Experimental studies and continuum modelling of vibro-fluidized granular beds

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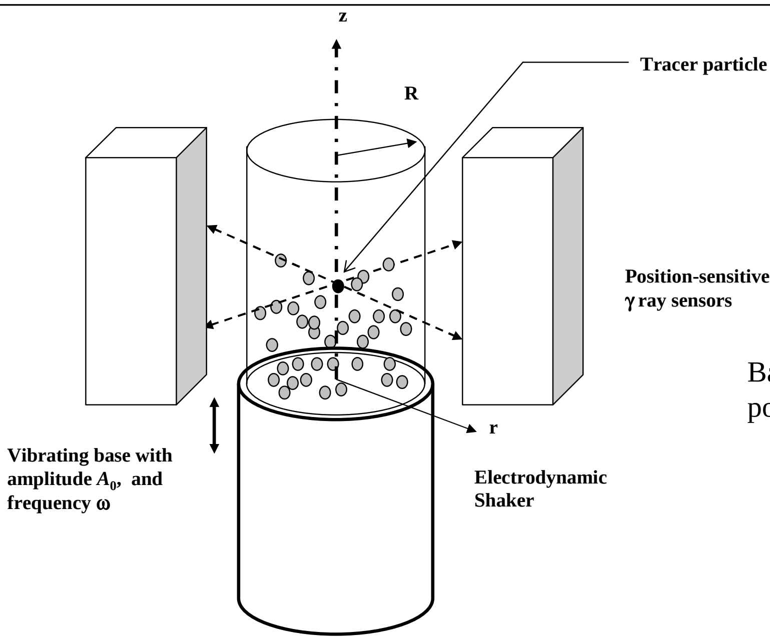
Outline

1. Time-averaged 3-D measurements using Positron Emission Particle Tracking (e.g. *PRE* **62** 3826 (2000), *PRE* **63** 061311 (2001), *PRL* **86** 3304 (2001))
2. 1-D hydrodynamic model (to appear in *JFM*)
   - Comparison of heat flux models
3. 3-D axisymmetric hydrodynamic model
4. Phase-resolved 3-D velocity measurements using dynamic nuclear magnetic resonance
5. Summary

Positron Emission Particle Tracking (PEPT)*

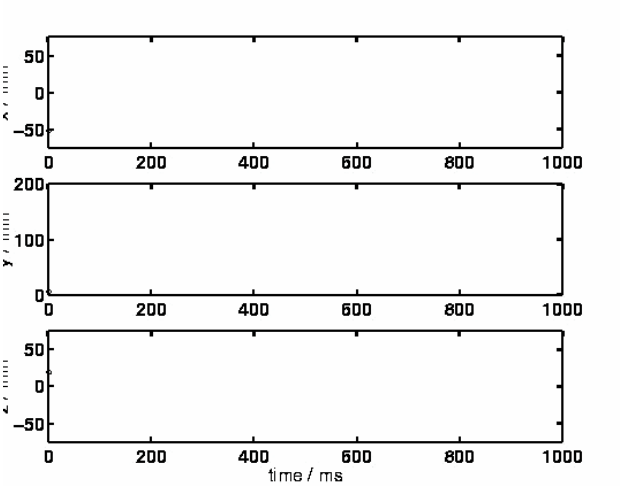


Back-to-back 511 keV photons from positron-electron annihilation

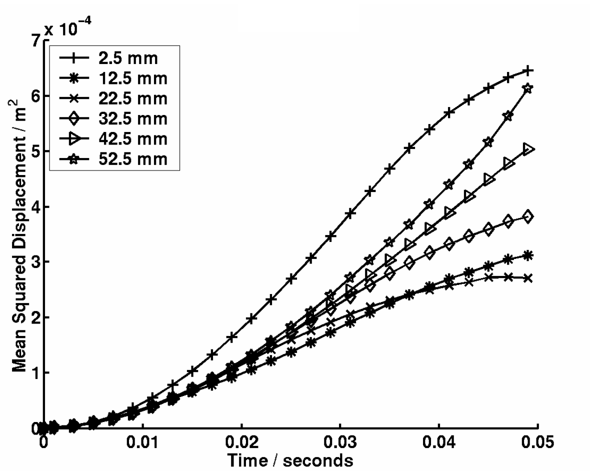
* D J Parker, University of Birmingham UK

