Gravitational Transport of Grains

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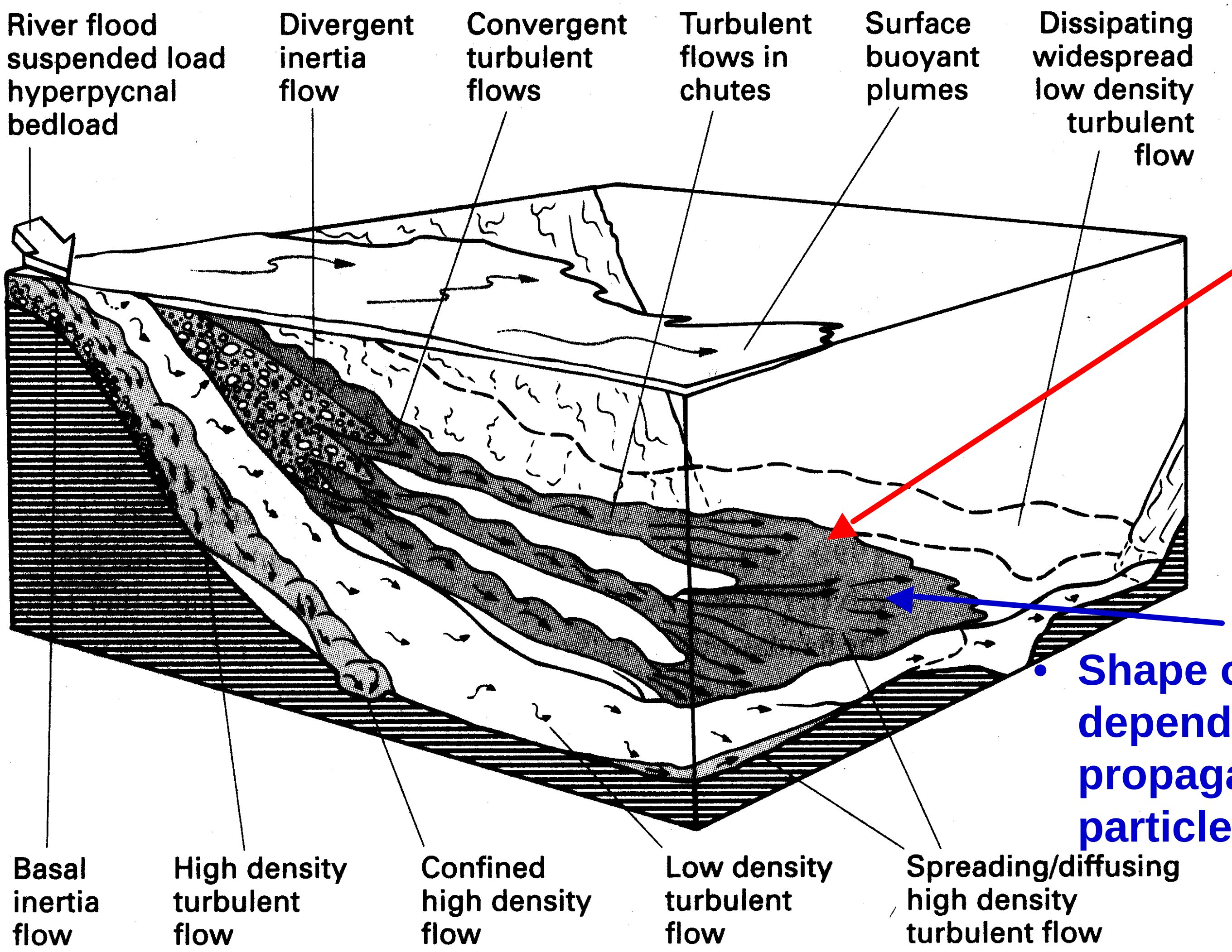
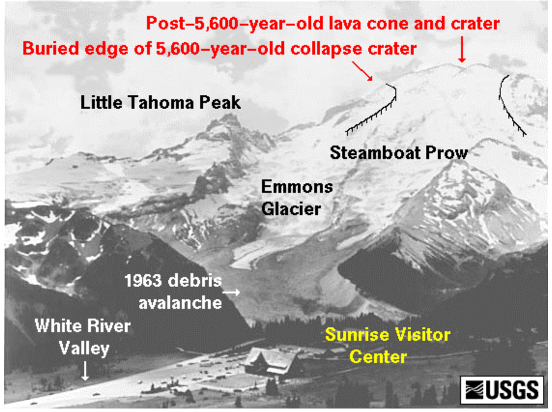
Formation of Off-Shore Reservoirs from Sediments

