KITP Workshop on Physics of Glasses

Activated Hopping, Dynamic Heterogeneity, and Mechanical Response in Glassy *Particle* Fluids and Suspensions

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Coworkers : 2003-present

Hard Spheres: *Erica Saltzman*, Vladimir Kobelev, Daniel Sussman
Soft Colloids: *Jian Yang*
Colloid-Polymer Gels: Yeng-Long Chen, Vladimir Kobelev
Molecular Colloids & Liquids: Mukta Tripathy, Galina Yatsenko, *Rui Zhang*
Polymer Melts & Glasses: Kang Chen, Erica Saltzman

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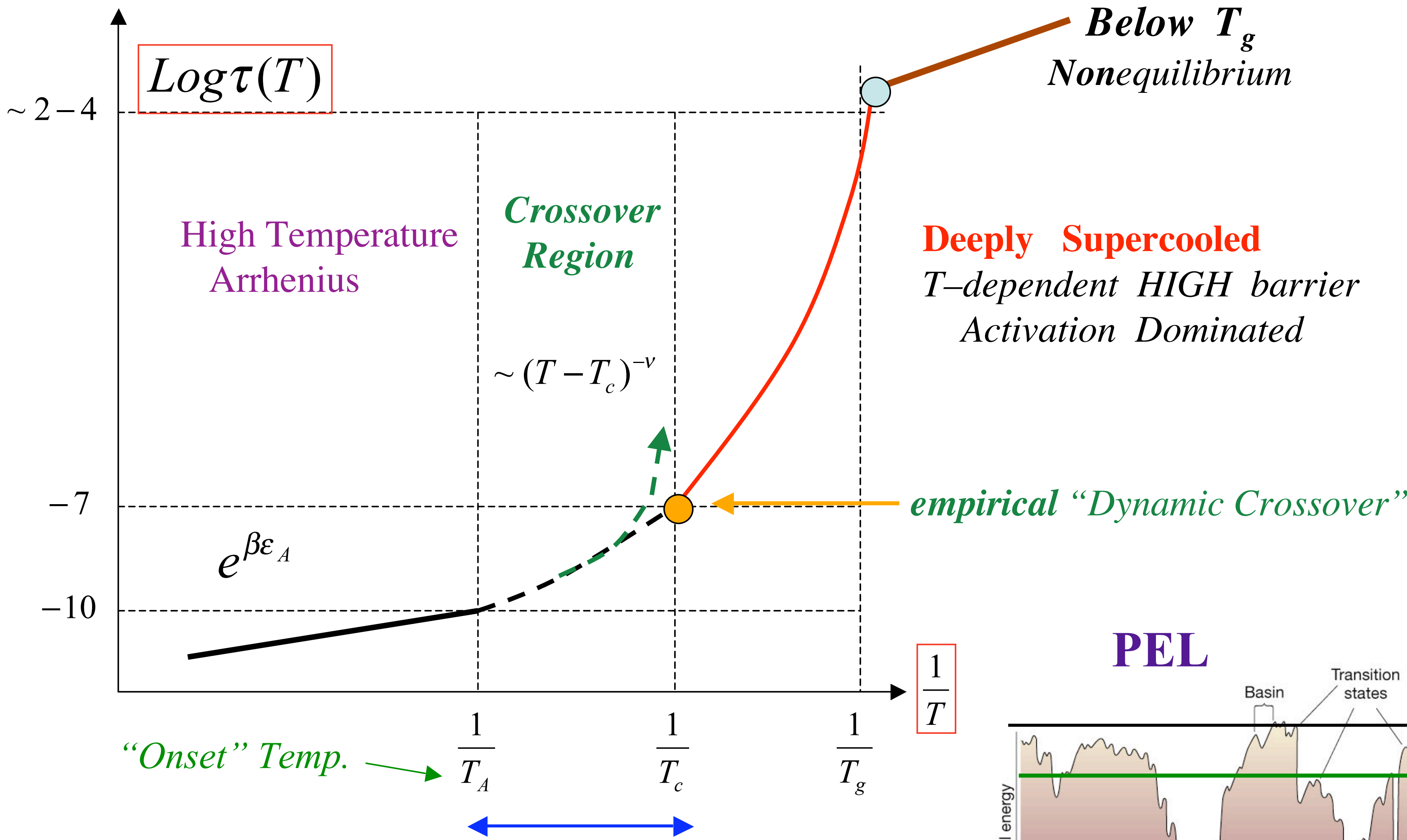
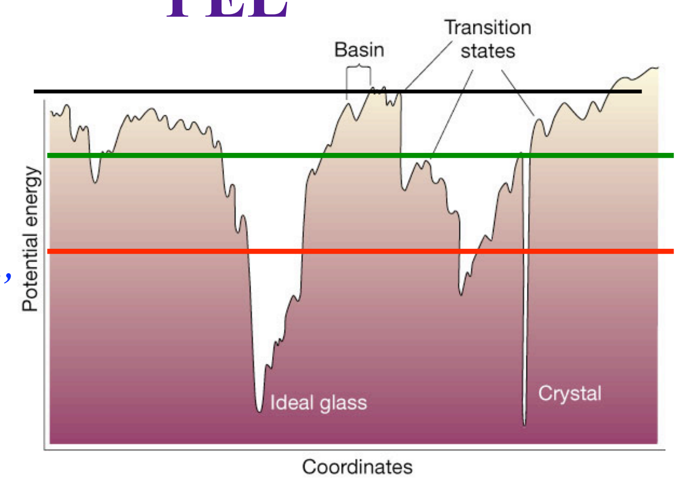
NSF-NIRT (w/ Mark Ediger, Juan dePablo, Jim Caruthers)
DOE-BES FS-MRL Soft Materials Cluster

Alpha Relaxation Map & Regimes



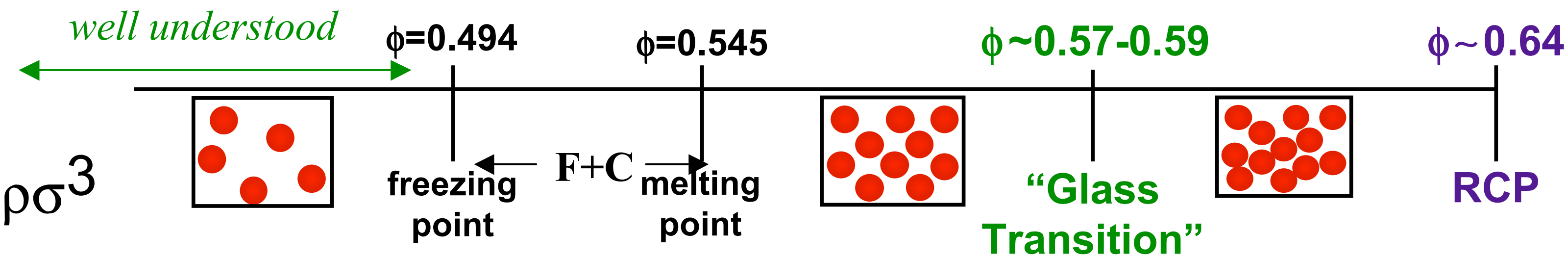
mode coupling effects + low barriers: "landscape influenced"

PEL

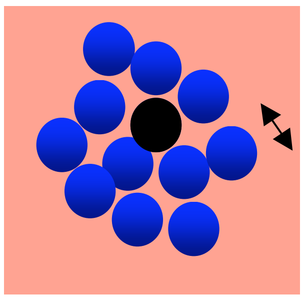


COLLOIDS: **µm vs nm**…."glassy" dynamics probed only over ~ 3-5 orders magnitude…..***ala simulations***

"Athermal" **HARD SPHERE Suspensions** (and fluids)



$\phi \equiv \frac{\pi}{6}\rho\sigma^3$

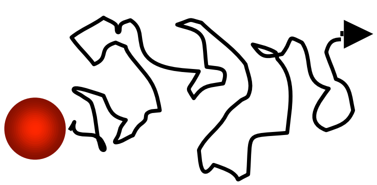


σ ~ **100 nm -2 μm**

Dilute Brownian time: $\tau_0 = \sigma^2 / D_0 \sim$ **0.01-30** sec

Kinetically "Vitrify": Relaxation Time > Expt time scale ~ 10,000 secs

CONFOCAL Microscopy & Simulations

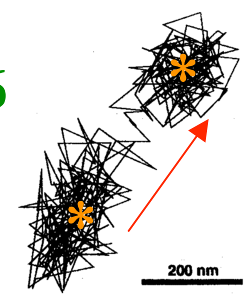


"smooth, hydrodynamic like"
Collective, small steps, ~Gaussian

"High" volume fraction

"Solid - Like"…intermittent hopping

$\phi = 0.56$

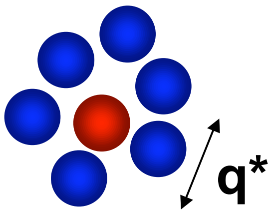


~σ/2

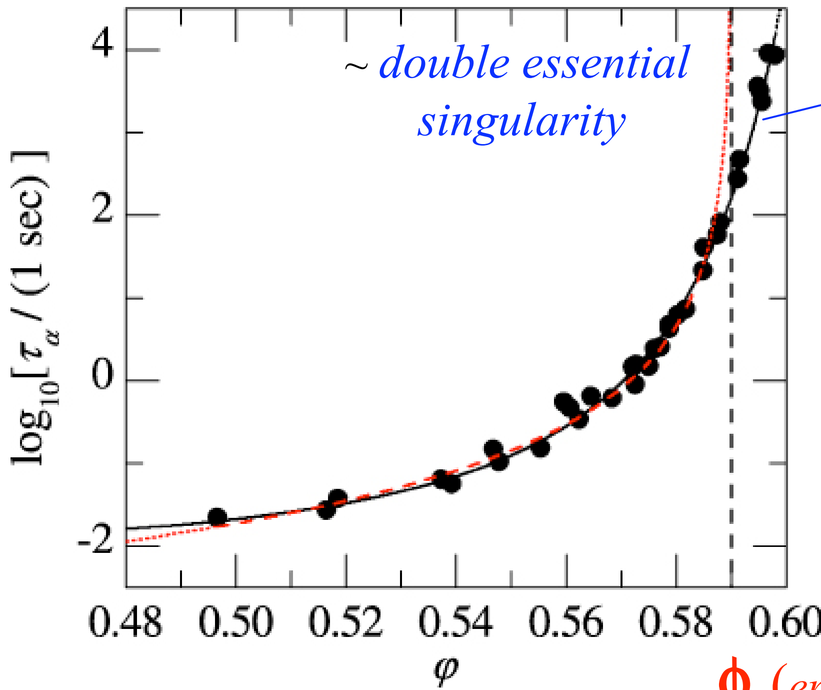
Weeks Weitz

Colloid Experiments & Computer Simulations

Alpha Relaxation



Cipelletti, Berthier, et al, PRL, 2009



$$\propto \exp\left(\frac{B}{(\phi_{RCP}-\phi)^2}\right)$$

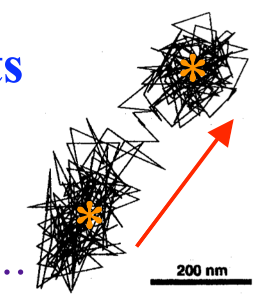
MCT ***fit :*** $\propto (\phi_c - \phi)^{-2.5}$

works over ~ 3 order magnitude

ϕ_c (*empirically deduced*)

***But even in regime where can fit MCT, see large NONgaussian effects**

Nongaussian parameter, Decoupling of diffusion & relaxation,
Exponential tails in van Hove function, Growing dynamic length scale,…..



…..**suggests large amplitude, intermittent activated processes important**

GOAL: Predictive Microscopic Theory @ level of Forces

build on Ideal MCT: *retain Structure, Forces, Slow Dynamics connection*

*.... allows **NONuniversal chemical/materials** aspects to be addressed*

BUT go beyond to treat **Activated Intermittent Dynamics**
at ***Single Particle level*** **:** ***"theory of simulation or confocal microscopy trajectories"***

→ *restores long time ergodicity, destroys "ideal" MCT glass transition*

allows treatment of some space-time Dynamic Heterogeneity effects

can generalize to NONlinear Viscoelasticity

Diverse Material Classes:

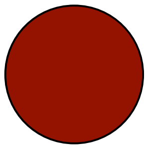
Particle Suspensions : hard/soft, sphere/nonspherical, glass/gel/Janus

Atomic & Molecular Liquids

Polymers …..including nonequilibrium "plastics"

Avoid Fitting & Adjustable Parameters….1st Principles

DENSE Colloidal & Nanoparticle Brownian Suspensions

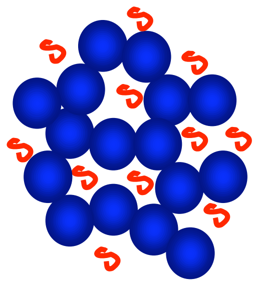


Hard **SPHERE**

Tunably SOFT : many arm stars; microgels,….

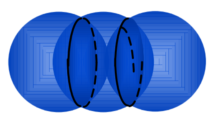
metals ?

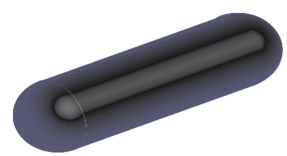
Sticky **GELS**

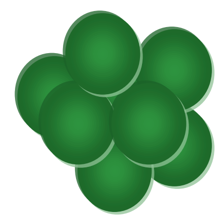


NONspherical

(***molecular liquids***)







Coupled Translate-Rotate

TODAY :

I. Hard Spheres : *basic concepts, Mean & Fluctuation phenomena*

II. Soft Spheres (microgels)….*role of highly variable soft repulsion*

III. Uniaxial hard particle….*role of shape, rotation*

IV. Nonlinear Rheology of hard spheres (likely no time)

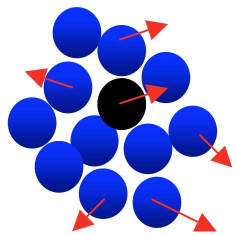
Saltzman & KSS
JCP, 2003

Nonlinear Langevin Eqn Theory

DERIVATION : KSS, JCP, 2005

Seek Stochastic Equation of Motion *NOT closed equation for time correlation functions*

$$\hat{\rho}_s(\vec{r},t)=\delta\left(\vec{r}-\vec{r}_i(t)\right)$$



r(t) = *scalar* ***displacement of a particle*** *from initial position*

$\mathbf{D_s}$ *: dissipative, short time, "bare" process*

Formally:
$$\frac{\partial\hat{\rho}_s(\vec{r},t)}{\partial t} = D_s\nabla^2\hat{\rho}_s(\vec{r},t) + D_s\nabla\hat{\rho}_s(\vec{r},t)\int d\vec{r}\,'\,\hat{\rho}(\vec{r}\,',t)\nabla V(\vec{r}-\vec{r}\,') + \eta_i\nabla\hat{\rho}_s(r,t)$$

Physical Ideas & Technical Approx.