Wildman et al., *Phys Rev E* **62** 3826-3835 (2000)

x, y, z versus t



Mean-square displacement

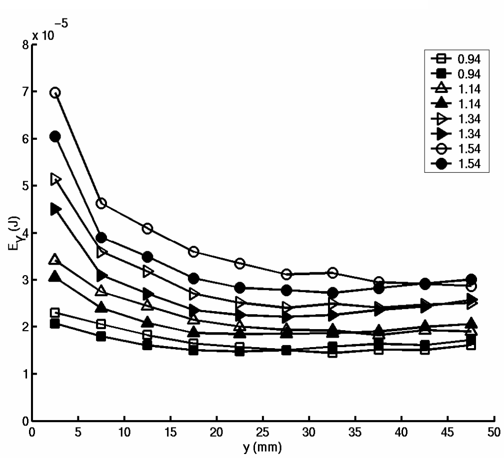


N = 350

Example: convection in vertically vibrated bed

PRL **86** 3304 (2001)

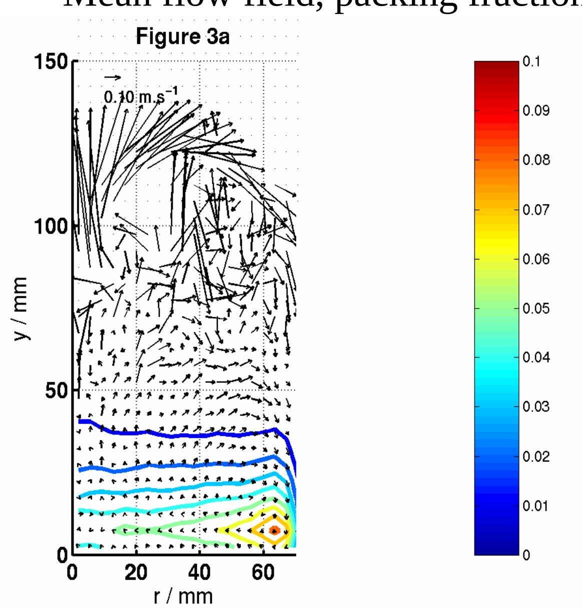
Granular temperature



Open symbols: averaged over $r < 30$ mm

Closed symbols: averaged over $30 < r < 60$ mm

Mean flow field, packing fraction



1-D hydrodynamic model of vibro-fluidized bed

[to appear in *J. Fluid Mech.*]

Force balance (*z* is vertical):

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g$$

Equation of state:

$$P = \frac{6}{\pi d^3}\frac{\eta(1+\eta+\eta^2-\eta^3)}{(1-\eta)^3}T$$

(Carnahan and Starling 1967)

Energy flux:

$$J = -\kappa\frac{\mathrm{d}T}{\mathrm{d}z} - \mu\frac{\mathrm{d}\eta}{\mathrm{d}z}$$

Energy dissipation:

$$\frac{\mathrm{d}J}{\mathrm{d}z} = -\gamma_{gg} - \frac{2}{R}\gamma_{gw}$$

grain-grain grain-wall

Hence set of 3 coupled first order ODEs

Energy flux expressions, boundary conditions

Brey J J, Dufty J W, Kim C S, & Santos A *Phys. Rev. E* **58**, 4638-4653 (1998)

$$\kappa = \kappa^*(e)\kappa_0, \qquad \mu = \mu^*(e)\frac{T}{\eta}\kappa_0 \qquad \kappa_0 = \frac{75k_B}{64d^2}\left(\frac{k_B T}{\pi m}\right)^{1/2}$$

Jenkins J T In *Physics of Dry Granular Media*, Kluwer Academic 353-370 (1999)

$$\kappa = \frac{45}{8}\frac{\eta}{\sqrt{\pi}d^2}\left(\frac{T}{m}\right)^{1/2}\left[\frac{5}{24}\frac{1}{G}+1+\frac{6}{5}\left(1+\frac{32}{9\pi}\right)G\right], \quad \mu = 0, \qquad G = \frac{\eta(2-\eta)}{2(1-\eta)^3}$$

Boundary conditions: require $\eta\big|_{z=0}, T\big|_{z=0}, \left.\frac{\mathrm{d}T}{\mathrm{d}z}\right|_{z=0}$

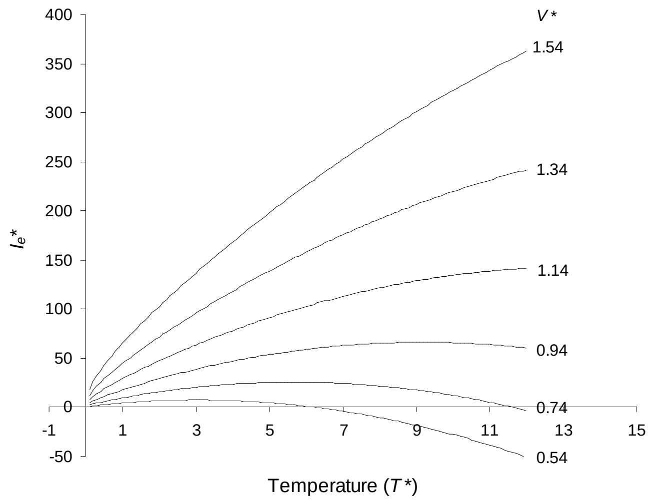
Know: $J\big|_{z\to\infty} = 0, \qquad \int_0^\infty \eta\,\mathrm{d}z \propto N$

