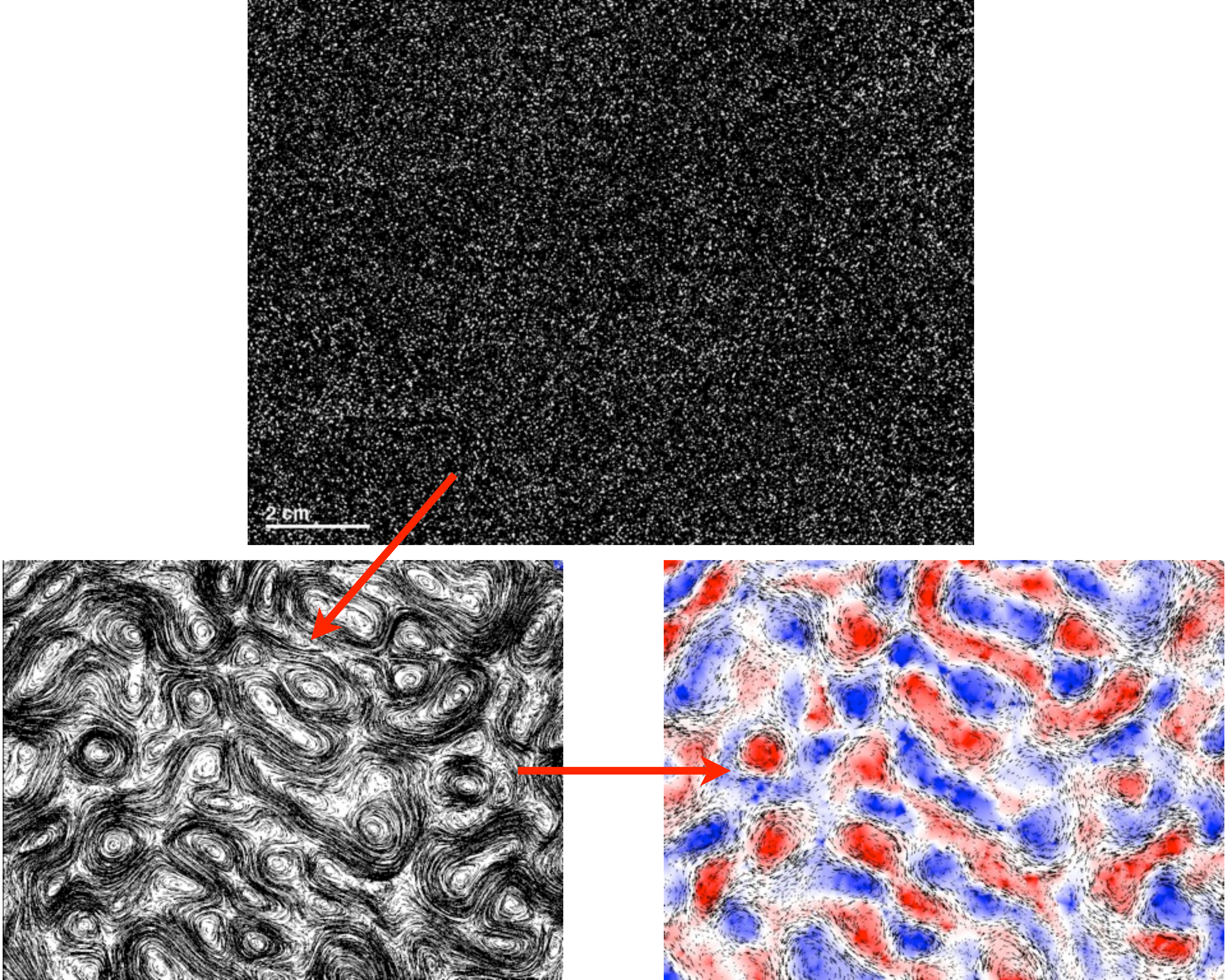
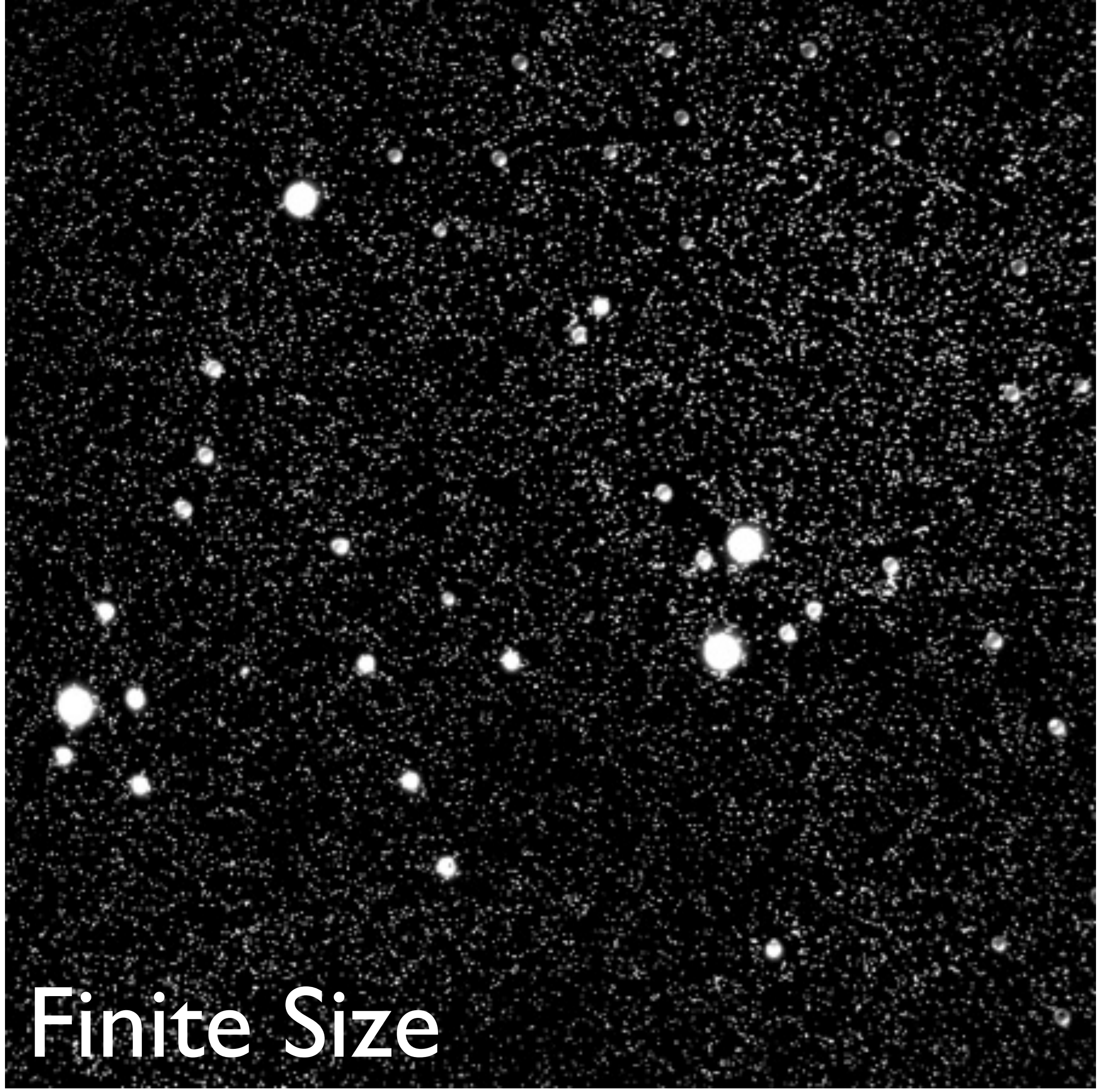
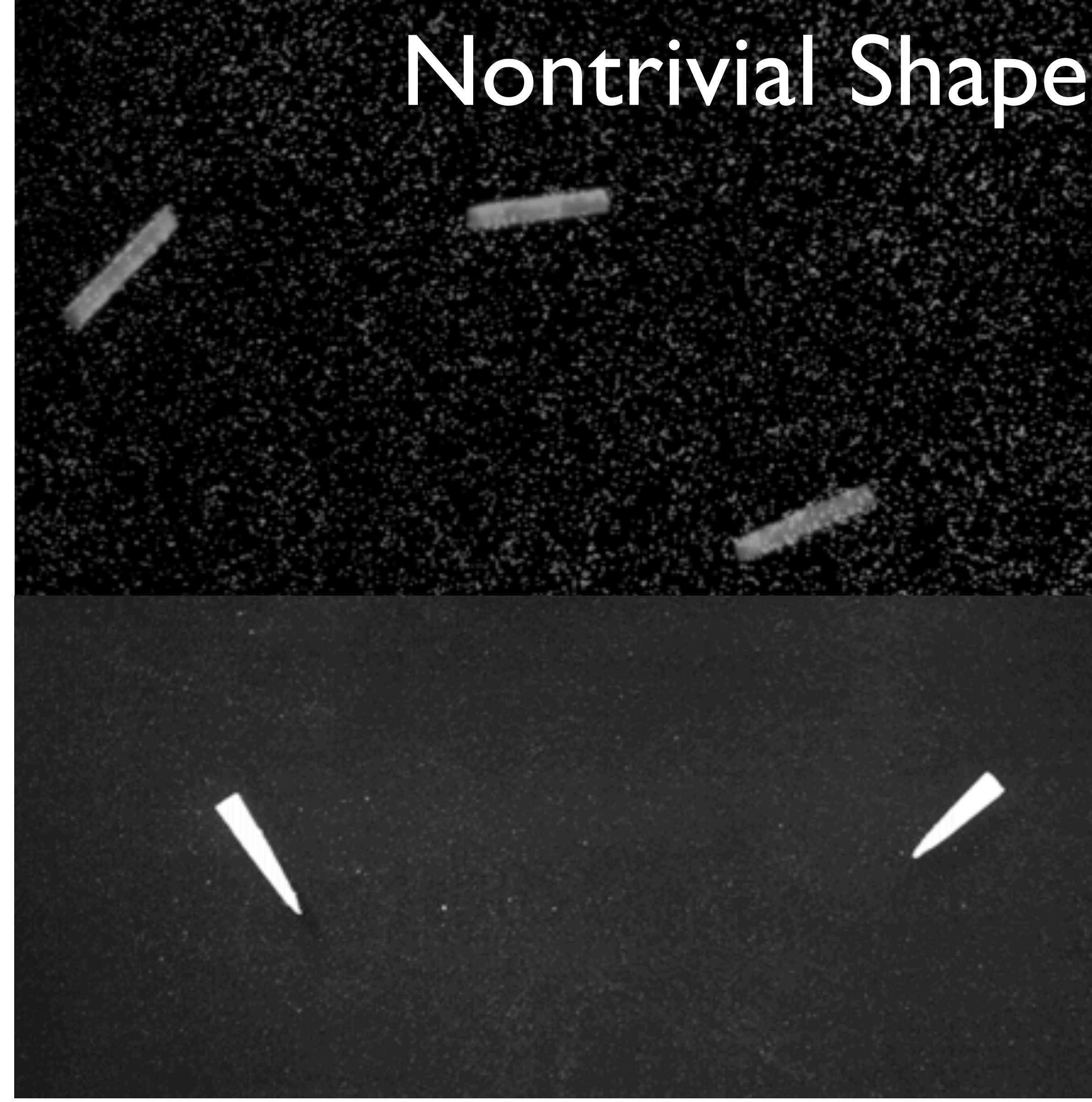
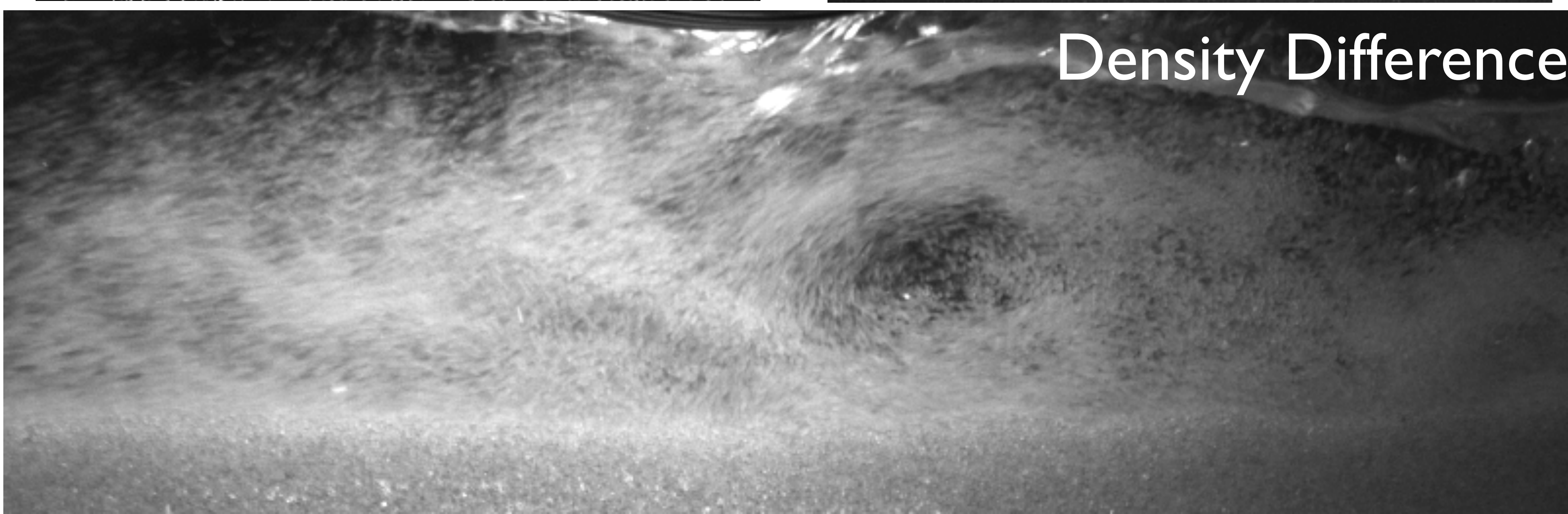
Advection of Active Particles

N.T. Ouellette
Mechanical Engineering & Materials Science
Yale University

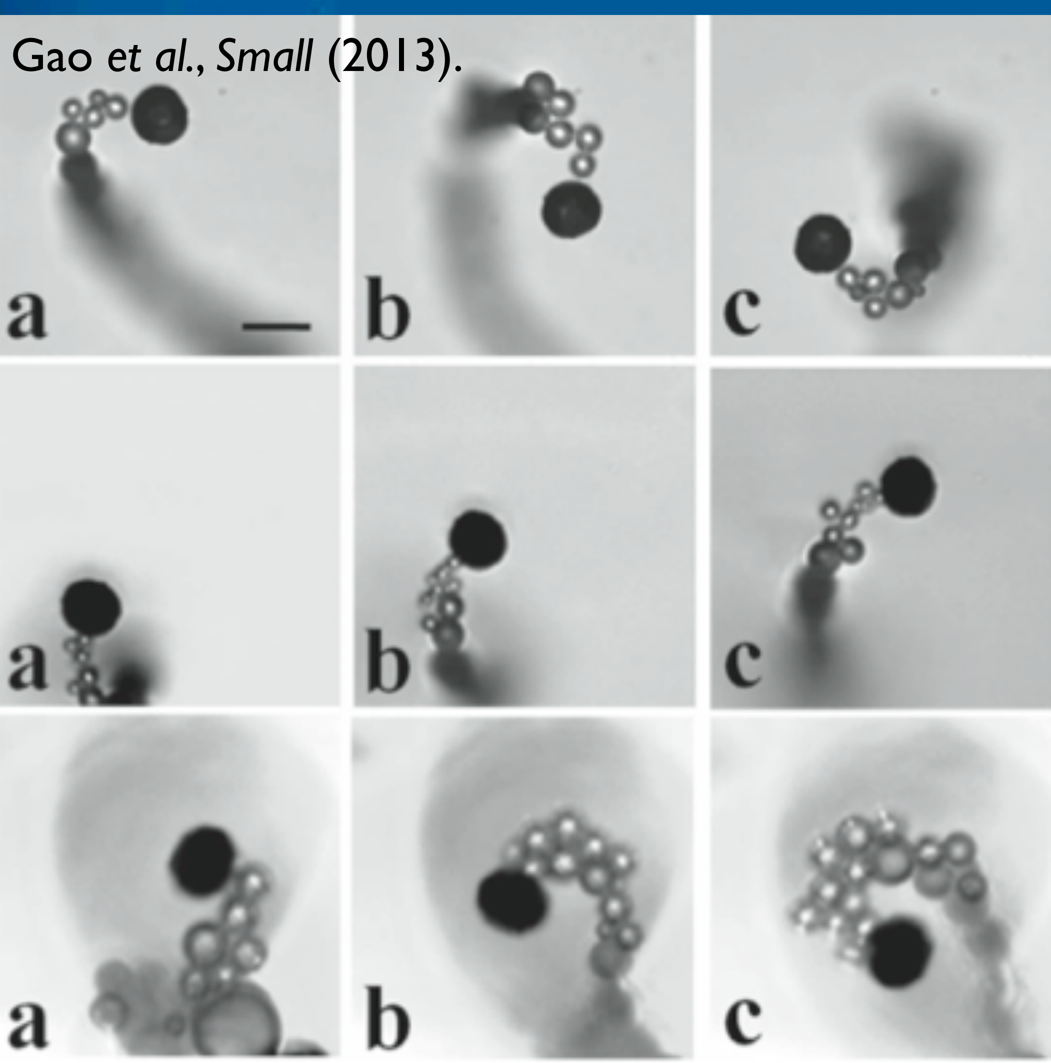

2 cm


Finite Size

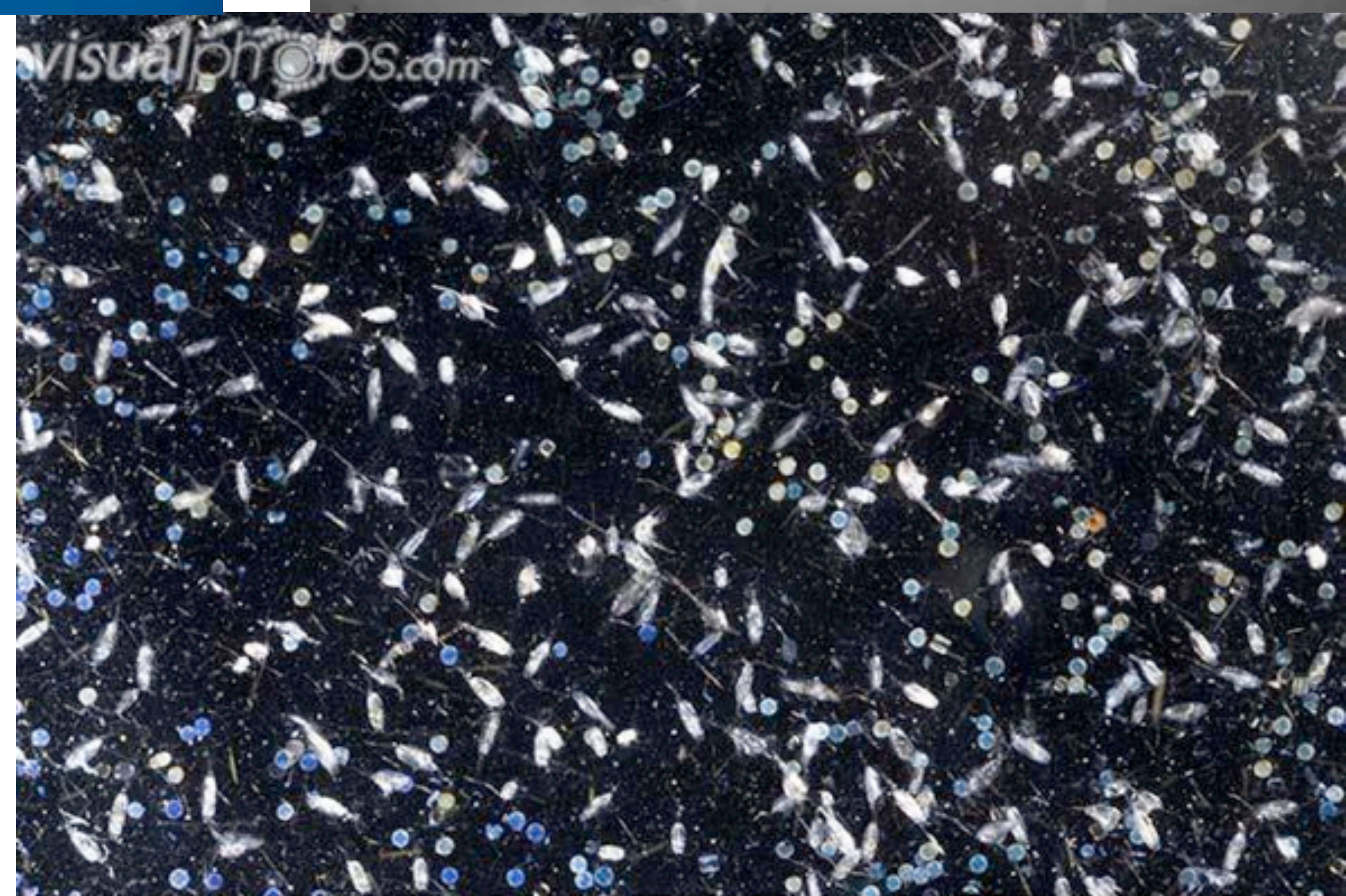

Nontrivial Shape


Density Difference

National Geographic



Gao *et al.*, *Small* (2013).

