimensionless depth

$$Pe=\frac{L}{\ell_d}\frac{1}{S}$$ Peclet number = advective / diffusive bedload

Numerical solution

2 explicit parameters : Pe and channel depth

Pe = 2

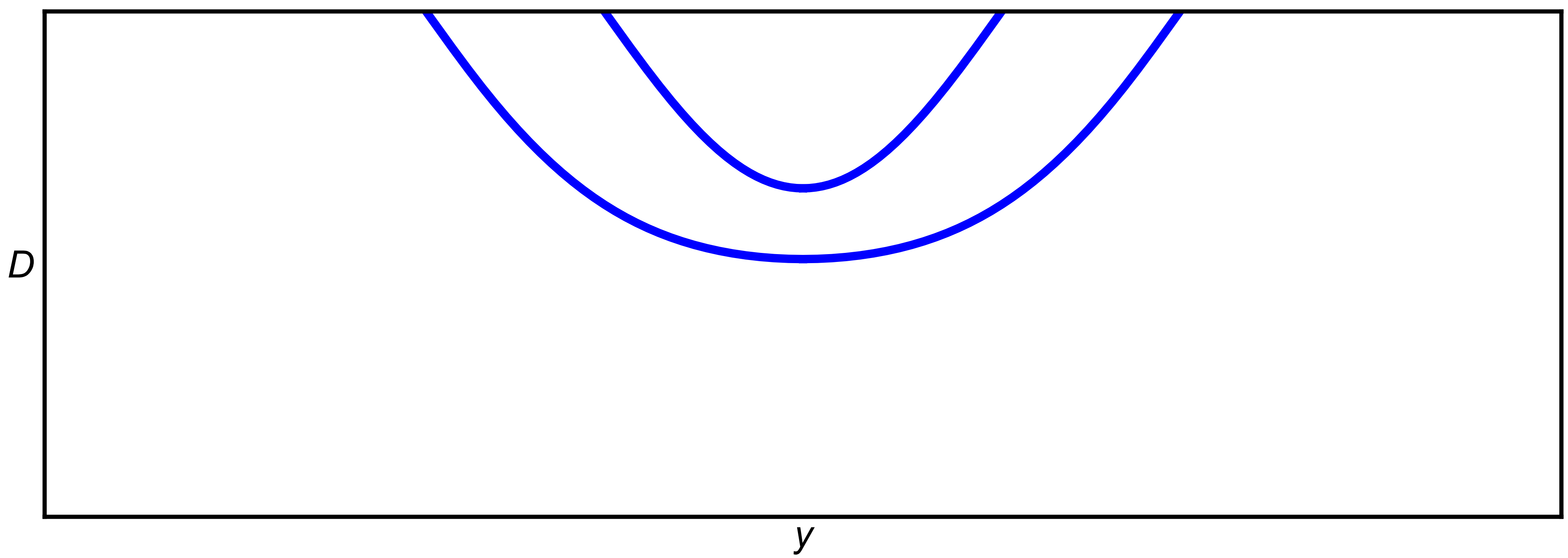


cosine solution for the 0 transport case

Numerical solution

2 explicit parameters : Pe and channel depth

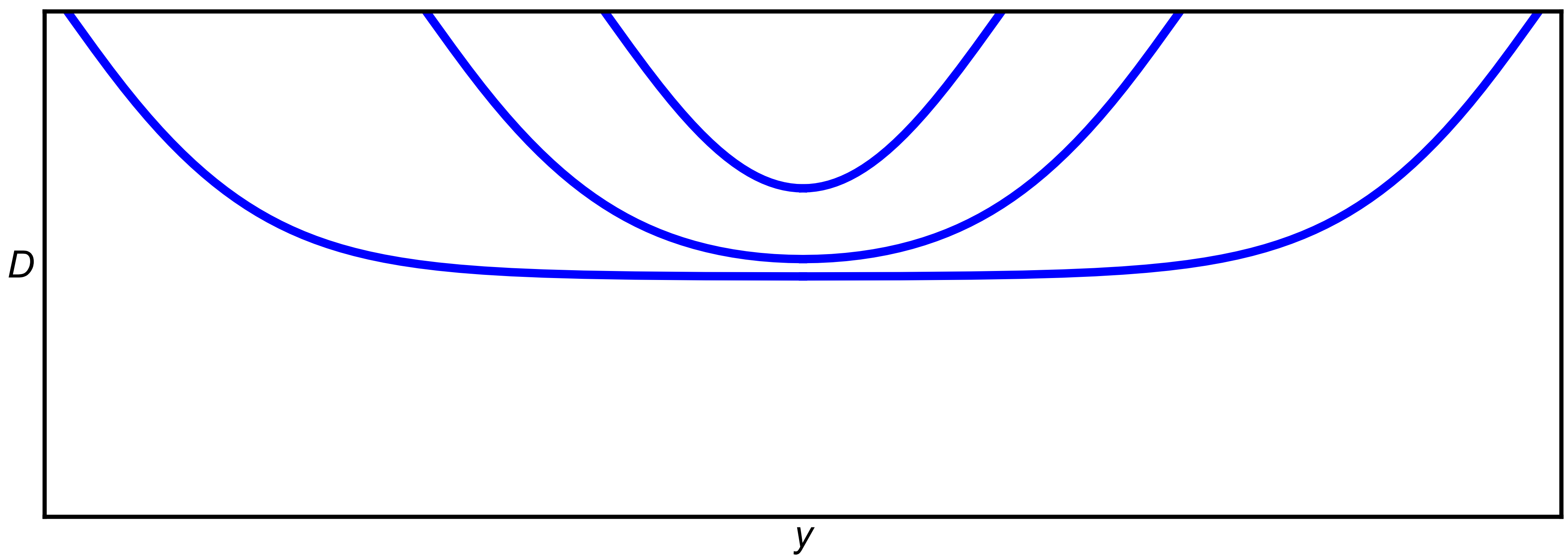
Pe = 2



Numerical solution

2 explicit parameters : Pe and channel depth

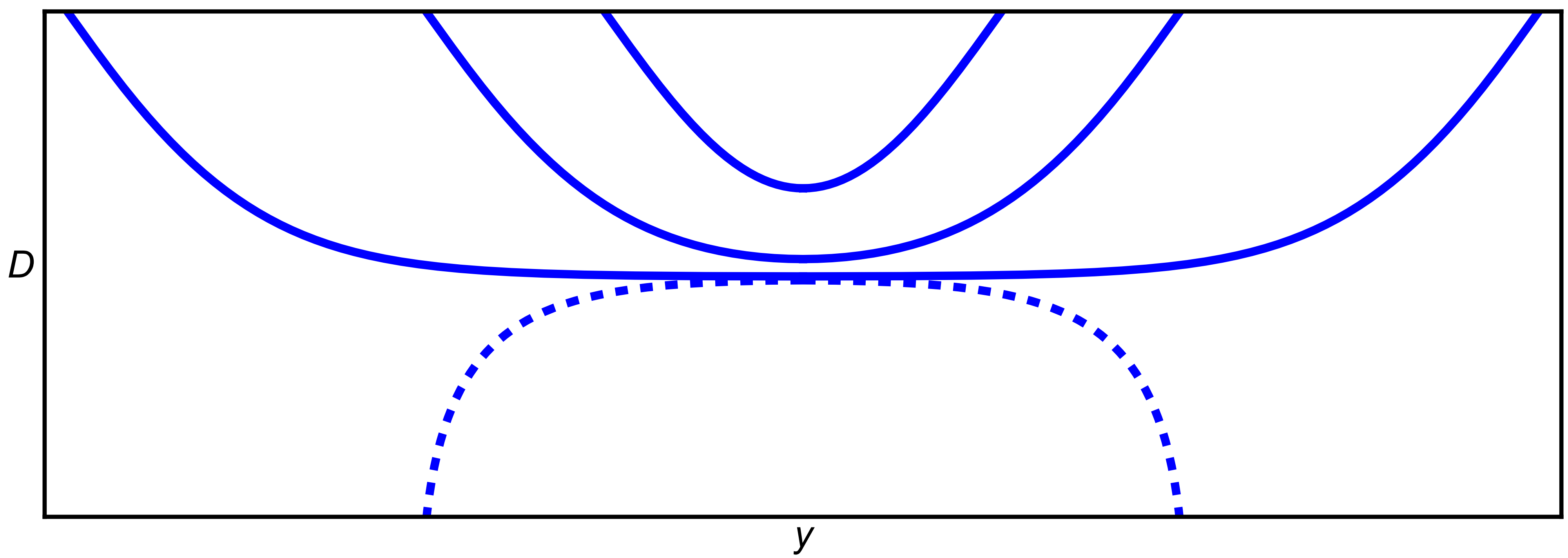
Pe = 2



Numerical solution

