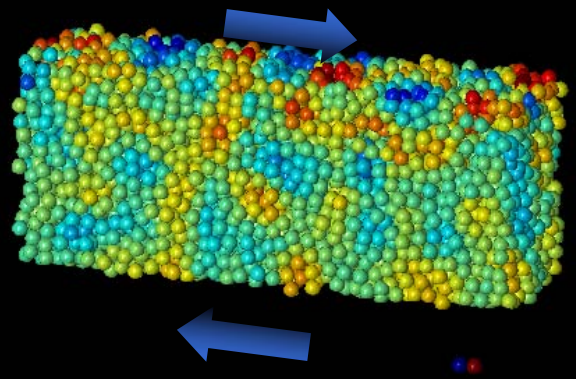
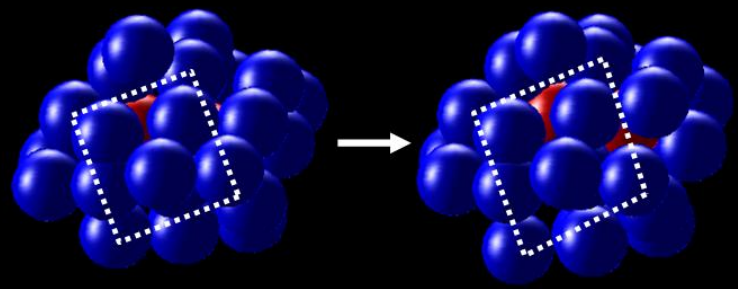


Colloidal Glasses

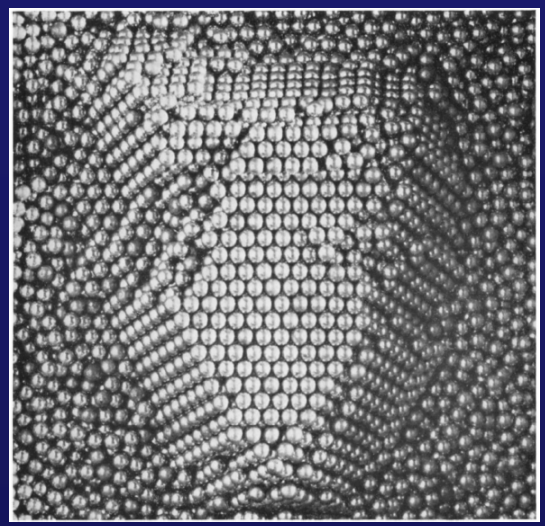
3D Analogue Computers for studying Flow in Amorphous Solids

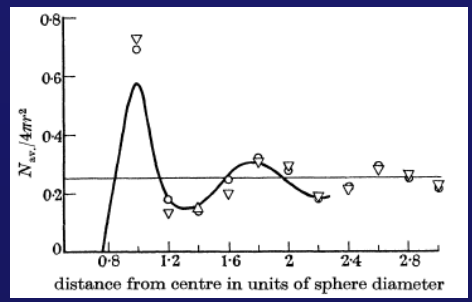
Peter Schall, *University of Amsterdam*



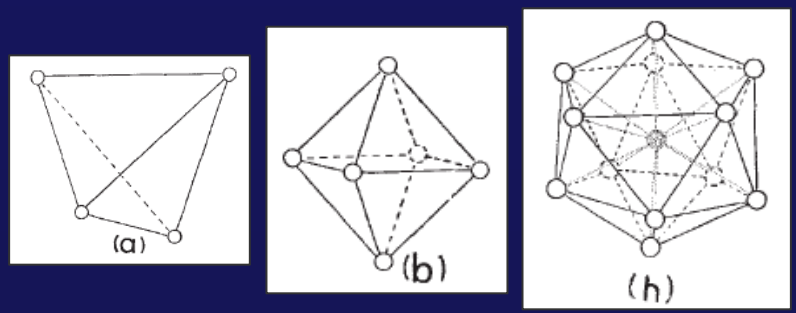
Hard spheres

Bernal, Finney
The structure of liquids *et al.* 1960-70





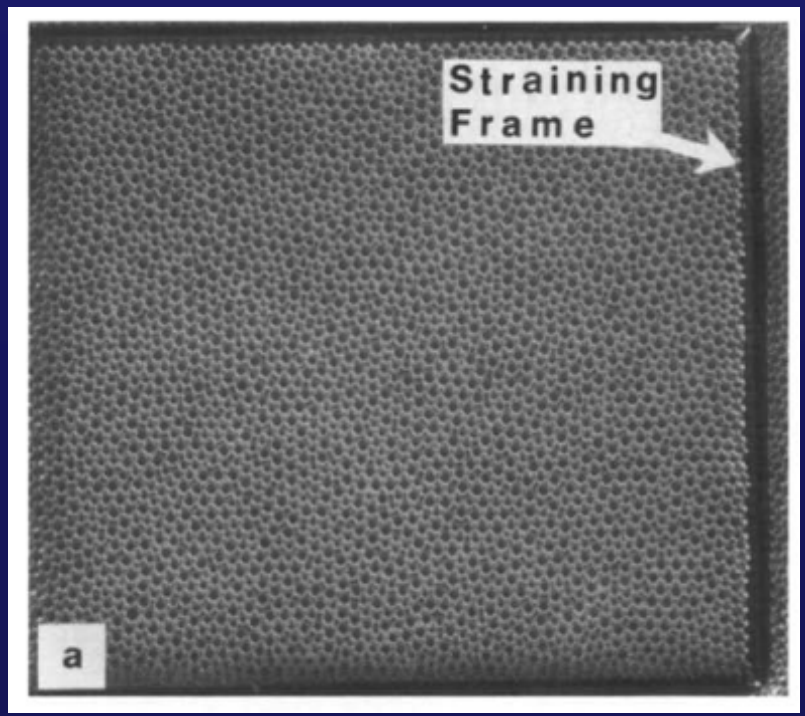
Pair correlation function



Canonical Holes

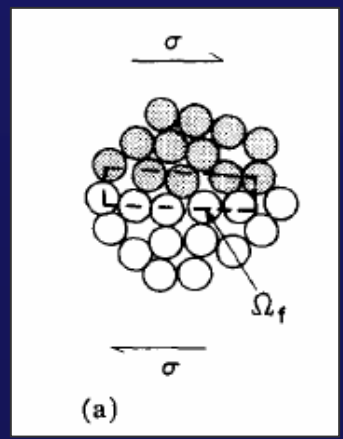
Bubble raft experiments

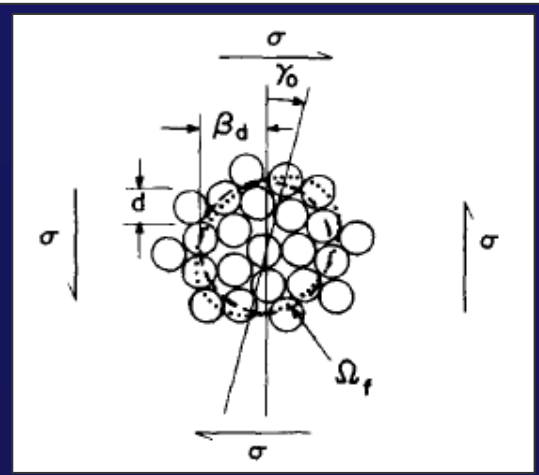
- Bragg-Nye Bubble raft:
  Dislocations and Dislocation motion in crystals (1950‘s)
- Disordered Bubble raft (Argon, Kuo 1979)
  Shear transformation zones in glasses



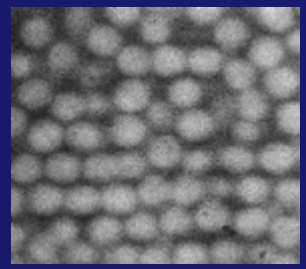
Plastic Flow in a Disordered Bubble Raft (an Analog of a Metallic Glass)

A. S. ARGON and H. Y. KUO*
Massachusetts Institute of Technology, Cambridge, Mass. 02139 (U.S.A.)
(Received November 1, 1978; in revised form December 13, 1978)

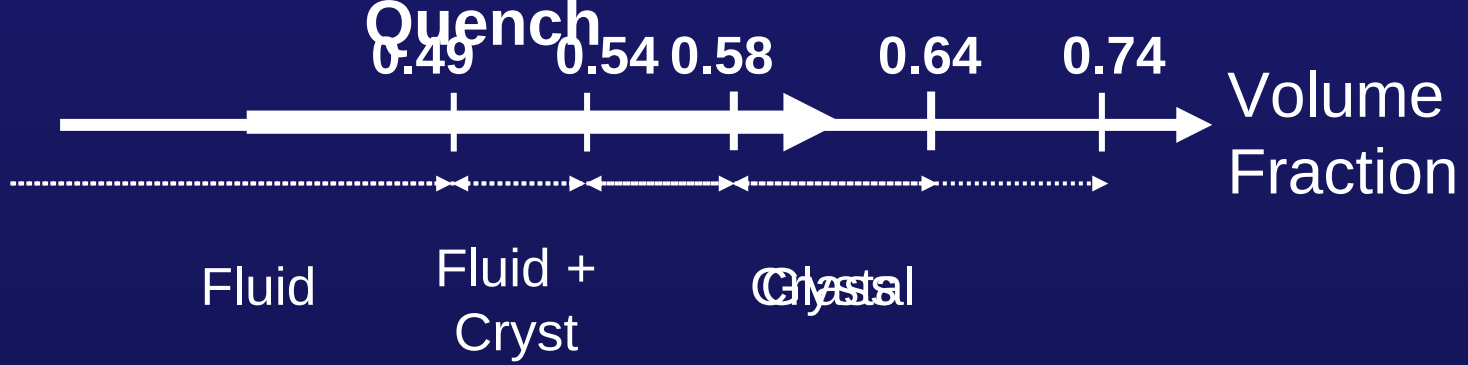




Colloidal Hard Spheres

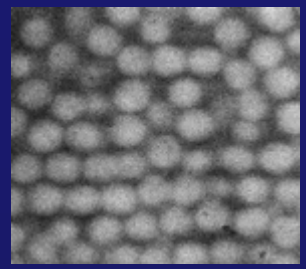


Hard-sphere Phase Diagram



(Alder, Wainwright 1957)

