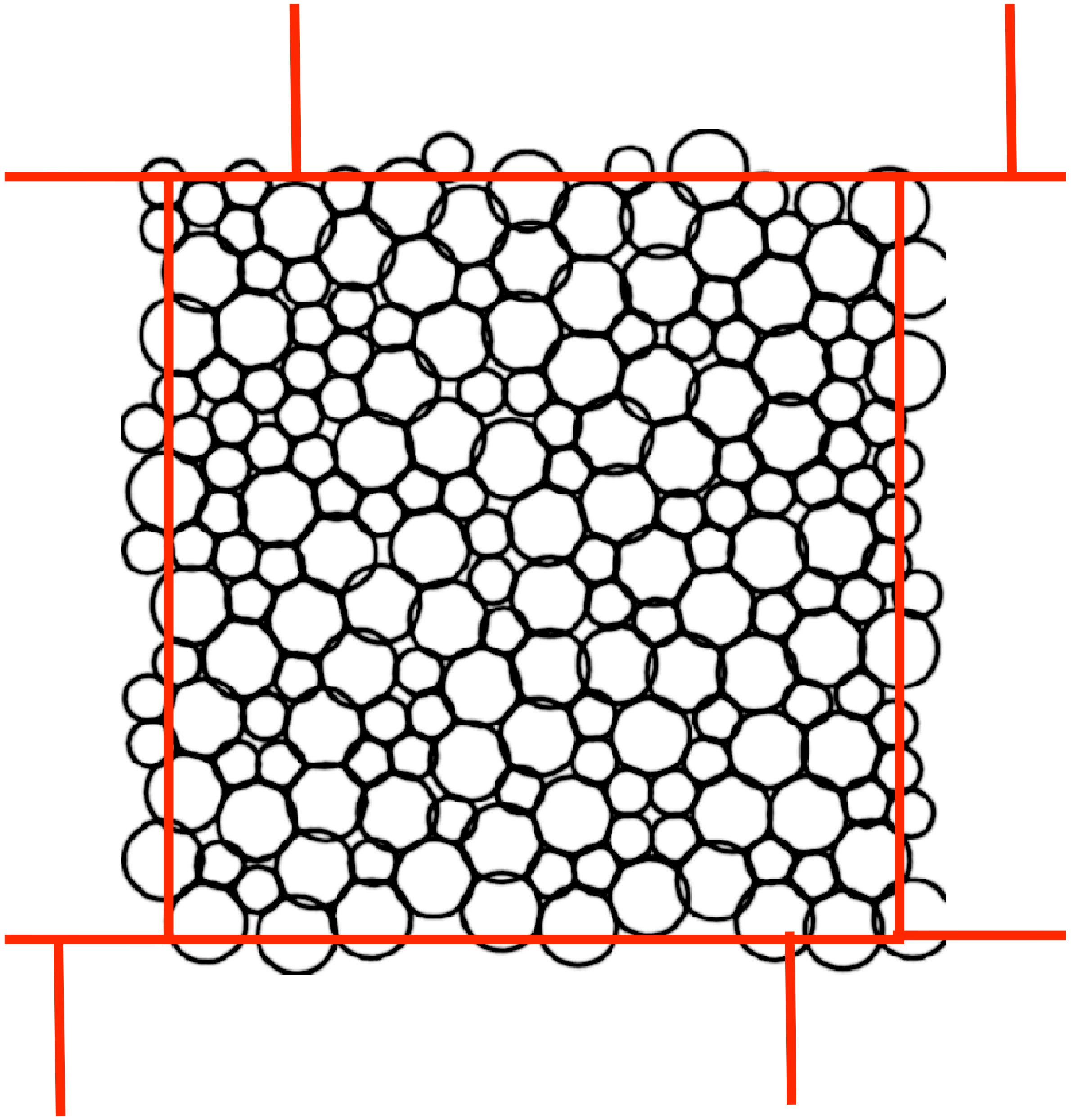
Avalanches and Diffusion in Amorphous Solids Under Athermal, Quasistatic Shear



Craig Maloney

Collaborators:
M. O. Robbins (Hopkins)

Funding:
NSF DMR-0454947 and
PHY99-07949

KITP

June 2010

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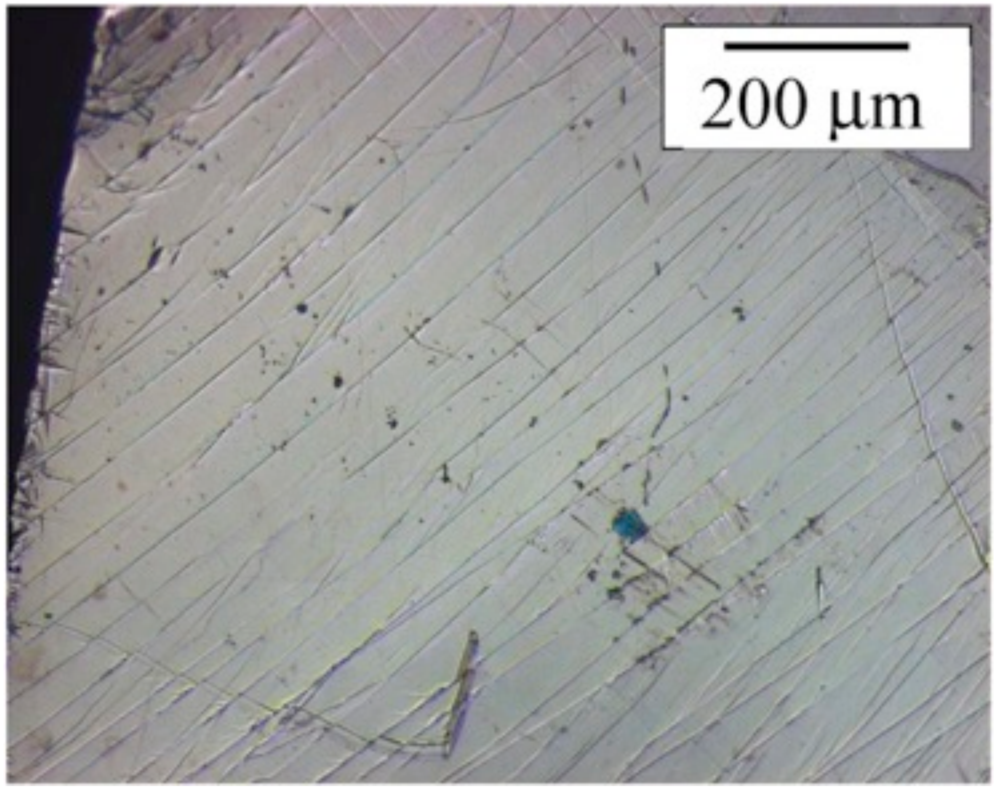
Outline

- Amorphous solids
  - Types
  - Athermal quasistatic shear (AQS)
- Slip lines in Lennard-Jones solids
  - CEM + M.O. Robbins (J. Phys 2008, PRL 2009)
  - Spatial structure of plasticity
  - Effective diffusion
- Jamming
  - (CEM. PRL Submitted)
  - Bubble model / critical scaling near jamming
  - Effective diffusion
  - Avalanches

The question(s) I am asking

- For "simple" amorphous solids in AQS:
  - What is the elementary mechanism(s) which accommodates applied shear?
  - How are they organized in space and time?
  - ~~(How does this impact visco-plastic rheology)~~?



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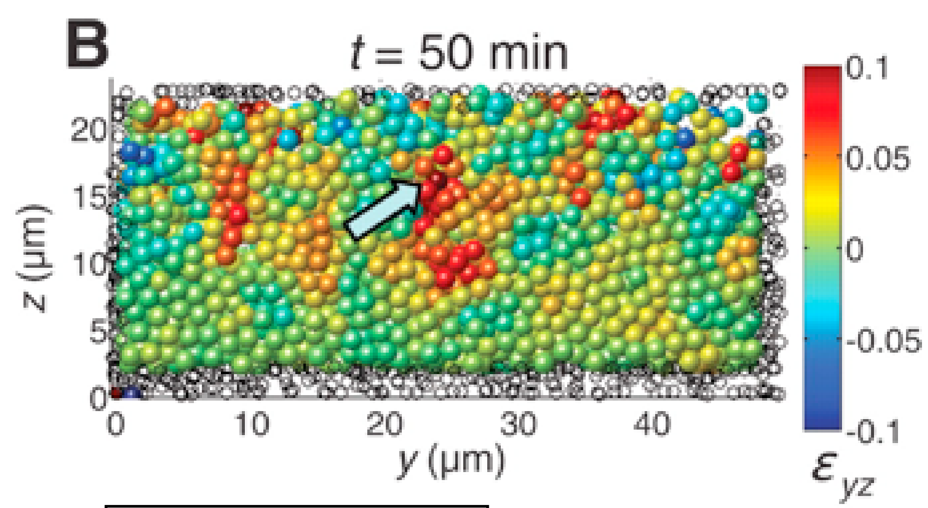
Types of amorphous solids

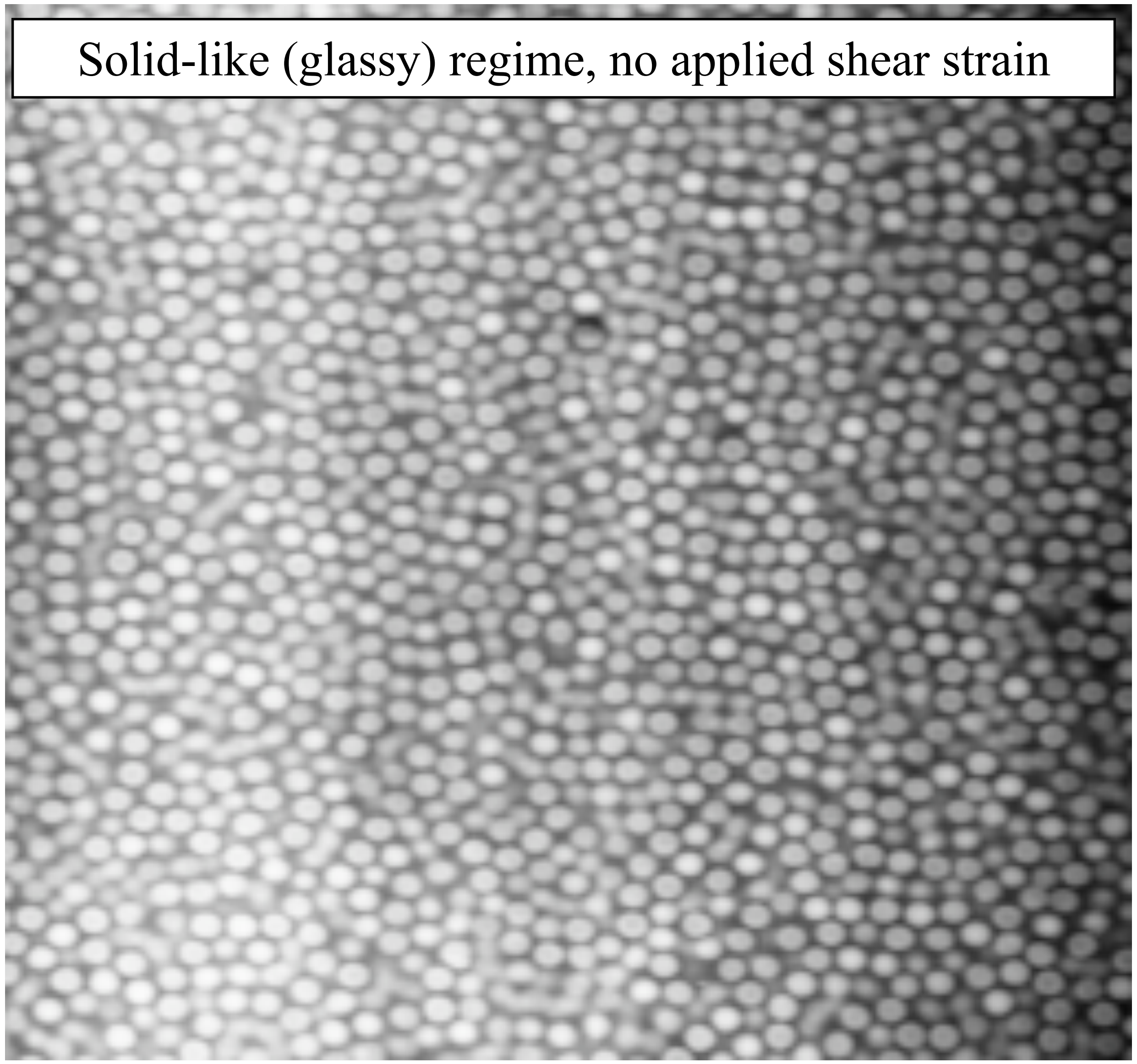
- Types
  - Emulsions / Foams
  - Granular packings
  - Colloidal suspensions
  - Atoms / Molecules

Local shear strain under driving:



(Schall et. al.)

Polydisperse PMMA spheres in density-matched solvent



(Weeks et. al.)

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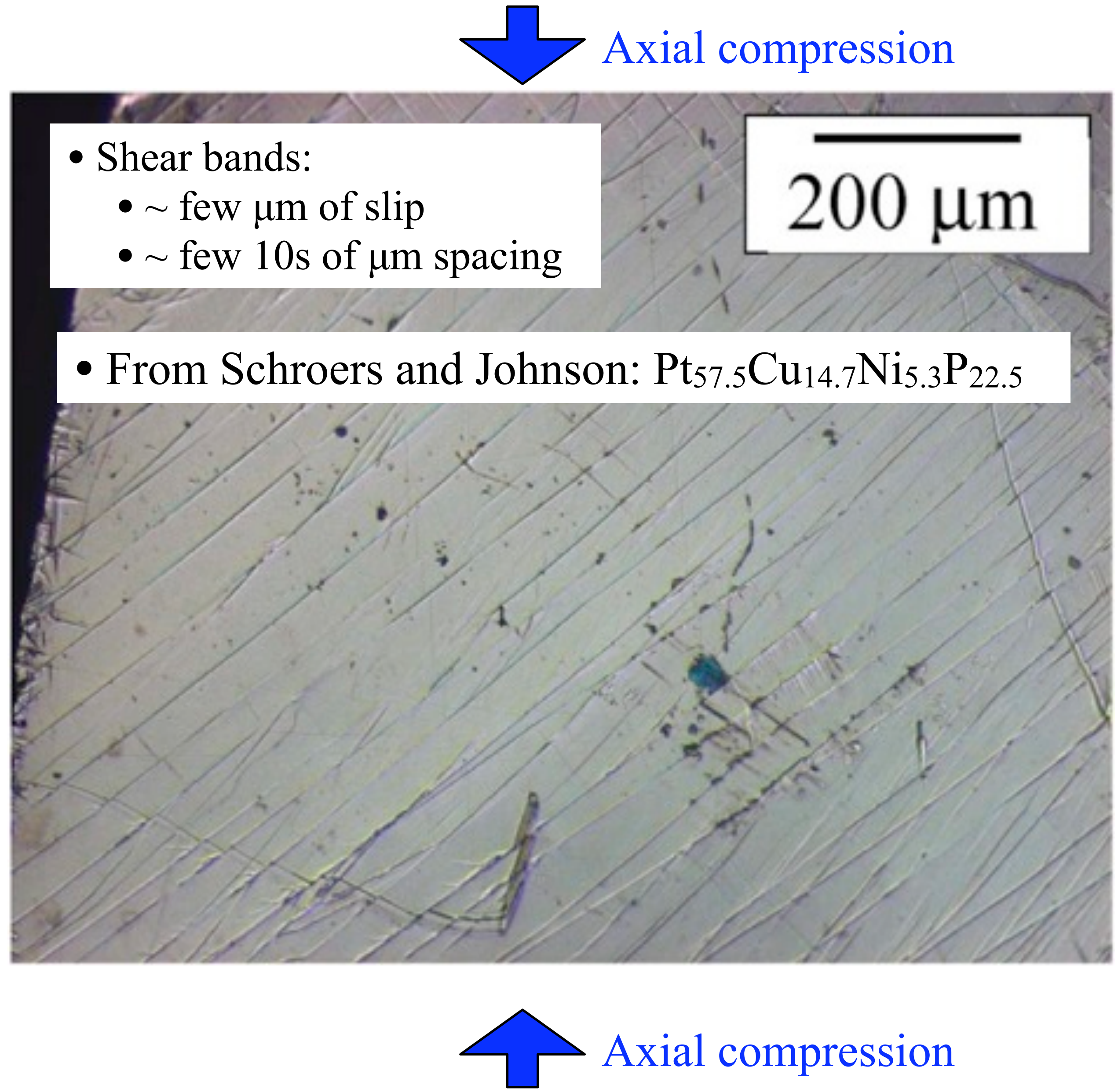
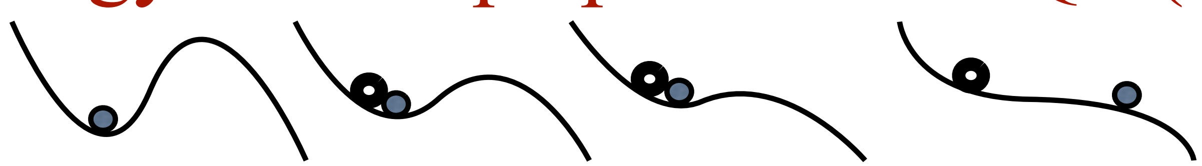
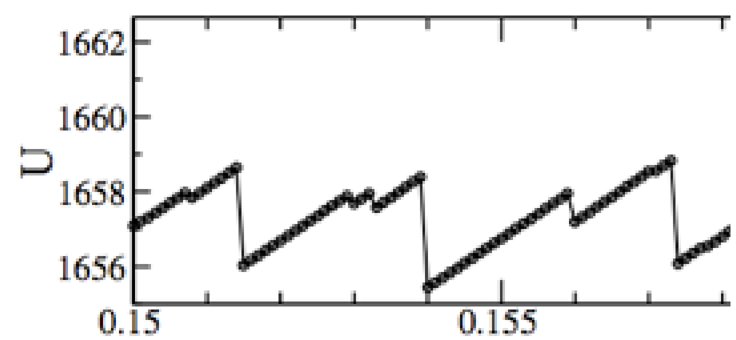


Athermal, quasistatic shear (AQS)

- Differences in particle-scale physics (do they matter?):
  - Inertial or overdamped?
  - “Real” temperature
  - Dissipation mechanisms / hydrodynamics
  - Attractive forces / adhesion
  - Coulomb friction / covalent bonding

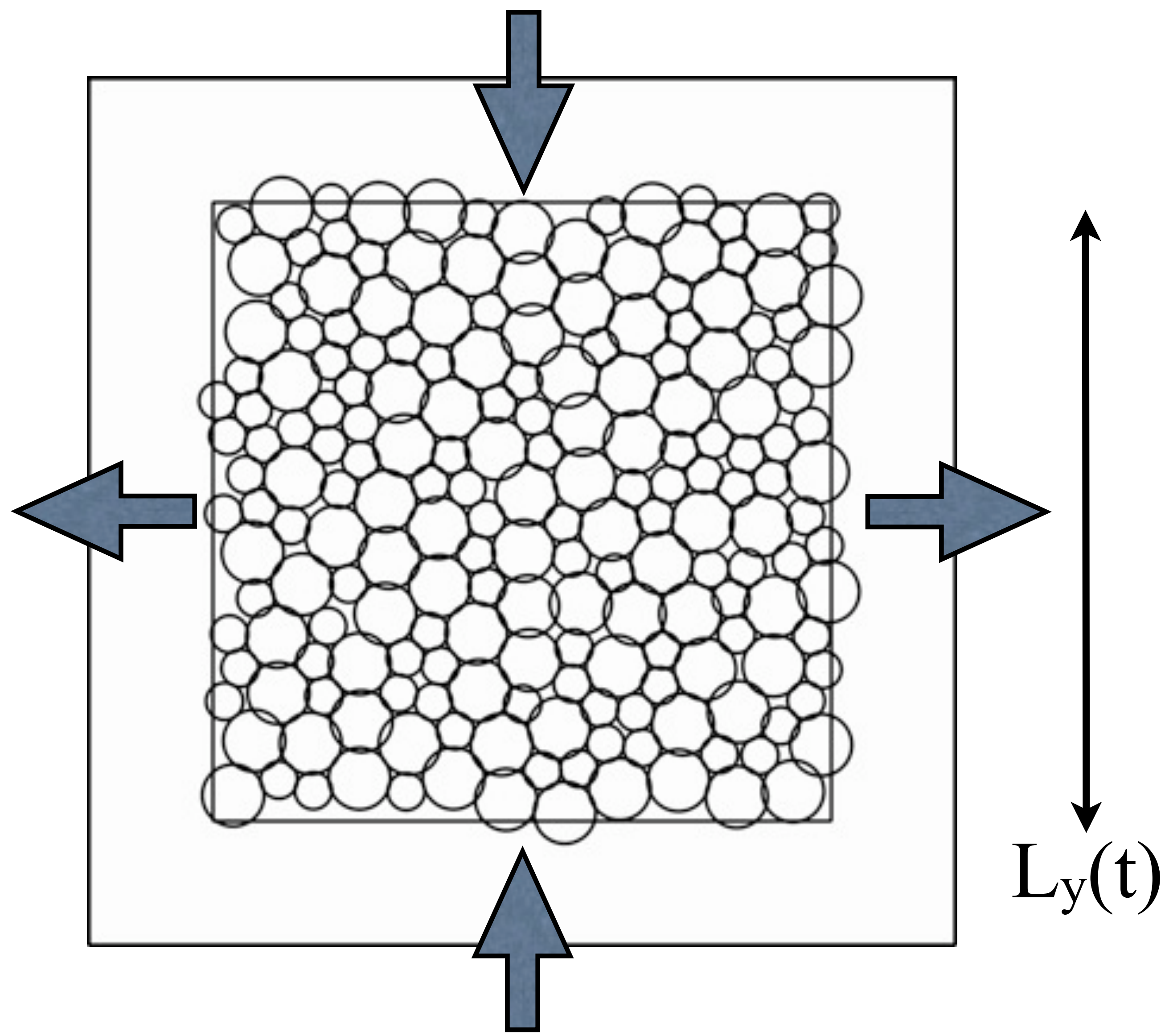
- Energy landscape picture of AQS (Malandro and Lacks)
  - 

    Increasing shear →

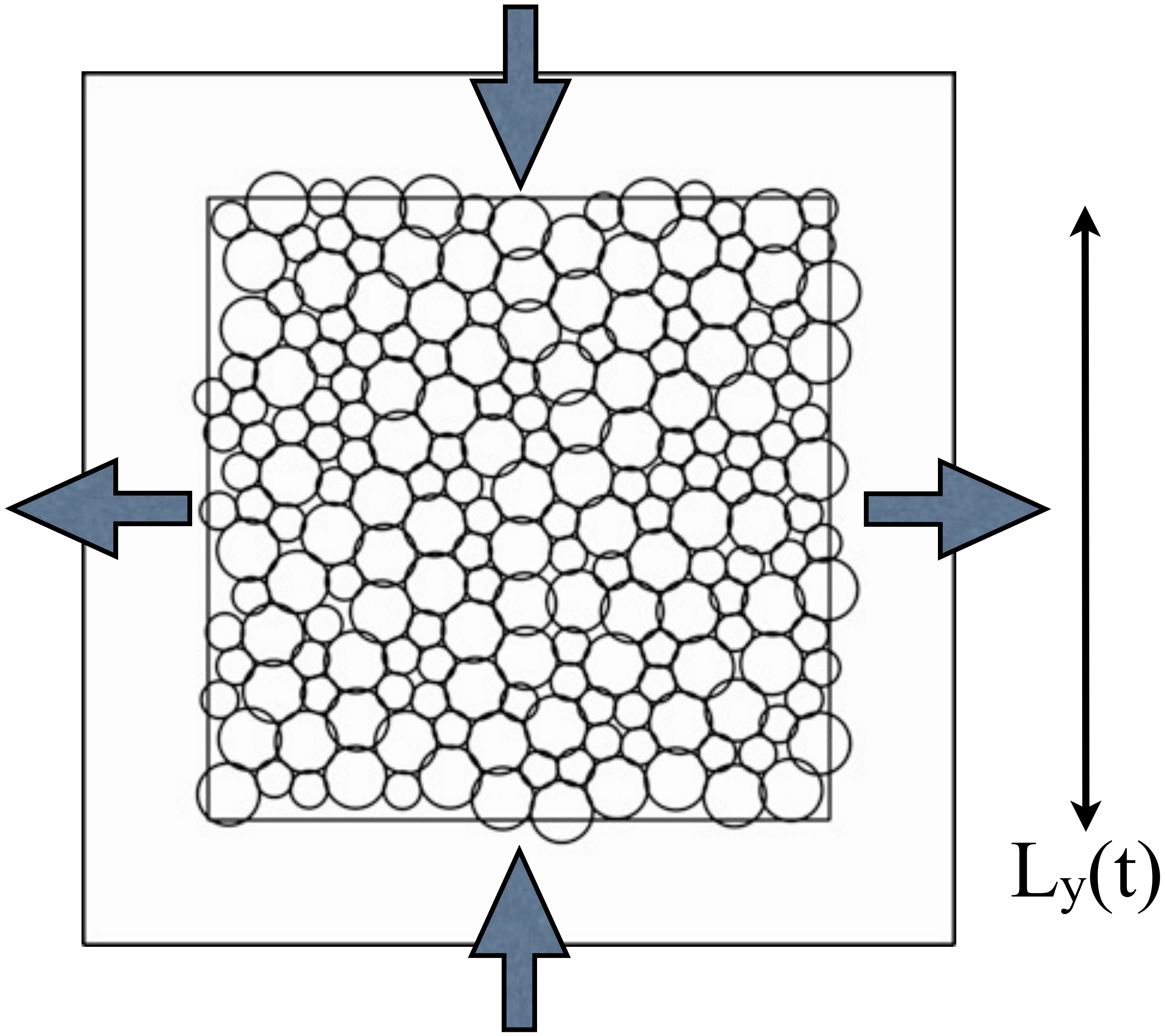
  - $t_{thermal} >> t_{shear} >> t_{rearrange}$
  - first Temperature to zero, then shear rate to zero.

Zero temperature molecular dynamics



Zero temperature molecular dynamics

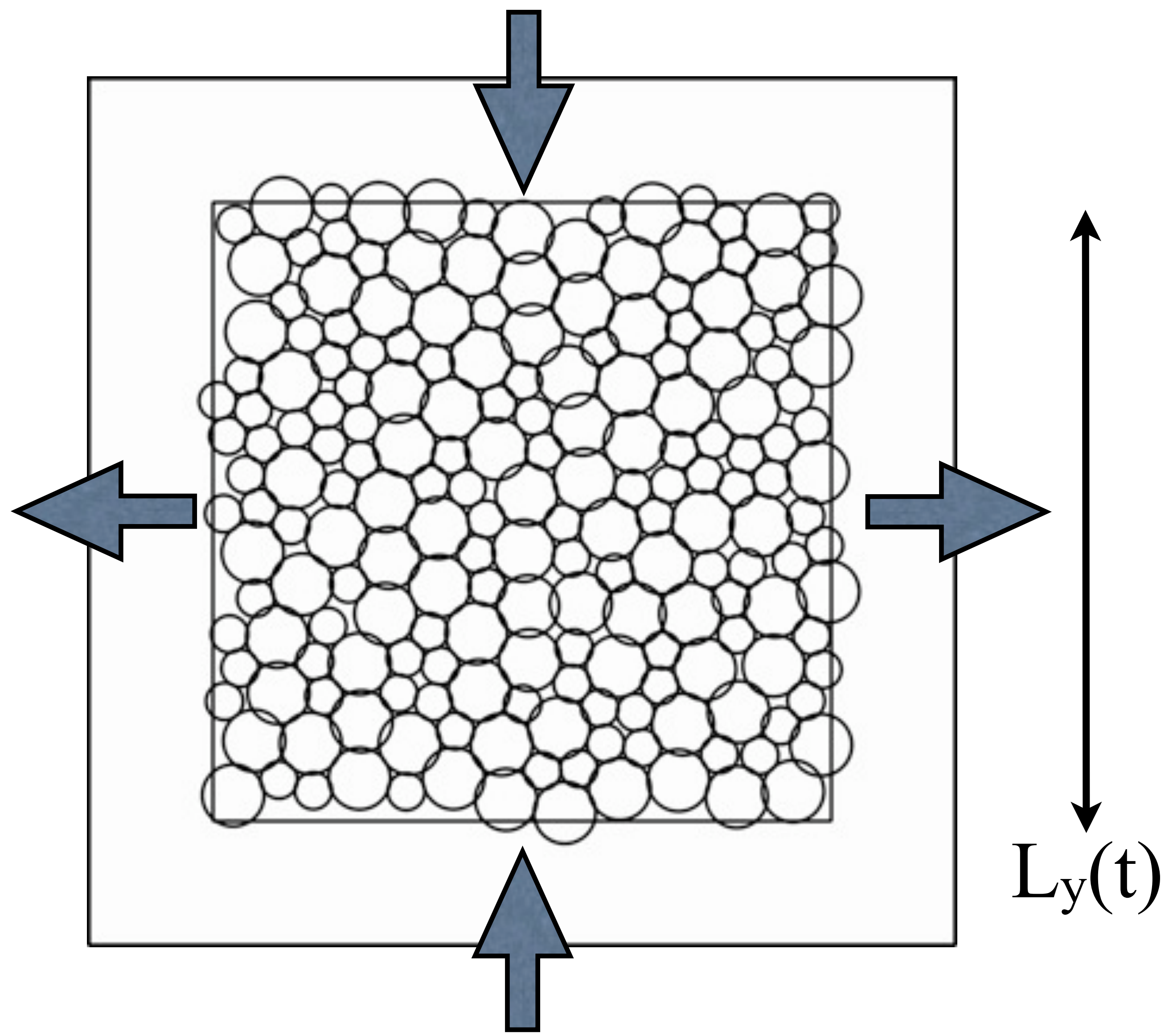
- 2D Molecular Dynamics:



Zero temperature molecular dynamics

- 2D Molecular Dynamics:
- binary Lennard-Jones

control: $L_y(t)$, $L_x(t)$
conserve area



Zero temperature molecular dynamics

- 2D Molecular Dynamics:
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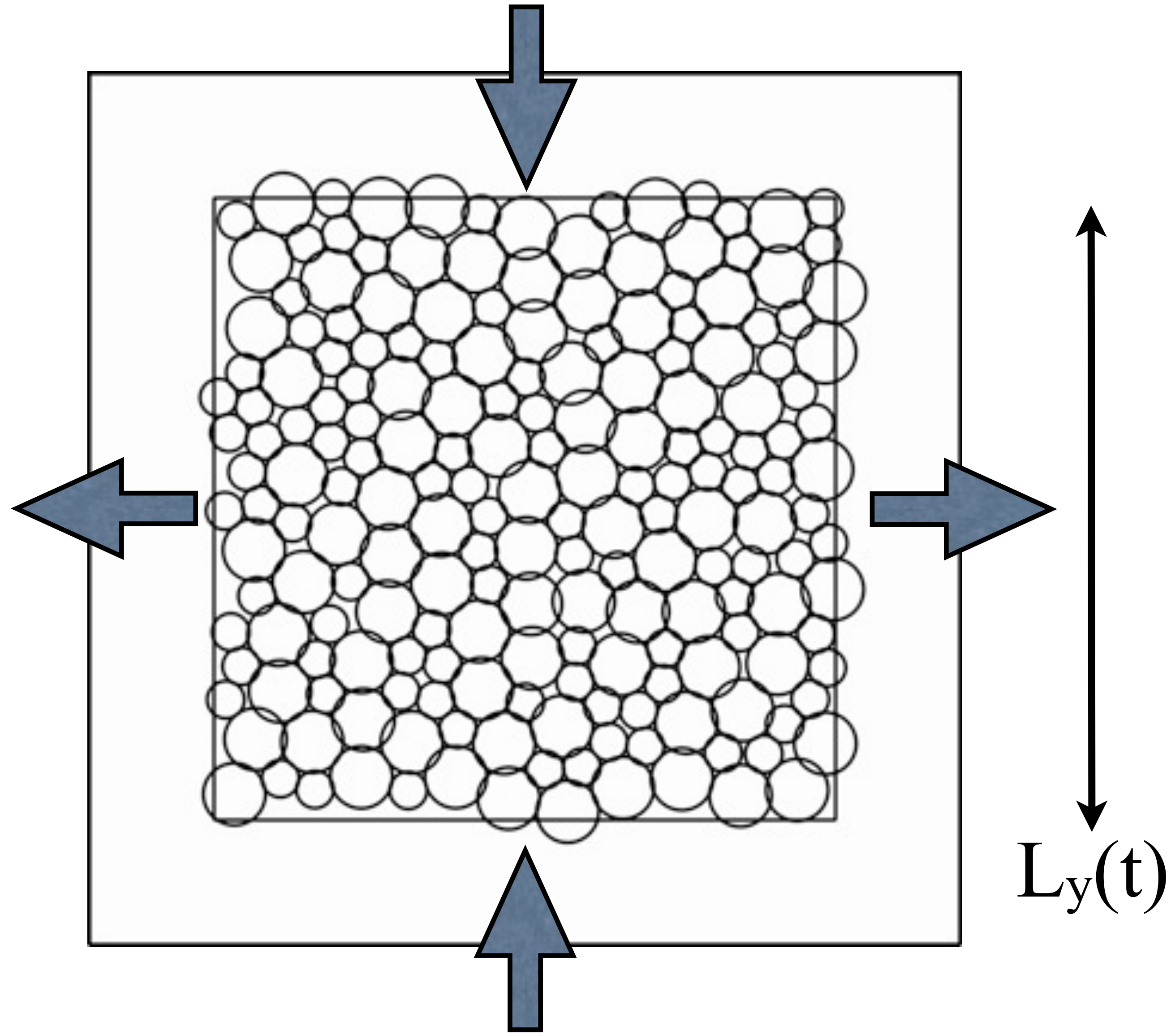

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Zero temperature molecular dynamics

- 2D Molecular Dynamics:
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- quenched at Pressure=0
- damp relative velocity (Kelvin/ DPD)

control: $L_y(t)$, $L_x(t)$
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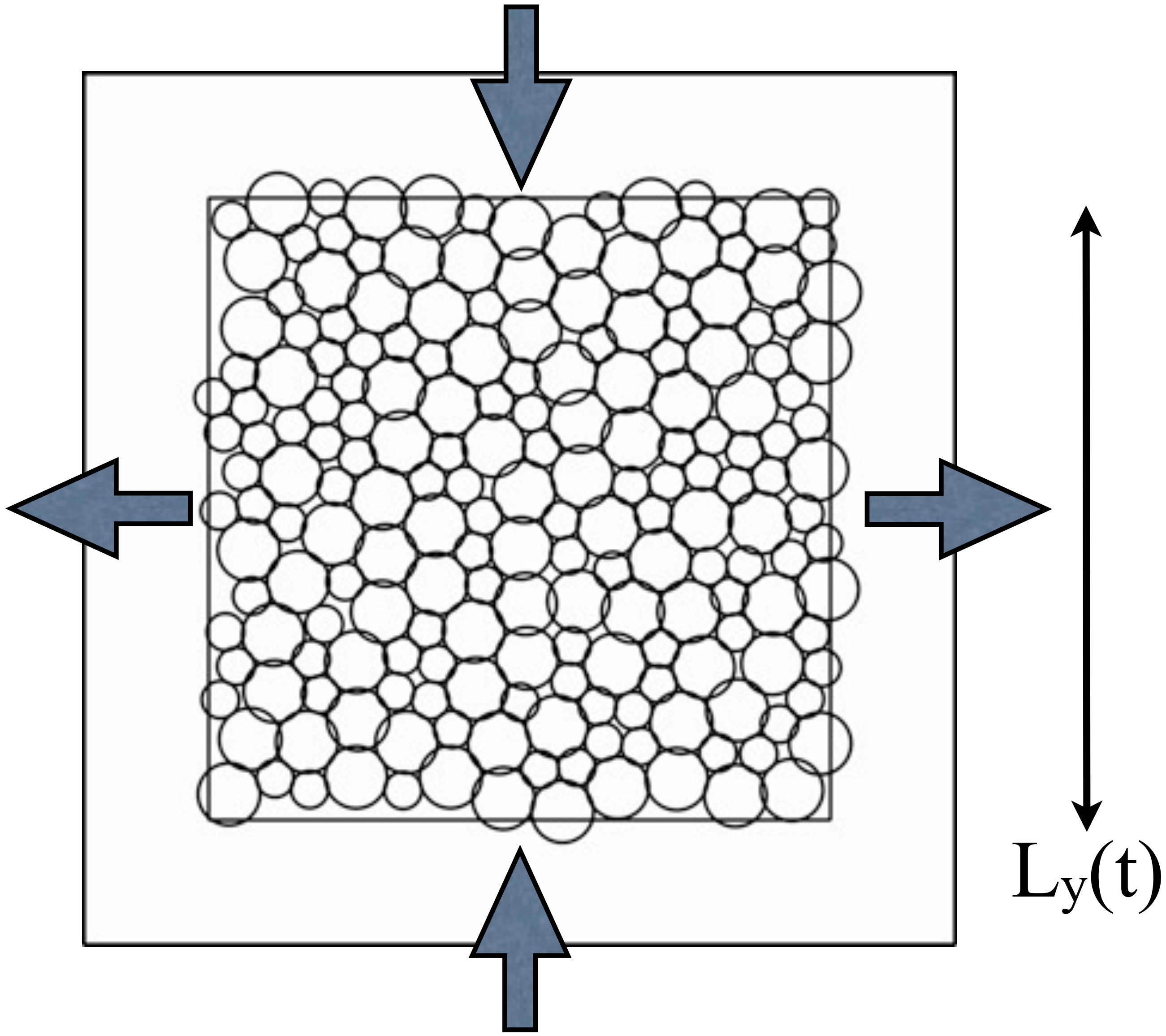

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Zero temperature molecular dynamics

- 2D Molecular Dynamics:
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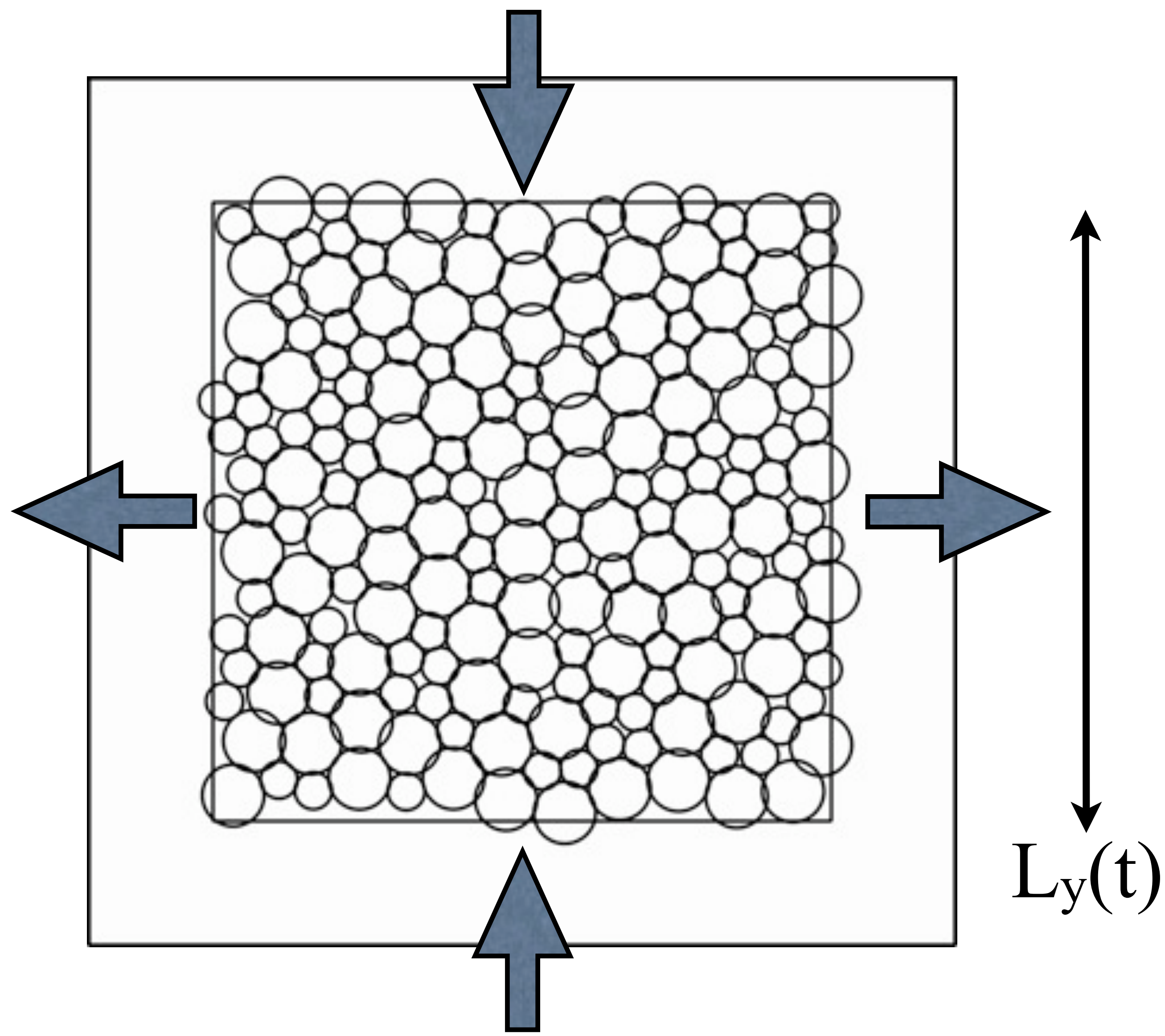

Zero temperature molecular dynamics

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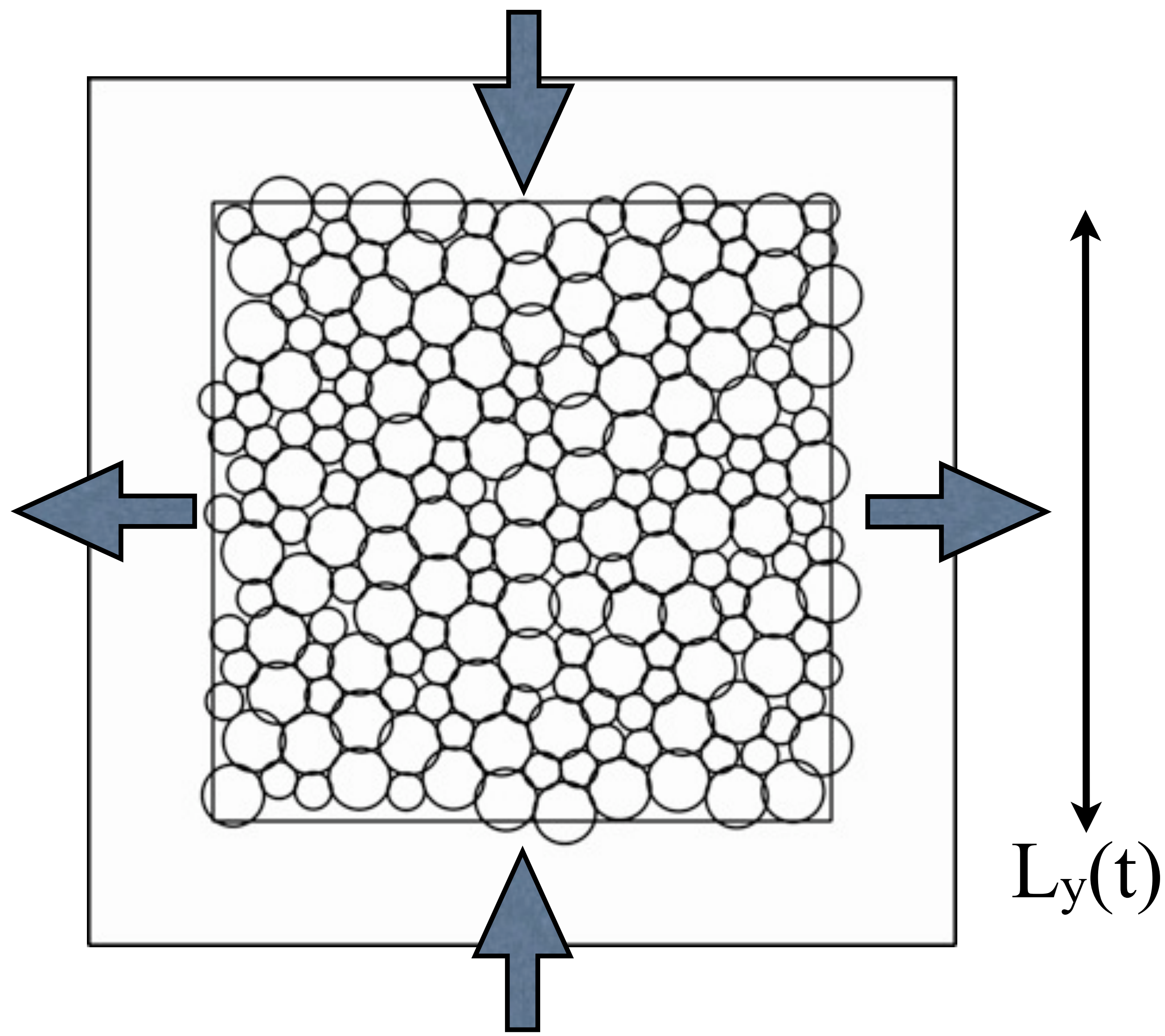

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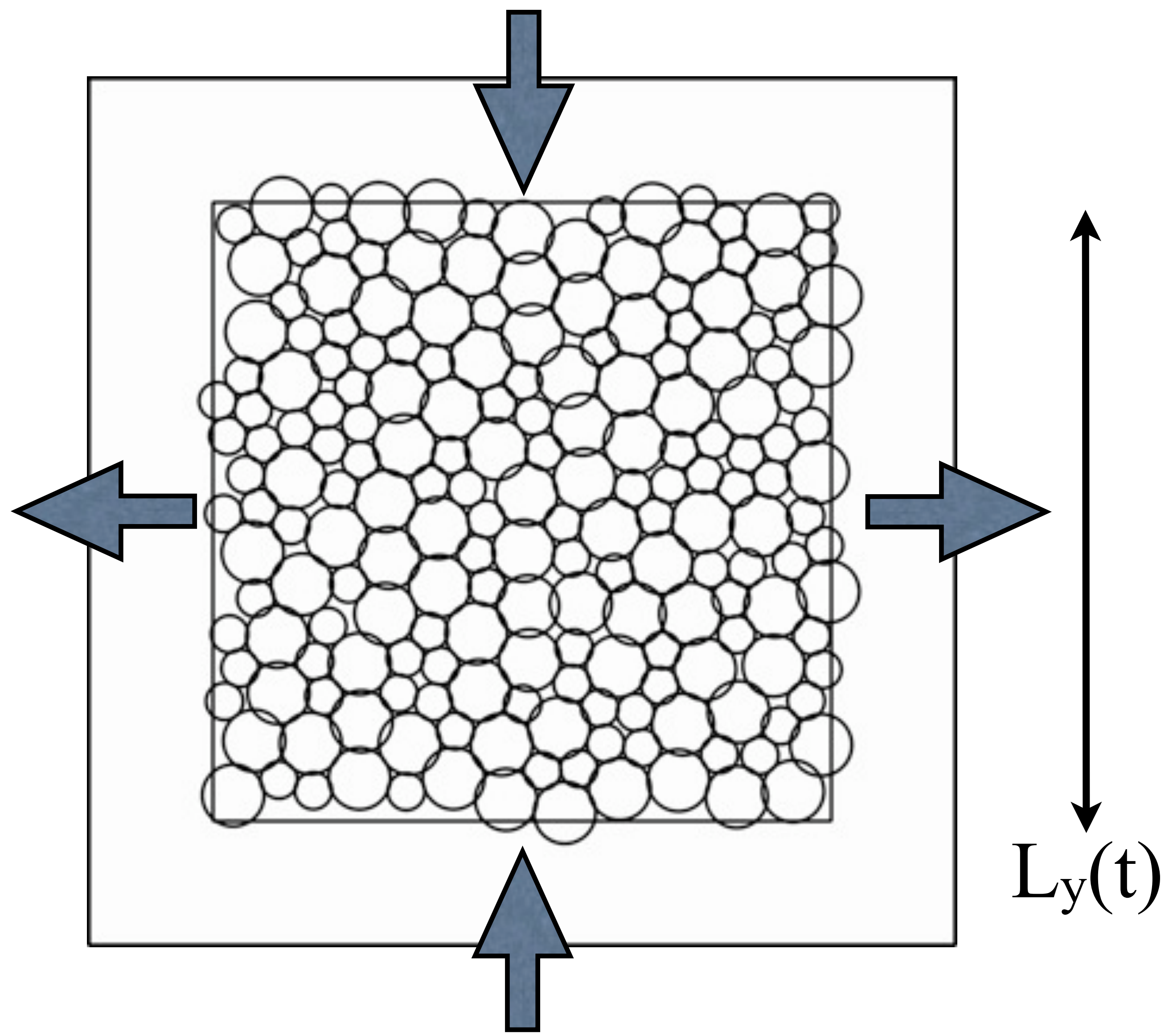

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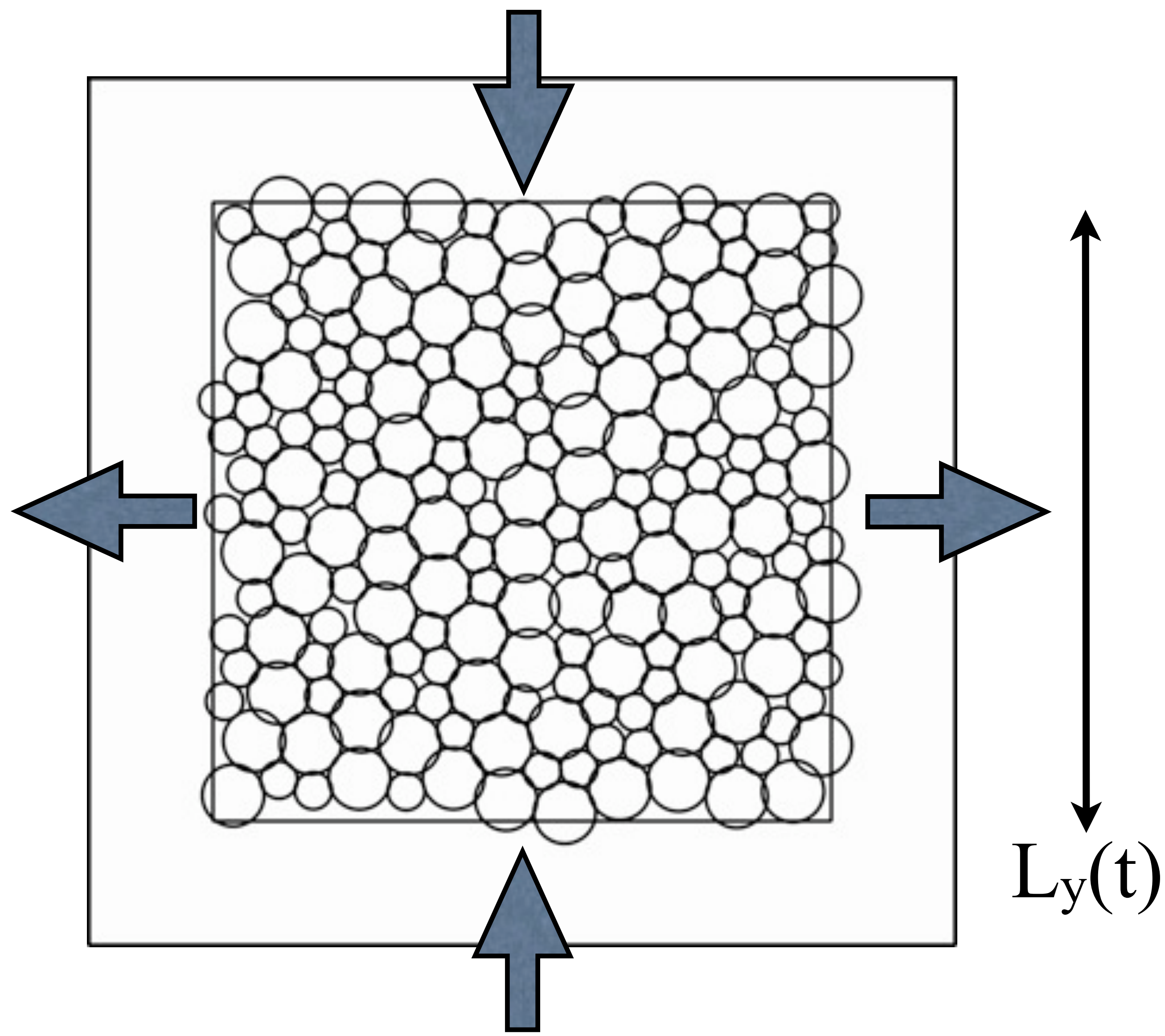

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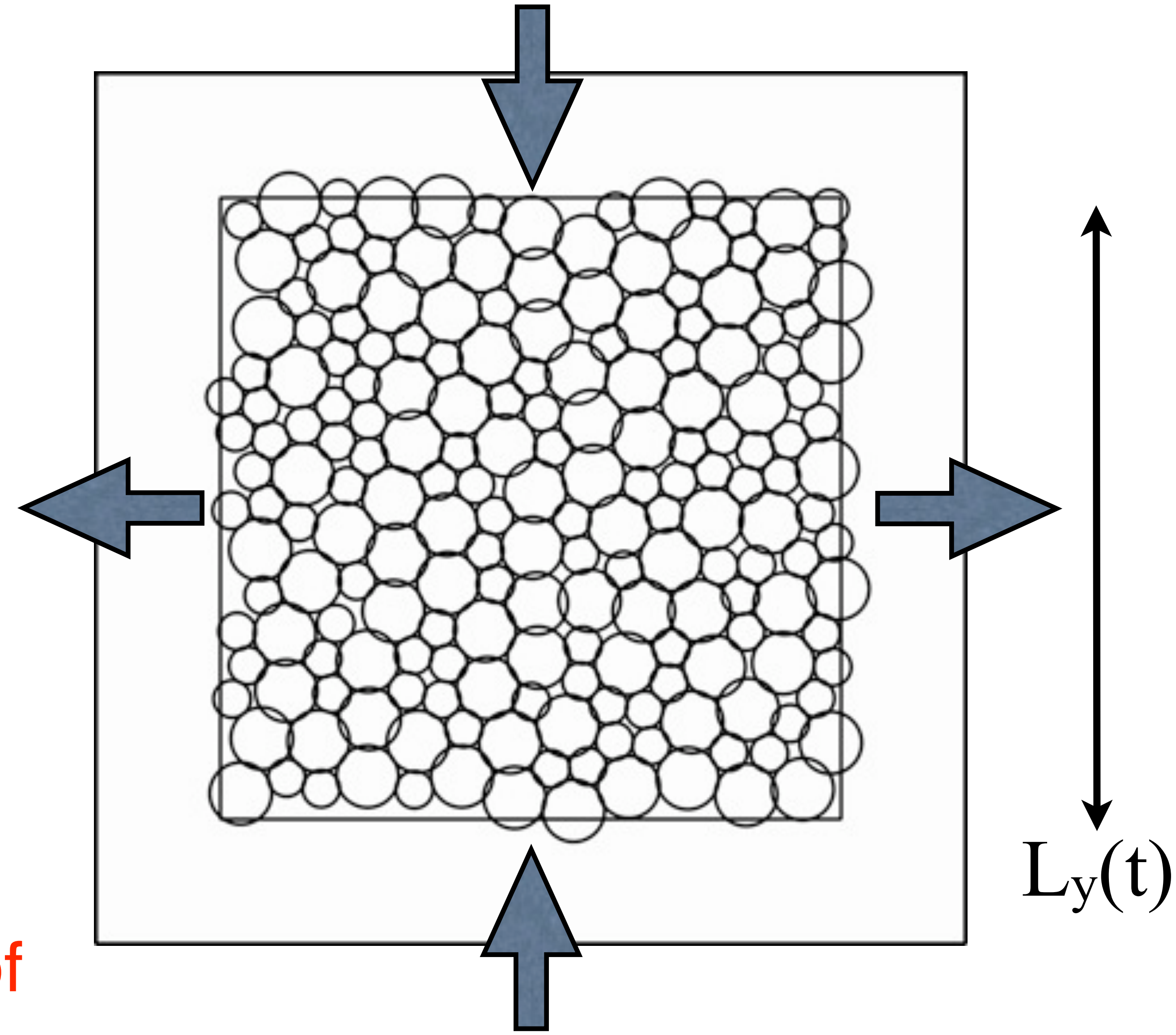
- Quasi-static limit (about 500 CPU days / run)
- Strain window, $\Delta\gamma$, plays role of time!