2 explicit parameters : Pe and channel depth

Pe = 2

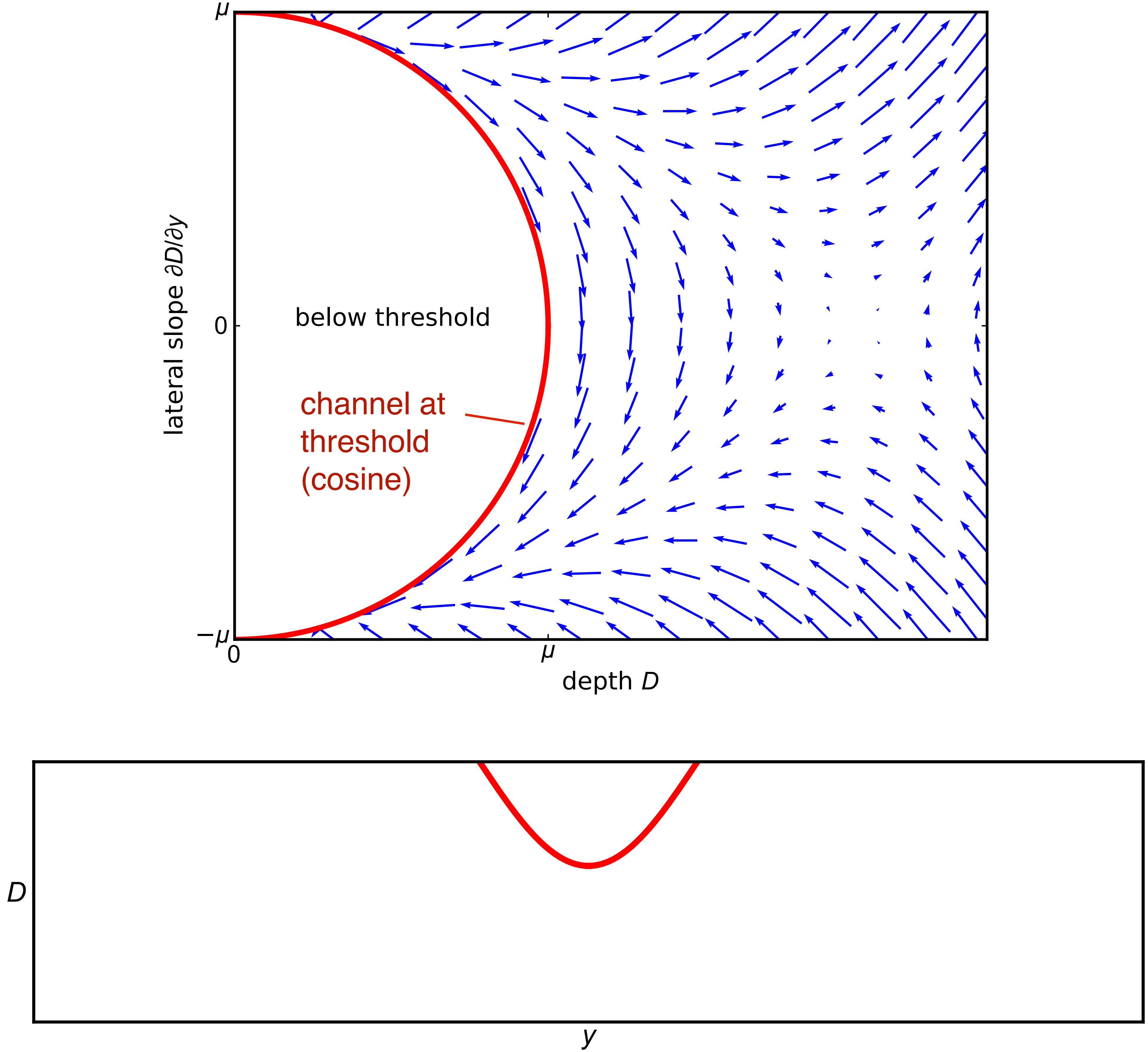


Numerical solution

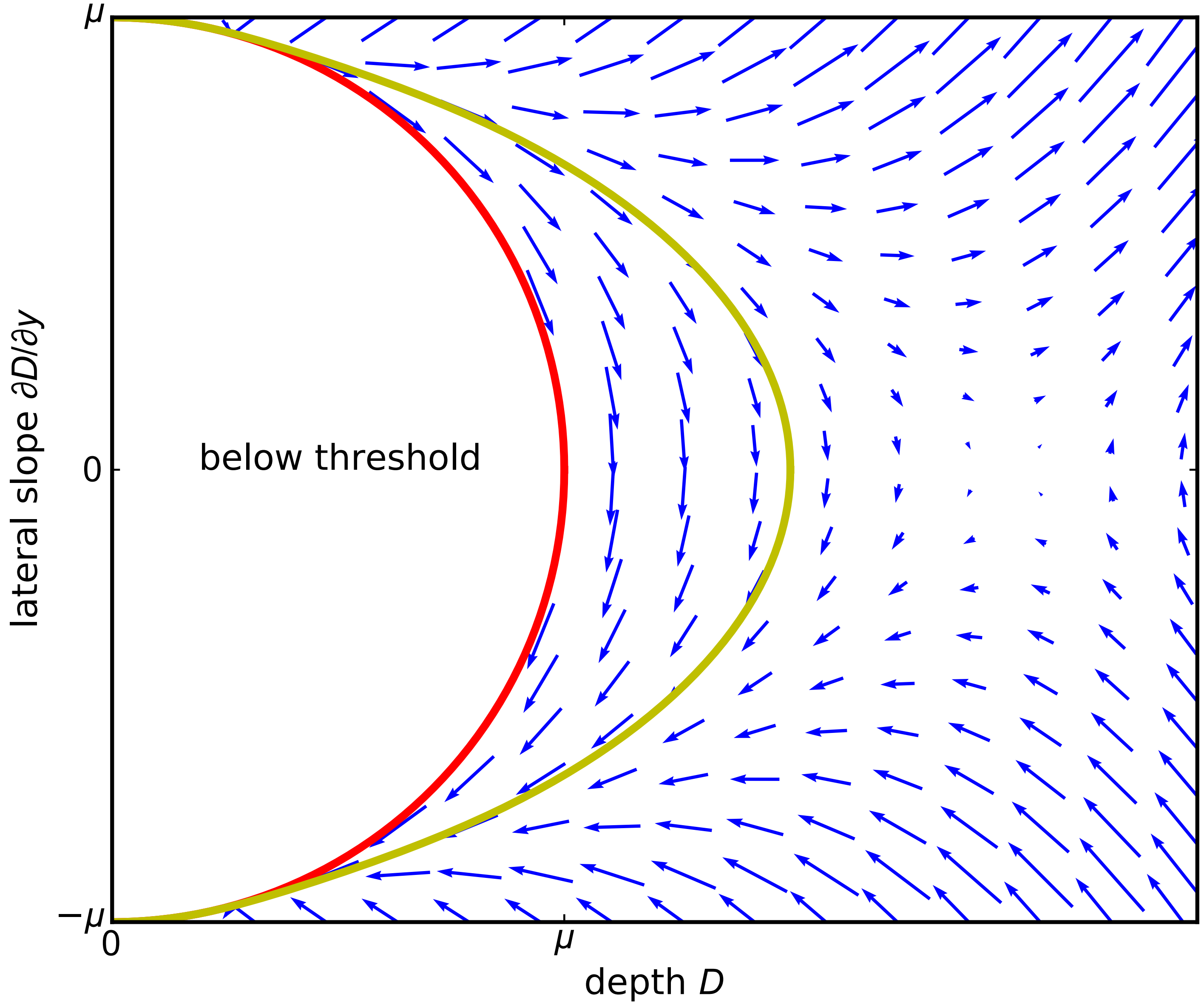
The theoretical solution depends explicitly on Pe and the channel depth ....

whereas we are interested in the evolution of the river morphology as a function of the flow and sediment discharges!

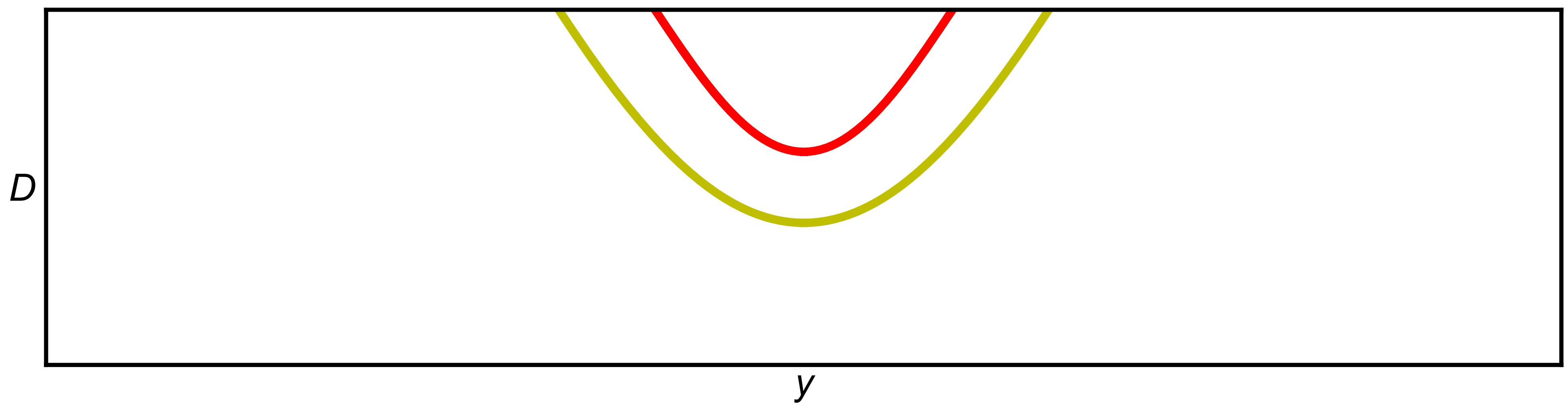
Phase space


μ
0
−μ
lateral slope ∂D/∂y
below threshold
channel at
threshold
(cosine)
0
μ
depth D
D
y

