Unearthing Fractional Statistics via a Hanbury Brown-Twiss set-up

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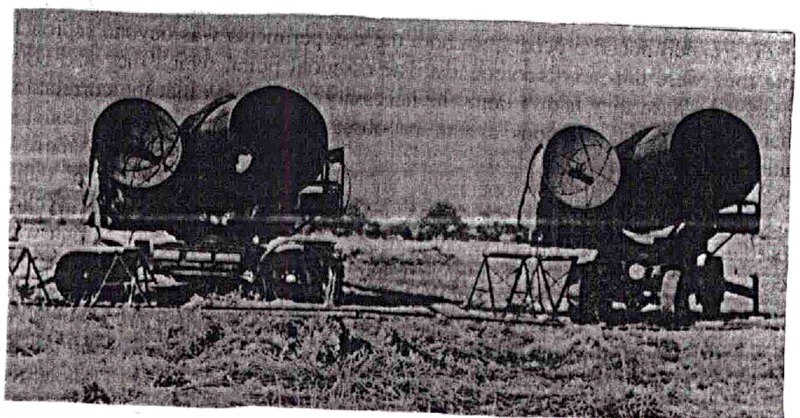
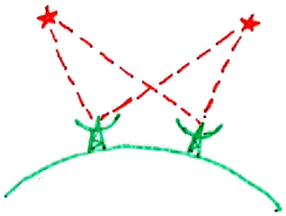


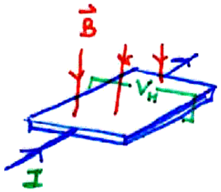
Courtesy Boffin, by R. Hanbury Brown

Outline

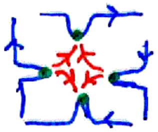
- **Hanbury Brown-Twiss Experiment**
  - History
  - Correlation Functions
  - Fermions and Bosons



- **Anyons**
  - Fractional Quantum Hall System
  - Charge and Statistics



- **A HBT set-up in the FQH system**

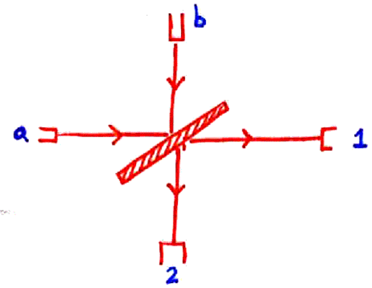


Many warm thanks to E. Ardonne, G. Baym, M.P.A. Fisher, E. Fradkin, P. Goldbart, C. Kane, E. Kim, T.C. Wei
& many others........

②

HBT History-

Quantum Optics



Prob. of 1 particle at detector 1:

$$\boxed{P_k(1,i) = \int dx\, dx'\, e^{ikx} e^{-ikx'} \times \left\{ \varphi_i^*(x)\, S_1(x,x')\, \varphi_i(x') \right\}}$$

$S \to$ detector response

Prob of 1 particle in detector 1, & 1 particle in 2:

$$P_k(1,2;i,j) = \int dx\, dx'\, e^{ik(x-x')} S_1(x,x') \int dx''\, dx'''\, e^{ik(x''-x''')} S_2(x'',x''') \times$$
$$\left(\varphi_i^*(x)\varphi_j^*(x'') + \varphi_j^*(x)\varphi_i^*(x'')\right)\left(\varphi_i(x')\varphi_j(x''') + \varphi_j(x')\varphi_i(x''')\right)$$

$$\boxed{P_k(1,2;i,j) = P_k(1,i)P_k(2,j) + P_k(2,i)P_k(1,j) + \text{exchange terms}}$$

2-particle correlation function

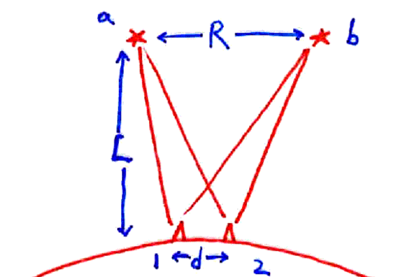
$$\langle \psi_i^\dagger(x)\, \psi_j^\dagger(x'')\, \psi_j(x''')\, \psi_i(x') \rangle$$

(HBT, Nature 177, 27 '56 ; 178, 1046, '57
E. Purcell, Nature 178, 1449 ; G. Baym Act. Phys. Pol B 29, 1, etc)

④

HBT History-

Radio Interferometry



Amplitudes

$$A_1 = \frac{1}{L}\left(\alpha e^{ikr_{1a} + i\varphi_a} + \beta e^{ikr_{1b} + \varphi_b}\right)$$

Intensities

$$I_1 = A_1^2$$

$$\langle I_1 \rangle = \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2\right)$$

Intensity Correlations

$$\boxed{\frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} = 1 + 2\frac{|\alpha|^2|\beta|^2}{(|\alpha|^2 + |\beta|^2)^2}\cos\left(k(r_{1a} - r_{2a} - r_{1b} + r_{2b})\right)}$$

