

# Shear Banding in Amorphous Solids

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Johns Hopkins University*

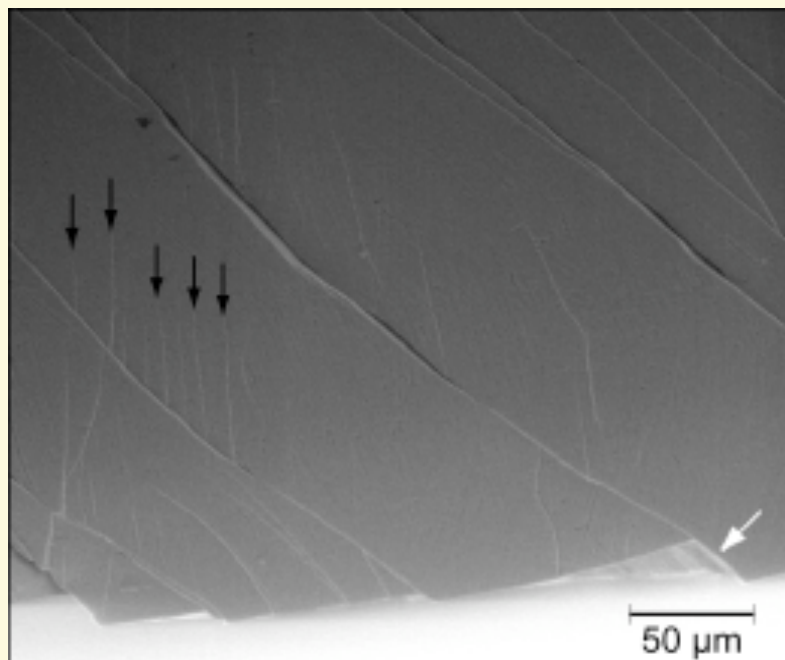
***Yunfeng Shi***

*Materials Science and Engineering  
Rensselaer Polytechnic Institute*

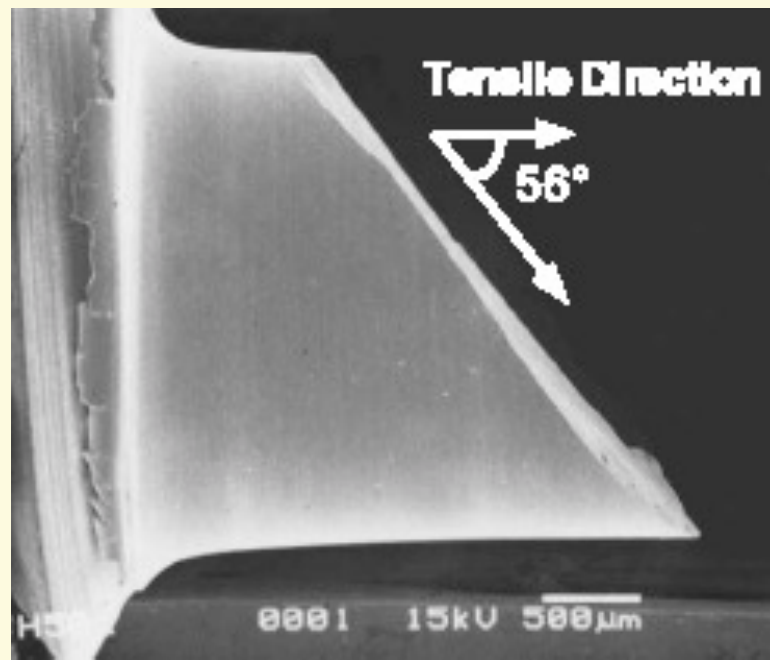


# Shear Bands in Metallic Glass

**strain localization (shear banding) is the primary failure mode**

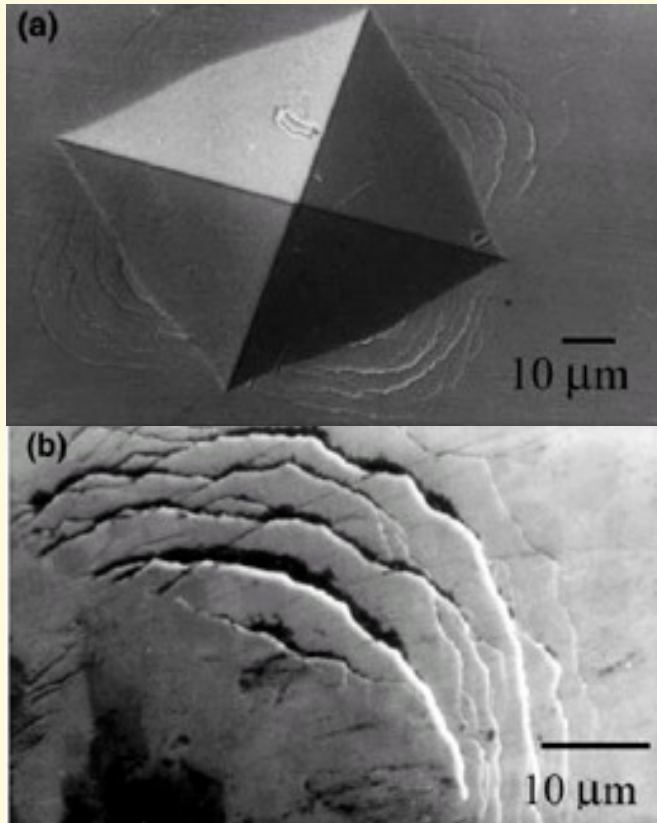


**Electron Micrograph of Shear Bands Formed  
in Bending Metallic Glass**  
Hufnagel, El-Deiry, Vinci (2000)

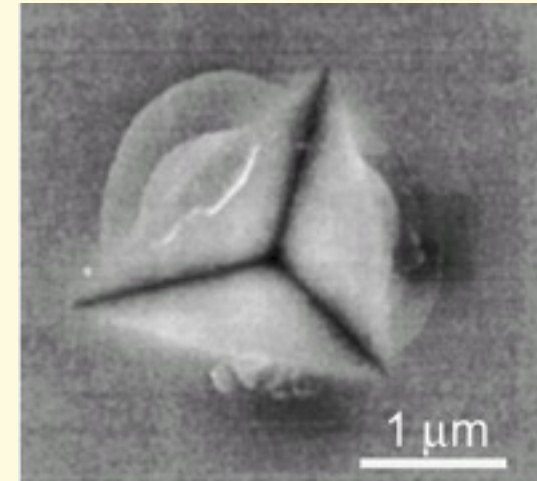


**Quasistatic Fracture Specimen**  
Mukai, Nieh, Kawamura, Inoue, Higashi  
(2002)

# Indentation Testing of Metallic Glass

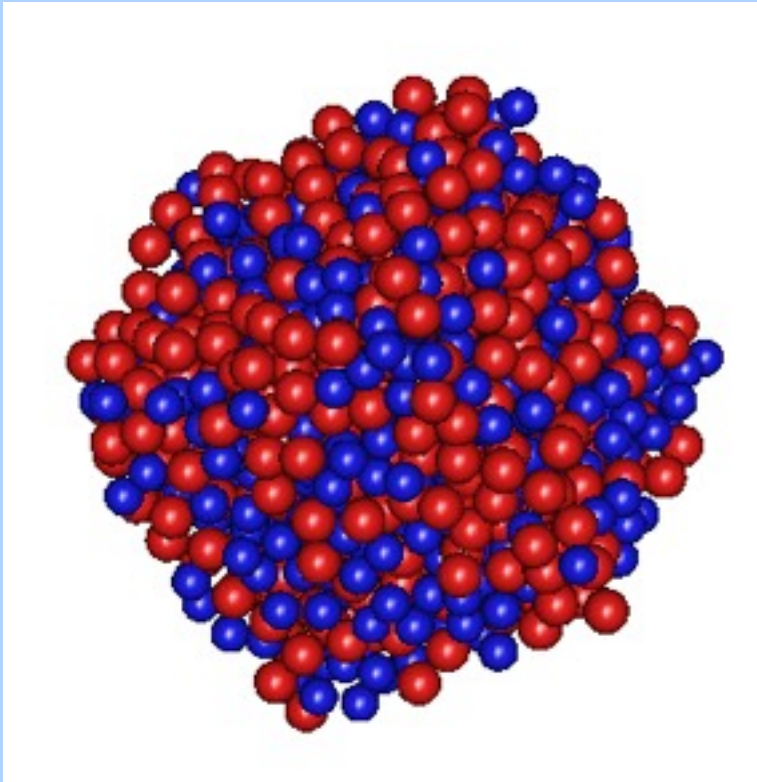


**“Hardness and plastic deformation in a bulk metallic glass”**  
Acta Materialia (2005)  
U. Ramamurty, S. Jana, Y. Kawamura, K. Chattopadhyay



**“Nanoindentation studies of shear banding in fully amorphous and partially devitrified metallic alloys”**  
Mat. Sci. Eng. A (2005)  
A.L. Greer, A. Castellero, S.V. Madge, I.T. Walker, J.R. Wilde

# Simulated System: 3D Binary Alloy

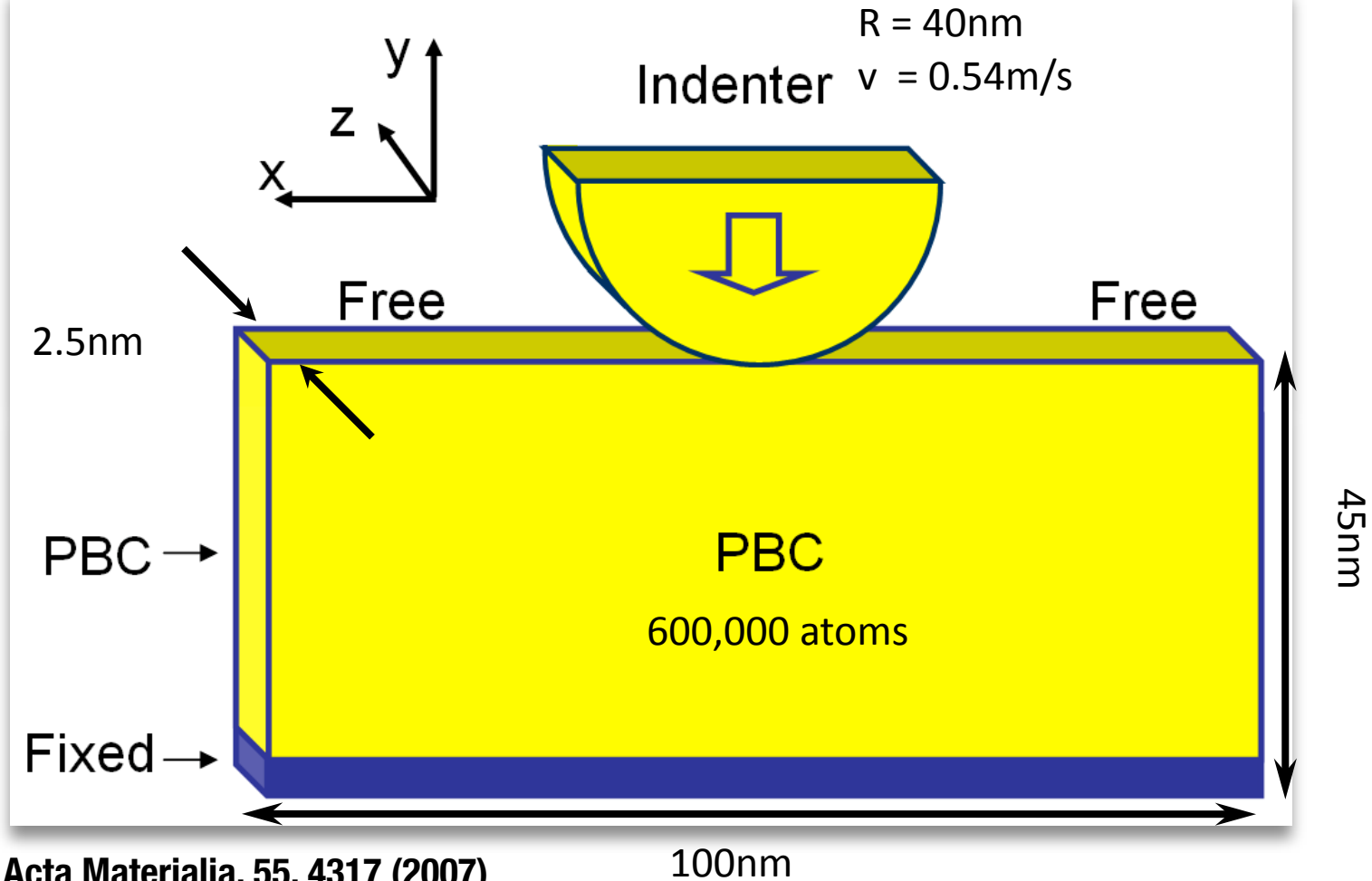


- Wahnstrom Potential (PRA, 1991)
- Rough Approximation of  $\text{Nb}_{50}\text{Ni}_{50}$
- Lennard-Jones Interactions
- Equal Interaction Energies
- Bond Length Ratios:
  - $a_{\text{NiNi}} \sim \frac{5}{6} a_{\text{NbNb}}$
  - $a_{\text{NiNb}} \sim \frac{11}{12} a_{\text{NbNb}}$
- $T_g \sim 1000\text{K}$
- Studied previously in the context of the glass transition (Lacevic, *et. al.* PRB 2002)

