Granular flows

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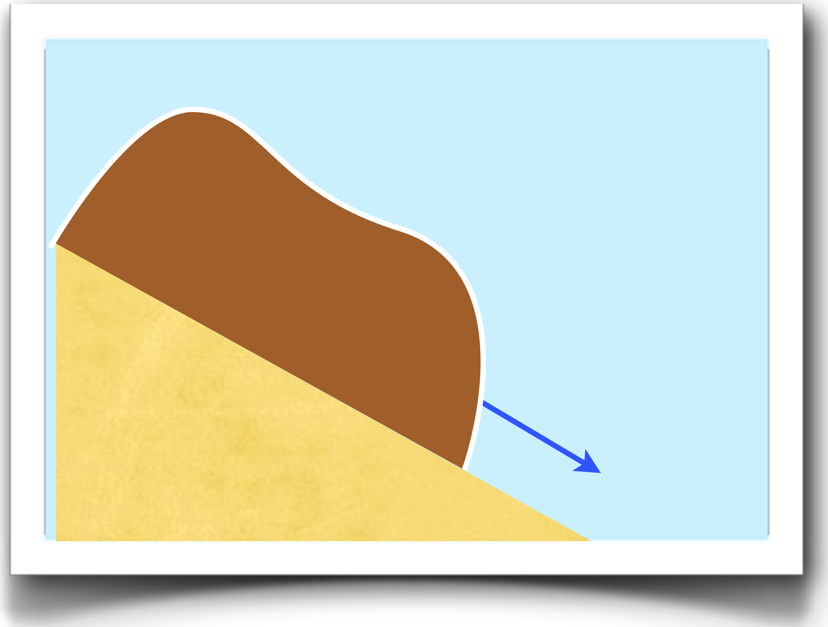
debris flows,
pyroclastic flows,
rock avalanches...,

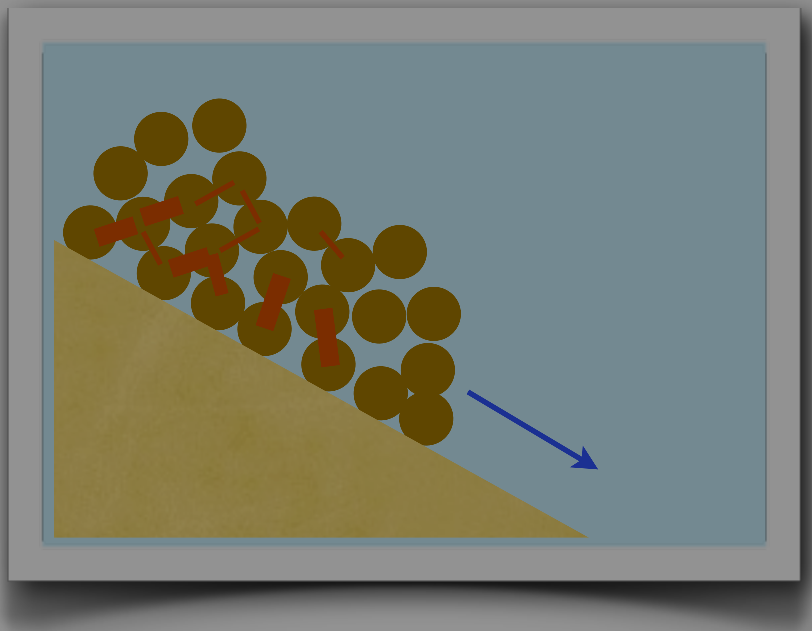
Questions:

continuum description
of granular media
==> phenomenology



In this talk:
a fluid mechanics point of view

understanding the microscopic dynamics.



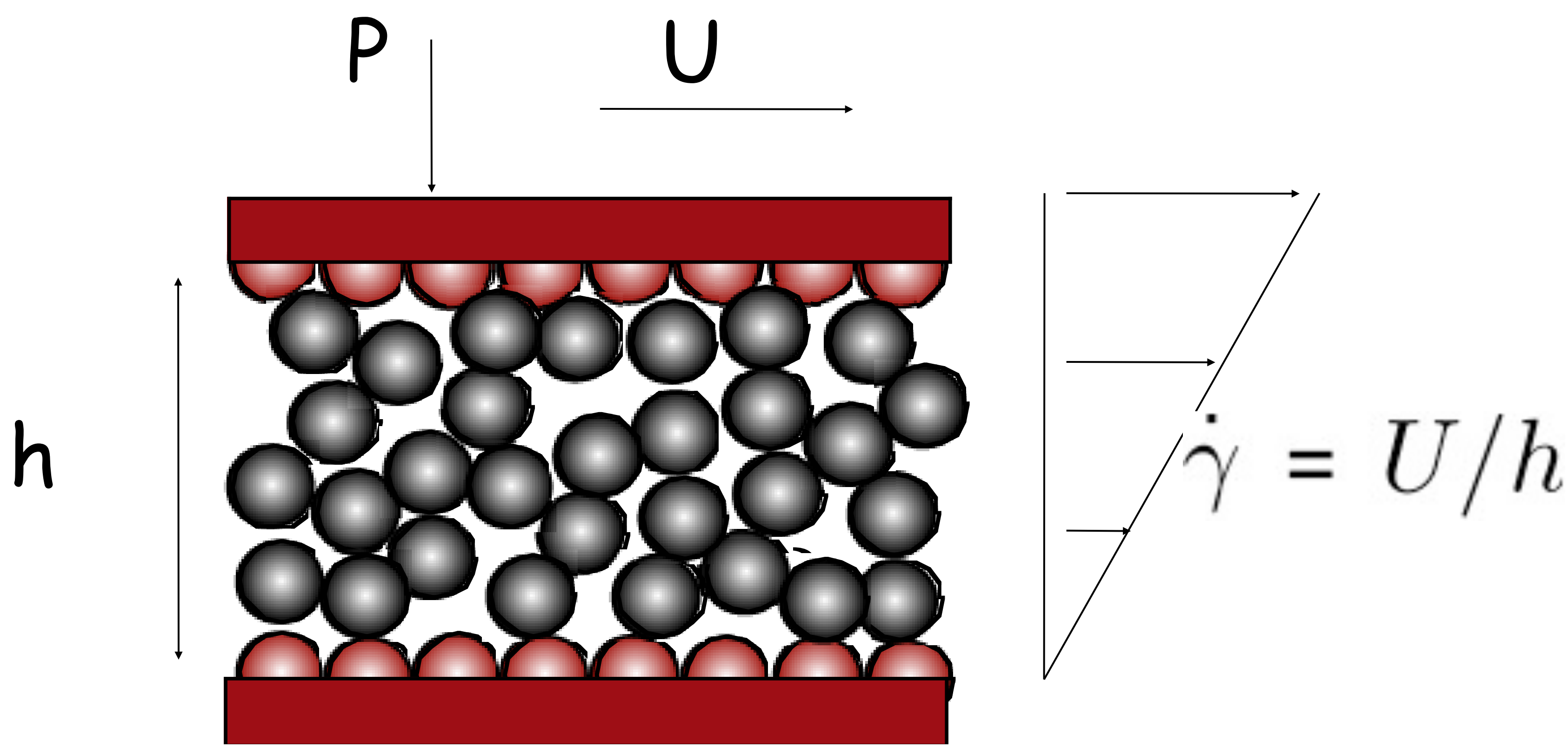
1) Rheology of dry granular flow

2) Rheology of immersed granular flow

3) dragging objects in a granular medium...

Dry granular flows

plane shear under controlled normal stress



Lois et al 2005
Da Cruz et al, PRE 05
GdR Midi, Eur. Phys. J 04

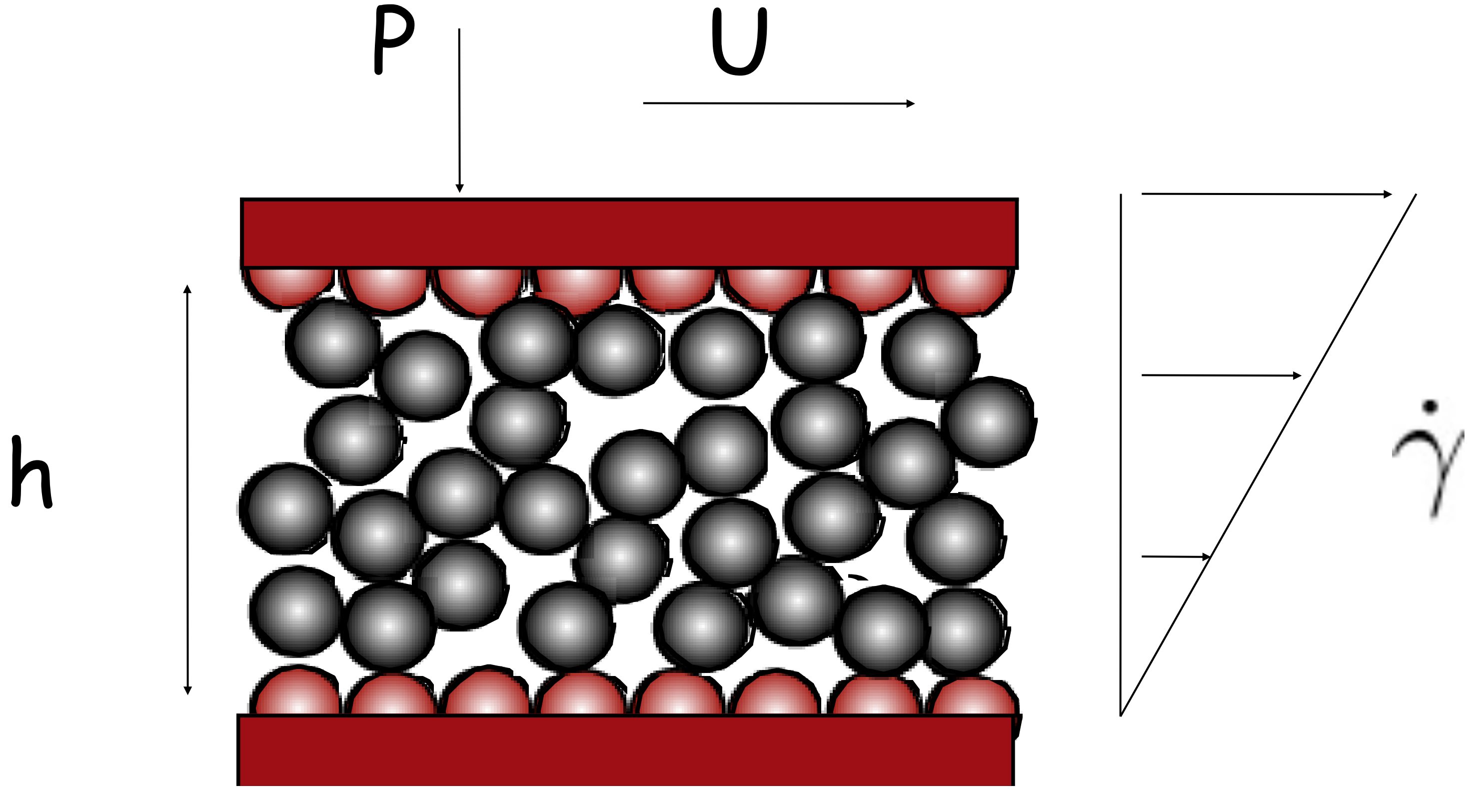
One imposes P and $\dot{\gamma}$

Shear stress τ?
Volume fraction ϕ?

A single dimensionless number
(inertial number)

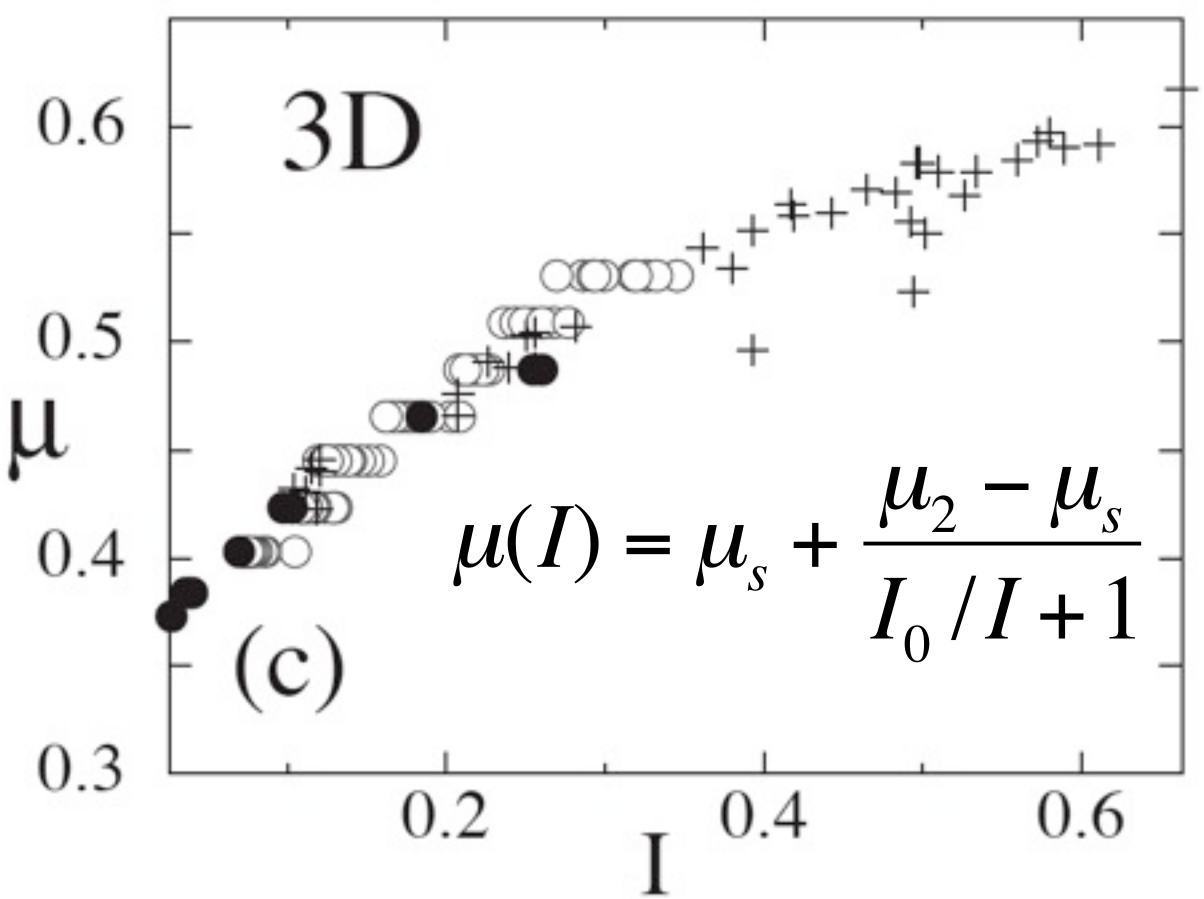
$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

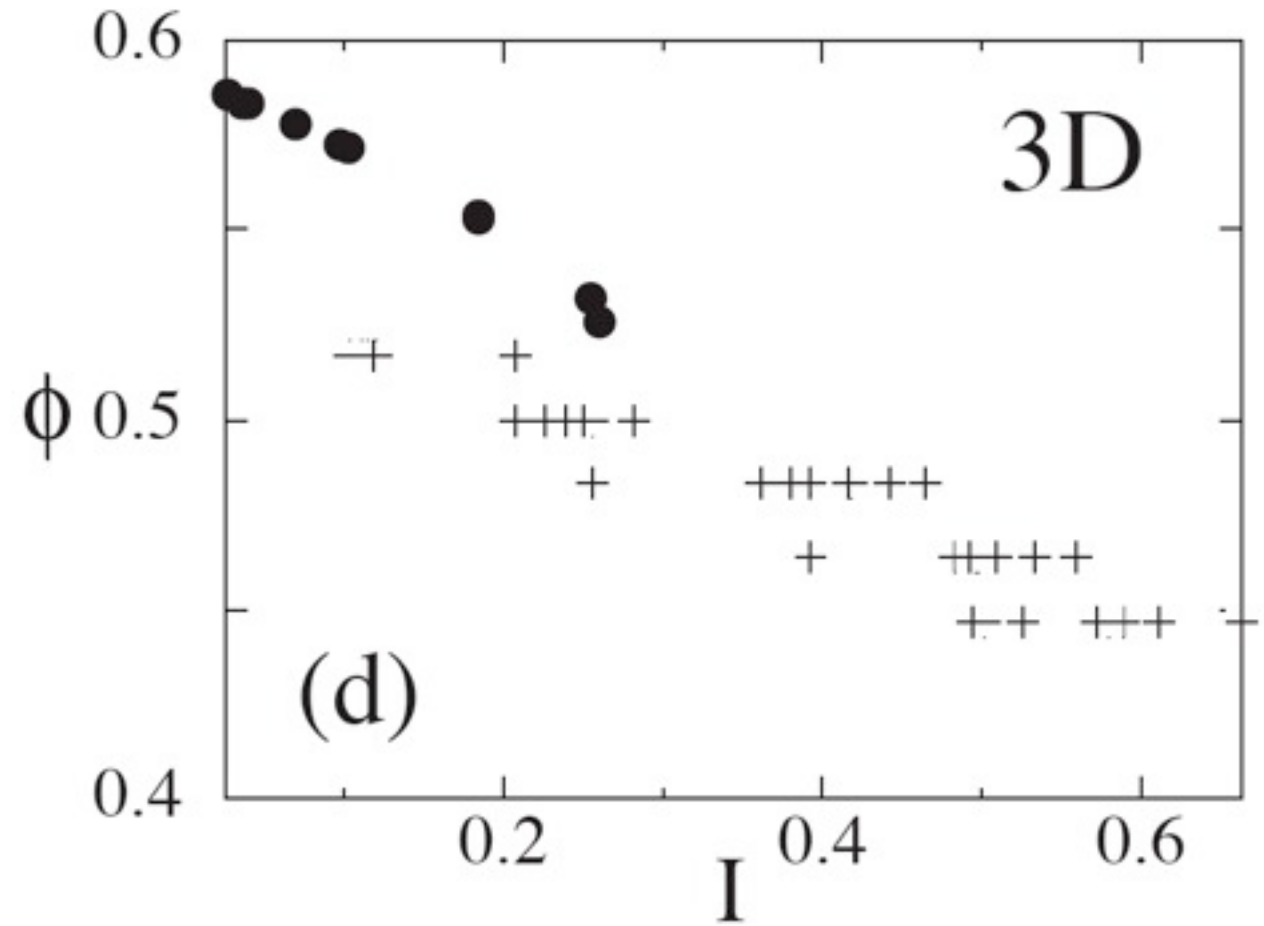
(Savage 84,
Ancey et al 99)



$$\tau = \mu(I)P$$

$$\Phi = \Phi(I)$$





3D generalisation of the friction law : granular flows as a viscoplastic fluid

(Jop et al Nature 06)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0,$$

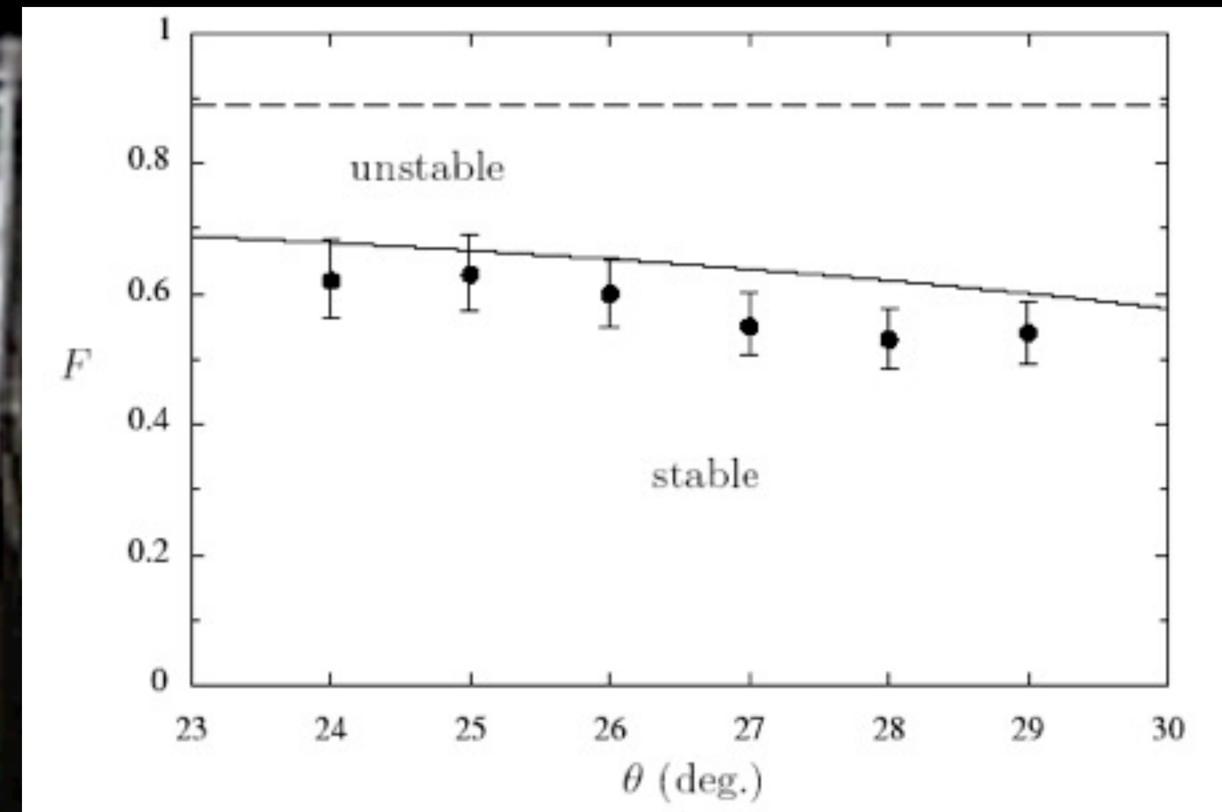
$$\rho_s \phi \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right) = \rho_s \phi g \sin\theta - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},$$

$$\rho_s \phi \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right) = -\rho_s \phi g \cos\theta - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z},$$

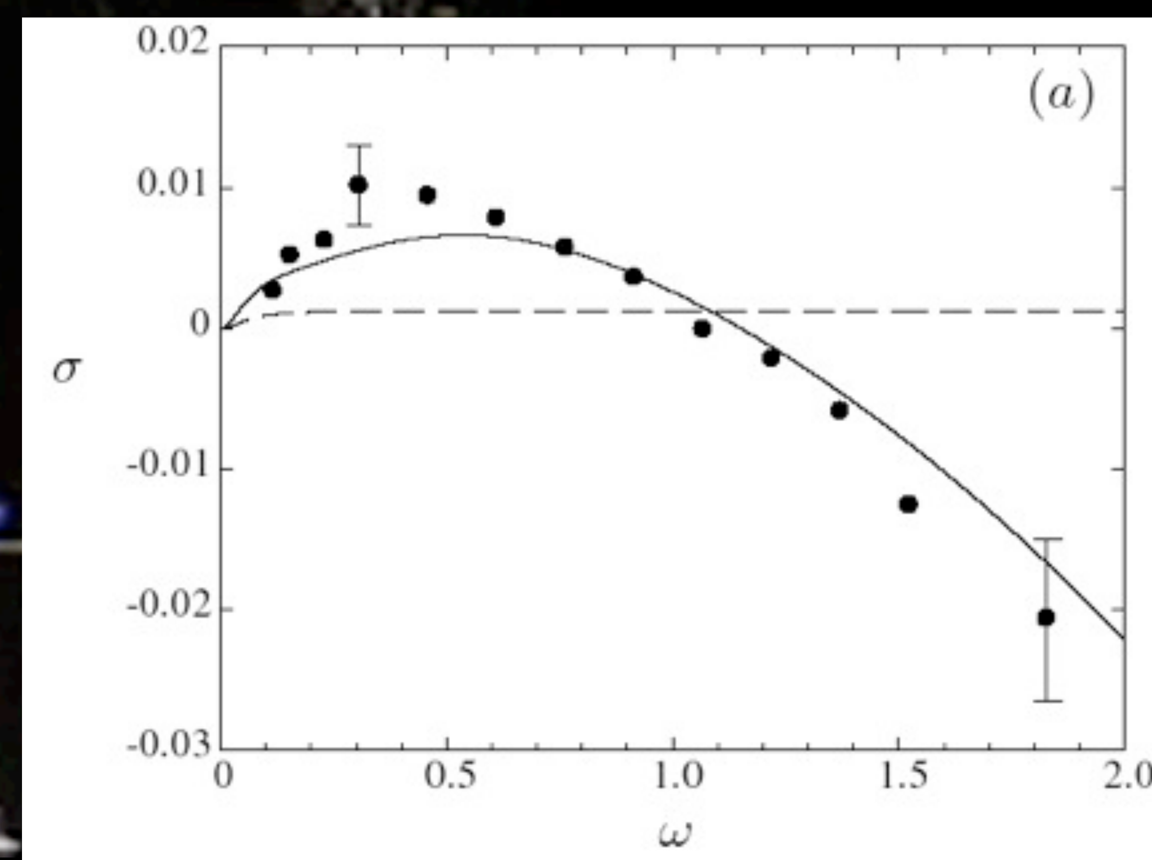
$$\tau_{ij} = \frac{\mu(I) P}{\dot{\gamma}} \dot{\gamma}_{ij} \qquad I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