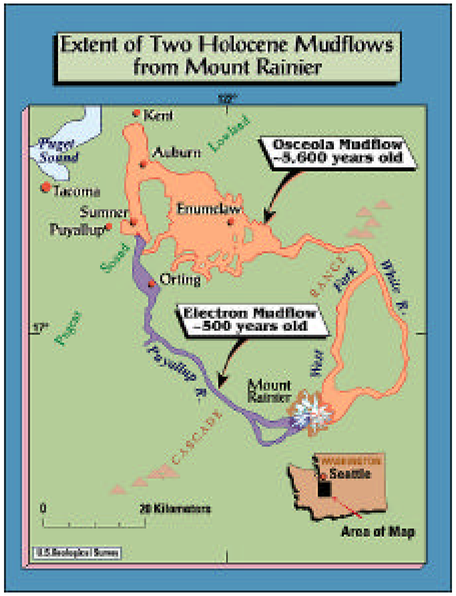


Erosion & deposition by sediment flow

- **Shape of formation depends on the propagation of particle-laden flows**

Debris Flows - Mt. Rainier

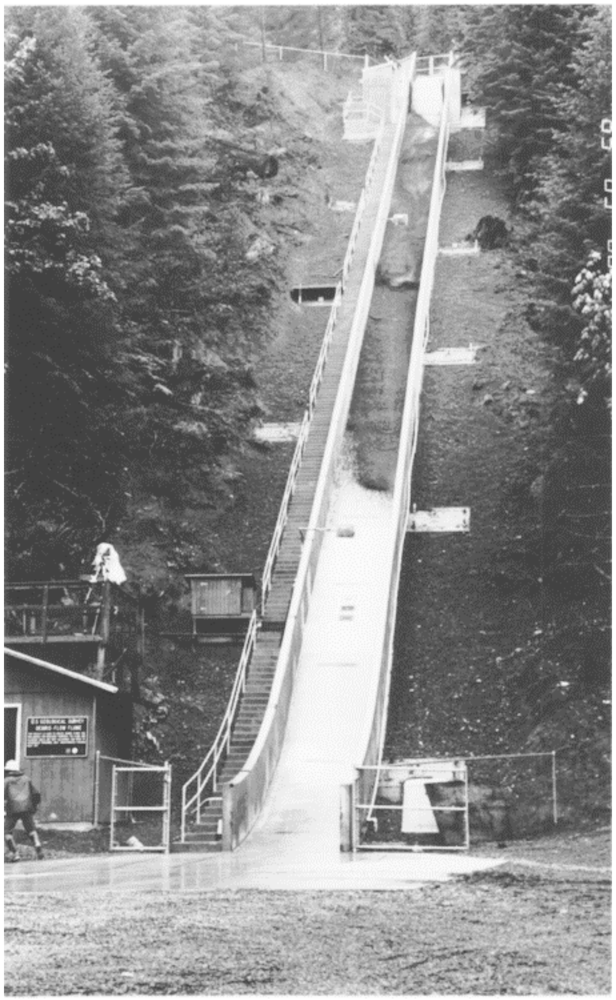




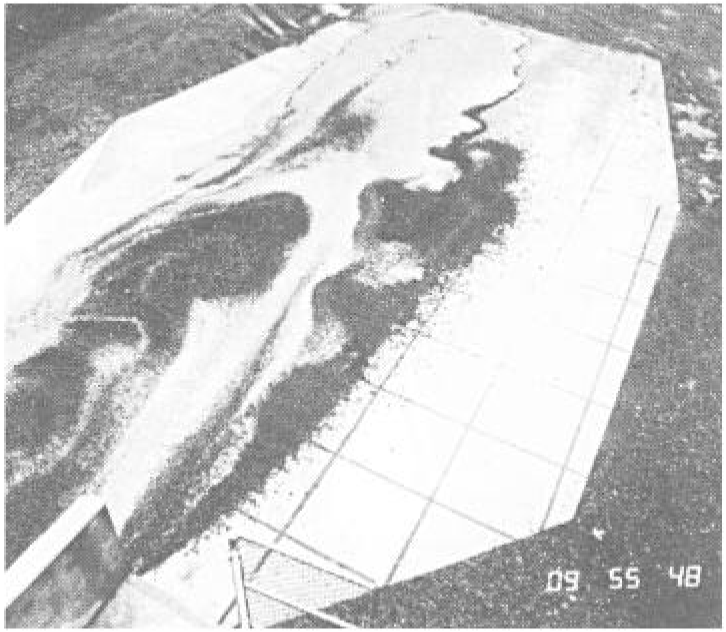