control: $L_y(t)$, $L_x(t)$
conserve area



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Local vorticity, ω

For each triangle:

$$\frac{\partial u_i}{\partial x_j} = F_{ij}$$

$$\epsilon_1 = \frac{F_{xx} - F_{yy}}{2}$$

$$\epsilon_2 = \frac{F_{xy} + F_{yx}}{2}$$

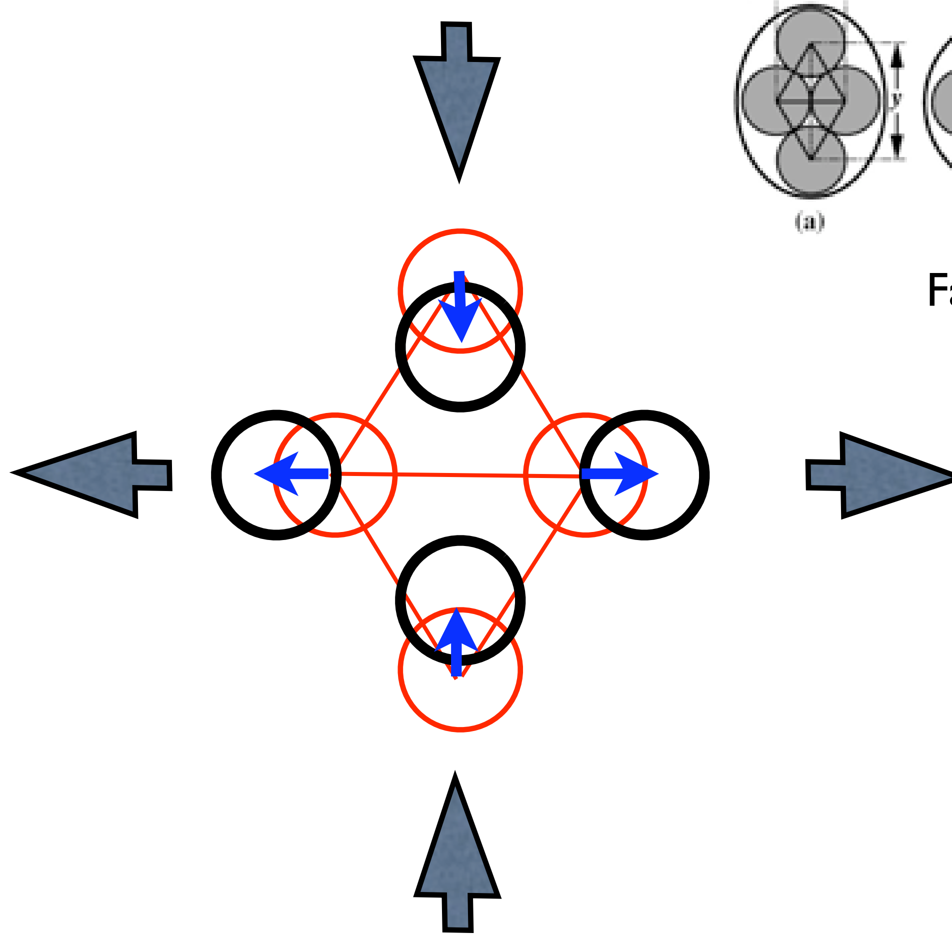
Invariants:

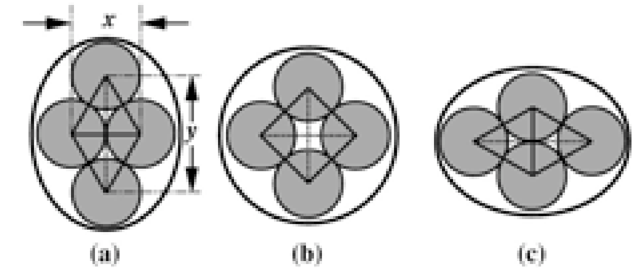
$$\epsilon = \sqrt{\epsilon_1^2 + \epsilon_2^2}$$

$$\omega = F_{xy} - F_{yx}$$

"Canonical" atomistic Eshelby shear transformation:

pure shear $\epsilon_1>0$ $\epsilon_2=0$ $\omega=0$





Falk PRB 1998

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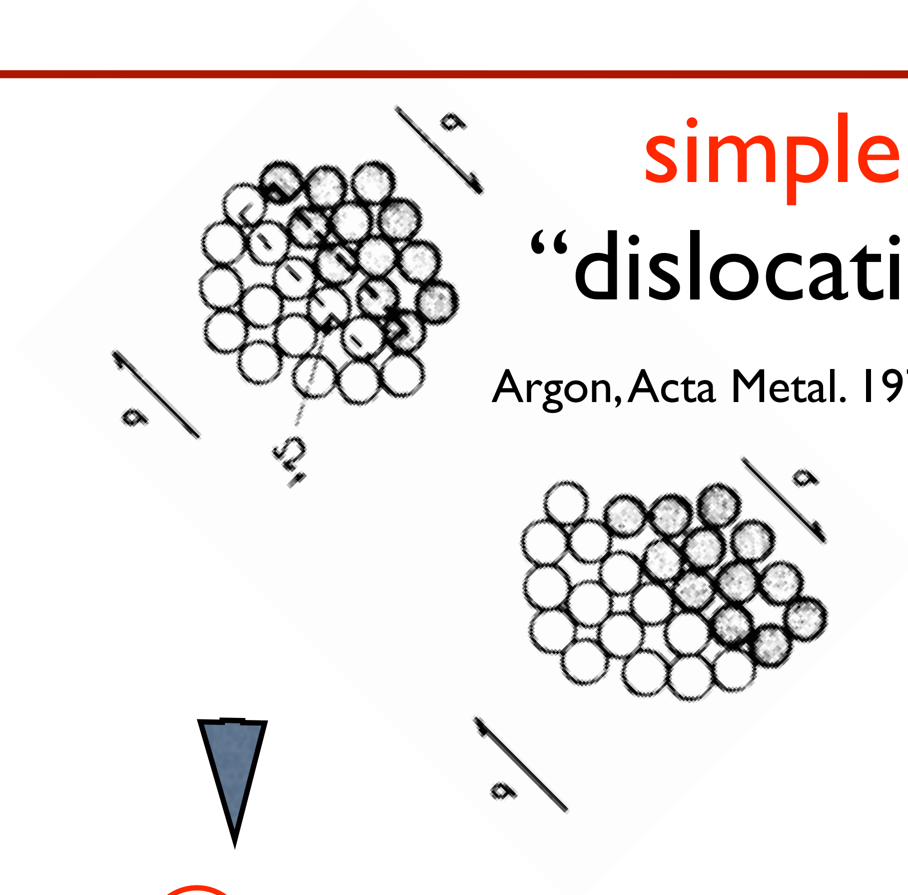
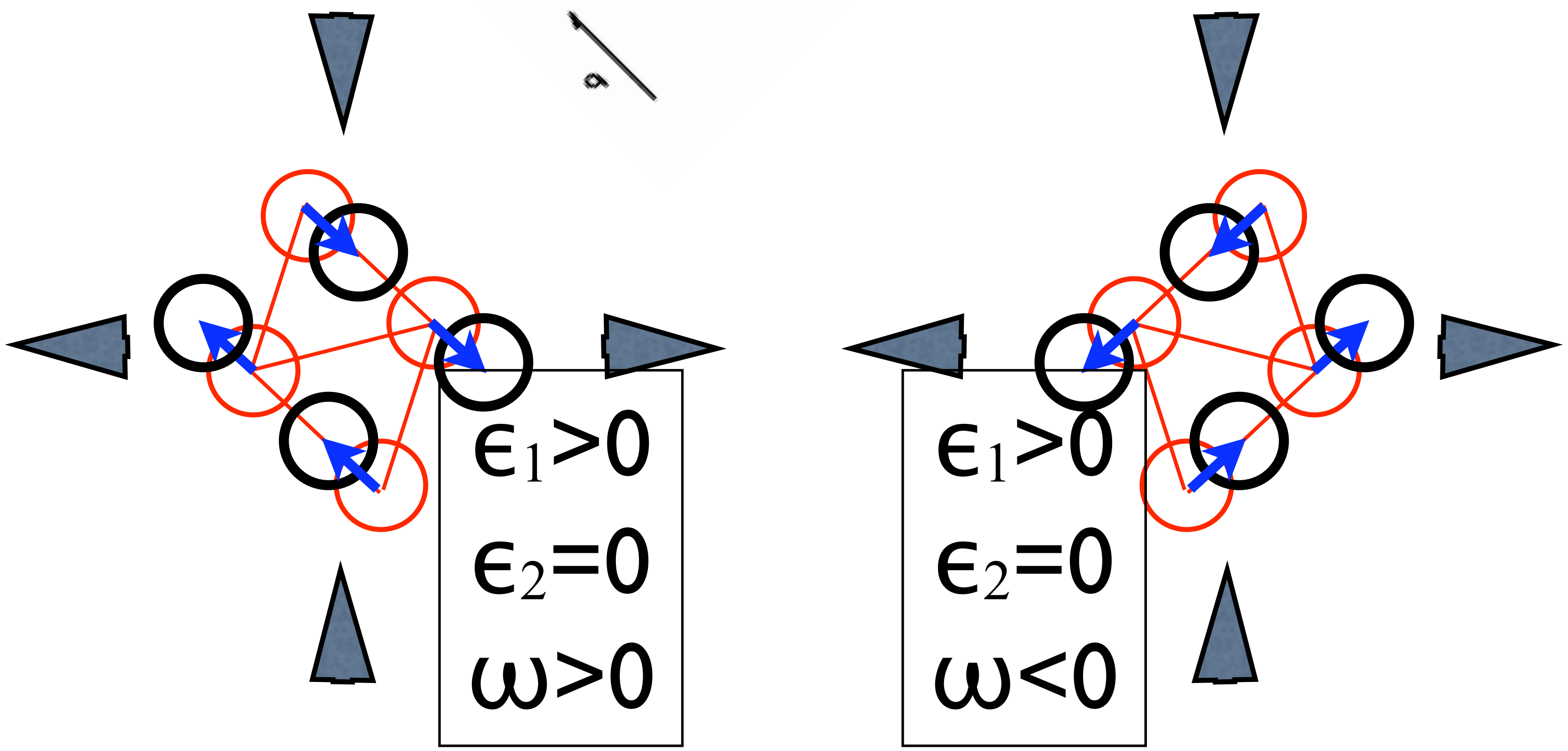


simple shear, "dislocation loop"

Argon, Acta Metal. 1979



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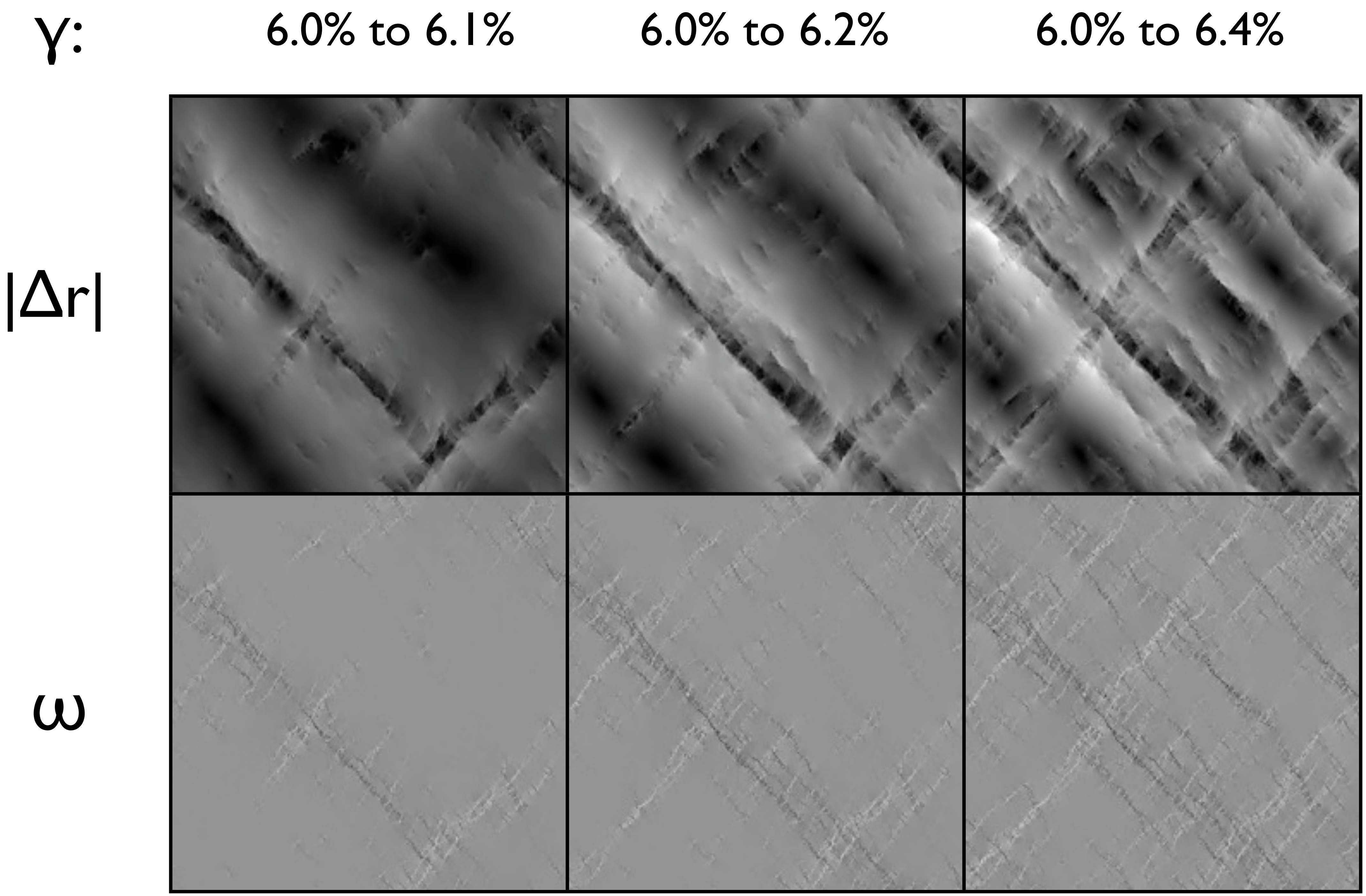
Correlations in steady state



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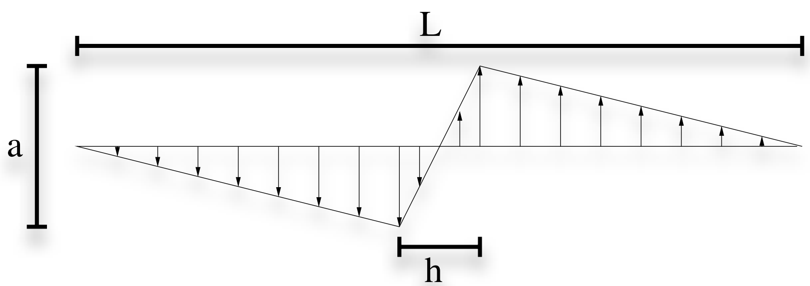
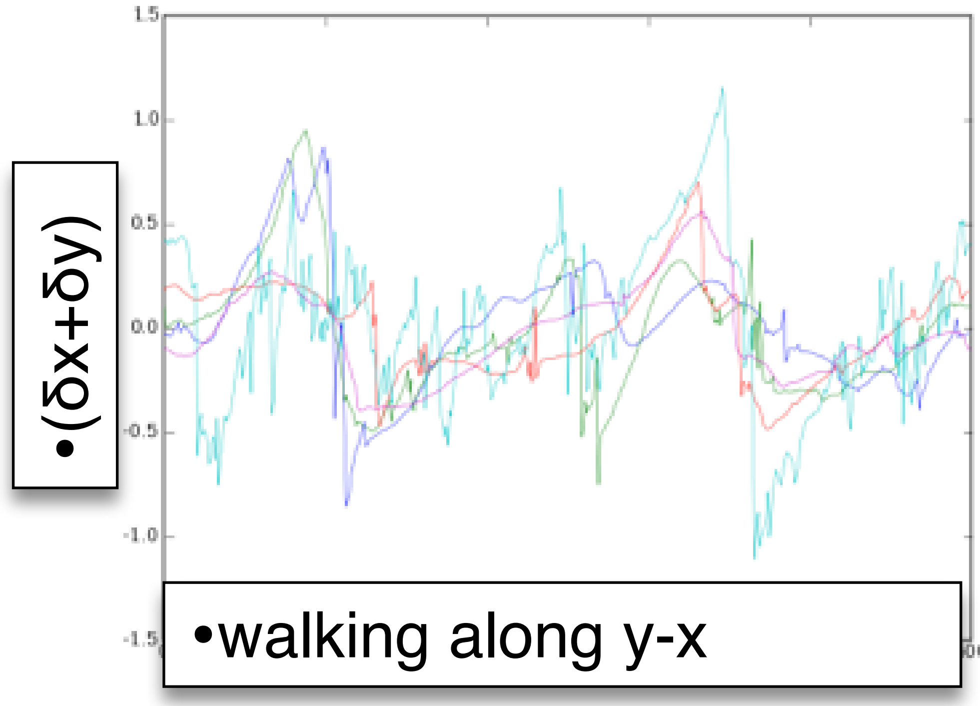
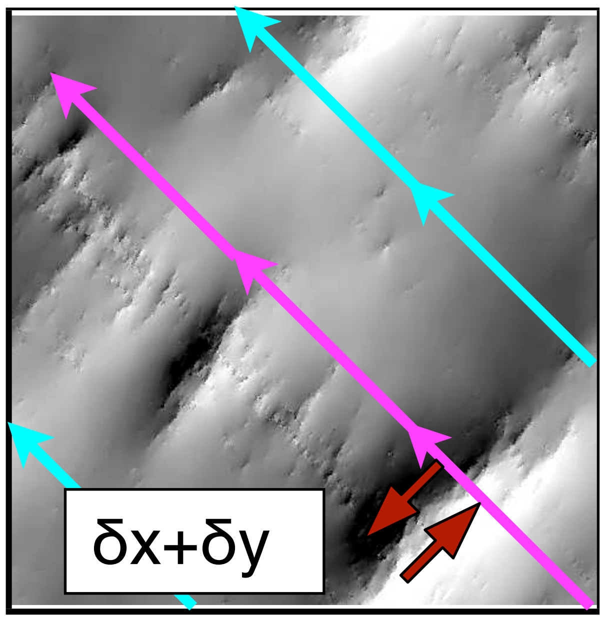
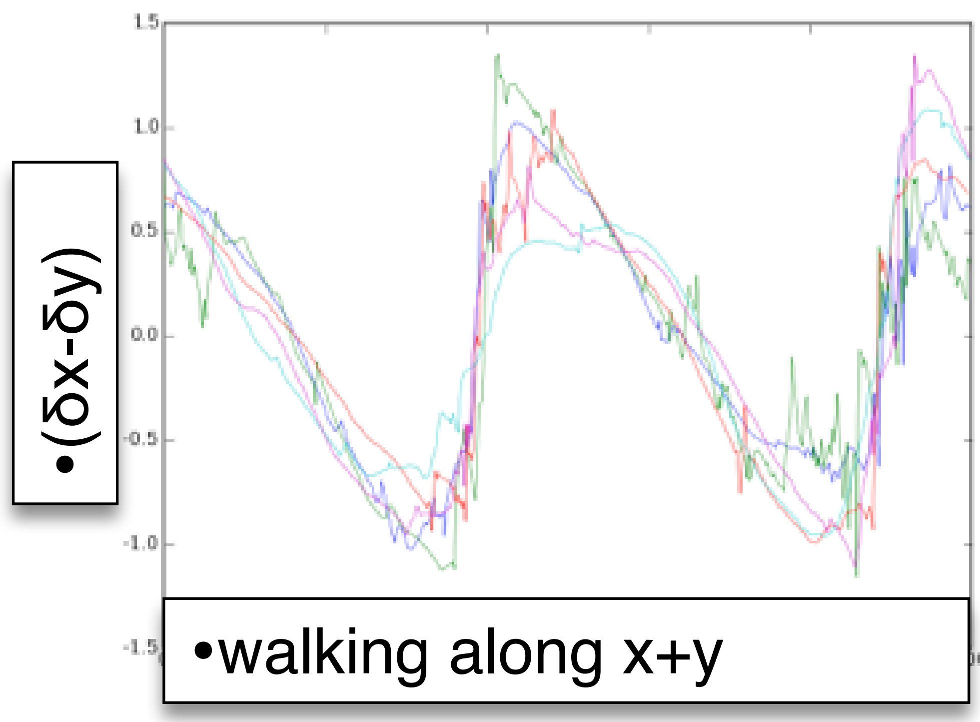
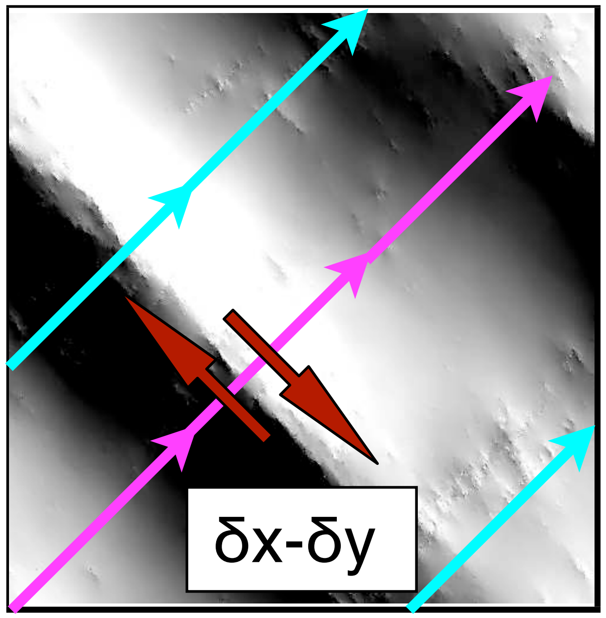
Transverse displacement traces (6.1% -> 6.2%)





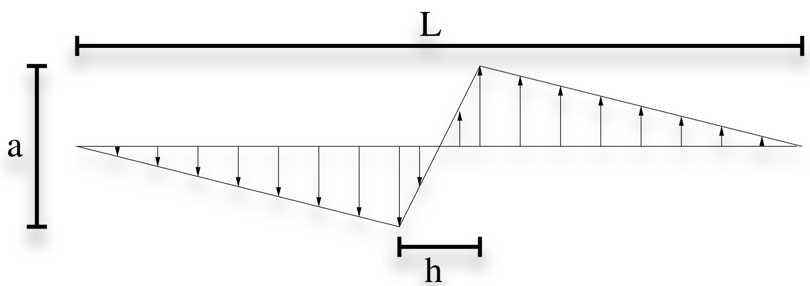






Transverse displacement traces (6.1% -> 6.2%)

L~1000σ_0

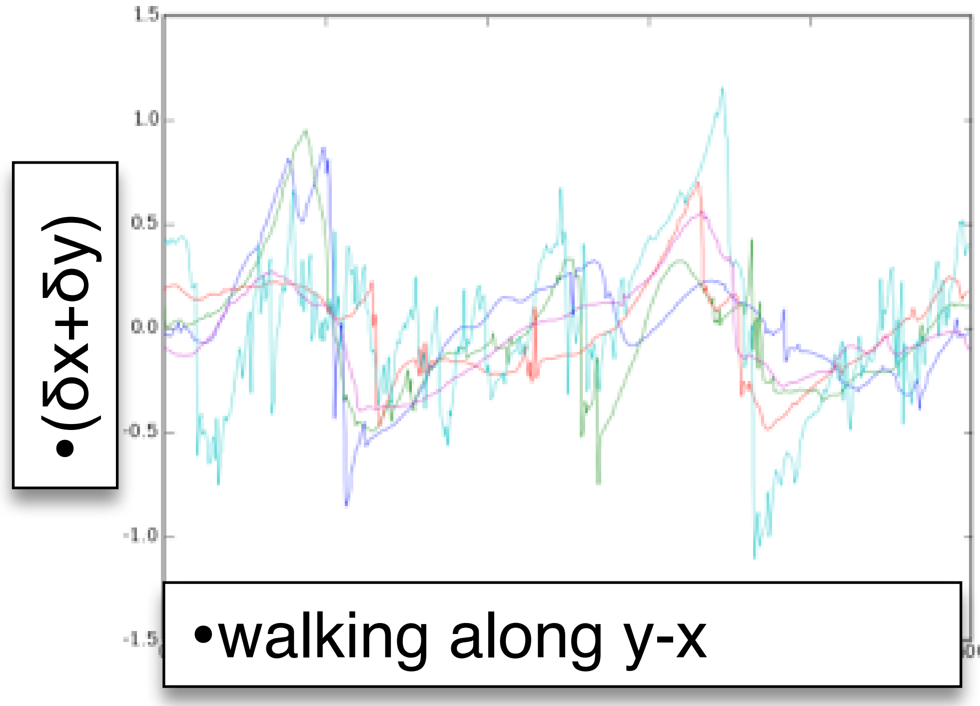


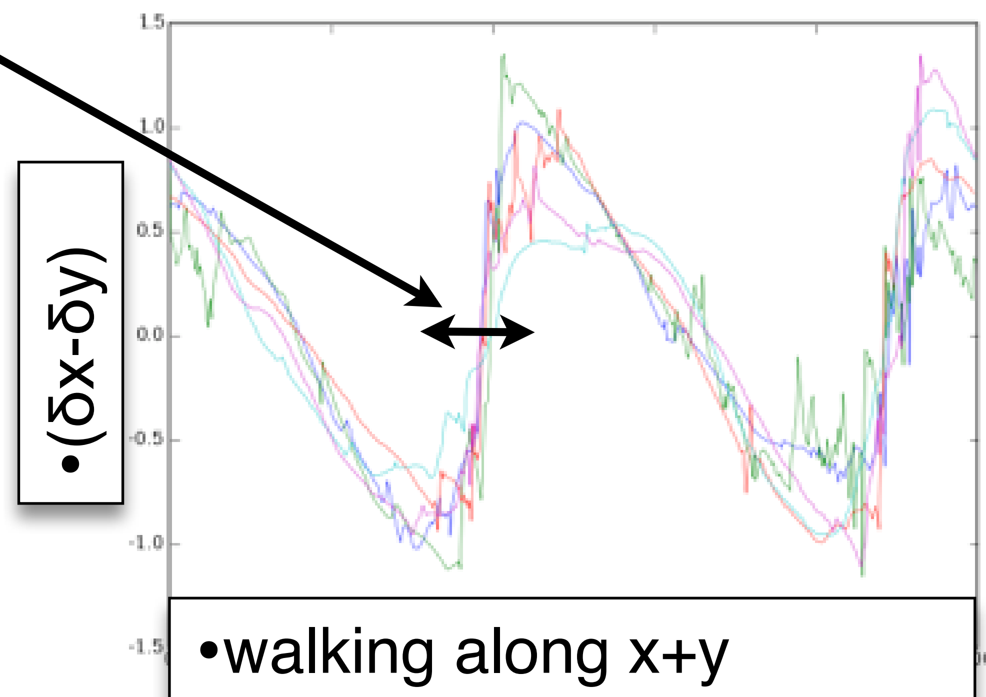
width, h~50σ_0

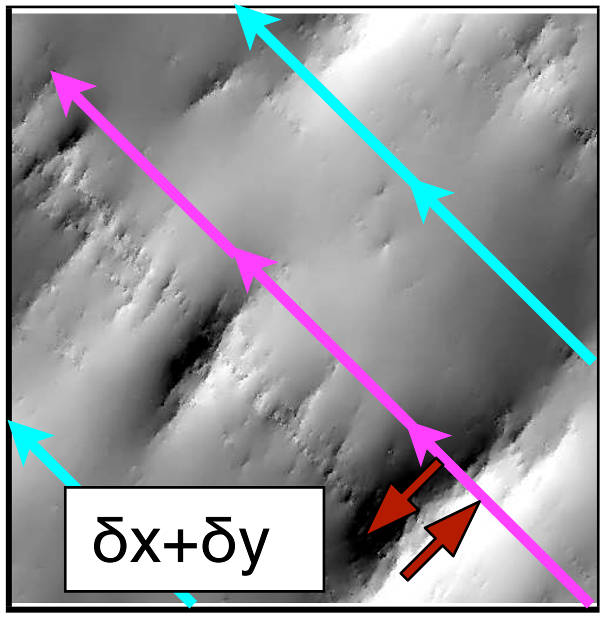
strain in shear zone ~2% to 4%

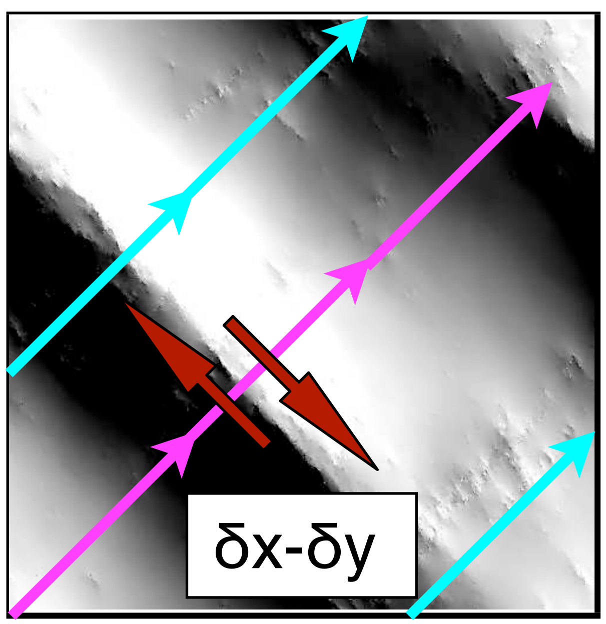
a~1 to 2 σ_0

unloaded ~ a/L ~ 0.1% to 0.2% globally









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P(δx,δy) for 8 consecutive Δγ=0.001 windows

- System is either:
  - active along δx=δy
  - active along δx=-δy
  - or quiescent

+2

+1

δy

-1

-2

-2 -1 δx +1 +2

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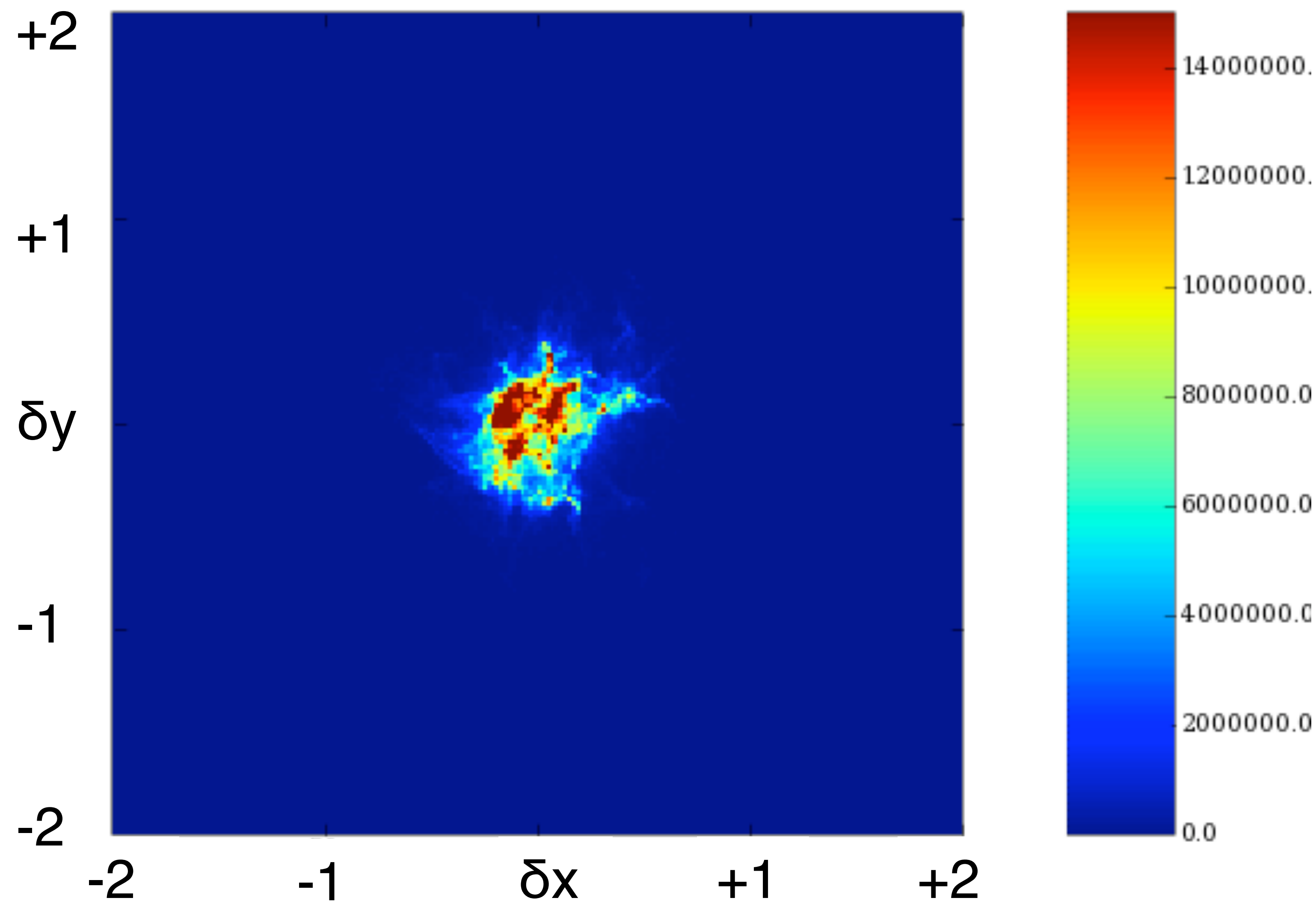

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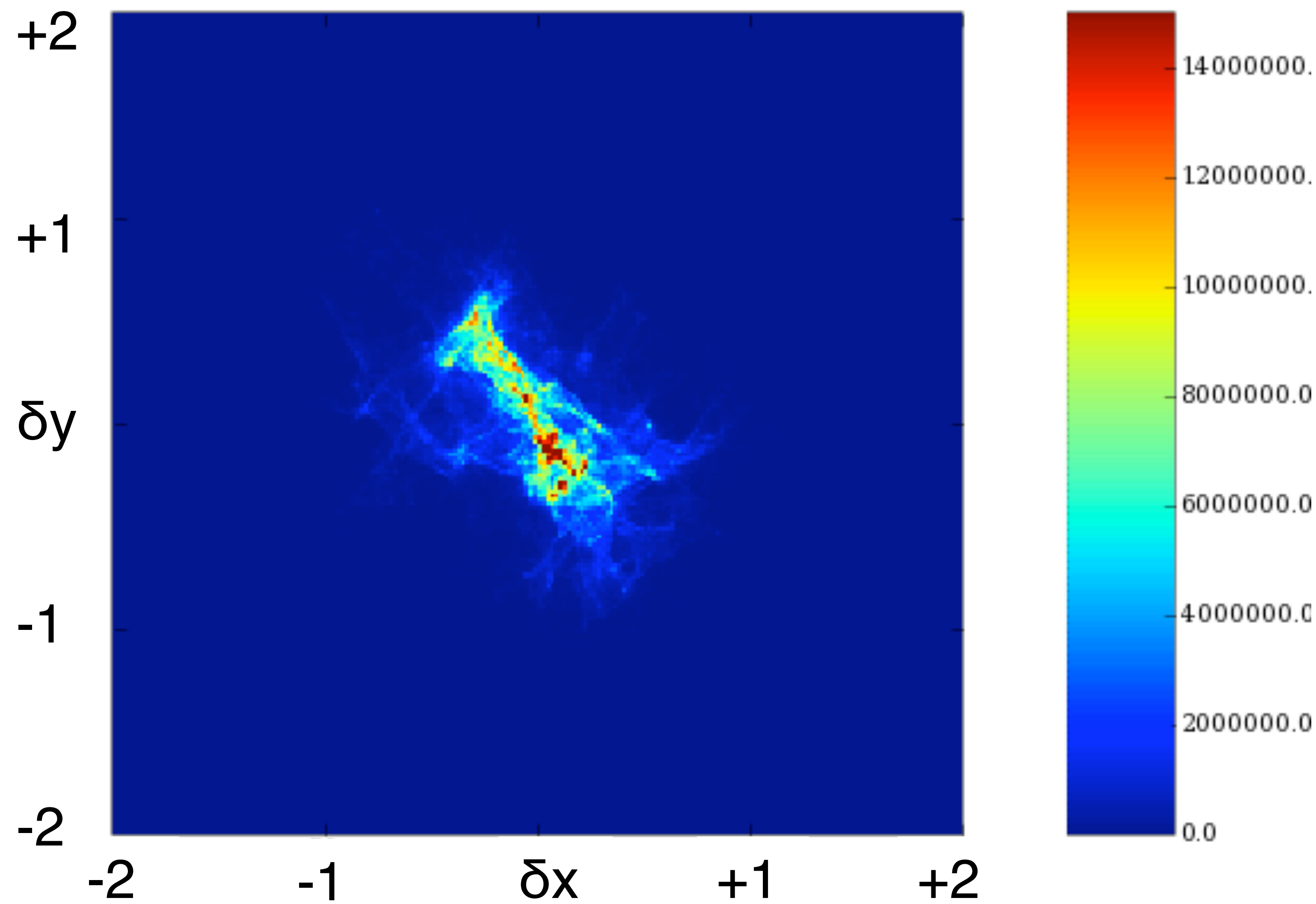

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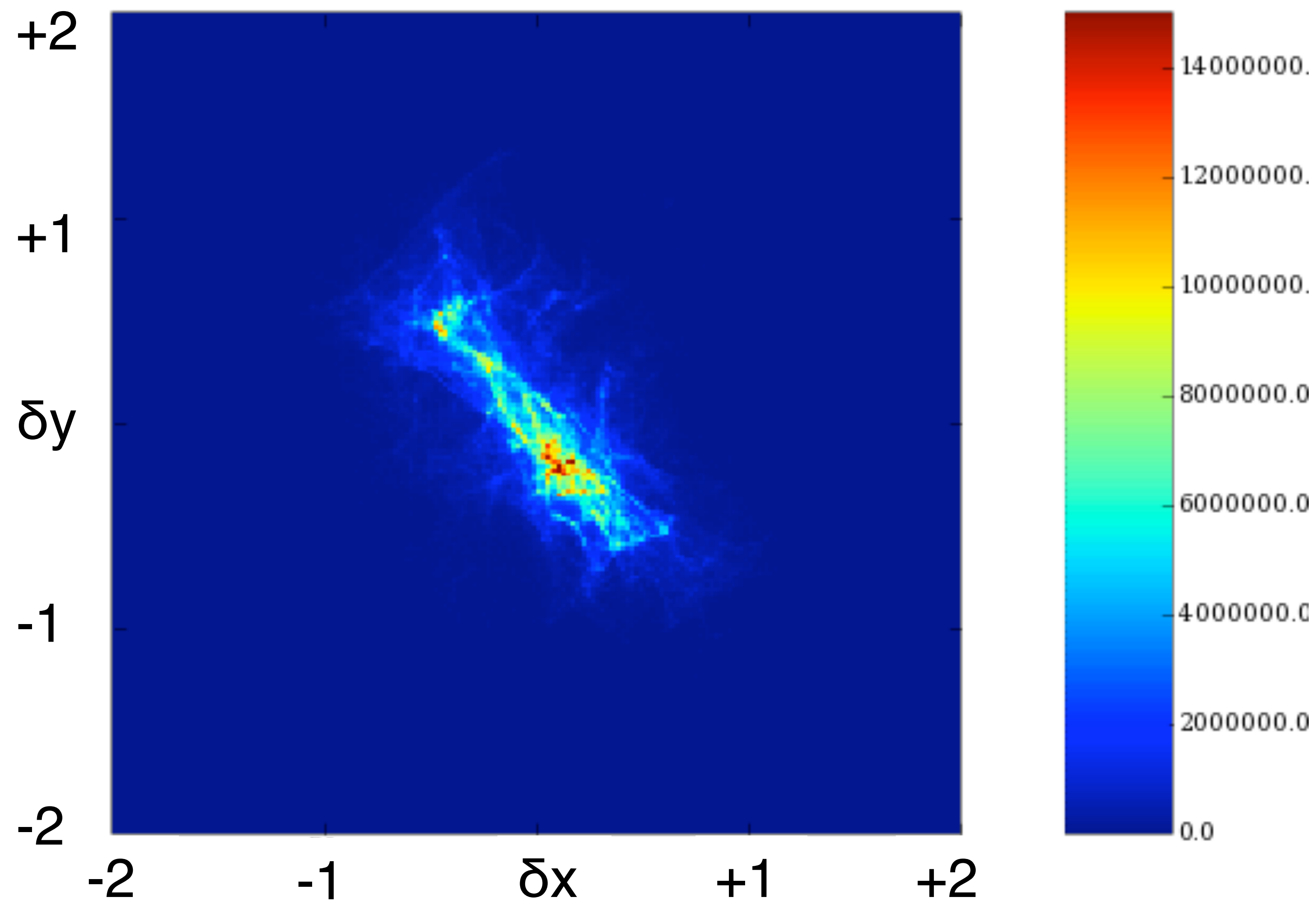

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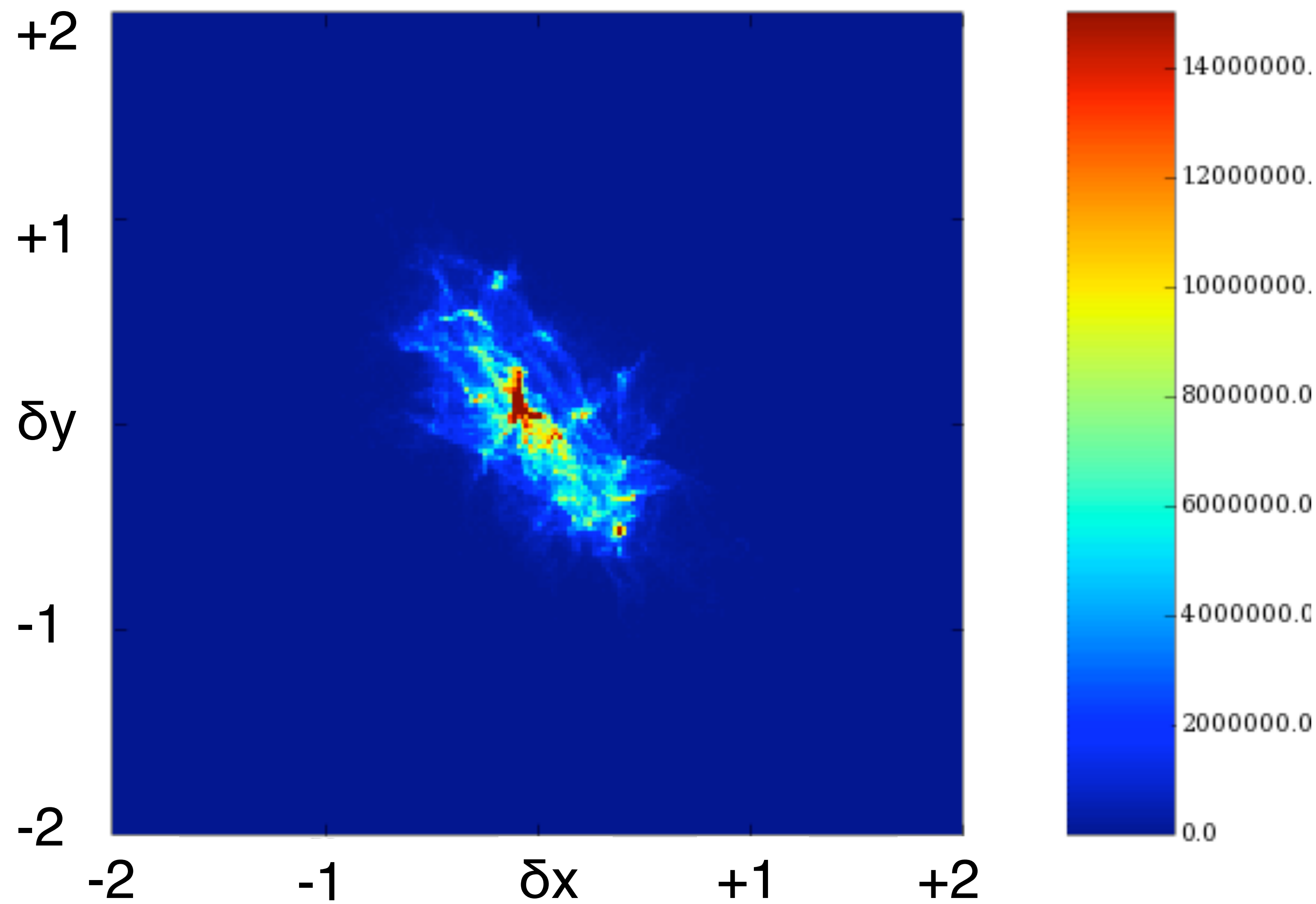

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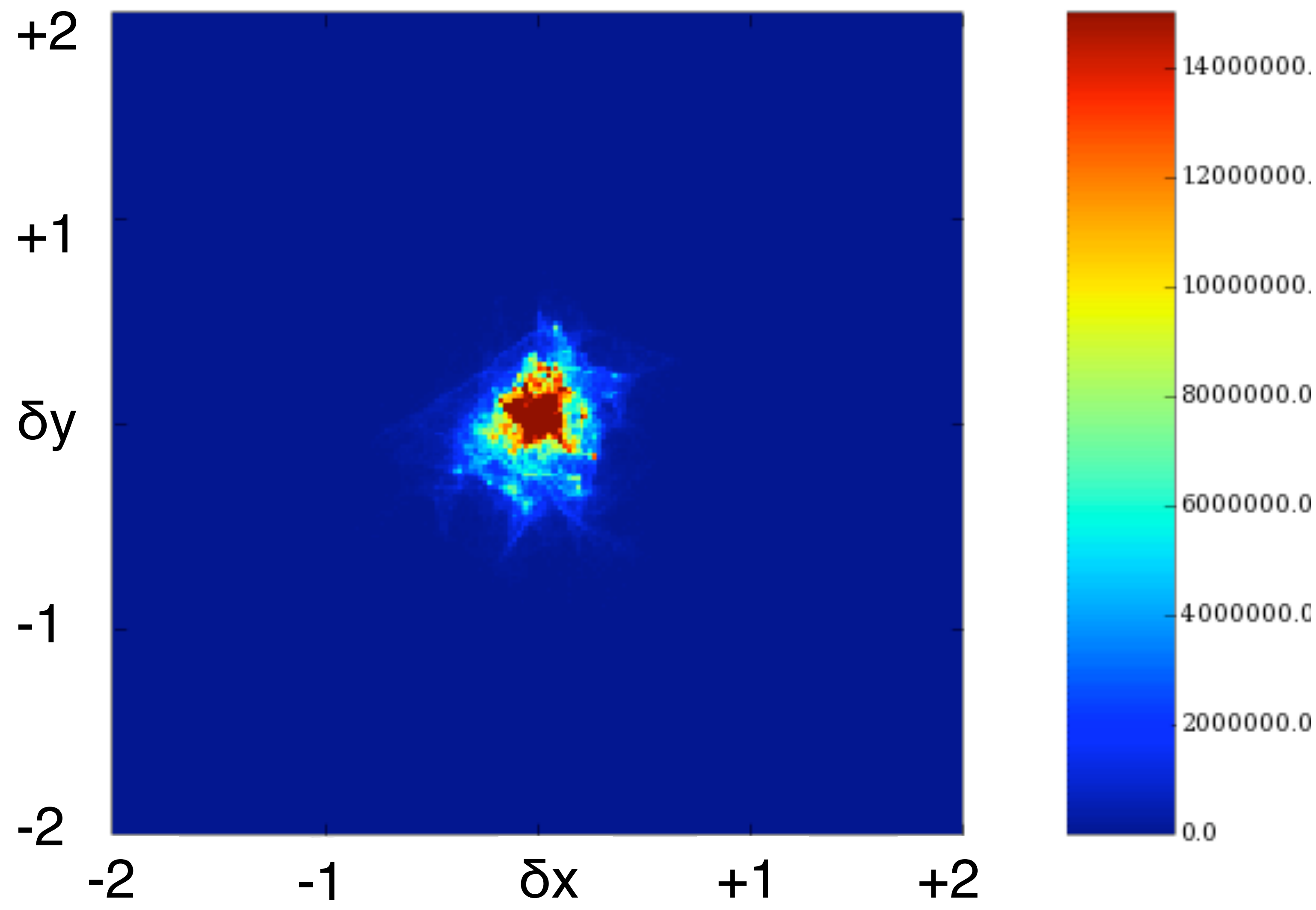

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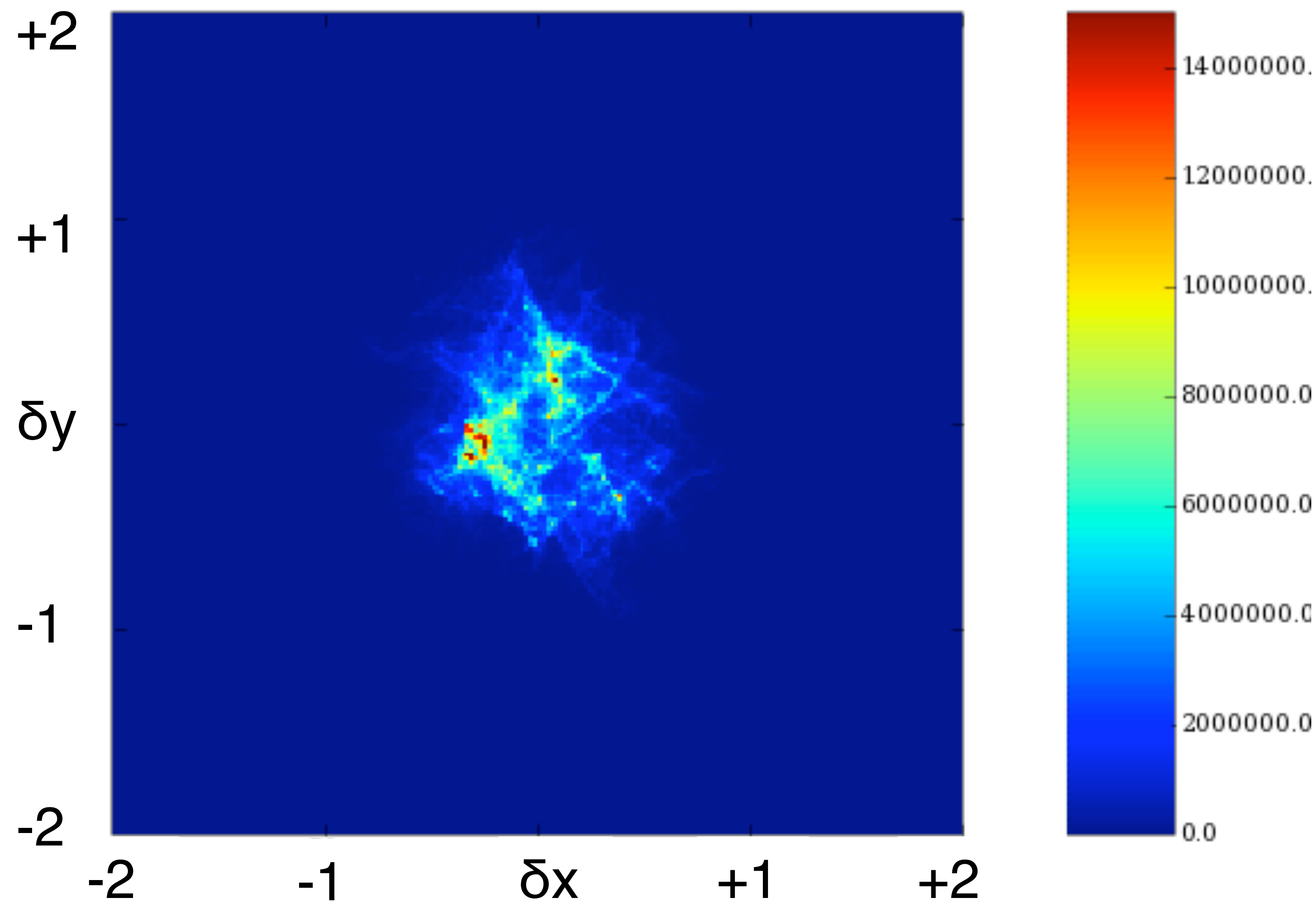

