Does the Functional RG makes (any) sense ?

T=0: *coupling is function* $g\ \phi^4 \ \rightarrow\ R(u)$

perturbative expansion in $\epsilon = 4 - d$

$\tilde{R}_l(u) \rightarrow \tilde{R}^*(u) = O(\epsilon)$ DSFisher 86

BUT! $\tilde{R}_l(u)$ is non analytic at u=0

$\Rightarrow$ AMBIGUOUS diagrams in

- RG functions at two loop $O(\epsilon^2)$
- correlation function at one loop

$R'''(0^+) \neq R'''(0^-)$

many metastable states, system preparation

Q: how to lift ambiguities ?

T>0: $T_l \sim L^{-\theta}$ irrelevant BUT smoothes the cusp

$\tilde{R}_l(u)$ remains analytic

$R_l''''(0) \sim L^{\theta}$

Q: does it remain perturbative in 4-d ?

droplet picture,creep

first principles exact RG methods

Collaborations

- FRG 2 loop T=0 depinning, statics	Kay Wiese, P. Chauve	(3)
- FRG large N +3,..loops, anis. depin.,rand fields..	Kay Wiese	(2)
- T>0 FRG first principles for equilib. dyn.,statics	Léon Balents	(1)
- exact RG methods	G. Schehr, P. Chauve	(2)
- RSRG, d=0 toy model (+Sinai model, 1D RFIM)	Cécile Monthus +D S Fisher	(3)

And also... FRG, creep + … T. Giamarchi

Coulomb gas RG + disorder random Dirac — D Carpentier, B Horovitz, H Castillo

Mean field dynamics — L. Cugliandolo, J. Kurchan

OUTLINE

- depinning
  - one loop FRG T=0
    - statics, depinning (not fully consistent)
  - difficulties beyond one loop
  - solution for N=1 depinning
    - renorm field theory from 1st principles
    - roughness $\zeta = \frac{\epsilon}{3}(1 + 0.14\epsilon + ..)$
    - simulations, contact line experiments

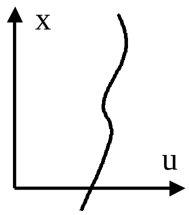
Kay's talk: T=0 statics, large N

- $T > 0$
  - statics : field theory of droplets
    - d=0 model
  - eq. dynamics

Elastic Manifold in Random Potential

$u(x) \in \mathbb{R}^N$

$x \in \mathbb{R}^d$



$$H = \int d^d x \ \frac{c}{2}(\nabla u)^2 + V(x, u(x))$$

$$\overline{V(x,u)V(x',u')} = \delta^d(x-x') \ R(u-u')$$

$R(u)$ is

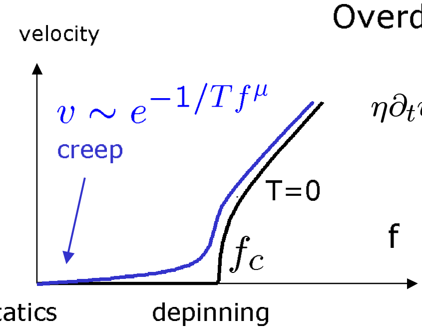
SR	random bond	*magnetic DW*
LR	random field	*interfaces*
periodic	*CDW, vortex lattice(Bragg glass)*	

$cq^2 \to c|q|$ LR elasticity *contact line* $+c_4(\nabla u)^4$

Pinned T=0 $\overline{(u(x)-u(0))^2} \sim |x|^{2\zeta}$ few univ class

$T_l \sim L^{-\theta}$ $\zeta(N, d, ..)$

Overdamped dynamics



$v \sim (f - f_c)^{\beta}$

$$\eta \partial_t u(x,t) = -\frac{\delta H}{\delta u(x,t)} + \xi(x,t) + f$$

Q: coarse grained model?
potential vs non pot.

$$\overline{F(x,u)F(x',u')} = \delta^d(x-x')\Delta(u-u')$$

$F(x,u) = \partial_u V(x,u)$

- potential equilib dyn $\eta \partial_t u = \nabla^2 u + F(x,u)$ $\quad \Delta(u) = -R''(u)$

- non potential 1) $F(x,u) \neq \partial_u V(x,u)$ $\quad \int \Delta(u) \neq 0$

$v > 0$ 2) $+\lambda(\nabla u)^2$ KPZ

$v \to 0^+$ at quasi-static depinning ??

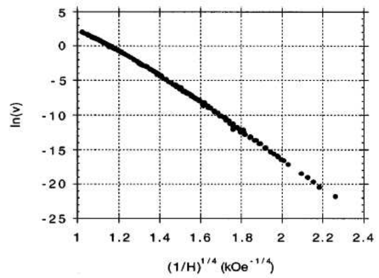


FIG. 3. Natural logarithm of MDW velocity as a function of $(1/H)^{1/4}$ (room temperature, $H \leq 955$ Oe).

