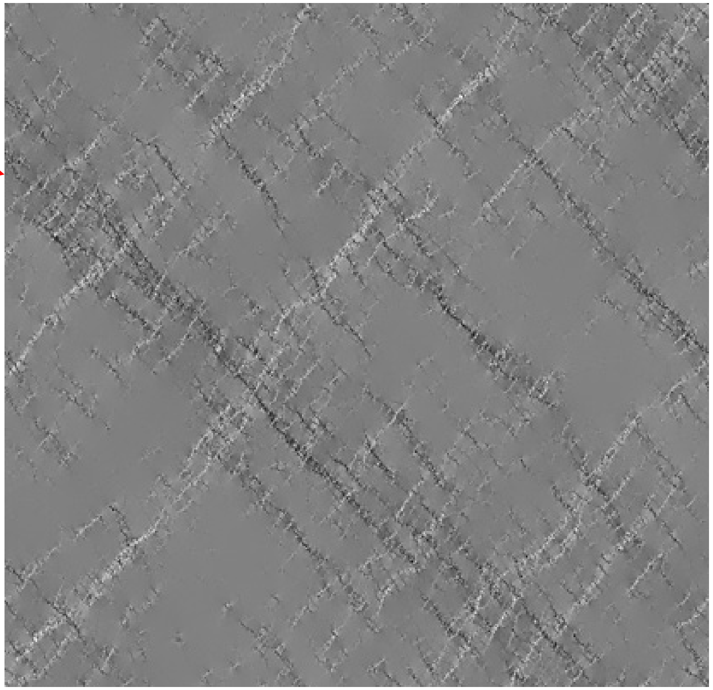
Power Law Correlations and Avalanche Distributions in Quasistatic Shear

M. O. Robbins & K. M. Salerno, Johns Hopkins University
C. Maloney, Carnegie Mellon Univ.; MASM 5, 11/20/2009

How do microscopic displacements accommodate total global strain?



Is there a typical size of plastic event?

How are "earthquakes" distributed in energy, location and time?

How does inertia affect deformation?
Many systems critical without inertia.
Here critical with inertia.

Motivation

Find power law distribution of events, avalanches, earthquakes in a wide variety of systems and on wide range of scales as long as they are driven slowly

- Charge-density wave motion
- Fluid invasion of porous media, contact line motion
- Magnetic domain switching (Barkhausen noise)
- Deformation of solids
  (acoustic emission, dislocation bursts, cracks, earthquakes)
- Granular media, foams, ….

For large objects or events temperature may be irrelevant

Dynamics are also generally underdamped at large scales

⇒Focus on quasistatic, T=0, momentum conserving dynamics

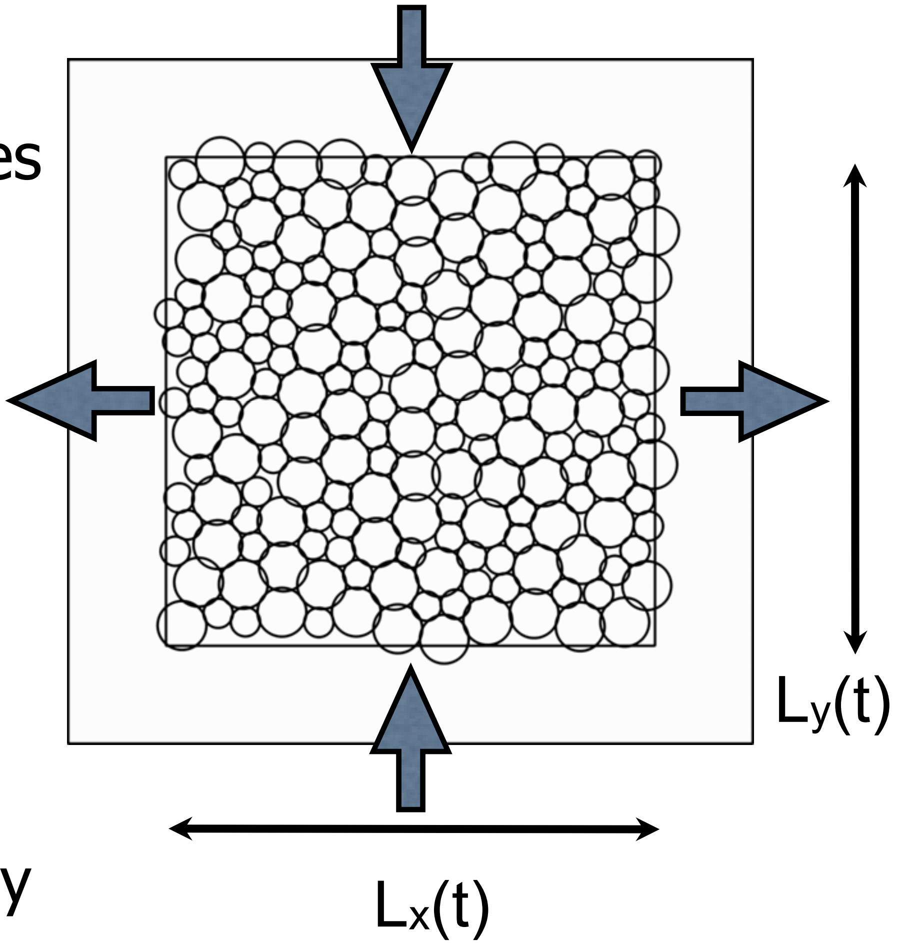
Quasistatic Athermal Simulations

2D Molecular Dynamics (usually)

- Binary Lennard-Jones (or harmonic)
  Mean diameter σ, binding energy ε
- Quenched at pressure p=0
  protocol not important
- T=0, Underdamp relative velocities
  Gallilean invariant (Kelvin/DPD)
- Periodic boundaries (usually)
- Axial, fixed area strain or
  simple shear, const. rate
- Quasi-static limit $\Rightarrow$ low rate
  controlled by $\Delta\gamma$ not Δt

Different than saying motion of all atoms is always slow,
i.e. overdamp or minimize energy

Prescribed $L_y(t)$, $L_x(t)$ to conserve area



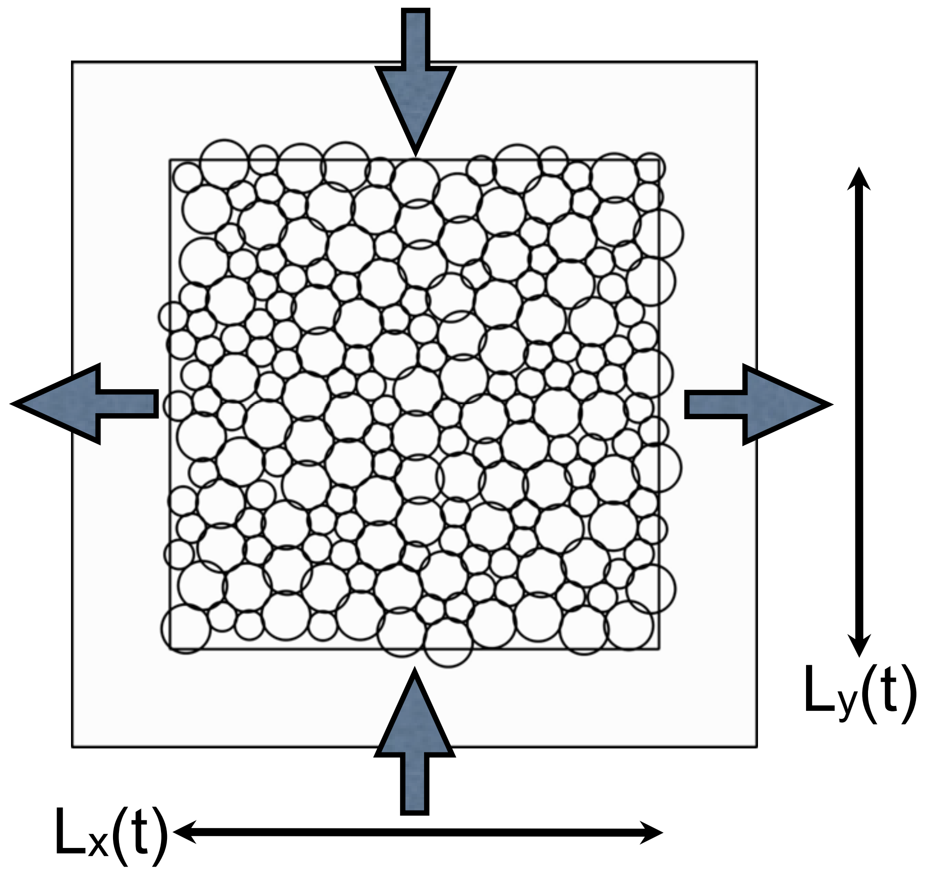
Nonaffine Particle Displacements

Non-affine displacement u = deviation from mean motion

Integrate affine displacement along trajectory rather than using value for initial position.

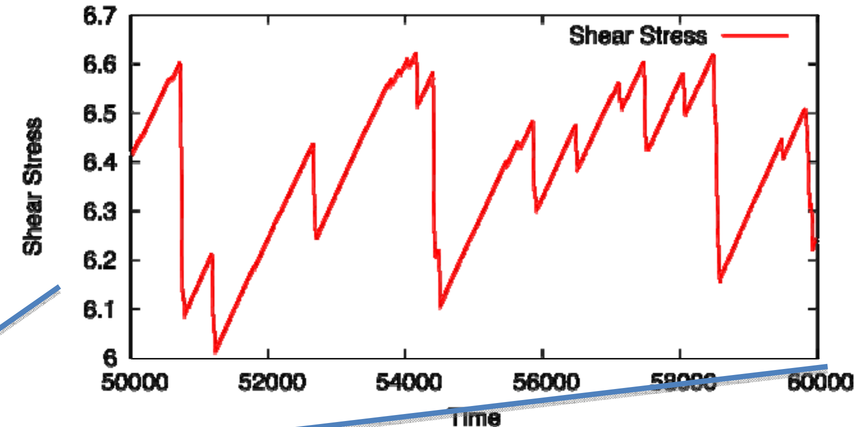
For each particle at each time step find distance moved relative to affine displacement at instantaneous position.

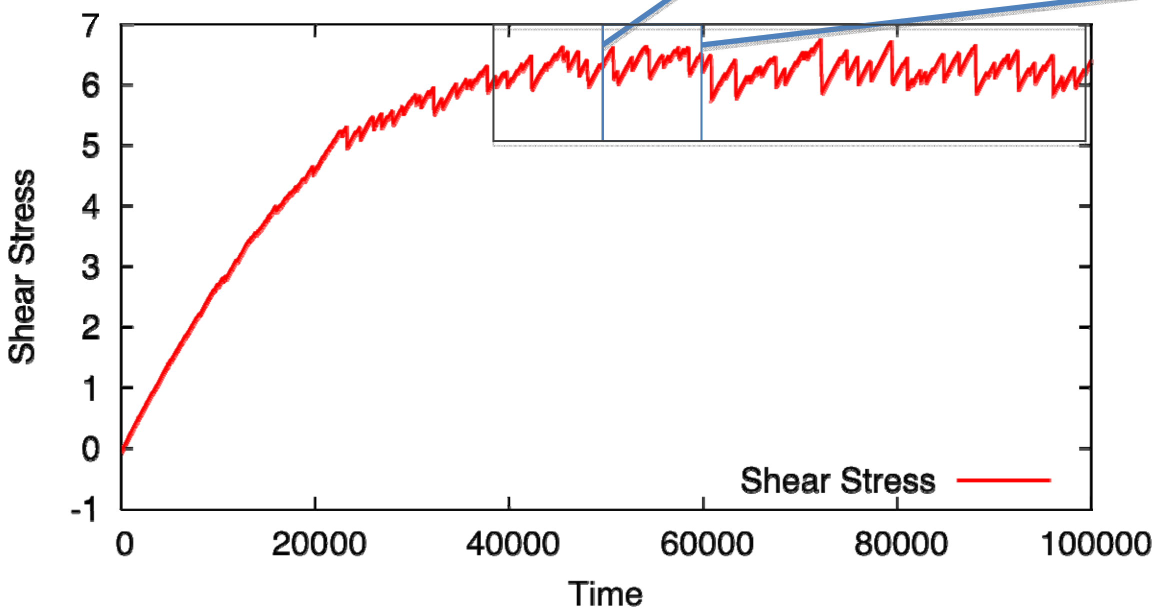
Eliminates terms analogous to Taylor diffusion in simple shear

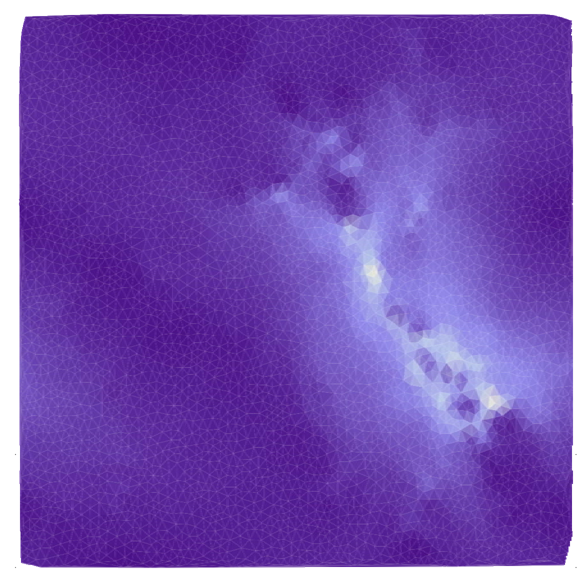


Quasi-static, Steady-State Shear

- Shear rate low enough that independent (not causally related) events are separated in time. Typically $10^{-6}\, t_{LJ}^{-1}$ or less
- Number independent of rate
- Statistical properties independent of initial configuration
- Atomic velocities fast, only shear rate low

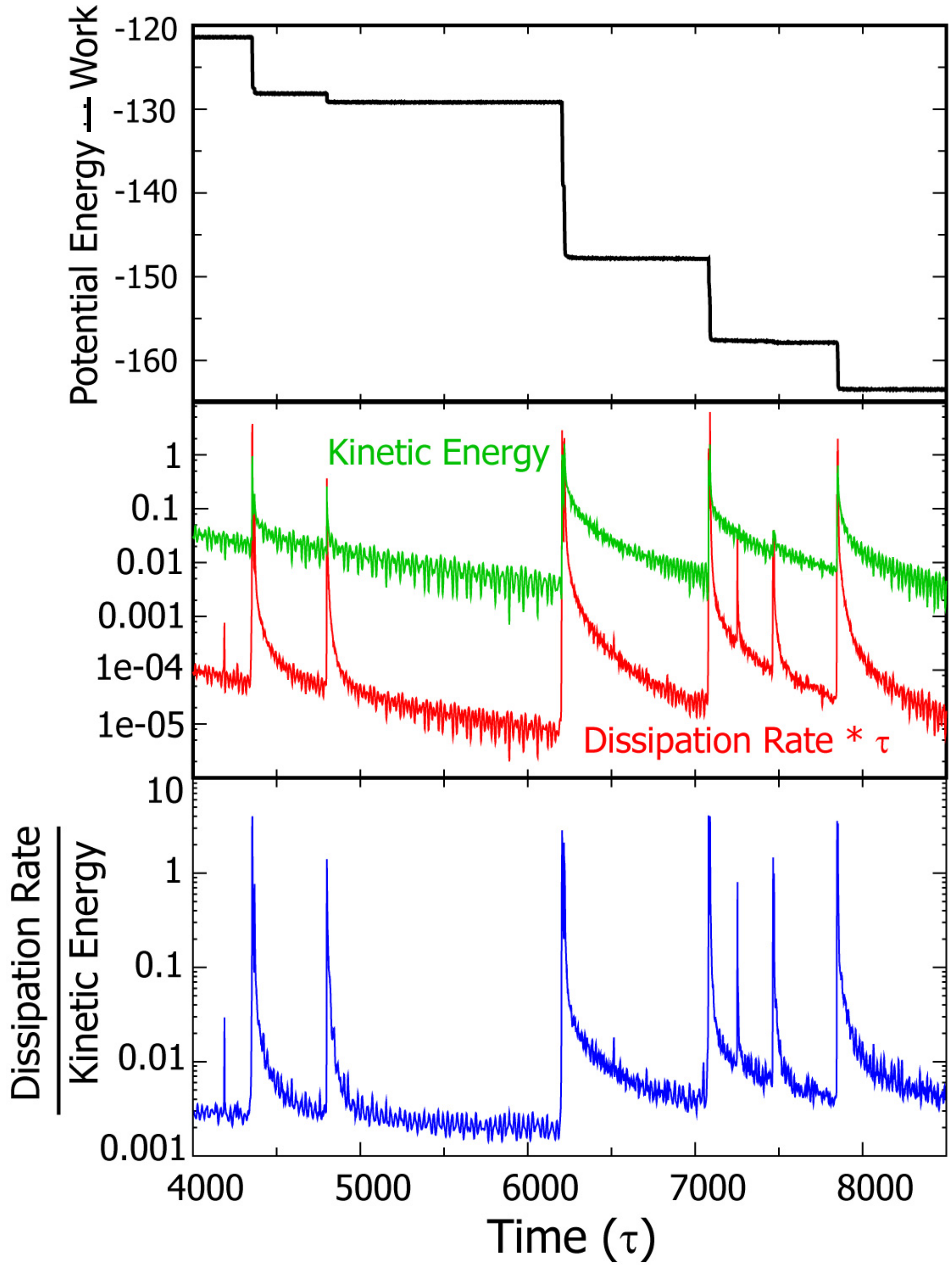






Defining Avalanches

- Deformation through series of avalanches
- Find energy dissipated = change in potential energy - work done by system.
- Drops have wide distribution, 0.03 to 20 here
- Identify by sharp rises in dissipation rate.
- Dissipation rate/Kinetic energy drops to constant as energy moves to longest wavelengths.
- Ratio measures wavelength where have kinetic energy