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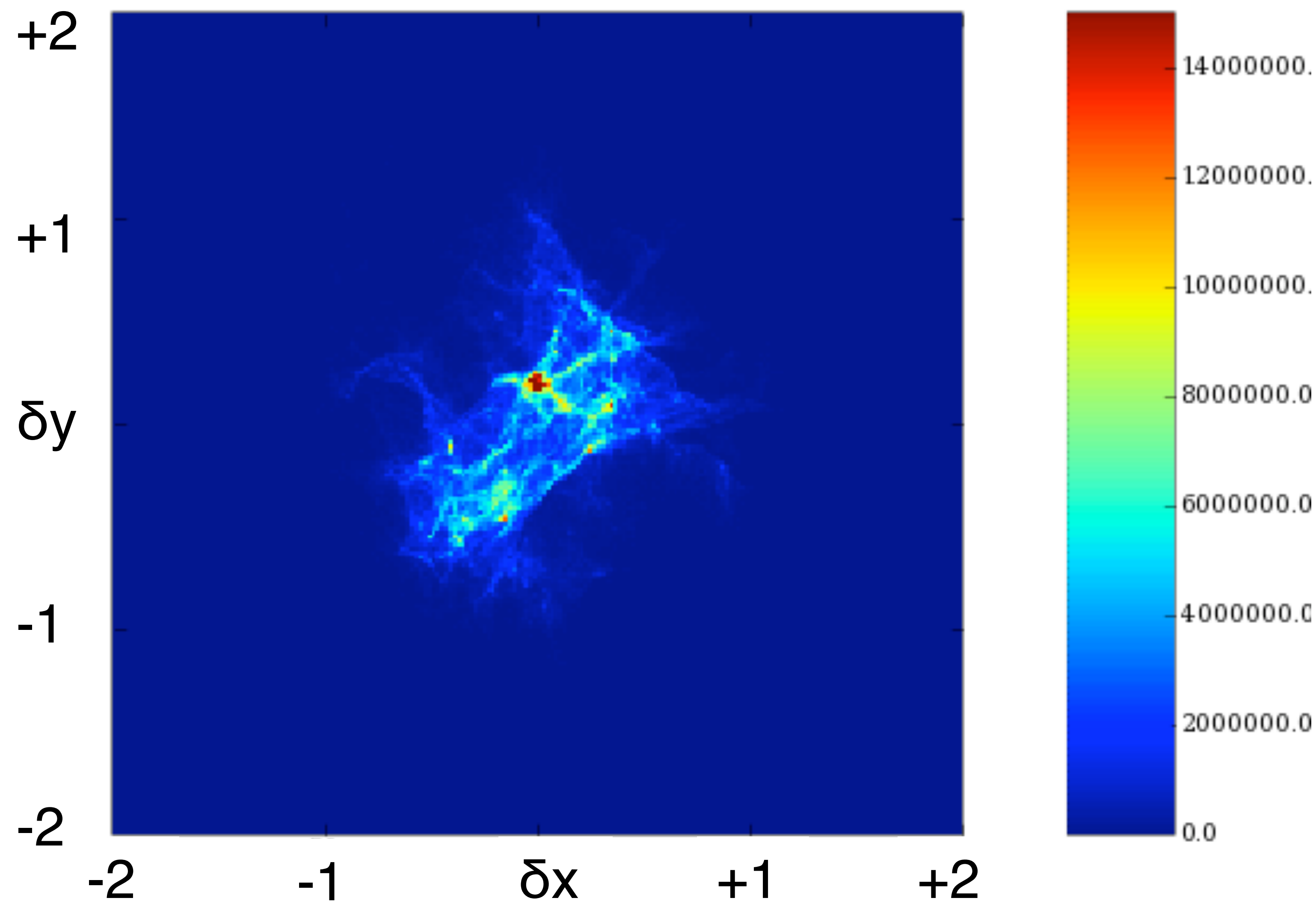

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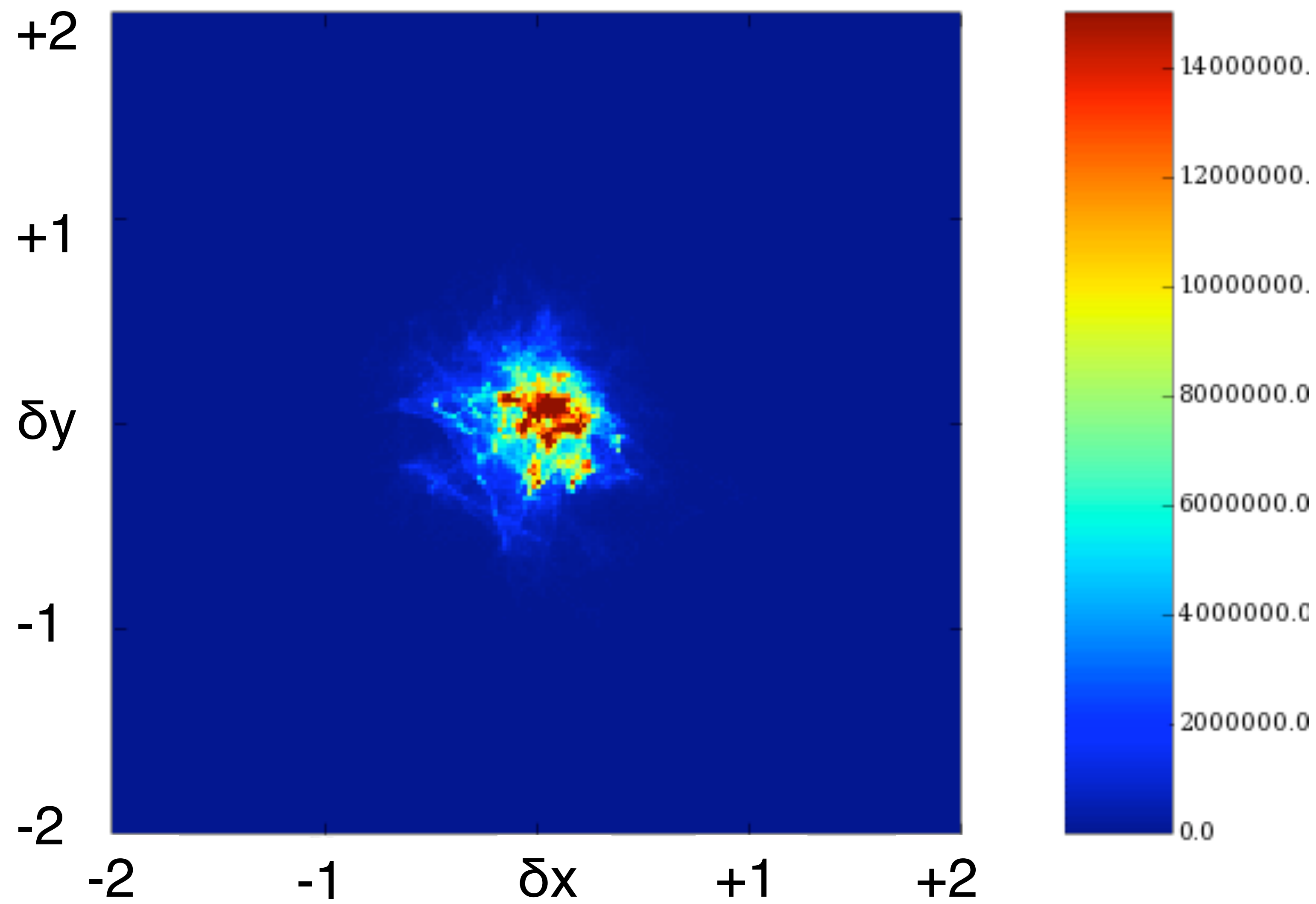

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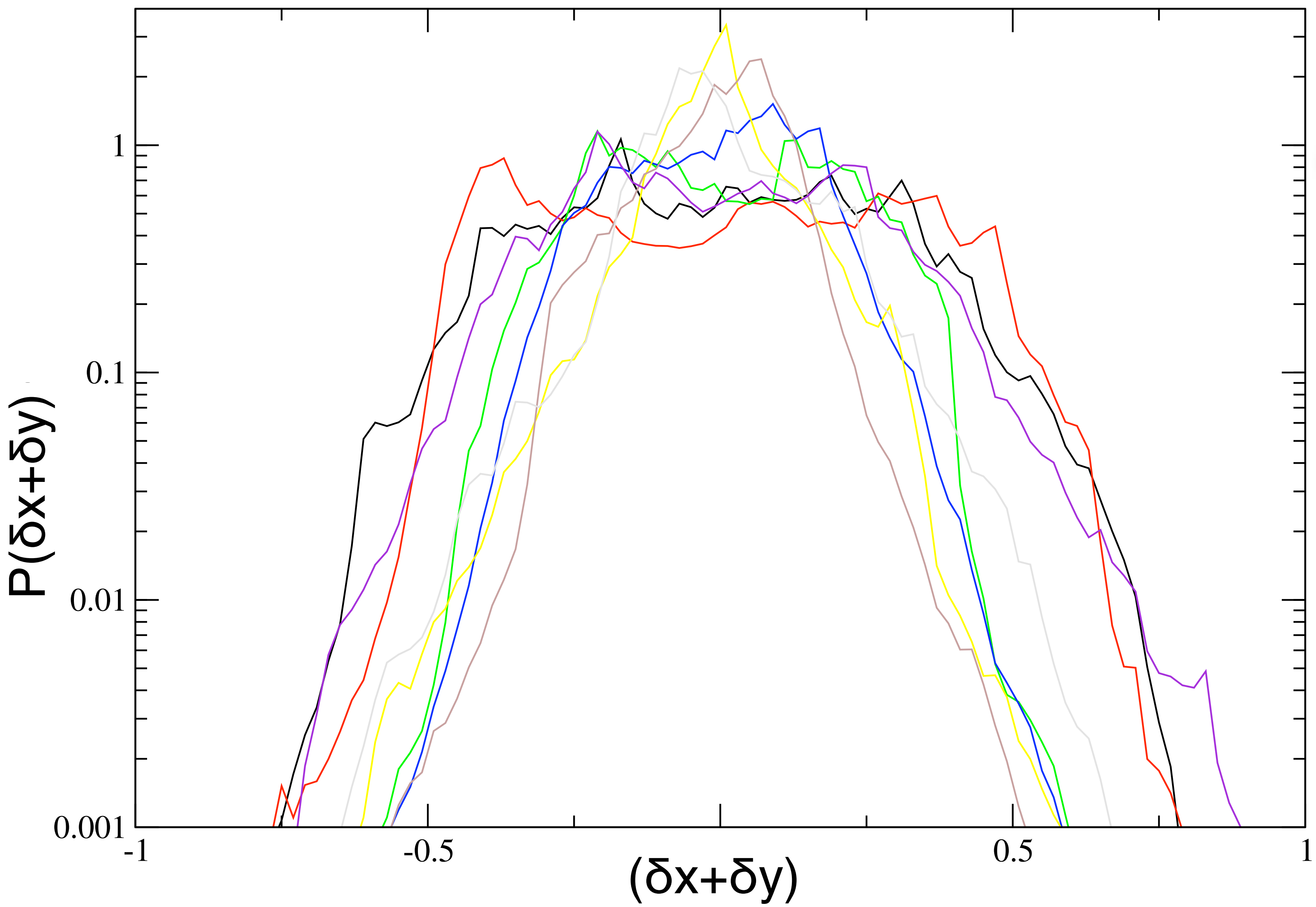
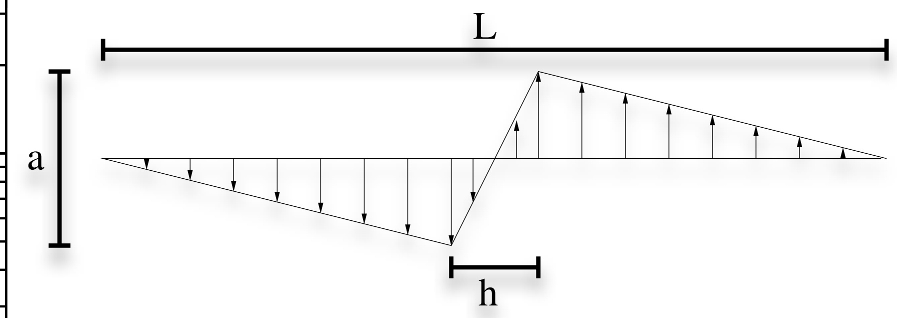
P(δx+δy) for 8 consecutive Δγ=0.001 windows





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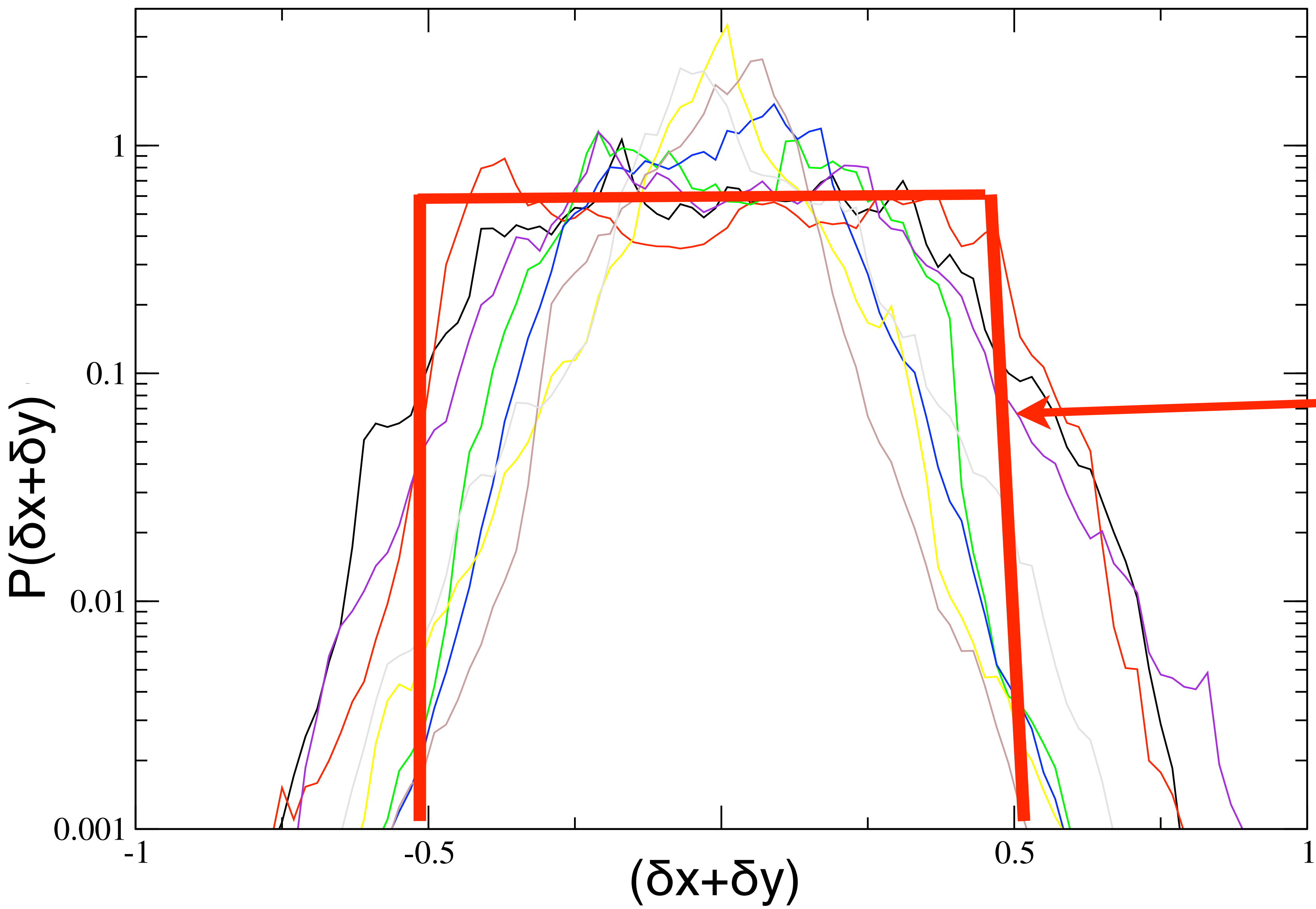
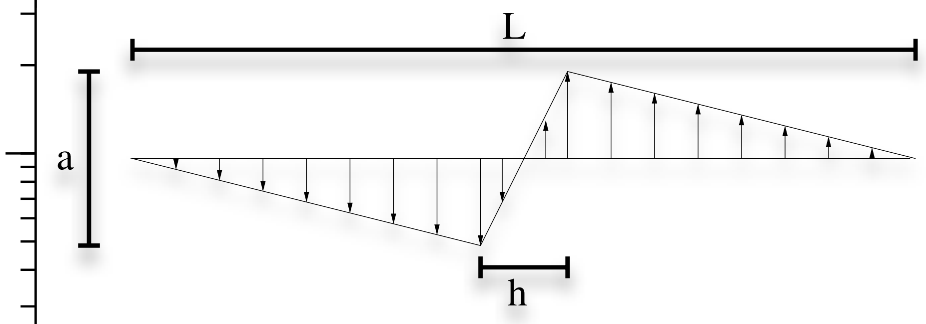
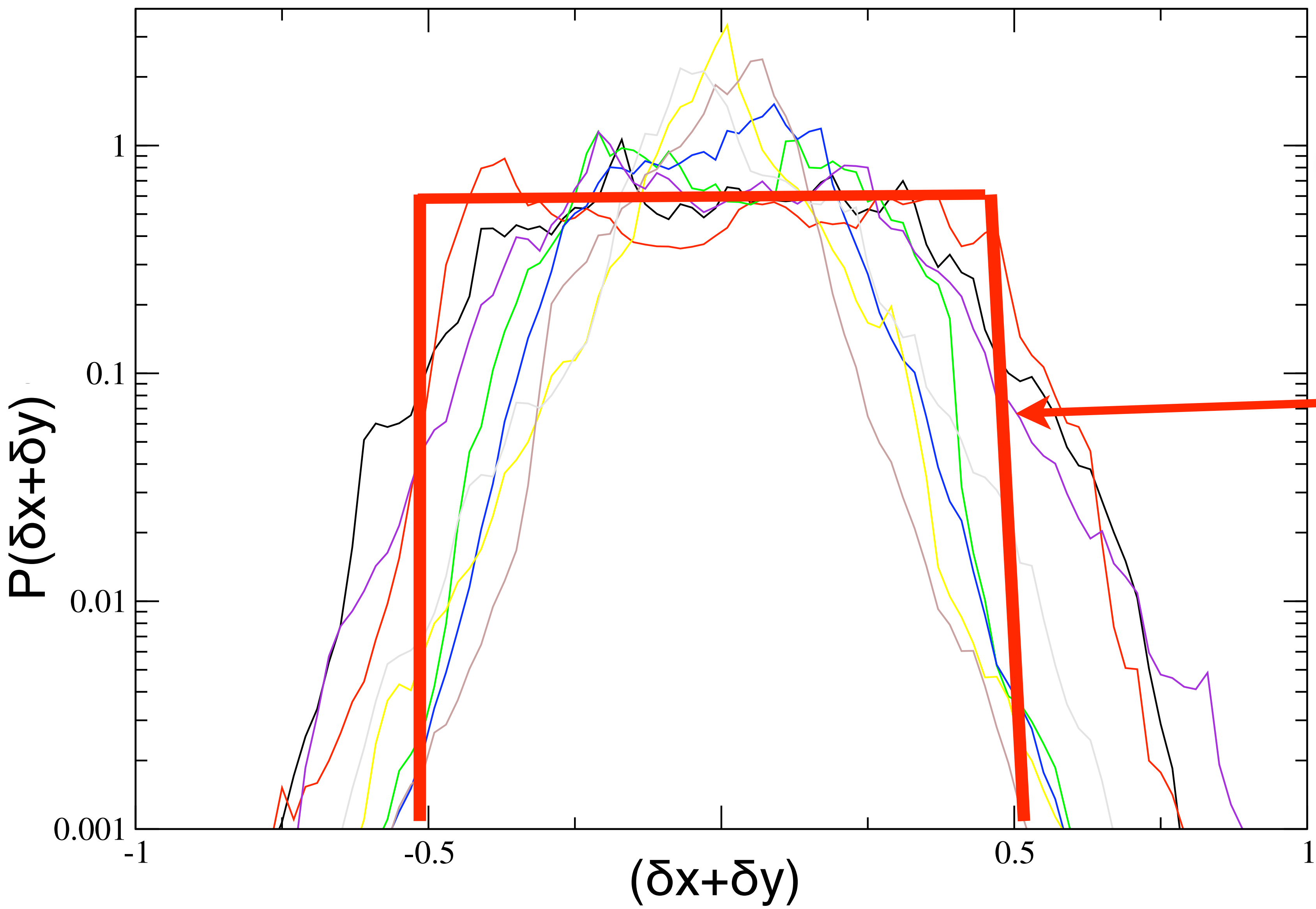
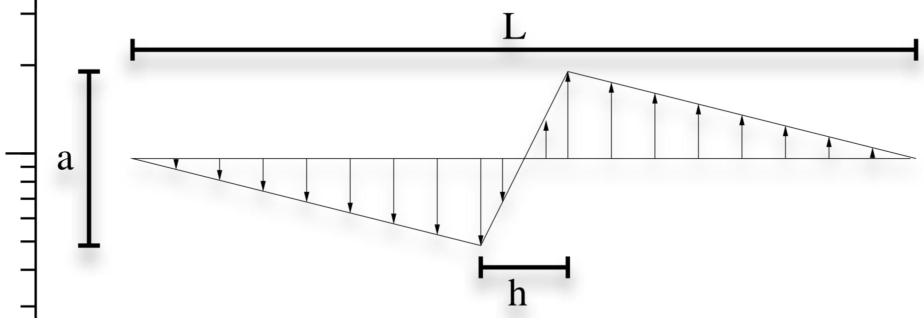




active:
P(δx+δy) flat up to "a"

P(δx+δy) for 8 consecutive Δγ=0.001 windows





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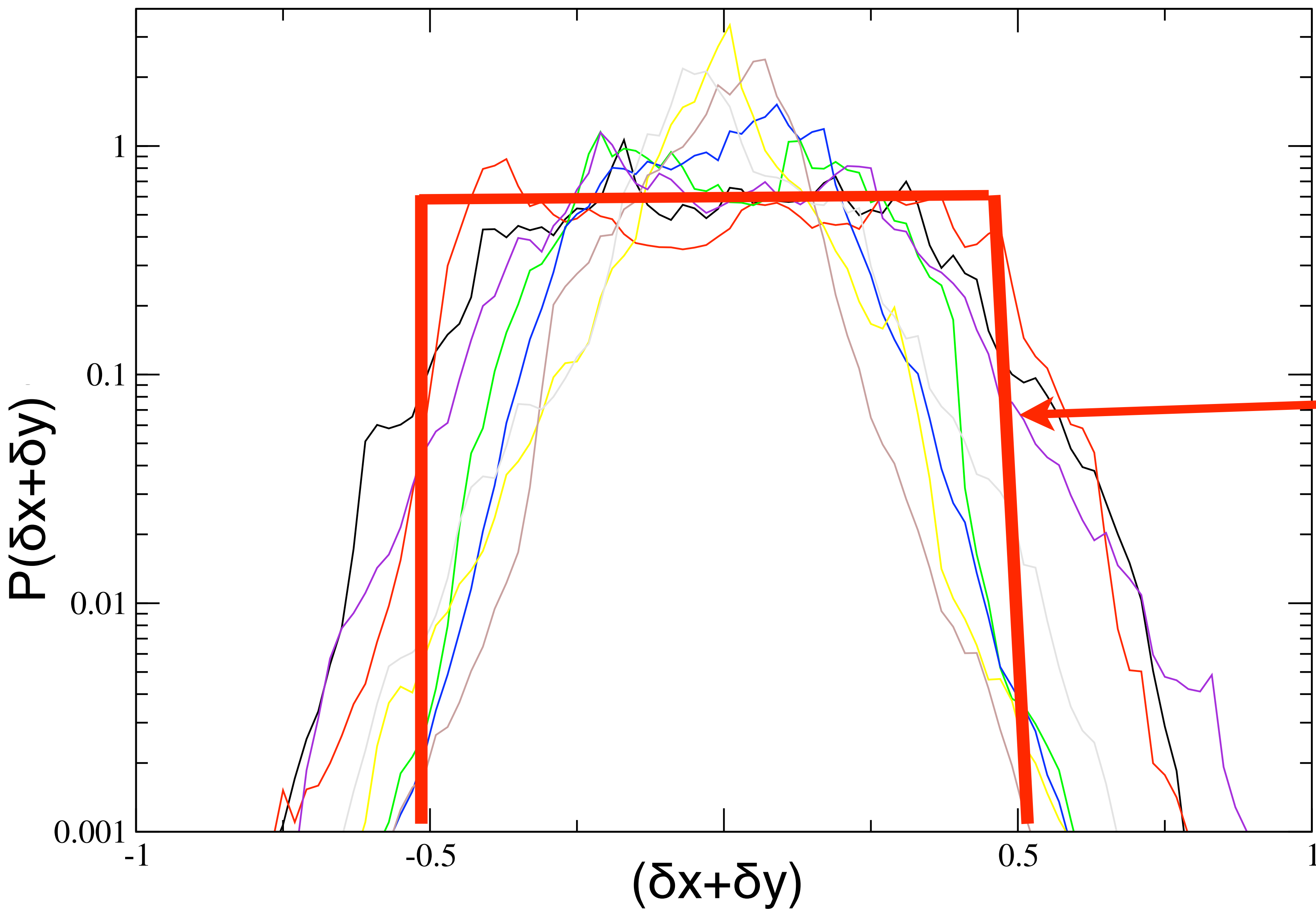
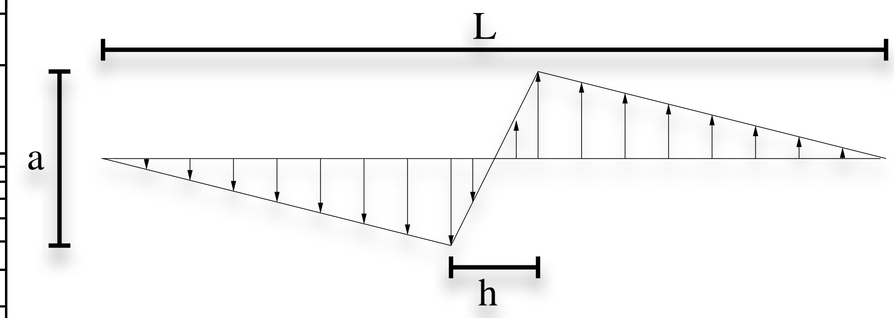
Elementary slip lines have $\langle \Delta r^2 \rangle_{elem.} \sim a^2/12$

P(δx+δy) for 8 consecutive Δγ=0.001 windows





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Elementary slip lines have $\langle \Delta r^2 \rangle_{elem.} \sim a^2/12$

At larger Δγ, these add incoherently

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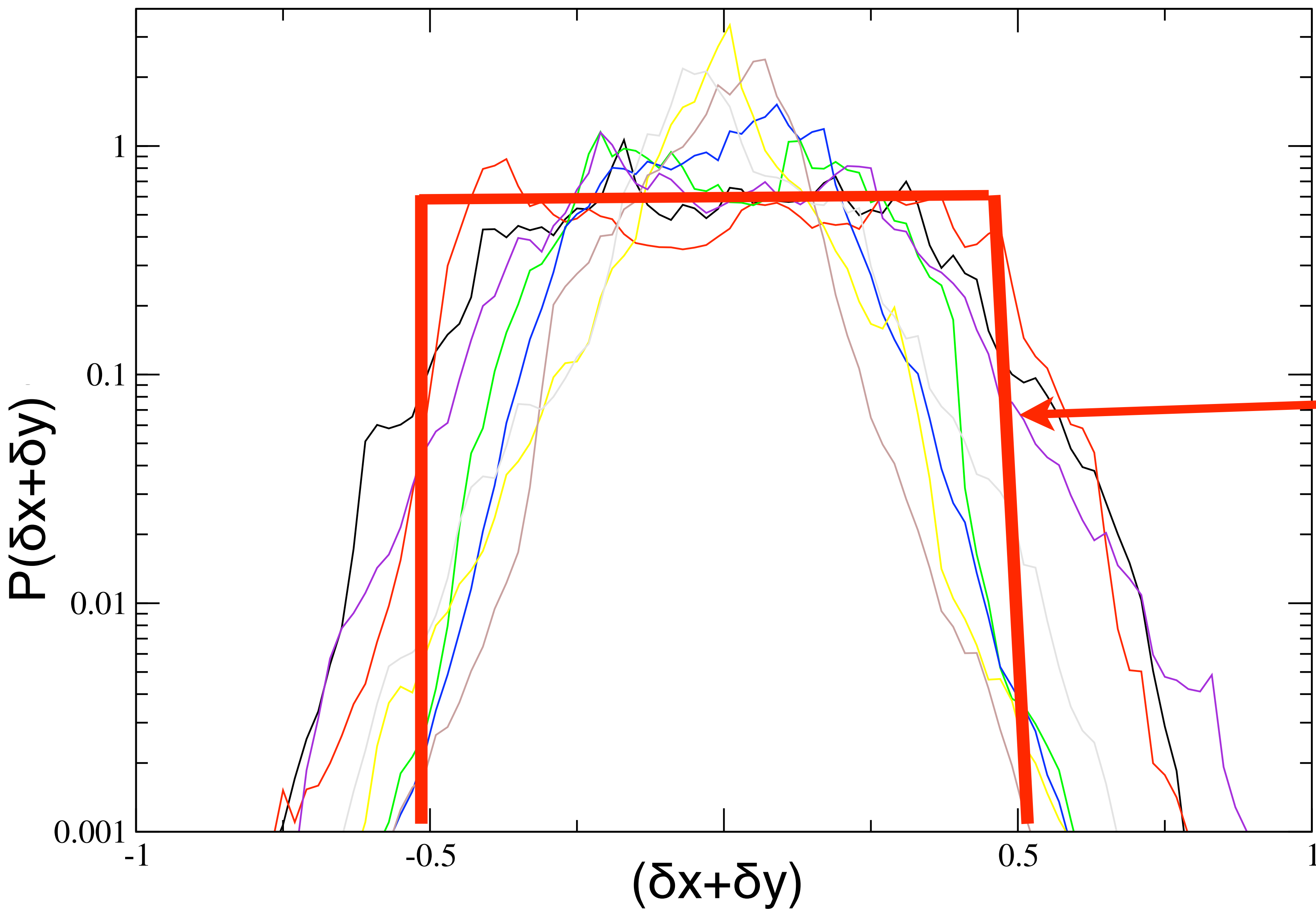

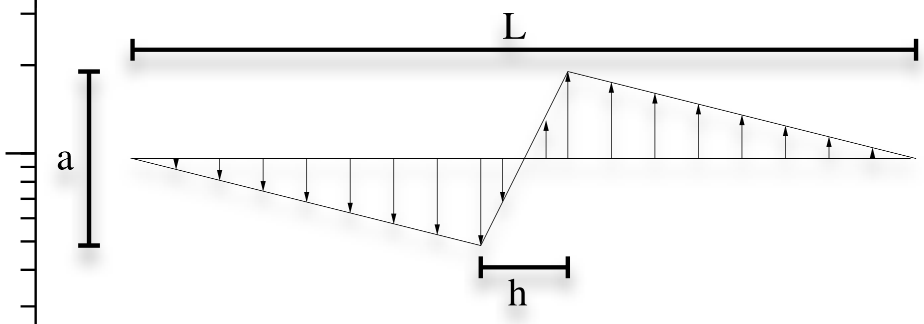

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At larger Δγ, these add incoherently

$\langle \Delta r^2 \rangle = \{N_{events}\} \{\langle \Delta r^2 \rangle_{elem.}\} = \{\Delta\gamma/(a/L)\} \{a^2/12\} = La/12\ \Delta\gamma$

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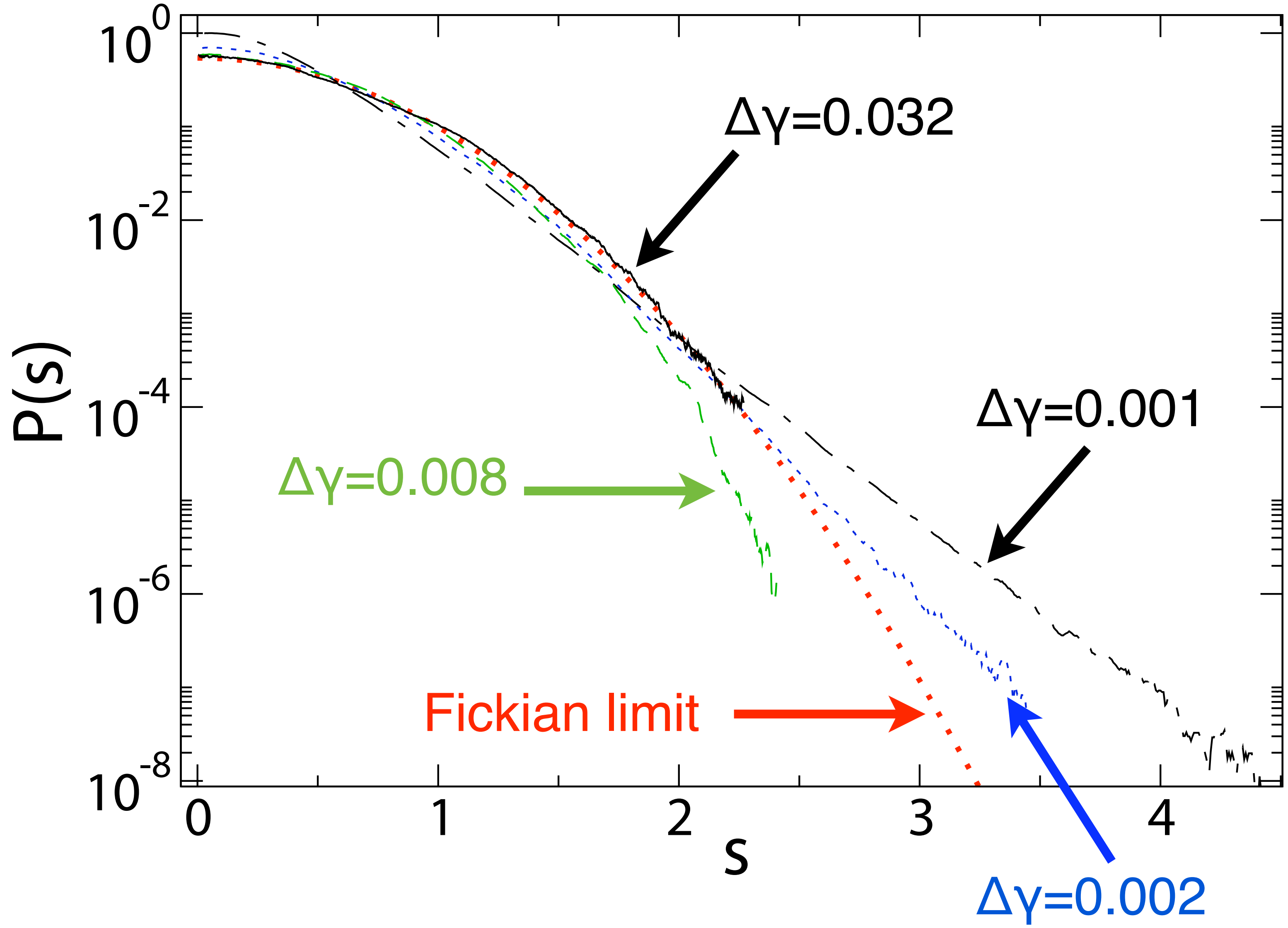
P(Δr) for various Δγ



All distributions rescaled by Fickian expectation:

$s = \langle \Delta r^2 \rangle / \Delta\gamma$

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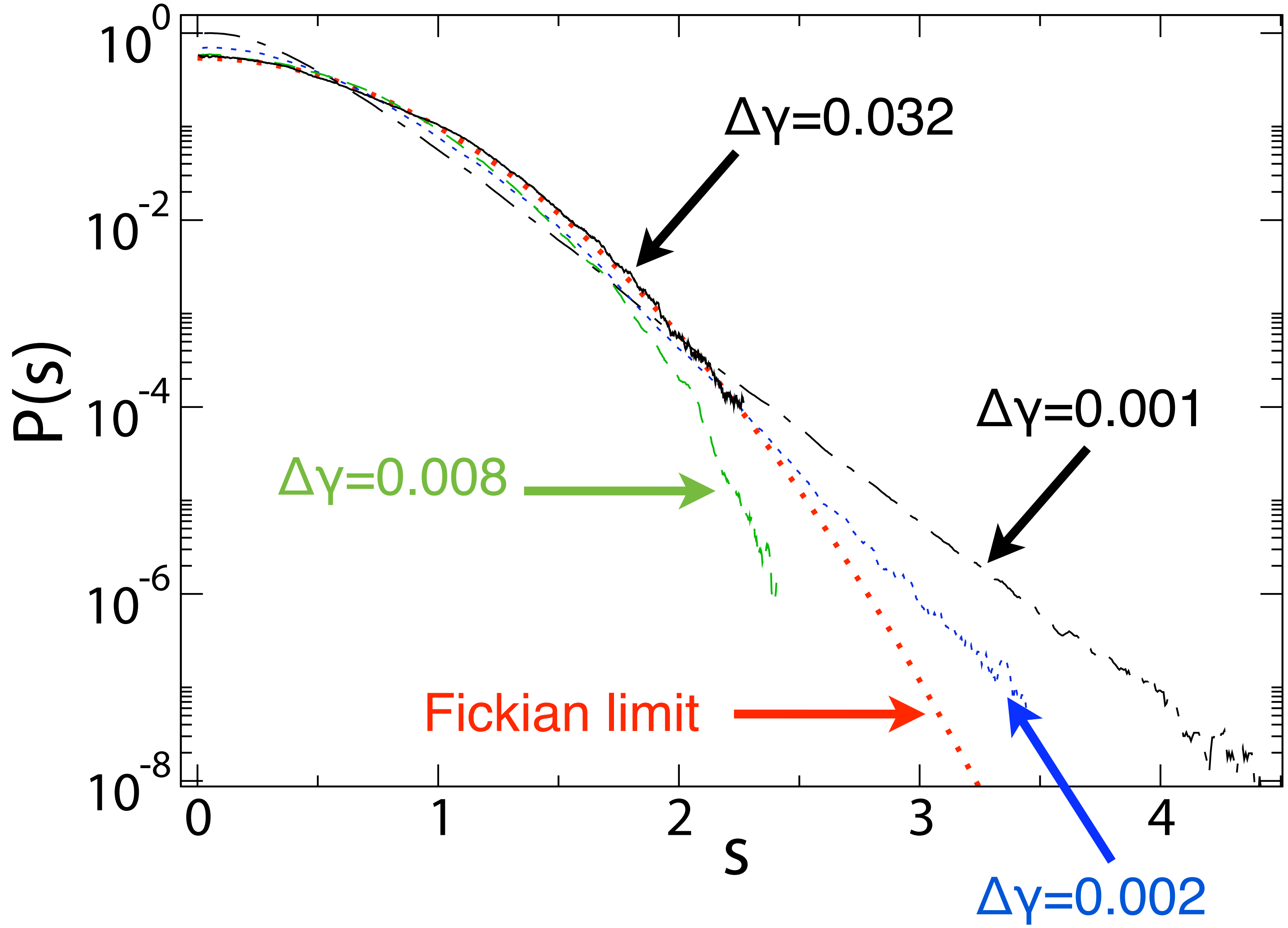
P(Δr) for various Δγ



All distributions rescaled by Fickian expectation:

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Looks Fickian but:
- spatial correlations
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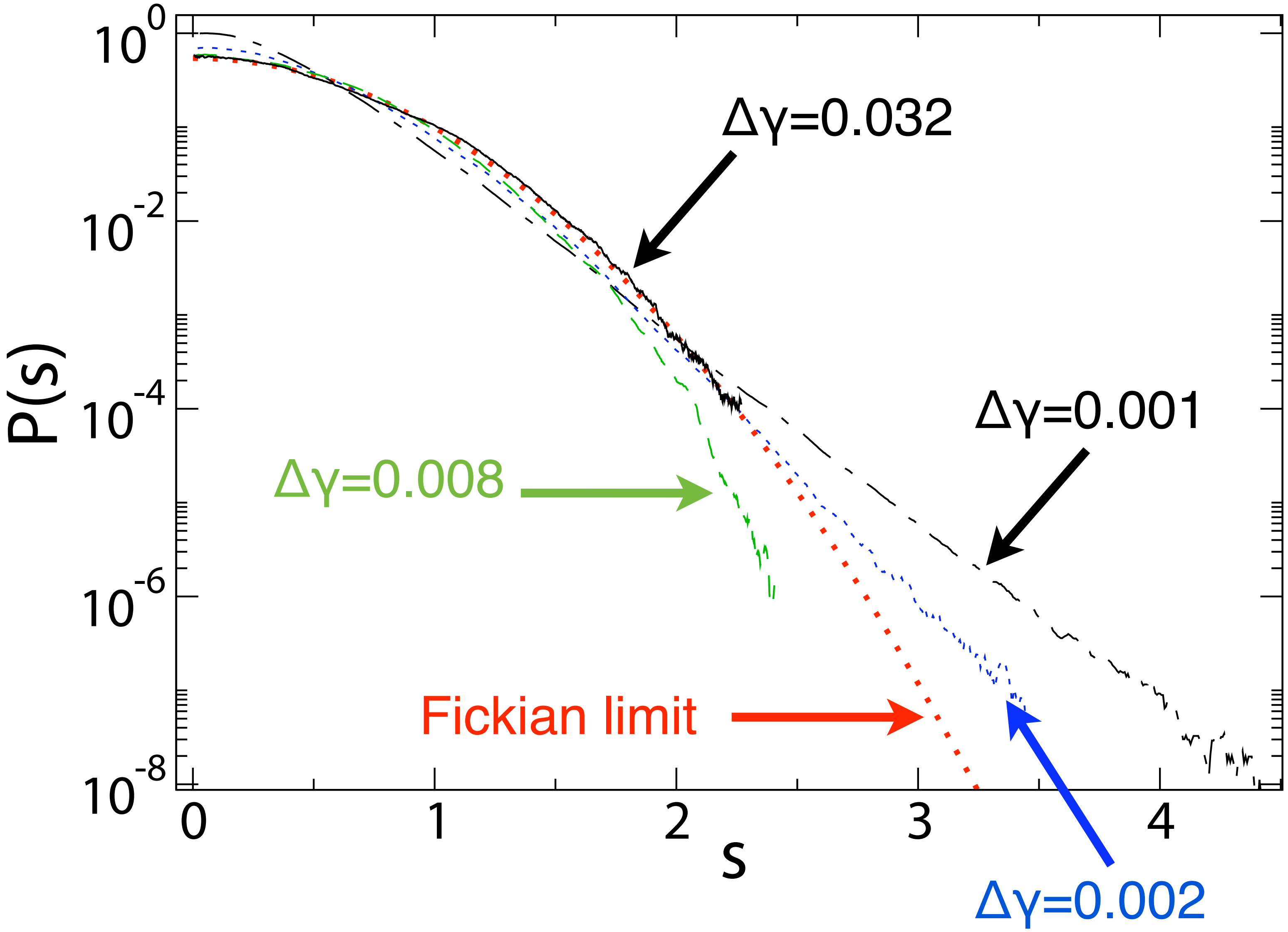
P(Δr) for various Δγ



All distributions rescaled by Fickian expectation:

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Crossover to Fickian (Δγ~0.032) consistent with thick bands filling space

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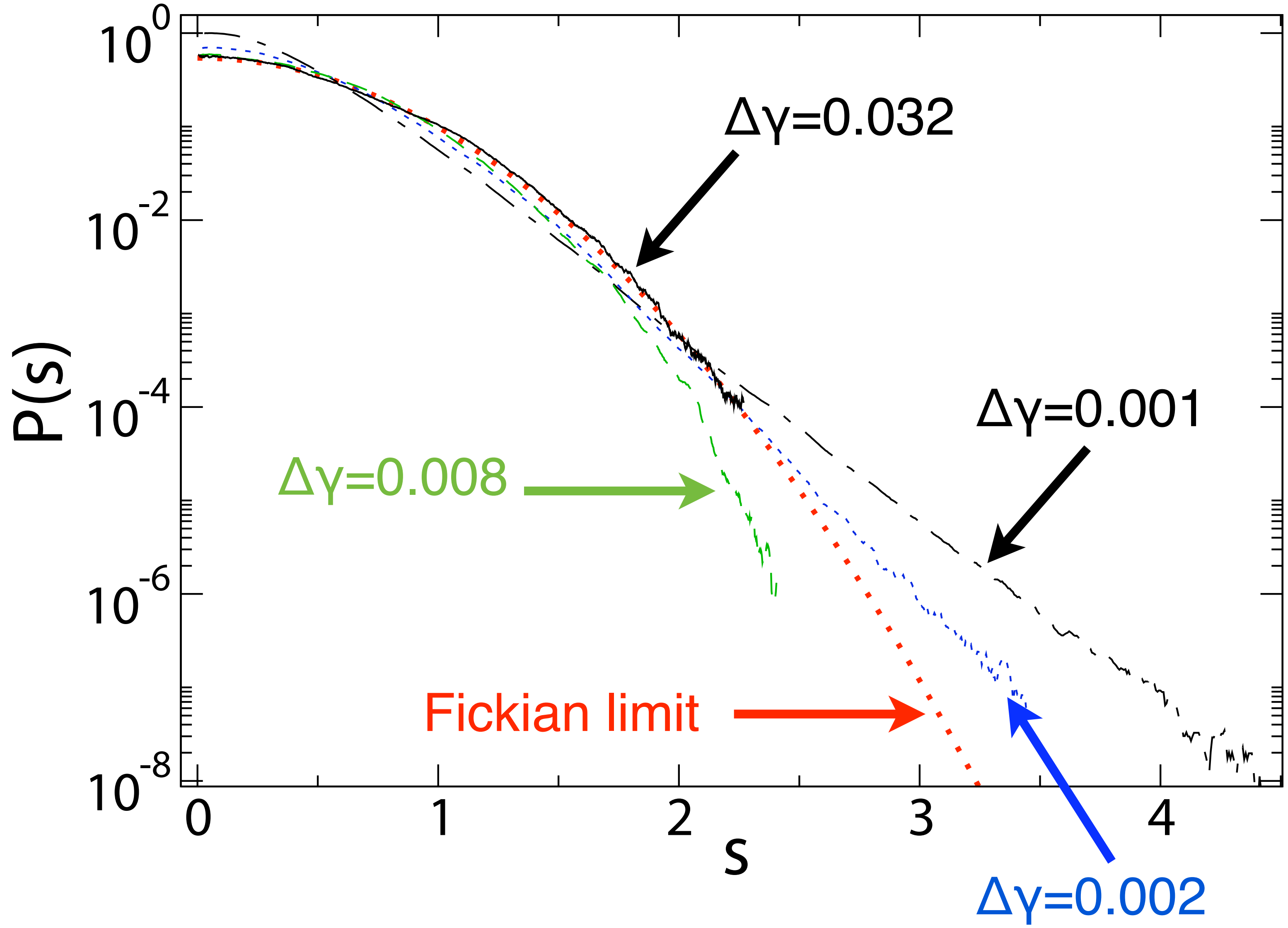

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- Slip line argument:

$a=(12s/L) \sim 0.7\sigma_0$

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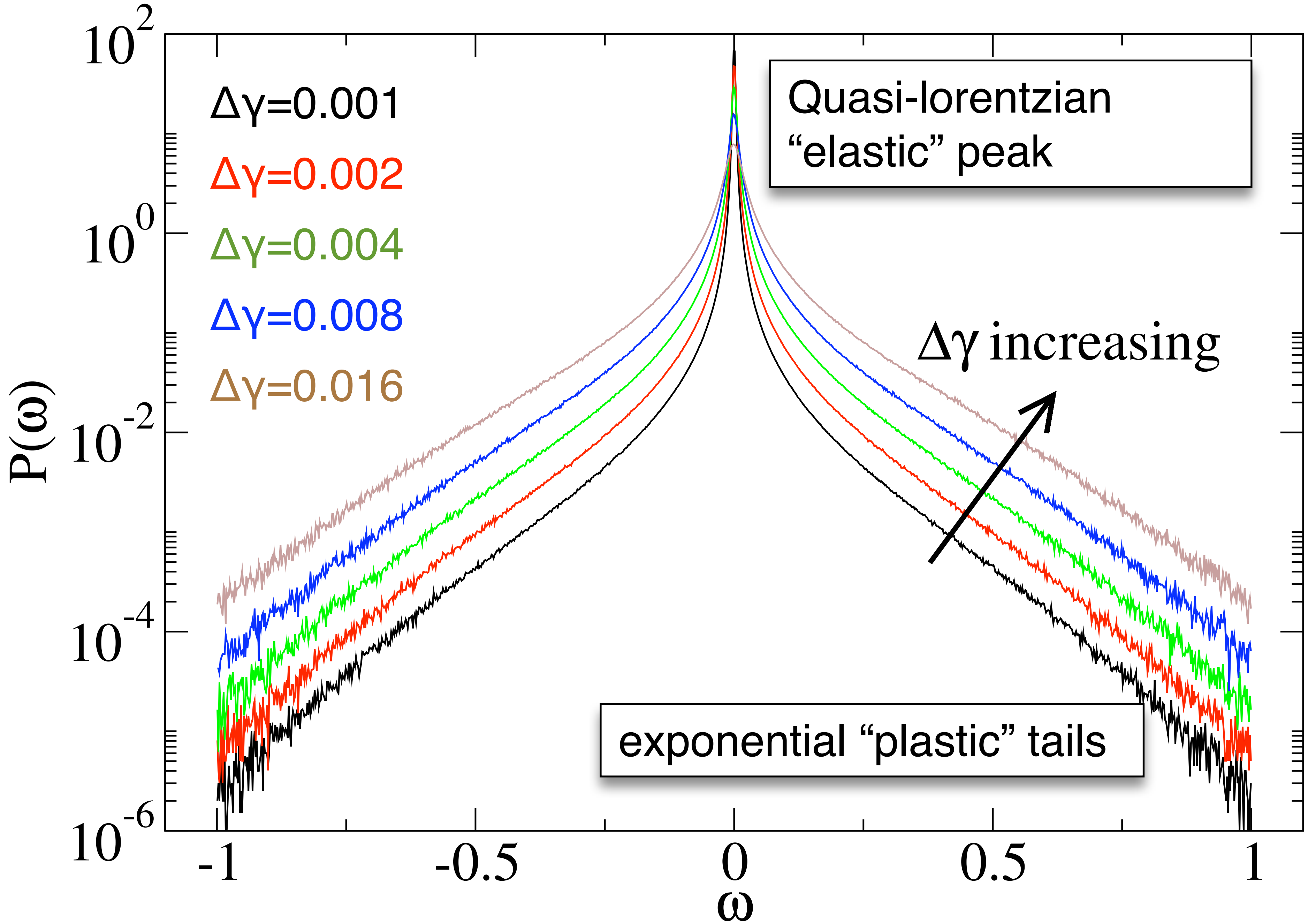
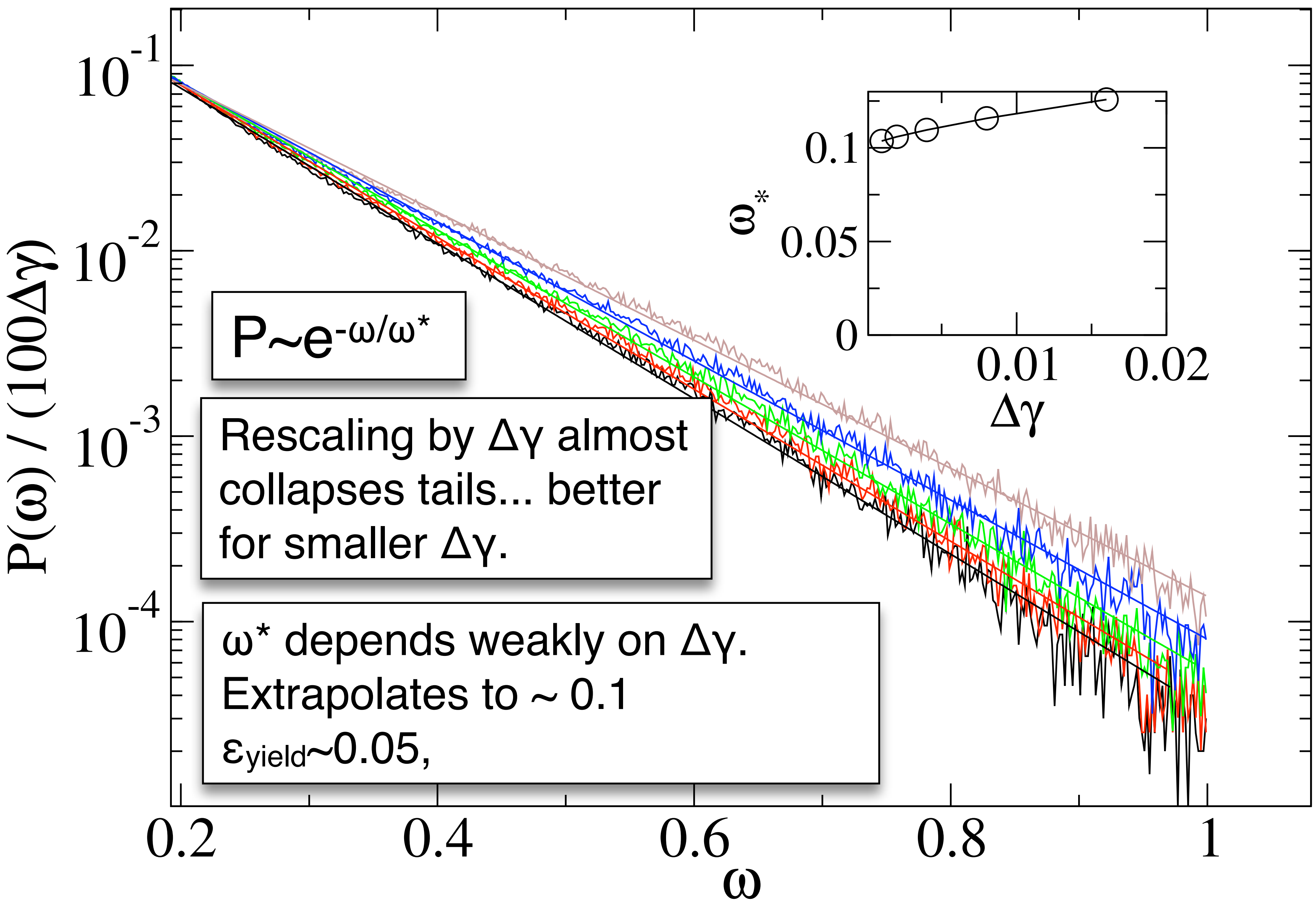
P(ω;Δγ)



Δγ=0.001
Δγ=0.002
Δγ=0.004
Δγ=0.008
Δγ=0.016
Quasi-lorentzian "elastic" peak
Δγ increasing
exponential "plastic" tails
P(ω)
ω
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
-1
-0.5
0
0.5
1