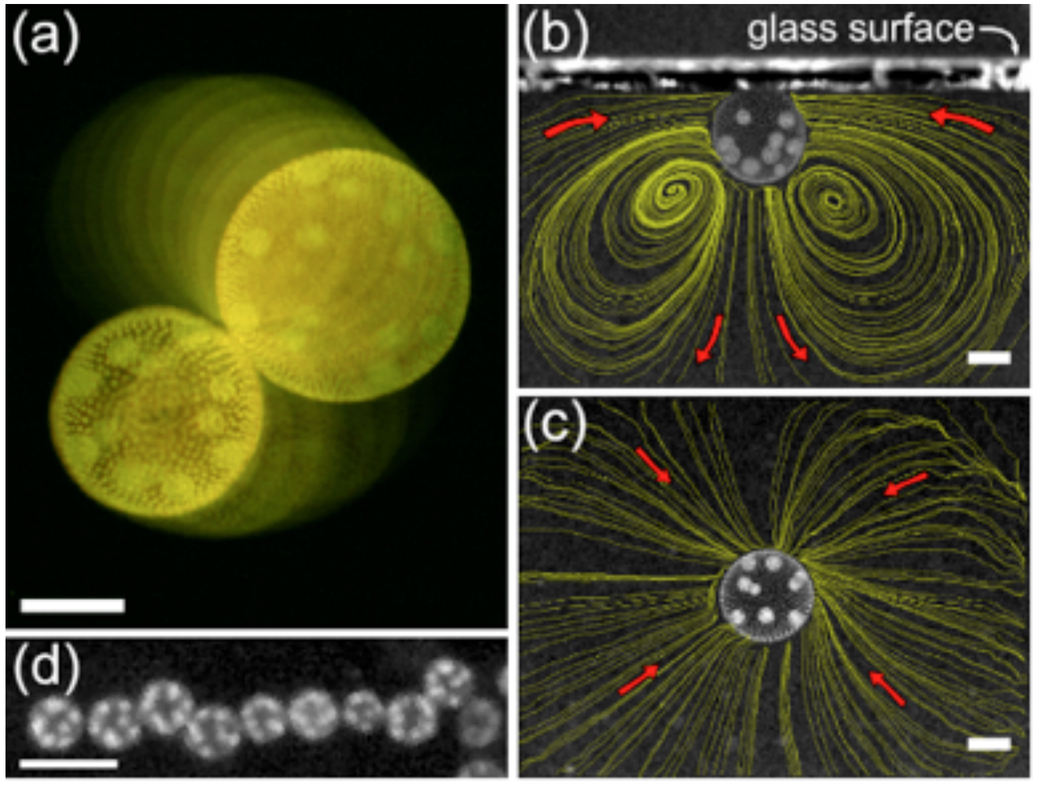


Swimmers

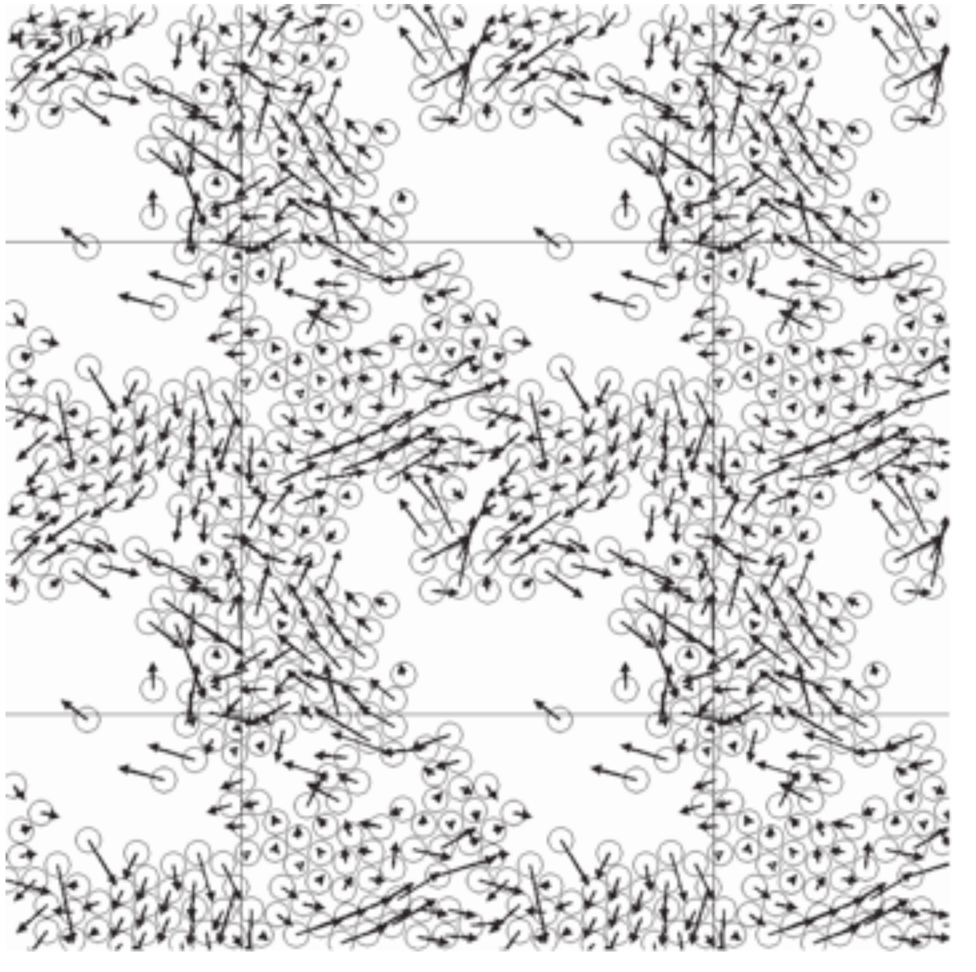
Microorganisms in quiescent fluids

Hydrodynamic interactions and structure formation

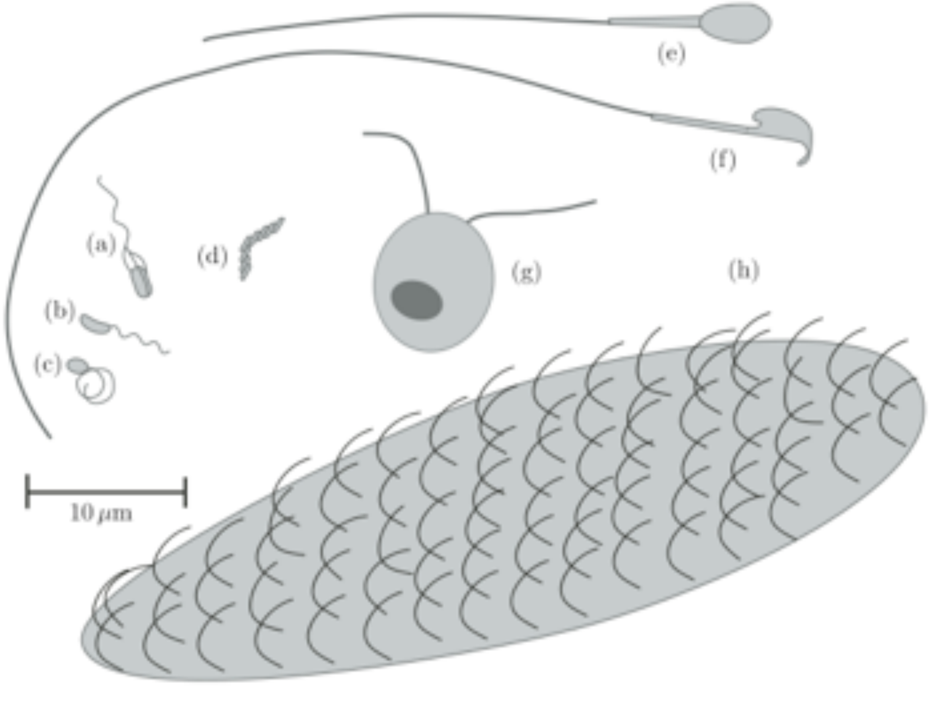
Propulsion mechanisms



Drescher *et al.*, *PRL* (2009)

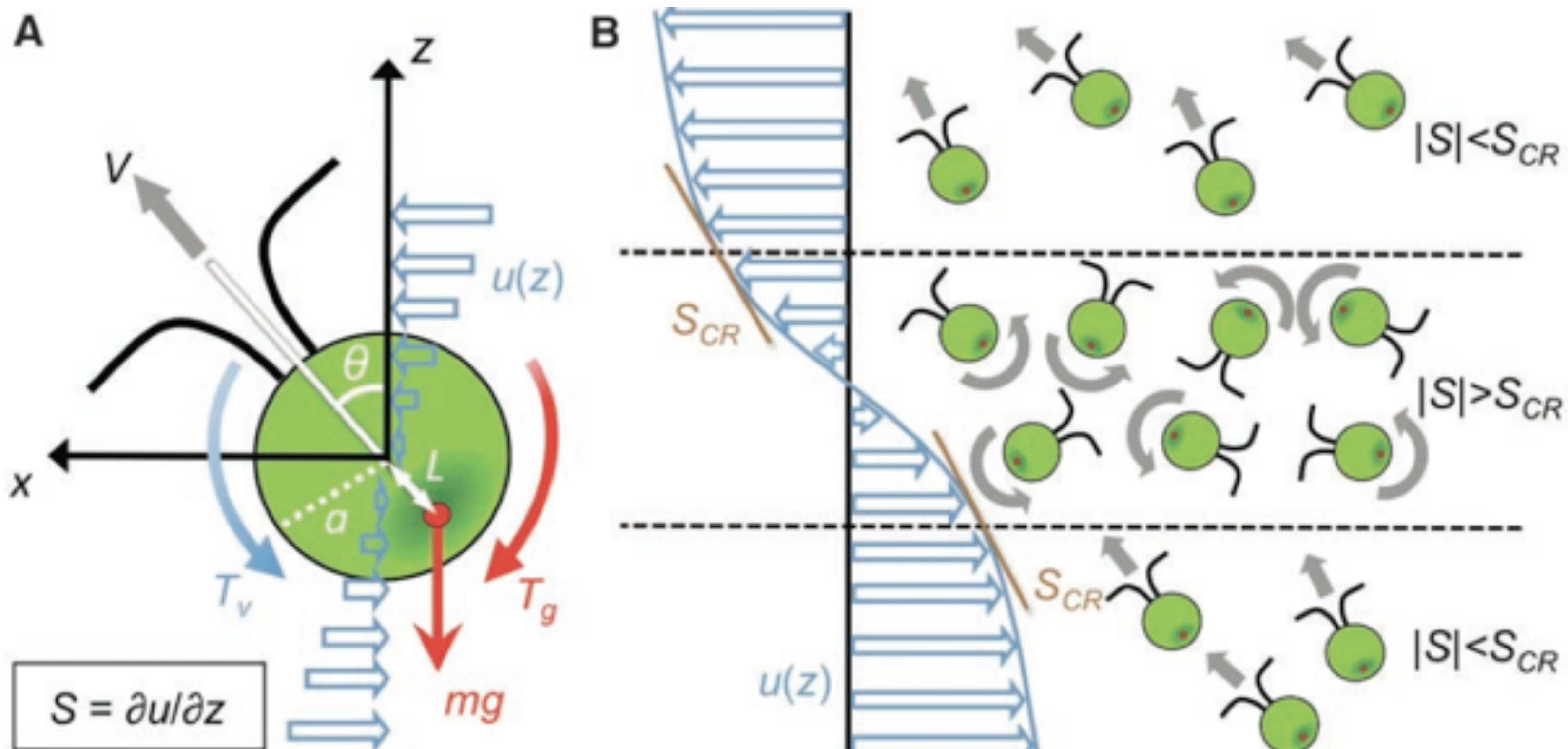


Ishikawa & Pedley, *PRL* (2008)

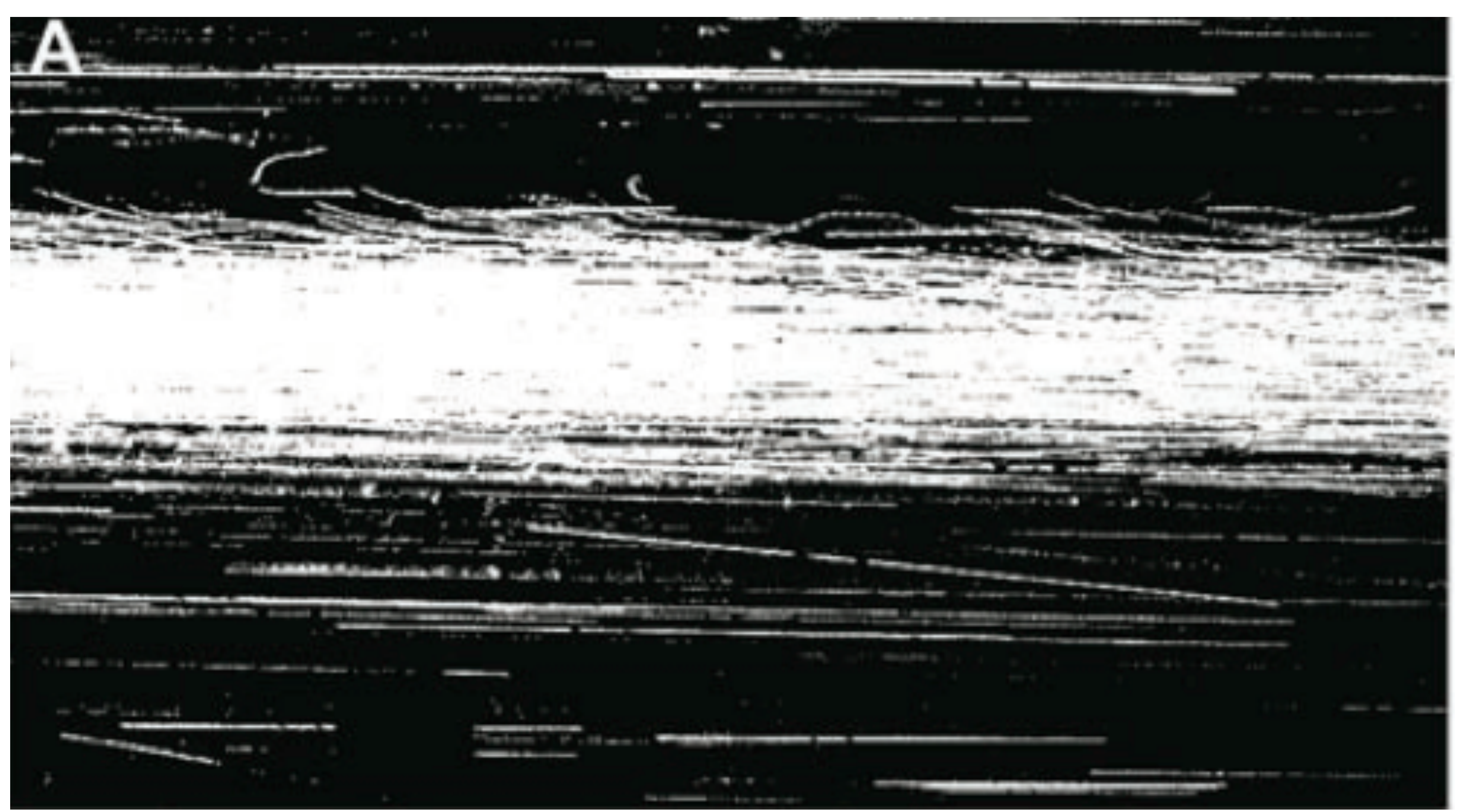


Lauga & Powers, *Rep. Prog. Phys.* (2009)

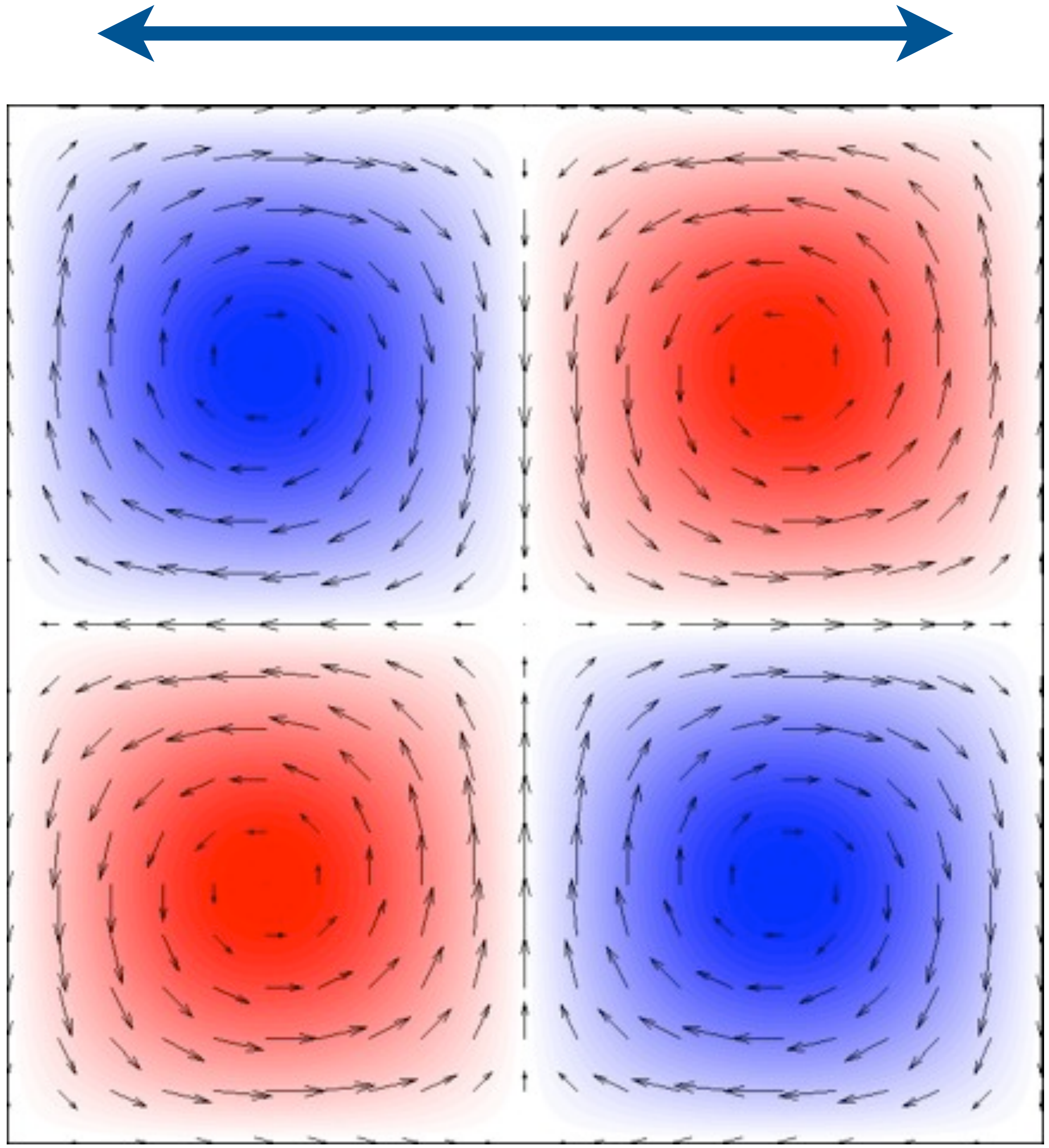
What happens to swimmers in nontrivial flow fields?