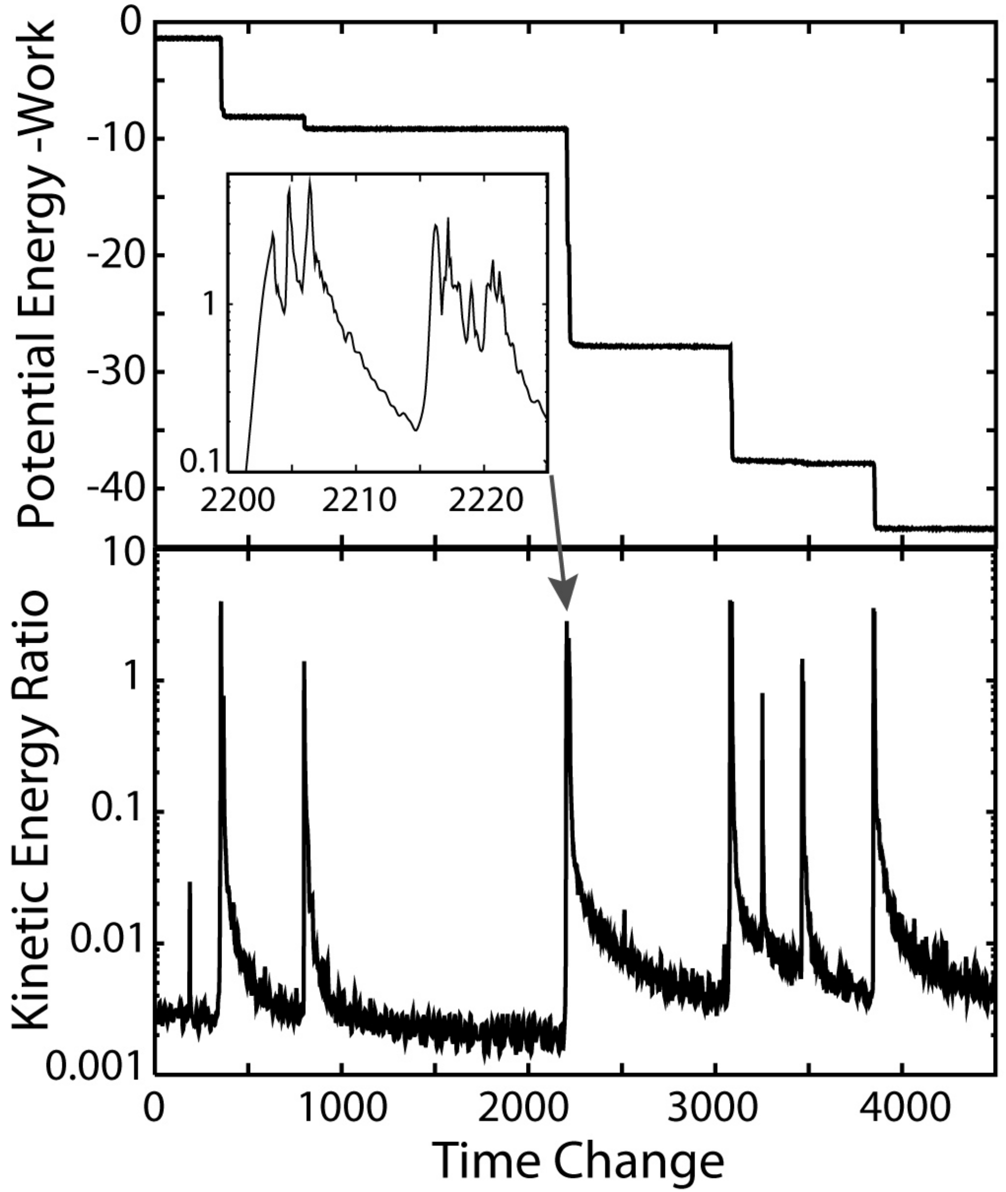
Avalanche Distribution

Amplifying scale shows earthquakes can have complex structure with many spurts of activity.

Find kinetic energy ratio gives a constant number of events per unit strain in the limit of low rates

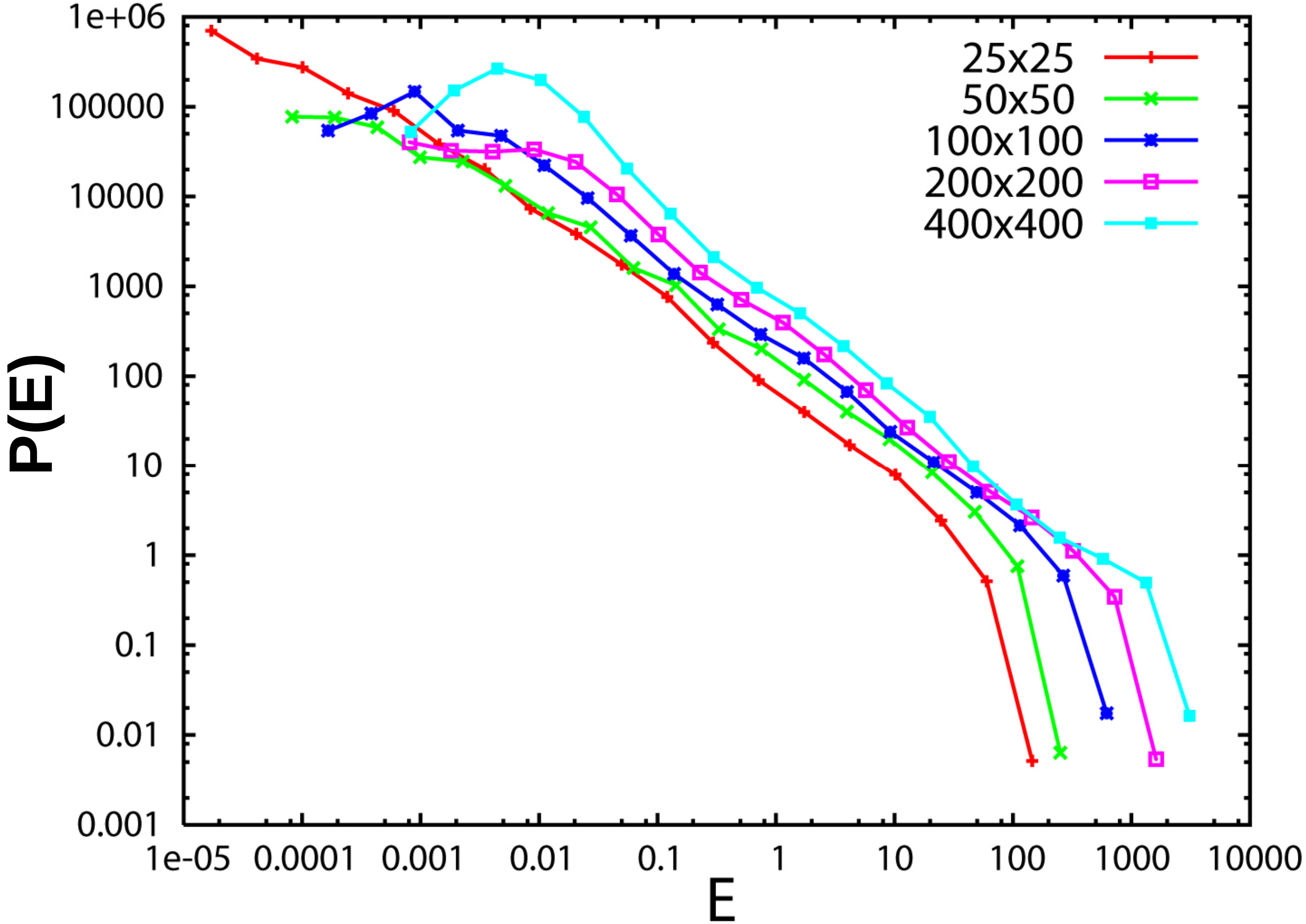
Have also tested that stopping strain during event gives same statistics

$\Rightarrow$ quasistatic



P(E) = Number of Events per Unit Strain per E

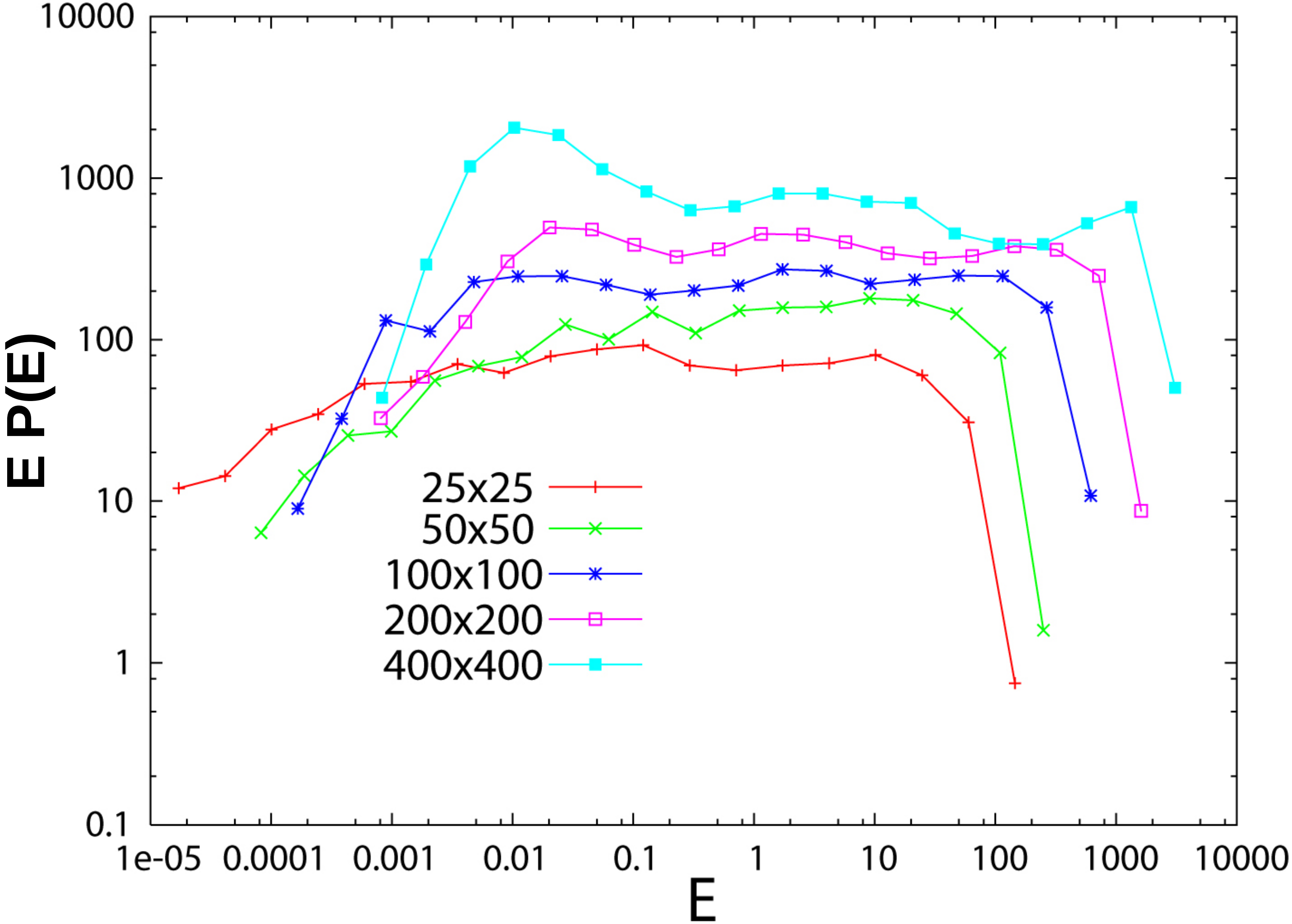
Reduce strain rate so quasi-static, only affects small events.
Find power law P(E)~1/E over 5 decades.
P(E) and maximum E increase with system size



E P(E) ~ Constant

Find P(E) ~ E^{-1}

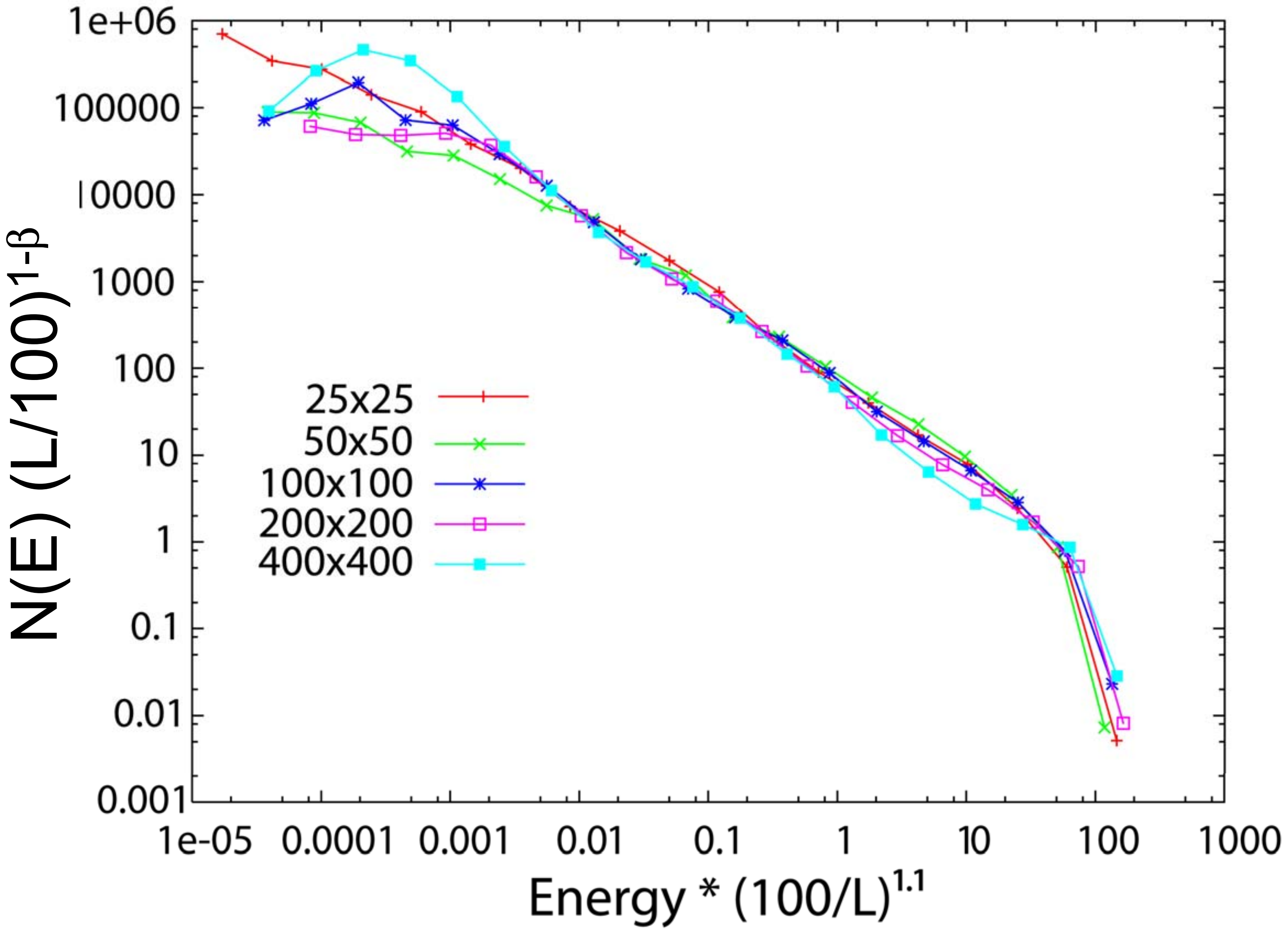
Maximum event size grows more rapidly than length L