Phase space

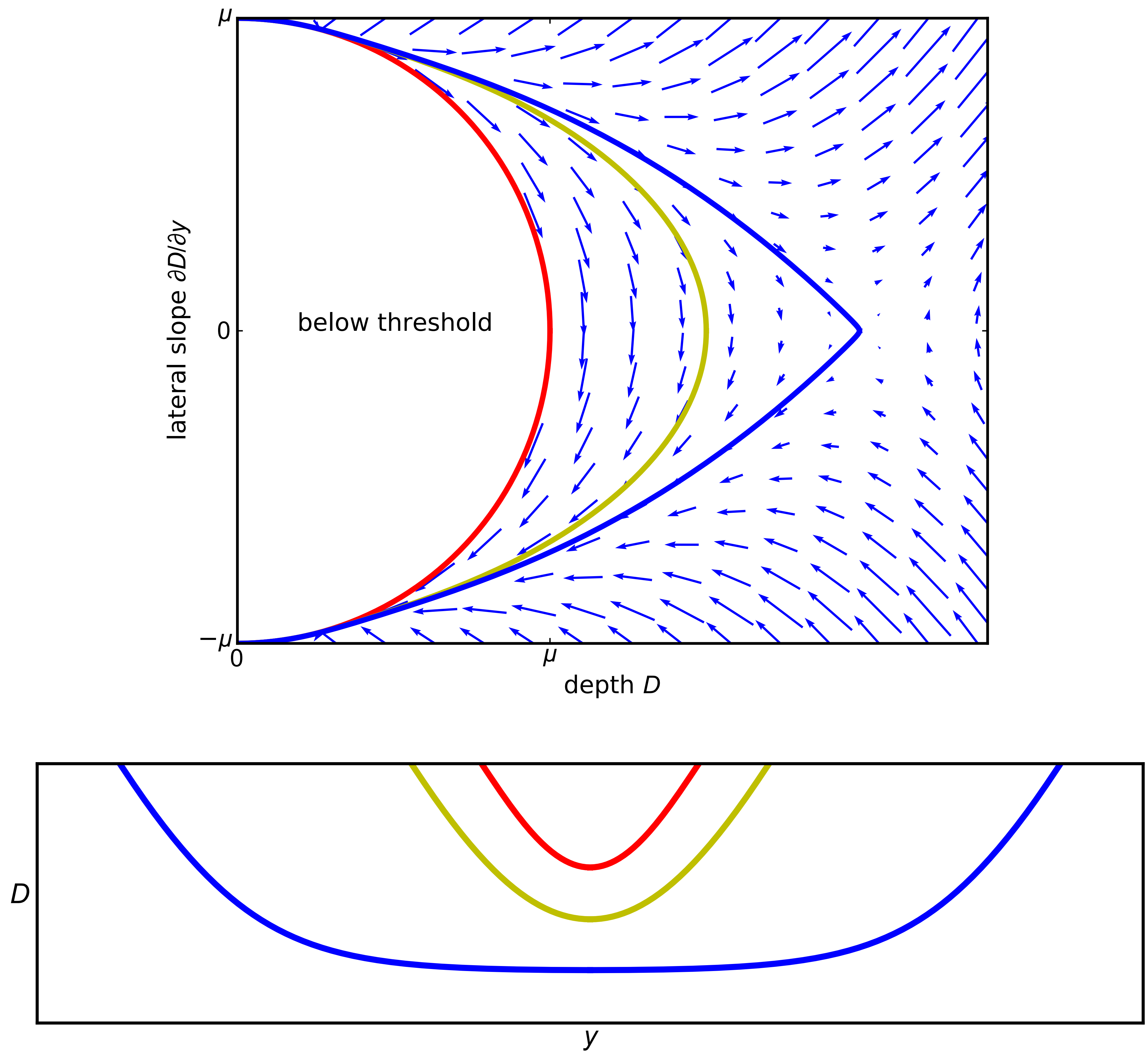


μ
0
$-\mu$
lateral slope $\partial D/\partial y$
below threshold
0
μ
depth D

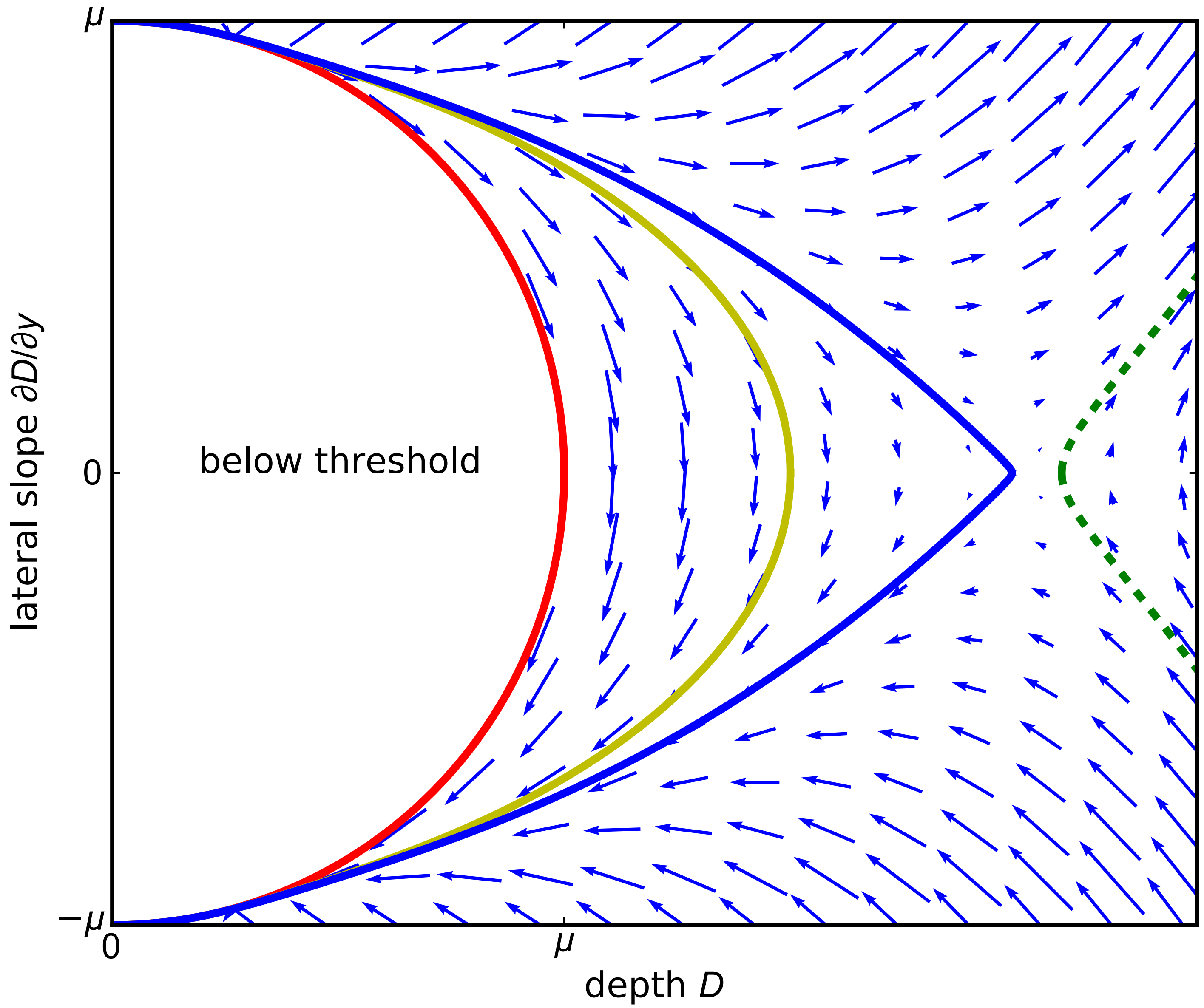


D
y

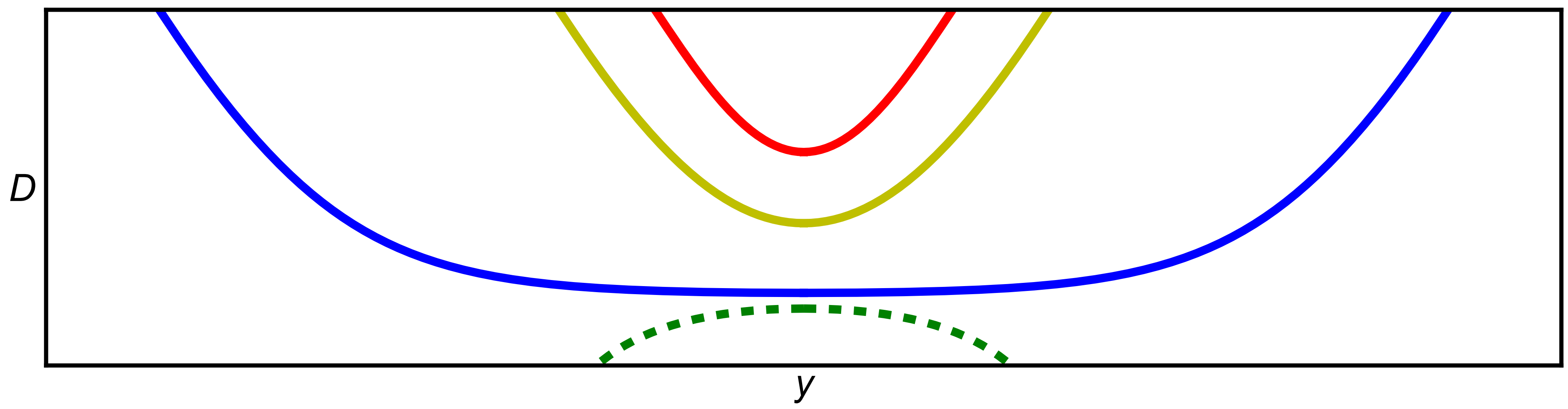
Phase space



Phase space

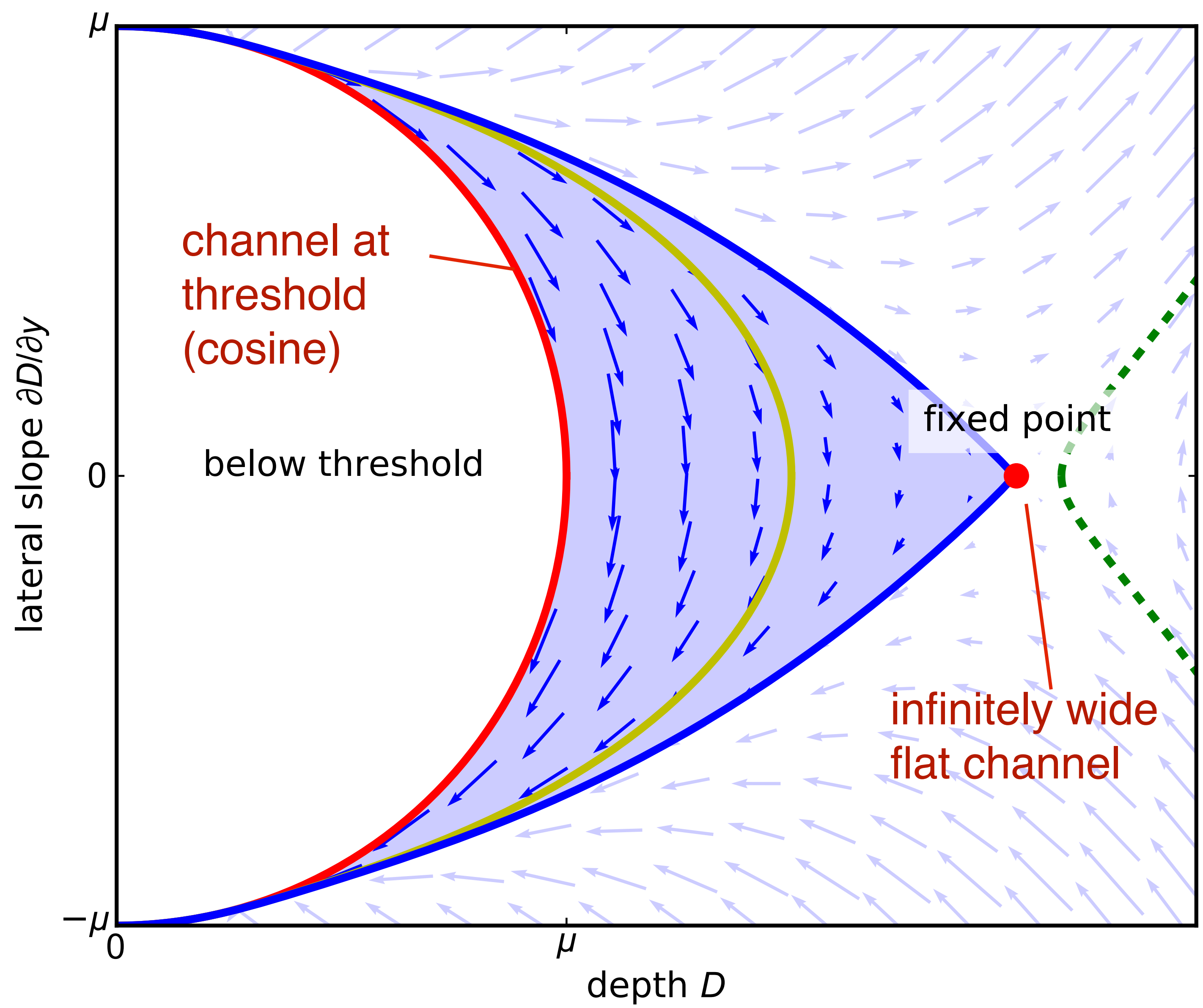


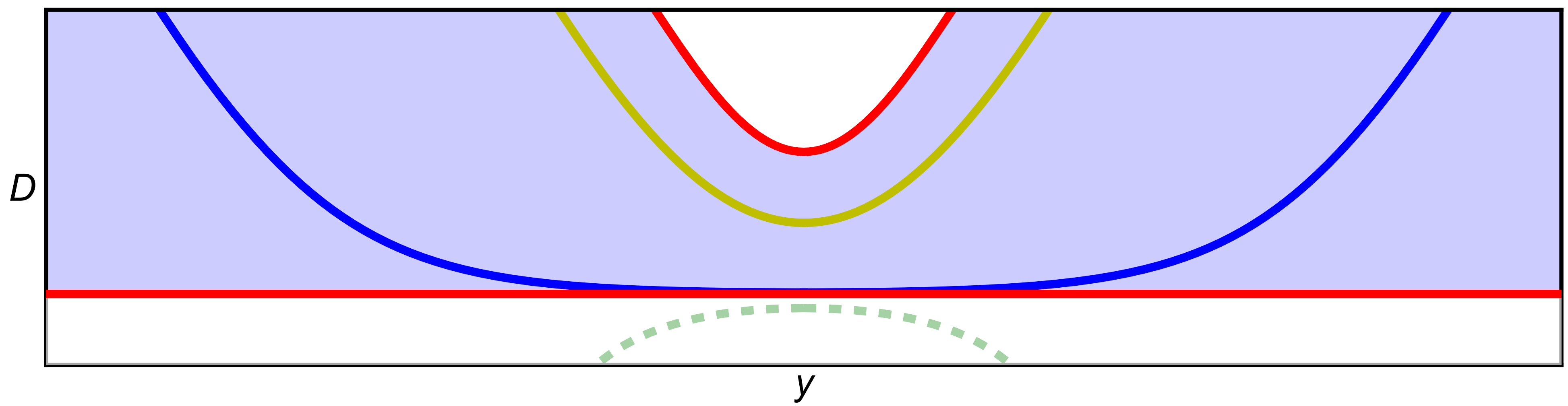
μ
0
$-\mu$
lateral slope $\partial D/\partial y$
below threshold
0
μ
depth D