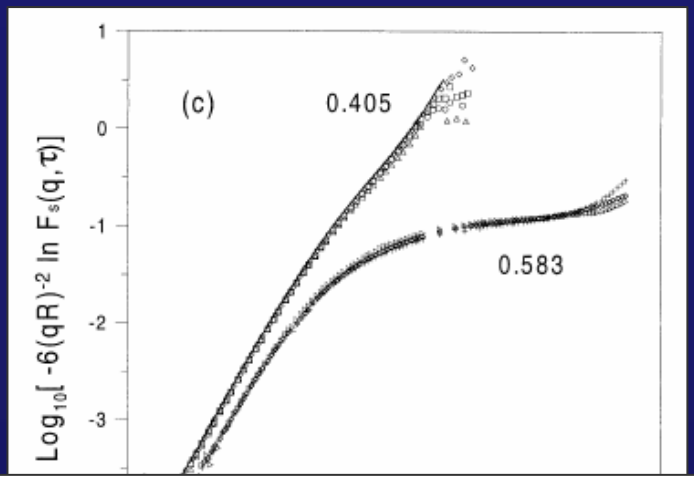
Colloidal Hard Spheres

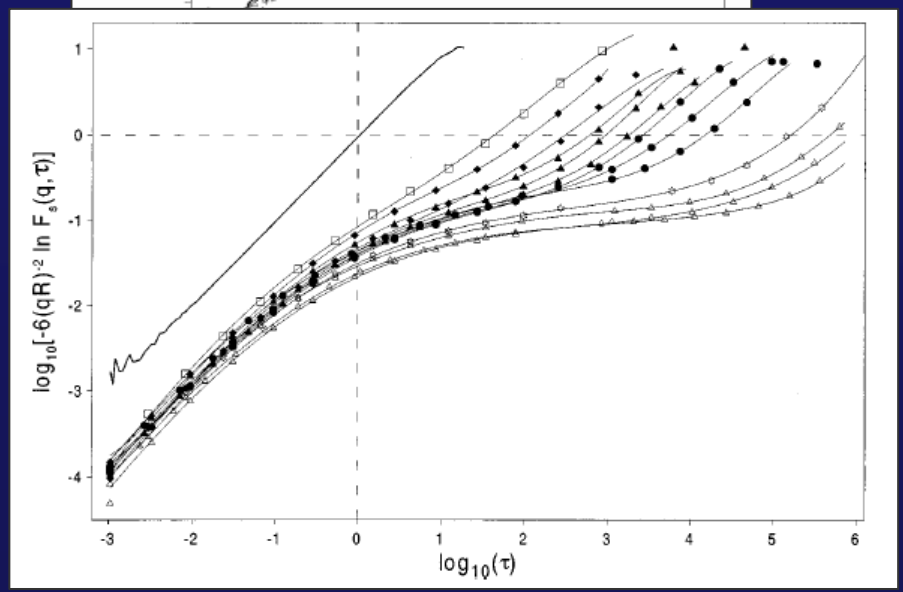


Scaling of the Moduli

Elastic Modulus ~ Energy / a^3

Colloidal Glasses





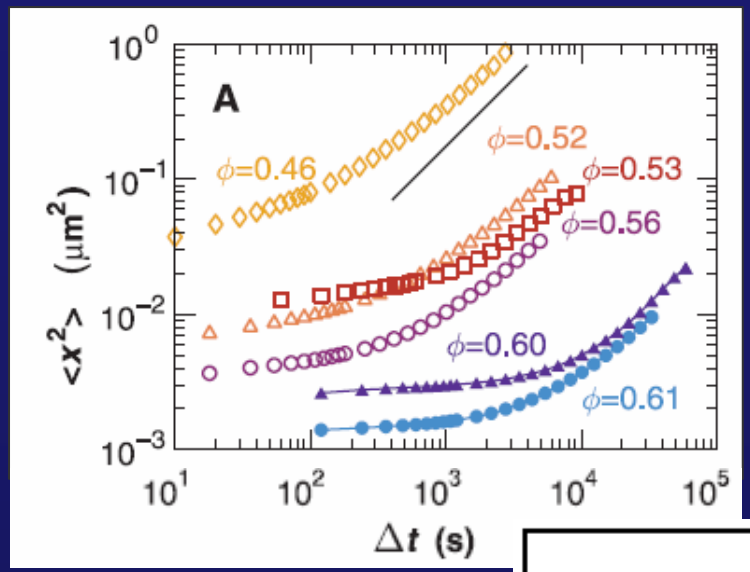
- Light scattering

van Megen *et al.* (PRE 1998)

Colloidal glass transition

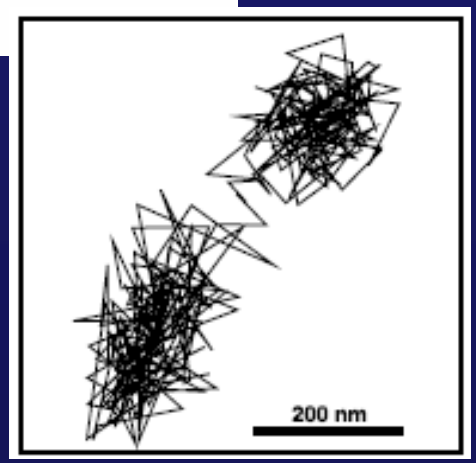
$\phi_g \sim 0.57$

Colloidal Glasses

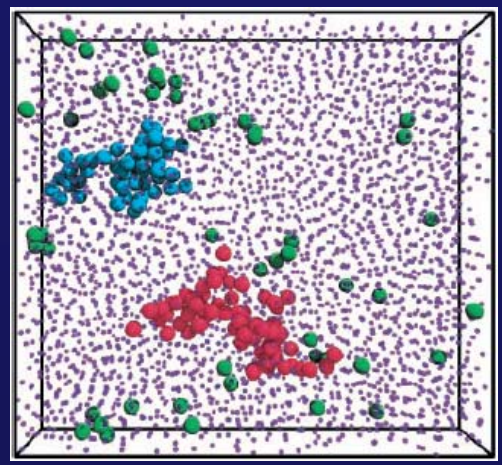


- Microscopy

Weeks, Weitz *et al.* (Science, 2000)



