Sand Ripples and Dunes

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Outline

Examples of sand patterns

- Aeolian ripples (Sahara, Mauritania) :

$$Wavelength \approx 5 - 10\, cm\, ;\, Amplitude \approx 1 - 2\, cm$$

Examples of sand patterns

- Mega-ripples (Sahara, Mauritania) :

$$Wavelength \approx 50 - 100\ cm$$

Examples of sand patterns

- Aquatic ripples (Beach) :

$$Wavelength \approx 5 - 10\, cm \, ; \, Amplitude \approx 1 - 2\, cm$$

Examples of sand patterns

- Barchan dune (Mauritania) :

$$Length \approx 20 - 500\ m\ ;\ Amplitude : \approx 2 - 50\ m$$

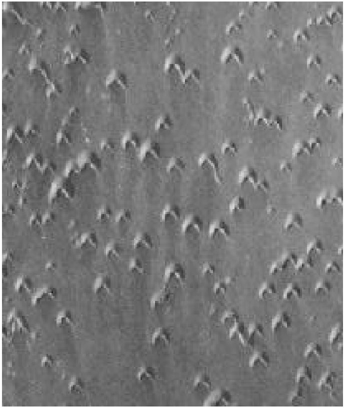
Examples of sand patterns

- Mega barchan dune (Mauritania) :

$$Length \approx 1000\ m$$

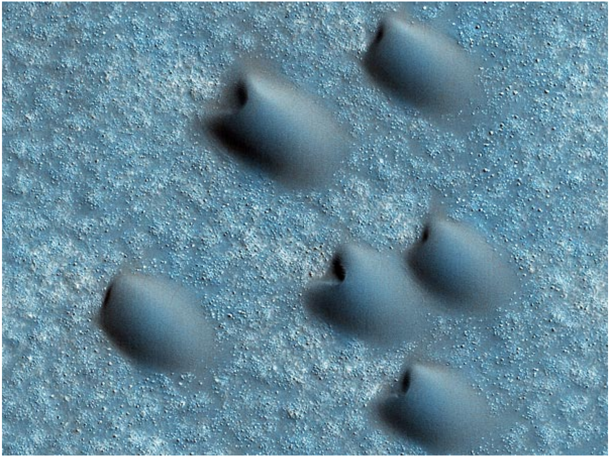
Examples of sand patterns

- Barchan field (Marocco) :



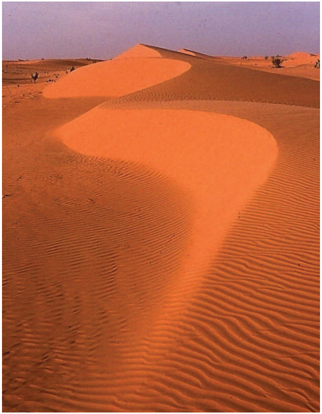
Examples of sand patterns

- Martian barchan dunes



Examples of sand patterns

- Linear Dunes (Seif) : Bidirectional wind



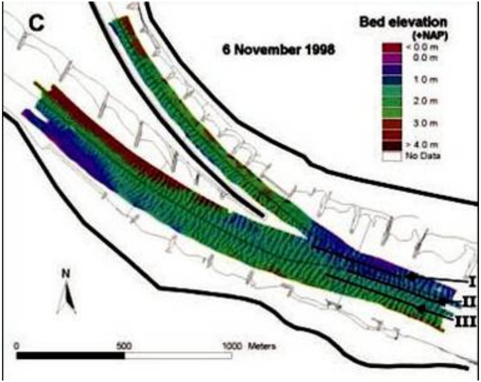
Examples of sand patterns

- Star dune : Multidirectional wind

Examples of sand patterns

- River dunes (Rhein, Nederland) :

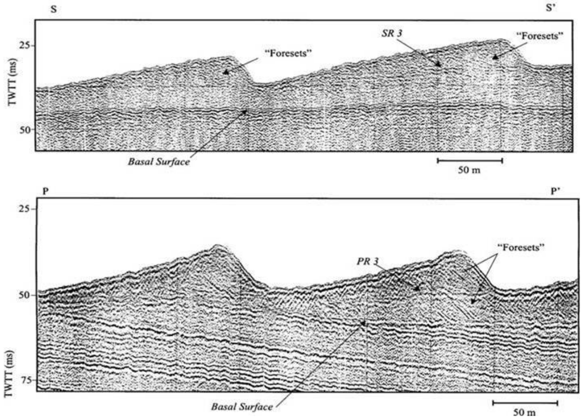
$$Wavelength \approx 10\ m$$



Examples of sand patterns

- Submarine dunes (English Channel) :

$$Wavelength \approx 200\ m$$



Societal and Economical Issues

- Submarine and River Dunes : hindrance to ship traffic
- Desert Dunes : sand encroachment (roads, villages . . .)