Debris flow at Tahoma Creek, July 26, 1988.

Source: USGS

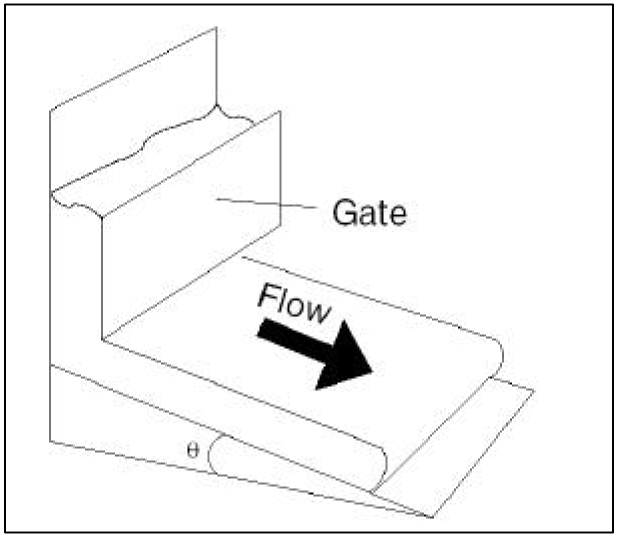
Debris Flow Flume

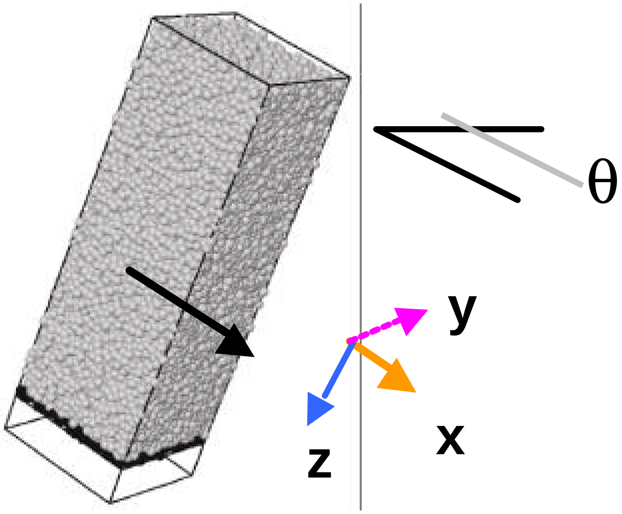


- *R.M. Iverson, J.E. Costa, and R.G. LaHusen, 1992, H.J. Andrews Experimental Forest, Oregon: USGS*