Solid State View

CONTRACT to lowest level, **r(t)**

* **Key "slow variable" :** ***density fluctuations*** **….**ala MCT

* **Average over local packings**: dynamical caging constraints via **S(q)**

…**Effective** interparticle ***pair force*** : $\vec{f}(r)=k_BT\vec{\nabla}C(r)$ **….**from Structure (ala MCT)

** **Local Equilibrium Approx**: *relate 1 and 2 body dynamics*

Dynamic "closure" ala Einstein solid or Vineyard

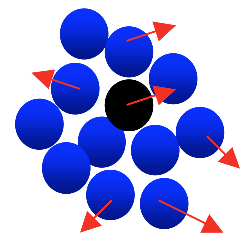
$$\frac{\rho^{(2)}\left(\vec{r},\vec{r}\,';t\right)}{\rho^{(1)}\left(\vec{r};t\right)}\approx\rho g\left(|\vec{r}-\vec{r}\,'|\right)$$

→ Nonlinear Langevin Eqn Theory

...force balance in overdamped regime

$$\zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial r} F_{eff}(r(t)) + \eta(t)$$

white noise $\langle \eta(t)\,\eta(0) \rangle = 2\zeta_S k_B T\,\delta(t)$



$$r(t=0)=0$$

Instantaneous Force due to surroundings

"*Dynamic* Free Energy" = ***Spatially-resolved, Time Local Displacement-Dependent "Field"***

$$\beta F_{eff}(r) = -3\ln(r) - \frac{1}{3}\int \frac{d\vec{q}}{(2\pi)^3} C^2(q)\,\rho S(q)\, e^{-q^2 r^2 \left(1+S^{-1}(q)\right)/6} \equiv F_{ideal} + F_{cage}$$

Structure ← ***Mean square Caging Force***

$$S^{-1}(q) = 1 - \rho C(q)$$

Favors :	**Delocalized** ***Liquid***	**Localized** ***Solid***

competition

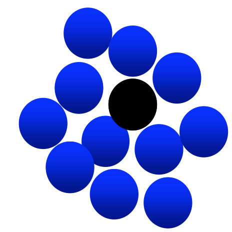
Reduction to simplified Ideal MCT

$$\zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial r} F_{eff}(r(t)) + \eta(t)$$

* **RECOVER** ***Naïve MCT*** *Transition of Kirkpatrick-Wolynes* IF :

Dynamical Gaussian approximation for $\langle \mathbf{r^2(t)} \rangle$

Mean Localization Length $r^2_{LOC} \equiv \left\langle r^2(t \to \infty) \right\rangle$



$$\frac{1}{r^2_{LOC}} = \frac{1}{18\pi^2} \int_0^\infty dq\, q^2 q^2 C^2(q) \rho S(q)\; e^{-\frac{q^2 r^2_{LOC}}{6}(1+S^{-1}(q))}$$

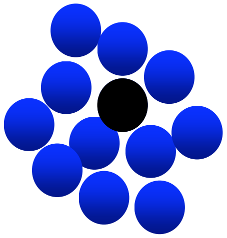
Einstein solid
Debye-Waller

$$\left\langle \vec{f}(0) \bullet \vec{f}(t \to \infty) \right\rangle$$

"uphill" thermally activated hopping NOT allowed → ***IDEAL GLASS*** @ ϕ_c, $\mathbf{T_c}$

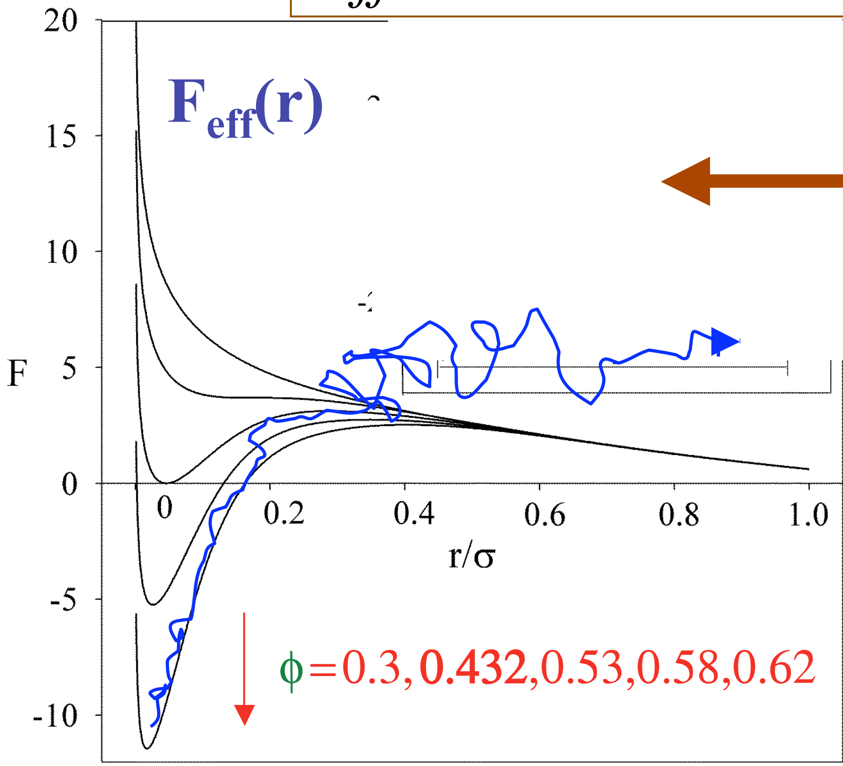
Reality : MCT "transition" = Dynamical Crossover

I. Dynamic Free Energy: Hard Spheres

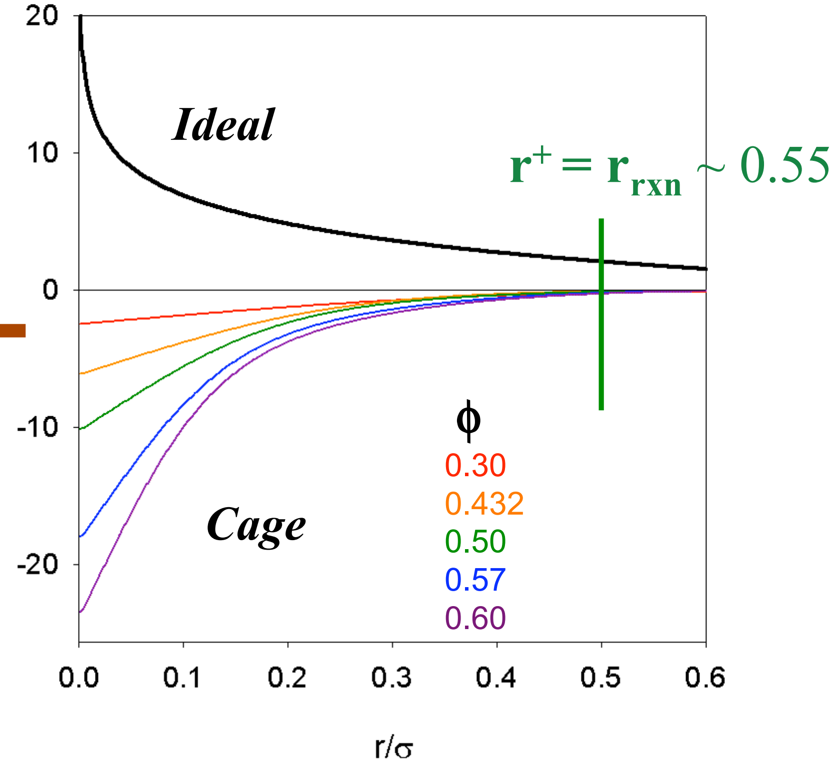


* Naïve MCT "ideal glass transition" at $\phi_C \sim 0.432$

$$F_{eff}(r) = F_{ideal} + F_{cage}$$



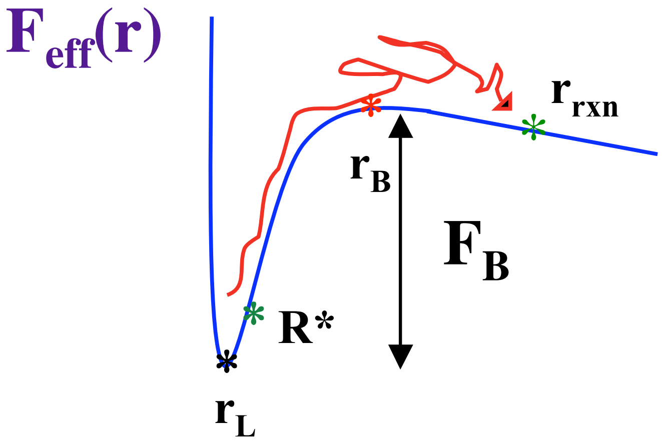
Displacement, r(t)



Reaction Pt: Cage Escape, Onset of IRReversibility ….negligible localizing ***force***

$$\zeta_{tot} = \zeta_s + \zeta_{HOP} \quad ; \quad D_{HOP} = r^{+2} \langle \tau_{rxn}^{-1} \rangle / 6 \equiv k_B T \zeta_{HOP}^{-1}$$

Source of Rich Physics :Many Relevant Energy and Length Scales



as ϕ increases :

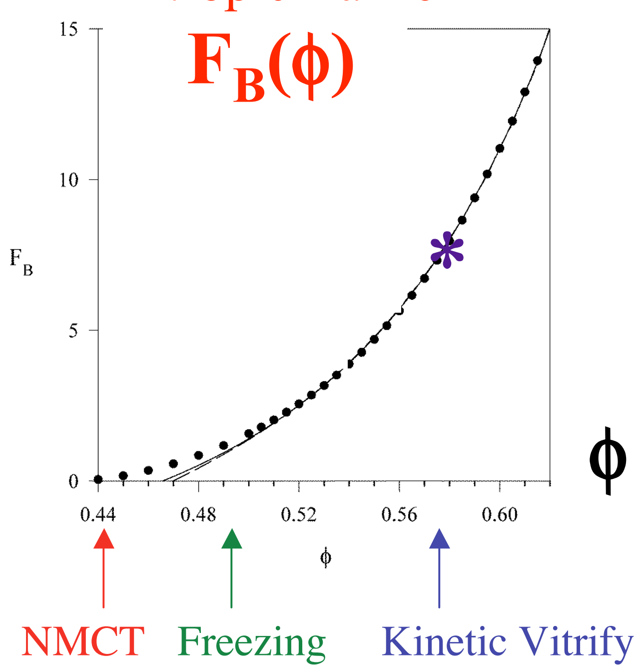
r_L~ 0.18--> 0.03 R* *decrease*

r_B ~ 0.25 --> 0.35 *ala "interstitial"* r_{rxn}~ 0.55

Localization well & Barrier **curvatures**
Entropic **barrier height**
Maximum restoring force, f*

ALL increase

Entropic Barrier



ANALYTICS : Kramers, Green-Kubo

$$\frac{\bar{\tau}_{hop}}{\tau_0} = \frac{2\pi\,(\zeta_s/\zeta_0)}{\sqrt{K_0\,K_B}}\,e^{F_B}$$

~ **mean** alpha time

Shear Modulus, Diffusion constant,…..

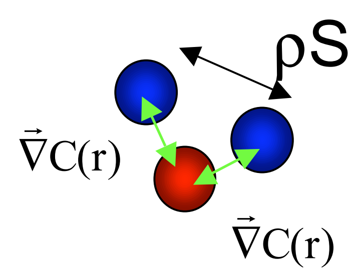
Limiting Analytic Analysis : **Real Space Picture & "Universality"**

KSS, JCP, 2007

Caging Force

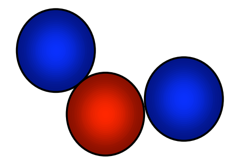
Fourier-resolved mean square force

$$-\frac{\partial F_{eff,cage}(r)}{\partial r} \propto -r\int_0^{\infty} \frac{d\vec{k}}{(2\pi)^3}\,[kC(k)]\rho S(k)[kC(k)]\; e^{-\frac{k^2r^2}{6}\left(1+S^{-1}(k)\right)}$$



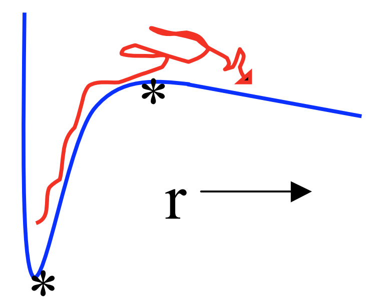
Single
"coupling constant"
controls **entire** F_{eff}!

$$V_{\infty} \equiv \phi g^2(\sigma) \propto F_B$$



"contacts"

→ Predicts connections between slow dynamics on different time & length scales : e.g., **late β /early α** vs. **final α**



e.g.,

$$G_{glass} \propto \phi \frac{k_B T}{\sigma\, r_{loc}^2} \qquad F_B \propto \frac{\sigma}{r_{loc}}$$

"SOLID" only at RCP
Jamming

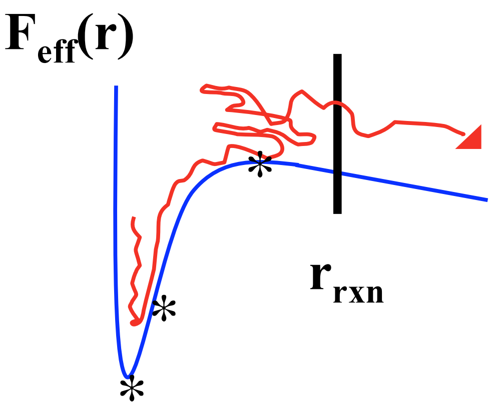
$$F_B \propto \phi g^2(\sigma) \propto \left(\phi_{RCP} - \phi\right)^{-2} \rightarrow \infty$$