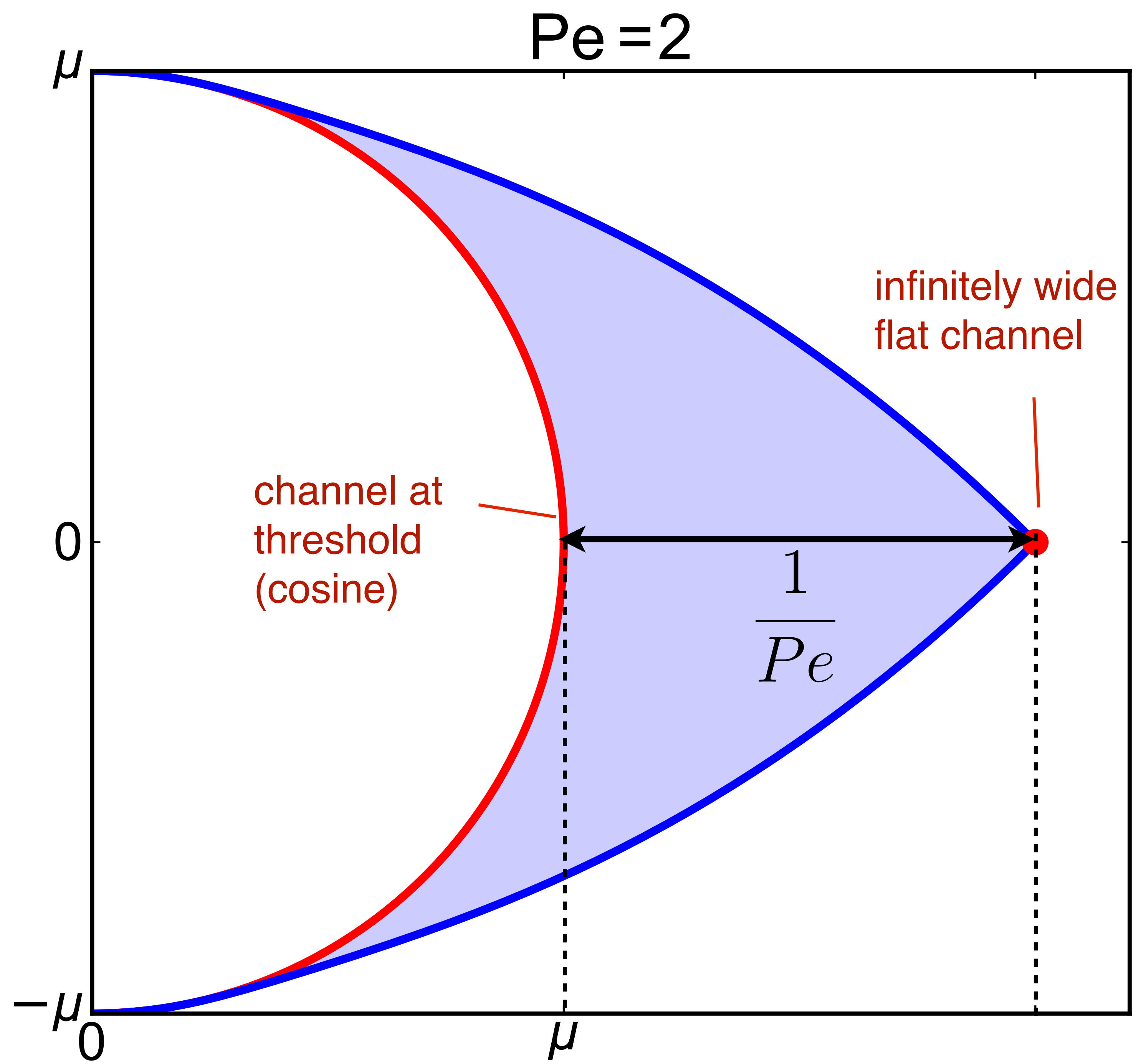
D
y

Phase space


μ
0
−μ
0
μ
lateral slope ∂D/∂y
depth D
channel at
threshold
(cosine)
below threshold
fixed point
infinitely wide
flat channel

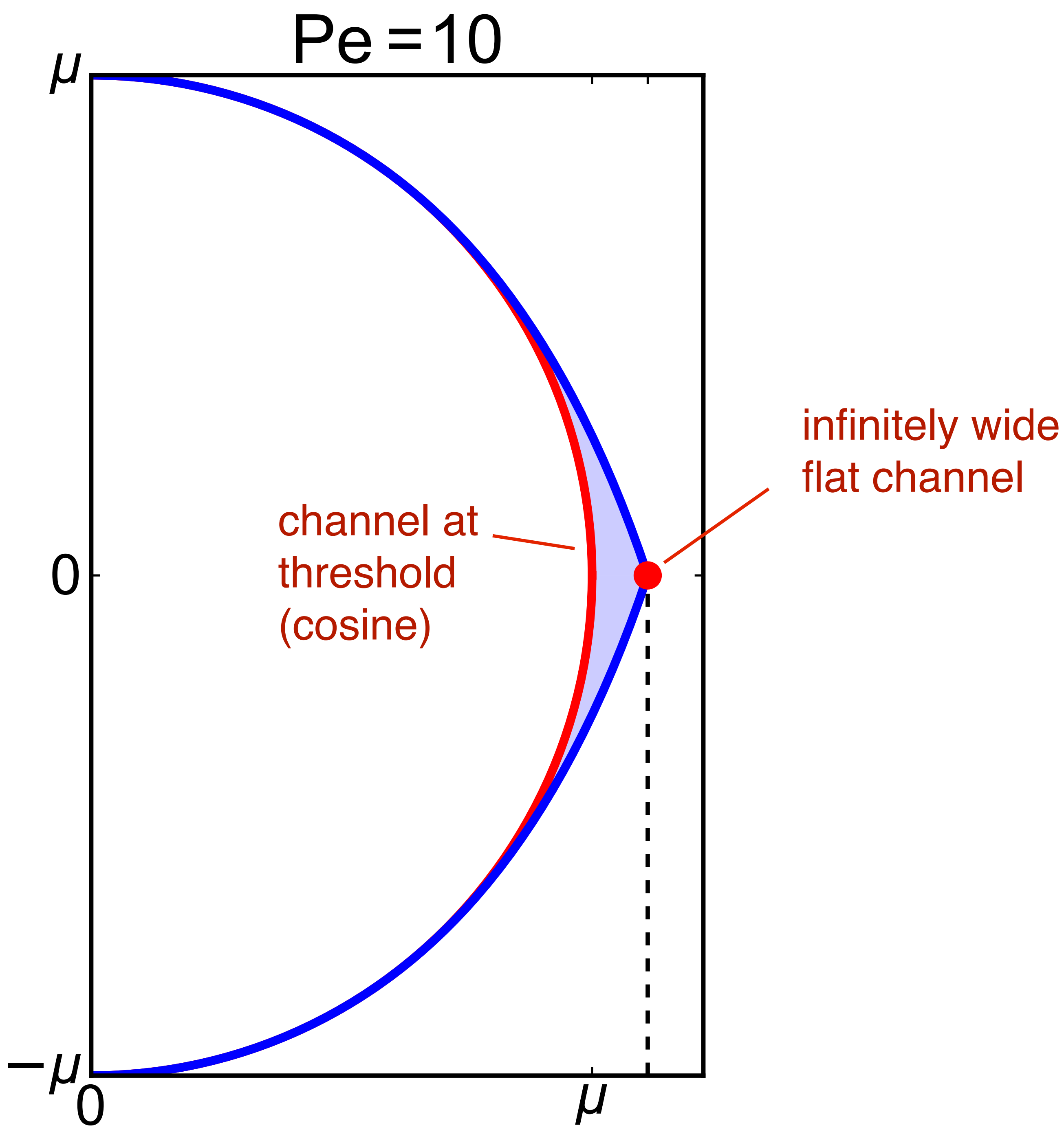

D
y

Phase space



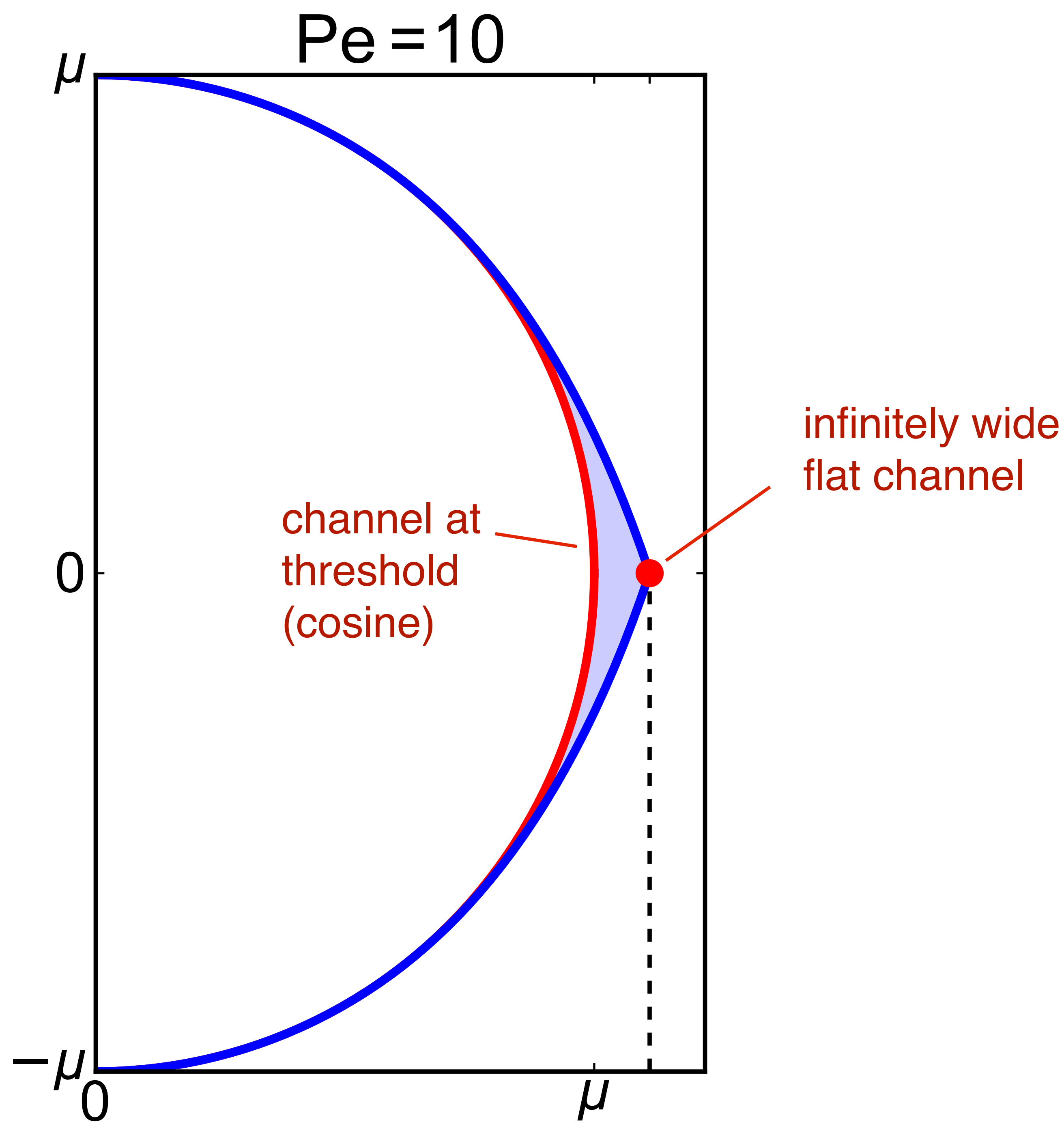
Pe =2
μ
0
$-\mu$
0
μ
infinitely wide
flat channel
channel at
threshold
(cosine)
$\frac{1}{Pe}$