Pressure dependent viscosity

rolls waves

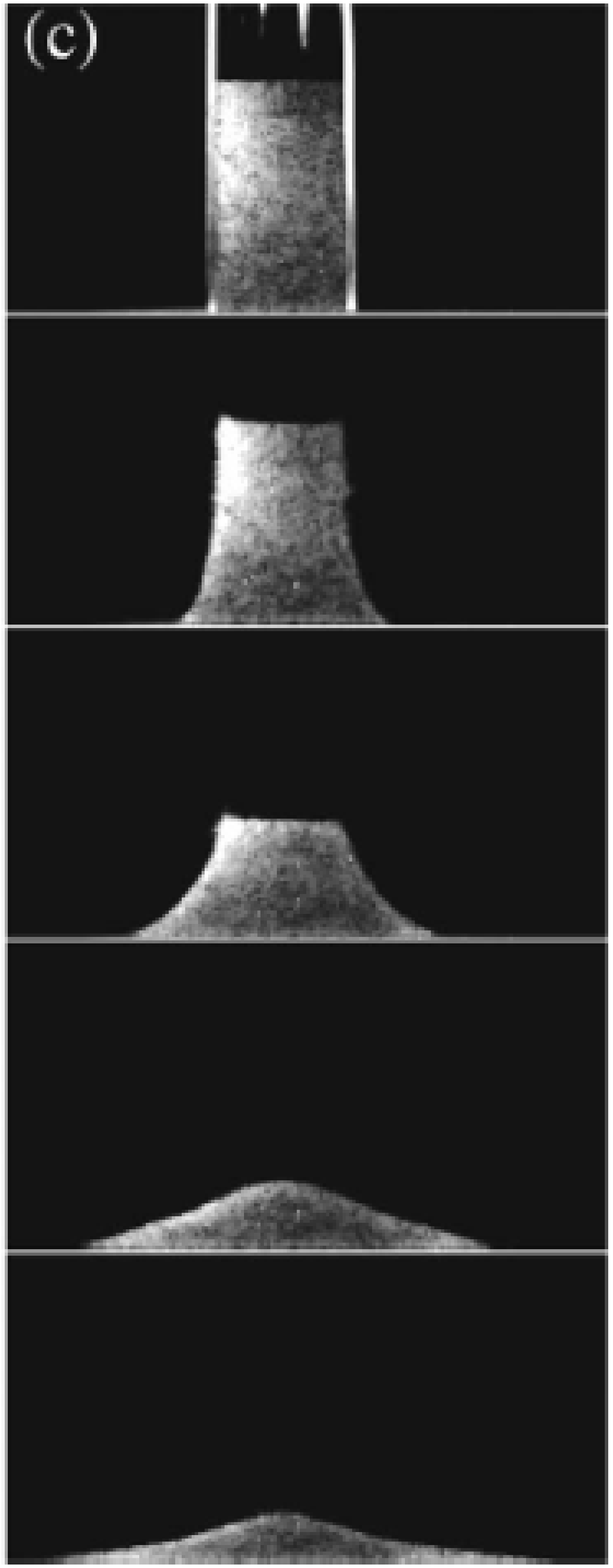


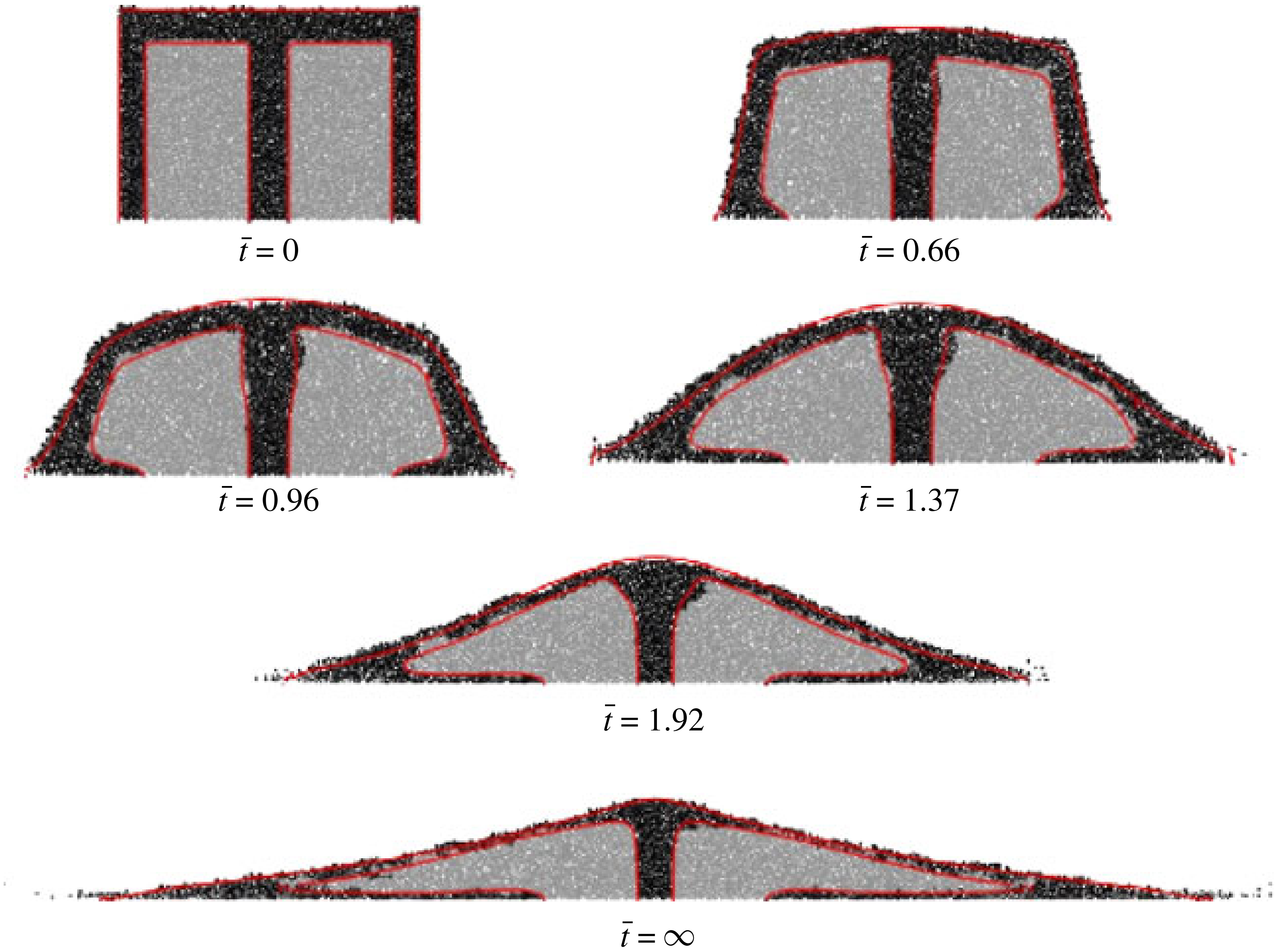
1
0.8
0.6
0.4
0.2
0
unstable
stable
F
23
24
25
26
27
28
29
30
θ (deg.)



0.02
0.01
0
-0.01
-0.02
-0.03
(a)
σ
0
0.5
1.0
1.5
2.0
ω

granular collapse



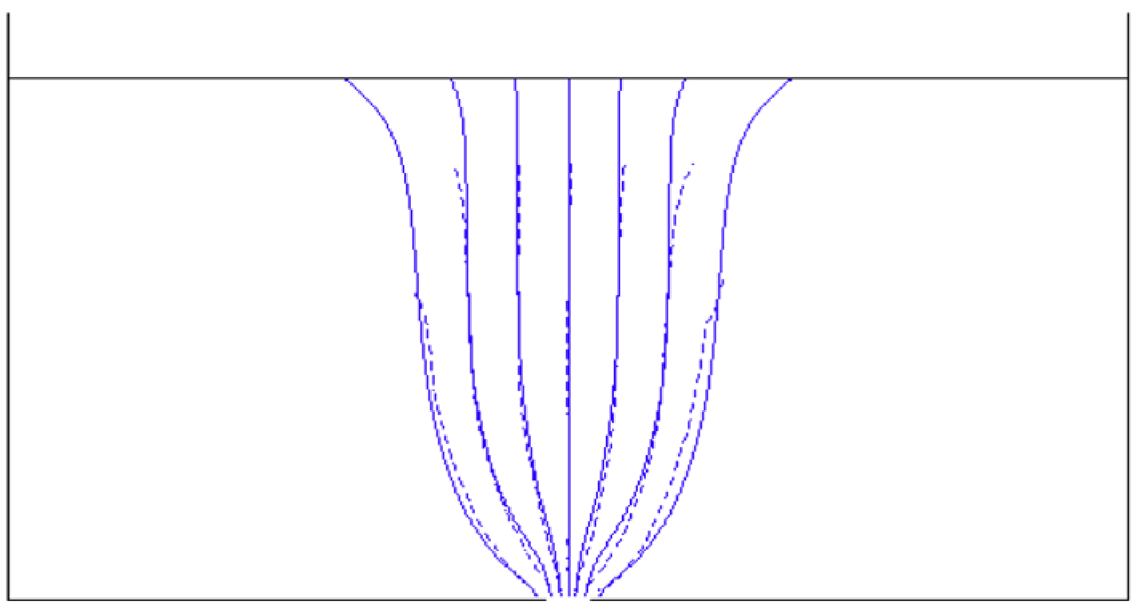


Lagree et al 2011

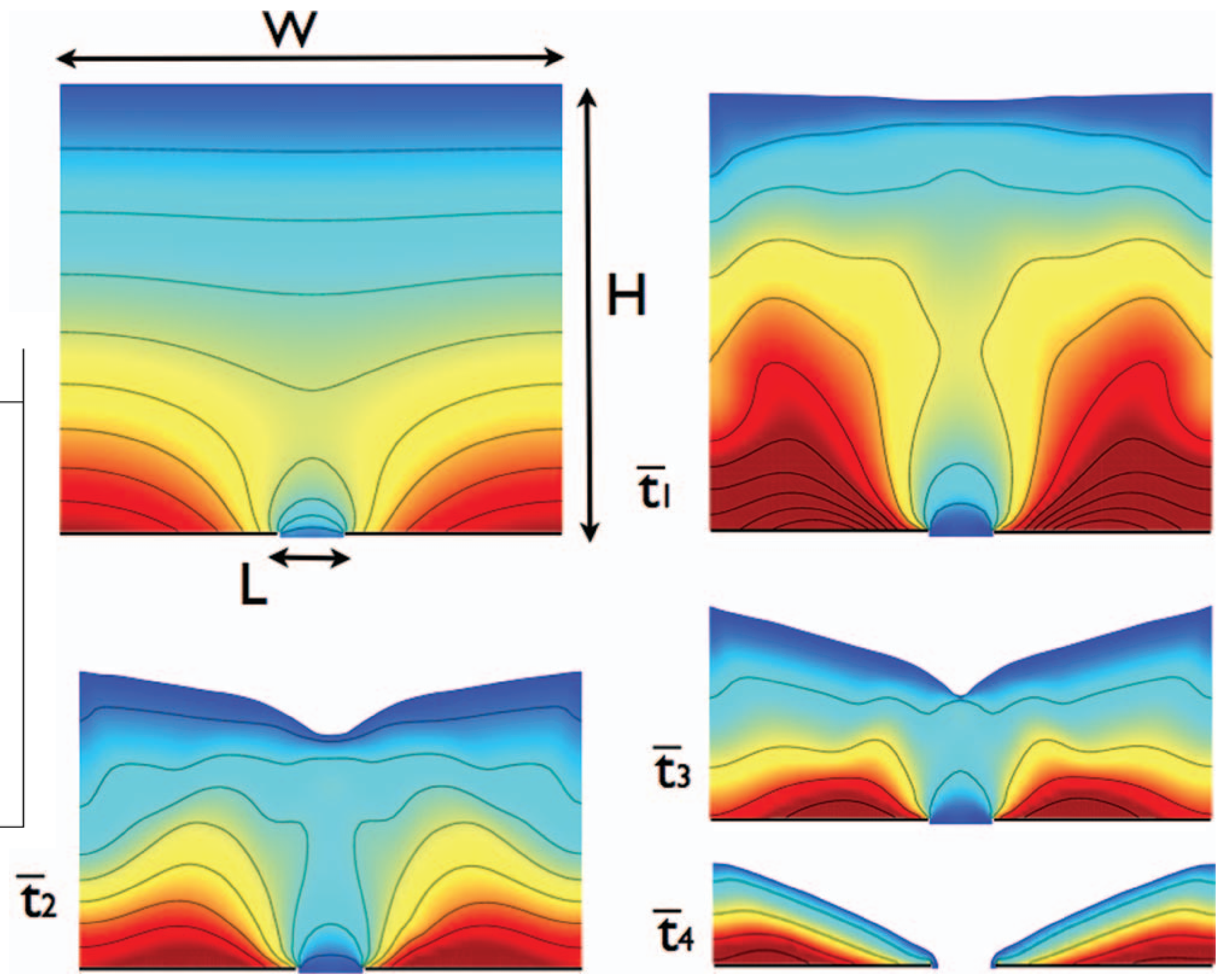
Gerris code..

lajeunesse et al 2005

flow in a silo



Kamrin 2010



Staron et al 2012

Particles + liquid
in a dense regime

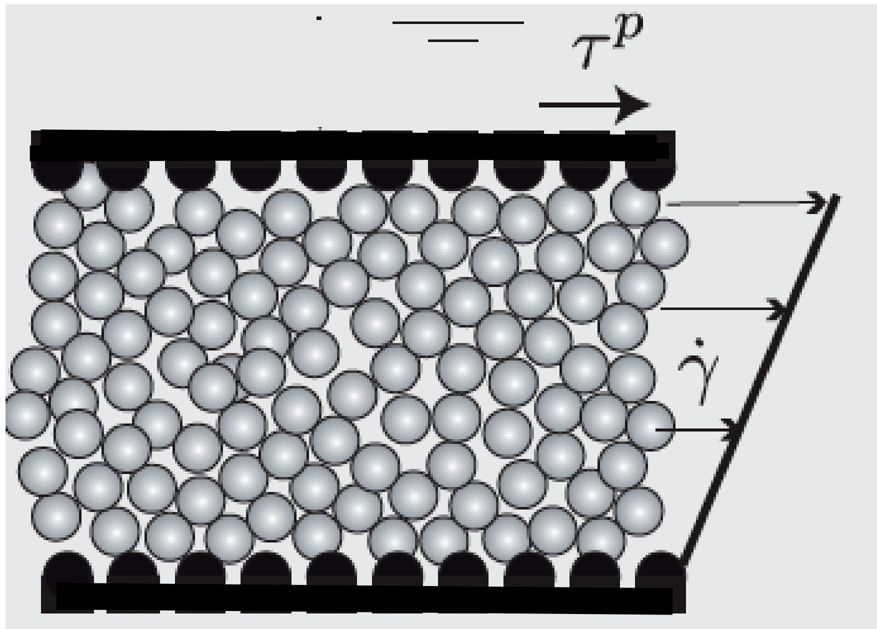
Difficulties:

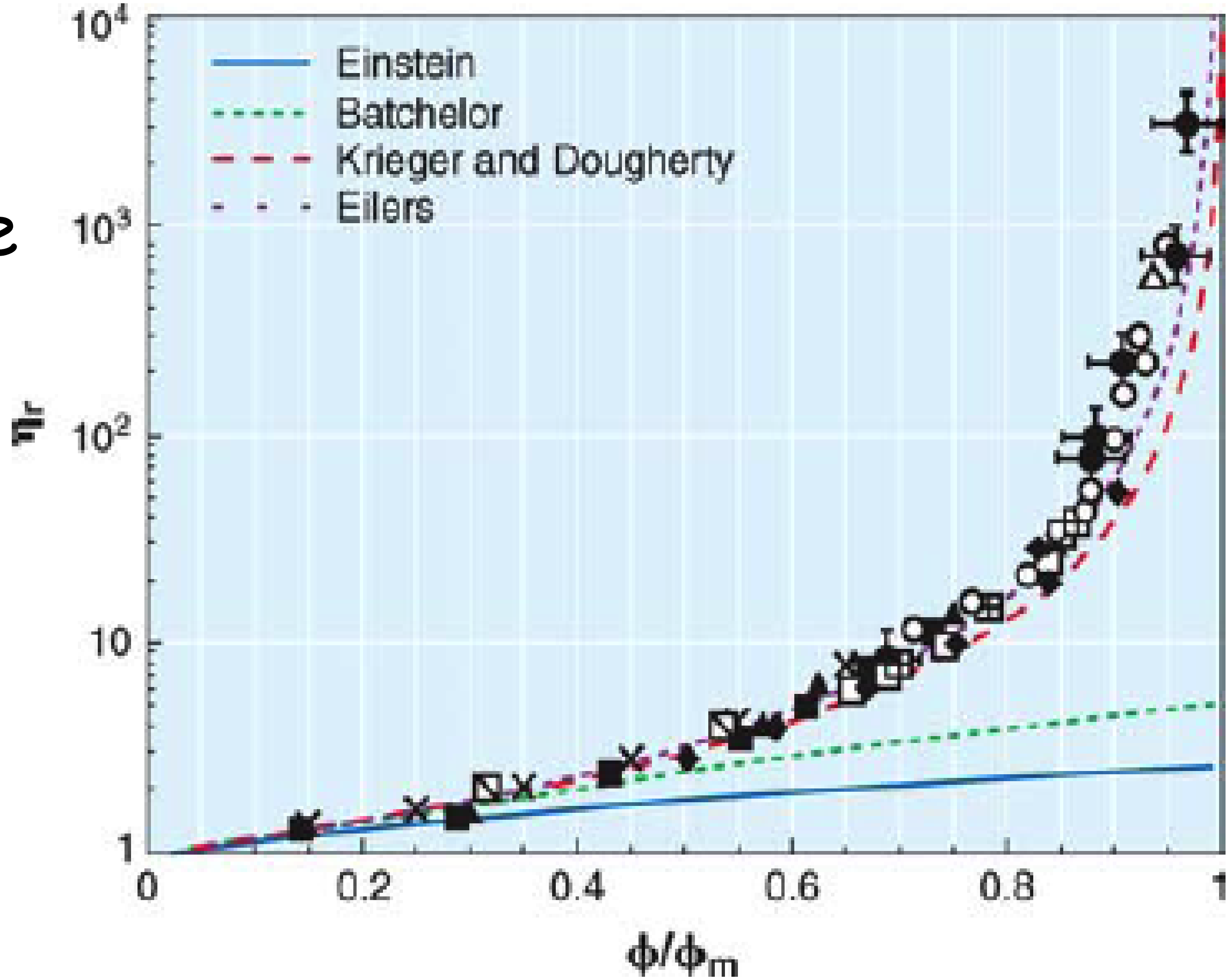
multi body problem with :

Hydrodynamic interactions
+
Contact between grains

the big picture of suspensions

- Non brownian rigid spheres
- incompressible and newtonian fluid
- no cohesion, no attractive forces...





$$\tau = \eta_s(\phi)\eta_f\dot{\gamma}$$

Ovarlez *et al.*, *J. Rheol.* 2006

Bonnoit *et al.*, *J. Rheol.* 2009

A brief history of viscous suspensions

dilute suspension | dense suspension | jamming

0 → ϕ_m ϕ

1906 | 1970,72 | 1988 =>... | 2007 =>... | time

A very brief history of viscous suspensions

dilute suspension

dense suspension

jamming

0 ϕ_m ϕ

1906
Einstein

Single particle

$$\eta = \eta_f(1 + \frac{5}{2}\phi + \alpha\phi^2..)$$

1970,72
Batchelor
Green

Pair interaction

1988 =>...
Brady, Bossis
Nott, Morris,..

hydrodynamic interactions

towards Constitutive equations

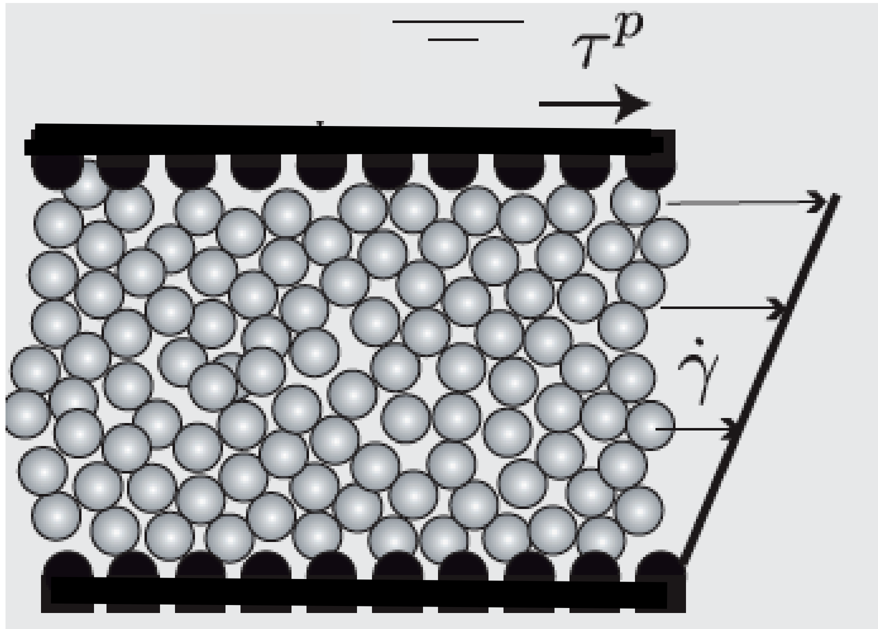
2007 =>...
Olson teiltel07, Tighe et al 09,
Heussinger, Andreotti et al 10,12
Wyart 12, Berthier12,...

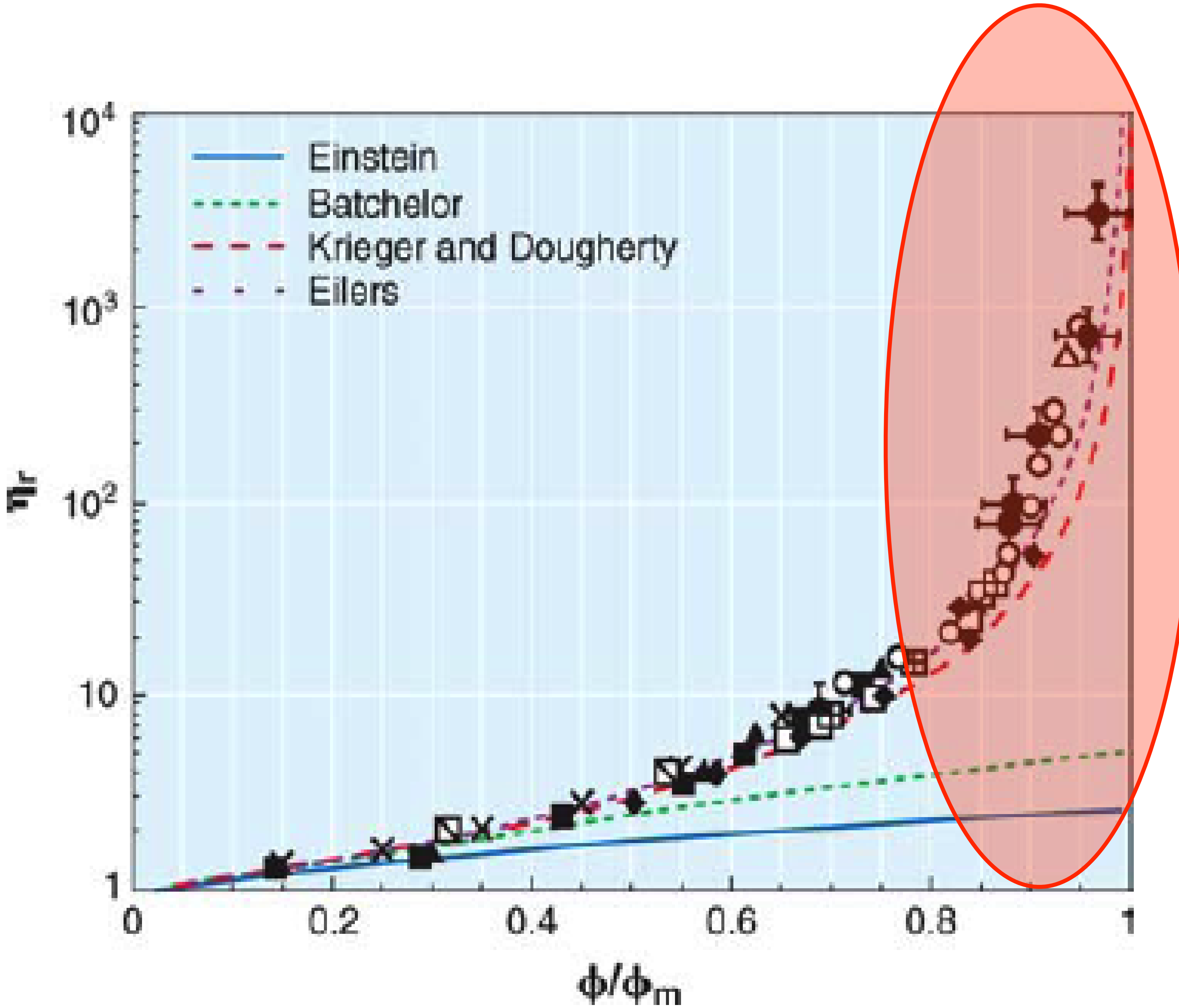
steric/elastic interactions

understanding the divergence

time

link with granular media?