– Variation over $\frac{\lambda L}{R}$ (obtain $\theta = R/L$)

(see Nature, Dec '52 vol 170 ;
G. Baym, Acta Phys. Pol. B 29, 1 '98; PS1, Course 217B, Phys., UCSB '99)

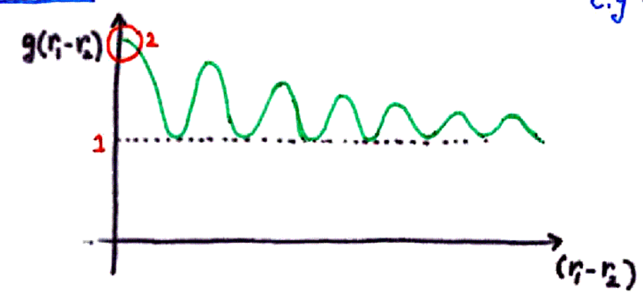
③

Density-density Correlation

$$\boxed{g(r_1, r_2) = N(N-1)\int dr_3 \dots dr_n |\Psi(r_1, r_2 \dots r_n)|^2}$$

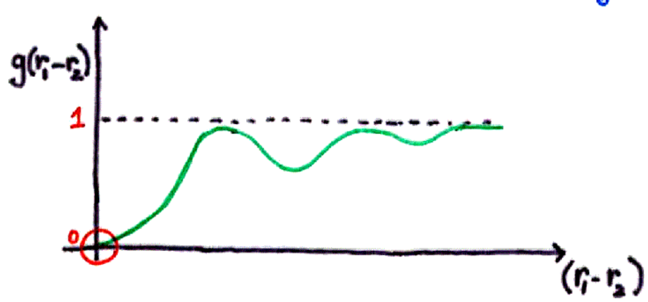
Statistics

Bosons



E.g: Photons from two different sources

Fermions



E.g. Non-interacting Fermi gas of spinless electrons

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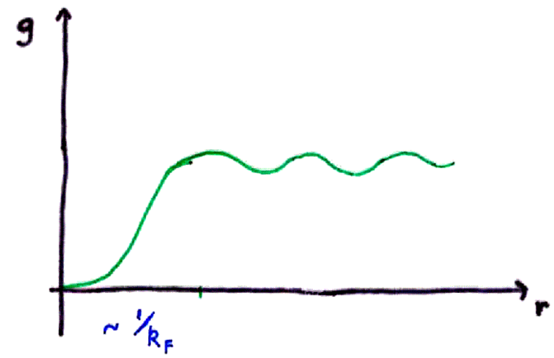
Density-density Correlation

(contd.)

Other Information



E.g 1-d non-interacting Fermi gas

$$g = \left[1 - \left(\frac{\sin k_F r}{k_F r}\right)^2\right]$$

Characteristics of System

- Oscillations
  Position / Momentum
  Energy / Time
- Decay

HBT Correlation Regime

$$\Delta x \ll (\Delta k)^{-1}$$
$$\Delta t \ll \omega^{-1}$$
$$kT \ll \Delta E$$

(E.g G. Baym, Act. Phys. Pol B, 29 1)

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January 7, 1956 NATURE 29

COMPARISON BETWEEN THE THEORETICAL AND EXPERIMENTAL VALUES OF THE CORRELATION

Cathodes superimposed ($d = 0$)		Cathodes separated ($d = 2a = 1\cdot8$ cm.)	
Experimental ratio of correlation to r.m.s. deviation $S_e(0)/N_e$	Theoretical ratio of correlation to r.m.s. deviation $S(0)/N$	Experimental ratio of correlation to r.m.s. deviation $S_e(d)/N_e$	Theoretical ratio of correlation to r.m.s. deviation $S(d)/N$
+7·4	+8·4	−0·4	≃ 0
+6·6	+8·0	+0·5	≃ 0
+7·6	+8·4	+1·7	≃ 0
+4·2	+5·2	−0·3	≃ 0

correlation is virtually zero. The ratio of $S_e(d)$, the counter reading after 90 minutes, to N_e, the root mean square deviation, is shown in the third column of Table 1.

The results shown in Table 1 confirm that correlation is observed when the cathodes are superimposed but not when they are widely separated. However, it may be noted that the correlations observed with $d = 0$ are consistently lower than those predicted theoretically. The discrepancy may not be significant but, if it is real, it was possibly caused by defects in the optical system. In particular, the image of the arc showed striations due to imperfections in the glass bulb of the lamp; this implies that unwanted differential phase-shifts were being introduced which would tend to reduce the observed correlation.

This experiment shows beyond question that the photons in two coherent beams of light are correlated, and that this correlation is preserved in the process of photoelectric emission. Furthermore, the quantitative results are in fair agreement with those predicted by classical electromagnetic wave theory and the correspondence principle. It follows that the fundamental principle of the interferometer represented in Fig. 1b is sound, and it is proposed to examine in further detail its application to visual astronomy. The basic mathematical theory together with a description of the electronic apparatus used in the laboratory experiment will be given later.