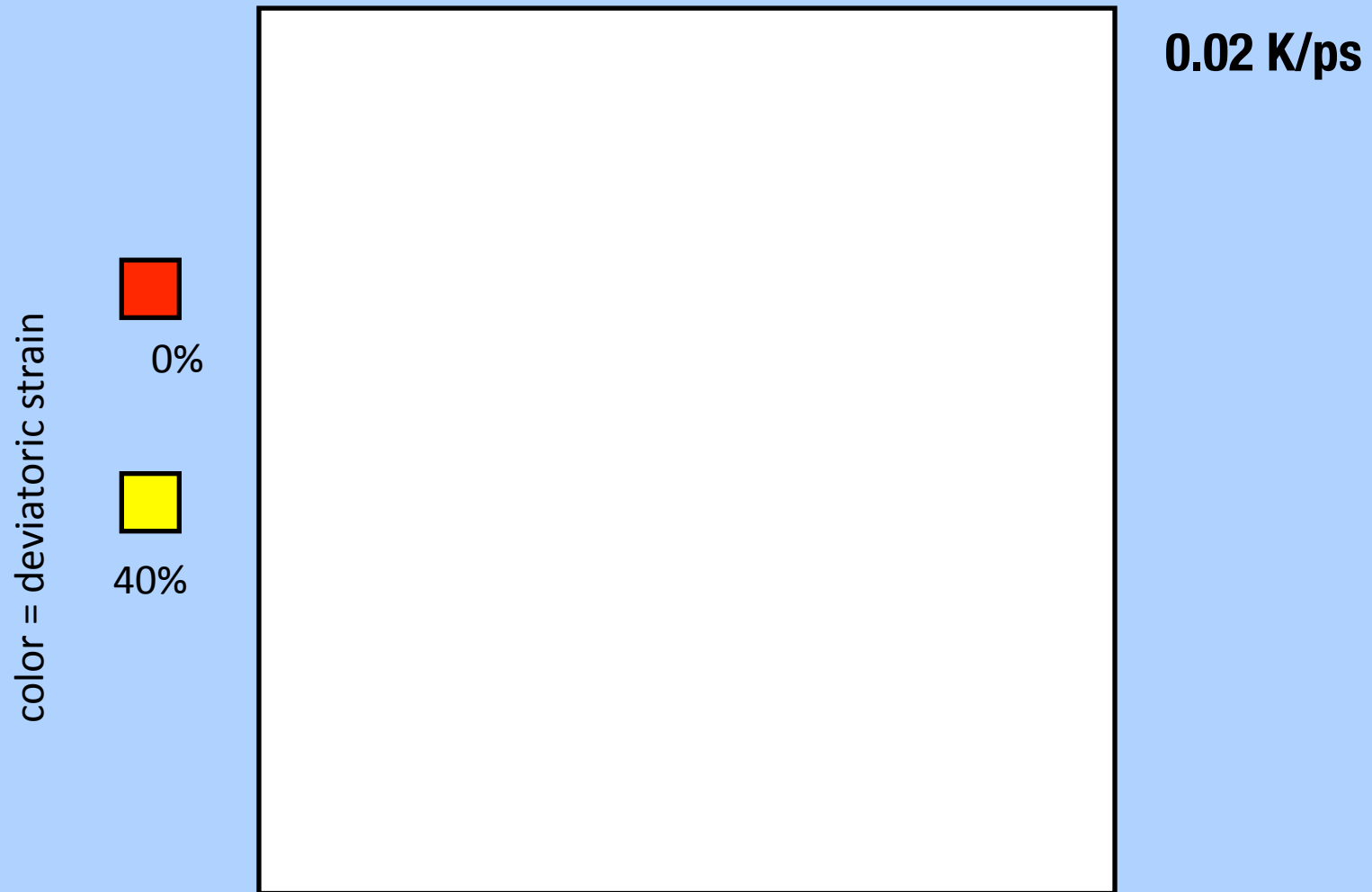
- Unlike crystalline systems, it is not possible to skip simulating the processing step
- Glasses were created by quenching at 3 different rates: 50K/ps, 1K/ps and 0.02 K/ps

# Metallic Glass Nanoindentation

Simulations performed using molecular dynamics code across 64 nodes of a parallel cluster

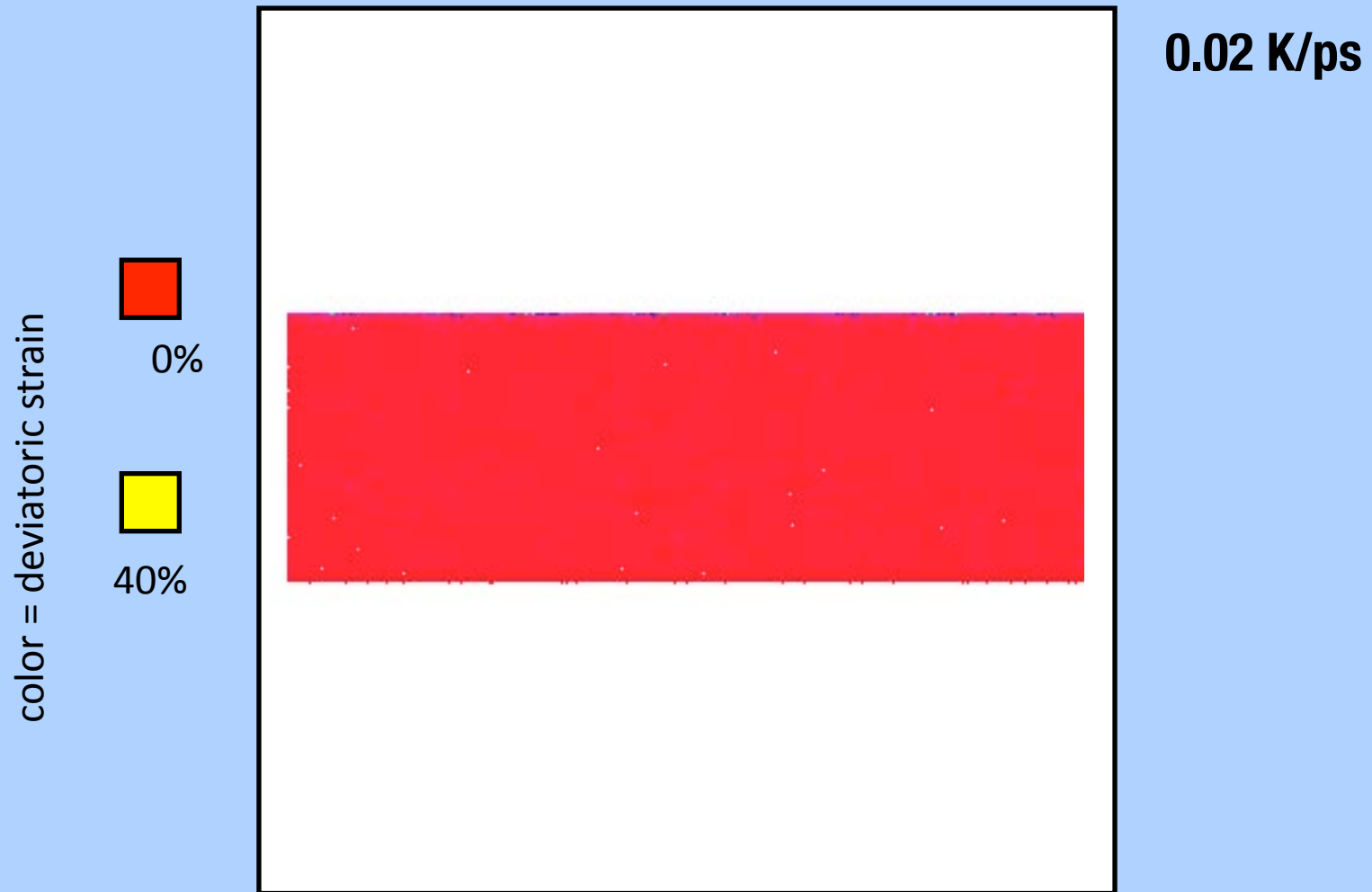


# Metallic Glass Nanoindentation



Y. Shi, MLF, *Acta Materialia*, 55, 4317 (2007)

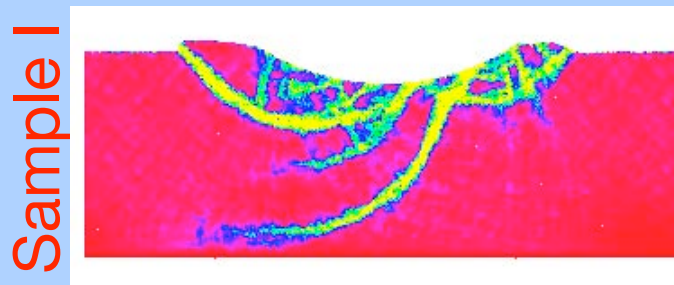
# Metallic Glass Nanoindentation



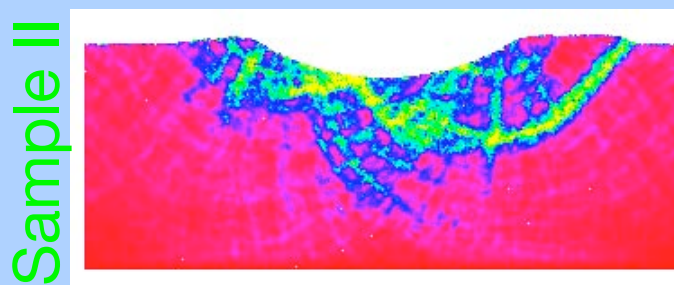
Y. Shi, MLF, *Acta Materialia*, 55, 4317 (2007)