$P(\omega;\Delta\gamma)$. Scale by $\Delta\gamma$, fit to $e^{-\omega/\omega^*}$



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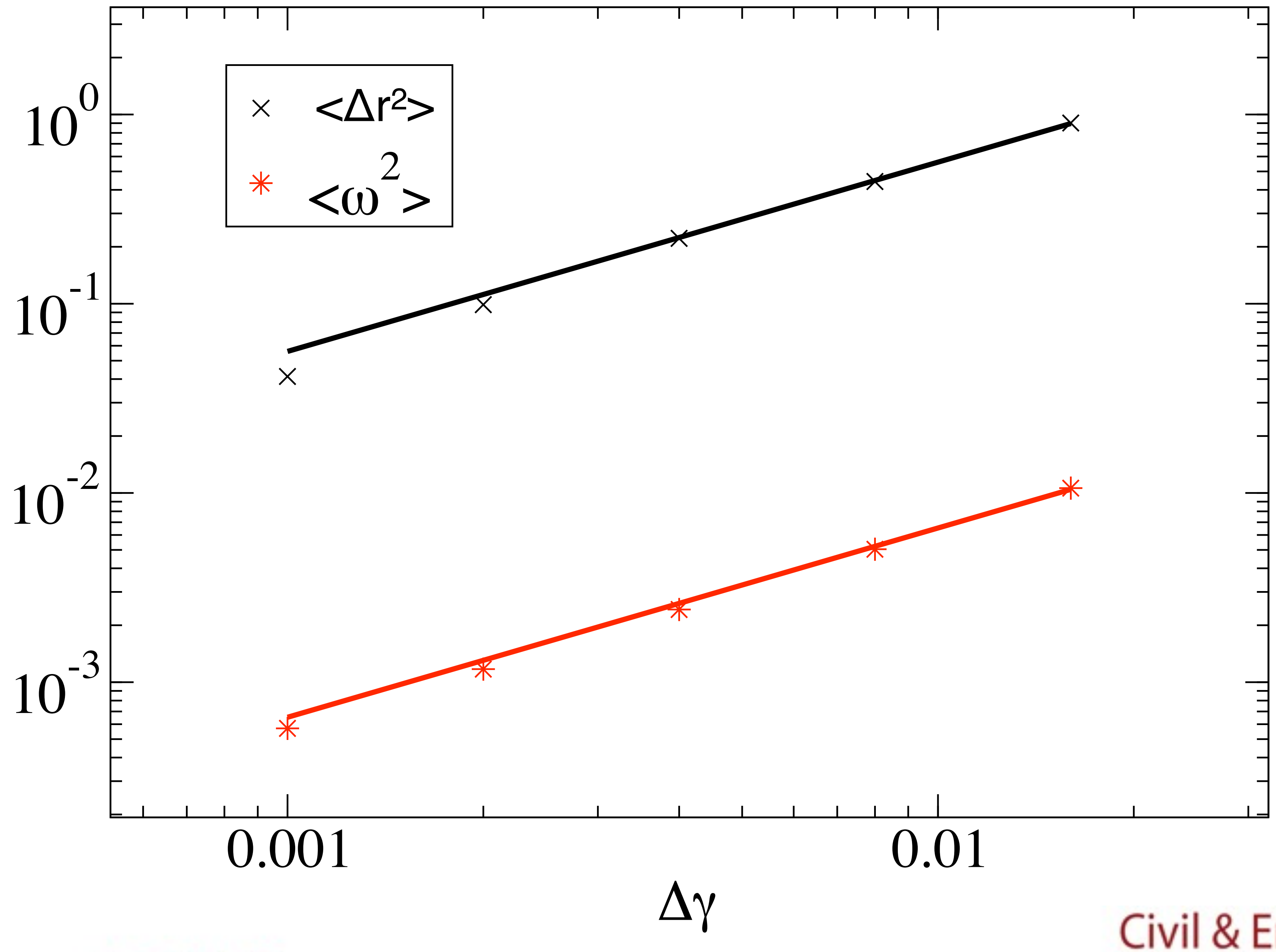
RMS ω vs Δγ



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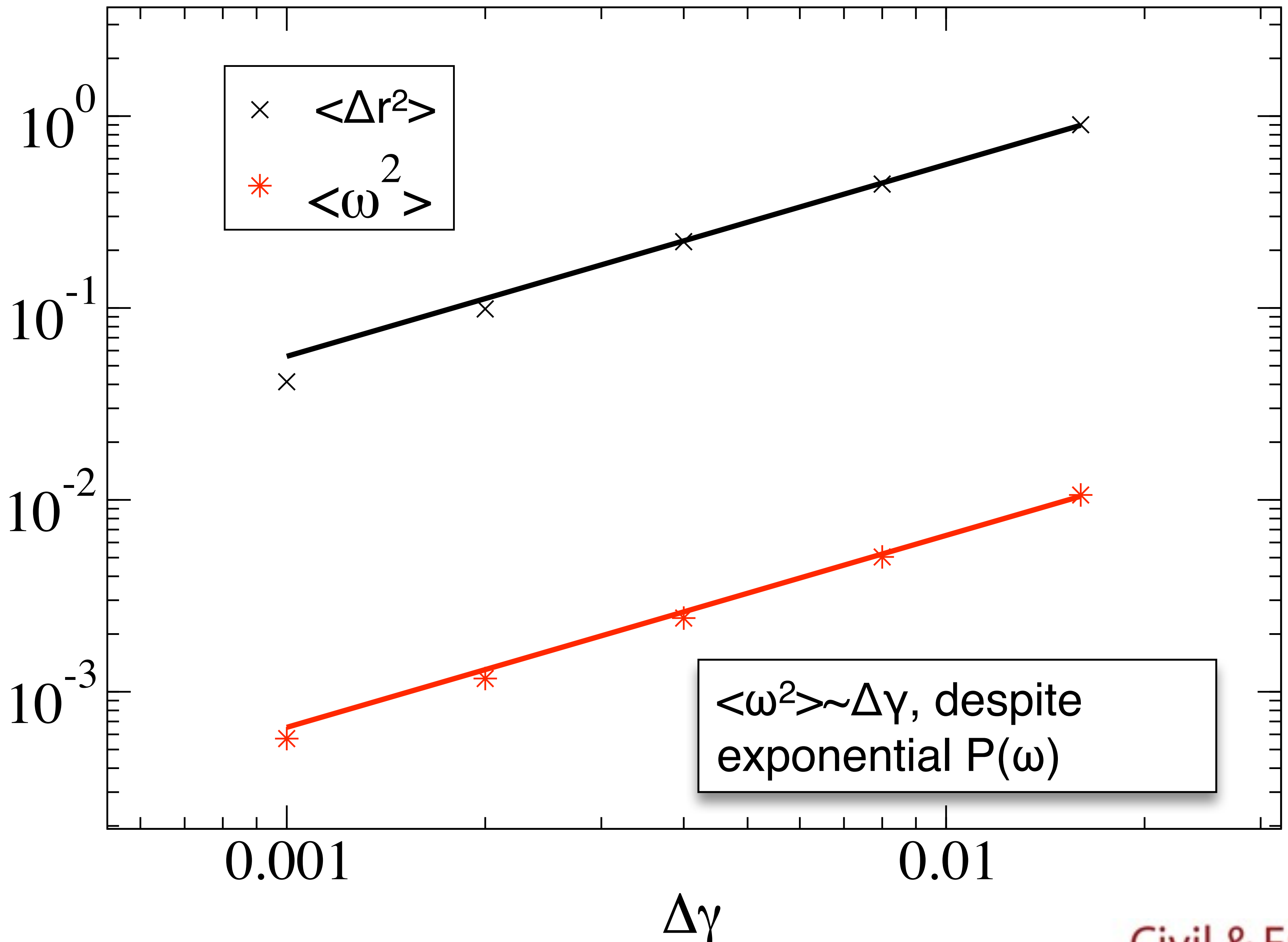

<Δr²>
<ω²>
<ω²>~Δγ, despite exponential P(ω)
10^0
10^-1
10^-2
10^-3
0.001
0.01
Δγ

Effective diffusion in Lennard-Jones

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- Slip in bands: $a \sim \sigma_0$, $h \sim 50\sigma_0$, $\gamma_{band} \sim 1\%$ (for $L \sim 1000$)

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- $\omega^* \sim 0.1$ compatible with yield strain $\varepsilon_{yield} \sim 0.05$

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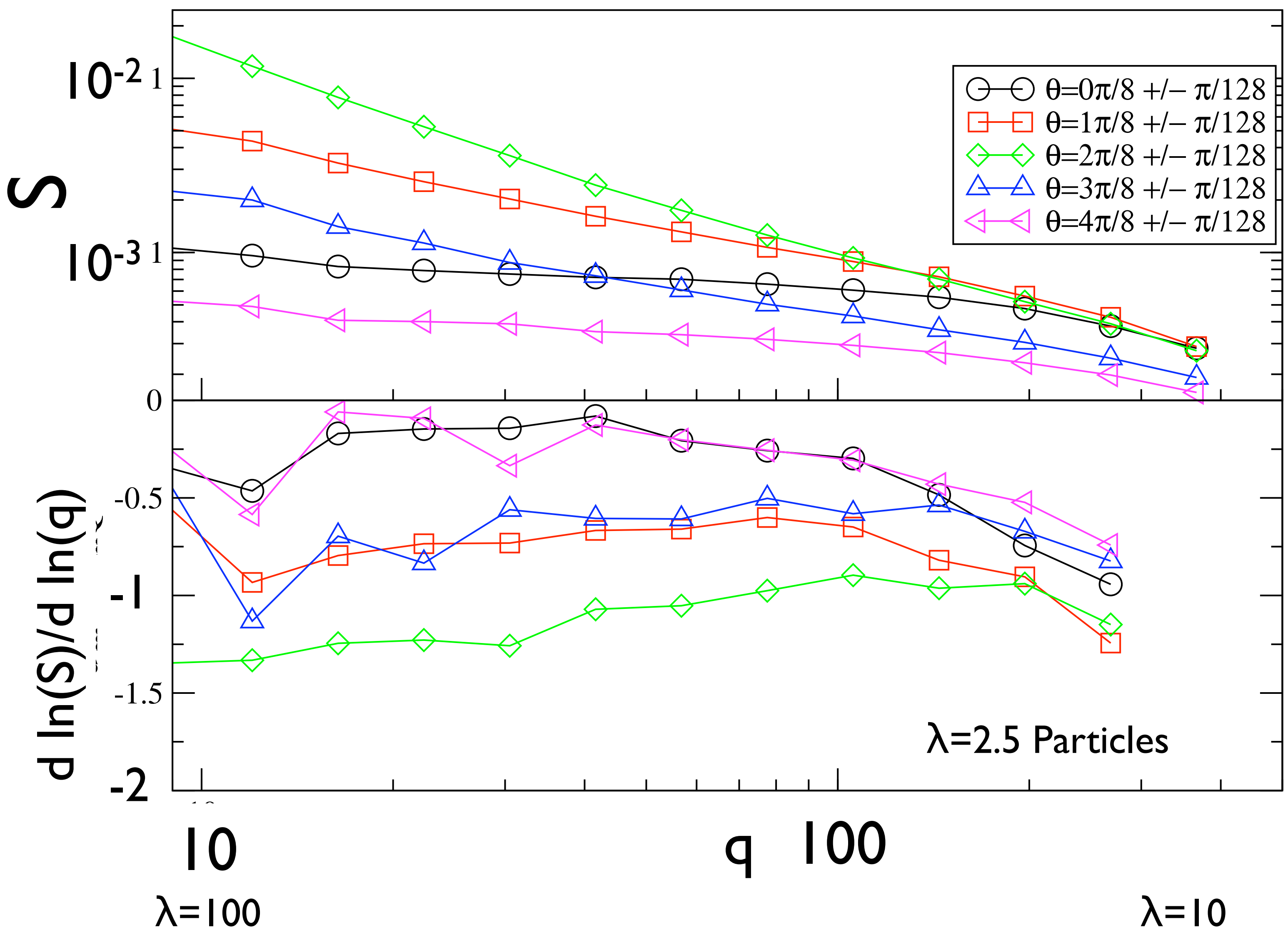
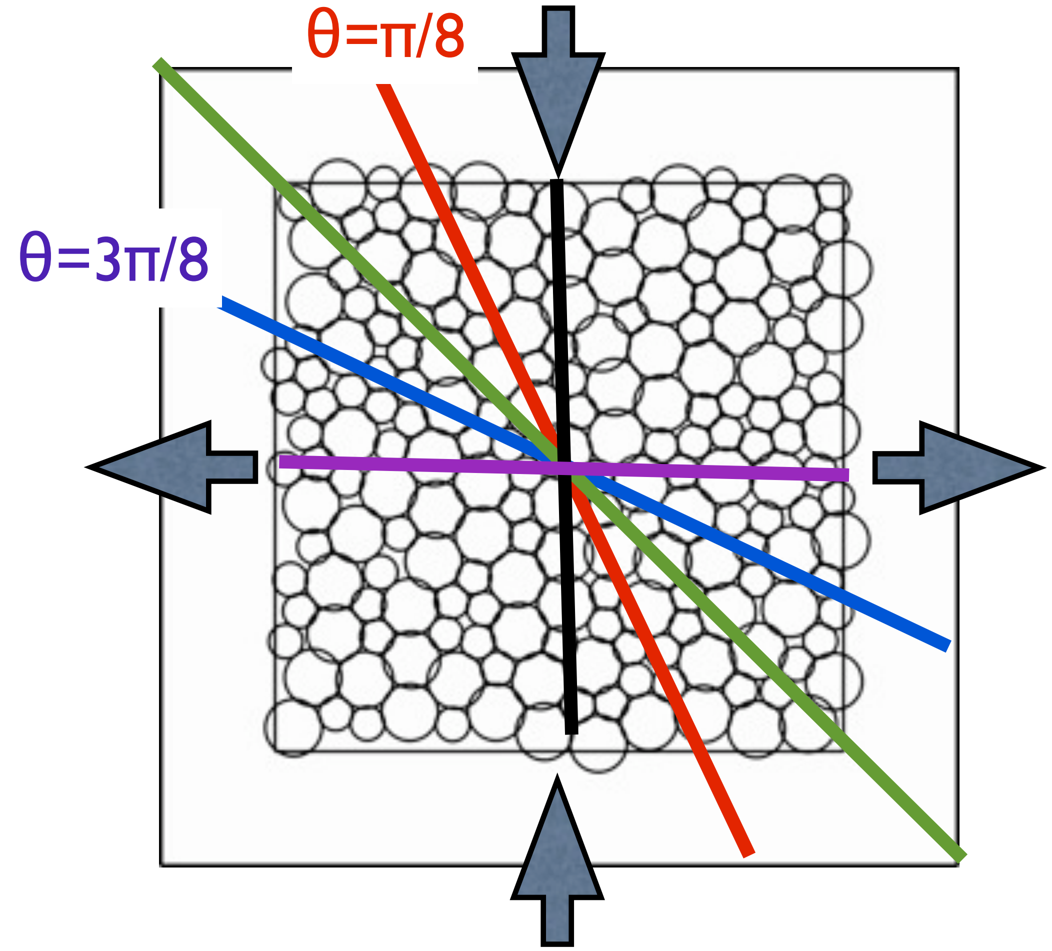
Structure factor for Δγ=0.04 $S(\vec{q}) = \left| \int \omega(\vec{r}) exp[i\vec{q} \cdot \vec{r}] dr \right|^2$





θ=π/8 and θ=3π/8 have same shear stress, different normal stress.

$S(q;\theta)=A(\theta)q^{-\alpha(\theta)}$

α depends on angle!

α: has “shear” symmetry

θ: does not

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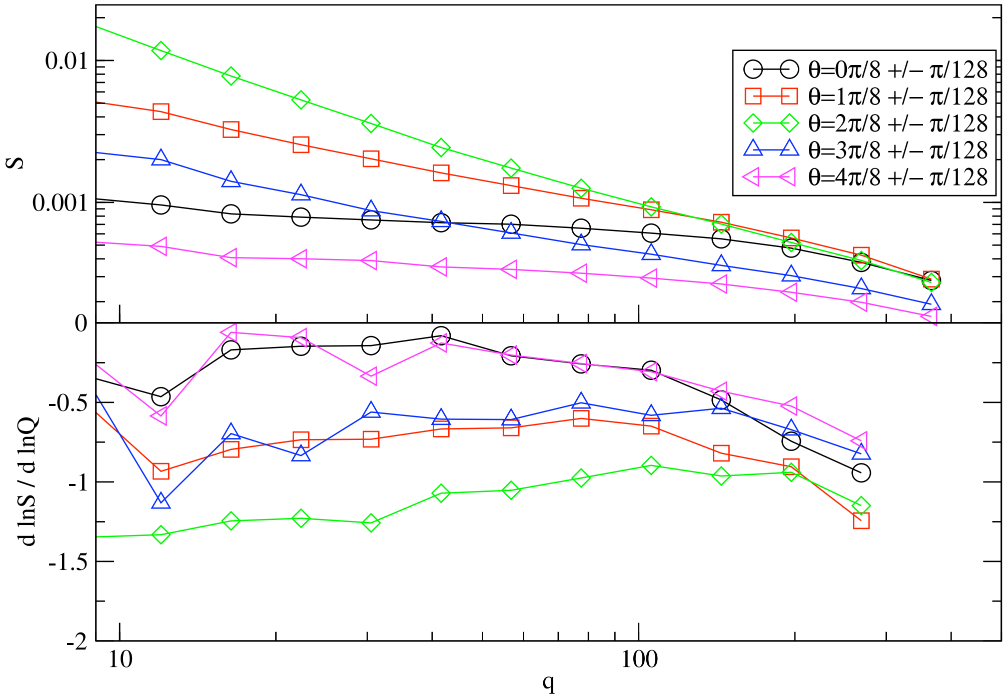
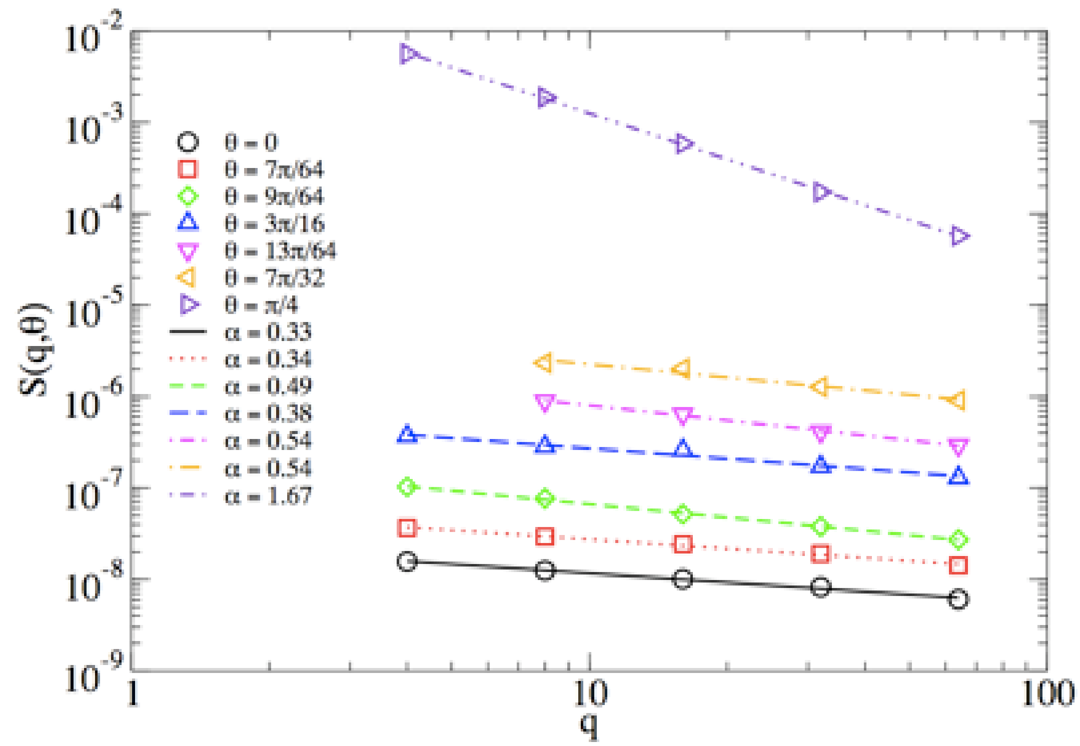
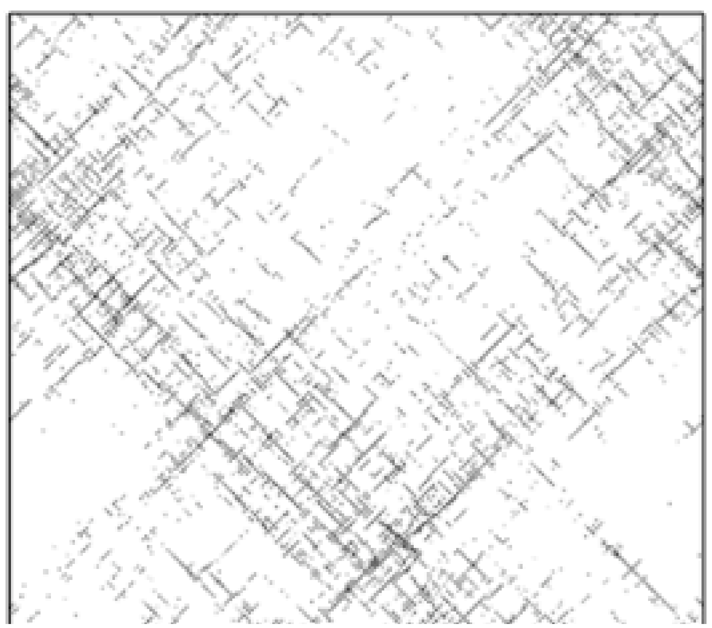
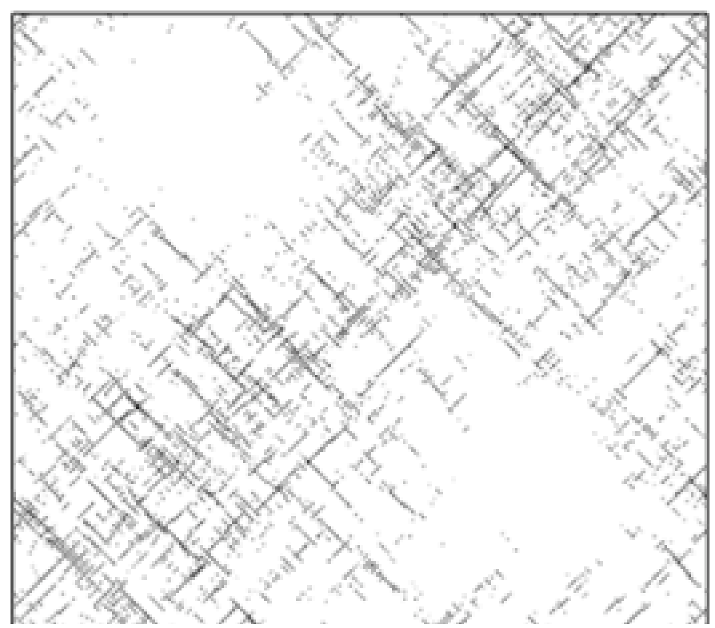
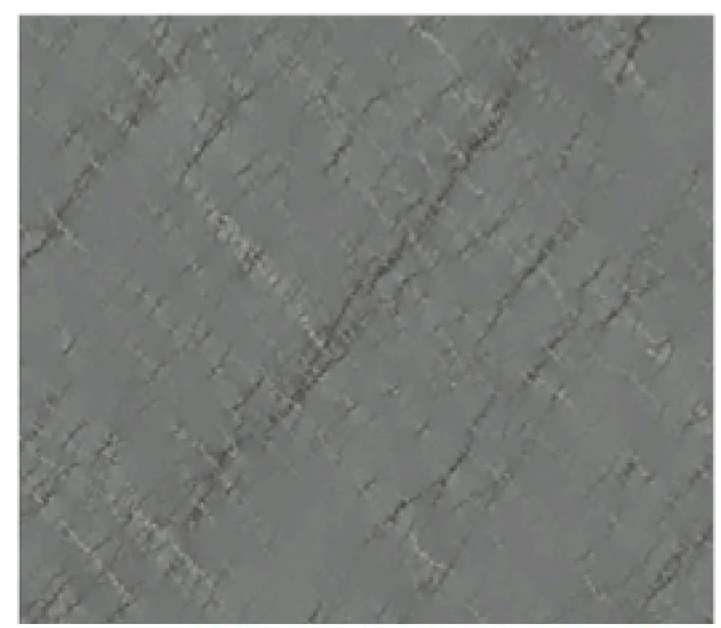
Compare to Talamali et. al. (Vandembroucq talk)











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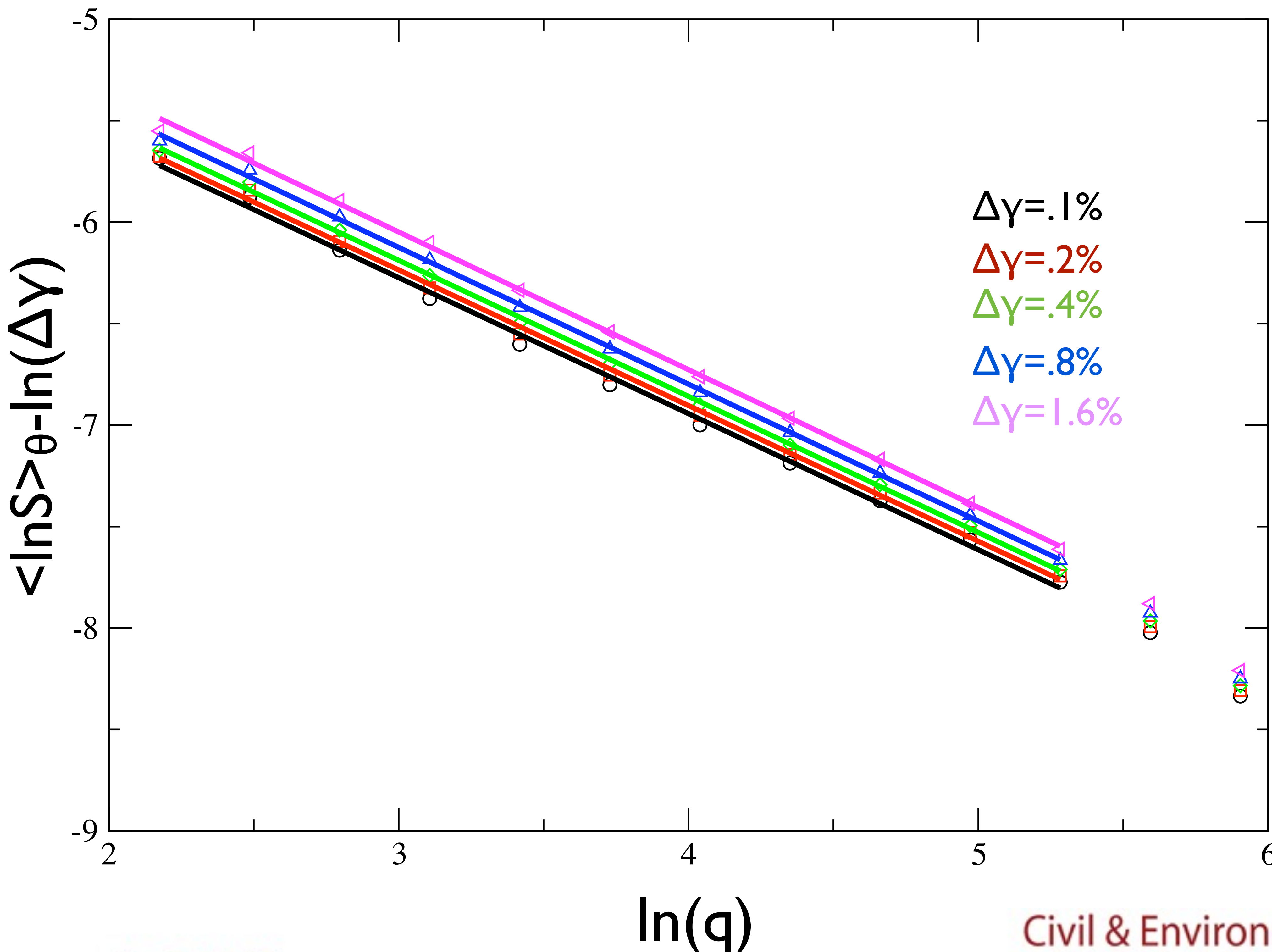
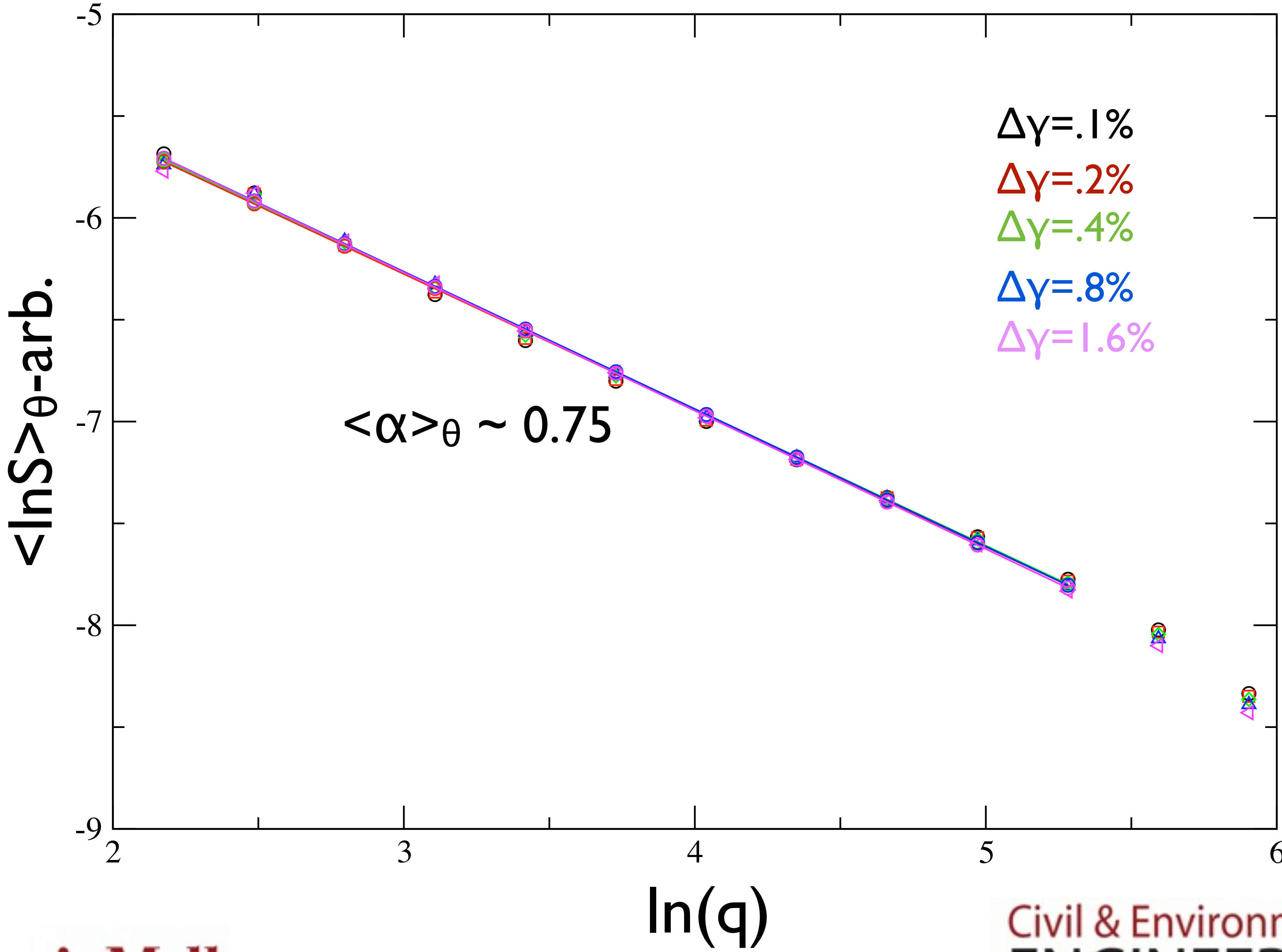
$\langle \mathrm{LogS} \rangle_\theta$ scaled by $\Delta\gamma$

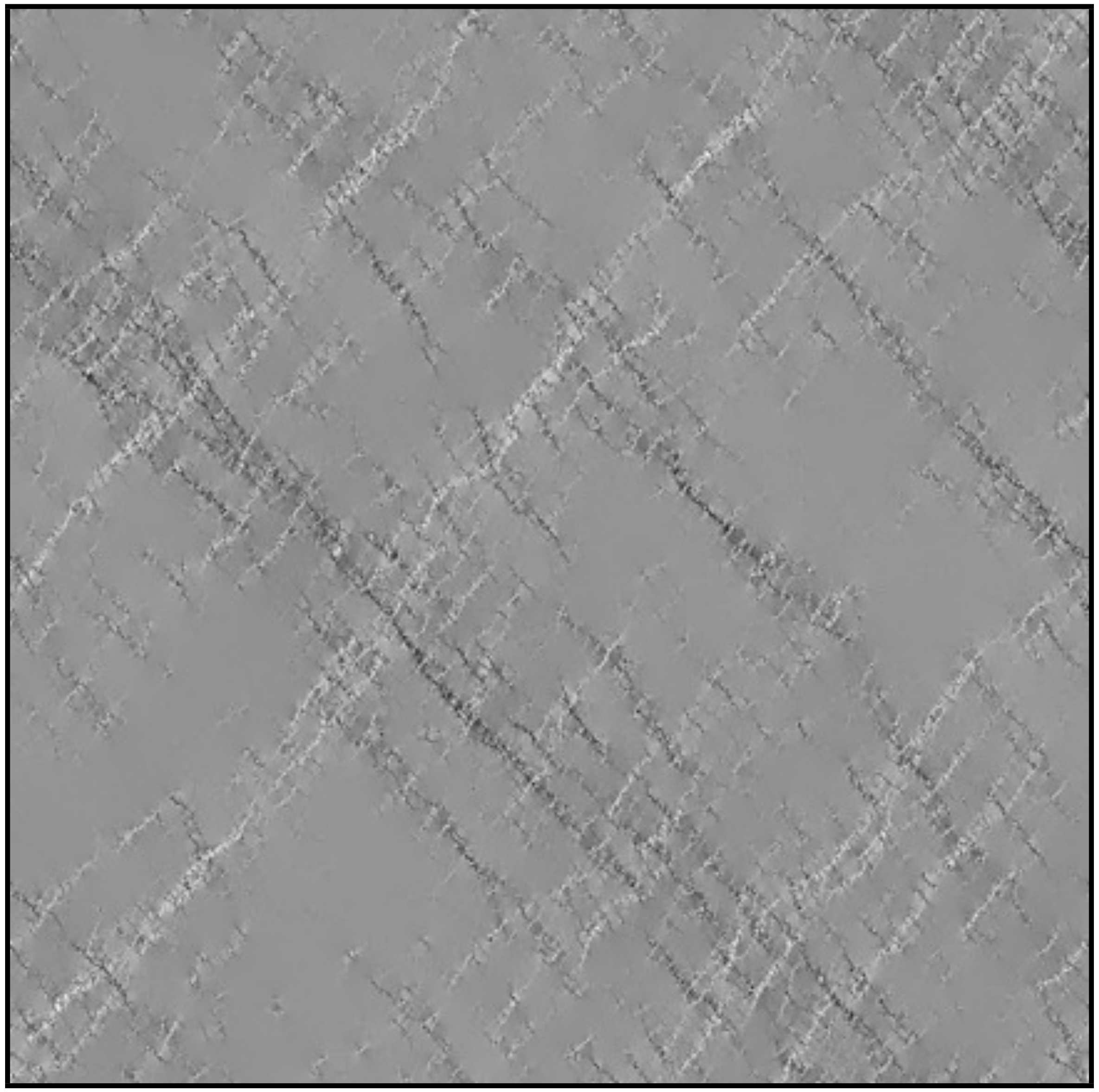


<LogS>$_\theta$ best-rescaling



Summary: Spatial structure of strain

- Measured vorticity, ω, for various, Δγ
- In steady state, $S(q,\theta)=A(\theta)q^{\alpha(\theta)}$
- α has "shear symmetry"
- A(θ): Mohr-Coulomb effect
- S/Δγ collapse implies: ω is de-correlated



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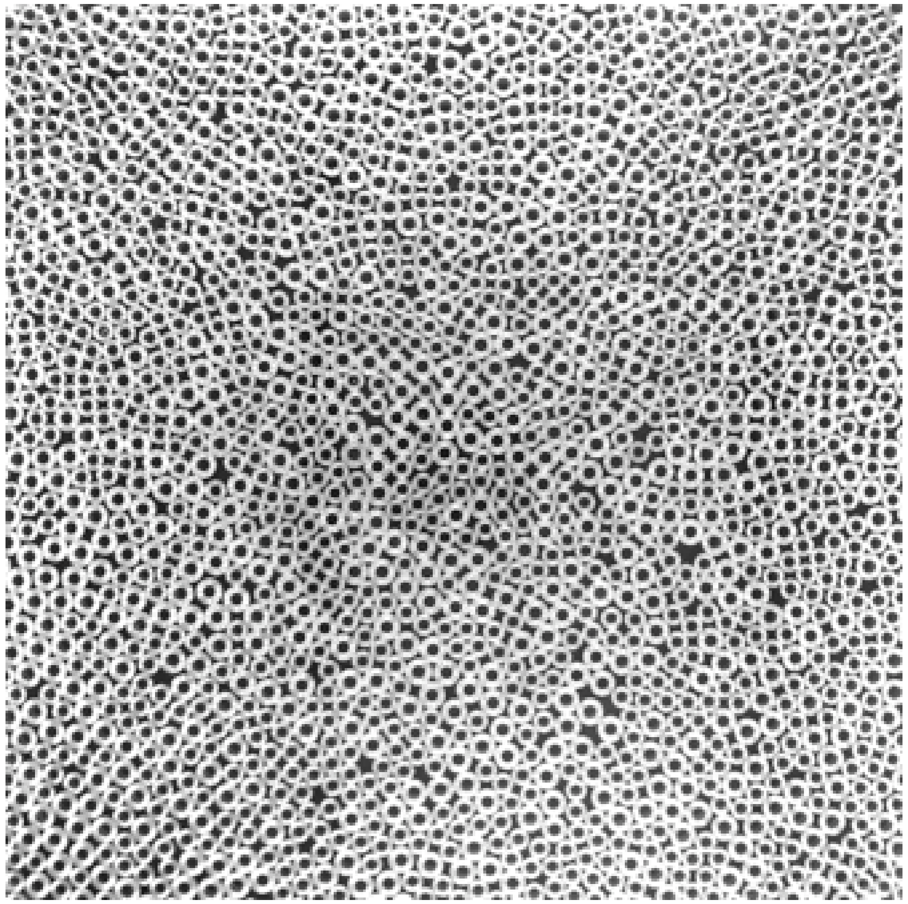
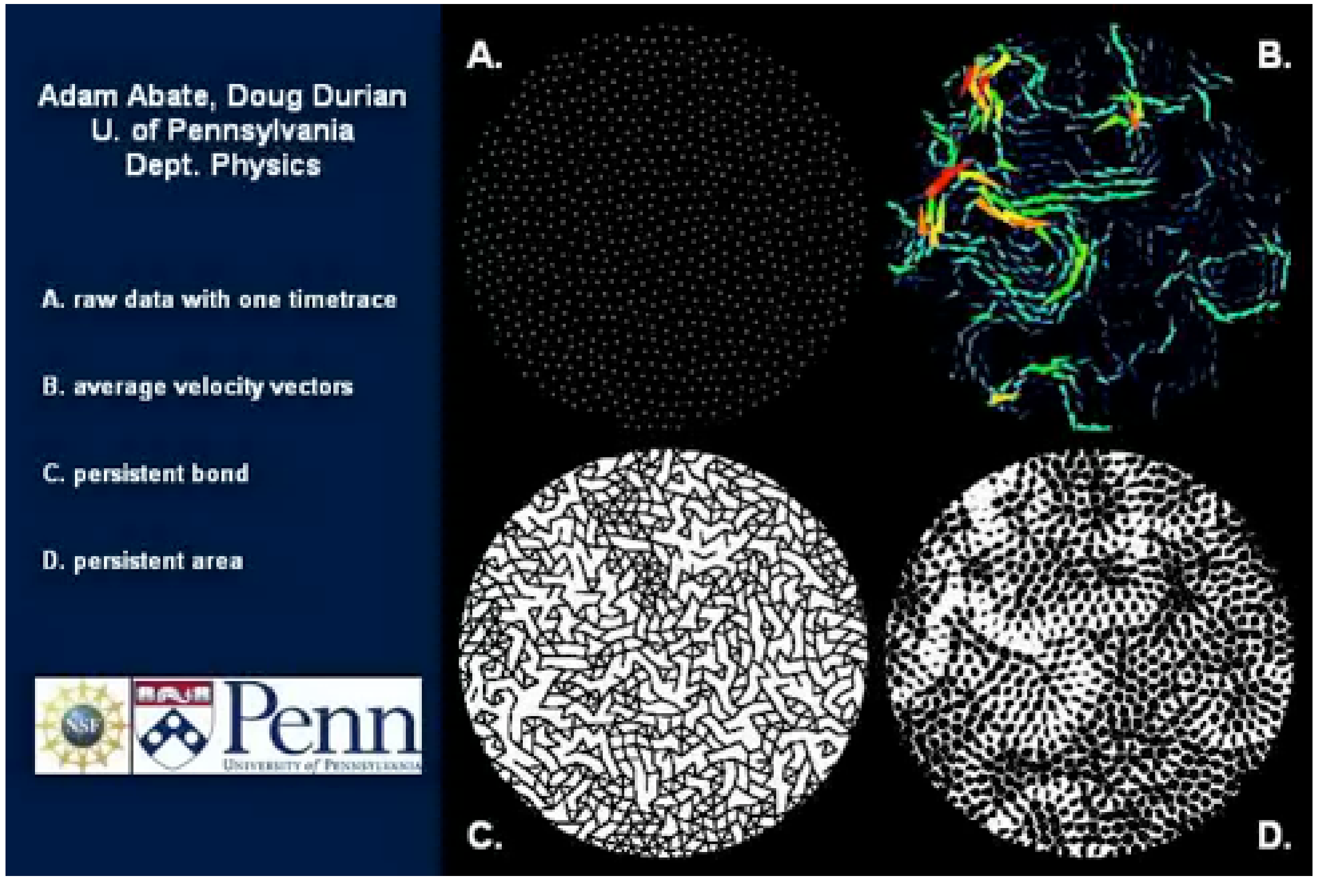
Jammed systems



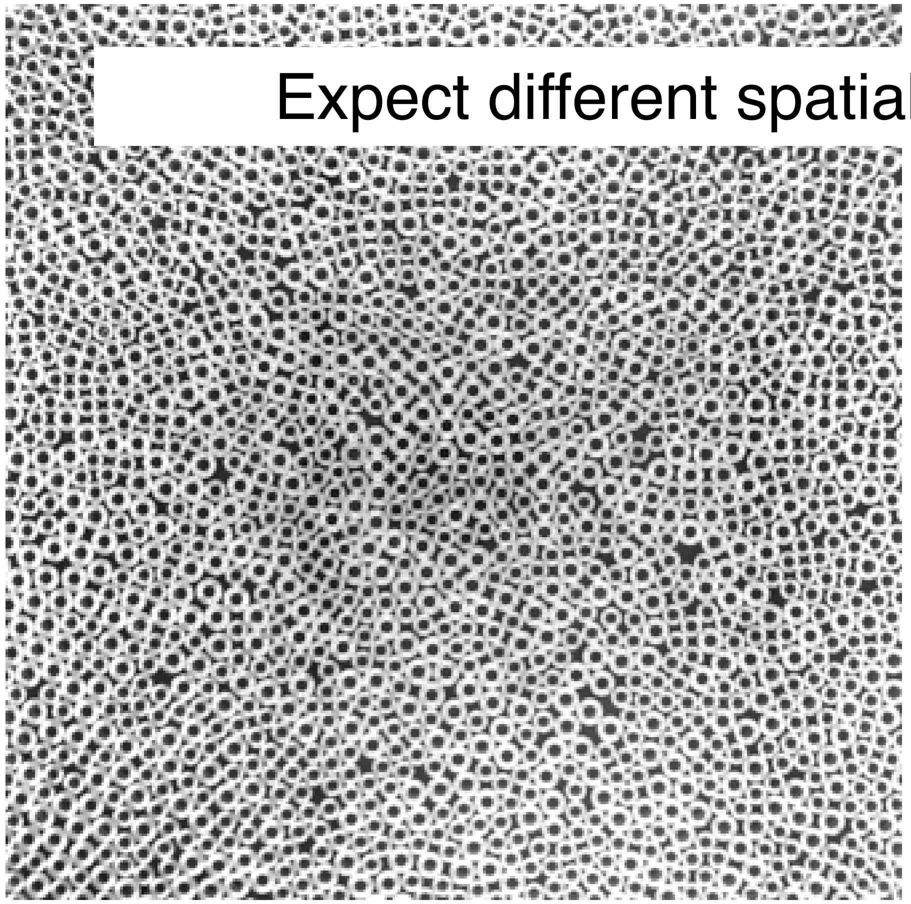
From F. Lechenault



From A. Abate

Jammed systems

Expect different spatial heterogeneity in steadily sheared systems!



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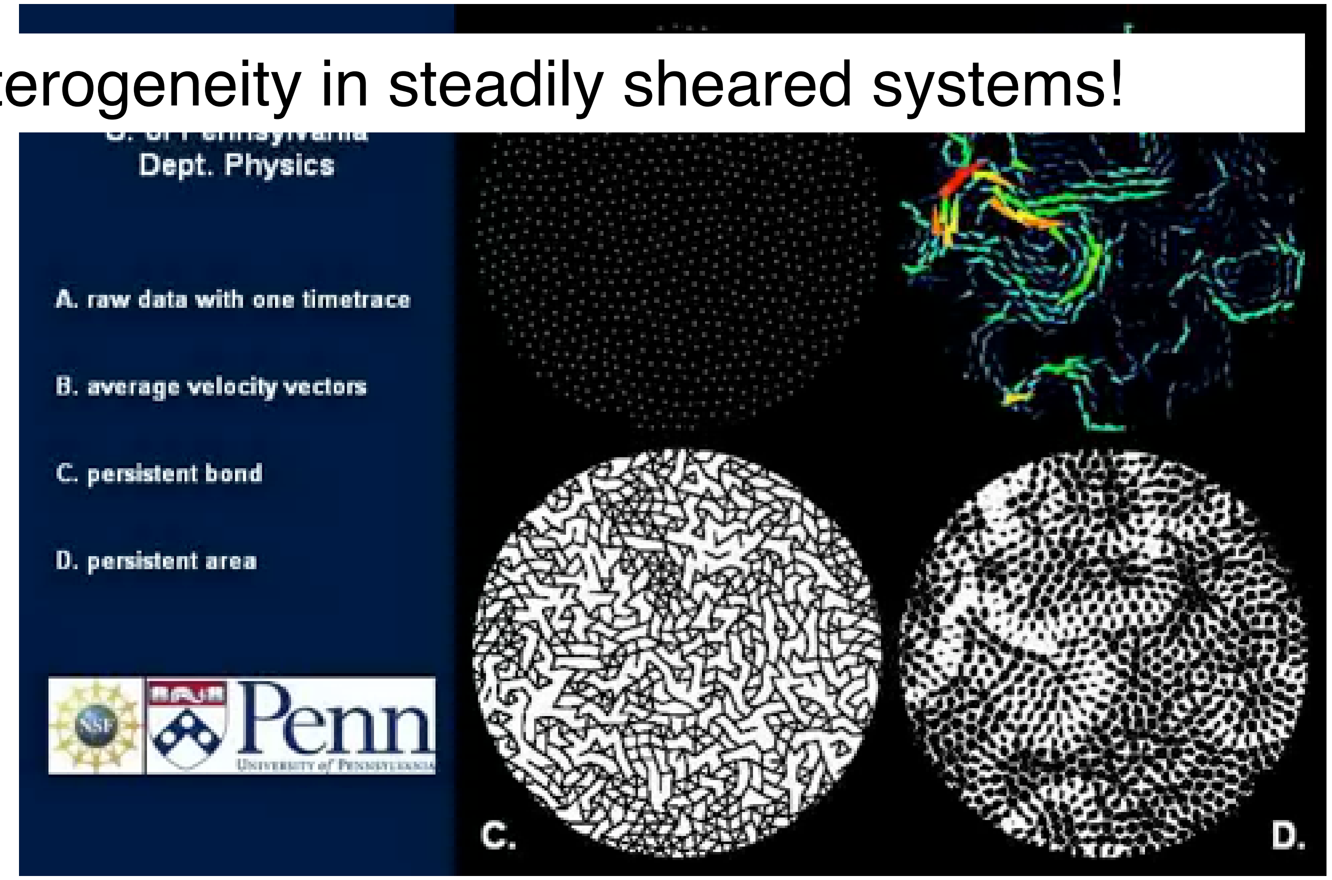

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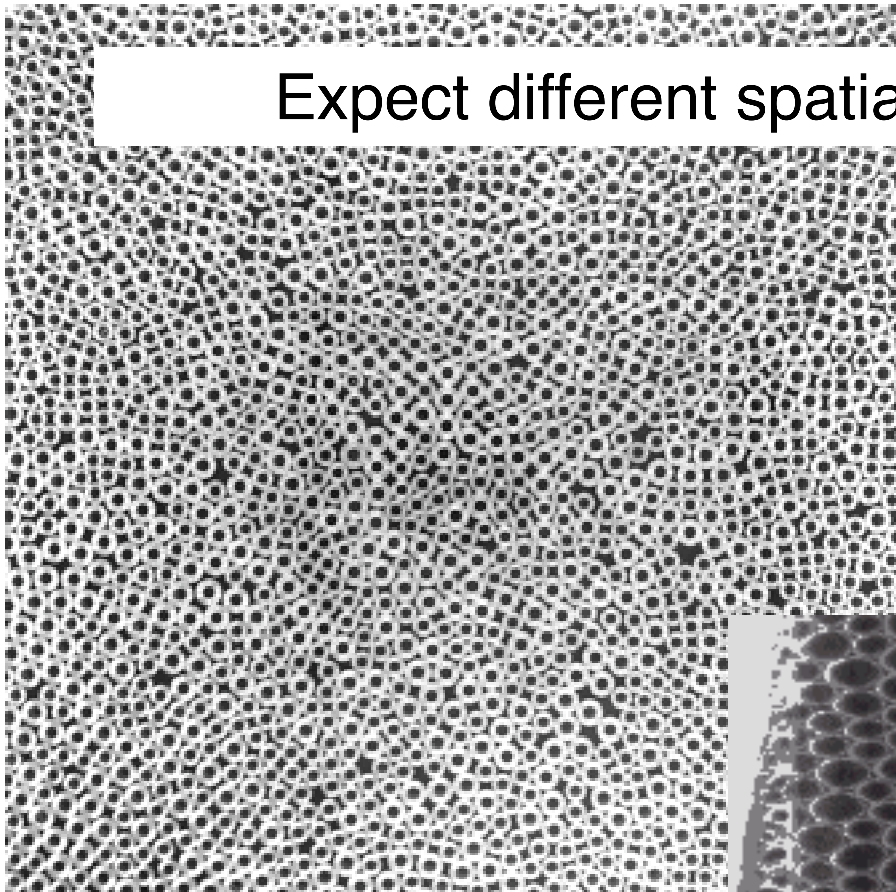

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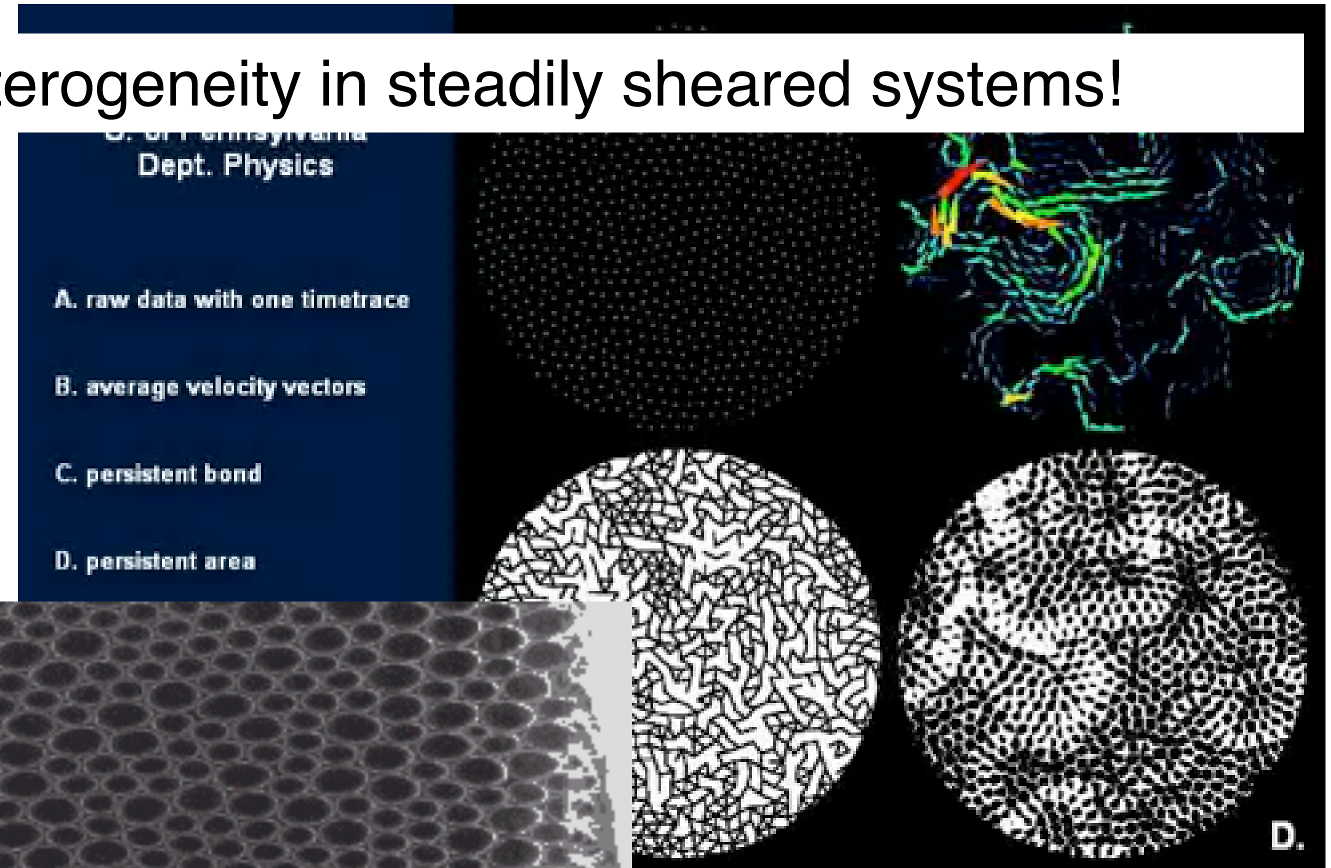