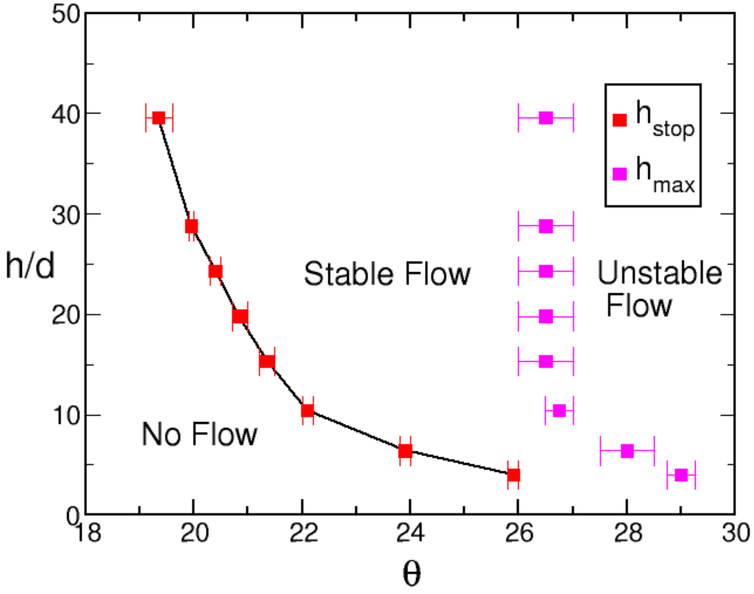
Chute Flow



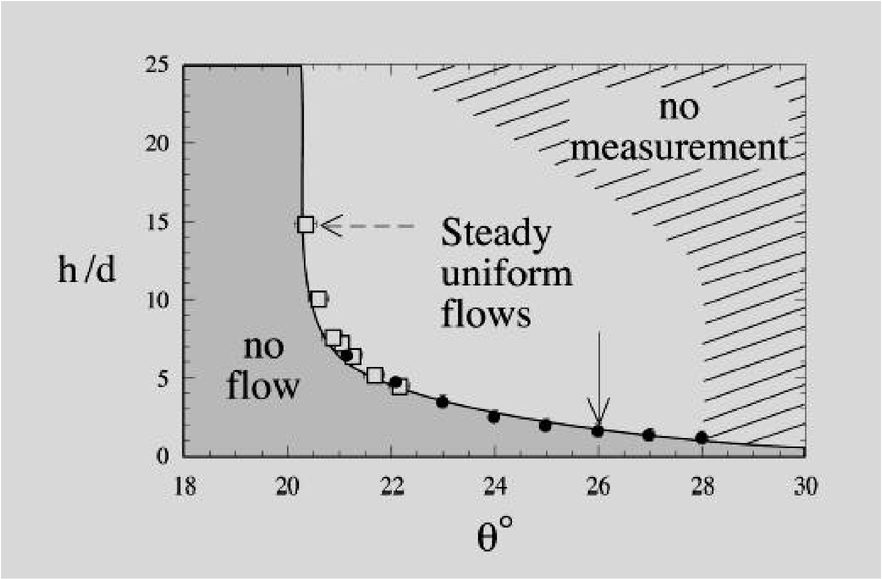


- Study chute flow -- flow of a dry granular medium down an inclined plane
- Fundamental geometry for geophysical applications
  - Debris flows
- Concentrate mainly on three dimensions
  - Two-dimensional variation of parameters
  - In 2-d, crystallization creates significant hysteresis, boundary effects

