Kavli Institute for Theoretical Physics
University of California, Santa Barbara

Fluid-Mediated Particle Transport in Geophysical Flows

Sep 23, 2013 - Dec 20, 2013

Kinetic theory applied to debris flow

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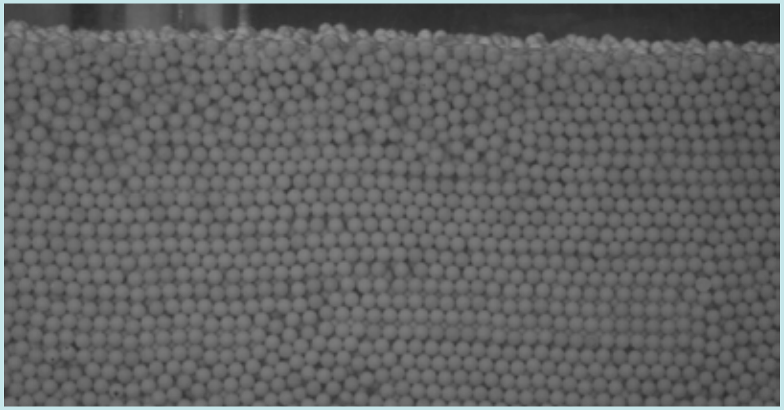
October 24, 2013

Ingredients

Nature

Experiments



Theory

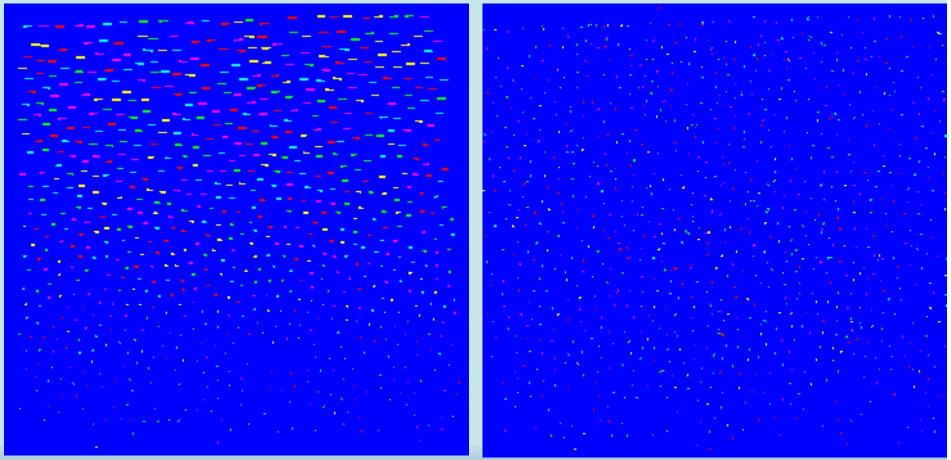
$$velocity = \ldots$$

$$flow\ \ depth = \ldots$$

$$concentration = \ldots$$

$$total\ volume = \ldots$$

Experimental methods



Ingredients

Nature

Experiments

Aronne Armanini, Univ. of Trento

Luigi Fraccarollo, Univ. of Trento

Theory

Jim Jenkins, Cornell University

Experimental methods

Hervé Capart, National Taiwan Univ.

Benoit Spinewine, Univ. catholique de Louvain

Experiments

Recirculating flume

Journal of Fluid Mechanics 532, 269-319 (2005)

Journal of Hydraulic Research 45, 59-71 (2007)

Granular Matter, 9, 145-157 (2007)

Powder Technology, 182: 218-227 (2008)

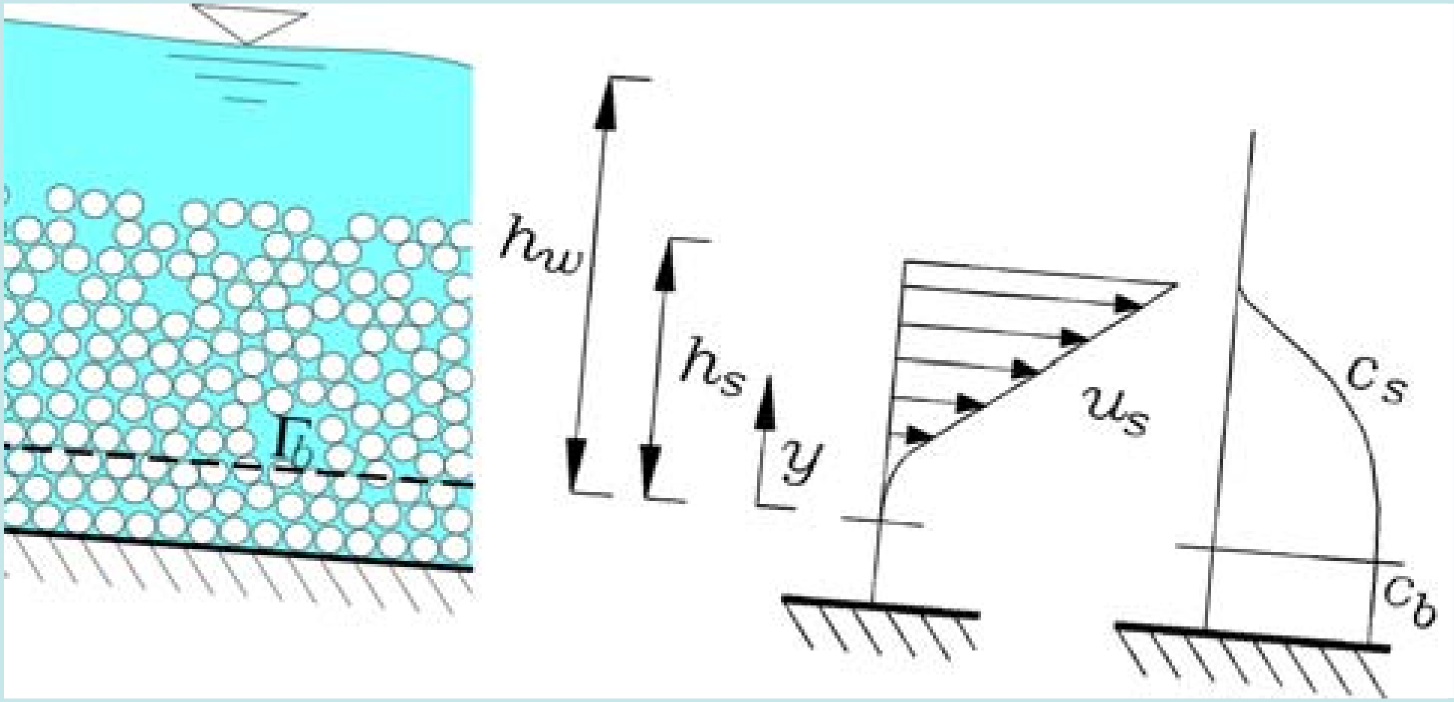
Physical Review E, 79, 051306 (2009)

Experiments

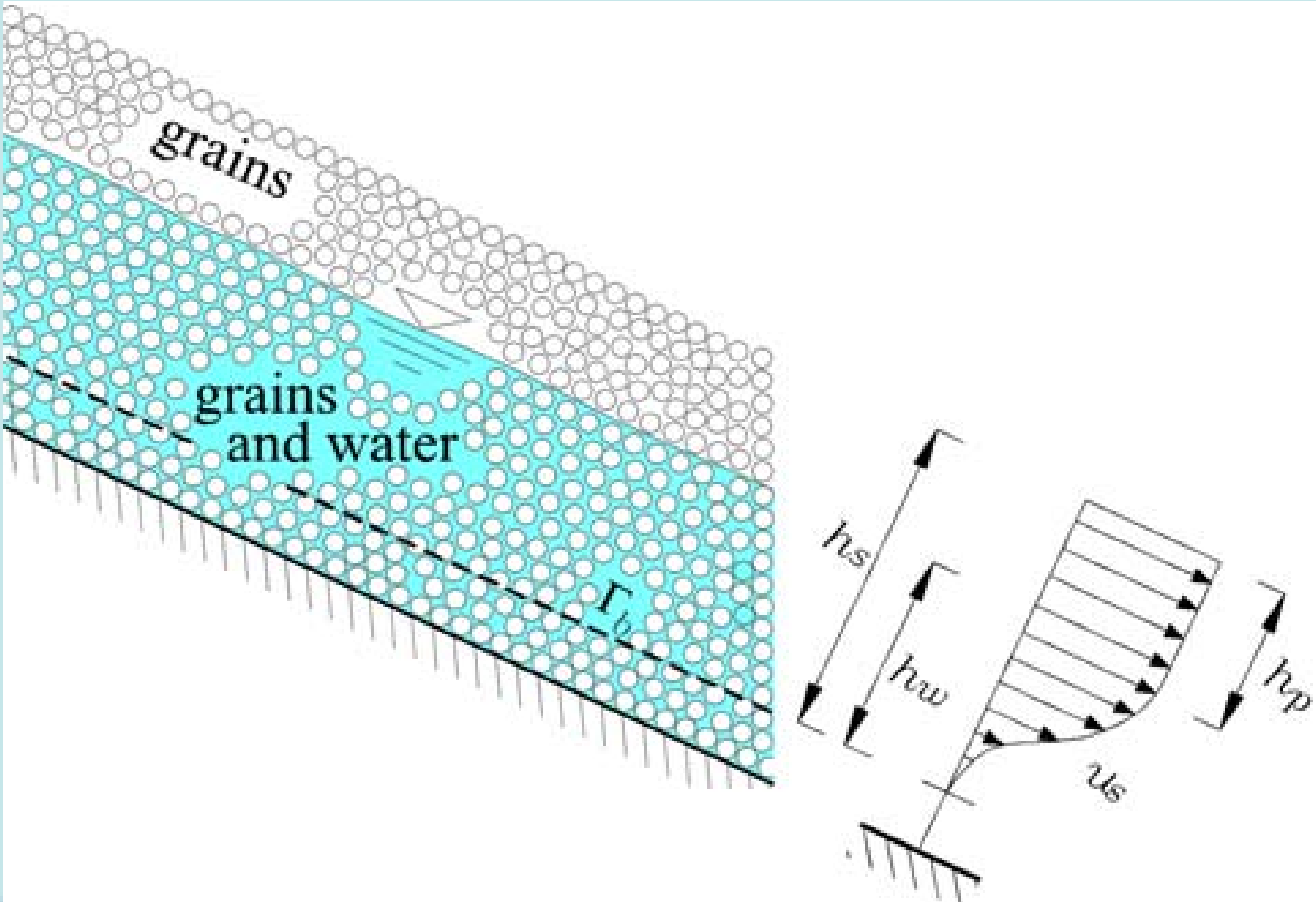
Steady, Fully-Developed Flow over a mobile bed

