GRANULAR UNSTEADINESS

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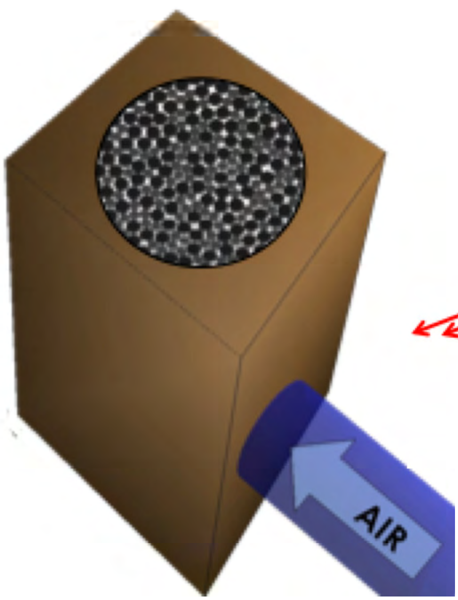
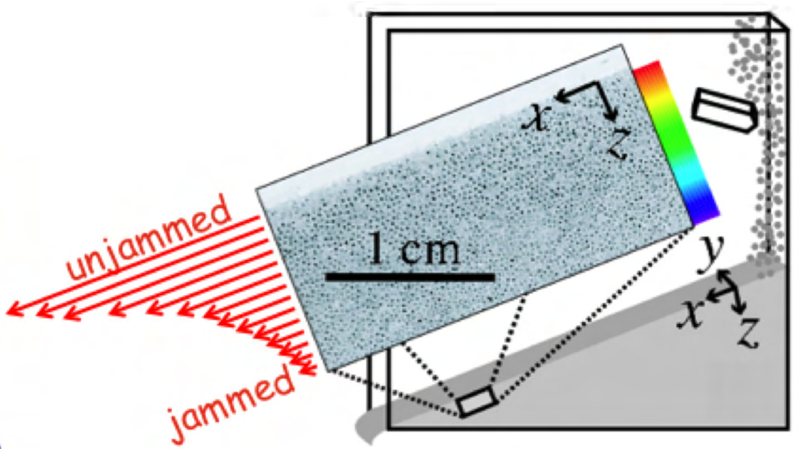
U Penn Physics

- Steady input of energy can provoke an unsteady response

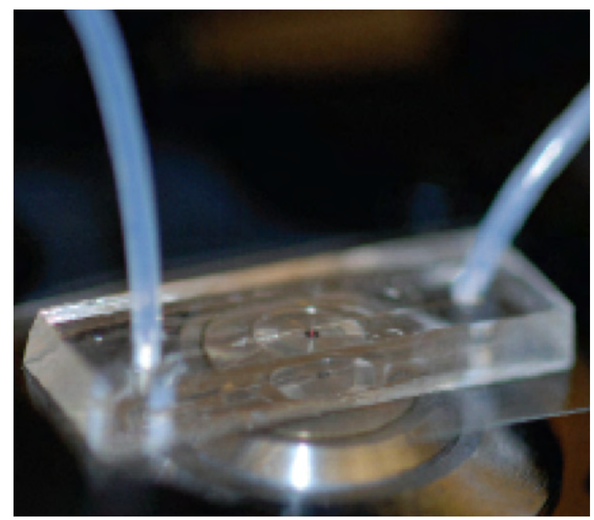
  eg: *dynamical heterogeneities* for three systems near jamming:



gas-fluidized monolayer



steady heap flow



soft colloids in a μchannel

CMMP-2010 decadal study

- What happens far from equilibrium, and why?
  - exemplified by granular media and jamming:

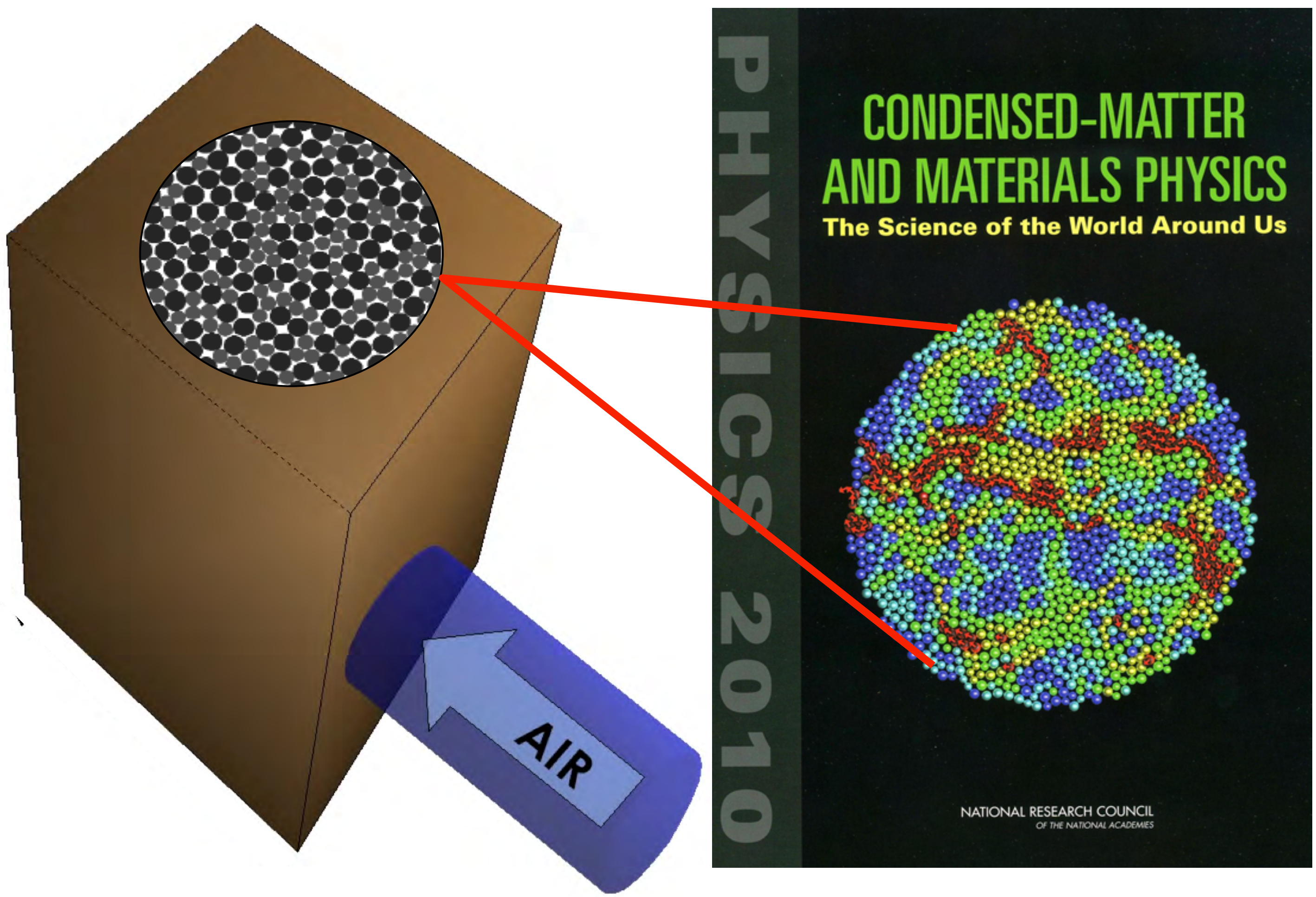
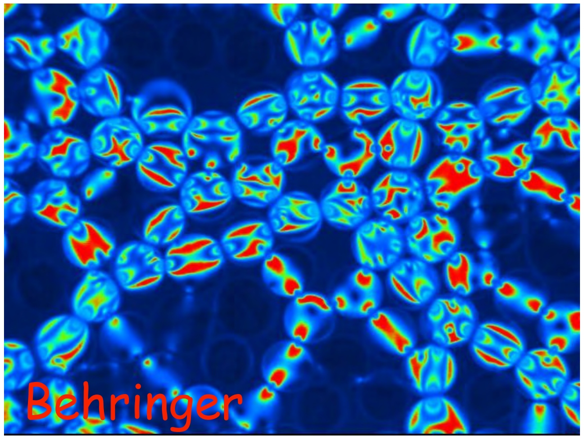
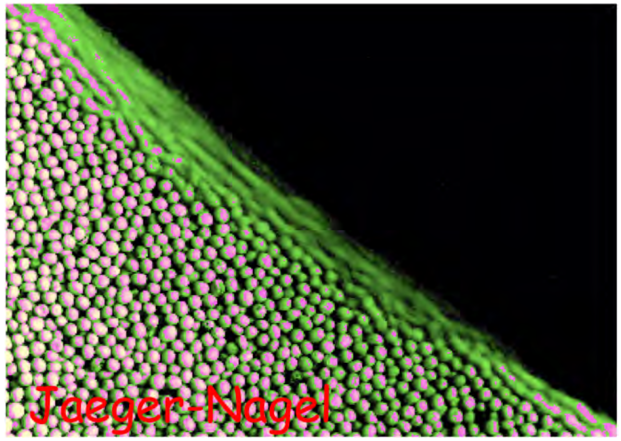
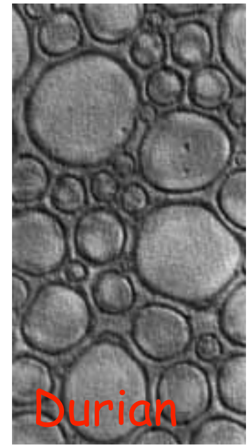


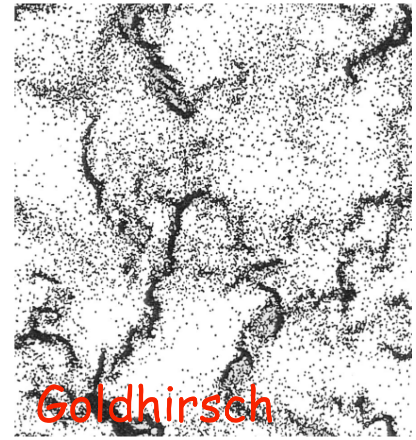
image: Keys - Abate - Glotzer - Durian

No basis for usual intuition









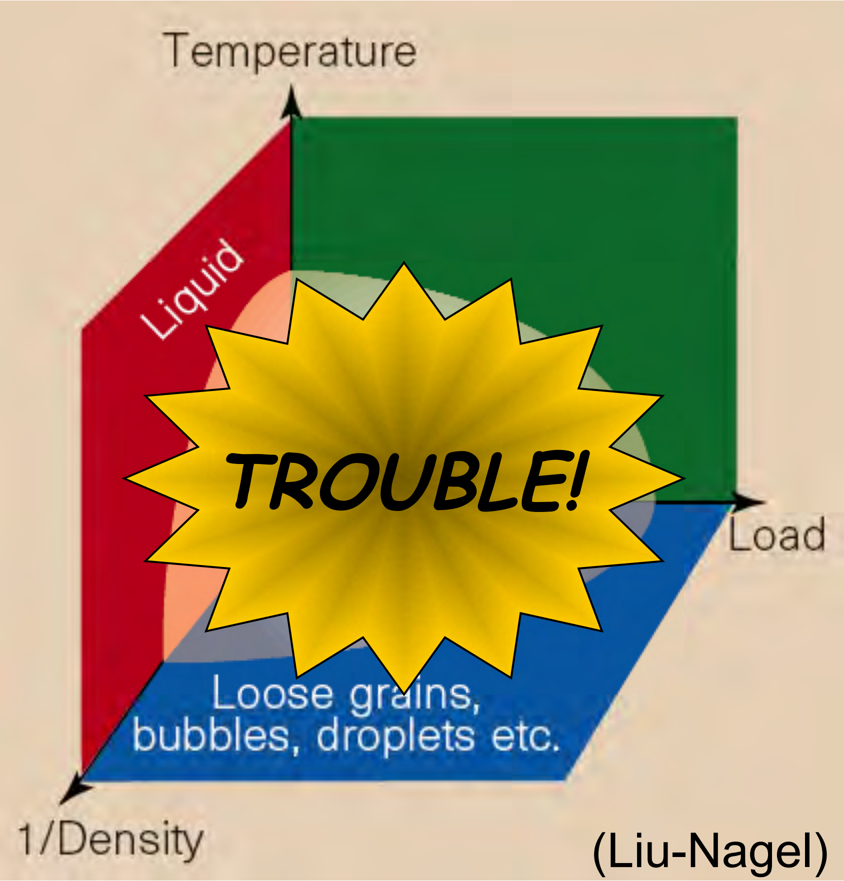
- Grains, bubbles, colloids, cells, tectonic plates,…

disordered / heterogeneous:	*no symmetries*
$k_B T$<<interaction energy:	*far-from-equilibrium*
flow beyond threshold:	*nonlinear response*

hard problems = new physics!
unreliable / inefficient engineering practices!

On approach to jamming…

- …effects of disorder, non-equilibrium, and nonlinearity all become more and more important



  - eg response to steady driving becomes more unsteady…

Unsteady response to steady driving

- Intermittency…

  …avalanches, rearrangements, mudslides, earthquakes

  …force chains in shear and impact/penetration

  …clogging / arching over an orifice

- Convection, size segregation, pattern formation, compaction, phase separation, in vibrated systems

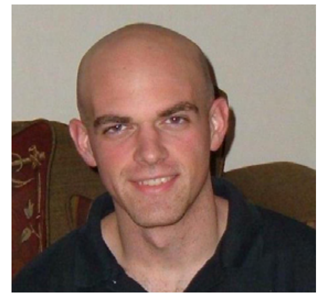
- Clustering and finite-time singularities in a freely cooling inelastic gas

- Swarming, density waves, giant #fluctuations for self-propelled particles and rods

- *And, of course, dynamical heterogeneities…*

1. Monolayer of air-fluidized balls

- upflow of gas randomly kicks the grains, without causing levitation
- 50:50 mixture with 1:1.4 diameter ratio, to prevent crystallization