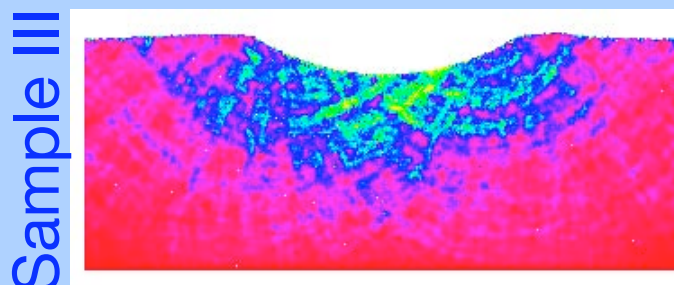
# Metallic Glass Nanoindentation



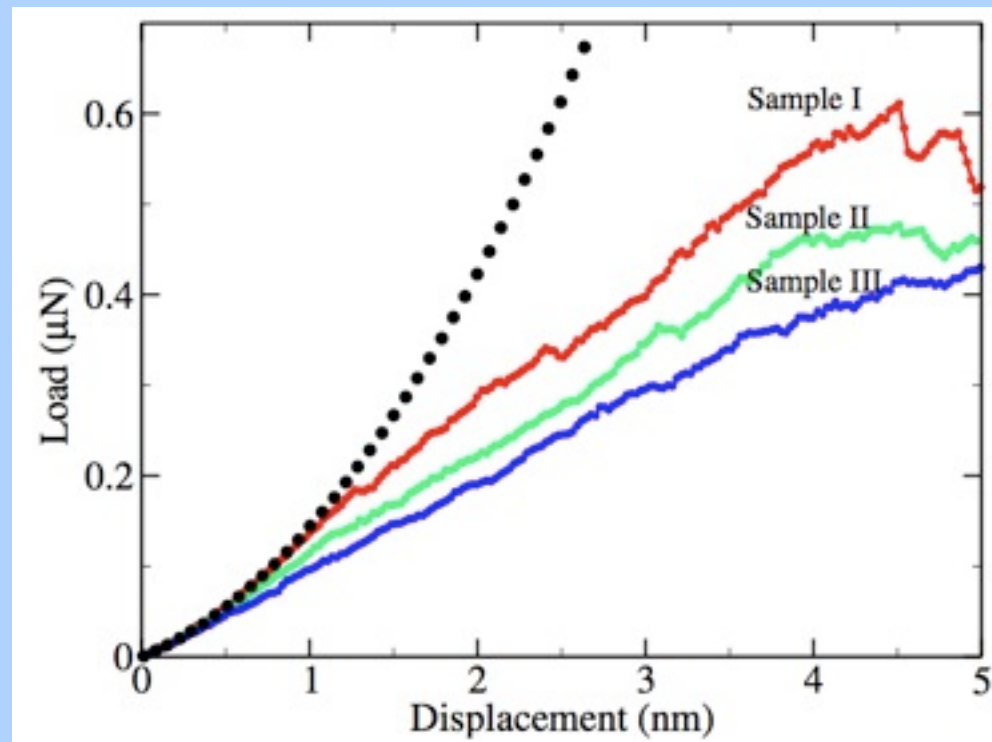
0.02 K/ps



1K/ps



50K/ps

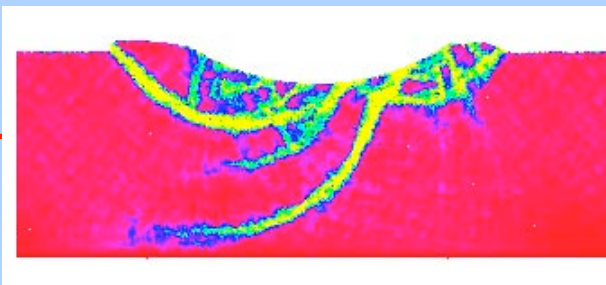


Y. Shi, MLF, Acta Materialia, 55, 4317 (2007)

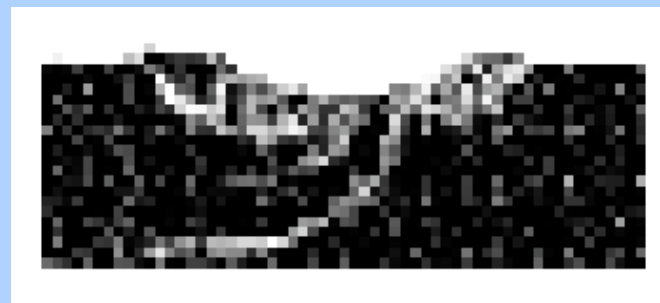


# Metallic Glass Nanoindentation

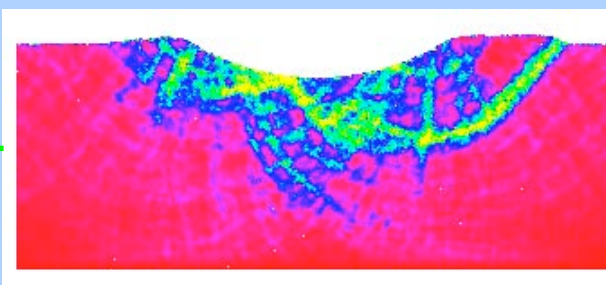
Sample I



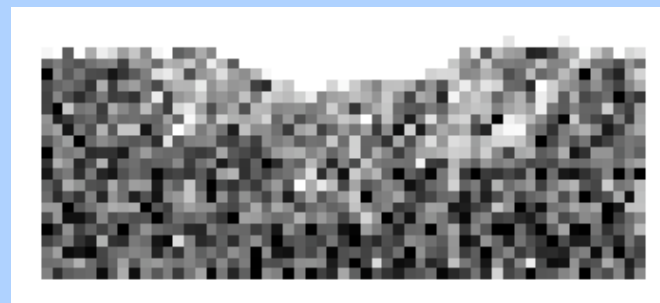
0.02 K/ps



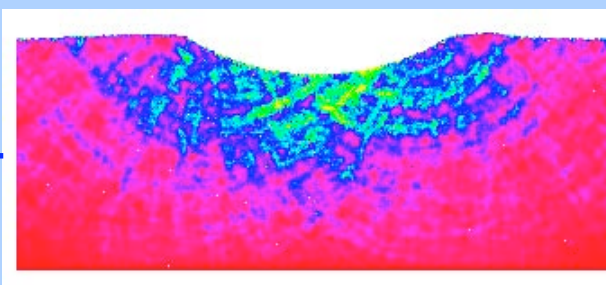
Sample II



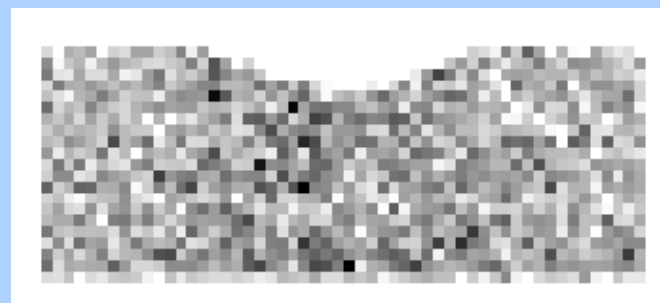
1K/ps



Sample III

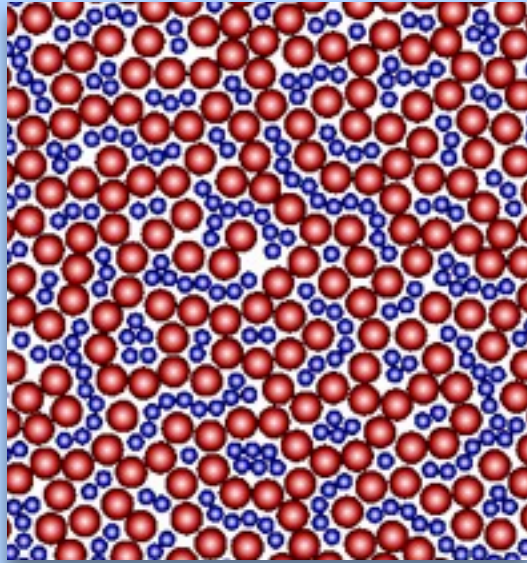


50K/ps

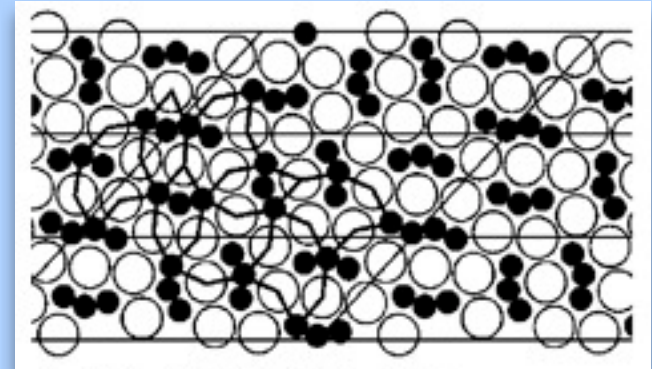


Y. Shi, MLF, *Acta Materialia*, 55, 4317 (2007)

# 2D Simulation System

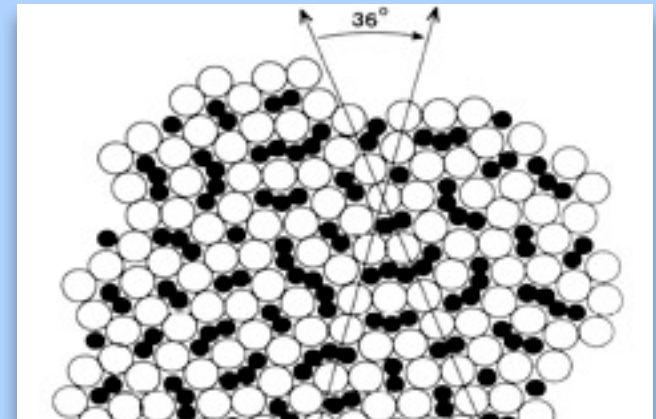


(Lancon et al, Europhys. Lett, 1986)



Lee, Swendsen, Widom (2001)

- **2D binary Lennard-Jones 12-6 potential**
- **Binary system with quasi-crystalline packing**
- **45:55 composition, 20,000-80,000 atoms**
- **$T_{MCT} \approx 0.325$**



Widom, Strandburg, Swendsen (1987)

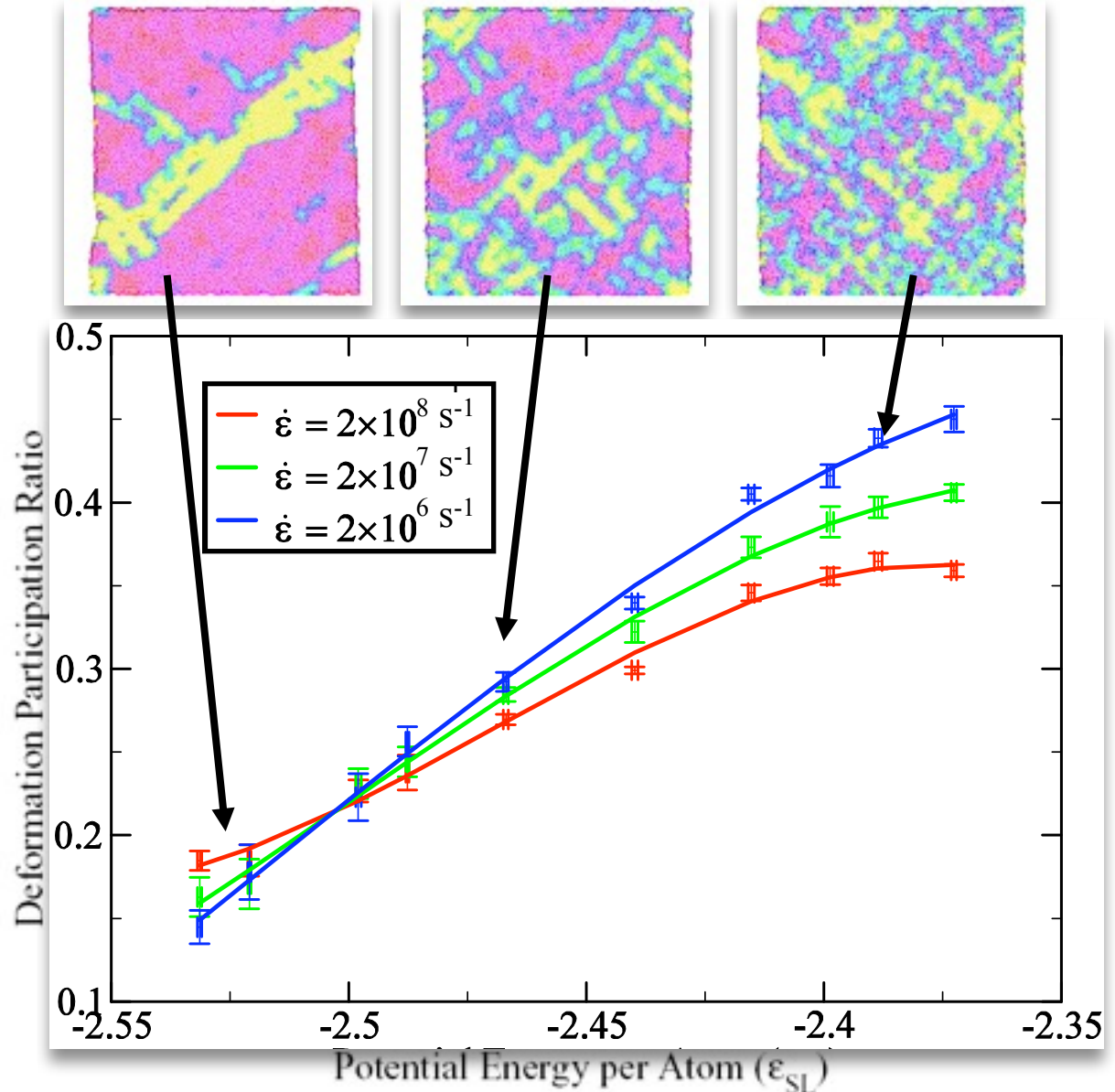
# Quantifying the Dependence of Localization on Quench Rate (2D)

- Performed **756** individual 2D uniaxial tensile test simulations at  $0.1 T_g$
- 10 different quench schedules starting from equilibrium liquids
- 6-10 samples at each quench schedule
- Each of these 84 specimens was tested at 9 different strain rates spanning 2 orders of magnitude

# Quantification of Shear Localization

## Deformation Participation Ratio

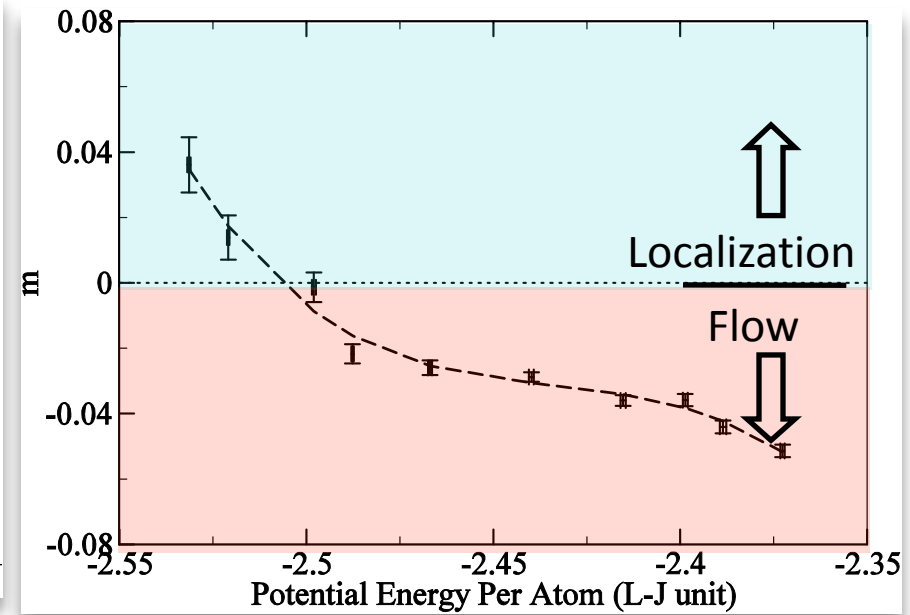
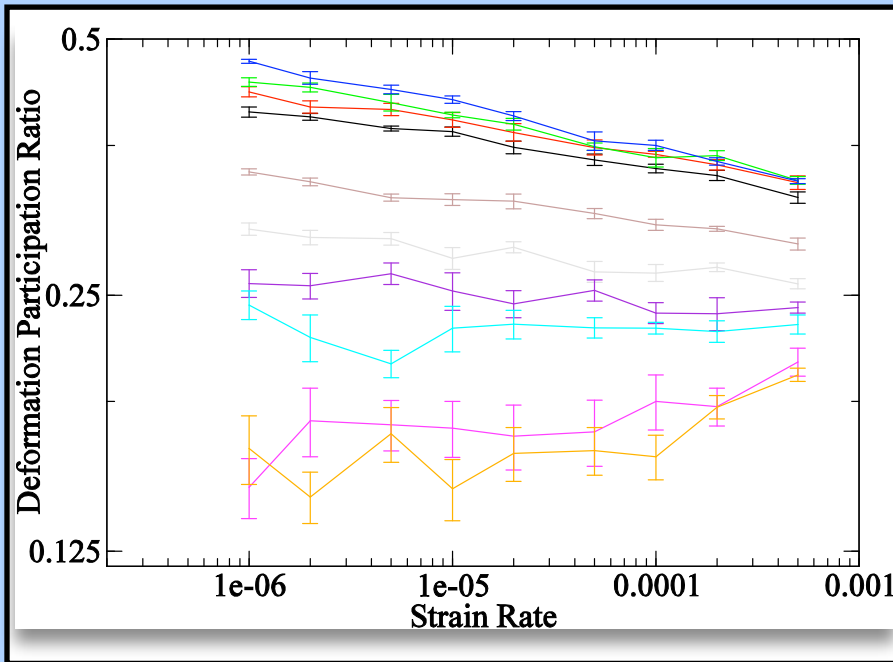
- Participation Ratio: Percentage of material with a local shear strain larger than the nominal strain
- Low strain rate favors homogenous deformation in instantaneously quenched samples
- Low strain rate favors inhomogeneous deformation in gradually quenched samples.