- Over-saturated: water depth greater than grain depth
- Saturated: water depth equal to grain depth
- Under-saturated: water depth less than grain depth

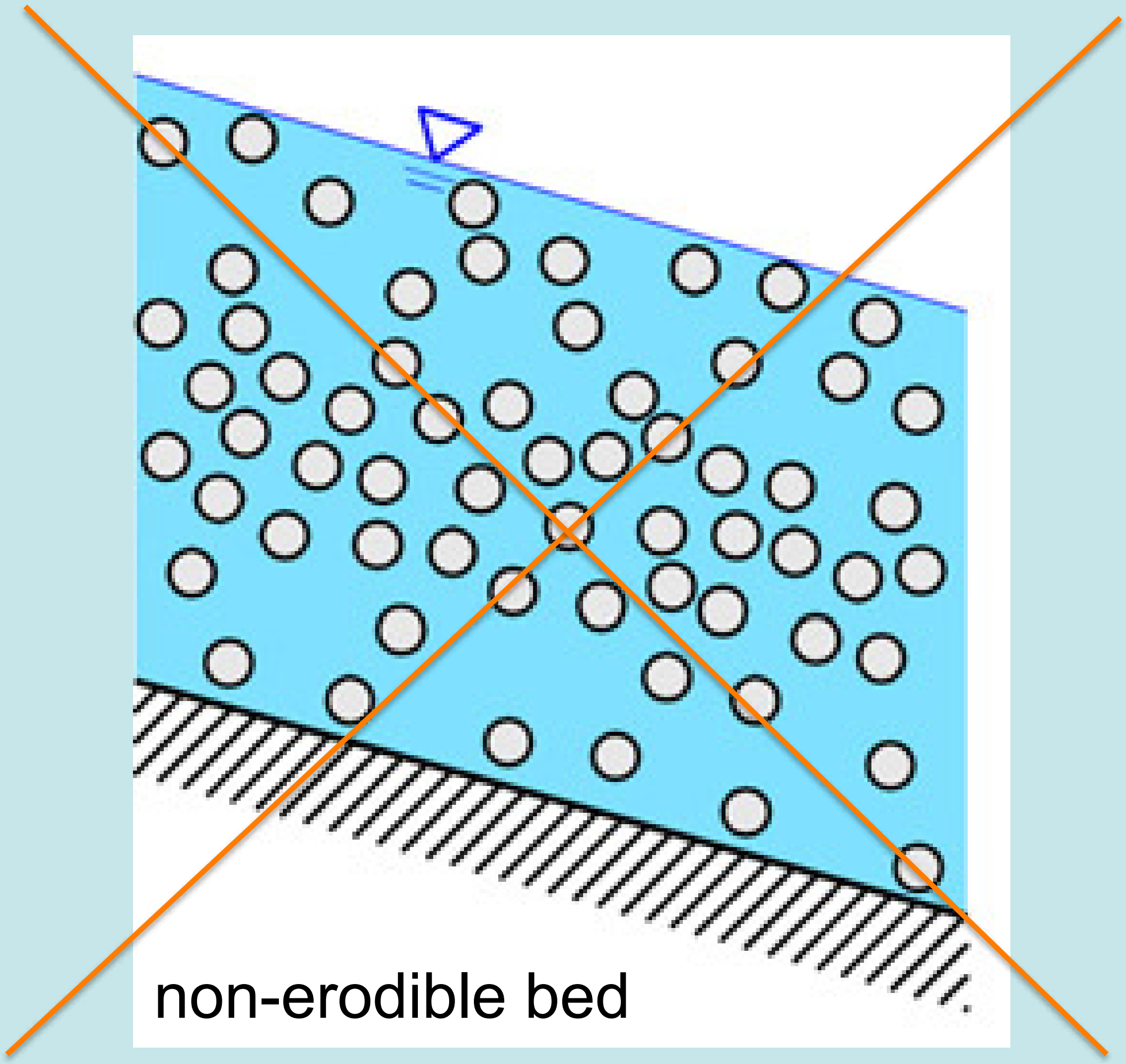
Experiments



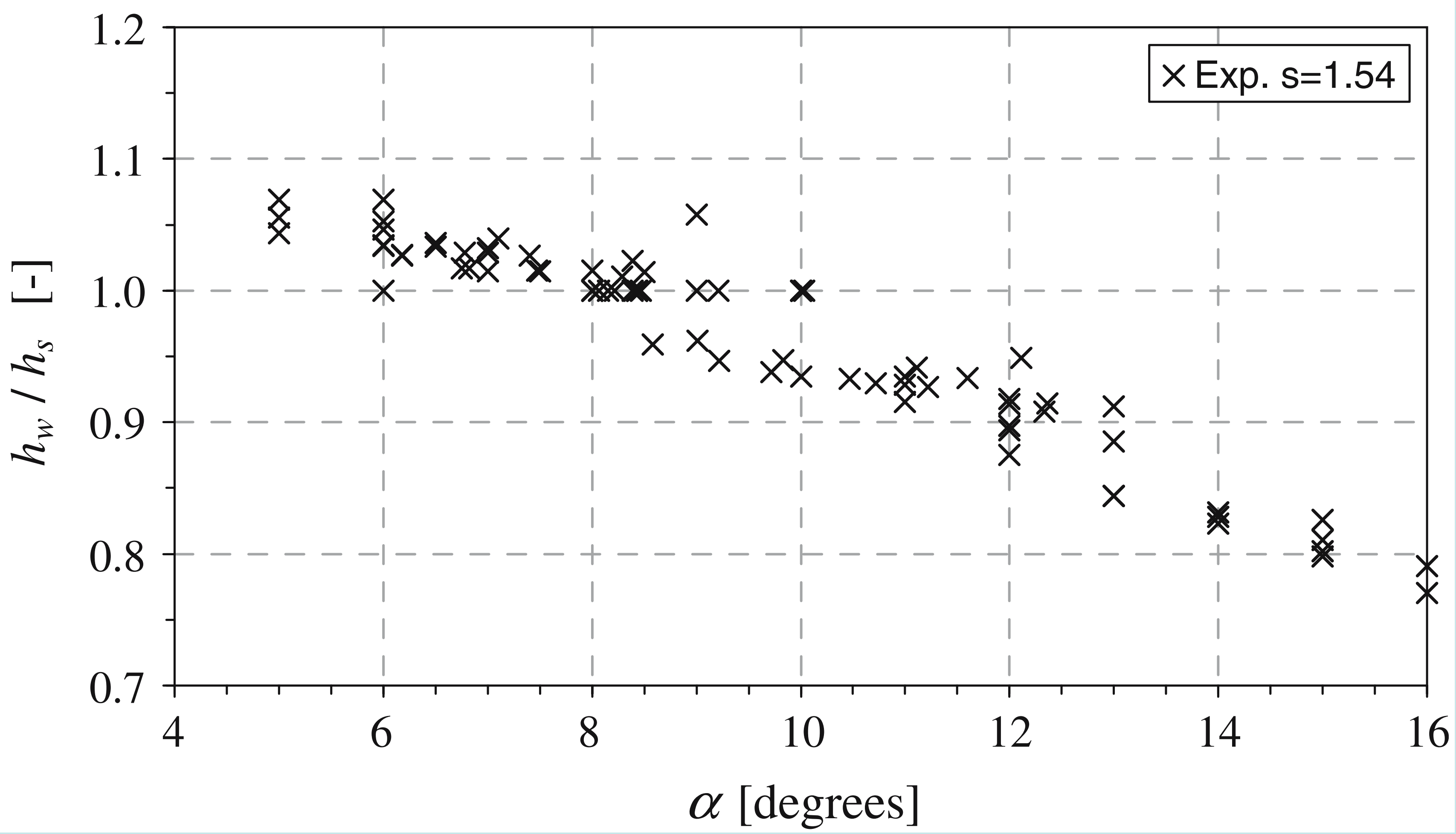
Experiments



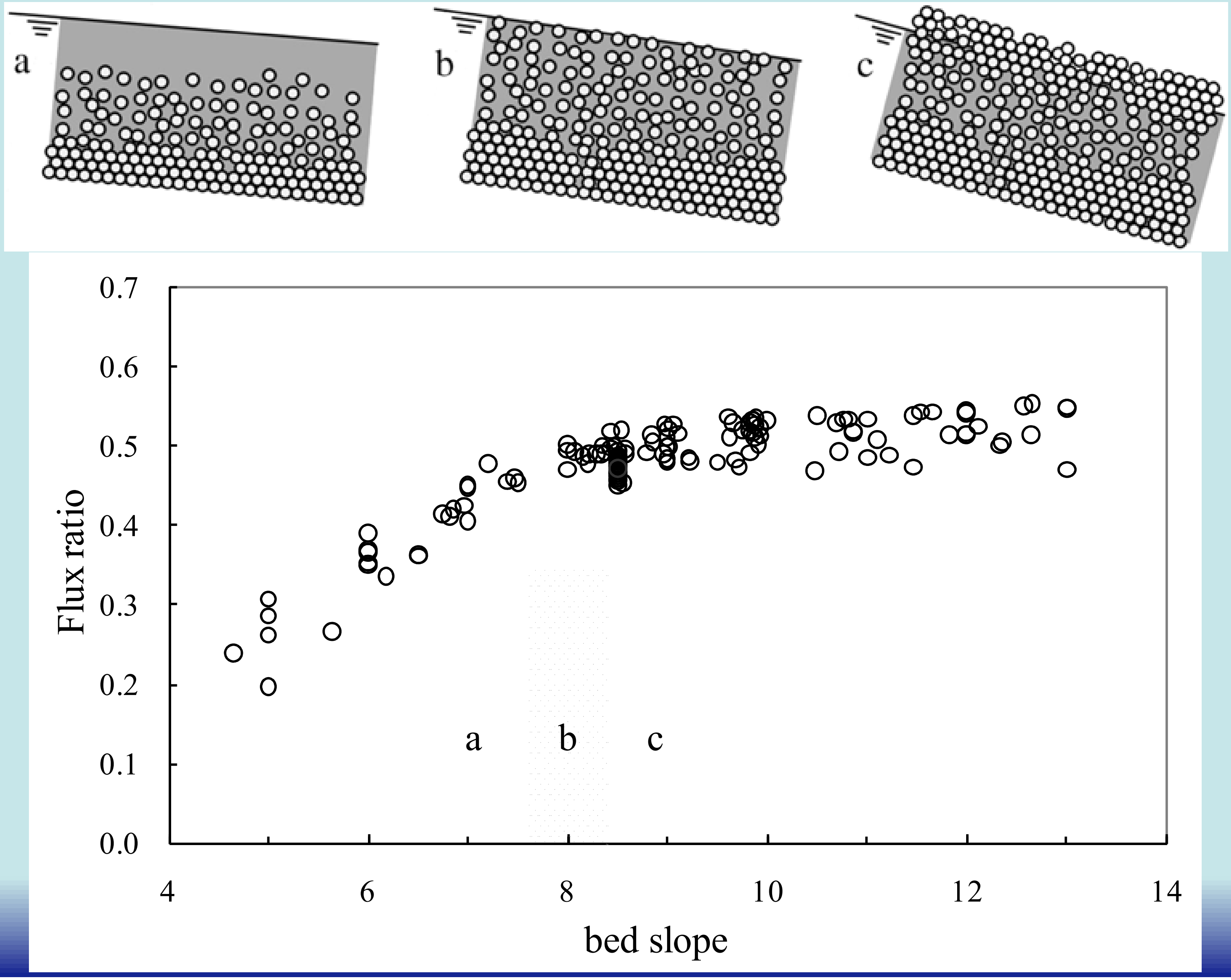
Experiments



