COMMON FEATURES IN THE DYNAMICS OF GLASSY SYSTEMS

(Classical, quantum & optimisation proc.)

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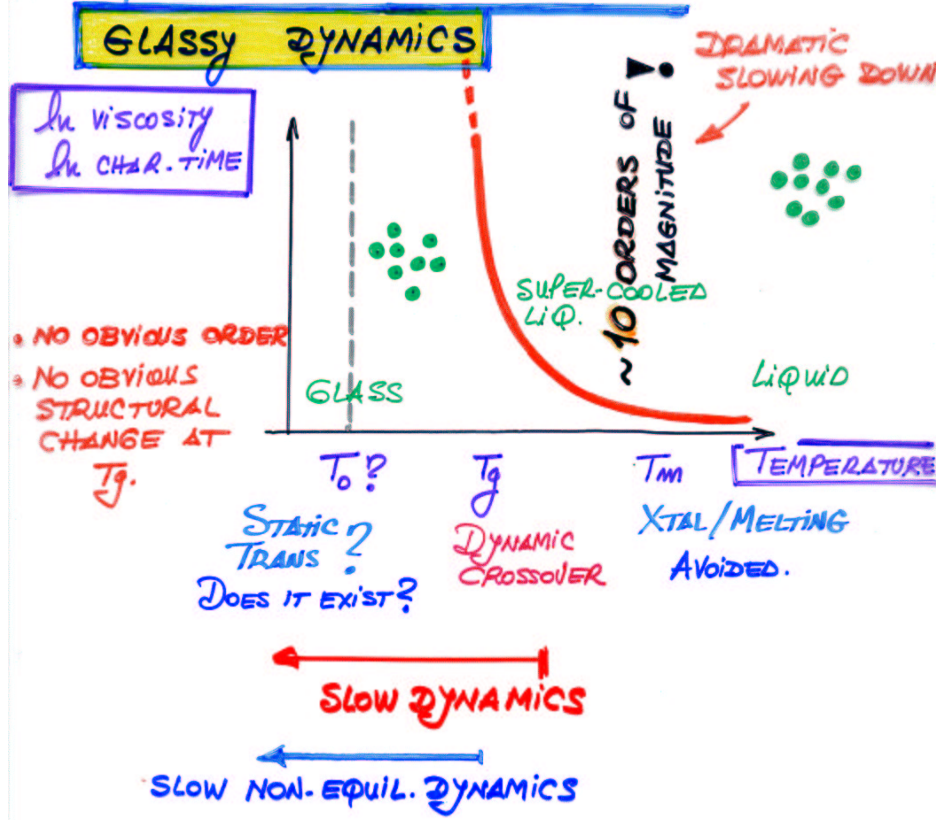
REVIEW COND-MAT/0210312
LES HOUCHES LECTURE NOTES.

PROBLEMS & QUESTIONS

PARTICLES IN INTERACTION, THERMOSTATED SYST.

- LIQUID-GLASS "TRANSITION"



OTHER GLASSES: SPINS IN INTERACTION VIA QUENCHED RANDOM COUPLINGS.

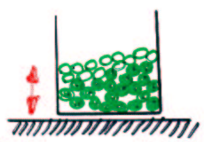
PROBLEMS & QUESTIONS

- GLASSY PROBLEM:

  SLOWING DOWN $\longrightarrow$ NON-EQUIL. RELAXATION

- DRIVEN PHYSICAL SYSTEMS

  OTHER NON-EQUIL. MACROSCOPIC SYSTEMS :



WEAKLY TAPPED GRANULAR MATTER

$k_B T \ll mgd$

NO TEMPERATURE

PURELY DRIVEN SYST.

RHEOLOGY OF COMPLEX LIQUIDS

THERMOSTATTED

INTERPLAY BETWEEN T AND DRIVING FORCE

- QUANTUM FLUCTUATIONS IN LOW T_g GLASSES

  SYSTEM IN CONTACT WITH AN ENVIRONMENT

  QUANTUM DYNAMICS $\hbar \neq 0$

MODELS

ENERGY FUNCTION - HAMILTONIAN

$$\boxed{H_J[\vec{s}] = \sum_{i_1 < i_2 < \ldots < i_p} J_{i_1 \ldots i_p} \, s_{i_1} s_{i_2} \ldots s_{i_p}}$$

INTERACTIONS BETW. p-UPLETS

QUENCHED RANDOM EXCHANGES

$P(J_{i_1 \ldots i_p})$ GAUSSIAN

ISING OR SPHERICAL

$s_i = \pm 1$ $\quad \sum_i s_i^2 = N$

SPINS.