Shi and Falk, PRL (2005)

18 May 2010

# Strain-rate sensitivity of DPR



$$DPR \approx A \dot{\epsilon}^m$$

For  $\dot{\epsilon} \rightarrow 0$  and system size  $\rightarrow \infty$

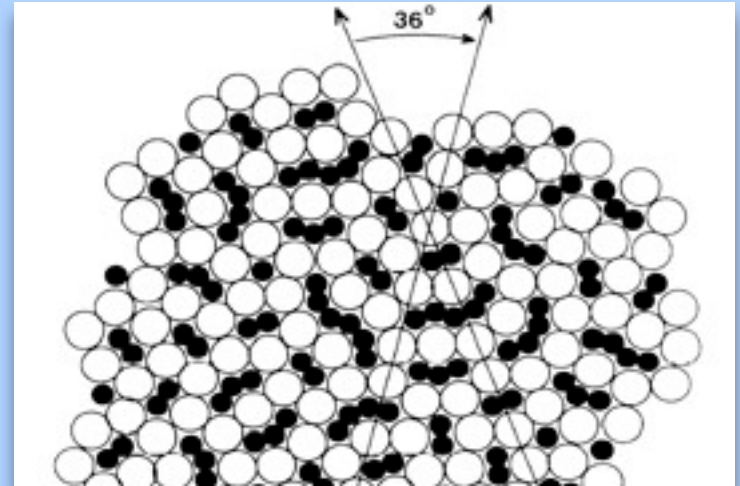
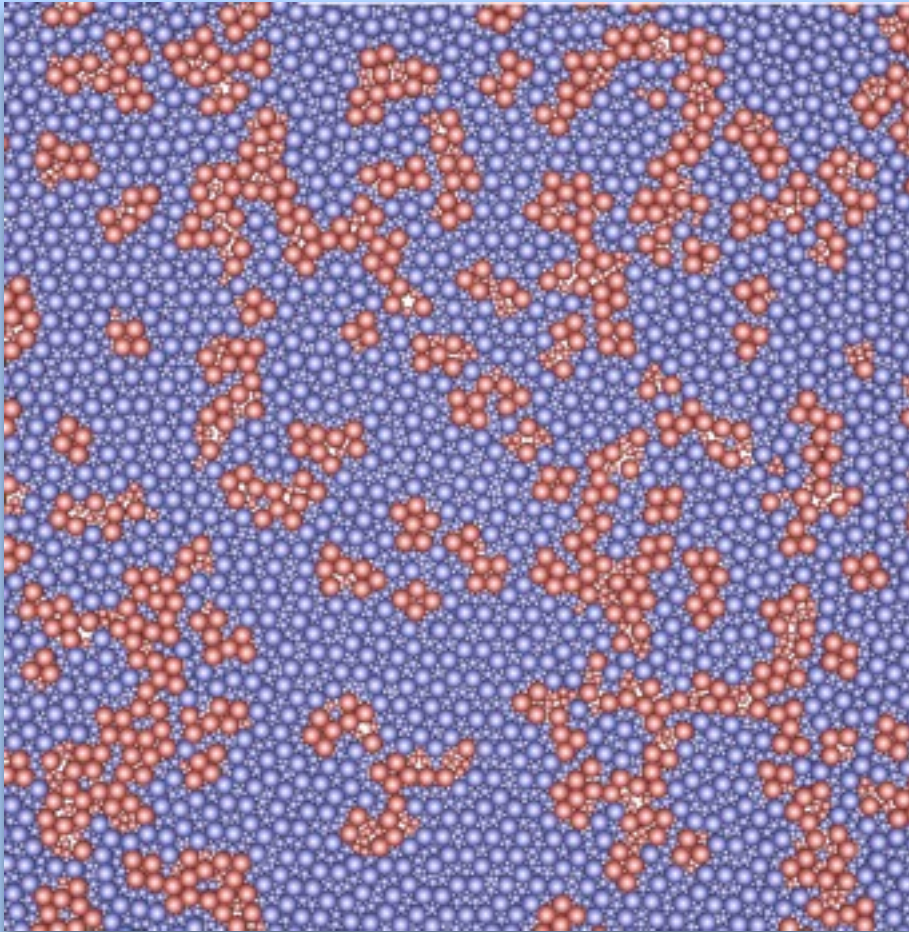
$m < 0$ : homogenous deformation

$m \geq 0$ : localized deformation

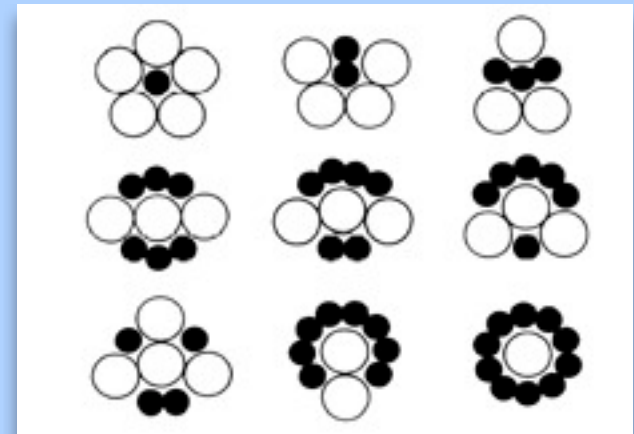
**Shi and Falk, Scripta Mat (2005)**



# Local Structural Analysis



*Widom, Strandburg, Swendsen (1987)*

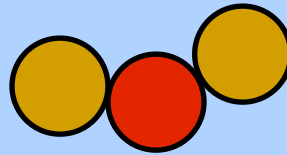


*Complete set of low-energy local environments (Widom, 1987)*

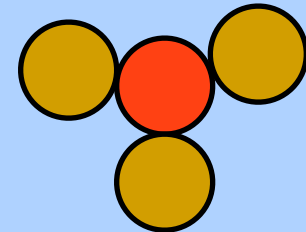


# K-core Percolation of SRO

Serves as a simple approximation of rigidity percolation

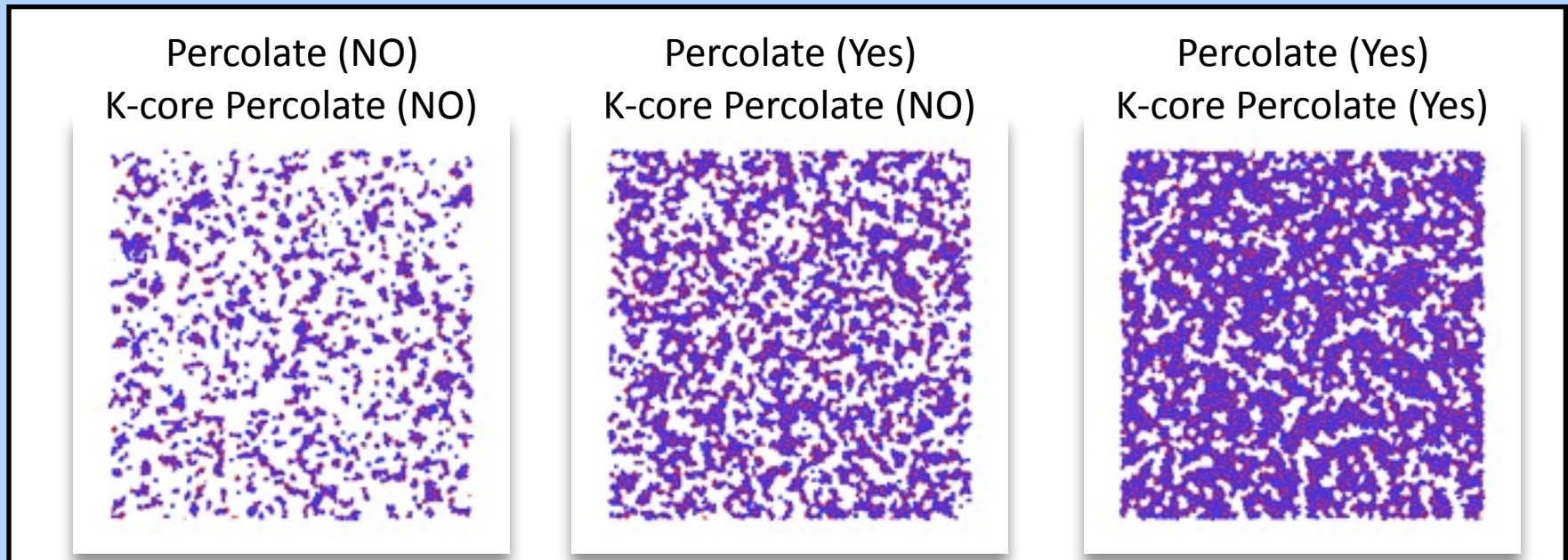


Mechanically unstable

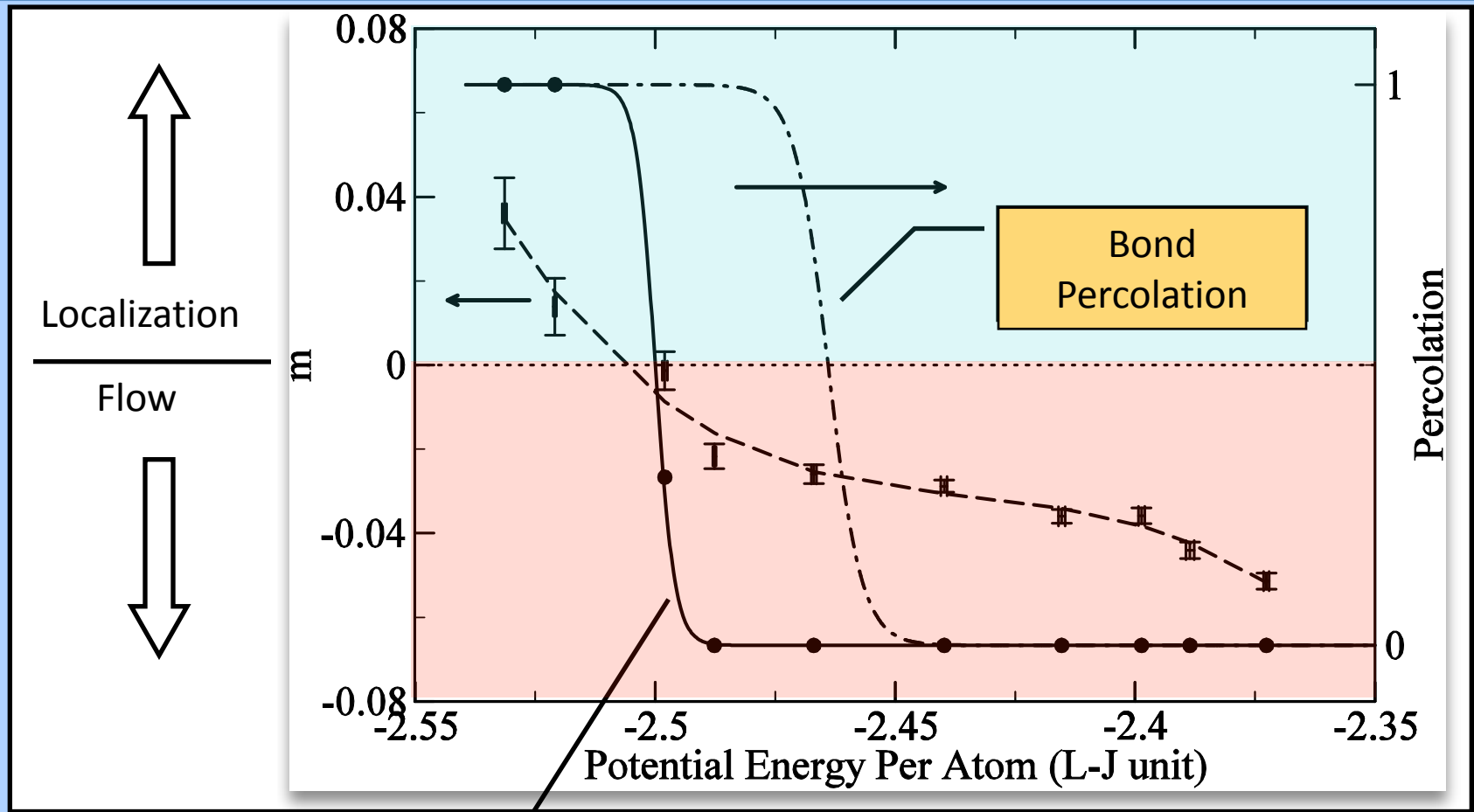


Mechanically stable

Schwarz, Liu and Chayes, arXiv:cond-mat/0410595, 2004



# K-Core percolation and

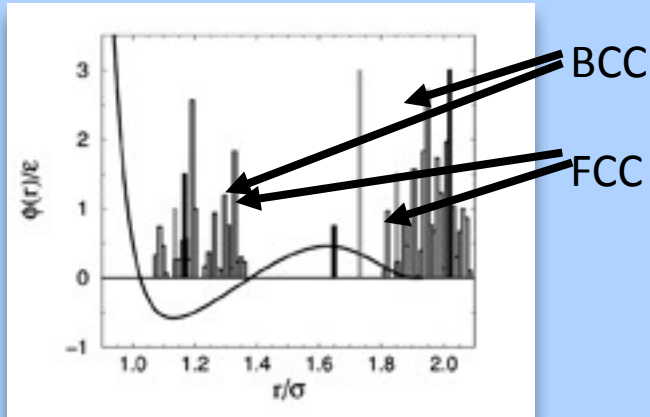


K-core  
Percolation

*Y. Shi and M.L. Falk, PRL, 95, 095502 (2005)*

# 3D Simulation Potentials

## Dzugutov Potential

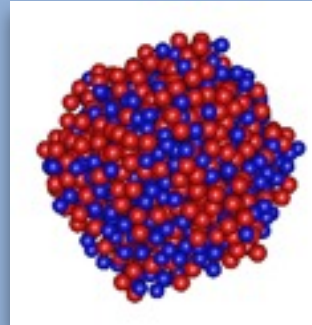


*Roth and Denton, PRE (2000)*

- 3D Monoatomic
- Energy penalties for crystalline phases
- Dodecagonal quasicrystal
- $T_{MCT} @ 0.4$

*Zetterling et al., JNCS (2001)*

## Wahnstrom LJ Binary



Bond length      Bond strength

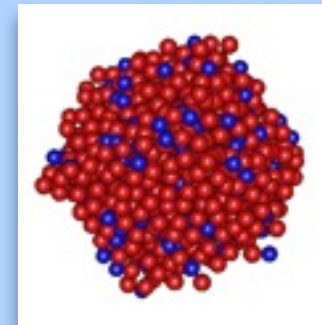
AA 1.000	AA 1.0
AB 0.917	AB 1.0
BB 0.833	BB 1.0