Double Pole

Full Numerical Soln: Includes Dynamic Fluctuation Effects

JCP & PRE
2006 & 2008

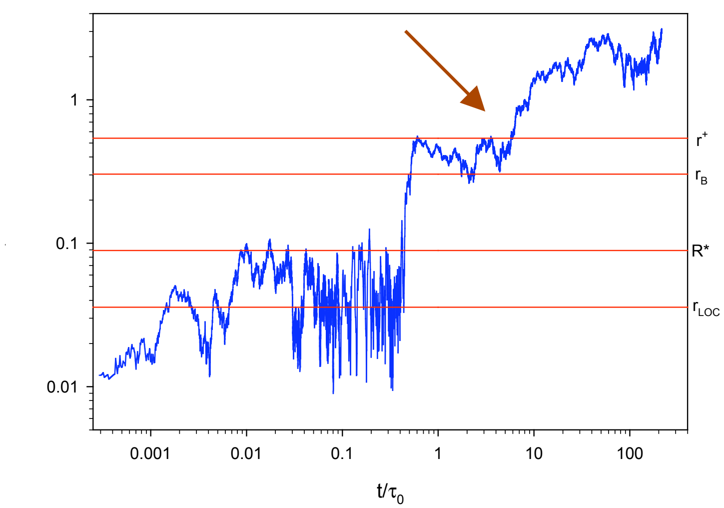
$$\zeta_s \frac{\partial r(t)}{\partial t} = -\frac{\partial}{\partial r} F_{eff}(r(t)) + \eta(t)$$



Noise-Driven Trajectory Fluctuations → ***Heterogeneous Dynamics***

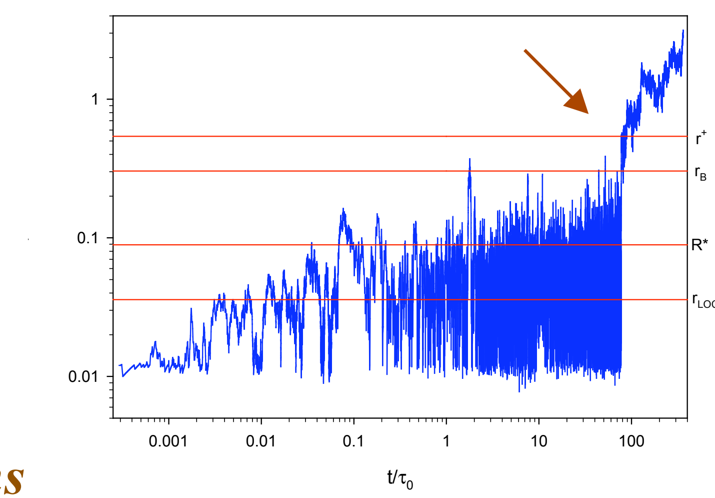
r(t)/σ trajectories

ϕ=0.55 ; Barrier ~ 5



Reaction point
Barrier
Maximum force
Localization length

Re-crossings
"back-hops"
Large Fluctuations



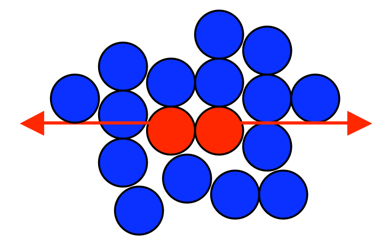
Limitations & Possible Caveats

*** Full Dynamics ~ Sequence of Independent "local events"***

evidence for weak space-time correlation of **rare** "hops" :
Joerg Rottler simulations: EPL, 2009; PRL, 2010
successes of simple CTRW,…

*** Single Particle vs. Cage vs. Stress Relaxation time ?***

Daniel Sussman & KSS

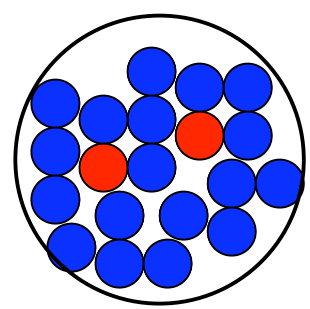


evidence closely correlated from simulation:
Yamamoto-Onuki; Rottler ;……
and experiment

*** Single particle Dynamic Heterogeniety vs. Many particle space-time ?***

e.g. $\chi_4(t)$

Dasgupta & Sastry
Szamel
many others



dynamic length scale ξ

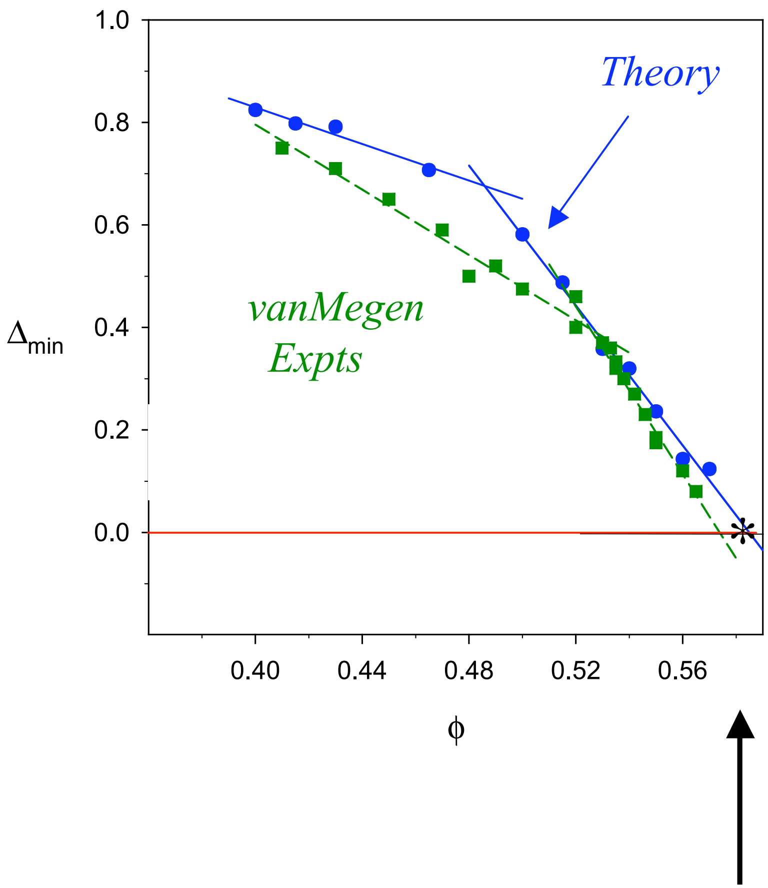
expect connected if hopping controlled

We do find explicit connections

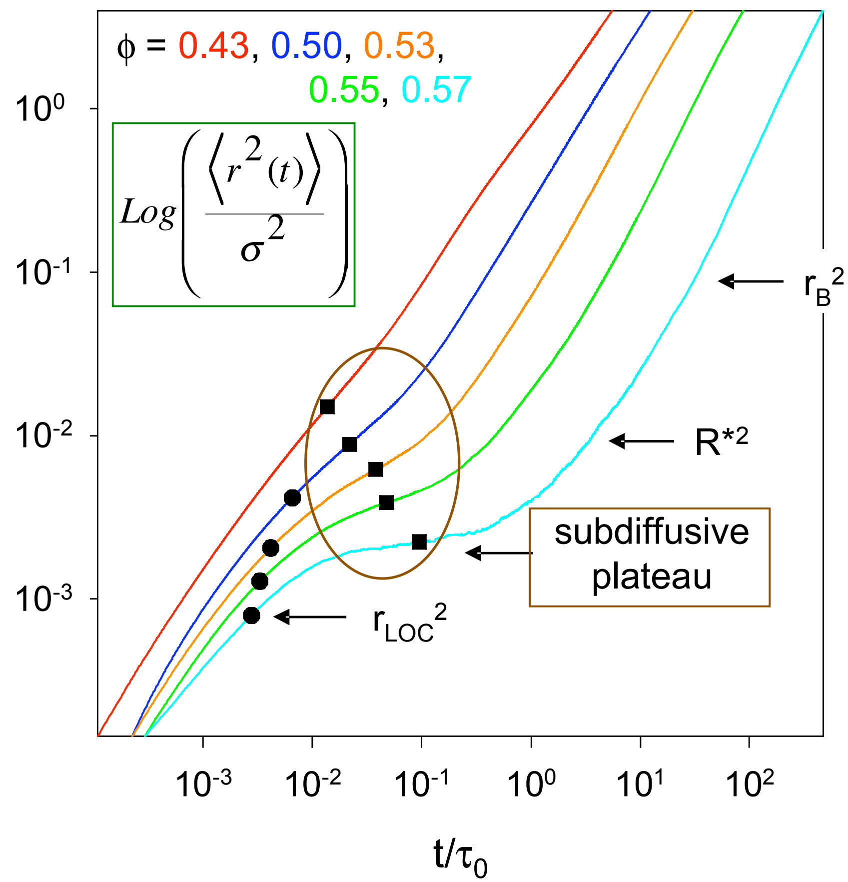
Mean Square Displacement & Anomalous Diffusion

Maximum non-Fickian

$$\langle r^2(t) \rangle \propto t^{\Delta_{min}}$$



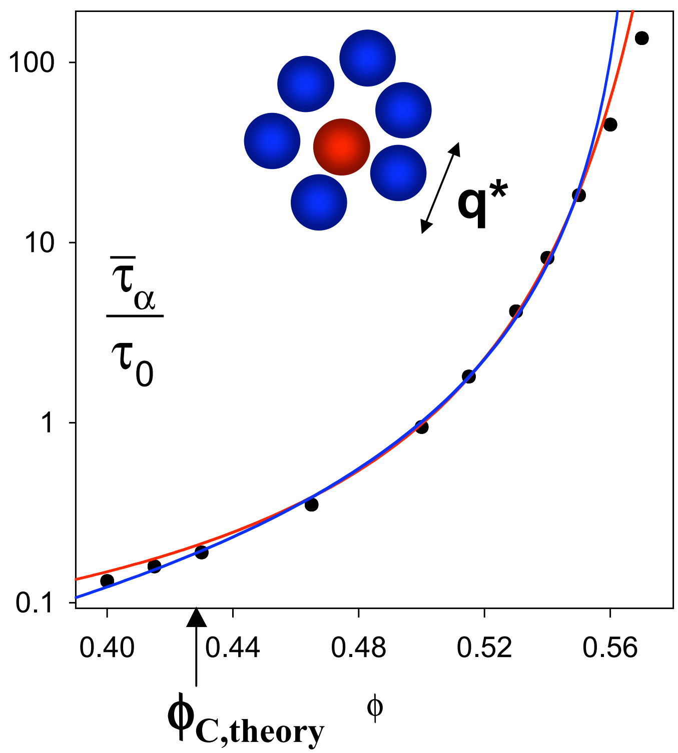
2nd moment



Extrapolate: $\phi_c \sim 0.58$ ~ *Experimental result based on fits to MCT*

Alpha (cage scale) Relaxation

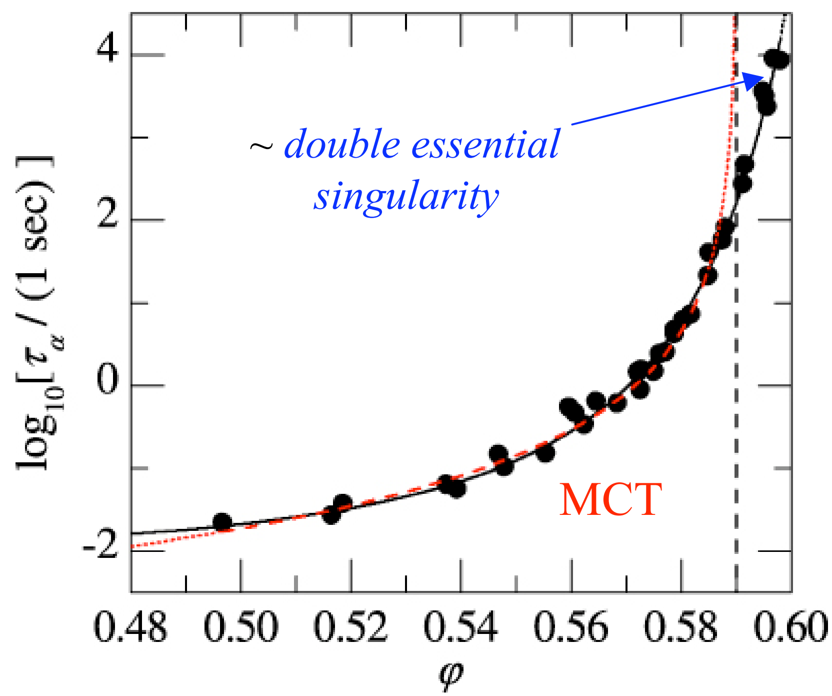
$F_s(q^*,t)$



$\sigma = 1\ \mu m$	
τ_α	ϕ
5x10^4 s (14 hrs)	**0.57**
5 months *"glass"*	**0.61**

Cipelletti et al, PRL, 2009

extra 2 orders magnitude



MCT critical power law fits the NLE THEORY & EXPT over ~**3 orders of magnitude..**.... *then breaks down (no singularity)*