We thank the Director of Jodrell Bank for making available the necessary facilities, the Superintendent of the Services Electronics Research Laboratory for the loan of equipment, and Mr. J. Rodda, of the Ediswan Co., for the use of two experimental phototubes. One of us wishes to thank the Admiralty for permission to submit this communication for publication.

[Oct. 5

[1] Hanbury Brown, R., and Twiss, R. Q., *Phil. Mag.*, 45, 663 (1954).
[2] Jennison, R. C., and Das Gupta, M. K., *Phil. Mag.* (in the press).

CORRELATION BETWEEN PHOTONS IN TWO COHERENT BEAMS OF LIGHT

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRI

By R. HANBURY BROWN
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AND
DR. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

December 29, 1956 NATURE

and Ferguson[1] have reported experimental results which they believe to be incompatible with the observation by Hanbury Brown and Twiss[2] of correlation in the fluctuations of two photoelectric currents evoked by coherent beams of light. Brannen and Ferguson suggest that the existence of such a correlation would call for a revision of quantum theory. It is the purpose of this communication to show that the results of the two investigations are not in conflict, the upper limit set by Brannen and Ferguson being in fact vastly greater than the effect to be expected under the conditions of their experiment. Moreover, the Brown–Twiss effect, far from requiring a revision of quantum mechanics, is an instructive illustration of its elementary principles. There is nothing in the argument below that is not implicit in the discussion of Brown and Twiss, but perhaps I may clarify matters by taking a different approach.

Consider first an experiment which is simpler in concept than either of those that have been performed, but which contains the essence of the problem. Let *one* beam of light fall on *one* photomultiplier, and examine the statistical fluctuations in the counting-rate. Let the source be nearly monochromatic and arrange the optics so that, as in the experiments already mentioned, the difference in the length of the two light-paths from a point A in the photocathode to two points B and C in the source remains constant, to within a small fraction of a wave-length, as A is moved over the photocathode surface. (This difference need not be small, nor need the path-lengths themselves remain constant.) Now it will be found, even with the steadiest source possible, that the fluctuations in the counting-rate are slightly greater than one would expect in a random sequence of independent events occurring at the same average rate. There is a tendency for the counts to clump'. From the quantum point of view this is not surprising. It is typical of fluctuations in a system of bosons. I shall show presently that this extra fluctuation in the single-channel rate necessarily implies the cross-correlation found by Brown and Twiss. But first I propose to examine its origin and calculate its magnitude.

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and it has b Now $\alpha\overline{P}$ is correlation approximat width $\Delta\nu$; half intens $\tau_0 = (\pi\Delta\nu)^-$ gular densi fractional i the 'norma about equa interval 1/ much smal this way, d If one i packets an of the extr given. But as a rule, t of wave pa sequence. such train they inter loosely) f between a ference in experimen photons a interferenc fluctuation

Model Systems - Bosons

Optics

Table-top apparatus with 2 detectors

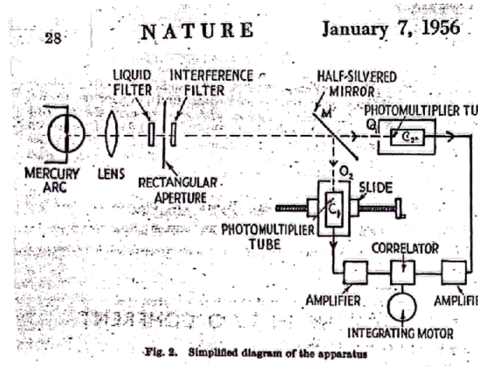
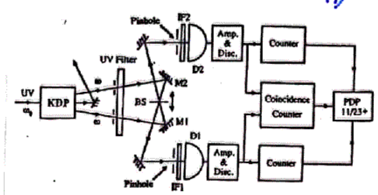


Fig. 2. Simplified diagram of the apparatus

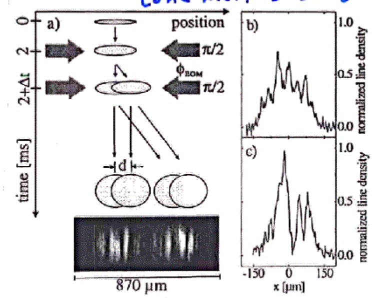
(L. Mandel, RMP 71, S274, '99)

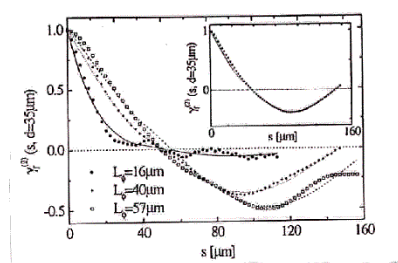


Bose-Einstein Condensates

D. Hellweg et al, cond-mat/0303308

Measuring the spatial correlation function





2 interfering spatially displaced BEC's

- Scattering: 3-body recombination rates for condensed vs uncondensed bosons E. Burt et al, PRL 79, 337 '97