Phase Diagram for Flow



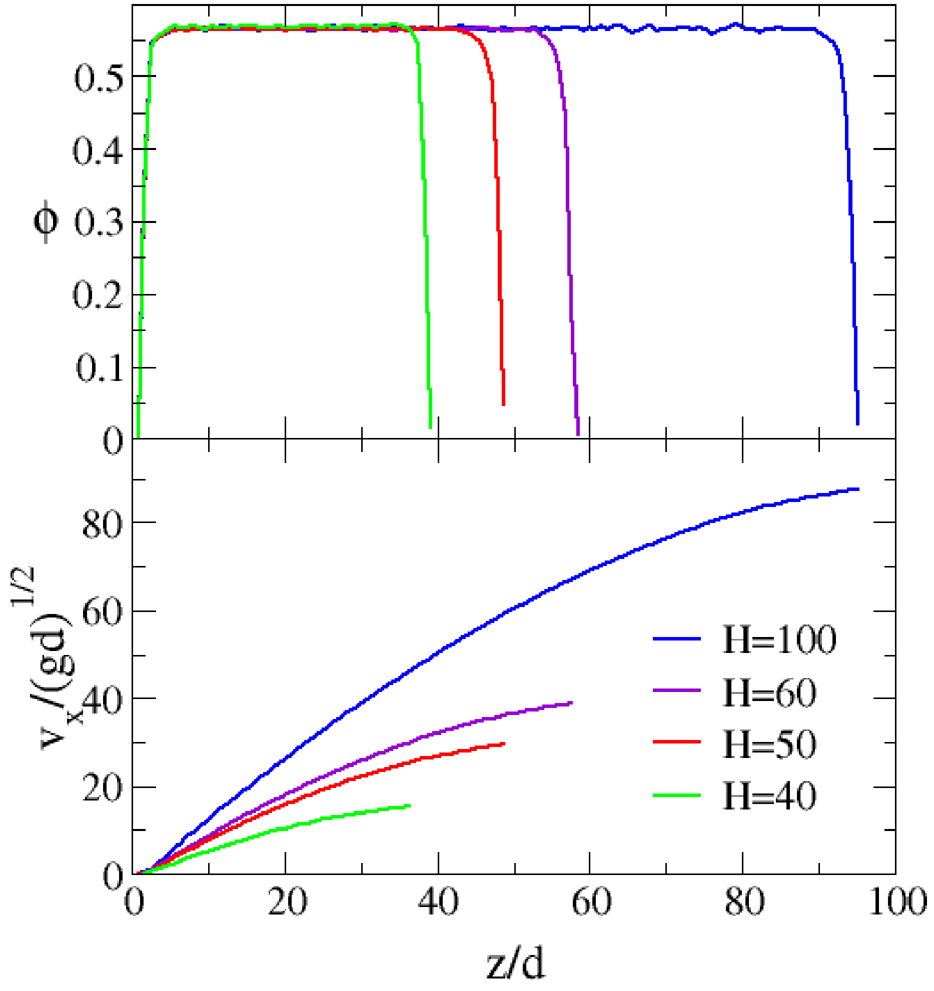
Numerical Results



Experimental Results Pouliquen, '99

- **Phase diagram shows three regions:**
  - **No flow**
  - **Stable flow**
  - **Accelerating (unstable) flow**
- **For h large, angle of repose is ~19.5^o**

Kinematics of Chute Flow



- **Constant density profile observed with depth**
  - **Density drops near surface**
- **Velocity obeys 3/2 power law with depth**
  - **Best fit to power law with exponent of 1.52**
  - **Agrees with Bagnold scaling (next slide)**
- **Other kinematic variables also suggest inverse strain rate as only apparent time scale**
- **Normal stresses in shear plane approximately equal**

$$S_{xx} \approx S_{zz} > S_{yy}$$

Viscosity Length and Bagnold Rheology

- **Typically for liquid-like shear flow, we expect shear stress to obey**

$$s_{xz} = r\,n\dot{g}$$

- **On dimensional grounds, define viscosity length scale** $l_\nu(\rho)$

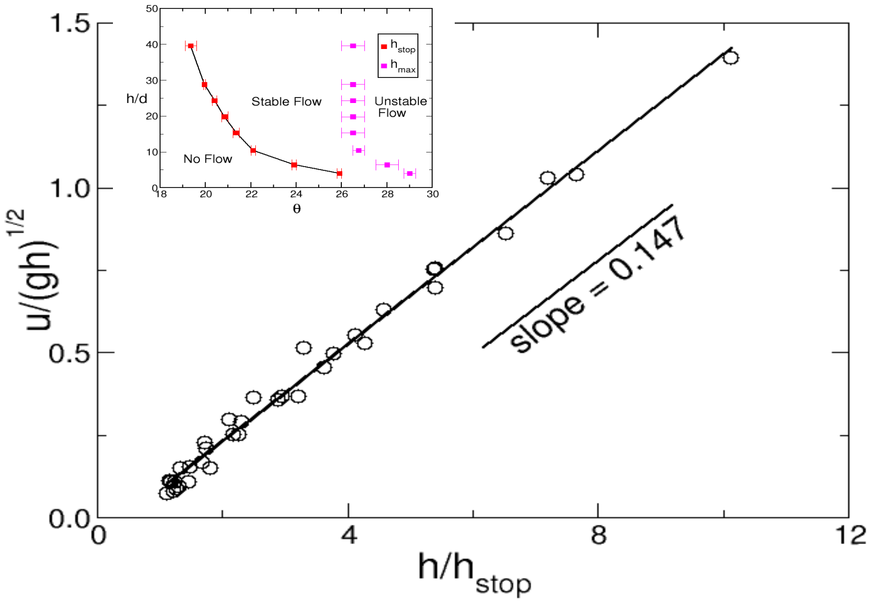
$$n \equiv l_n^2\,\dot{g}$$

- **For chute flow, if ρ = const, “Bagnold rheology”**

$$s_{xz} = r\,gz\sin q = r\,l_n^2\dot{g}^2 \Rightarrow \begin{cases} \partial_z \mathrm{v}_x = A_{Bag}\sqrt{z} \\ A_{Bag} = \dfrac{\sqrt{g\sin q}}{l_n} \end{cases}$$