Durham, Kessler, & Stocker, *Science* (2009)



Model Flow



2D oscillating cellular flow

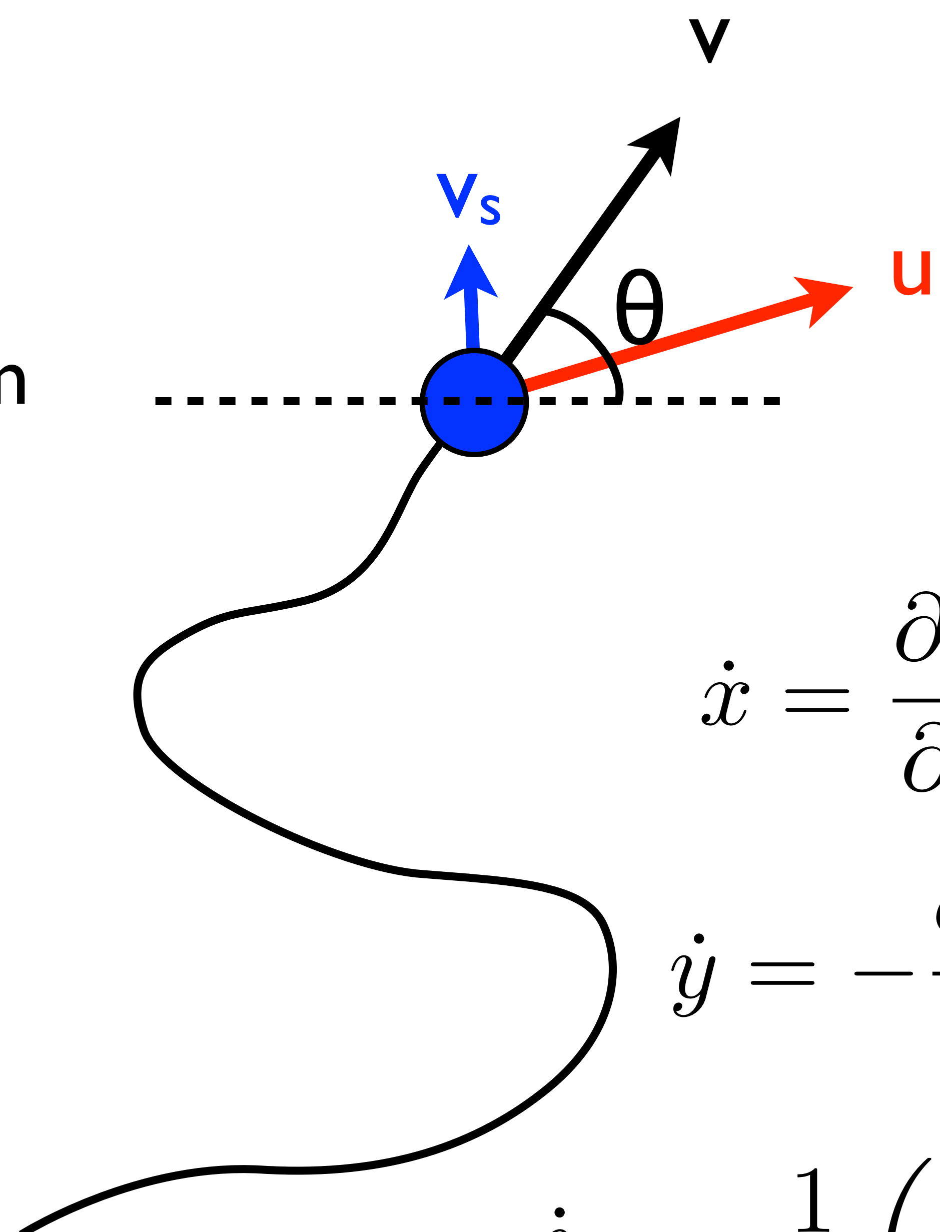
Simple, well characterized

Hamiltonian

$$\psi(x, y, t) = \frac{U}{k} \sin[k(x + B \sin \Omega t)] \sin ky$$

Solomon & Gollub, *Phys. Rev. A* (1988)

Swimmer Model



Swimmer speed is vector sum of flow speed and intrinsic velocity

Swimmers rotate with vorticity

One-way coupling

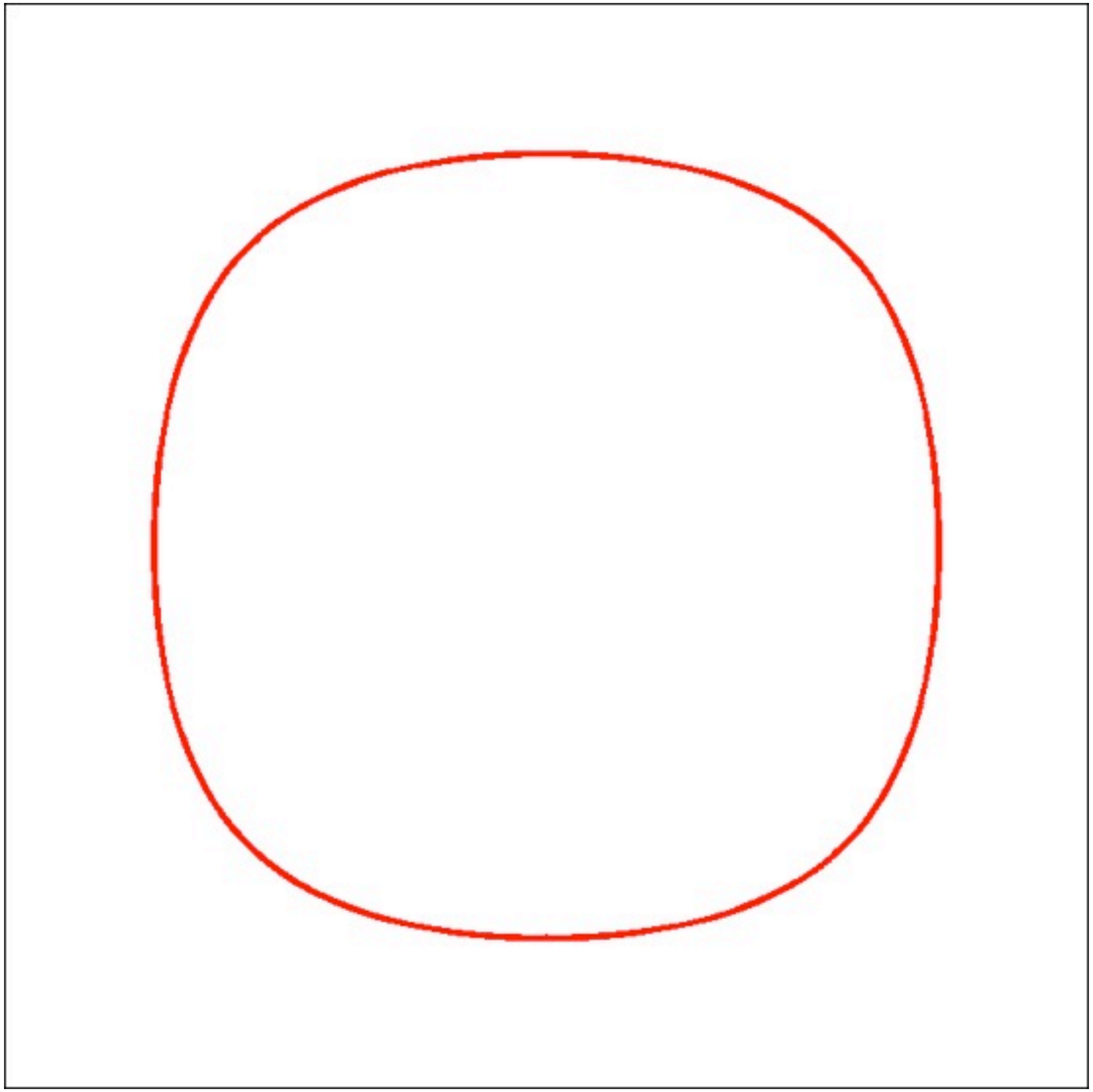
$$\dot{x} = \frac{\partial \psi}{\partial y} + v_s \cos\theta$$

$$\dot{y} = -\frac{\partial \psi}{\partial x} + v_s \sin\theta$$

$$\dot{\theta} = -\frac{1}{2}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

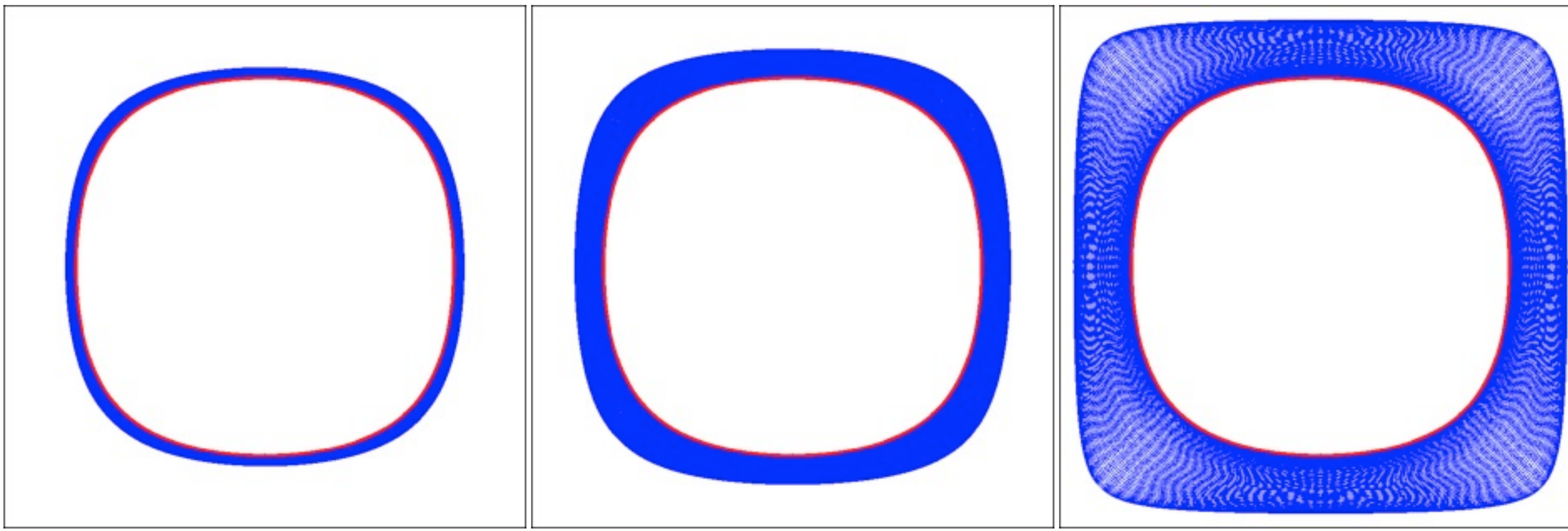
Torney & Neufeld, *PRL* (2007)