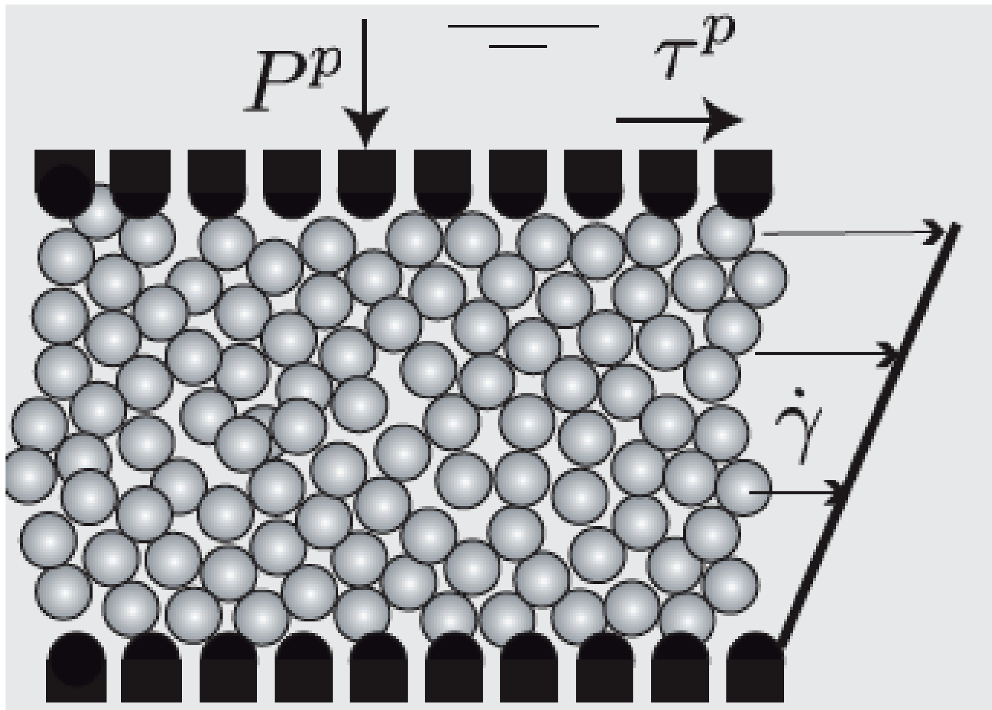
$$\tau = \eta_s(\phi)\eta_f\dot{\gamma}$$

Ovarlez *et al.*, *J. Rheol.* 2006

Bonnoit *et al.*, *J. Rheol.* 2009

The granular approach of suspensions!!

If inertia is negligible...



One imposes P and $\dot{\gamma}$

Shear stress τ?

Volume fraction φ?

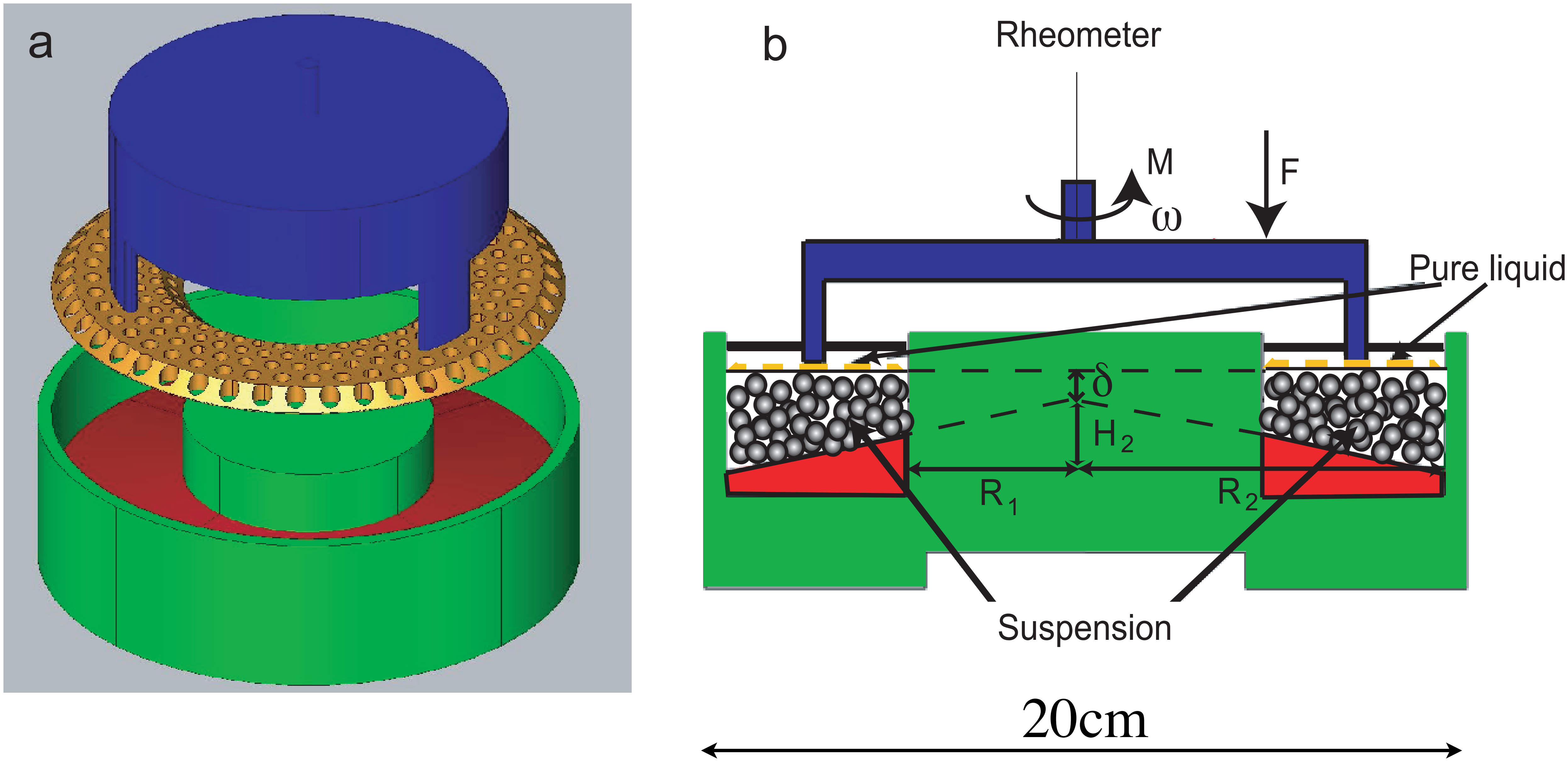
$$t_{micro} = \frac{\eta_f}{P^p}$$

$$I_v = \frac{\eta_f \dot{\gamma}}{P_p}$$

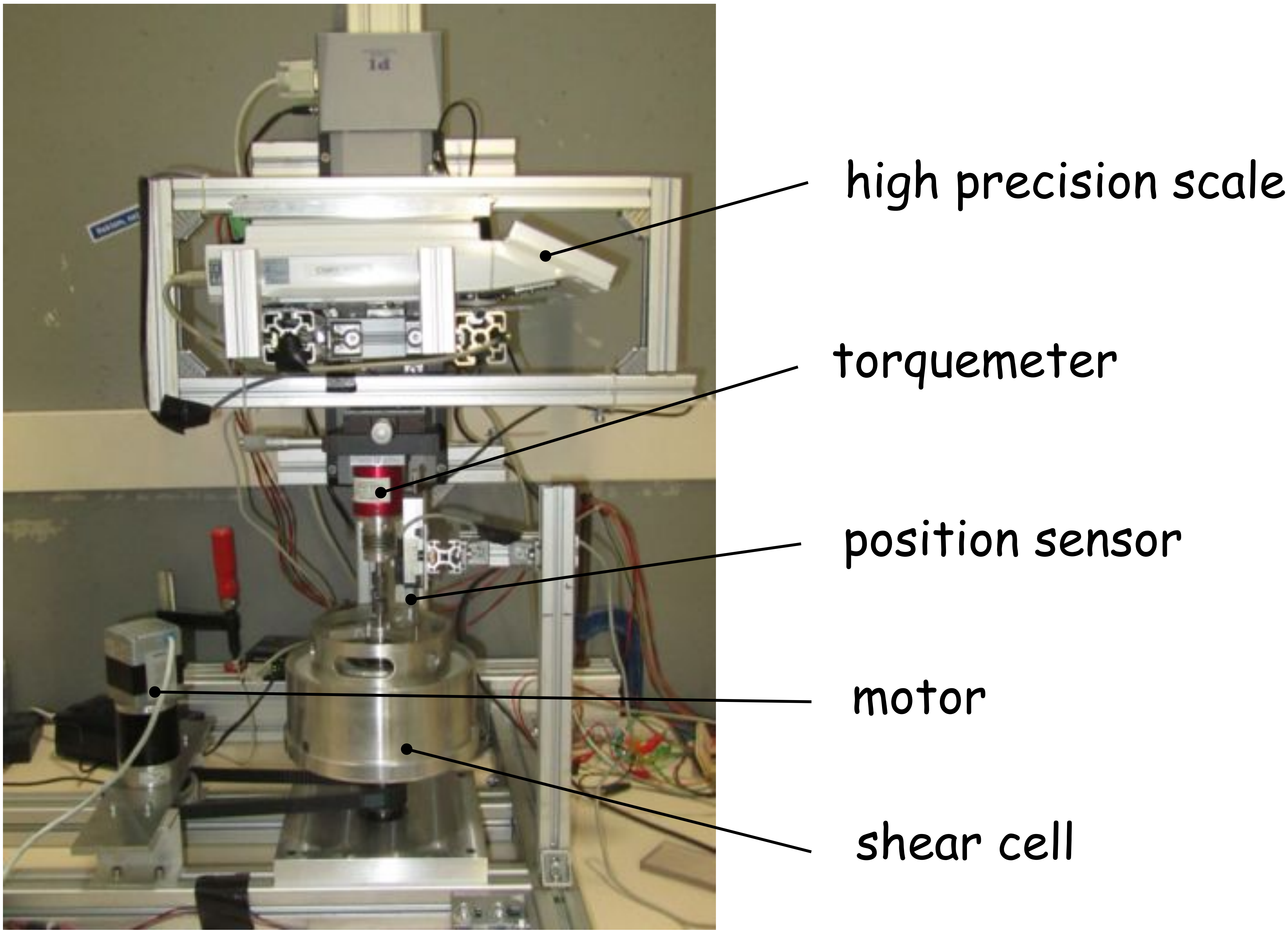
$$\tau = \mu(I_v) P_p$$

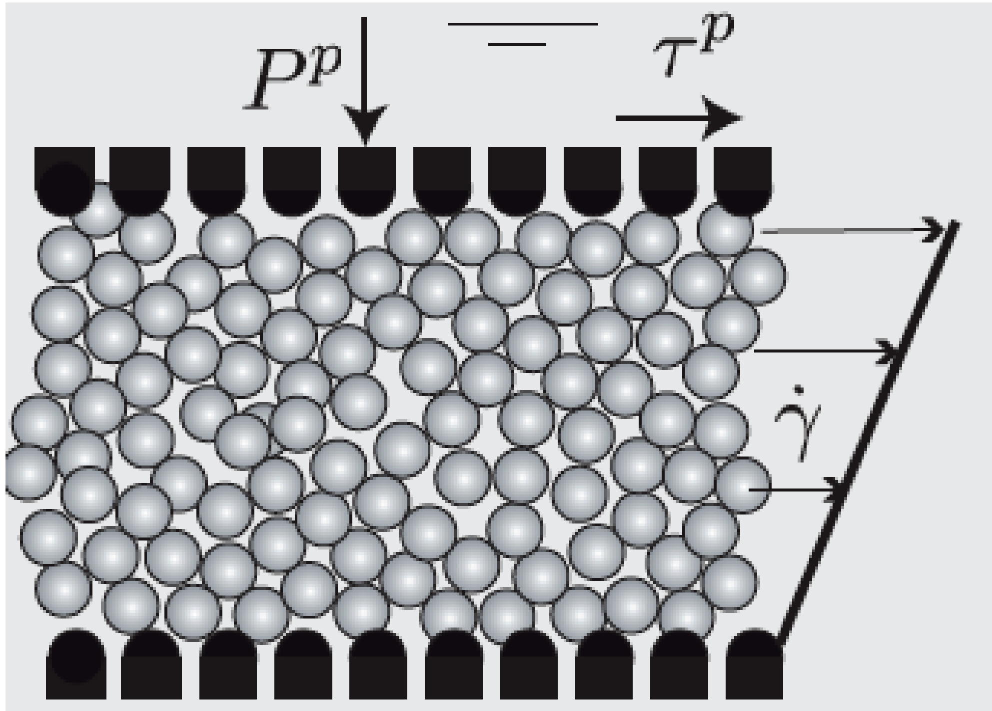
$$\phi = \phi(I_v)$$

François Boyer's Phd...

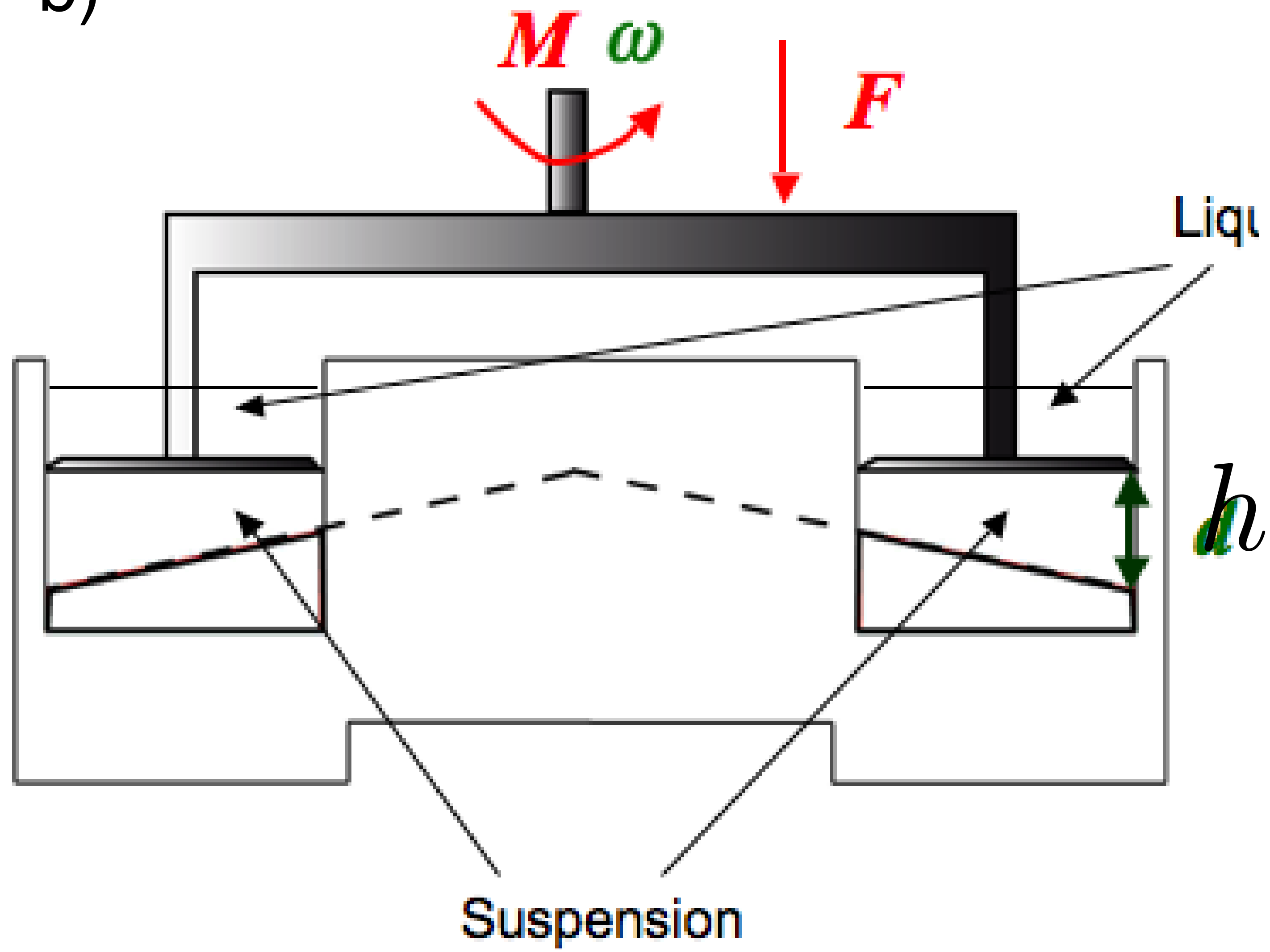


580 μm and 1mm polystyrene beads in viscous fluid (Polyethyline glycol-n) of the same density (1.05g/cm3)


high precision scale
torquemeter
position sensor
motor
shear cell



b)



Imposed $\begin{cases} F \implies P_p \\ M \implies \tau \end{cases}$

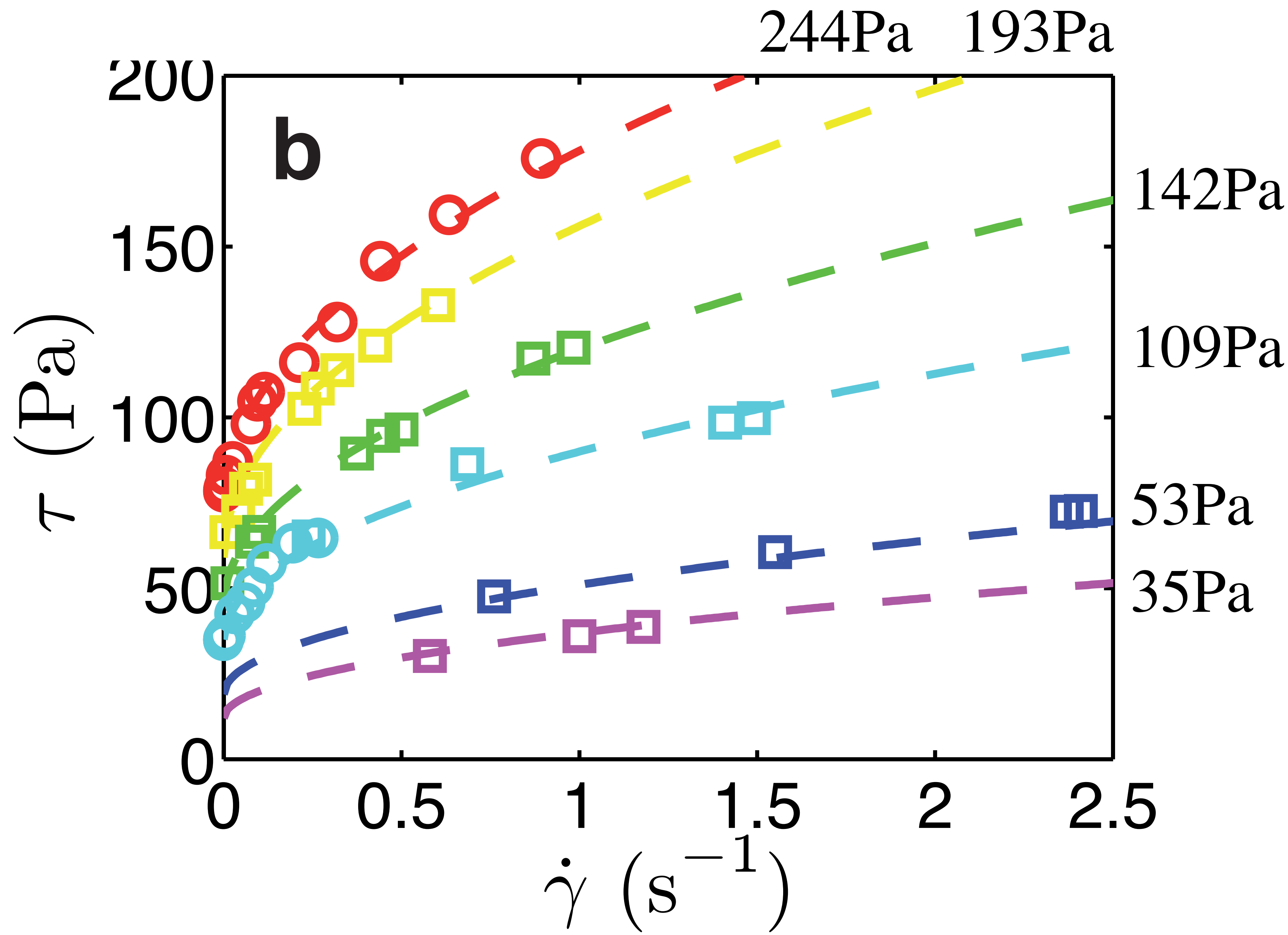
Measured $\begin{cases} \omega \implies \dot{\gamma} \\ h \implies \phi \end{cases}$