← Ising magnetic film Co

Lemerle Ferre et al. 98

domain wall creep, d=1+1, RB

$\mu = 1/4$

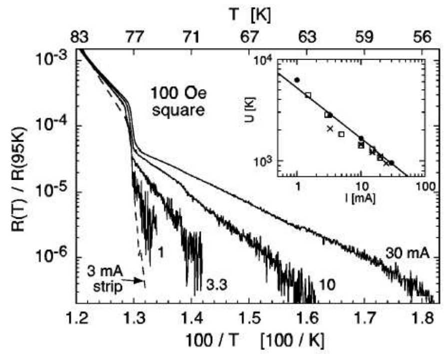
Zeldov et al. BSCCO

Bragg glass →

$\mu = 1/2$

$$R \sim v \sim e^{-U(j)/T}$$

$$U(j) \sim j^{-\mu}$$



Functional RG to one loop

T=0

(T irrelevant)

statics $$H_{rep} = \int_x \frac{1}{2} \sum_a (\nabla u_a)^2 + \frac{1}{2T} \sum_{ab} R(u_a(x) - u_b(x))$$
$$+T^{-2} \sum_{abc} S(u_a, u_b, u_c) + ..$$

$$\overline{u^q u^{-q}} = \langle u_a^q u_b^{-q} \rangle = \frac{-R''(0)}{q^4}$$ to all orders..

u dimless d=4

$R_l(u)$

$l = \ln(1/m)$

$\sim \ln L$

$$\partial_l R(u) = (\epsilon - 4\zeta) R + \zeta u \partial_u R + \frac{1}{2} R''^2 - R'' R''(0)$$

$\Rightarrow$ RB,RF,RP fixed points

$\zeta_{RF} = \epsilon/3, \zeta_{RB} = 0.208\epsilon$

dynamics

depinning

$+f \hat{u}_{xt}$

$$S = \int_{xt} \hat{u}_{xt} (\eta \partial_t - \nabla_x^2) u_{xt} - \frac{1}{2} \int_{xtt'} \hat{u}_{xt} \hat{u}_{xt'} \Delta(u_{xt} - u_{xt'})$$

$\Delta(u)$ force correlator

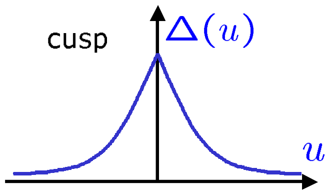
$$\partial_l \Delta(u) = (\epsilon - 2\zeta) \Delta + \zeta u \partial_u \Delta - \Delta'^2 - \Delta''(\Delta - \Delta(0))$$

$$= \frac{d^2}{du^2} \partial_l R(u)$$

$\Rightarrow$ to one loop

$\Delta(u) = -R''(u)$

Problems with FRG approach ?



$f_c \sim |\Delta'(0^+)|$

Nattermann et al. 92
Narayan Fisher

quasi-static depinning $v \to 0^+$

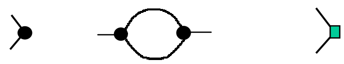
one loop theory   not fully consistent

- β -function for force correlator same as statics → where is irreversibility ?
- RB $\neq$ RF   how many univ class?
- conjectures $\zeta_{dep} = \epsilon/3$ (NF)
  numerics ?

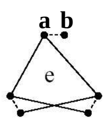
$$\langle u_a^q u_b^{-q} \rangle = \frac{1}{q^4}[-R''(0) + R'''(0)^2 I(q) - R'''(0^+)^2 I(0)]$$

cusp → ambiguities
lifted by renormalizability ?

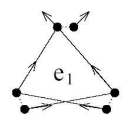
$$I(q) = \int_k \frac{1}{k^2(k+q)^2}$$



$\delta R =$



$R''(u)R'''(0)^2$



$\Delta''(u)\Delta'(0)^2$

Solution for N=1 depinning (from first principles)

NAFT
non analytic field theory

$$\Delta(u) = \Delta(0) + \Delta'(0^+)|u| + \frac{u^2}{2}\Delta''(0^+) + ..$$

Wick → $\langle \mathrm{sgn}(u_{xt} - u_{xt'}) \cdots \rangle$

BUT for N=1 $\partial_t u_{xt} > 0$ sgn(t-t') $v \to 0^+$

no passing rule
Middleton theorems

$\Rightarrow$ renormalizable $1/\epsilon^2$ cancel

$$\partial_l \Delta(u) = (-\frac{\Delta^2}{2} + \Delta\Delta(0))'' + \frac{1}{2}(\Delta'^2(\Delta - \Delta(0))'' + \frac{\lambda}{2}\Delta'(O^+)^2\Delta''(u)$$