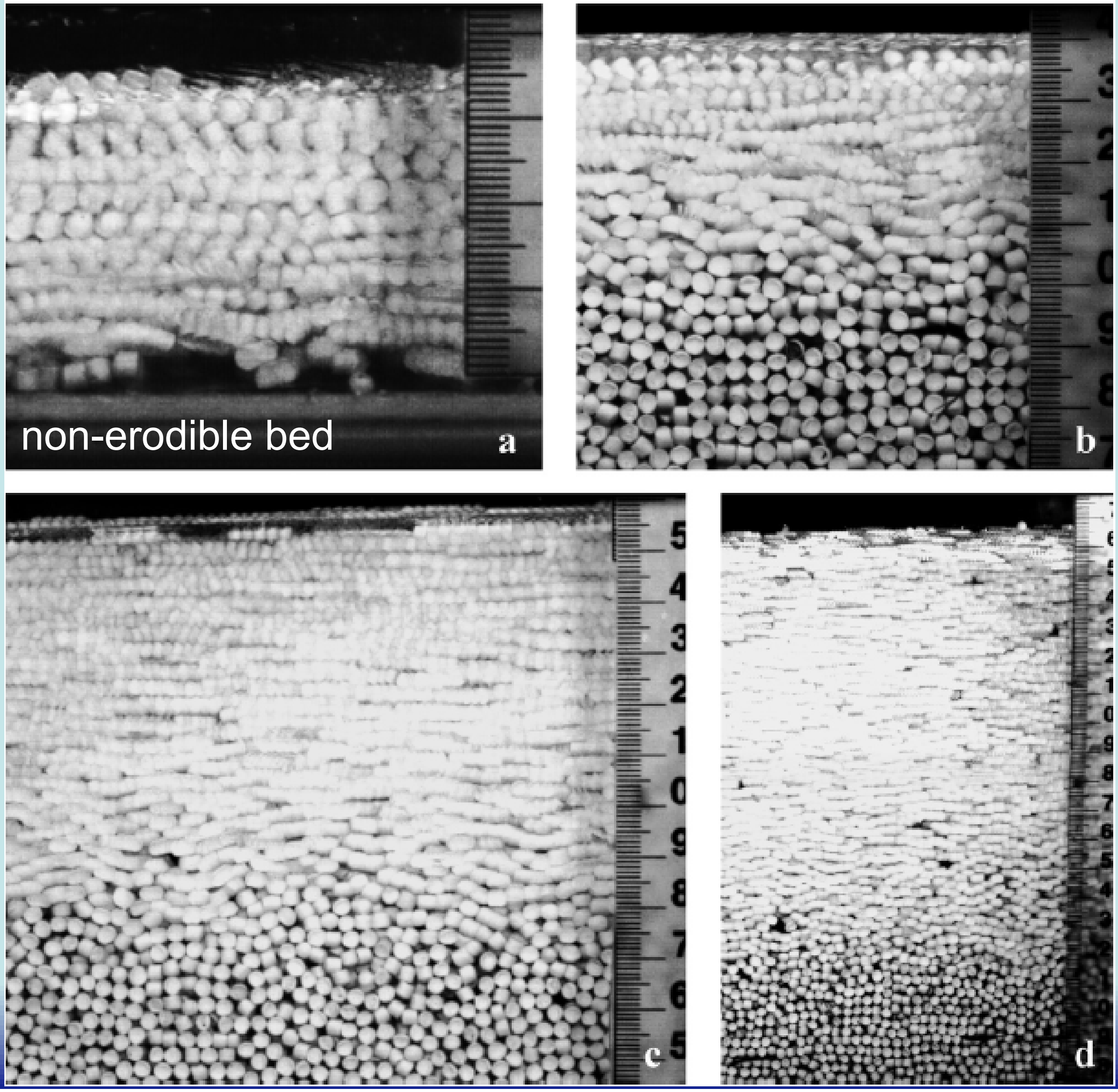
Experiments



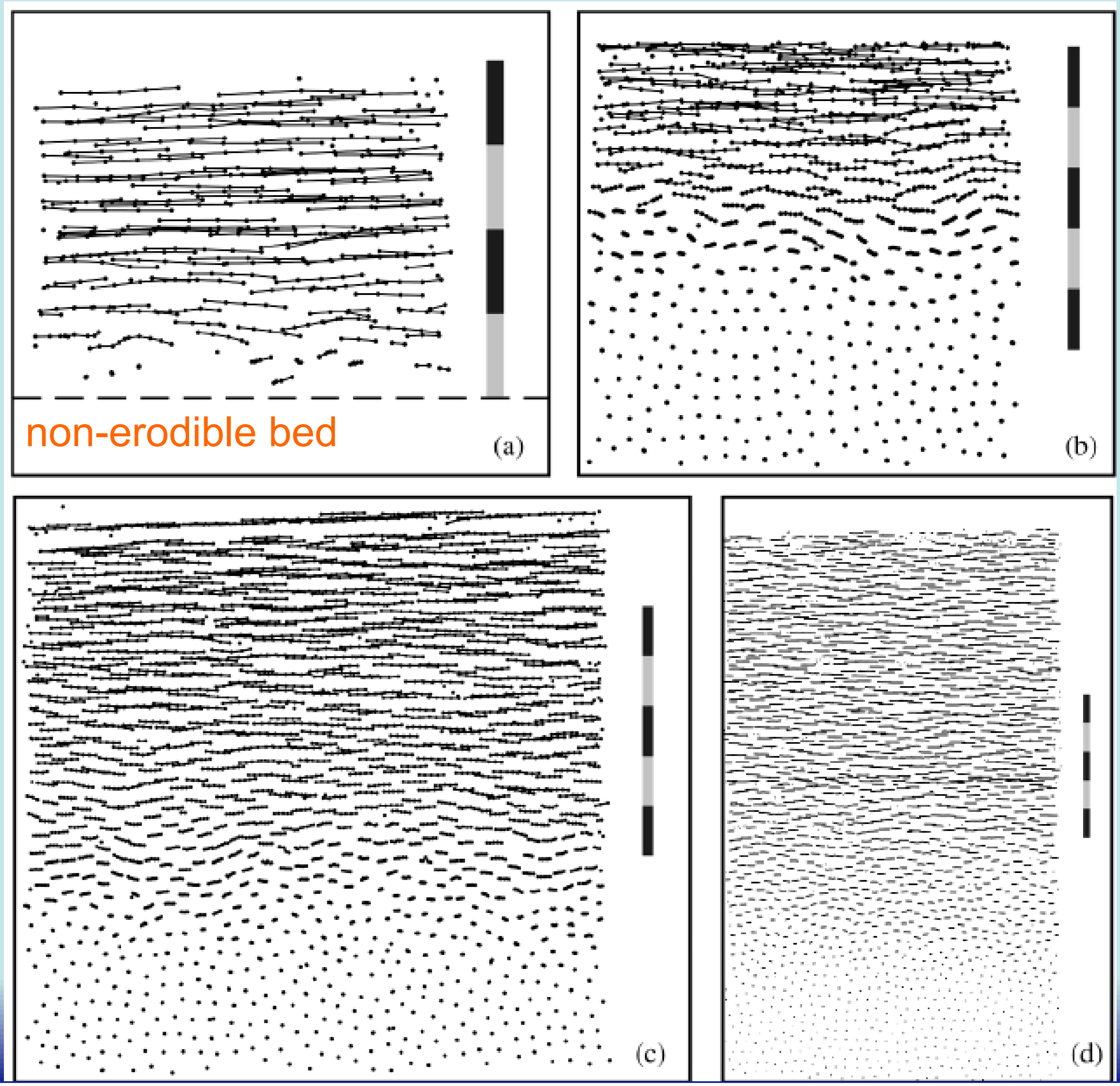
Experiments



Experiments

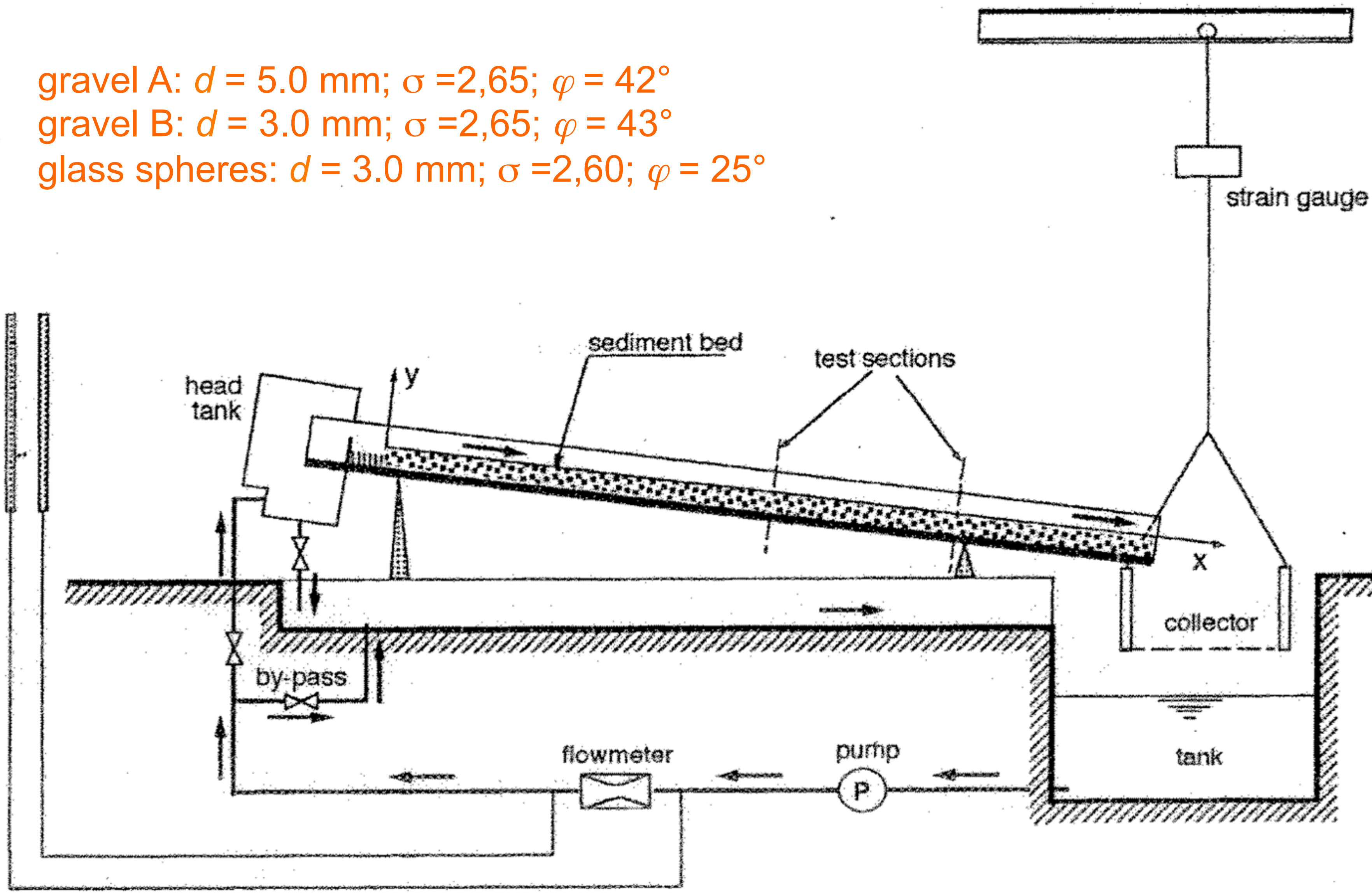


Experiments



Other experiments

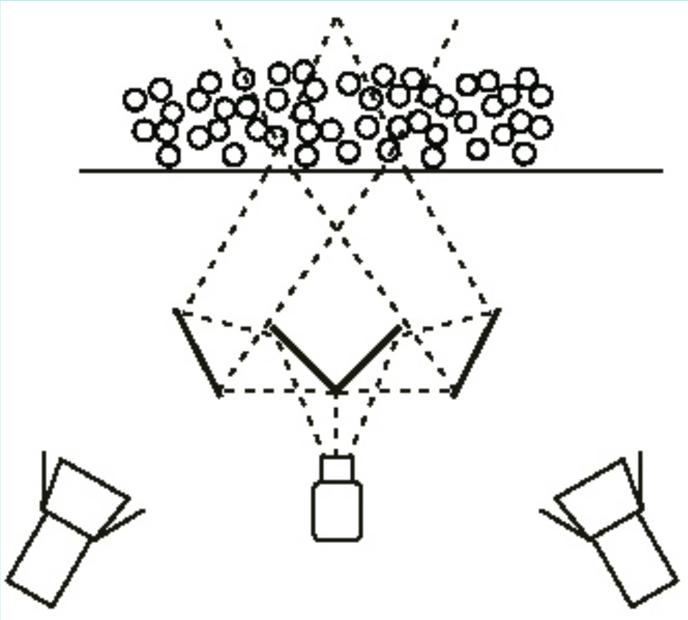
gravel A: d = 5.0 mm; σ =2,65; φ = 42°
gravel B: d = 3.0 mm; σ =2,65; φ = 43°
glass spheres: d = 3.0 mm; σ =2,60; φ = 25°