3rd boundary condition: incorporate model for energy transfer from the vibrating base

Comparison of base energy input rates (1)

Richman M W, *Mech. Mat.* **16** 211-218 (1993)

$$I_e^* = 8\left(T^* + V^{*2}\right)^{1/2}\left[2V^{*2} - (1-e_n)\left(T^* + V^{*2}\right)\right]T^{*1/2}$$



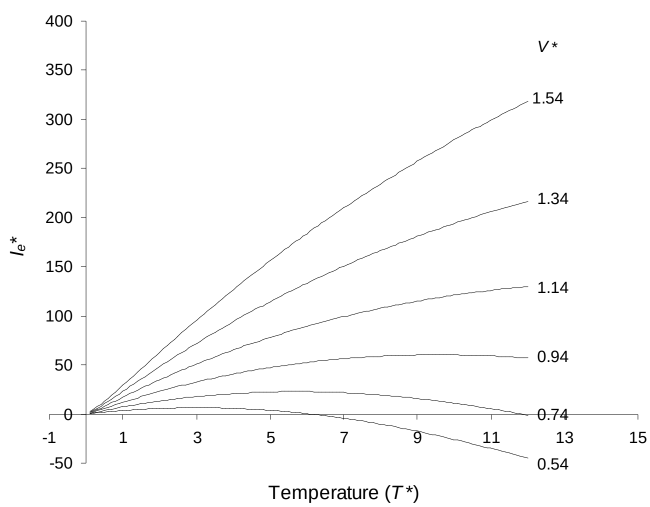
- Assumes Maxwellian distribution
- Calculates exact expressions for source and dissipation terms
- Closed form expression not possible in general - lowest order term in series expansion shown above left
- * denotes non-dimensional quantitities, e.g.

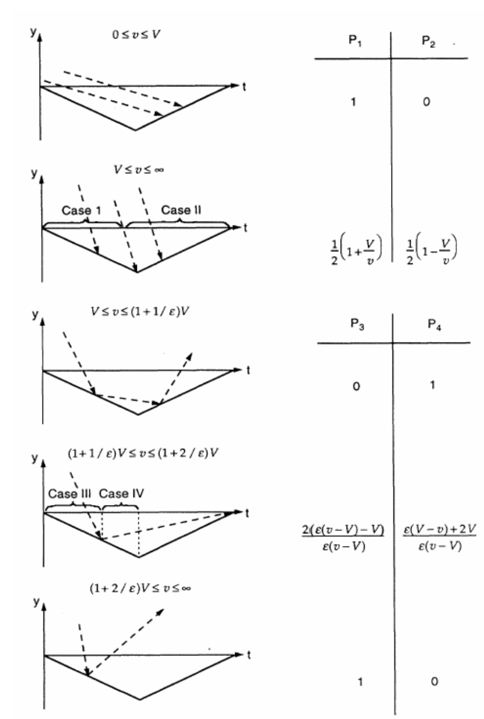
$$T^* = \left\langle \left(v_z - \langle v_z \rangle\right)^2 \right\rangle \Big/ gd$$

d = particle diameter,
g = acceleration due to gravity
V^* = peak base velocity
e_n = grain-base restitution coefficient

Comparison of base energy input rates (2)

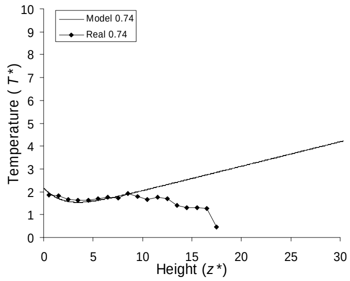
Warr S and Huntley J M, *Phys. Rev. E.* **52** 5596-5601 (1995)