- 3D binary LJ 12-6 potential
- 50:50 composition, 144,000 atoms
- $T_{MCT} @ 0.57$

*Wahnstrom, PRA, 1991*

*Lacevic et al., PRB, 2002*

## Kob-Andersen LJ Binary



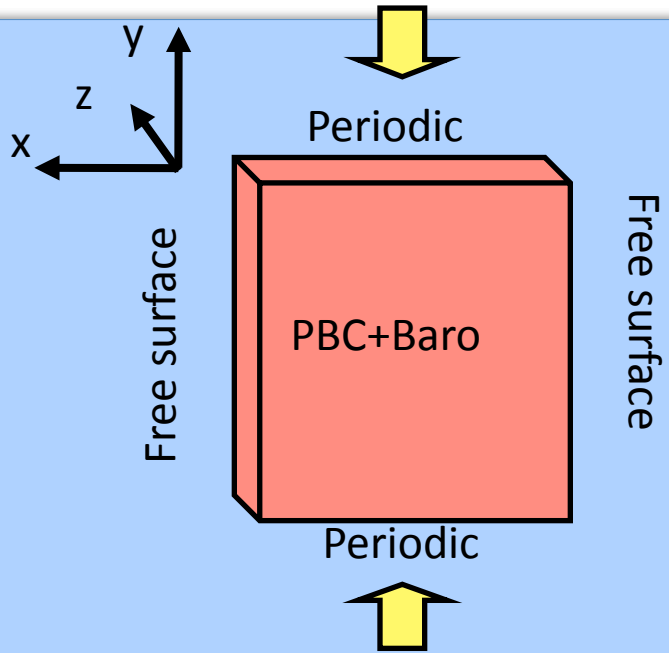
Bond length      Bond strength

AA 1.00	AA 1.0
AB 0.80	AB 1.5
BB 0.88	BB 1.0

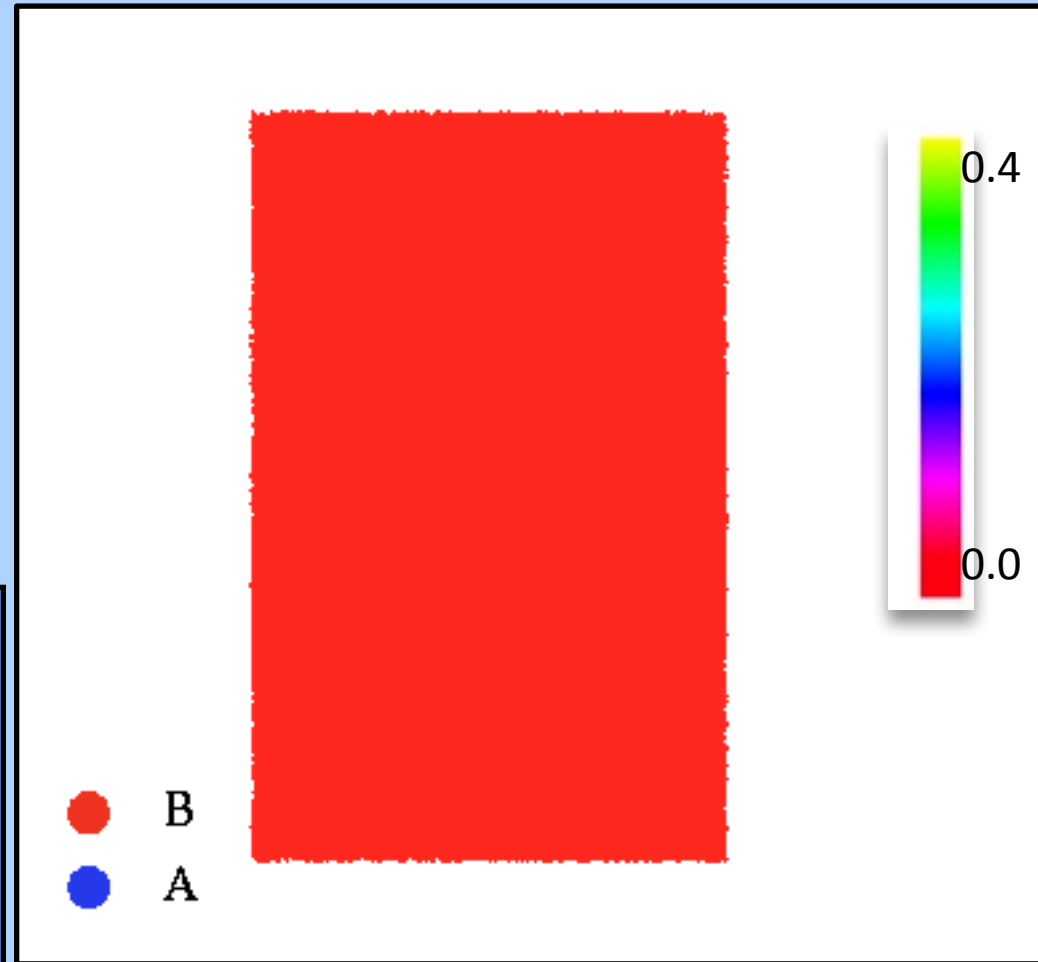
- 3D binary LJ 12-6 potential
- 80:20 composition, 144,000 atoms
- $T_{MCT} @ 0.435$

*Kob and Andersen, PRE 1995*

# 3D Uniaxial Compression Test

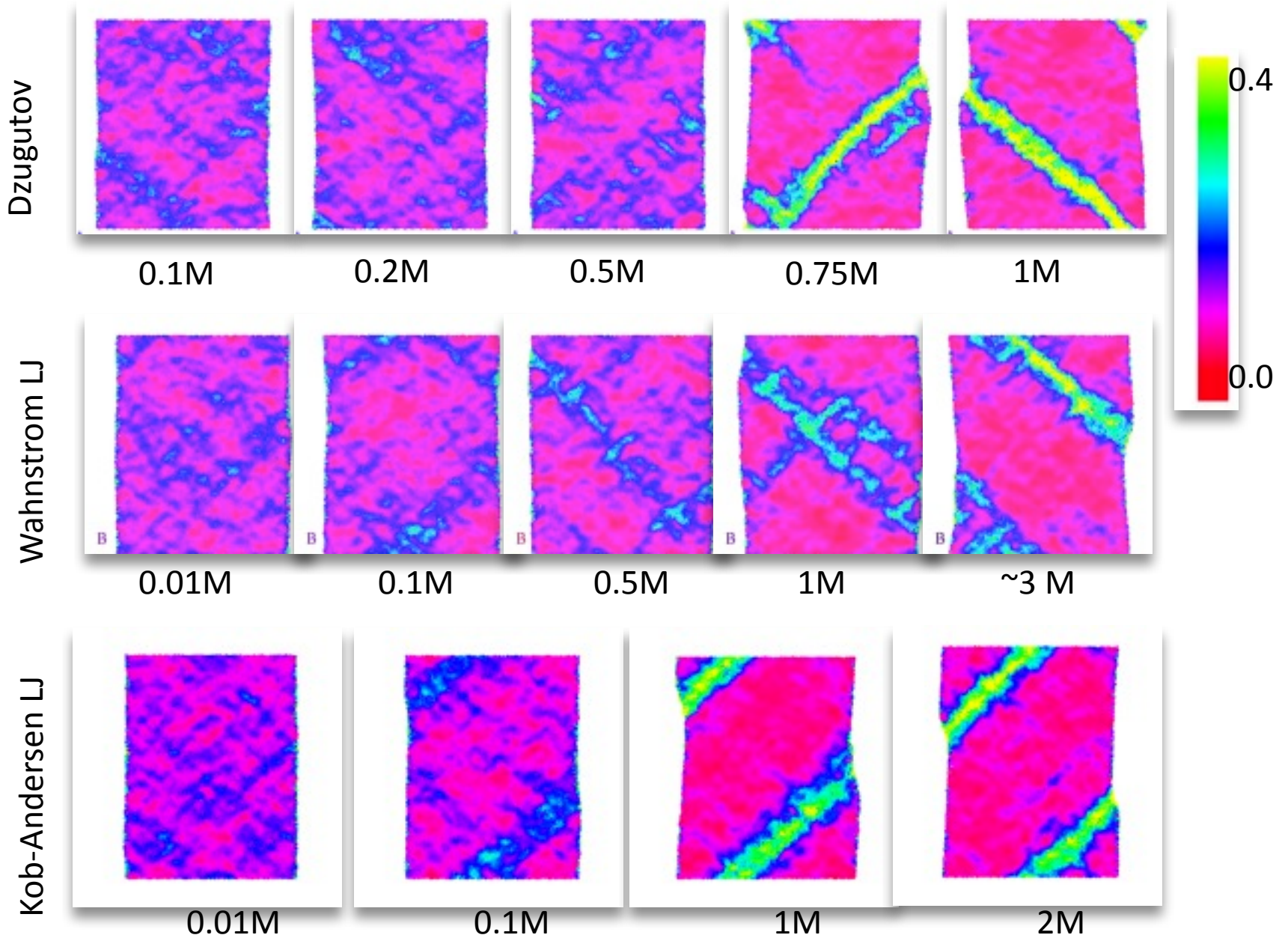


- Thin slab geometry to maximize in-plane spatial dimension  
75×110 ×15: 140,000 atoms
- Free surfaces in Y-Z
- PBC in X-Y and Y-Z
- Plane Strain: Average  $\sigma_{zz}$  zero



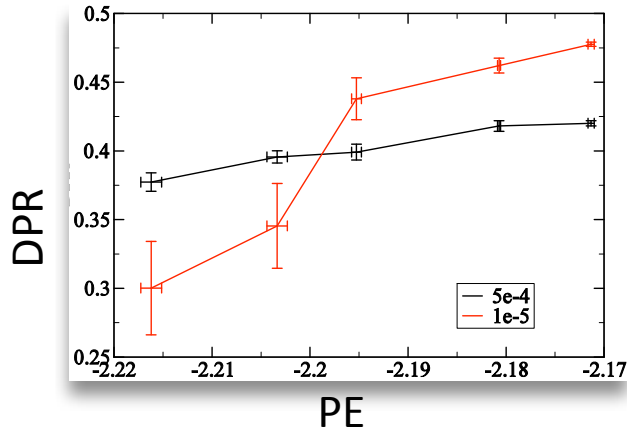


# 3D Uniaxial Compression Various Quench Times

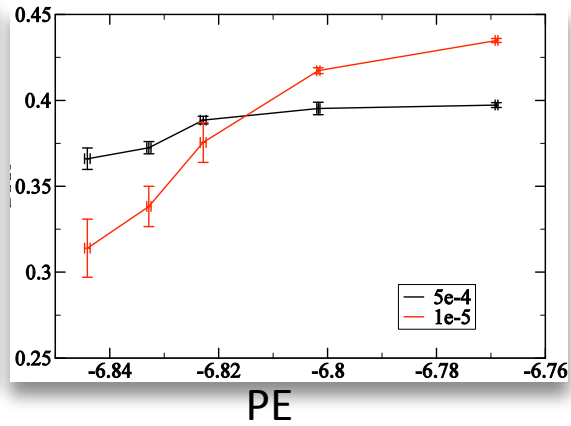


# DPR and Strain Rate Sensitivity

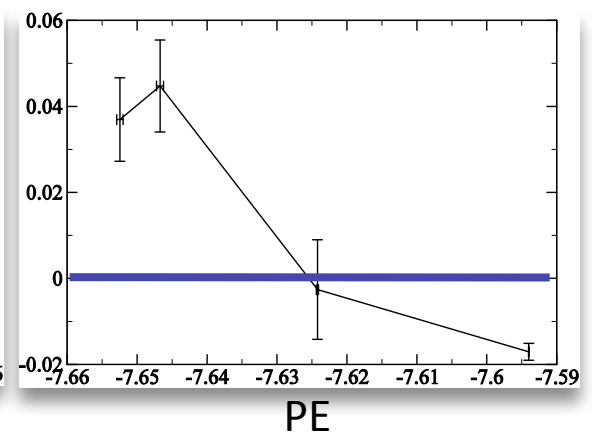
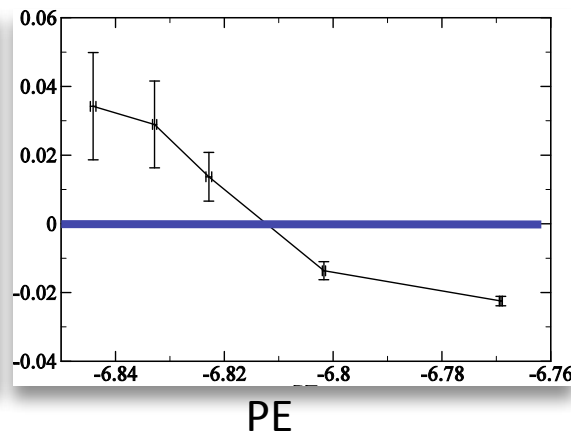
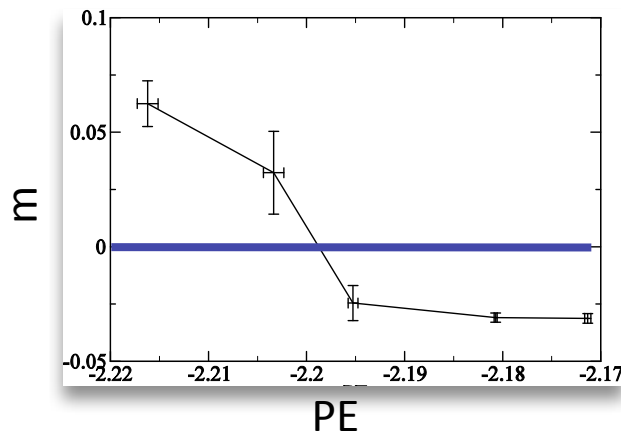
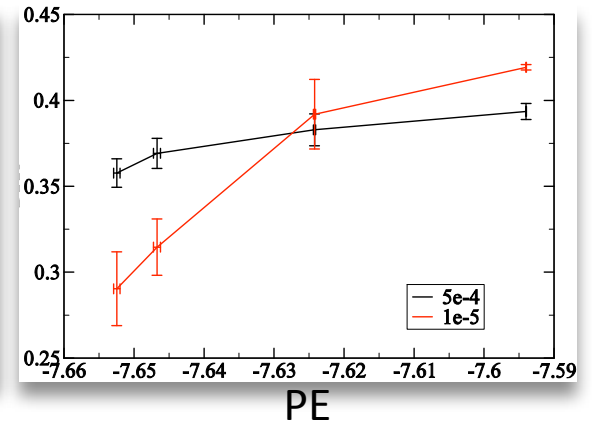
### Dzugutov System



### Wahnstrom LJ



### Kob-Andersen LJ





# Triangulated Coordination Shell Analysis of SRO

Triangulated Coordination Shells: Bonds by atoms within the coordination shell form only triangles. The center atom and the triangle has to form a space dividing tetrahedral.