Scaling Collapse of N(E) and E

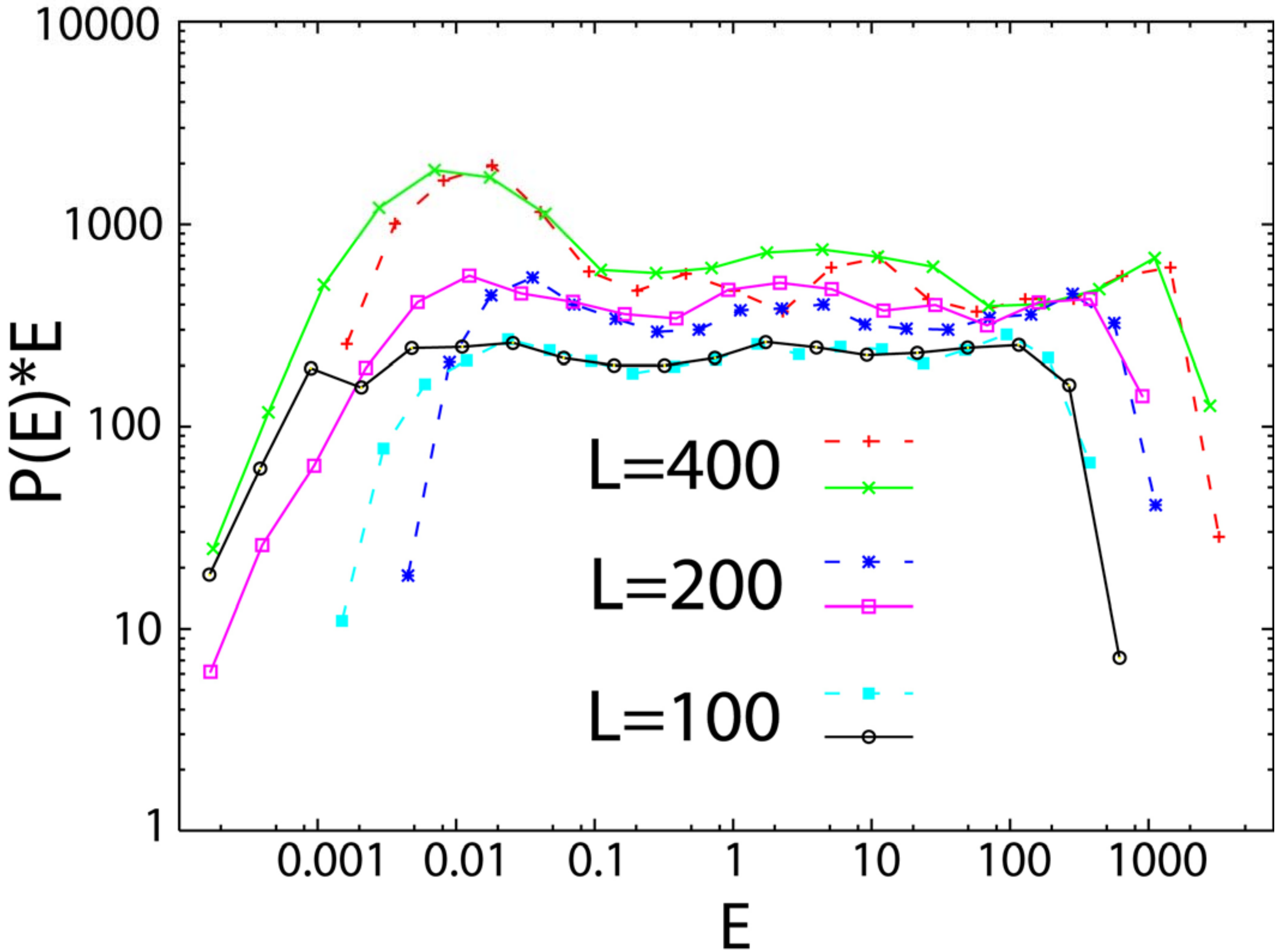
Scale E by $E_{max} \sim L^{\beta}$.   Find $\beta \sim 1.1$

$N(E/E_{max})$ must scale as $L^{1-\beta}$ to maintain energy balance

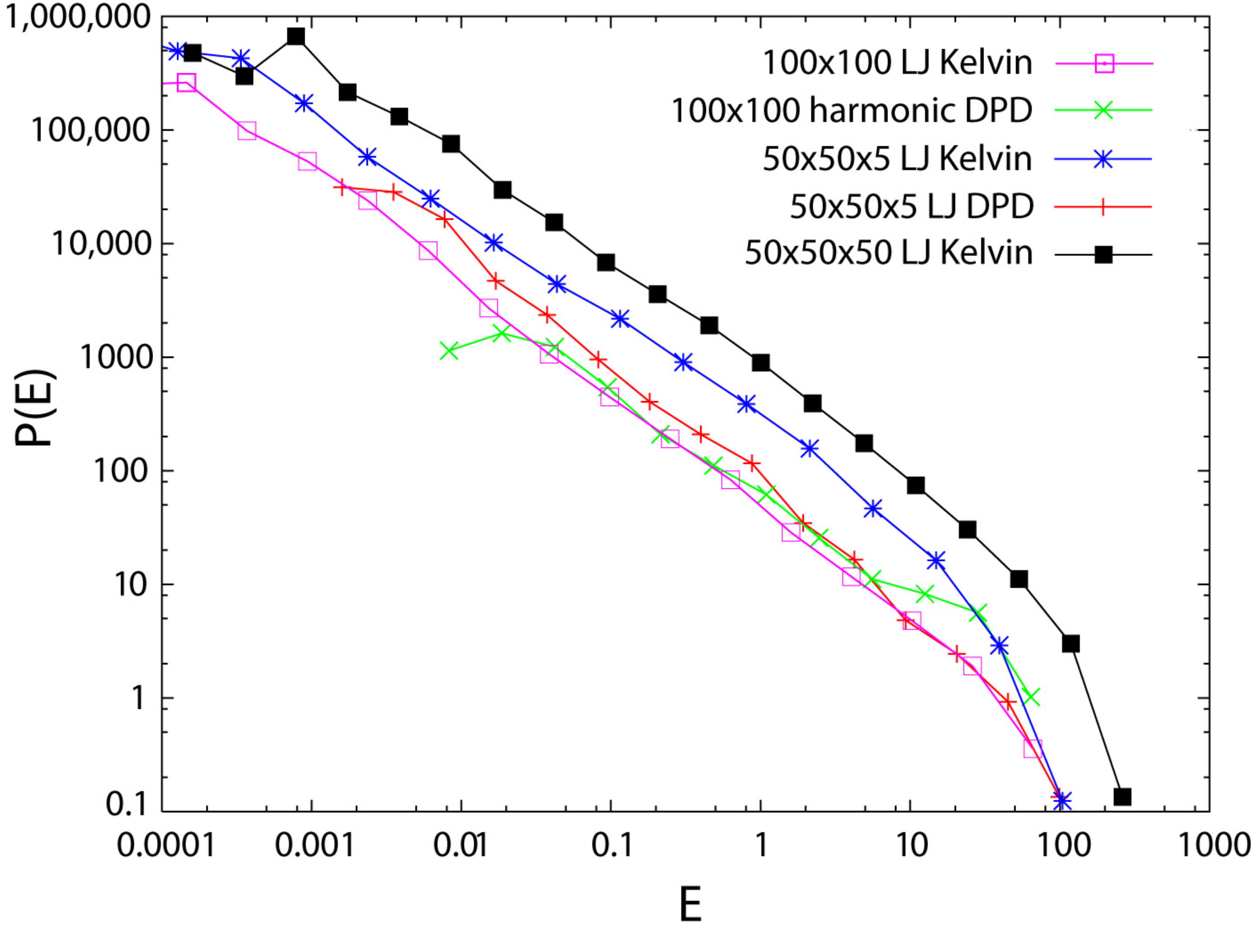


Slow Enough?

Stop straining when detect event
Same distribution (dashed curves) as continuous strain (solid)



Power Law Independent of Potential, Geometry and Thermostat

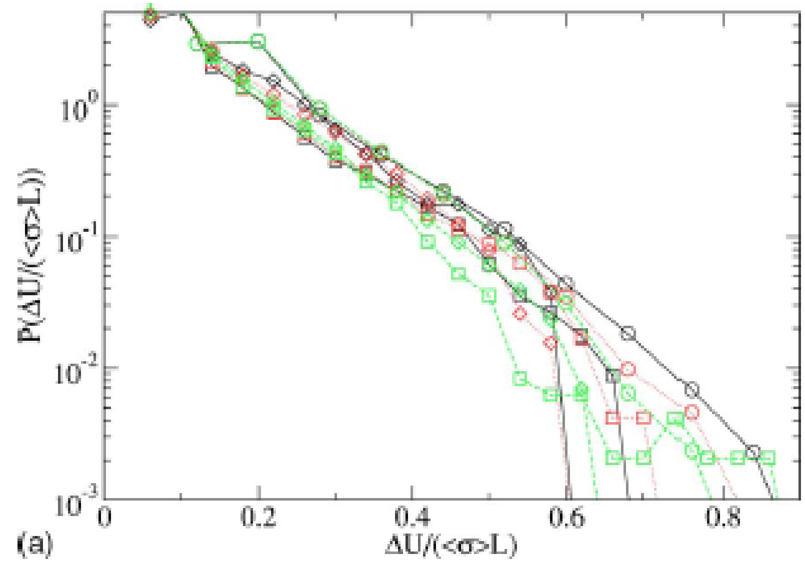


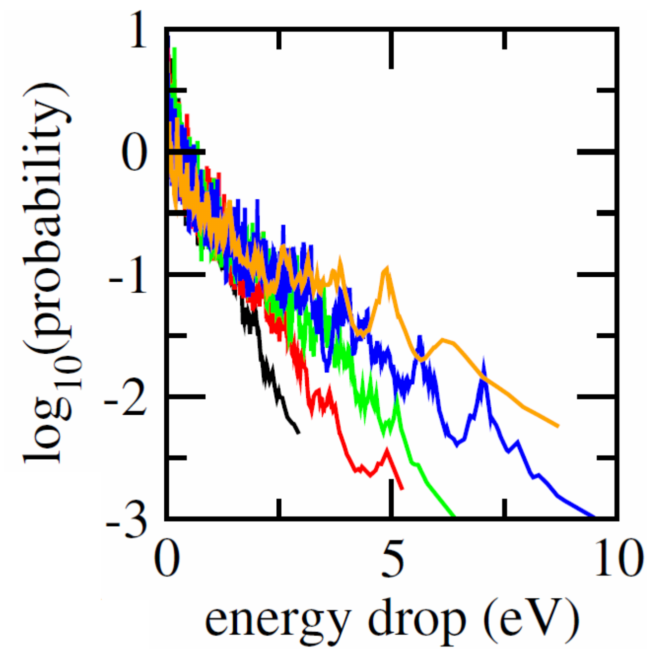
Exponential Scaling of P(E) in Other Models

Same system but with energy minimization, not dynamics

Largest event ~3ε vs. 200ε

(Maloney & Lemaître PRE 74, 016118, '06)





3D amorphous metal

$N(E) \sim \exp[-E/L^x]$ x=1.4

(Bailey, Schiøtz, Lemaître & Jacobsen, PRL 98, 095501, '07).

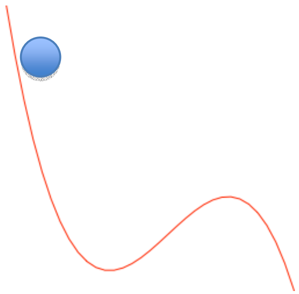
Do see power laws in overdamped systems with quenched disorder

AND

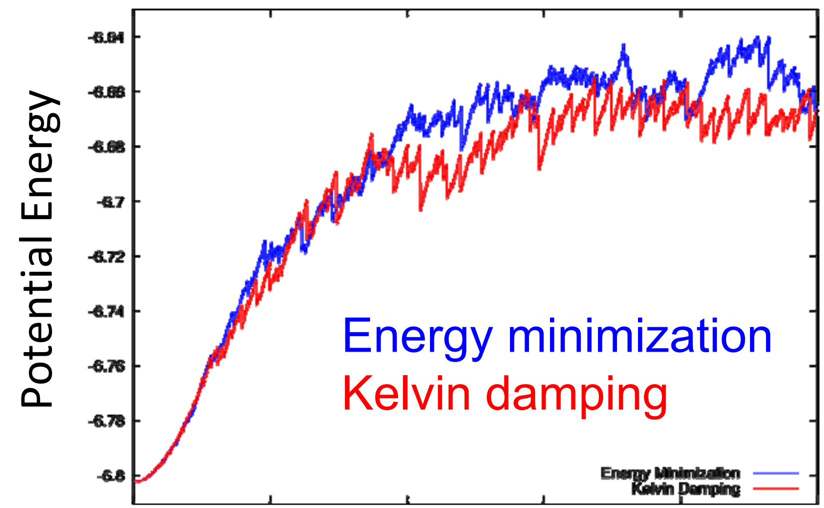
Inertial models without disorder

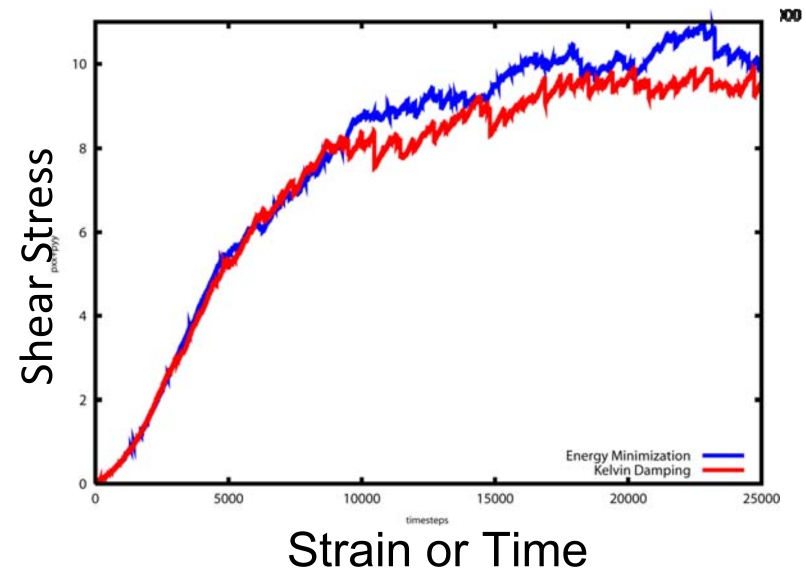
(Carlson, Langer, Shaw – Burridgge-Knopof)