Phase space



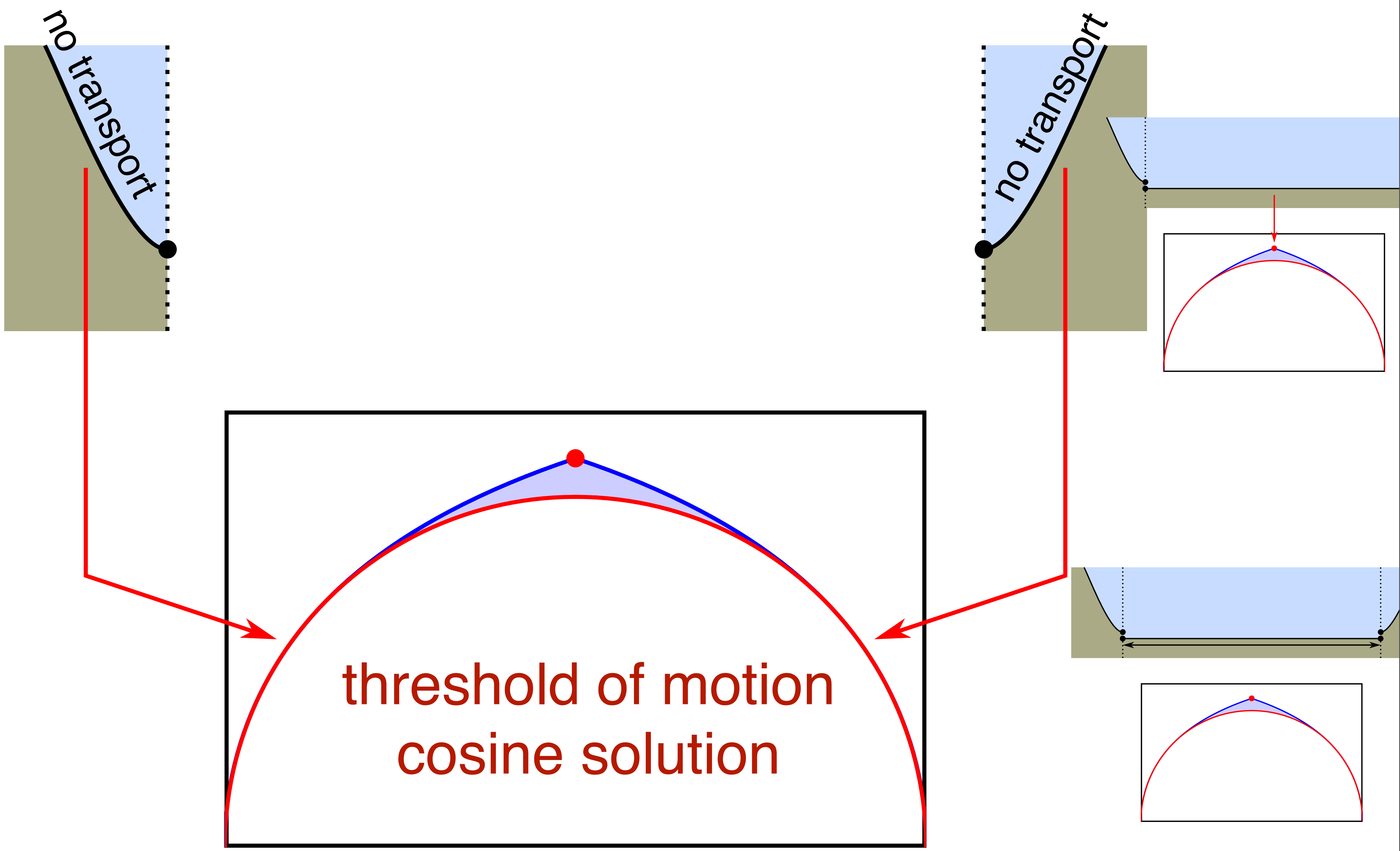
Pe = 10
μ
0
−μ
0
μ
infinitely wide
flat channel
channel at
threshold
(cosine)

Phase space

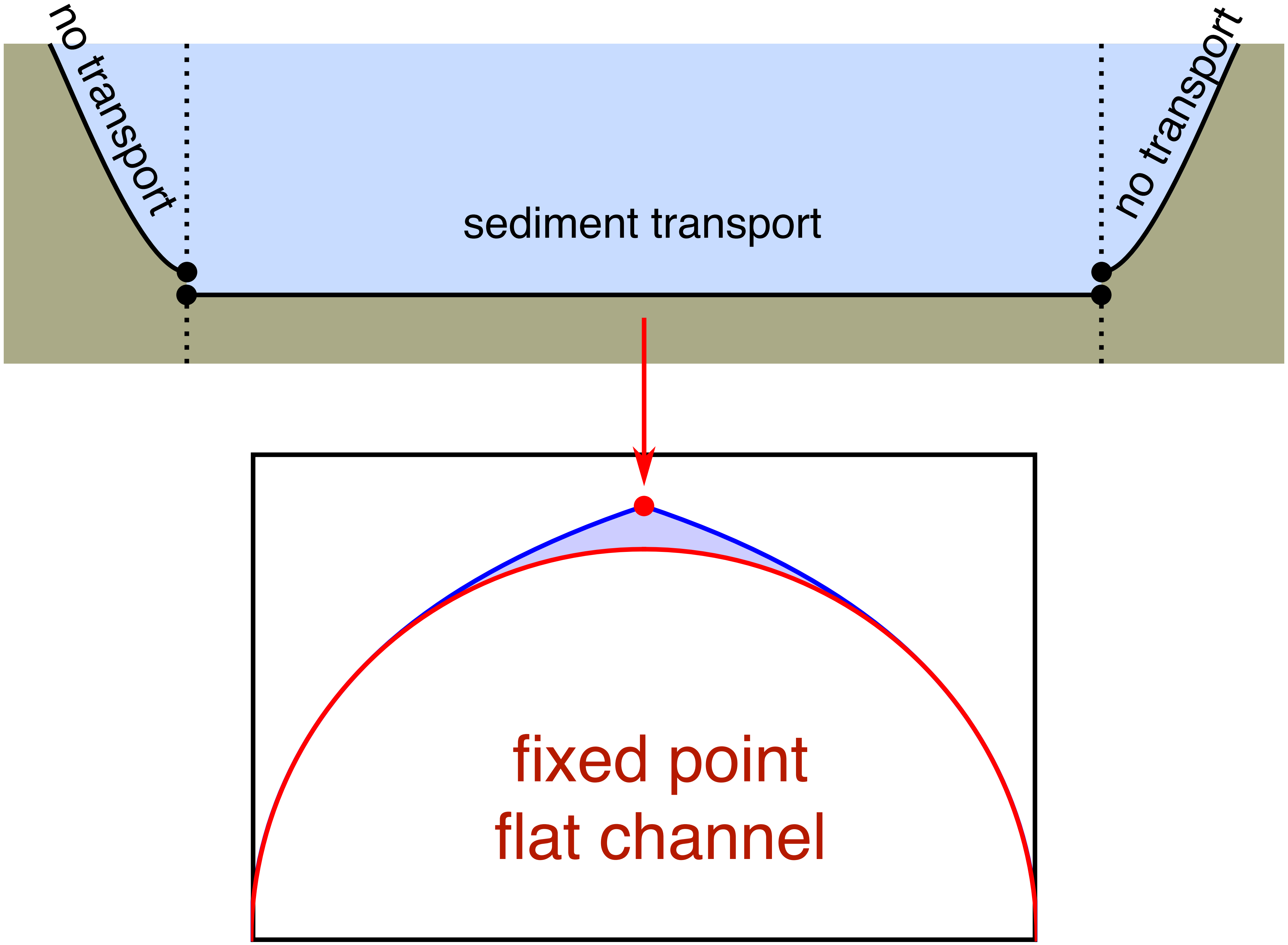


Experiments → $Pe = \frac{L}{\ell_d}\,\frac{1}{S} \sim \frac{d_s}{10^{-2}\,d_s}\,\frac{1}{10^{-2}} \sim 10^4$

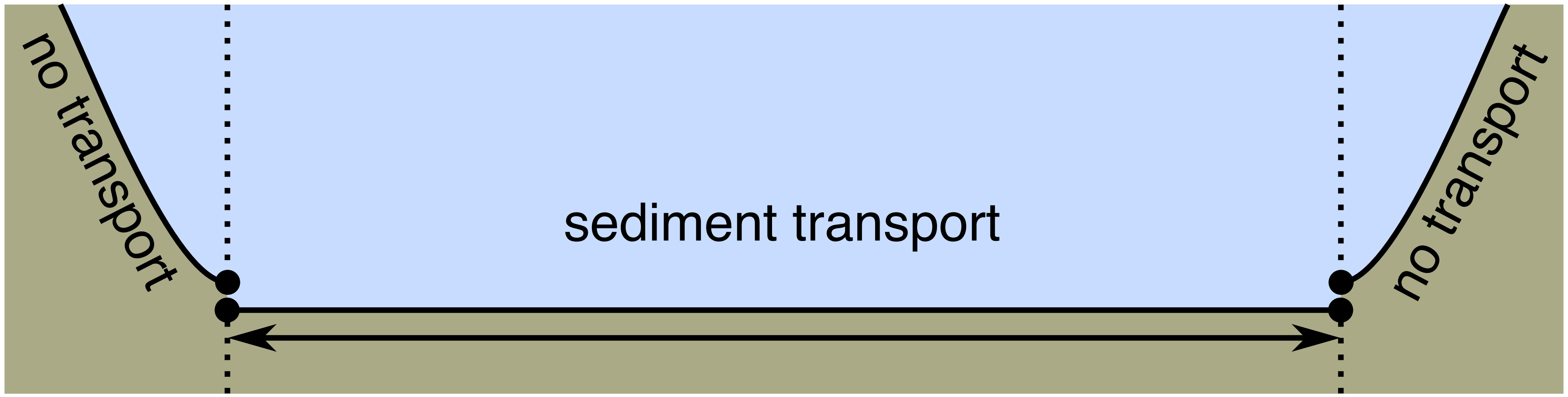
‘flat river’ approximation’