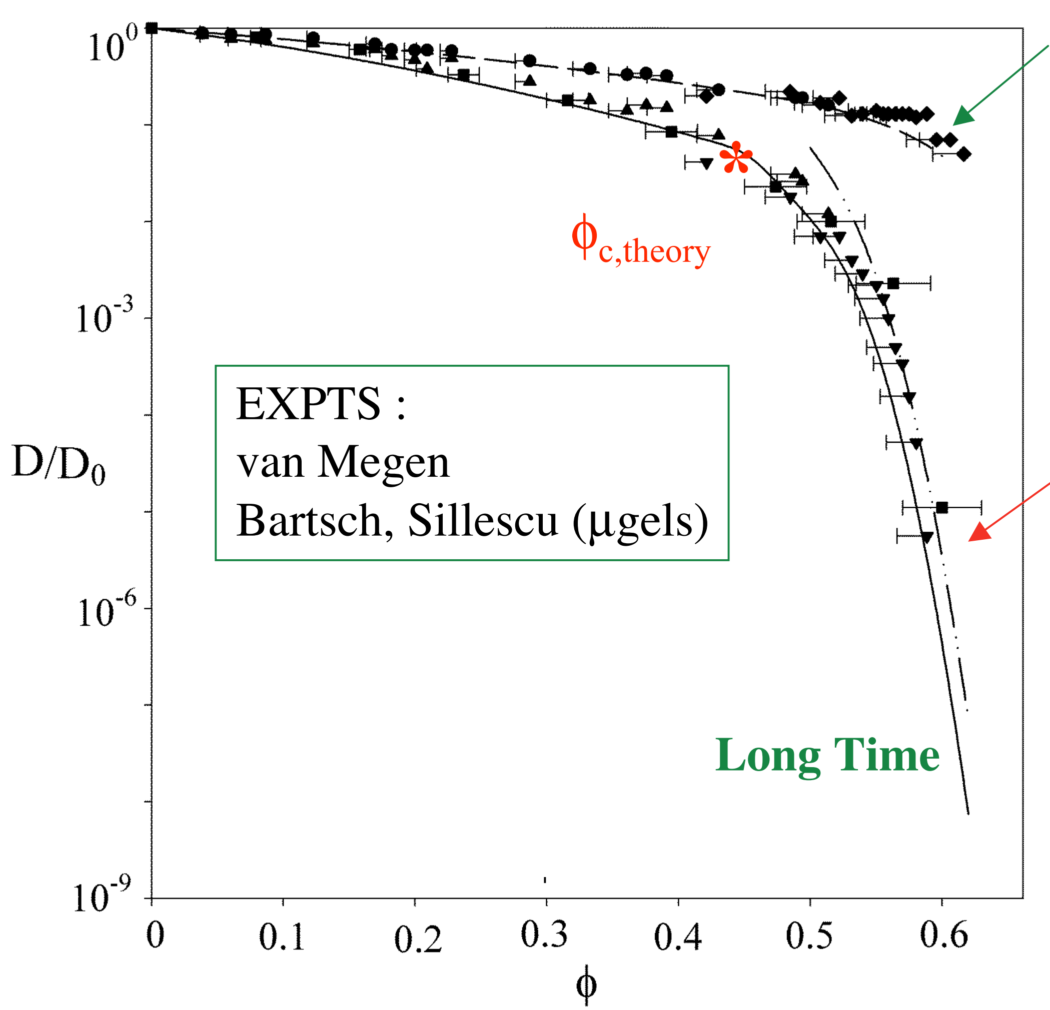
NLE Prediction (JCP, 2007)

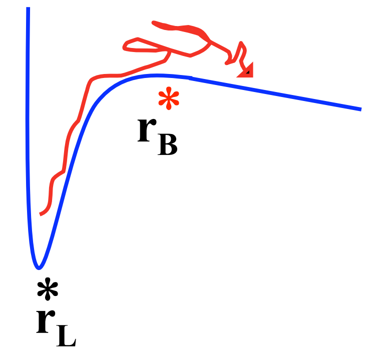
$$\tau^* / \tau_0 \propto e^{F_B(\phi)} \underset{\text{approach RCP}}{\propto} \exp\left(\frac{B}{(\phi_{RCP} - \phi)^2}\right)$$

ala new expts

Self-Diffusion Constant

NLE theory + Green-Kubo +… (JCP, 2003)





$$D_{HOP} \equiv \frac{(r_B - r_L)^2}{6\bar{\tau}_{hop}}$$

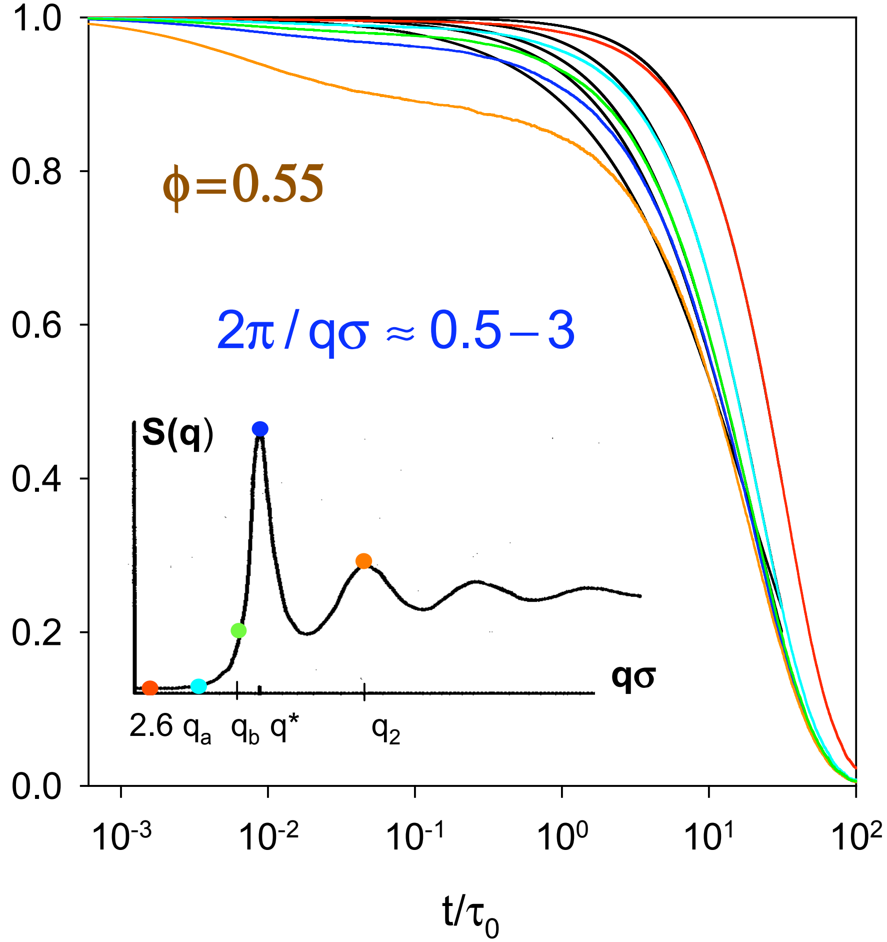
NO divergences below RCP

Barriers important for $\phi > 0.5$

Fluctuation consequences ?

NONgaussian Spatial α–Relaxation: *a signature of hopping*

$$F_s(q,t) = \langle \exp[i\vec{q}\cdot\vec{r}(t)] \rangle = F.T.\langle \delta(r - r_1(t)) \rangle$$



q-dependent relaxation :

Grossly NONgaussian

$$F_s(q,t) \neq \exp(-q^2 Dt)$$

WHY ?

Intermittent Hopping ?

Growing NonFickian length scale ?

Growing Dynamical Length Scale

$$F_s(q,t) \equiv \exp(-D(q)q^2 t) \equiv \exp(-t/\tau(q))$$

Define: $R(q) \equiv q^2 D\tau(q) \to 1$, Gaussian $\approx$ MCT

IF activated, Numerics described by:

$$\frac{1}{\tau(q)} = \frac{q^2 D}{1+(q\xi_D)^2} \equiv q^2 D(q)$$

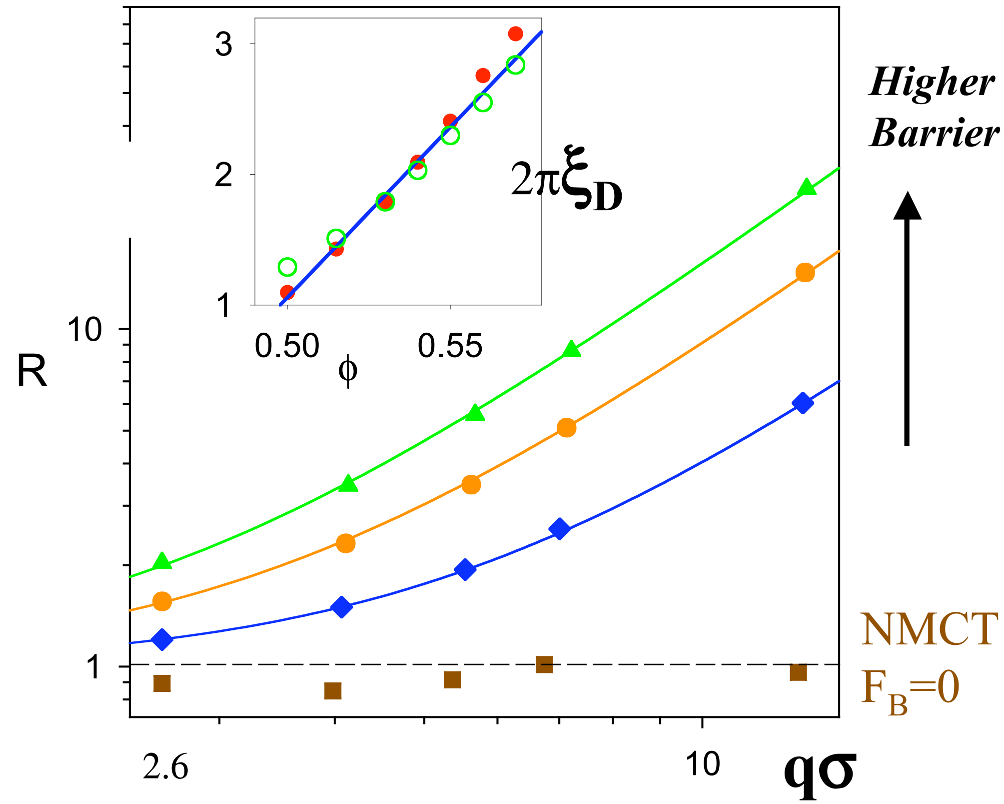
$$D(q) \approx D\left(q\xi_D\right)^{-2}, \quad q\xi_D >> 1$$

$$\tau(q) \approx q-\text{independent}$$

Growing length scale for recovery of Fickian diffusion

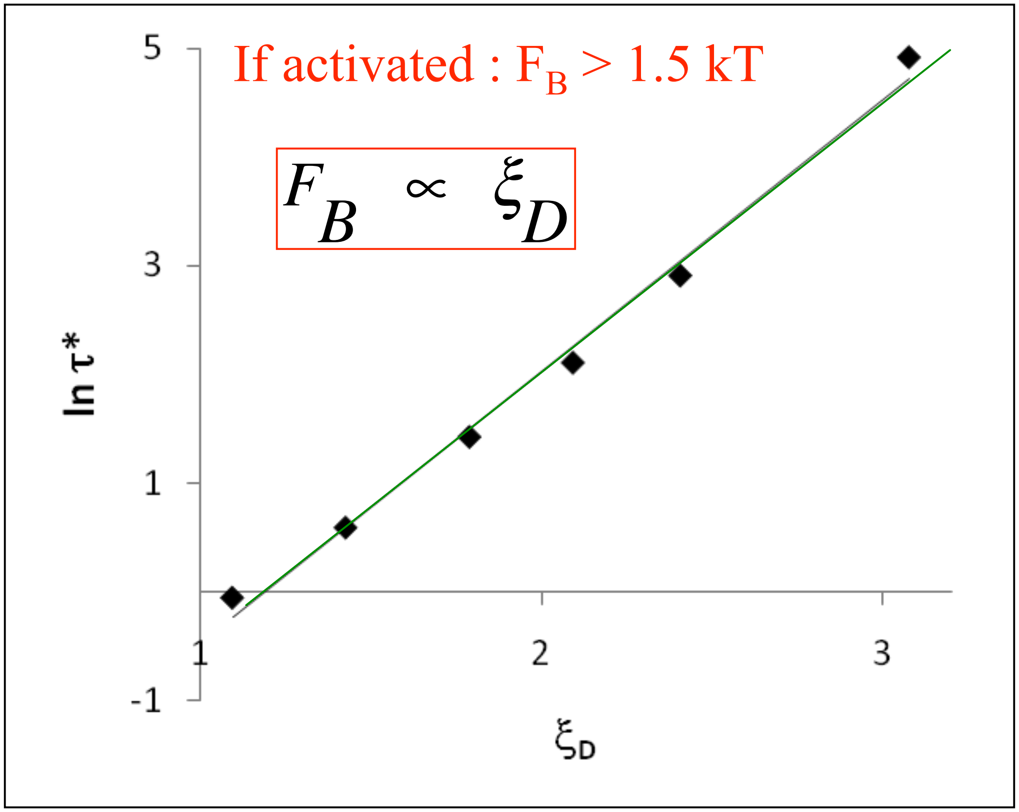
$$\xi_D(\phi)$$

ϕ = **0.43**, 0.5, 0.53, 0.55



Consistent with BLJM Simulations (Szamel ; Berthier)

Connection of Alpha Time and Growing Length Scale



Very different scaling than :

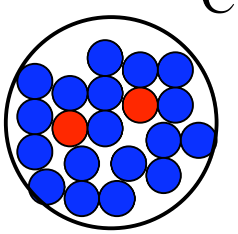
Naive Adams-Gibbs

Inhomogenous-MCT

other thermo-based theories

Close to Dasgupta-Sastry BLJM simulations: (PNAS, 2009)