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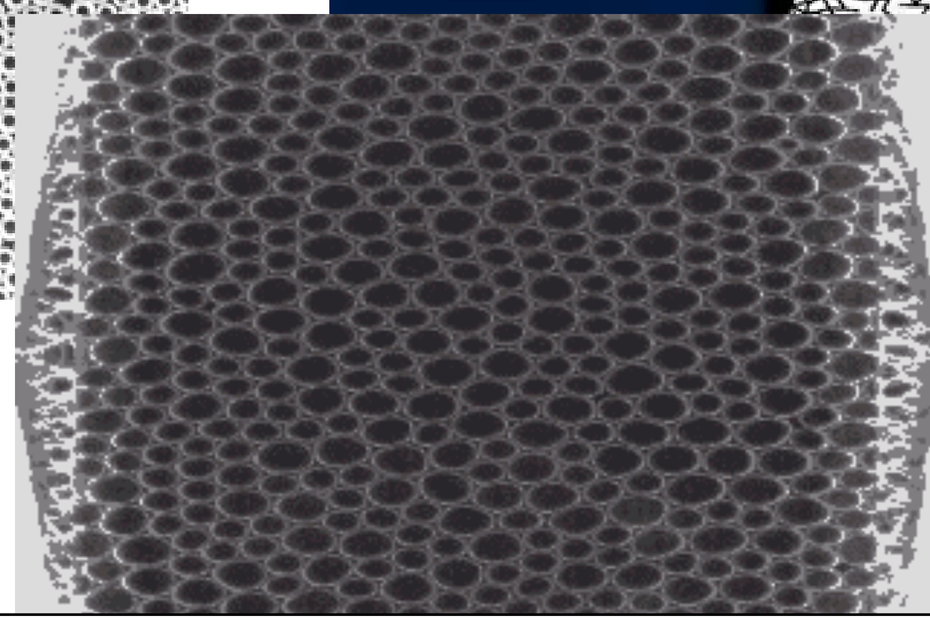
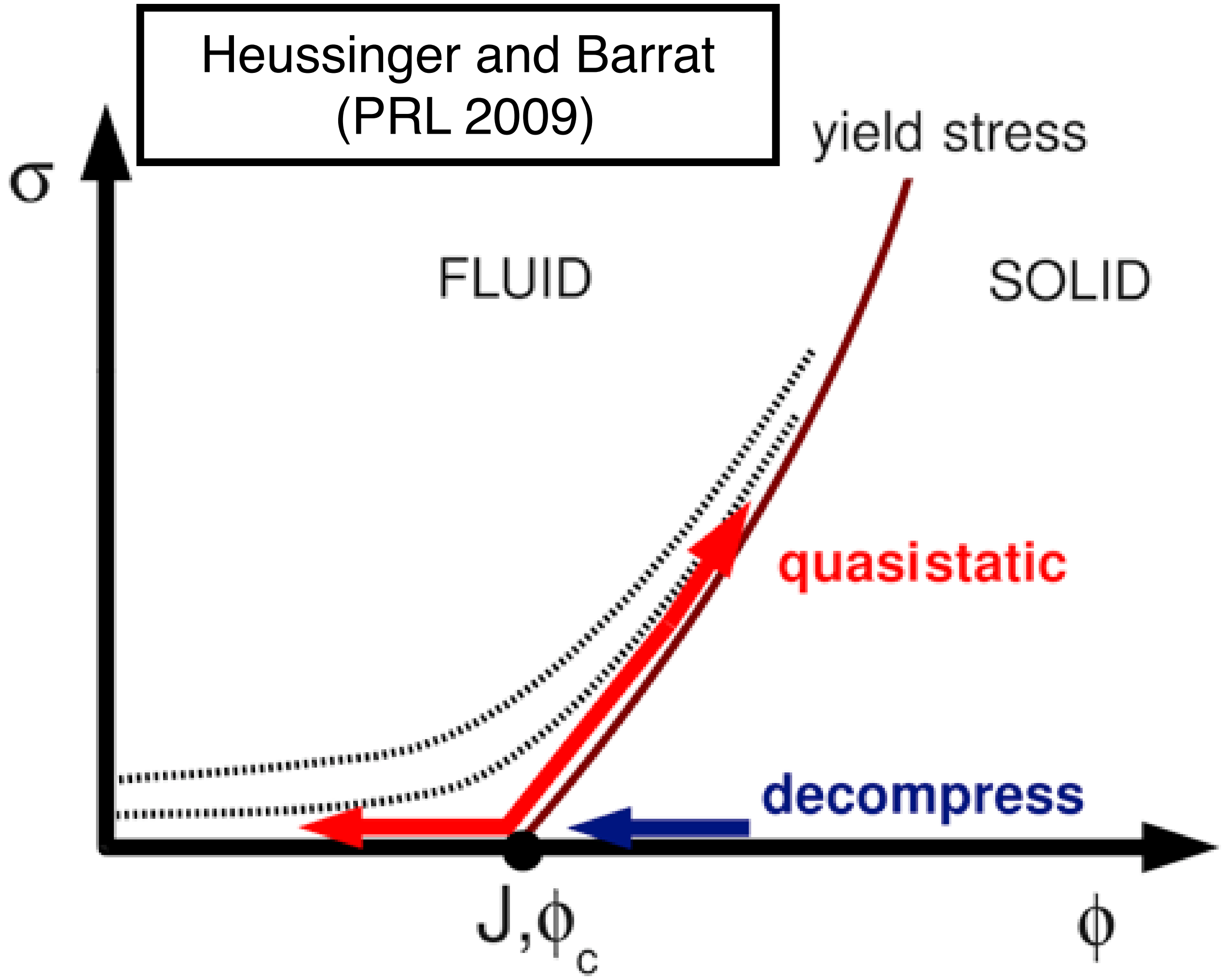


Layer of polydisperse soap bubbles on water

vanHecke group

Jamming and critical scaling at φ_c



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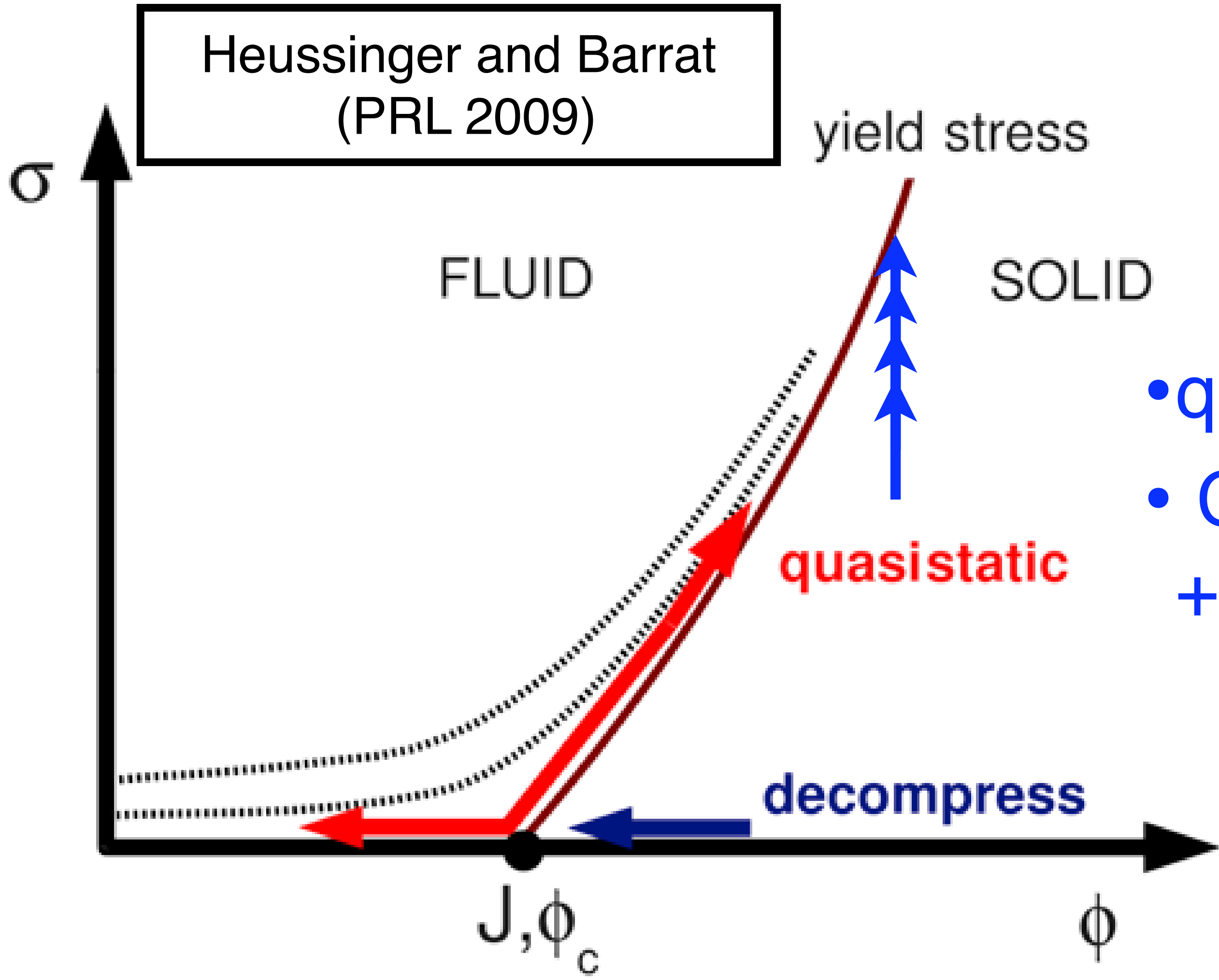
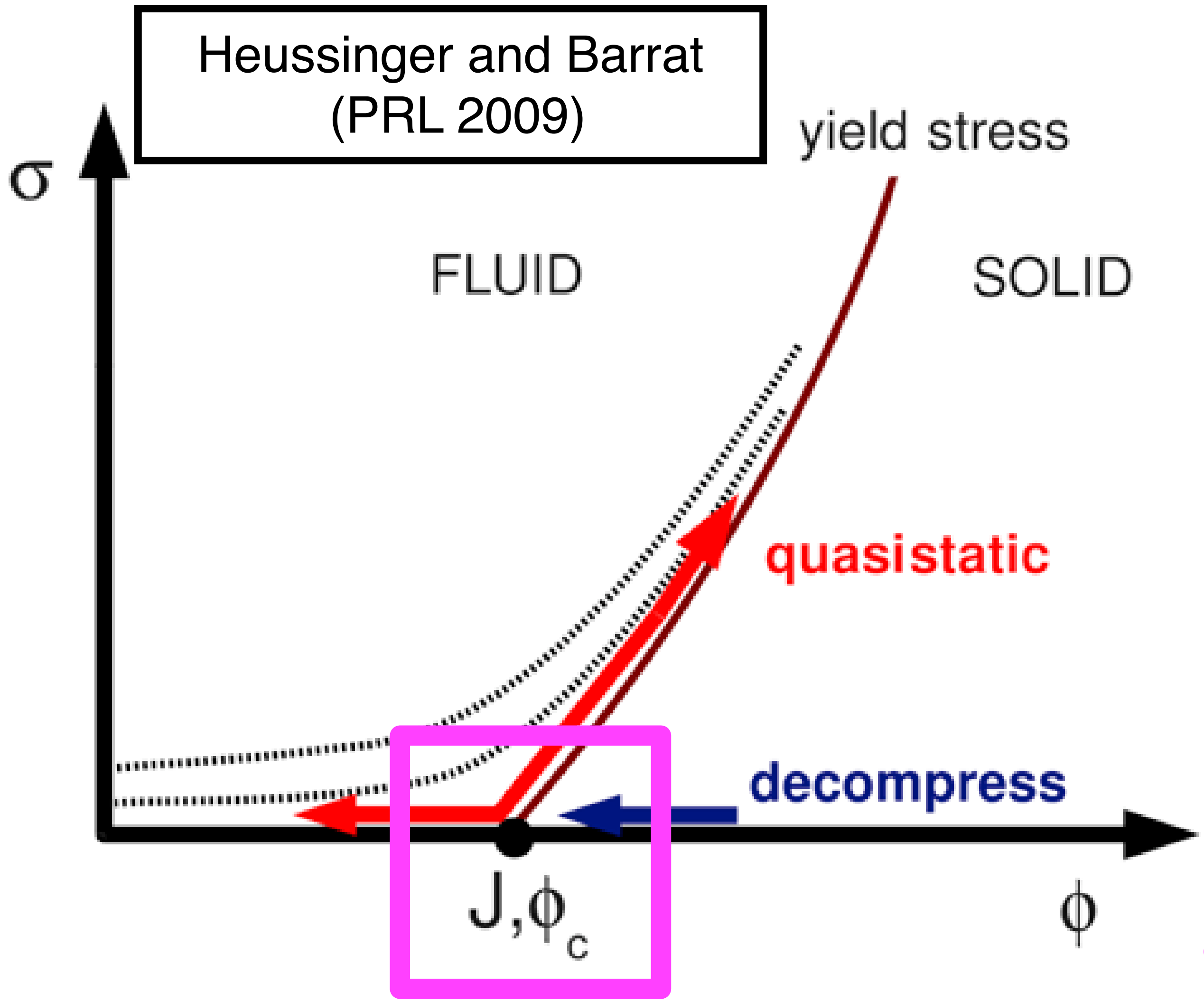


- quasistatic avalanches
- CEM+Robbins / Lemaître +Caroli

Jamming and critical scaling at φ_c



•Olsson and Teitel PRL 2008

- ϕ,σ rheology scaling near “point J”
- Olsson and Teitel (bubbles), Hatano (grains)...

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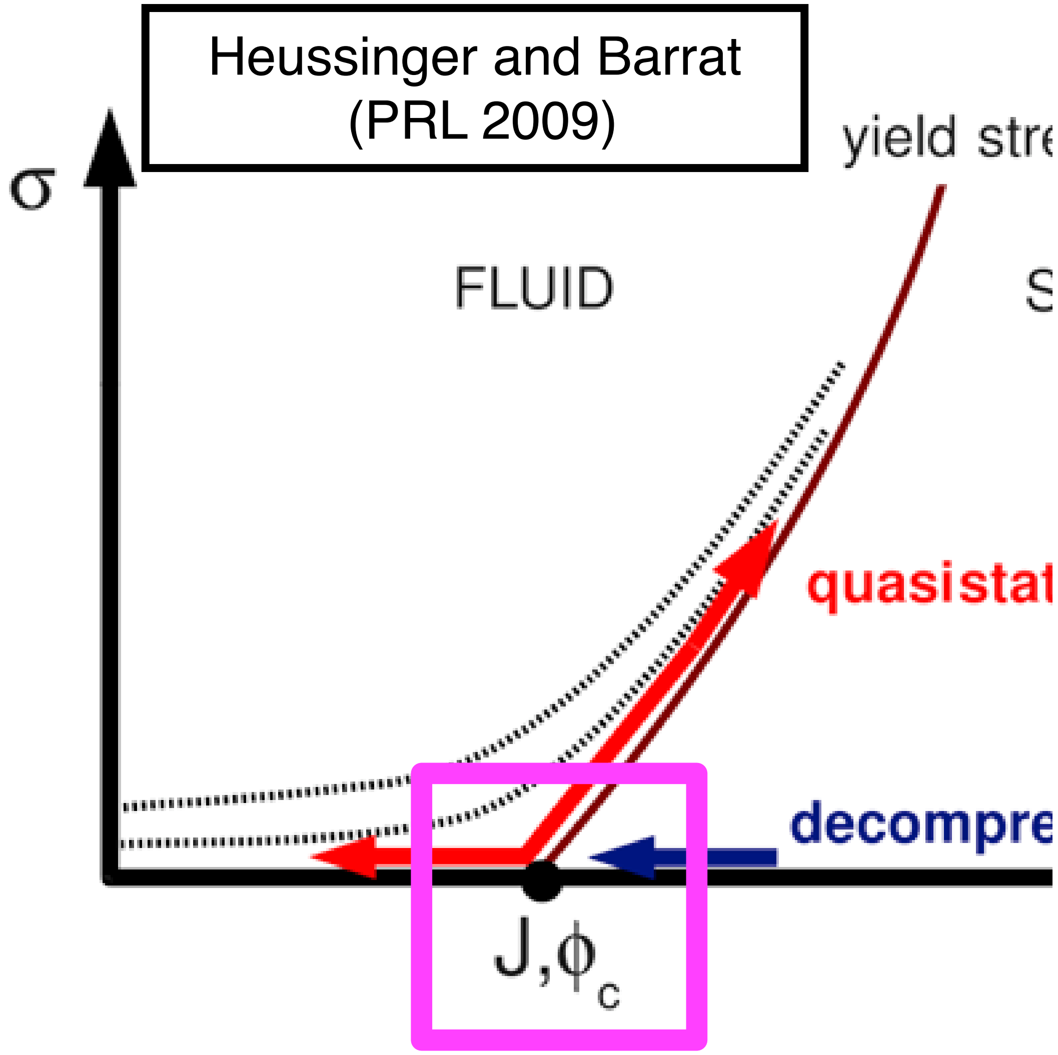
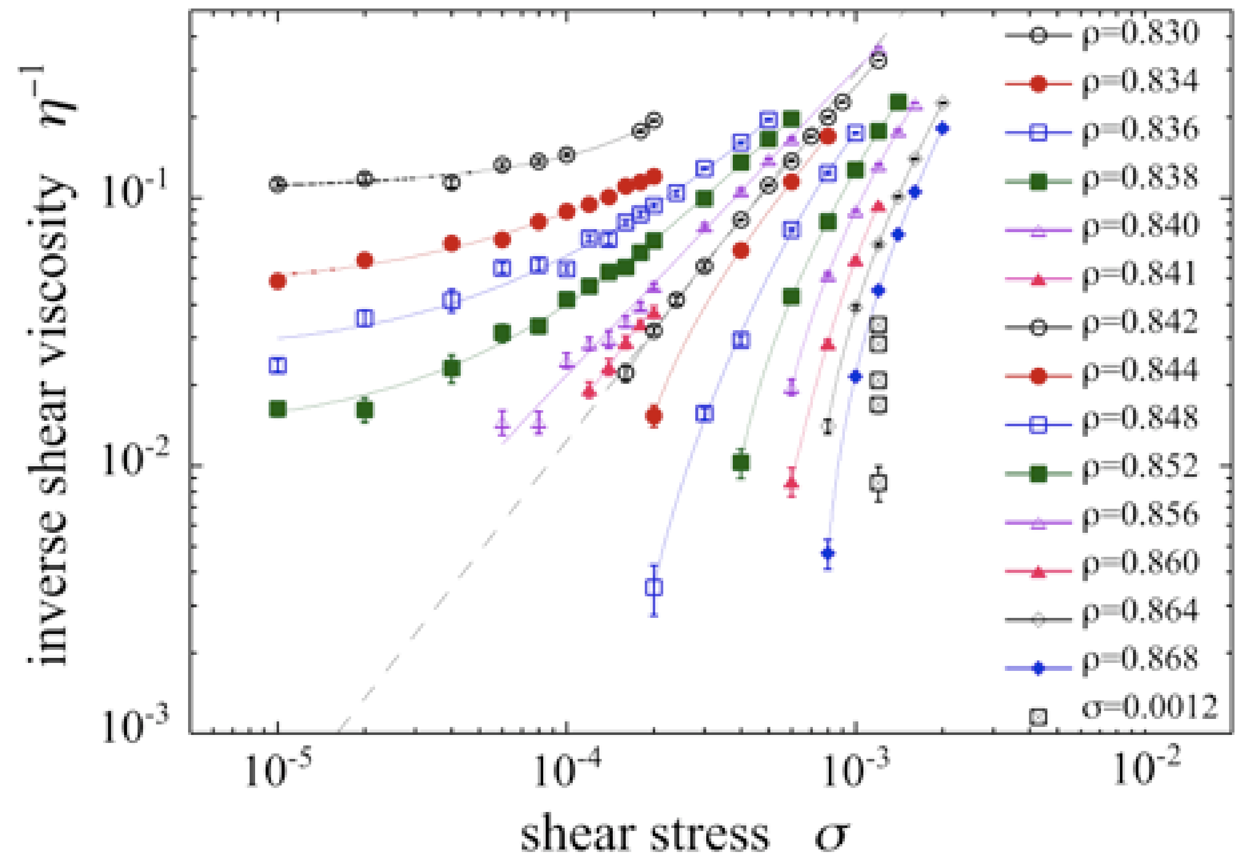
Jamming and critical scaling at φc





ψ

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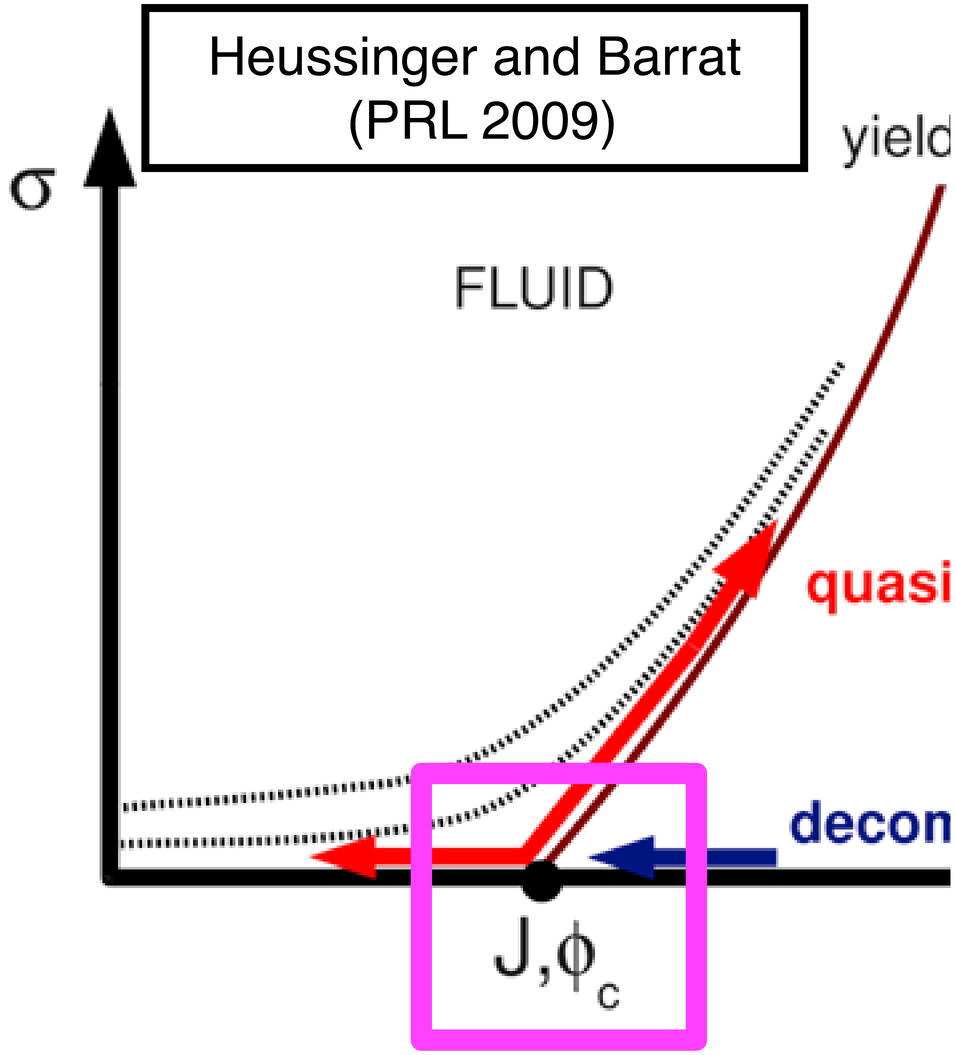

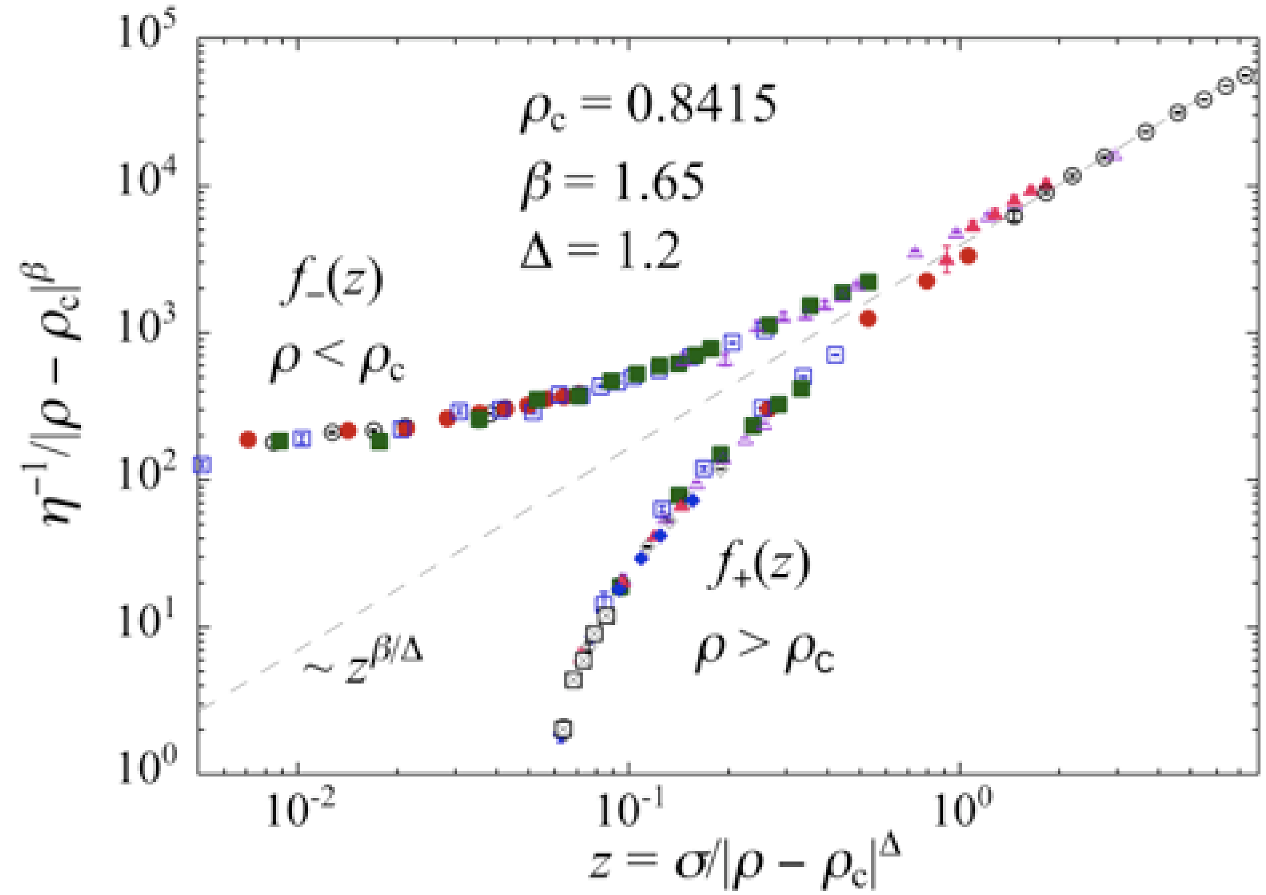

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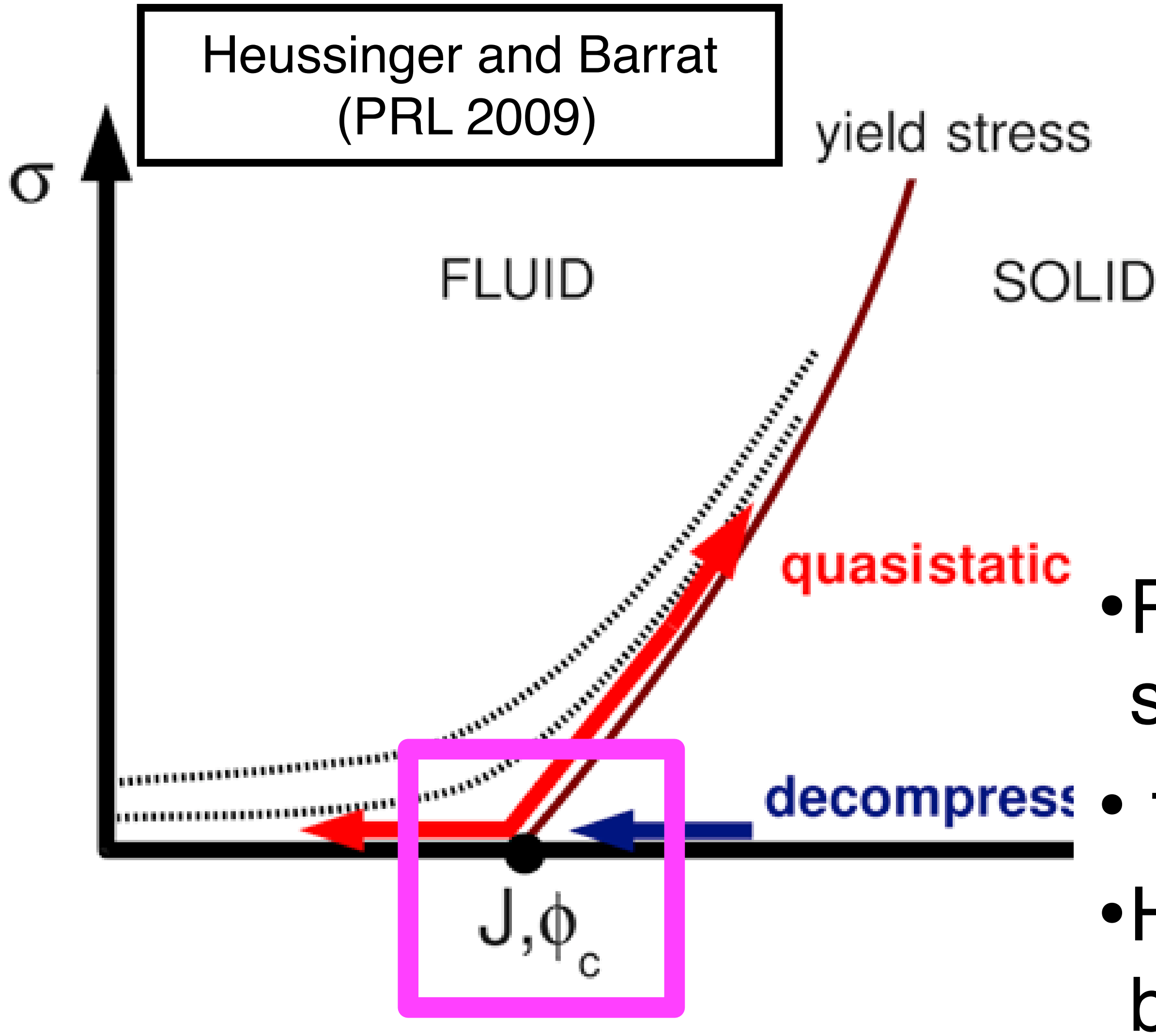
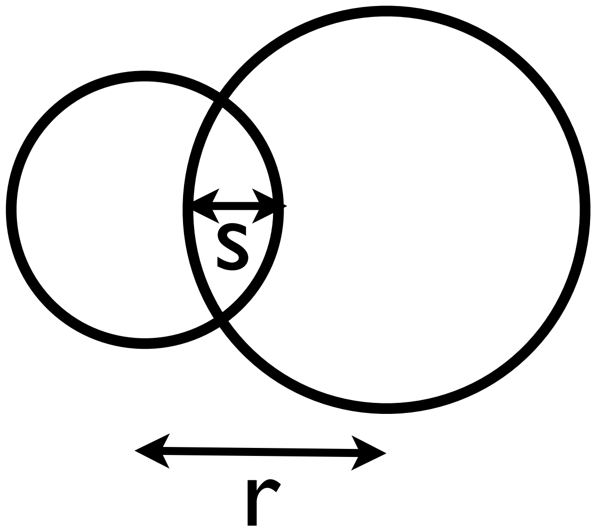


- Point J scaling implies: ϕ-ϕ_c sets stress and time scale.
- $\tau_J=(\phi-\phi_c)^{\Delta-\beta}$
- How does this affect quasistatic behavior?

- ϕ,σ rheology scaling near "point J"
- Olsson and Teitel (bubbles), Hatano (grains)...

Bubble model

$$\delta\vec{v}_i = \vec{F}_i/D; \quad \delta\vec{v}_i = \vec{v}_i - y_i\dot{\gamma}\hat{x}; \quad \dot{\vec{r}}_i = \vec{v}_i$$



$$a = \frac{(R_i + R_j) - r_{ij}}{R_i + R_j}$$

$$U = \frac{\epsilon}{2}a^2$$

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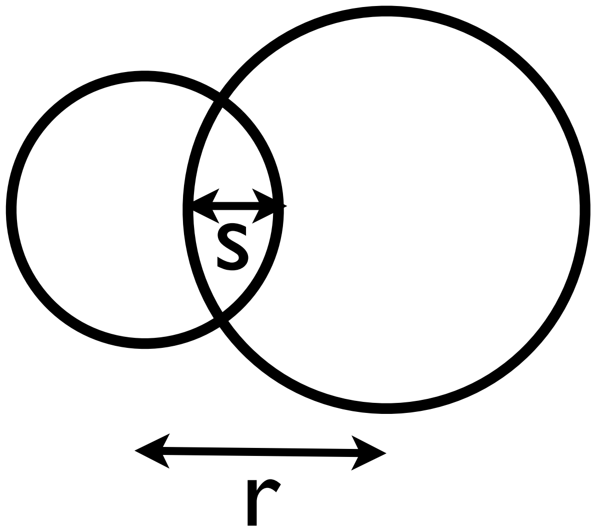

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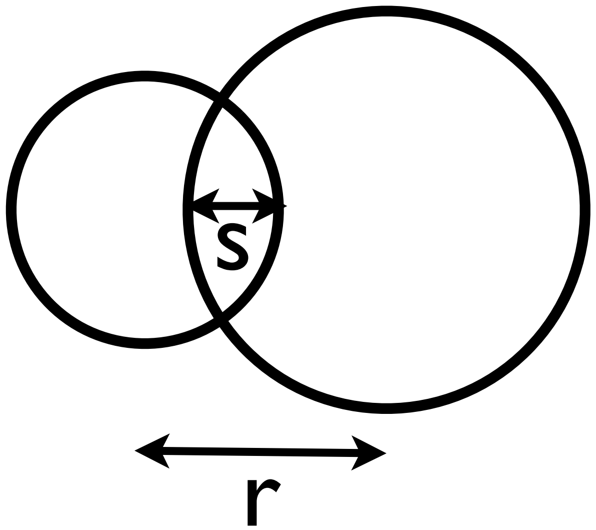

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- R_large = 1.4 R_small = 1.4 σ_0

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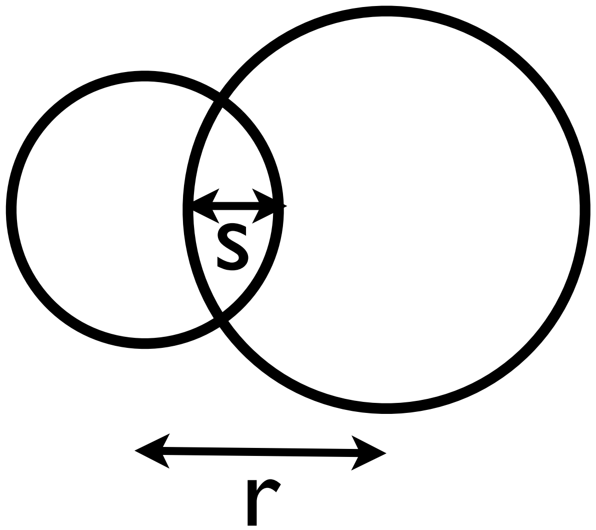

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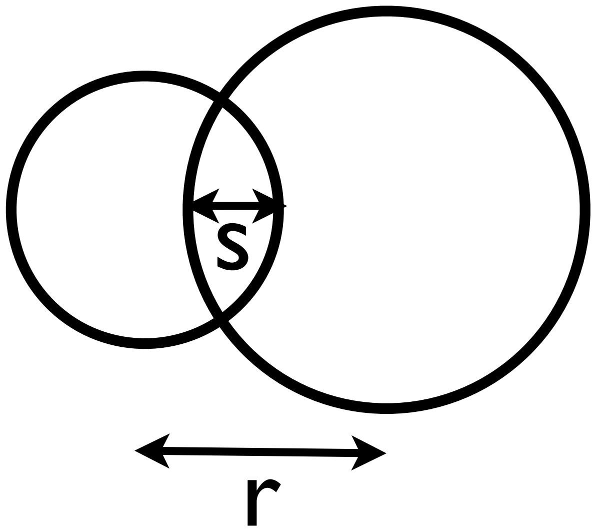

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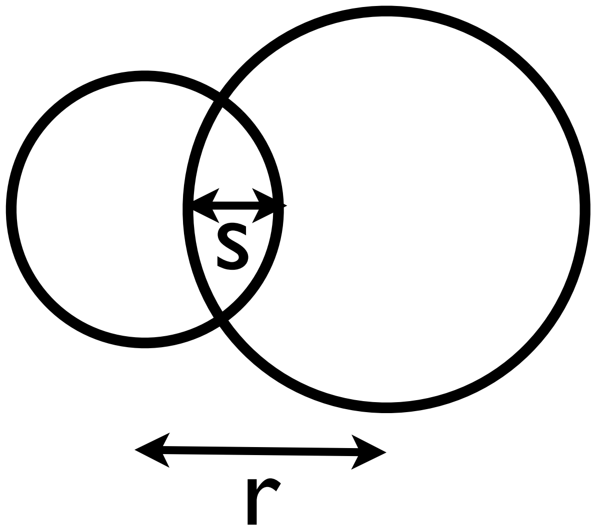

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- Only single timescale in model:

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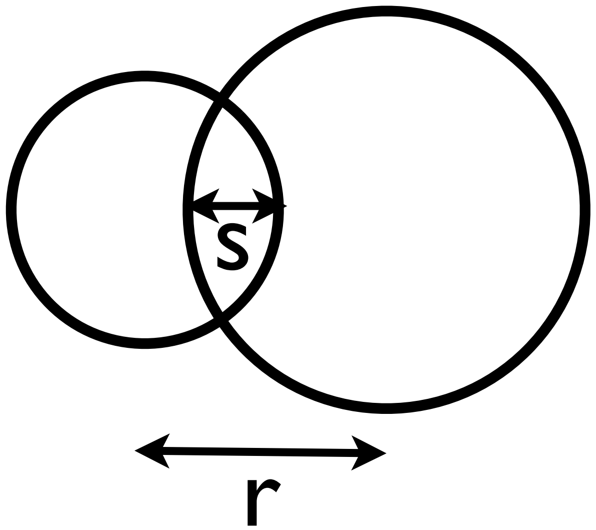

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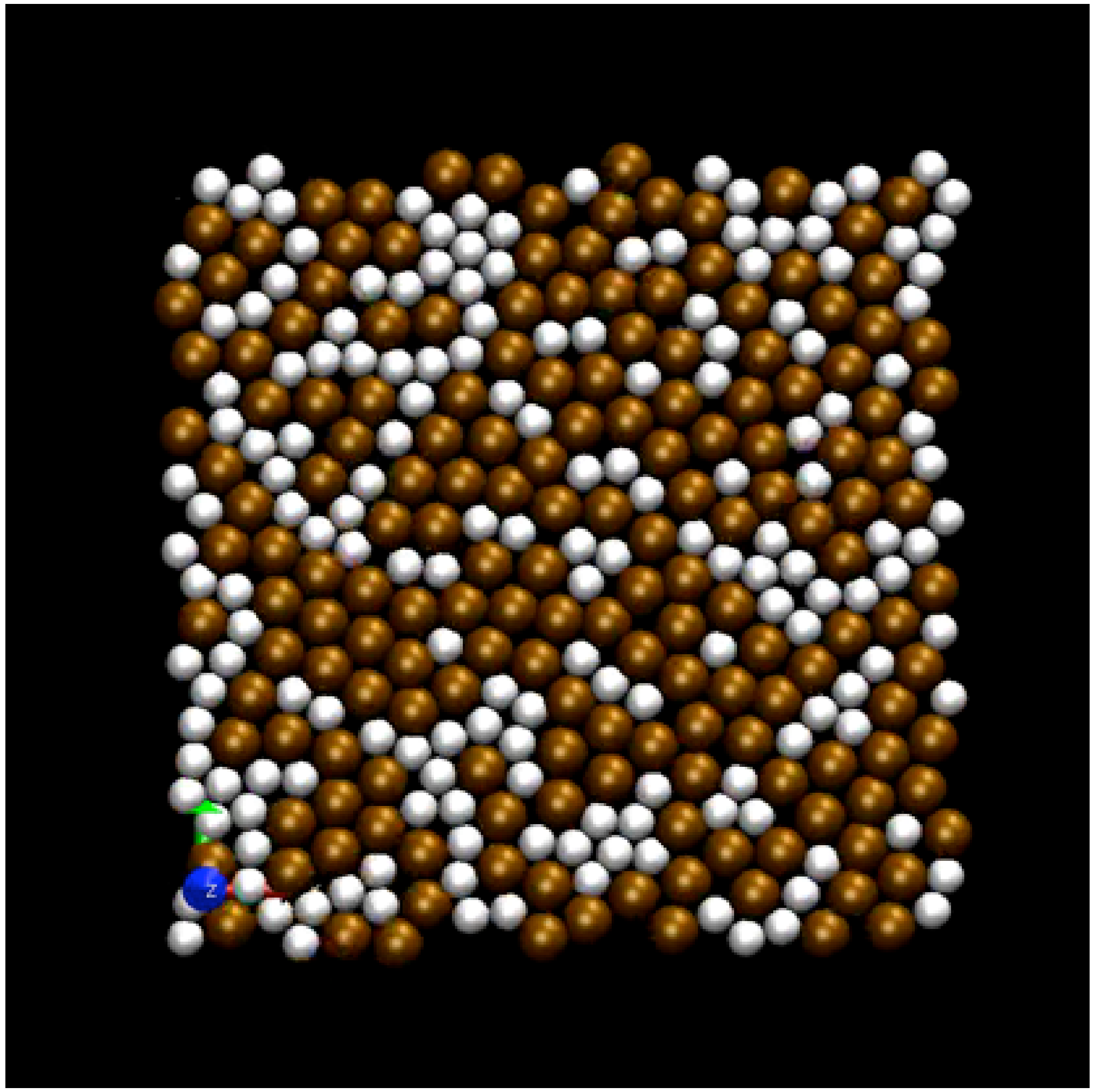
- R_large = 1.4 R_small = 1.4 σ_0

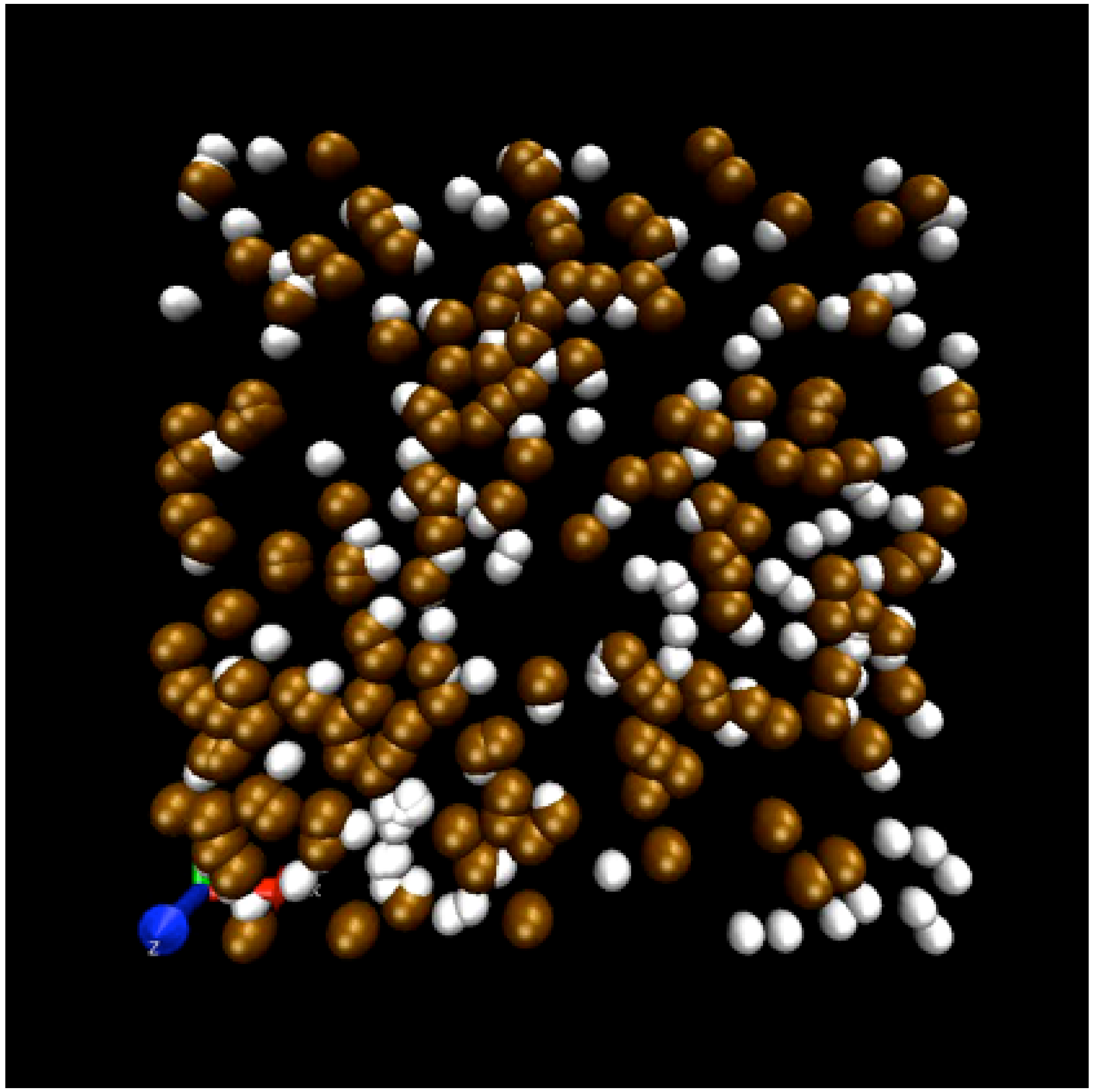
- Only single timescale in model:

$$\tau_D \doteq D\sigma_0^2/\epsilon$$

- Drag force, Dδv, proportional to motion w/r/t homogeneous flow
- Must balance potential force, F

"Slow" shear at various density

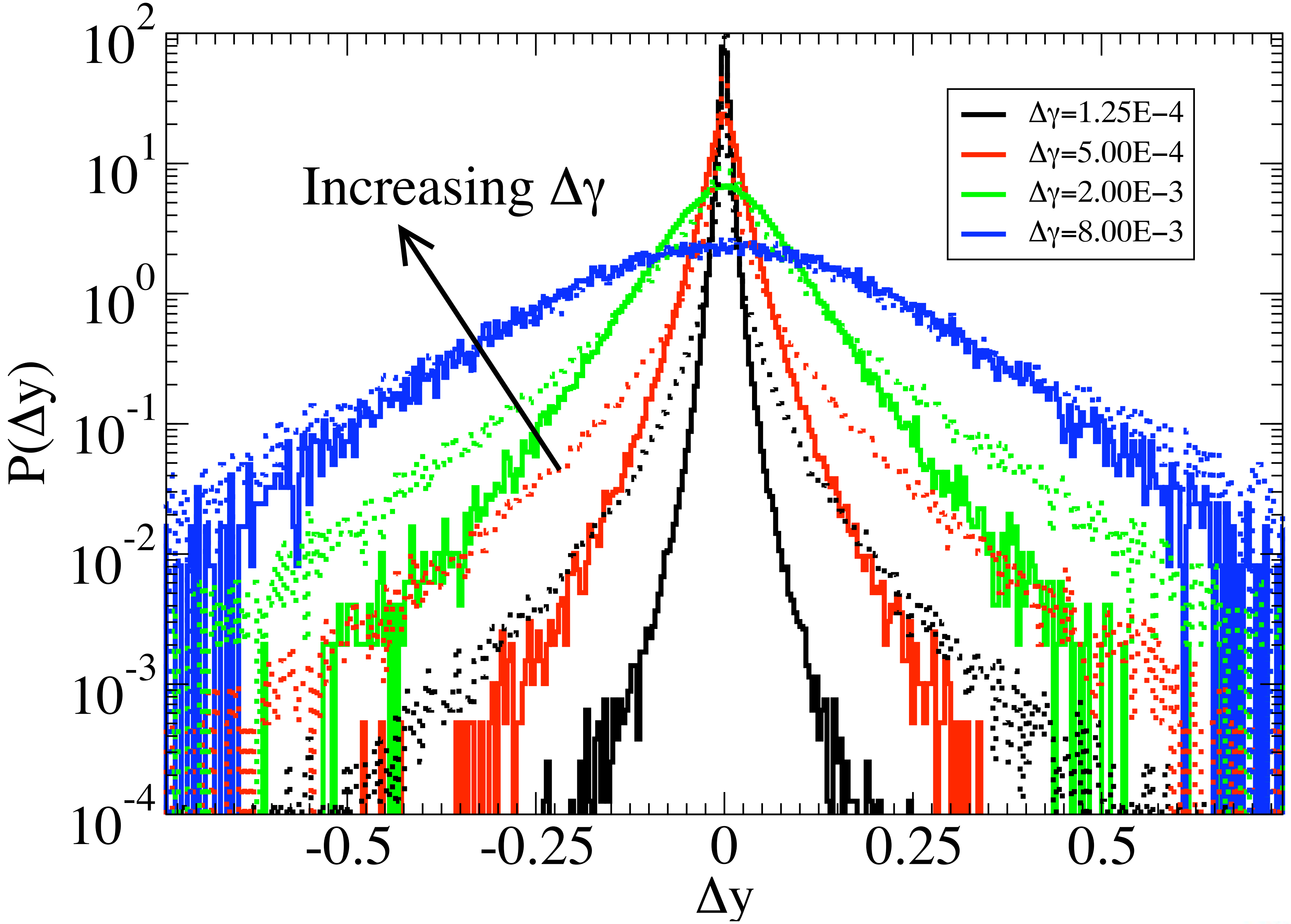




ϕ=1.0 $d\gamma/dt$=1.25x10^{-6} ϕ=0.85

How are they different?

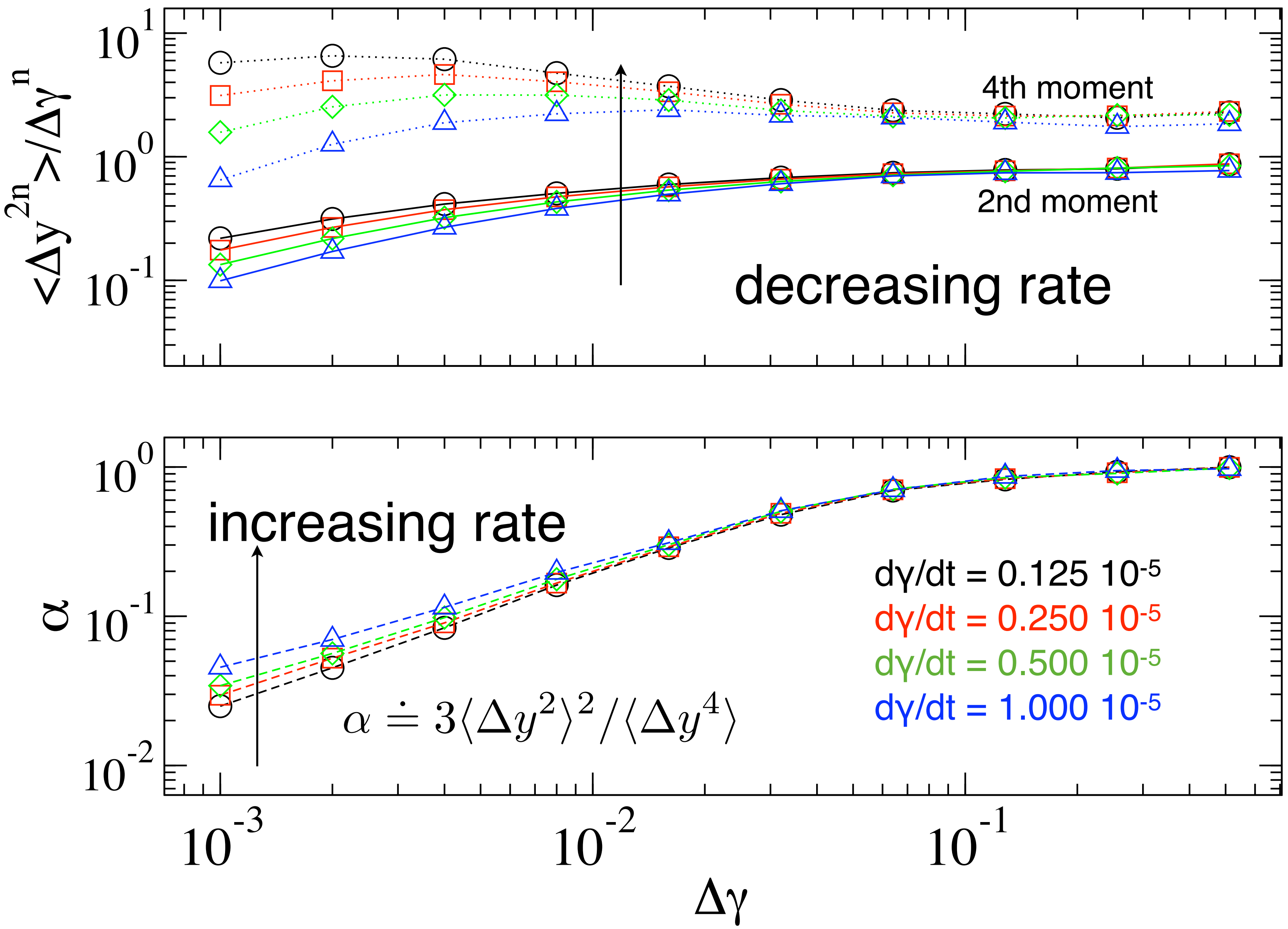
Transverse displacement distribution



P(Δy) much broader for ϕ=1.0 than ϕ=0.85 at early Δγ

P(Δy) similar for ϕ=1.0 and ϕ=0.85 at late Δγ

2nd and 4th moments (φ=1.0)



no rate dependence at plateau, we're quasistatic!