Caging



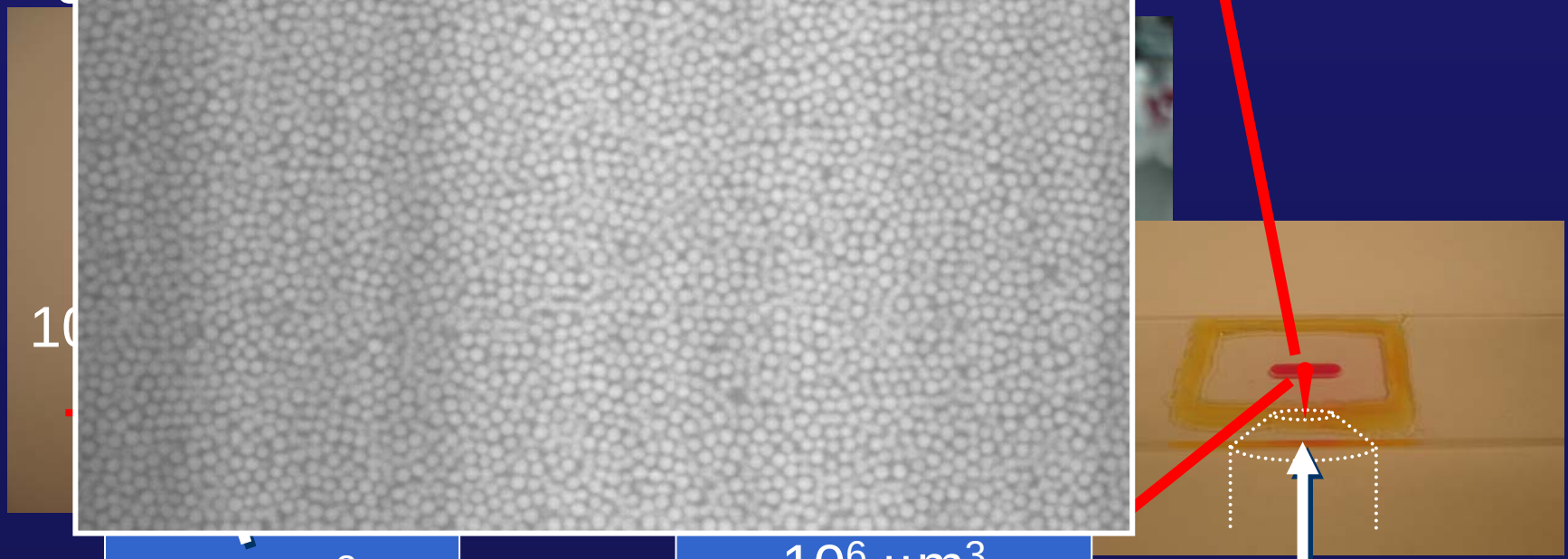
Dynamic heterogeneity

ters

Lig

$\phi \sim 0.59$

Diameter 1.3 μm

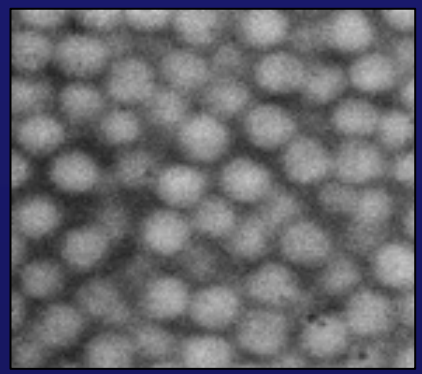


1mm^3
10^9 particles

10^6 μm^3
$2x10^5$ particles

Colloids: 3D Analogue Computers

Colloidal Systems
Time Scale

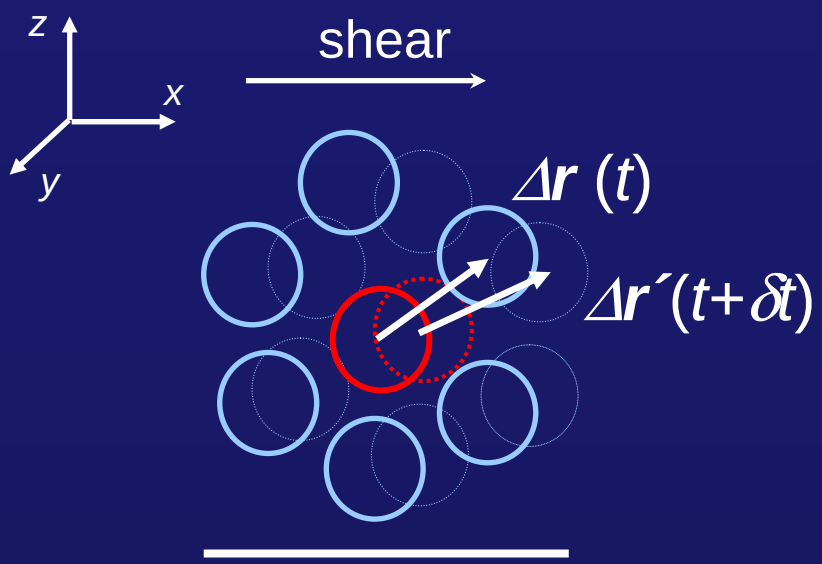


Collision time
τ = (1/100) s

Total “simulation“ time : 10 h $\approx$ 3million τ

Time increment : 1 min $\approx$ 6000 τ

Strain and non-affine displacements

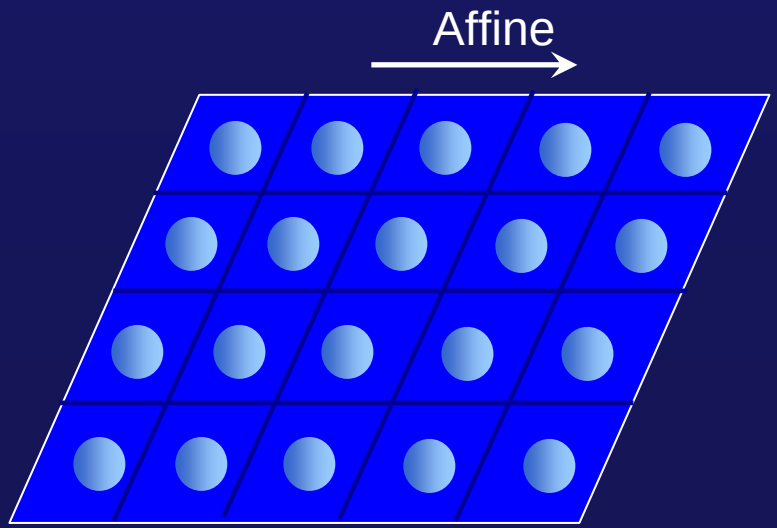


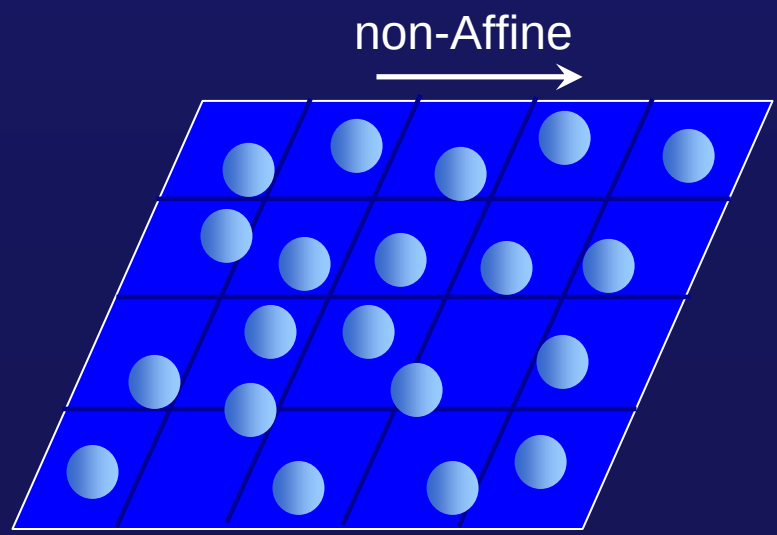
Affine transformation : γ

$$\Delta \mathbf{r}' = \Delta \mathbf{r} + \gamma \Delta \mathbf{r}$$

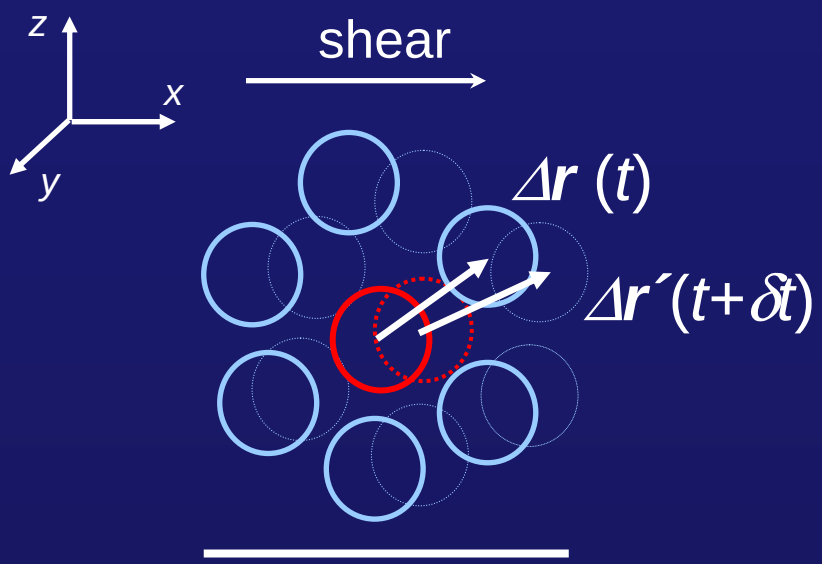
$$D^2_{\min} = \sum^{\text{neighbors}} (\Delta \mathbf{r}' - \gamma \Delta \mathbf{r})^2$$

Falk and Langer, *PRE* 57, 1998.





Strain and non-affine displacements



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$$D^2_{\min} = \sum^{\text{neighbors}} (\Delta \mathbf{r}' - \gamma \Delta \mathbf{r})^2$$

Falk and Langer, *PRE* 57, 1998.

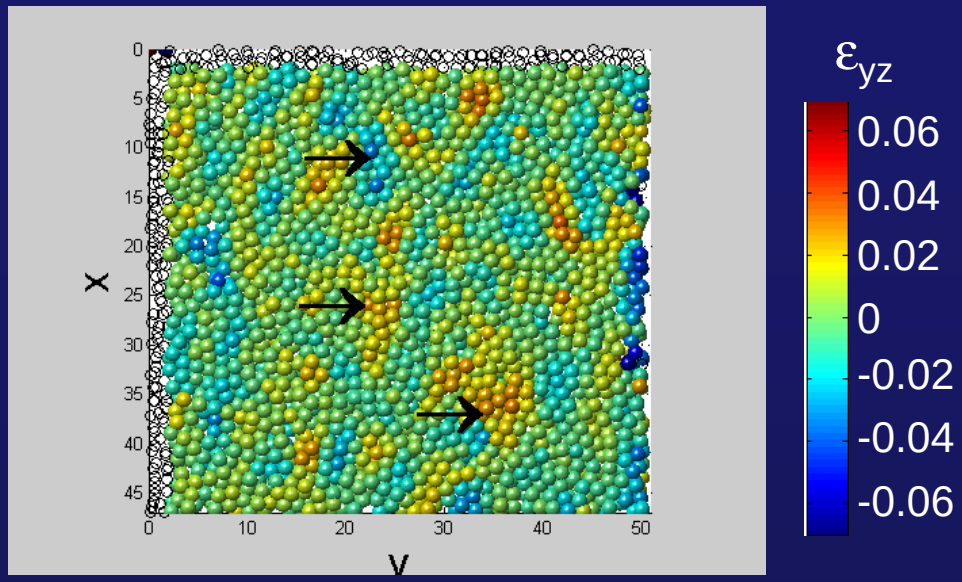
Symmetric part of γ

Strain tensor $\varepsilon_{ij} =$

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

Colloidal Glass - quiescent

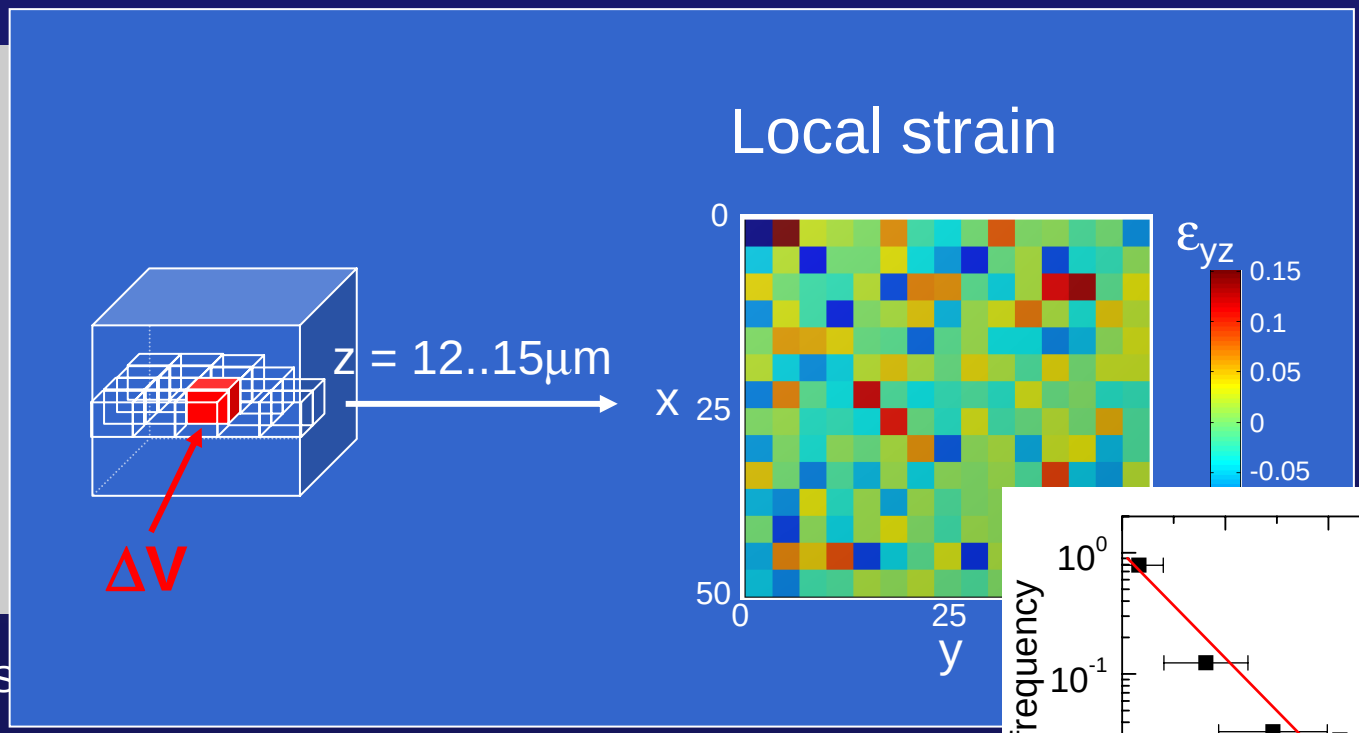
Thermal fluctuations

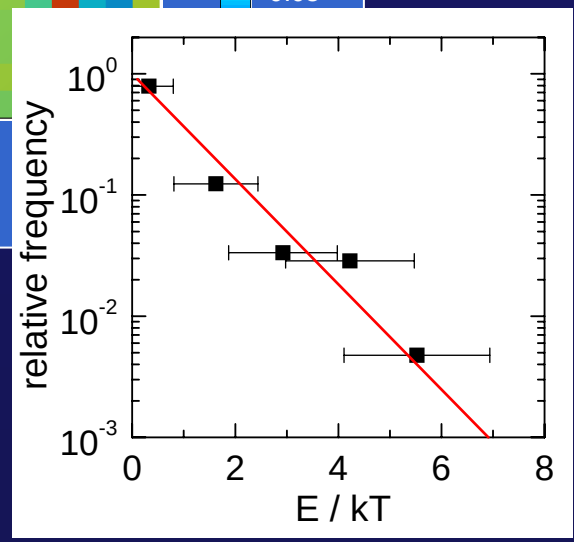


Strain distribution in subsequent 2.5 min intervals.