Inertia Matters



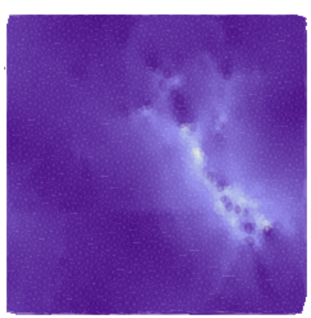
Energy minimization finds
nearest minimum
Energy lower with inertia



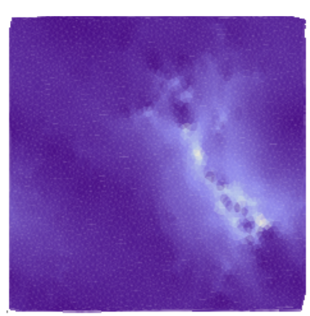


Inertial overshoots -> weaker regions

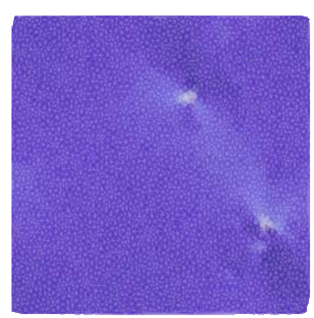
Overdamped



Inertial



Difference

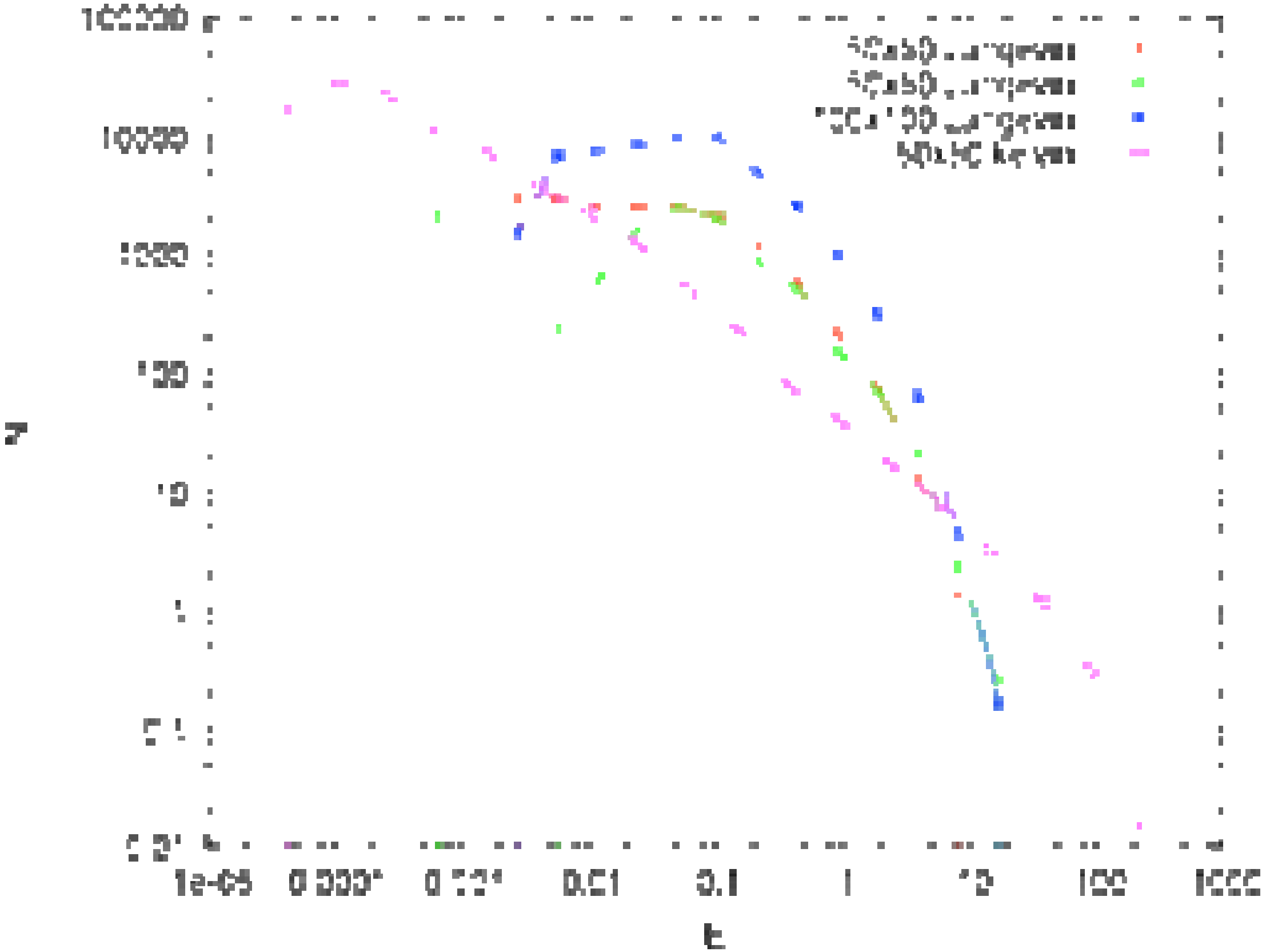


white is larger displacement of particles

Langevin Damping Changes Distribution

Overdamped systems ⇒ fewer events at small and large E

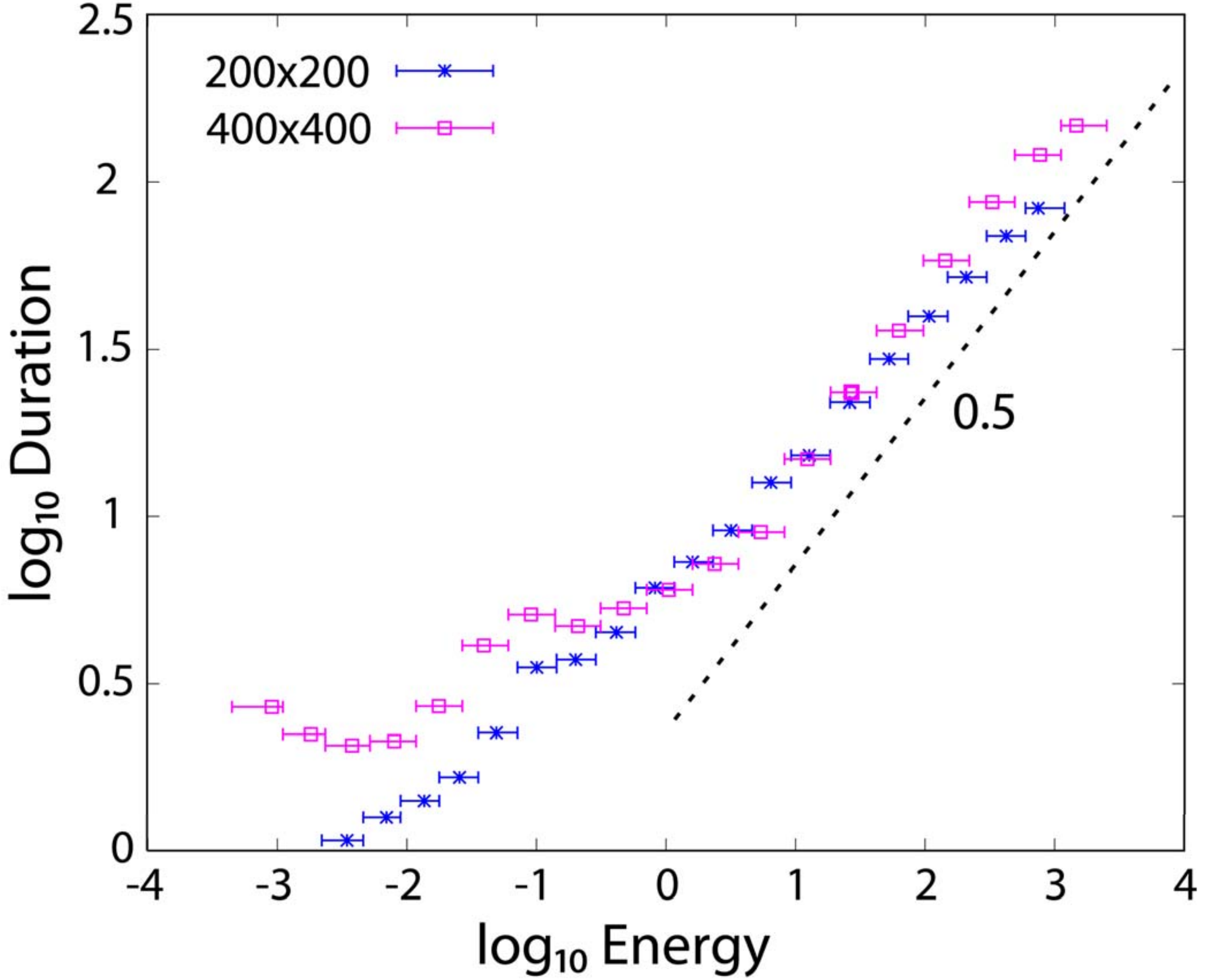
Tail at large E ~ exponential



Relation Between Duration and Energy

Duration ~ square root of energy

Consistent with area~energy and duration ~ length = area$^{0.5}$

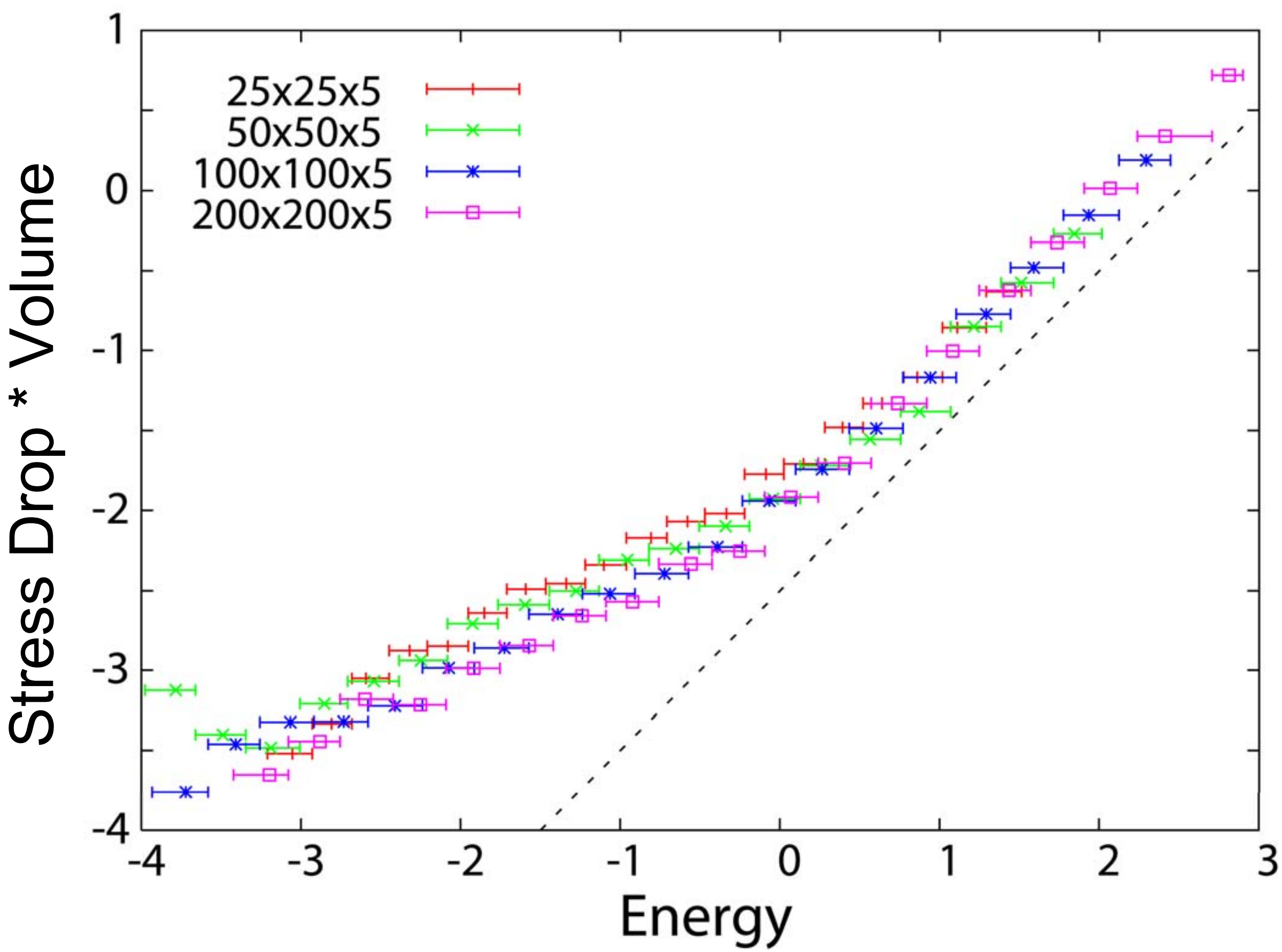


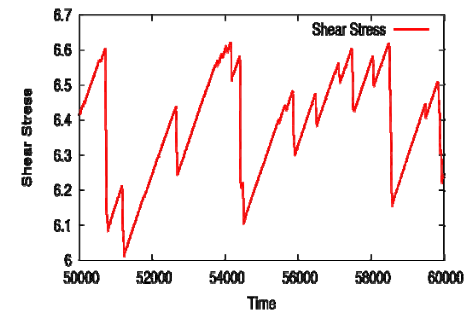
Relation Between Stress Drop and Energy

Mean stress over whole volume * volume ~ moment

Find moment ~ energy for large events (dashed line)