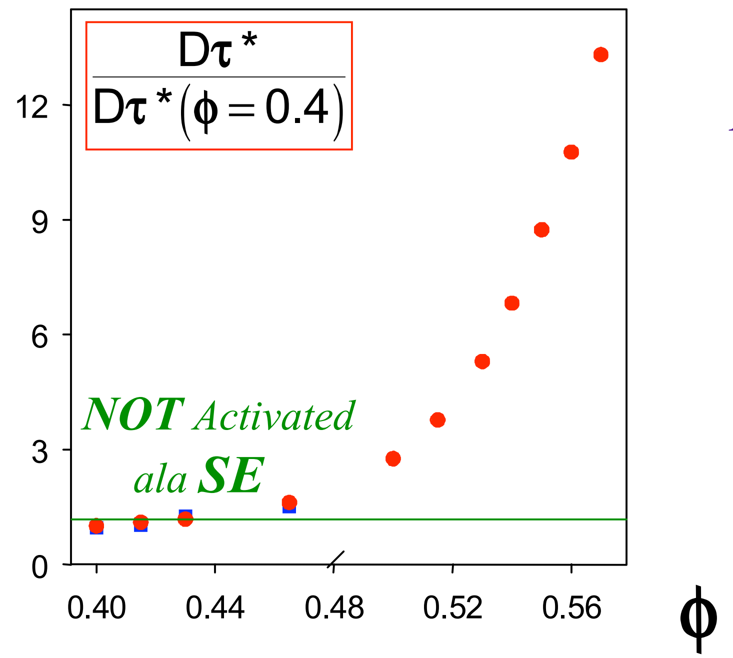
$$\ln(\tau_4) \propto (\xi_4)^{0.7}$$



4-point "susceptibility" $\chi_4(t)$: *time scale & dynamic correlation length*

"Decoupling" of Self-Diffusion & Alpha Relaxation

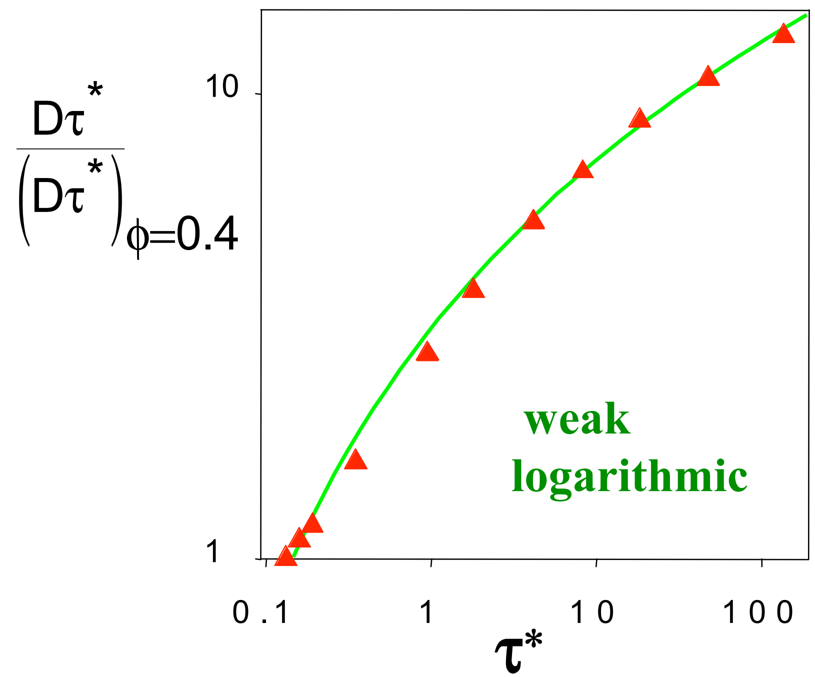
…failure of Stokes-Einstein behavior



Mass Transport ENHANCED
@ fixed "relaxation time"

$$\sim \boxed{\frac{D\tau^*}{(D\tau^*)_0} \approx 10-20\,;\ \phi = 0.58-0.59}$$

Sanat Kumar; Tom Truskett
PD-Hard Sphere SIMS



"Decoupling length"

$$L_d \equiv \sqrt{D\tau^*} \propto \xi_D \propto \ln(\tau^*)$$

WHY ?

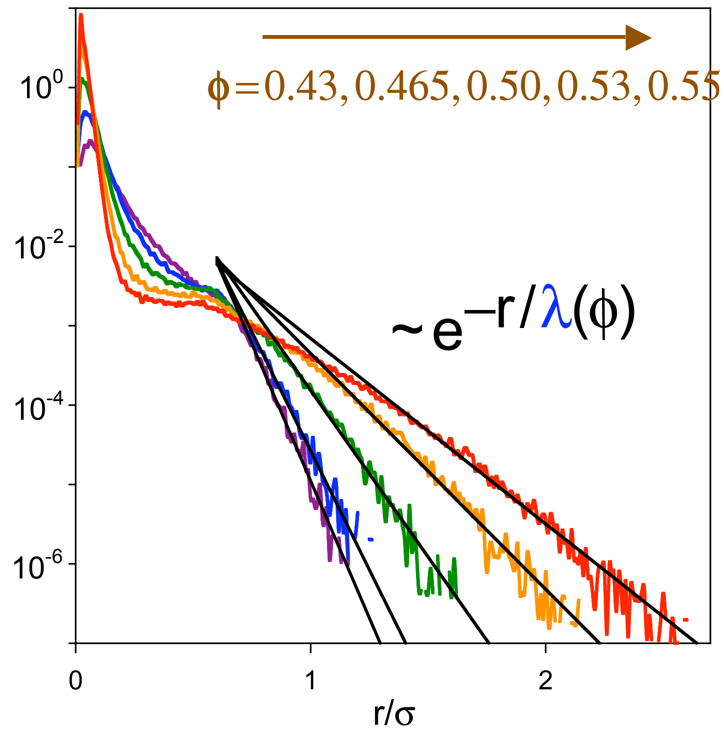
Mobility Bifurcation and Exponential Tails

PRE, 2008

Real Space Van Hove

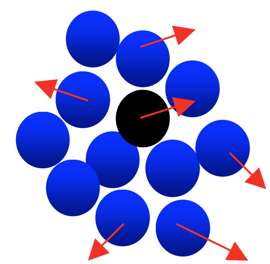
$$G_S(r,t) = \left\langle \delta(r - r_1(t)) \right\rangle$$

@ α–time : $\text{Log } \mathbf{G_s(r,t=\tau_\alpha)}$



Exponential tail

"fast hoppers"



$$\lambda \propto \sqrt{t}$$

in α–regime

λ="Jump Length"

grows with ϕ

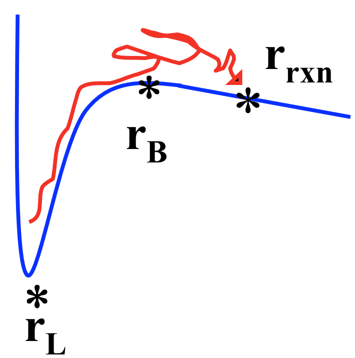
directly correlated with

Fickian Crossover

and

Decoupling lengths

$$L_d \equiv \sqrt{D\tau^*} \propto \lambda \propto \xi_D$$



Mobility = function of length scale

Relaxation more local process (barrier, r_B, slow) ***than***

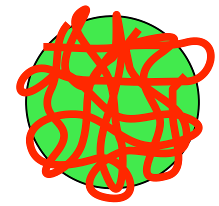
Diffusion (reaction pt, r_{rxn}, faster)

…akin to "facilitation" ? (Chandler,Garrahan)

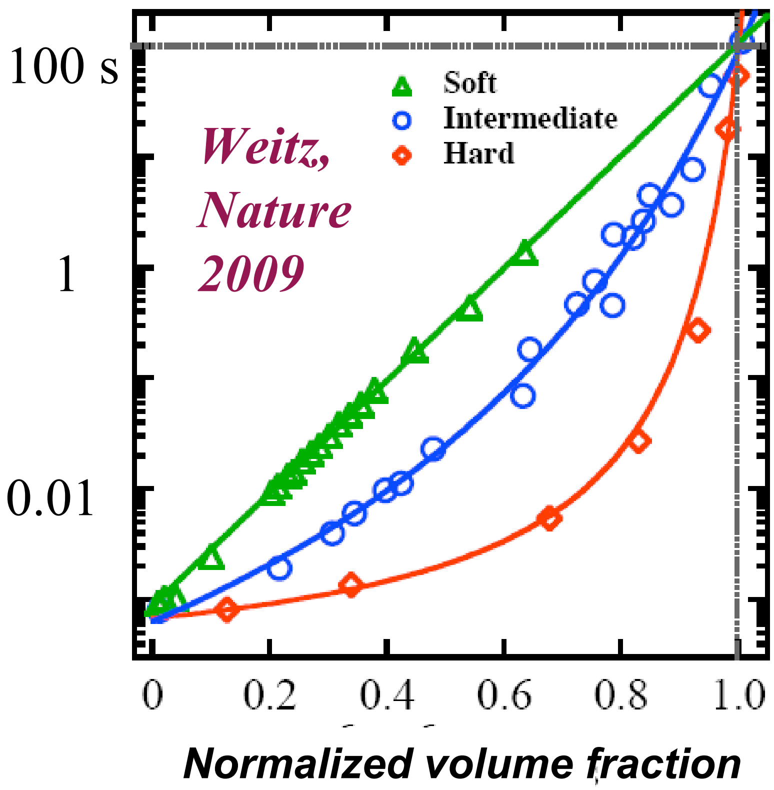
II. Soft Repulsive Spheres ~ MICROGELS…important materials !

Vary Single Particle Stiffness (*crosslinks*) ….interparticle repulsion strength

→ **Massive Change in Dynamic Fragility**



Relax Time (ϕ) from DLS



"glass"

finite range

Hertzian Contact Model :

$$V(r) = \frac{4}{15} E^{*} \sigma^{3} \left(1 - \frac{r}{\sigma}\right)^{5/2} \quad , r \le \sigma$$

$$= \quad 0 \quad , \quad r > \sigma$$

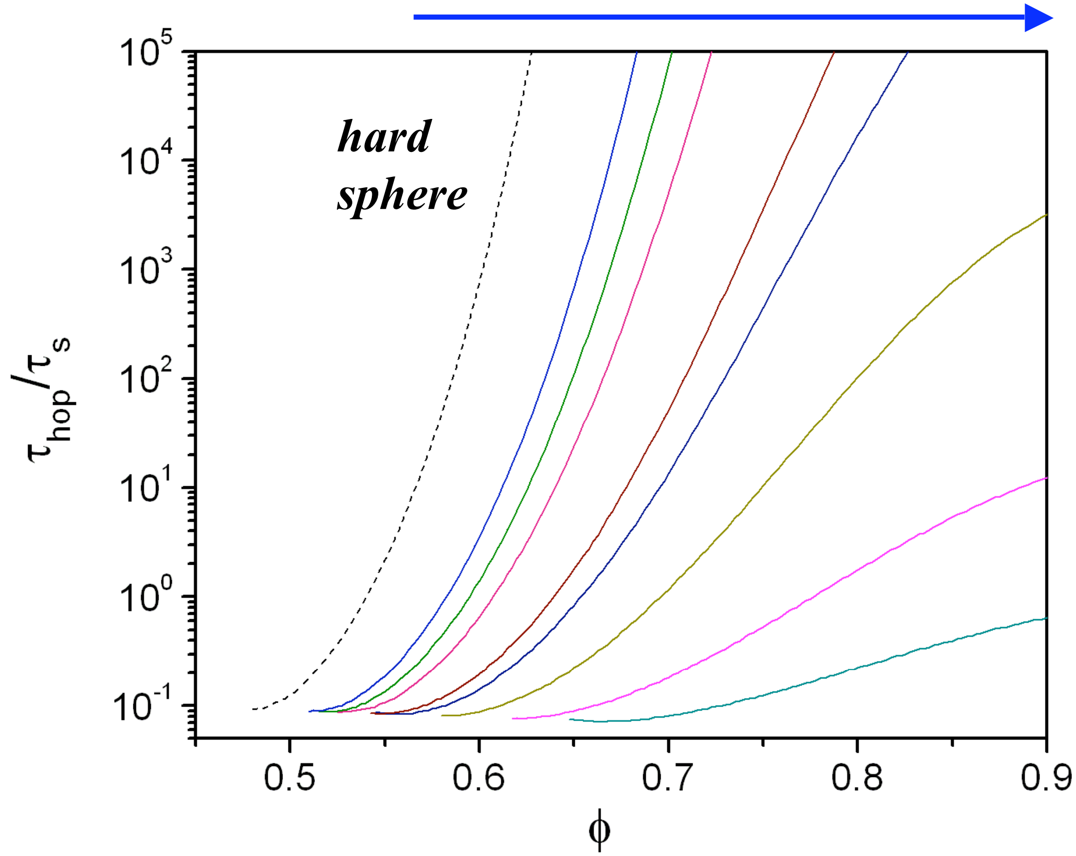
Packing Complexity
as function of **ϕ and E***

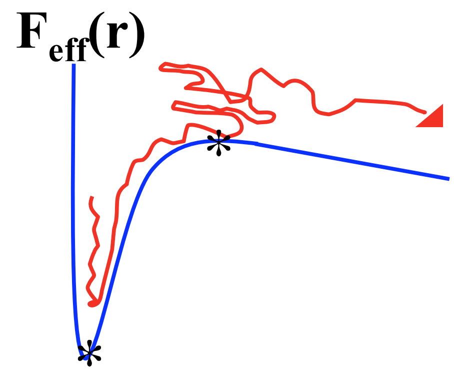
$$g(r)$$

NLE Theory: Activated Kramers Time

Yang & KSS submitted

Fix **E*** = infinity, (5,3,2,1)10^4, (8,5,3,2) 10^3

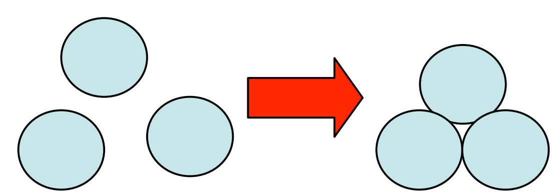




NONexponential Growth
non-Arrhenius

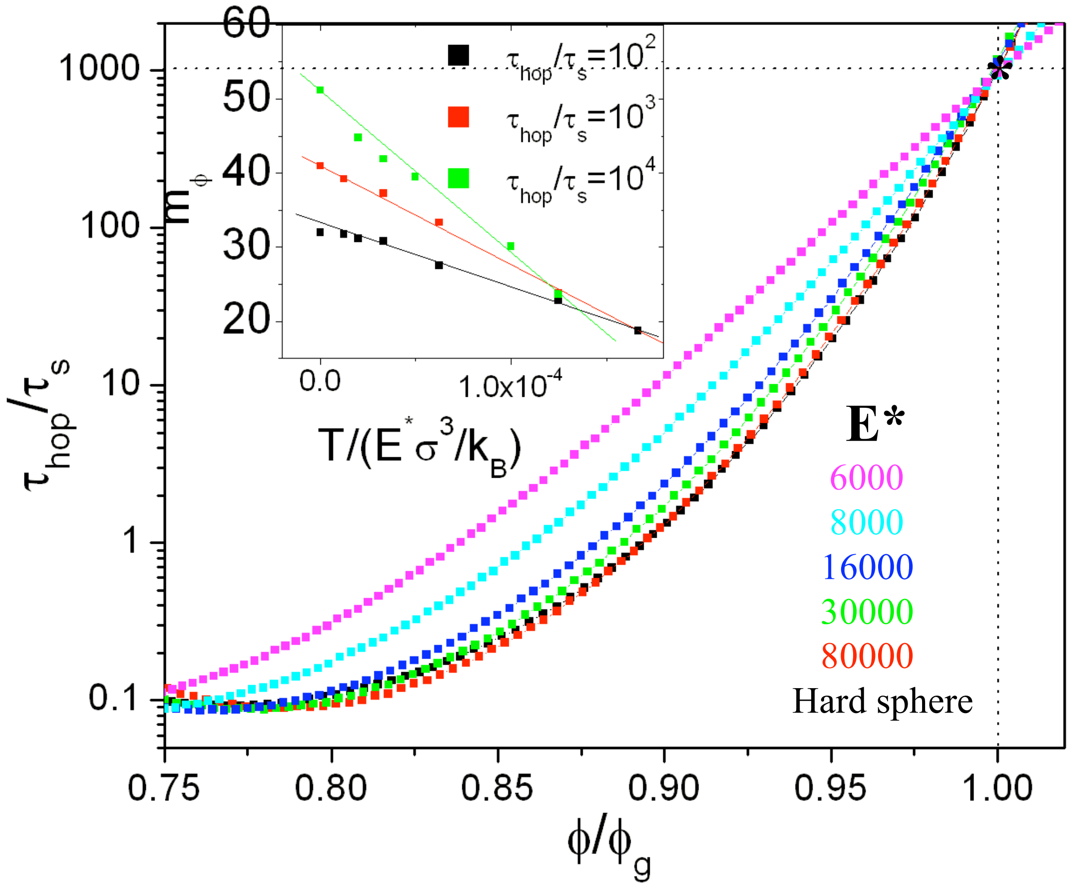
More "Fragile" as
Single Particle **Stiffens**