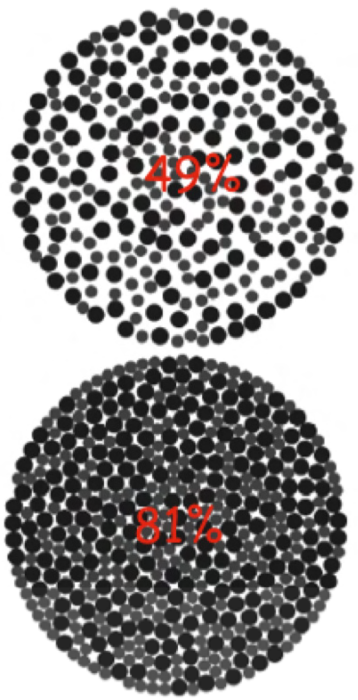


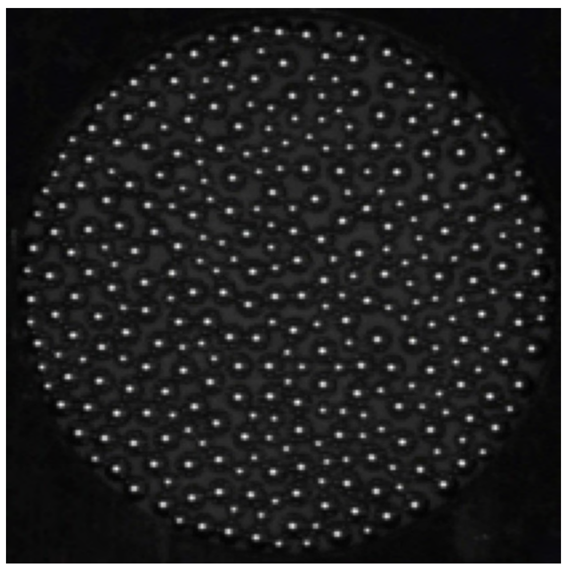
Adam Abate

Two approaches to jamming

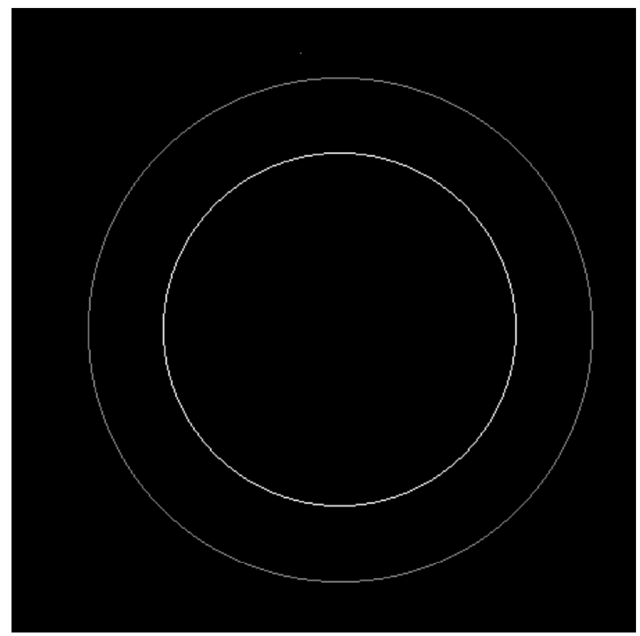
vs packing fraction

vs gas speed





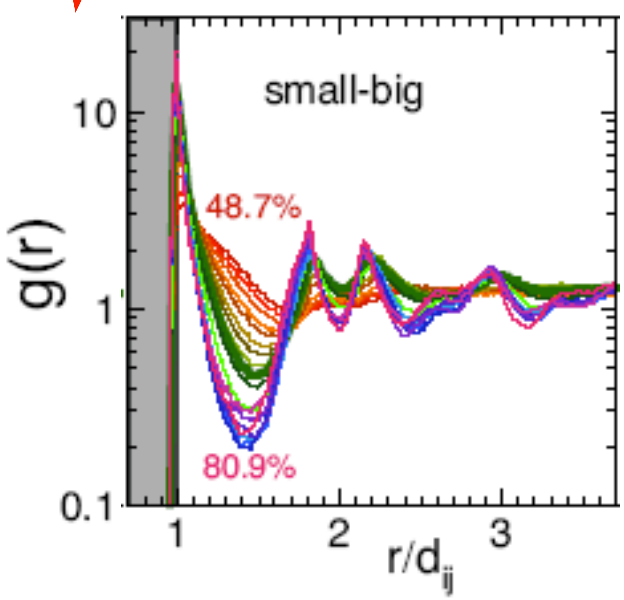
12" diameter sample

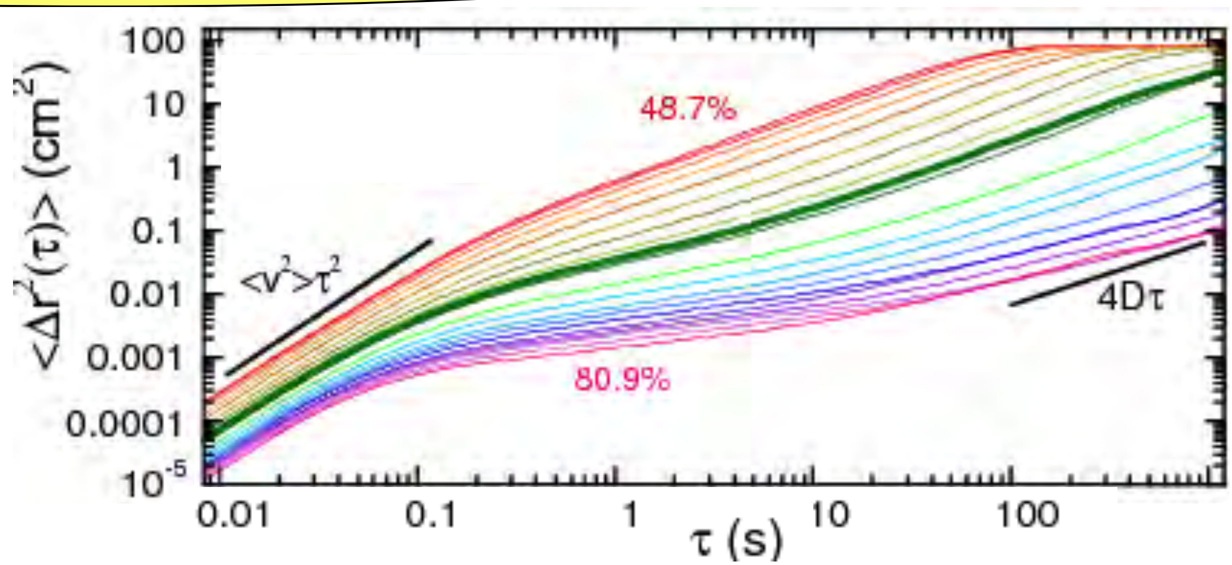


Approaches jamming like a glass

(Abate & Durian, PRE '06)

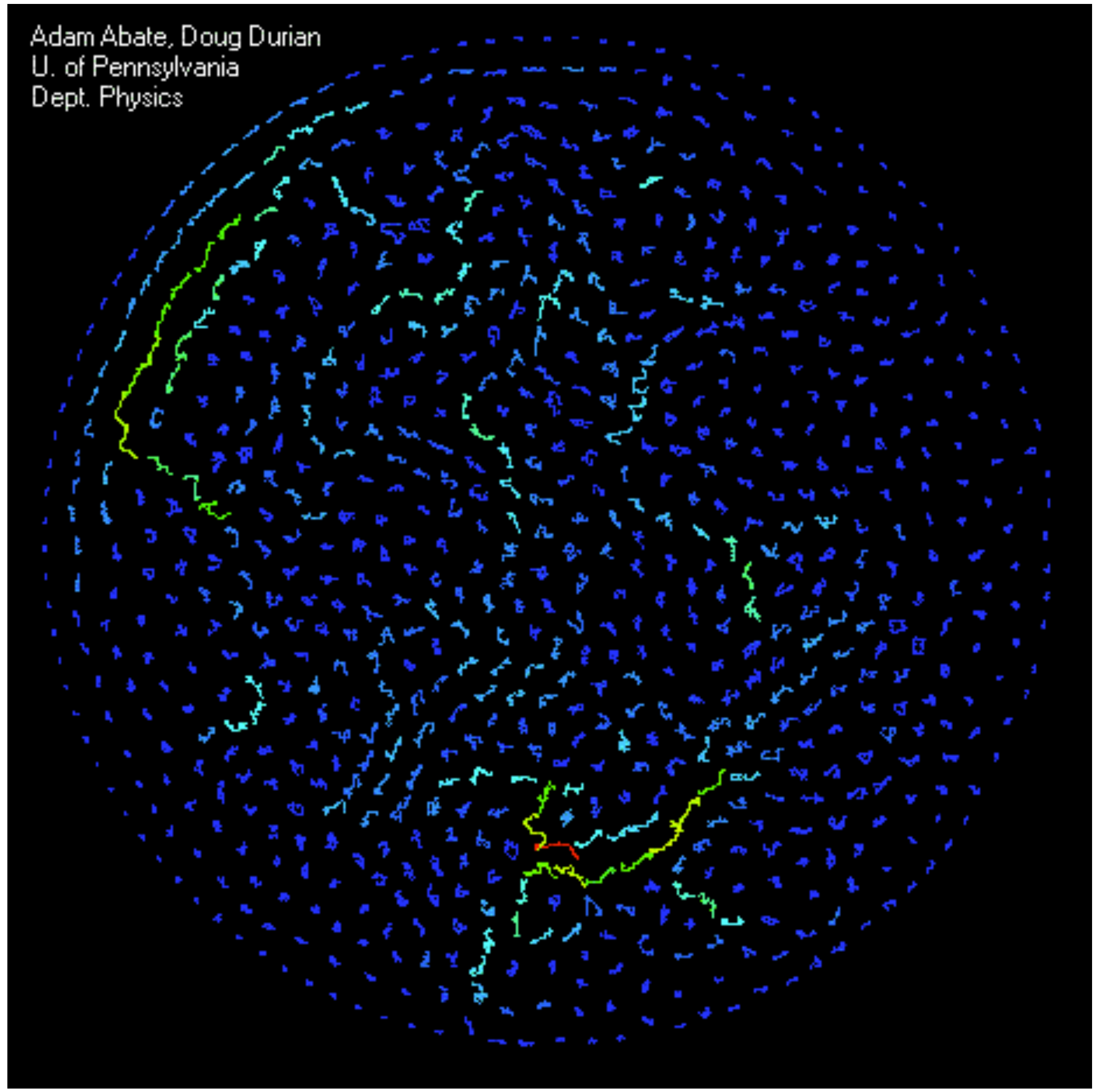
- Subtle structural changes:
  - growing 1st peak and split 2nd peak in g(r)
  - vanishing number of four-fold coordinations
- Dramatic dynamical changes:
  - Growing region of subdiffusive / caged motion
  - Displacement distribution becomes non-Gaussian
  - Dynamics become spatiotemporally heterogeneous





String-like intermittent swirls

{~movie of bead velocities, averaged over time τ = cage breakout time}



Lay tracks
from t to $t+\tau$

color-code
by avg. speed =
length of $\Delta r/\tau$

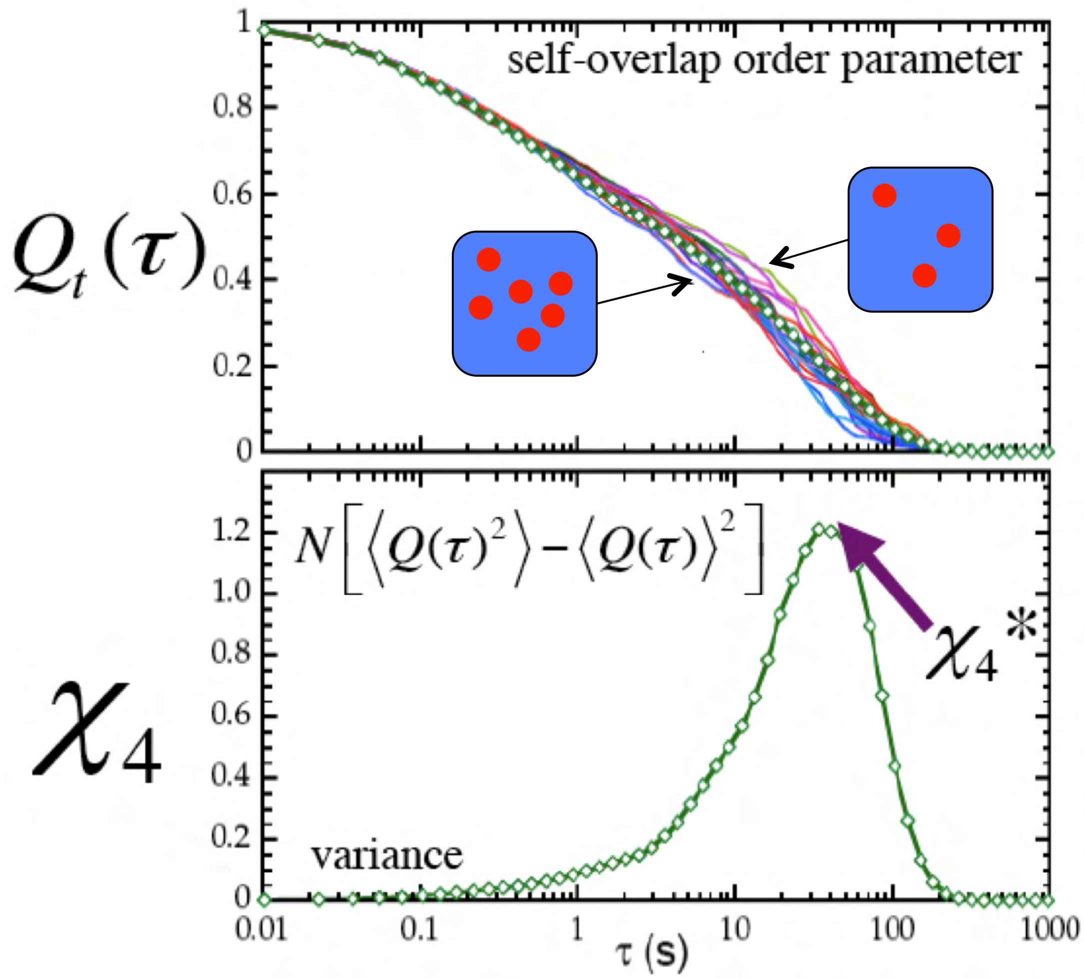
RED: fast/mobile
O
Y
G
BLUE: slow

To quantify heterogeneities…

…use $\chi_4(l,\tau)$, a four-point dynamic susceptibility:

variance in decay of dynamical order parameter

Faster decay of $Q_t(\tau)$ when there are more fast/mobile regions



χ_4^* ~ n^*, the size of the heterogeneities

$\chi_4 \rightarrow$ flucts in number of "fast" regions

(Abate & Durian, PRE '07)

- Define…

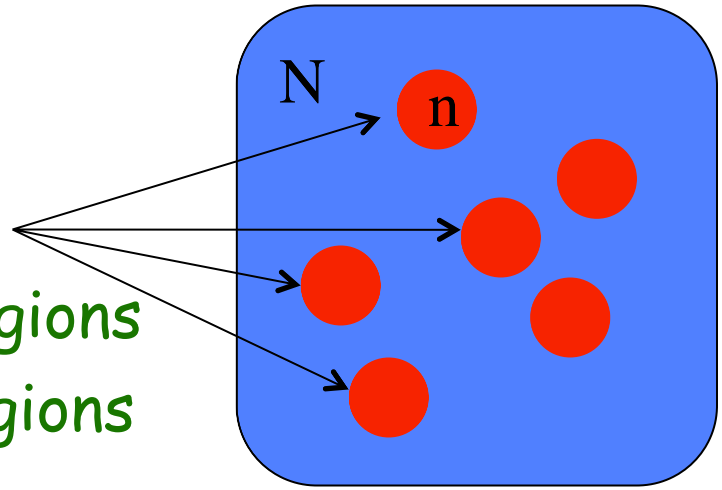
  N number of particles in system

  n particles in each fast region

  $M+/-M^{\frac{1}{2}}$ number of fast regions

  Q_0 = order parameter in fast regions

  Q_1 = order parameter in slow regions



- Compute average Q and its variance…

  $Q^* = [Q_0 (nM) + Q_1 (N-nM)]/N$

  $\chi_4^* = N(\Delta Q^*)^2$

  #particles in heterogeneity: $n^* = \chi_4^* / [(Q_1-Q_0)(Q_1-Q^*)]$

Flucts Δn in domain size are harmless:

(Abate & Durian, PRE '07)

- Define…

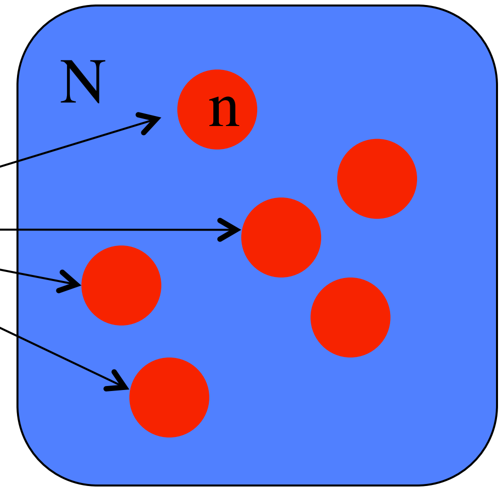
  N number of beads in system

  n+-Δn beads in each fast region

  M+/-$M^{\frac{1}{2}}$ number of fast regions

  Q_0 = order parameter in fast regions

  Q_1 = order parameter in slow regions

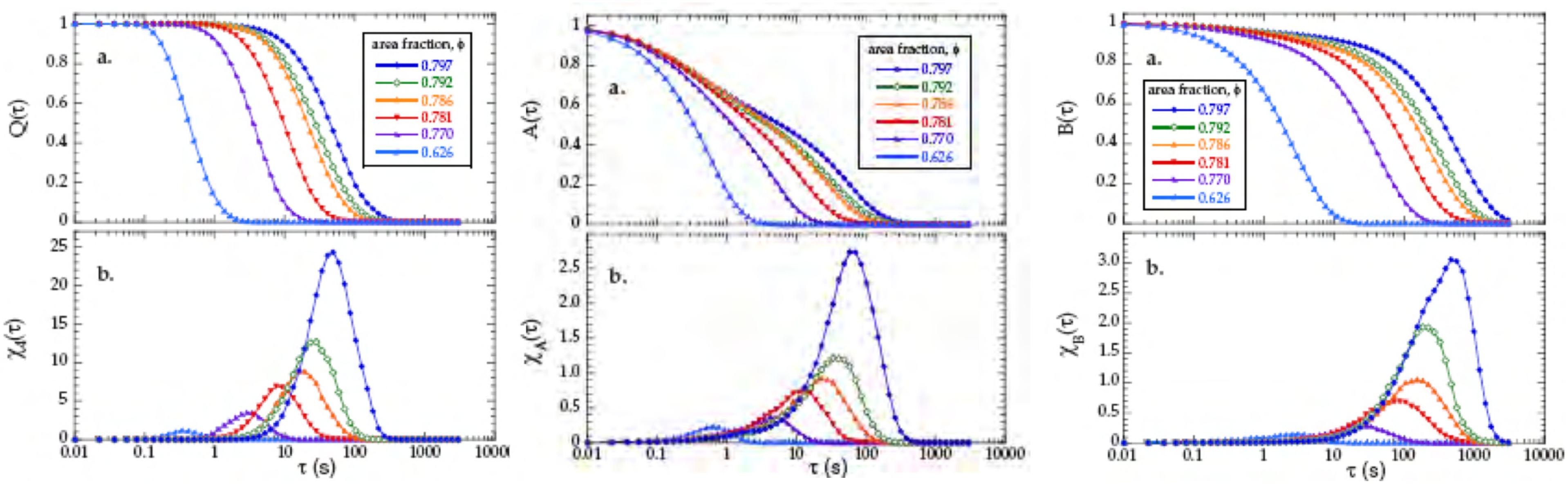


→ $n^* [1 + (\Delta n/n^*)^2] = \chi_4^*/ [(Q_1-Q_0)(Q_1-Q^*)]$

Constant airspeed, increasing ϕ

- Slower decays, increasing peak heights

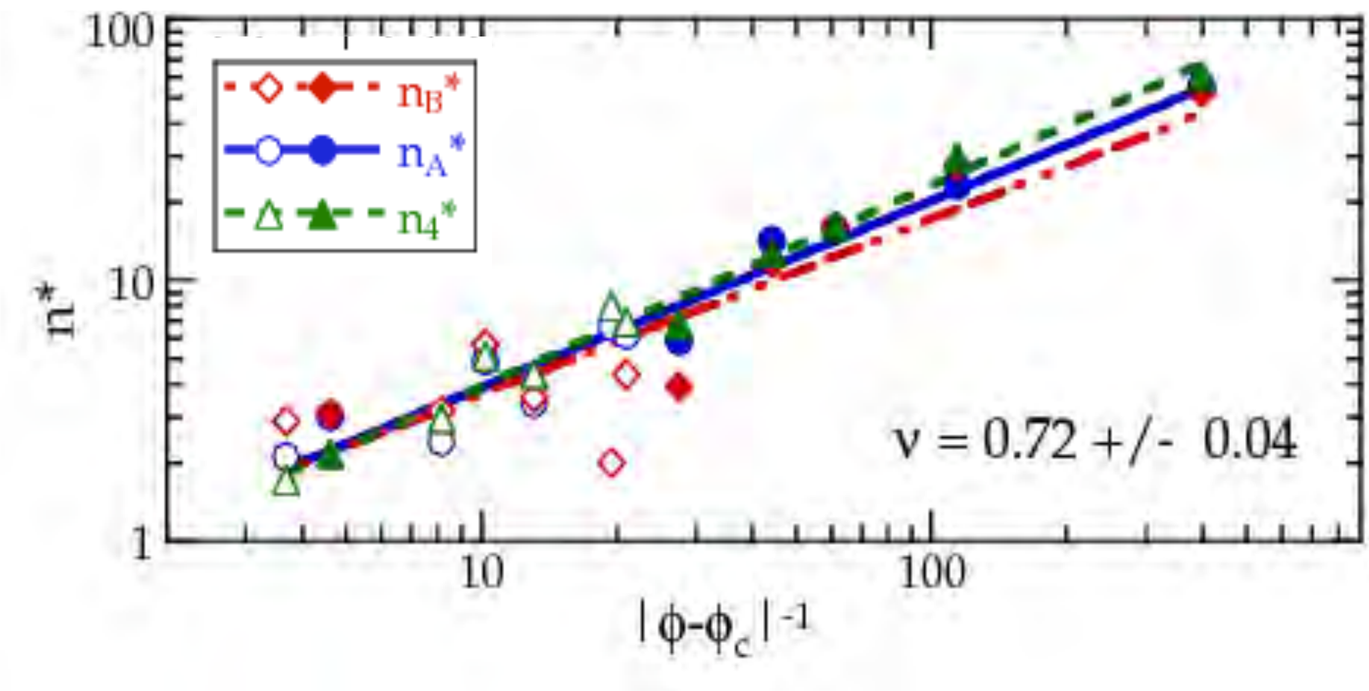
overlap — persistent area — persistent bond



Three different decay times & peak heights, and arbitrary choice of overlap cutoff function, ***but…***

…same size n* of heterogeneities

- Power-law growth on approach to jamming from below:



  - Exponent ν=0.72+-0.04 is curiously consistent with simulations:
    - OHern-Silbert-Liu-Nagel (2003) finite size scaling
    - Drocco-Hasting-Olsen-Reicchardt (2005) perturbation around object
    - Olsson-Teitel (PRL 2007) velocity correlations
- *But $\langle v^2\rangle$ decreases as $(\phi-\phi_c)$ so "T" isn't constant…??!!*

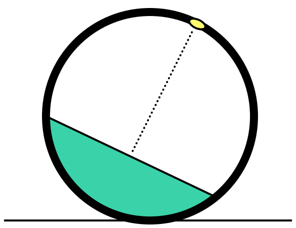
Three Effective Temperatures

(2) Granular: $T_g = \frac{1}{2} m \langle v^2 \rangle$

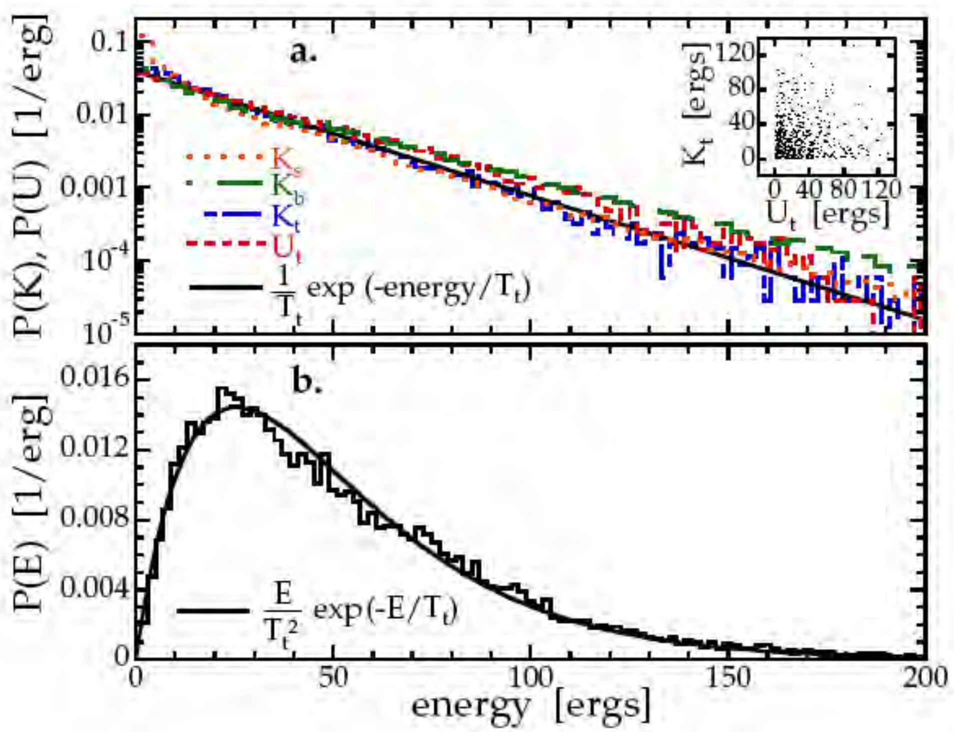
(2) Weighted-ball: $T_t = \langle KE \rangle = \langle PE \rangle$

(1) Einstein: $T_e = D / \mu$

(Abate & Durian, PRL ‘08)

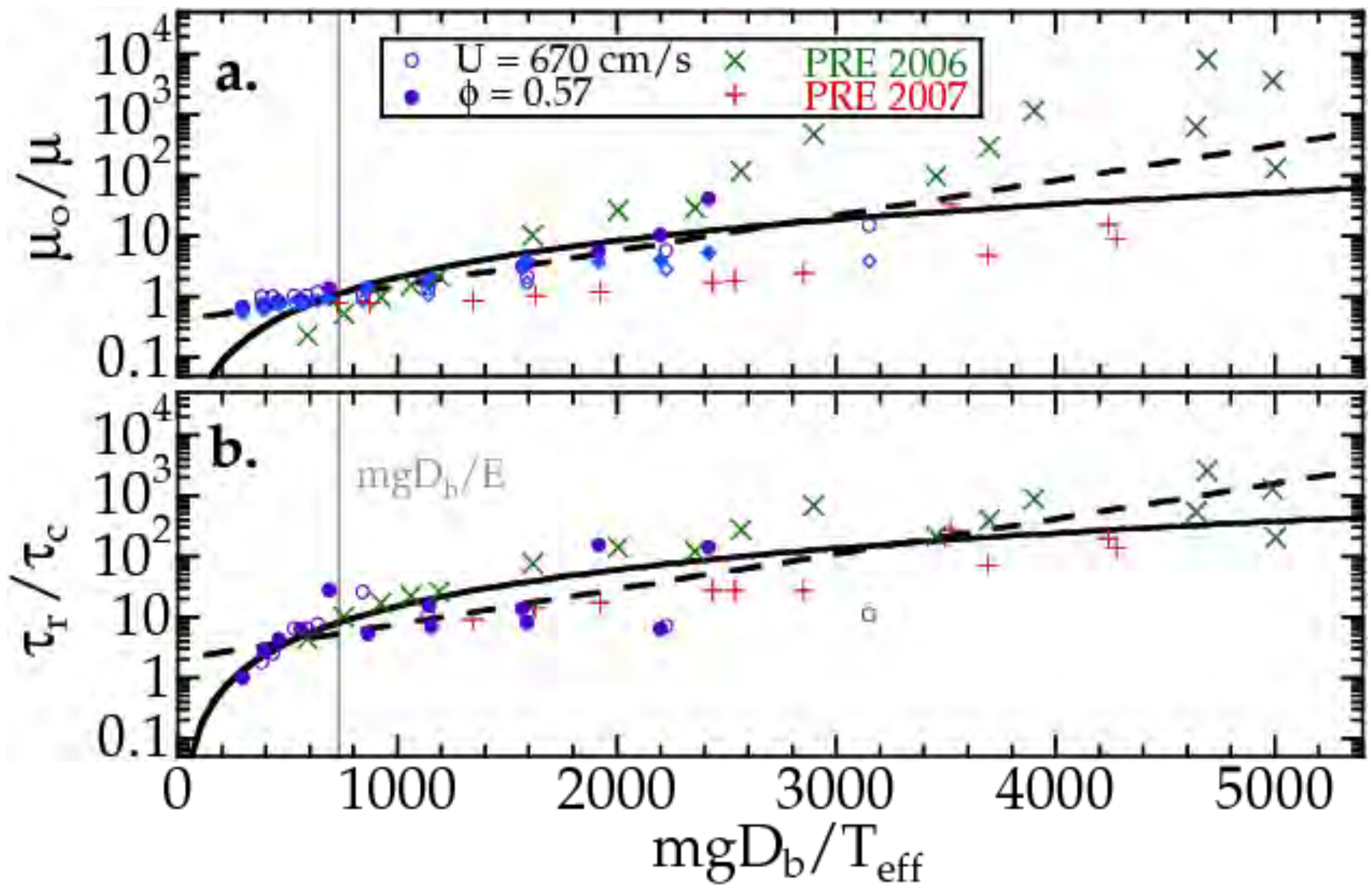


- Energies are thermally-distributed
- Near agreement of all five effective temps