Steady Flow, Fluid Particle

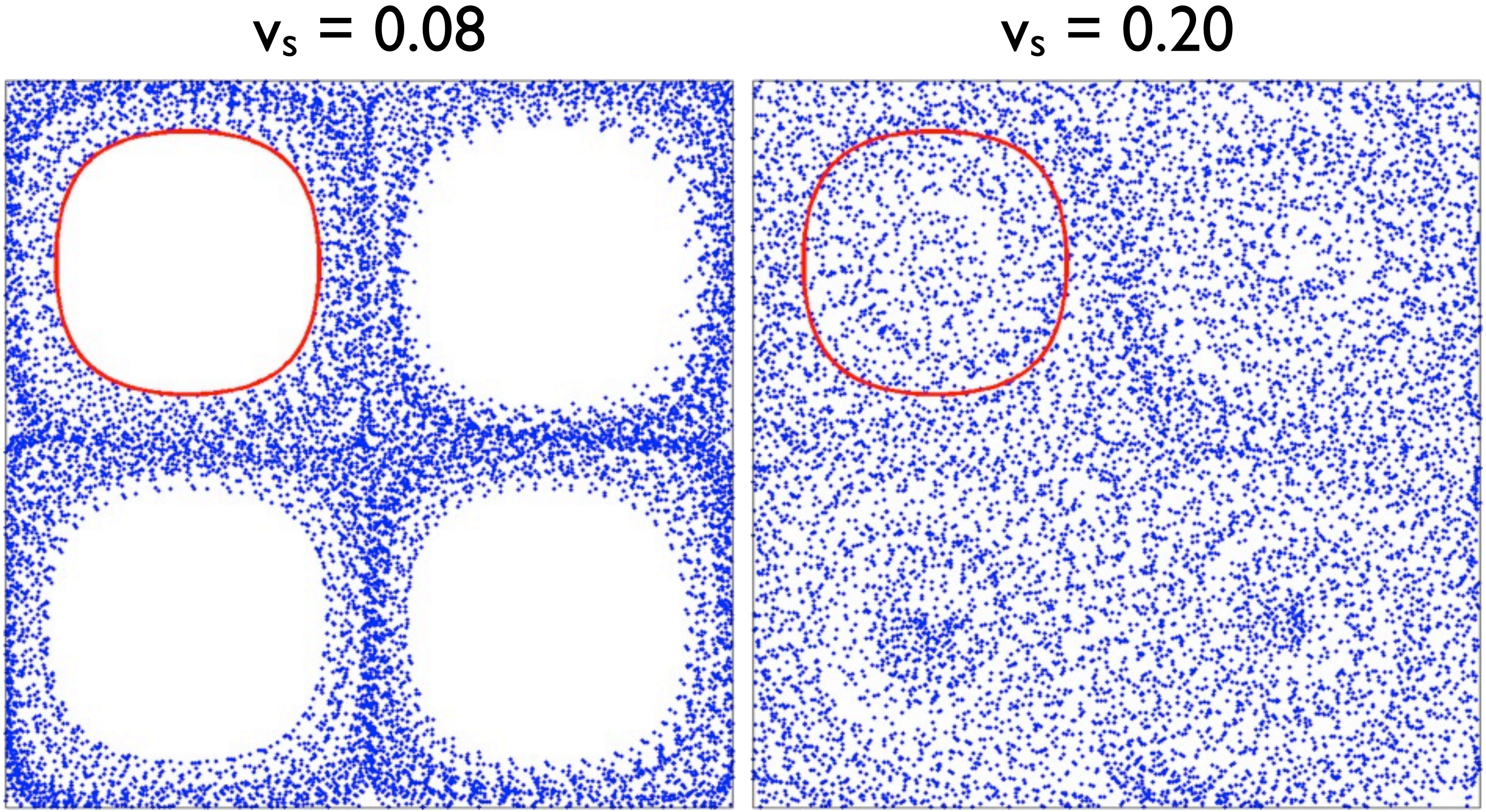


Steady Flow, Swimmer

$v_s = 0.01$ $v_s = 0.03$ $v_s = 0.07$



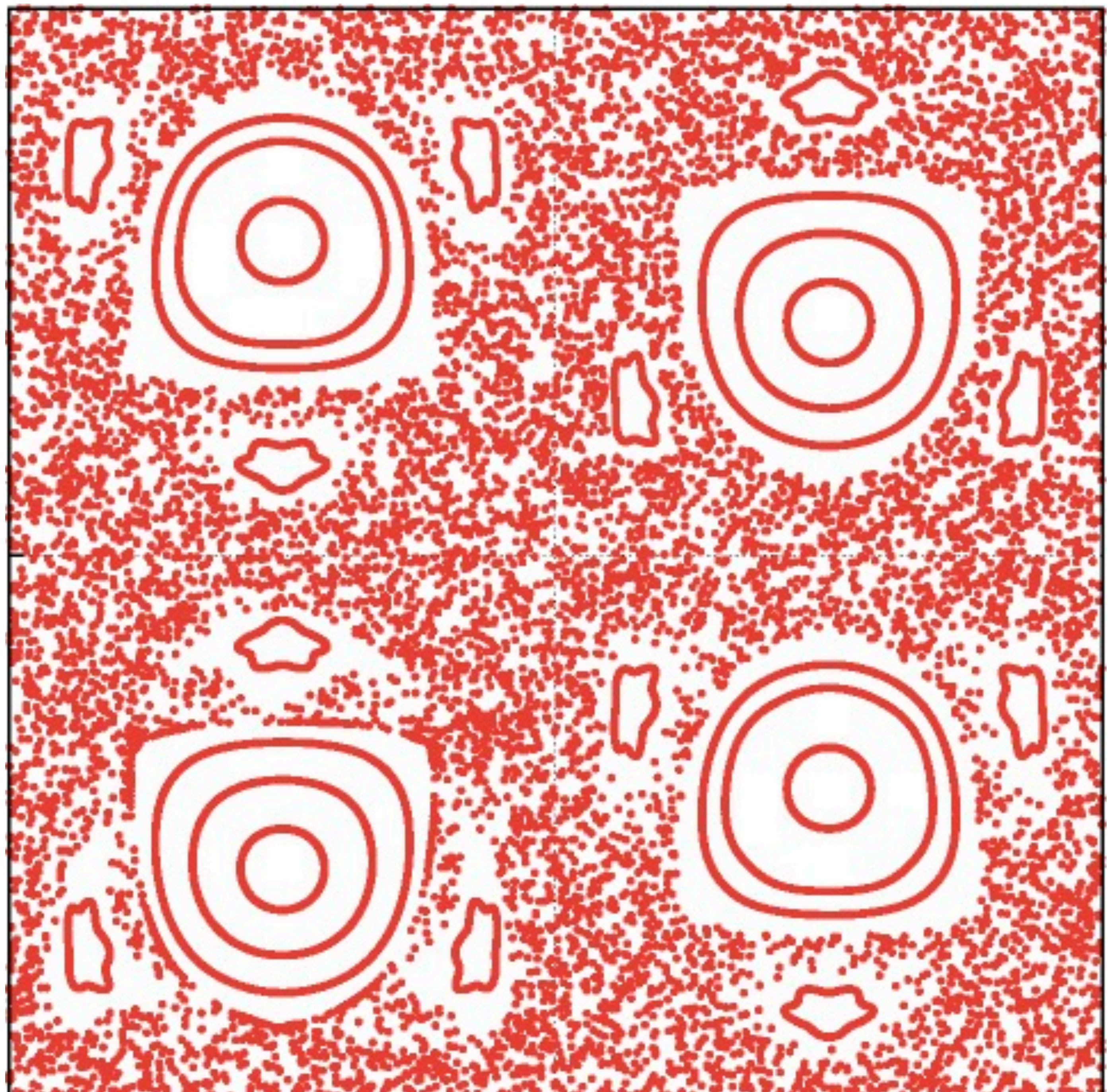
Steady Flow, Swimmer



Oscillating Flow, Fluid Particles

$$\psi(x, y, t) = \frac{U}{k} \sin[k(x + B \sin \Omega t)] \sin ky$$

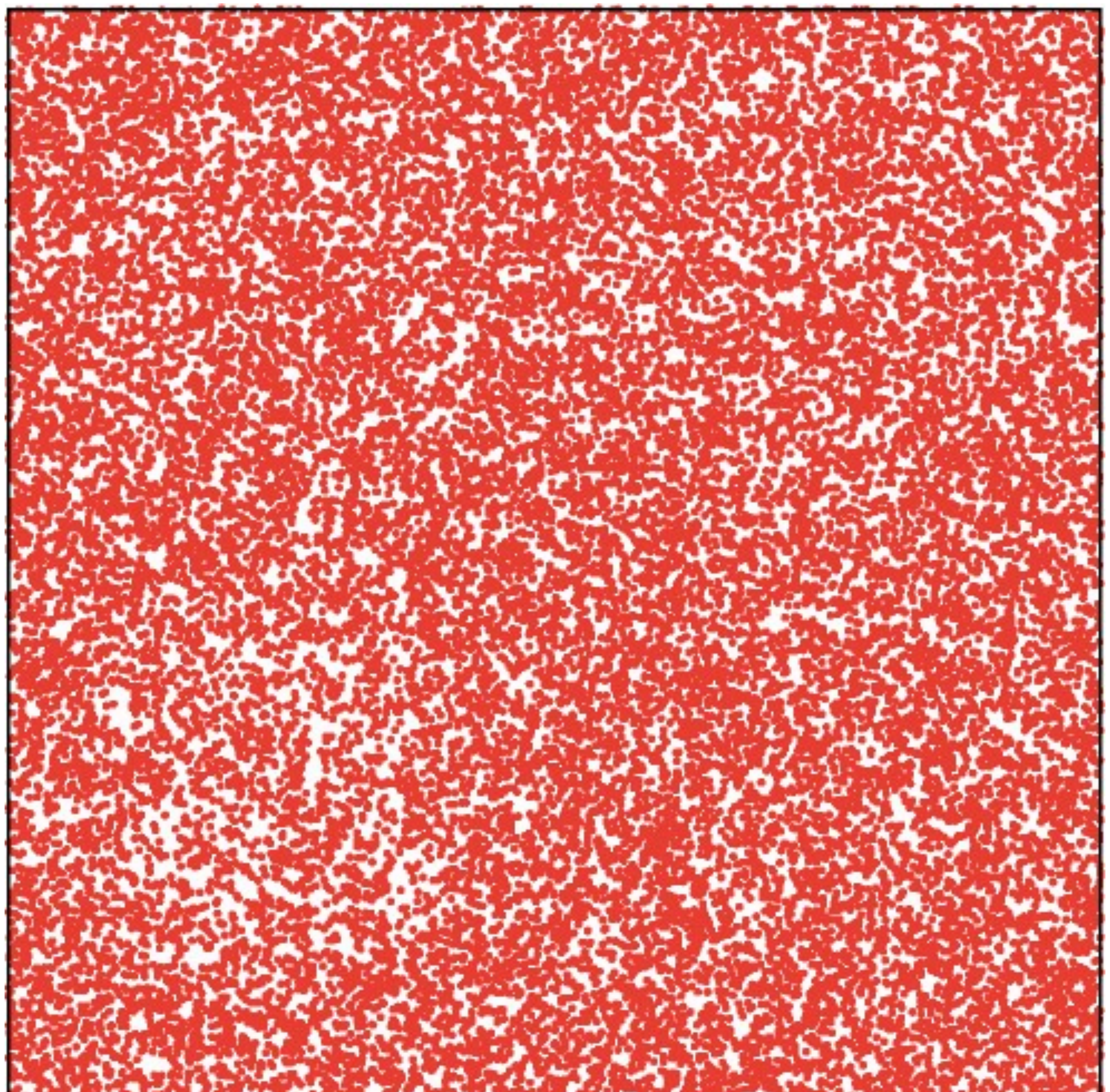
$B = 0.12$
$\Omega = 6.28$



N. Khurana, J. Blawzdziewicz, & NTO, *PRL* (2011)

Oscillating Flow, Swimmers

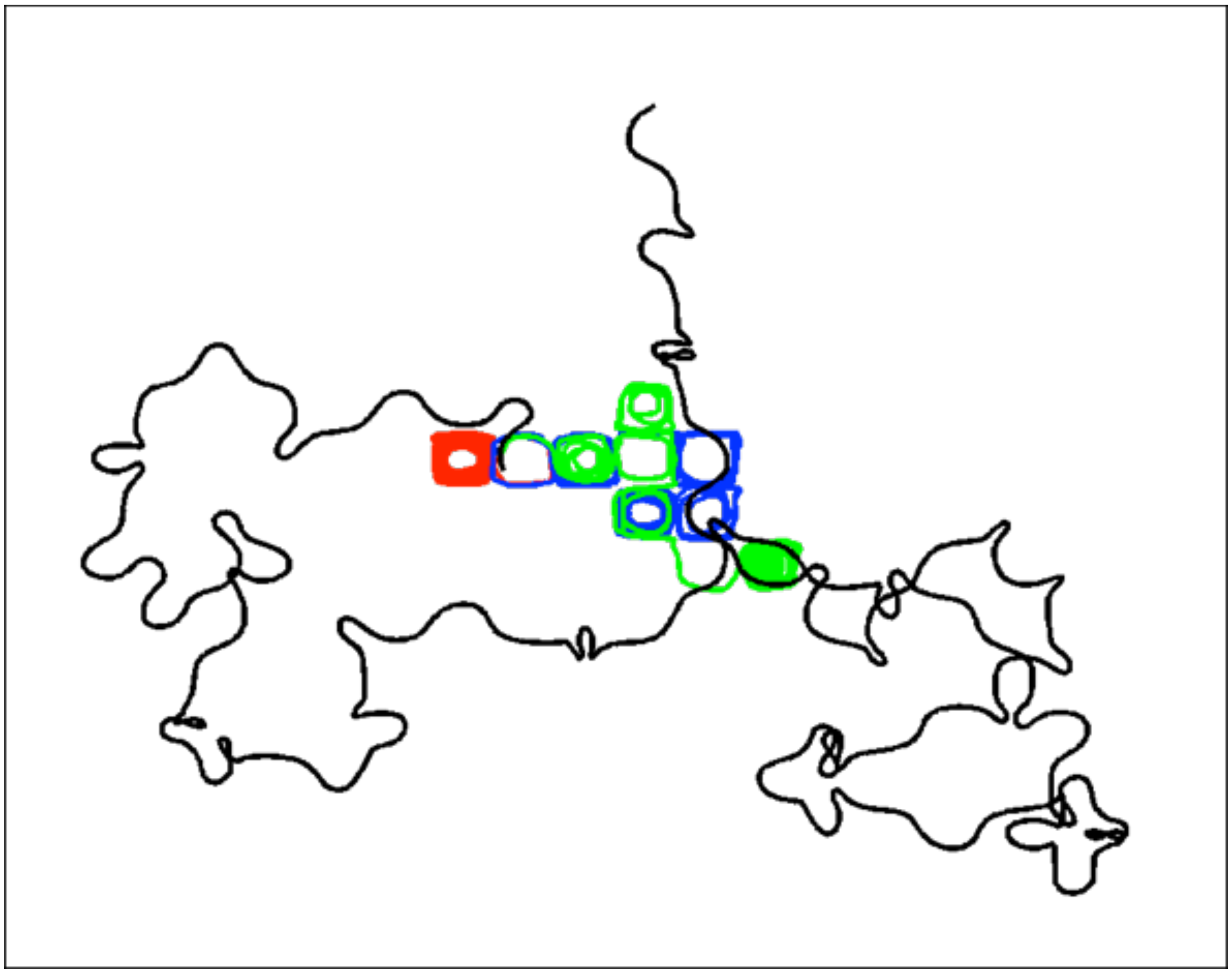
$v_s = 0.05$



Swimmers break transport boundaries

N. Khurana, J. Blawzdziewicz, & NTO, *PRL* (2011)

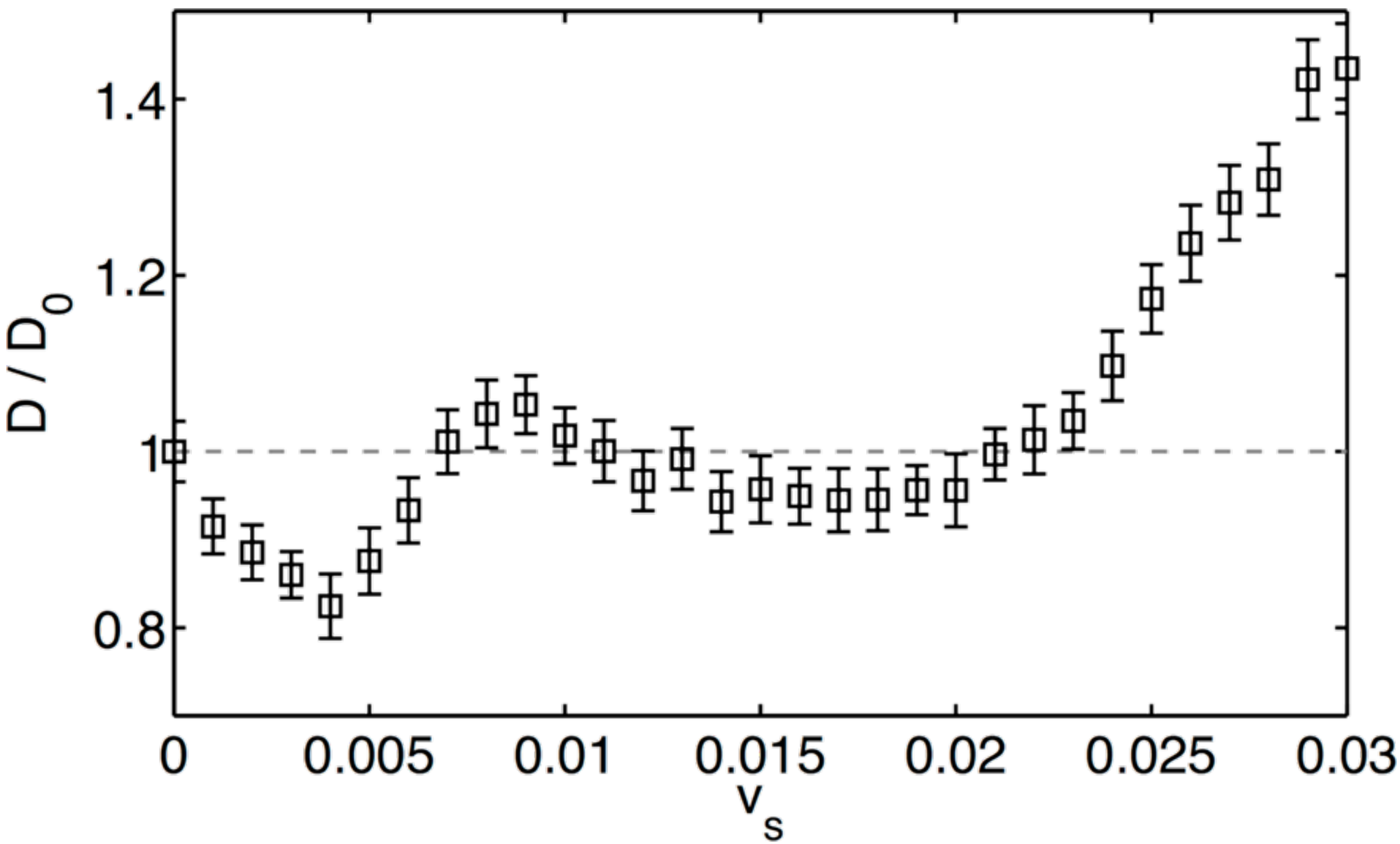
Transport?



