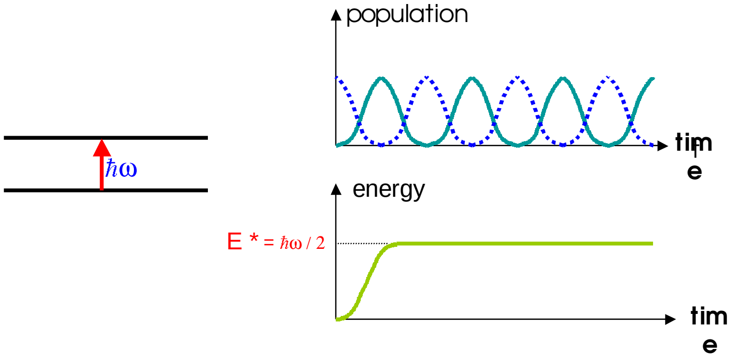
Dynamic localization in quantum dots: analytical theory

D.M.Basko[1] ,   **M. A. Skvortsov**[2], **and V. E. Kravtsov**[1,2]

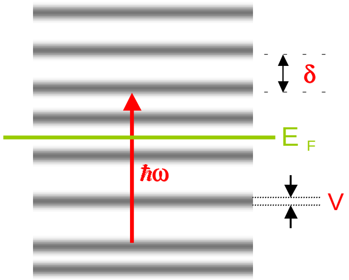
[1]*Abdus Salam International Centre for Theoretical Physics, Trieste*
[2]*Landau Institute for Theoretical Physics, Moscow*

Localization in a two-level system



Population oscillates (Rabi oscillations), energy saturates

Infinite system: the case of small perturbation



$H = H_0 + V\cos(\omega t)$

$V \ll \delta$

Probability to fall in resonance **(V/δ)**

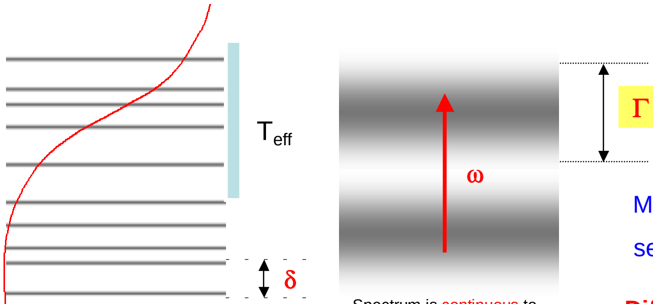
Number of allowed active initial levels (ω/δ)

Saturation energy of a resonant pair $\hbar\omega$

For $t > t^* = \hbar/\delta$ the total energy of an infinite system saturates at
$E^* \sim V\,(\omega/\delta)^2$

Wilkinson, Austin, 1992

Infinite system: the case of large perturbation



$\Gamma = V^2/\delta > \delta$

Spectrum is continuous to the first approximation: resonance does not play any role

Many levels involved ⇨ semiclassical picture

Diffusion in the energy space:

$l \rightarrow \omega\,;\ \tau \rightarrow 1/\Gamma$

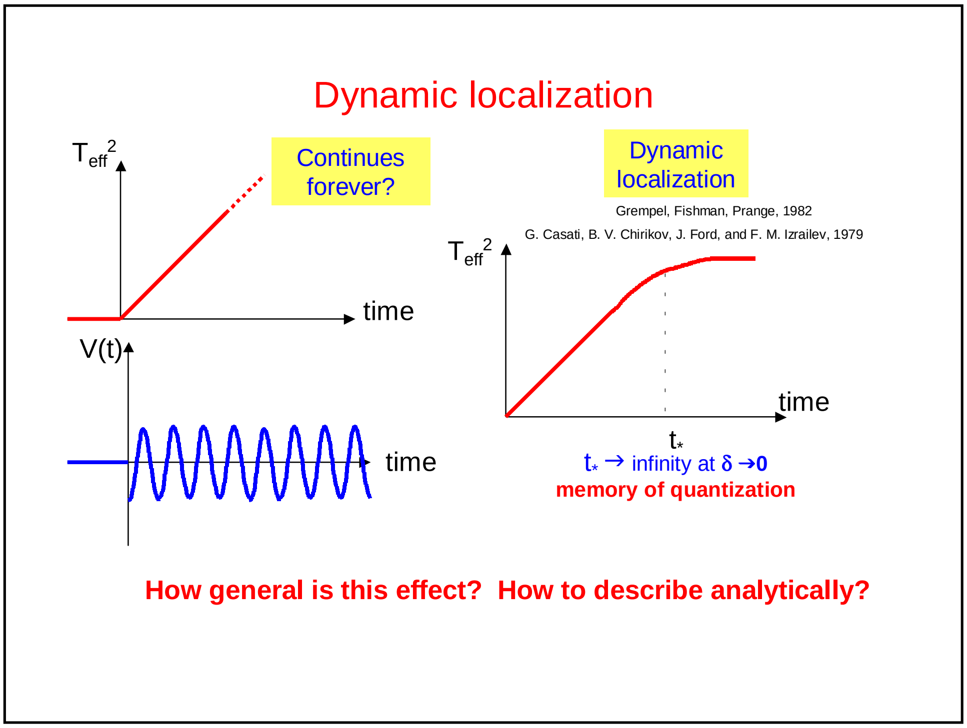
$D = l^2/\tau \rightarrow \Gamma\omega^2$

$\boxed{T^2_{eff} = D\,t}$

Total energy:

$$E = const + \int \varepsilon\left[f(\varepsilon) - \theta(\varepsilon)\right]\frac{d\varepsilon}{\delta} \propto \frac{T_{eff}^{\;2}}{\delta}$$

$dE/dt = D/\delta = const$



Dynamic localization

Continues forever?

Dynamic localization

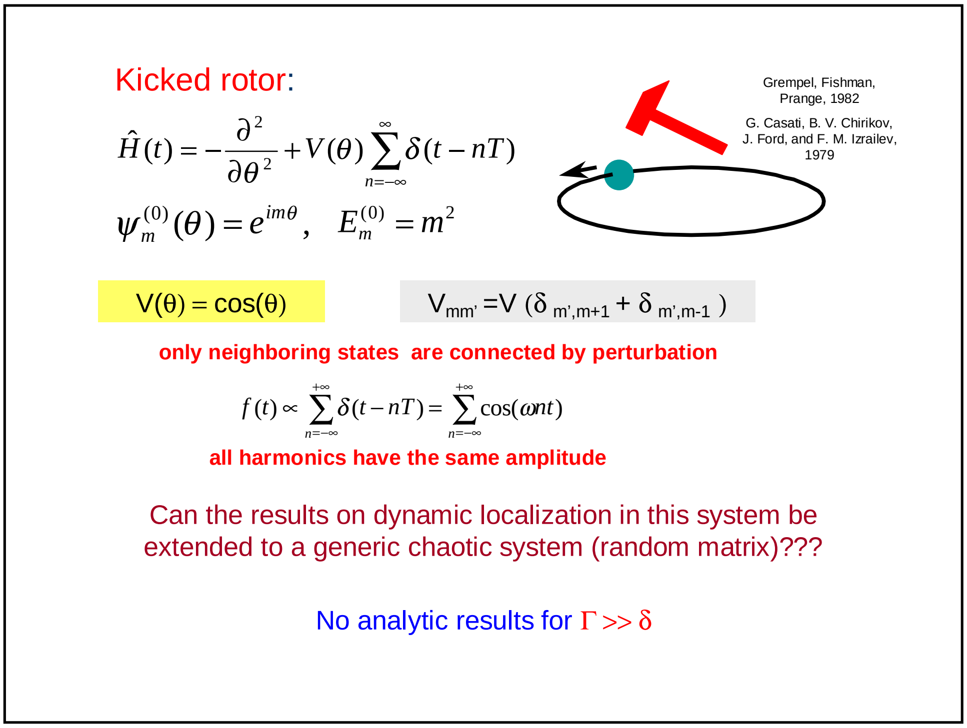
Grempel, Fishman, Prange, 1982

G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979

$t_* \rightarrow$ infinity at $\delta \rightarrow 0$

memory of quantization

How general is this effect? How to describe analytically?