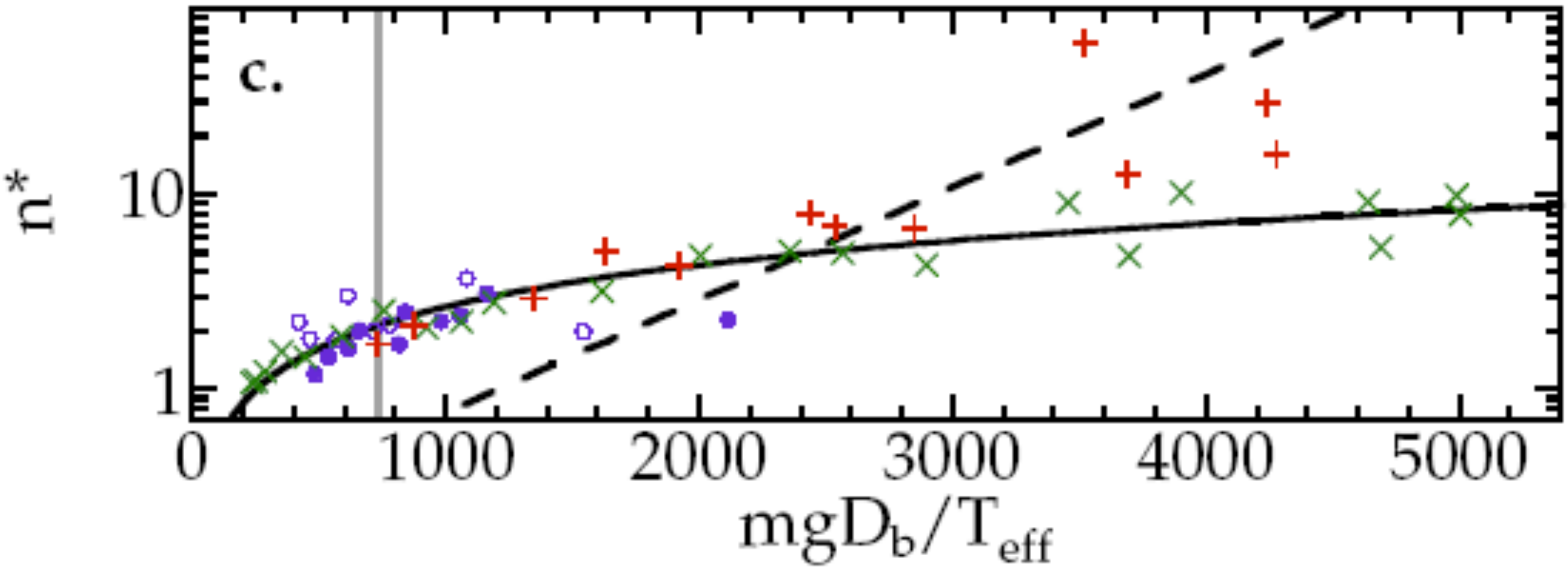
Activated dynamics

- 1/mobility and relaxation time ~ exp[E/T_{eff}]
  - collapse and same energy barrier, E, for both trajectories



Growth of heterogeneities vs T_{eff}

- Again, find good collapse for different ball sizes, packing densities, and gas-speeds:



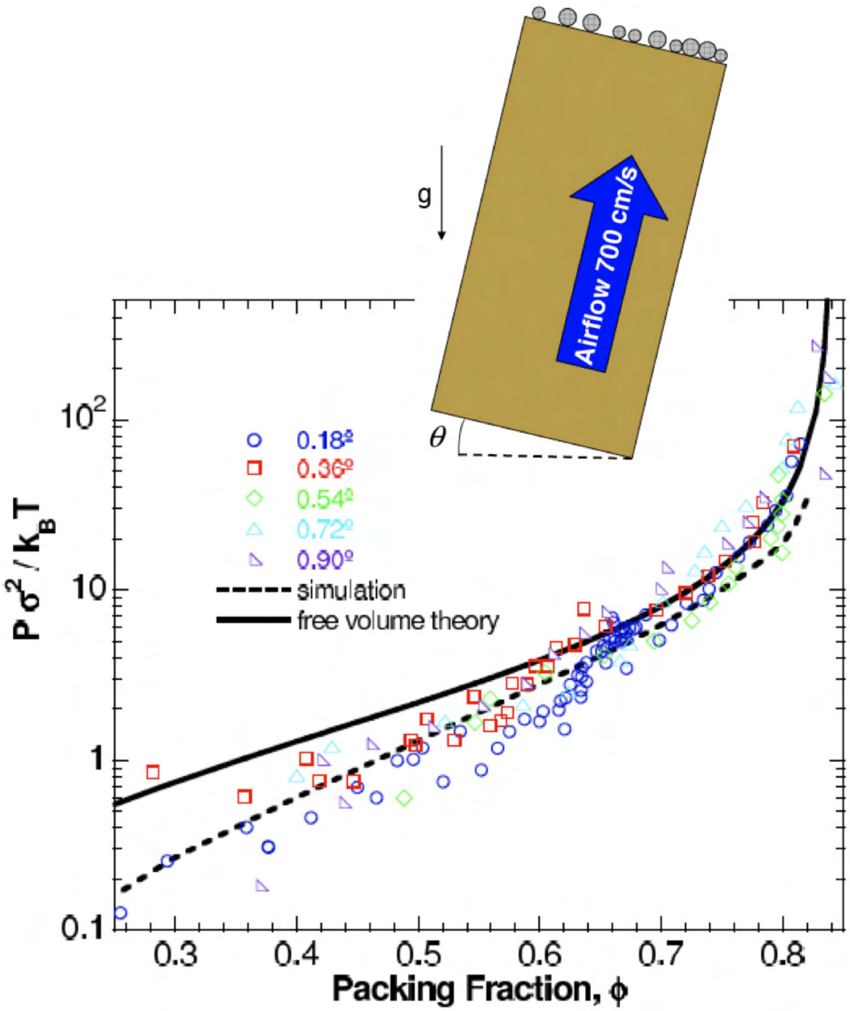
solid curve: power law, with exponent 0.7+-0.2

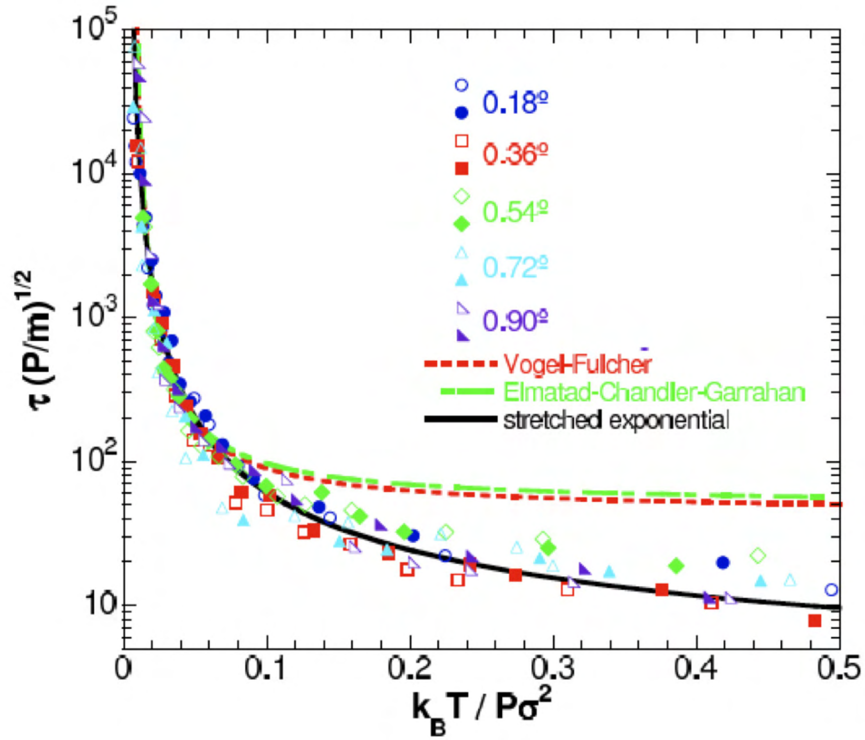
dashed line: activated, with same barrier as τ, μ

In progress…

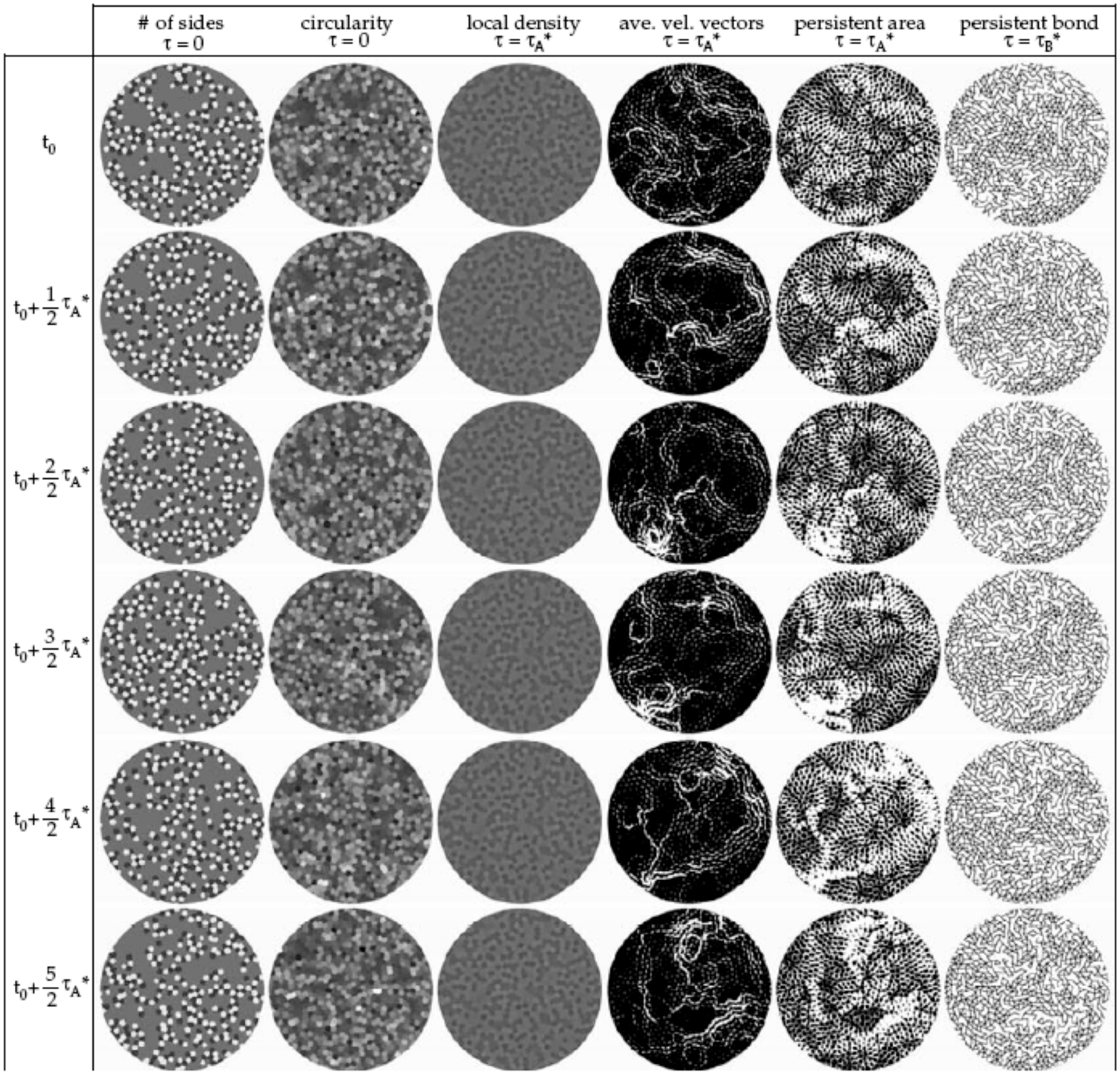
- Measure pressure, and scale temperature by $P\sigma^2$:

(Lynn Daniels)





No correlation with structure



[2] Steady granular heap flow

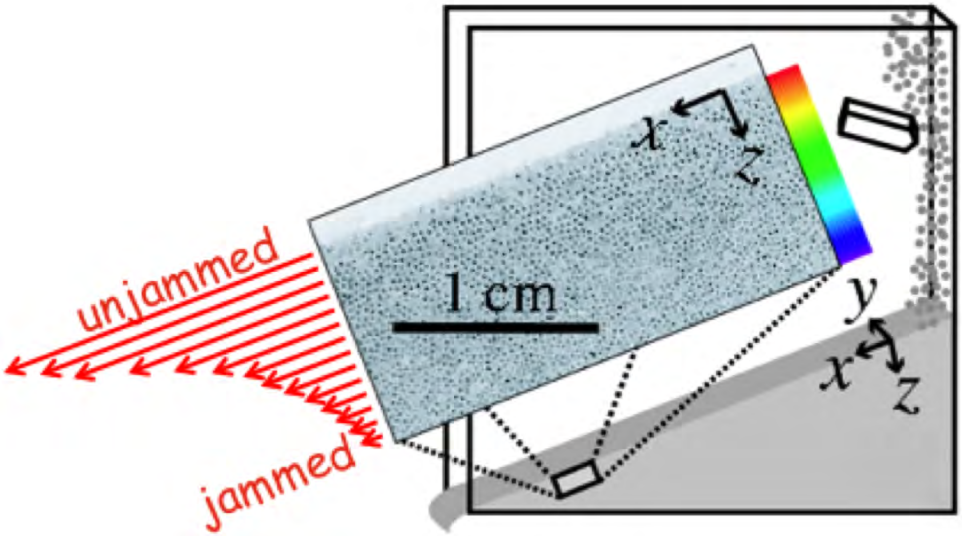
300 micron glass beads, 1 cm wide x 30 cm long heap (Soft Matter, '10)

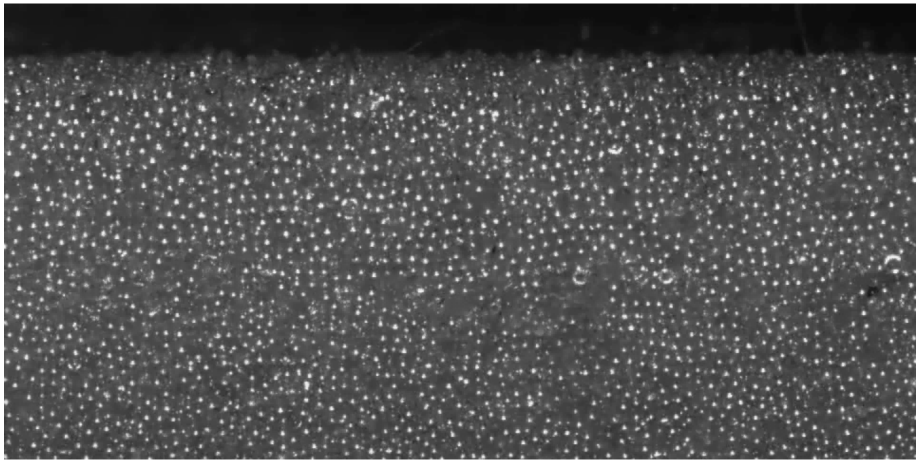
- Continuous flow near surface; no flow down deep