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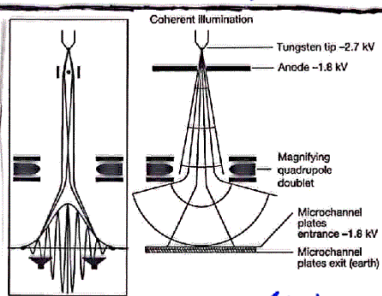
Model Systems - Fermions

Free Space

Observation of Hanbury Brown–Twiss anticorrelations for free electrons

Harald Kiesel, Andreas Renz & Franz Hasselbach

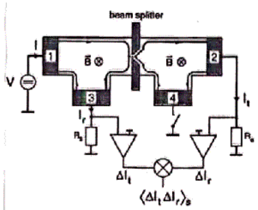
Nature 418, 392 '02

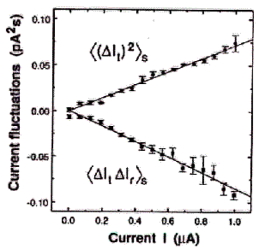


(Anti)Correlations in arrival times at detectors.

(Interactions?)

Semi-conductor Devices



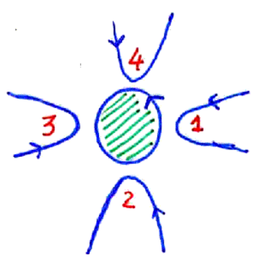


Beam splitter set-ups

(Eg. W. Oliver et al Sci 284, p299 '99; M. Henny et al, Sci 284, p296, '99)

A Quantum Hall set-up

M. Buttiker PRB 46, 12485 '92; Blanter et al, Phys. Rep 336, 1 '00.



Statistics explicitly seen in

$$\langle \Delta I_{12} \Delta I_{34} + \Delta I_{14} \Delta I_{32} \rangle$$

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A qtm Hall set-up

$$\langle \Delta I_2 \Delta I_4 \rangle_c = \langle \Delta I_2 \Delta I_4 \rangle_A + \langle \Delta I_2 \Delta I_4 \rangle_B$$

$$\mp 4 \Delta\nu \frac{e^2}{h} \int dE\, (f - f_0)\, T_1 T_2 T_3 T_4 \cdot R_2 R_3 / |Z|^4$$

$$Z = \left[1 - \sqrt{R_1 R_2 R_3 R_4}\, e^{i\chi}\right], \quad \chi = \sum_{i=1}^{4} (\phi_i + \Delta\phi_i)$$

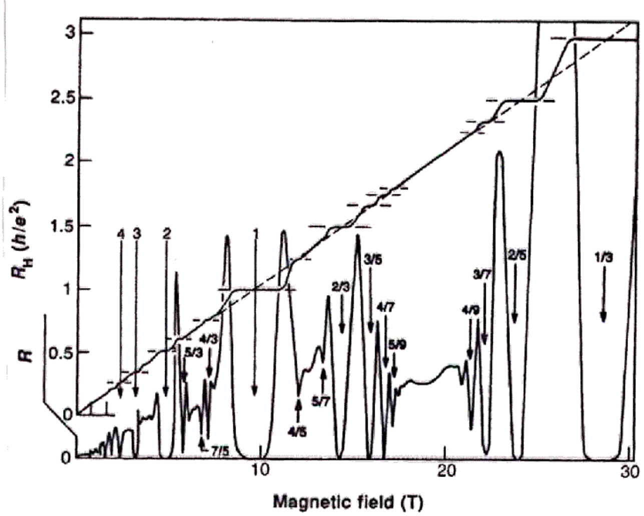
– M. Buttiker, PRB 46, 12485, '92

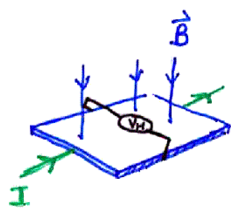
Anyons- fractional statistics

$$\boxed{\Psi(r_1, r_2, r_3, \ldots, r_n) = e^{\pm i\pi\alpha}\Psi(r_2, r_1, r_3, \ldots, r_n)}$$

($\alpha = 1 \longrightarrow$ Fermions
$\alpha = 0 \longrightarrow$ Bosons)

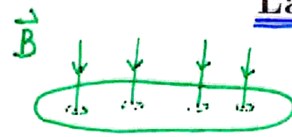
Fractional quantum Hall effect





(9) (for e.g. H.L. Stormer, Physica B177, 401 '92 ; R.Laughlin, RMP 71, 863 '99)

Laughlin States



(e.g. R.Laughlin, RMP 71, 863 '99)

N states (flux q'ta)