Colloidal Glass - quiescent

Thermal fluctuations

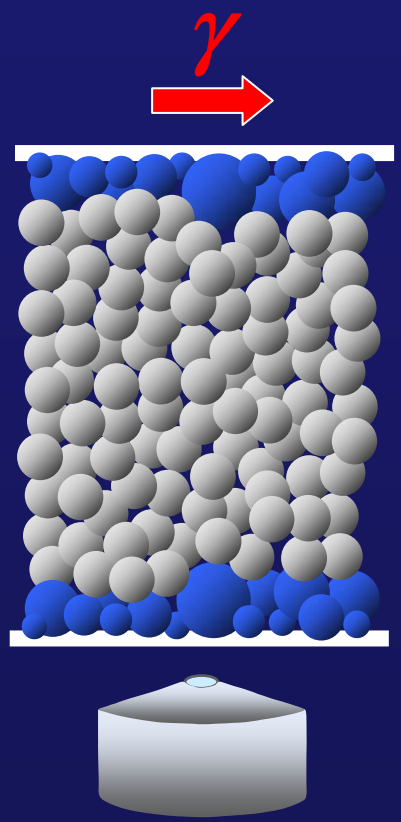




Energy $W = \frac{1}{2} \mu (2\varepsilon_{yz}^{2}) \Delta V$

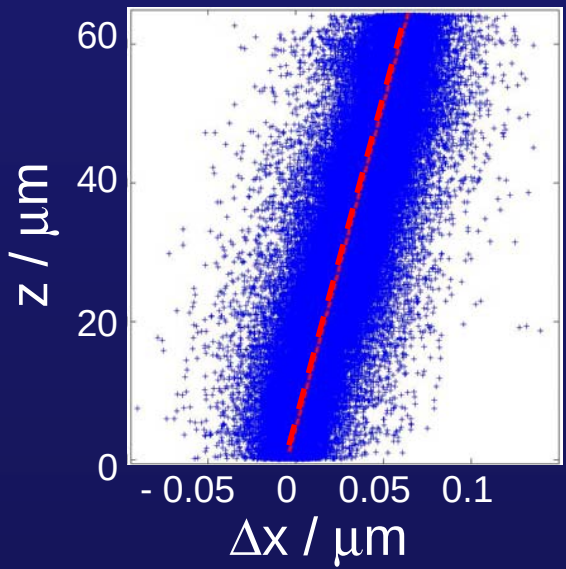
⇨ Shear Modulus $\mu = 0.05$ Pa

Shear experiments



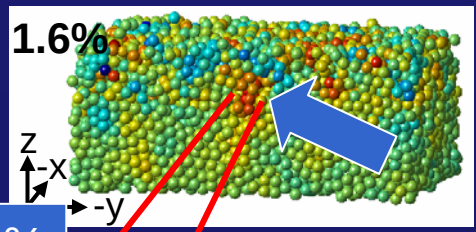
Confocal microscopy

Particle displacements



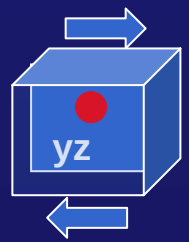
Shear transformation zones

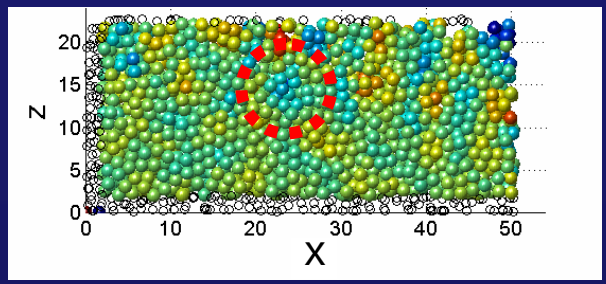
Incremental strain

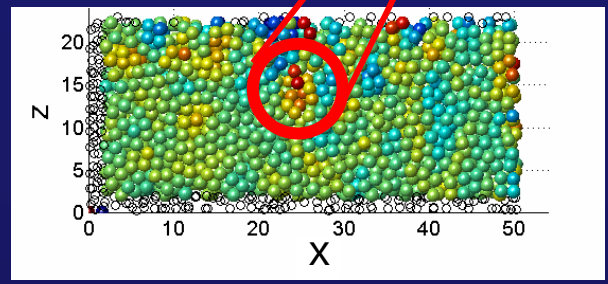


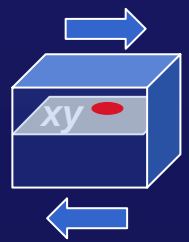
1.2->1.4 %

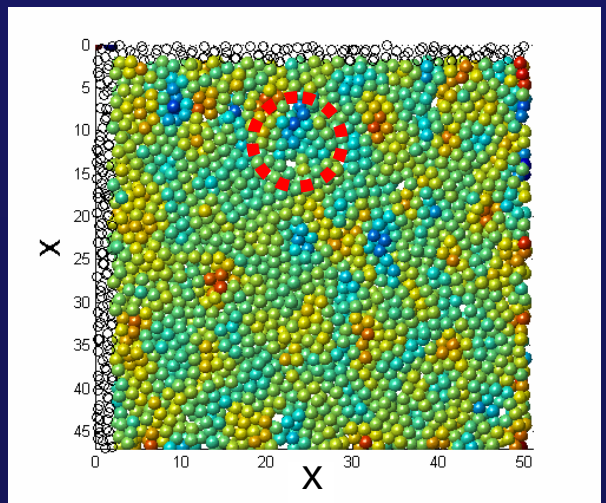
1.4->1.6 %

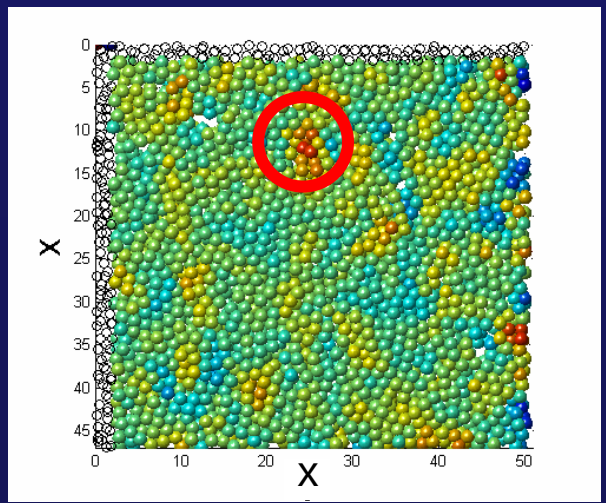




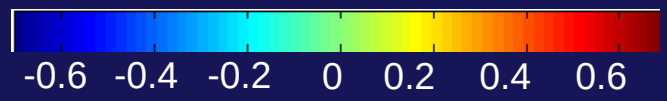






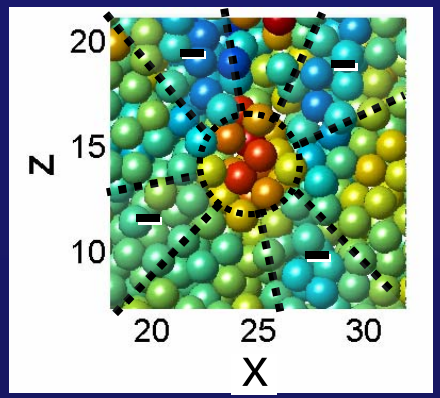


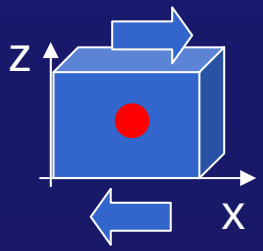
(PS, Weitz, Spaepen, Science 2007)

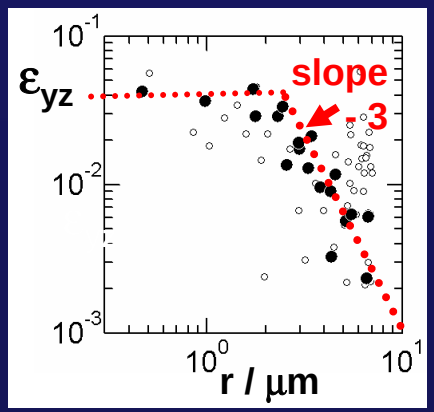


Shear transformation zones

Incremental strain



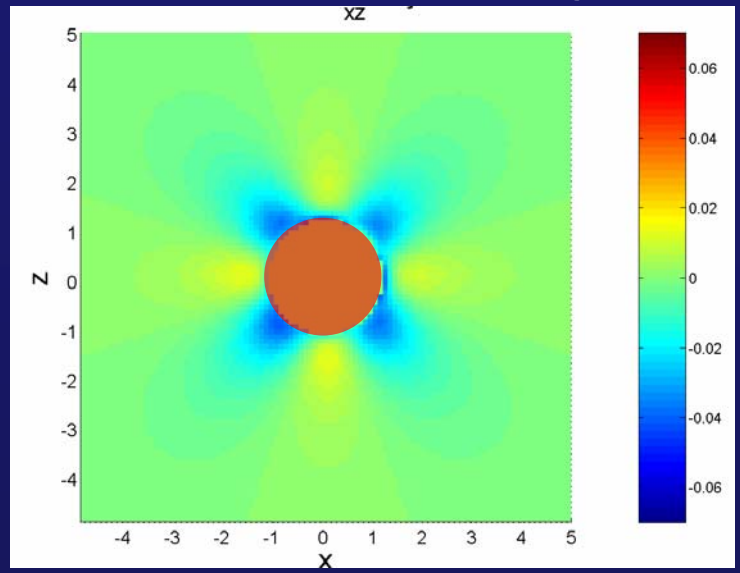




$$\varepsilon_{yz} \sim \frac{1}{r^3}$$

(PS, Weitz, Spaepen, Science 2007)

Continuum elasticity

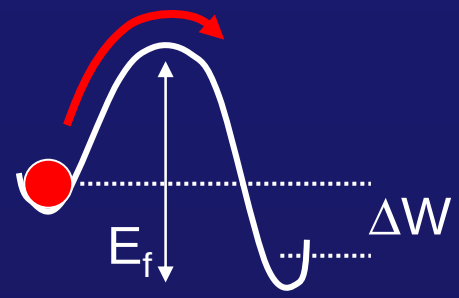


Thermally activated formation of shear zones

Formation of Shear events

Rate of formation

$$J = N f_0 \exp\left(-\frac{E_f - \Delta W}{k_B T}\right)$$

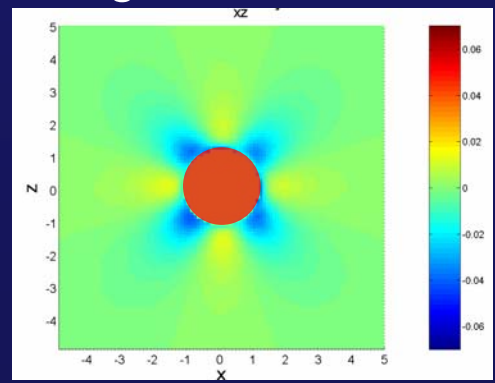


Formation Energy $E_f = \frac{1}{2} \int (2\mu\varepsilon_{ij}^2 + \lambda\varepsilon_{kk}^2)\, dV$