$v_s = 0$
$v_s = 0.01$
$v_s = 0.1$
$v_s = 1$

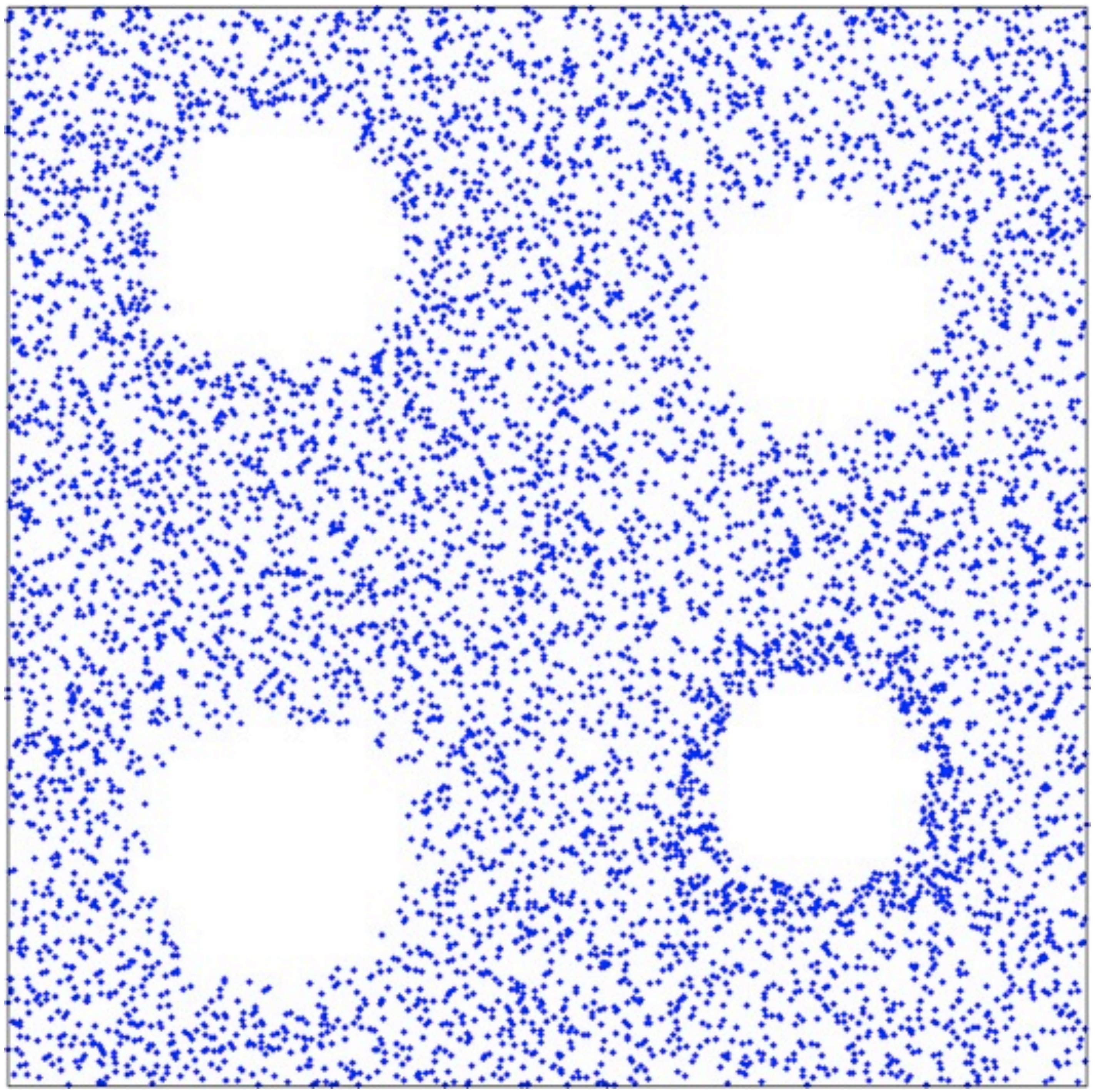
Long-time dynamics are diffusive

Chaotic Diffusion



N. Khurana, J. Blawzdziewicz, & NTO, *PRL* (2011)

Single Swimmer Dynamics

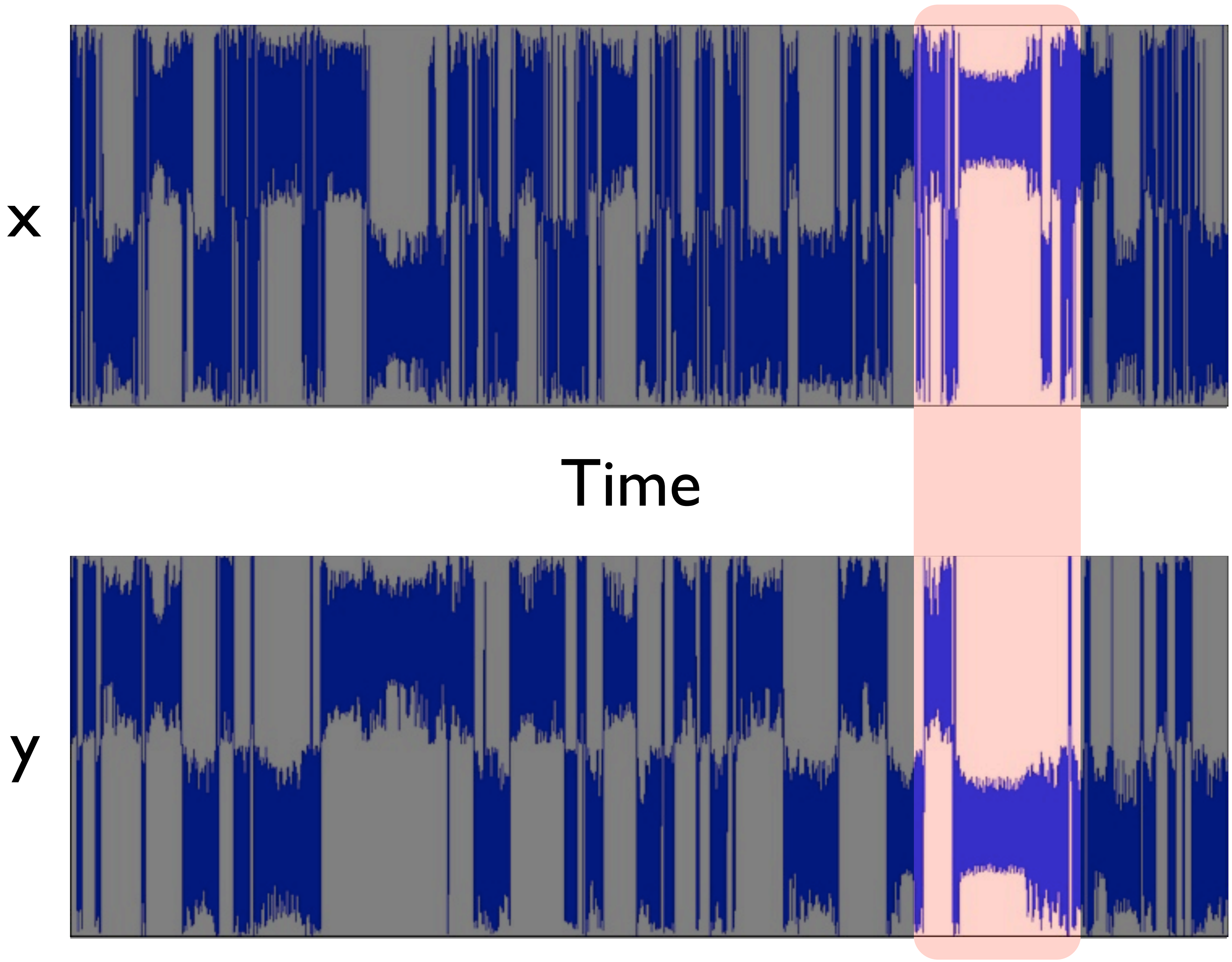


v_s = 0.01

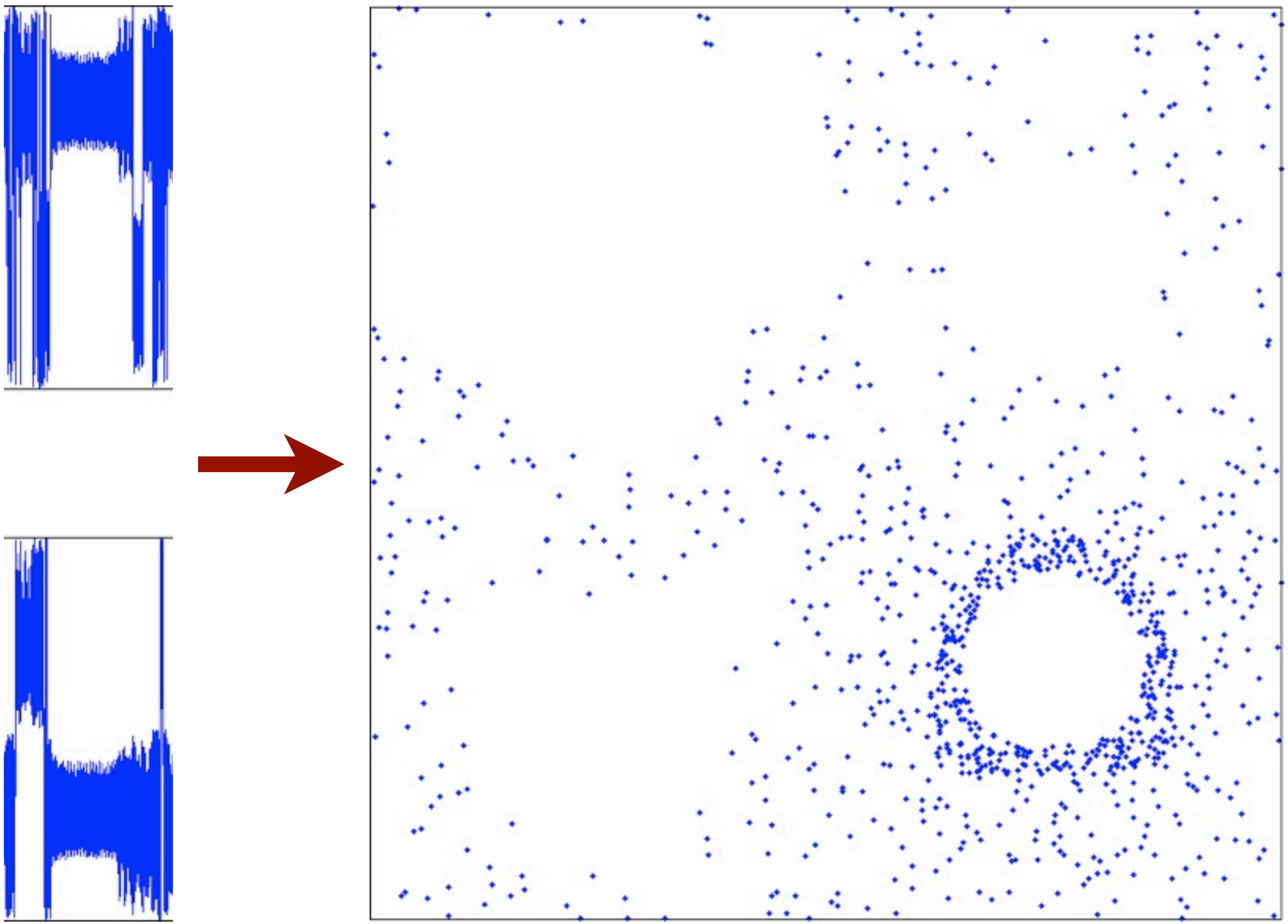
Period 3 islands are gone

Overdensity around
period 1 island

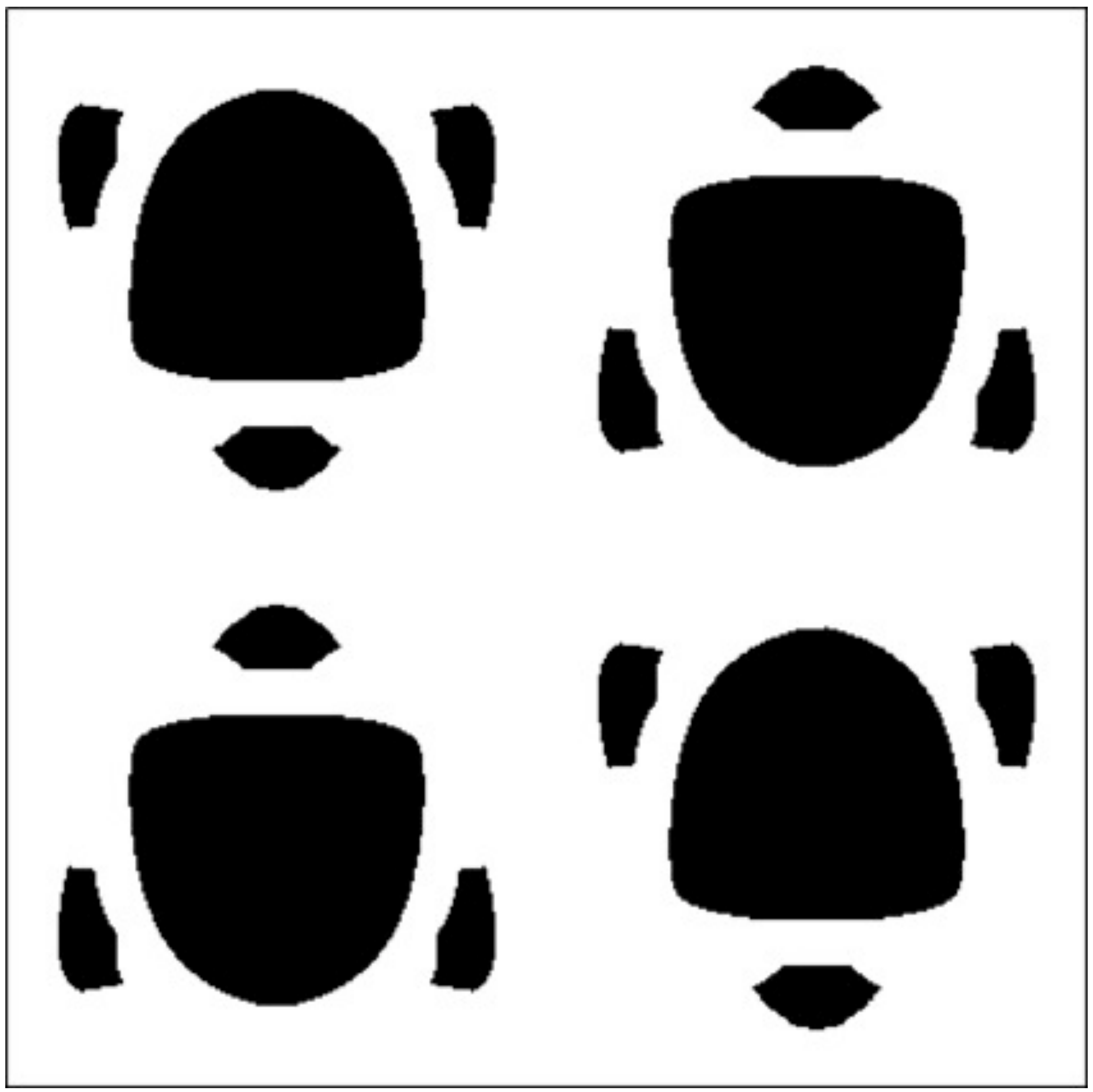
“Sticky” Regions



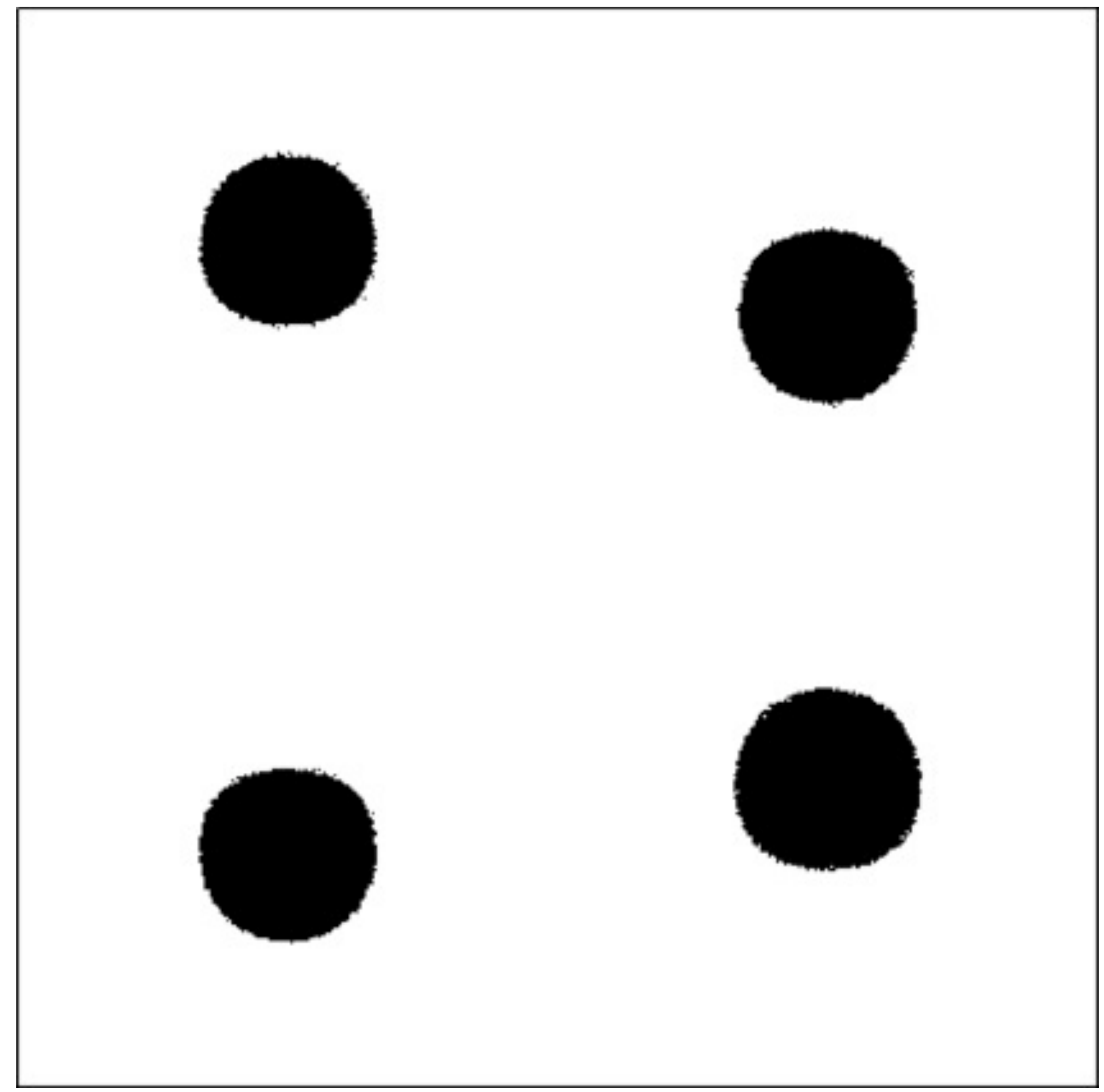
Trapping



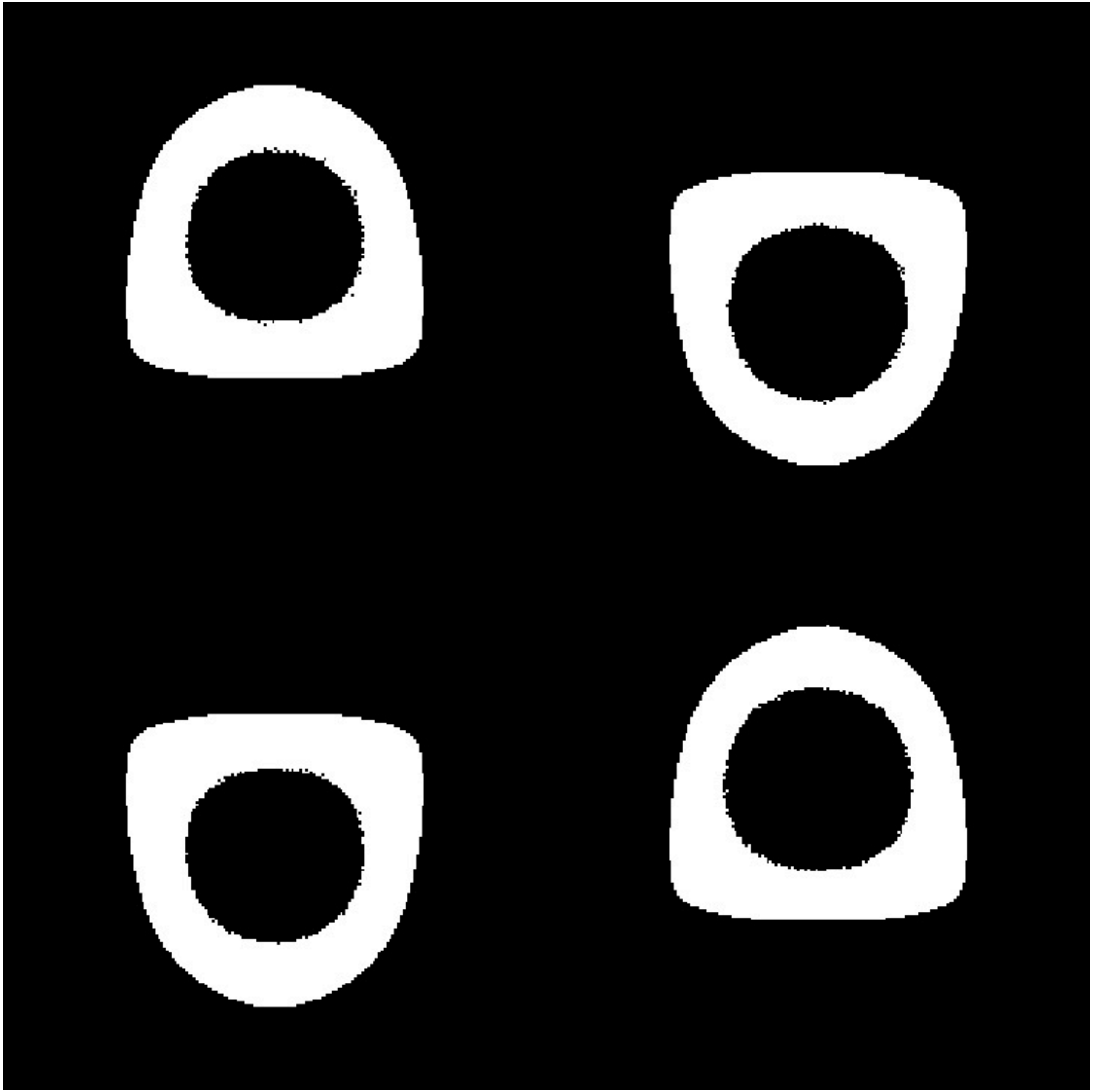
Traps form in newly accessible regions

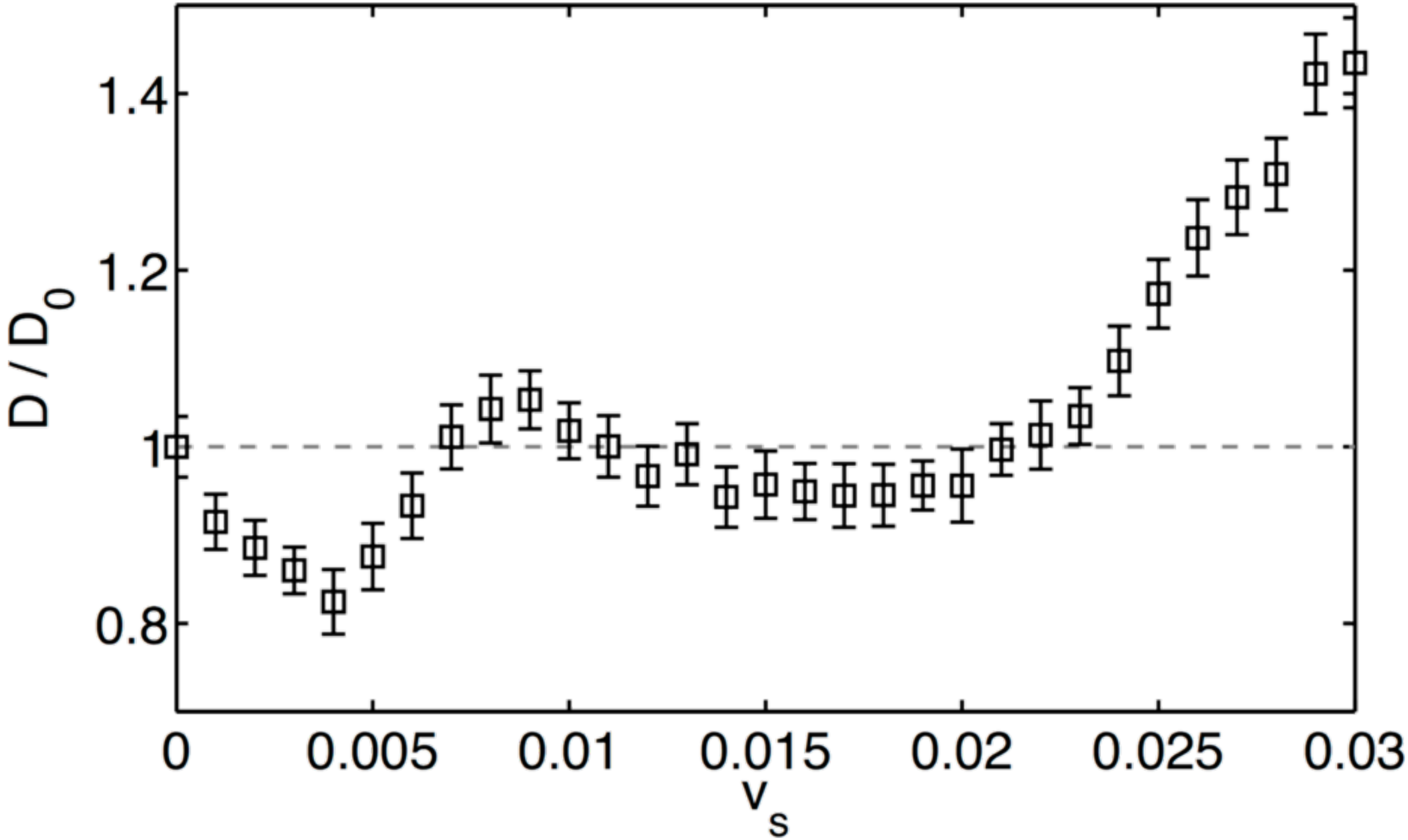


$v_s = 0$



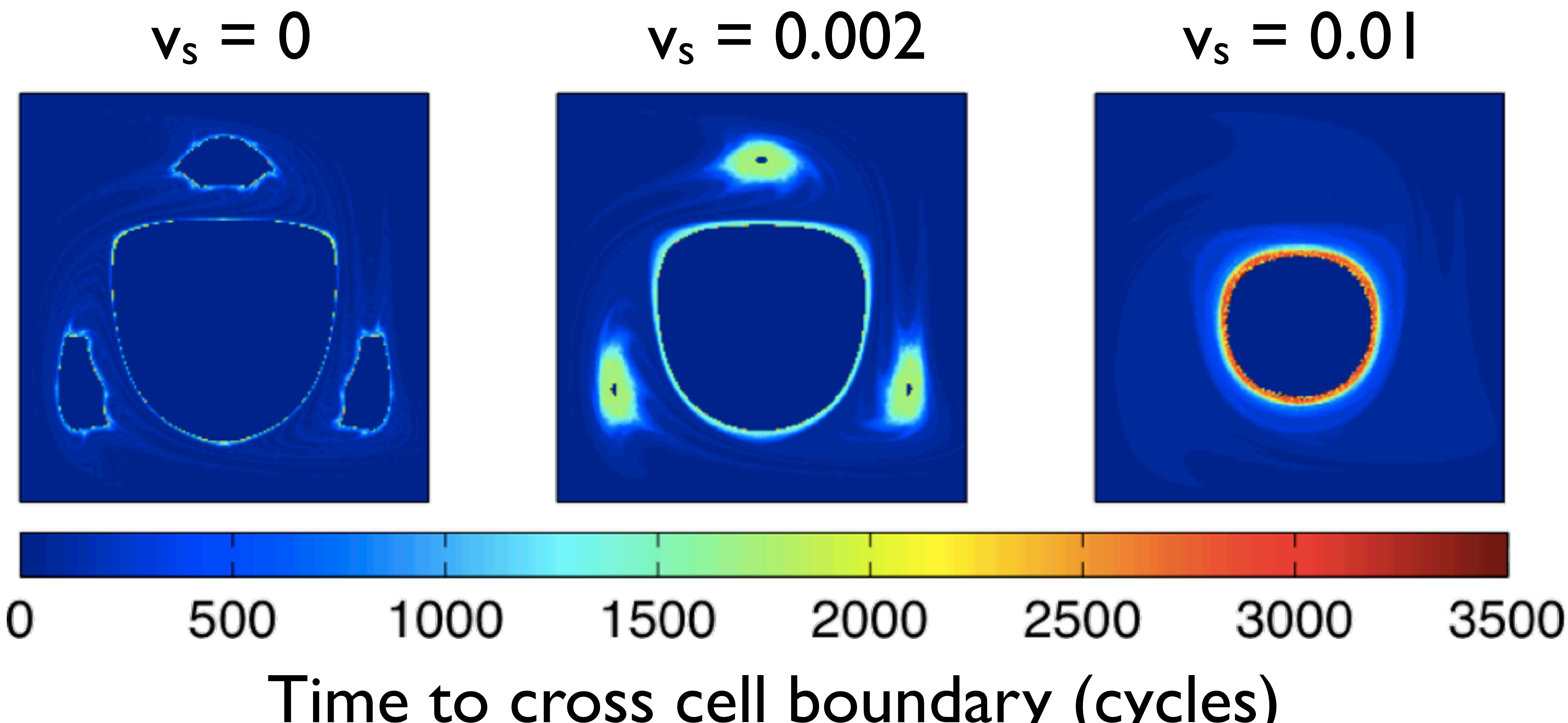
$v_s = 0.01$



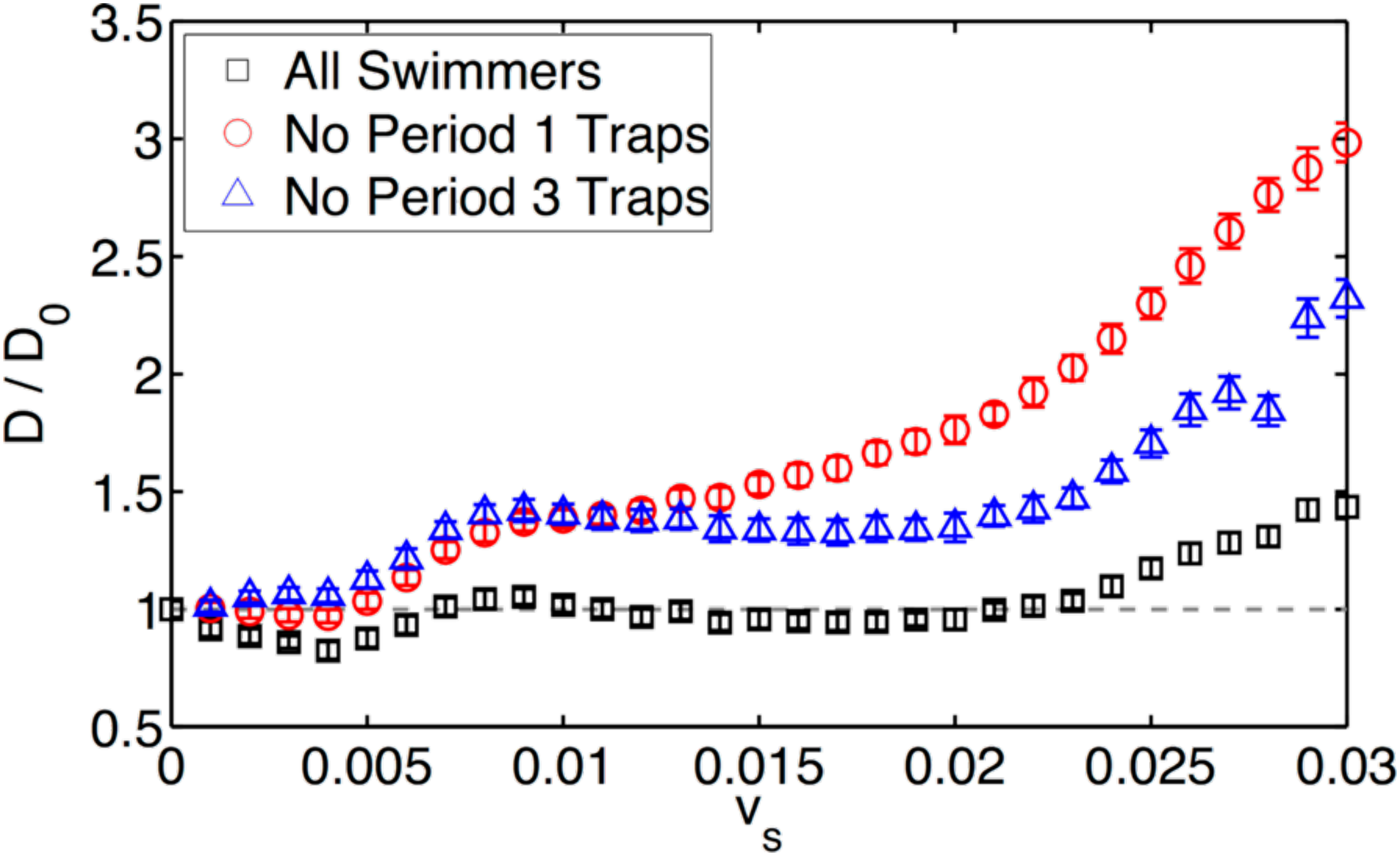


1.4
1.2
1
0.8
D / D_0
0
0.005
0.01
0.015
0.02
0.025
0.03
v_s

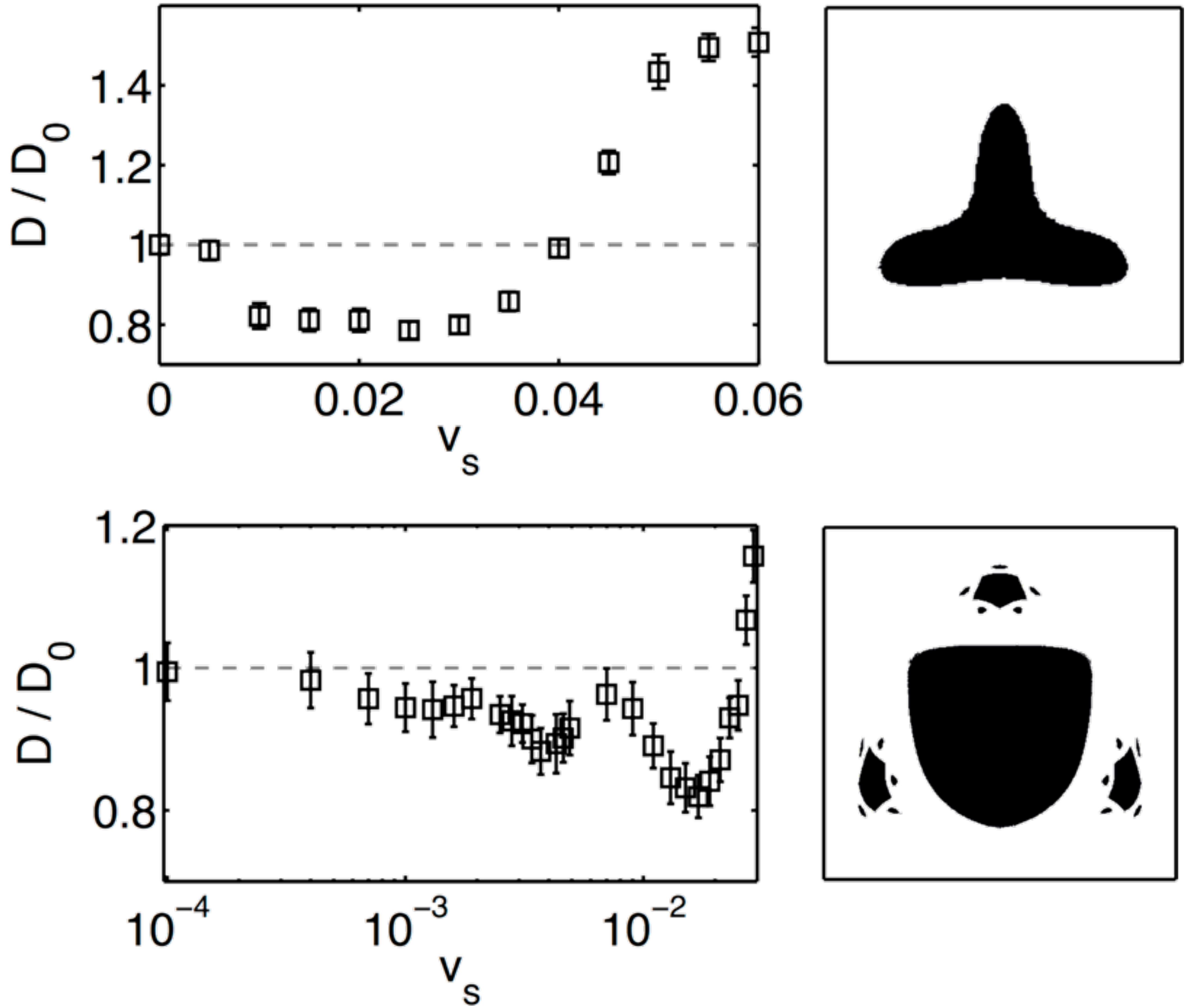
Escape Times



N. Khurana, J. Blawzdziewicz, & NTO, *PRL* (2011)



N. Khurana, J. Blawzdziewicz, & NTO, *PRL* (2011)