"Bends over" as "soft jamming" approached
due to qualitative change of packing



Dynamic Fragility: Tunable via Particle Softness

Angell Fragility Plot based on ***Kinetic Glass Criterion***



$$m_\phi \equiv \frac{\partial}{\partial(\phi/\phi_g)}(\log\tau_{hop})\big|_{\phi_g}$$

varies by factor ~ 3 - 4

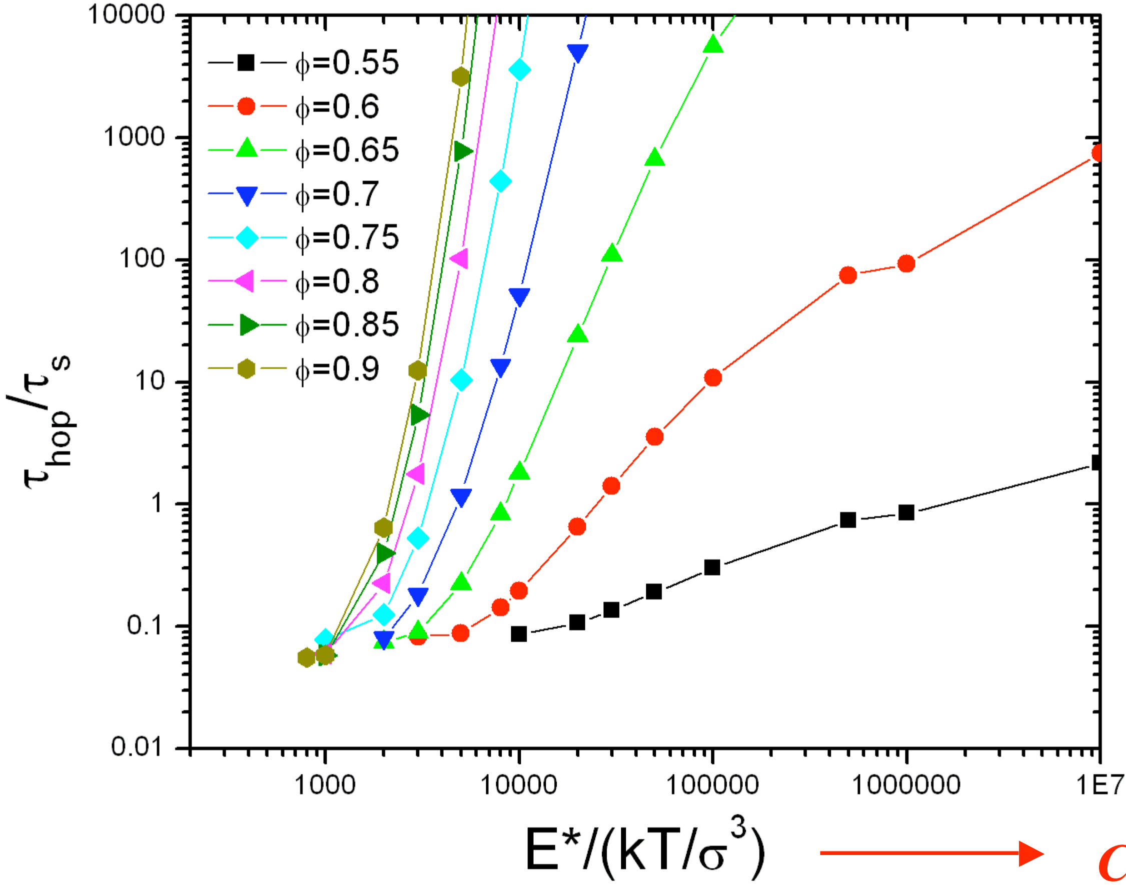
Decreases
LOGARITHMICALLY
as Particle softens

↓

"Soft Particles Make STRONG GLASSES"
~ Arrhenius

ala Weitz et al, Nature, 2009

"Thermal Fragility" at Fixed Volume Fraction



"Two φ–Regimes"

ala Simulations with harmonic repulsions

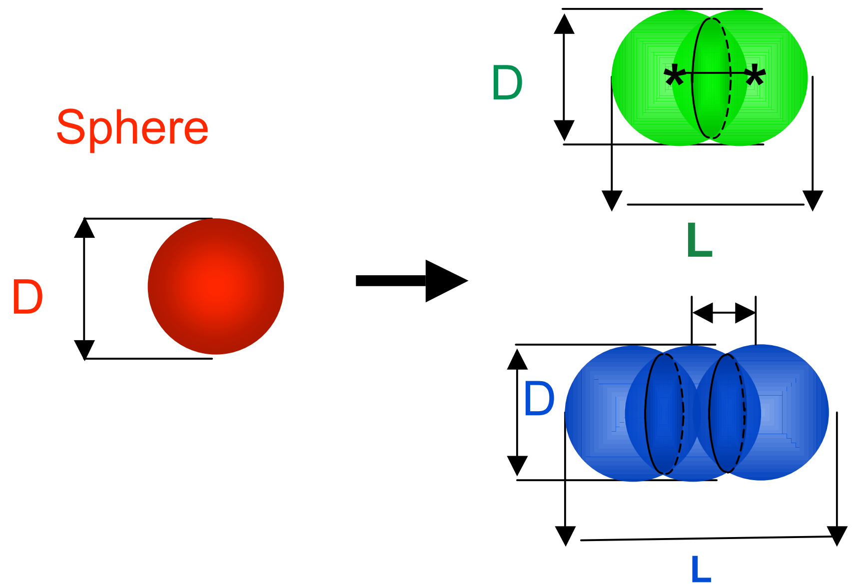
Berthier & Witten
EPL; PRE, 2009

→ ***Cool or Stiffen Particle***

MASSIVELY Enhanced Thermal Fragility as Volume Fraction grows

PHYSICS: below vs. above HS "jamming" per Berthier-Witten scaling argument

BEYOND SPHERES : Hard *Uniaxial* Particles



Diatomic

1≤L/D ≤2

Triatomic

“Molecular Colloids”

A frontier of particle science and engineering

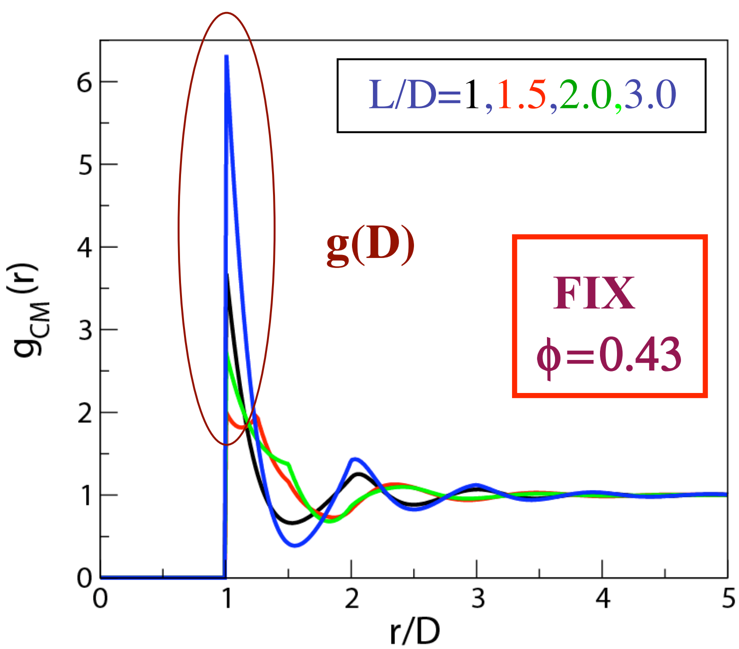
Site-Site Hard Core Repulsions

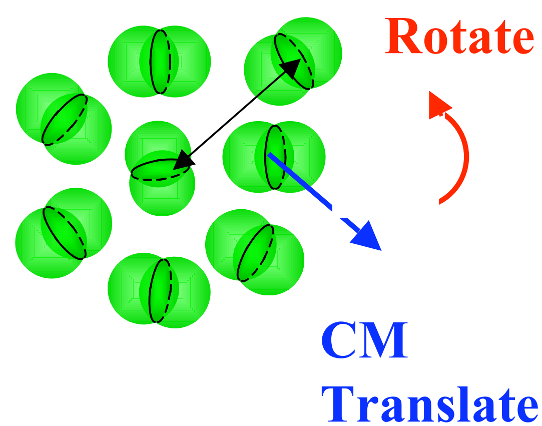
NON-monotonic local order as vary ASPECT RATIO

L/D

incommensurate

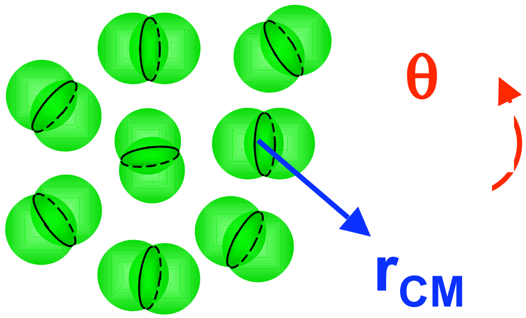
Packing Frustration





COUPLED Translation-Rotation Dynamics

Zhang & KS
PRE 2009



Cumulative angular rotation

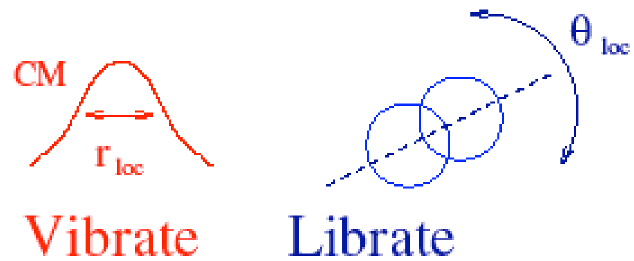
$$|\vec{\theta}(t)| = \left| \int_0^t dt' \vec{\omega}(t') \right|$$

Center-of-Mass displacement

Naïve MCT

CM Force & Torque

Time correlations



2 coupled self-consistent localization eqns

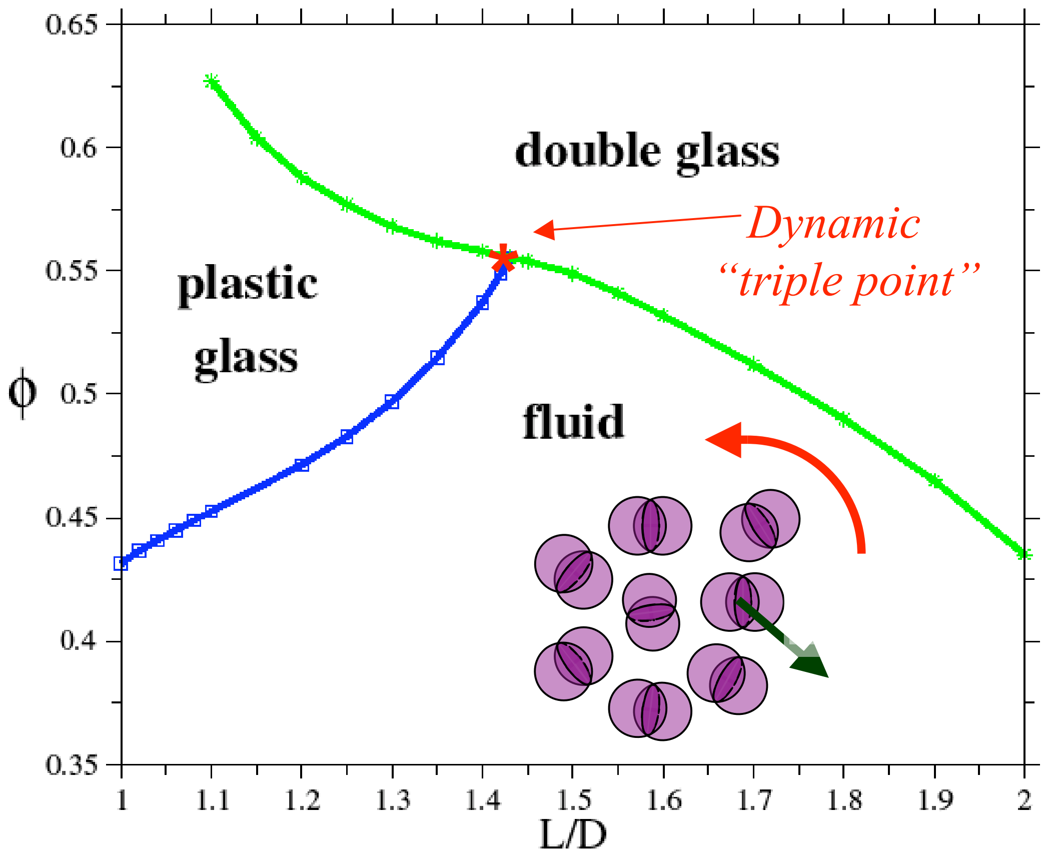
NLE Activated Dynamics Theory

Dynamic Free Energy
SURFACE

$$-\zeta_T \frac{d}{dt} r_{CM} - \frac{\partial}{\partial r_{CM}} F_{eff}(r_{CM}, \theta) + \delta f_T = 0$$

$$-\zeta_R \frac{d}{dt} \theta - \frac{\partial}{\partial \theta} F_{eff}(r_{CM}, \theta) + \delta T_R = 0$$