CLASSICAL GLASSES

+ STOCHASTIC DYNAMICS

$$\gamma \, \dot{s}_i(t) = - \frac{\delta H_J[\vec{s}]}{\delta s_i(t)} + \xi_i(t) + f_i(t)$$

$$\langle \xi_i(t) \xi_j(t') \rangle = 2\gamma T \delta_{ij} \, \delta(t-t') \qquad \text{THERMAL NOISE}$$

$f_i(t)$: EXTERNAL FORCES, e.g. NON. POT $f_i \neq \frac{\delta H}{\delta s_i}$

OR TIME - DEPEND.

RHEOLOGY, GRANULAR MATTER.

QUANTUM FLUCTUATIONS.

$$H_J[\vec{s}] \longrightarrow H_J[\vec{s}] + \sum_i \frac{\Pi_i^2}{2M} \qquad [\Pi_i, s_j] = i\hbar \delta_{ij}$$

& ENVIRONMENT

$$+ H_{INT}[\vec{s}, \vec{x}^{\alpha}] + H_{ENV.}$$

QUANTUM GLASSES

MODELS - FULLY CONNECTED

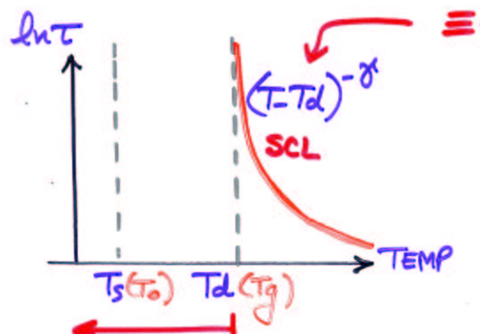
$\sum_{i_1<\dots<i_p} \equiv$ SUM OVER ALL p-UPLETS

WHY ARE THESE MODELS INTERESTING?

ANALYTICALLY SOLVABLE STATICS & DYNAMICS

- $p=2$ ISING SPINS SK MEAN-FIELD SPIN-GLASS.
- $p>2$ ISING OR SPHERICAL MF MODEL FOR (FRAGILE) GLASSES.*

DISCONTINUOUS 2nd ORDER TRANSITION(S)
(Also called Random 1st order scenario)



AT T_d: PROLIFERATION OF METASTABLE STATES

AT T_s: ENTROPY CRISIS.

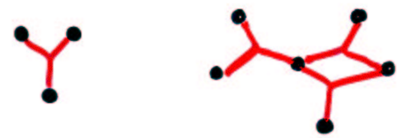
NON-EQUIL. DYNAMICS** & AGING PHENOMENA

** L.F.C. & J. KURCHAN, PRL 71, 173 (93).
* T.R. KIRKPATRICK, D. THIRUMALAI, P. WOLYNES (80s).

MODELS: DILUTE LIMIT

e.g. $p=2$ (Pairwise Int) FULLY-CONNECTED

e.g. $p=2$ FIXED CONNECTIVITY $C=2$

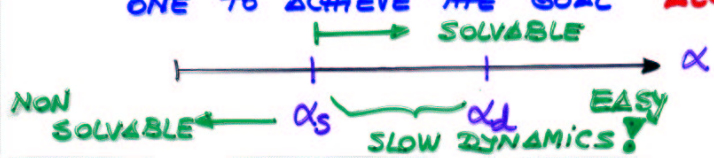


e.g. $p=3$ HYPER-GRAPH

APPLICATIONS: OPTIMISATION PROBLEMS

e.g. CODES, K-SAT, ETC.