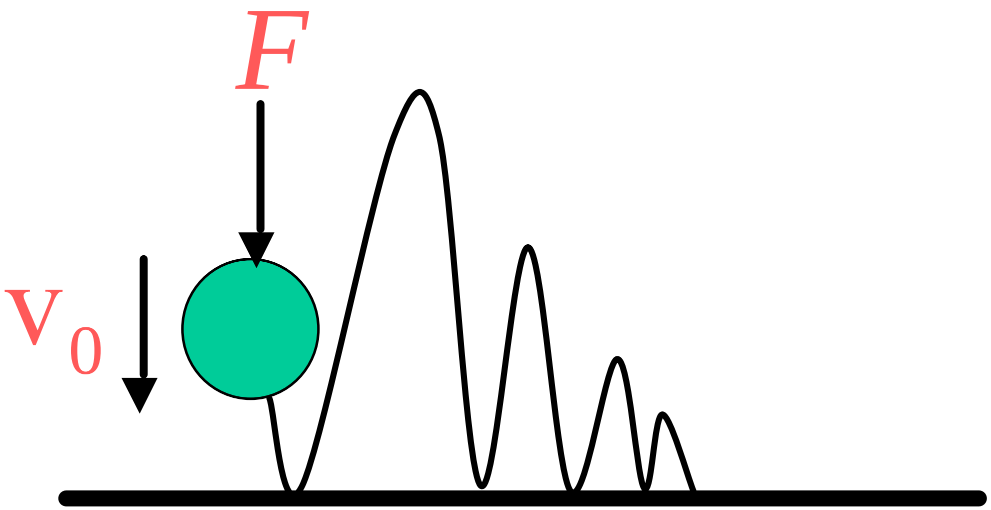
Pouliquen Flow Rule

$$\frac{u}{\sqrt{gh}} = b\,\frac{h}{h_{stop}(q)}$$



- **Pouliquen flow rule summarizes much of the phenomenology of chute flows**
  - **Relates average velocity *u* to depth *h***
  - **Connects depth of arresting pile $h_{stop}(\theta)$ with rheology (l_ν)**
  - **Unstable flow line is approximately independent of flow depth**

Inelastic Collapse of Nearest Neighbors



$$\left.\begin{aligned} v_i &= e^i v_0 \\ t_i &= \frac{2m v_i}{F} \end{aligned}\right\} \Rightarrow t_b = \sum_{i=1}^{\infty} t_i = \frac{2m v_0}{F}\frac{e}{1-e}$$

$$\text{Granular Flow:}\quad t_{nnc} = \frac{\rho d^2 \dot{\gamma}}{P} f(e)$$

- **Inelastic ball ($\varepsilon < 1$) pushed to a surface with force F comes to rest in finite time τ_b**
- **In dense granular media, depletion force**

$$F \sim P / d^2$$

- **Initial collision velocity**

$$v_0 \sim d\dot{\gamma}$$

- **Expect more complicated dependence on ε in granular flow due to disorder, friction, angular averaging and presence of other neighbors**

Correlated Motion of Grains

- **Anticipate that neighboring particle motions will become correlated if they collide sufficiently many times before shearing off:**

$$t_{nnc} = \frac{r\,d^{2}\dot{g}}{P}\,f(e) < \dot{g}^{-1}$$

- **Time for a correlated region surrounded by constant pressure P to grow to size l :**

$$t_c(l,P) \sim \left(\frac{l}{d}\right)^{2} t_{nnc} = \frac{r\,l^{2}\dot{g}}{P}\,f(e)$$