Barchan crossing a road (Mauritania)

House burried under sand (Niger)

Societal and Economical Issues

- Dune fixation : fences, plant cover . . .

Square grid (Mauritania)

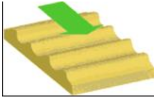
Bofix method (Meunier, Mauritania)

Fondamental Physical issues

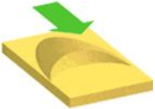
- What drives the formation process of sand dunes ?
  - Hydrodynamical instability ?
  - Aeolian dunes vs Aquatic bedforms
  - Turbulent flow versus Laminar Flow
- What determines the morphology of sand patterns ?
  -Sand availability
  -Flow directionality (bidirectional, multi-directional)
  -Flow symmetries (uni-directional, oscillating, . . .)
- What selects the size of the sand patterns ?
  - Uni-directional flow versus oscillating
  - Shallow flow vs Deep flow

Classification of Aeolian Dune patterns

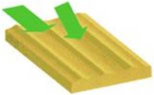
- Transverse dune : *high availability of sand and unidirectional wind*



- Barchan dune : *low availability of sand and unidirectional wind*



- Longitudinal dunes (seif) : *high availability of sand and bidirectional wind*



- Star dune : *high availability of sand and multi-directional wind*

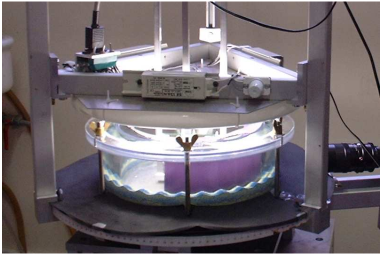
How to Improve our Understanding of Aeolian Sand Dunes ?

- Field observation
  -Temporal and spatial monitoring of dune field (Satellite Images, Lidar . . .)
- Laboratory experiments
  - Wind-tunnel experiment (sand transport, aeolian ripples)
  - Aquatic ripples and dunes in flumes
- Theoretical and Numerical modeling
  - Continuum models
  - Numerical simulations : Particle/Fuid hybrid models (D.E.M + Fluid Solver)

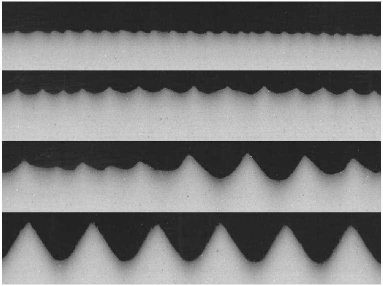
Laboratory Experiments

- Aquatic ripples under oscillating flow :

Couette Cell (Rousseaux, 2003)



Bedform evolution (Rousseaux, 2003)

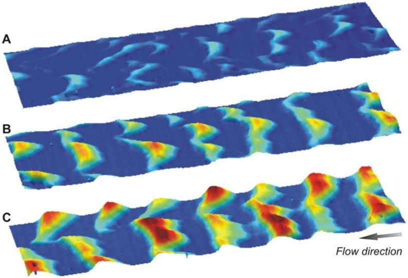


Laboratory Experiments

- Aquatic ripples formed under unidirectional flow :

Dimensions : 3m long, 10cm wide and 3cm high (Dreano et al., ESPL 2010)

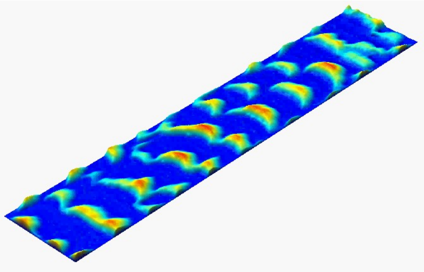
Bedform evolution



Laboratory Experiments

- Aquatic ripples formed under unidirectional flow (Dreano et al., ESPL 2010) :