Dynamic Crossover Diagram *(naive MCT "ideal glass")*



3 dynamical states

Fluid

Plastic Glass

…*CM localized, Rotation Ergodic*

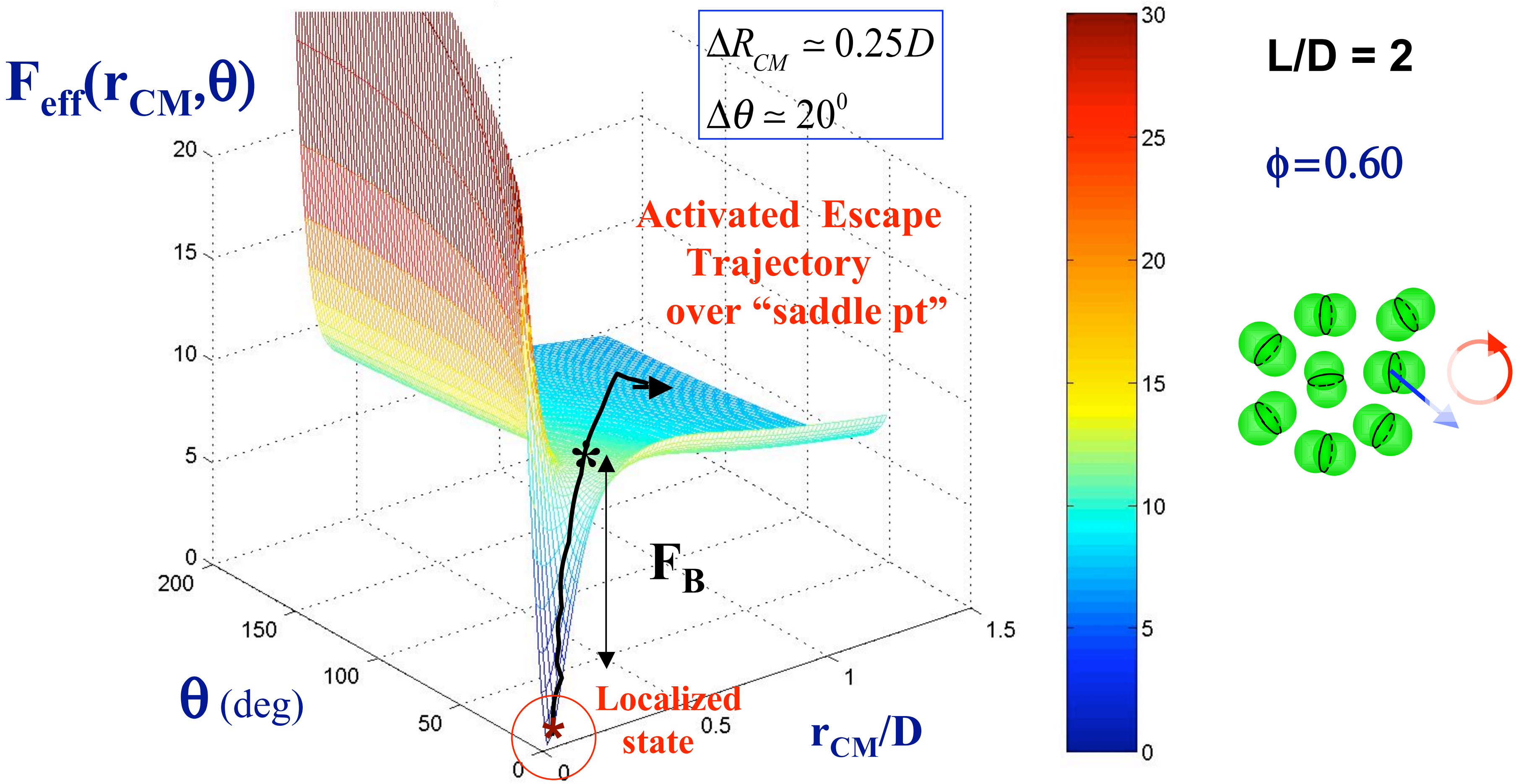
Fully Arrested **"Double Glass"**

Qualitatively ala Full site-site MCT
(Chong & Gotze, PRE, 2002)

"Most Difficult to Vitrify" state….*analogous to granular jamming* !

Physical Mechanism: "packing frustration"….*weakest short range caging order*

Dynamical Free Energy Surface *(double glass regime)*



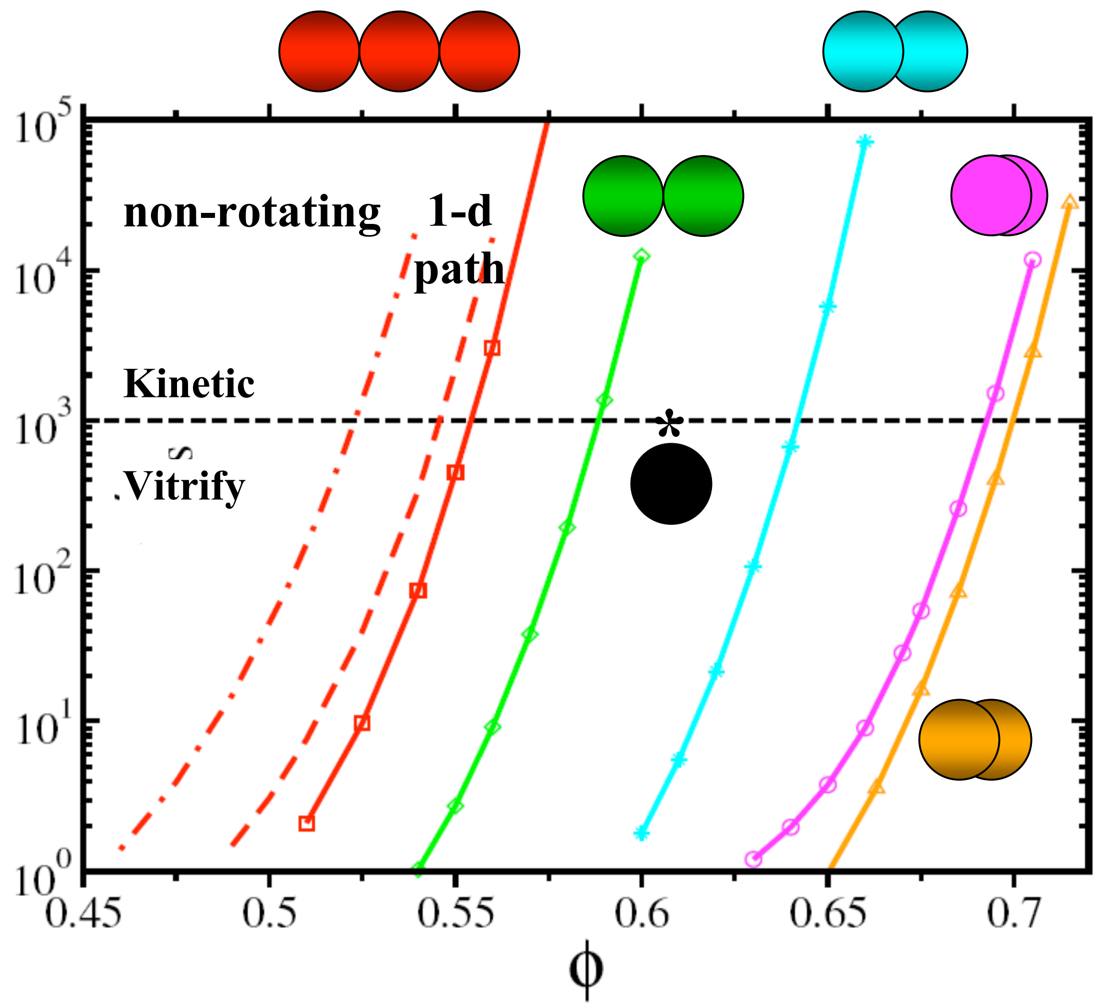
Cooperative Translate-Rotate Activated Path……*barrier varies with "eigenvector"*
depends mainly on particle shape

Mechanistic picture of Alpha Relaxation ala chemical reaction

Relaxation Rate: Multi-Dimensional Kramers-Langer Theory

$$\frac{\tau}{\tau_s} = \frac{2\pi}{\lambda^+}\left(\frac{|\det \mathbf{K}_B|}{\det \mathbf{K}_o}\right)^{1/2} \exp(F_{B,SP})$$

Saddle Trajectory + local fluctuations



- **Supra-Arrhenius Growth**
- **NON-monotonic**
- **Less rotation @ saddle**

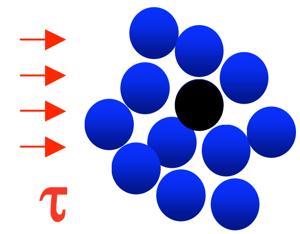
L/D ↑

L/D	ϕ_g
1	0.60
1.25	0.693
1.43	0.700
1.8	0.642
2	0.588
3	0.554

Connection *between ideal MCT and activated NLE*

Nonlinear Viscoelasticity: *Simple Stress Perspective*

Classic *Idea: External Deformation Reduces Barriers to Flow*



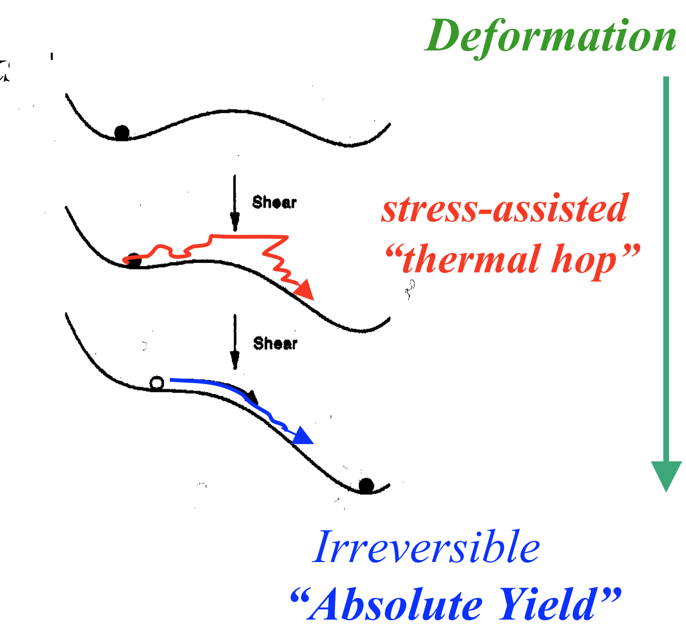
* ***Eyring*** *(1936)* *Arrenhius viscoplastic flow*

Frenkel (crystals)

Mechanical Work $E_B(\tau) \approx E_B(0) - \tau V_A$

* ***Potential Energy Landscape simulations…*** *Dan Lack*

stress reduces and ultimately destroy barriers



*** Macroscopic Rheology ⟷ cage scale physics**

Micro*rheology concept*

↑

Simulation Support *: Yamamoto ; dePablo ; Rottler ; … …*

↓

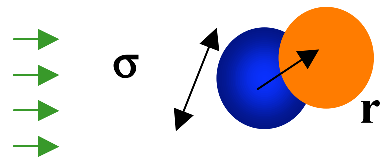
* *Dynamics ~* ***Isotropic*** *on CAGE scale*

Incorporation of Stress in NLE Theory

Kobelev+KSS
PRE 2005

External force on particle

τ = Applied Stress

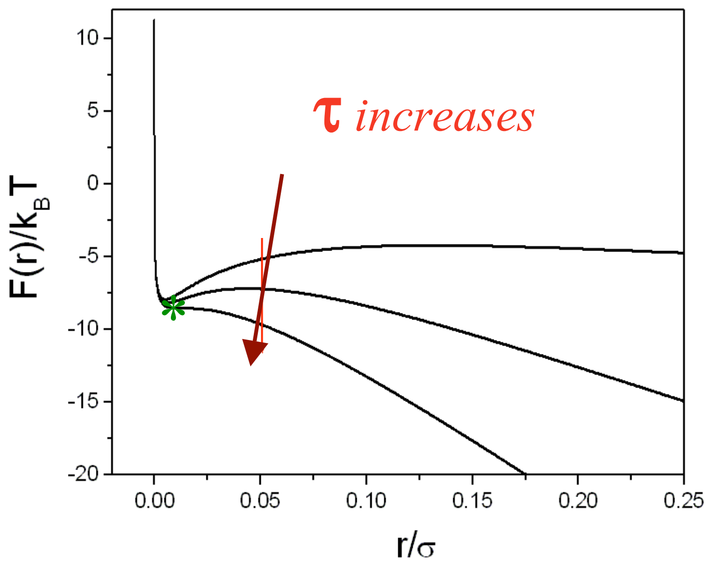


Mechanical Work

ala Eyring @ "instantaneous dynamical variable" level

$$F(r;\tau) = F(r;\tau=0) - \#\sigma^2 \tau\, r$$

"tilted landscape"



STRESS : Reduces Modulus
Accelerates Relaxation

"Absolute YIELD" → Barrier destroyed

$$\frac{\bar{\tau}_{hop}}{\tau_0} = \frac{2\pi\, g(\sigma)}{\sqrt{K_0(\tau)\, K_B(\tau)}}\, e^{F_B(\tau)}$$

$$G'(\tau) = \frac{1}{60\pi^2} \int_0^\infty dq\, q^4 \left(\frac{\partial \ln S(q)}{\partial q} \right)^2 e^{-q^2 r_{LOC}^2(\tau)/3S(q)}$$