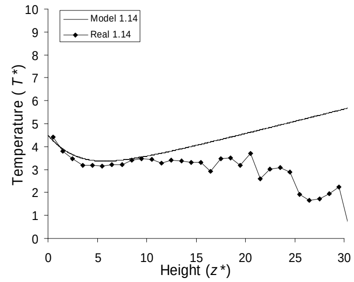


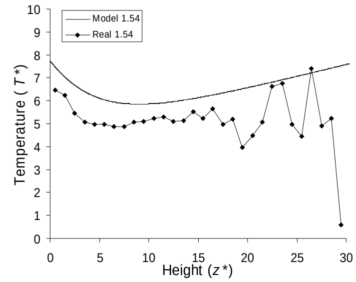


Comparison of solutions with experiment (Brey)

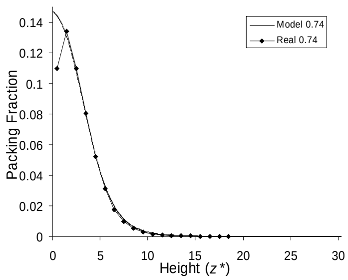
Temperature

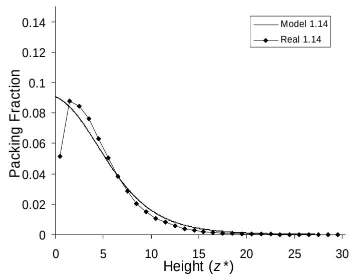


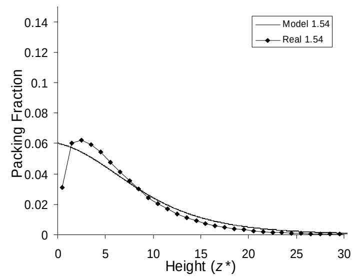




Packing fraction

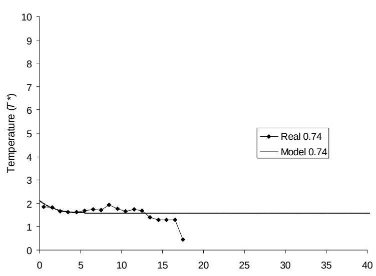


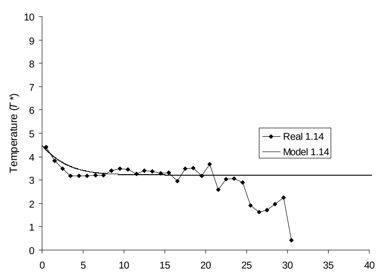


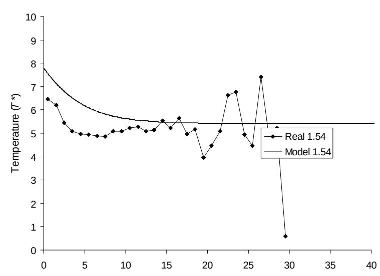


Comparison of solutions with experiment (Jenkins)