Kicked rotor:

Grempel, Fishman, Prange, 1982

G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979

$$\hat{H}(t) = -\frac{\partial^2}{\partial\theta^2} + V(\theta)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$\psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2$$

$V(\theta) = \cos(\theta)$

$V_{mm'} = V\,(\delta_{m',m+1} + \delta_{m',m-1})$

only neighboring states are connected by perturbation

$$f(t) \propto \sum_{n=-\infty}^{+\infty}\delta(t-nT) = \sum_{n=-\infty}^{+\infty}\cos(\omega nt)$$

all harmonics have the same amplitude

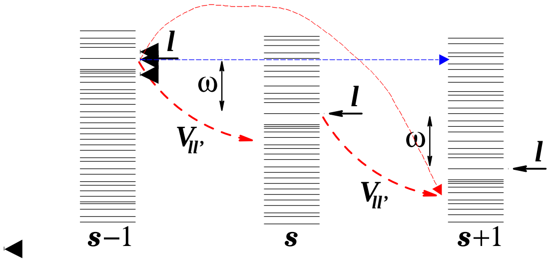
Can the results on dynamic localization in this system be extended to a generic chaotic system (random matrix)???

No analytic results for $\Gamma >> \delta$

Perturbation by periodic δ–function (kicks) $H = H_0 + Vf(t);$

$$f(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2\pi n/\omega) = \sum_{n=-\infty}^{+\infty} \cos(\omega n t)$$

All sites are directly connected



Kicked rotor :

$V_{mm'} = V(\delta_{m',m+1} + \delta_{m',m-1})$

only neighboring orbitals are connected: remote sites are out of resonance → dynamic localization

Kicked random matrix : $$\langle V_{ll'}^{2} \rangle = const$$

NO DYNAMIC LOCALIZATION

All orbitals are connected: resonance between remore orbitals on arbitrary remote sites is possible

---

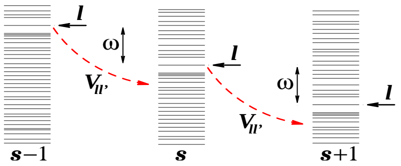
Random matrix with almost harmonic perturbation

$$H = H_0 + Vf(t);$$

GOE GOE

$$f(t) = \sum_n A_n \cos(\omega_n t + \varphi_n)$$

Few harmonics are relevant: $A_n < \frac{1}{n^{3/2}}$



Few sites are connected

DYNAMIC LOCALIZATION IS POSSIBLE

Weak dynamic localization

Basko, Skvortsov, V.E.K, 2003

$$\partial_t E = W^{(0)} + \frac{\Gamma}{2\pi^2} \int_0^t \partial_t f(t)\, \partial_t f(t-\xi)\, \mathcal{C}_{t-\xi/2}(\xi,-\xi)\, d\xi$$

Classical diffusion

Quantum interference correction

Altshuler, Aronov, Khmelnitskii, 1982

Yudson, Kanzieper, V.E.K. 2001

Vavilov, Aleiner, 2001

diffuson: $$\mathcal{D}_\eta(t,t') = \theta(t-t') \exp\left[-\Gamma \int_{t'}^{t} [f(t_1 + \overset{\gamma_d}{\eta}/2) - f(t_1 - \eta/2)]^2\, dt_1\right]$$

cooperon: $$\mathcal{C}_t(\eta,\eta') = \theta(\eta-\eta') \exp\left[-\frac{\Gamma}{2} \int_{\eta'}^{\eta} [f(t + \overset{\gamma_c}{t_1}/2) - f(t - t_1/2)]^2\, d\eta_1\right]$$