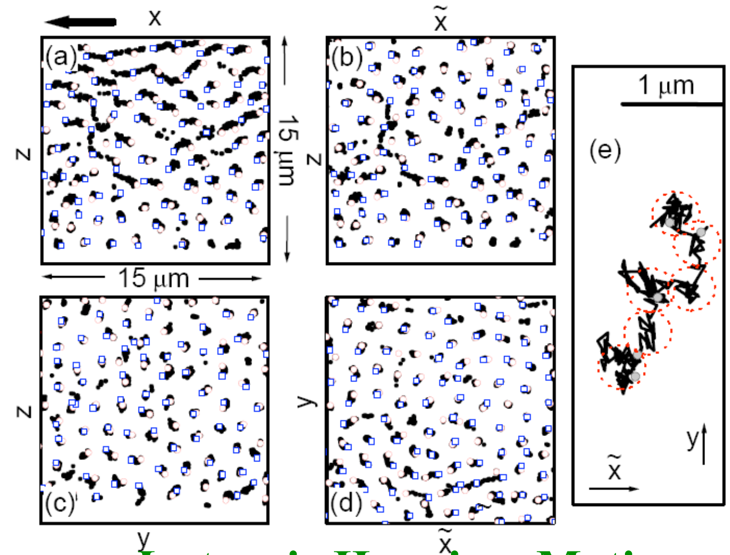
Viscosity, Flow Curve, Shear Thinning,…

Constitutive eqn: Chen + KSS, Macromolecules, 2008

SELF-Motion Under Shear

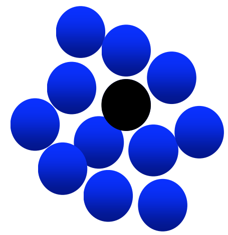
Besseling, Weeks, Poon, PRL, 2007

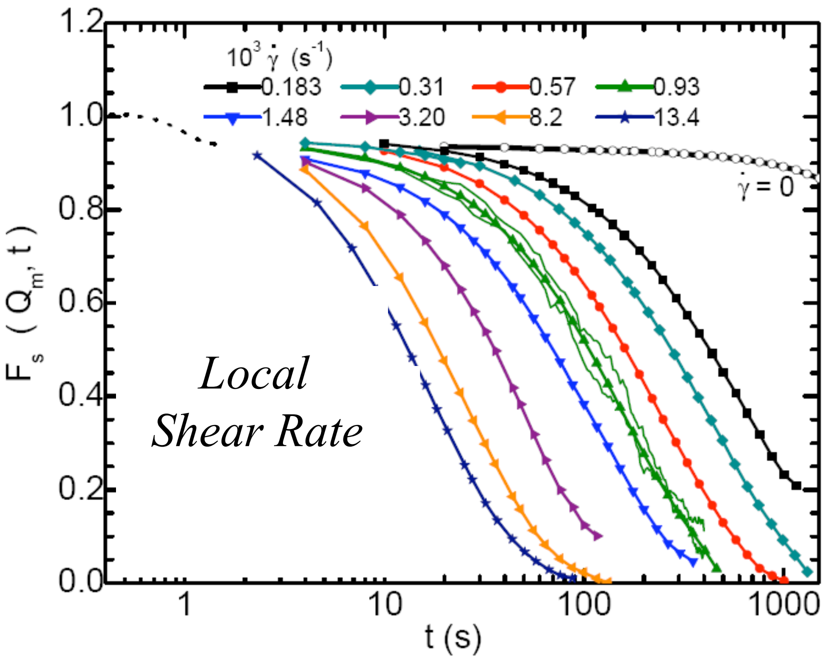
***Confocal :** direct microscopic probe of theory*



~ Isotropic Hopping Motion

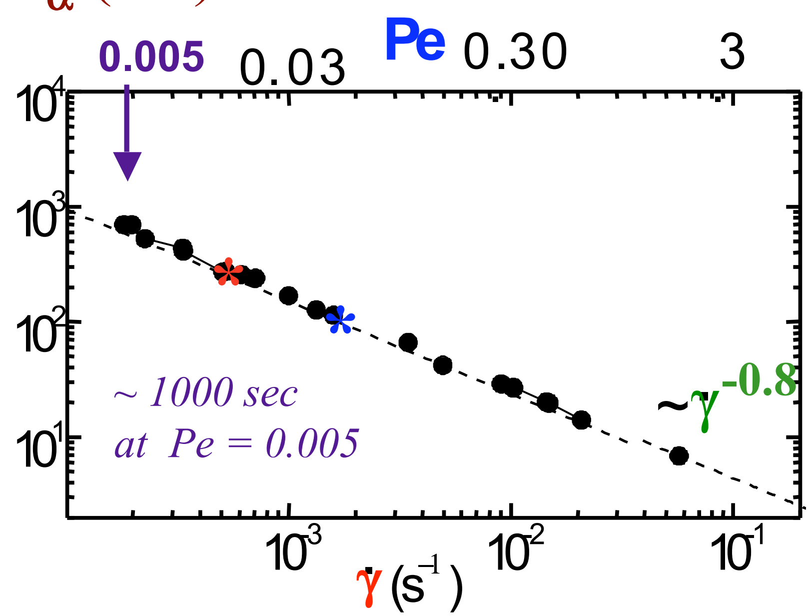
$F_s(q^*,t)$ $\phi = 0.62$





Exponential Relaxation

τ_α (sec)



$\tau_\alpha \sim$ 1/(shear rate)$^{0.8}$

Steady State NLE Theory *Predictions*

PRE, 2005
JPCM,2008

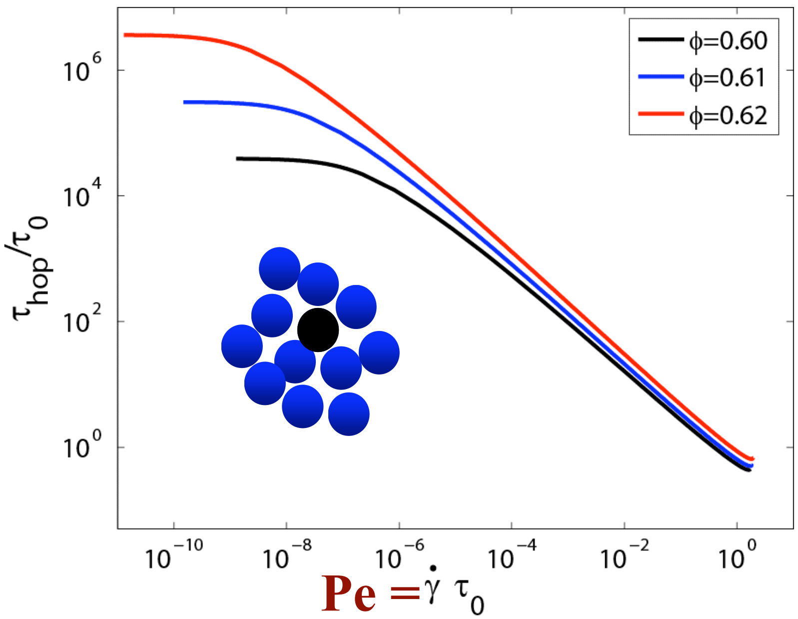
$$\tau = \eta(\tau)\dot{\gamma} = G'(\tau)\tau_{\alpha}(\tau)\dot{\gamma}$$

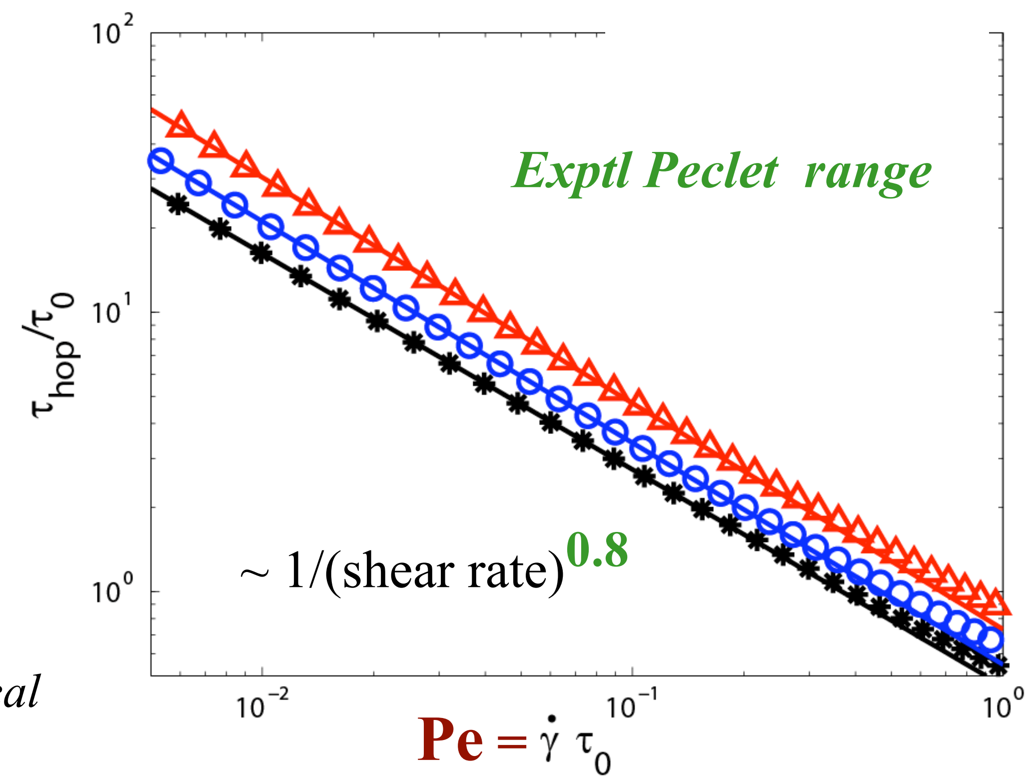
Hopping

τ_0 ~ 30 secs → **τ_α ~ 60 million secs**
ϕ=0.62 **~ 2 Years**

AT *lowest* **Pe = 0.005 : 900 secs ~ EXPT**

"shear thins" by ~5 orders of magnitude !





agrees with EXPT

Entropic Barriers NOT Zero

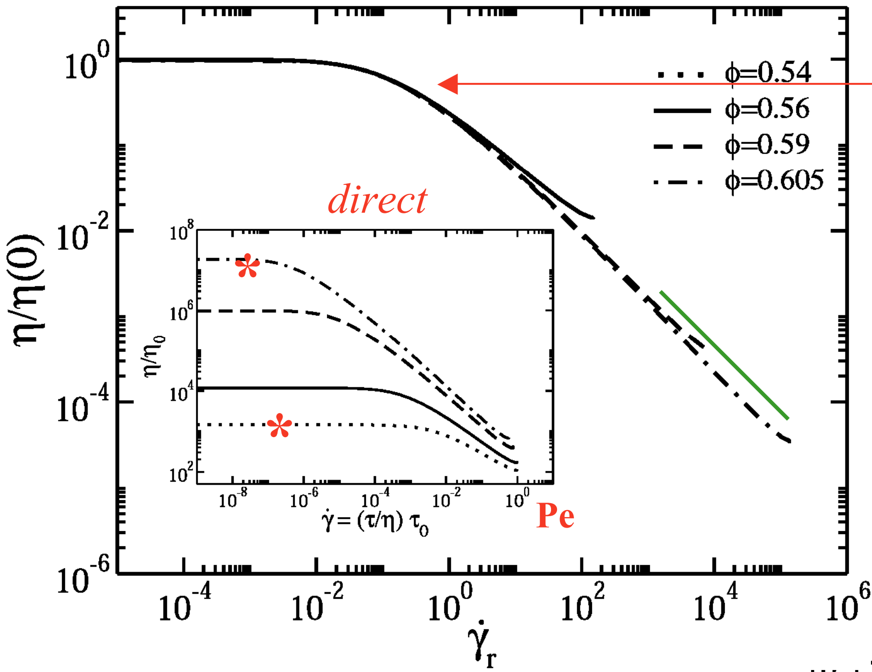
per Intermittent Hopping seen in confocal

Viscosity Thinning & Flow Curves

ϕ
0.54
0.56
0.59
0.605

η/η_s

10^3-10^7



$\dot{\gamma}_{r,crit} \approx 0.3 - 0.4$ ~ **Expts**

$$\dot{\gamma}_r \equiv \dot{\gamma}\left(\beta R^3 \eta(0)\right)$$

Dressed Peclet

"stress relaxation time"

Near collapse

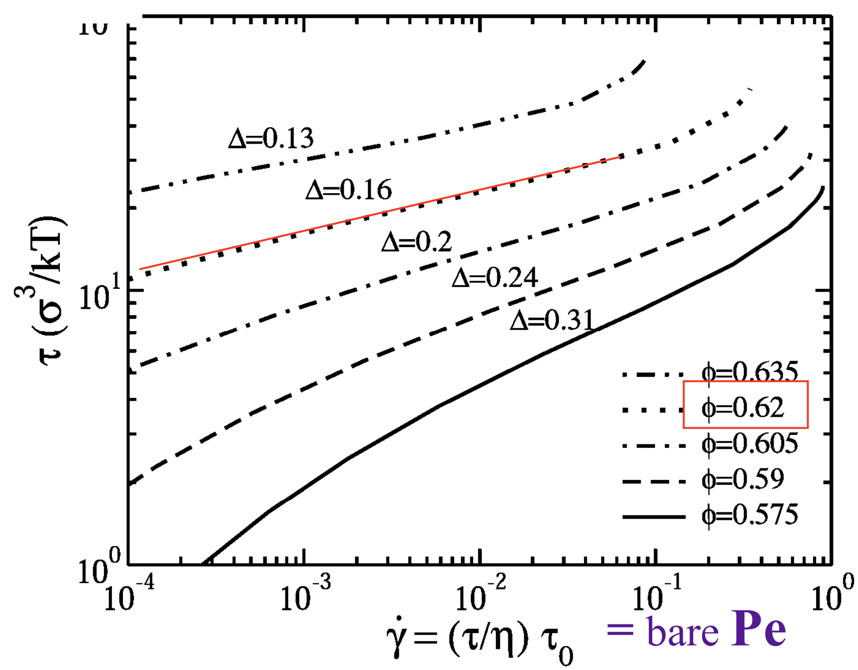
FLOW CURVES

No rigorous plateau (hopping)

Apparent power law regime

$$\tau \equiv \dot{\gamma}^{\Delta}$$

exponents ~ **0.1 - 0.3**



= bare **Pe**