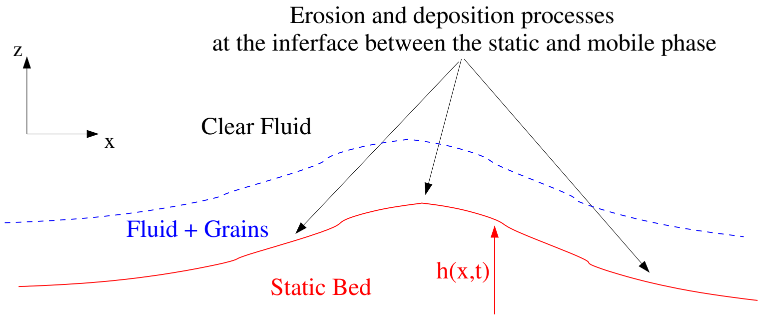
Low supply of sediment



High supply of sediment

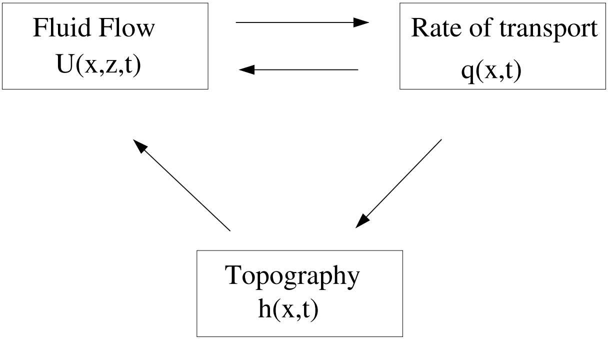
Theoretical Modeling

- Two-phase flow : particles + air
- Three different regions



Theoretical Modeling

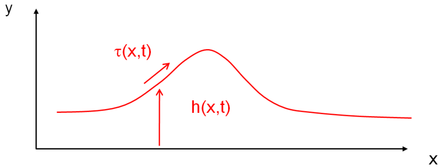
- Intricate interactions between the flow, the sediment and the topography



1D Dune Model

- Equation for Dune Evolution :

$$\frac{\partial h}{\partial t}(x,t) = -\frac{1}{\rho_{bed}}\frac{\partial Q}{\partial x}(x,t)$$



- Transport law : $Q(x,t) = Q_{sat}\left[\tau(x,t)\right] \propto \tau^{\alpha}(\tau - \tau_c)^{\beta}$

- Computational need :

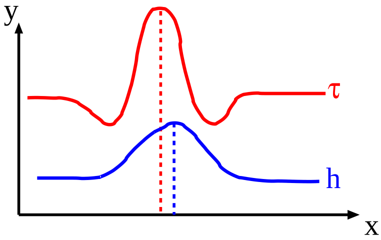
  -Calculation the shear stress τ above the bedform
  -Quasi-static approximation : bed is assumed to be fixed
  $t_{dune} >> t_{flow}$

1D Dune Model

- Calculation of the turbulent shear stress over a small hill from RANS equations (Jackson and Hunt, 1975)
- Hypothesis : H/L (Linear analysis)

$$\Rightarrow \tau_{JH} = \tau_0 + \tau_1 = \tau_0 \left(1 + A \int_{-\infty}^{\infty} dx' \frac{\partial_x h(x')}{x - x'} + B\, \partial_x h \right)$$

$$\Rightarrow \hat{\tau}_1 = \tau_0 \left(A|k| + iBk \right) \hat{h}$$



Positive phase shift to due fluid inertia ⇒ Instability

1D Dune Model : Morpholigical Instability

- We seek solutions of the form : $h(x,t) \propto e^{ikx+\omega t}$

$$\frac{\partial h}{\partial t}(x,t) = -\frac{1}{\rho_{bed}} \frac{\partial Q}{\partial x}(x,t)$$

$$Q = Q_{sat} \propto \tau^{\alpha} \, (\tau(x) - \tau_c)^{\beta} \quad \text{with} \quad \tau(x) = \tau_{JH}$$

- Dispersion Relation $\omega = \omega(k)$

$$\Rightarrow Re(\omega) \propto B\, k^2$$

All modes are unstable and the most dangerous one has an infinitively small wavelength !

Non-Equilibrium Transport Model

- Hypothesis :
  the actual transport rate Q does not reach instantaneously its saturated value Q_{sat}

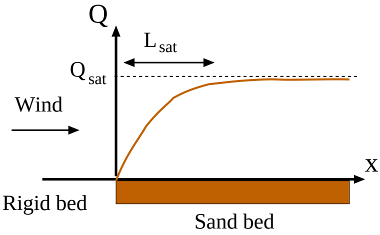
- Relaxation law

Herrmann et al, 2001

$$\frac{\partial Q}{\partial x} = -\frac{Q(Q - Q_{sat})}{Q_{sat} L_{sat}}$$

Andreotti et al, 2002

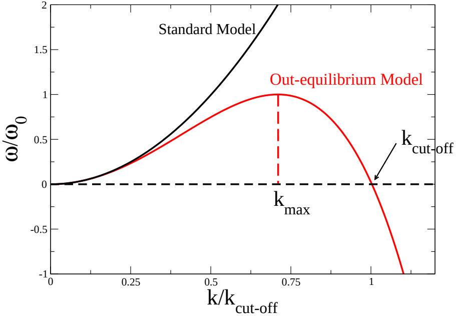
$$\frac{\partial Q}{\partial x} = -\frac{Q - Q_{sat}}{L_{sat}}$$