From LJ slip-line arguments: $D_{eff} \sim La/12$

a~0.8σ

Δγ*~a/L~.05

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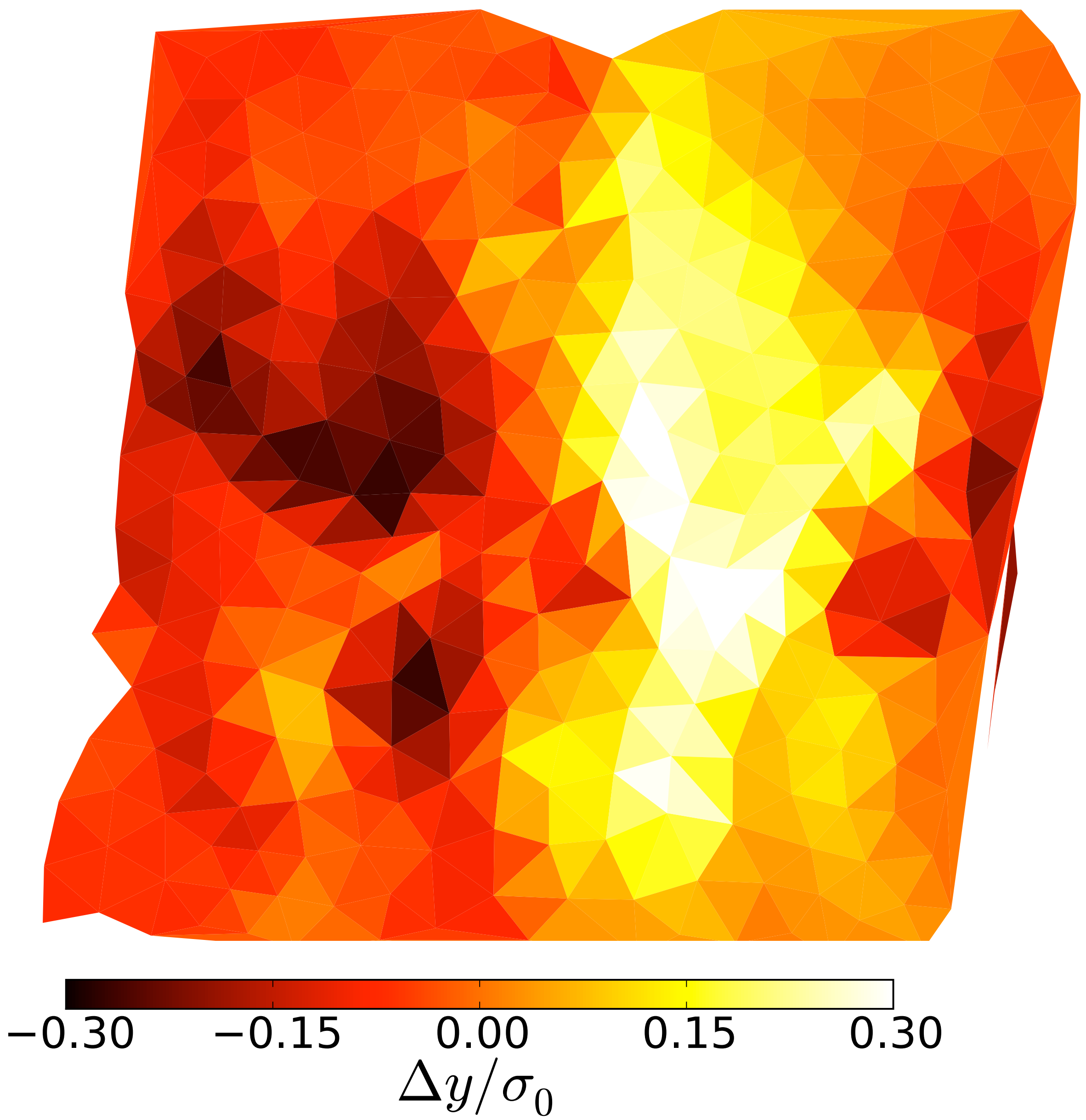
Typical displacement over Δγ~0.05

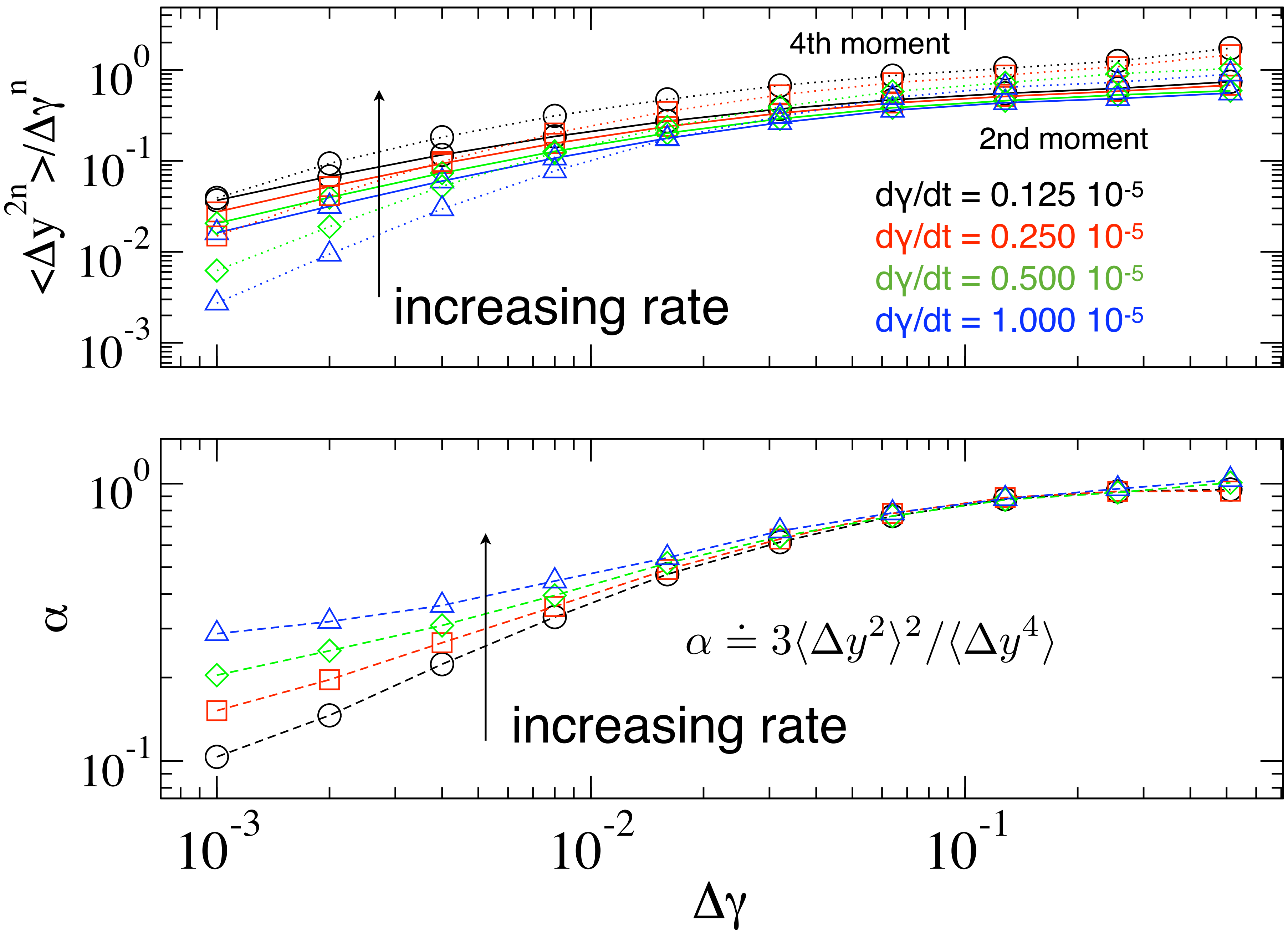


From LJ slip-line arguments:
D_{eff}~ La/12

a~0.8σ

Δγ*~a/L~.05

2nd and 4th moments (φ=0.85)



slight rate dependence at plateau

at slowest rate, D_{eff} within 10% of D_{eff} for ϕ=1.0

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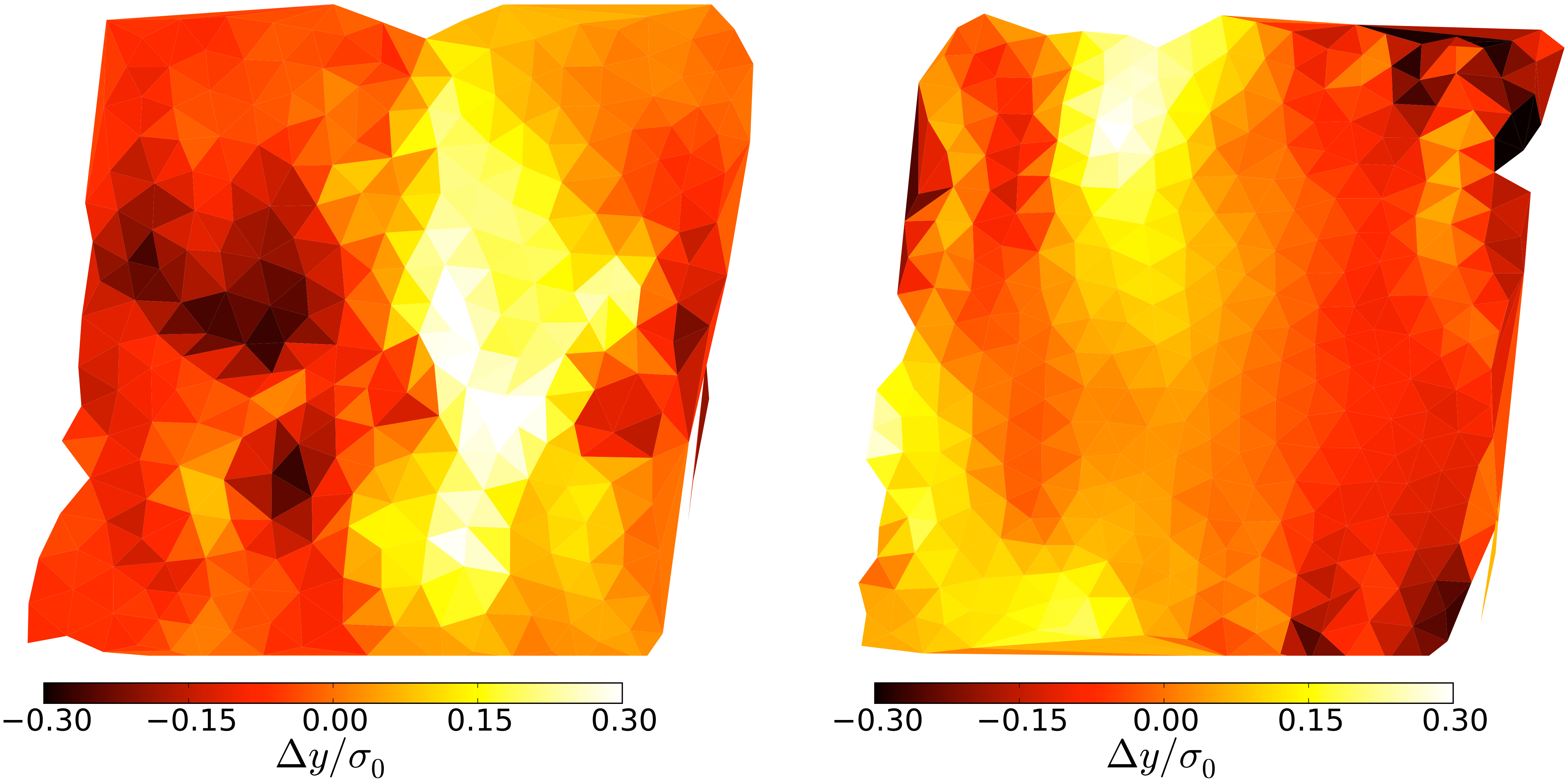
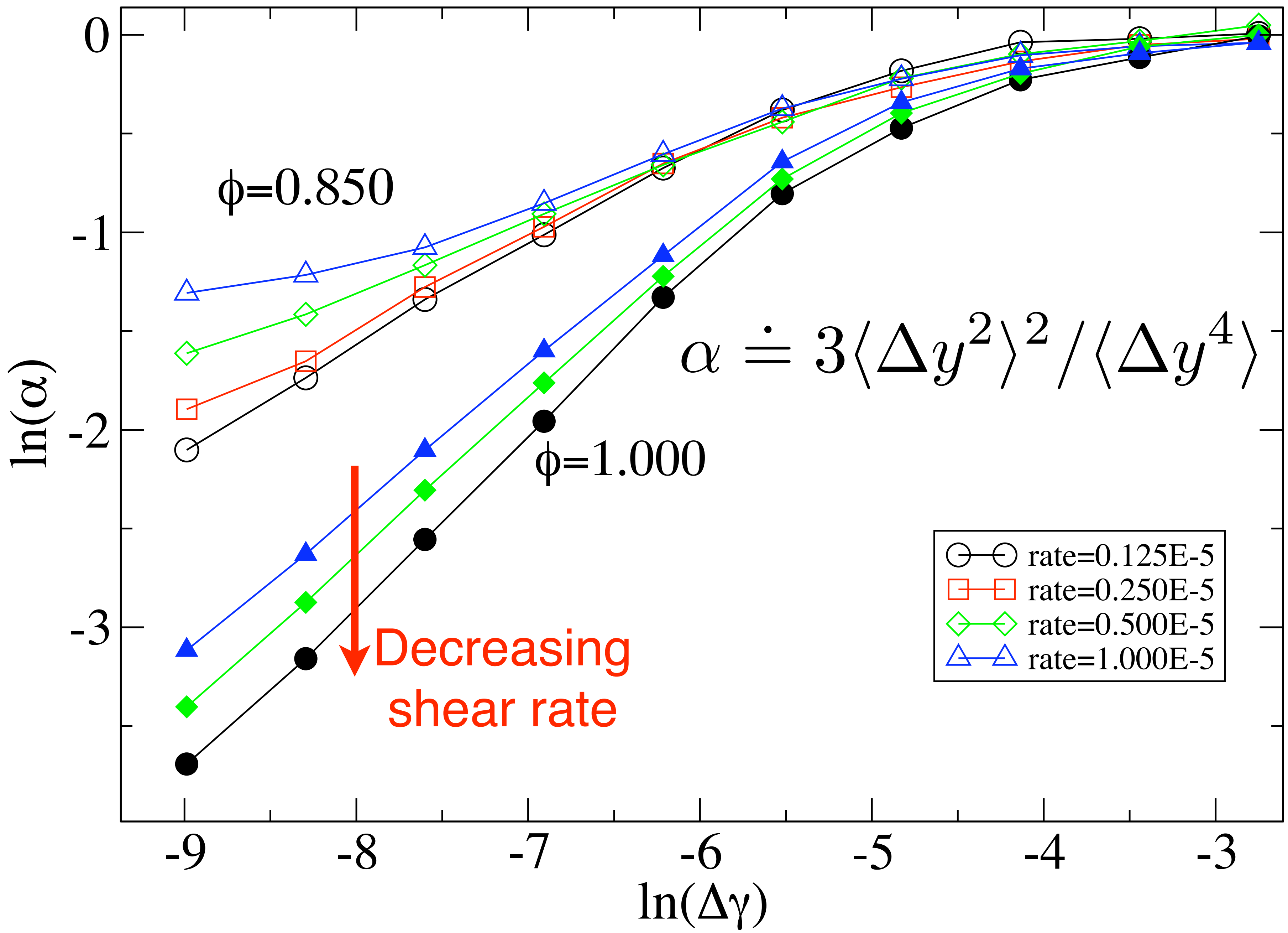
Typical displacement over Δγ~0.05



ϕ=1.0

ϕ=0.85

Non-gaussian parameter, α



cross-over to Gaussian is roughly independent of ϕ and dγ/dt.

Conclusion (Diffusion)

- Slip lines argument gives:
  slip amplitude = a ~ 0.8σ
  strain quantum = $\Delta\gamma_*$ ~ a/L ~ 0.05
- Displacement fields at $\Delta\gamma$~0.05 look like slip lines with consistent slip amplitude
- Seems surprisingly robust with respect to ϕ!
  - systems near ϕ_c much less intermittent at small $\Delta\gamma$
  - but surprisingly similar in Fickian regime!

Dissipation

$$\frac{dU}{dt} = \left.\frac{\partial U}{\partial \gamma}\right|_s \dot{\gamma} + \sum_i \frac{\partial U}{\partial \vec{s}_i}\dot{\vec{s}}_i = \sigma\dot{\gamma} - \sum_i \vec{F}_i \cdot \delta\vec{v}_i$$

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- Energy change under affine deformation = σ

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- Identify as input power

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- Energy change under affine deformation = σ

- Identify as input power
- Identify as dissipation rate

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$$\Gamma\dot{\gamma} = \sigma\dot{\gamma} - \frac{dU}{dt} = \sum_i \vec{F}_i \cdot \delta\vec{v}_i = D\sum_i \delta v_i^2$$

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Dissipation

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- Γ is energy dissipated per unit strain

Dissipation

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- Ono *et. al. PRE 2003*

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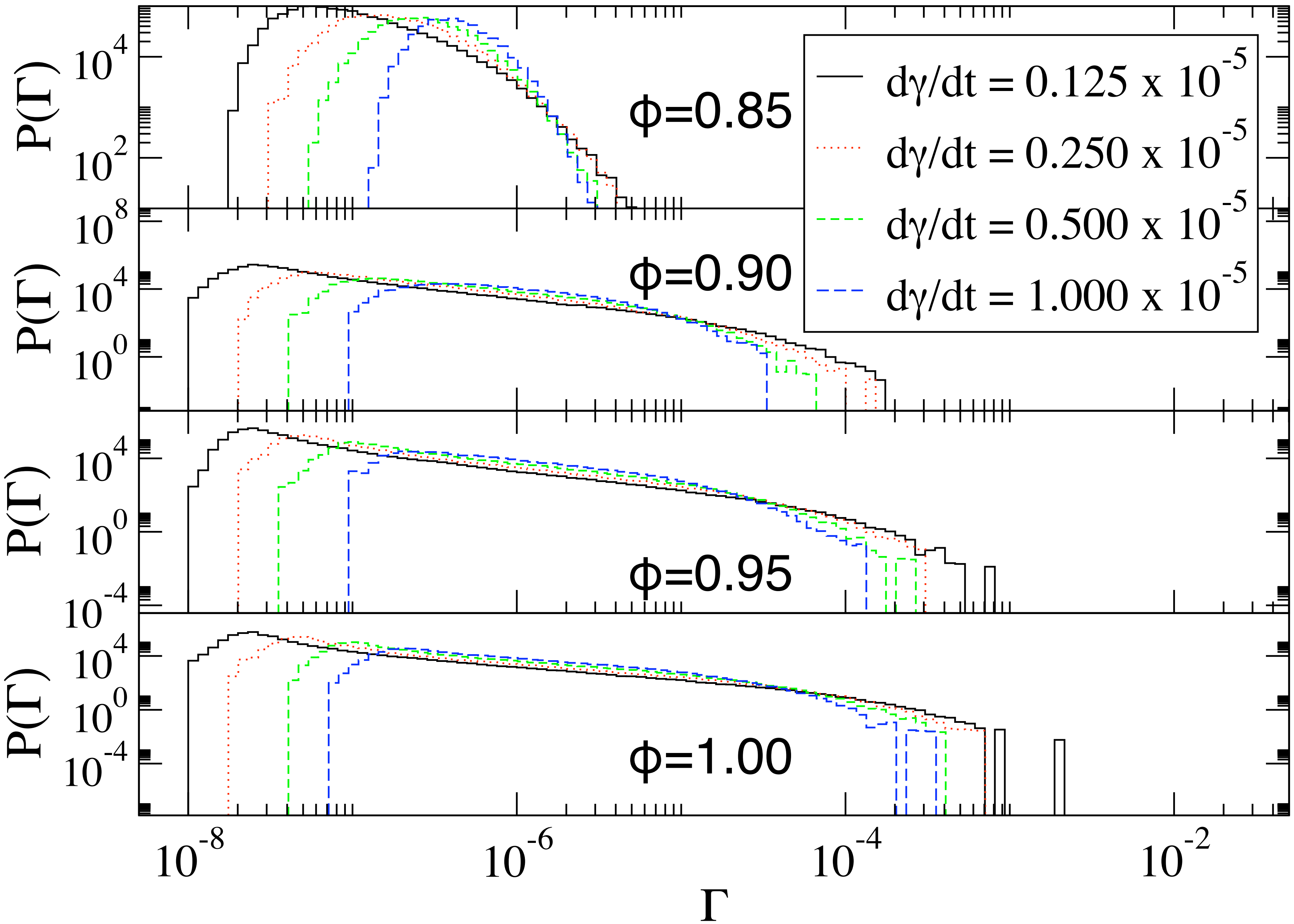
- Ono *et. al. PRE 2003*
- Rheology = fluctuations

- Γ is energy dissipated per unit strain

Γ distribution (like acoustic emission spectrum)



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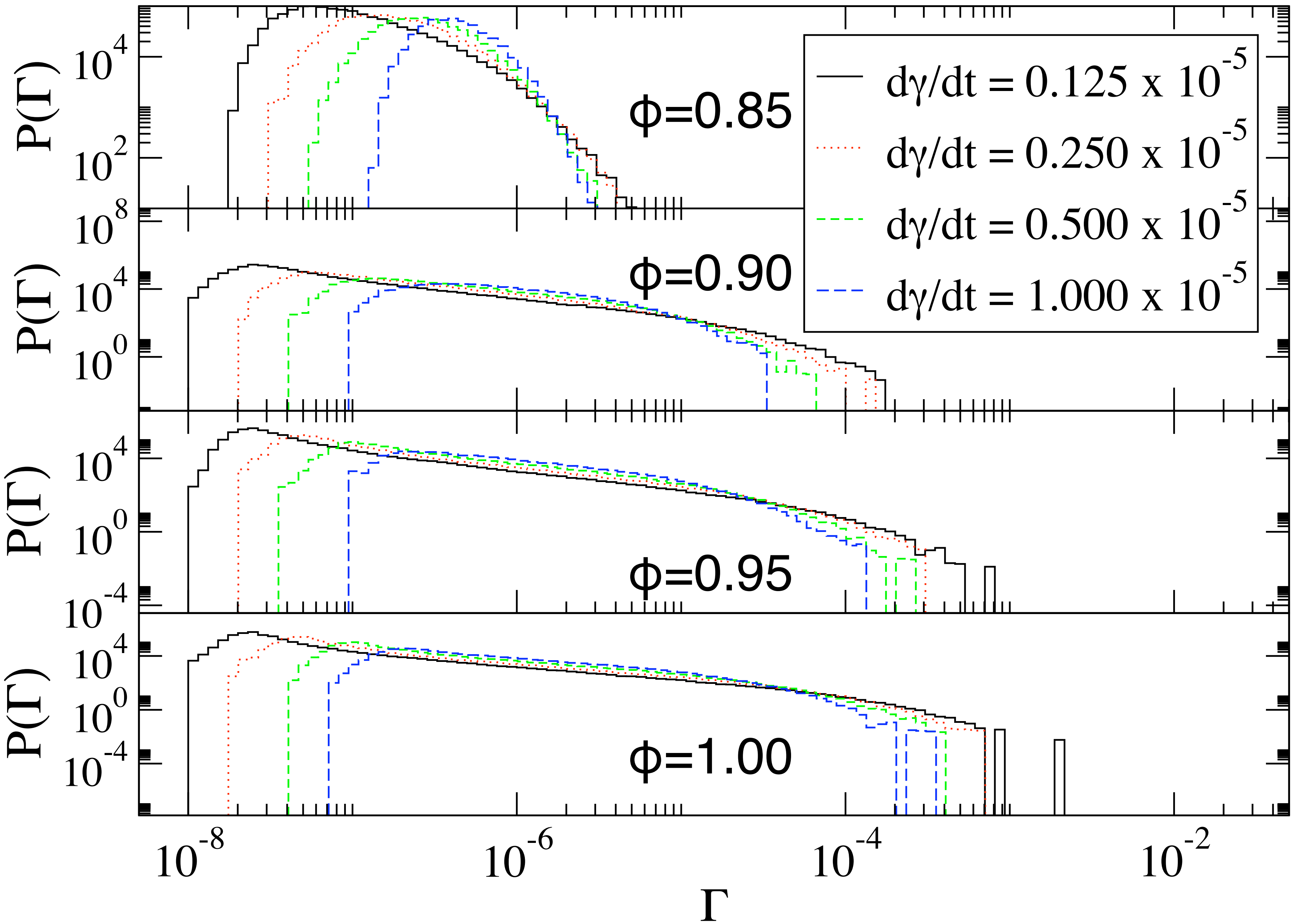

- power-law regime

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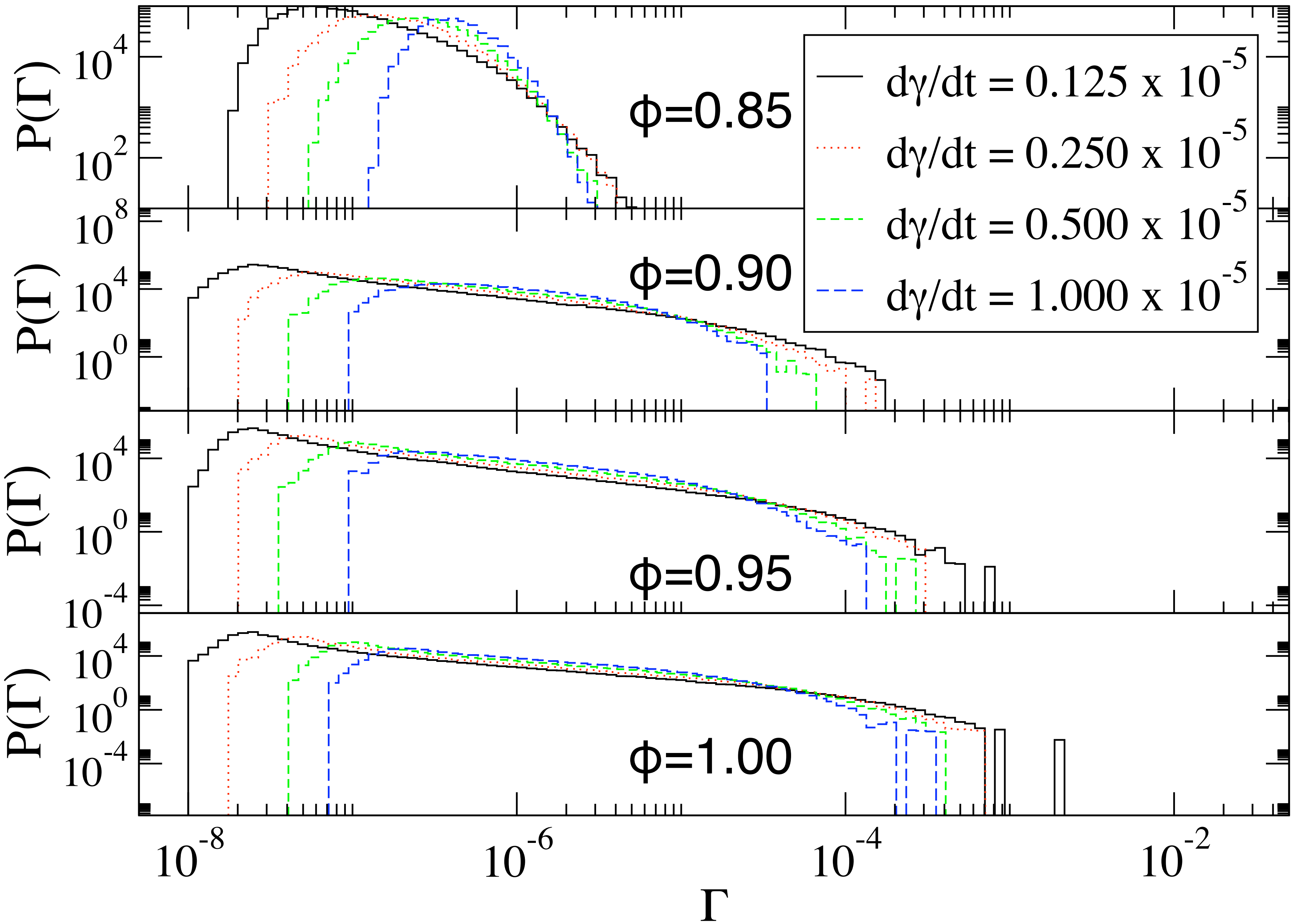

- power-law regime
- exponent~-1.2

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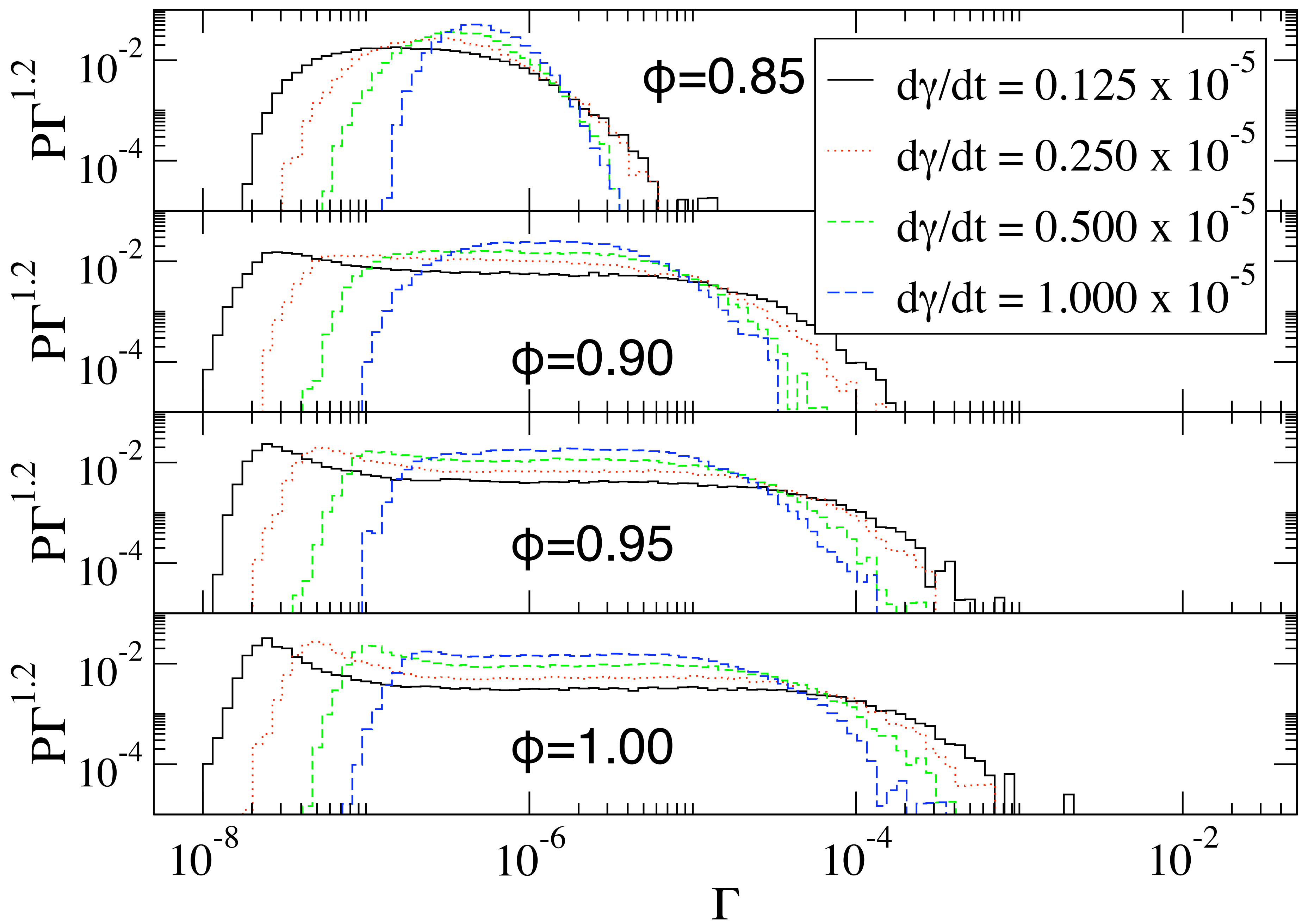
Γ distribution power-law rescaling



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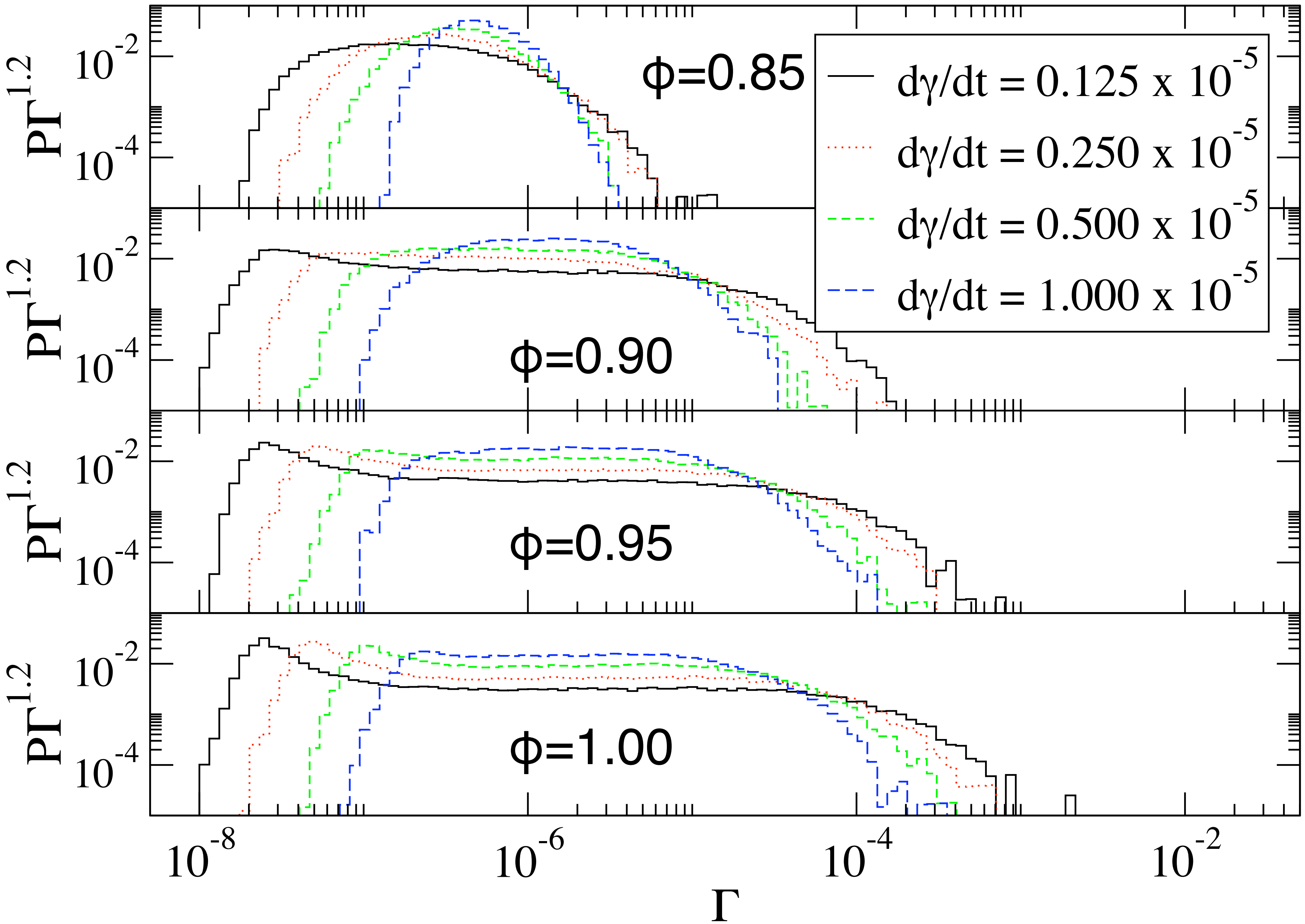

- Scale P by $\Gamma^{-1.2}$

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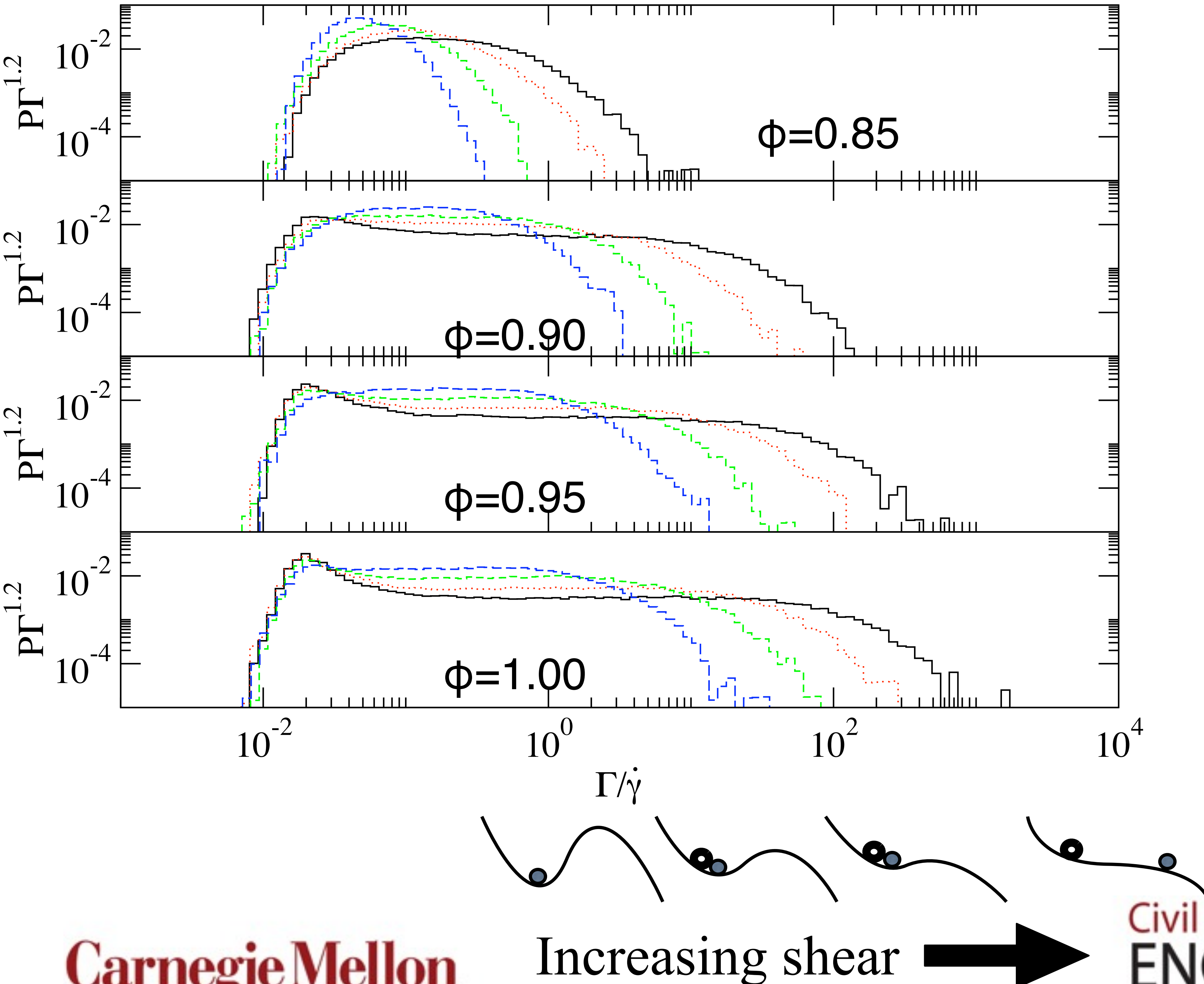
Γ distribution kinematic QS rescaling



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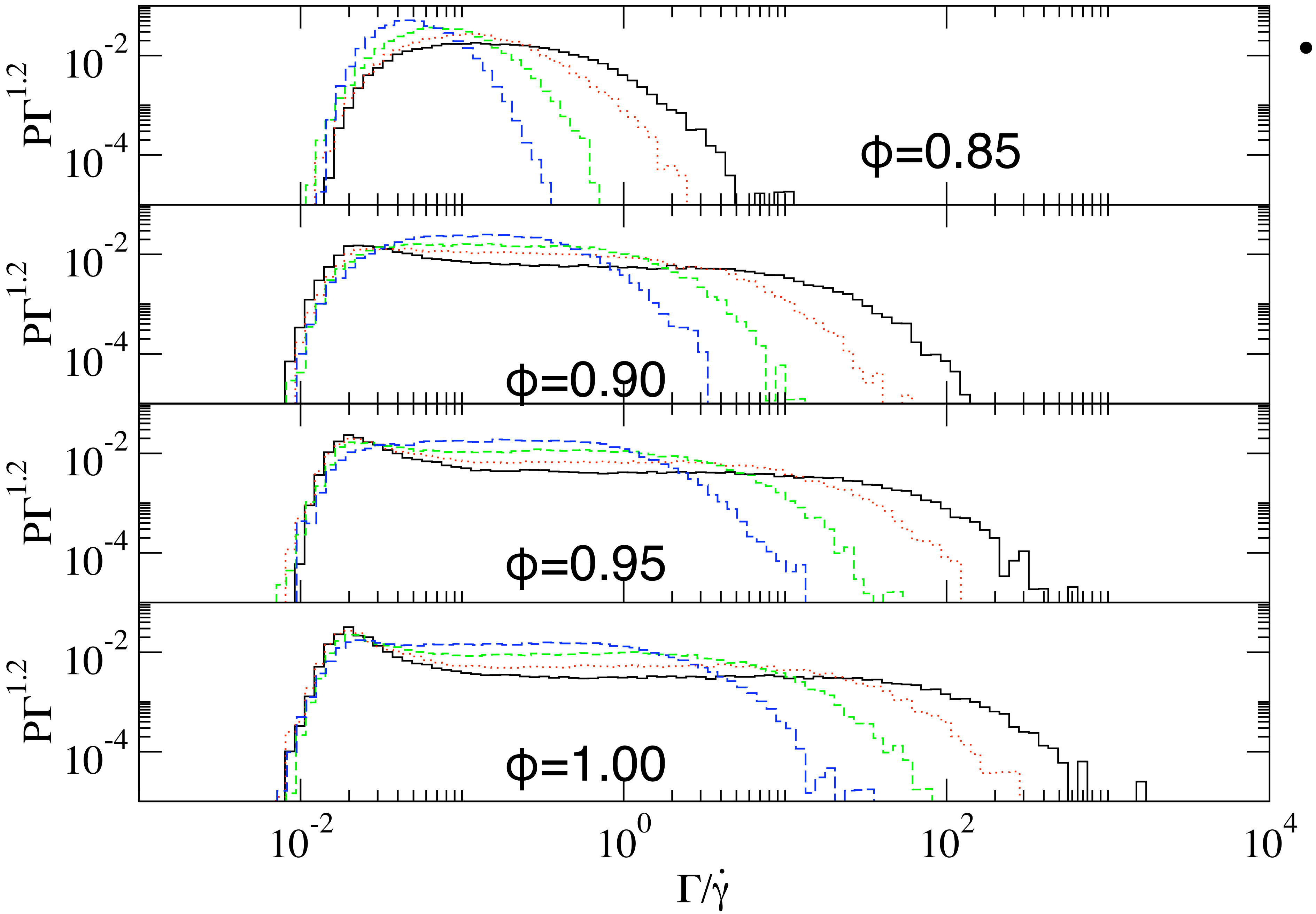
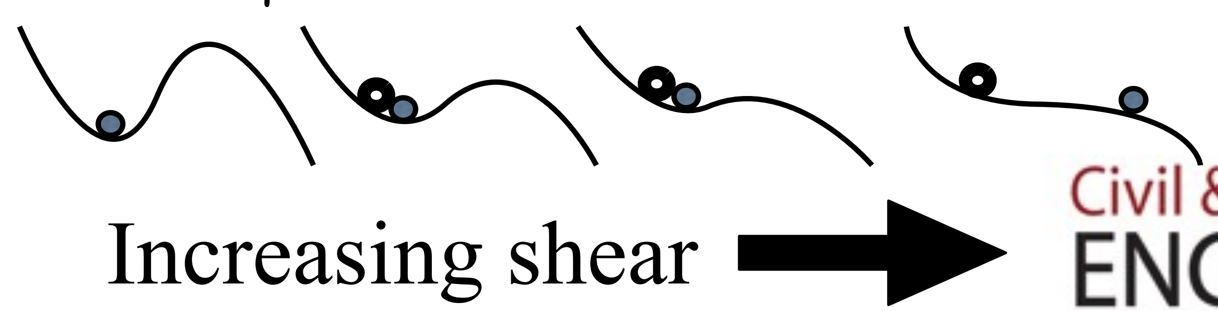


- QS scaling:



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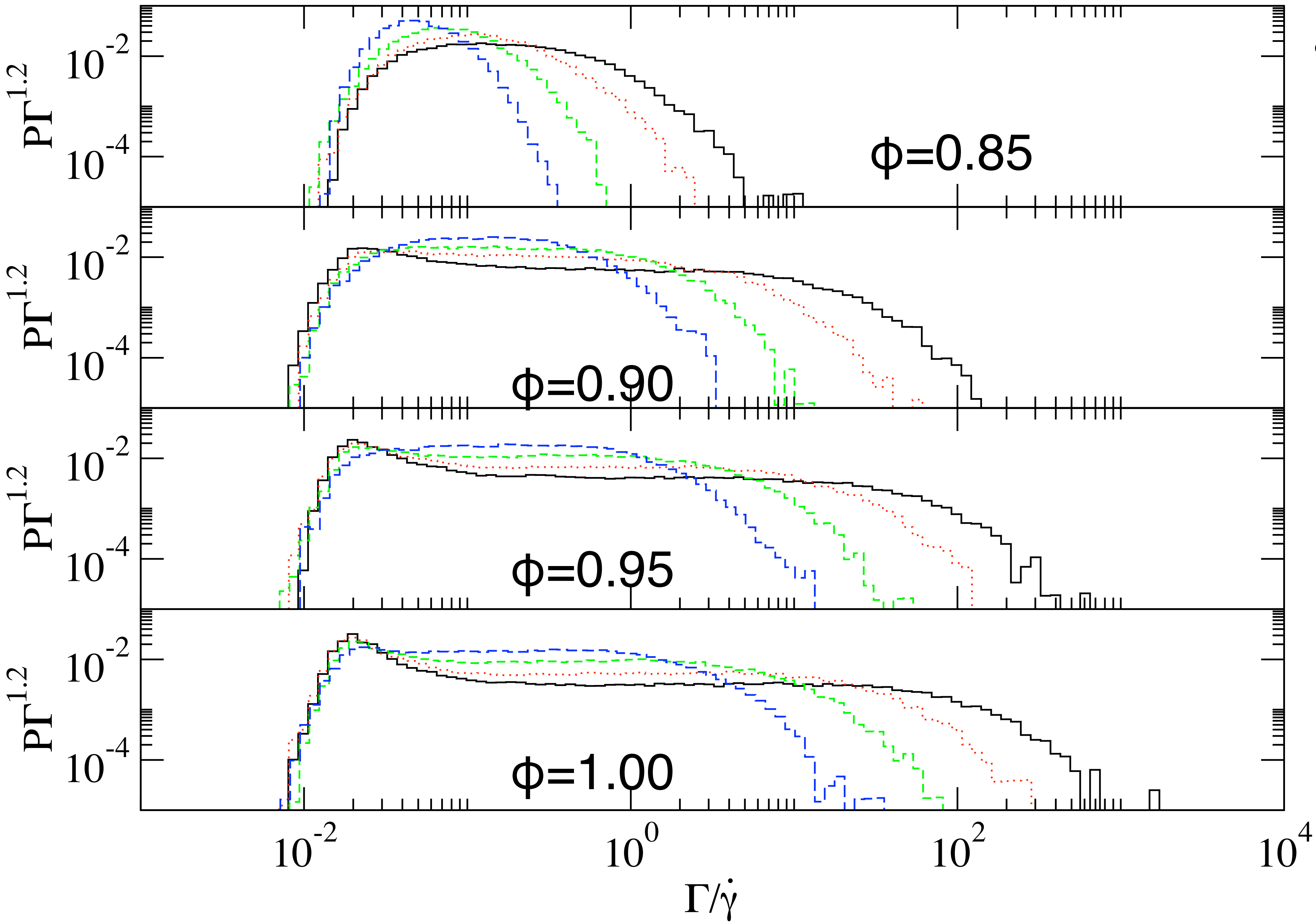
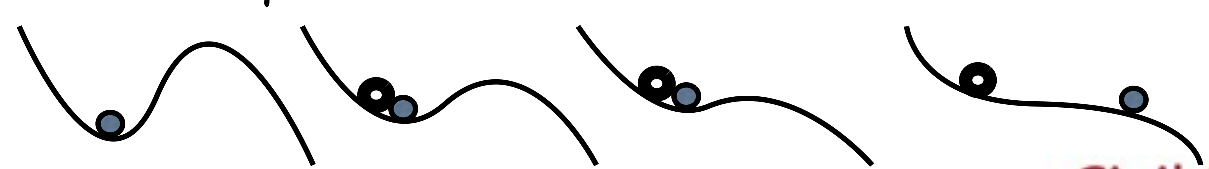


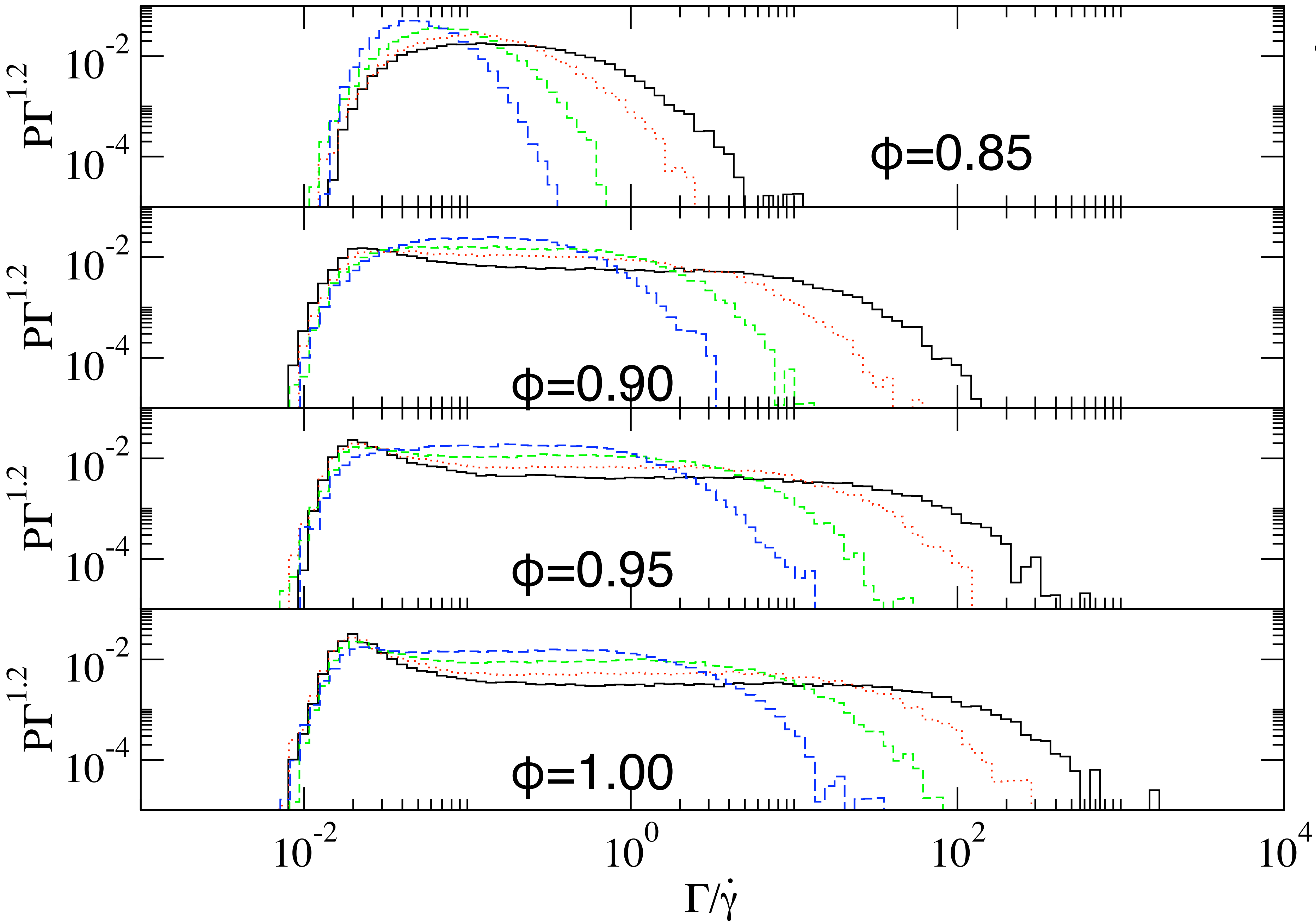
- QS scaling:

$$\Gamma = \frac{D}{\dot{\gamma}} \sum_i \delta v_i^2$$



Increasing shear →

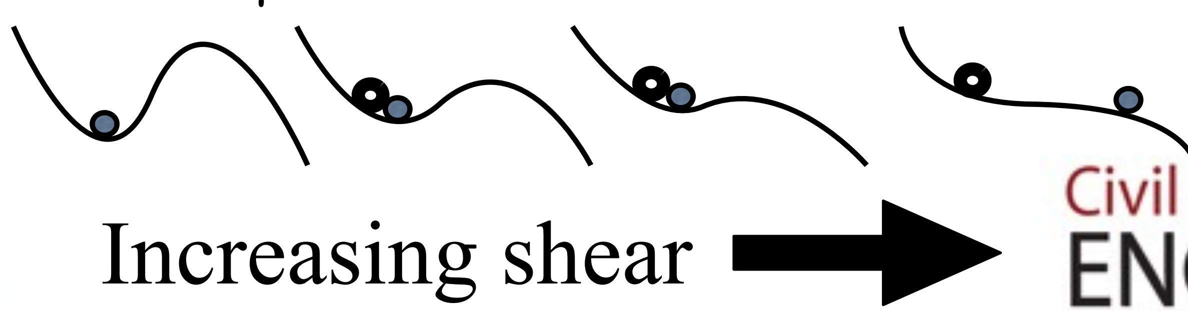
Γ distribution kinematic QS rescaling



- QS scaling:

$$\Gamma = \frac{D}{\dot{\gamma}} \sum_i \delta v_i^2$$

$$\delta v_i \propto \dot{\gamma}$$



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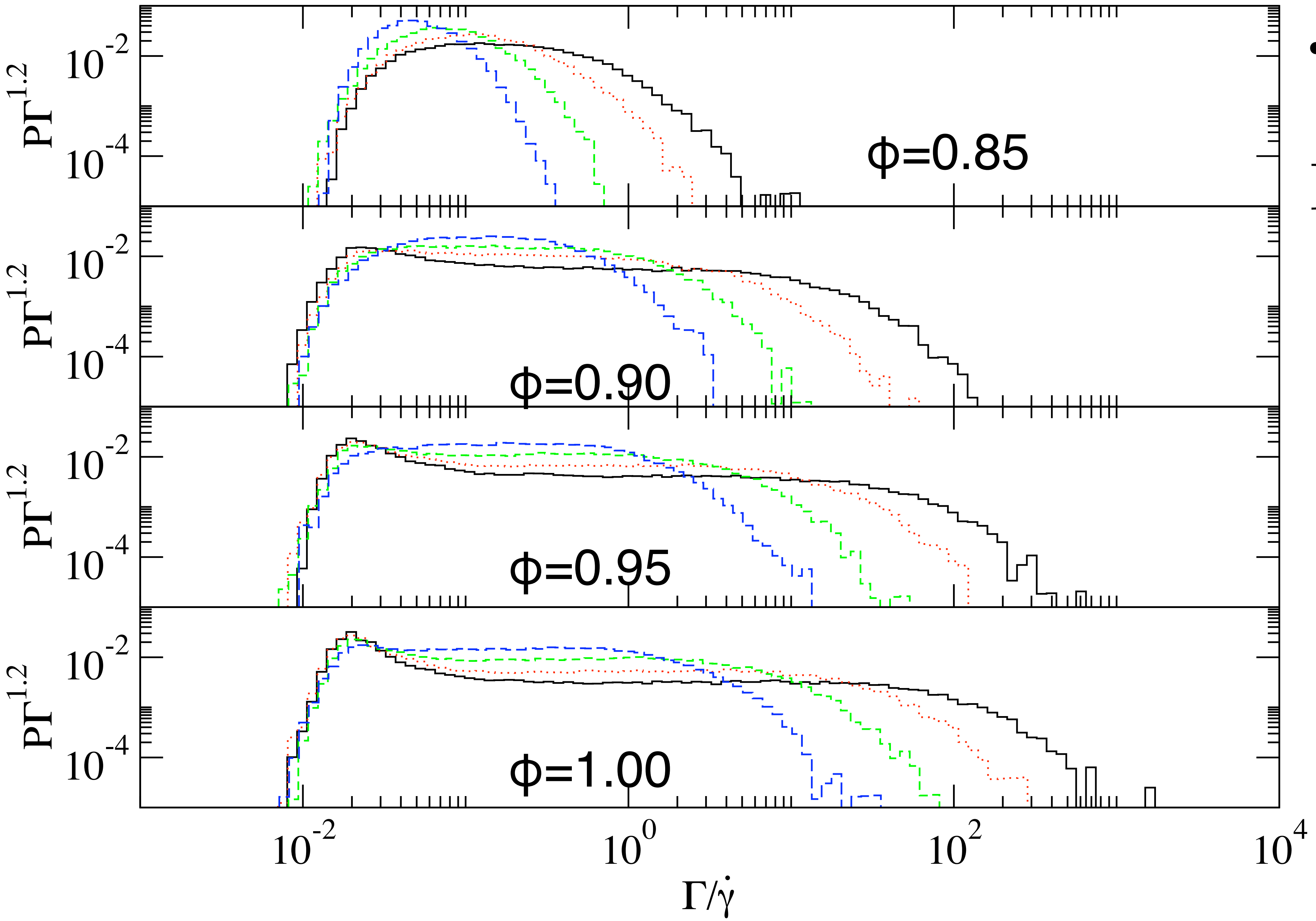
Saturday, June 12, 2010

Γ distribution kinematic QS rescaling

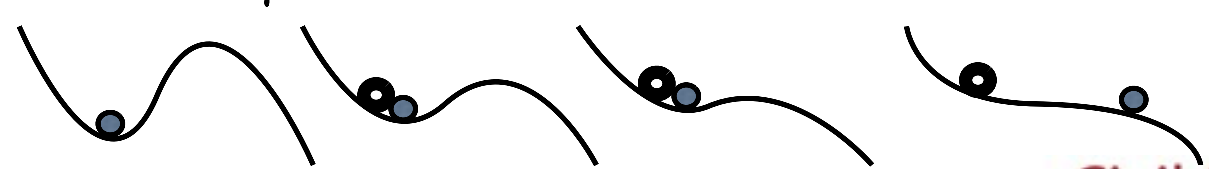


- QS scaling:

$$\Gamma = \frac{D}{\dot{\gamma}} \sum_i \delta v_i^2$$

$$\delta v_i \propto \dot{\gamma}$$

$$\Gamma \propto \frac{D}{\dot{\gamma}} N \dot{\gamma}^2 = N D \dot{\gamma}$$



Increasing shear →

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Conclusion (avalanches/dissipation)

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Conclusion (avalanches/dissipation)

- Instantaneous energy dissipation:

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Conclusion (avalanches/dissipation)

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  - $\phi > \phi_c$, $d\gamma/dt \to 0$:

Conclusion (avalanches/dissipation)

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- Questions:

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  - Slip line argument predicts $D_{eff} \sim L$. Can we see it?