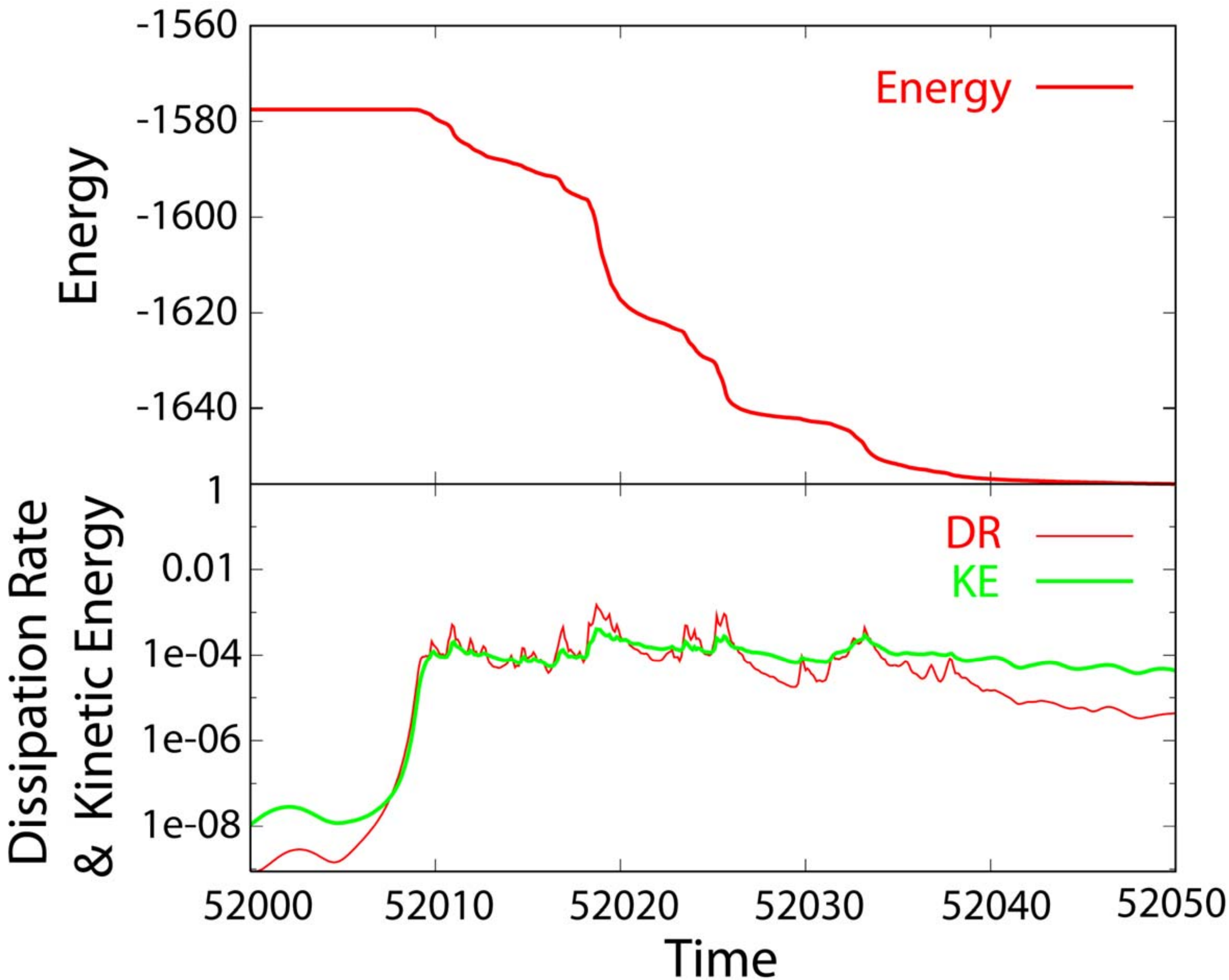
Large dispersion for small events





Different distribution if use stress minima and maxima to define event

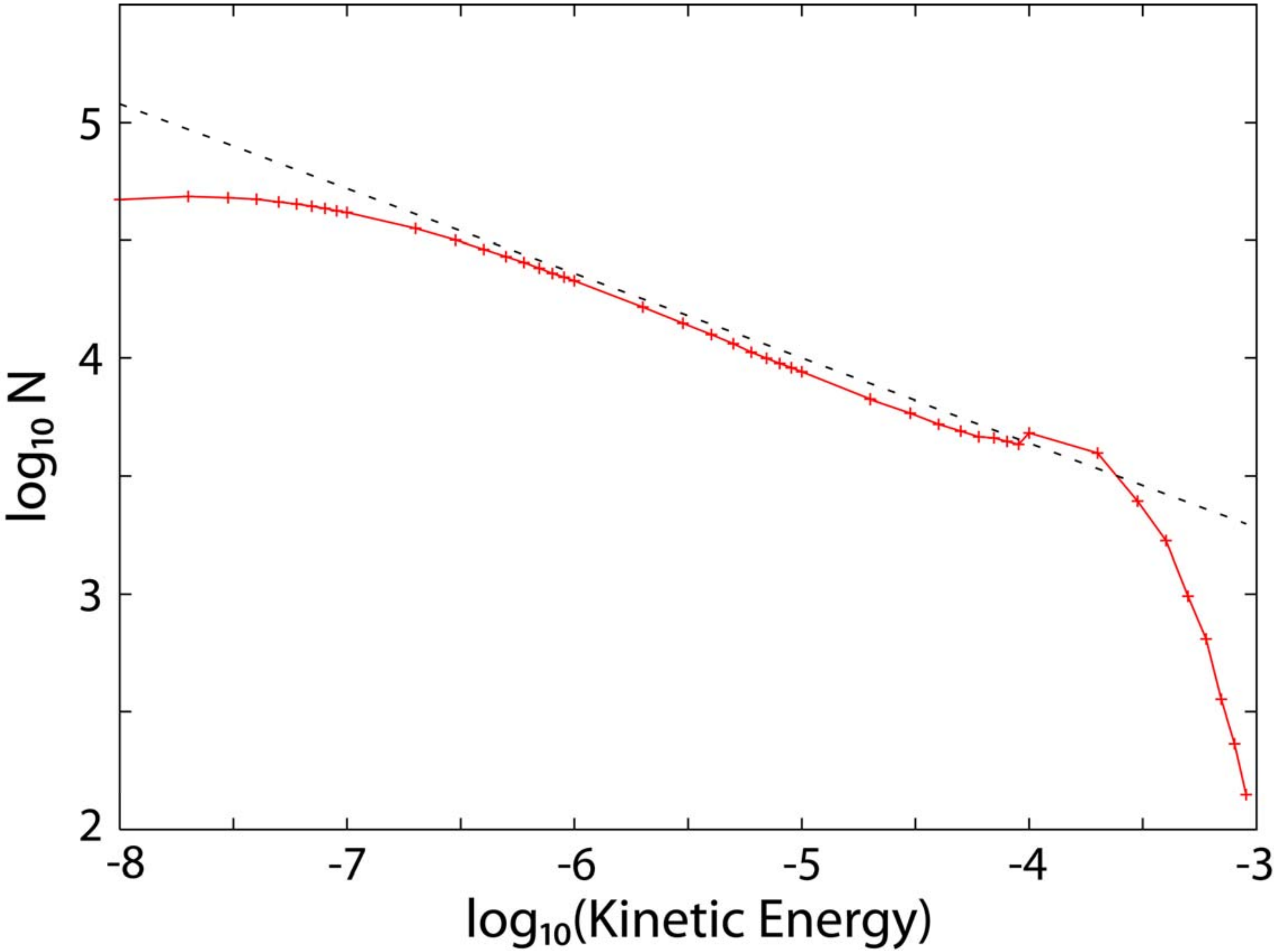
Our Quakes Include Fore and After Shocks How Does this Affect Statistics?



Integrated Density of Quakes Based on Kinetic Energy

Large events are broken up. Find integrated density ~ $E^{-0.4}$

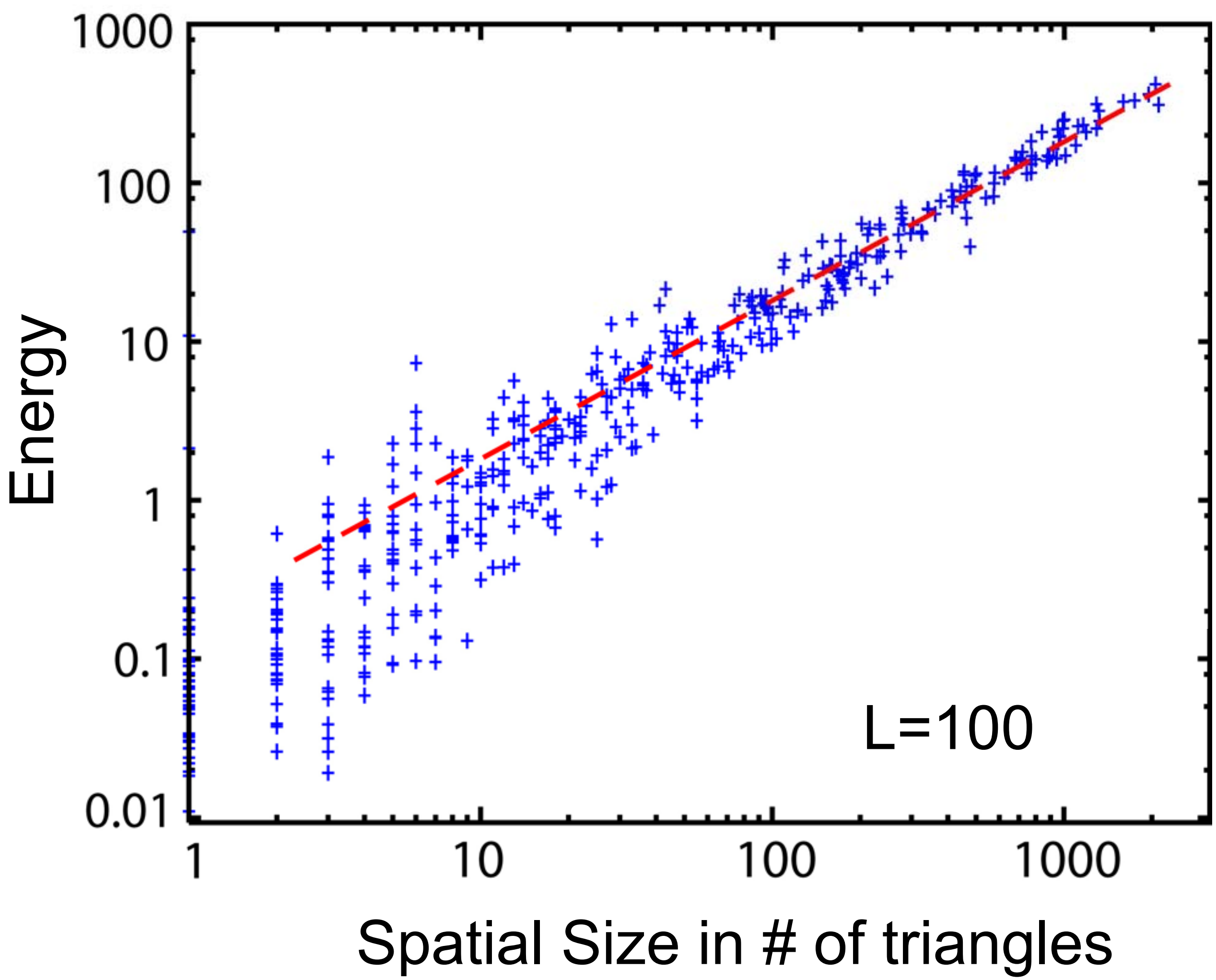
Gutenberg-Richter ~ $E^{-2/3}$



Relation Between Energy and Spatial Size

Size = # triangles where curl of displacement field has magnitude > 0.1 (limit of elastic deformations)

Find linear (dashed line) relation for large events



MR4

Defining Local Deformation

Shear related to local rotation $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, $\mathbf{u}$=displacement

Find Delaunay triangulation for initial particle centers

For each triangle:

$$\frac{\partial u_i}{\partial x_j} = F_{ij}$$

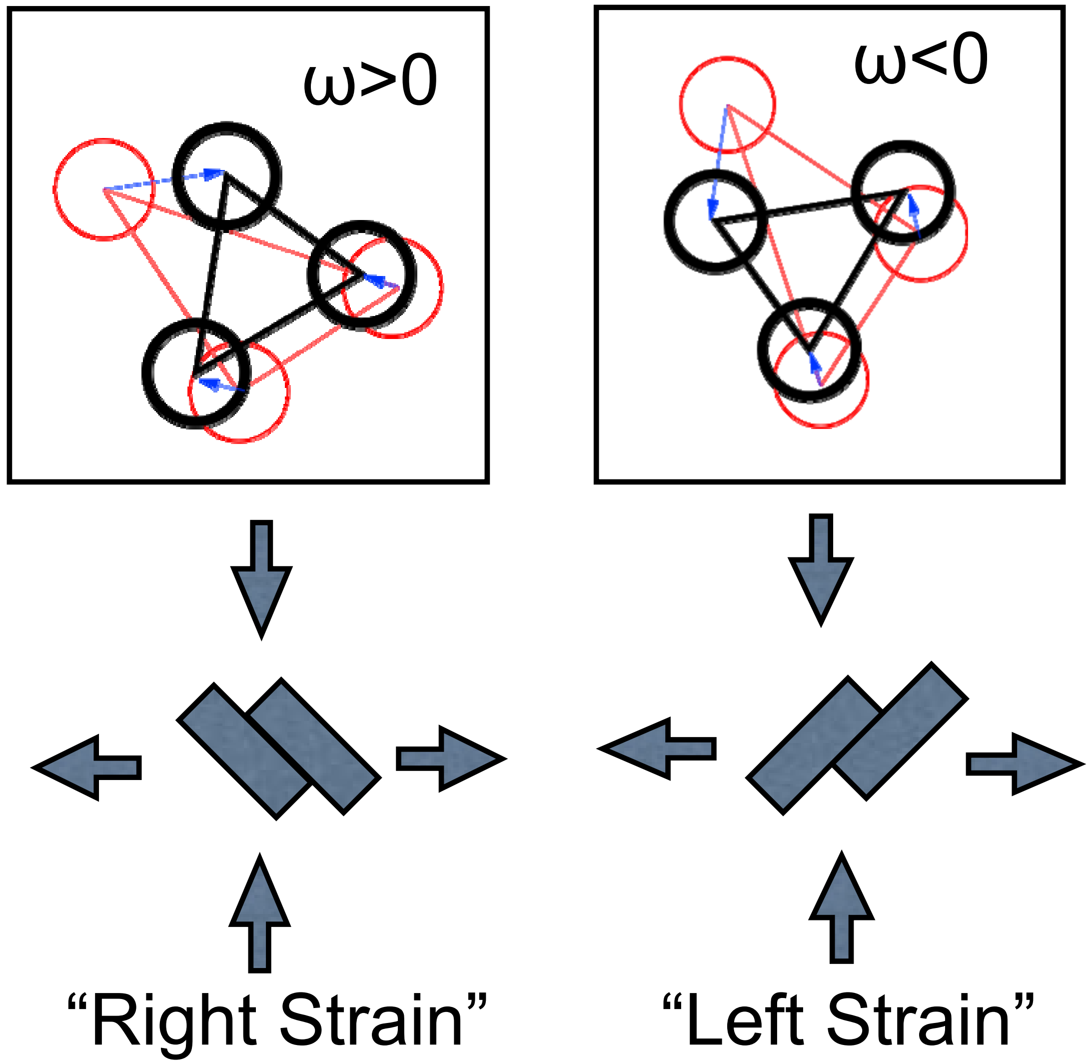
$$\epsilon_1 = \frac{F_{xx} - F_{yy}}{2}$$

$$\epsilon_2 = \frac{F_{xy} + F_{yx}}{2}$$

Invariants:

$$\epsilon = \sqrt{\epsilon_1^2 + \epsilon_2^2}$$

$$\omega = F_{xy} - F_{yx}$$



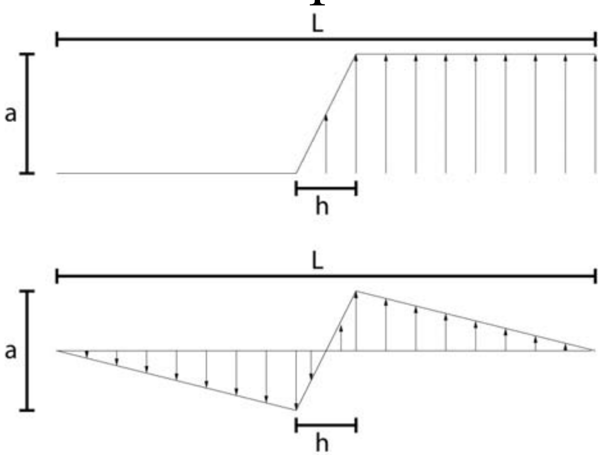
MR4 Mention coarse-graining property
Note we 2x strain
Mark Robbins, 10/30/2008