Non-equilibrium Transport Model

- Linear stability analysis :

$$\sigma = Re(\omega) = k^2 \frac{B - Akl_{sat}}{1 + (kl_{sat})^2} \quad \text{and} \quad c = \frac{Im(\omega)}{k} = \frac{A + Bkl_{sat}}{1 + (kl_{sat})^2}$$

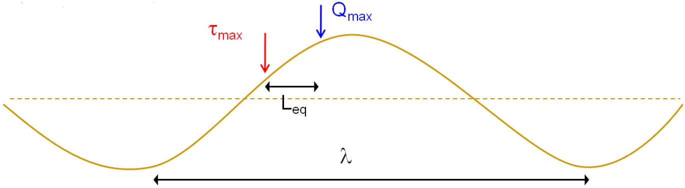


$$\lambda_{cut-off} = \frac{2\pi}{k_{cut-off}} \approx \frac{6A}{B} l_{sat}$$

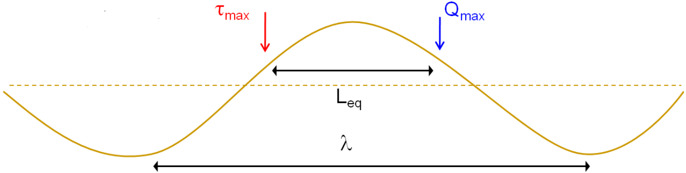
$$\lambda_{max} = \frac{2\pi}{k_{max}} \approx \frac{9A}{B} l_{sat}$$

Non-equilibrium Transport Model

- Instability Mechanism :
  $\lambda \gg l_{sat}$: Unstable mode



  $\lambda \lesssim l_{sat}$: Stable mode



Sand Dune Instability

- Instability driven by the aerodynamics ($\Rightarrow$ Same type of instability is expected for water flow)

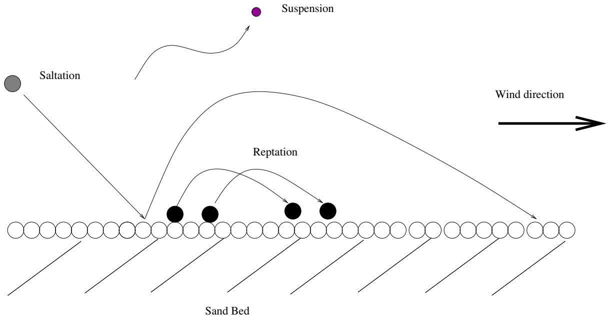
- Most unstable mode selected by the non-equilibrium transport process :

  The wavelength of the most unstable mode is governed by the saturation length and is independent of the mass flow rate

- If the saturation length is smaller than the grain diameter, we expect than other stabilizing mechanisms come into play

Aeolian dunes versus Aquatic ripples

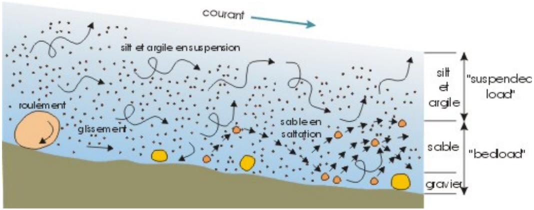
- Aeolian sand transport (Bagnold 1941) :
  Suspension, Saltation, Reptation



Unique feature of aeolian transport : erosion by mechanical impact

Aeolian dunes versus Aquatic ripples

- Aquatic transport :
  - *Suspended load*
  - *Bed load*



Erosion by impact inefficient : High dissipation due to lubrication force (Stokes $\ll$ *1)*

Aeolian dunes vs Aquatic Ripples

- Transport law
  - Aeolian transport :

    $Q_{sat} \propto (S - S_c)$ where $S = \tau/(\rho_p - \rho_f)gd$

    (Hungar and Haff, 1987 ; Creyssels et al, 2009 . . .)
  - Aquatic transport (bedload) :

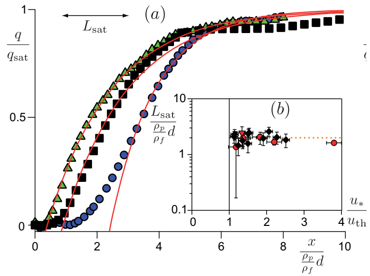
    $Q_{sat} \propto (S - S_c)^{3/2}$

    (Peter-Meyer and Muller, 1948 ; Claudin et al, 2012 . . .)