Dephasing factors

Harmonic perturbation with high frequency: $\omega \gg \Gamma \gg \delta$

$$\gamma_c(t - \xi/2) = \sin^2[\omega(t - \xi/2)]$$

No-dephasing windows near

$\omega(t - \xi_n/2) = \pi n$

make the main contribution

$$W = W^{(0)}\left[1 - \sqrt{\frac{t}{t_*}}\right]$$

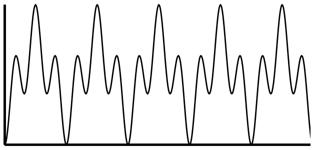
$t_* = \pi^3\Gamma/(2\delta^2)$

X.B.Wang, V.E.K.,2001

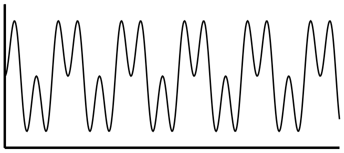
$$f(t + t_0) = f(-t + t_0)$$

$$f(t) = \sum_n A_n \cos(n\omega t + \varphi_n)$$

Average dephasing rate γ_c versus time:



Quasi-1d orthogonal:
$\delta W \sim -(t / t^*)^{1/2}$



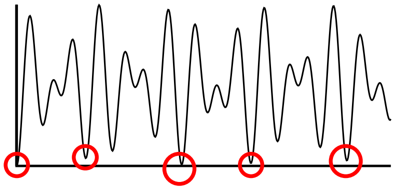
Quasi-1d unitary:
$\delta W \sim -(t / t^*)$

Monochromatic perturbation: *T*-symmetry always – a very special case

Incommensurate periods

$A_n^2 \sin^2(\omega_n t + \varphi_n)$

dephasing rate:



Number theory game seen in mesoscopic physics

X.B.Wang, V.E.K. 2001

Almost-no-dephasing points contribute:

$$W(t) - W_0 \sim -\omega^2 \int_{1/\Gamma}^{t} \frac{\Gamma\, dt_1}{\sqrt{(\Gamma t_1)^d}}$$

d-dimensional weak Anderson localization

Basko,Skvortsov, VEK, 2003;

Numerics for kicked rotor: Casati, Guarneri, Shepelyanskii, 1989

A glance at the reality

GaAs dot:

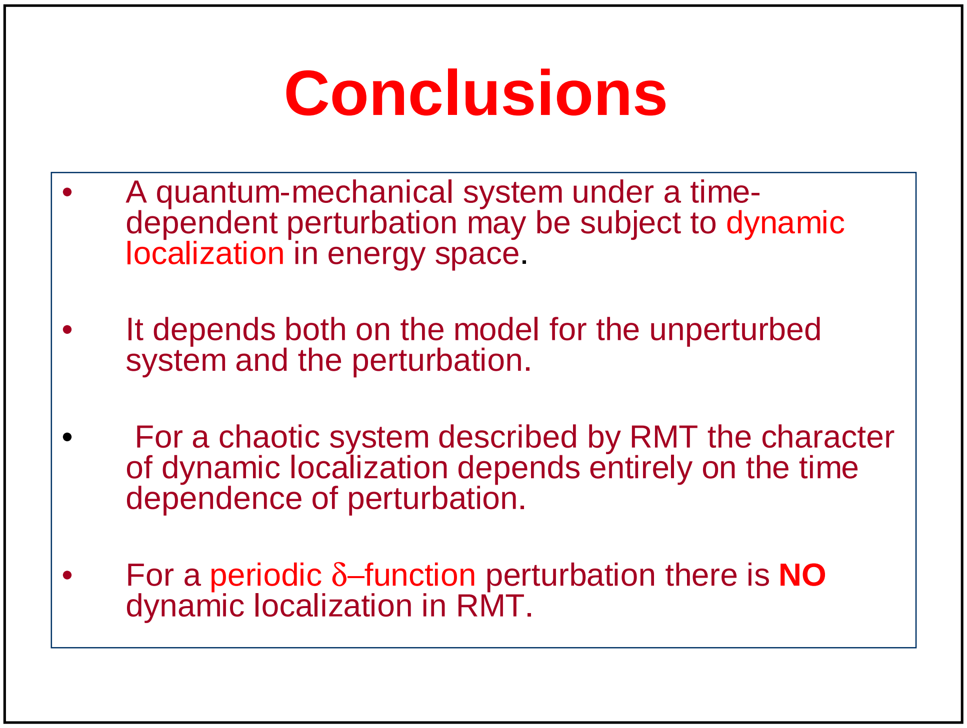
- size $L \sim 1\,\mu\mathrm{m}$
- mean level spacing $\delta \sim 1\,\mu\mathrm{eV}$
- Thouless energy $E_{Th} \sim 100-1000\,\mu\mathrm{eV}$
- dephasing time $t_\varphi \sim 1-10\,\mathrm{ns}$