- THERE IS NO T
- CONTROL PARAMETER α RELATED TO c.
- PROBLEM: FIND AN OPTIMAL CONFIGURATION THAT CORRESPONDS TO A GLOBAL MINIMUM OF AN ENERGY FUNCTIONAL $H_J[\vec{s}\,]$.
- DYNAMICS: NON-PHYSICAL JUST THE FASTEST ONE TO ACHIEVE THE GOAL ALGORITHMS

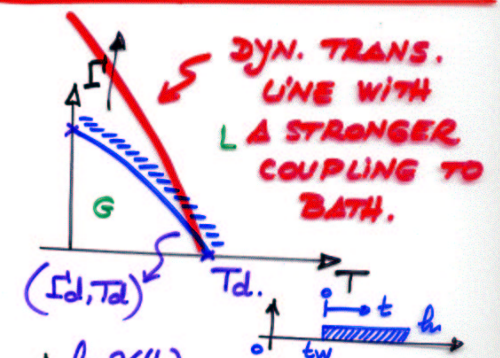


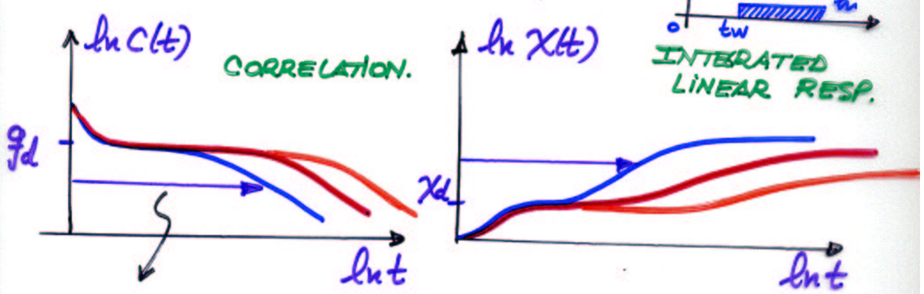
DYNAMICS OF PHYSICAL SYSTEMS

$p > 2$ MODELS

DYNAMICS CLOSE THE DYNAMIC CRITICAL LINE

(ON LIQUID SIDE)





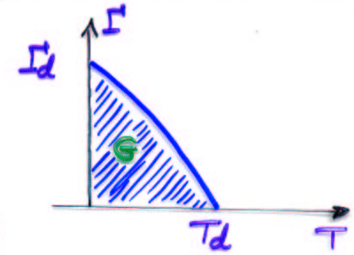
CHARACTERISTIC TIME $\tau(T, \Gamma)$ DIVERGES WHEN APPROACHING THE CRITICAL LINE.

- TWO-STEP RELAXATION : SUCCESS OF MCT FOR CLASSICAL SUPER-COOLED LIQUIDS.
- $T > T > T$
- C AND χ ARE RELATED BY (QUANTUM) FDT
- $q_d > 0$ DISCONTINUOUS 2nd ORDER TRANS.

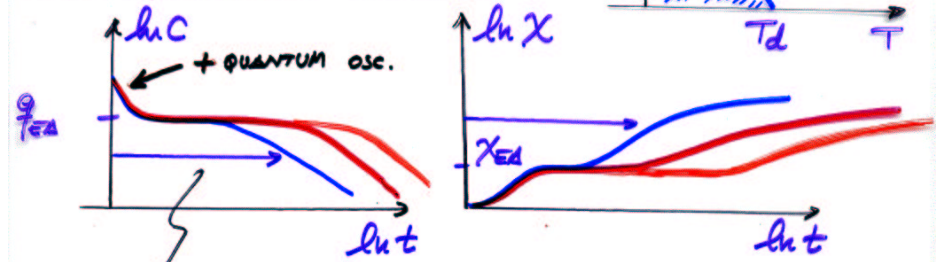
DYNAMICS OF PHYSICAL SYSTEMS

DYNAMICS "BELOW" T_d

(ON GLASSY SIDE)

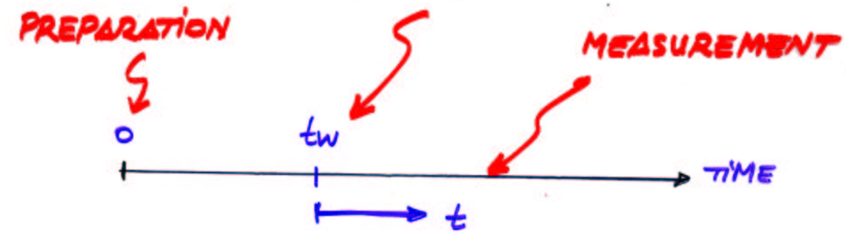


NON-EQUILIBRIUM EFFECTS:



$\tau(T, \Gamma, t_w)$ CHARACTERISTIC TIME AGING

$t_w < t_w < t_w$ WAITING-TIME DEPENDENT



- TWO-STEP RELAXATION EVEN IN THE FULL "GLASSY PHASE".
- SLOW RELAXATION FOR LONG t AS COMP. TO t_w.