Spatial Variation of Nonaffine Displacement u

Components of u during 0.2% strain

+σ white, -σ black

Strain localizes in plastic bands → step in total displacement

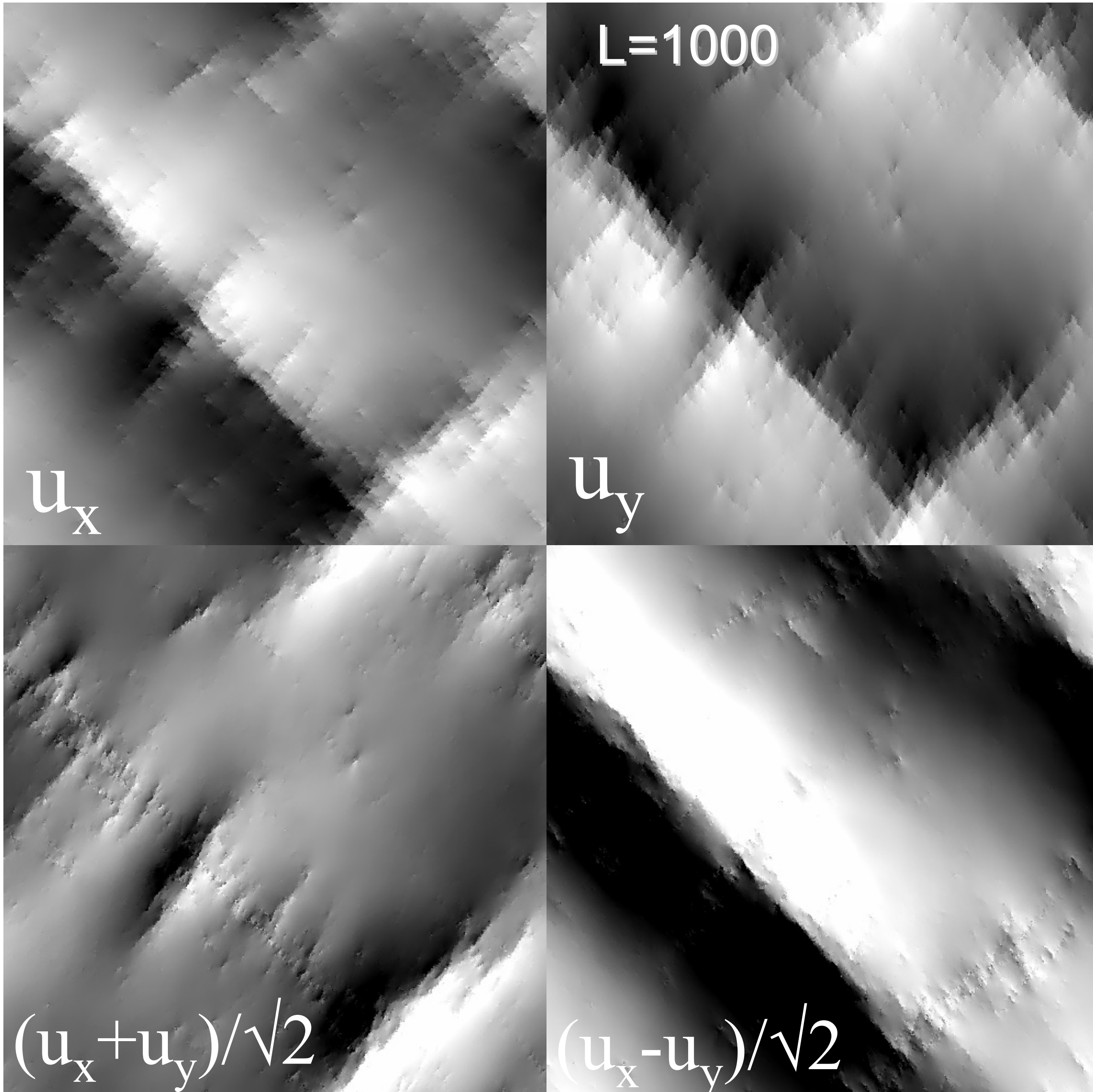


Largest projection of u along bands ±45°

Sign → rotation sense

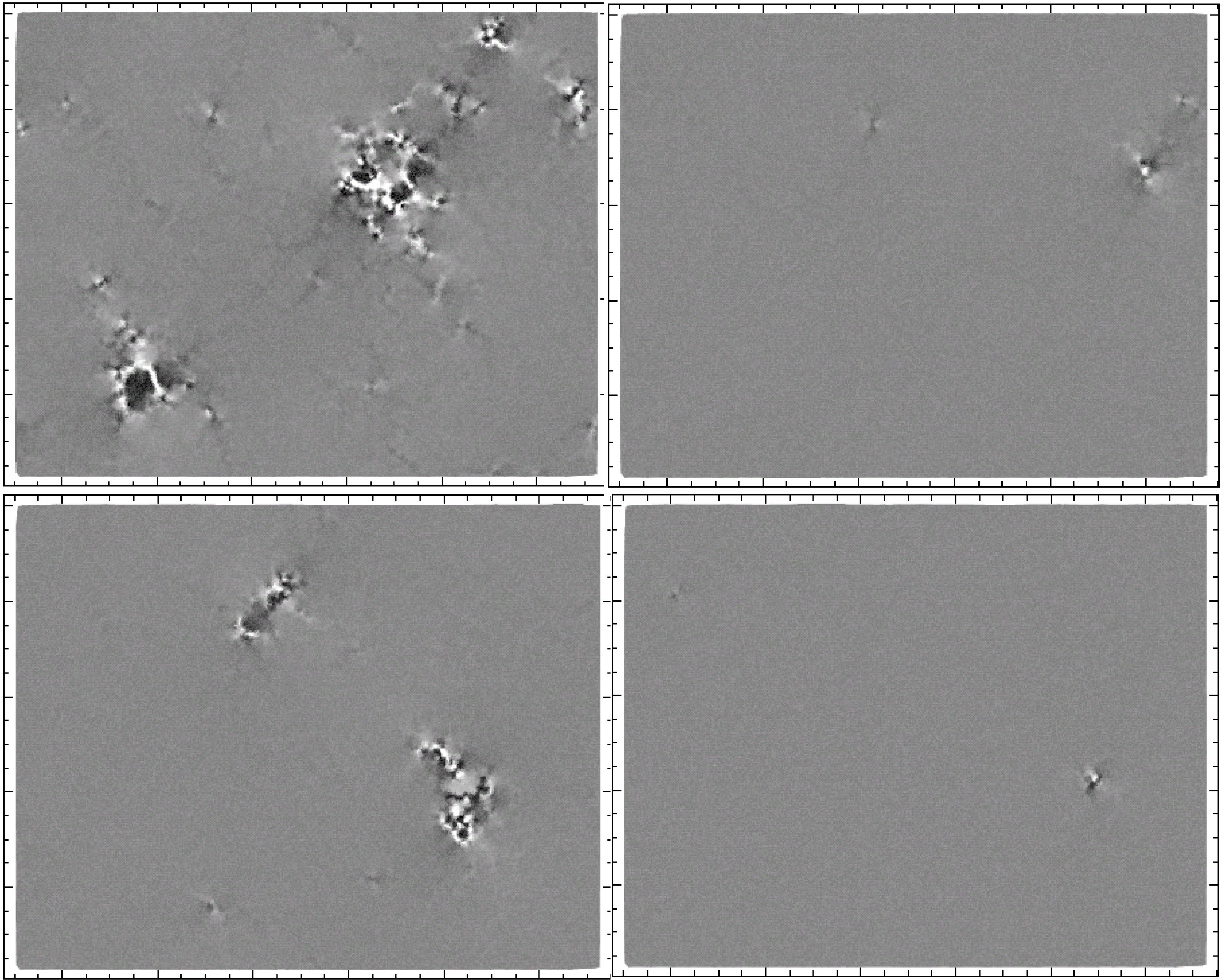
Length up to system

Typical a ~ σ



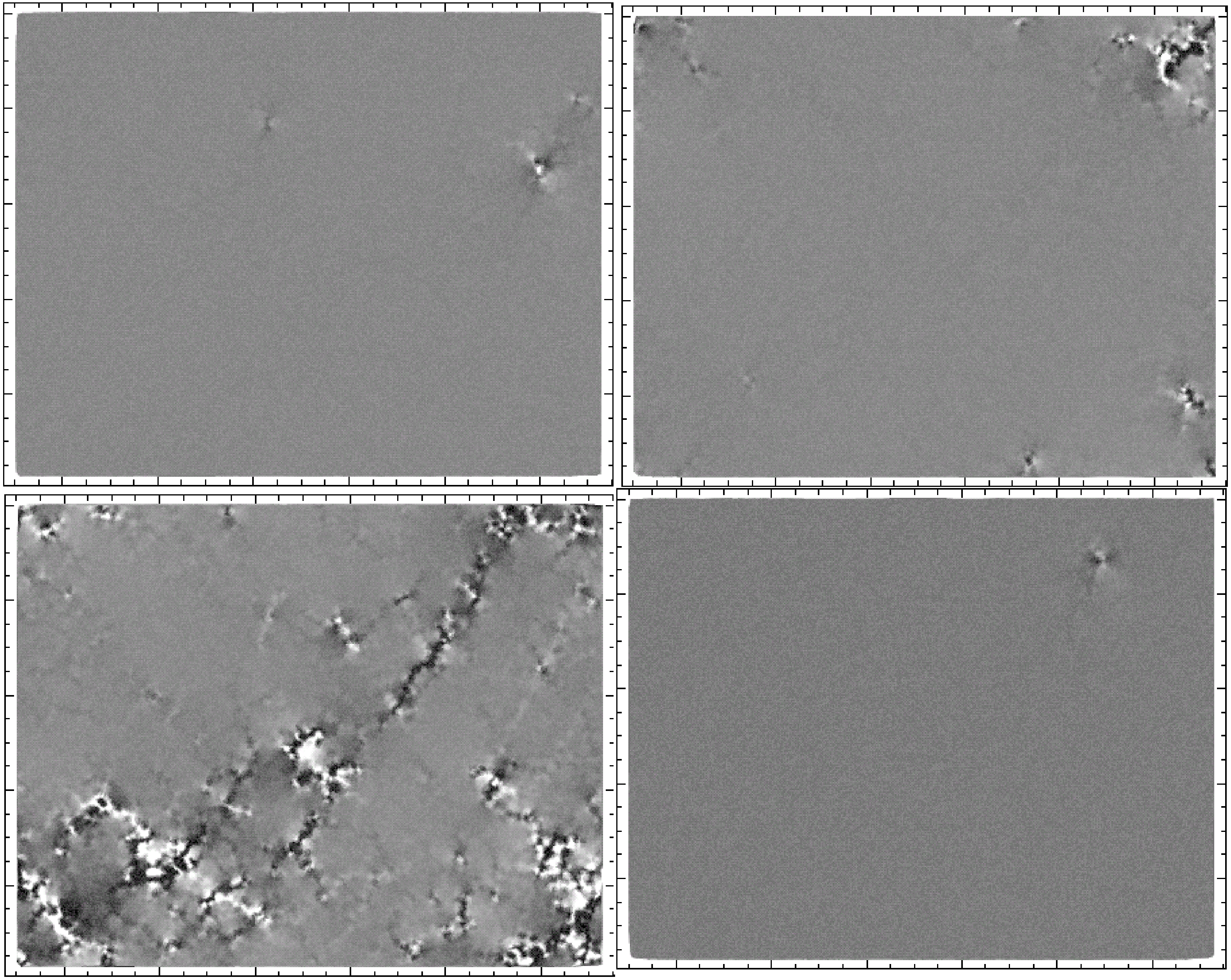
Earthquakes

Examples of events (111.065,24.5098,2.982,1.4181)



Earthquakes

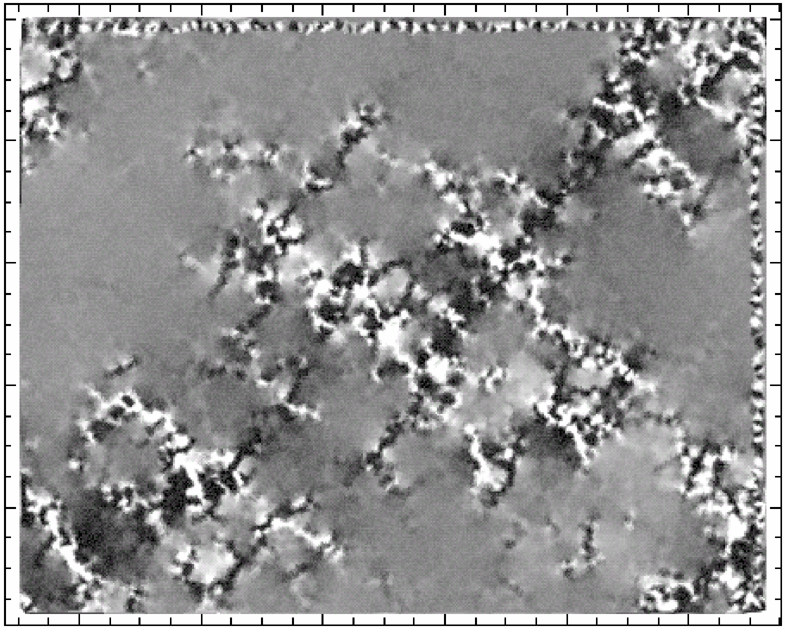
Examples of events (3.2643,301.448,21.3862,0.0794)



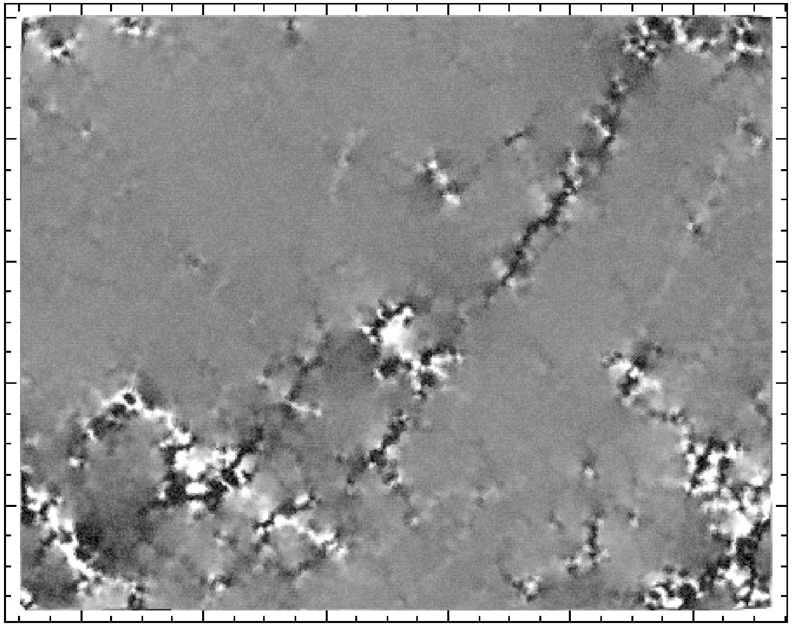
Earthquakes

Total integrated activity shows long range correlations

Activity over 1% strain



Single largest event



Displacement and ω in steady state

Most analysis looks at magnitude $\Delta r = |\vec{u}|$ → smallest in plastic band

Curl sharply localized in plastic zone, +/- regions correlate along -/+45°

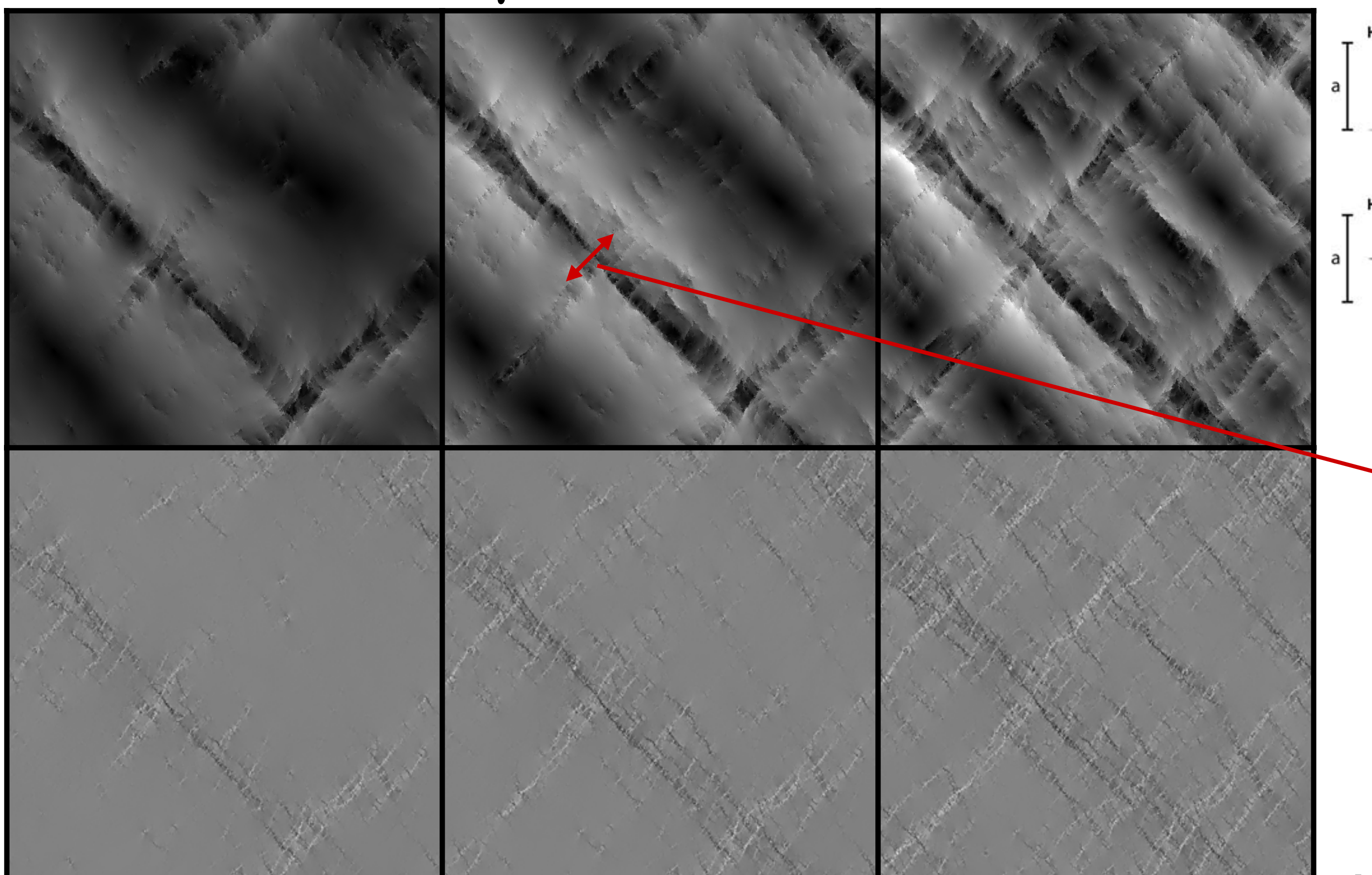
Strain accumulates to a~σ in plastic bands through many avalanches