Aeolian dunes vs Aquatic Ripples

- Saturation length for aeolian transport

(Andreotti et al, EPSL 2010)



$$l_{sat} \propto \frac{\rho_p}{\rho_f} d$$

$$\Rightarrow l_{sat} \approx 1\,m \text{ for } d = 0.2\,mm$$

$$\Rightarrow \lambda_{max}^{aeolian} \approx 20\,m$$

Aeolian dunes vs Aquatic Ripples

- Saturation length for aquatic transport : Few measurements
  - Bed load :

    -Grain inertia : $l_{sat} \propto \frac{\rho_p}{\rho_f} d$ (*Claudin et al., EPSL 2006*)

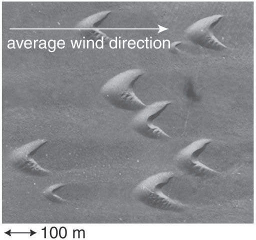
    -Deposition : $l_{sat} \propto \frac{U}{v_{fall}} d$ (*Charru et al., JFM 2006*)
  - Suspension :

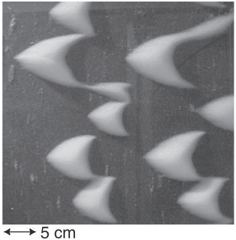
    -Suspension : $l_{sat} \propto \frac{U}{v_{fall}} D$ (*Andreotti et al., JFM 2011*)

Aeolian dunes vs Aquatic Ripples

- Aeolian barchan versus Aquatic barchan

$$l_{sat} \propto \frac{\rho_p}{\rho_s} d \quad \Rightarrow \quad l_{sat}^{water} = \frac{l_{sat}^{air}}{1000} \approx 1\ mm$$





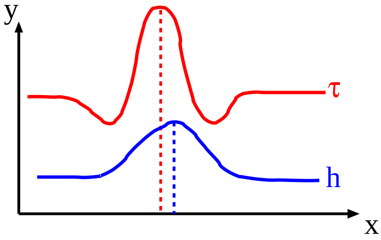
(Hersen et al., PRL 2002)

Turbulent flow vs Laminar flow

- Shear stress over a deformed sand bed in a viscous flow (Charru & Hinch, JFM 2000)

$$\tau = \tau_0 + \tau_1 = \tau_0 \left(1 + \int_{-\infty}^{x} dx' \frac{h_x(x')}{(\nu/\dot{\gamma})^{1/3} (x - x')^{1/3}} \right)$$

$$\Rightarrow \hat{\tau}_1 \propto \tau_0 \left[(1/2)k^{1/3} + (\sqrt{3}/2)ik^{1/3} \right] \hat{h}$$



Positive phase shift due to competition between fluid inertia and viscous force

$\Rightarrow$ *Bed instability*

Nonlinear evolution (Laminar flow)

Weakly non-linear analysis (Valance, PRE 2011)

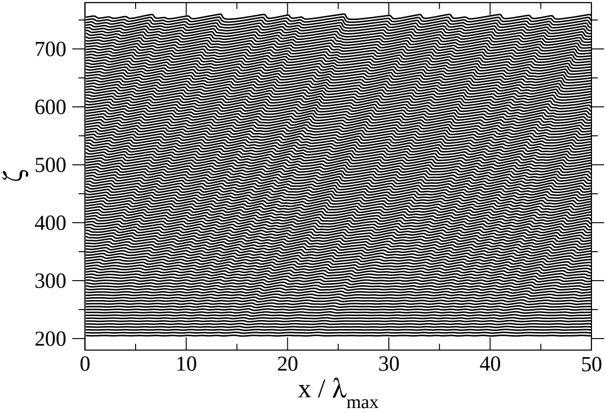
- Hypothesis :
  - nonlinear effects dominated by the nonlinearity of the transport law
  - Infinite flow depth

$$\frac{\partial h}{\partial t} = - \frac{\partial}{\partial x}\left\{\left(\int_{-\infty}^{x} dx' \frac{h_x(x')}{(\nu/\dot{\gamma})(x-x')^{1/3}} - h_x\right) + C\left(\int_{-\infty}^{x} dx' \frac{h_x(x')}{(\nu/\dot{\gamma})(x-x')^{1/3}} - h_x\right)^2\right\}$$