Criterion: (From Euler's formula)

$$\sum_q (6 - q)v_q = 12$$

$q$  is the surface coordination number (from 3 to 8 for now)

$v_q$  is the count of neighbors has surface coordination number  $q$

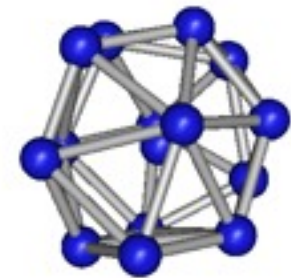
Glassy samples with lowest quenching rate

	TCS	Icosahedra
Dzugutov	25%	12%
Wahnstrom	13%	10%
K-A	3%	0.1%

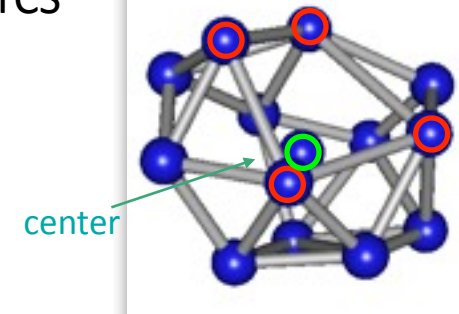
18 May 2010

Kavli Institute for Theoretical Physics

TCS

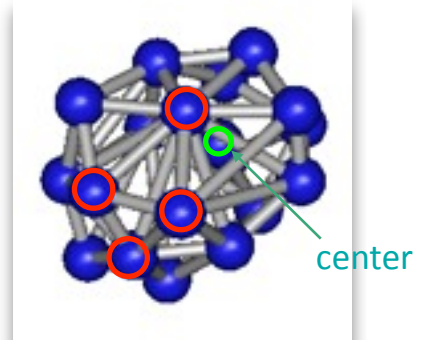


NOT TCS



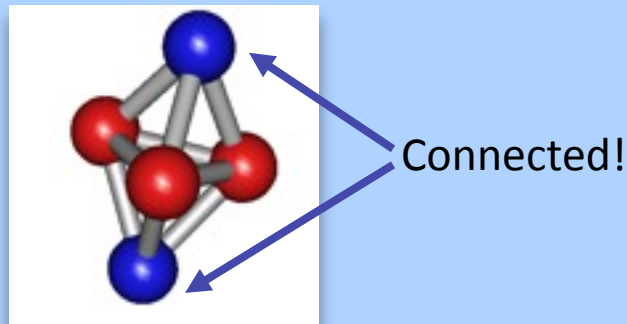
The four red atoms are forming a non-planar quadrilateral not a triangle

NOT TCS



The tetrahedra formed by 4 red atoms does not include the center atom (green)

# 3D Percolation Analysis



Two atoms sharing at least three atoms are “connected”

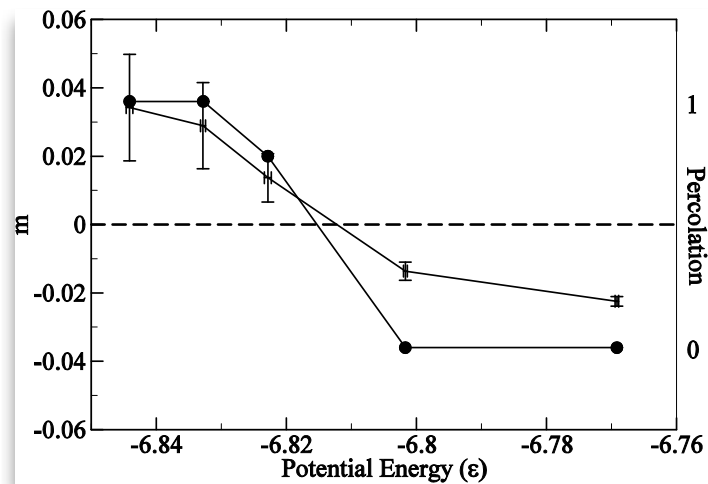
The cage of two atoms have to interpenetrate or sharing faces

Similar to Zettering, et al, JNCS, 2001

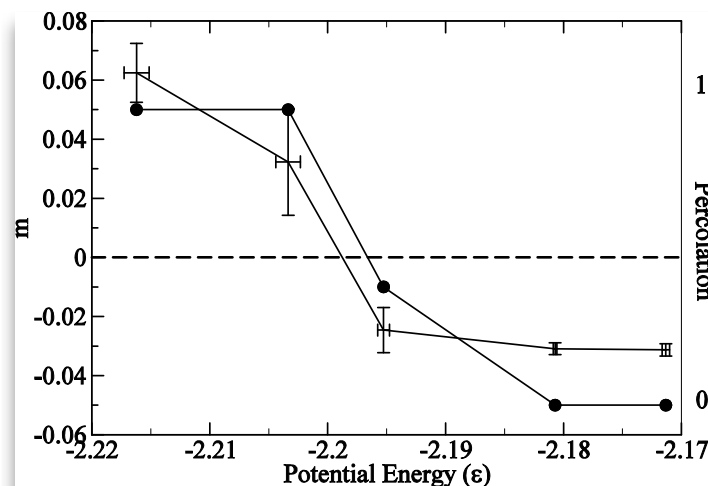
DZ: (all percolate)

KA: (none percolate)

WA system (TCS SRO)



DZ system (Icosahedral SRO)



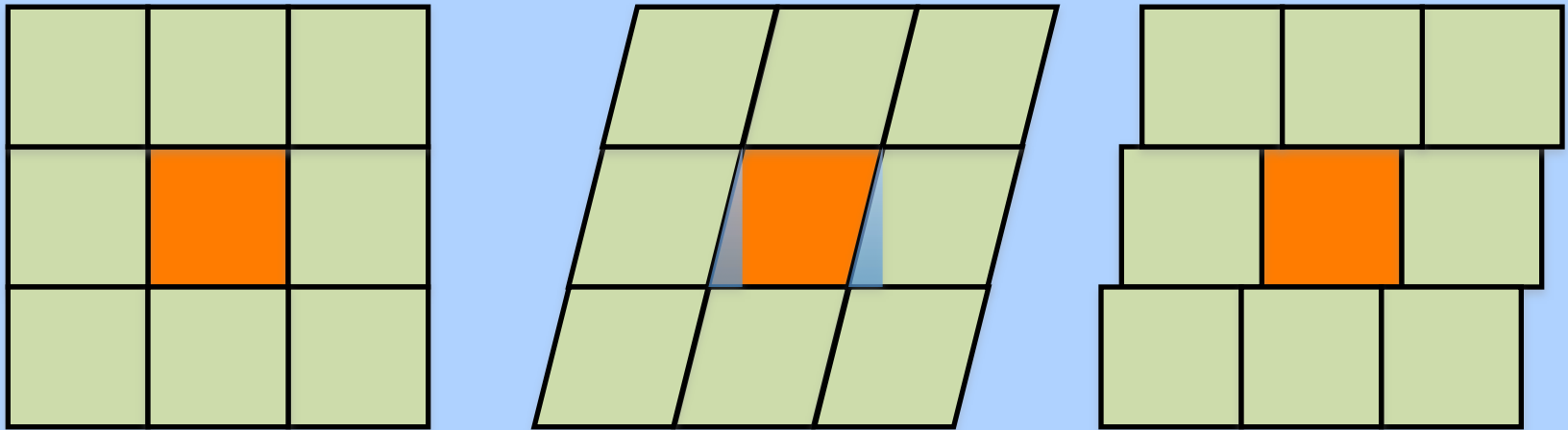
# Short Range Order and Shear Bands

- Simulated glasses with higher degrees of topological SRO demonstrate a stronger tendency to localize strain.
- In more rapidly quenched samples localization **decreases** at lower strain rates.
- In more slowly quenched samples localization **increases** at lower strain rates.
- The transition from homogeneous to localized deformation in the quasi-static limit appears to correspond to the **percolation of a backbone of SRO**.
- How to unambiguously define the appropriate measure of SRO or MRO for a given system **remains an open question**.

*Y. Shi and M.L. Falk,  
Physical Review Letters, 95, 095502 (2005)  
Physical Review B, 73, 214201 (2006)  
Acta Materialia, 55, 4317 (2007)*

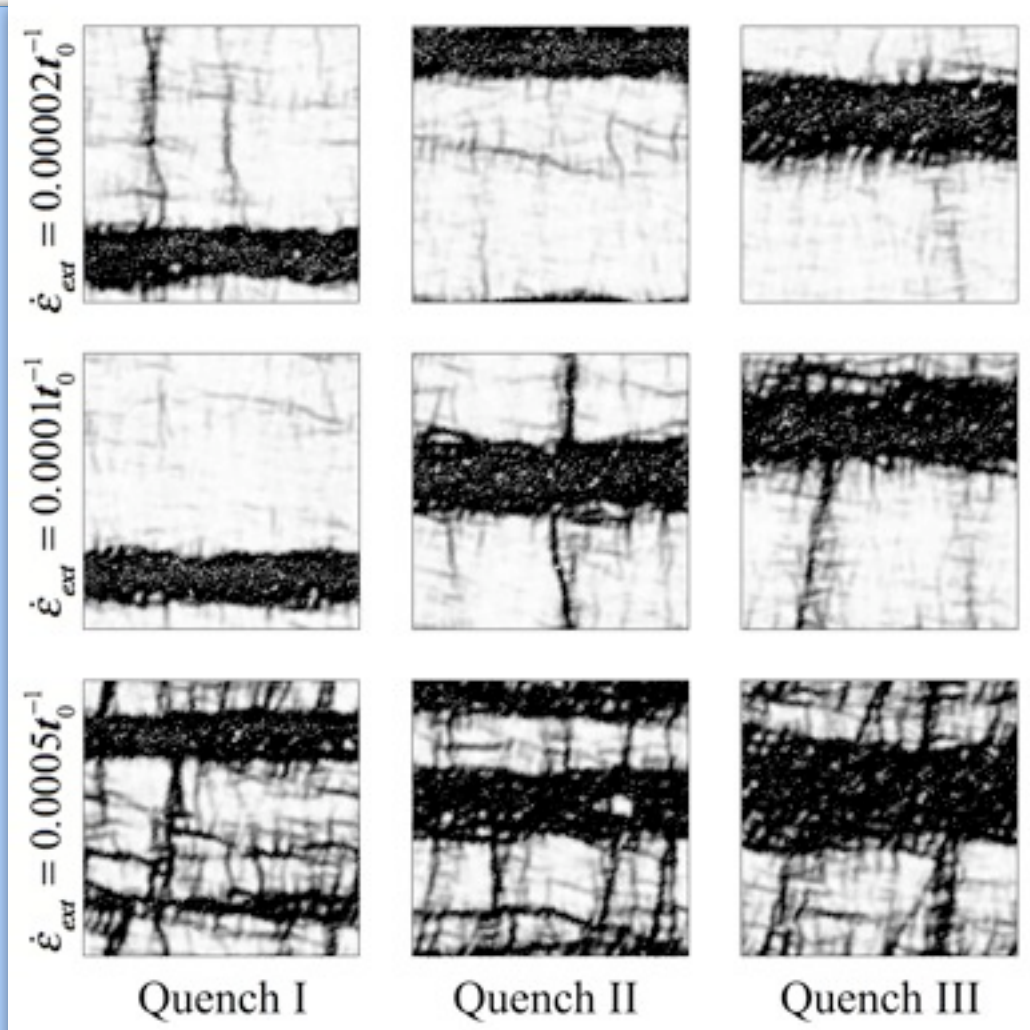
# MD with Periodicity

- Simple shear is imposed maintaining periodicity using Lees-Edwards boundary conditions



- Simultaneously couple to a heat bath throughout so  $T$  is always much less than  $T_g$ .