==>

$$I_v = \frac{\eta_f \dot{\gamma}}{P_p}$$

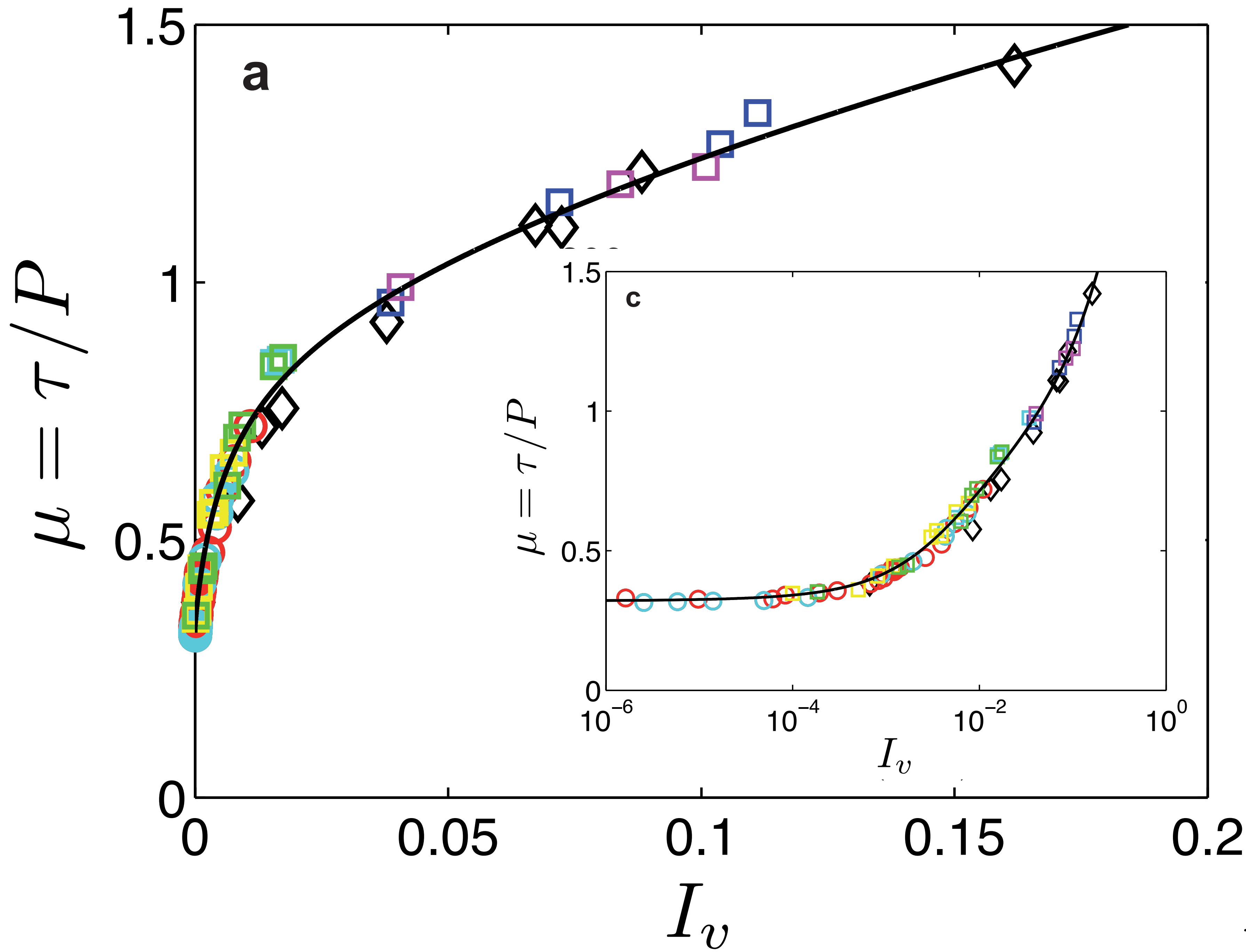
$$\mu = \frac{\tau}{P_p}$$

$$\phi$$



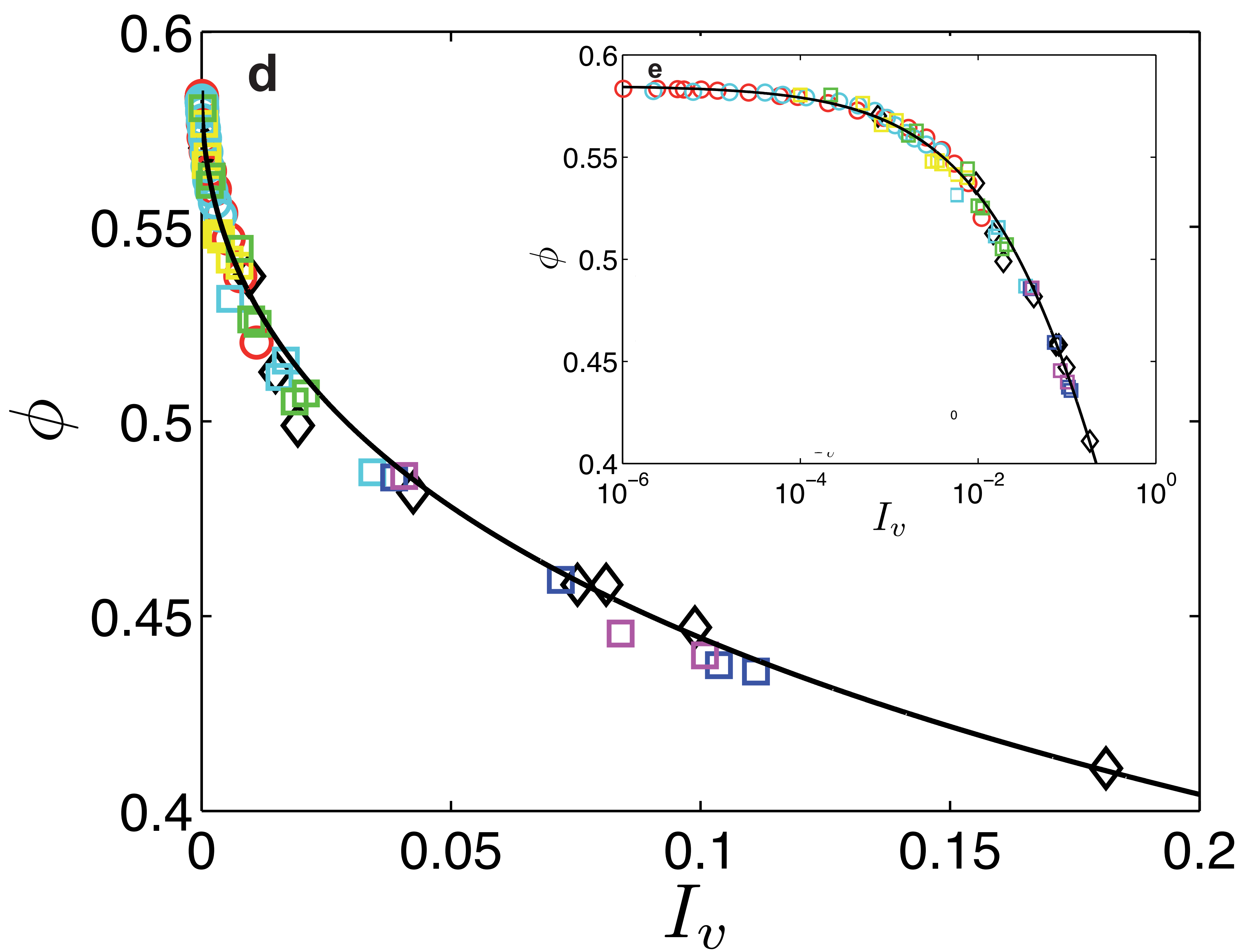
b
244Pa
193Pa
142Pa
109Pa
53Pa
35Pa
τ (Pa)
γ̇ (s⁻¹)
0
50
100
150
200
0
0.5
1
1.5
2
2.5

a friction law $\mu(I_v)$: different viscosities, pressure, shear rate, particle size,..



$$\mu_c = 0.32$$

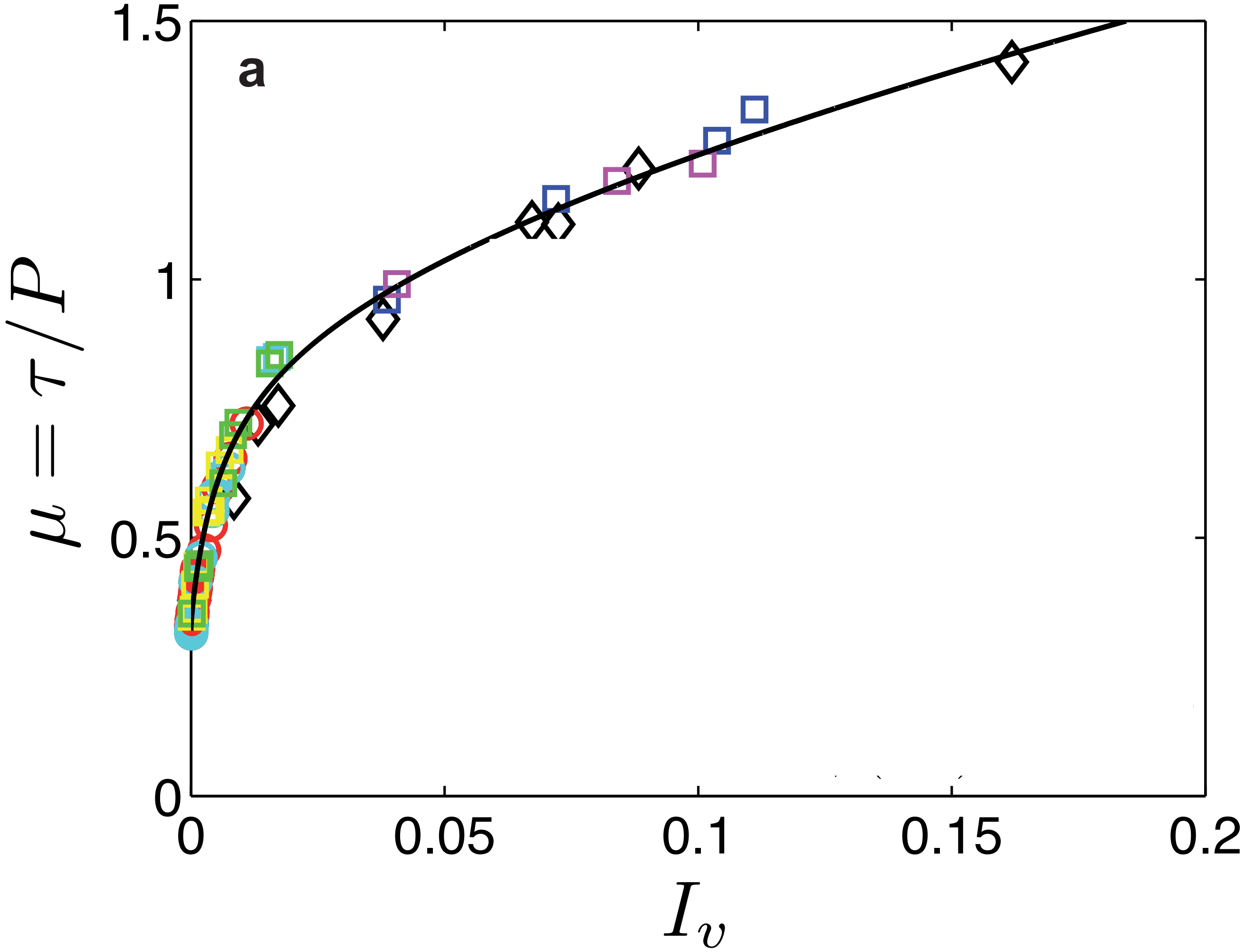
$$I_v = \frac{\eta_f \dot{\gamma}}{P_p}$$

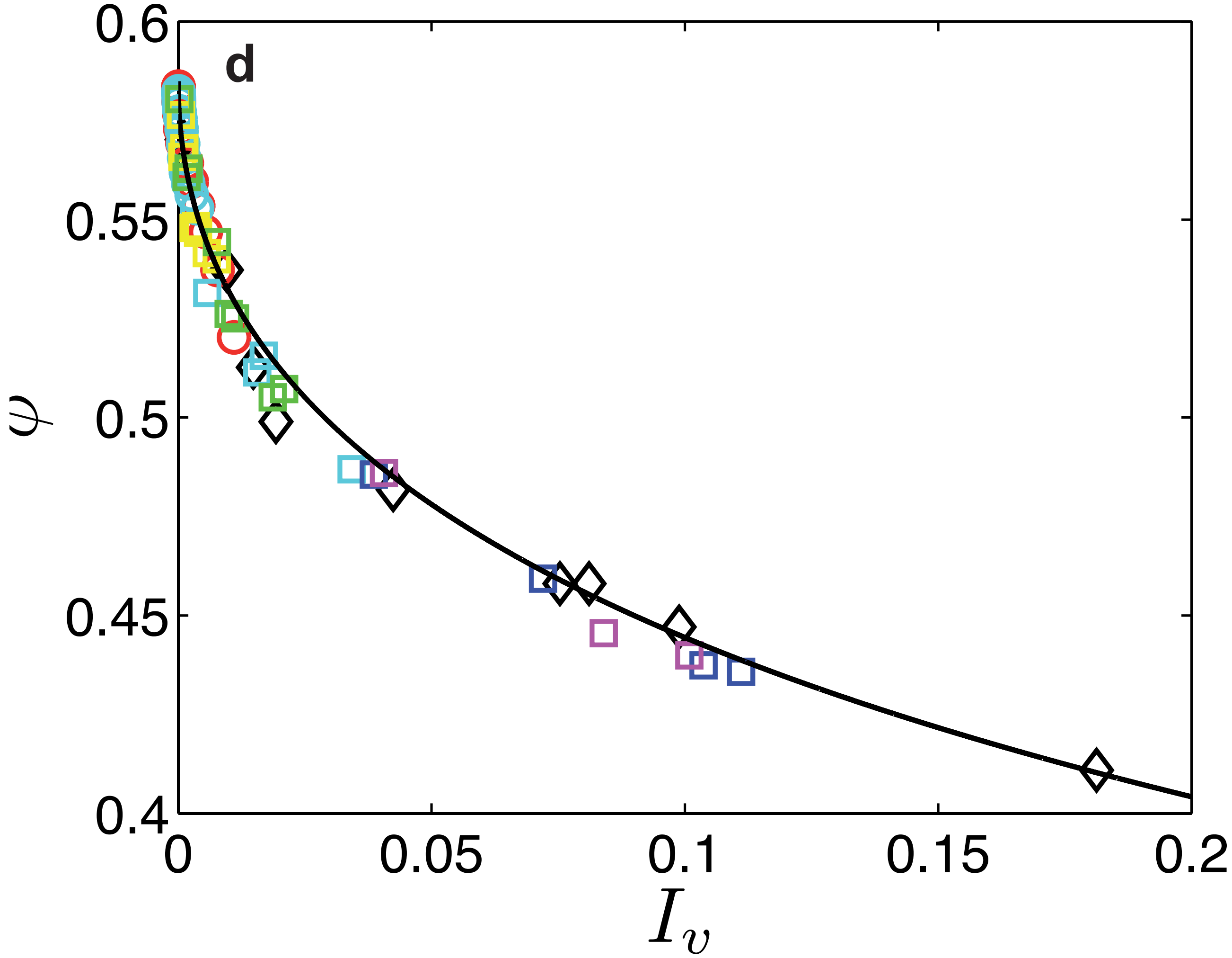


$$\phi_c = 0.585$$

a phenomenological rheology:

$$\tau = \mu(I_v) P_p$$
$$\phi = \phi(I_v)$$





$$\mu(I_v) = \mu^c(I_v) + \mu^h(I_v) \text{ with}$$
$$\mu^c(I_v) = \mu_1 + \frac{\mu_2 - \mu_1}{1 + I_0/I_v}$$
$$\mu^h(I_v) = I_v + \frac{5}{2}\phi_m I_v^{1/2}$$

$$\phi(I_v) = \frac{\phi_m}{1 + I_v^{1/2}}$$

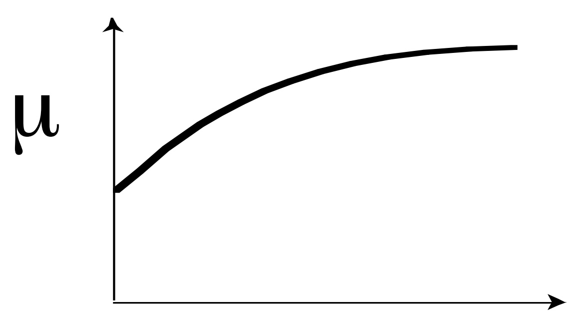
Link with the rheology of dense suspensions :

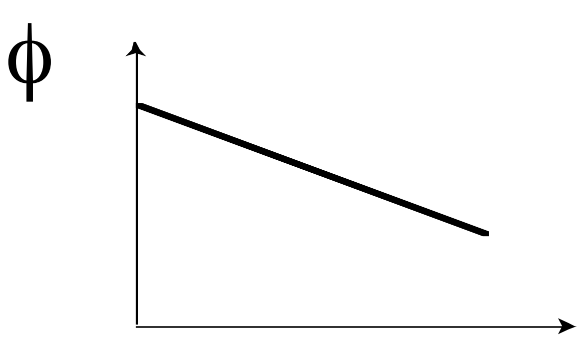
imposed pressure

$$\frac{\tau}{P_p} = \mu(I_v)$$

$$\phi = \phi(I_v)$$

$$I_v = \frac{\eta_f \dot{\gamma}}{P_p}$$





Or

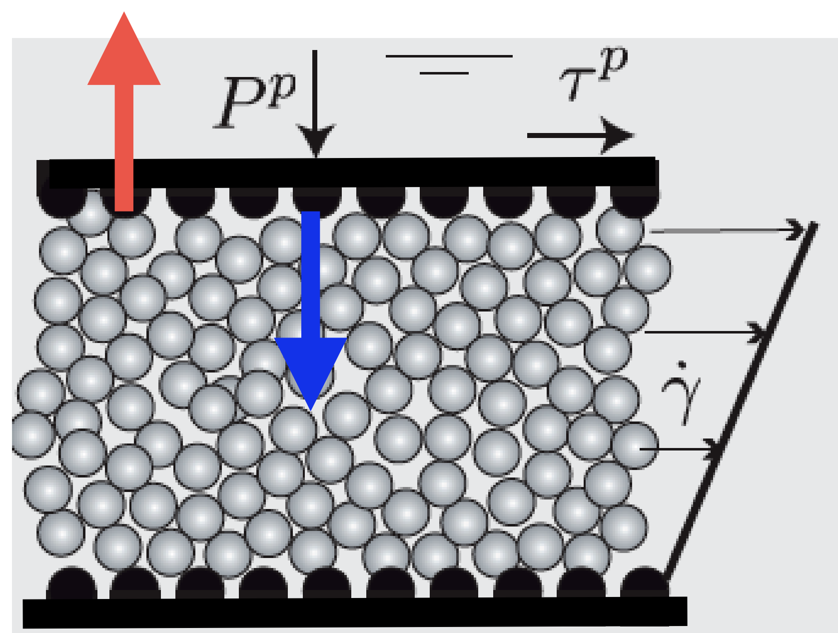
volume imposed

$$\tau = \eta_s(\phi)\eta_f\dot{\gamma}$$

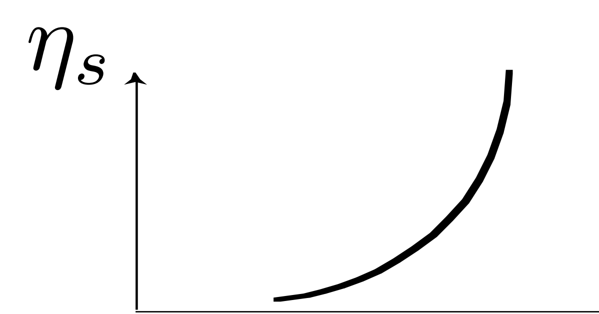
$$P_p = \eta_n(\phi)\eta_f\dot{\gamma}$$

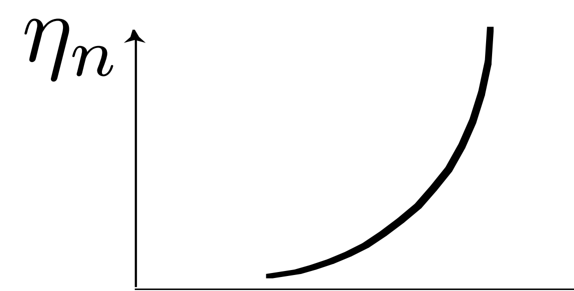
(Morris and Boulay 99)

Particles push on the wall

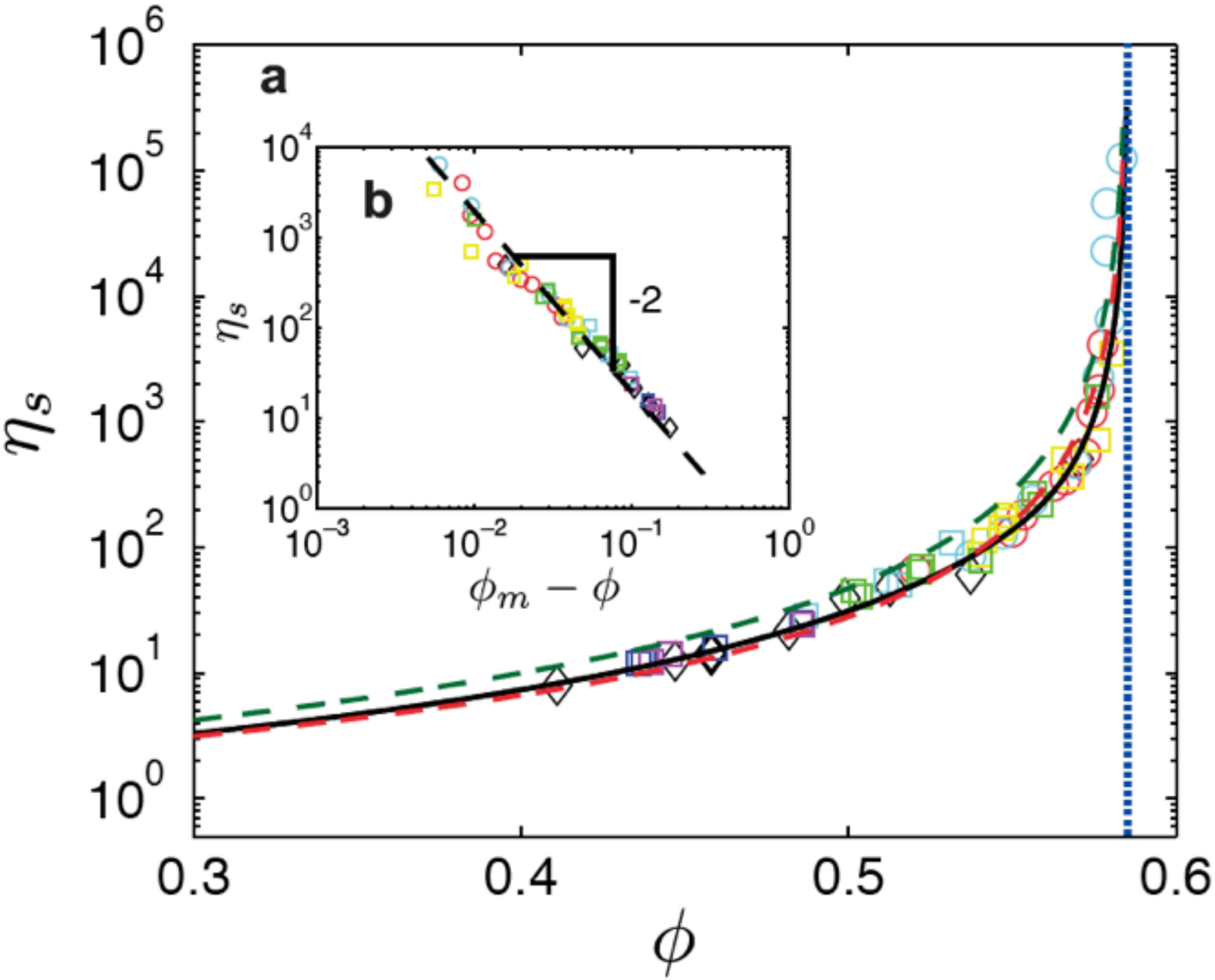


the wall pulls on water





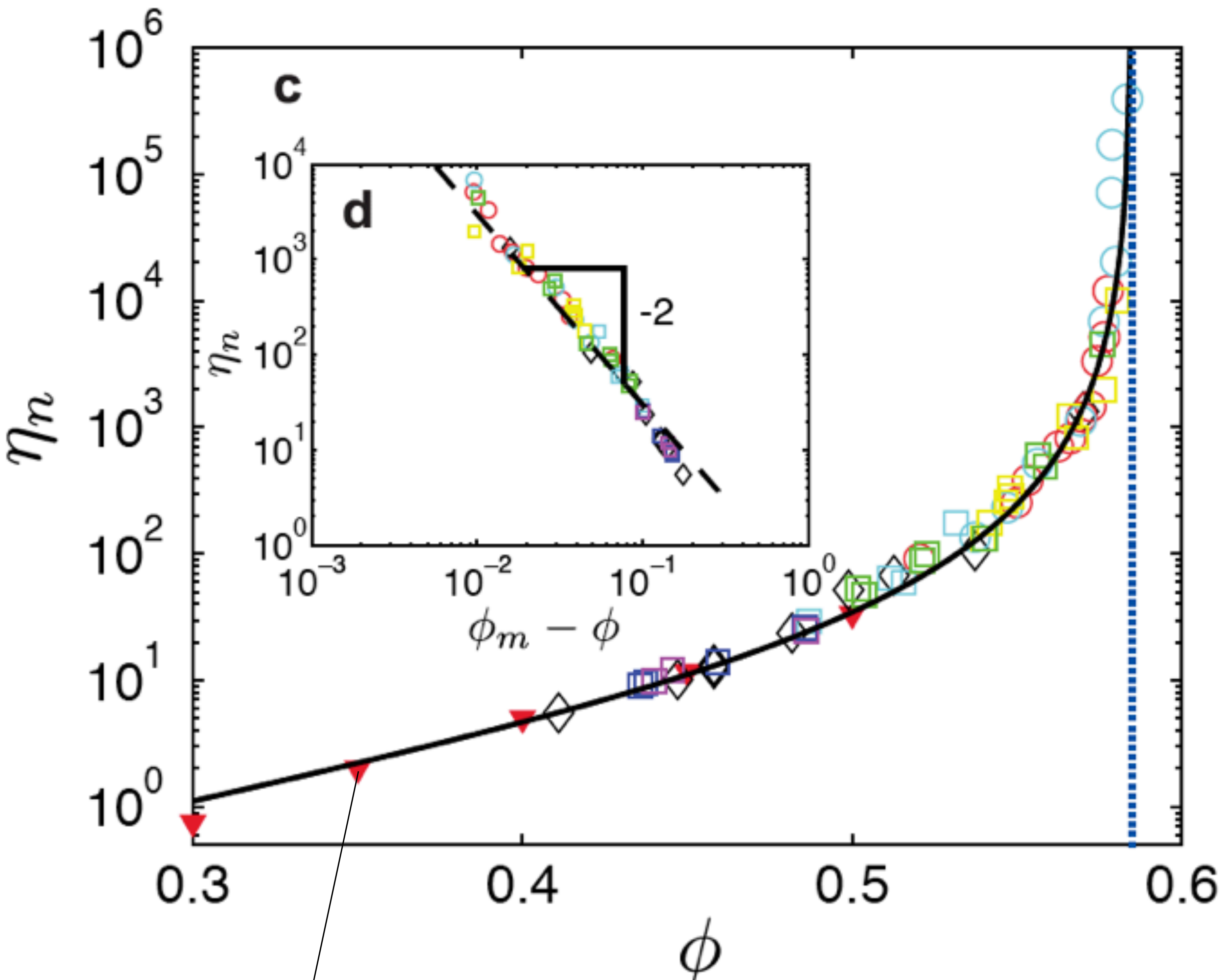
Link with dense suspensions??



$$\tau = \eta_s(\phi)\eta_f\dot{\gamma}$$

$$\eta_s(\phi)^{-1} \sim \frac{\mu_1}{\phi_m^2}(\phi_m - \phi)^2$$

Link with dense suspensions??



$$P_p = \eta_n(\phi)\eta_f\dot{\gamma}$$

Deboeuf et al PRL 2010

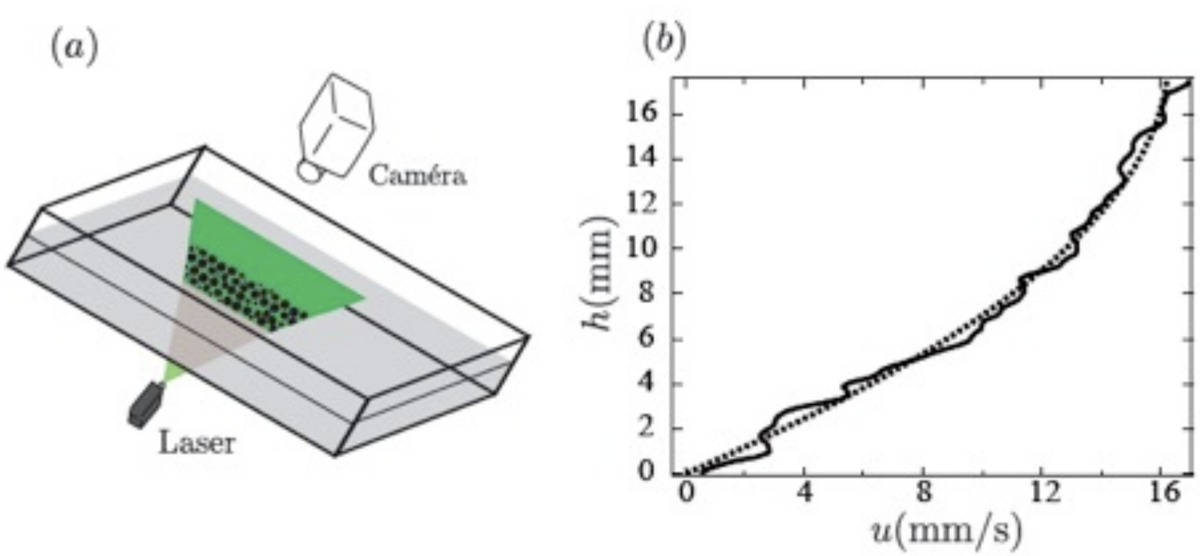
the pressure imposed rheology is interesting :

1) From a rheological point of view:

easy to be close to the maximum volume fraction :
precise measure of ϕ_m , study of the divergence...