Shear rate : $\dot{\gamma}$
Kinematic viscosity : ν

Nonlinear evolution (Laminar flow)

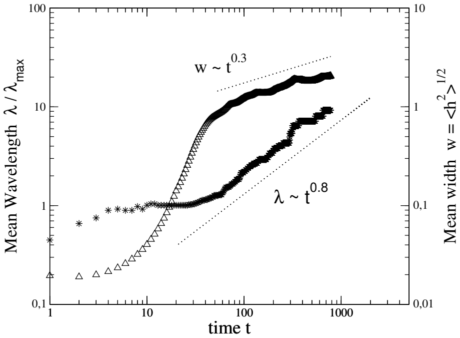
- Spatio-temporal diagram (Valance, PRE 2011)



Indefinite Coarsening

Nonlinear Evolution (Laminar flow)

- Scaling Exponents



- Transient Regime :
  $\lambda \approx \lambda_{max}$
  $A \sim e^t$
- Coarsening Regime :
  $\lambda \sim t^{0.9}$
  $A \sim \lambda^{1/3} \sim t^{0.3}$

Nonlinear Evolution : Infinite vs Finite flow depth

- Interrupted coarsening in flow of depth (Valance, PRE 2011)
  - Laminar flow :

    $\lambda_{final} \propto \frac{\dot{\gamma}}{\nu} D^3$

    $A_{final} \propto D$

  - Turbulent flow :

    $\lambda_{final} \propto D$

    $A_{final} \propto D$

Free surface flow

- Examples :

  -Submarine dunes
  -River dunes
  -Giant Aeolian dunes

- Additional control parameter : $Fr = U/\sqrt{gD}$
  - $Fr < 1$: Stabilisation of long wavelength modes ($\lambda > D$)
  - $Fr > 1$: Resonance with the free surface ($\Rightarrow$ Anti-dunes)

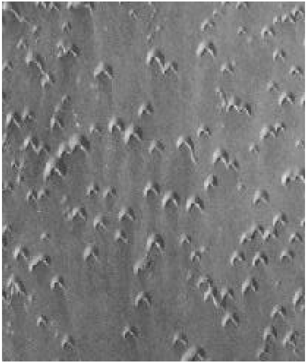
  (See for more details : Fourriere et al, JFM 2010 ; Charru et al., ARFM 2013)

Open issues

- Determination of the saturation length for aeolian and aquatic transport in wider contexts *(spatially heteregeneous boundary conditions at the bed, bimodal sand)*
- Nonlinear dynamics :
  - Indefinite and Interrupted Coarsening ?
  - Mechanisms interrupting the coarsening process ?
  - Nonlinear interactions between the flow and the bedform (flow recirculation, secondary flows . . .)
  - Secondary instabilities (i.e., in the spanwise direction)
- Unsteady flow regimes

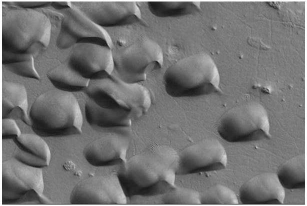
Open issues

- Stability of a barchan field (lateral winds, Intermittency, dune collisions . . .) ?



Open issues

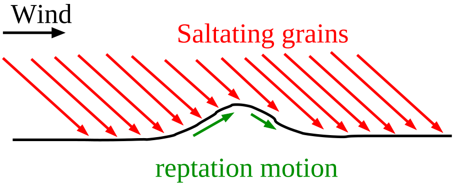
- Earth vs Mars



- Mega-dunes and star dunes

Ripples Instability Mechanism

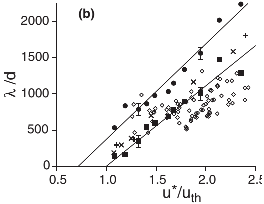
- Heuristic explanation of Bagnold (1941)



Aeolian Ripple Formation

- Wavelength

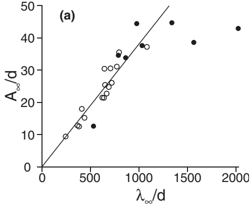
(Andreotti et al., PRL 2006)



$$\lambda_{initial} \simeq 500d(u^* - u_c^*)/u_c^*$$

$$\lambda_{final} \simeq 250d + 500d(u^* - u_c^*)/u_c^*$$

- Equilibrium ripples



Aspect ratio : $A_{final}/\lambda_{final} \simeq 1/30$
Stoss slope : 15°
Lee slope : $25^\circ <$ avalanche angle