M. Tubino, S. Lanzoni, Excerpta 7: 201–236 (1993).

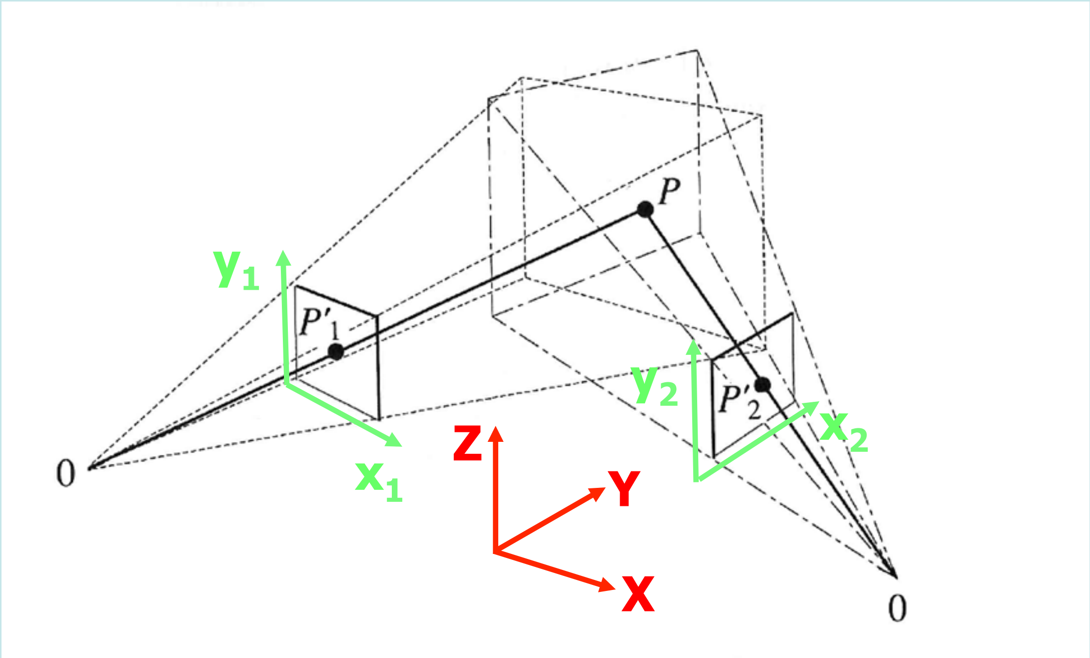
Experimental methods

Voronoï 3D tracking velocimetry



Experiments in Fluids 32: 121-135 (2002)
Experiments in Fluids 34: 227-241 (2003)

Experimental methods

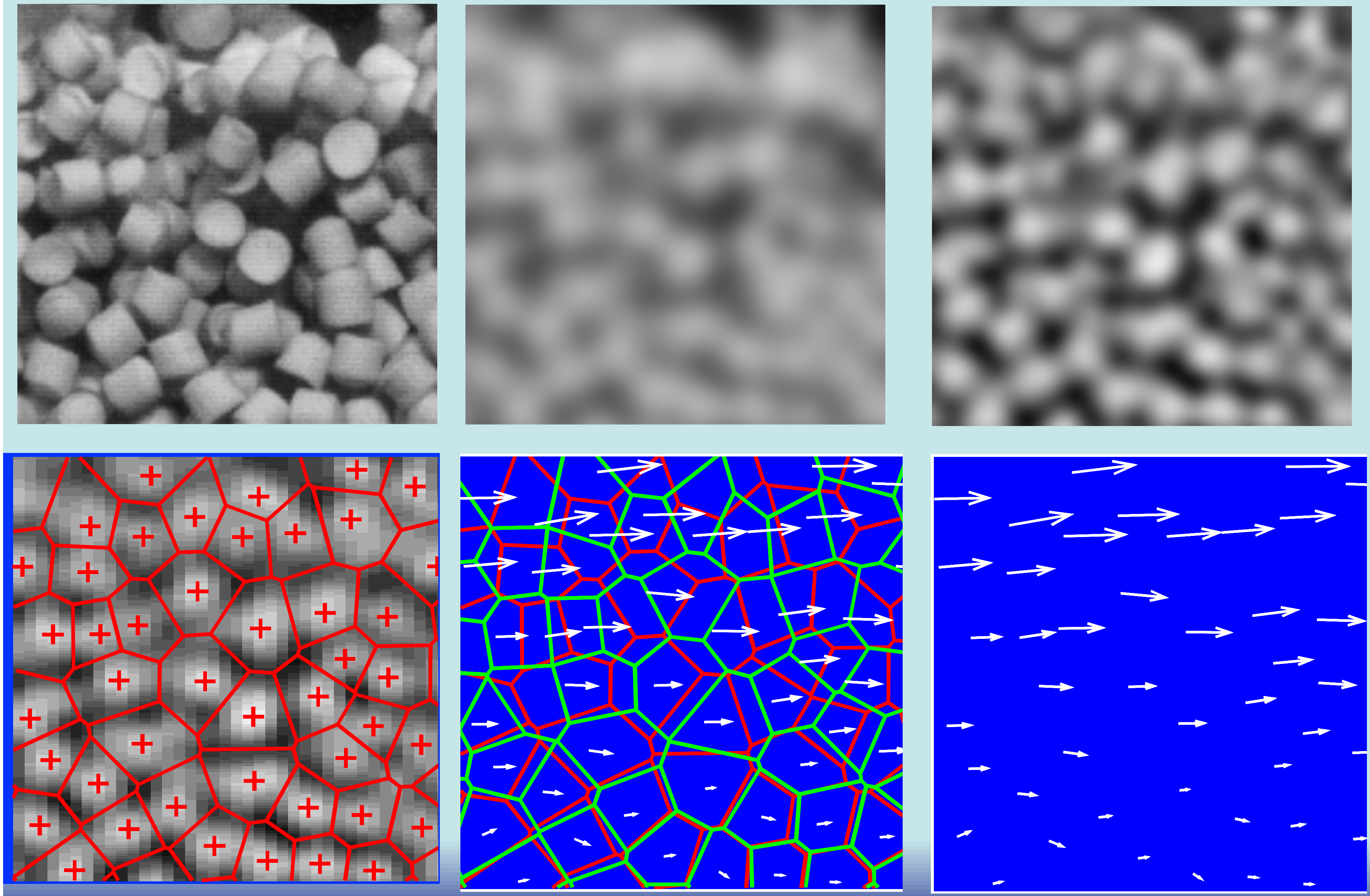


(x_1, y_1) (x_2, y_2) : Image coord.