N. Khurana, J. Blawzdziewicz, & NTO, *PRL* (2011)

Additional Complexities

Add true stochasticity

→ model imperfect response

Vary particle shape

→ allow coupling to strain field

→ permit formation of attractors

N. Khurana & NTO, *Phys. Fluids* 2012

Include particle/particle interactions

Use a 3D, turbulent flow

N. Khurana & NTO, *New J. Phys.* 2013

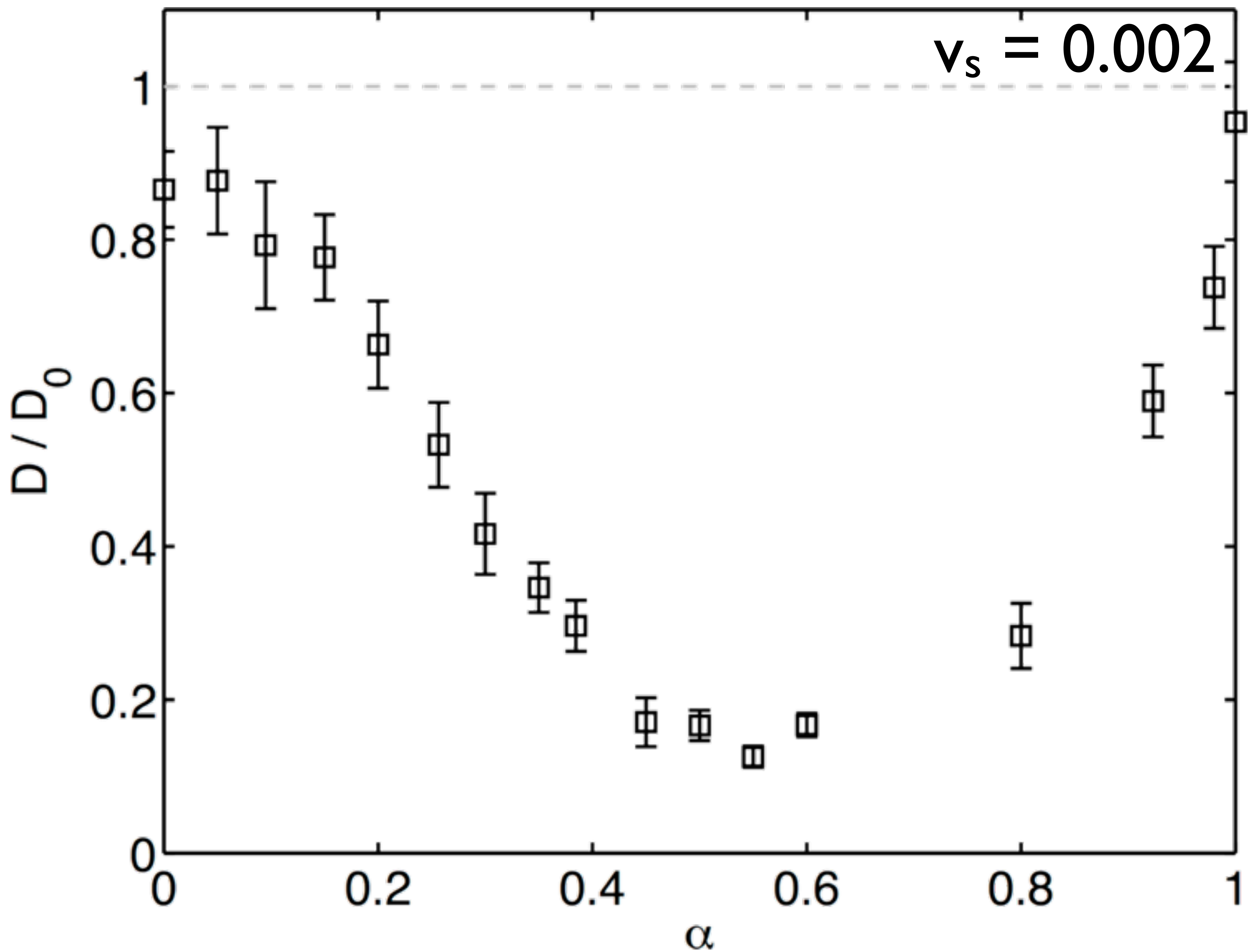
Ellipsoidal Swimmers

$$\dot{x} = \frac{\partial \psi}{\partial y} + v_s \cos\theta$$

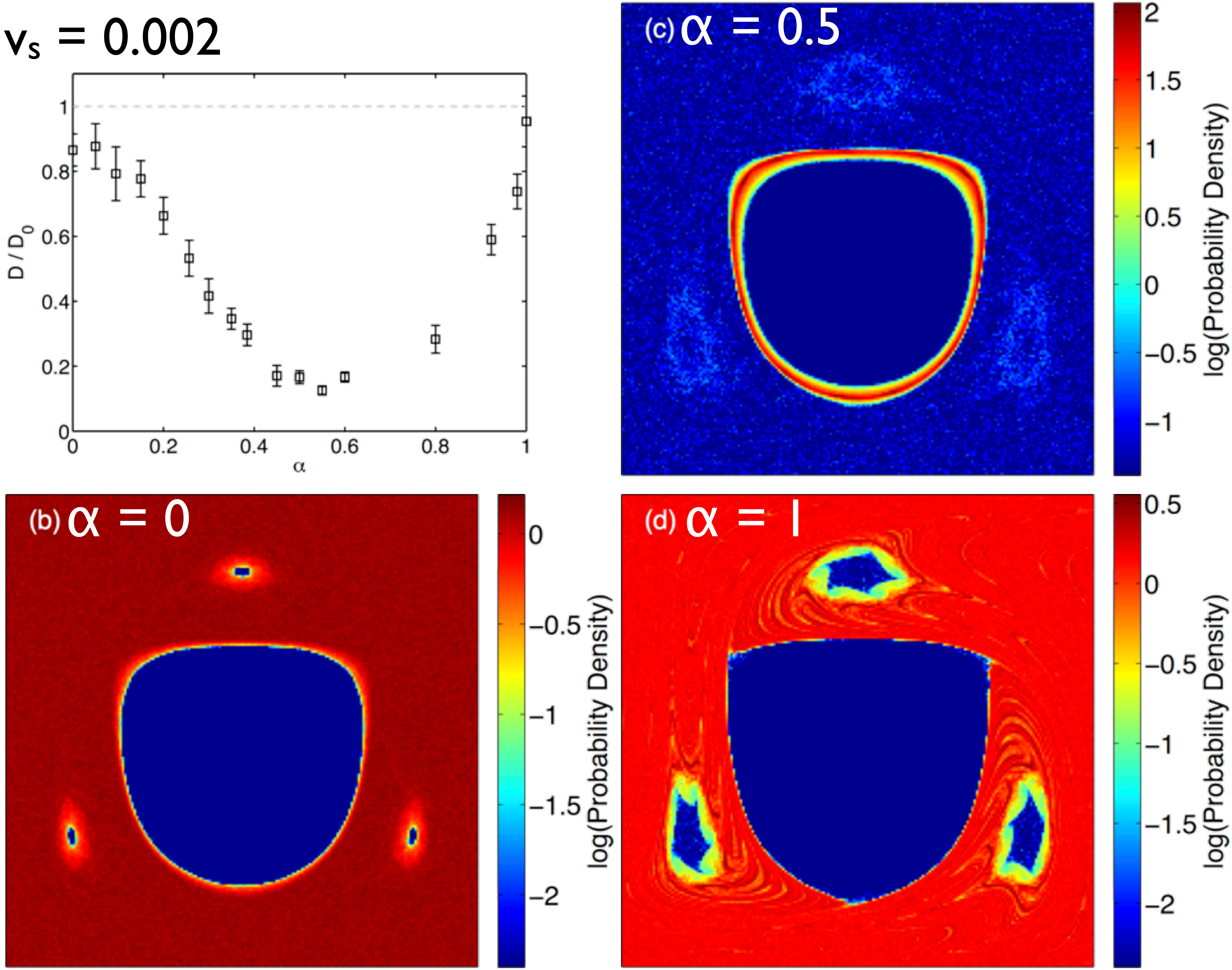
$$\dot{y} = -\frac{\partial \psi}{\partial x} + v_s \sin\theta$$

$$\dot{\theta} = \alpha \left[\frac{1}{2} \left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right) \cos 2\theta - \frac{\partial^2 \psi}{\partial x \partial y} \sin 2\theta \right] - \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)$$

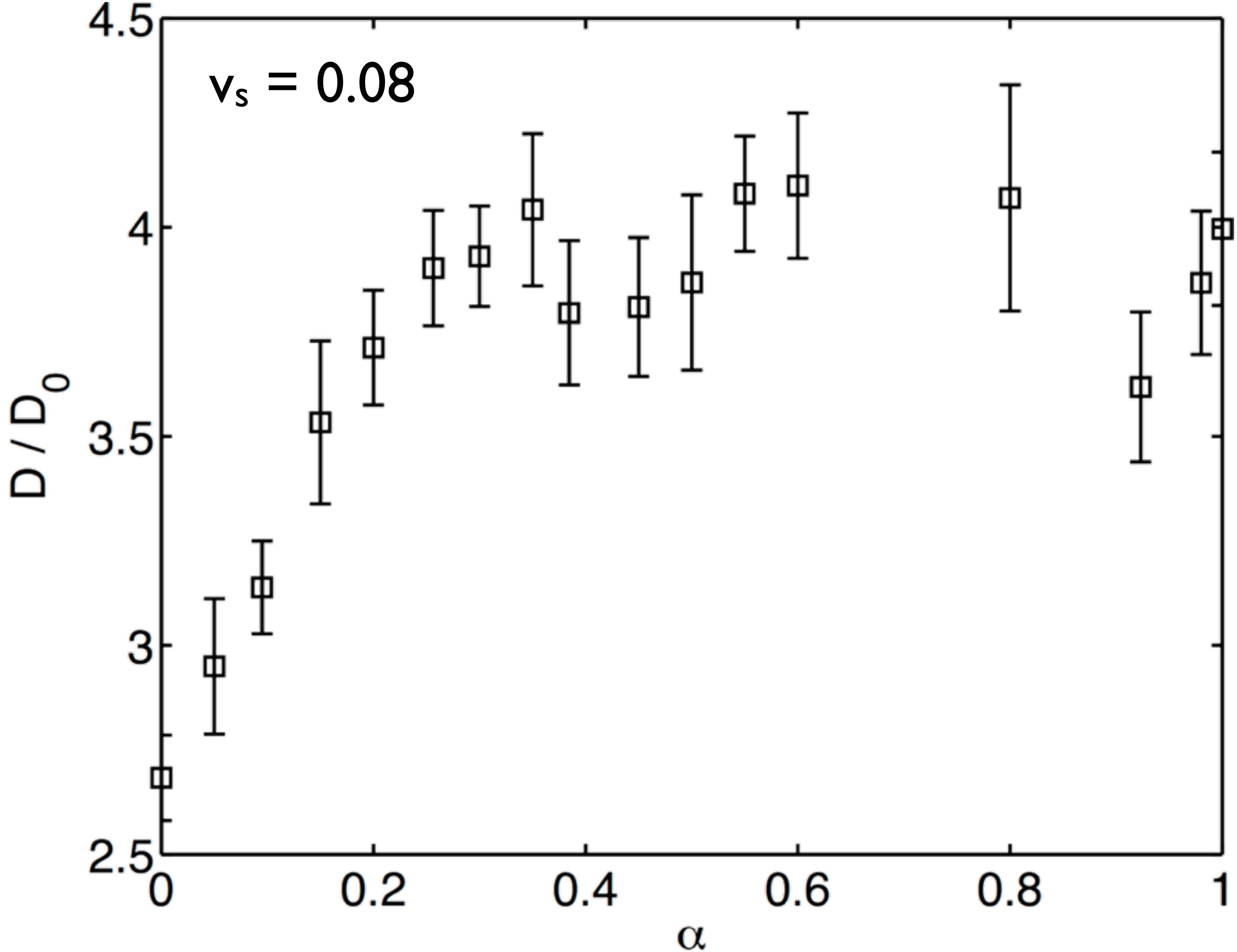
Ellipsoidal Swimmers



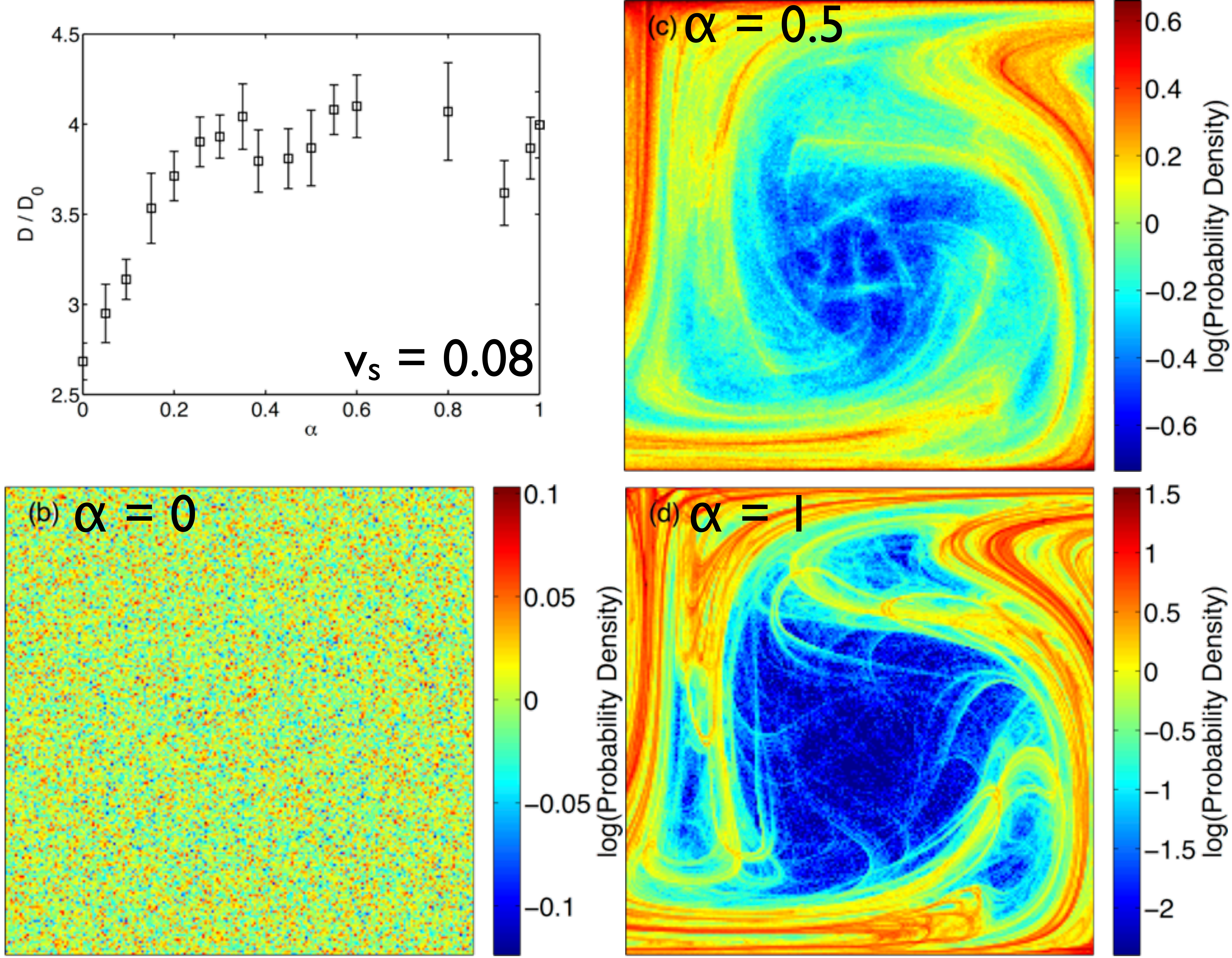
N. Khurana & NTO, *Phys. Fluids* (2012)