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  - How does combined rate/size dictate Fickian cross-over?

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Conclusion (avalanches/dissipation)

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  - $\phi > \phi_c$, $d\gamma/dt \to 0$:
    - Quasistatic peak
    - Power law regime with exponent ~ 1.2

- Questions:
  - Slip line argument predicts $D_{eff} \sim L$. Can we see it?
  - How does combined rate/size dictate Fickian cross-over?
  - Is physics the same at the same $\tau_J \, d\gamma/dt$?

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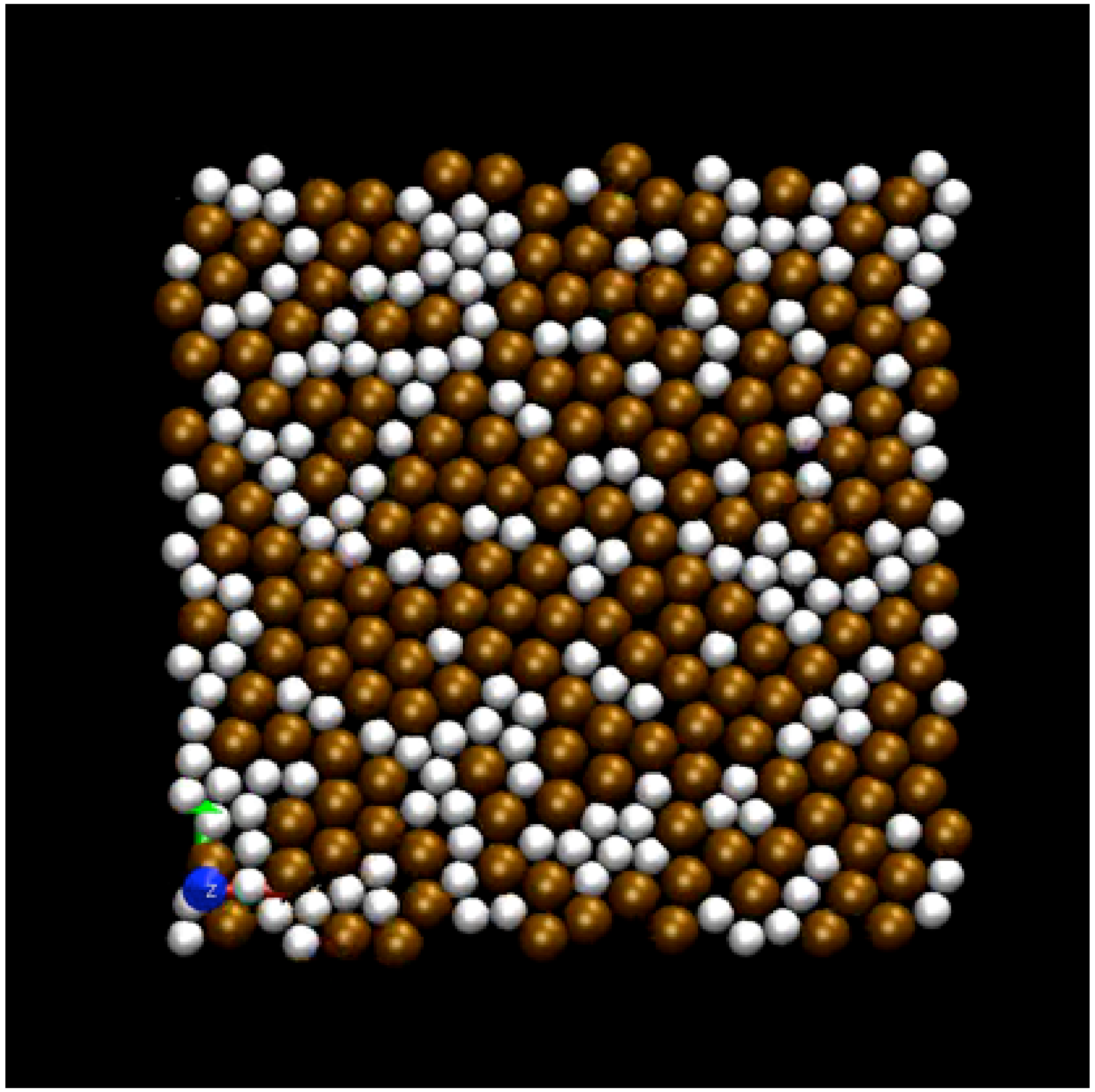
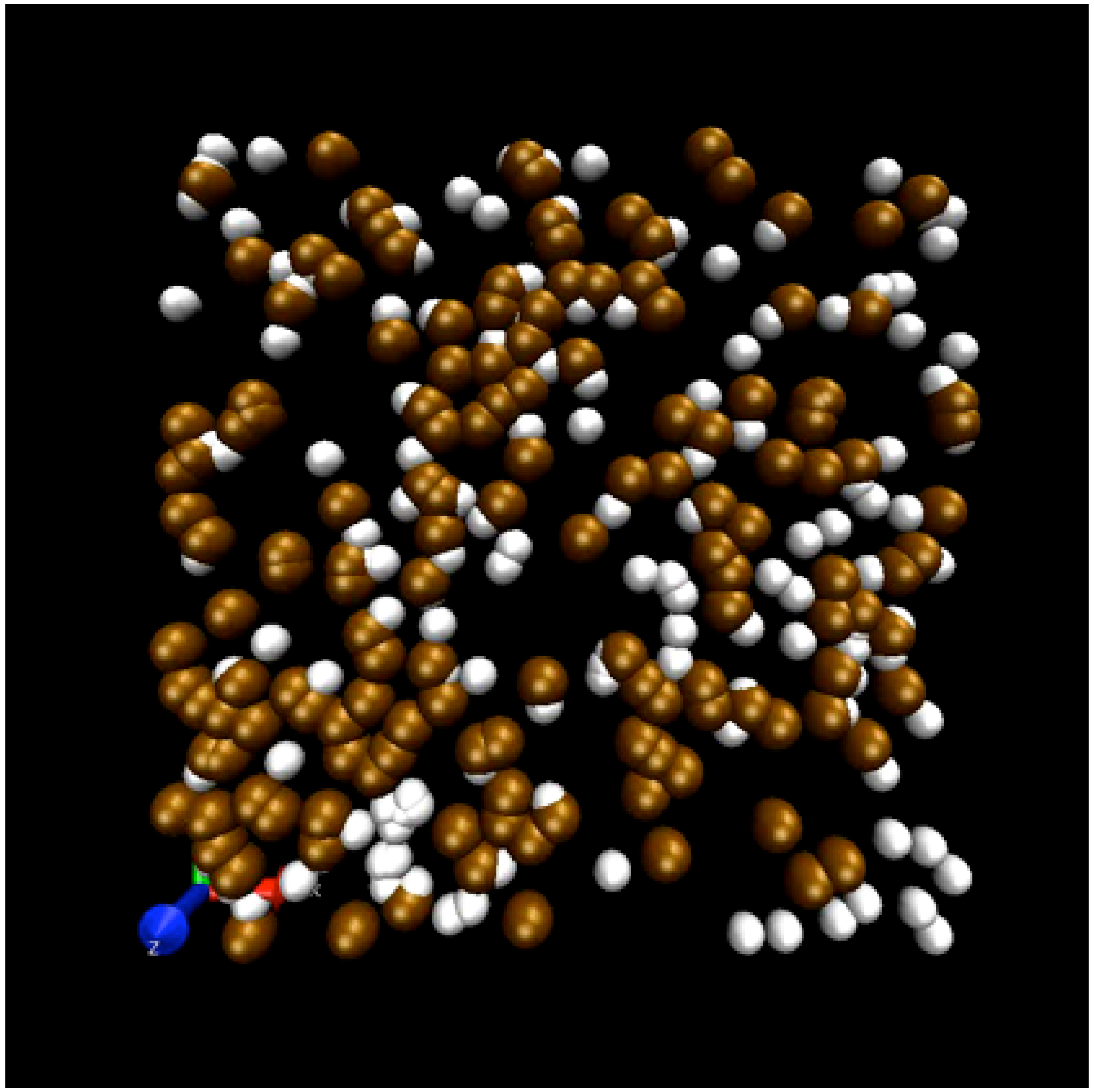
THE END





Thanks!

Numerical models / algorithms

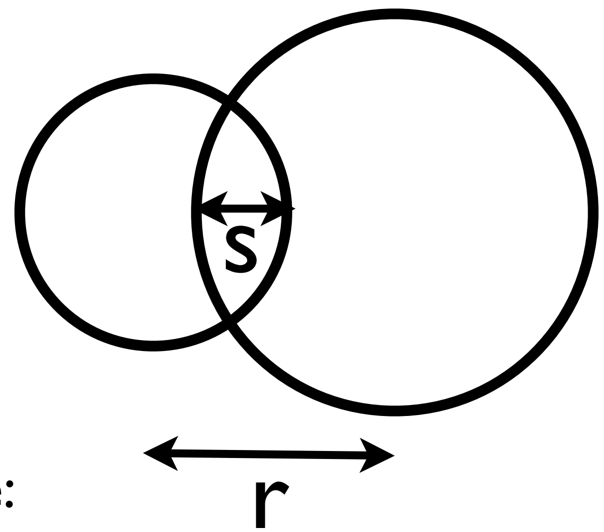
Various interaction potentials:

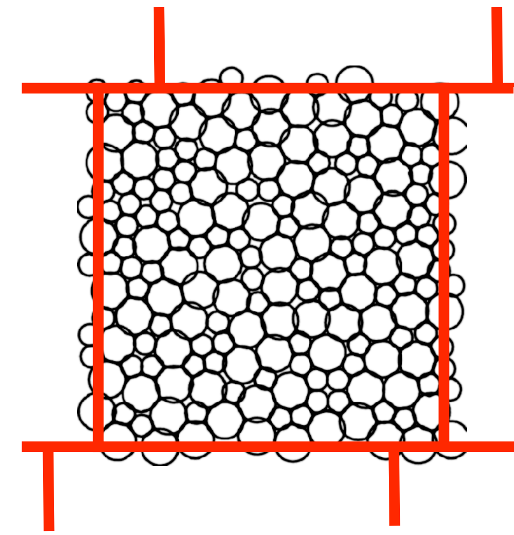
$U_{harm}=(\epsilon/2)\, s^2$

$U_{hertz}=\epsilon\, s^{5/2}$

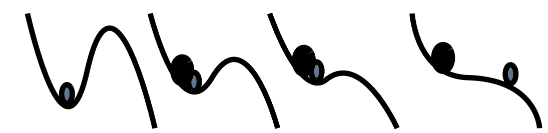
$U_{Lennard\text{-}Jones}=\epsilon(r^{-12}-r^{-6})$

Binary distribution in 2D





Landscape picture:



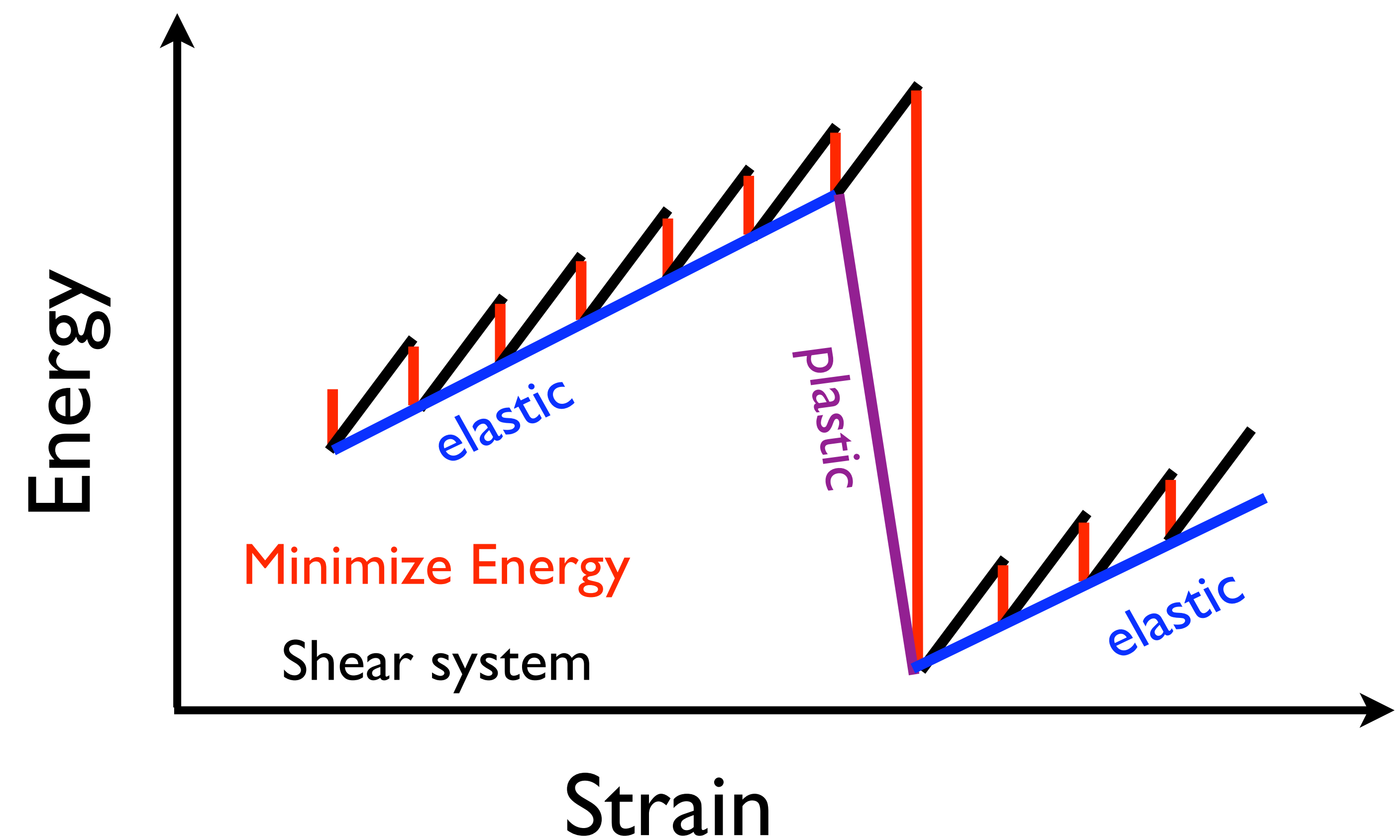
(Malandro and Lacks)

Athermal, Quasistatic Procedure:

- Minimize potential energy
- Shear boundaries and particles
- Repeat

Represents: $\tau_{pl} << \tau_{dr} << \tau_{th}$

- Bulk metallic glass in the zero temperature, zero strain rate limit
- Granular material or emulsion in zero strain rate limit



Behavior:

- Discrete plastic jumps separate smooth, reversible elastic segments

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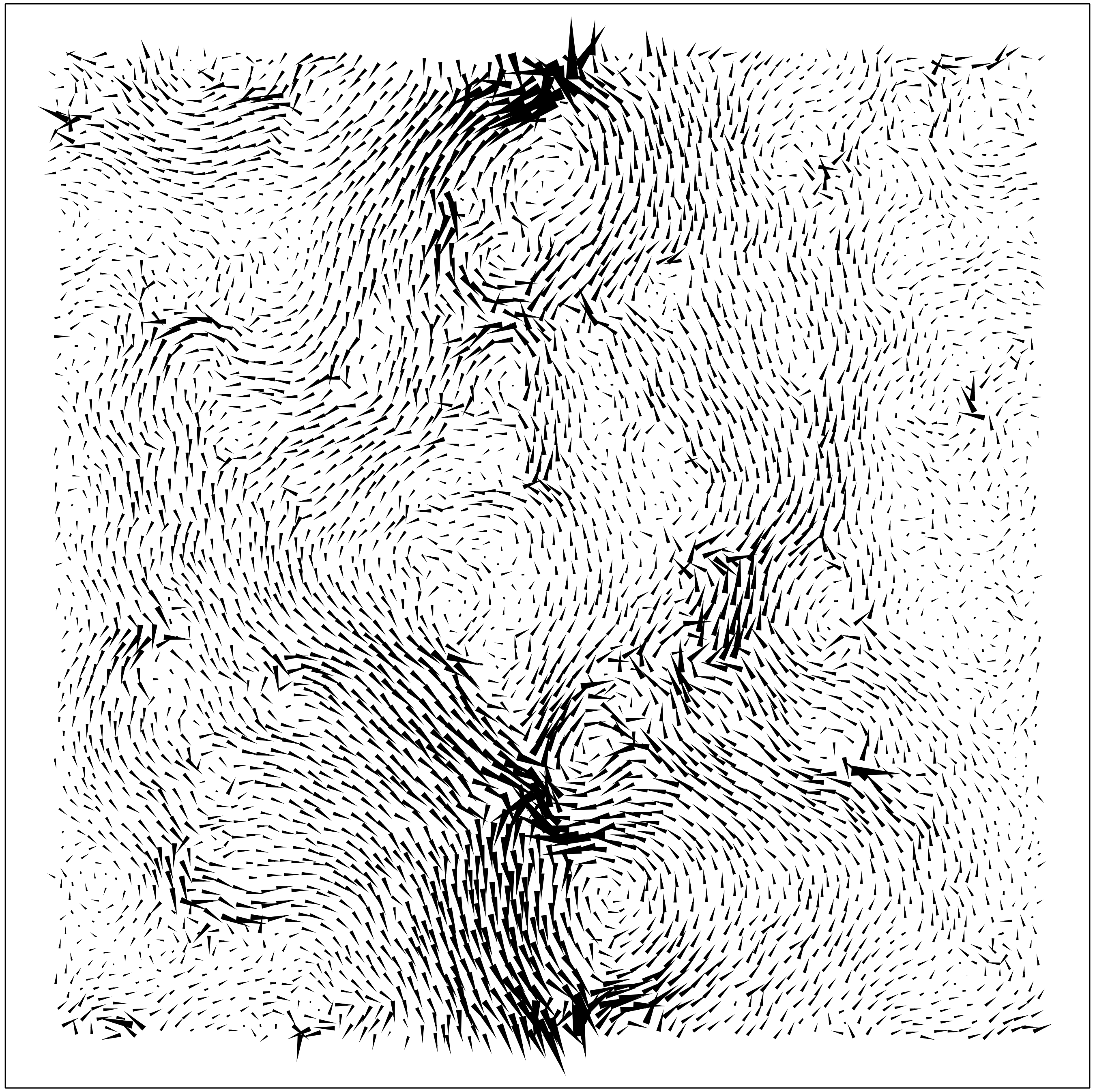
Linear elastic response (at zero temperature)

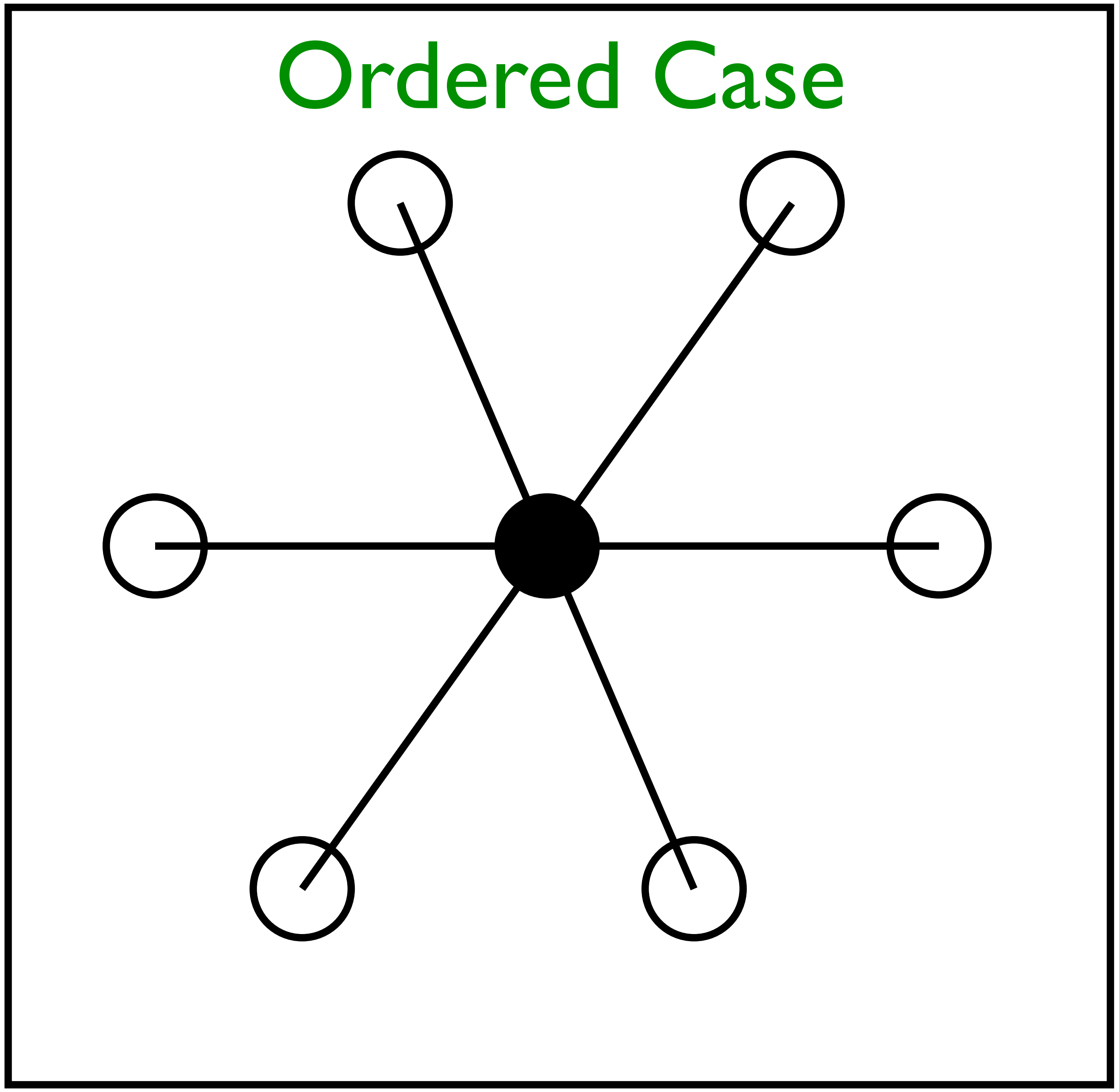
- Take a binary Lennard-Jones system
- Quench instantaneously from T=infinity to T=0
- Apply infinitesimal shear strain
- Compute deviations from homogeneous shear
- Note vortex-like patterns... lengthscale?

A. Tanguy et. al. PRB 2002



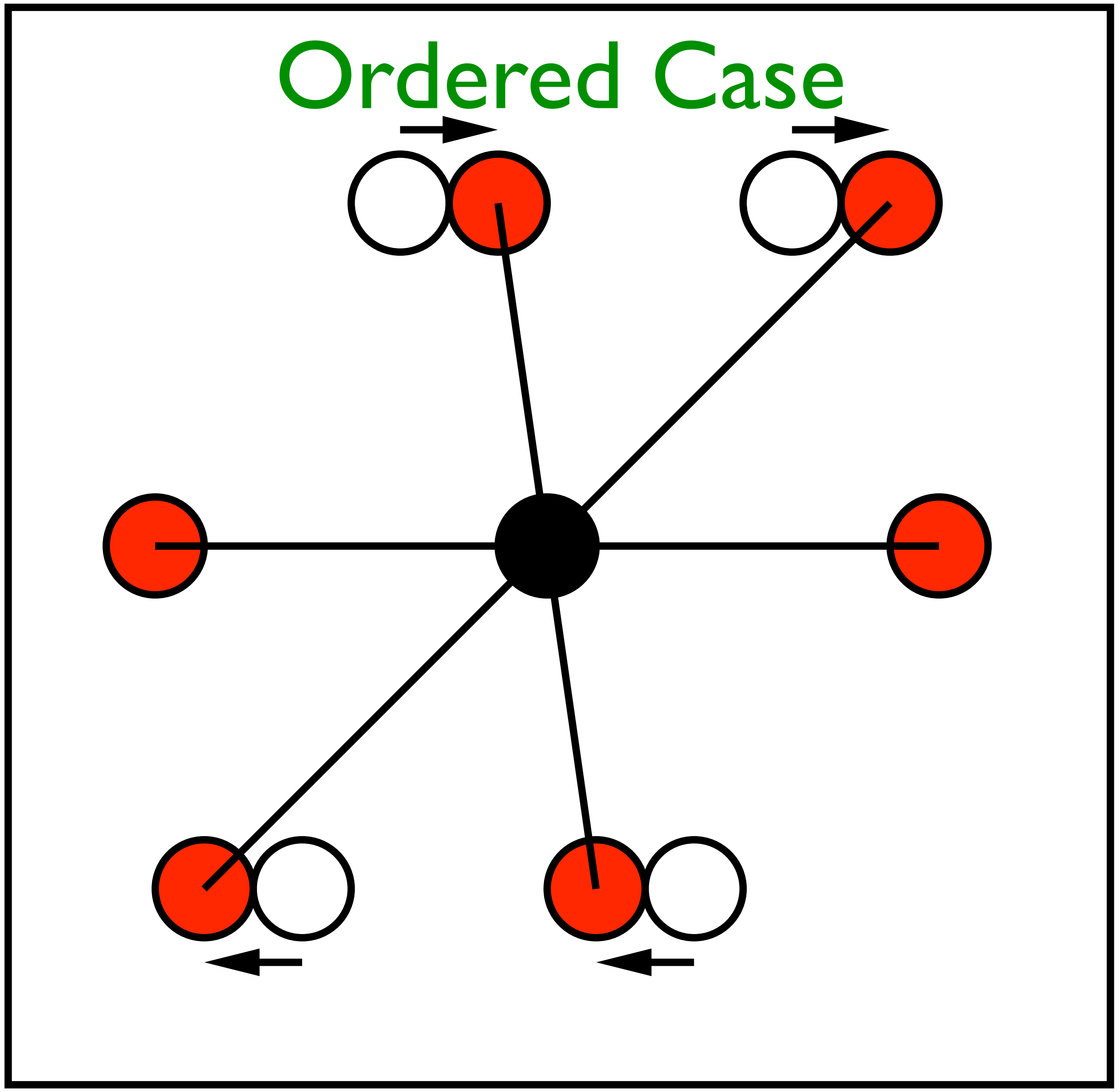
Computing linear response

- Single particle toy problem:
  - Start at F=0



Computing linear response

- Single particle toy problem:
  - Start at F=0
  - Apply affine shear
  - Forces remain zero
  - No correction necessary



Computing linear response

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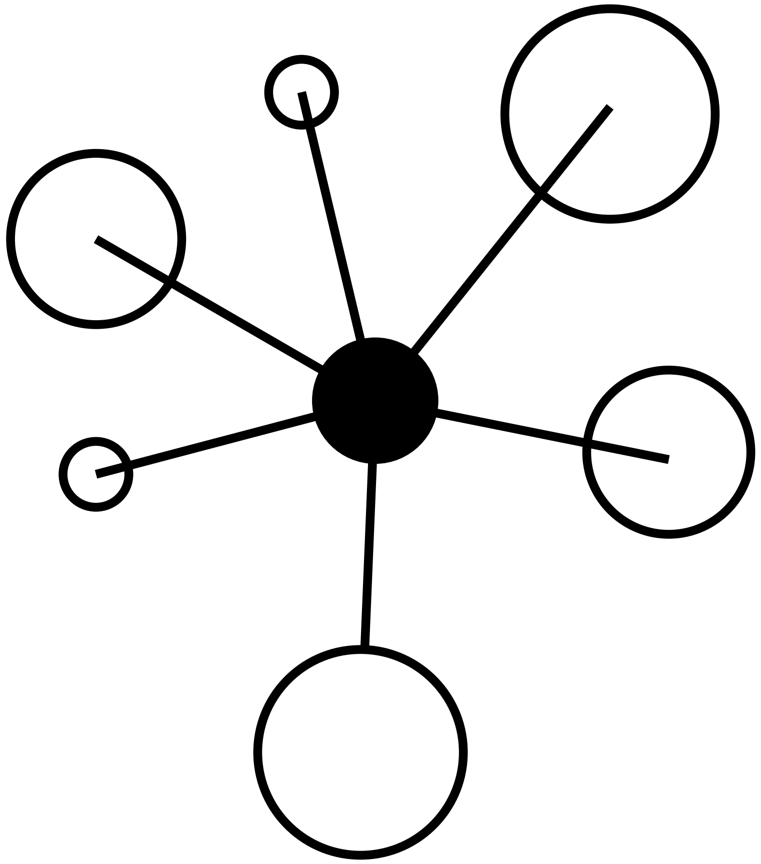
Disordered Case



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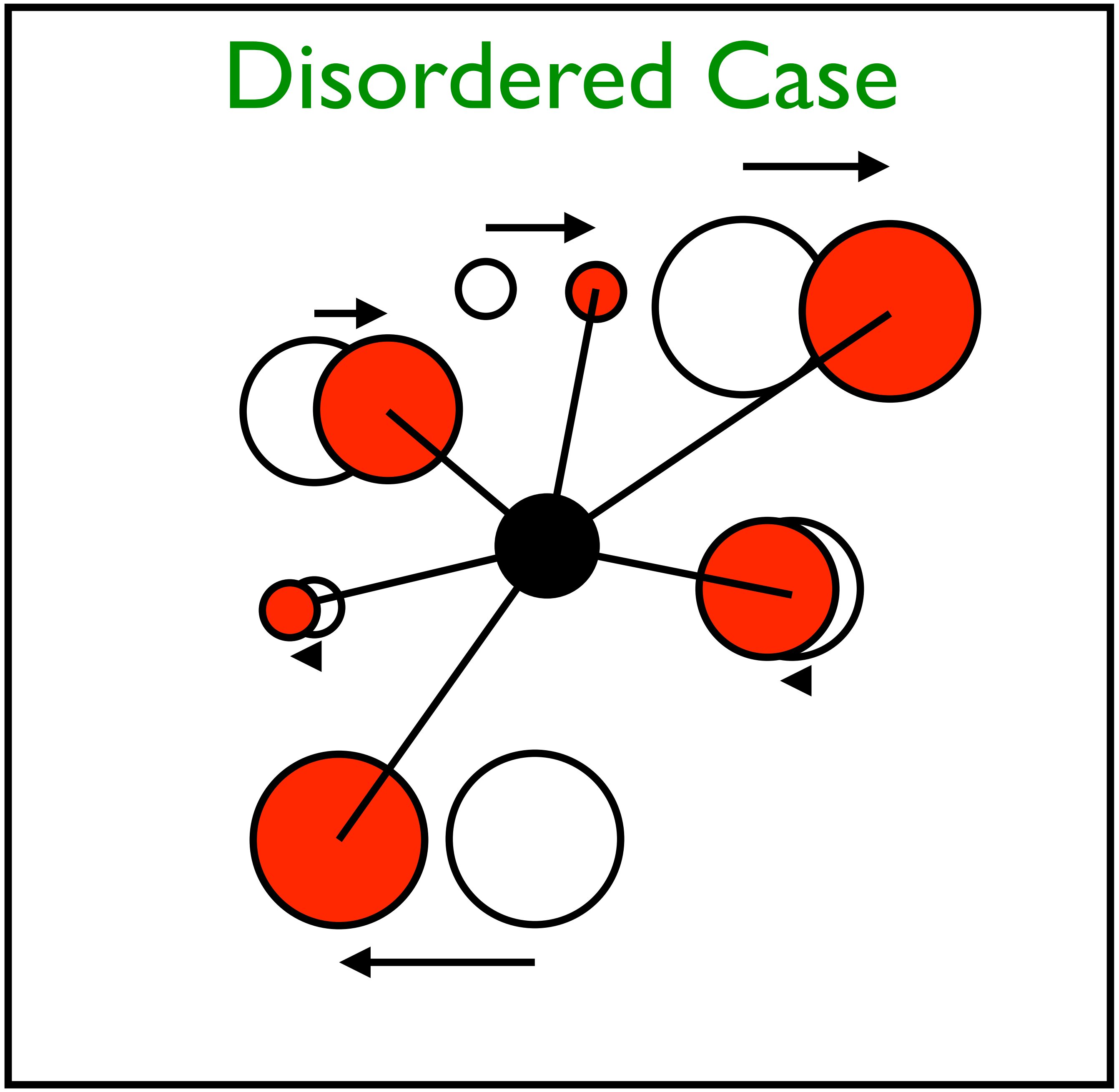
Computing linear response

- Single particle toy problem:
  - Start at F=0
  - Apply strain

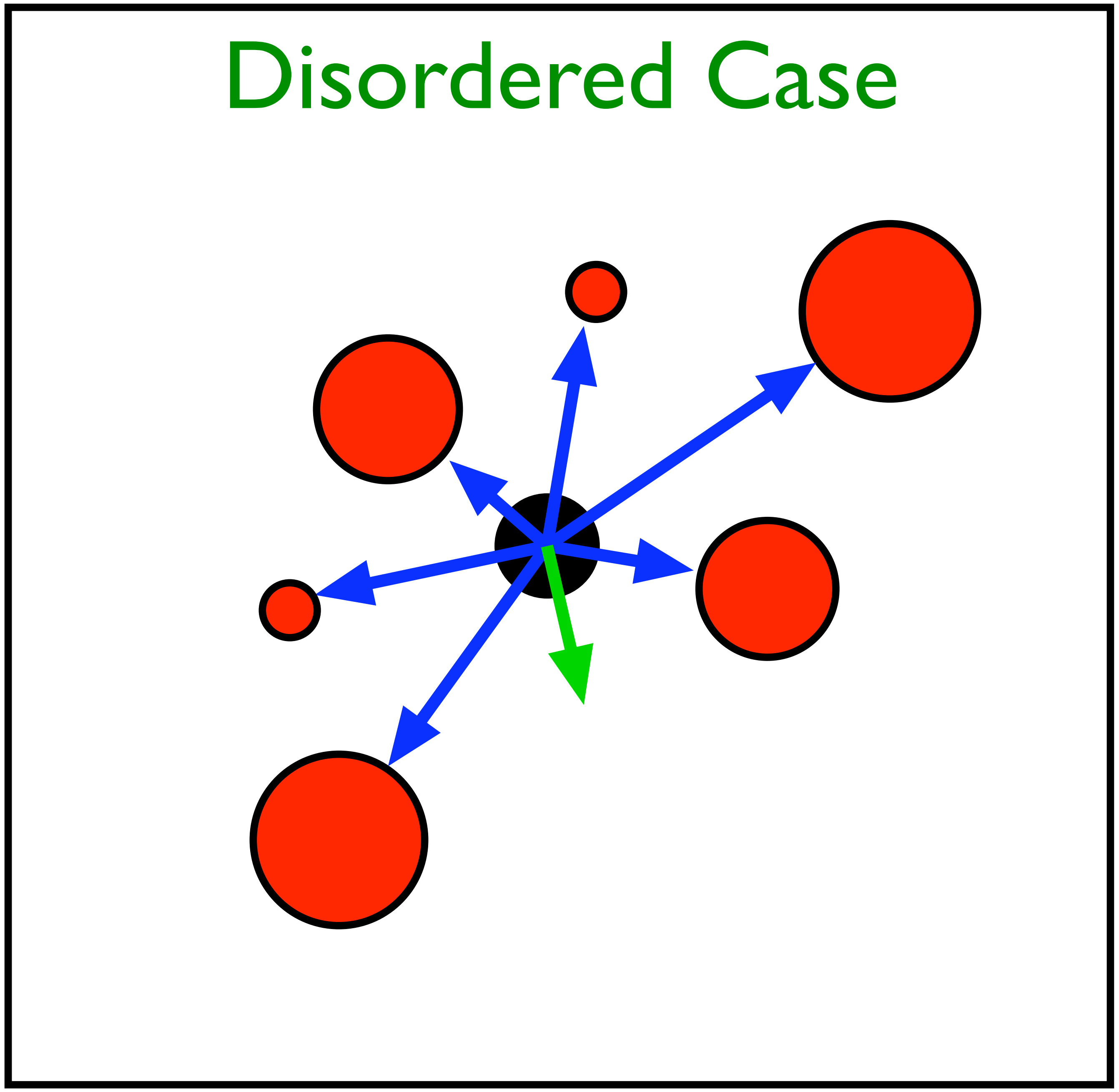


Computing linear response

- Single particle toy problem:
  - Start at F=0
  - Apply strain

Use Hessian to compute "Affine force"

$$\vec{\Xi}_i = \gamma \sum_j \mathbf{H}_{ij} \hat{\mathbf{x}} \delta y_j$$



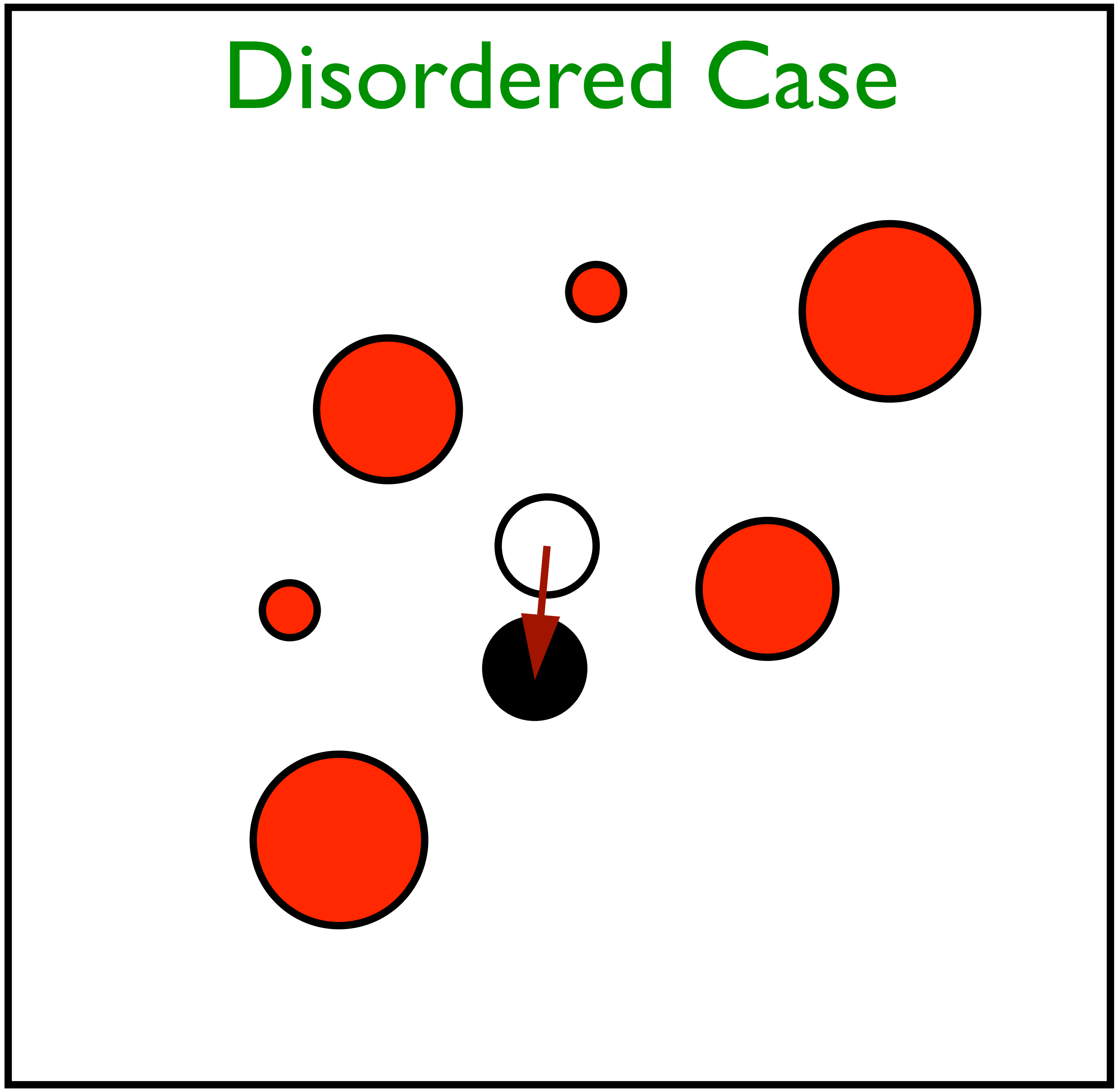
Computing linear response

- Single particle toy problem:
  - Start at F=0
  - Apply strain

Use Hessian to find position correction

$$\vec{\Xi}_i = \mathbf{H}_{ii} \vec{dr}_i$$

$$\vec{dr}_i = \mathbf{H}_{ii}^{-1} \vec{\Xi}_i$$

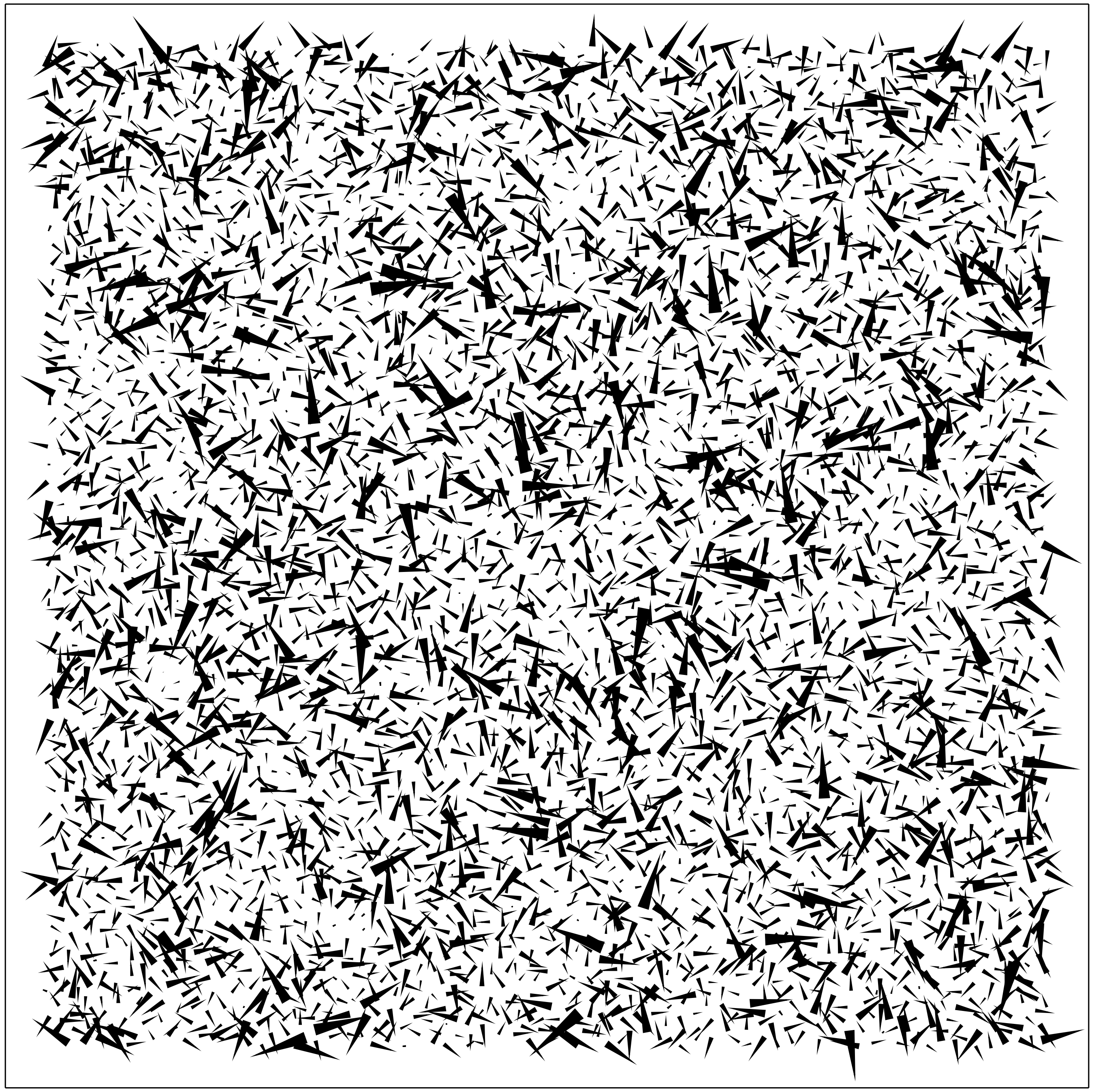


Computing linear response

- Back to full assembly:

$$\vec{\Xi}_i = \gamma \sum_j \mathbf{H_{ij}} \hat{\mathbf{x}} \delta y_{ij}$$

  - Measure of local disorder.
  - Only short range correlations in our samples.

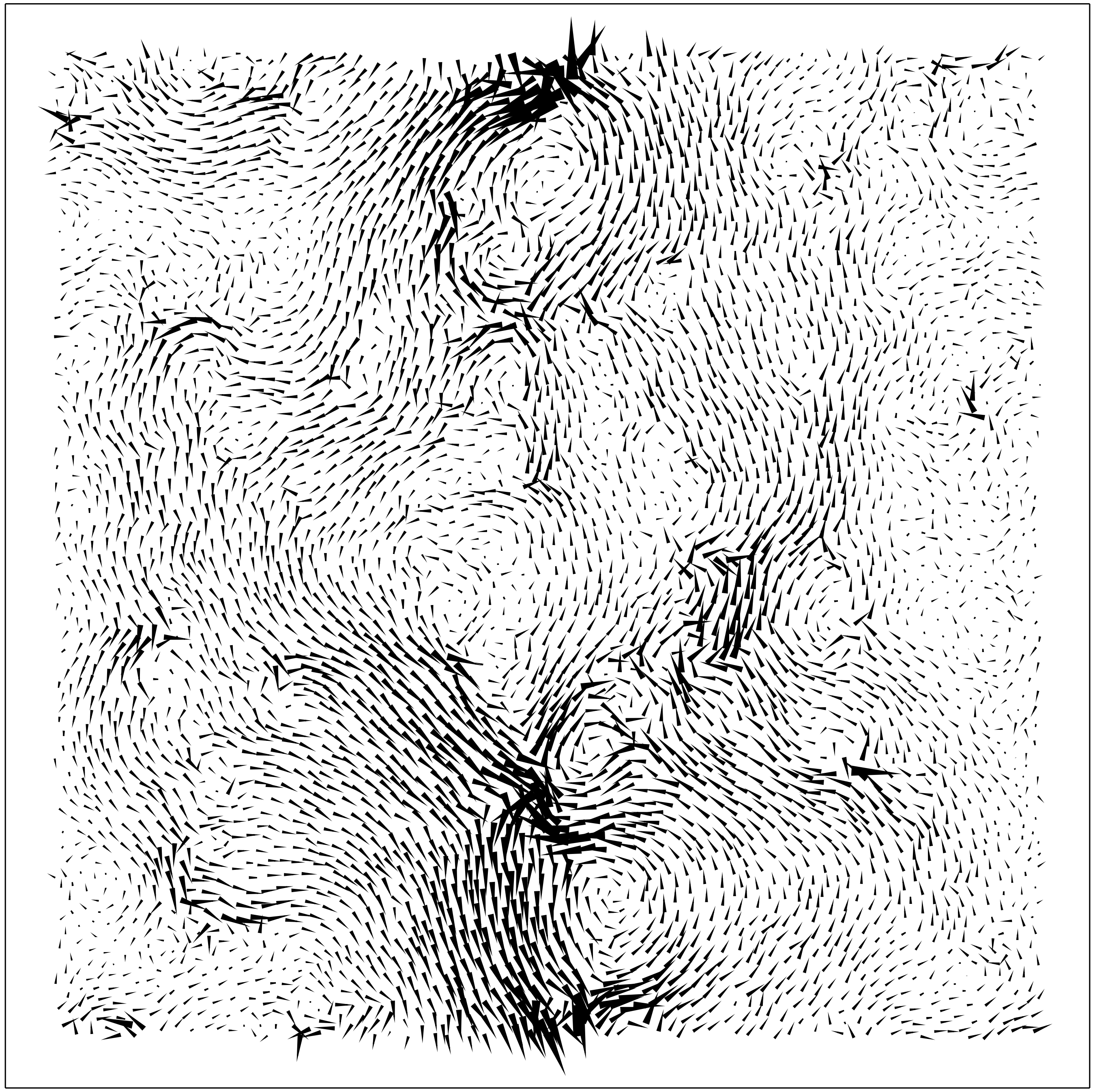


Computing linear response

- Back to full assembly:

$$\vec{dr}_i = \gamma \sum_j \mathbf{H}_{\mathbf{ij}}^{-1} \vec{\Xi}_j$$

Force balance:
Affine forces, Ξ, must be balanced by correction forces, $H^{-1}_{ij}dx_j$



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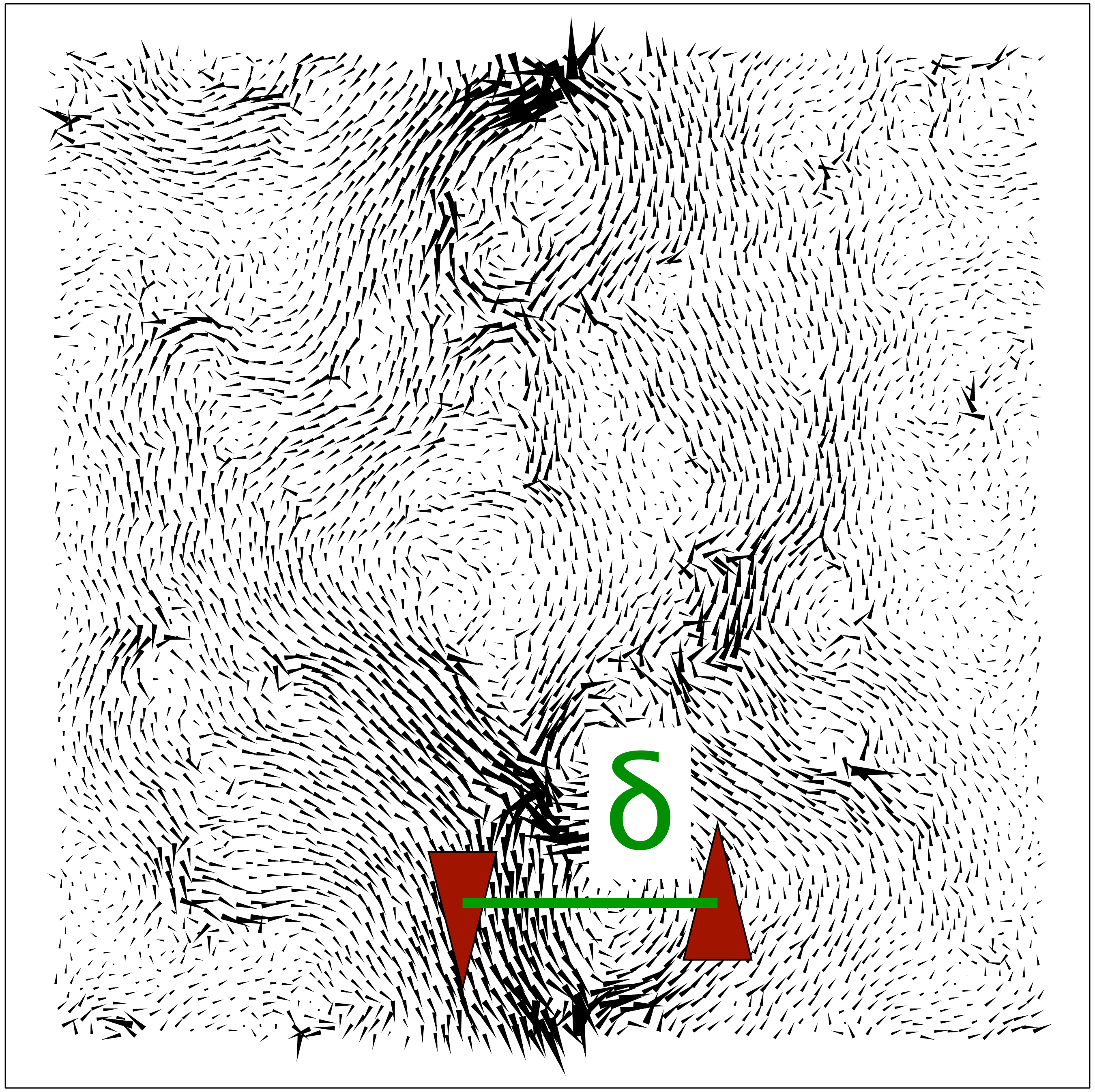
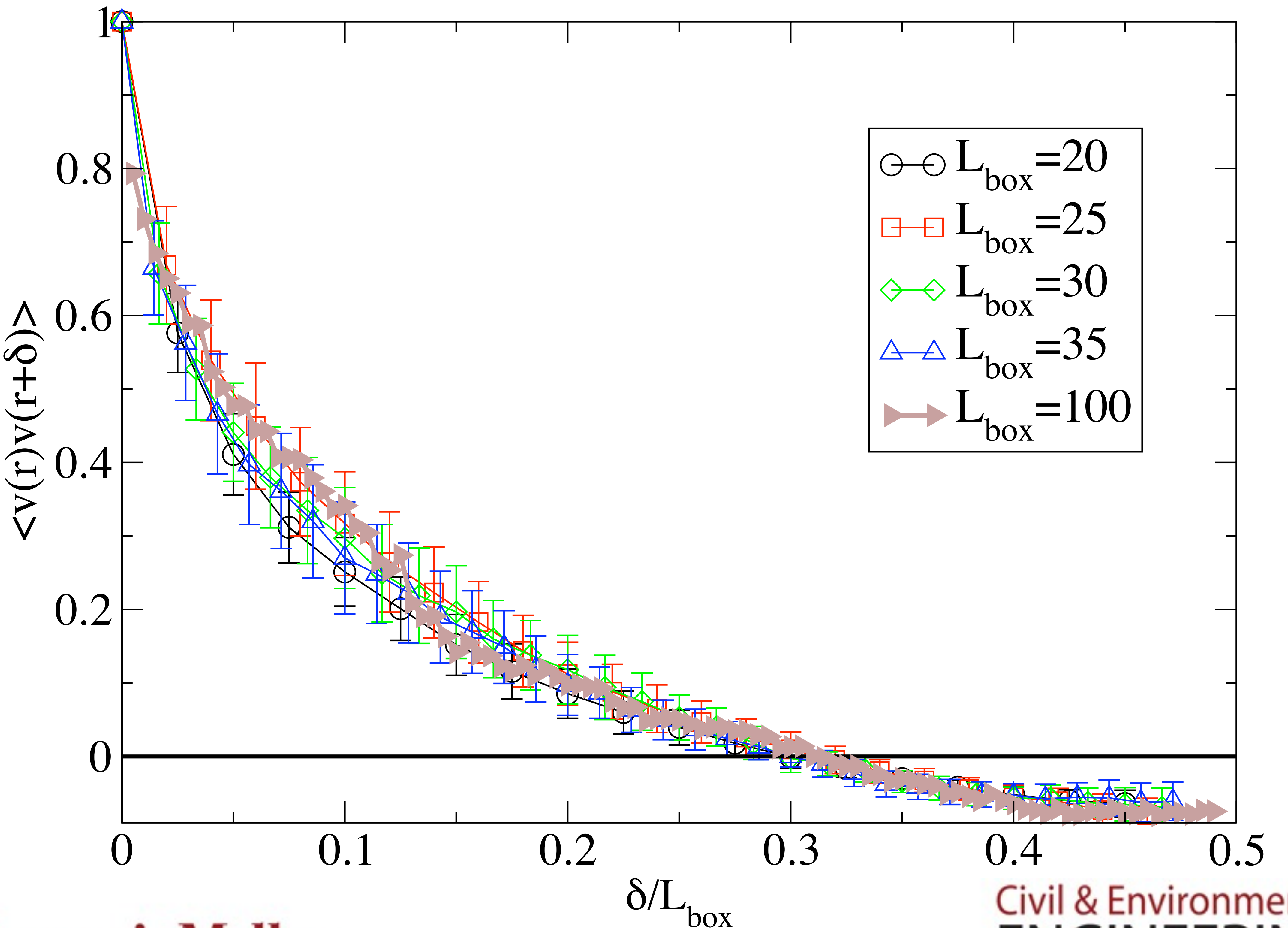
Spatial autocorrelation function g(δ)

$$g(\vec{\delta}) \doteq \int \vec{v}(\vec{r}) \cdot \vec{v}(\vec{r} + \vec{\delta}) d\vec{r}$$

- Usual autocorrelation
- Measures "vortex size"
- Characteristic length?