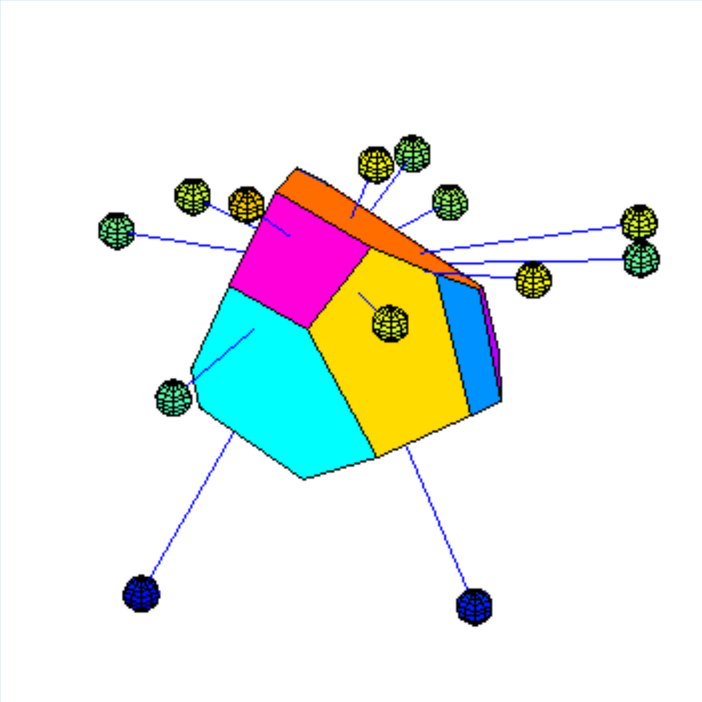
(X,Y,Z) : World coordinates

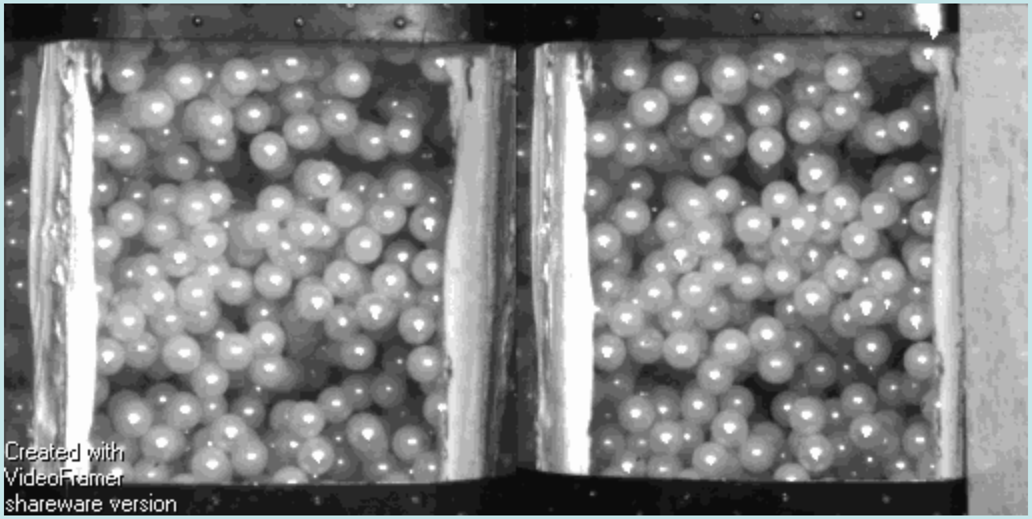
$$[x \ \ y]' = P.[X \ \ Y \ \ Z \ \ 1]'$$