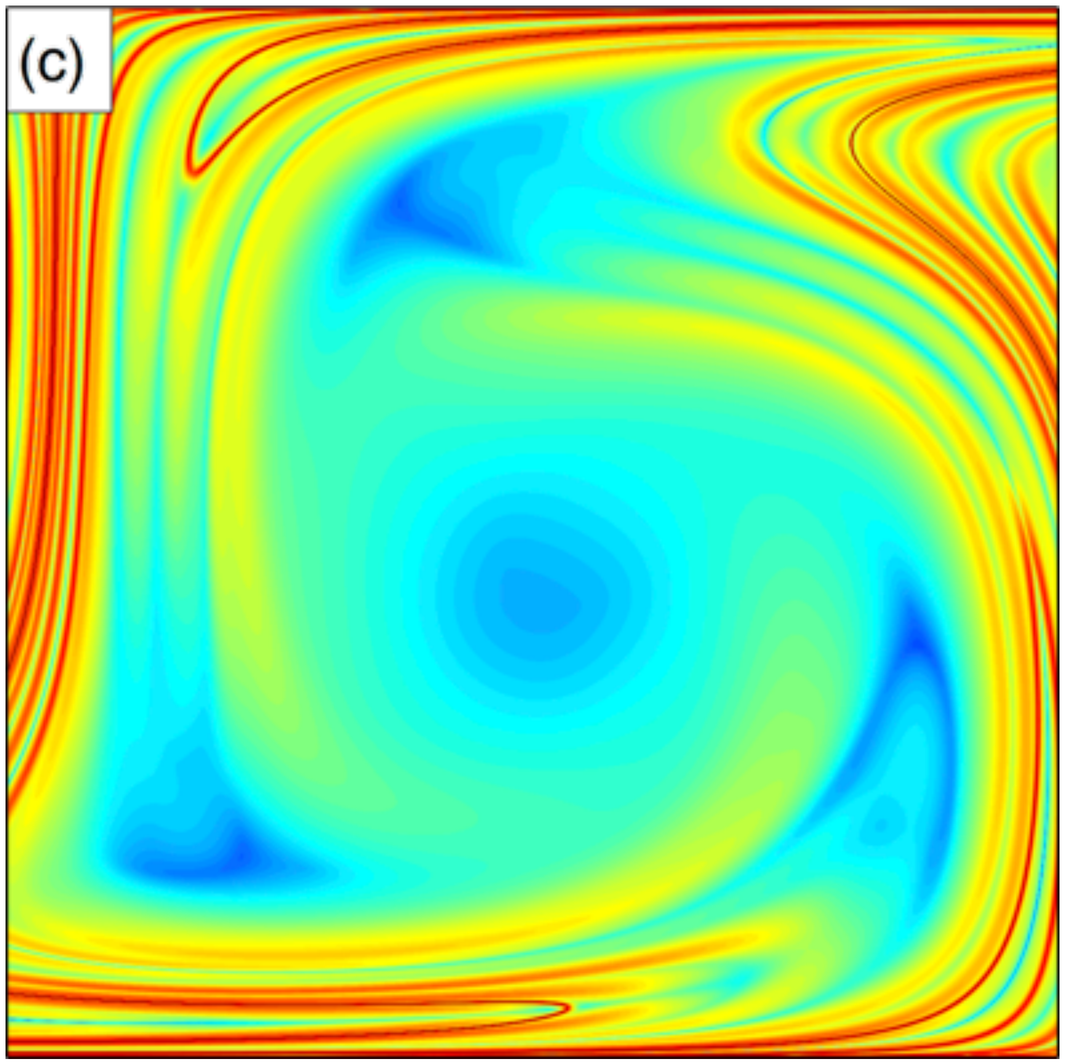
$v_s = 0.002$
D / D0
α
(b) α = 0
(c) α = 0.5
(d) α = 1
log(Probability Density)



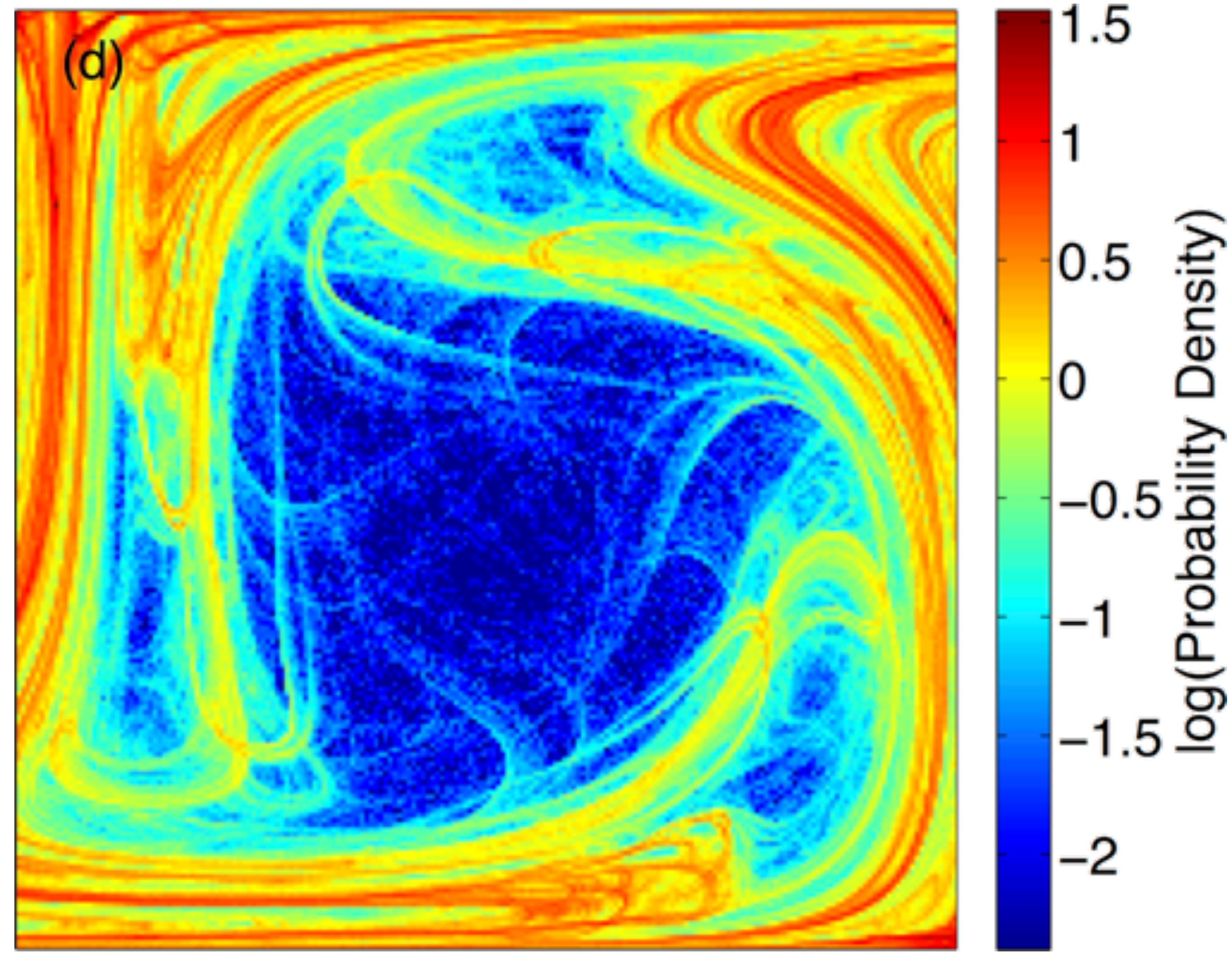
N. Khurana & NTO, *Phys. Fluids* (2012)



4.5
4
3.5
3
2.5
D / D0
0
0.2
0.4
0.6
0.8
1
α
vs = 0.08
(c) α = 0.5
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
log(Probability Density)
(b) α = 0
0.1
0.05
0
−0.05
−0.1
log(Probability Density)
(d) α = 1
1.5
1
0.5
0
−0.5
−1
−1.5
−2
log(Probability Density)



FTLE
(Lagrangian Strain)

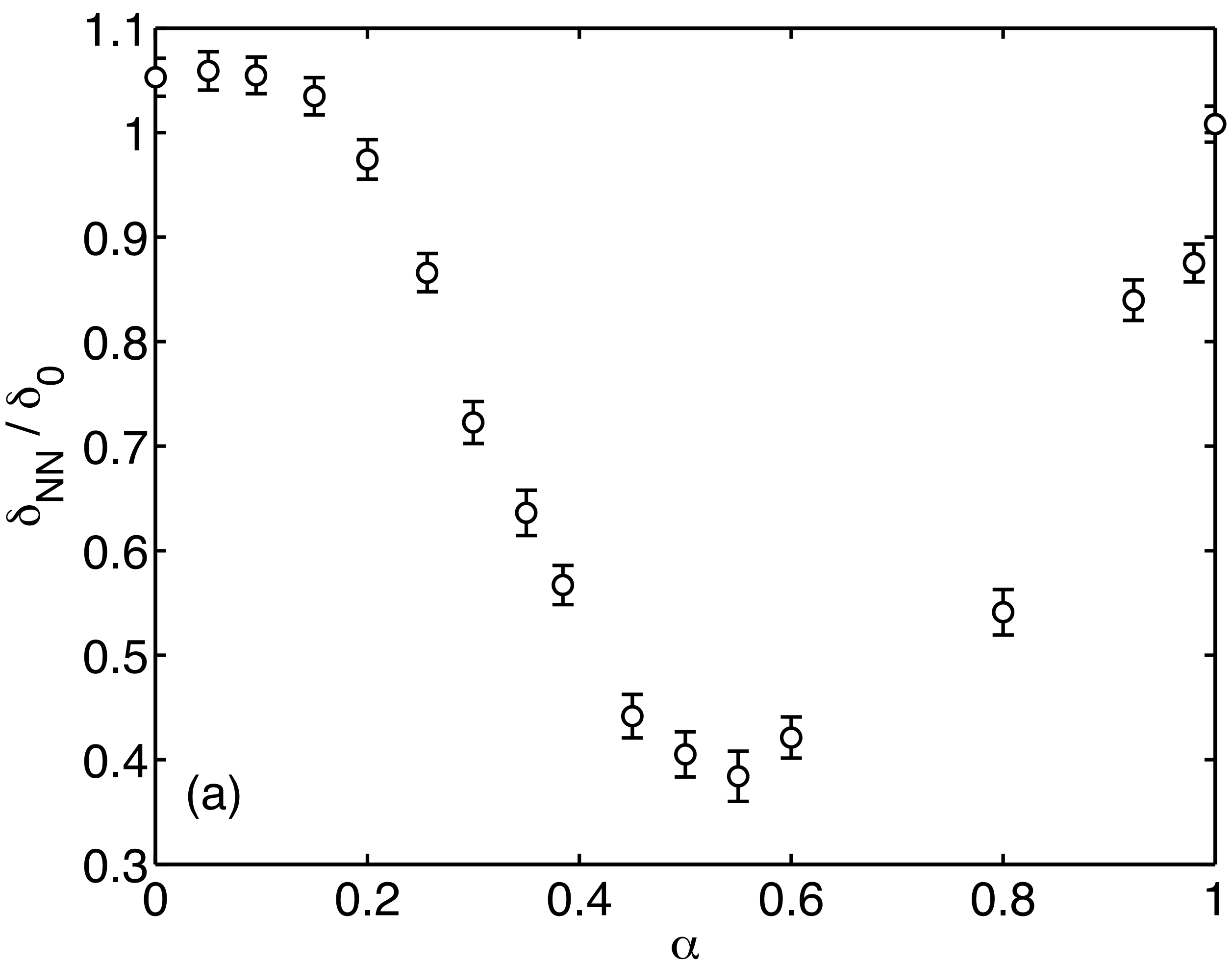


$\alpha = 1$
$v_s = 0.08$
Position PDF

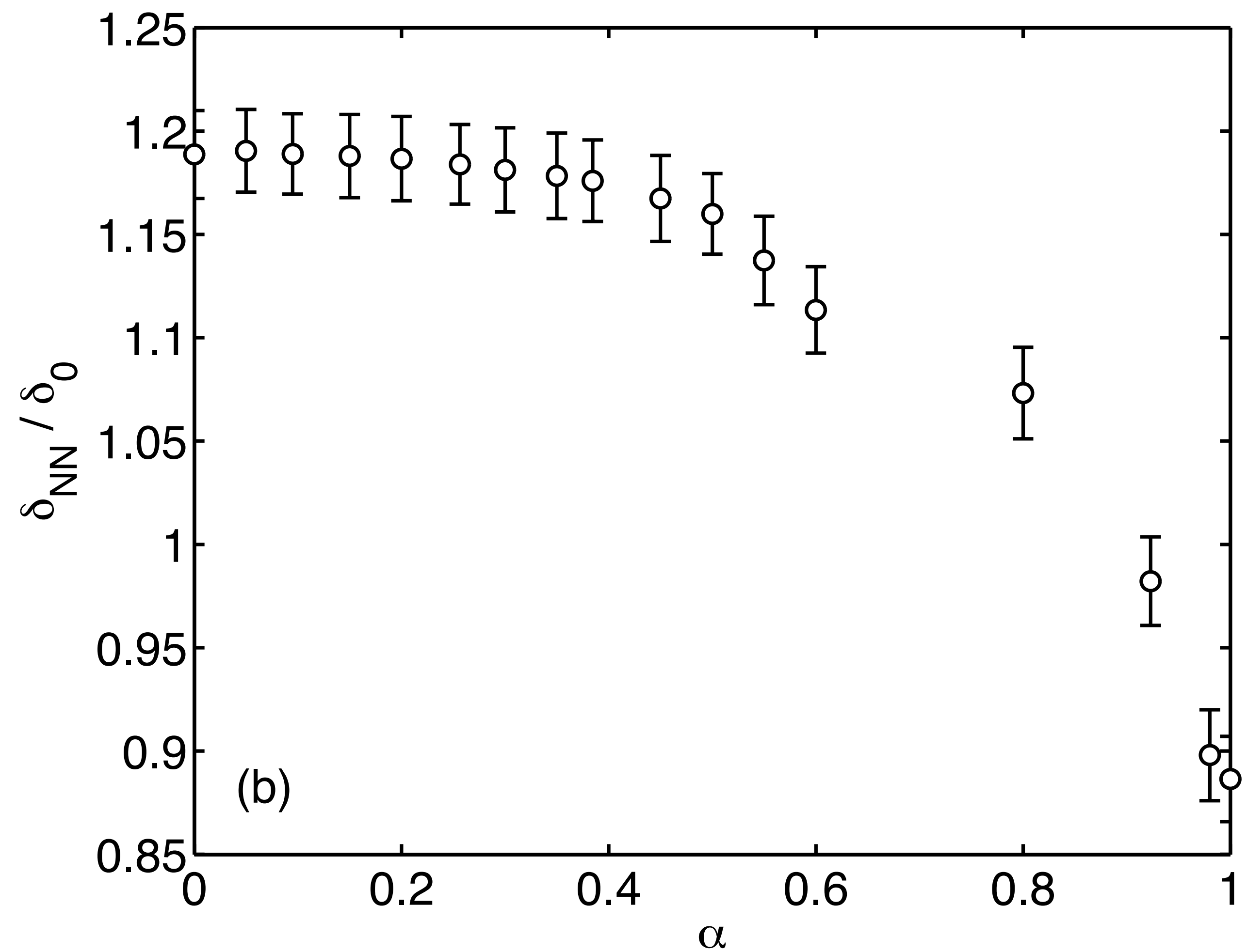
N. Khurana & NTO, *Phys. Fluids* (2012)

Encounter Rates?