Anderson Model (1980)

- Model equations :

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho_{bed}} \frac{\partial Q}{\partial x}$$

$$Q = Q_{Saltation} + Q_{Reptation}$$

- Hypotheses :
  - $Q_{saltation} = constant$
  - Reptation hop : $l_r = constant$

  $\Rightarrow Q_{reptation}(x) = \int_{x-l_r}^{x} dx' \, \phi_{ej}(x')$

  ϕ_{ej} is the vertical flux of ejected grains

$$\Rightarrow \frac{\partial h}{\partial t} = -\frac{1}{\rho_{bed}} \left(\phi_{ej}(x - l_r) - \phi_{ej}(x)\right)$$

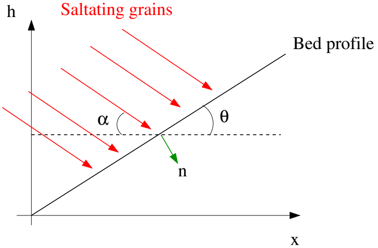
Anderson Model (1980)

- Vertical flux of ejected grains ϕ_{ej}

  $$\phi_{ej}(x) = n_0 \phi_{impact}(x)$$

  n_{ej} : number of ejected grains per impact ; ϕ_{imp} : rate of impacting grains (per unit time and surface)

- Slope effect



$$\phi_{impact}(x) = \phi_0 \cos\theta \left(1 + \frac{1}{\tan\alpha} \partial_x h\right)$$

Anderson Model (1980)

- Linear stability analysis (Rough calculation)

$$\phi_{ej} \propto (1 + \frac{\partial_x h}{\tan \alpha})$$

$$\Rightarrow \partial_t h \propto -\left(\phi_{ej}(x - l_r) - \phi_{ej}(x)\right)$$

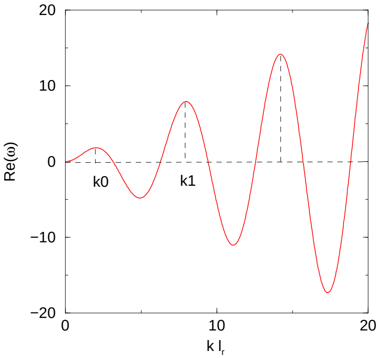
$$\Rightarrow \partial_t h \propto (\partial_x h(x - l_r) - \partial_x h(x)) \propto -\partial_{xx} h(x)$$

If $h \sim \cos(kx)\, e^{\omega t} \quad \Rightarrow \omega \propto k^2 > 0$. All modes are unstables

Anderson Model (1980)

- Linear stability analysis (Exact calculation)

$$\omega \propto k \sin(kl_r) - ik\,[1 - \cos(kl_r)]$$



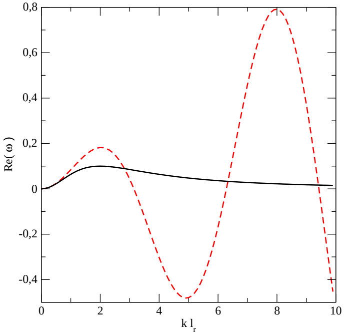
The most unstable mode diverges !

Anderson Model (1980)

- Dilemna Resolution

  Distribution of reptation hop : $P(l_r) = \frac{4l_r}{\bar{l}_r^2} \exp\left(-2l_r/\bar{l}_r\right)$

- New dispersion relation

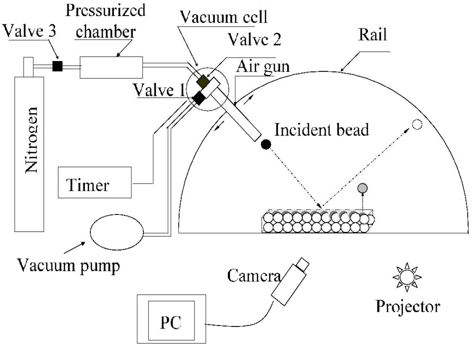


$$Re(\omega) = \frac{k^2\bar{l}_r^2}{1+k^2\bar{l}_r^2/4}$$

$$k_{max} = \sqrt{2}/\bar{l}_r \text{ and } \lambda_{max} \approx 4\bar{l}_r$$

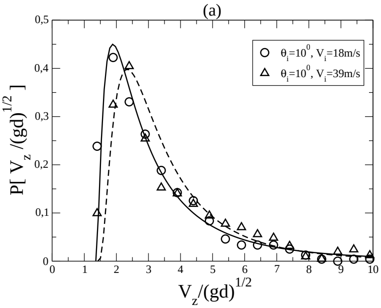
Estimation of the reptation hop length

- Model Collision experiment (Beladjine et al, PRE 2007)