$\lambda_{dep} = 1$

- different from statics: irreversibility recovered

$\lambda_{stat} = -1$

- single FP for RB,RF $\int \Delta \neq 0$

$$\zeta_{dep} = \frac{\epsilon}{3}(1 + 0.1433\epsilon + ..) > \zeta_{NF}$$

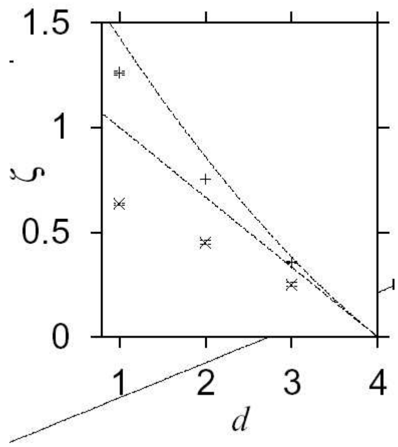
d	analytic		ζ_{Δ^2}	ζ_{Δ^4}
	one-loop	two-loop		
1	1	1.44	1.26 ± 0.01	0.635 ± 0.005
2	2/3	0.86	0.753 ± 0.002	0.45 ± 0.01
3	1/3	0.38	0.355 ± 0.01	0.25 ± 0.02

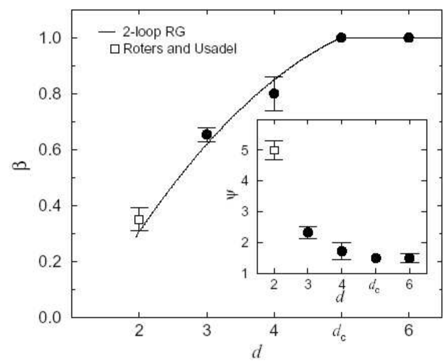
Rosso and Krauth 02

TABLE I: Roughness exponents as a function of dimension d for the $(d+1)$–dimensional driven manifold problem, both for the Δ^2–model (harmonic elastic energy) and the Δ^4–model.

$$\nu = 1/(2-\zeta)$$

$$\beta = \nu(z-\zeta)$$





Roters and Usadel

contact line, cracks $\qquad \zeta_{exp} \approx 0.55$

for long range elasticity $\qquad cq^2 \to c|q|$

$$\zeta_{1loop} = 0.33$$

we find $\qquad \zeta_{dep} = \frac{\epsilon}{3}(1 + 0.397\epsilon + ..)$ $\qquad \epsilon = 2 - d$

$\longrightarrow \qquad \zeta_{2loop} = 0.47$

BUT $\qquad$ Rosso Krauth $\qquad \zeta = 0.390 \pm 0.002$

is this elastic overdamped model

the correct one for these systems ?

Contact line depinning

viscous fluid water,glycerol

Moulinet,Guthmann,Rolley 2002

overdamped, quasi-static } checked

$$\zeta = 0.51 \pm 0.03$$

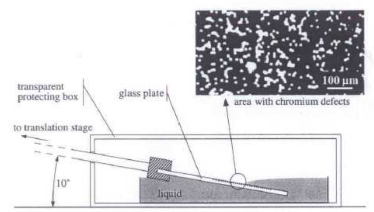


Fig. 2. Sketch of the experimental setup. Inset: photograph of the disordered substrate, the chromium defects appear as white square spots.

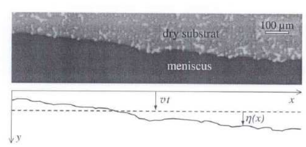


Fig. 1. Upper part: image of the contact line obtained with an ordinary CCD camera. Lower part: the position $\eta(x,t) \equiv y(x,t) - vt$ of the CL is defined with respect to its average position vt.

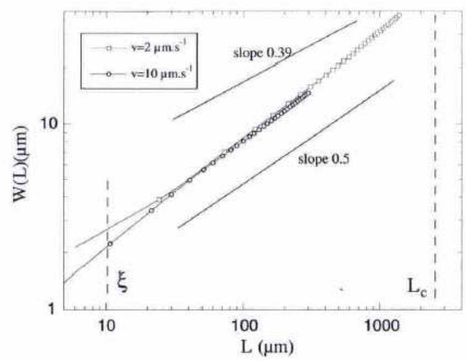
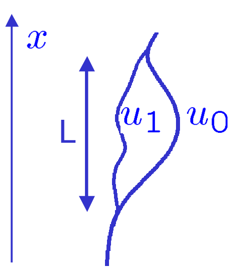


Fig. 3. Roughness W as a function of distance L for different drift velocities. The upper (respectively, lower) graph corresponds to data obtained with water (respectively, water-glycerol mixture). For both graphs, the data ∘ have been obtained with a larger magnification (resolution 2.1 μm) than the others (resolution 6.1 μm).