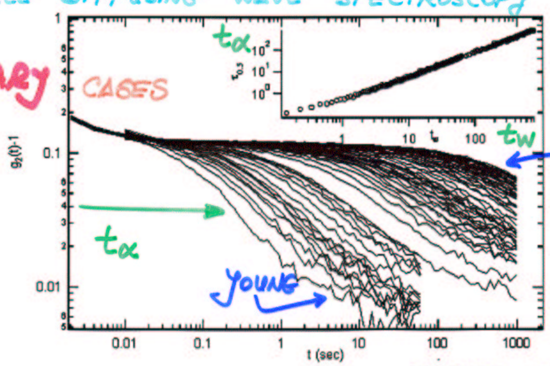
SOFT CONDENSED MATTER

$$g_2(t,t_w) = \langle I(t+t_w)\, I(t_w)\rangle / \langle I(t_w)\rangle\langle I(t+t_w)\rangle$$

COLLOIDAL SUSPENSIONS

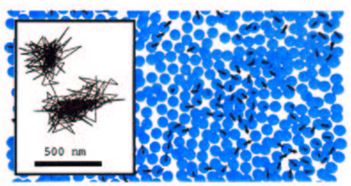
MULTI SPECKLE DIFFUSING WAVE SPECTROSCOPY

STATIONARY CASES



NON STATION.

STRUCTURAL RELAXATION



V. ViASNOFF, F. LEQUEUX
PHY. REV. LETT. 89
065701 (2002).

E. WEEKS, D. WEITZ
CONFOCAL MICROSCOPY.

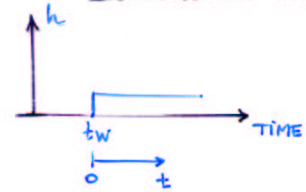
- RELAXATION
  WAITING-TIME t_w
- RHEOLOGY
  SHEAR RATE $\dot{\gamma}$

$t_\alpha(t_w)$	AGING
$t_\alpha(\dot{\gamma})$	STATIONARY

RELATION BETWEEN C AND R

FDT VIOLATIONS

L.F. CUGLIANDOLO & J. KURCHAN; PRL 71, 173 (93).



$$\chi(t+t_w,t_w) \equiv \int_{t_w}^{t+t_w} d\tau\, R(t+t_w,\tau)$$

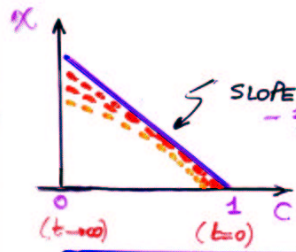
SUSCEPTIBILITY

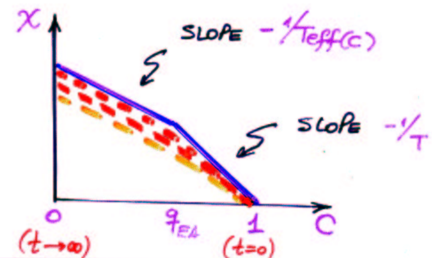
CONSTRUCTION OF χ VS C PLOT

TAKE $t_{w_1} < t_{w_2} < t_{w_3} < \dots < t_{w_k}$ AND

PLOT χ vs C USING t AS A PARAMETER $t: 0 \to \infty$.

$T > T_d$	$T < T_d$
EQUILIBRIUM	NON EQUILIBRIUM





$$\lim_{\substack{t_w \to \infty \\ C(t+t_w,t_w)=C}} \chi(t+t_w,t_w) = \chi(C)$$

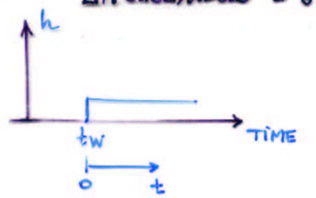
PREDICTION:

THE LIMIT EXISTS & CAN BE NONTRIVIAL IF $T < T_d$

RELATION BETWEEN C AND R

FDT VIOLATIONS

L.F. CUGLIANDOLO & J. KURCHAN; PRL 71, 173 (93).