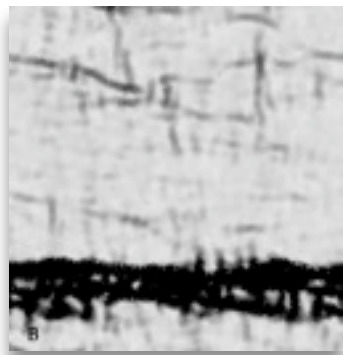
# Simulations in Simple Shear (2D)



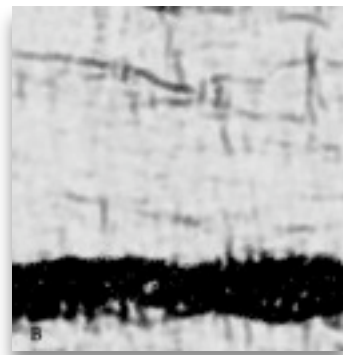
Cumulative strain up to 50% macroscopic shear

Y Shi, MB Katz, H Li, MLF, PRL, 98, 185505 (2007)

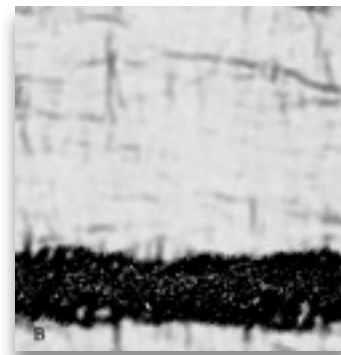
# 2D Simple Shear: Broadening



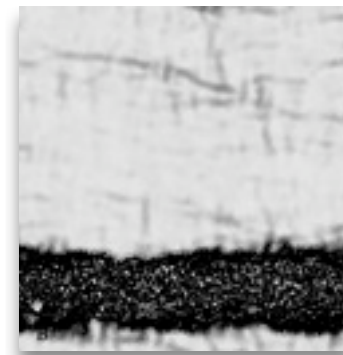
10%



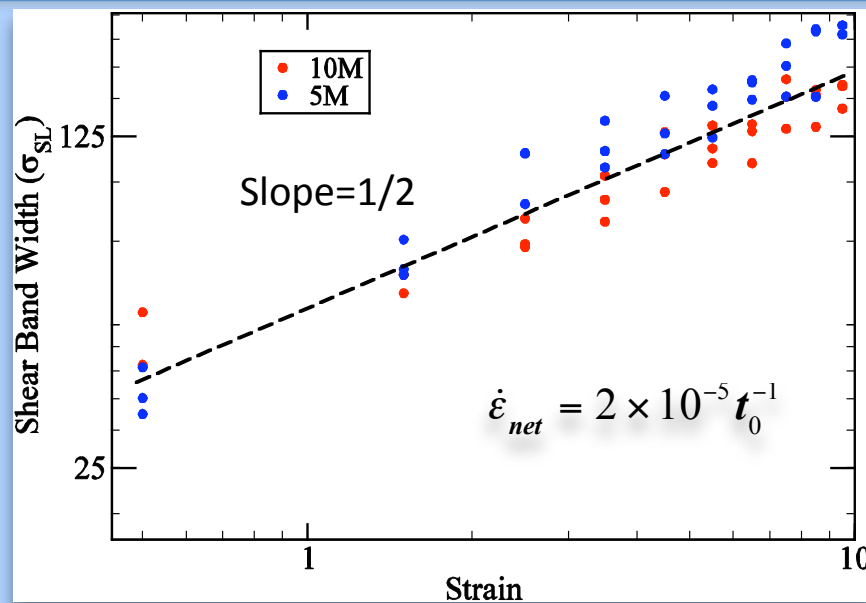
20%



50%



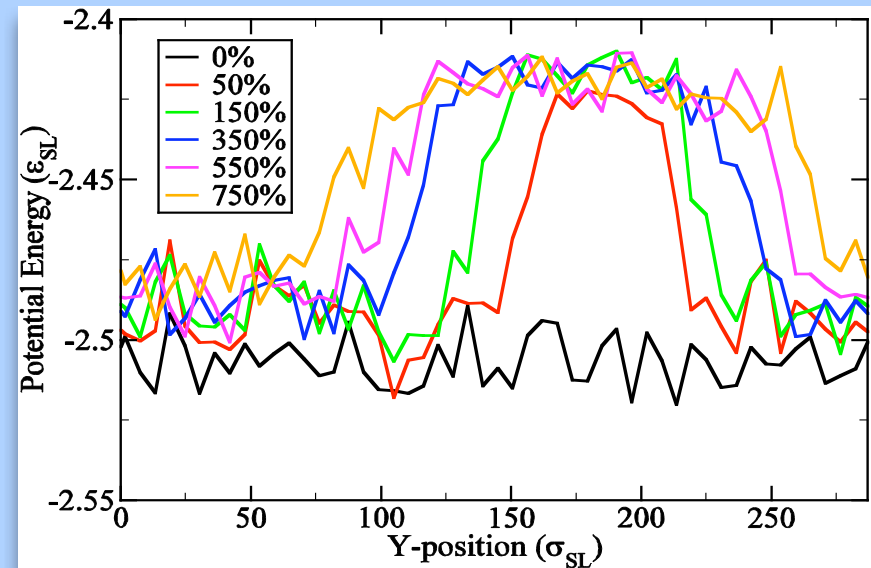
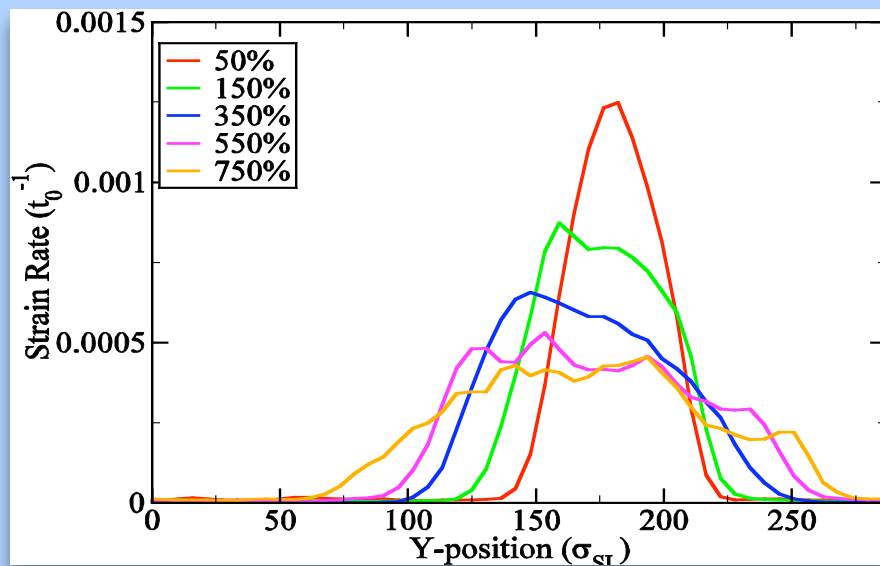
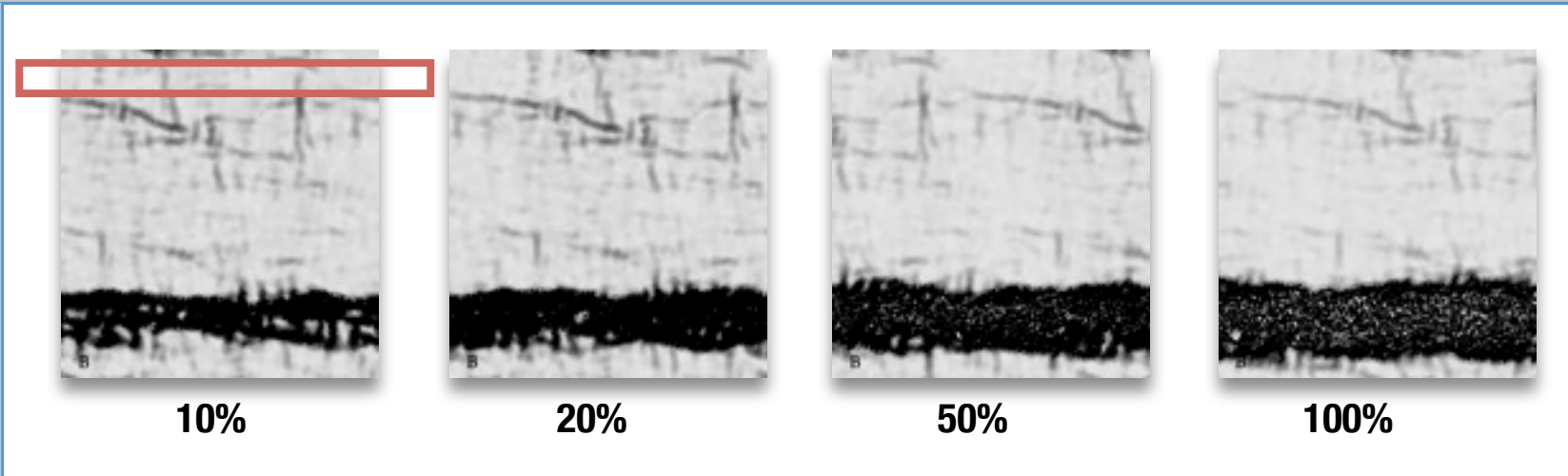
100%



Y Shi, MB Katz, H Li, MLF, PRL, 98, 185505 (2007)



# Development of a Shear Band



Y Shi, MB Katz, H Li, MLF, PRL, 98, 185505 (2007)

# Testing Theories of Plastic Deformation

(Falk and Langer (1998), Falk, Langer and Pechenik (2004), Heggen, Spaepen, Feuerbacher (2005), Langer (2004), Lemaitre and Carlson (2004))

- Is there an intensive thermodynamic property (called  $\chi$  here) that controls the number density of deformable regions (STZs)?

$$n_{STZ} \propto e^{-1/\chi}$$

- This would be an “disorder temperature” that characterizes structural degrees of freedom quenched into the glass.

Disorder Temperature  $T_d$

$$\chi \equiv \frac{kT_d}{E_Z}$$

Free Volume  $v_f$

$$\chi \equiv \frac{v_f}{V^*}$$

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$$\dot{\epsilon}_{ij}^{pl} = e^{-1/\chi} f_{ij} (s_{kl})$$

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$$c_0 \dot{\chi} = 2s_{ij} \dot{\epsilon}_{ij}^{pl} (\chi_{\infty} - \chi) - \kappa(T) e^{-\beta/\chi}$$

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$$c_0 \dot{\chi} = \underbrace{2s_{ij} \dot{\epsilon}_{ij}^{pl}}_{\text{mechanical disordering}} (\chi_{\infty} - \chi) - \kappa(T) e^{-\beta/\chi}$$

mechanical  
disordering

Disorder Temperature  $T_d$

$$\chi \equiv \frac{kT_d}{E_Z}$$

Free Volume  $v_f$

$$\chi \equiv \frac{v_f}{V^*}$$

# Testing Theories of Plastic Deformation

(Falk and Langer (1998), Falk, Langer and Pechenik (2004), Heggen, Spaepen, Feuerbacher (2005), Langer (2004), Lemaitre and Carlson (2004))

- Is there an intensive thermodynamic property (called  $\chi$  here) that controls the number density of deformable regions (STZs)?

$$n_{STZ} \propto e^{-1/\chi}$$

- This would be an “disorder temperature” that characterizes structural degrees of freedom quenched into the glass.

$$\dot{\epsilon}_{ij}^{pl} = e^{-1/\chi} f_{ij}(s_{kl})$$

$$c_0 \dot{\chi} = \underbrace{2s_{ij} \dot{\epsilon}_{ij}^{pl}}_{\text{mechanical disordering}} (\chi_{\infty} - \chi) - \underbrace{\kappa(T) e^{-\beta/\chi}}_{\text{thermal annealing}}$$

mechanical  
disordering

thermal annealing

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# Relating $\chi$ to the microstructure

- Consider a linear relation between the  $\chi$  parameter and the local internal energy

$$C_1 \chi = PE - PE_0$$
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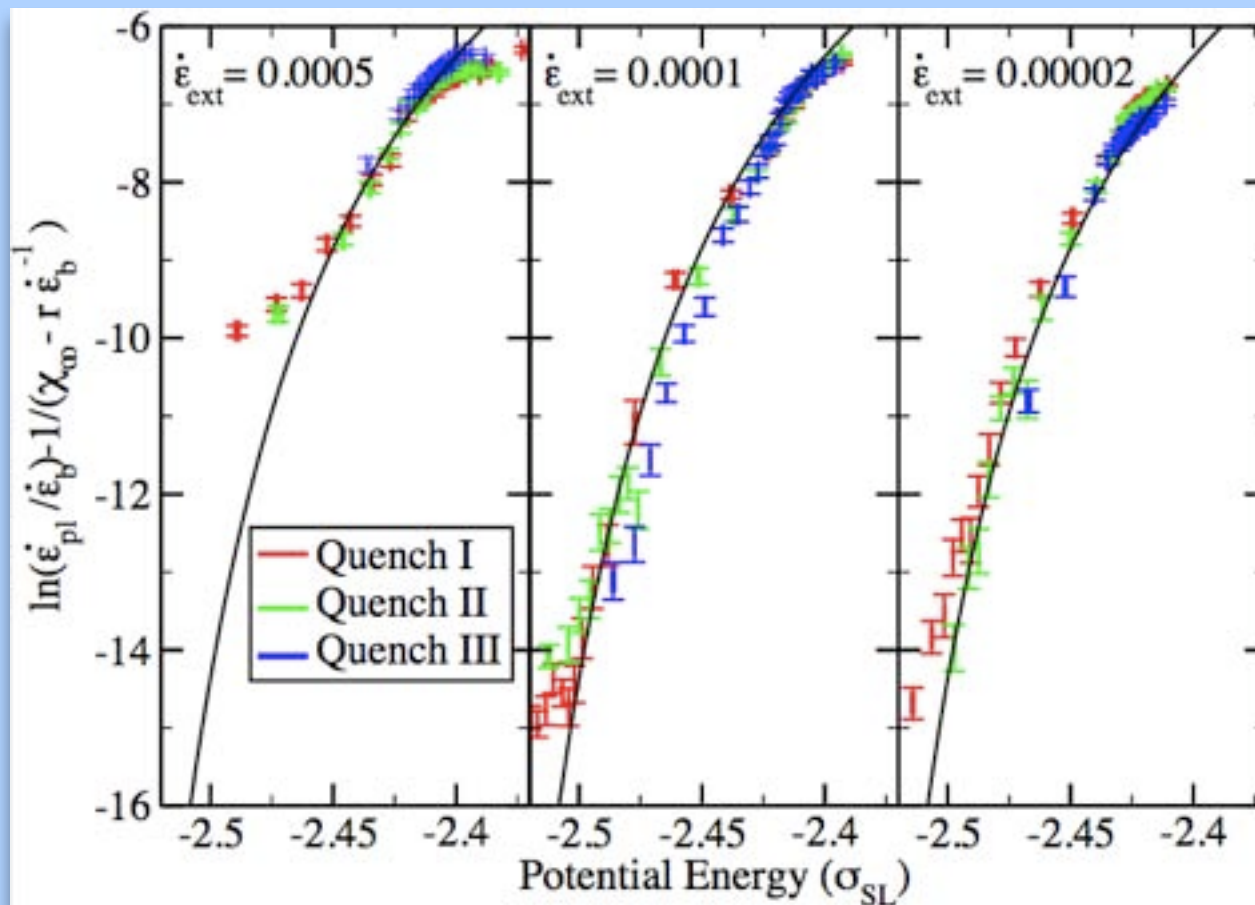
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$$\ln \left[ \frac{\dot{\epsilon}_{pl}(y)}{\dot{\epsilon}_b} \right] - \frac{1}{\chi_\infty - r \dot{\epsilon}_b^{-1}} = - \frac{C_1}{PE - PE_0}$$