→Displacement ~σ allows all regions to find new metastable state

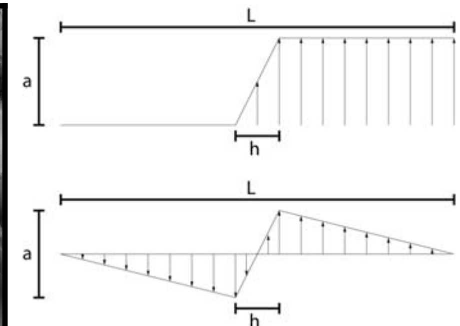
Strain over intervals Δγ > ~σ/L occurs in uncorrelated locations

|Δr|

ω



h~L/20



Black =clockwise

White = counter-clockwise

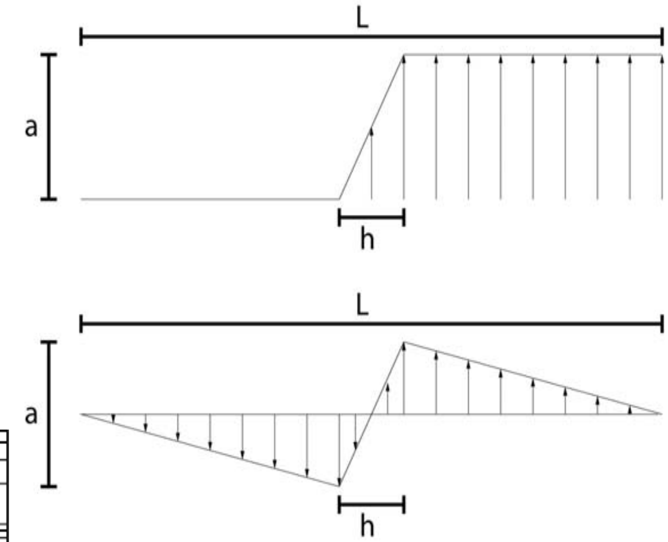
Strain γ: 6.0% to 6.1% 6.0% to 6.2% 6.0% to 6.4%

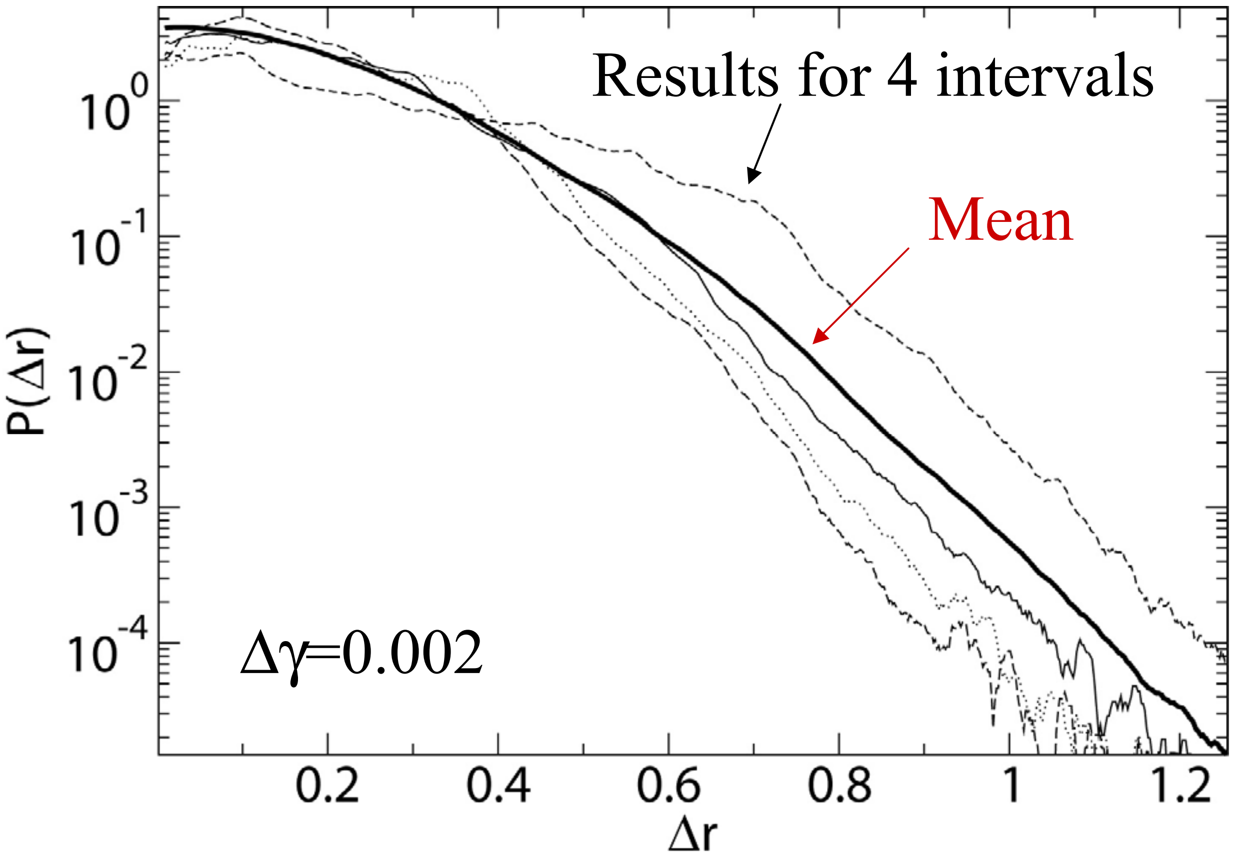
L=1000

Distribution of Displacement Magnitude Δr = rms u

If displacement across band = a,
flat distribution P(Δr) up to a/2

Roughly consistent with observed P
Large fluctuations → long-range correlations



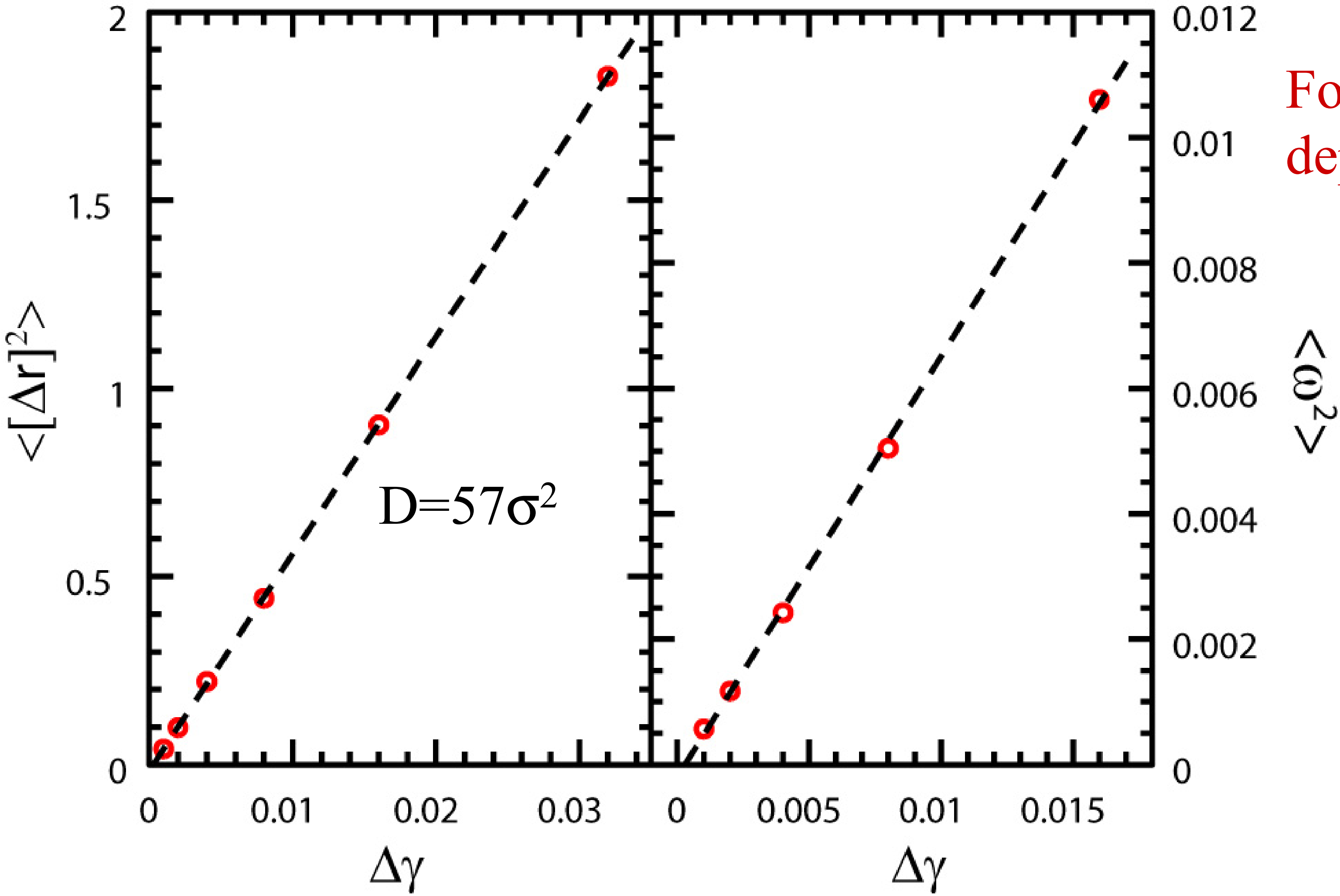


Displacements Diffusive with D∝L

Plastic band formed in each strain~a/L gives $|\Delta r|<a/2$, add incoherently

$$\Delta r^2 \sim \Delta\gamma/(a/L)\ a^2/12 \sim D\ \Delta\gamma \ \rightarrow\ D \sim L\ a/12$$

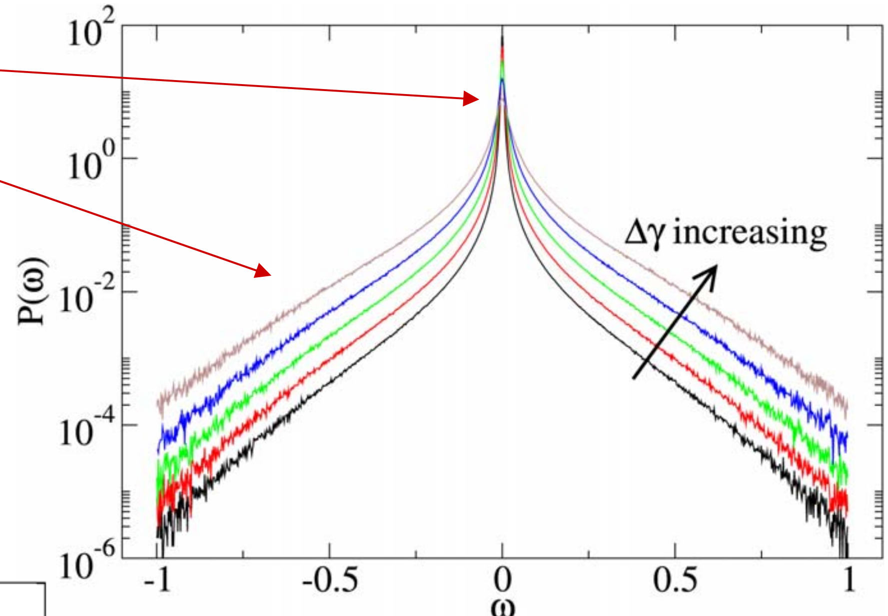
Consistent with observed $D=57\sigma^2$ for $a=0.7\sigma$ and for L down to 40

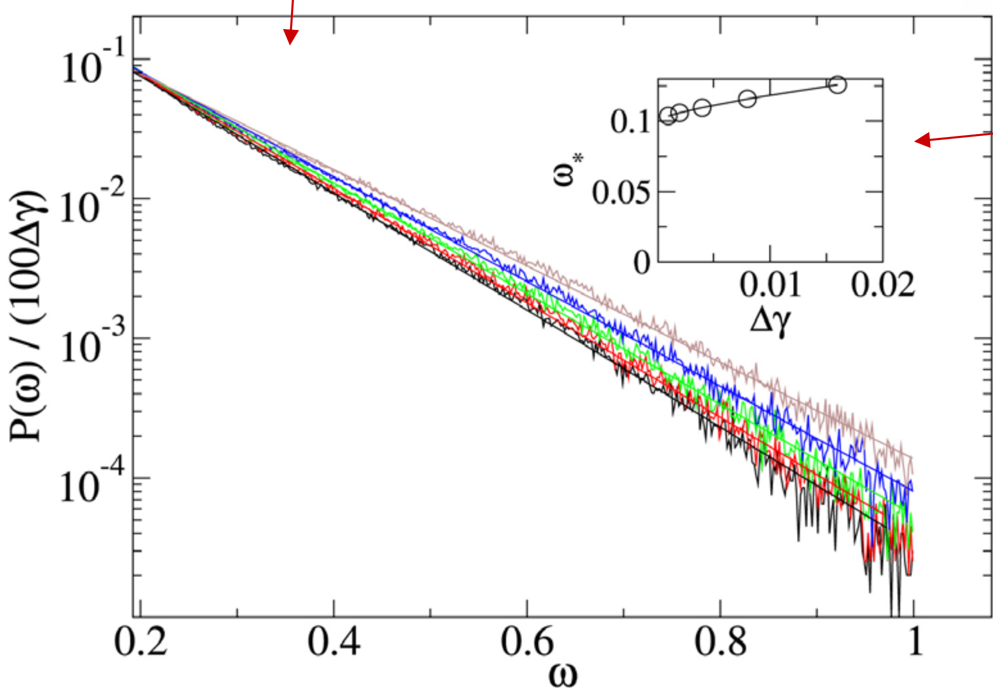


For ω, no L dependence

Distribution of Vorticity ω

Sharp elastic peak at small ω
Exponential tails at large ω
Weight in tails grows
~linearly with strain