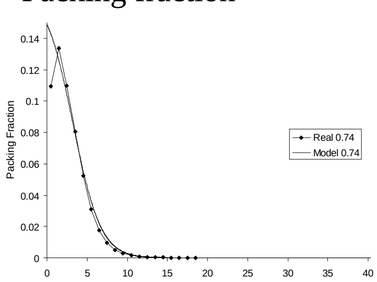
Temperature

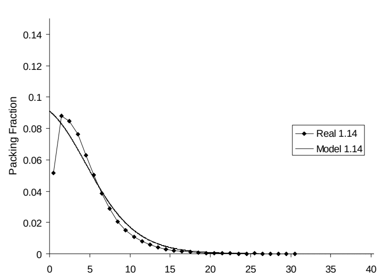


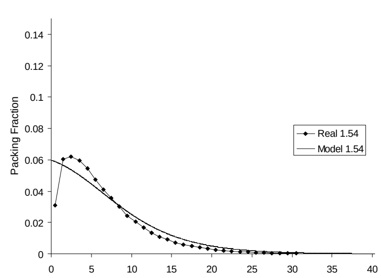




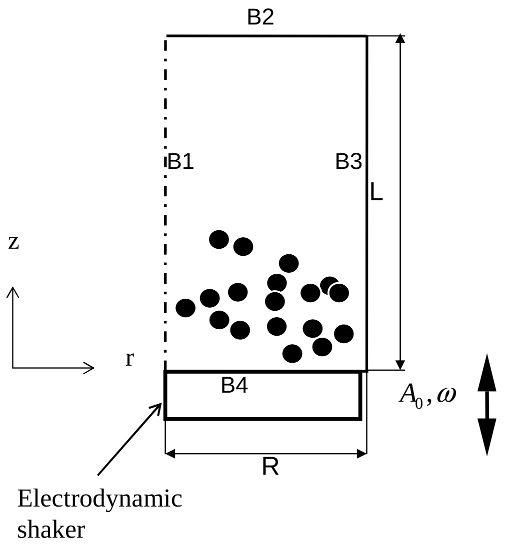
Packing fraction







Axisymmetric 3-D model

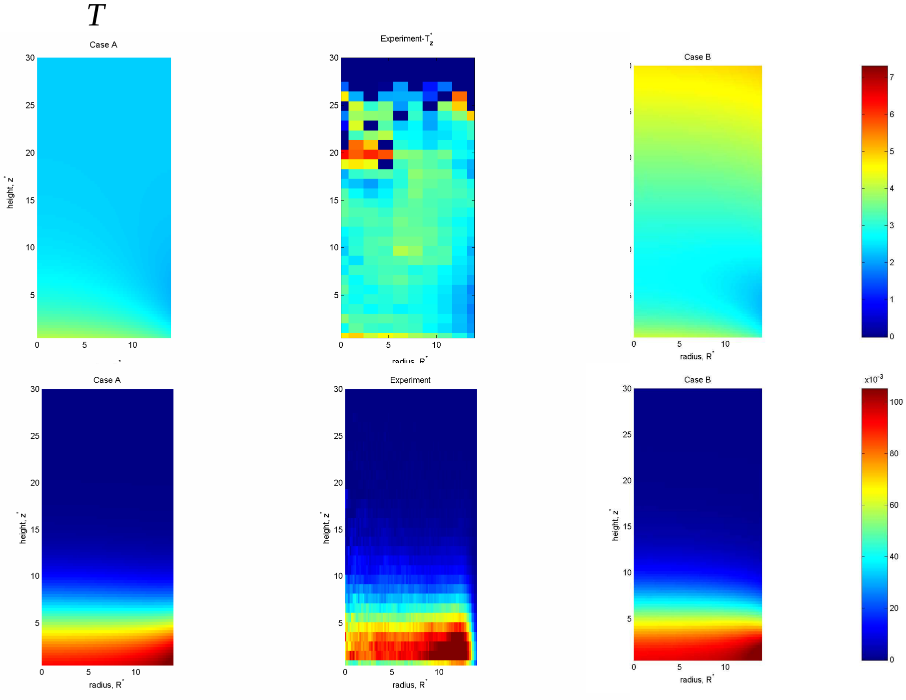


- Solve for *P*, *T* fields – post-process for η
- Finite element method implemented in FEMLAB
- Boundary conditions:

Boundary	*P*	*T*
B1	$\frac{\partial P}{\partial r} = 0$	$\frac{\partial T}{\partial r} = 0$
B2	$\frac{\partial P}{\partial z} = 0$	$J_z = 0$
B3	$\frac{\partial P}{\partial r} = 0$	$\frac{\partial T}{\partial r}$ (Richman)
B4	$P = Nmg / \pi R^2$	$\frac{\partial T}{\partial z}$ (Richman)