Supplied Work $\Delta W = \tau \int 2\varepsilon_{xz}\, dV$

ε_{ij}: Strain tensor
λ: Lamé constant
μ: shear modulus

Integrate strain field



Fundamental Transition

Elastic solid
affine
Elastic Modulus μ

Viscous Liquid
non-affine, diffusive
Viscosity η

Symmetry change

Spatial symmetry
Temporal symmetry

Elastic

$F(-t) = F(t)$

Energy storage

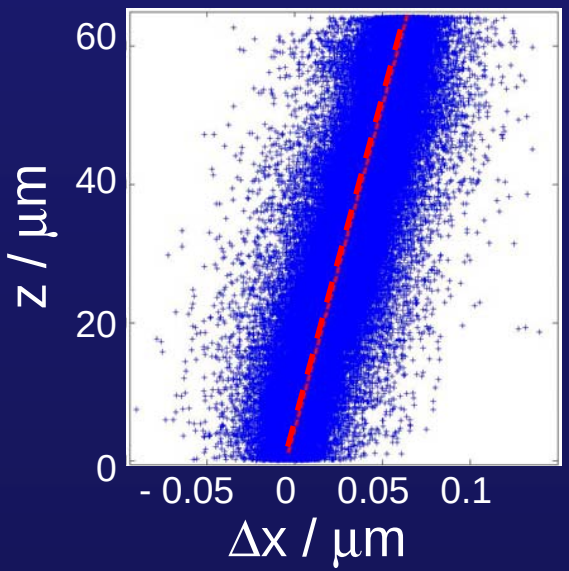
Plastic

$F(-t) = -F(t)$

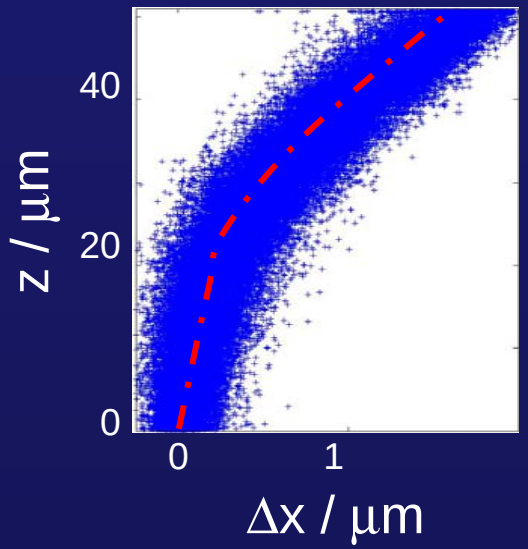
Energy loss

Solid-Liquid transition: Shear banding

Homogeneous



Inhomogeneous



$\dot{\gamma}_c$

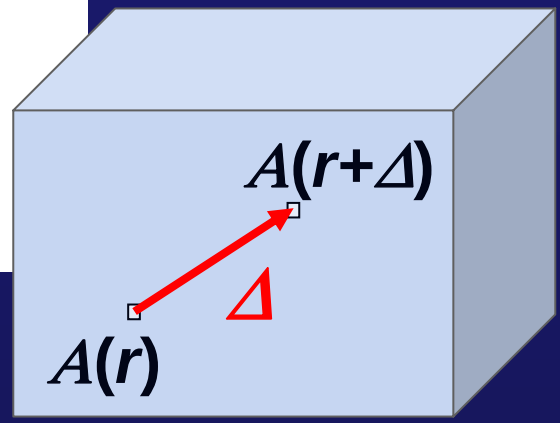
$\dot{\gamma}$ (s^{-1})

Spatial Correlations

$$C_A(\Delta) = \frac{\langle A(\bar{r})A(\bar{r}+\Delta)\rangle - (\langle A\rangle)^2}{\langle (A)^2\rangle - (\langle A\rangle)^2}$$

Δ : *difference vector*

$\langle\ \rangle$: *spatial average*

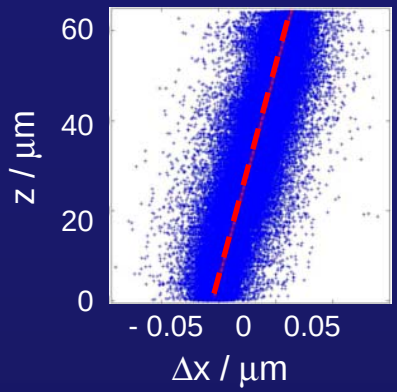


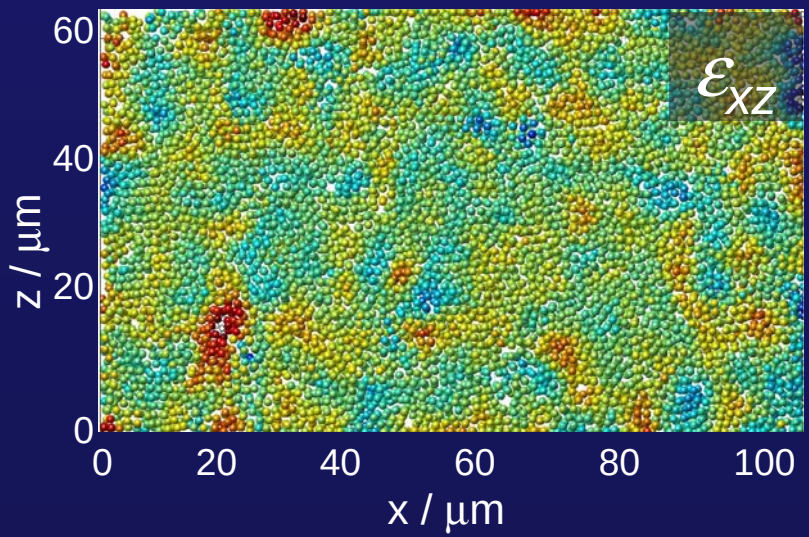
Strain correlation : $A = \varepsilon_{xz}$

Non-affine correlation : $A = D^2_{min}$

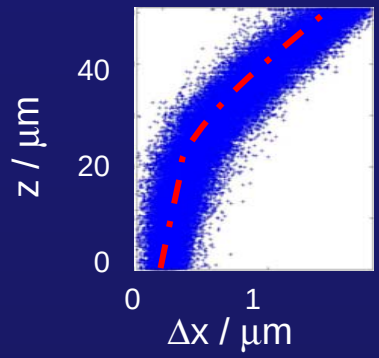
Shear banding transition

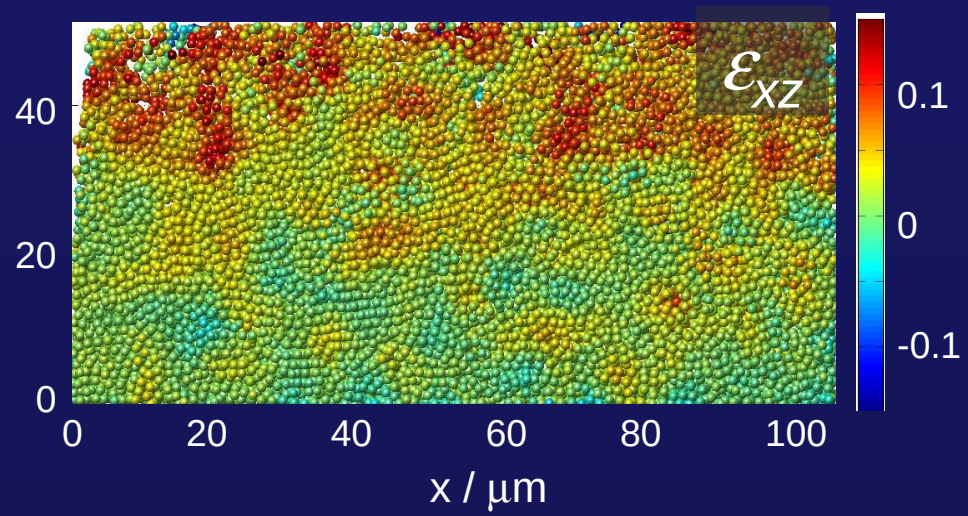
Homogeneous





Inhomogeneous



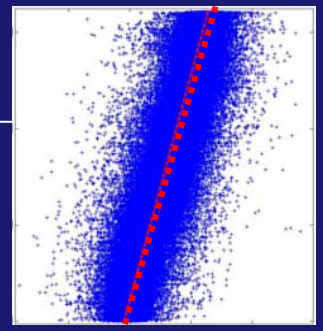


$\dot{\gamma}_c$

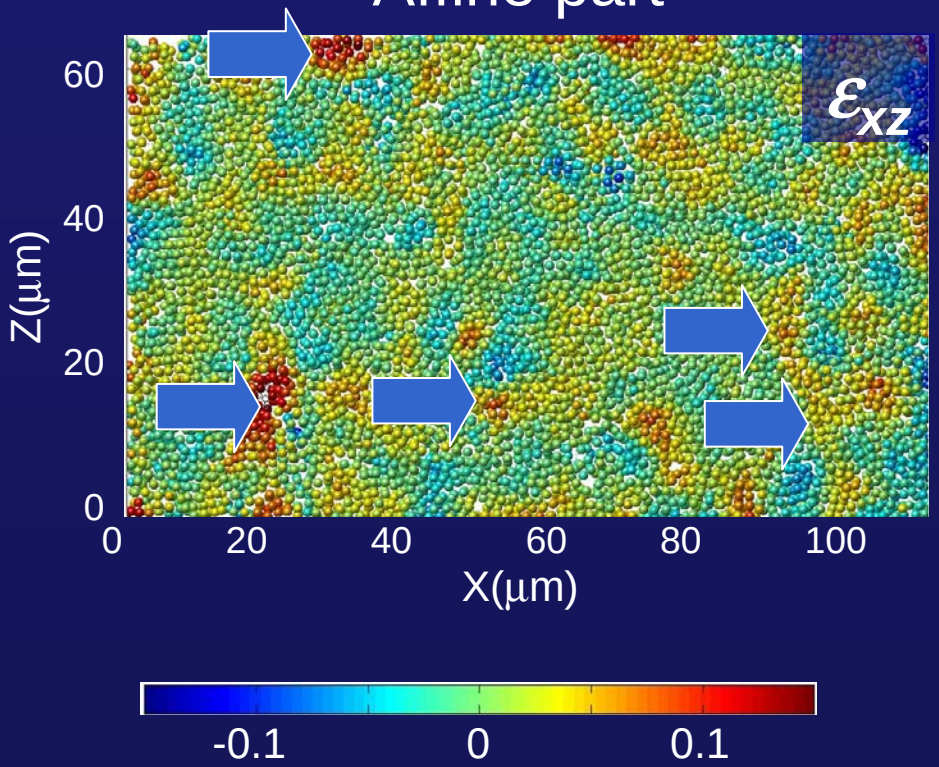
$\dot{\gamma}$ (s^{-1})