$$\chi(t+t_w,t_w) \equiv \int_{t_w}^{t+t_w} d\tau \; R(t+t_w,\tau)$$

SUSCEPTIBILITY

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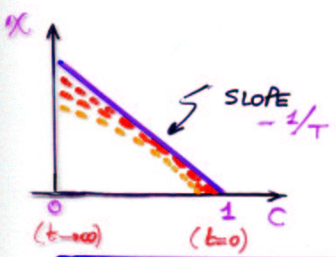
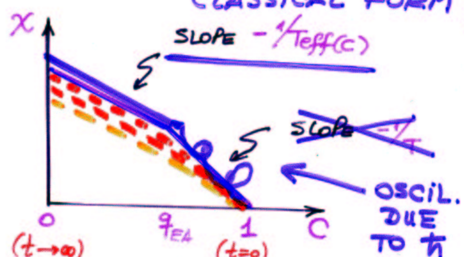
$T > T_d$	$T < T_d$
EQUILIBRIUM	NON EQUILIBRIUM

CLASSICAL FORM !





L.F.C. & G. LOZANO PRL (98).

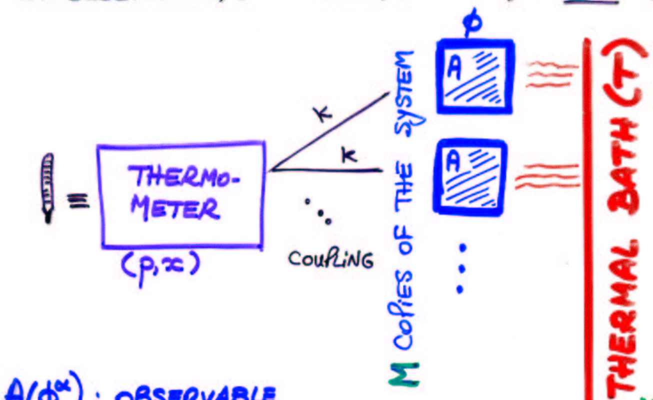
$$\boxed{\lim_{\substack{t_w\to\infty \\ C(t+t_w,t_w)=C}} \chi(t+t_w,t_w) = \chi(C)}$$

PREDICTION:

THE LIMIT EXISTS & CAN BE NONTRIVIAL IF $T<T_d$

EFFECTIVE TEMPERATURES

L.F. CUGLIANDOLO, J. KURCHAN, L. PELITI, PRE 55 3898 (1997).



$A(\phi^\alpha)$: OBSERVABLE

$$E_{TOT} = \underbrace{\frac{m\dot{x}^2}{2} + \frac{\omega_0 m x^2}{2}}_{\text{THERMOMETER}} + \underbrace{E(\phi^\alpha)}_{\text{ISOLATED SYSTEMS}} - \underbrace{\frac{k}{\sqrt{M}}\, x \sum_{\alpha=1}^{M} A(\phi^\alpha)}_{\text{COUPLING}}$$

EQ OF MOTION FOR THE THERMOMETER

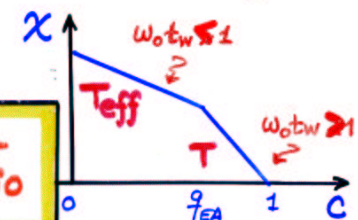
$$m\ddot{x}(t) = -\frac{\partial V(x)}{\partial x(t)} - \frac{k}{\sqrt{M}} \sum_{\alpha=1}^{M} A(\phi^\alpha)(t)$$

EQUIVALENT TO A **LANGEVIN - EQUATION**

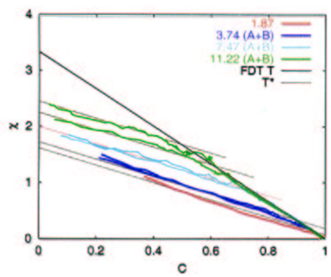
$$m\ddot{x}(t) = -\frac{\partial V(x)}{\partial x(t)} + k^2 \int_0^t dt' \; R_{AA}(t,t')\, x(t') + \eta(t)$$

$$\langle \eta(t)\eta(t')\rangle = k^2\, C_{AA}(t,t')$$

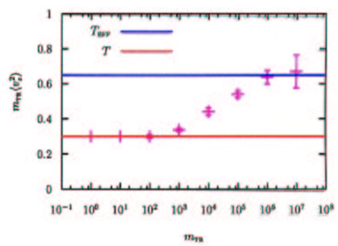
$$\boxed{\langle E_{THERMO}\rangle = \frac{\omega_0\, \tilde{C}_{AA}(\omega_0,t_w)}{\chi''_{AA}(\omega_0,t_w)}}$$ FDT RATIO