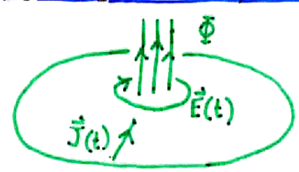
νN e^-

Filling Fraction

$\nu = 1/(\text{odd integer})$

Quasiparticles/holes

(e.g. S.Girvin in Les Houches lectures, '98)



Fractional Charge

1 extra flux qtm h/e $\nearrow$ $e^* = \nu e$

Anyons

(e.g. E.Fradkin in Qtm.Phys.at Mesoscopic Scales)

$\Delta\theta = 2\pi\nu$

Fractional Statistics

$e^{\pm i\pi\nu}$ upon exchange

Phase factor picked up by one q.h. encircling another

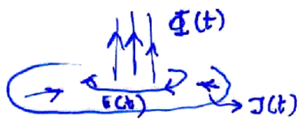
(D. Arovas et al., PRL 53, 722 '84
F.D.M. Haldane, PRL 67, 937 '91)

- Correlations in bulk ?
- Filling up states ?

(10)

Laughlin States

Fractional charge



$$\oint d\vec{r}\cdot\vec{E} = -\frac{1}{c}\frac{\partial\Phi}{\partial t}$$

$$\vec{E} = \rho_{xy}\,\vec{J}\times\hat{z}$$

$$\rho_{xy}\oint\vec{J}\cdot(\hat{z}\times d\vec{r}) = -\frac{1}{c}\frac{d\Phi}{dt}$$

$$\rho_{xy}\frac{dQ}{dt} = -\frac{1}{c}\frac{d\Phi}{dt} \longrightarrow Q = \frac{1}{c}\sigma_{xy}\Phi_0 = \frac{h}{e}\sigma_{xy}$$
$$= \nu e$$

Quasiholes (Arovas et al.)

Grd st: $\Psi_m = \prod_{j<k}(z_j - z_k)^m \exp\left(-\frac{1}{4}\sum_\ell |z_\ell|^2\right)$

$\downarrow$

$$\Psi_m^{+z_0} = N_+\prod_i (z_i - z_0)\Psi_m$$

- Charge: Calculate change of phase γ upon encircling one flux qtm $\left(\frac{d\gamma}{dt} = i\langle\Psi(t)|d\Psi(t)/dt\rangle\right)$

- Statistics: Phase accumulated by 1 particle/hole encircling another $\left(\Psi_m^{a,b} = N_{ab}\prod(z_i - z_a)\times(z_i - z_b)\Psi_m\right)$
  $\Delta\gamma = 2\pi\nu$

($\Delta\gamma$ can be accounted for by including statistical vector potential $\vec{A}_\varphi$, $e^*/\hbar c\oint\vec{A}_\varphi\cdot d\ell = \Delta\gamma$,
$\vec{A}_\varphi(\vec{r}-\vec{r}_b) = \phi_0\,\hat{z}\times(\vec{r}-\vec{r}_b)/2\pi|\vec{r}-\vec{r}_b|^2$)

Quantum Hall Edge-States

Gapless Edge Excitations

- Chiral Luttinger liquid

$$H = \frac{1}{4\pi\nu}\int(\partial_x\phi)^2dx$$

$$[\phi(x),\phi(x')] = i\pi\nu\,\mathrm{sign}(x - x')$$

$\delta\rho\sim\partial_x\phi$

Edge-State Quasiparticles

- Charge νe, fractional statistics

$$\boxed{\psi^\dagger(x) = \kappa_j e^{-i\phi(x)}}$$

(e.g. X.G. Wen, PRB 43, 11025 '91; Adv. Phys. 44, 405 '95; E. Fradkin in Qtm. Phys. at Mesoscopic Scales)

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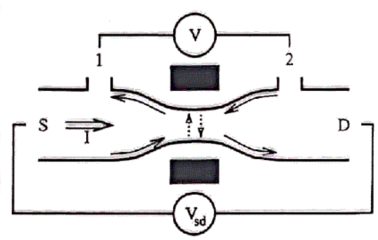
Measuring Fractional Charge

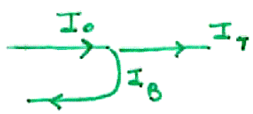


- Current Noise S_I

- Shot noise form at $T=0$:

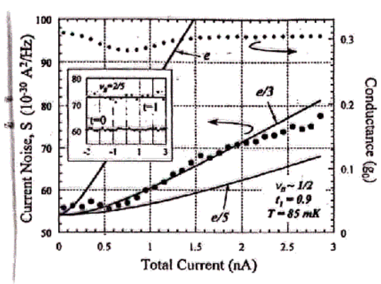
$$\boxed{S_I(\omega=0) = e^* \langle I_B \rangle}$$

for weak backscattering ($e^* = \nu e$)