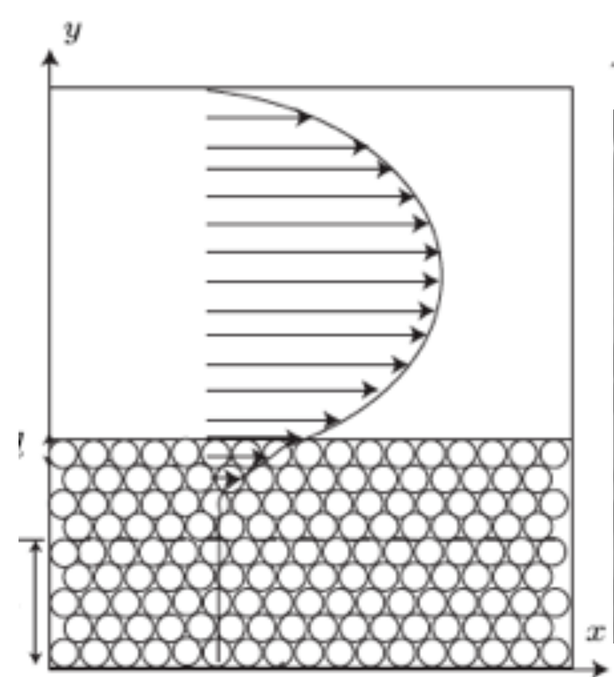
2) the natural description for some configurations:

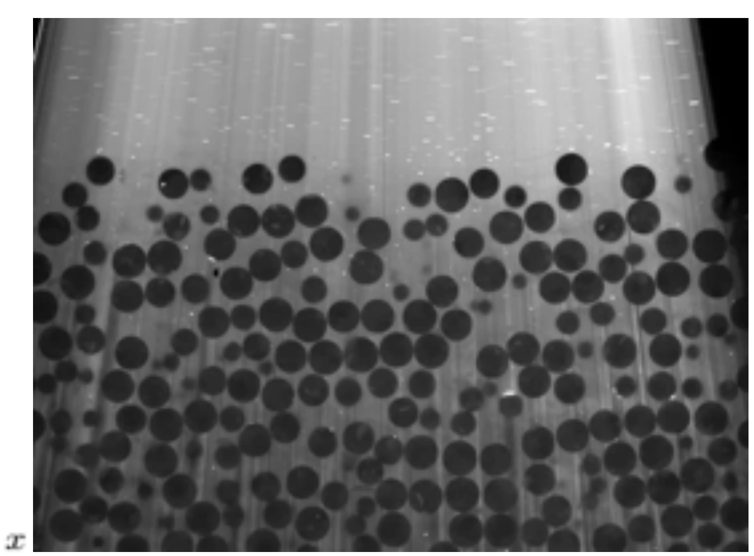
-submarine avalanches



-sediment transport

Ouriemi, Aussillous, Guazzelli et al 08,09
Pailha et al, 12



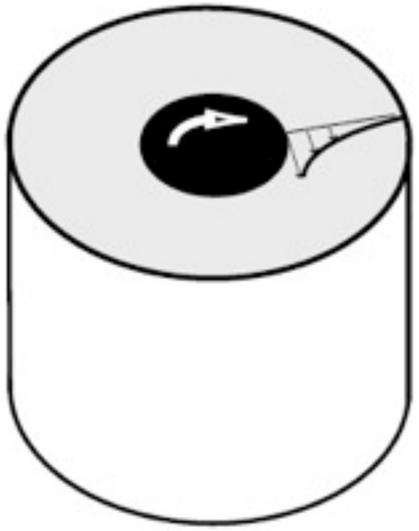


For both dry and immersed granular media, a visco-plastic description is relevant and captures the first order of the viscous nature of the flows.

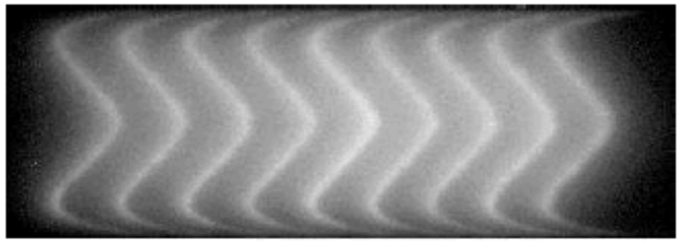
Beyond $\mu(I)$ ...

1) Quasistatic flows (shear band, finite size effe

A need for non local approach...

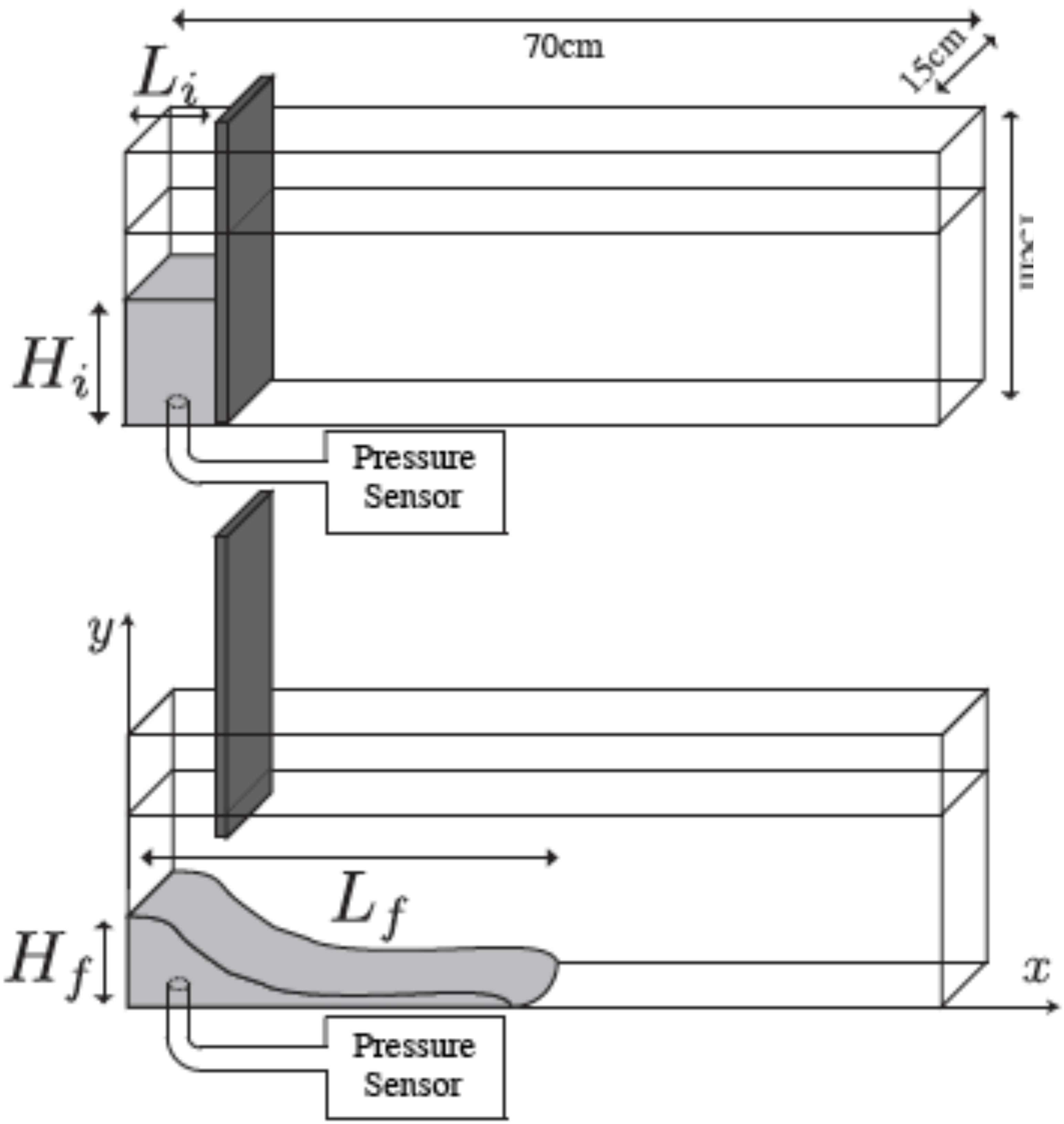


2) Normal stress differences



3) Transient flows when preparation plays a crucial role:

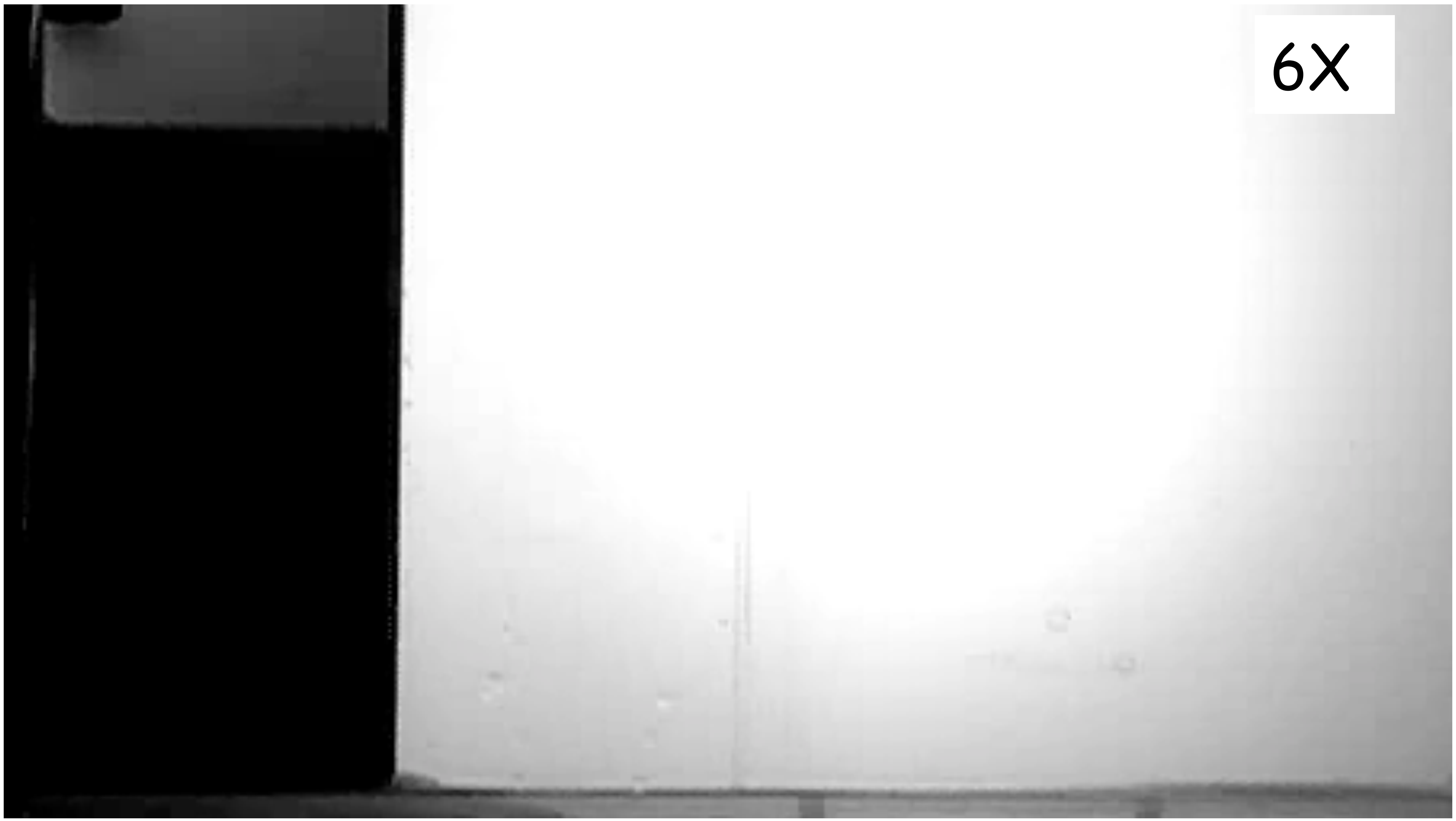
Role of the preparation on submarine avalanches: Dam break problem (L. Rondon Phd).



- glass beads, $d = 225\,\mu m$ in mean diameter
- Liquid : mix of Ucon-oil and water, 23 times more viscous than water
- Initial aspect ratio $\mathcal{A} = \frac{H_i}{L_i}$
- $0.55 < \phi_i < 0.62$

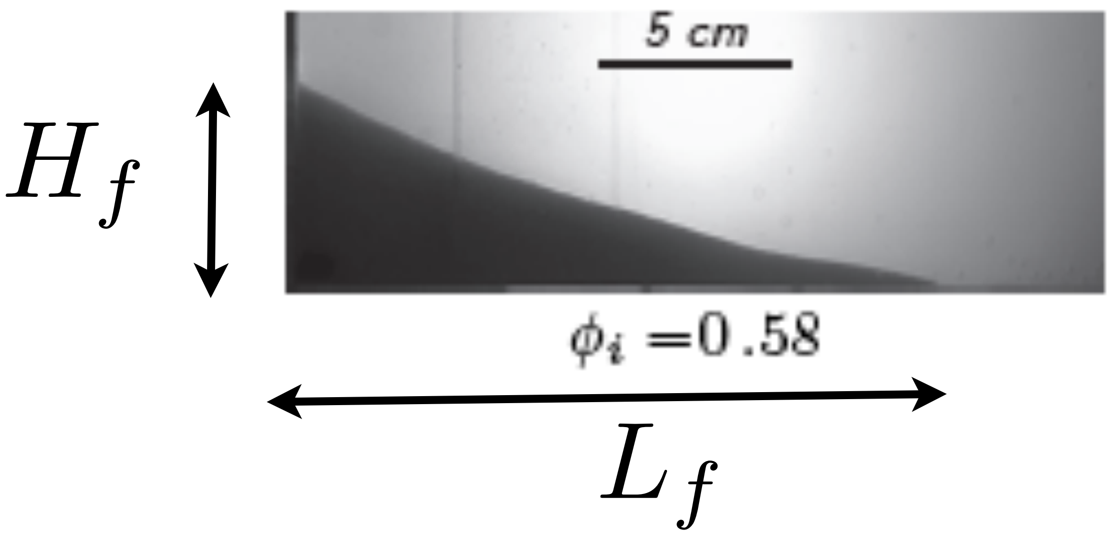
Loose $\phi_i = 0,55$

dense $\phi_i = 0,61$

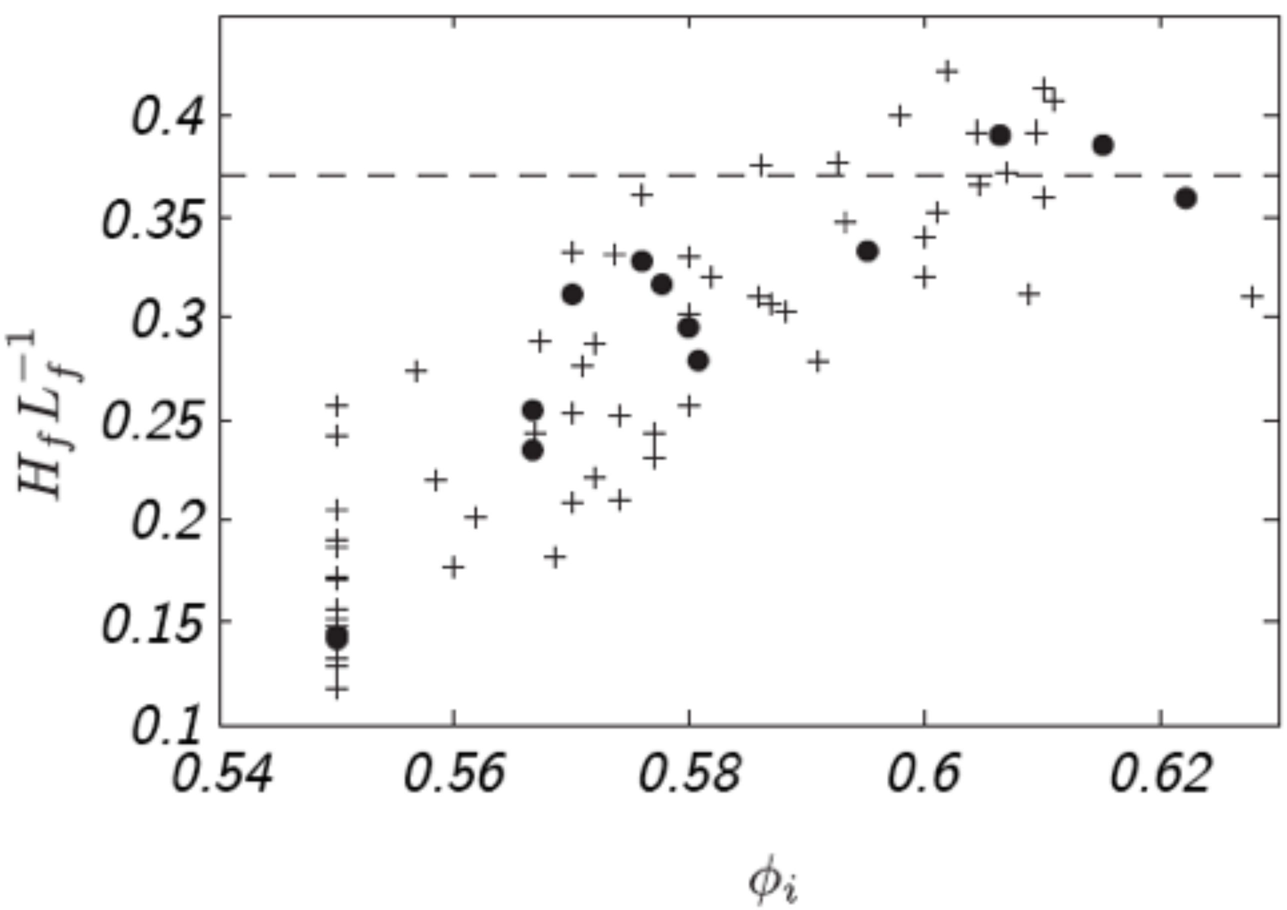


Slope

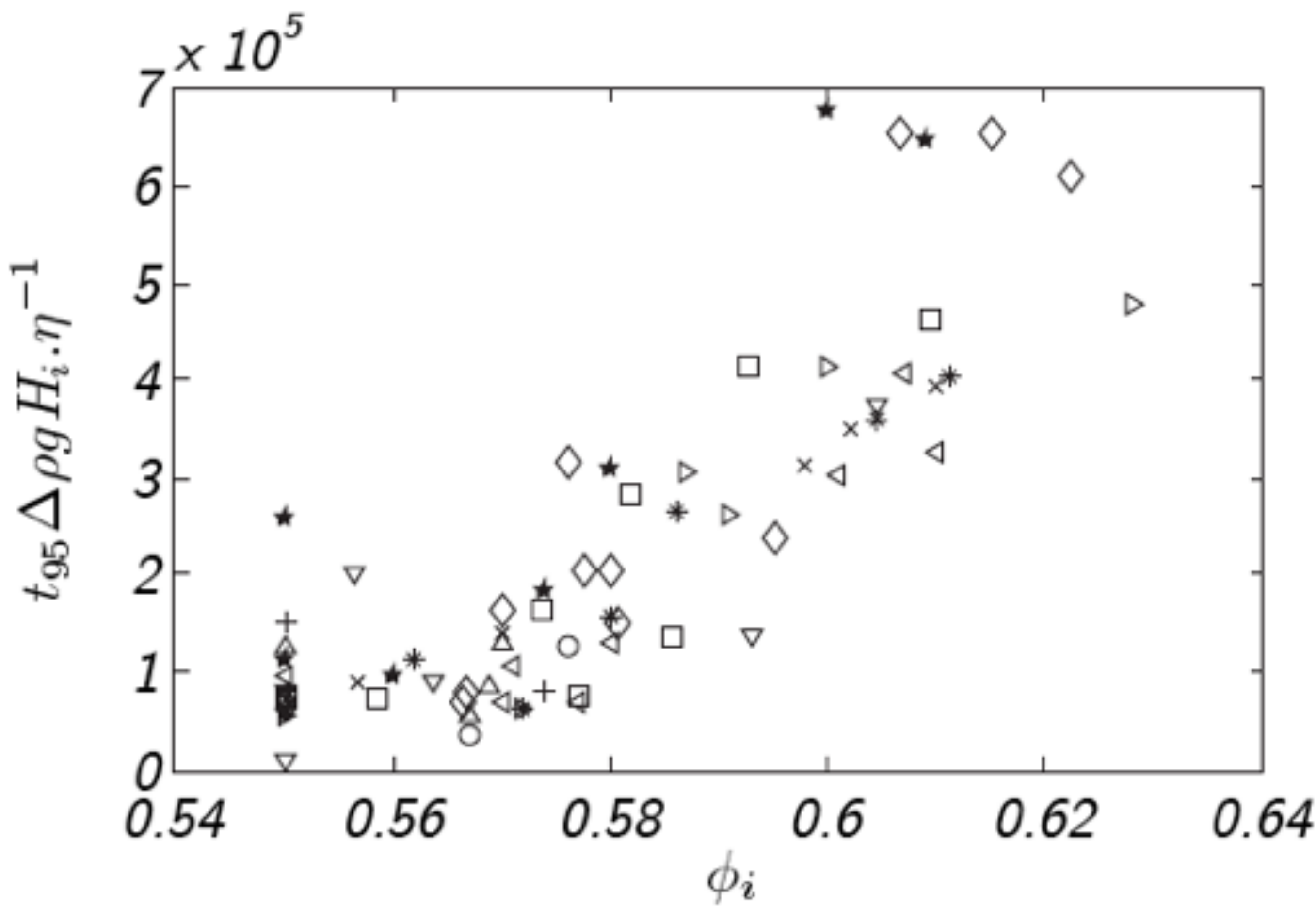
time



5 cm
H_f
ϕ_i =0.58
L_f



$H_f L_f^{-1}$
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.54
0.56
0.58
0.6
0.62
ϕ_i



x 10^5
$t_{95} \Delta \rho g H_i . \eta^{-1}$
7
6
5
4
3
2
1
0
0.54
0.56
0.58
0.6
0.62
0.64
ϕ_i

Unsteady flows => variation of ϕ

=> relative motion between grains and fluids

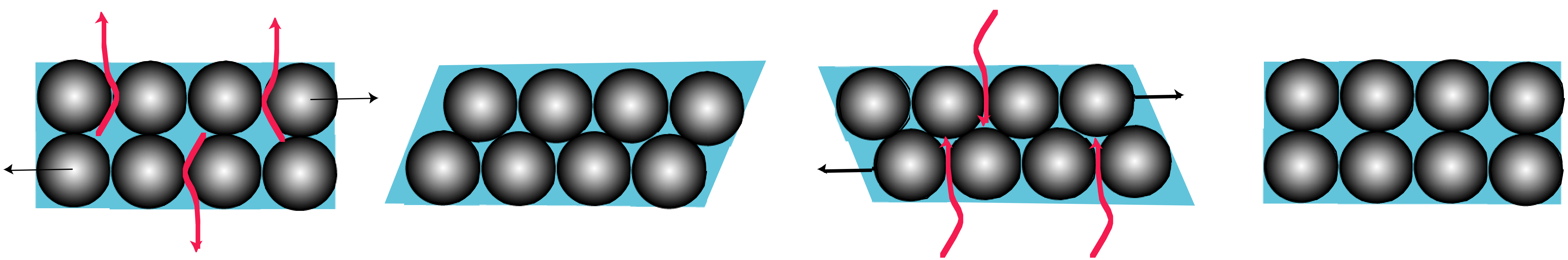
=> additional stress...