Experiment versus models – V^*_{rms}=1.14

T



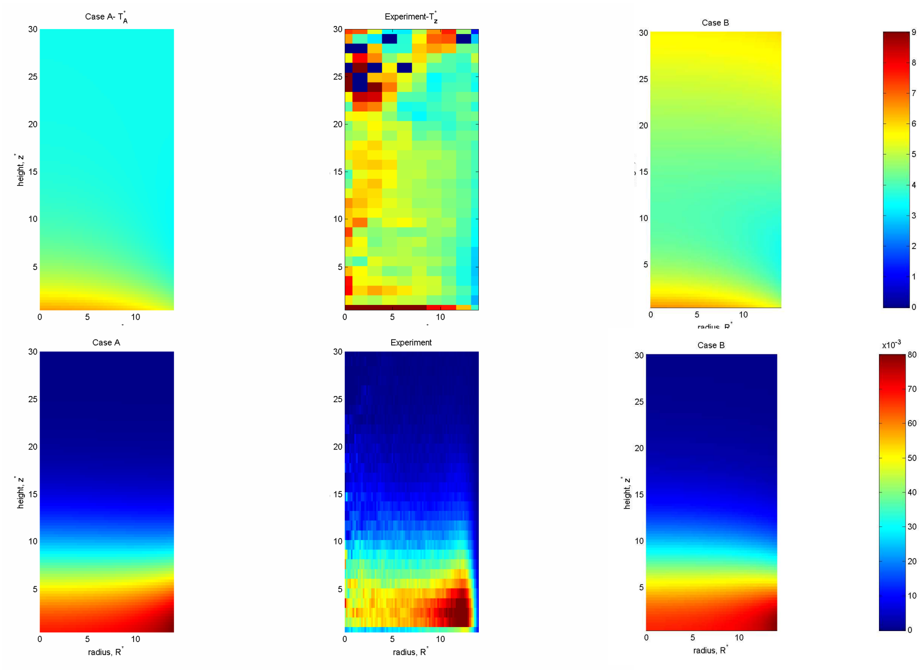
T

η

Case A: Jenkins

Case B: Brey

Experiment versus models – V^{*}_{rms}=1.54



T

η

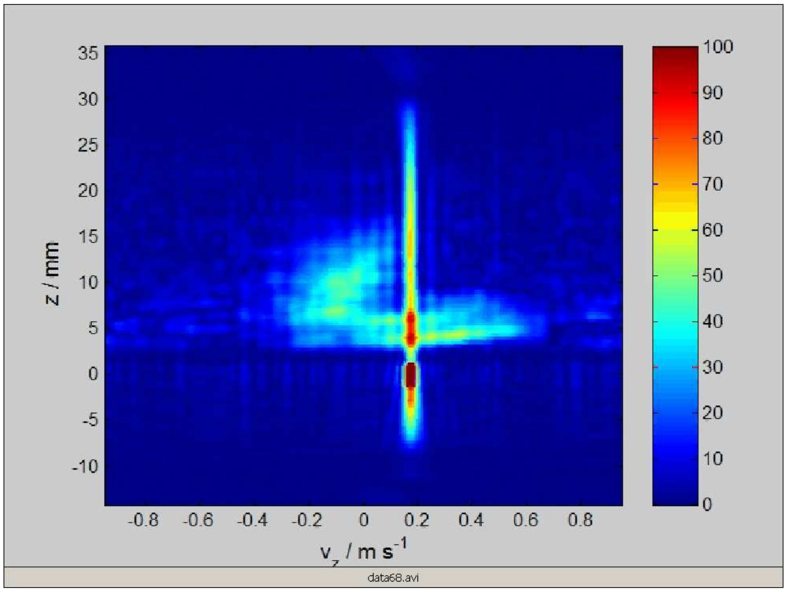
Case A: Jenkins

Case B: Brey

NMR experiments – University of Cambridge

- Cell
  - Permanently anti-static co-polymer acetal
  - Glass base insert
  - Internal diameter 18 mm
  - Evacuated to reduce air drag
- Mustard seed
  - Restitution coefficients:
    - Mustard - plastic 0.60 ± 0.03
    - Mustard - glass 0.58 ± 0.05
    - Mustard - mustard 0.68 ± 0.10
  - Diameter: 2.04 ± 0.23 mm
  - Aspect ratio: 1.17 ± 0.09
  - Mean mass: 5.66 mg
- Measures vertical velocity Probability Density Functions (PDFs):
  – 64 velocities (v_z)
  – 128 slices (z)
  – 12 phases in driving cycle (α)

Phase-resolved velocity PDF (1)



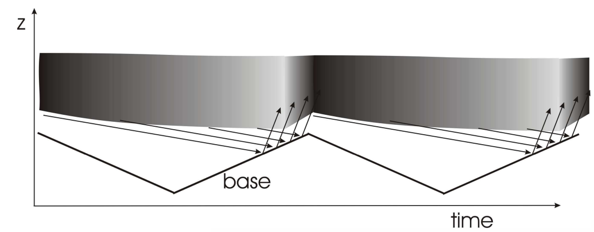
f = 31.8 Hz
A_0 = 1.68 mm
V* = 1.68
N_g = 55

Some interesting features

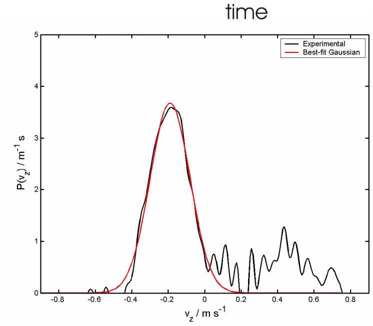
Large deviations from steady-state behaviour