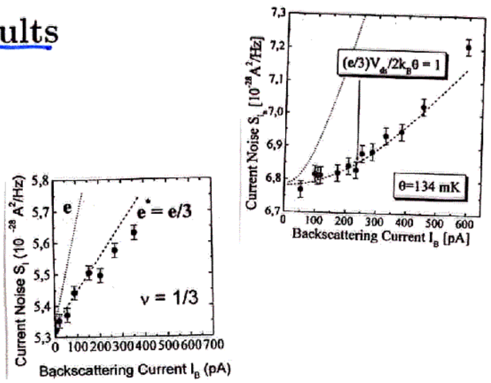


C. Kane & M.P.A. Fisher, PRL 72, 724 '94

Experimental Results



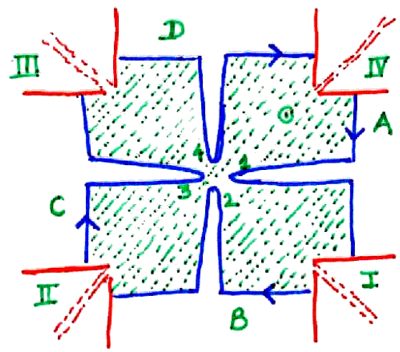
M. Reznikov et al., Nature 399, 238 '99



L. Saminadayar et al, PRL 79, 2526 '97

(12)

Measuring Fractional Statistics



A-B - Edge States

I-IV - Leads

1-4 - Tunneling points (controlled by gates)

Statistics

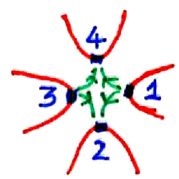
Between tunneling q.p. :

$$\boxed{\psi_j^\dagger \psi_k^\dagger = e^{-i\pi\nu} \psi_k^\dagger \psi_j^\dagger}$$

$j<k$ for $j=1,2,3$

$k=1$ for $j=4$

Weak Tunneling



$$\boxed{\mathcal{H}^{jk} = u_{jk} \psi_j^\dagger \psi_k + \text{h.c.}}$$

(S.V. cond-mat/0304568 ; I. Safi et al, PRL 86, 4628, '01)

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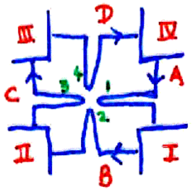
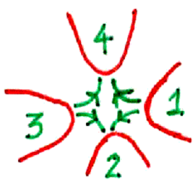
Sources, Sinks, Currents



- Edges A, C at voltage V w.r.t. B, D

- $\boxed{\begin{array}{l}\text{Sources : } m = 1, 3 \\ \text{Sinks : } n = 2, 4\end{array}}$



- Currents

$$\boxed{\begin{array}{l} I_{I} = \frac{2e^2}{h}V - I_{12} - I_{14} \; ; \quad I_{II} = I_{12} + I_{32} \\ I_{III} = \frac{2e^2}{h}V - I_{32} - I_{34} \; ; \quad I_{IV} = I_{34} + I_{14} \end{array}}$$

Tunneling Currents

$$\boxed{I_{mn}(t) = \frac{ie^*}{\hbar}(u_{mn}\psi_m^\dagger\psi_n e^{i\tilde{V}t} - \text{h.c.})}$$

$\tilde{V} = \frac{e^* V}{\hbar}$

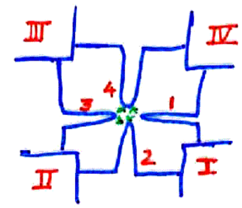
Perturbative treatment :

$$\frac{d\langle I_{mn}\rangle}{dV} = V^{2\nu-2} f(e^* V/kT)$$

(non-eq. finite T technique)

(14)

HBT Correlator

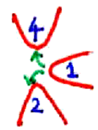


Given

$$\langle \Delta I_{II} \Delta I_{IV} \rangle = \langle\langle (\Delta I_{12}\Delta I_{34} + \Delta I_{12}\Delta I_{14} + \Delta I_{32}\Delta I_{34} + \Delta I_{32}\Delta I_{14}) \rangle\rangle$$

$(\Delta I = I - \langle I \rangle)$

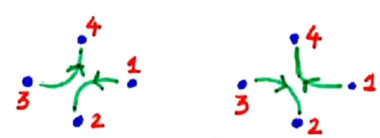
Subtract off



$\langle \Delta I_{12} \Delta I_{14} \rangle$, $\langle \Delta I_{32} \Delta I_{34} \rangle^{\star}$ to get

(measured by turning off sources m=3 & m=1 respectively, by using gates)

Interesting piece :



$$\boxed{C(t-t') = \langle \Delta I_{12}(t)\Delta I_{34}(t') + \Delta I_{14}(t)\Delta I_{32}(t') \rangle}$$