# Scaling verifies the hypothesis



- Assuming,  $\chi_{\infty} = \frac{kT_g}{E_z}$ ,  $E_z = 1.9\epsilon$

Y Shi, MB Katz, H Li, MLF, PRL, 98, 185505 (2007)

# Implications for Constitutive Models

- To model the band a length scale must enter the constitutive relations

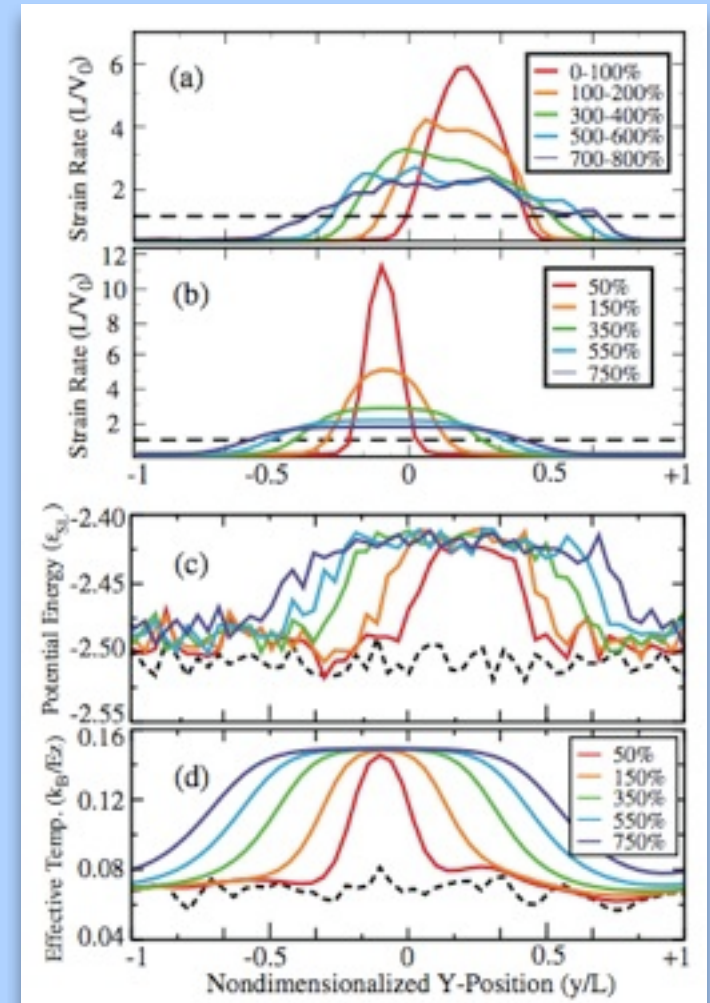
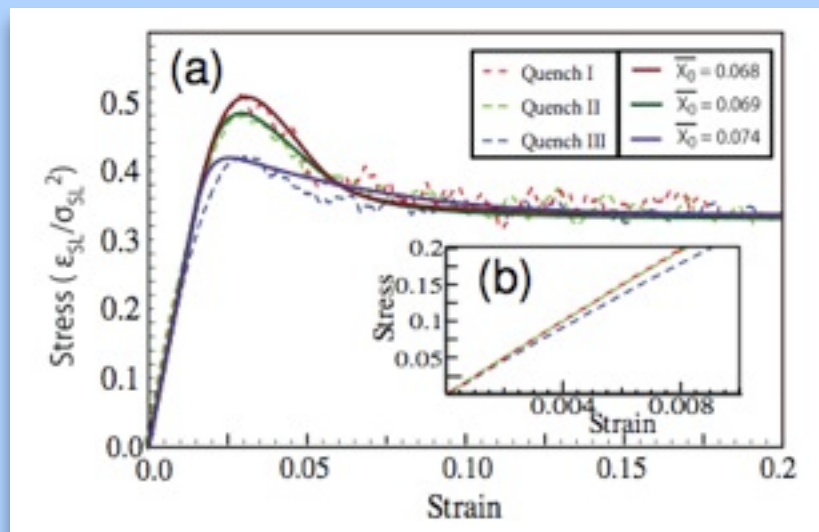
$$\partial_t \chi = \frac{2s\dot{\epsilon}_{pl}}{c_0} (\chi_\infty - \chi) \quad \Rightarrow \quad \partial_t \chi - \mathbf{D} \partial_x^2 \chi = \frac{2s\dot{\epsilon}_{pl}}{c_0} (\chi_\infty - \chi)$$



# Numerical Results

M Lisa Manning and JS Langer, PRE, 76, 056106(2007)

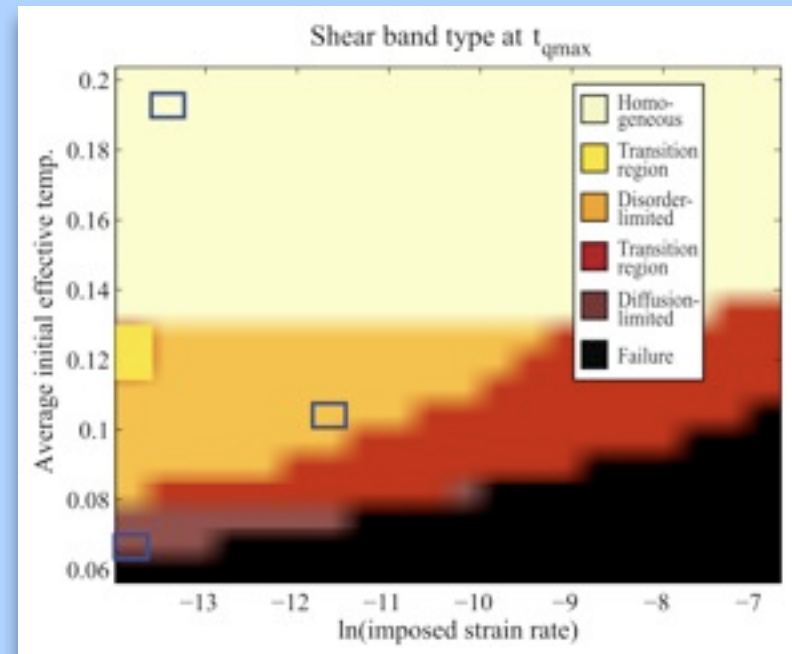
- These equations closely reproduce the details of the strain rate and structural profiles during band formation



# More Analysis by Manning, et al.

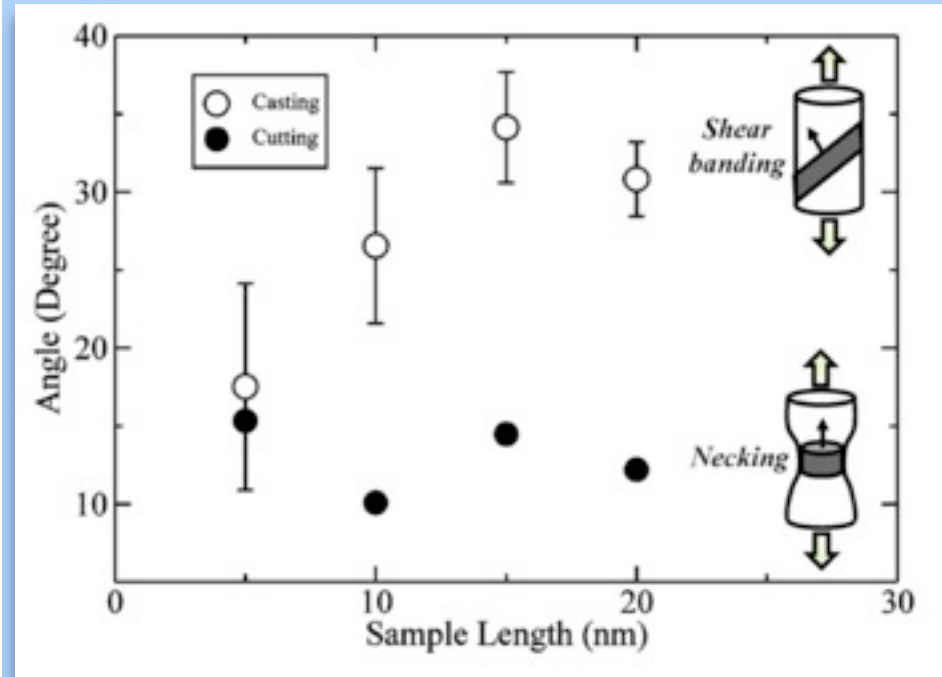
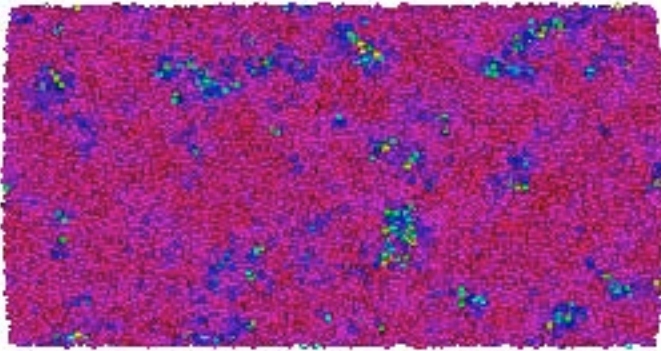
Manning, Daub, Langer, Carlson, Phys. Rev. E 79, 016110 (2009)

- Incorporates the Haxton-Liu effective temperature dynamics and shear rate dependent diffusivity.
- Identifies 3 failure modes:
  - Diffusion limited bands
  - Disorder limited bands
  - Failure/Fracture/Melting



# Effect of Surface Preparation

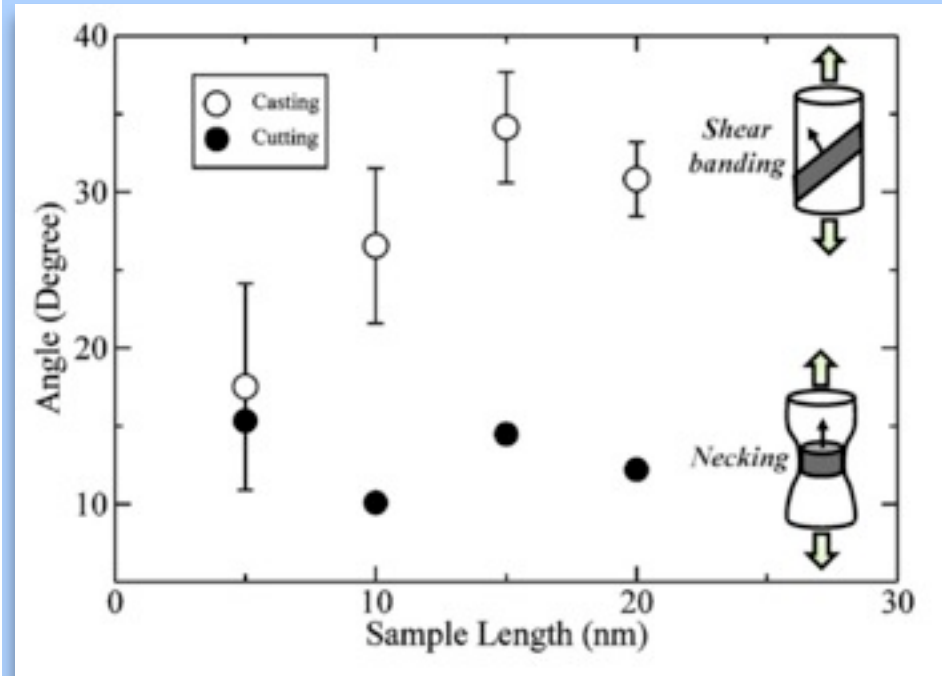
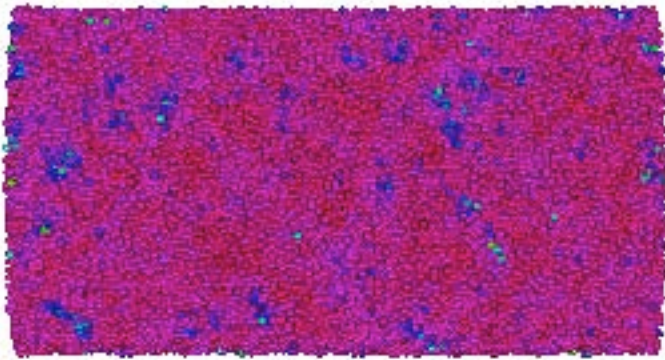
Yunfeng Shi, APL 96, 121909 (2010)



**Created by  
"Cutting"**

# Effect of Surface Preparation

Yunfeng Shi, APL 96, 121909 (2010)



**Created by  
"Casting"**

# Summary

- Shear bands in metallic glasses arise due to **mechanical softening** caused by disordering.
- A **percolating backbone of short range order** appears to be necessary for localization to dominate at low shear rates.
- No unique means exists for characterizing the geometric character of this short range order for a known alloy description.
- Analysis of the transition from flow to jammed material in a shear band reveals that **potential energy per atom may be a good measure of "effective temperature"**.
- The **proportionality of strain rate to  $\exp(-1/\chi)$**  has been tested and appears to hold.
- The data also indicates that the **energy to create an STZ is about 2 bonds per STZ**.