Homogeneous flow

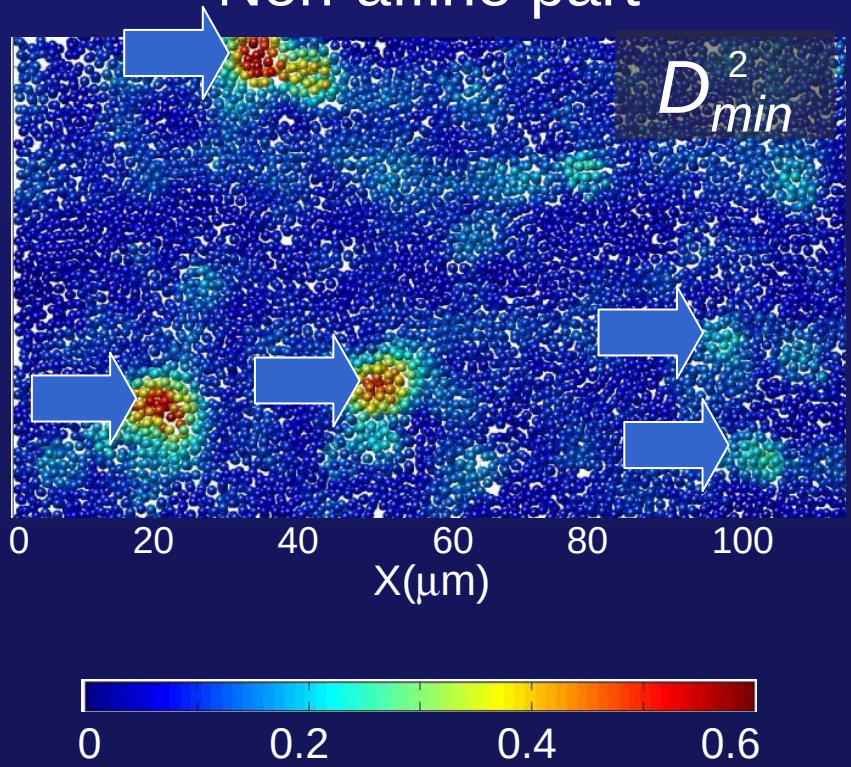
$\dot{\gamma} \sim 1 \times 10^{-5}\ s^{-1}$