- **Characteristic correlation length l_e due to cutoff time imposed by strain rate, given initial collision velocity :**

$$\tilde{a}\,\frac{r\,l_e^{2}\dot{g}}{P}\,f(e) = \dot{g}^{-1}$$

Hypothesis of Collapsed Flow

- **We postulate that apart from finite-size corrections, the viscosity length scale is set by the characteristic correlation length:**

$$l_n^2 = l_e^2(1+\tilde{b}\,\frac{d}{l_e}+\ldots)$$

- **Solving for** l_e **and** $\dot{g}$

$$l_e = \frac{\tilde{b}\,d\tan q_R}{\tan q - \tan q_R} \sim \frac{d}{q-q_R}, \quad \tan q_R = [\tilde{a}f(e)]^{-1}$$

and

$$\dot{g} = A_{Bag}\sqrt{z}, \quad A_{Bag} = \frac{\sqrt{g\sin q_R}}{l_e} \sim \frac{\sqrt{g}}{d}(q-q_R)$$

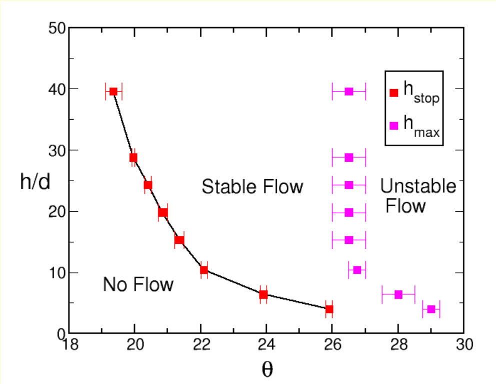
Phase Diagram

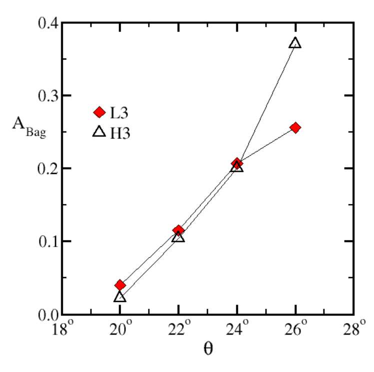
- **Expect rheology to break down if cluster size is comparable to total flow depth (flow arrest), or size of one particle (unstable flow)**
- **Predicts Pouliquen flow rule with**

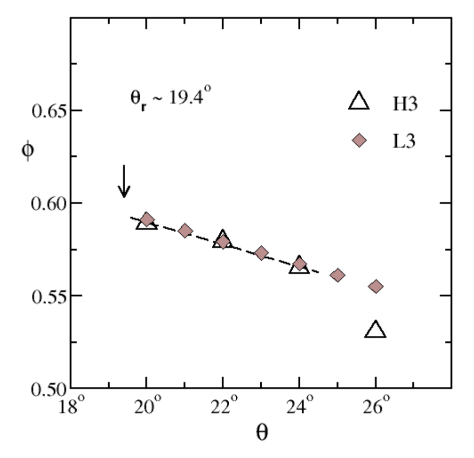
$$h_{stop} \propto \frac{d \tan q_R}{\tan q - \tan q_R}$$

- **Also predicts depth-independent unstable flow criterion--correlated motion a <u>necessity</u> for stable flow**
- **Predicts A_{Bag} linear in tilt angle**
- **Consistent with**