Jamming transition is versus depth

- Structure: no obvious changes
- Dynamics…

Hiroaki Katsuragi





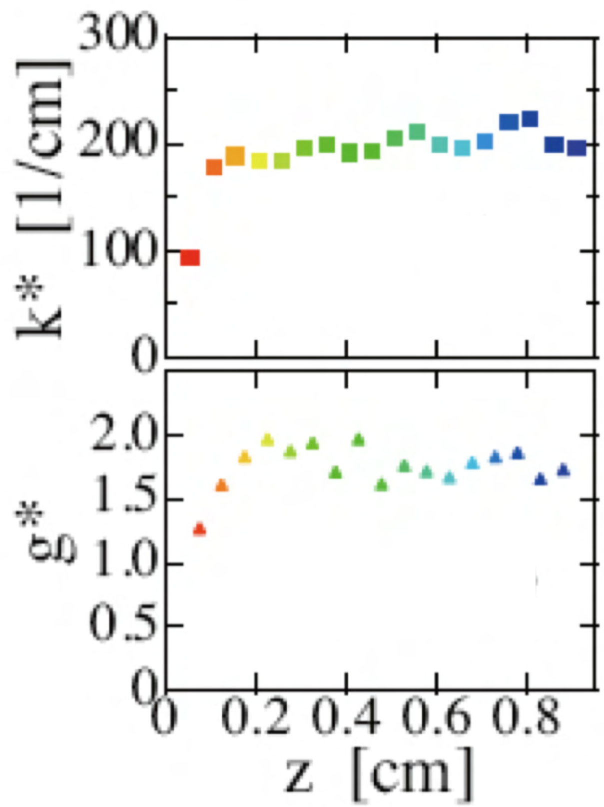
Structure vs depth

{at Q=2.5 g/s, well into the continuous regime}

- No evident variation, except for top 2-3 layers, just as in the glass transition:

Location of 1st peak of S(k): {and density}

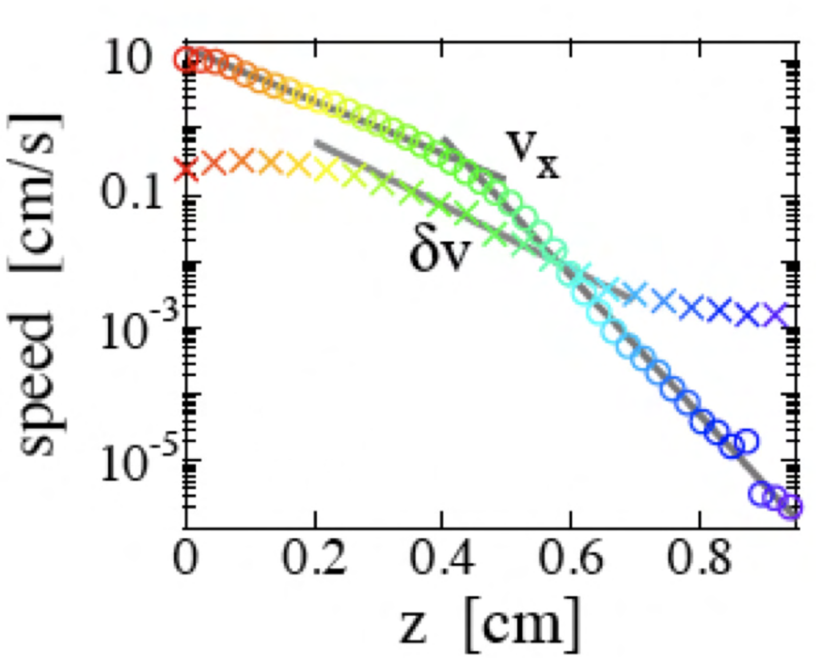
Value of 1st peak of g(r):



Flow speeds vs depth

{at Q=2.5 g/s, far into the continuous regime}

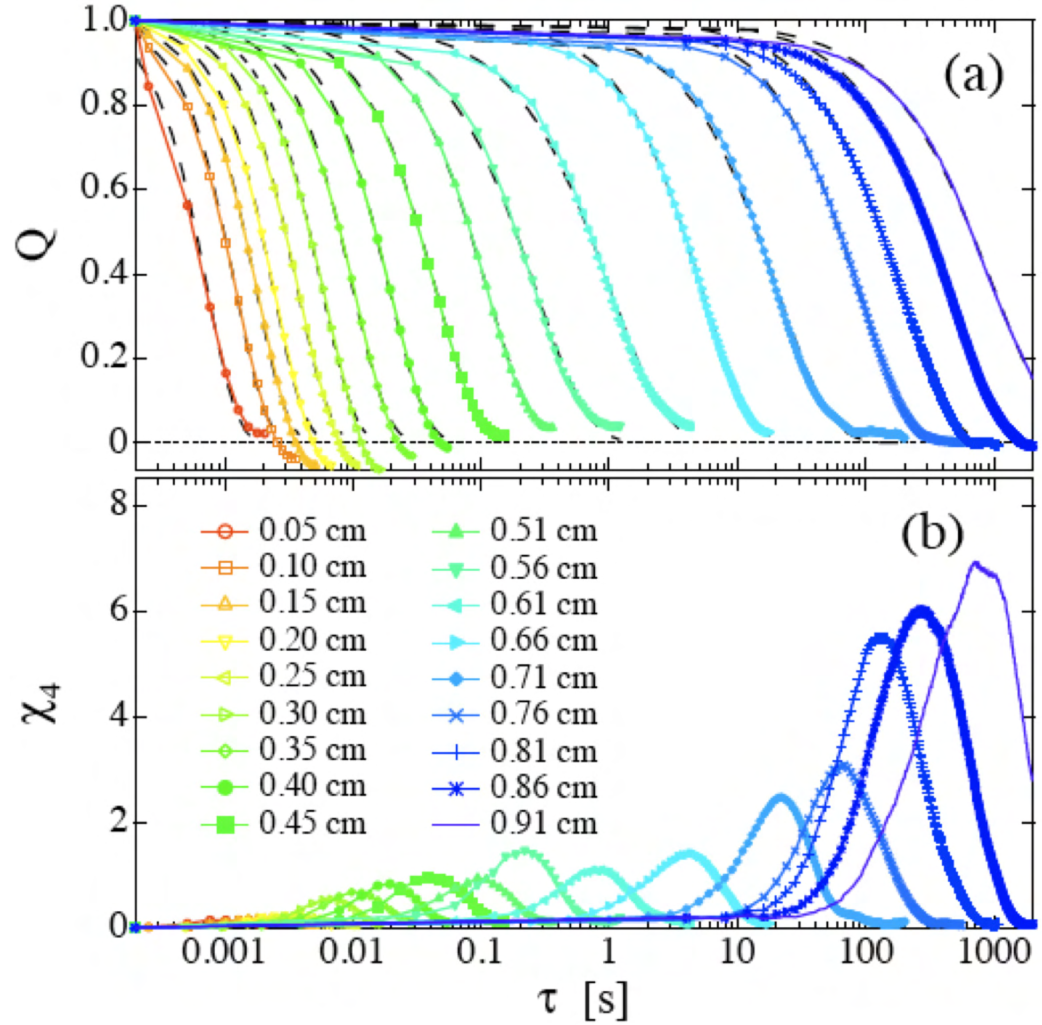
- $v_x(z)$ looks like a double exponential
- $v_x > \delta v$ only near the top
- Fluctuations dominate down deep, near jamming



Deeper → more heterogeneous

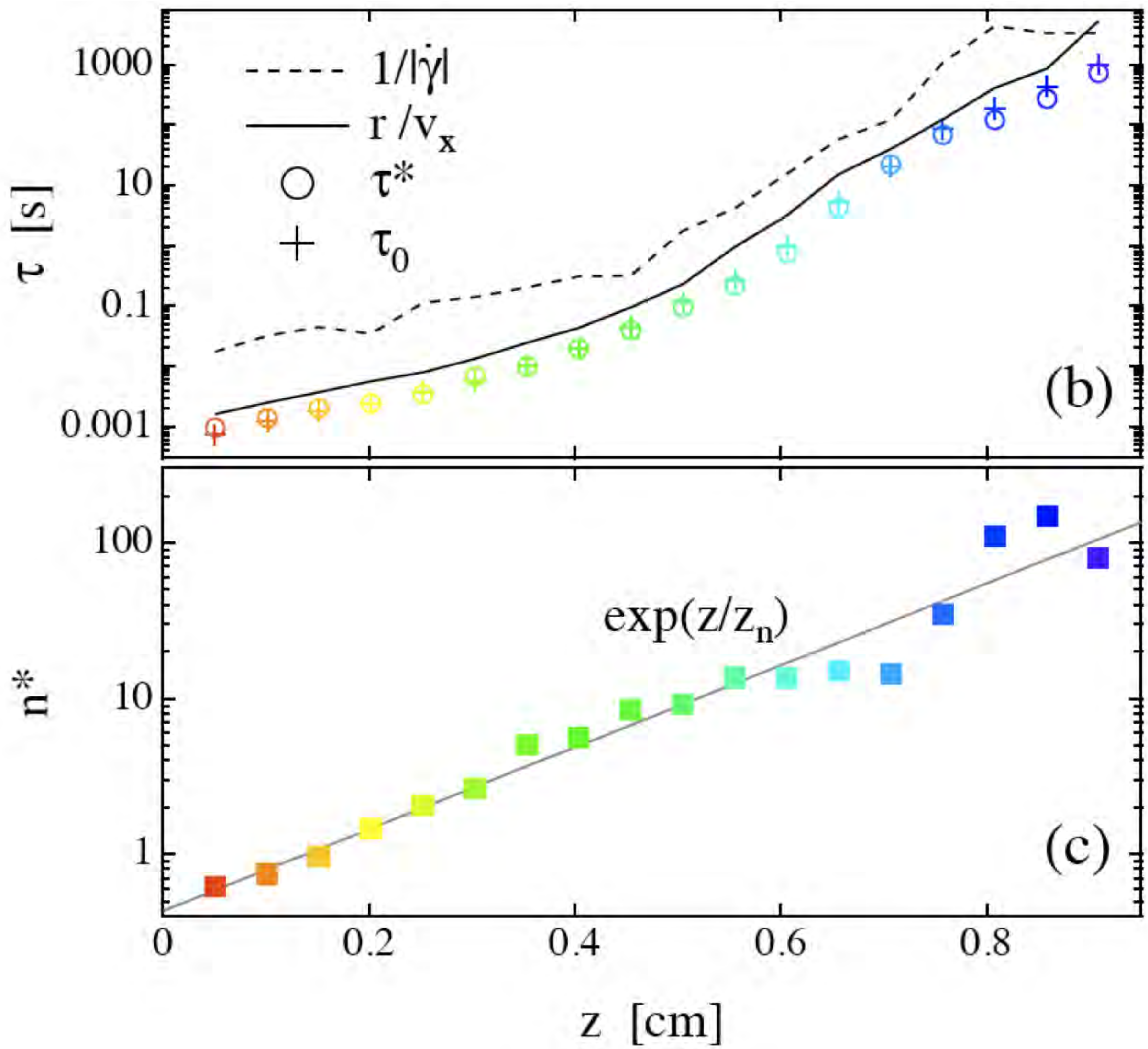
- Compute overlap parameter $Q(t)$ and $\chi_4 = N\sigma^2(t)$ by image-strip correlations (*not* particle tracking)

- Approach to J…
  - $Q(\tau)$ vs τ:
    - decays slower, both in time and in shape
  - Peak of χ_4:
    - moves to longer times
    - increases in height



Diverging time and length scales

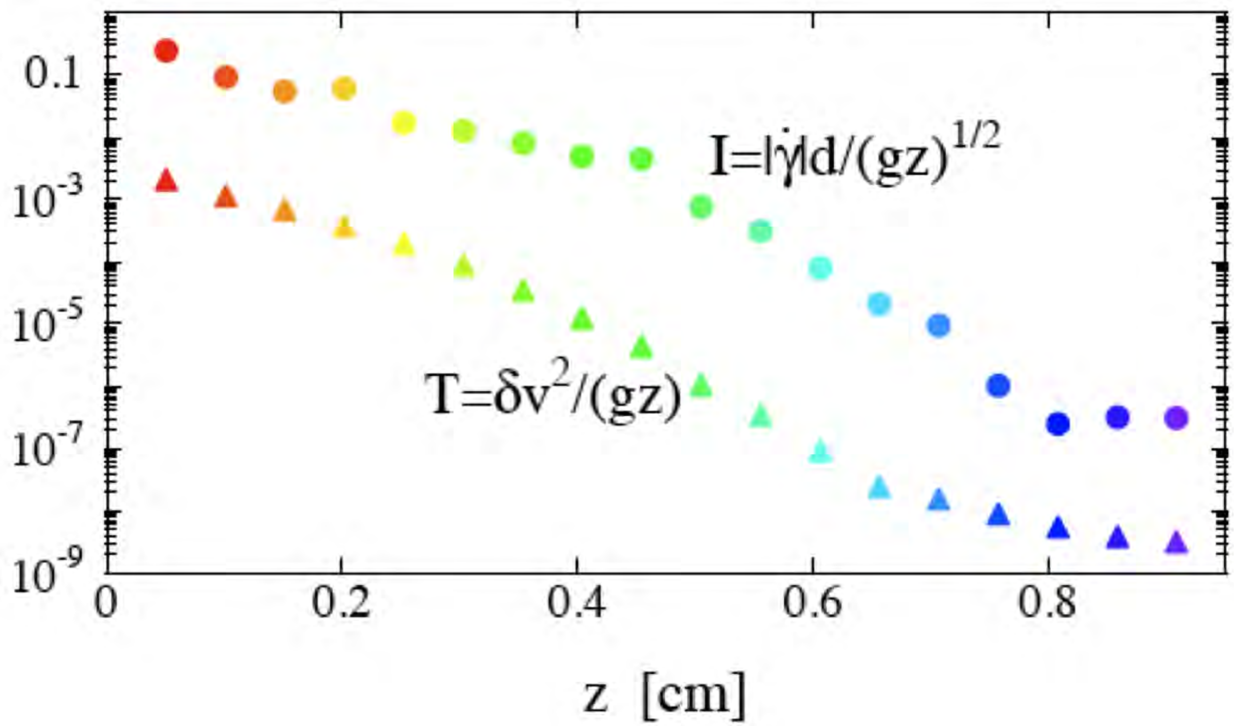
~Exponential with depth, just like velocity profile



Control parameter

- Dimensionless temperature or strainate ??