Experimental methods

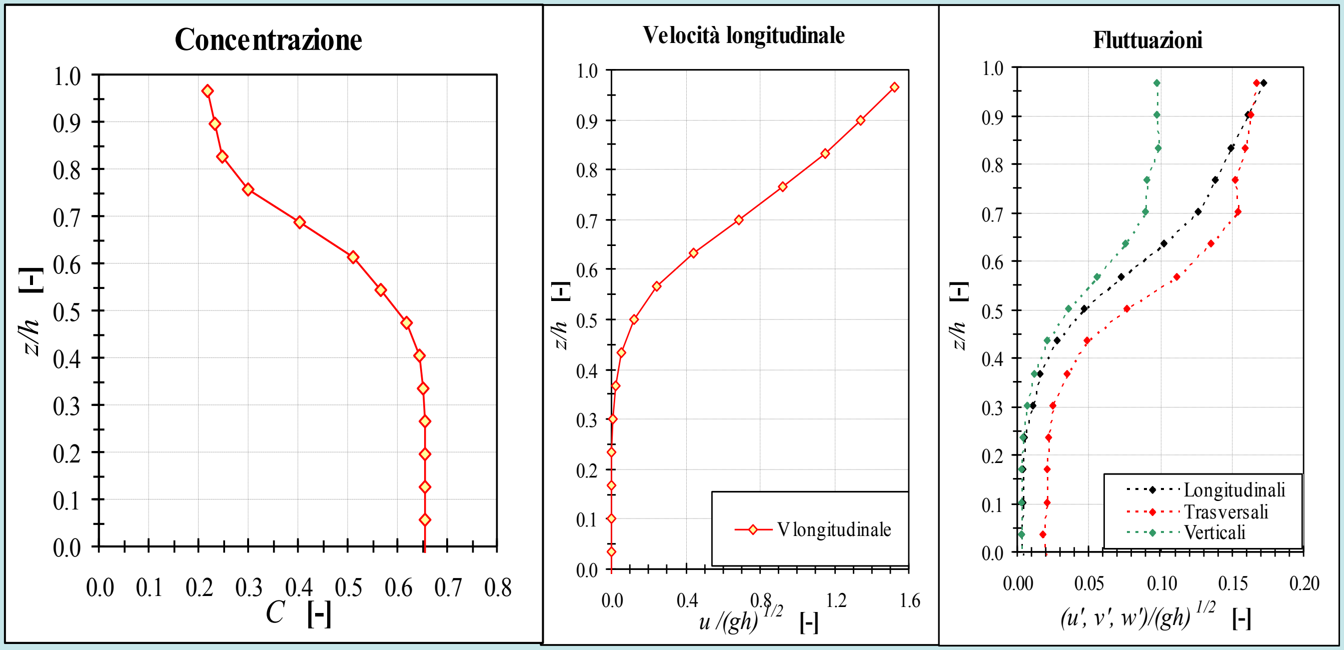


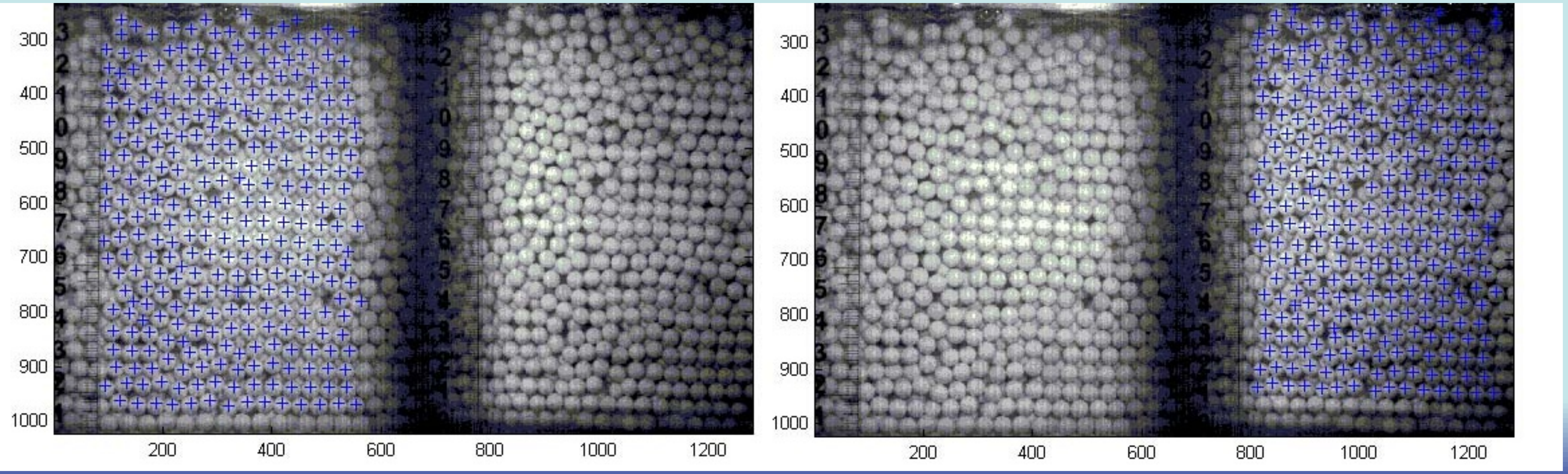
Experimental methods





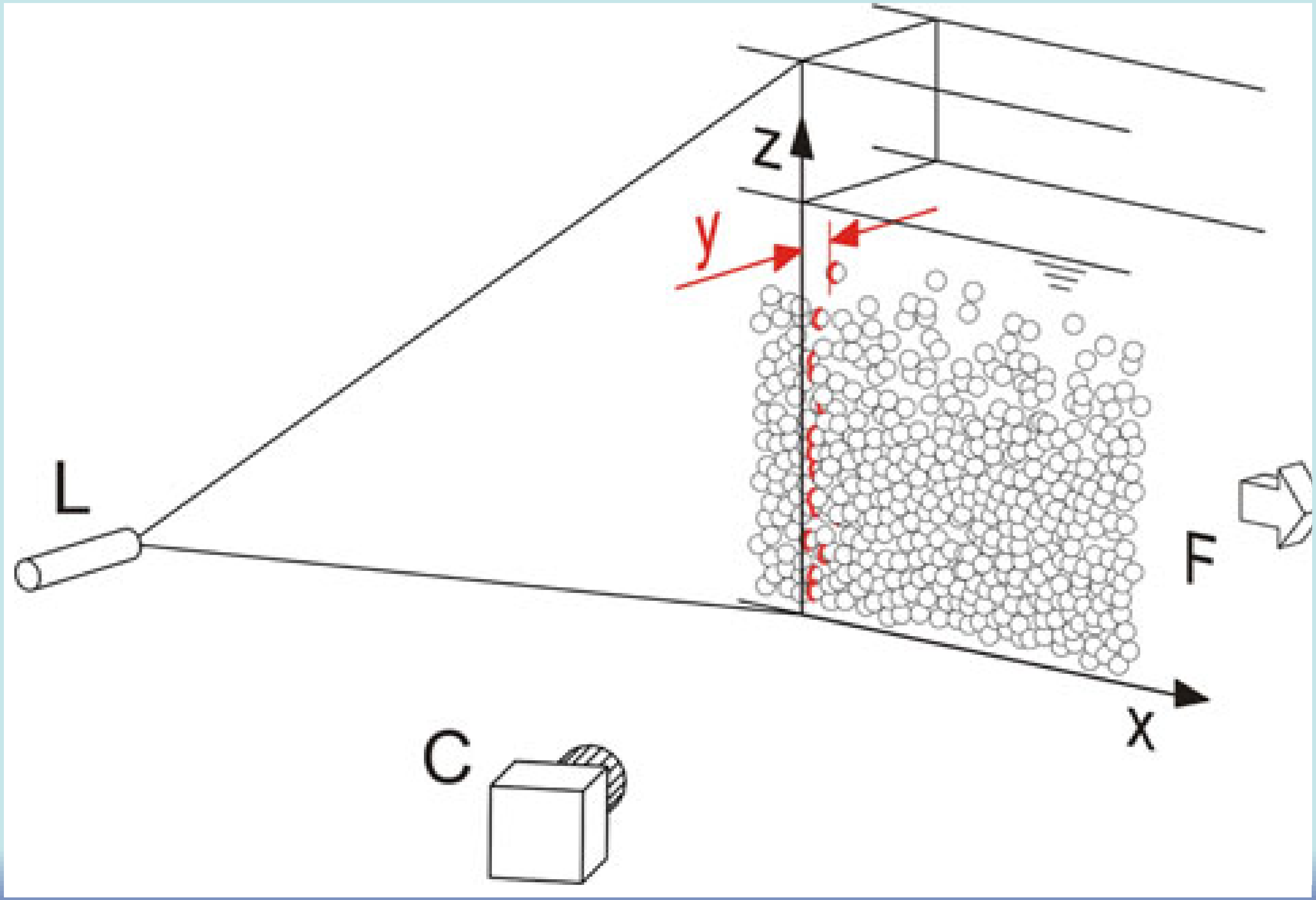
Experimental methods



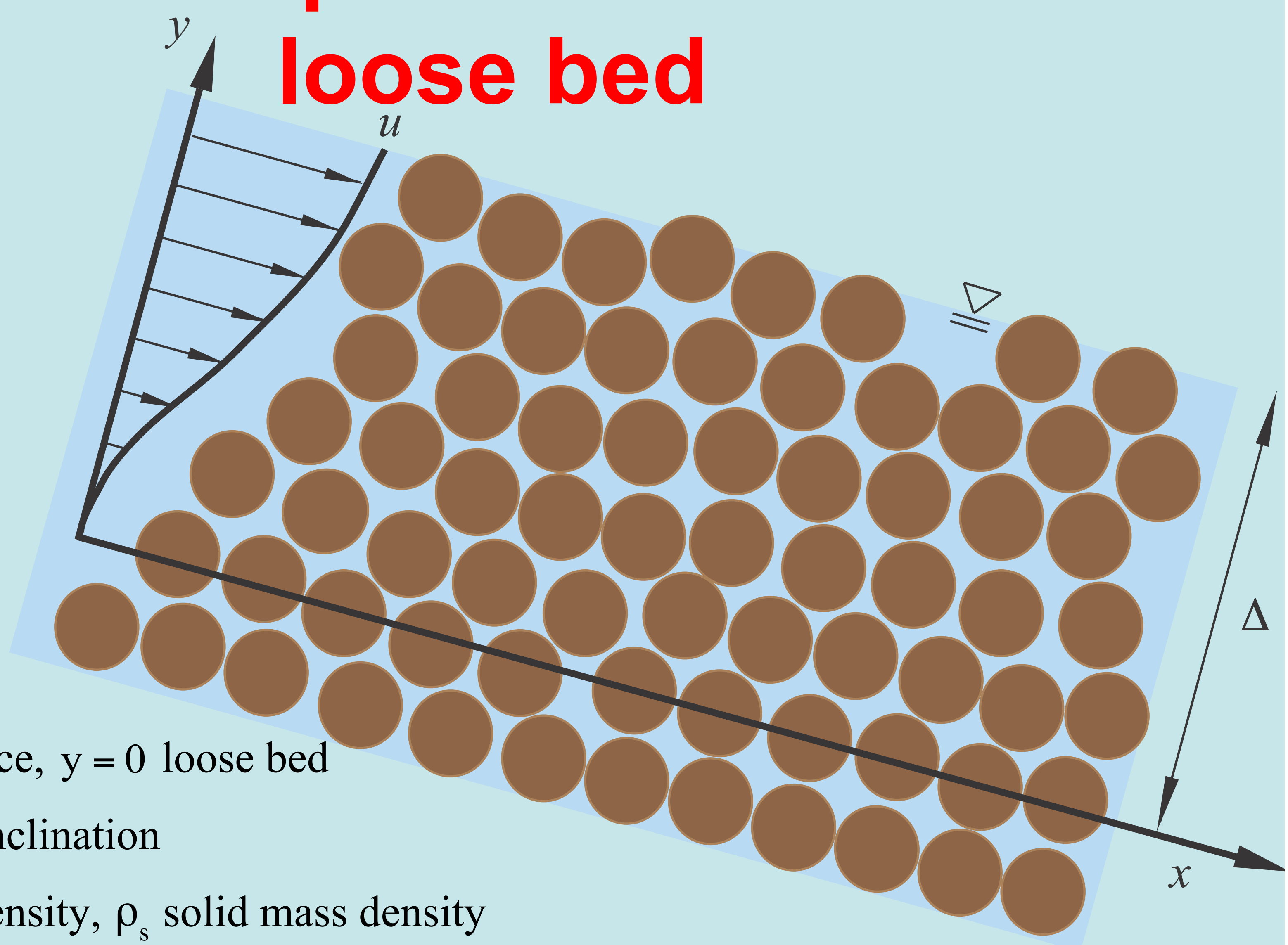


Experimental methods

Laserstripe method



Saturated particle flow over a loose bed



$y = \Delta$ free surface, $y = 0$ loose bed

θ free surface inclination

ρ_f fluid mass density, ρ_s solid mass density

$\sigma = \rho_s / \rho_f$ grain specific mass, ν grain concentration

g gravitational acceleration, $\hat{g} = (\sigma - 1) g / \sigma$ buoyant gravity

Advances in Water Resources (submitted)

Fluid

U fluid velocity, η fluid viscosity

$R = \rho_f d(\hat{g}d)^{1/2} / \eta$ Reynolds number

Particles

u grain average velocity

C grain fluctuation velocity

$T = \langle C^2 \rangle / 3$ granular temperature

$St = \sigma T^{1/2} R / 9(\hat{g}d)^{1/2}$ Stokes number

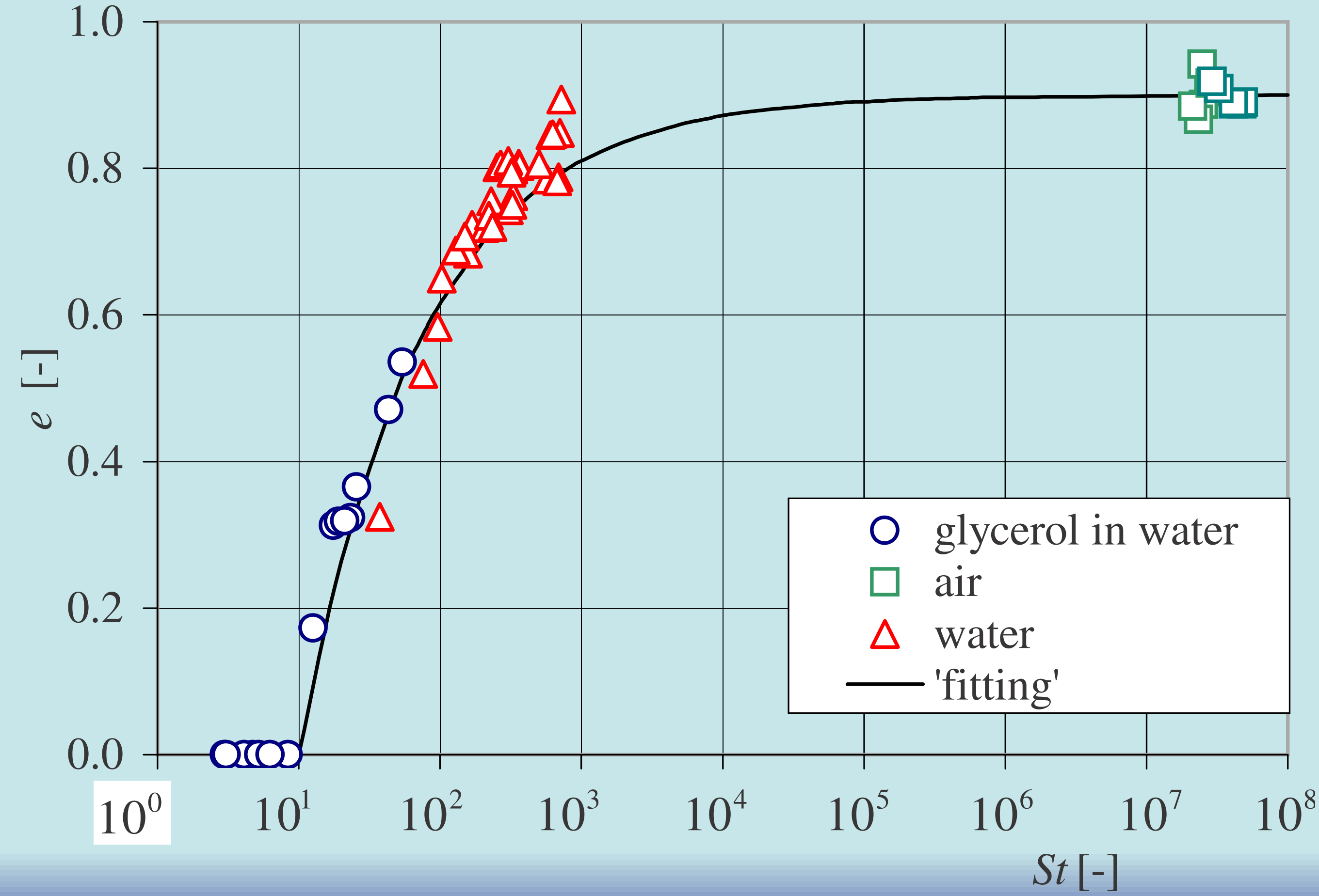
ε dry restitution coefficient

α bed yield stress ratio

Make lengths dimensionless by grain diameter d, velocities by $(\hat{g}d)^{1/2}$, and stresses by $\rho_s \hat{g} d$.