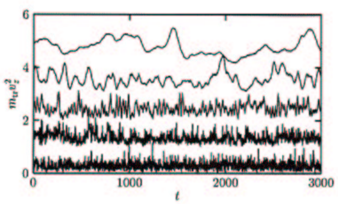
Test of partial equilibrations in a sheared Lennard-Jones mixture

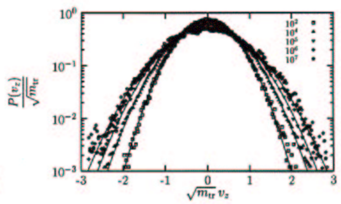


Parametric plot



Av. transv. velocity of a tracer





Transverse velocity fluctuations of the tracer

L. Berthier and J-L Barrat
Phys. Rev. Lett. 89, 095702 (2002) & J. Chem. Phys. 116, 6228 (2002)

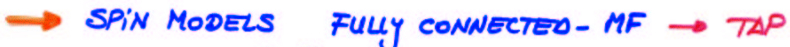
LANDSCAPES

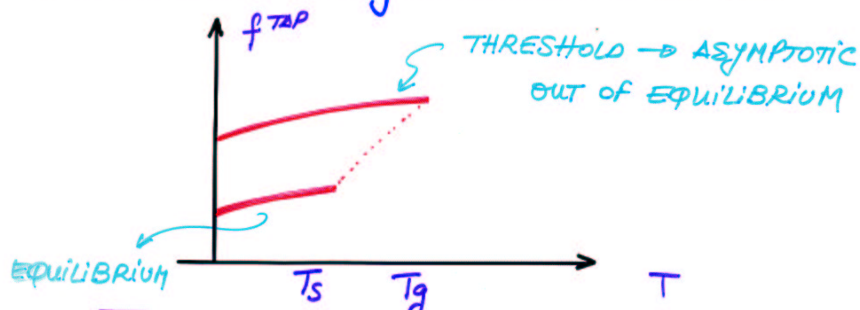
MODIFICATIONS OF FDT $\Longleftrightarrow$ EFFECTIVE TEMPERATURES

APPEAR IN: GLASSES, RHEOLOGY, VIBRATED GRANULAR MATTER, DRIVEN VORTEX SYSTEMS ...

RELATED TO THE ORGANISATION OF

$\underbrace{\text{METASTABLE STATES}}_{\text{TO BE DEFINED !}}$ IN THE $\underbrace{\text{FREE-ENERGY LANDSC}}_{\text{TO BE DEFINED !}}$





$$\boxed{\begin{aligned} \Sigma(\beta,f) &= \frac{1}{N}\ln \#\ \text{metast states}(\beta,f) \\ \frac{1}{T_{\rm eff}} &= \left.\frac{\partial \Sigma(\beta,f)}{\partial f}\right|_{f_{\rm threshold}} \end{aligned}}$$

cfr. EDWARDS. INHERENT STRUCT $\mathcal{E}$

FLUCTUATIONS

H. CASTILLO, C. CHAMON, L.F.C., J.L. IGUAIN, M.P. KENNETT

- **FULLY CONNECTED MODELS**

THE EXACT DYN. EQS. FOR $\{C(t_1,t_2), \chi(t_1,t_2)\}$ ACQUIRE AN (APPROXIMATE) TIME-REPARAM. INVARIANCE

$$\begin{cases} t_1 \to h(t_1) \\ t_2 \to h(t_2) \end{cases} \qquad \begin{matrix} C \to C \\ \chi \to \chi \end{matrix}$$

IN THE SLOW REGIME. (SOMPOLINSKY 80's)

- **FINITE d SYSTEMS.**

THE ACTION FOR THE SLOW DECAY OF COARSED-GRAINED $(C_r(t_1,t_2), \chi_r(t_1,t_2))$ WE ARGUE IS INV. UNDER GLOBAL $t_{\frac{1}{2}} \to h(t_{\frac{1}{2}})$

(APPROX. SYMM. REALIZED IN THE ASYMPT. LIMIT)

SAME

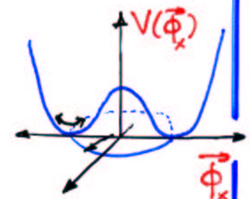
$$\boxed{C^{\text{slow}}(t_1,t_2) \simeq f\left(\frac{\tilde{h}(t_2)}{\tilde{h}(t_1)}\right) \longrightarrow C_r^{\text{slow}}(t_1,t_2) \simeq f\left(\frac{h_r(t_2)}{h_r(t_1)}\right)}$$

GLOBAL $\simeq$ SADDLE-POINT ↑ FLUCT.