$$r_c - r \sim l_e^{-1}$$





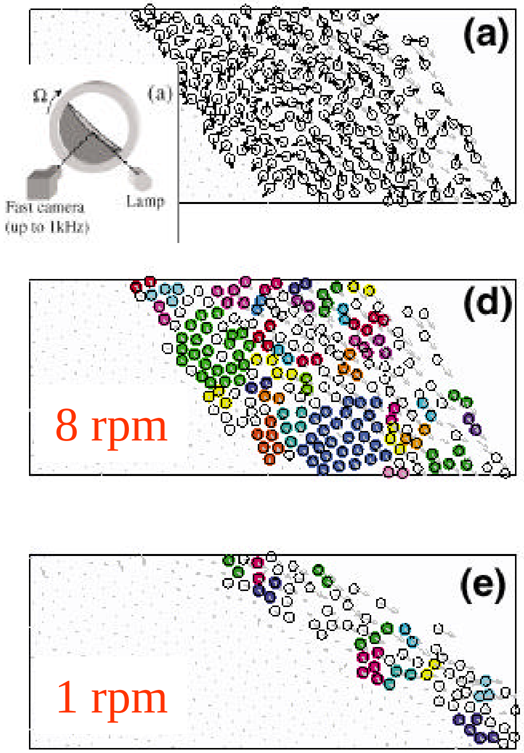


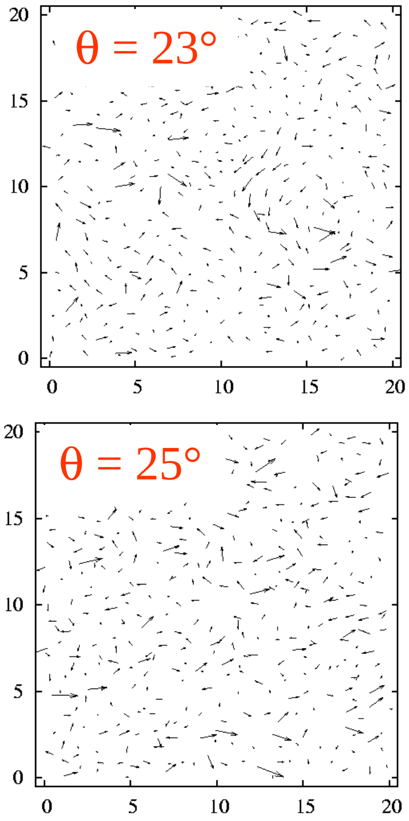
Evidence of Correlated Motion

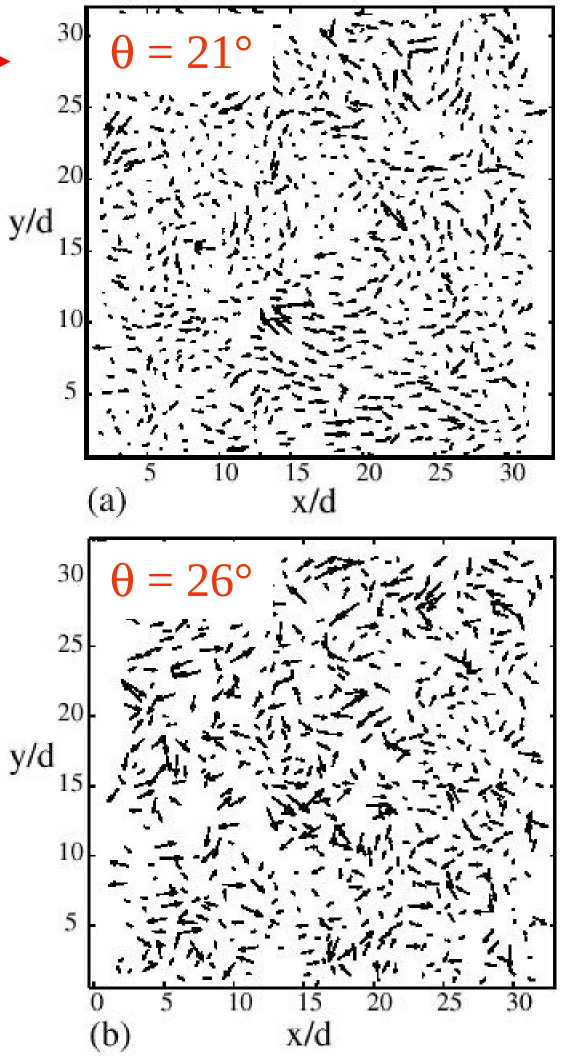
- **Free Surface Velocity Fluctuations**
  - O. Pouliquen, PRL **93**, 248001 (2004)
  - MD Simulations (2005)

- **Clusters in Rotating Drum**

D. Bonamy et al., PRL **89**, 034301 (2002)







Summary

- Chute flow obeys “Granular Liquid” kinematics with Bagnold rheology
- Hypothesis of correlated motion accounts semi-quantitatively for notable aspects of phenomenology
  - ✓ Velocity profile with depth, tilt angle
  - ✓ Pouliquen flow rule
  - ✓ Phase diagram
- Recent hints on the nature of the correlated motions
  - ? Kinetic theory incorporating velocity correlations

- Next Challenge: Underwater flows through implementation of lubrication forces