Mean particle speed ($\sqrt{<v_z^2>}$) is less than peak speed of base V

Rebounding particles have speed higher than V



Bimodal velocity distributions

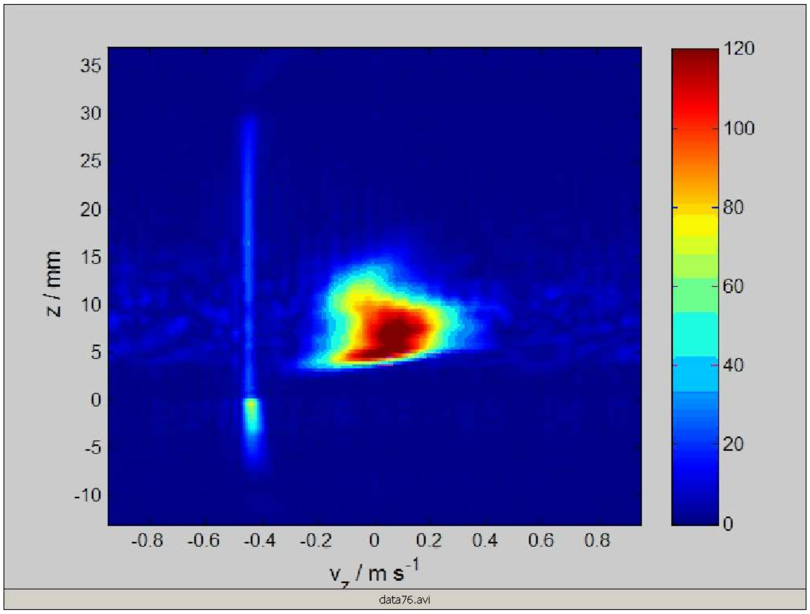


z = 4 mm

Particles cannot reach base for at least first half of cycle

- Granular gas expands into ‘vacuum’
- Negative bias to velocity distribution close to base

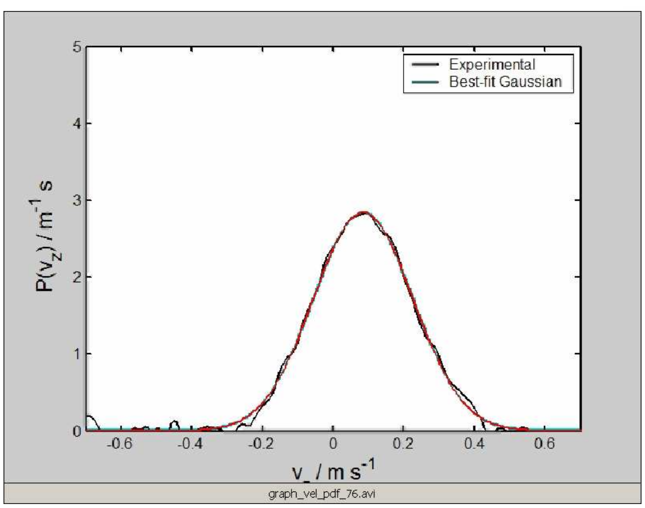
Phase-resolved velocity PDF (2)



f = 38.2 Hz
A_0 = 1.84 mm
V* = 2.2
N_g = 110

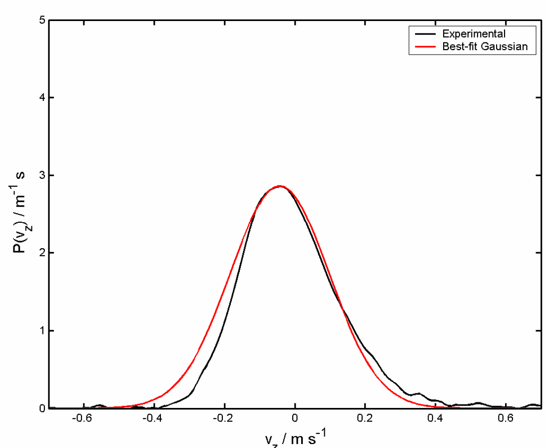
Phase-resolved velocity PDFs: z = 7.8 mm