Spatial autocorrelation function g(δ)



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g(δ): theoretical form

Recall: $\vec{dr}_i = \gamma \sum_j \mathbf{H_{ij}^{-1}} \vec{\Xi}_j$

g(δ): theoretical form

Recall: $\vec{dr}_i = \gamma \sum_j \mathbf{H_{ij}^{-1}} \vec{\Xi}_j$

Then: $\vec{dr}_i = \gamma \sum_p \left(\frac{\Xi_p}{\lambda_p} \right) \vec{\psi}_{ip}$

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g(δ): theoretical form

Recall: $\vec{dr}_i = \gamma \sum_j \mathbf{H}_{\mathbf{ij}}^{-1} \vec{\Xi}_j$

Then: $\vec{dr}_i = \gamma \sum_p \left(\frac{\Xi_p}{\lambda_p} \right) \vec{\psi}_{ip}$

- Assume:
  - Ξ is a random dipole field
  - Ψ_p are plane waves
  - $\lambda_p = k_p^2$; $\Xi_p = k_p$

g(δ): theoretical form

Recall: $\vec{dr}_i = \gamma \sum_j \mathbf{H}_{\mathbf{ij}}^{-1} \vec{\Xi}_j$

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- Assume:
  - Ξ is a random dipole field
  - Ψ_p are plane waves
  - $\lambda_p = k_p^2$; $\Xi_p = k_p$

Approximate dr_i as random sum of plane waves:

$$\vec{dr}_i \sim \sum_{k=(m,n)} \phi_{mn} \frac{e^{2\pi i \vec{k} \cdot \vec{x}_i / L}}{|\vec{k}|}$$

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g(δ): theoretical form

Recall: $\vec{dr}_i = \gamma \sum_j \mathbf{H}_{\mathbf{ij}}^{-1} \vec{\Xi}_j$

Then: $\vec{dr}_i = \gamma \sum_p \left(\frac{\Xi_p}{\lambda_p} \right) \vec{\psi}_{ip}$

- Assume:
  - Ξ is a random dipole field
  - Ψ_p are plane waves
  - $\lambda_p = k_p^2$; $\Xi_p = k_p$

Approximate dr_i as random sum of plane waves:

$$\vec{dr}_i \sim \sum_{k=(m,n)} \phi_{mn} \frac{e^{2\pi i \vec{k}\cdot\vec{x}_i/L}}{|\vec{k}|}$$

Then g(δ) is:

$$g(\vec{\delta}) \sim \sum_{k=(m,n)} \frac{\cos(2\pi\vec{k}\cdot\vec{\delta}/L)}{k^2}$$

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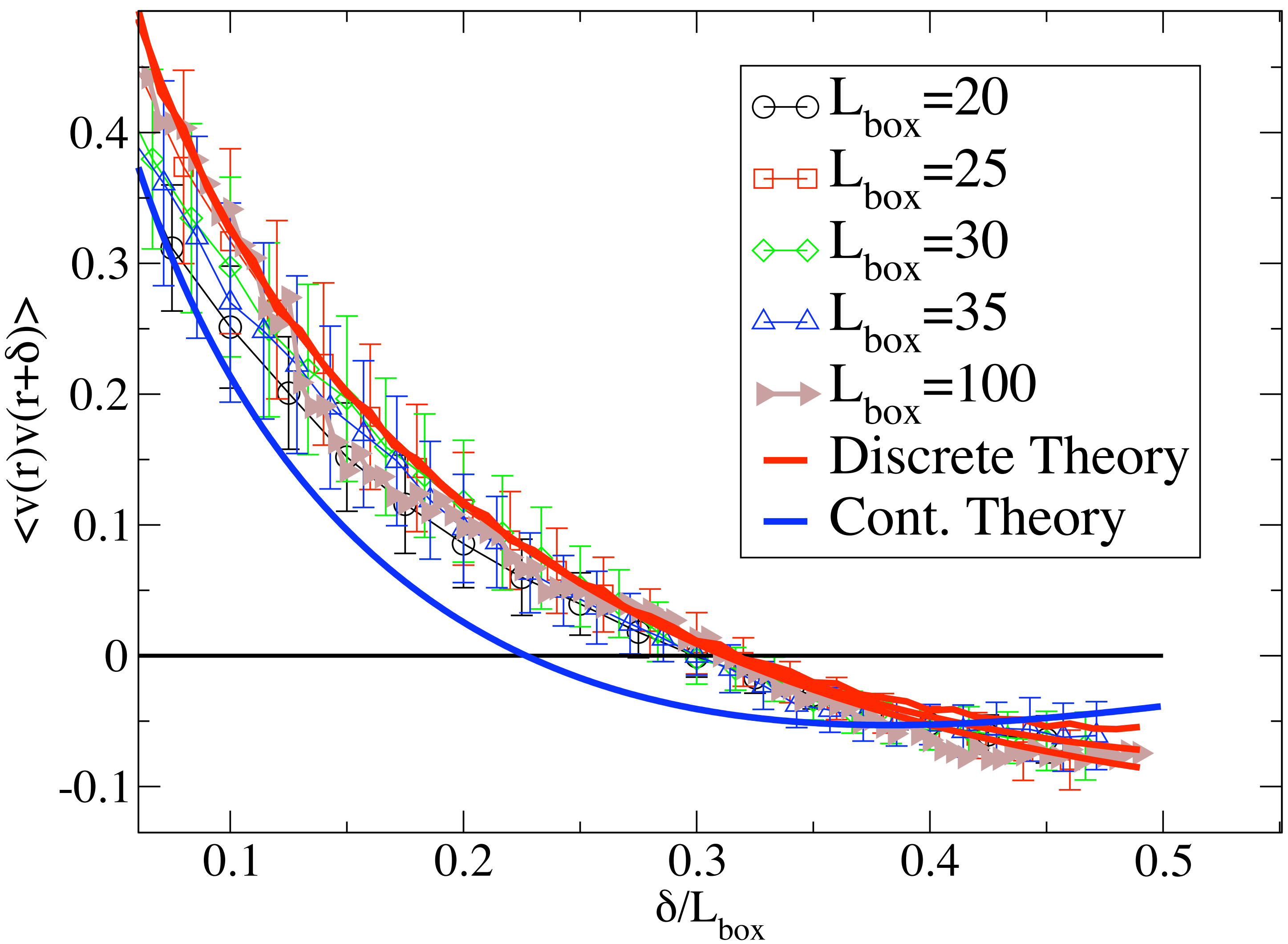
g(δ): theoretical form

$$g(\vec{\delta}) \sim \sum_{k=(m,n)} \frac{\cos(2\pi \vec{k} \cdot \vec{\delta}/L)}{k^2}$$



Similar to DiDonna +Lubenksy,

- g(k)~1/k²

but:

- Fully discrete derivation

Blue curve: Semi-continuum

Red curve(s): Partial sum (n=40) 3 different angles

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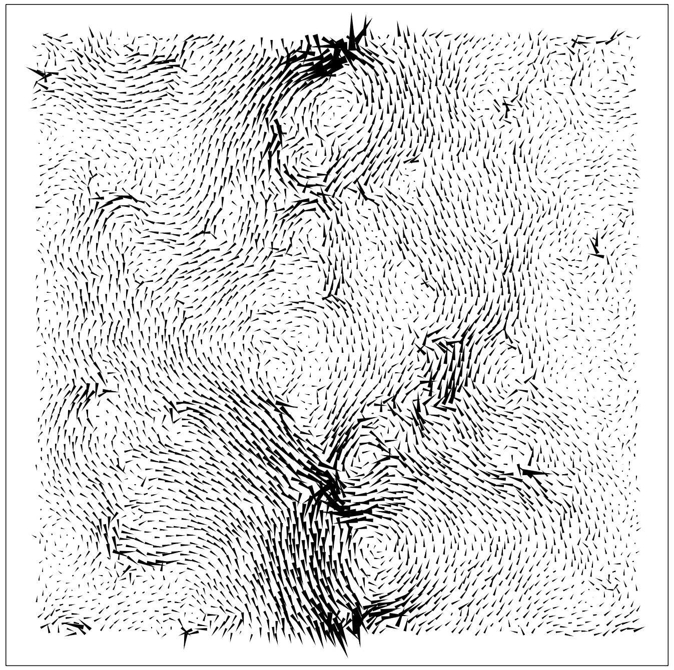
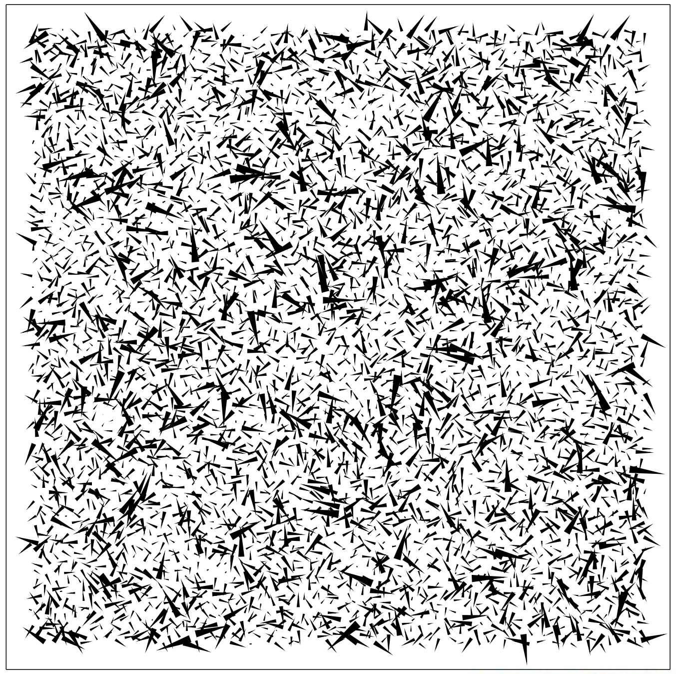
Summary: Elastic response

- Linear elastic (zero temperature) response is inhomogeneous.
- Displacement fluctuations appear as vortices
- Size scales with system size... no characteristic length
- “Affine forces”: a new measure of local disorder.
- Fluctuations derived from approximating eigenmodes as plane waves and affine forces as a random dipoles.





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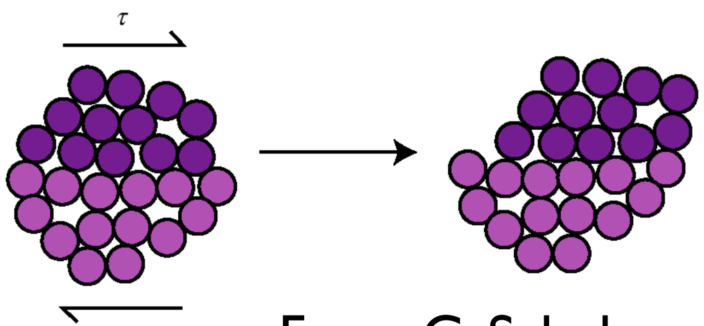
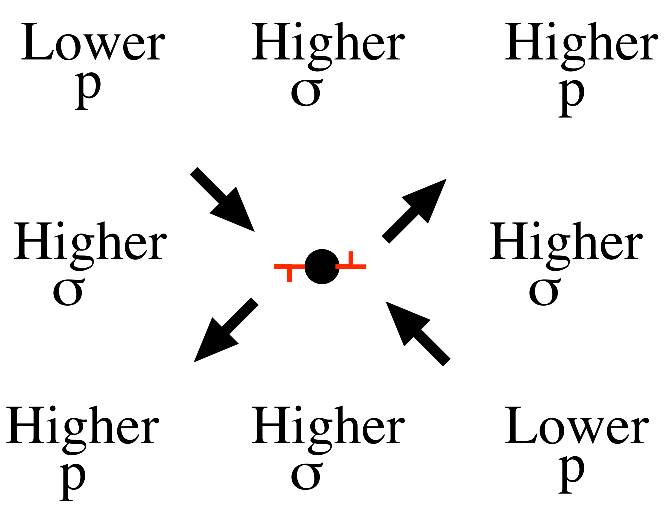
Plastic response (Shear Transformation Zones)

No crystal... so no dislocations...
but then what controls plasticity?



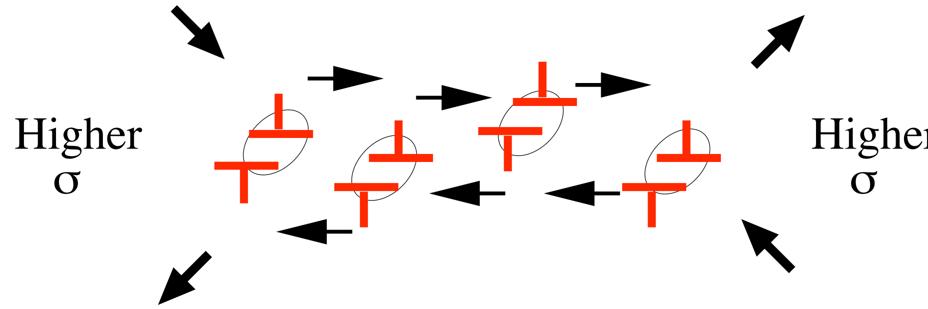
From C. Schuh

Elastic consequences:

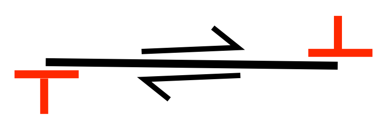


- Shear Transformation Zone (STZ) Mechanism:
  - Argon and Kuo: bubble raft experiments
  - Maeda and Takeuchi: computer simulations
  - Bulatov and Argon: banding mechanism
  - Falk and Langer: mean field theory

Cascade mechanism:



Analogous to dislocation glide:



What are the consequences of organization of local shear zones?

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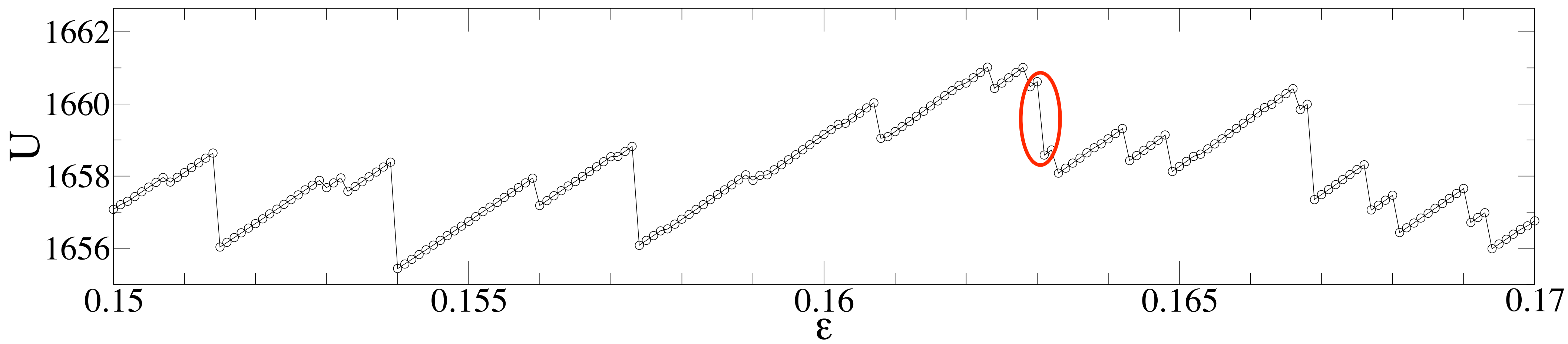
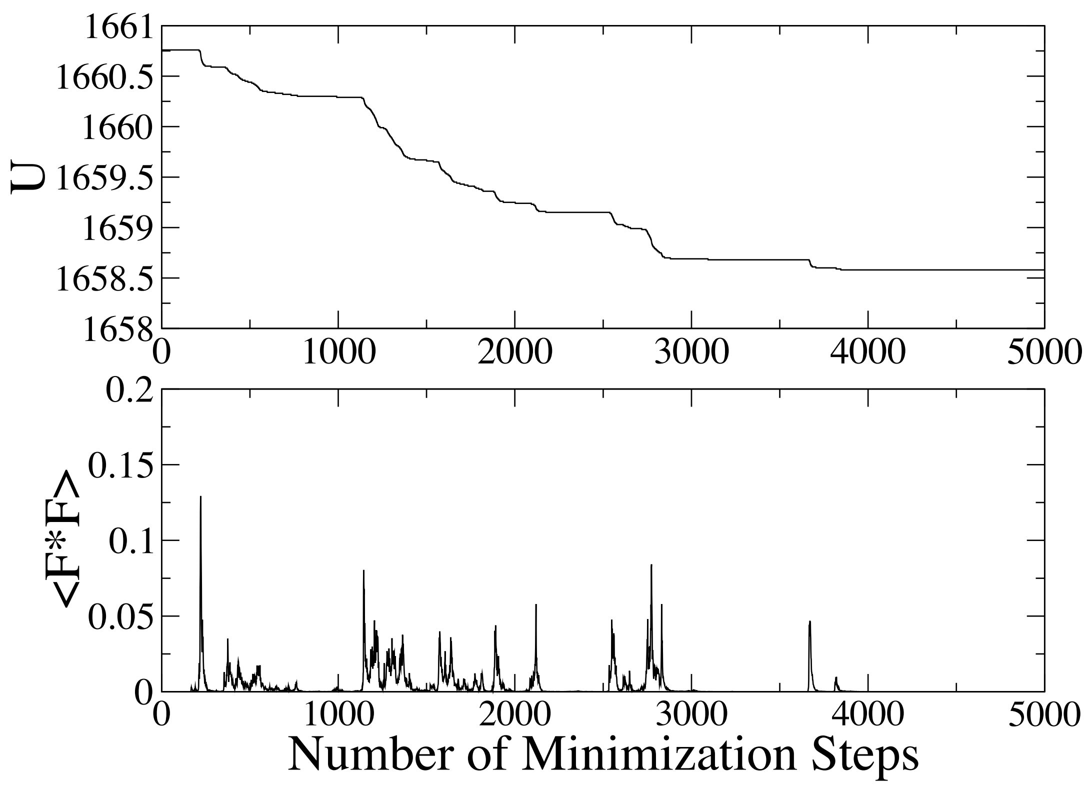
Saturday, June 12, 2010

Typical plastic cascade





- Protocol: shear, relax...
- Single typical plastic event
- All relaxation at one strain
- "Number of minimization steps" analogous to time $\langle F^2 \rangle \sim dU/dt$
- Descent is intermittent...

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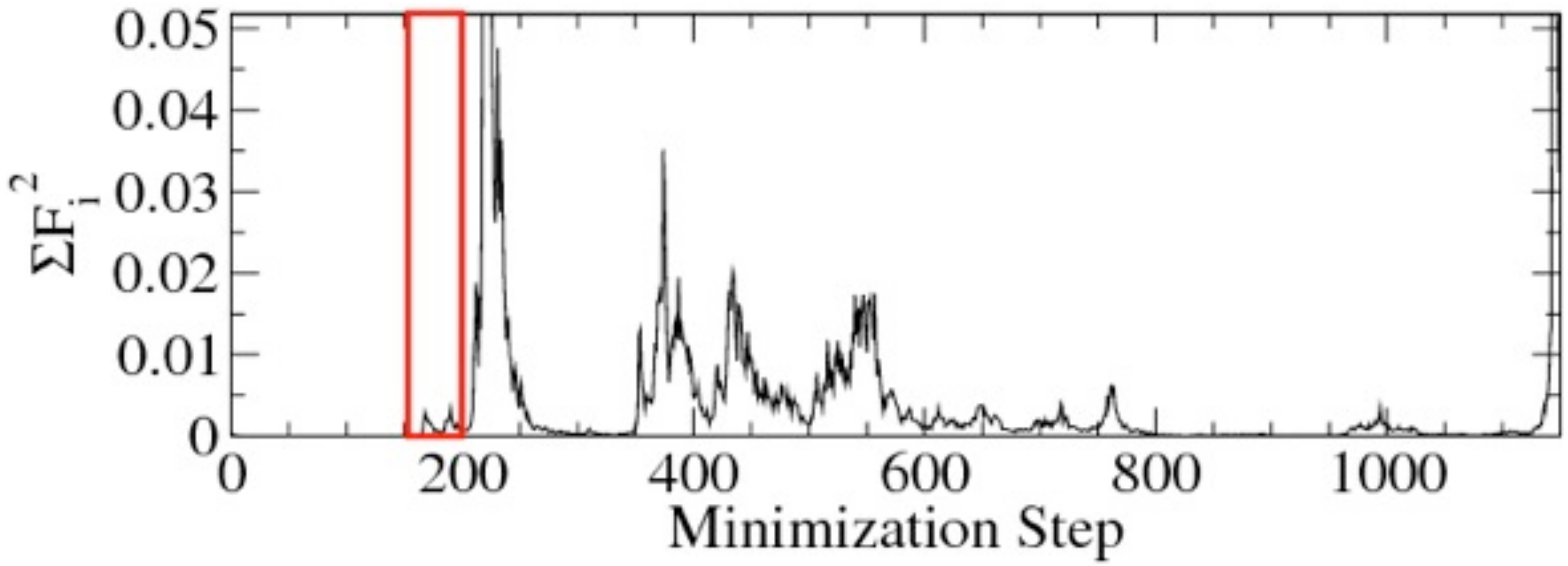
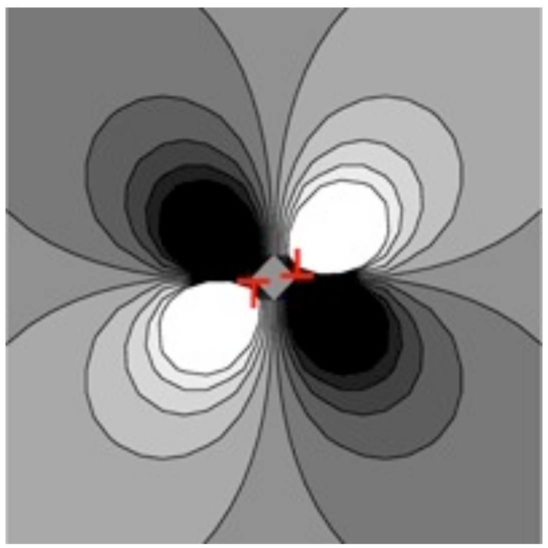
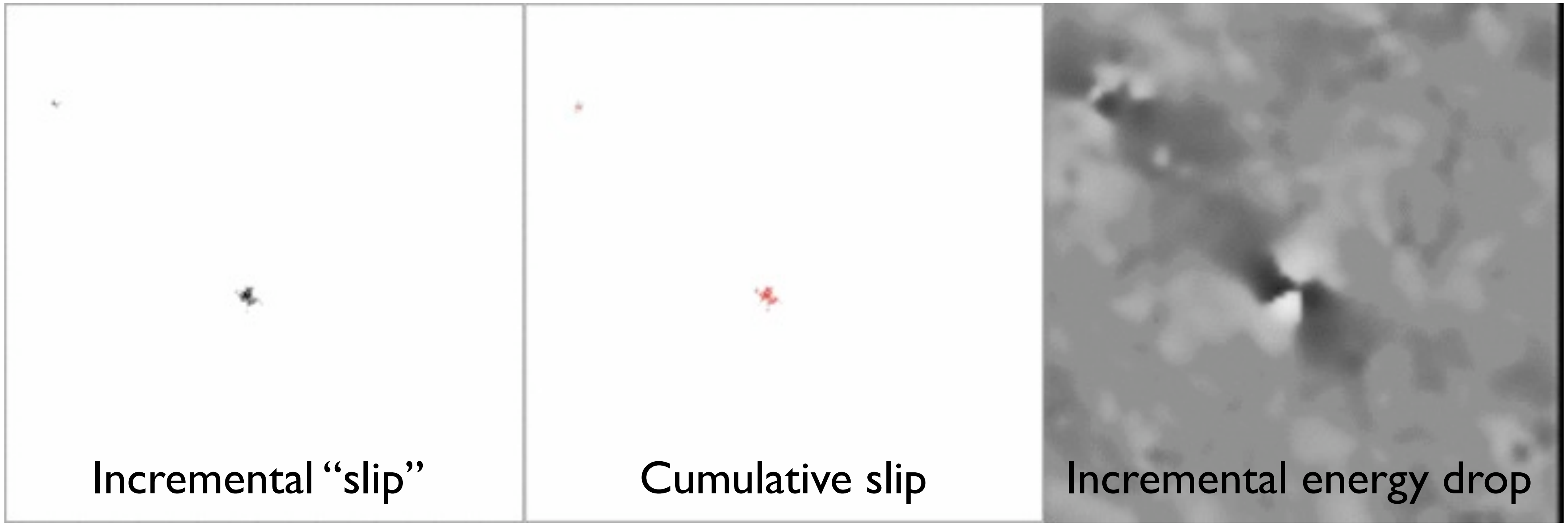
Typical plastic cascade

Initial portion of descent from previous slide:



Expected energy change after nucleation of localized slip:





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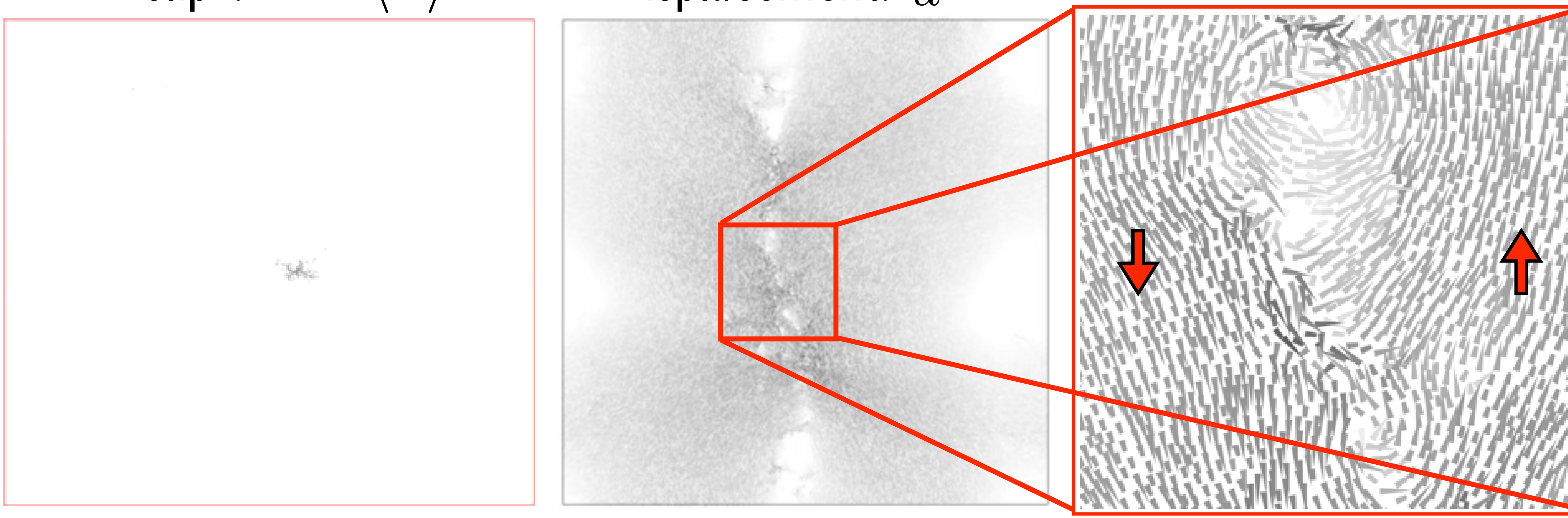
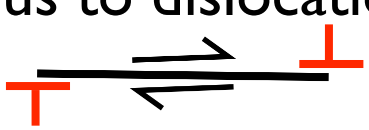
Typical plastic cascade

At the end of the whole cascade, we are left with a slip line:

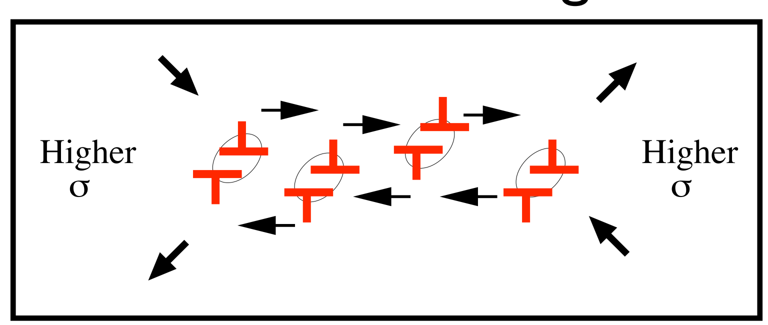
"Slip": $\vec{u} - \langle \vec{u} \rangle$ Displacement: $\vec{u}$



Analogous to dislocation glide:



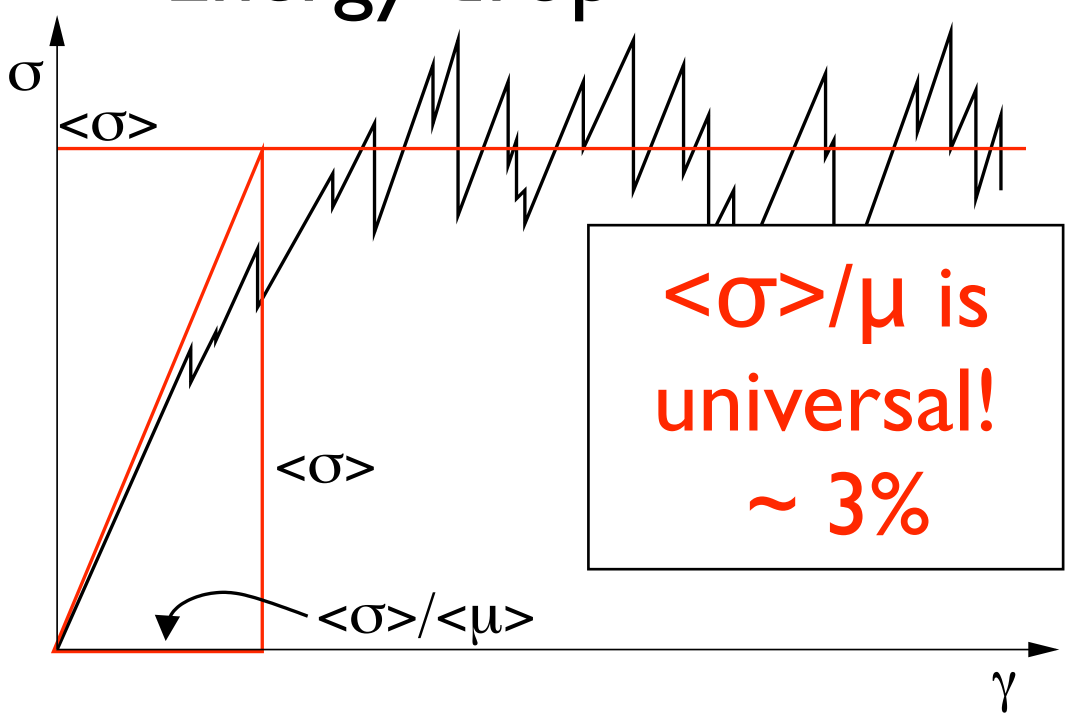
But with local shearing zones:



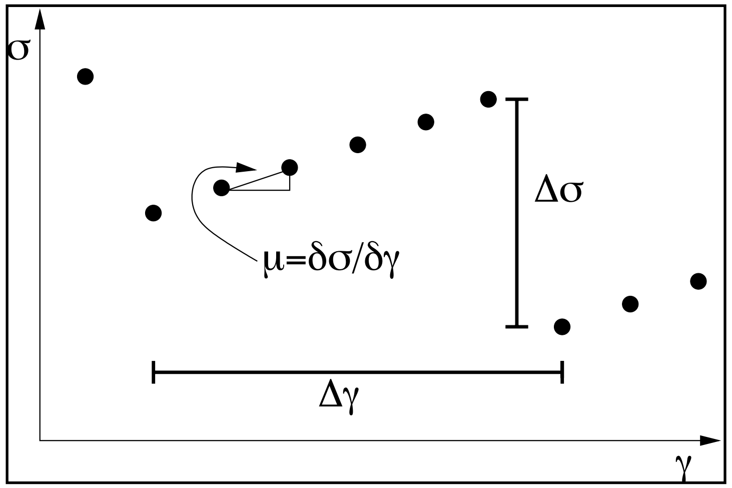
Statistics and size scaling

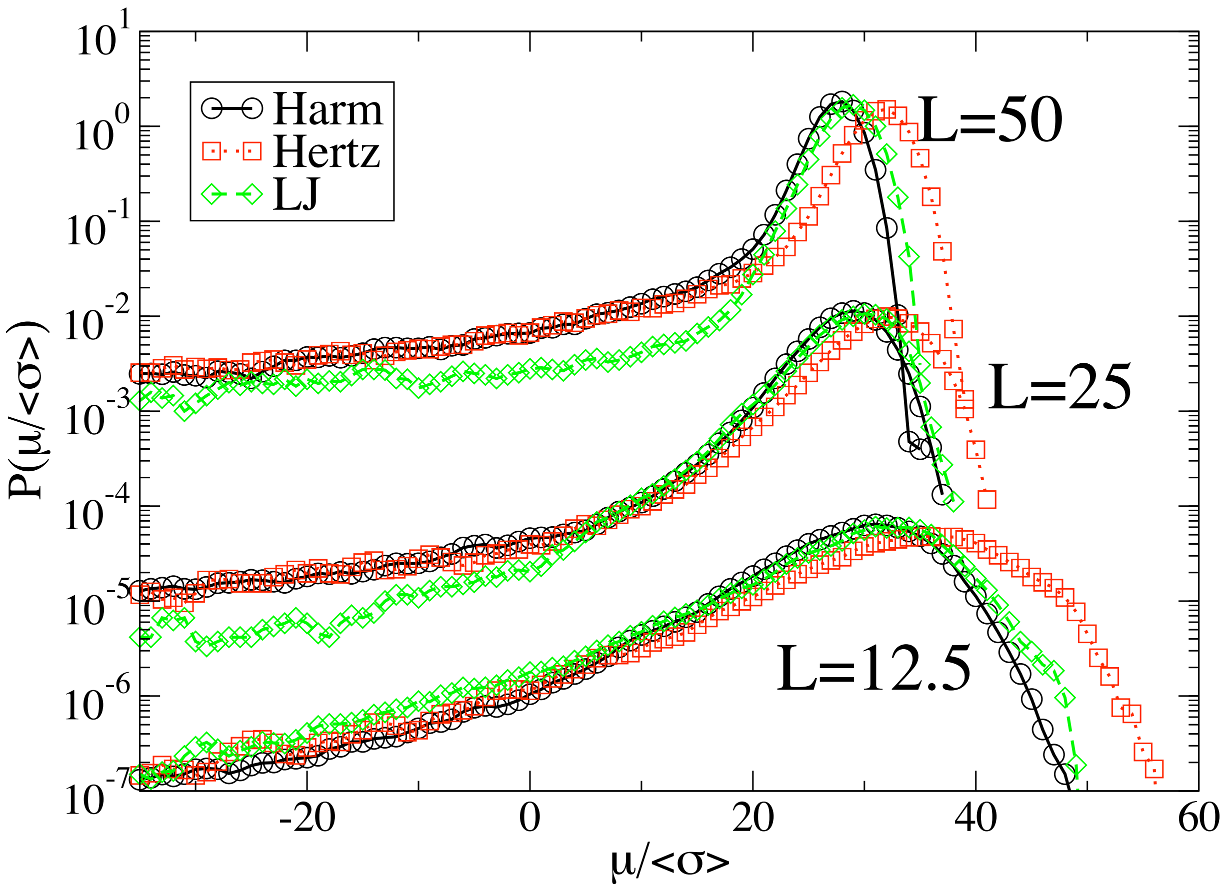
Collect statistics for different system size and interaction potentials:

- "Modulus"
- Elastic interval
- Stress drop
- Energy drop



<σ>/μ is universal!
~ 3%

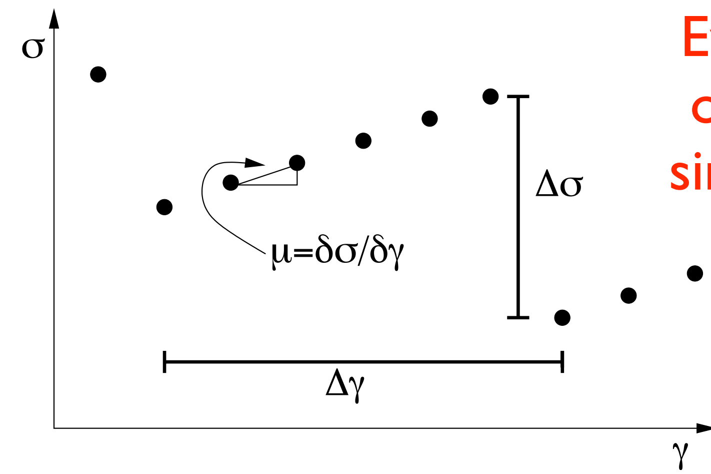




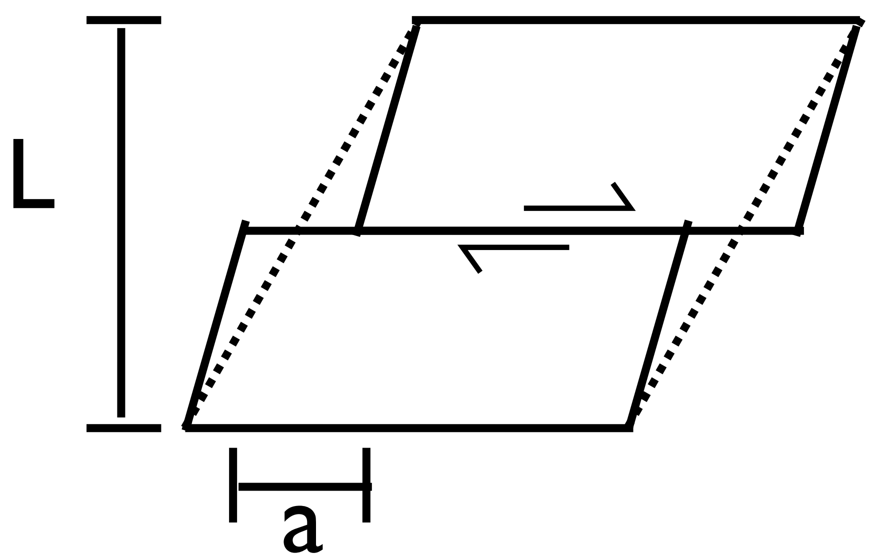
Statistics and size scaling

Collect statistics for different system size and interaction potentials:

- "Modulus"
- Elastic interval: $\Delta\gamma$
- Stress drop: $\Delta\sigma$
- Energy drop: ΔU



Event size independent of potential and scales simply with system size!

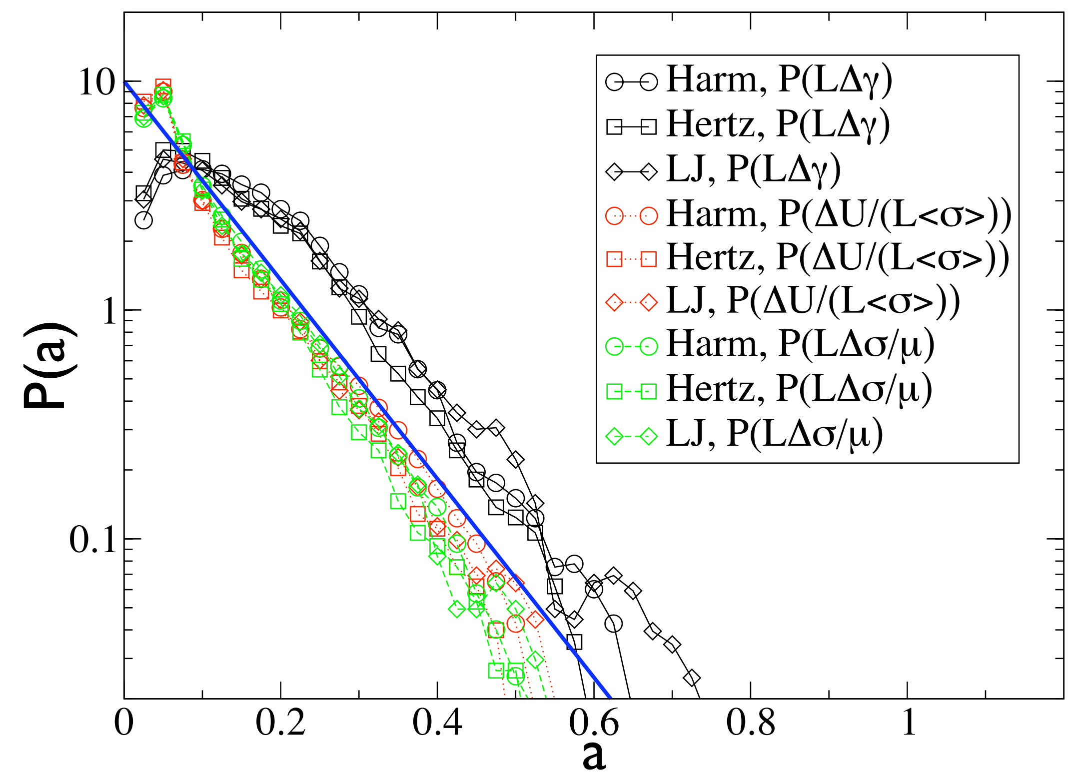


$\Delta\gamma \sim a/L$

$\Delta\sigma \sim \mu\Delta\gamma \sim \mu a/L$

$\Delta U \sim (L^2/\mu)\langle\sigma\rangle\Delta\sigma$
$\sim aL\langle\sigma\rangle$

Scaled distributions of $\Delta\gamma$, $\Delta\sigma$, ΔU



Scaling argument: slip by length "a"

Summary: Plastic response

- Plastic response is intermittent with large, system-spanning events (avalanches)
- Avalanches composed of clusters of local slip (STZs)
- STZs interact elastically
- Universal yield strain $\varepsilon \sim 3\%$... agrees with experiments
- Universal slip amplitude $a \sim .1$ particle diameters... experiments difficult

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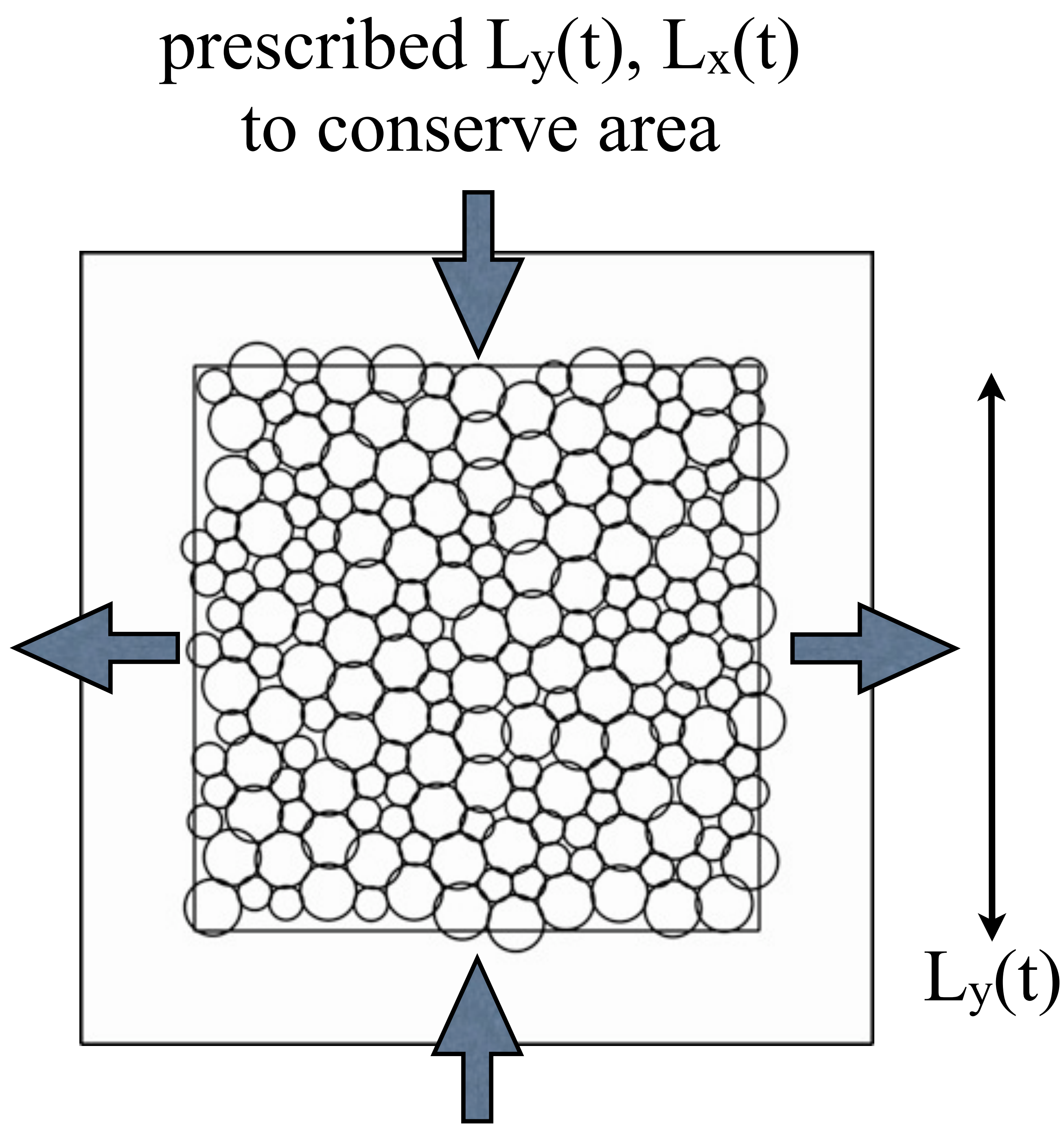
Zero temperature molecular dynamics

- 2D Molecular Dynamics:
  - binary Lennard-Jones quenched at Pressure=0
  - relative velocity damping (Kelvin/DPD)
  - axial, fixed area strain
  - periodic boundaries
  - system sizes up to 3000x3000 ~ 10M particles
  - Quasi-static limit (about 500 CPU days / run)



Local vorticity, ω

For each triangle:

$$\frac{\partial u_i}{\partial x_j} = F_{ij}$$

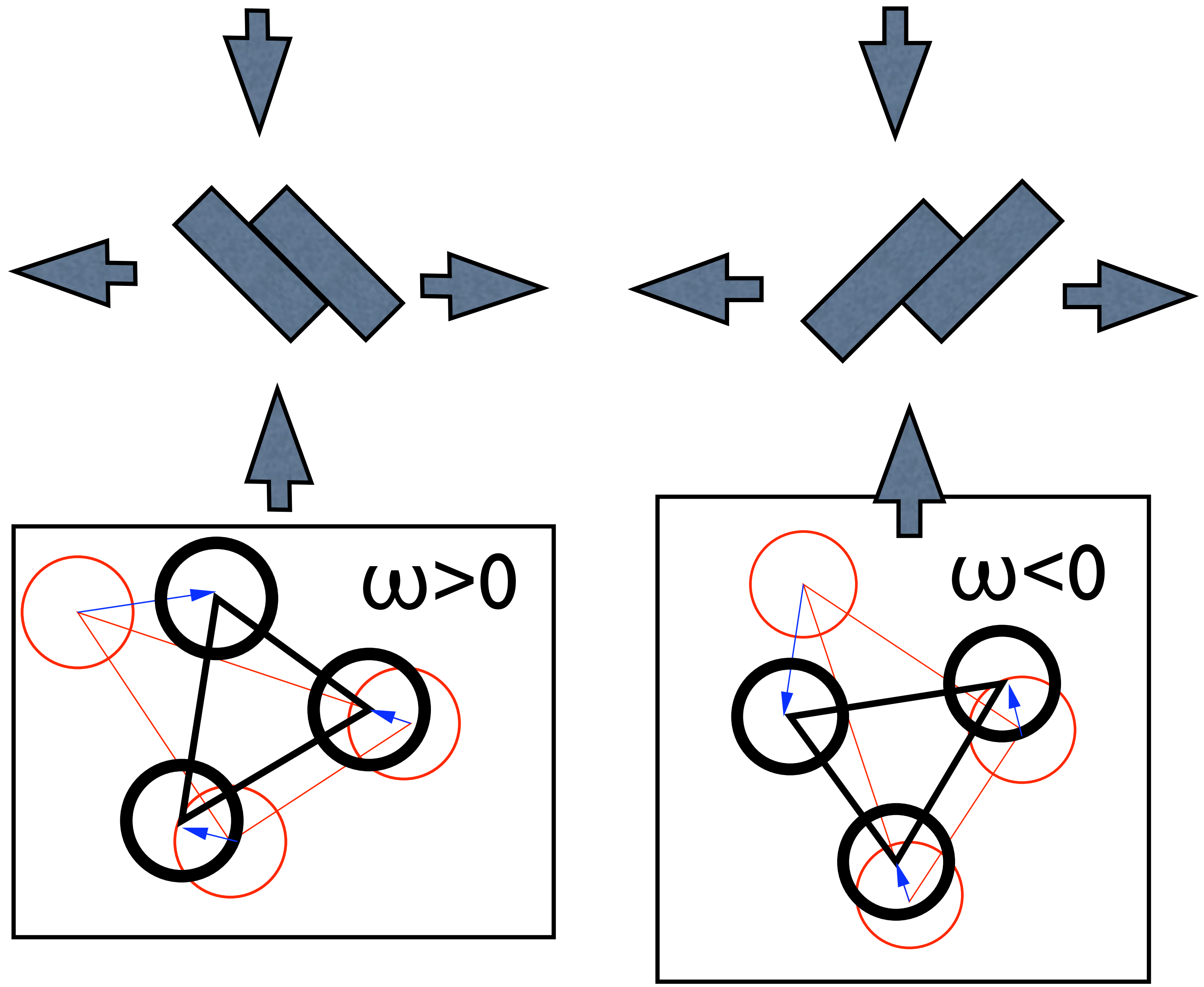
$$\epsilon_1 = \frac{F_{xx} - F_{yy}}{2}$$

$$\epsilon_2 = \frac{F_{xy} + F_{yx}}{2}$$

Invariants:

$$\epsilon = \sqrt{\epsilon_1^2 + \epsilon_2^2}$$

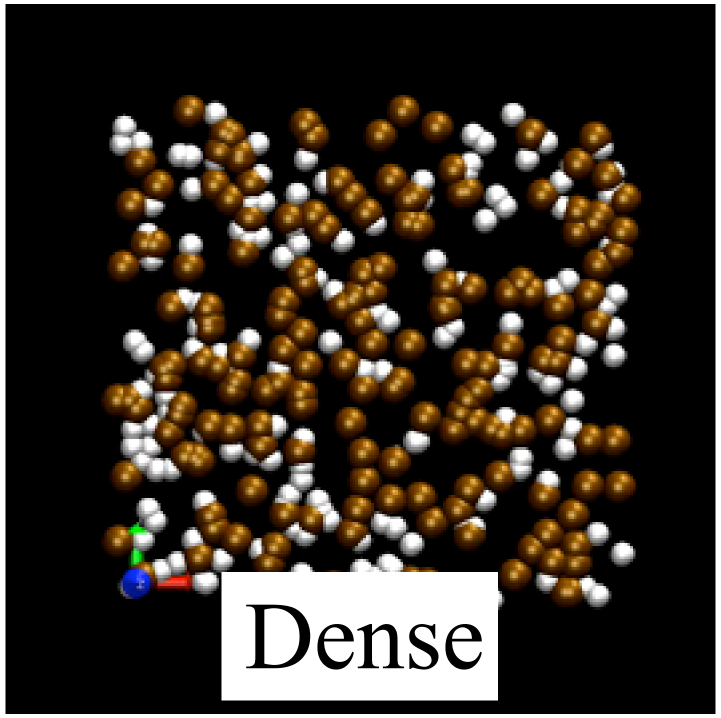
$$\omega = F_{xy} - F_{yx}$$



Future directions

•Recall:

- Differences in elementary physics:
  - Inertial or overdamped?
  - "Real" temperature
  - Dissipation mechanisms / hydrodynamics
  - Coulomb friction
  - Attractive forces / adhesion



•How do microscopic details affect the intermittency, slip avalanches, elasticity, rheology, and yield?

- Currently looking at:
  - densities near random close packing (RCP)
  - massless (mean field bubbles) and massive (frictionless granular DEM) models.