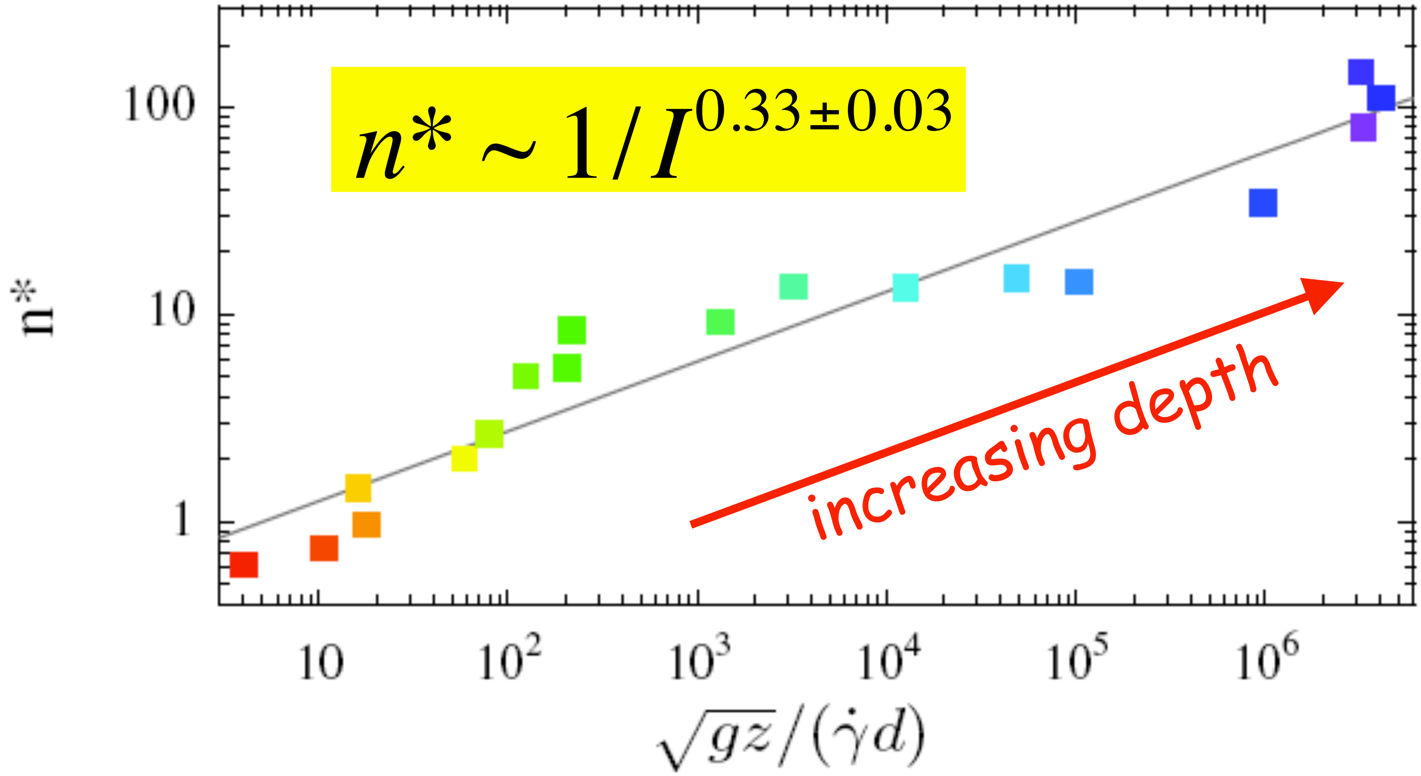
$(P = \rho g z)$ $T = \rho \delta v^2 / P$ $I = \dot{\gamma} d / \sqrt{P/\rho}$



NB: $T = I/100 \rightarrow \delta v = 0.1\ [\ (\dot{\gamma} d)(gz)\ ]^{1/2}$

n* scaling

- Set by inertia number: $I = \dot{\gamma} d / \sqrt{P/\rho}$



Compares well with simulations of velocity correlations

(3d) Takahiro Hatano: $P \sim \dot{\gamma}^{1/2}, \xi \sim \dot{\gamma}^{-1/4} \rightarrow \mathrm{I} \sim \dot{\gamma}^{3/4}$ and $\xi \sim I^{-1/3}$

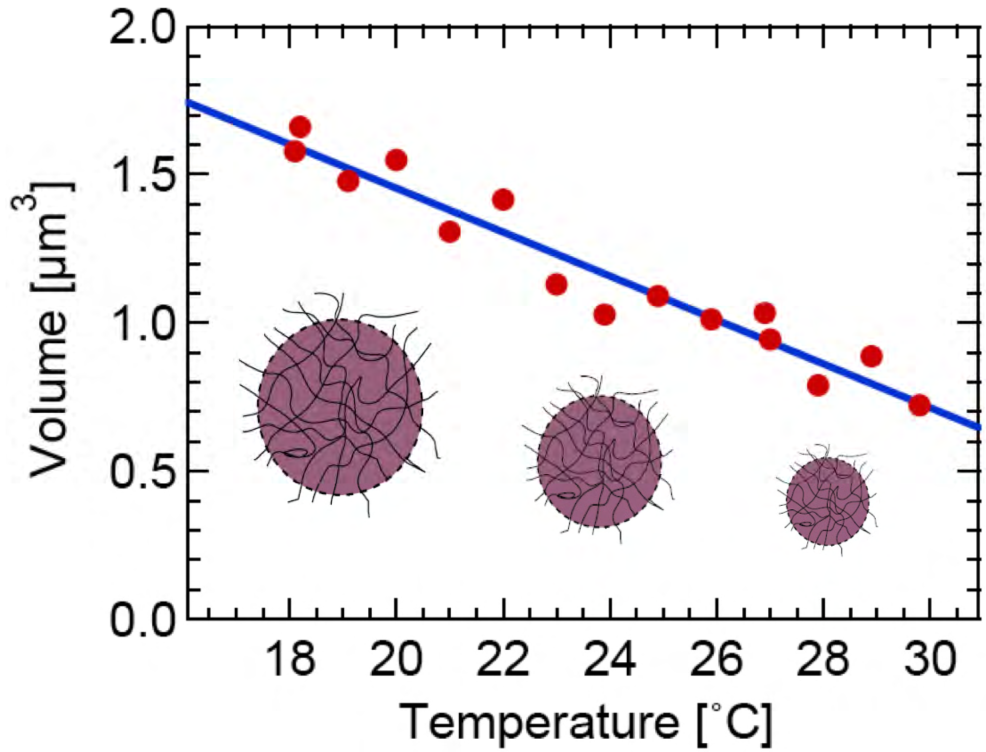
(2d) Lydie Staron, Lagree, Josserand, Lhuillier (preprint)

[3] NIPA microgel beads

- N-isopropylacrylamide (Yodh group) (preprints ‘10)

Soft: can be compressed above close packing

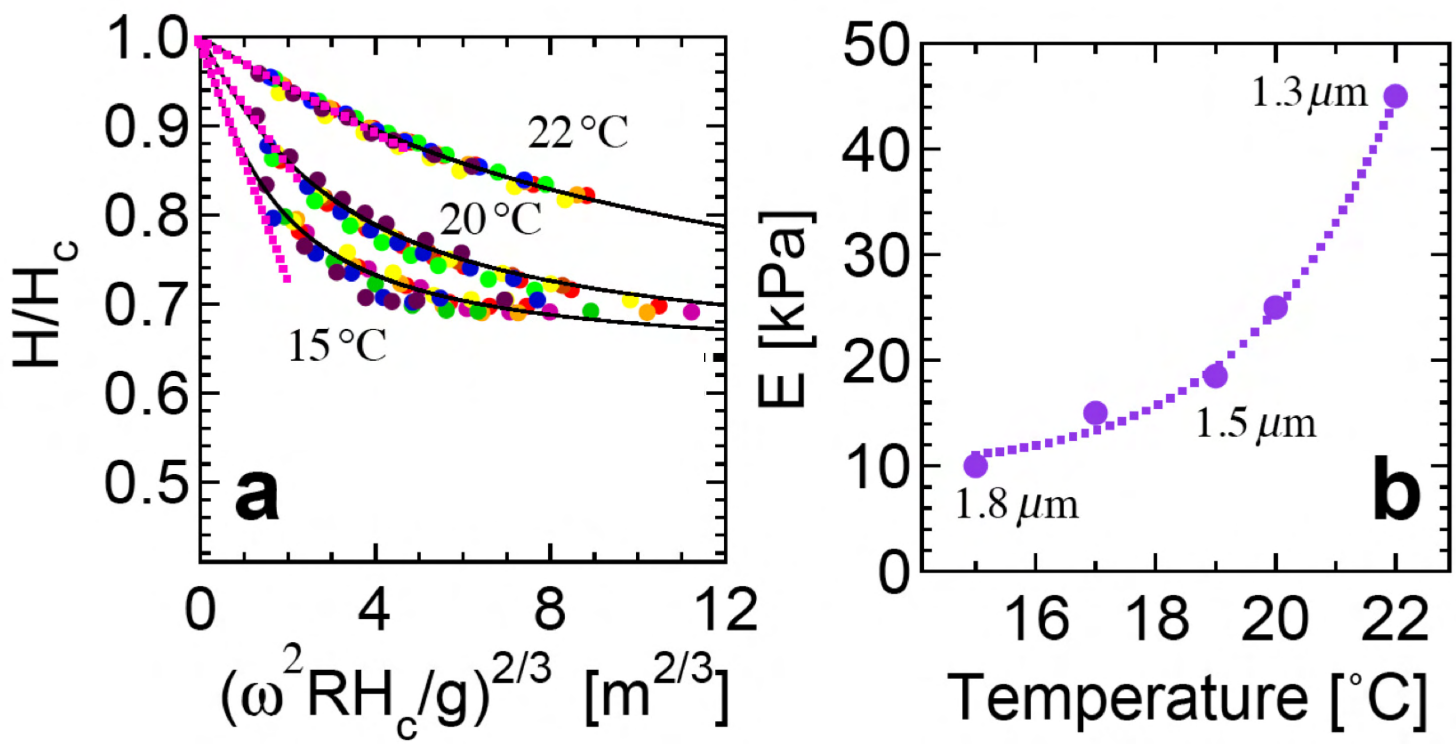
Thermoresponsive: size & hence packing fraction can be varied *in-situ* via temperature



Kerstin Nordstrom

Particle Mechanics

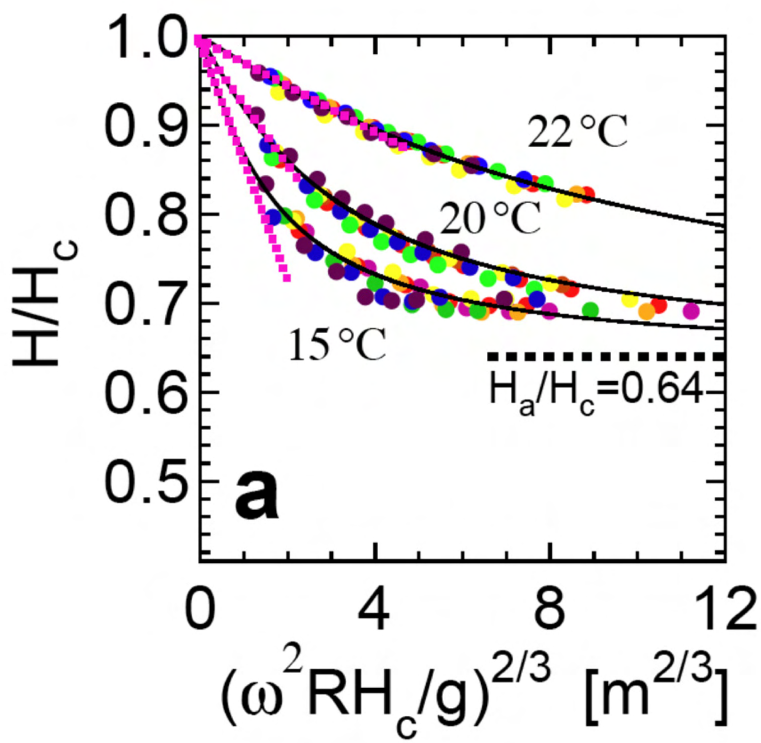
- Compress NIPA suspensions in a centrifuge
  - Low RPM: Hertzian behavior, deduce particle modulus E
  - E ~ 1/(particle volume)^3, must be scaled out in rheology



Particle Mechanics II

High RPM: height asymptotes toward a constant, H_a

- $H_a/H_c = (1/\phi_a) / (1/\phi_{rcp}) = 0.64 \rightarrow \phi_a=1$
- particles deform without deswelling!



Microfluidic Shear Rheology

- Pressure-driven flow in a PDMS microchannel

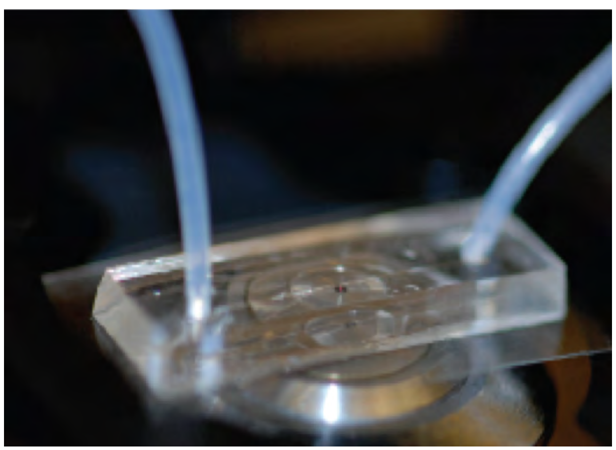
  L = 2 cm long, 100 μm tall x 25 μm wide

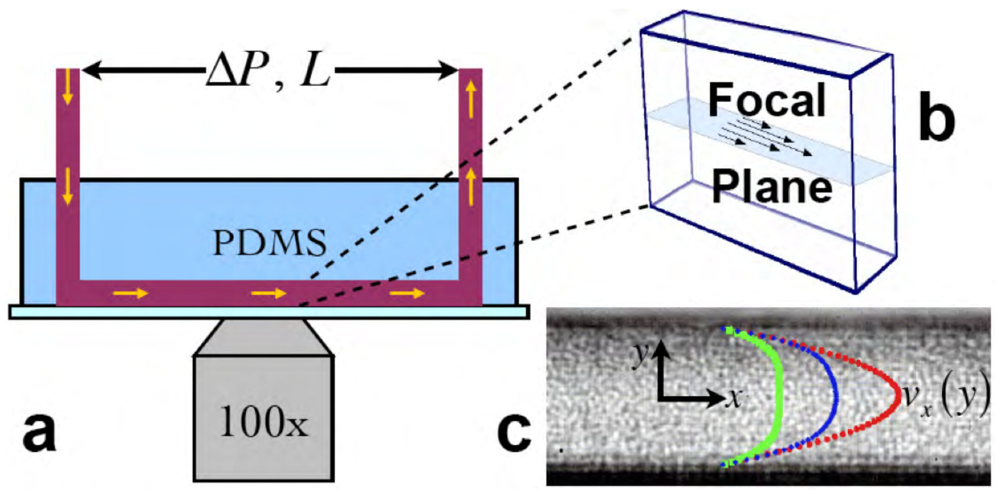
- Principle:

  [1] measure pressure gradient, deduce stress $\sigma=(\Delta P/L)x$ vs x

  [2] measure velocity profile, deduce strainrate $\dot{\gamma}=dv/dx$ vs x

  [3] combine for stress vs strainrate





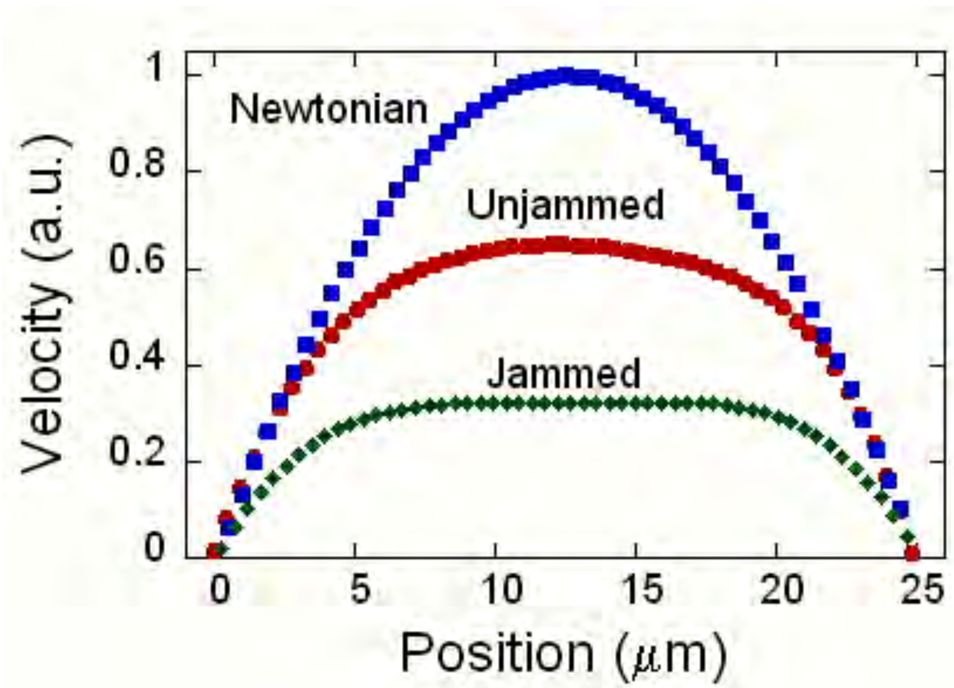
D. K. Schwartz, C. M. Knobler, and R. Bruinsma, Physical Review Letters **73**, 2841 (1994).