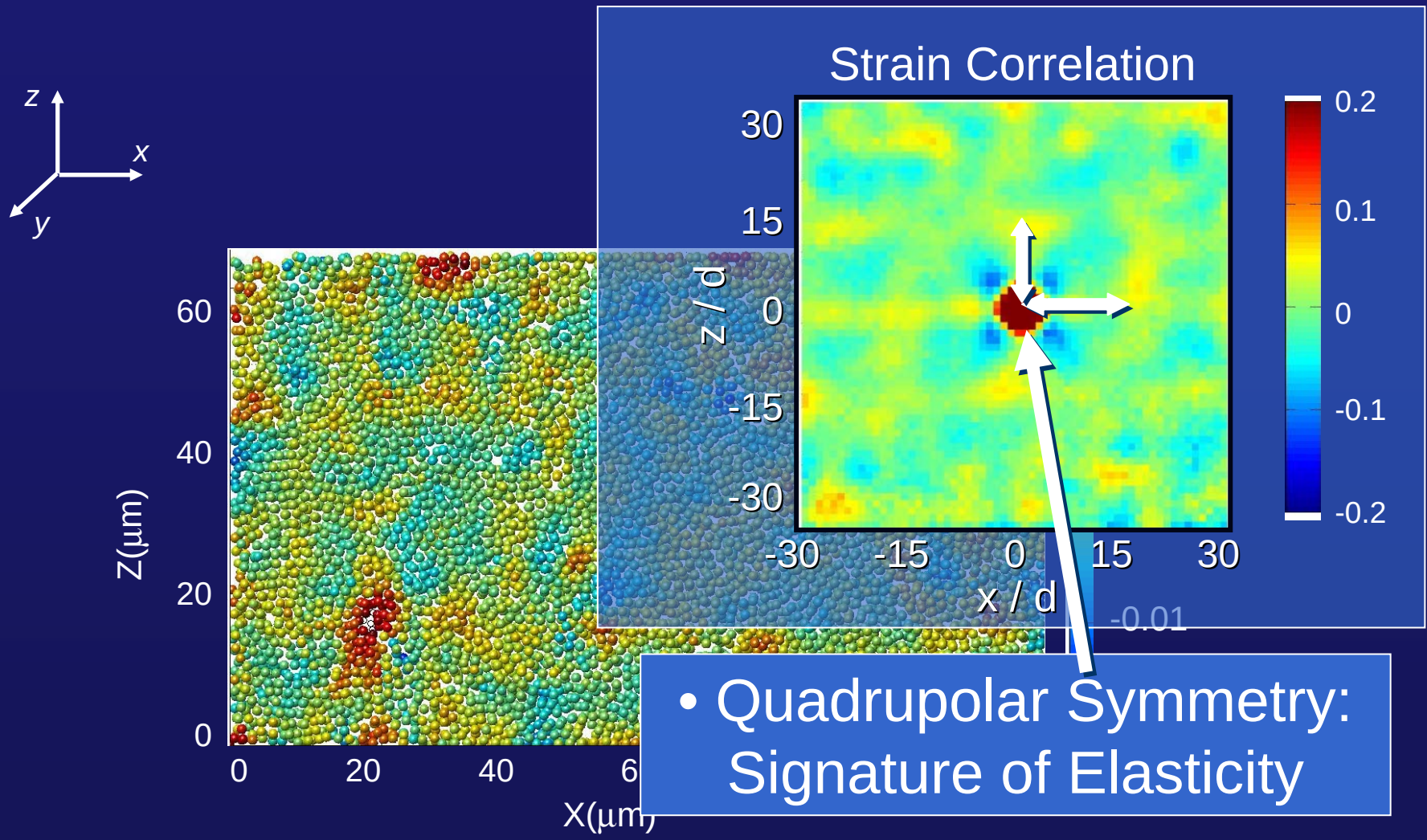
Affine part



Non-affine part

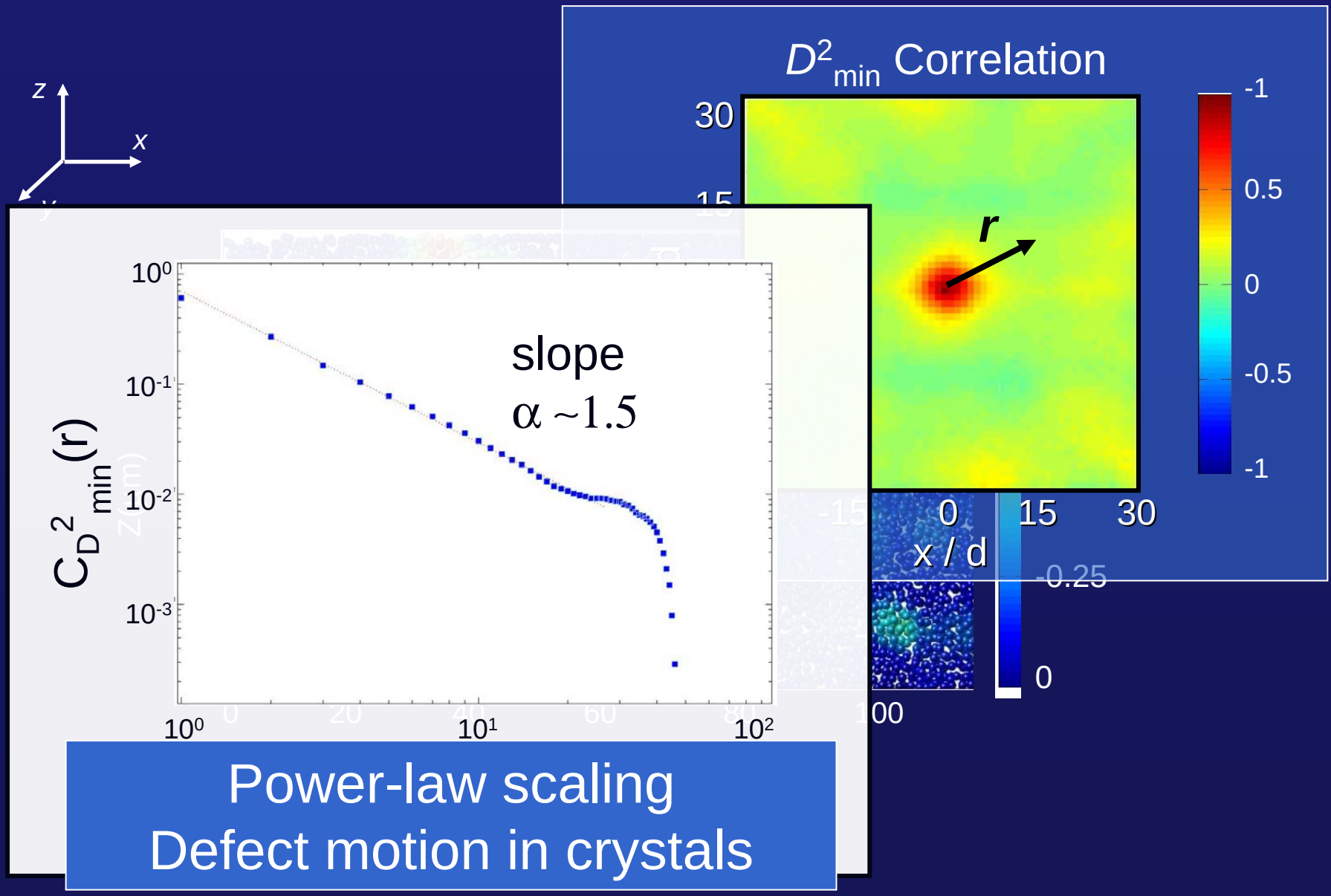


Shear Strain ε_{xz}



- Quadrupolar Symmetry: Signature of Elasticity

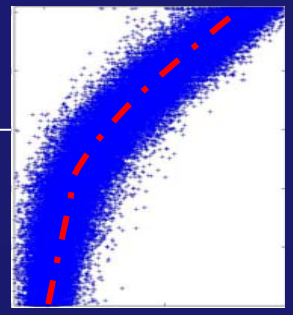
D^2_{min}

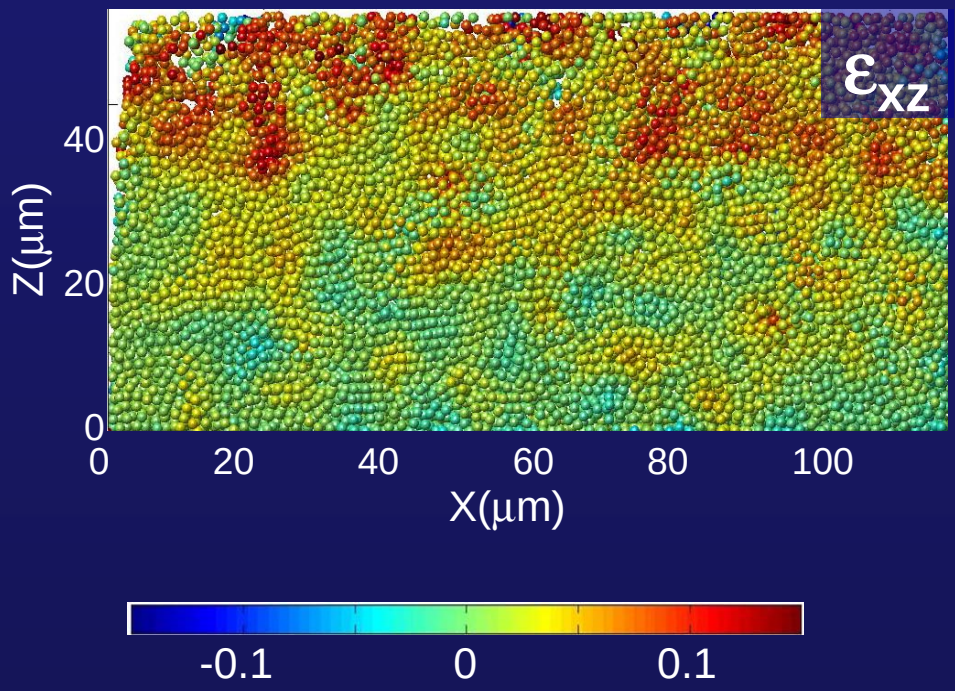


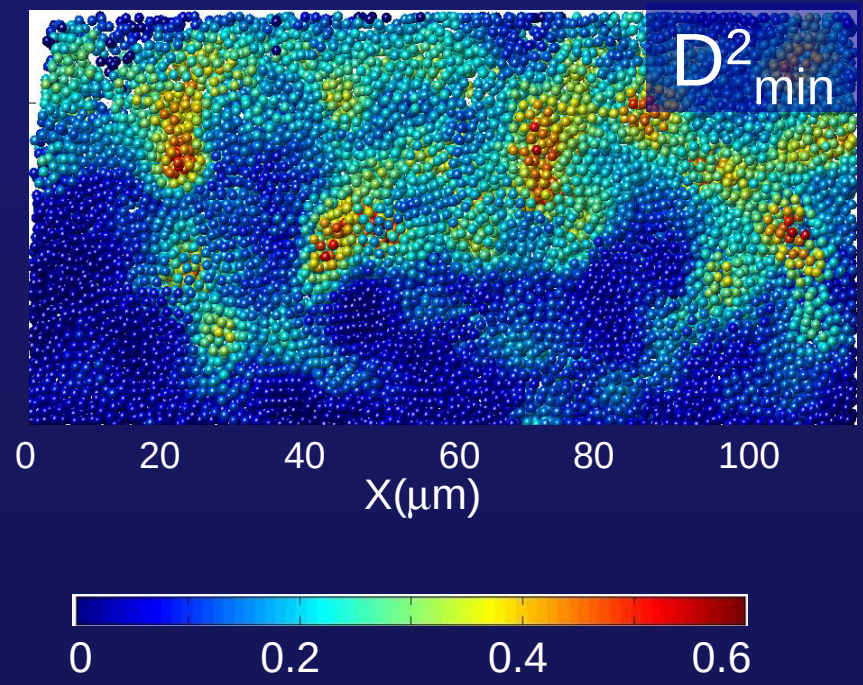
Power-law scaling
Defect motion in crystals

Shear banding

$\dot{\gamma} \sim 1 \times 10^{-4}\ s^{-1}$



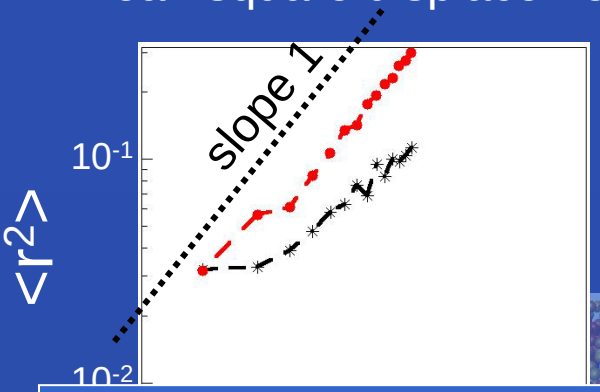


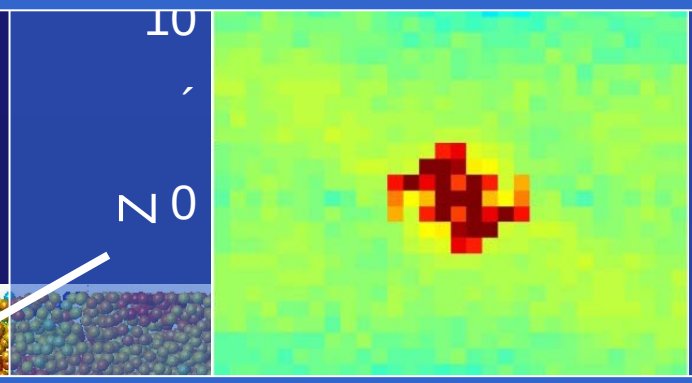


Strain Correlation functi

No quadrupolar symmetry
liquid like

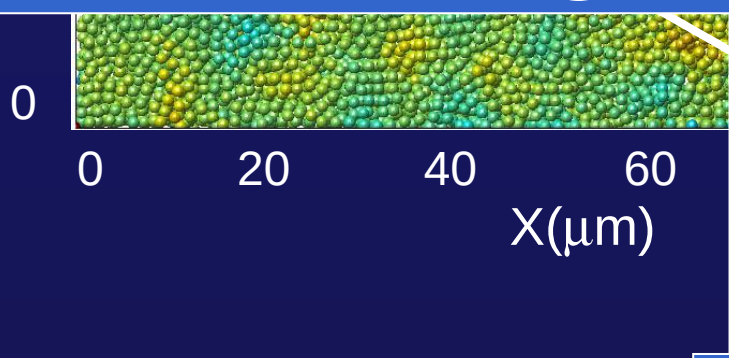
Mean square displacement

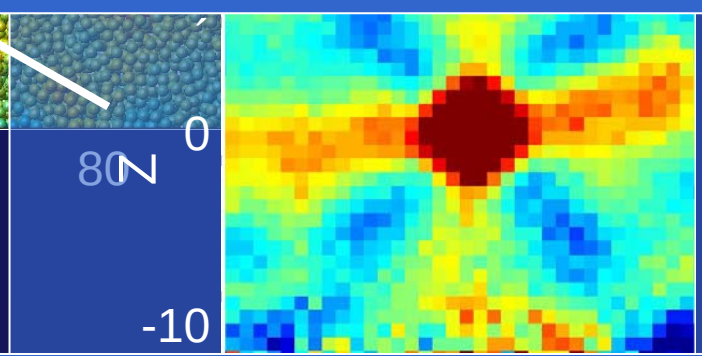




Fundamental Solid → Liquid transition

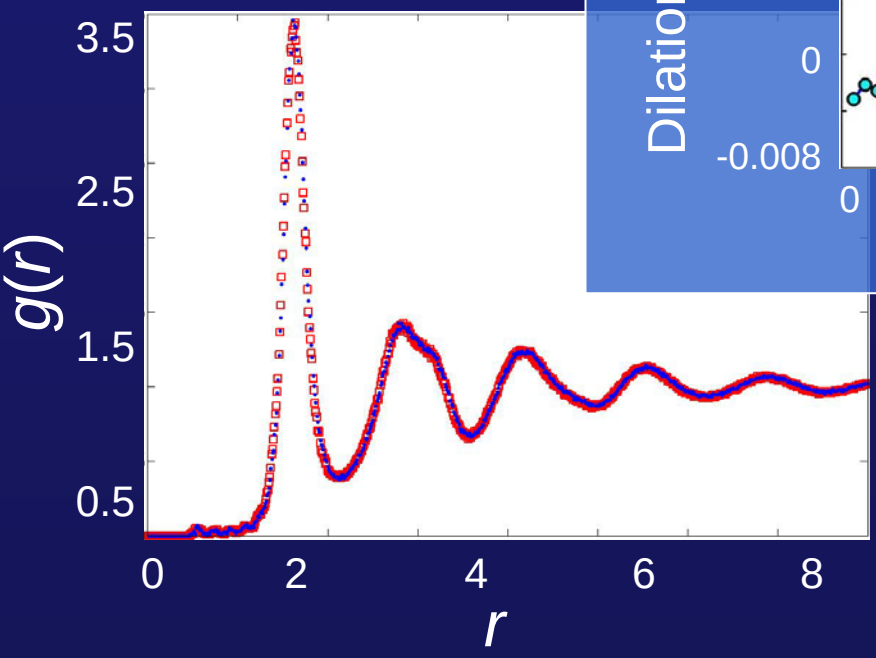
Origin ?