Microwave field:

- *V*~ several μeV (field ~ several 100 V/m)
- $\hbar\omega \sim 10-100\,\mu\mathrm{eV}\ (\sim 10^{10}\ \mathrm{Hz})$

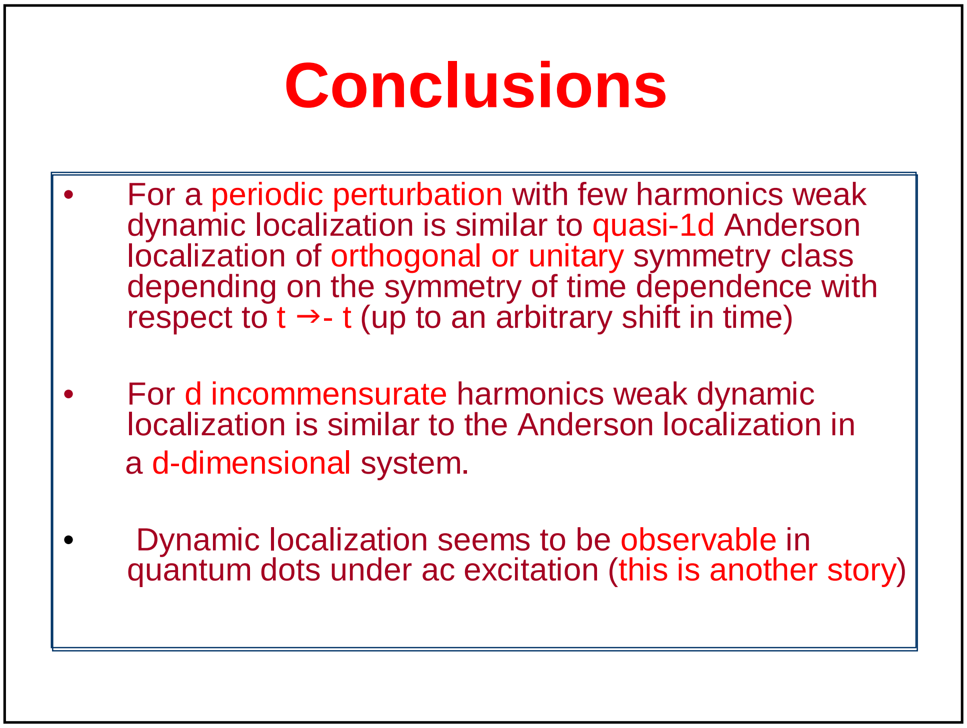
Dynamic localization:

- $t_{loc} \sim 10\,\mathrm{ns},\ E_{loc} \sim \sqrt{Dt_{loc}} \sim 100-1000\,\mu\mathrm{eV} \sim 1-10\,\mathrm{K}$



Conclusions

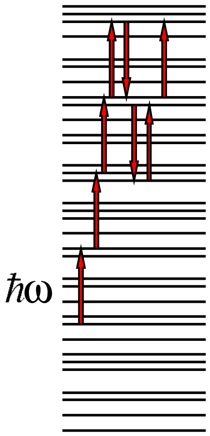
- A quantum-mechanical system under a time-dependent perturbation may be subject to dynamic localization in energy space.
- It depends both on the model for the unperturbed system and the perturbation.
- For a chaotic system described by RMT the character of dynamic localization depends entirely on the time dependence of perturbation.
- For a periodic δ–function perturbation there is **NO** dynamic localization in RMT.