Estimation of the reptation hop length

- Velocity distribution of ejected grains



- Distribution independent of the impact speed and angle
- Only the number of ejected grains varies with the impact speed and angle

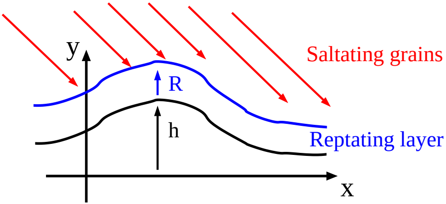
⇒ The reptation length is expected to be indepedent of the wind speed : $\bar{l}_r \approx 20d$

⇒ $\lambda_{max}^{Anderson} \approx 4\bar{l}_r \approx 80d$ independent of the wind speed. This is not in agreement with the experimental results.

Erosion-Deposition Model

- Model equations (Bouchaud et al, EPJB 1998, Valance et al. EPJB 99)

$$\partial_t R = -V_0 \partial_x(R) + \Gamma[R, h]$$
$$\partial_t h = -\Gamma[R, h]$$



V_0 : mean speed of the mobile grains

Γ : exchange rate between moving and immobile grains :

Erosion-Deposition Model

- Exchange Rate

$$\Gamma = \Gamma_{ejection} + \Gamma_{deposition}$$

with

$$\Gamma_{ejection} = \alpha_0(1 + \alpha_1 h_x + \dots)$$
$$\Gamma_{deposition} = -R\gamma_0(1 + \gamma_1 h_x + \gamma_2 h_{xx})$$

- Parameters

$\alpha_0 \propto n_{eje}\,\phi_{saltation}$: *Ejection rate*

$\gamma_0 = V_0/l_r$: *Deposition rate (l_r is the reptation length)*

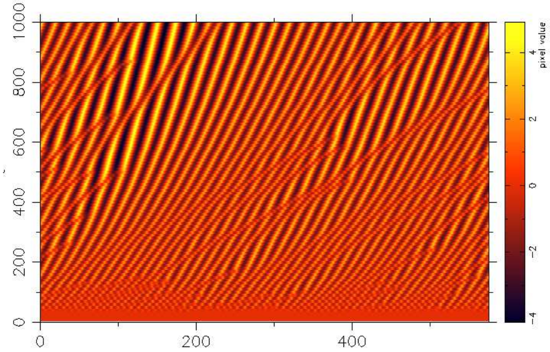
$\gamma_2 = l_c$: *Analog of a capillary length*

Erosion-Deposition Model

- Weakly nonlinear analysis (Valance et al, Physica D 1998)

$$\partial_t h = \partial_{xx} h + \nu \partial_{xxx} h + \partial_{xxxx} h + \mu \partial_{xx}(h_x^2) + \partial_x(h_x^3)$$

- Spatio-temporal Diagram



Erosion-Deposition Model

- Weakly nonlinear analysis (Valance et al, Physica D 1998)

$$h_t = -h_{xx} + \nu h_{xxx} - h_{xxxx} + \mu (h_x^2)_{xx} + (h_x^3)_x$$

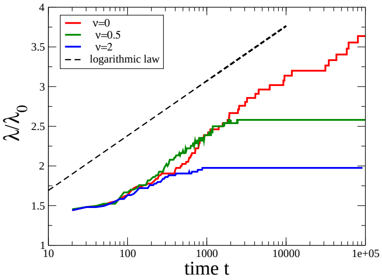
- $\nu = \mu = 0$

  Conserved Cahn-Hilliard equation
  $\Rightarrow$ logarithmic coarsening
  $\lambda \sim A \sim \ln t$

- $\mu = 0$ and $\nu \neq 0$

  $\lambda \sim \ln t$ when $\lambda < \lambda_c$

  $\lambda \simeq cst$ when $\lambda > \lambda_c$
  where $\lambda_c \approx 2\lambda_{max}/\nu$



Open issues

- What are the mechanisms that select the initial and final wavelength ?
- What are the mechanisms that interrupt the coarsening process ?
- Does the particle size dispersity play a role ? Is a perfectly unimodal bed unstable with respect to ripple formation ?
- Dynamics of Mega-ripples ?

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