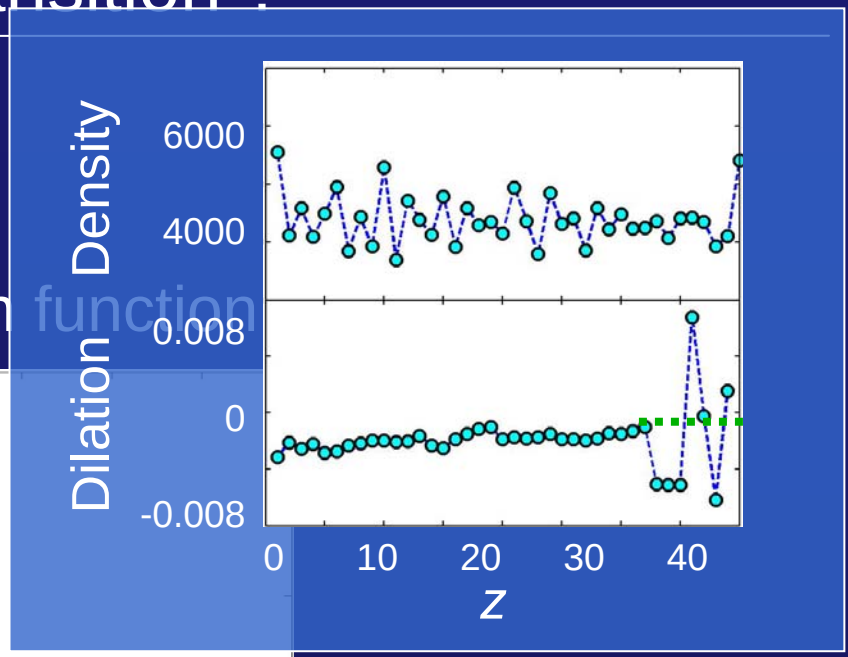




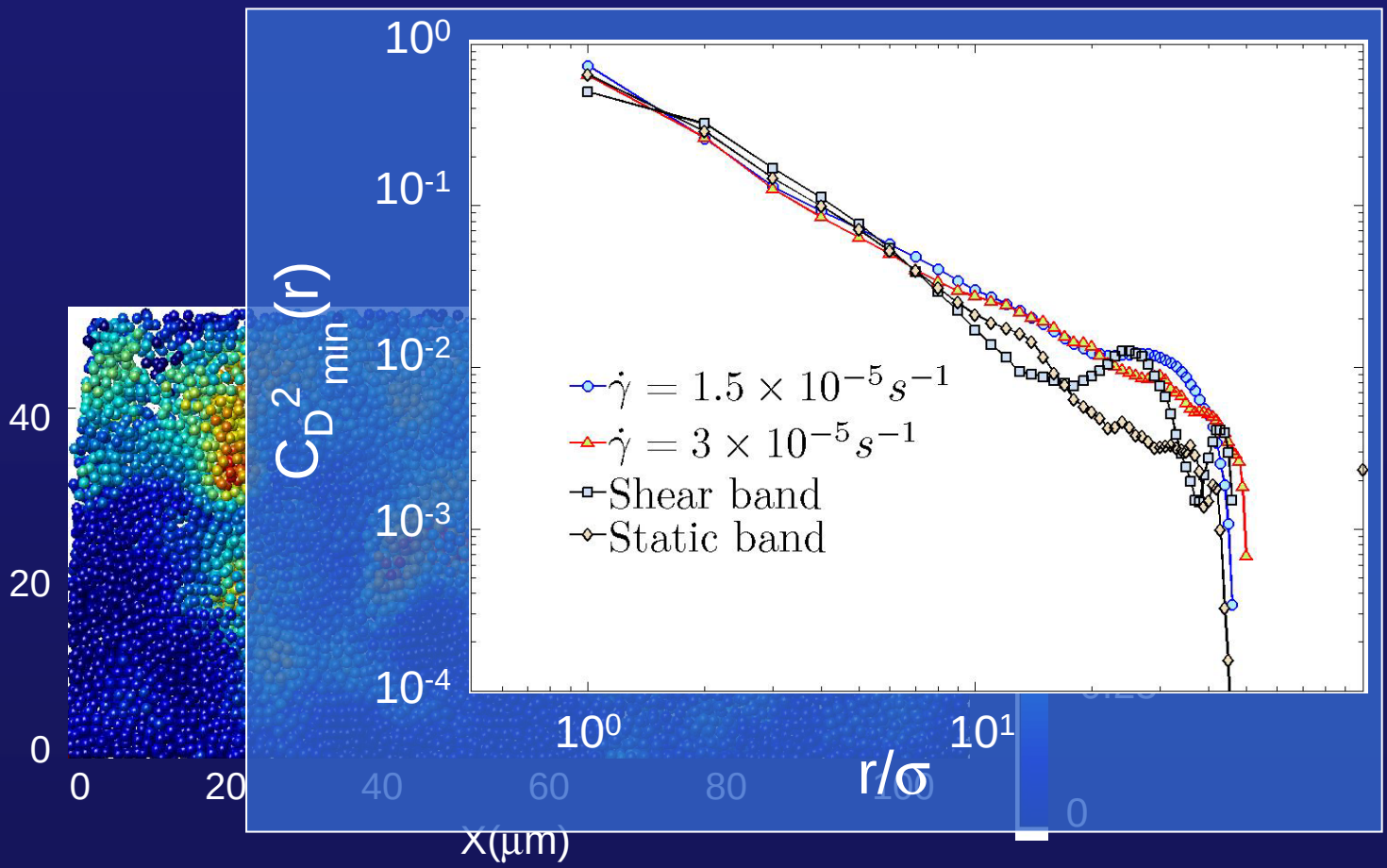
Quadrupolar symmetry
solid like

Structural transition ?

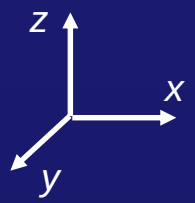




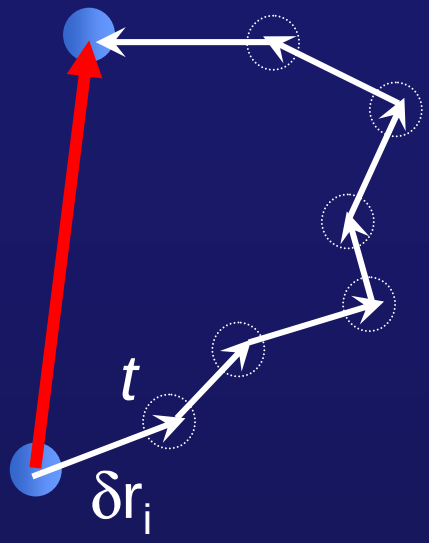
Spatial distribution of Flow ?



Particle trajectories



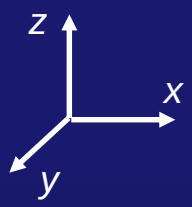
Dynamic Order Parameter ?

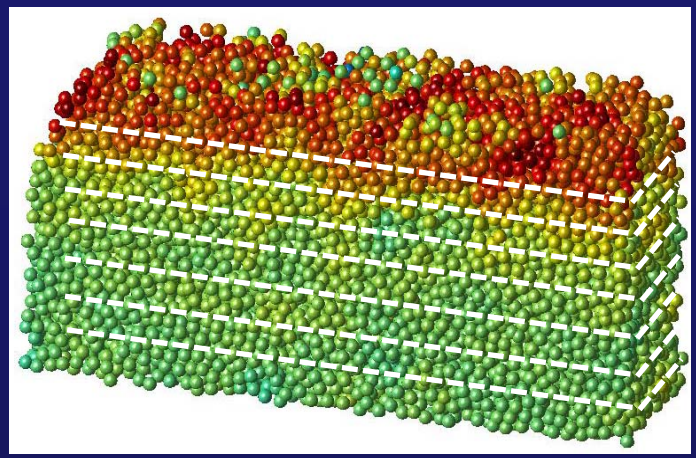


$$\mathbf{K} = \sum_{i=1}^{N} \sum_{t} |\delta \mathbf{r}_i(\Delta t)|^2$$

- Order parameter - extensive in space and time.
- Liquid region - large K. Solid region - small K.

Dynamic Order Parameter





Dynamic Order Parameter

$$K = \sum_{i=1}^{N} \sum_{t} |\delta r_i|^2$$

- Order parameter - extensive in space and time.
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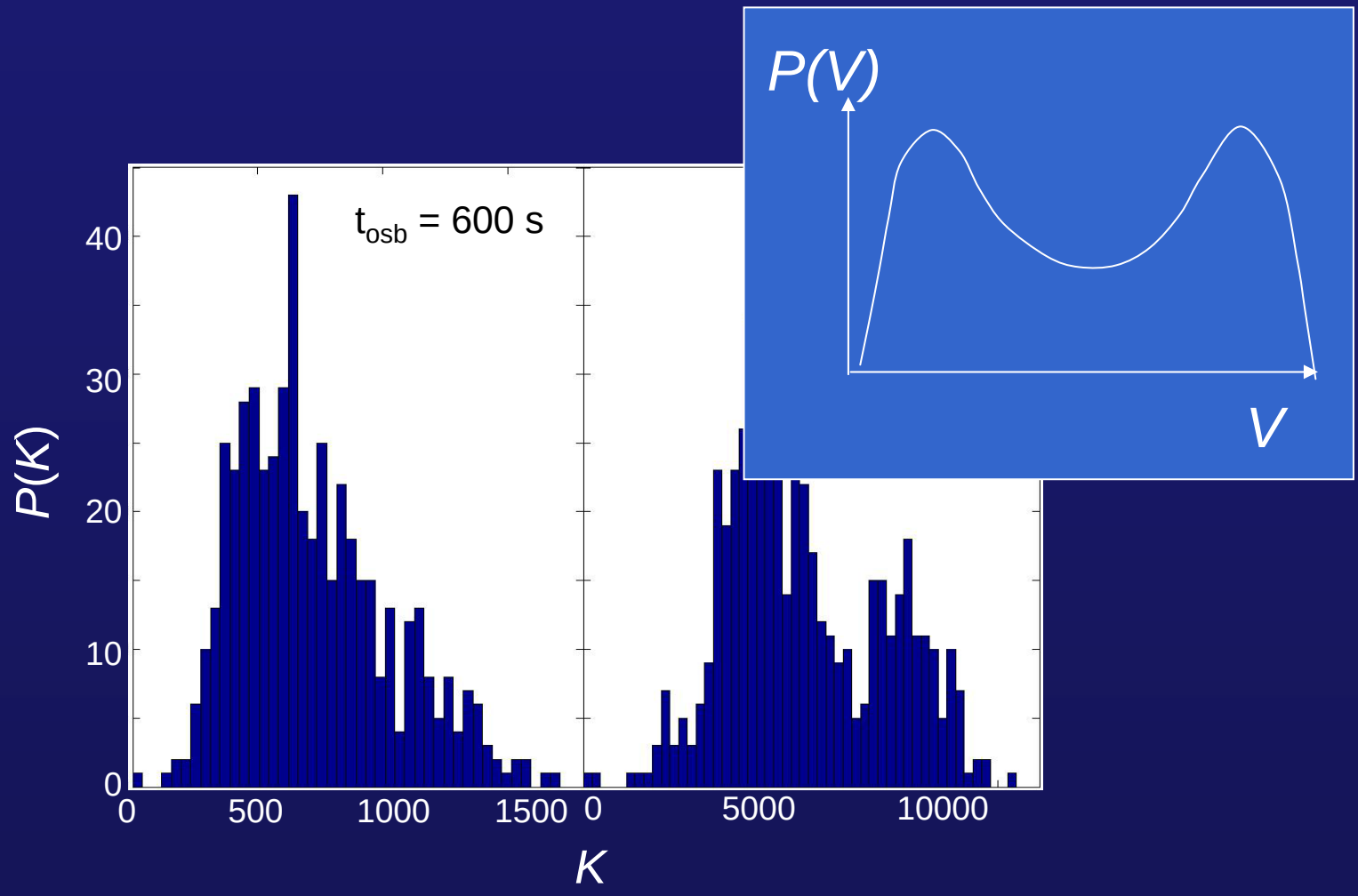
Distribution of K



Conclusions

- Colloidal Glasses :
  Insight into Flow and Shear Banding
- Thermally activated formation of STZ
- Spatial correlations in particle motion
- Shear banding: Dynamic Solid-Liquid Transition