Velocity dependent restitution

$$e = \max\left(\varepsilon - 6.9\frac{1+\varepsilon}{St}, 0\right)$$



Physical Review E, 79, 051306 (2009)

Goals

Given the angle of inclination, predict the **depth** of a saturated flow, the **total volume flux**, and the **range of inclinations** for which saturated flows are possible.

Parameters

$\sigma = 1.54$ $d = 0.37$ cm $\varepsilon = 0.50$ $W = 54d$

$\mu_w = 0.15$ $c = 0.52$ $\alpha = 0.60$

Fluid Momentum Balance

transverse:

$$P' = -\frac{1-\nu}{\sigma-1}\cos\phi$$

flow:

$$S' = -\frac{1-\nu}{\sigma-1}\sin\phi\,\frac{\sigma}{\sigma-1} + \frac{\nu D}{\sigma}\left(U-u\right)$$

$$D = \frac{1}{(1-\nu)^{3.1}}\left(0.3\left|U-u\right| + \frac{18.3}{R}\right)$$

Grain Momentum Balance

transverse:

$$p' = -\nu \cos\phi$$

flow:

$$s' = -\nu \sin\phi \frac{\sigma}{\sigma - 1} + 2\mu_w \frac{p}{W} - \frac{\nu D}{\sigma}(U - u)$$

wall friction

chute width

$$D = \frac{1}{(1-\nu)^{3.1}}\left(0.3\left|U - u\right| + \frac{18.3}{R}\right)$$

Depth of flow

$$\frac{d}{dy}\left(s+S\right)=-\sin\phi\frac{\nu(\sigma-1)+1}{\sigma-1}+2\mu_w\frac{p}{W}.$$

$$p=\nu\cos\phi(\Delta-y)$$

neglect the fluid shear stress in the dense flow:

$$\frac{s}{p}=\tan\phi\frac{\nu(\sigma-1)+1}{\nu(\sigma-1)}-\frac{\mu_w}{W}\frac{p}{\nu\cos\phi}$$

obtain the dependence of s/p on y:

$$\frac{s}{p}=\tan\phi\frac{\nu(\sigma-1)+1}{\nu(\sigma-1)}-\frac{\mu_w}{W}(\Delta-y)$$

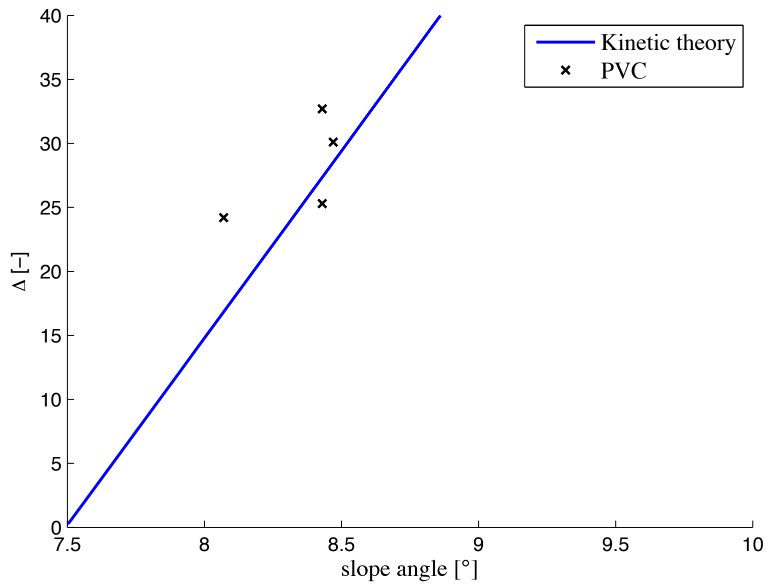
...depth of flow

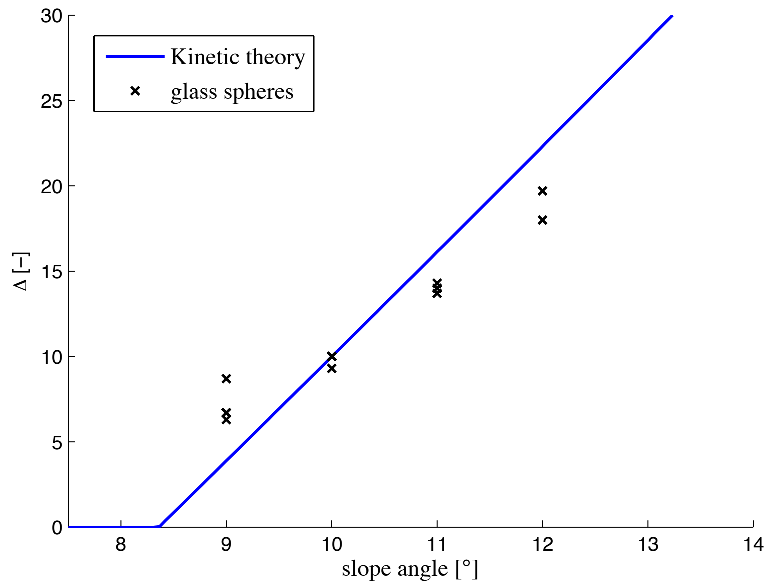
At the bed (y = 0), s/p = α, so:

$$\alpha = \tan\phi \frac{\nu(\sigma-1)+1}{\nu(\sigma-1)} - \frac{\mu_w}{W}\Delta$$

or:

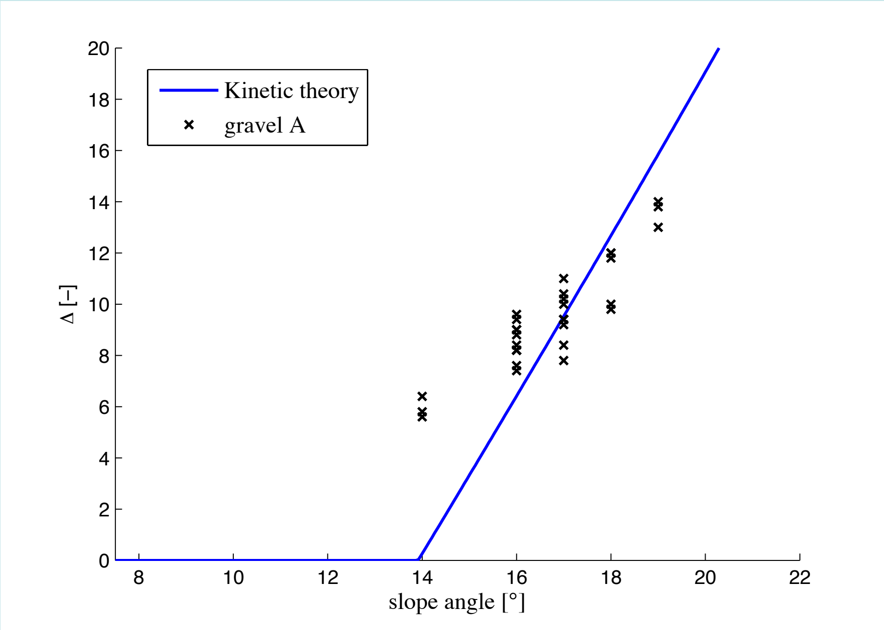
$$\Delta = \frac{W}{\mu_w}\left\{\tan\phi\left[1+\frac{1}{\nu(\sigma-1)}\right]-\alpha\right\}$$

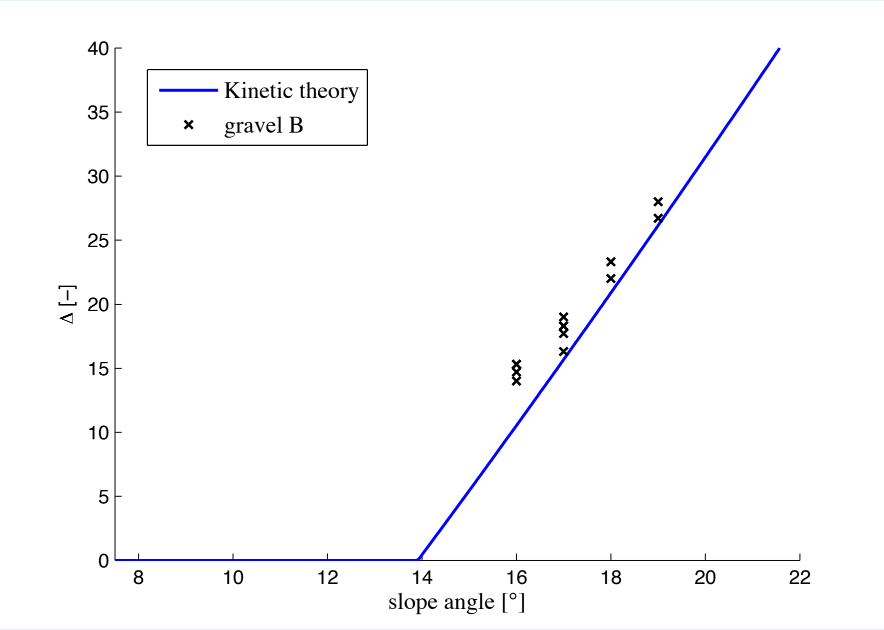




...depth of flow

$$\Delta = \frac{W}{\mu_w}\left\{\tan\phi\left[1+\frac{1}{\nu(\sigma-1)}\right]-\alpha\right\}$$





Particle Energy Balance

$$su' - \Gamma = 0$$

$$s = \mu u' \qquad\qquad p = 2(1+e)\nu G T$$

$$\mu = \frac{4J}{5\pi^{1/2}(1+e)}\frac{p}{T^{1/2}} \qquad J = \frac{(1+e)}{2} + \frac{\pi}{4}\frac{(3e-1)(1+e)^2}{\left[24-(1-e)(11-e)\right]} \qquad G = \frac{0.59c}{0.60-c}$$

collisional dissipation: $$\Gamma = \frac{12}{\pi^{1/2}}\frac{\nu G}{L}(1-e^2)T^{3/2}$$