★ Also carries statistical information : J. Safi et al, PRL 86, 4628 '01

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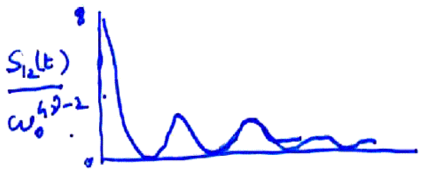
J. Safi, P. Devillard, T. Martin

$$S_{12}^{+}(t) = \frac{4\,|e^{*}\tau_0\,|^2}{(h\alpha)^4}\,\mathrm{Re}\int_{t_1,t_2}\sum_{\epsilon=\pm}\epsilon$$

$$\times \cos[\omega_0(t_1+\epsilon t_2)]\,(1-it_1)^{-2\nu}(1+it_2)^{-2\nu}(1-it)^{\epsilon\nu}$$

$$\times [1+i(t+t_2-t_1)]^{\epsilon\nu}[1+i|t+t_2|]^{-\epsilon\nu}(1-i|t_1-t|)^{-\epsilon\nu}$$

$$\times \exp\left(i\epsilon\frac{\pi}{2}\nu[\mathrm{sgn}(t+t_2)-\mathrm{sgn}(t-t_2)]\right)$$



$$\tilde{S}_{12}(\omega=0)\sim(\sin\pi\nu)^2$$

Calculating $\mathcal{C}$

(perturbation in tunneling)

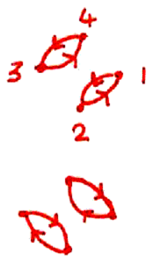
$$\mathcal{C} = \langle(\Delta I_{12}\Delta I_{34}+\Delta I_{14}\Delta I_{32})\rangle$$

4th order in u :

i) Separable

$\langle I_{12}I_{34}S_{12}S_{34}\rangle_0$, $\langle I_{12}I_{34}S_{34}S_{12}\rangle_0$

$\langle I_{14}I_{32}S_{14}S_{32}\rangle_0$, $\langle I_{14}I_{32}S_{32}S_{14}\rangle_0$



ii) Inseparable

$\langle I_{12}I_{34}S_{14}S_{32}\rangle_0$, $\langle I_{12}I_{34}S_{32}S_{14}\rangle_0$

$\langle I_{14}I_{32}S_{12}S_{34}\rangle_0$, $\langle I_{14}I_{32}S_{34}S_{12}\rangle_0$

$$\boxed{\begin{aligned}\mathcal{C}(\Delta t) = {}& \langle I_{12}(t)\rangle\langle I_{34}(t')\rangle + \langle I_{14}(t)\rangle\langle I_{32}(t')\rangle \\ & + \cos\pi\nu\;\; C_{\diamond}(\Delta t)\end{aligned}}$$

$\Delta t = t - t'$

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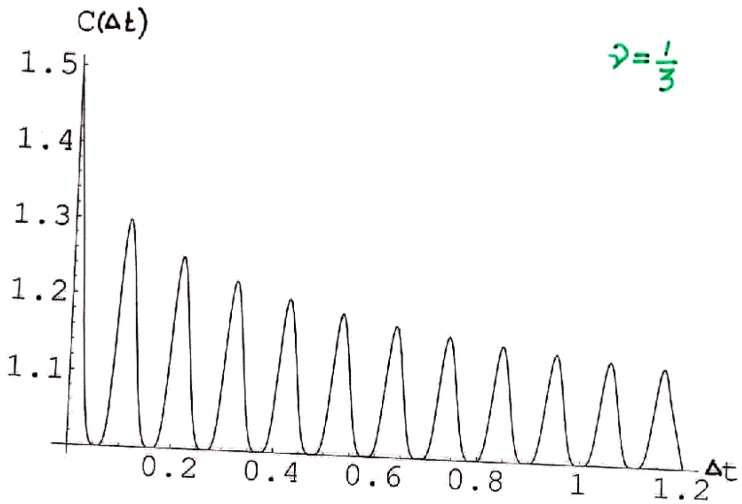
HBT Correlator (contd.)

$$\boxed{C_\diamond(\Delta t) = \left[\frac{4u^2 e^*}{\hbar^2}\left(\frac{V}{\epsilon_0}\right)^{2\nu}\frac{e^{-e^*V/\epsilon_0}}{V}\frac{\pi\cos\tilde{V}\Delta t/2}{\Gamma(2\nu)\Gamma(1-2\nu)}\right]^2 \times 2\left[\mathrm{Re}\left\{e^{-i\tilde{V}\Delta t/2}\int_0^\infty e^{-r} r^{-\nu}(r+i\tilde{V}\Delta t)^{-\nu}\right\}\right]^2}$$



C normalized by $2\langle I_{12}\rangle\langle I_{34}\rangle$

$\tilde{V} = 60$ (dimensionless units)

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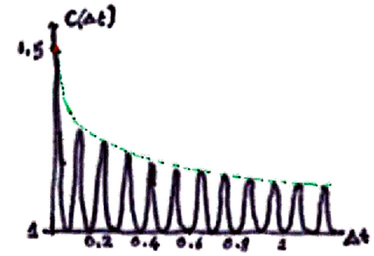
HBT Correlator (contd.)