$v_s = 0.002$



$v_s = 0.08$



Summary (so far)

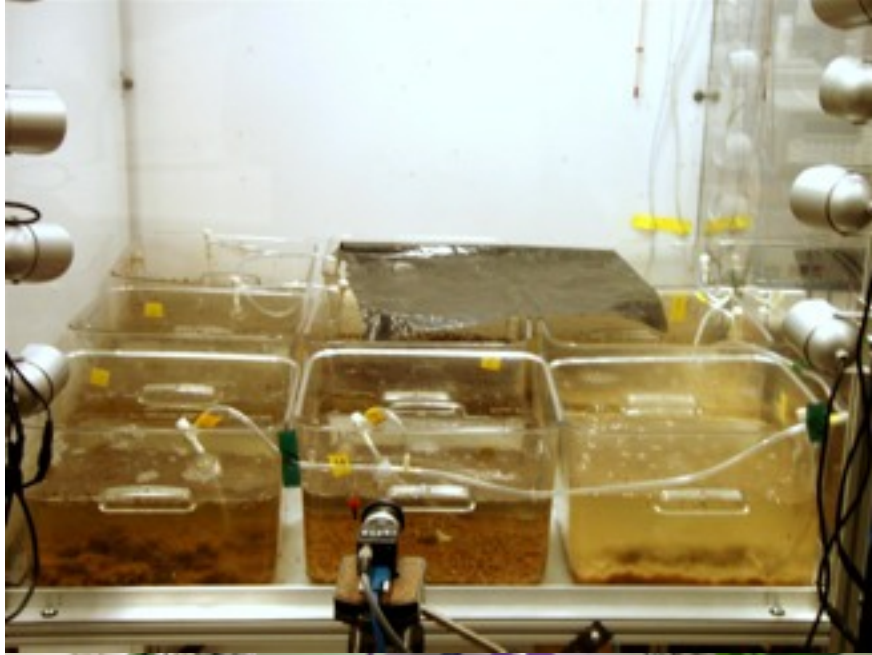
Swimming breaks flow transport barriers

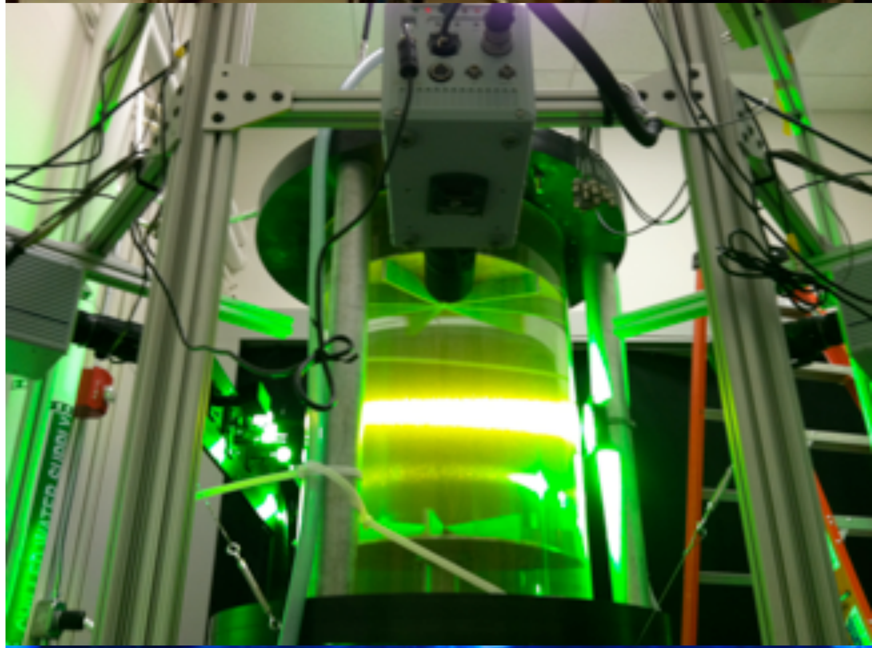
Transport may not be enhanced

Swimmers interact with flow structures

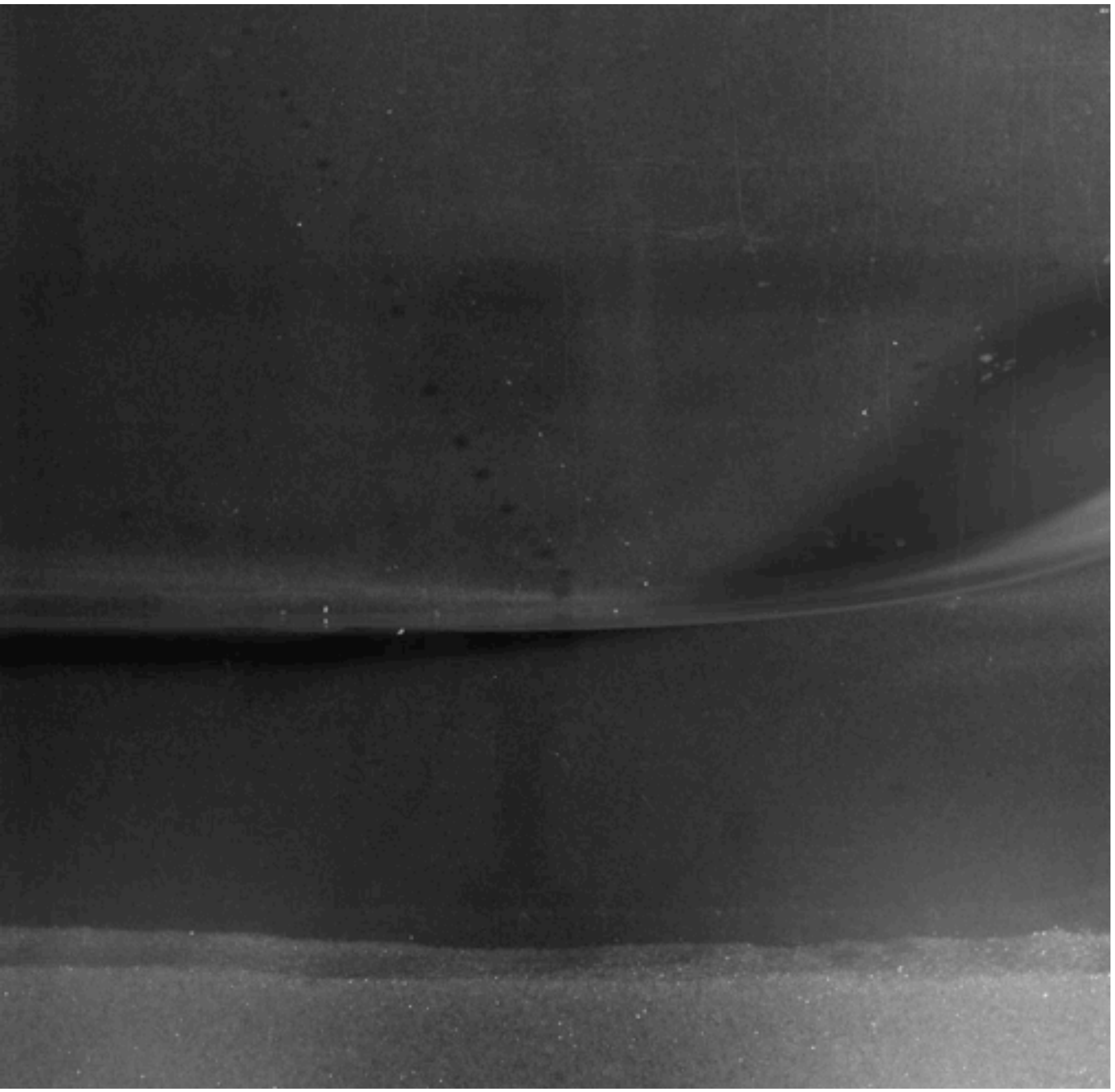
Particle *shape* plays a major role in dynamics

http://leviathan.eng.yale.edu





Erosion and Sediment Transport

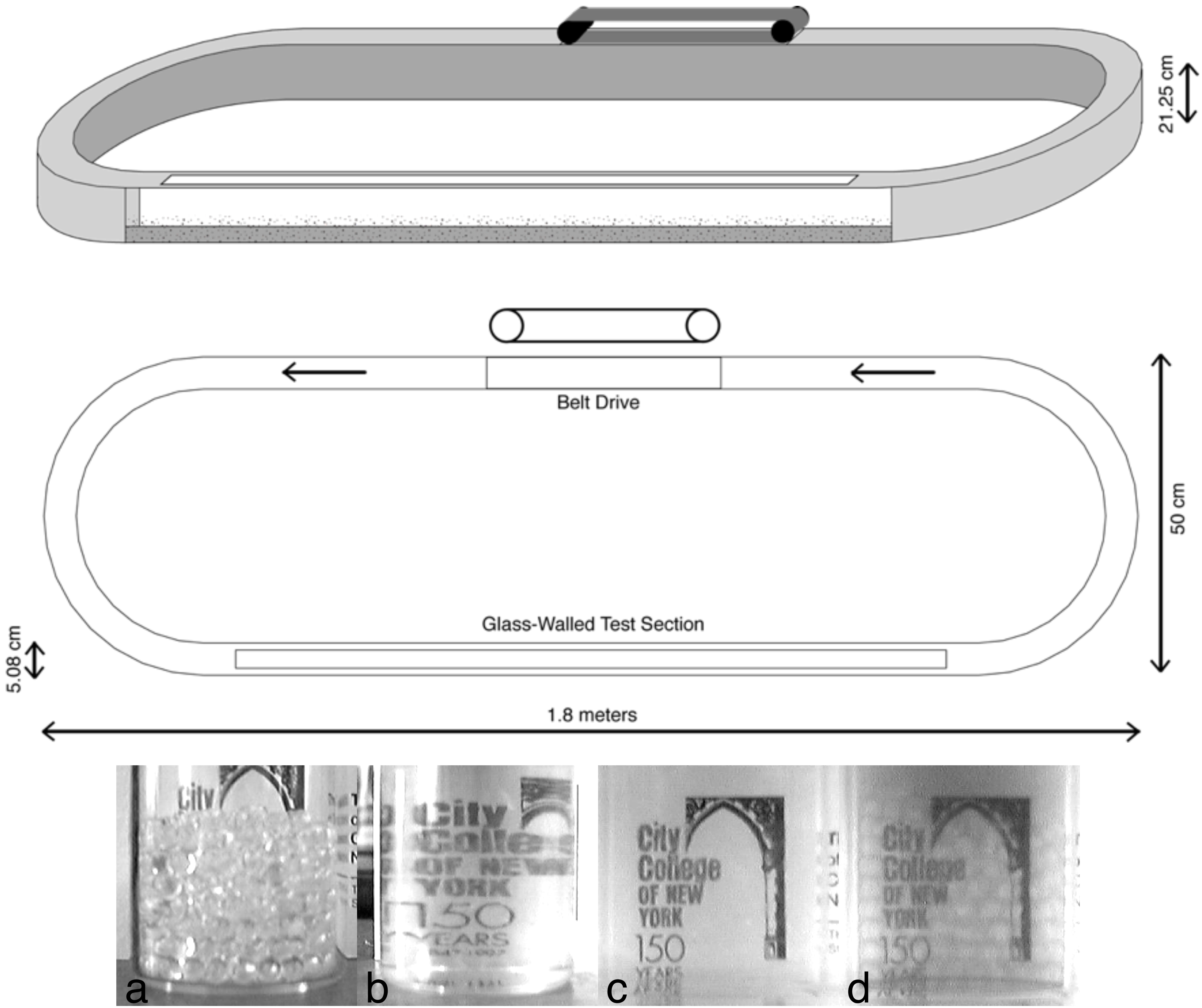


Complex flow interacting with granular material

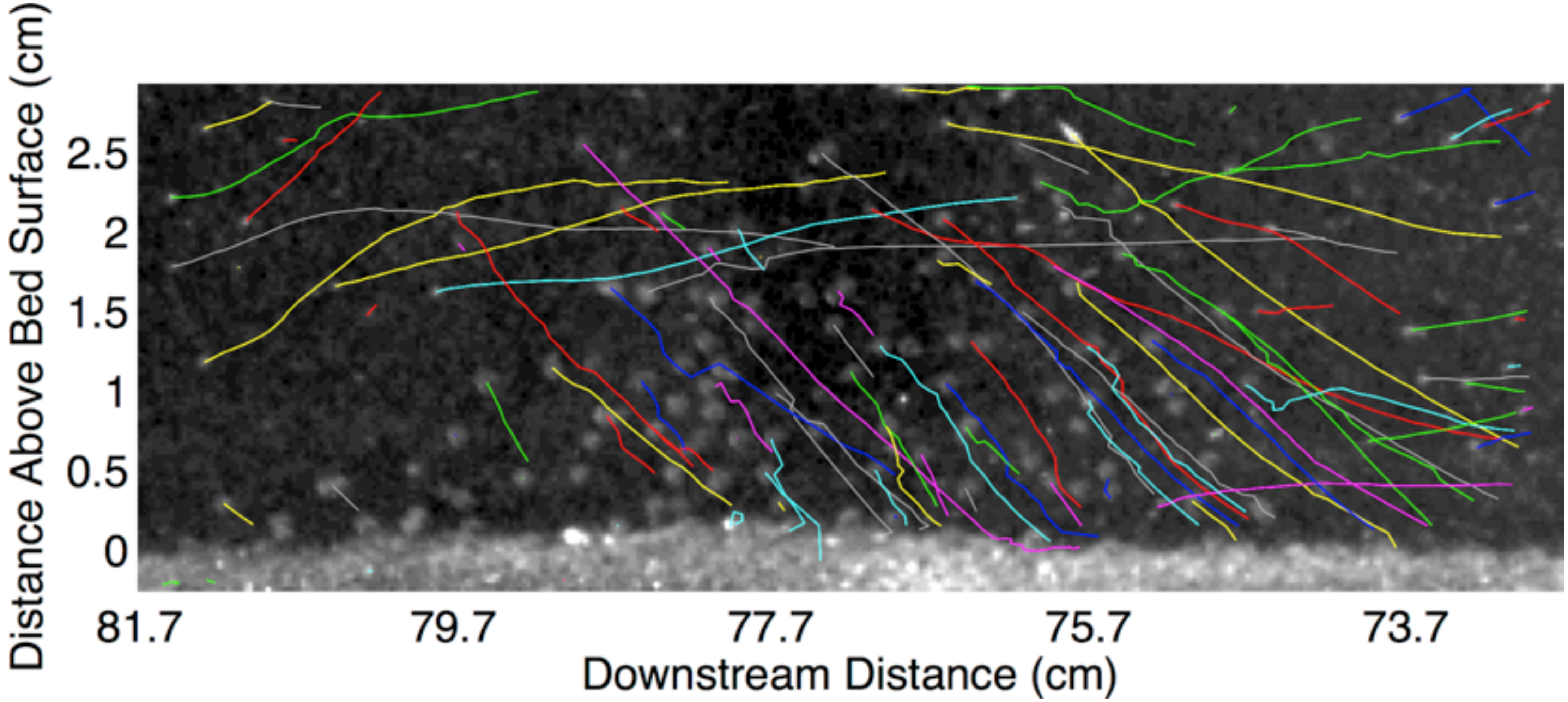
What factors are most important?

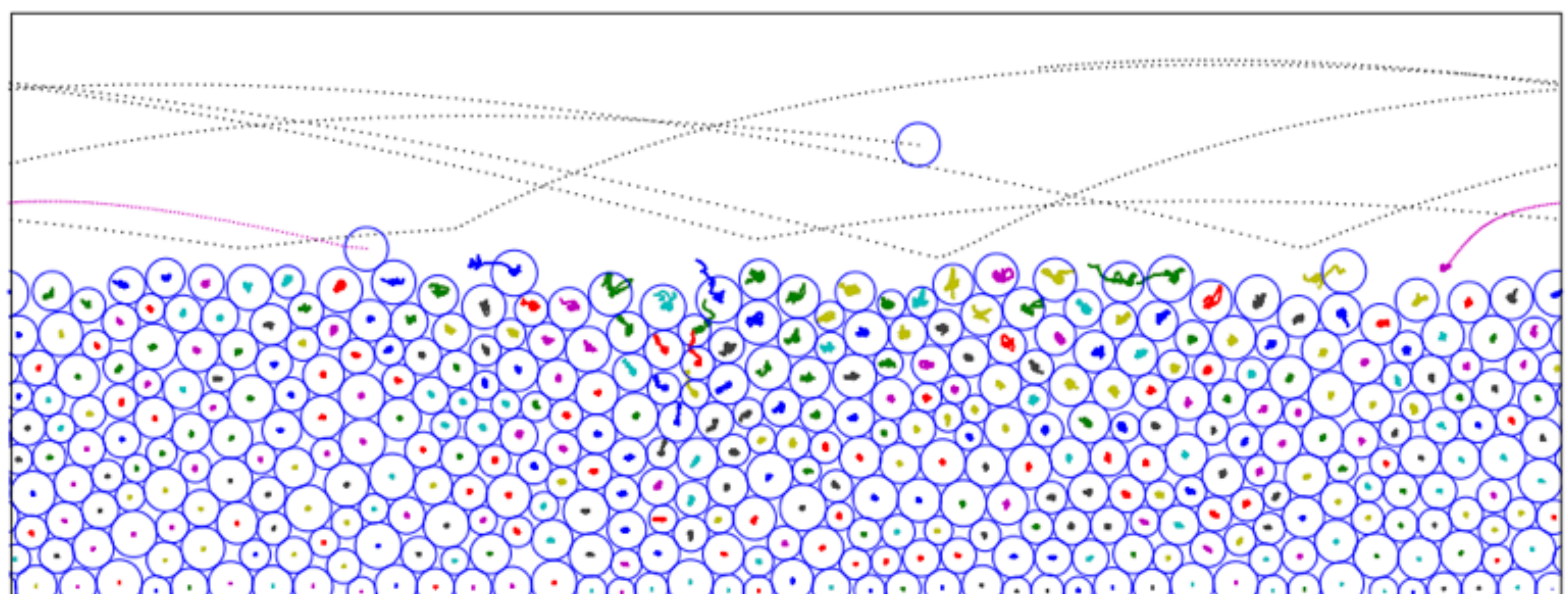
Role of bed structure?

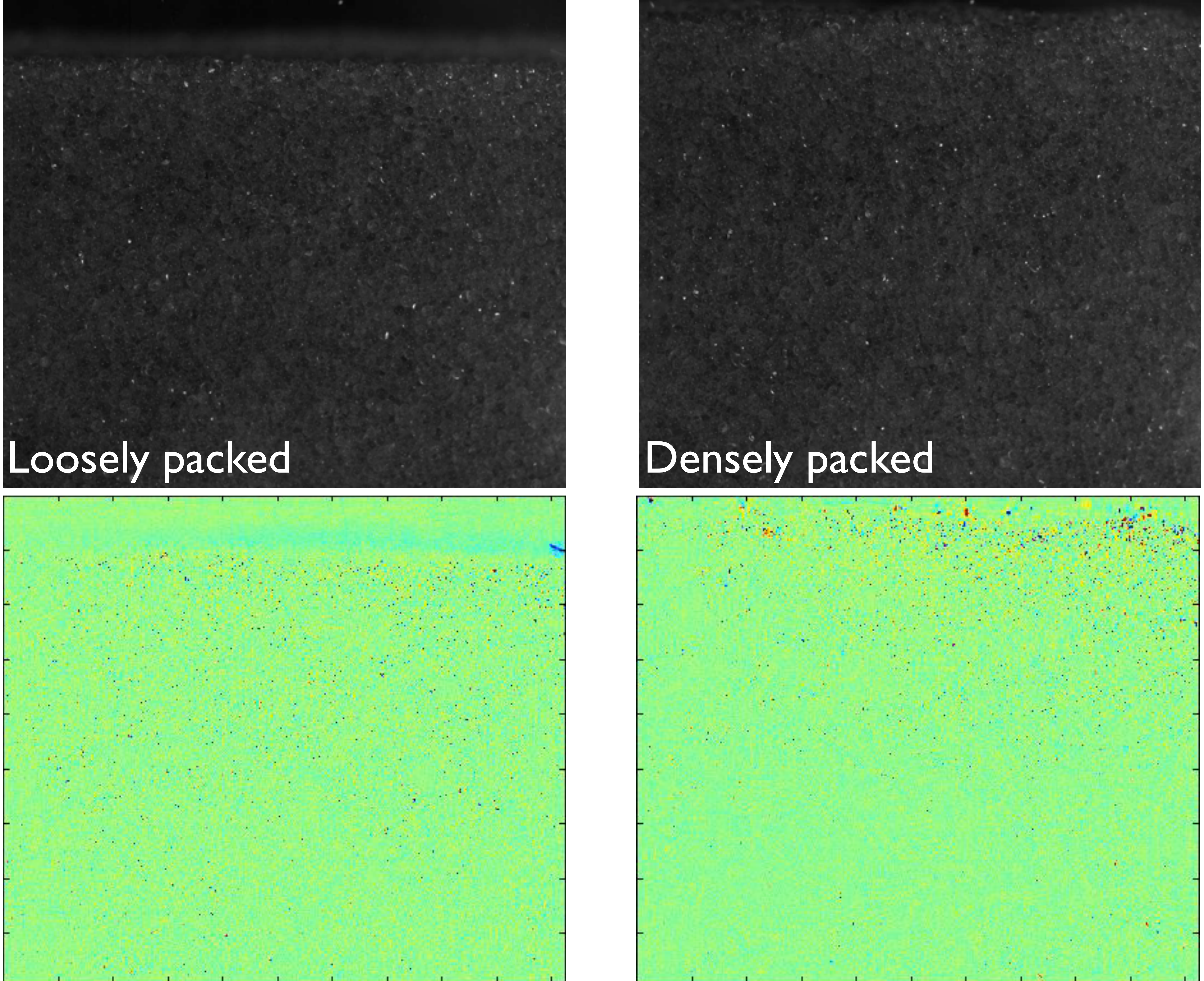
with C. O'Hern (Yale), M. Shattuck (CCNY), D. Jerolmack (Penn)


21.25 cm
Belt Drive
50 cm
Glass-Walled Test Section
5.08 cm
1.8 meters
City College OF NEW YORK 150
a
b
c
d

Complementary Experiments and Numerics







Loosely packed
Densely packed