Characteristic ω^* for decay ~0.1

Since ω ~ twice strain,
ω* → 5% strain
~ yield strain

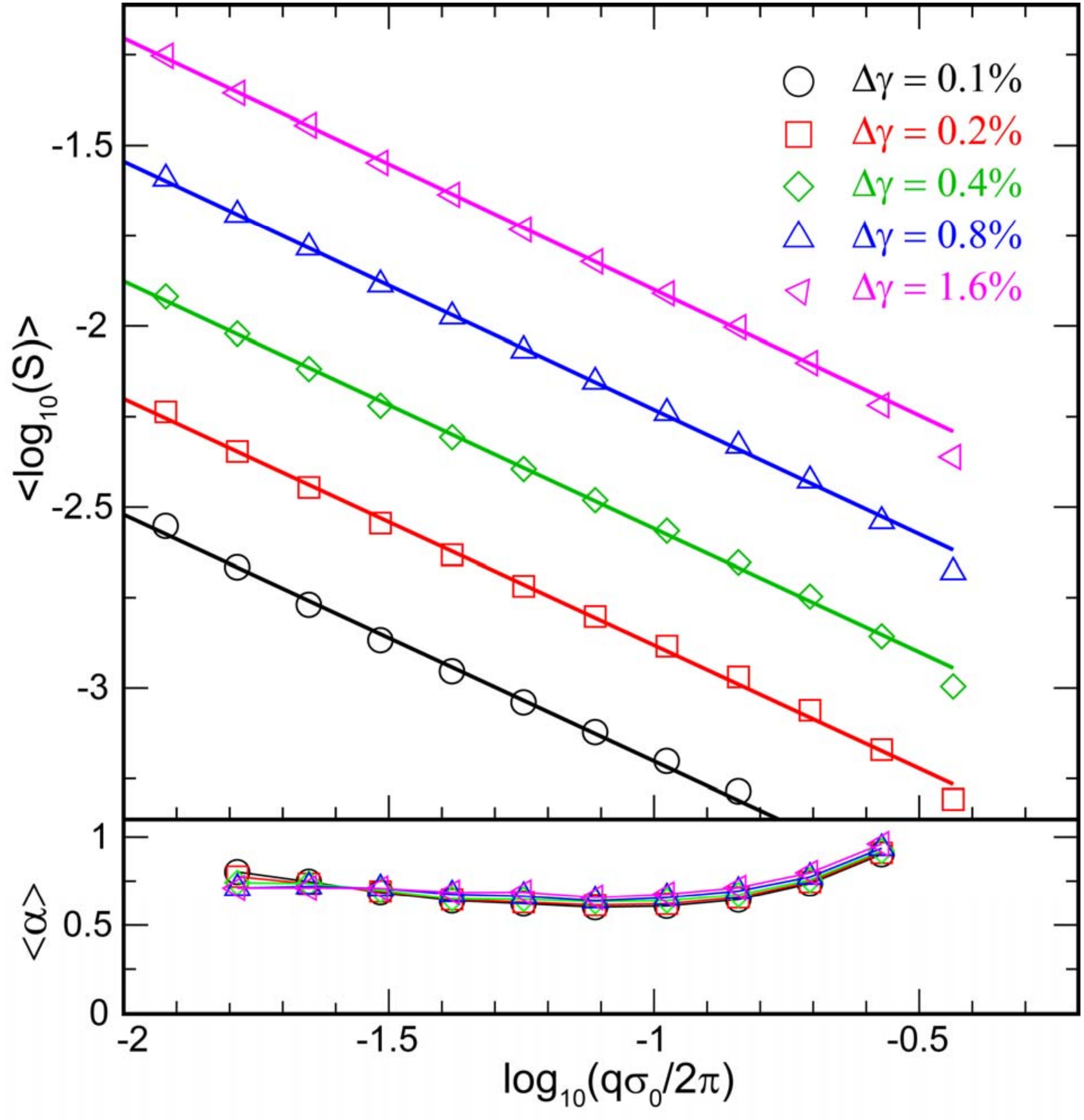
Plastic bands have ~10 sharper localized bands

Vorticity Correlation Function $S(\vec{q}) = \left| \int \omega(\vec{r}) exp[i\vec{q} \cdot \vec{r}] dr \right|^2$

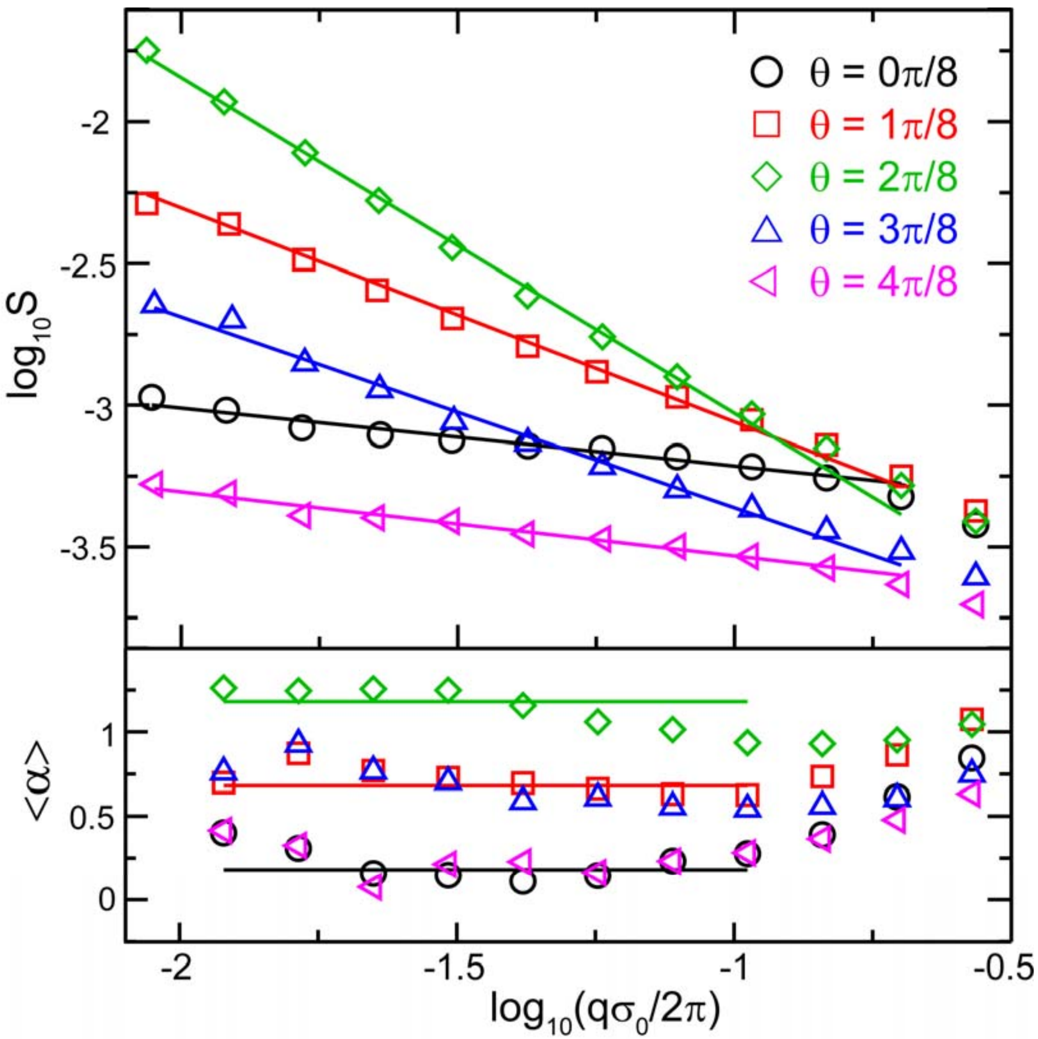
Mean of log S scales as power of wave vector.

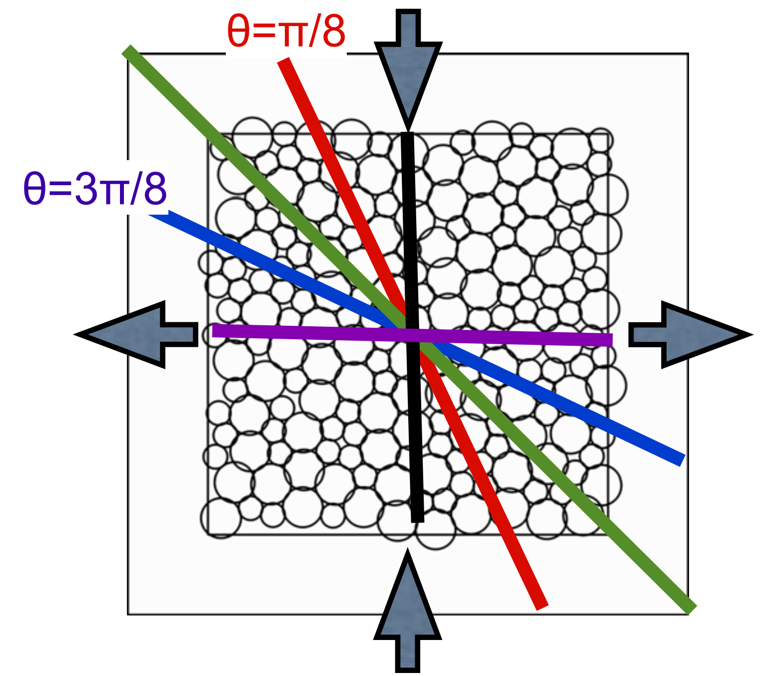
Prefactor linear in $\Delta\gamma$
$\rightarrow$Incoherent addition of successive intervals

BUT scaling highly anisotropic



Angle Dependence of Structure Factor, Δγ=0.1%





θ=π/8 and θ=3π/8 have same shear stress, different normal stress. Mohr-Coulomb predicts shift to π/8

$$S(q;\theta)=A(\theta)q^{-\alpha(\theta)}$$

$\alpha = a+b\cos(4\theta)$

$A = c+d\cos(2\theta)$

A: broken shear symmetry bigger for planes with low normal load as predicted by Mohr-Coulomb

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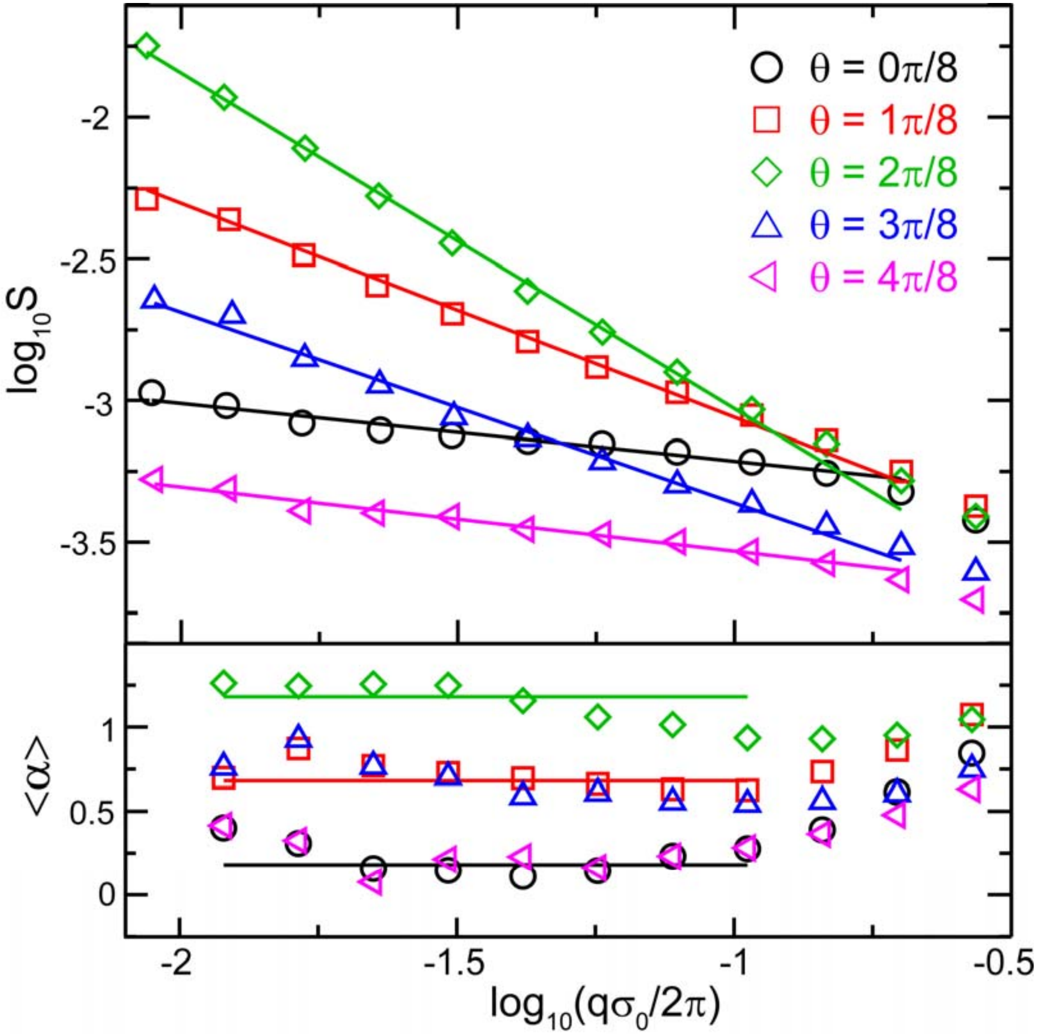

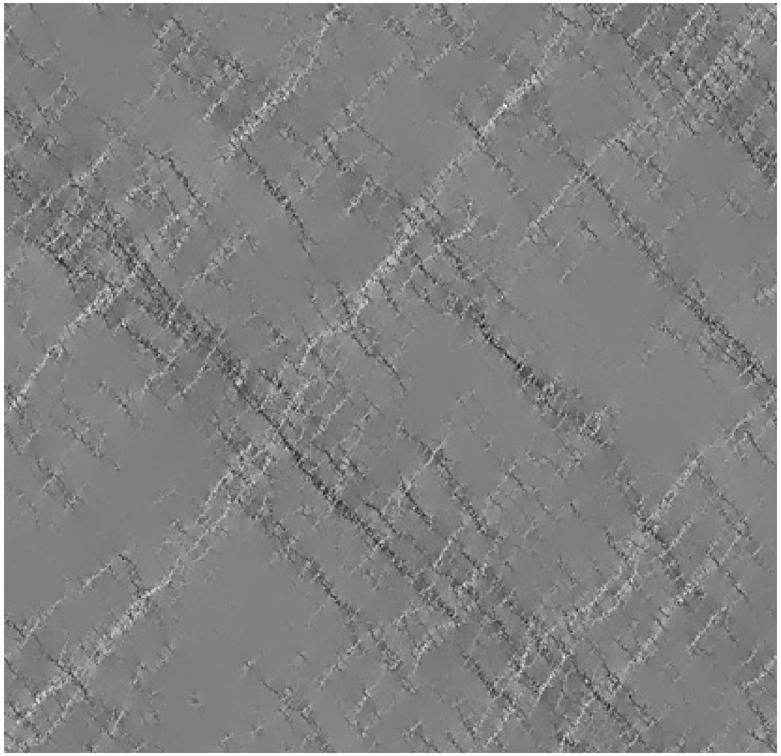

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bigger for planes with low normal load
as predicted by Mohr-Coulomb

Conclusions

- Earthquake probability P(E) ~ 1/E over ~ 5 decades
  Exponent independent of geometry, interactions
  May depend on how events are broken up
- Inertia leads to qualitative changes in deformation statistics.
- Strain localizes in plastic bands that extend across system
  Typical slip distance along band a~particle diameter
  Typical thickness h scales with system size ~L/20
  Several sharper features in h with strain ~5 – 10%
- Non-affine displacement $\Delta r^2 \sim \Delta\gamma/(a/L)\ a^2/12 \sim D\ \Delta\gamma$, $D \sim La/12$
- Strain over short intervals has anisotropic power law correlations
  $S(|q|,\theta) \sim A(\theta)\ |q|^{-\alpha(\theta)}$ where $\alpha = a+b\cos(4\theta)$, $A = c+d\cos(2\theta)$
  Breaking of symmetry for A $\rightarrow$ Mohr-Coulomb