- Fractional Statistics $(e^{\pm i\pi\nu})$

$$\boxed{C(\Delta t \to 0) = 2[1+\cos\pi\nu]\langle I_{12}\rangle\langle I_{34}\rangle}$$

- Fractional charge $(e^* = \nu e)$:

  Oscillations of period h/e^*V

  (Note : charge & statistics not simply related in higher states of hierarchy)

  E.g. Lopez et al., PRB 59, 15323 '99

- Chiral Luttinger liquid

  Power law decay $C_\diamond(\Delta t) \sim |\tilde{V}\Delta t|^{-2\nu}$, $|\tilde{V}\Delta t| \to \infty$

- Cross over to uncorrelated value at finite T : $kT \approx e^*V$

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From:
Universal structure of edge states of the FQH states — Ana Lopez & Eduardo Fradkin

Jain filling fraction $\left(\nu = p/(2np+1)\right)$ $p, n \in \mathbb{Z}$

$$\mathcal{L} = -\frac{1}{4\pi\nu}\left(\partial_1\phi_c\,\partial_0\phi_c - v\,\partial_1\phi_c\,\partial_1\phi_c\right) + \frac{1}{4\pi}\left(\partial_1\phi_N\,\partial_0\phi_N + \partial_1\phi_{N'}\,\partial_0\phi_{N'}\right)$$

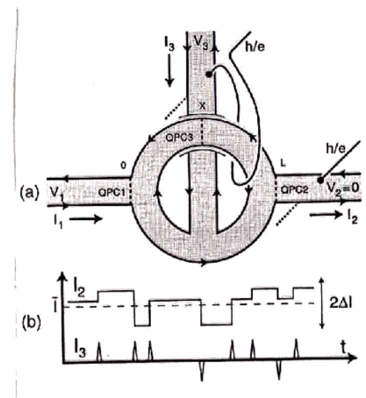
$c \rightarrow$ charge
$N, N' \rightarrow$ Neutral

$$\psi_{q,p} = e^{i\left[\frac{1}{p}\phi_c - \frac{1}{\sqrt{p}}\phi_N + \phi_{N'}\right]}$$

Other Proposals

(for frac. stat. in FQH systems)

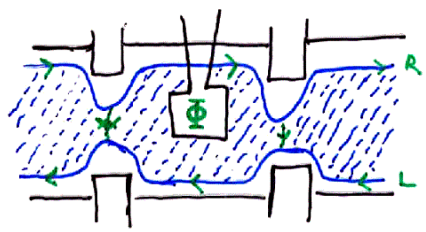
- Telegraph Noise



- Current-current correlations of I_2

C. L. Kane, cond-mat/0210621

- Two point-contact interferometer



→ Anomalous Aharanov-Bohm period due to fractional charge & fractional statistics.

C. Chamon et al., PRB 55, 2331 '97

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Telegraph noise C. Kane

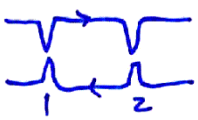
↳ Tunneling of Laughlin Q.P. from outer to inner circumference of ring → from 1 m-fold degen. grd. st. to another.

$I_2 = G_0 V_1 - I^b$

$$S(t) = \langle I_2(t_0) I_2(t_0+t) \rangle - \bar{I}^2$$

$$= \frac{\Delta I^2}{2} \mathrm{Re}\left[e^{-I_3|t|/e^*} \left(\coth\frac{e^* V_3}{2T} (1-\cos 2\theta_m) - i \sin 2\theta_m \right) \right]$$

Two point-contact interferometer Chamon, Freed, Kivelson, Sondhi, Wen



$$I_t = e^* |\Gamma_{eff}|^2 \frac{2\pi}{\Gamma(2g)} |\omega_J|^{2g-1} \mathrm{sgn}(\omega_J)$$

$$\omega_J = e^* V/\hbar$$

Electron tunneling

$\Gamma \to$ coupling

$$|\Gamma_{eff}|^2 = |\Gamma_1|^2 + |\Gamma_2|^2 + (\Gamma_1 \Gamma_2^* + \Gamma_1^* \Gamma_2) H_g\left(\frac{\omega_J a}{V}\right)$$

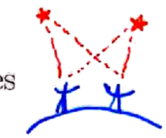
Has relative phase which

i) with B field, shows period Φ/Φ_0 if e^- density fixed

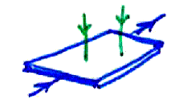
ii) with B field, " " Φ/Φ_0 if filling fraction fixed

In Conclusion,

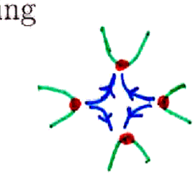
- The Hanbury Brown-Twiss experiment is an effective means of studying statistics of particles



- The fractional quantum Hall system could provide a playground for studying the physics of anyons



- Using the principles of the HBT experiment in the FQH system could well allow for unearthing fractional statistics



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