Pore Pressure feedback argument

(Iverson Rev. Geo. 97, JGR 05)

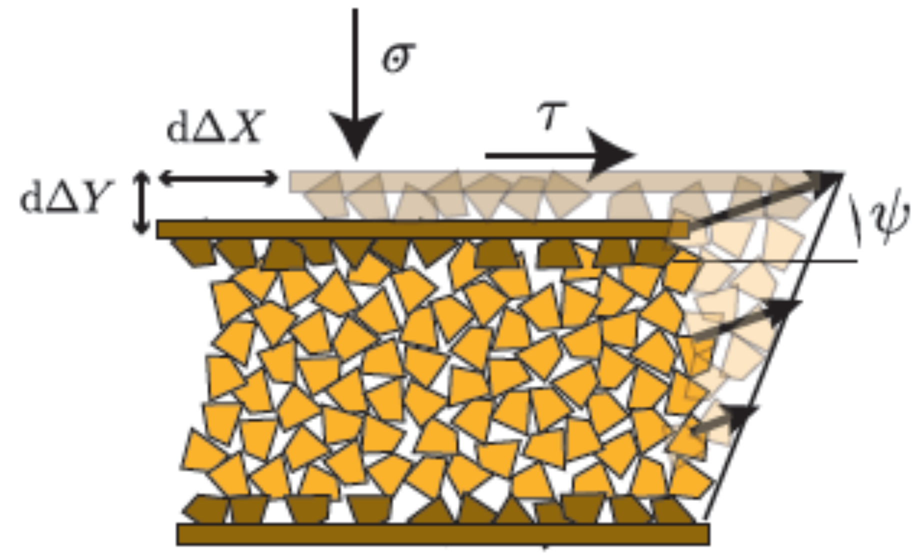
Loose case

Dense case



A shear rate dependent critical state theory

Dilatancy angle:



$$d\phi/dt = \dot{\gamma} \tan\psi$$

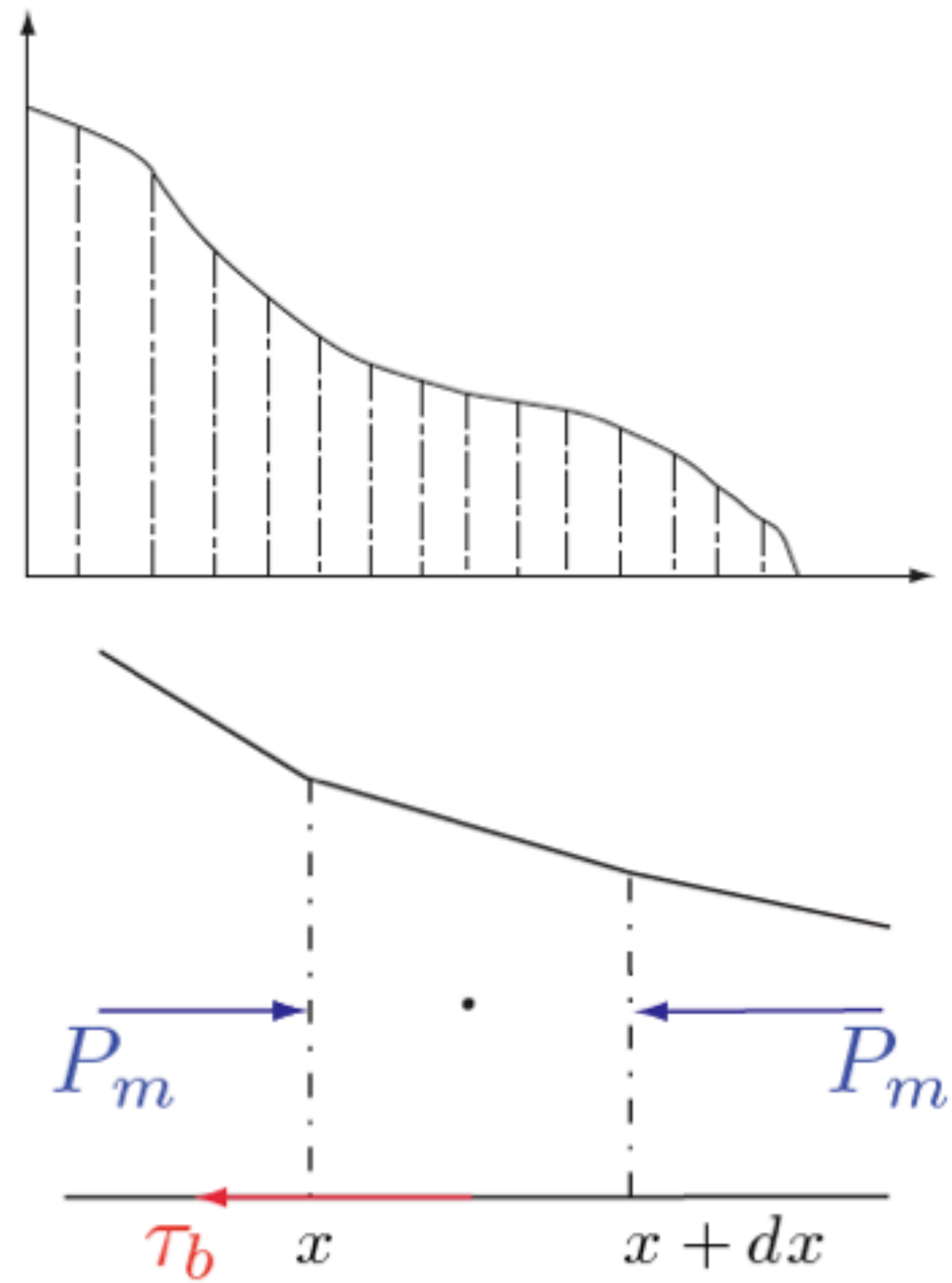
$$\tan\psi = K(\phi - \phi(I_v))$$

$$\tau_b = (\mu(I_v) + \tan\psi)P^p$$

The granular pressure:

$$P^p = \rho g h \cos\theta + \alpha \frac{\eta h}{d^2} u \tan\psi$$

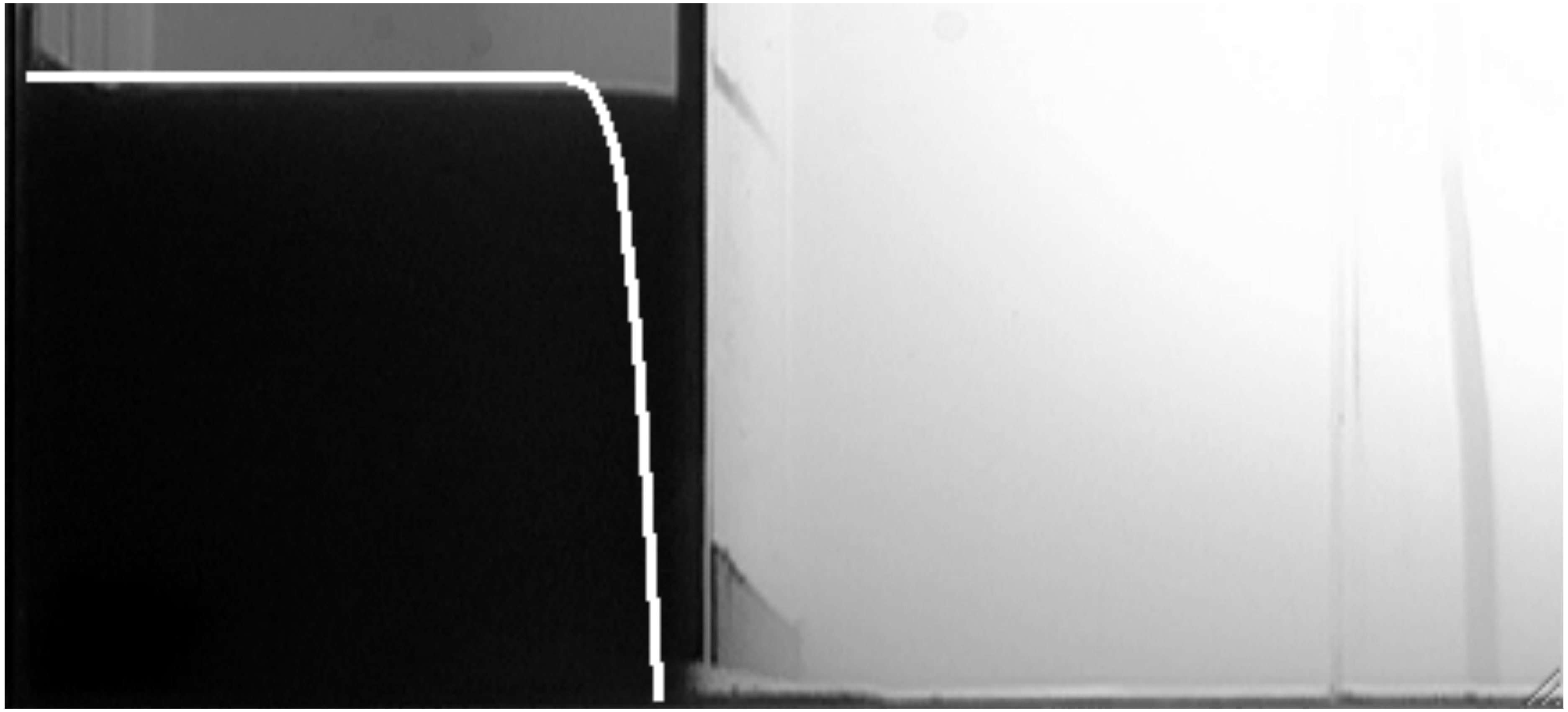
the steady
granular rheology
+
dilatancy
effects



in a depth averaged model

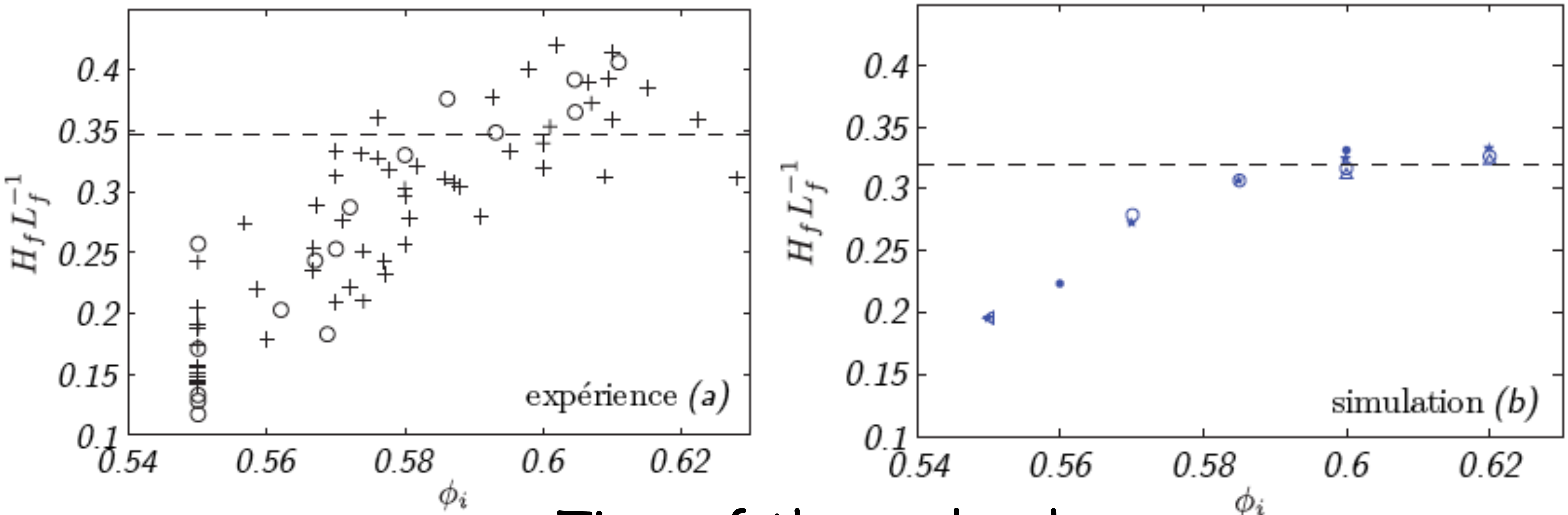
=> semi-quantitative predictions!

dense initial state

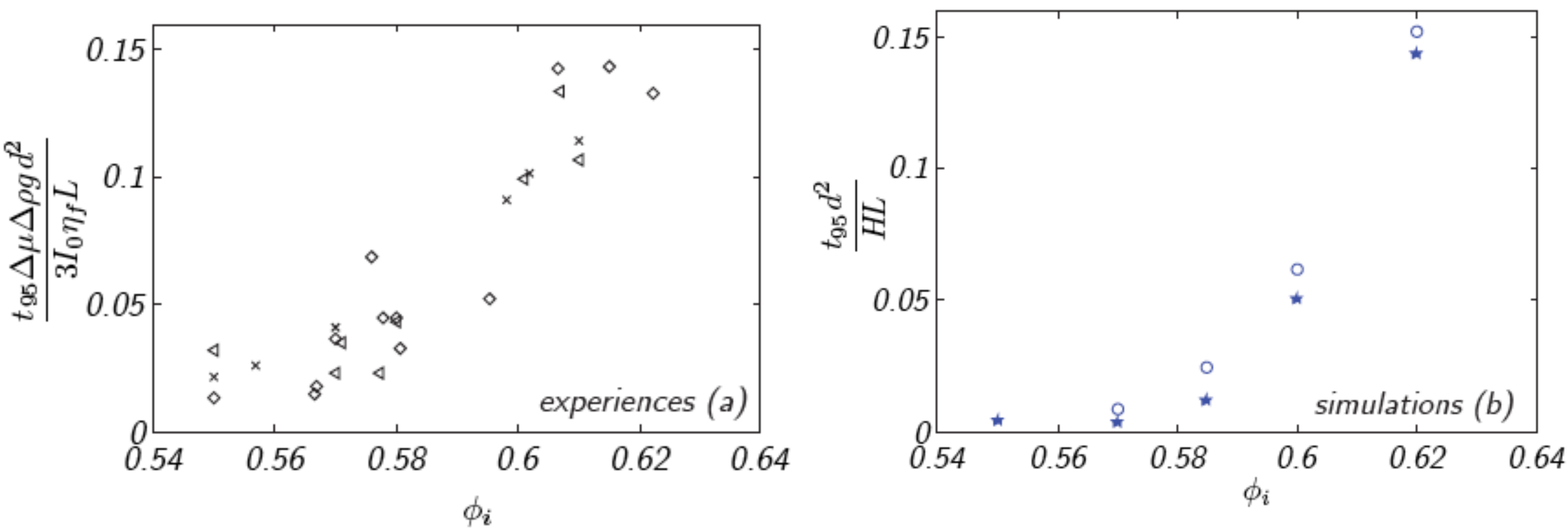


Loose initial state

final slope of the deposit



Time of the avalanche



Conclusions about the rheology...

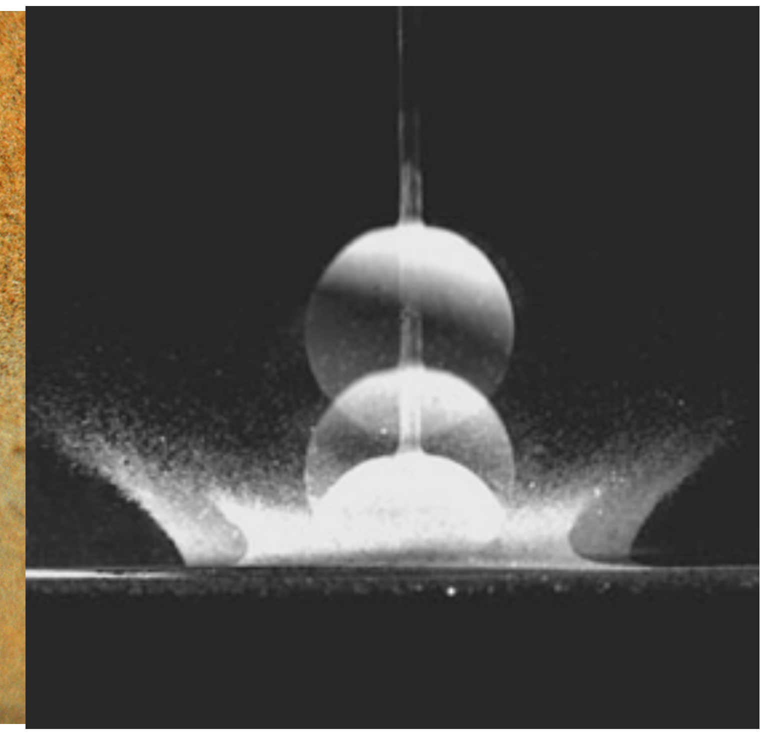
- pressure imposed versus volume imposed...

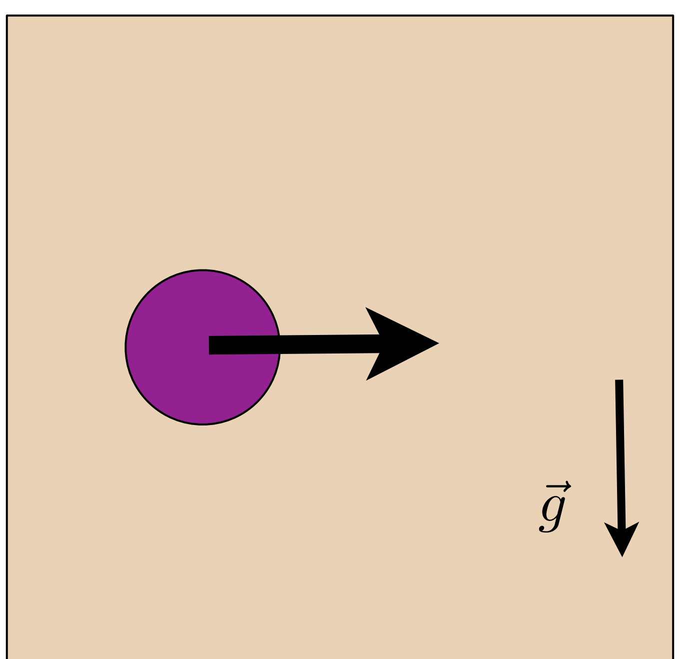
- A precise measure of rheology for very dense suspensions.

- the frictional description is relevant to describe flows under gravity (avalanches, sediment transport...) ...

A classical hydrodynamic problem: Forces on a moving object

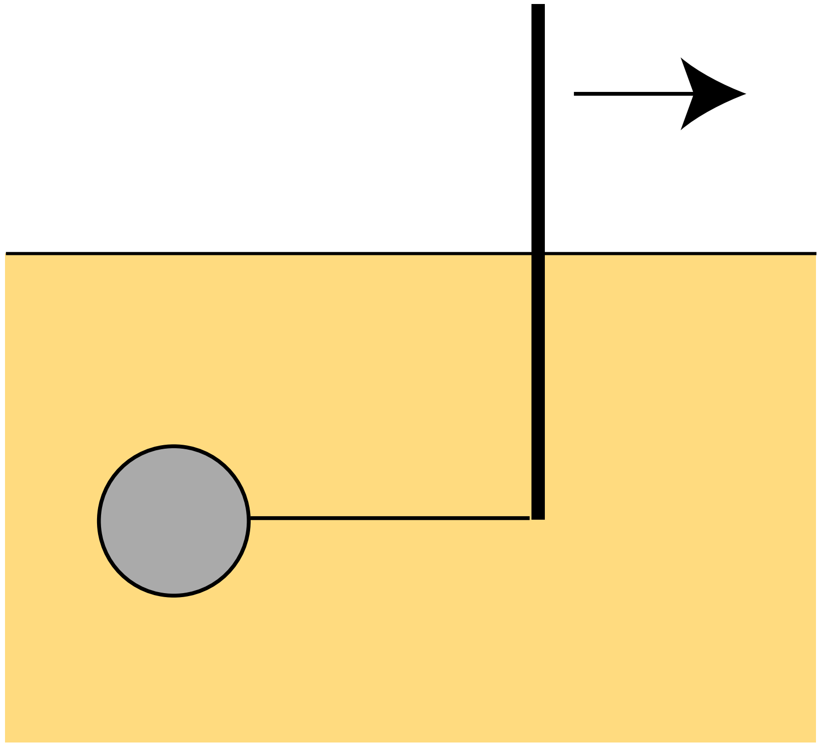






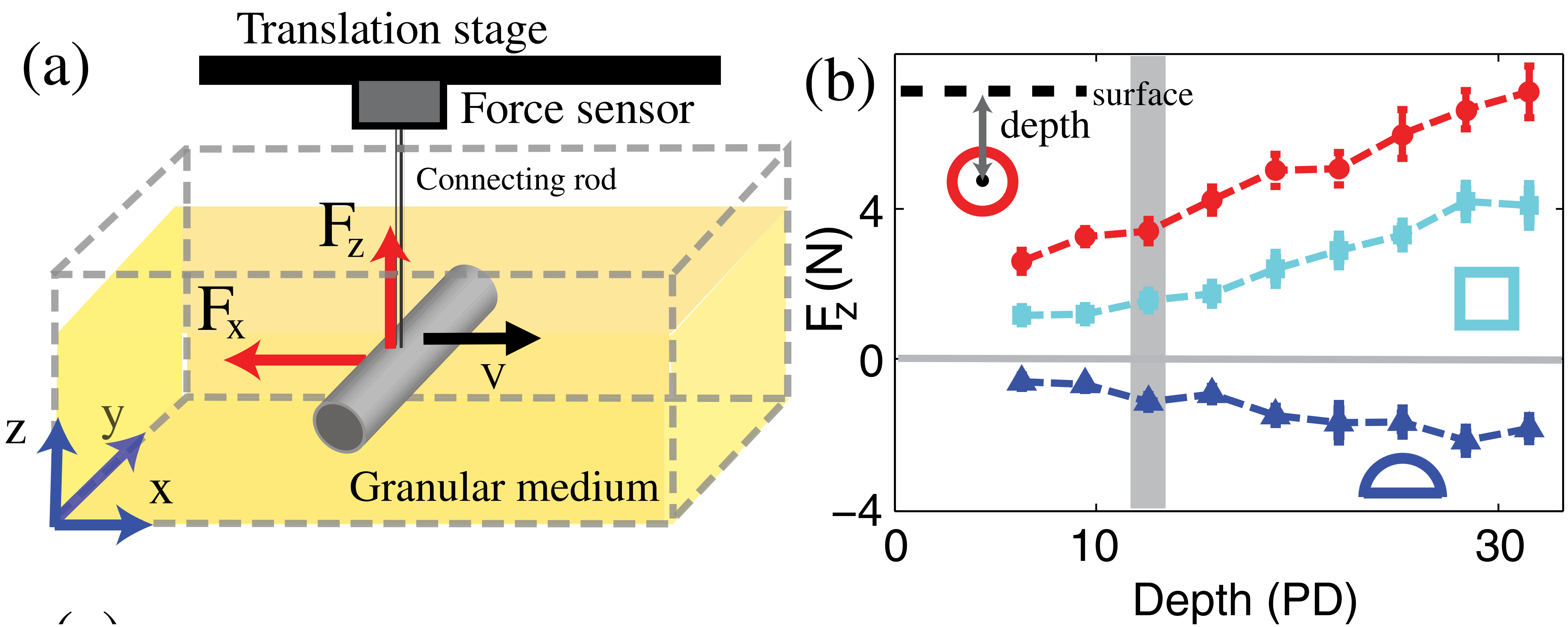
drag has been studied in details,...
what about lift ?

Evidence of lift force on a symetric object



a recent study on lift forces...