State degeneracies and thermal excitations

- droplet picture: a ground state $u_0(x)$



at each scale L proba that exists $u_1(x)$

$E[u_1] - E[u_0] \sim T$ is $p \sim T L^{-\theta}$

$P_L(\Delta F) \sim L^{-\theta} \Phi(\Delta F / L^{\theta})$ $\Phi(0) \neq 0$

$$\Rightarrow \overline{\langle (u - \langle u \rangle)^2 \rangle} \sim (T L^{-\theta}) L^{2\zeta} = T L^{d-2}$$

$(u - \langle u \rangle)_{typ} \approx 0$ rare events dominate thermal averages

- creep $\tau \sim e^{U_b/T}$ $U_b = L^{\psi} - f u L^d \sim_{sp} f^{-\mu}$

activated dynamics $\mu = \frac{\psi}{d + \zeta - \psi} = \frac{\theta}{2 - \zeta}$

- mean field distant states with $\Delta F \sim T$

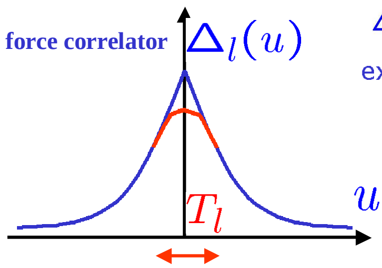
dynamics on saddles

T > 0 statics, equilib. dynamics: naive one loop approach

$$\partial_l \Delta(u) = \epsilon\Delta - \Delta'^2 - \Delta''(\Delta - \Delta(0)) + T_l \Delta''(u)$$

$\zeta = 0$

$$T_l = T_0 e^{-\theta l}$$



$\Delta_l(u) \to \Delta^*_{T=0}(u)$

except in thermal boundary layer $u \sim T_l$

$$\Delta_l(u) = \Delta^*(0) + T_l f(u/T_l)$$

$$f(x) = \sqrt{1 + \Delta'(0^+)^2 x^2} - 1$$

Balents,Chauve et al

- good points: $\frac{\partial_l \eta_l}{\eta_l} = \Delta_l''(0) \sim \frac{e^{\theta l}}{T_0}$ $\Rightarrow$ $\tau_L = \eta_l = e^{L^\theta / T_0}$

  $\to$ CREEP LAW $\psi = \theta$

- problems ?

  $\Delta_l''(0) \to \infty$ perturbative ? $T_l(..)'' \sim \frac{1}{T_l}$

  add loops gets more divergent

Boundary Layer Field Theory of droplets (statics)

- bad news: thermal boundary layer extends to ALL CUMULANTS

$$u \sim T_l = T_0 e^{-\theta l} \qquad R_l(u) = R^{*\prime\prime}(0)\frac{u^2}{2} + \frac{T_l^3}{\epsilon^2} r(\frac{u\,\epsilon}{T_l})$$

$$S^{(p)}(u_{a_1},..u_{a_p}) = f_p\, u_{a_1}..u_{a_p} + \frac{T_l^{p+1}}{\epsilon^2} s^{(p)}(\frac{u_{a_1}\epsilon}{T_l},..,\frac{u_{a_p}\epsilon}{T_l})$$

coupled FRG eq. for all $s^{(p)}, f_{2p}$ $\qquad f_{2p+1} = 0, f_{2p} = O(\epsilon^{2p})$

- bad news get worse: TBL all multilocal cumulants, appears non-perturbat.

GOOD NEWS: • physical content: we show TBL $\leftrightarrow$ droplets

$$\overline{\langle (u - \langle u \rangle)^{2n} \rangle} = c_n (T L^\theta) L^{2n\zeta} \qquad c_n \sim (s^{(p)}(0))^{(2n)}$$

- understand d=0 limit Matches with exact results RSRG for toy model

  $H = \frac{m^2}{2} u^2 + V(u)$ droplets exact

- exact RG methods hope to solve matching between T>0 and T=0 $\to$ { ϵ-expansion, unambiguous, 1st principles }

Conclusion

Hope to construct a field theory based on
Functional RG for this class disordered systems

but appears a very peculiar field theory !

T=0 fixed point : showed apparent ambiguities (non analytic)
at one loop (correlations) and 2 loop (beta function)

- N=1 depinning, unambiguous dynamics ⟶ renorm FT
  exponents,universal distrib, higher correlations,.. N>1 ?

- statics T=0: not 1st principles, proposed renorm FT 2,3 loops
  check predictions (numerics)
  checks from large N solution, 1/N.. around RSB ?

- T>0 VERY peculiar TBL F.T. mechanism rare events in F.T.
  some universality droplet TBL ⟶ 1st principle perturbative
  STILL WORKING !!!