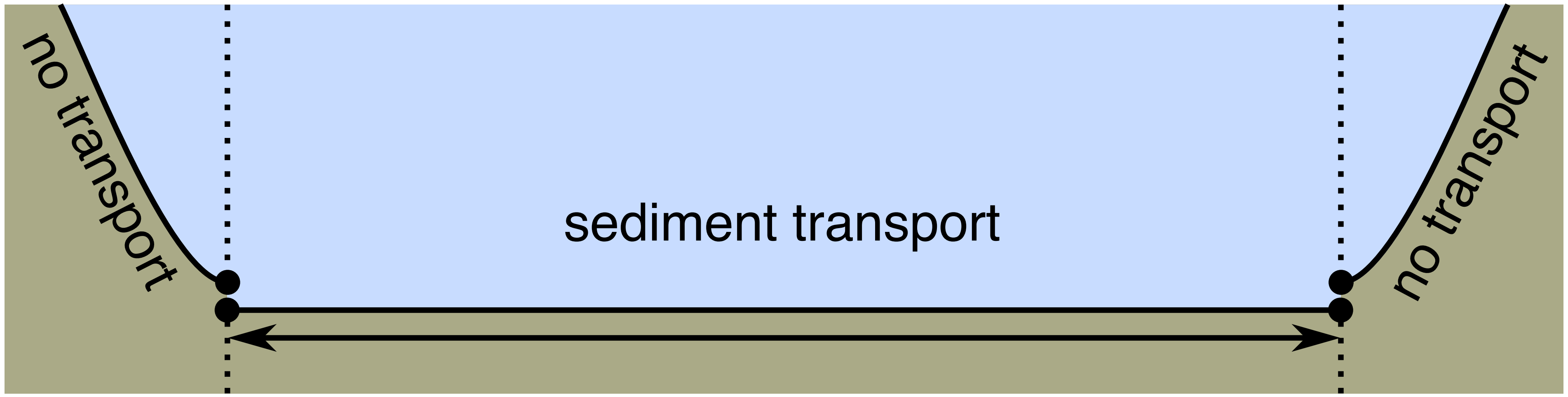
‘flat river’ approximation’



‘flat river’ approximation’



‘flat river’ approximation’



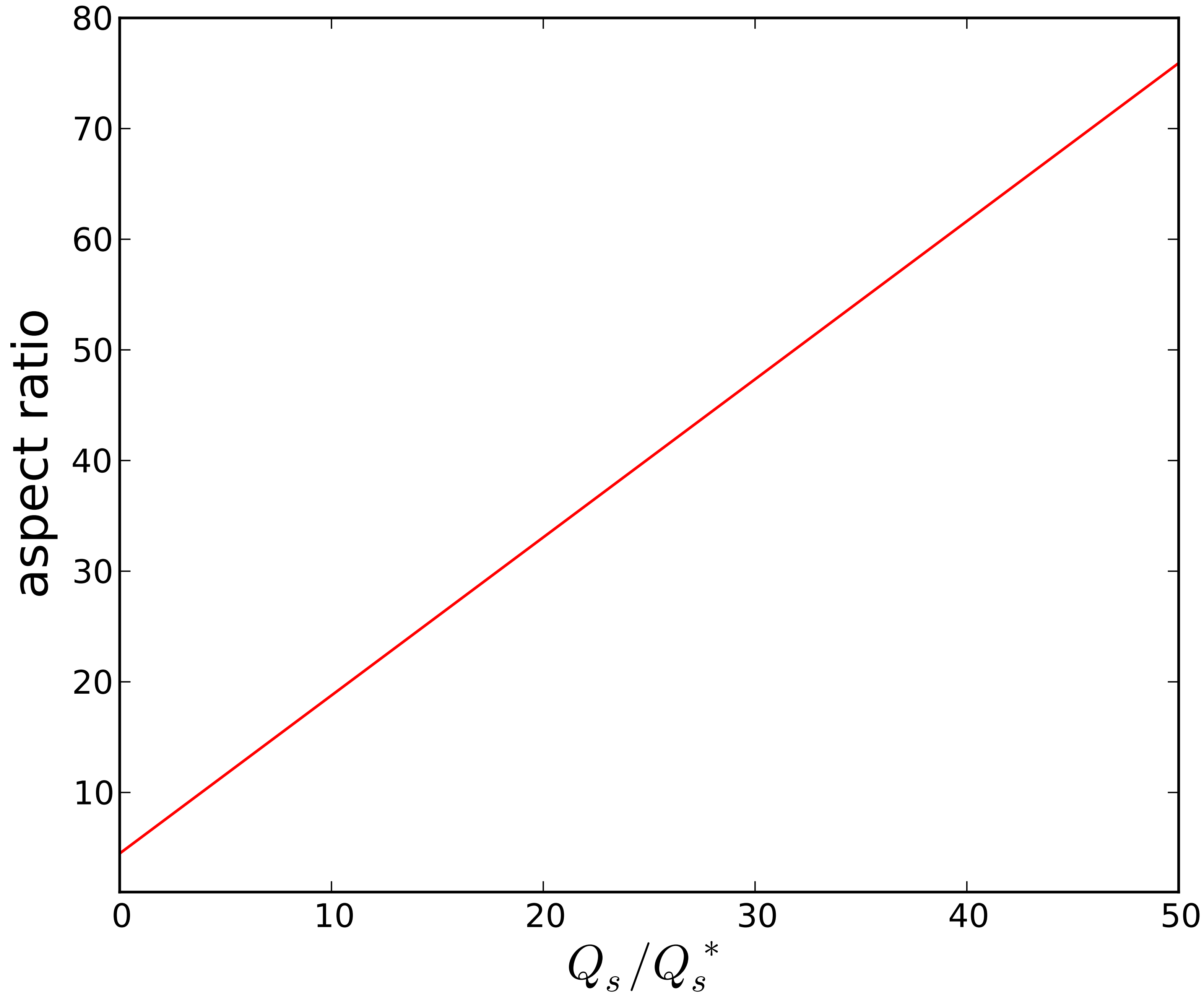
For a laminar flow :

$$S = \left(\frac{4g\mu^3 L}{9\nu}\right)^{1/3} L\, Q_w^{-1/3} \left(1 + \frac{3}{4}\frac{Q_b}{Q_b^*}\right)^{1/3}$$

$$W = \pi \left(\frac{4g\mu^3 L}{9\nu}\right)^{-1/3} Q_w{}^{1/3} \left(1 + \frac{1}{\pi}\frac{Q_b}{Q_b^*}\right)^{1/3} \left(1 + \frac{3}{4}\frac{Q_b}{Q_b^*}\right)^{-1/3}$$

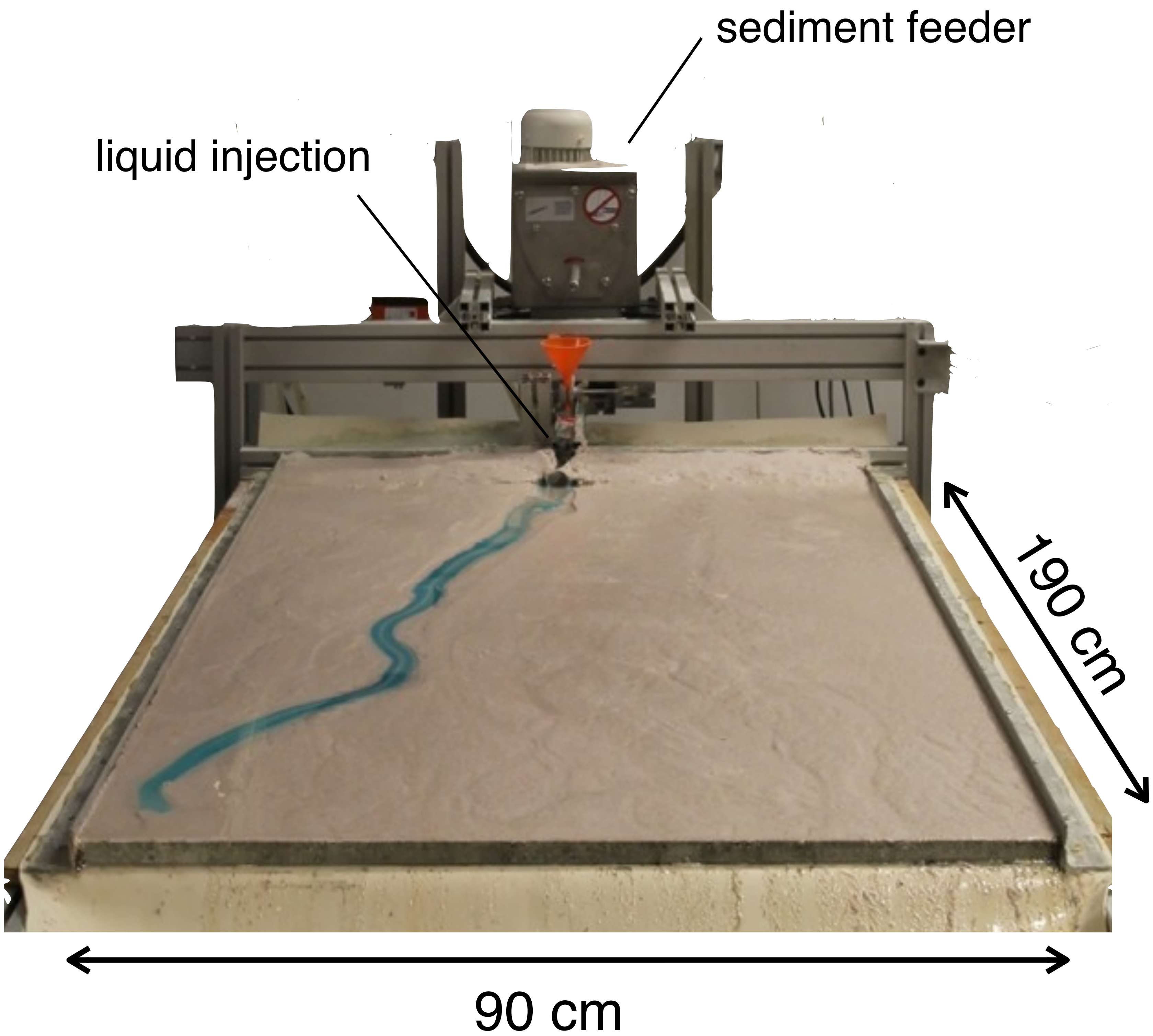
$$\frac{W}{D_0} = \frac{\pi}{\mu}\left(1 + \frac{1}{\pi}\frac{Q_b}{Q_b^*}\right) \qquad Q_b^* = \beta\,\frac{V}{d_s^2}\,\mu\,\ell_d$$

‘flat river’ approximation



$$\frac{W}{D_0} = \frac{\pi}{\mu}\left(1 + \frac{1}{\pi}\,\frac{Q_b}{Q_b^*}\right)$$