Ding et al. Phys. Rev. Lett. 2011

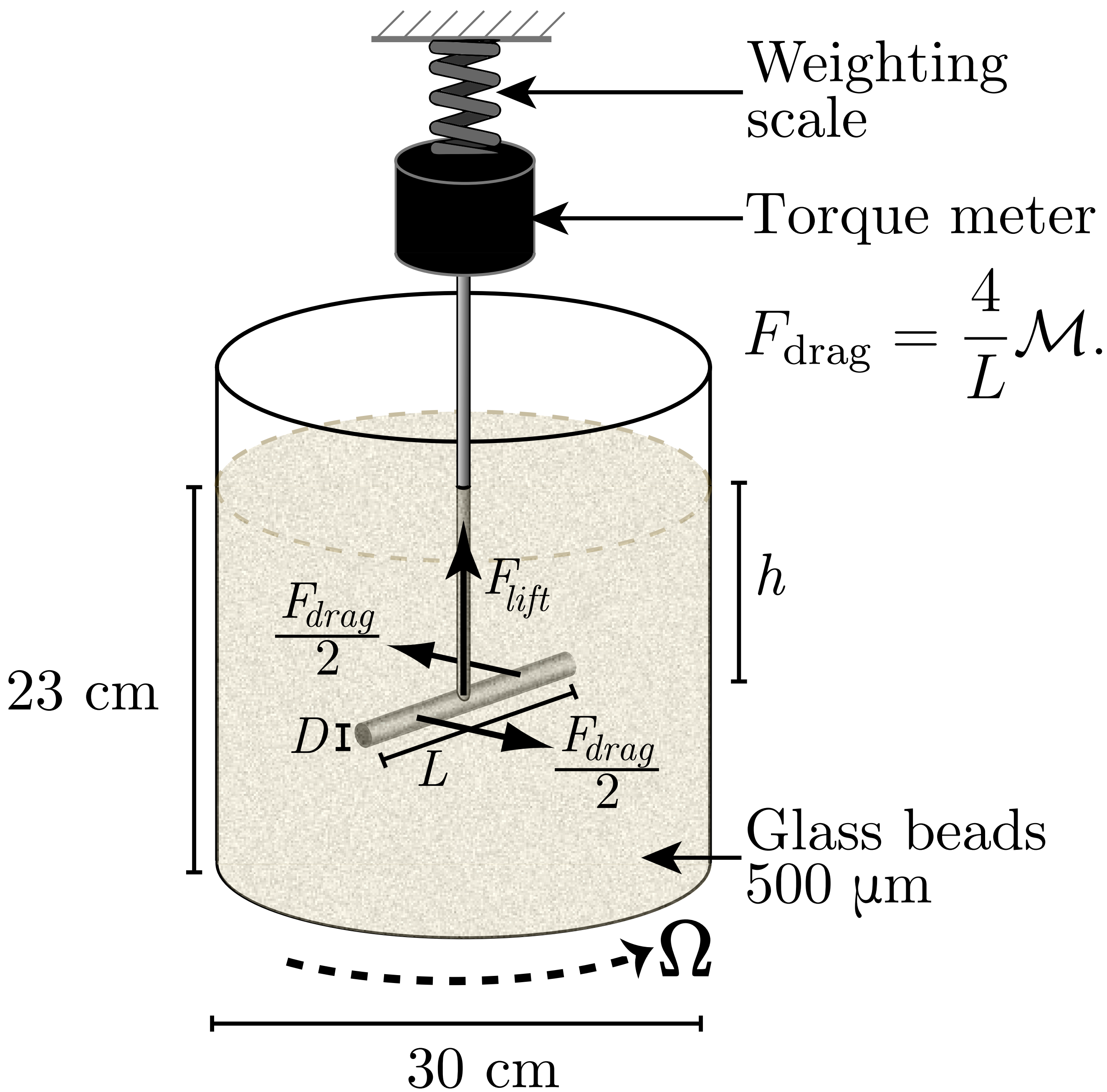


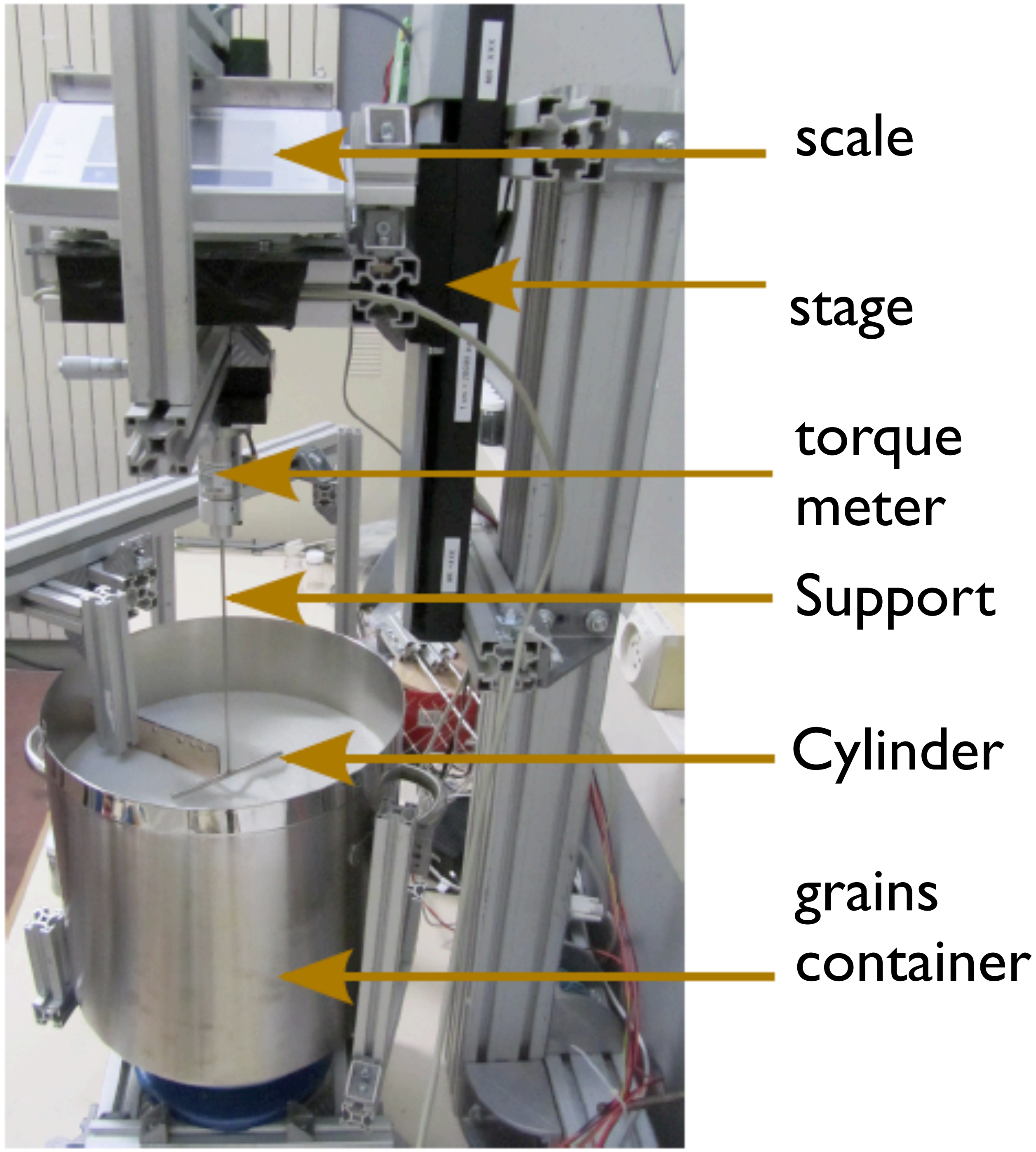
Lift on a symmetrical object which scales with depth...

physical origin, scaling ??

Experimental set-up: drag and lift on a rotating cylinder

Guillard et al. PRL 2013

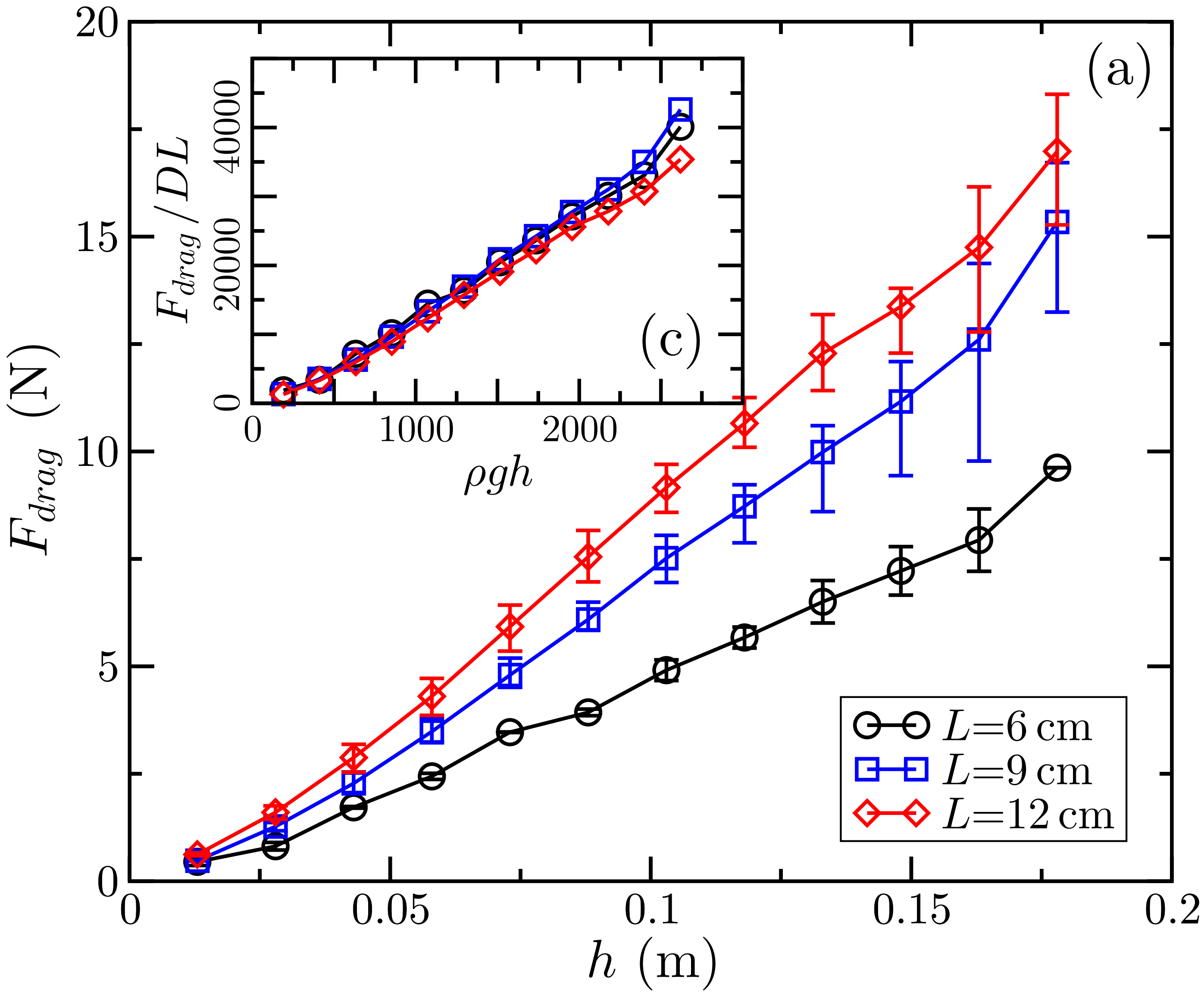




low velocity quasi-static regime : forces independent of velocity

measurements first half-turn...

Drag forces



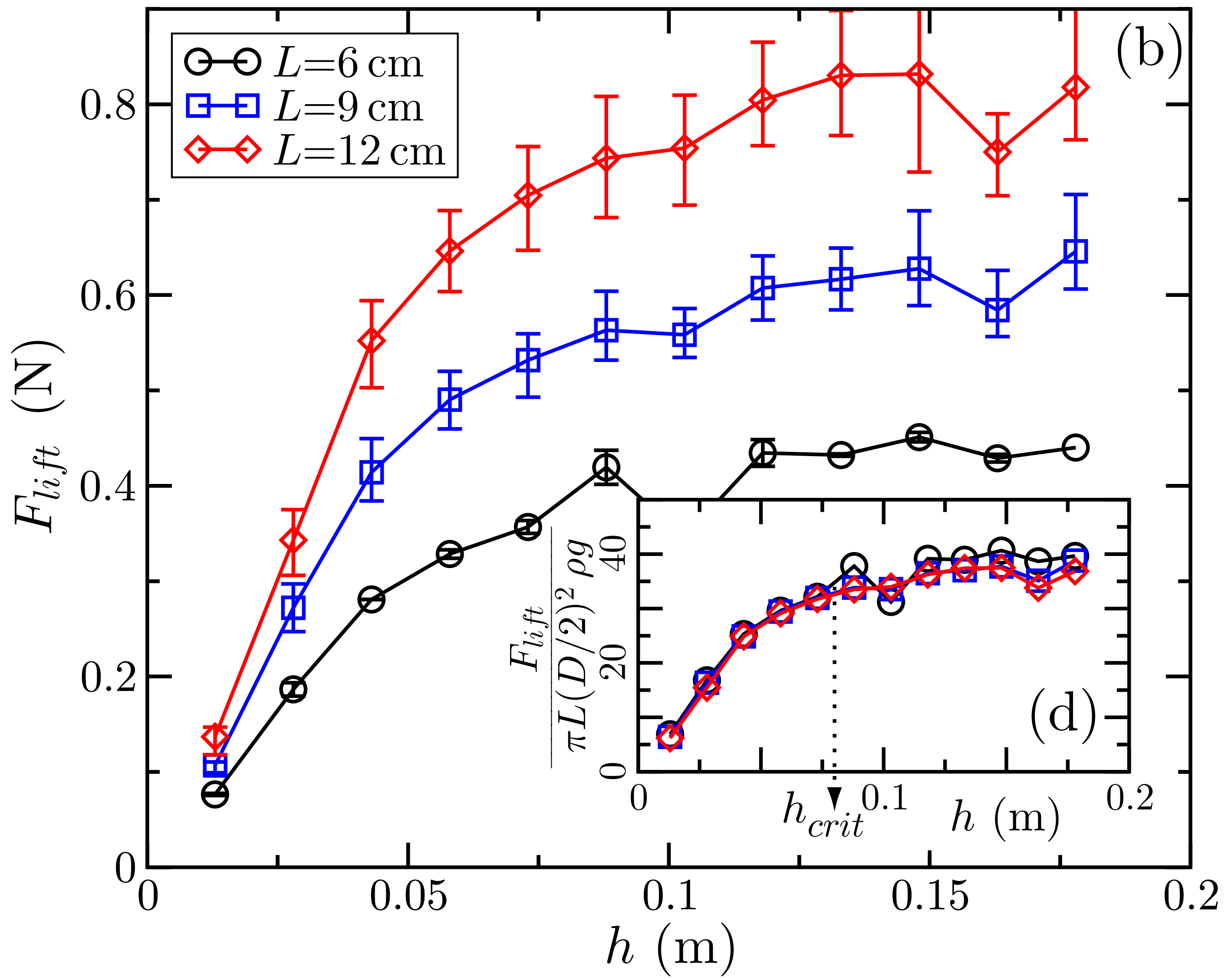
$D = 4\,\mathrm{mm}$

Drag forces proportional to depth (pressure)

$$F_{\mathrm{drag}} = C_d\,\rho g h\,DL \propto PS \qquad C_d \simeq 13$$

Albert et al. Phys. Rev. Lett. 1999

Lift forces

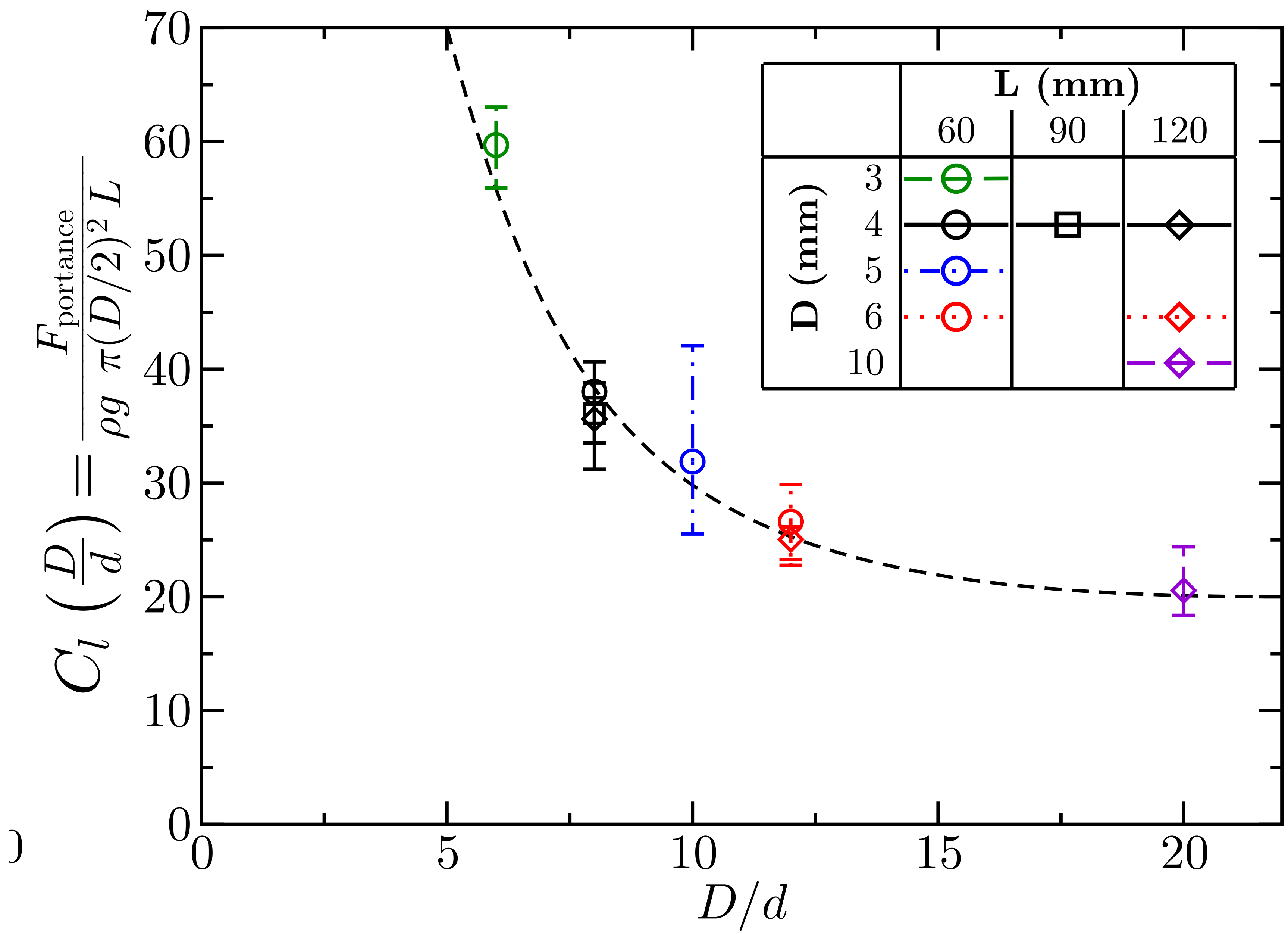


$D = 4\,\mathrm{mm}$

Saturation of the lift force with depth

$F_{lift} \sim 20\, F_{Archimede}$!!!

Lift force : general scaling

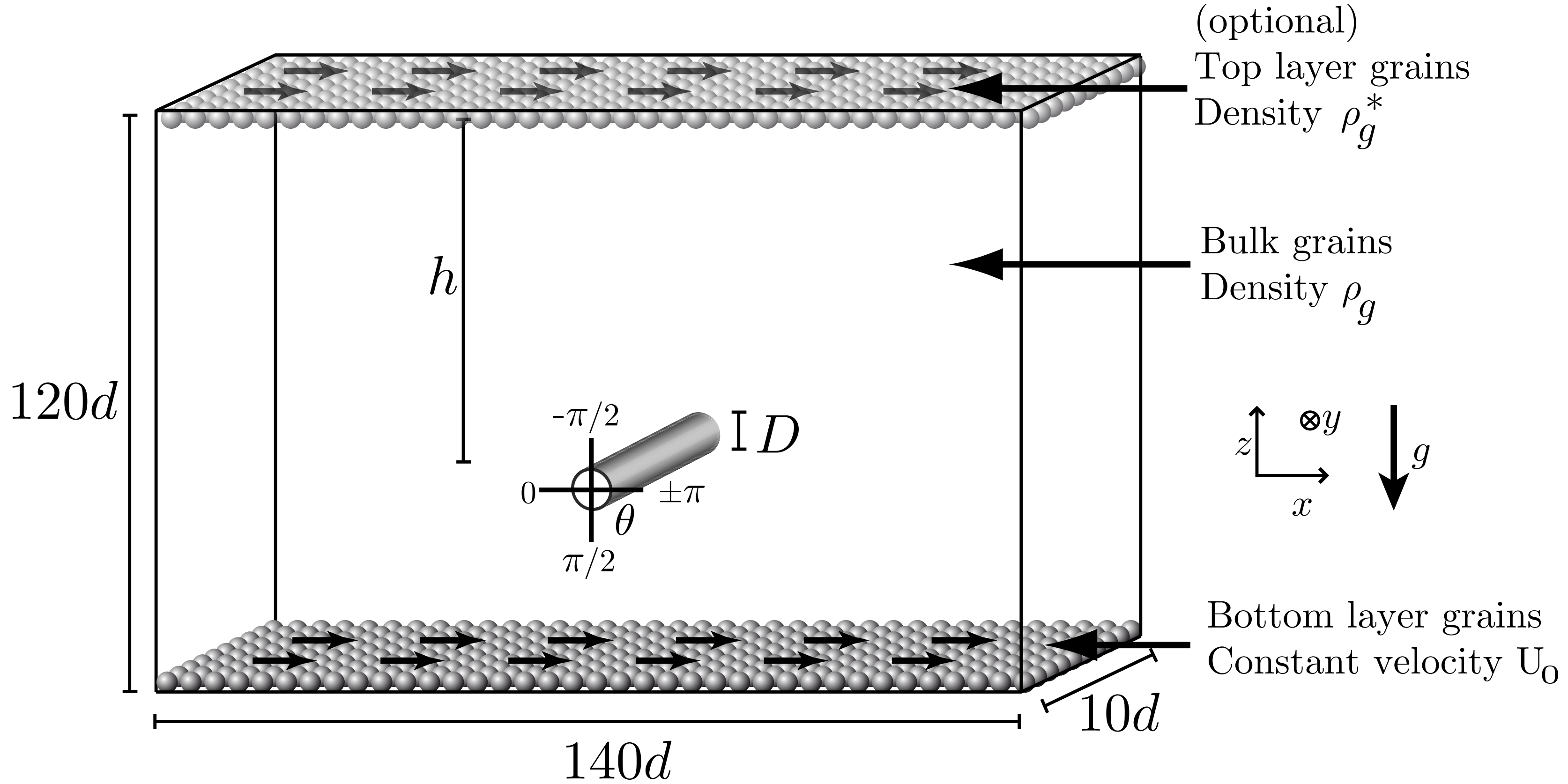


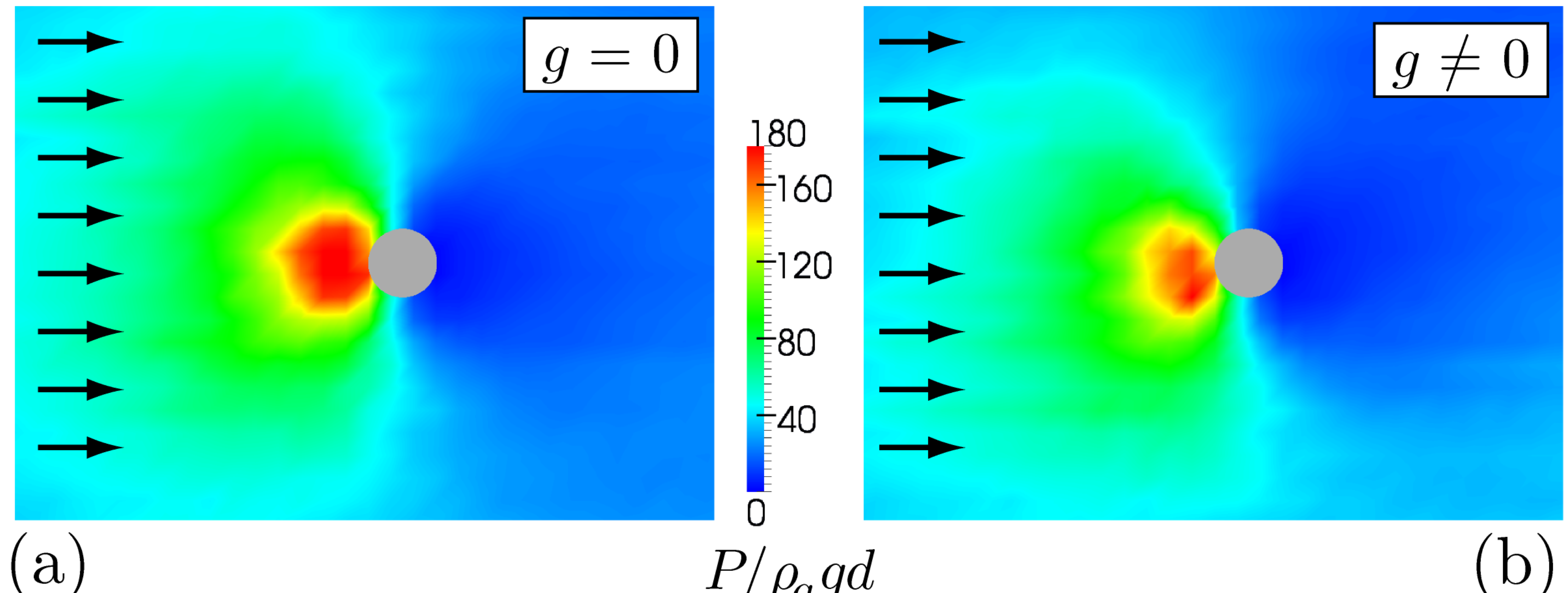
$$F_{\rm lift} = C_l(D/d)\,\rho g \pi \frac{D^2}{4} L \propto \nabla P \times V$$