Conclusions

- For a periodic perturbation with few harmonics weak dynamic localization is similar to quasi-1d Anderson localization of orthogonal or unitary symmetry class depending on the symmetry of time dependence with respect to $t \rightarrow - t$ (up to an arbitrary shift in time)
- For d incommensurate harmonics weak dynamic localization is similar to the Anderson localization in a d-dimensional system.
- Dynamic localization seems to be observable in quantum dots under ac excitation (this is another story)

What everybody knows…



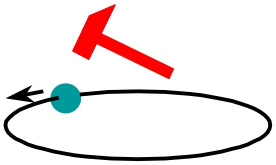
- $\hat{H} = \hat{H}_0 + \hat{V}\cos\omega t$
- (Quasi)continuous spectrum
- Absorption and emission of quanta $\hbar\omega$ – random walk up and down
- Diffusive evolution of the electron distribution function

What some people know…

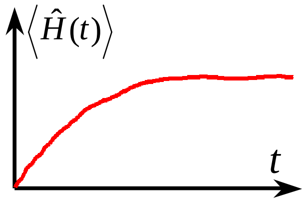
Kicked rotor:

$$\hat{H}(t) = -\frac{\partial^2}{\partial\theta^2} + V(\theta)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$\psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2$$



Dynamic localization in the energy space:
after some time the rotor stops absorbing



(G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979)

Historical developments

1. Quantum interference – analogous to the Anderson localization (Fishman, Grempel, and Prange, 1982)
2. Incommensurate periods T_1, T_2, T_3 – 3D localization (Casati, Guarneri, Shepelyansky, 1989)
3. Particle in a box: just $\psi(0)=\psi(2\pi)=0$ instead of the periodic $\psi(0)=\psi(2\pi)$ – no localization (Hu, Li, Liu, Gu, 1999)
4. Mapping to a quasi-*1d* σ-model (Altland, Zirnbauer, 1996)

What do these observations mean and how general are they?

Spatial localization

Quantum correction to the diffusion coefficient of electrons in disorder

mean free path

$$D-D_0 \sim -\frac{D_0}{\nu}\int_0^{1/l}\frac{d^d\vec{k}}{D_0k^2+1/t_\varphi}$$

density of states

dephasing time

Change variables $D_0k^2=1/t$:

$$D-D_0 \sim -\frac{1}{\nu}\int_\tau^{t_\varphi}\frac{D_0\,dt}{(D_0t)^{d/2}}$$

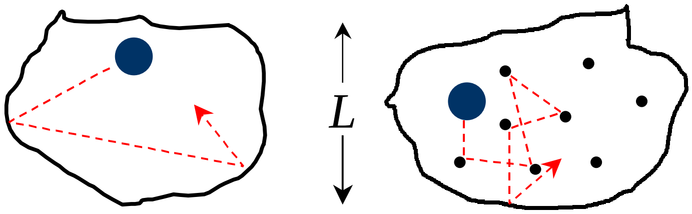
Localization: $d=1:\quad L_{loc}\sim \nu D_0 \sim l$

$d=2:\quad L_{loc}\sim l\exp(\nu D_0)$ (?)

$d\geq 3:$ no localization in weak disorder

Chaotic systems

Ballistic systems: Diffusive systems:



$\tau_{erg} = L / v_F$ ergodic time $\tau_{erg} = L^2 / D$

RMT is valid at low energies:

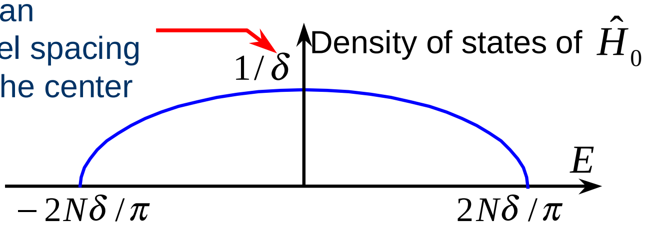
$E << E_{Th} = \hbar / \tau_{erg}$ (Thouless energy)