Jenkins (2006, 2007)

cluster size: $$L = \frac{1}{2}\hat{c}G^{1/3}\frac{u'}{T^{1/2}}$$

Concentration

Eliminate *L* from the energy balance and solve for $u'/T^{1/2}$:

$$\frac{u'}{T^{1/2}} = \frac{15}{J}\frac{1-e^2}{\hat{c}G^{1/3}}$$

Use this to eliminate $u'/T^{1/2}$ from the shear stress with *s*/*p*:

$$\frac{s}{p} = \left[\frac{192}{25\pi^{3/2}}\frac{J^2(1-e)}{\hat{c}(1+e)^2}\right]^{1/3}\frac{1}{G^{1/9}} = \tan\phi\,\frac{\nu(\sigma-1)+1}{\nu(\sigma-1)} - \frac{\mu_w}{W}(\Delta - y)$$

and substitute in:

$$\nu = \frac{0.63G}{0.60+G}$$

Temperature

Solve the pressure for T and obtain its variation with depth:

$$T = \frac{p}{2(1+e)\nu G} = \frac{\cos\phi(\Delta - y)}{2(1+e)G}$$

Velocity

Invert the shear stress and integrate (crudely, linear profile with $u_0 = 0$):

$$u' = \frac{5\pi^{1/2}}{4J}(1+e)\frac{s}{p}T^{1/2} \qquad u_\Delta = \frac{5\pi^{1/2}}{8}\frac{(1+e_0)T_0^{1/2}}{J_0}\alpha\Delta$$

Velocity: a more exact integration

$$\bar{G} = \left[\frac{192}{25\pi^{3/2}} \frac{\bar{J}^2(1-\bar{e})}{\hat{c}(1+\bar{e})^2(\overline{s/p})^3} \right]^3 \qquad \bar{e} = e_0/2 \qquad \overline{\frac{s}{p}} = \tan\phi \frac{\nu(\sigma-1)+1}{\nu(\sigma-1)} - \frac{\mu_w}{W}\frac{\Delta}{2}$$

equation to be integrated:

$$u' = \frac{5\pi^{1/2}}{4\bar{J}}(1+e)\frac{s}{p}T^{1/2}$$

velocity profile:

$$u(y) = \frac{5\pi^{1/2}}{4\bar{J}\sqrt{2\bar{G}}}\left(1+\frac{e_0}{2}\right)^{1/2}\left(\cos\phi\right)^{1/2}\left\{\frac{2}{5}\frac{\mu_w}{W}\left[(\Delta-y)^{5/2}-\Delta^{5/2}\right]-\frac{2}{3}\tan\phi\frac{\nu(\sigma-1)+1}{\nu(\sigma-1)}\left[(\Delta-y)^{3/2}-\Delta^{3/2}\right]\right\}$$

Total Volume Flux

Integrate again u_Δ:

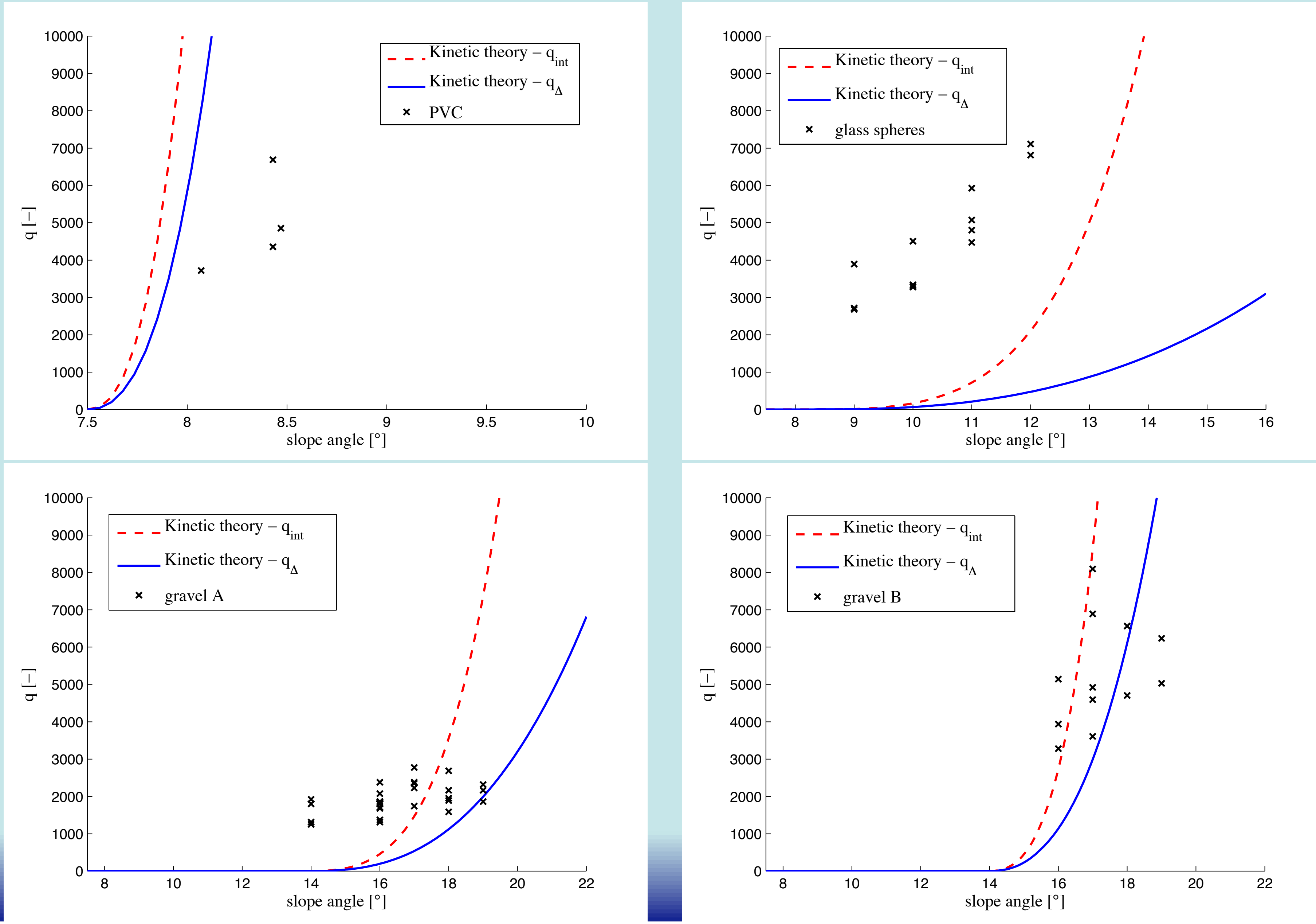
$$q_\Delta = \frac{1}{2} W u_\Delta \Delta$$

or, more exactly:

$$q_{\text{int}} = W \int_0^\Delta u(y)\,dy =$$

$$= \frac{5\pi^{1/2} W}{4\bar{J}\sqrt{2\bar{G}}} \left(1 + \frac{e_0}{2}\right)^{1/2} \left(\cos\phi\right)^{1/2} \left\{ \frac{2}{5}\left(1 + \frac{1}{\nu(\sigma - 1)}\right) \Delta^{5/2} \tan\phi - \frac{2}{7}\frac{\mu_w}{W}\Delta^{7/2} \right\}$$

Total Volume Flux



Extent of inclination

$$L = \frac{1}{2}\hat{c}G^{1/3}\frac{u'}{T^{1/2}}$$

$$u' = \frac{5\pi^{1/2}}{4J}(1+e)\frac{s}{p}T^{1/2}$$

$$\frac{s}{p} = \left[\frac{192}{25\pi^{3/2}}\frac{J^2(1-e)}{\hat{c}(1+e)^2}\right]^{1/3}\frac{1}{G^{1/9}}$$

$$L = \frac{24J}{5\pi}\frac{1-e}{1+e}\left(\frac{p}{s}\right)^2$$

Top:

$$L = 1$$

$$\mathrm{k} \equiv \frac{\mathrm{s}_\Delta}{\mathrm{p}_\Delta} = \left[\frac{24}{5\pi}\mathrm{J}_\Delta\frac{(1-\mathrm{e}_\Delta)}{(1+\mathrm{e}_\Delta)}\right]^{1/2}$$

Bed: Yield

$$\frac{s_0}{p_0} = \alpha$$

…extent of inclination

$$\frac{\nu(\sigma-1)}{1+\nu(\sigma-1)}\alpha \le \tan\phi \le \frac{\nu(\sigma-1)}{1+\nu(\sigma-1)}k$$

	observed	predicted
PVC:	$7.5° < \phi < 9°$	$7.5° < \phi < 10.2°$
sand:	$14° < \phi < 20°$	$14° < \phi < 22°$
glass:	$9° < \phi < 12°$	$8° < \phi < 21°$

Conclusion

A relatively simple theory based on a velocity-dependent coefficient of restitution and an extension of the kinetic theory to very dense, very dissipative grain interactions has the capability of reproducing the depth, volume flux, and range of inclination angle for dense, saturated, fluid-particle flows

Extend to under-saturated and oversaturated flows, mixtures, and unsteady, developing flows.

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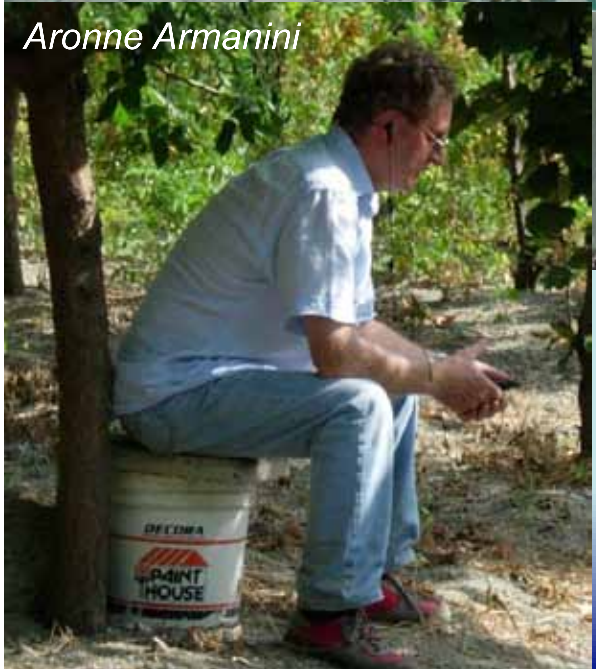
Benoit Spinewine


Jim Jenkins


Hervé Capart


Aronne Armanini

Thank you

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Luigi Fraccarollo