$\tilde{h}(t_1)$: THE ONE THAT IS SELECTED BY THE DYNAMICS.

PARALLELS → IDEA

O(N) MODEL - EQUIL.	GLASSY DYN. FINITE D
$H = \int d^d x \left[(\nabla \vec{\phi}_x)^2 + V(\vec{\phi}_x) \right]$	$H = \sum_{\langle ij \rangle} J_{ij} s_i s_j + H_{loc}(\vec{s})$
MEXICAN HAT, $\vec{\phi}_x^2 = 1$, N-DIM	+ DYNAMICS; $\lim_{t_w \to \infty} \chi_r = \chi_r(C_r)$ ↑ RpG INV.



RELEVANT VARIABLE / FIELD

$\vec{\phi}_x$	$Q_i^{ab}(t_1,t_2)$

ACTION INVARIANT UNDER

$\tilde{\phi}_x^\alpha = R^{\alpha\beta} \phi_x^\beta$	$\tilde{Q}_i^{ab}(t_1,t_2) = h_1^a h_2^b Q_i^{ab}(h,h)$
EXACT SYMMETRY	APPROX. SYMMETRY OF THE SLOW EFF. ACT.
A "PINNING" FIELD ⇒ FIXES THE DIRECTION	∂_t FIXES $h(t)$.

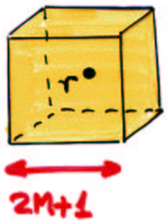
LOW COST LOCAL FLUCTUATIONS

DIFFERENT SITES x (i) CHOOSE DIFFERENT DIRECTIONS $\vec{\phi}_x$ / FUNCTIONS $h_i(t)$

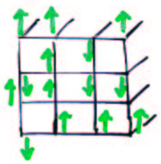
(SPIN WAVES) PARAMETRIZATION

TWO-TIME LOCAL CORRELATIONS

LOCAL INTEGRATED RESPONSE



$V_r \equiv (2M+1)^3$ COARSE-GRAINING



↑ RANDOM FIELD

$h_i \equiv h \cdot \varepsilon_i \qquad \varepsilon_i = \pm 1$

$[\varepsilon_i \varepsilon_j] = \delta_{ij}$

$$\chi_r(t+t_w, t_w) \equiv \frac{1}{N_{fields}} \sum_{k=1}^{N_{fields}} \frac{1}{V_r} \sum_{i \in V_r} \frac{\left(s_i^{\vec{h}^{(k)}} - s_i^{\vec{0}}\right)(t+t_w) \cdot \varepsilon_i}{h}$$

FIELD APPLIED FROM t_w ON.

AVERAGE OVER ≠ PERT. FIELD REALIZATIONS.

LOCAL CORRELATIONS

$$C_r(t+t_w, t_w) \equiv \frac{1}{V_r} \sum_{i \in V_r} s_i(t+t_w) \cdot s_i(t_w)$$