Random matrix theory

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t)$$

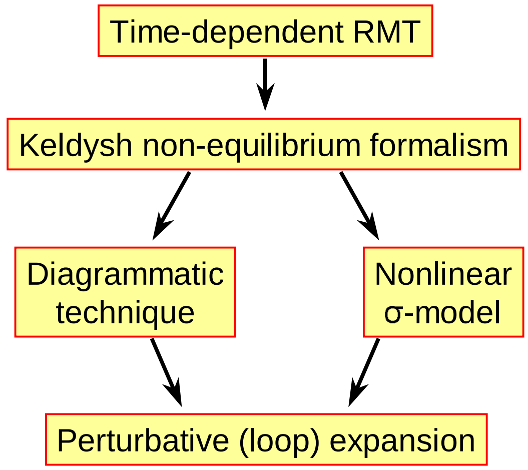
real symmetric
$N \times N$ Gaussian
random matrices
with statistically independent elements

mean
level spacing
at the center



In the end let $N \to \infty$

Technicalities



Time-dependent RMT

Keldysh non-equilibrium formalism

Diagrammatic technique

Nonlinear σ-model

Perturbative (loop) expansion

Zero order (diffusion)

$\Gamma \equiv \left\langle V_{ll'}^2 \right\rangle \big/ \delta$ – one photon absorption rate (measure of perturbation strength)

Long-time, period-averaged dynamics:

$$\left[\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial E^2}\right] f(E,t) = 0$$

time-dependent electron distribution (Wigner variables)

$D = \Gamma\overline{(d\phi/dt)^2}$ – energy diffusion coefficient

$$W_0 \equiv \frac{\partial}{\partial t}\int E\ f(E,t)\,dE = \frac{D}{\delta}$$

– energy absorption rate

One-loop correction

$$W(t)=\underbrace{\frac{D}{\delta}}_{\text{large zero-order}}+\underbrace{\frac{\Gamma}{\pi}\int_0^t \dot{\phi}(t)\,\dot{\phi}(t-\tau)\,C_{t-\tau/2}(\tau,-\tau)\,d\tau}_{\text{small (?) correction}}$$

Cooperon keeps track of the quantum interference:

$$C_t(\tau_1,\tau_2)\equiv\theta(\tau_1-\tau_2)\exp\left[-\int_{\tau_2}^{\tau_1}\underbrace{\frac{\Gamma}{2}[\phi(t+\tau/2)-\phi(t-\tau/2)]^2}_{\text{dephasing rate}}\,d\tau\right]$$

Periodic perturbation

$$\phi(t)=\sum_{n=1}^{\infty}A_n\cos(n\omega t-\varphi_n)\qquad W_0=\frac{\Gamma\omega^2}{2\delta}\sum_n n^2A_n^2$$

$$C_t(\tau_1,\tau_2)\approx\exp\left[-\Gamma(\tau_1-\tau_2)\sum_{n=1}^{\infty}A_n^2\sin^2(n\omega t-\varphi_n)\right]$$

If $\varphi_n=n\varphi$ the exponent can vanish at $t_k=\dfrac{\varphi+k\pi}{\omega}$

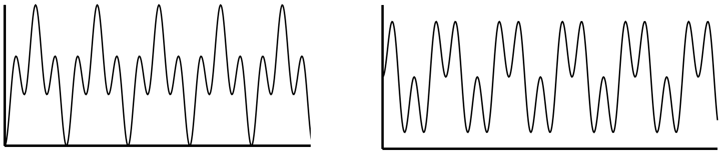
No-dephasing points give a large negative contribution to the integral:

$$W(t)-W_0\sim-\omega^2\sqrt{\Gamma t}$$

Time-reversal symmetry

$$\varphi_n = n\varphi \quad \Leftrightarrow \quad \phi(t-t_0) = \phi(-t-t_0)$$

Average dephasing rate versus time:



T-symmetry: yes

T-symmetry: no

Monochromatic perturbation: *T*-symmetry always –
a very special case

Two loops

There is a contribution from diffusons:

$$D_\tau(t_1,t_2) \equiv \theta(t_1-t_2)\exp\left[-\int_{t_2}^{t_1}\Gamma\,[\phi(t+\tau/2)-\phi(t-\tau/2)]^2\,dt\right]$$

For a periodic perturbation:

$$D_\tau(t_1,t_2) \approx \exp\left[-2\Gamma(t_1-t_2)\sum_{n=1}^{\infty} A_n^2 \sin^2 n\omega\tau\right]$$

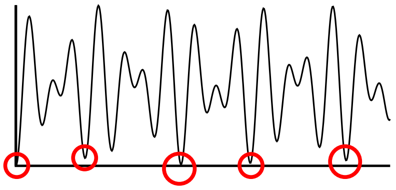
No-dephasing points are always present,
regardless of the time-reversal symmetry…

$$W(t) - W_0 = -\frac{\omega^2\delta}{24\pi^2}\,t$$

Incommensurate periods

$$f(t)=\sum_{n=1}^{d}A_n\cos(\omega_n t-\varphi_n) \qquad \gamma_c=\sum_n \sin^2(\omega_n t+\varphi_n)A_n^2$$

dephasing rate:



Phase relationships do not matter that much

Almost-no-dephasing points contribute:

$$W(t)-W_0\sim-\omega^2\int_{1/\Gamma}^{t}\frac{\Gamma\,dt_1}{\sqrt{(\Gamma t_1)^d}}$$

d-dimensional weak Anderson localization

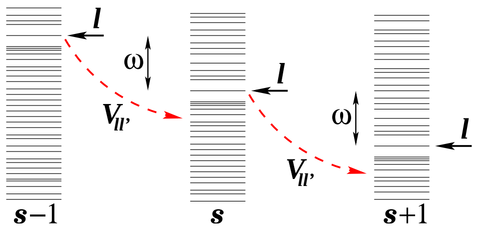
Conclusions…

1. A quantum-mechanical system under a time-dependent perturbation may be subject to dynamic localization in energy space.
2. It depends both on the model for the unperturbed system and the perturbation.
3. We have studied one-loop correction to the usual Fermi-Golden-Rule dissipation rate for a chaotic system described by RMT

…conclusions

4. For a perturbation with *d* incommensurate frequencies the correction can grow arbitrarily with time if *d=1,2* (analogously to spatial localization in *d*-dimensional disorder)
5. For commensurate frequencies phase relationships matter:
6. Time-reversal symmetry: the “dimensionality” is effectively lowered
7. No time-reversal: the correction is small

A stationary analogy



following directly from Schrödinger equation for $\hat{H}_0 + \hat{V}\cos\omega t$

- Take the original levels E_l of $\hat{H}_0$
- Replicate them into a lattice with a shift

  $E_{l,s} = E_l - s\hbar\omega$
- Couple neighboring sites with $\hat{V}$

Why RMT is not KQR

- Quantum rotor: $\psi_l = e^{il\theta}$, $V(\theta) = \cos\theta$,

  $V_{ll'} \propto \delta_{l',l\pm 1}$ – out of resonance

  $V(t) \propto \delta(t - nT)$ – all Fourier harmonics $V^{(s-s')}$

- Particle in a box: $\psi_l = \sin l\theta$, $V(\theta) \propto \cos\theta$,

  $V_{ll'} \propto 1/|l - l'|$ – long-range

- Random matrix: $V_{ll'} \propto const$ but

  we want few Fourier harmonics $V^{(s-s')}$

Spatial localization

Quantum correction to the diffusion coefficient of electrons in disorder

mean free path

$$D = D_0 - \frac{1}{\nu}\int_{1/L}^{1/l} \frac{d^d\vec{k}}{k^2}$$

density of states

sample size

$d = 1$: $L_{loc} \sim \nu D_0 \sim l$

$d = 2$: $L_{loc} \sim l \exp(\nu D_0)$ (?)

$d \geq 3$: no localization in weak disorder