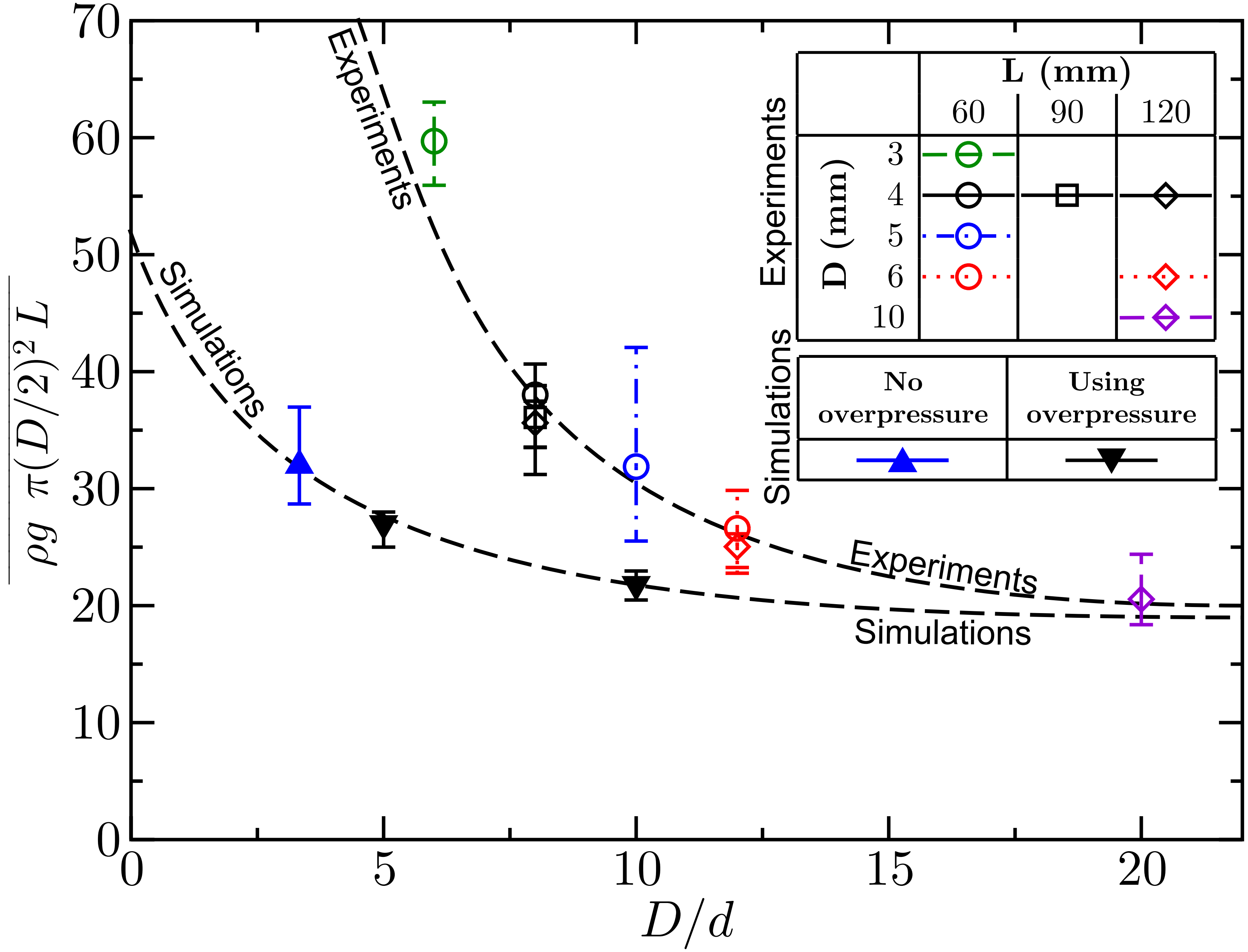
Archimedes-like scaling but finite size effects

very strong lift $C_l \approx 20$ when $D/d > 15$

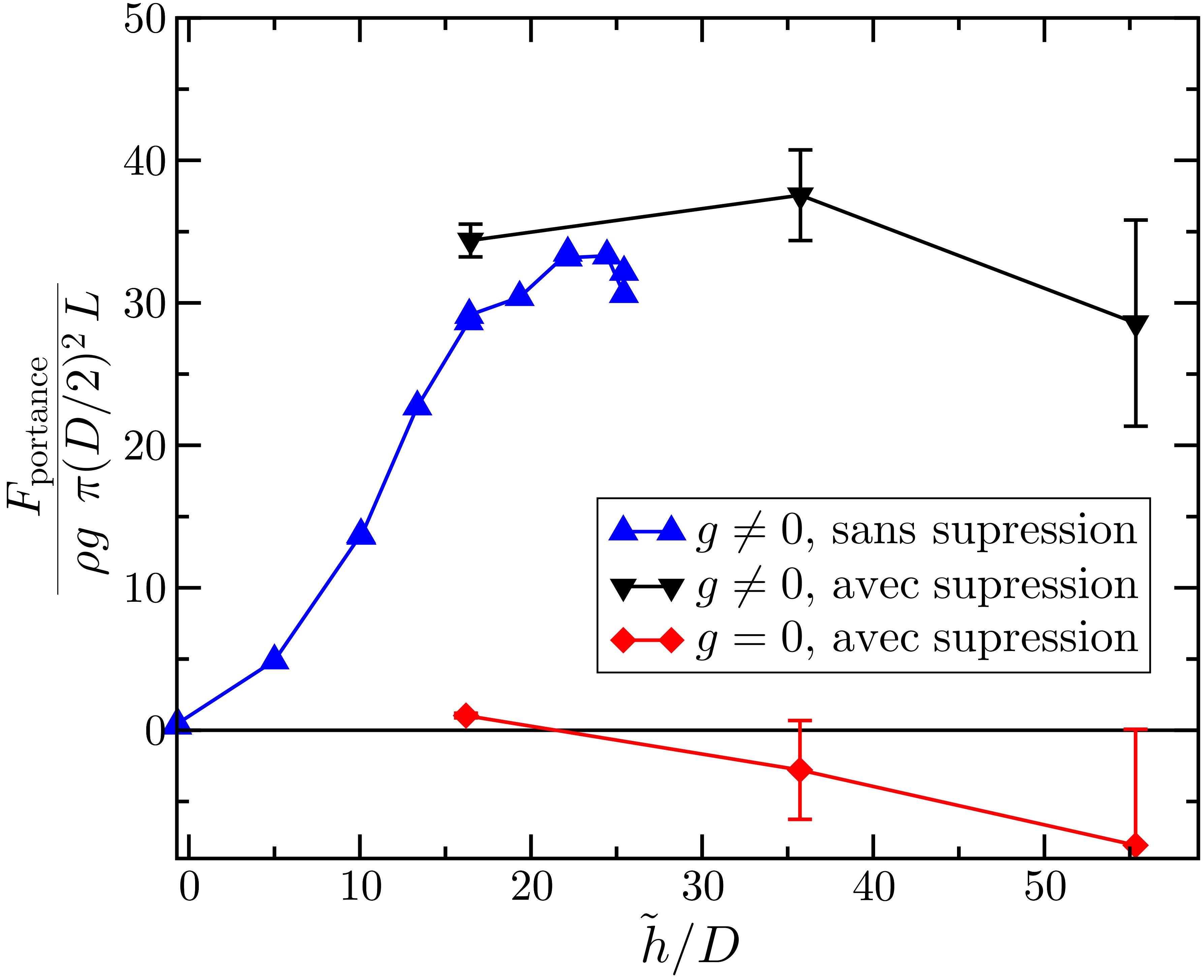
Molecular dynamics simulations (LIGGGHTS)





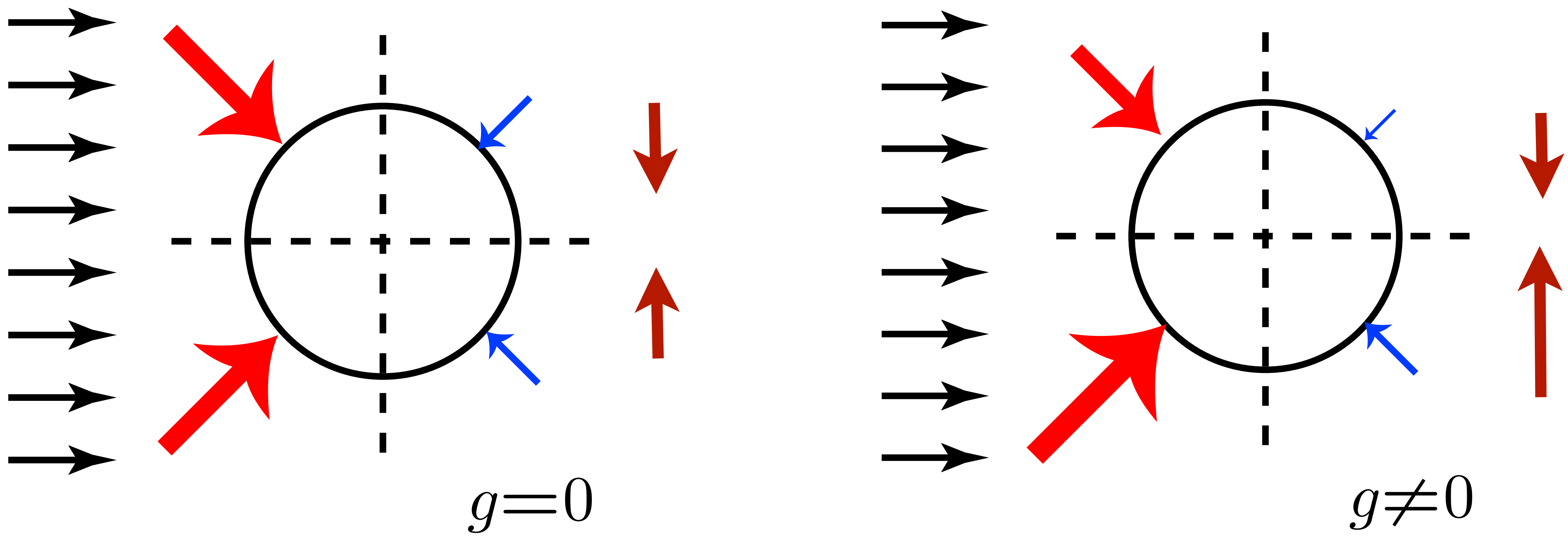


$\frac{F_{\mathrm{lift}}^{\mathrm{sat}}}{\rho g\ \pi(D/2)^2 L}$
D/d
0
5
10
15
20
0
10
20
30
40
50
60
70
Experiments
Simulations
Experiments
Simulations
Experiments
L (mm)
60
90
120
D (mm)
3
4
5
6
10
Simulations
No overpressure
Using overpressure



non pressure gradient => no lift

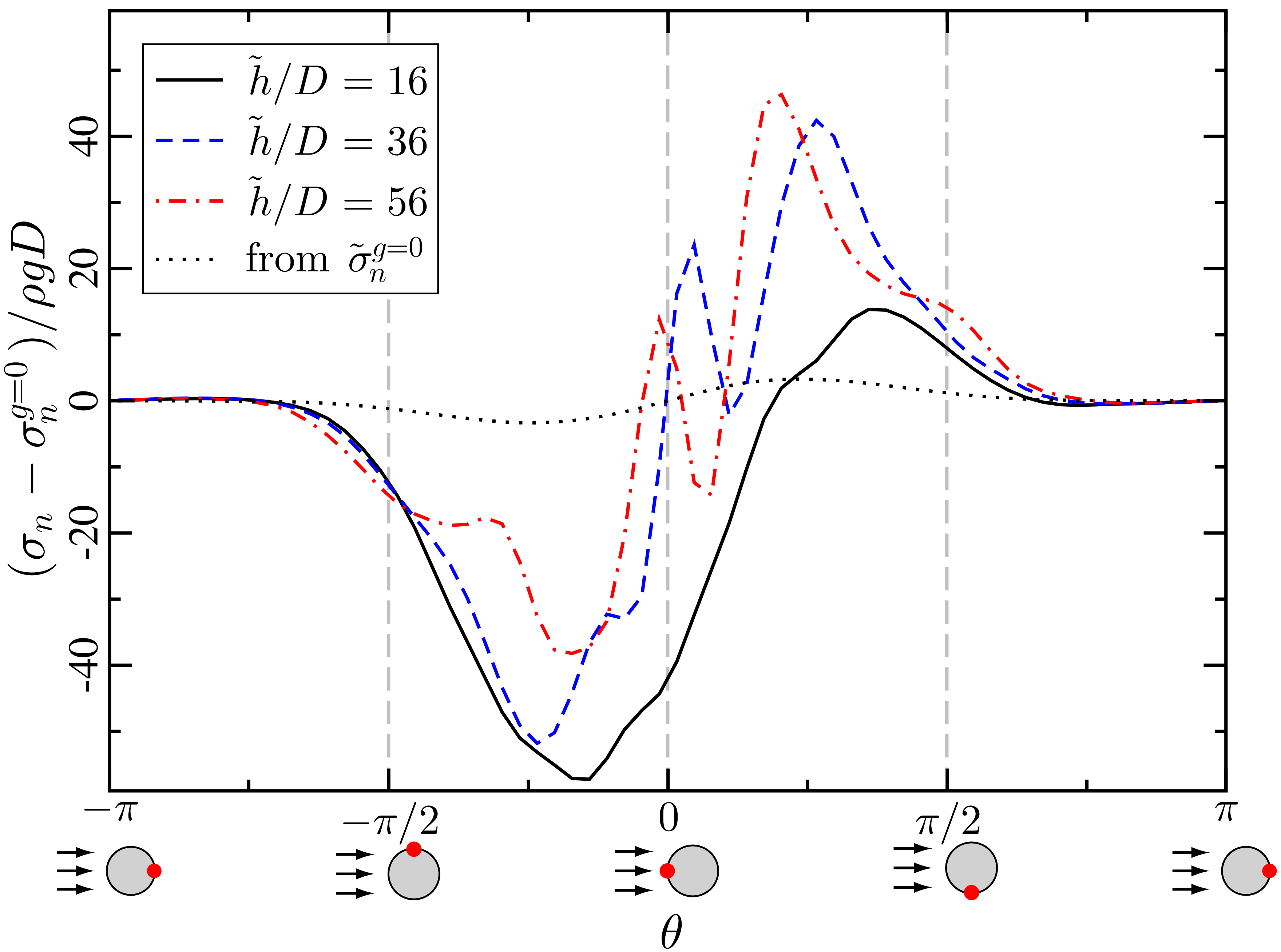
In granular media : strong left/right asymmetry



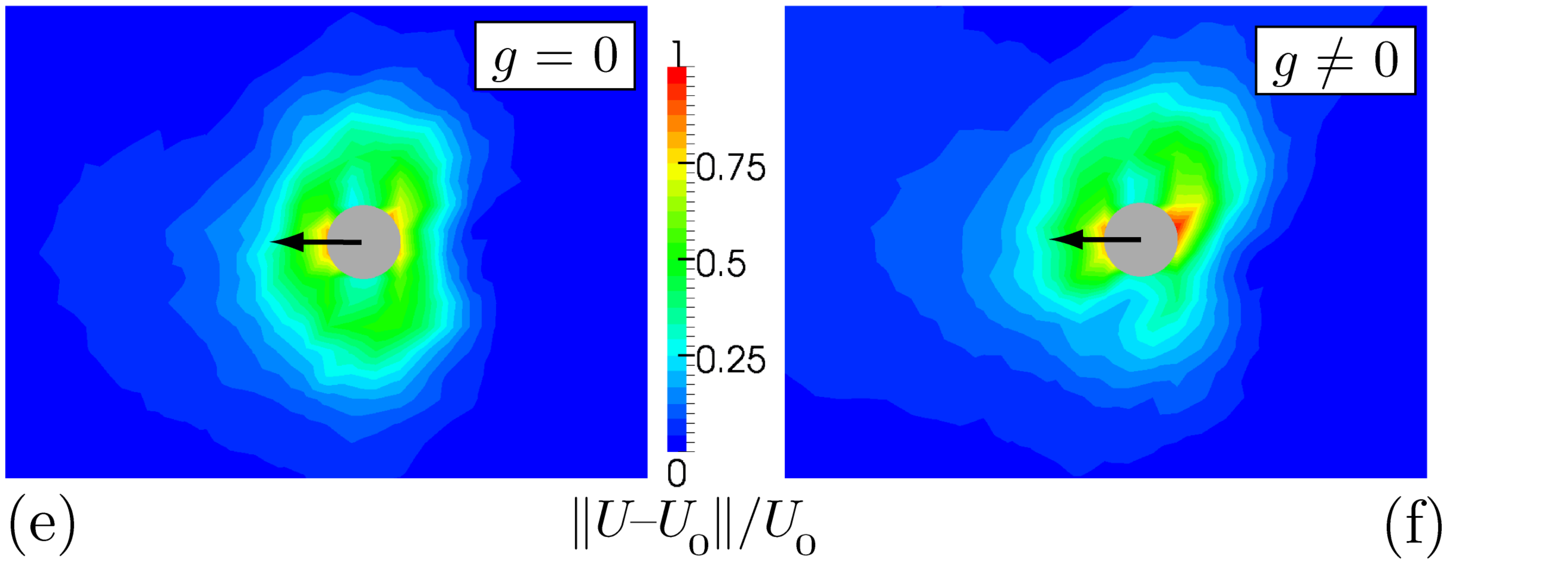
+ stresses proportional to pressure (friction)

$\longrightarrow$ in the presence of a pressure gradient : lift

$$F_{\text{lift}} = (\rho g L D^2/4) \int_{-\pi}^{\pi} \tilde{\sigma}_n^{g=0}(\theta) \sin^2\theta d\theta$$



$$F_{\text{lift}} = (\rho g L D^2/4) \int_{-\pi}^{\pi} \tilde{\sigma}_n^{g=0}(\theta) \sin^2\theta d\theta$$

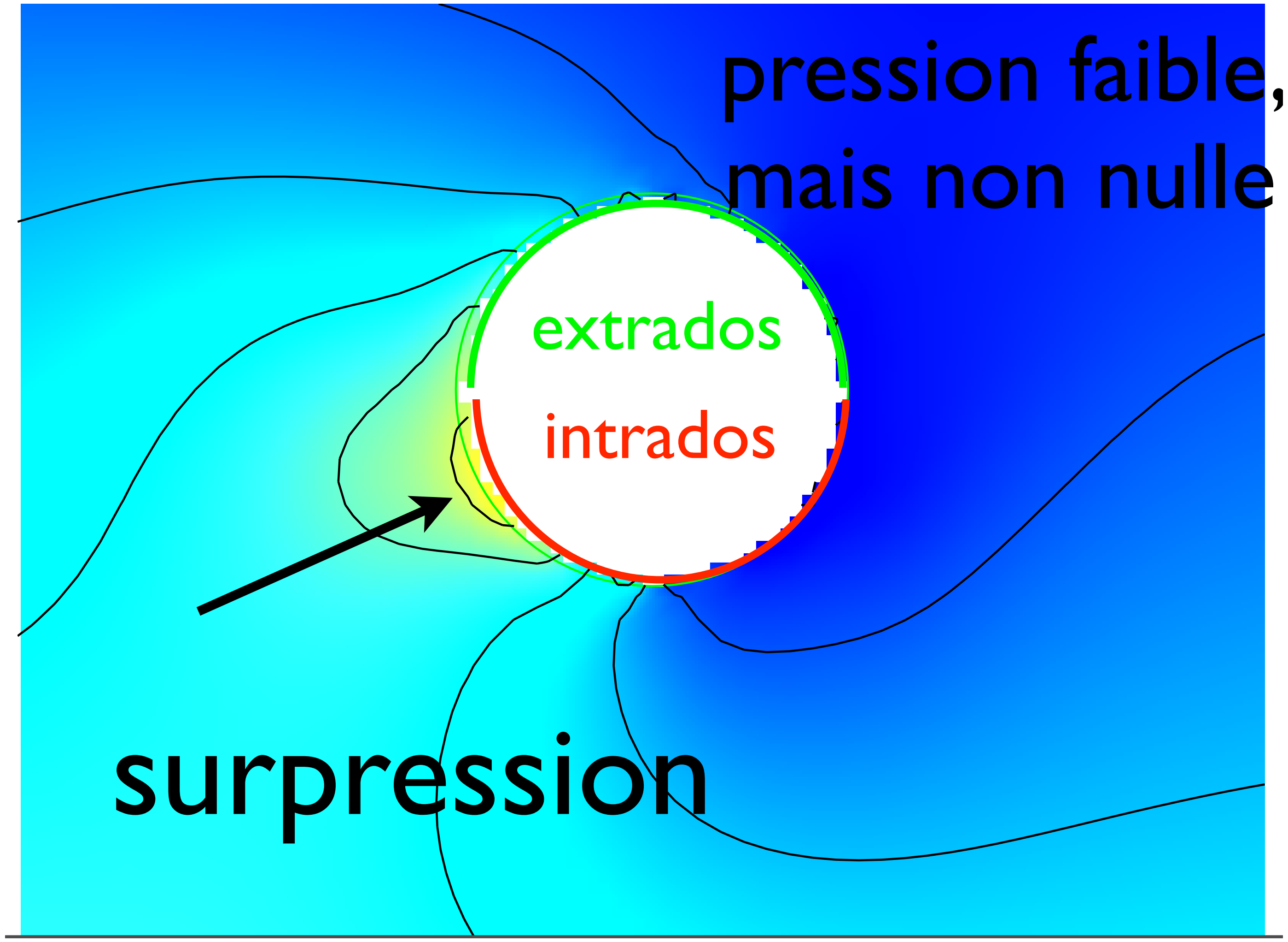


The flow itself is strongly modified in the presence of the pressure gradient

Guillard et al. preprint 2013

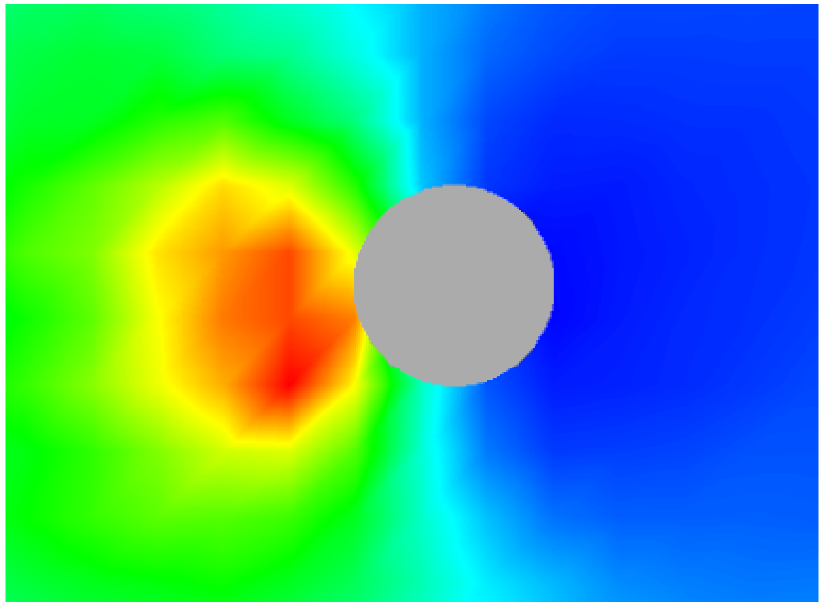
Simulation of the visco-plastic rheology (P. Y Lagree, M. Médale) :

simulation



a lift exists!!

DEM simulation



Conclusion:

-pressure imposed rheology an intersting approach

-visco-plastic frictionnal description captures many observed feature in dense granular flows

- Hydrodynamic approach

Questions:

-microscopic origin: role of contact? of fluctuations?

-link with quasi-static regime, and collisional regime?

thanks..

Suspensions

François Boyer

Etienne Couturier

Elisabeth Guazzelli

Granular media

François Guillard

Yoel Forterre

Granular collapse

Loic Rondon

Pascale Aussillous

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