Laboratory rivers

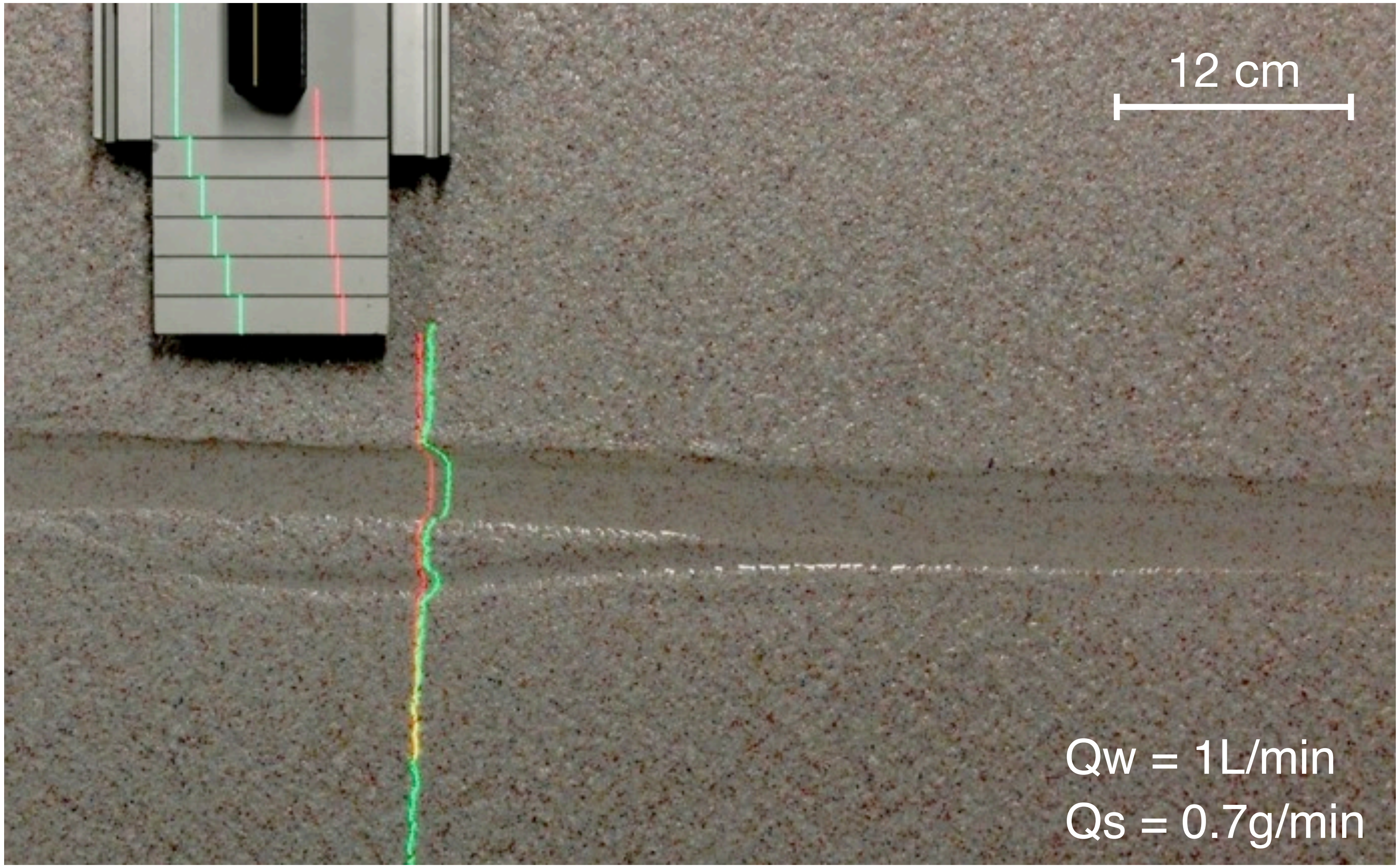


- constant discharge
- constant input sediment rate

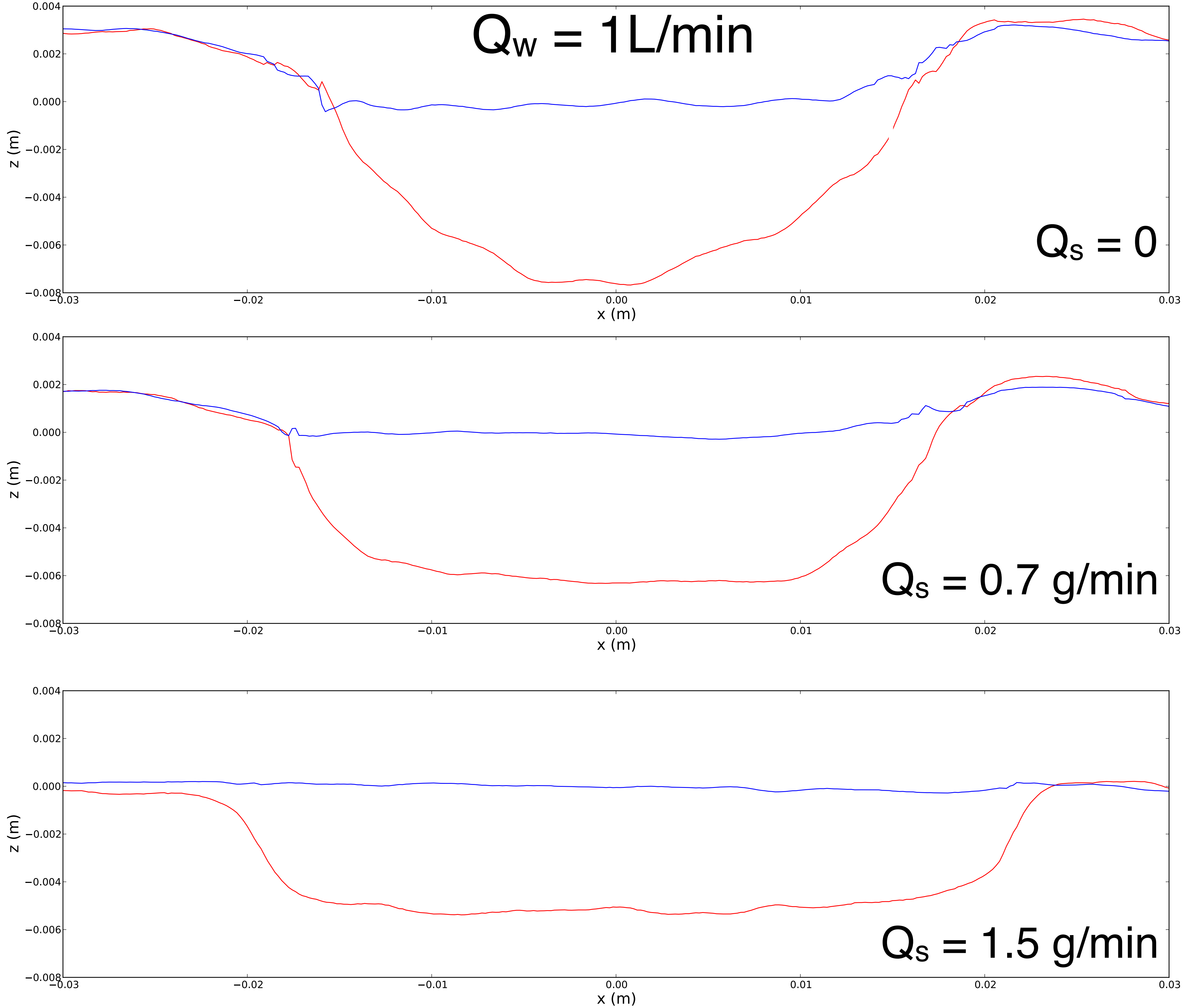
→ **steady-state**

Experiments

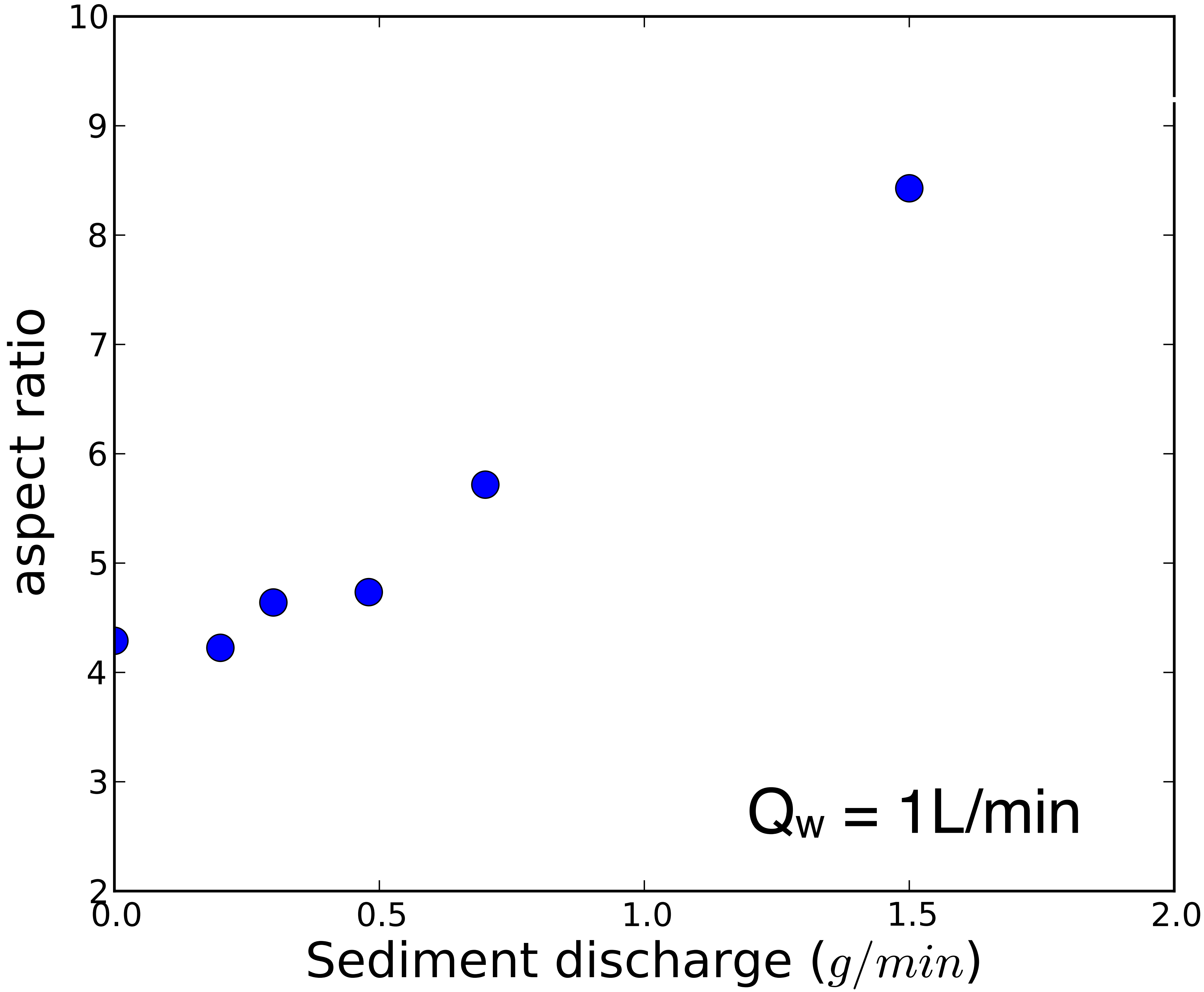
flow



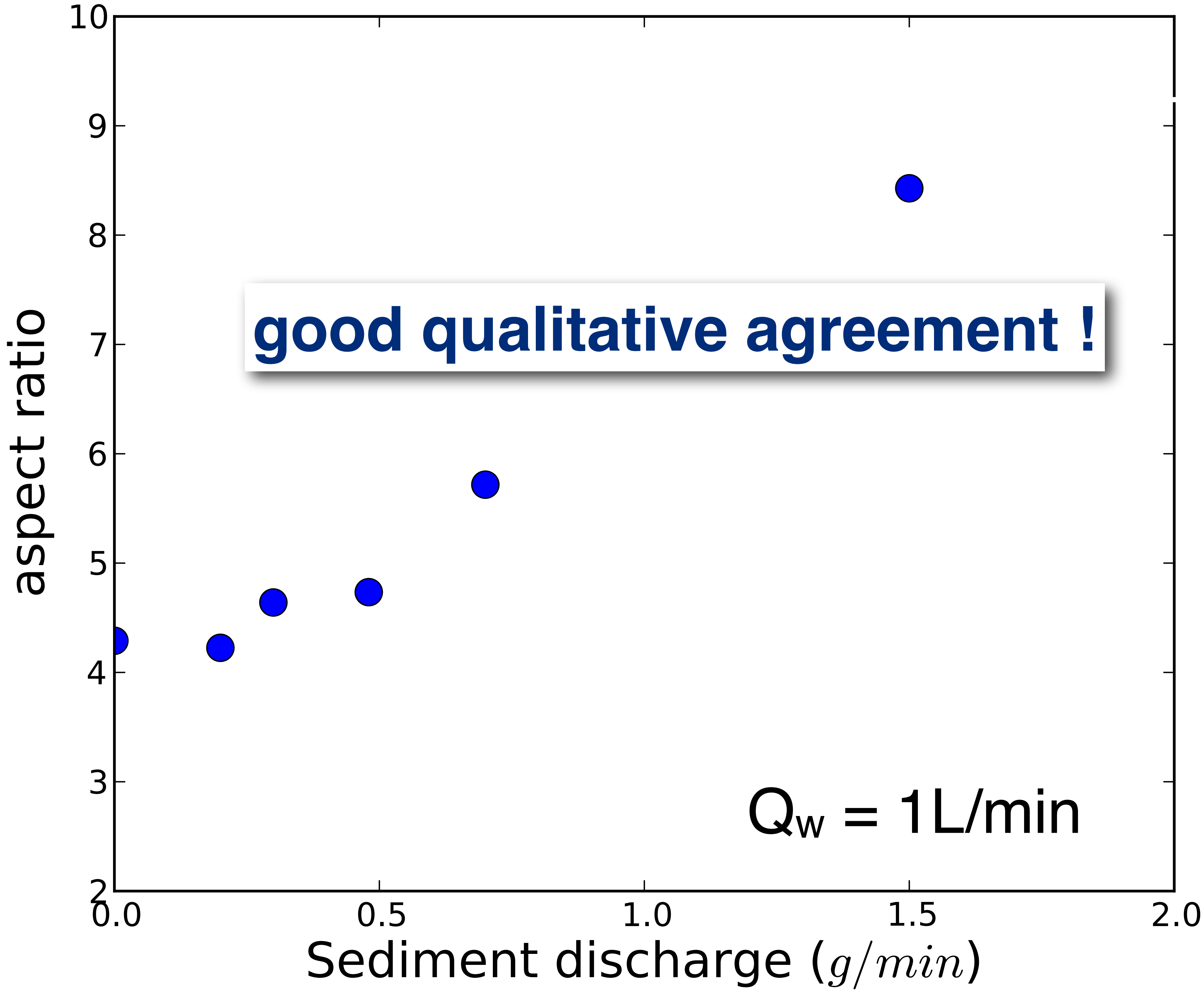
Experiments



Experiments



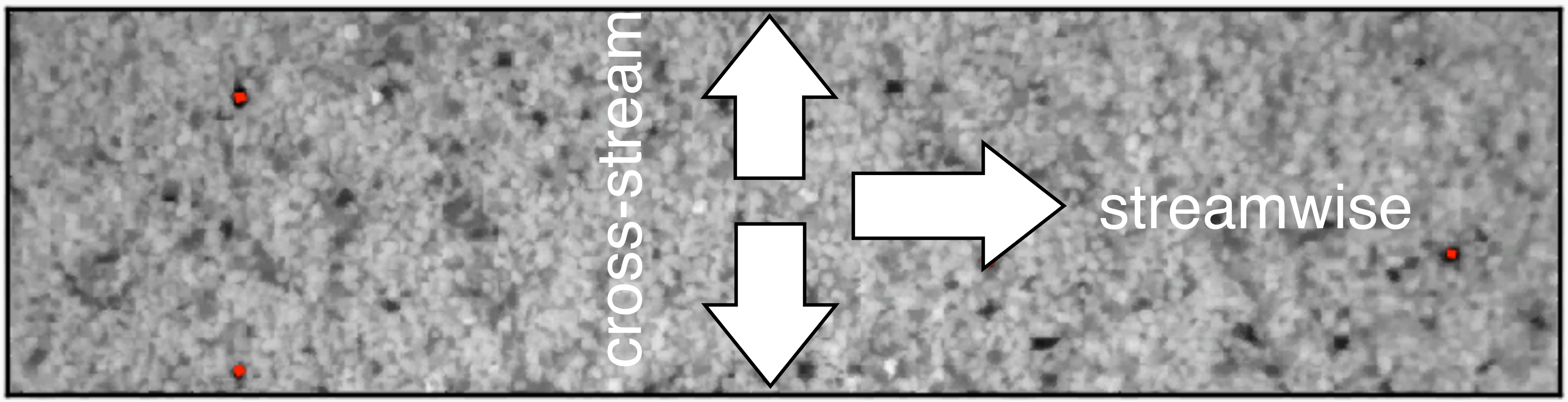
Experiments



good qualitative agreement !

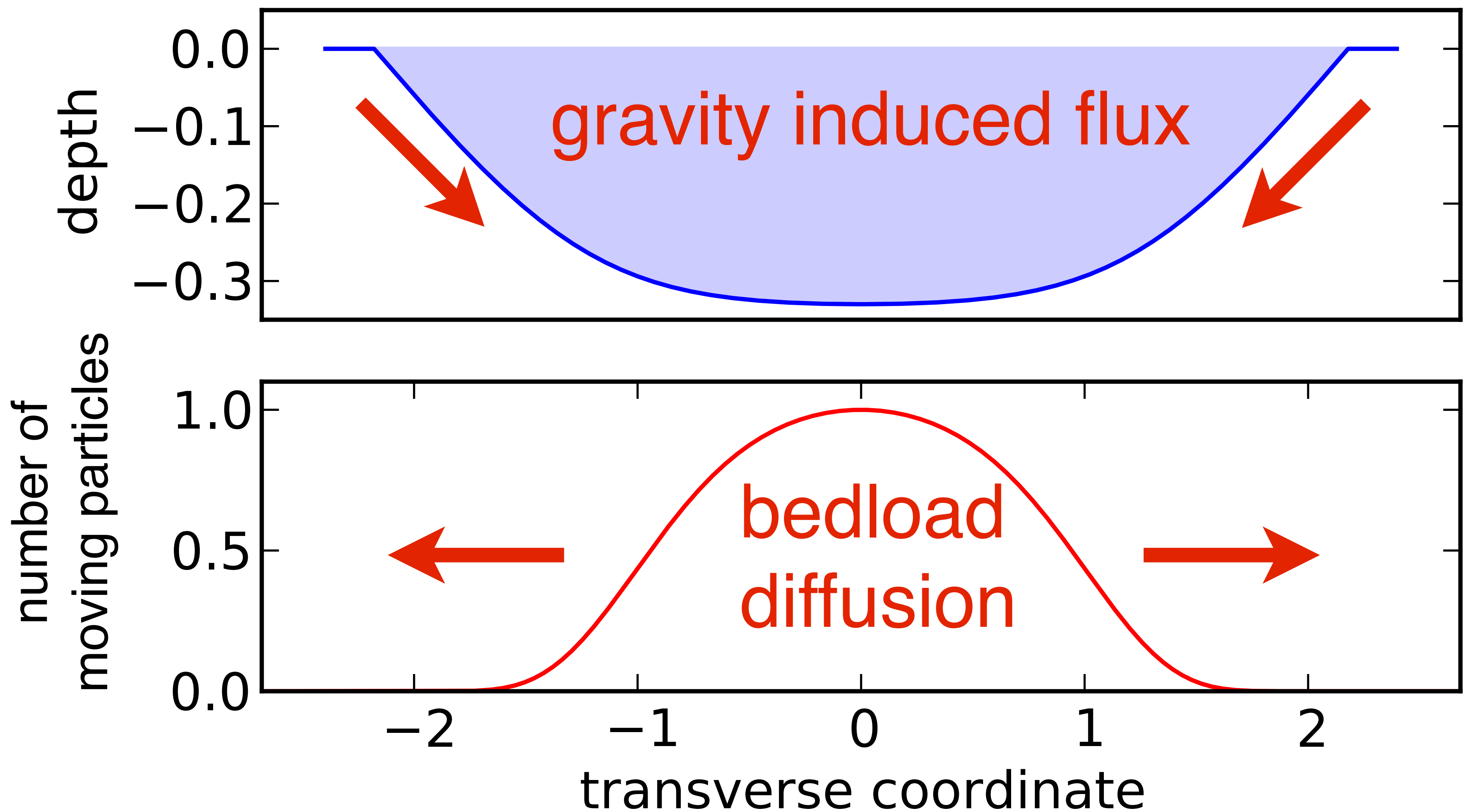
Summary

- **Bedload transport generates transverse diffusion.**



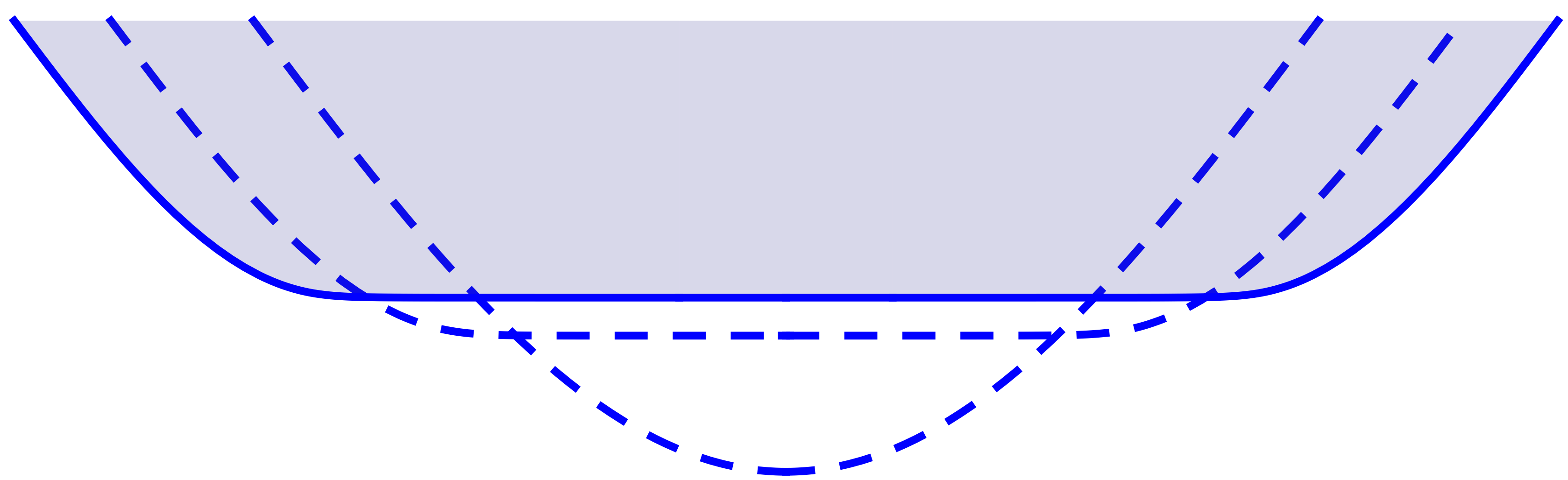
Summary

- **Bedload transport generates transverse diffusion.**
- **Equilibrium shape of rivers is selected by the balance between bedload diffusion and gravity induced flux.**



Summary

- **Bedload transport generates cross-stream diffusion.**
- **Equilibrium shape of rivers is selected by the balance between bedload diffusion and gravity induced flux.**
- **The width to depth ratio of the river increases with the sediment discharge.**



Summary

- Bedload transport generates cross-stream diffusion.
- Equilibrium shape of rivers is selected by the balance between bedload diffusion and gravity induced flux.
- The width to depth ratio of the river increases with the sediment discharge.
- Experimental (in)validation in progress !

How does this physical framework compare to field data ?

Field data

single thread gravel-bed rivers

Data compiled by Métivier & Barrier [2011]