G. Degré, P. Joseph, P. Tabeling, S. Lerouge, M. Cloitre, and A. Ajdari, Appl. Phys. Lett. **89**, 024104 (2006).

Example velocity profile data

[1] measure pressure gradient, deduce stress $\sigma=(\Delta P/L)x$ vs x
[2] measure velocity profile, deduce strainrate $\dot{\gamma}=dv/dx$ vs x

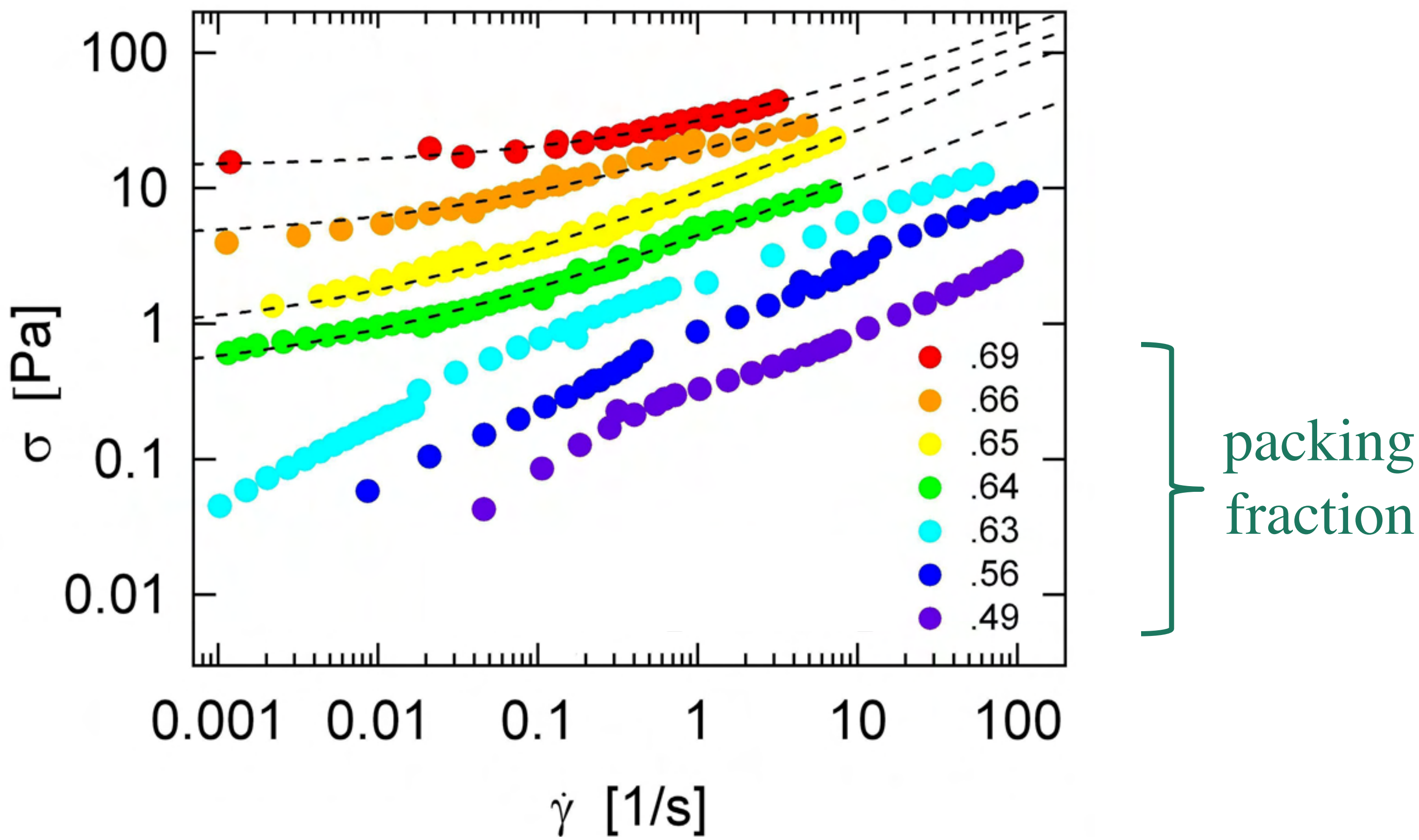
- Parabolic: dv/dx~x so viscosity = constant (Newt.)
- Plug: dv/dx=0 so stress = constant (yield stress)



μF Shear Rheology of NIPA

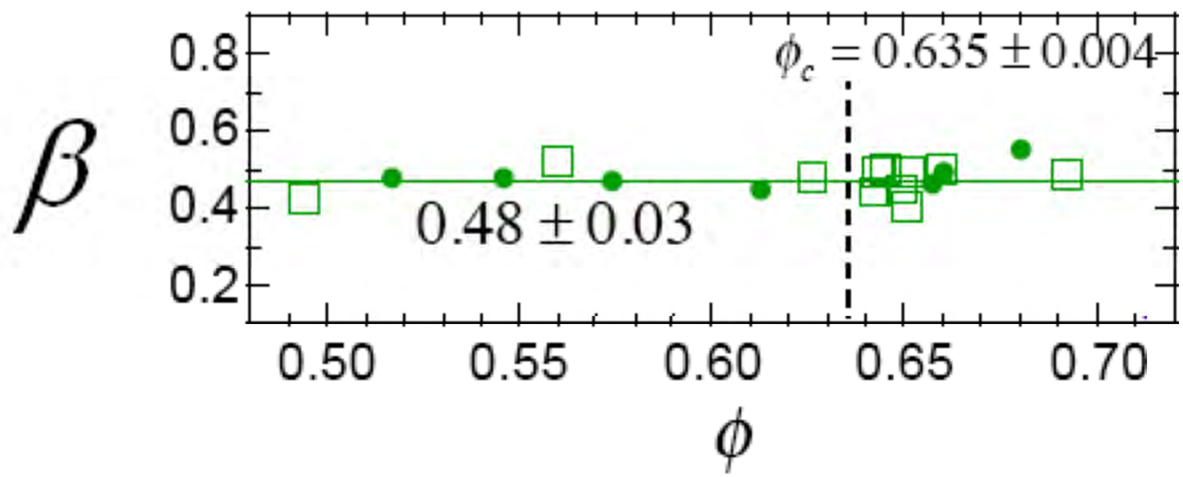
one sample: many tempertures → many packing fractions

- Below RCP: non-Newtonian power-law fluid
- Above RCP: Hershel-Bulkley behavior
  - yield stress + strainrate$^{\beta=1/2}$



$\sigma = \sigma_y \left[1 + (\dot{\gamma}\tau)^{\beta}\right]$ *Hershel-Bulkley fits*

- The power-law is consistent with β=1/2
  - good fits, constant β=1/2, even quite far from ϕ_c
  - same result for two different particle sizes



van Saarloos predicts β=2b/(b+3) if the drag between two moving particles is F ~ $-v^b$ {b=1 gives β=1/2}

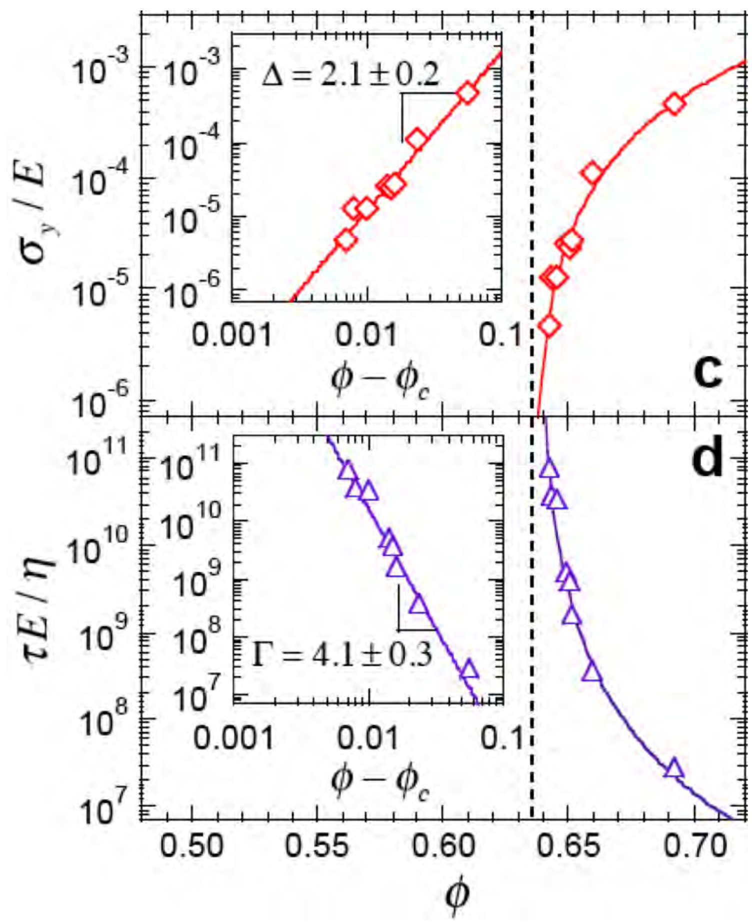
$$\sigma = \sigma_y \left[1 + (\dot{\gamma}\tau)^\beta\right]$$

Hershel Bulkely fits

- Yield stress vanishes & timescale diverges at RCP

$\sigma_y \sim (\phi-\phi_c)^\Delta$, $\Delta=2$

$\phi_c = 0.635 \pm 0.004$



vanSaarloos:

$\sigma_y \sim G\gamma_y$ and $\gamma_y \sim (\phi-\phi_c)$

Liu-Nagel:

$G \sim (\phi-\phi_c)^{\alpha-3/2}$ for potential $V \sim \text{overlap}^\alpha$

Hertzian:

$\alpha=5/2 \rightarrow \Delta=\alpha-\frac{1}{2} = 2$

$\sigma = \sigma_y\left[1+(\dot{\gamma}\tau)^{\beta}\right]$ *Hershel-Bulkley fits*

- Experimental results for exponents:

  $\sigma_y \sim (\phi-\phi_c)^{\Delta}$ $\quad$ $\Delta=2$

  $\tau \sim (\phi-\phi_c)^{-\Gamma}$ $\quad$ $\Gamma=4$

  $\beta=1/2$

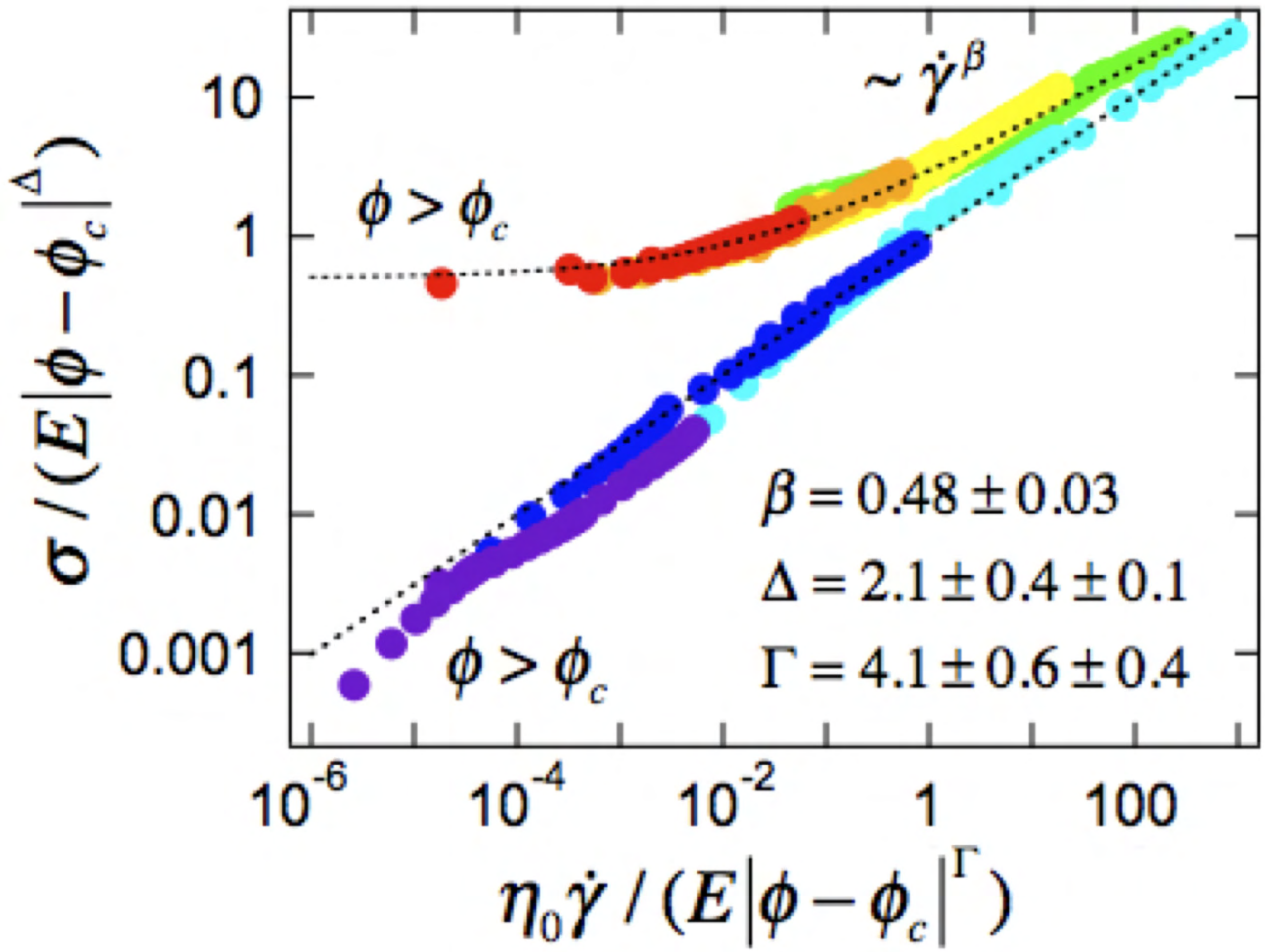
- Check:

  $\sigma_y(\dot{\gamma}\tau)^{\beta}$ must be finite and nonzero at ϕ_c: $\Delta - \Gamma\beta = 0$

$\sigma = \sigma_y \left[1 + (\dot{\gamma}\tau)^\beta\right]$ *Olsson-Teitel scaling*

- Collapse onto two branches, above & below ϕ_c

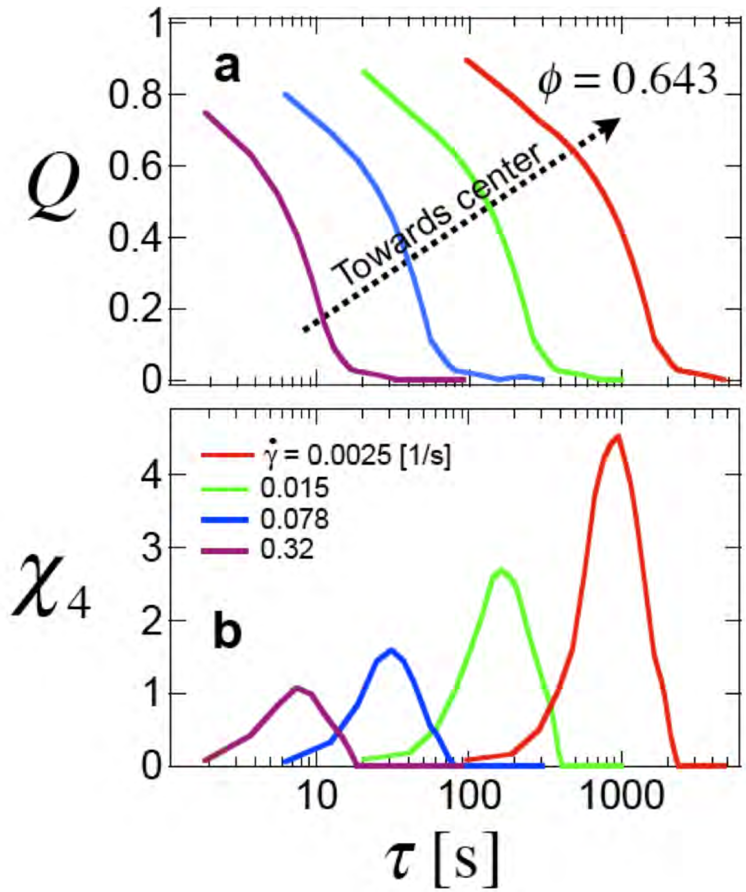
$\sigma_y \sim (\phi-\phi_c)^2$ $\tau \sim (\phi-\phi_c)^{-4}$ $\beta=1/2$



Slow shear → heterogeneous

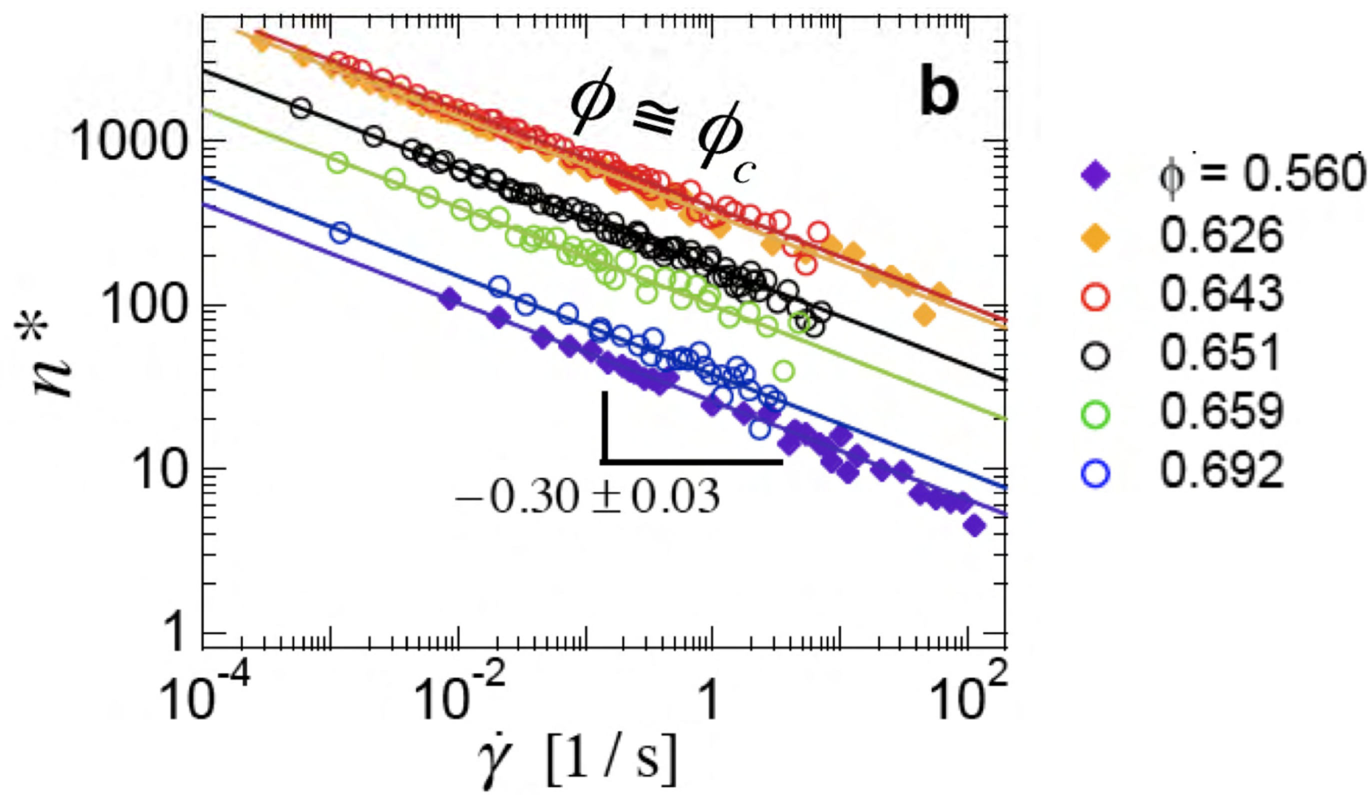
- Compute overlap parameter Q(t) and $\chi_4 = N\sigma^2(t)$ by image-strip correlations (*not* particle tracking)

- Decreasing strainrate…
  - Q(τ) vs τ:
    - decays slower
  - Peak of χ_4:
    - moves to longer times
    - increases in height



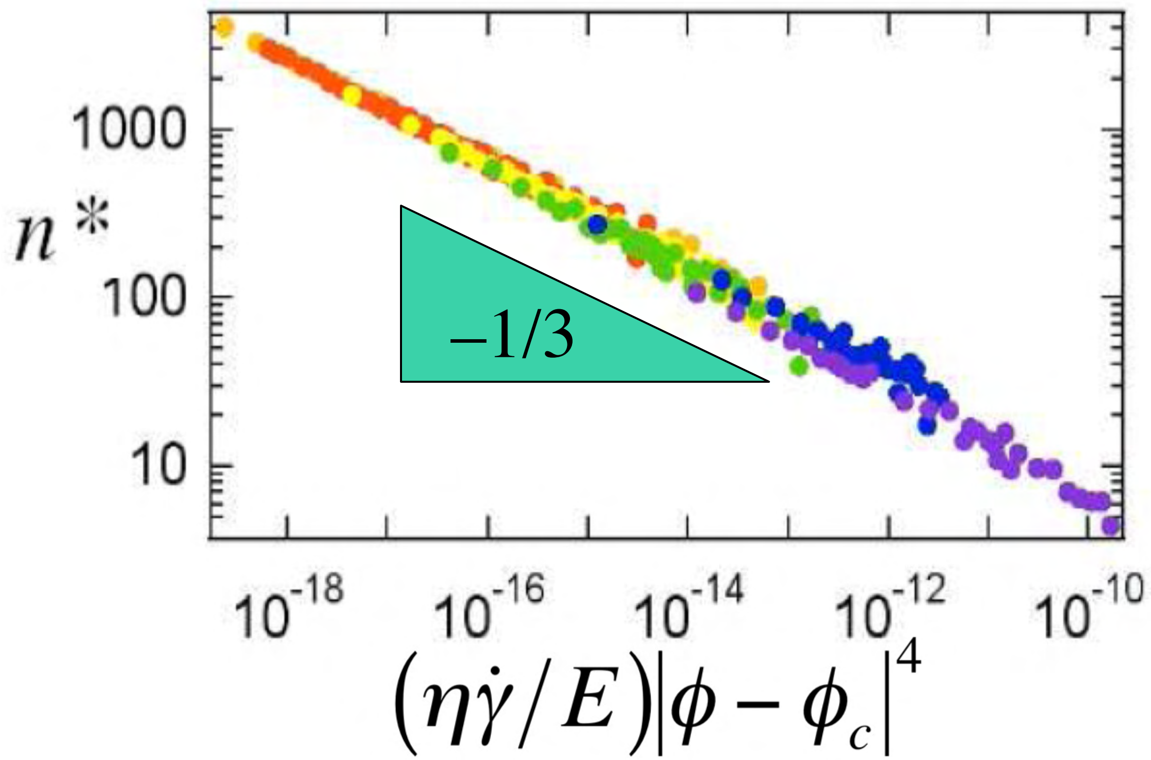
Size n* of heterogeneities

- Larger for…
  - low shear: n* ~ (strain rate)$^{-1/3}$, same as for heap flow
  - volume fractions closer to RCP



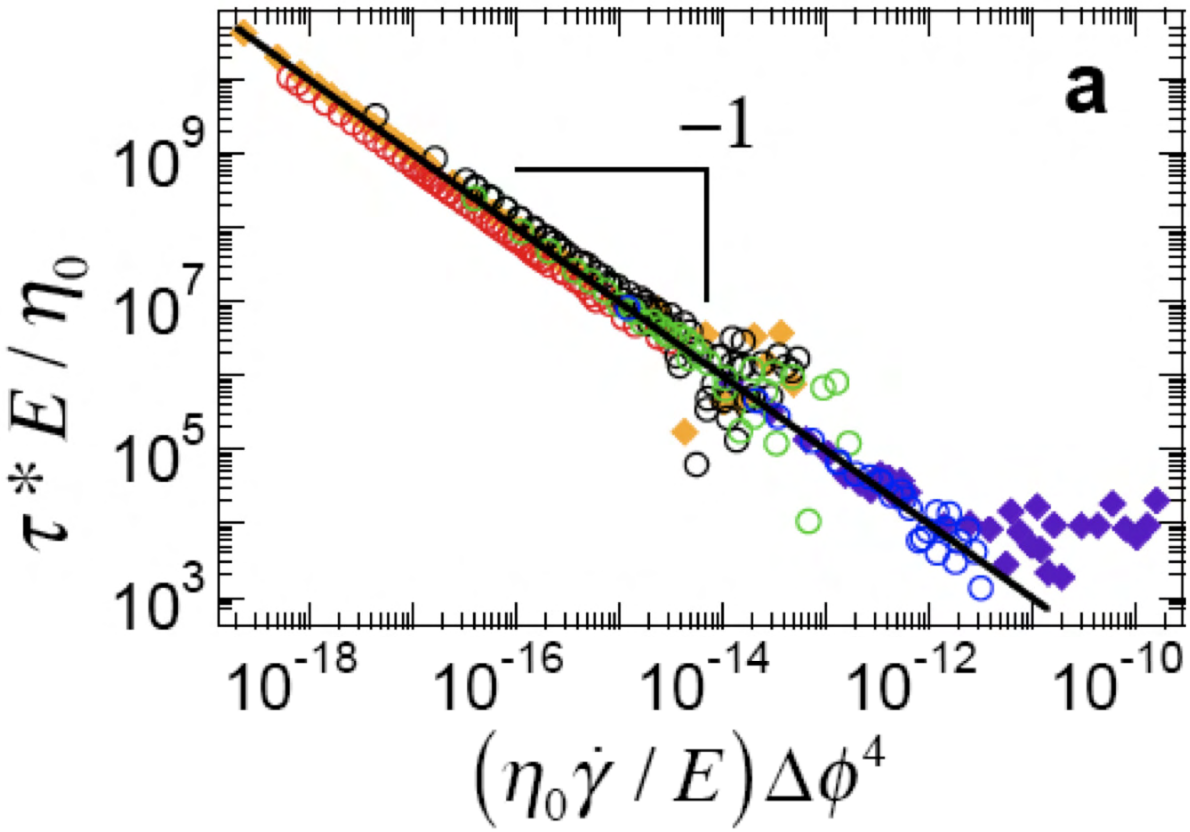
Full scaling of n for NIPA*

- $n^* \sim (\text{strainrate})^{-1/3} (\phi - \phi_c)^{-4/3}$
  what is the physical origin of these exponents?



Time scale diverges too

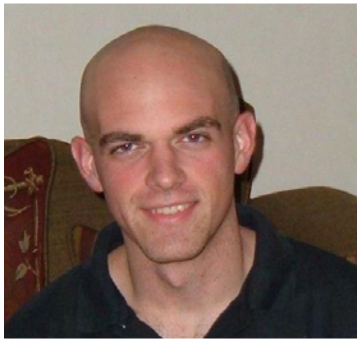
- $\tau^* \sim (\text{strainrate})^{-1} (\phi - \phi_c)^{-4}$



Conclusion

- The response to steady driving forces can be spatially and temporally heterogeneous, more so near jamming
  - vs packing, airspeed, shear, T/P, depth, discharge,…
- Size n* of dynamical heterogeneities
  - deduction from 4pt susceptibilities
  - growth on approach to jamming
    - Air-fluidized "thermal" balls: as $\phi \rightarrow \phi_c$ and $T/P \rightarrow 0$
    - Athermal hard grains: as strainrate$\rightarrow$0
    - Athermal soft colloids: as strainrate$\rightarrow$0 and $\phi \rightarrow \phi_c$
- Meaningful effective temperatures
- Macroscopic rheology

Many thanks to



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  Wim van SAARLOOS

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  Ahmed Alsayed, Anindita Basu, Zexin Zhang

THE END.