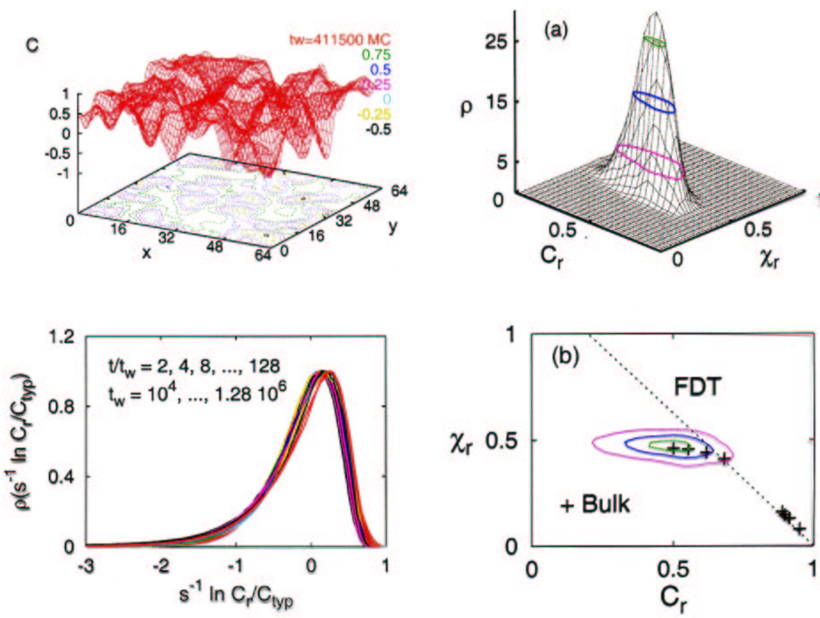
FIXED DISORDER & THERMAL HISTORY

Beyond mean-field: heterogeneities

Spin model on a lattice

$$C_r(t+t_w, t_w) = \frac{1}{V_r} \sum_{i \in V_r} s_i(t+t_w) s_i(t_w)$$

$$\chi_r(t+t_w, t_w) = \frac{1}{V_r} \sum_{i \in V_r} \left. \frac{\delta s_i(t+t_w)}{\delta h_i(t_w)} \right|_{h=0}$$



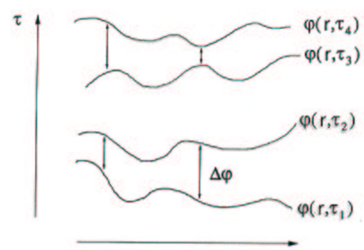
C. Chamon, M. P. Kennett, H. E. Castillo & L. F. Cugliandolo, Phys. Rev. Lett. 89, 217201 (2002).
H. E. Castillo, C. Chamon, L. F. Cugliandolo & M. P. Kennett, Phys. Rev. Lett. 88, 237201 (2002).

FLUCTUATIONS.

Effective random surface theory

$C(t_1,t_2) \simeq f\left(\frac{\tilde{h}(t_2)}{\tilde{h}(t_1)}\right)$ $\quad t_2 < t_1.$ GLOBAL

$C_r(t_1,t_2) \simeq f\left(\frac{\tilde{h}(t_2)}{\tilde{h}(t_1)}\, e^{-(\varphi_r(t_2)-\varphi_r(t_1))}\right)$ LOCAL



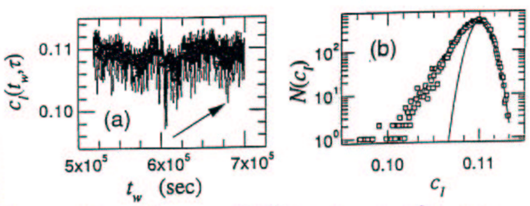
FLUCTUATIONS PARAMETRIZED BY $\varphi_r(t)$

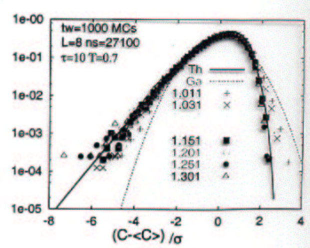
PHEN. ACTION, BASED ON REP. INV.

$S[\varphi_r(t)] = \gamma q_{EA}^2 \int d^dr \int d\tau \, (\nabla \dot{\varphi}_r(\tau))^2 \qquad \tau = \ln \tilde{h}(t)$

γ STIFFNESS REL TO $f(x)$

L. CIPELLETTI ET AL.





DISTRIBUTION $\rho(C_r(t_1,t_2))$ MODIFIED GUMBEL.

SUMMARY & CONCLUSIONS.

- CONSISTENT MEAN-FIELD THEORY FOR GLASS "TRANSITION" AND GLASSY DYN.

→ Two steps decay alpha & beta relax.

→ Aging phenomena

→ Cooling rate effects could be included

etc.

PREDICTIONS:

→ Modification of FDT

→ Relation to EFFECTIVE TEMPERATURE

→ Relation to NUMBER OF METASTABLE STATES

etc.

AND BEYOND MEAN-FIELD

→ FLUCTUATIONS.

TIME REPARAMETRIZATION INVARIANCE guiding symmetry to construct a SIGMA MODEL for the slow dynamics.