Phase-resolved



$\overline{T^*} = 0.788$

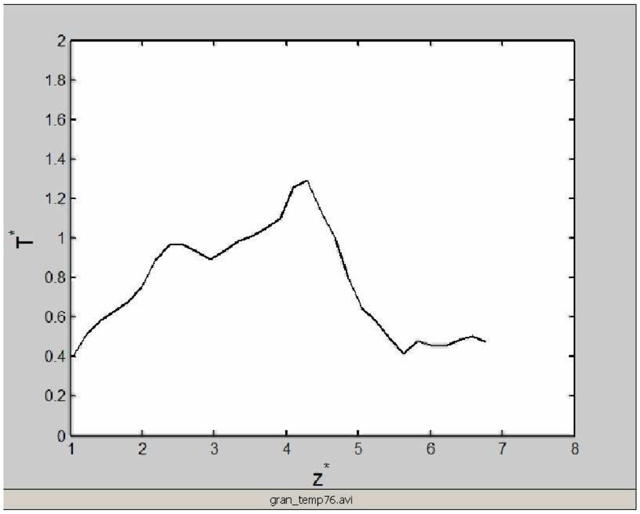
Time-averaged



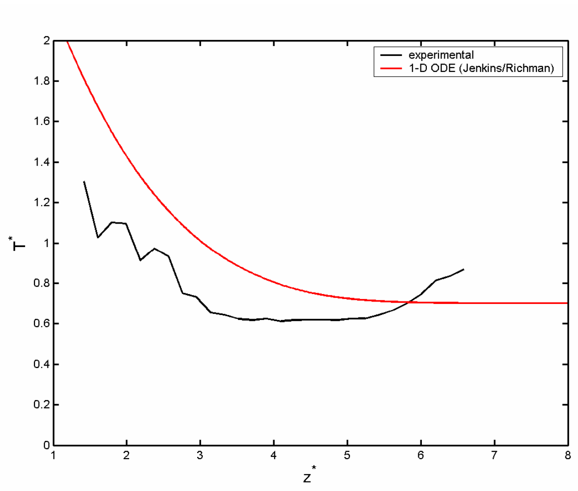
$\overline{T^*} = 0.972$

Granular temperature distributions

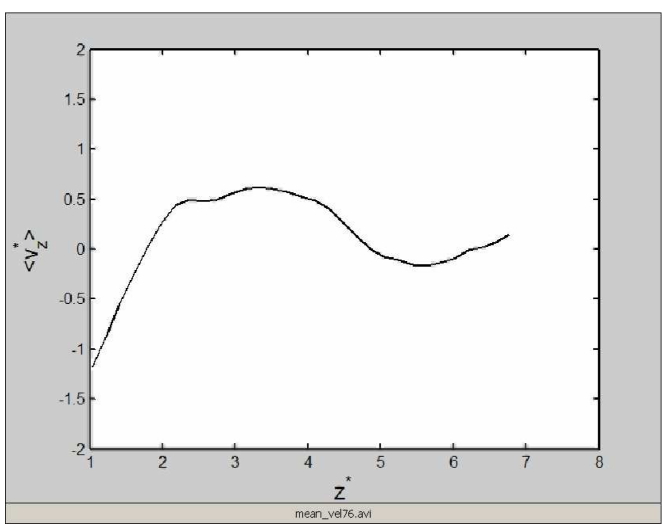
Phase-resolved



Time-averaged, compared to
1-D Jenkins/Richman ODE solution



Mean velocity distribution

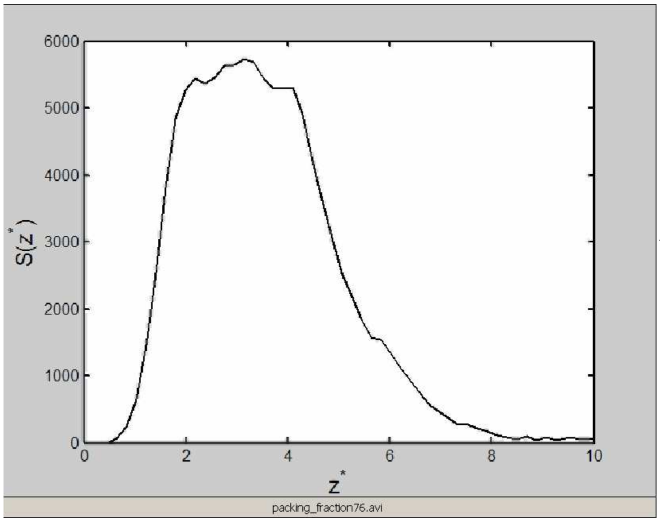


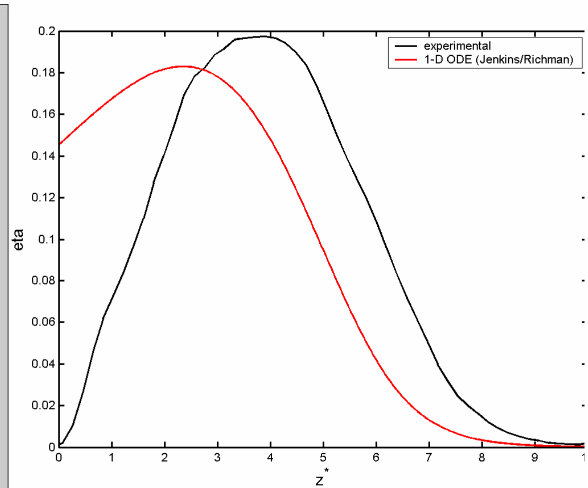
- Bed is expanding for most of the cycle
- Short intense compressional wave is not fully resolved

Packing fraction distributions

Phase-resolved signal (not normalized)

Time-averaged η





Summary

- Experimental time-averaged and through-thickness-averaged 3-D PEPT data agrees reasonably well with 1-D ODE solutions using both Brey and Jenkins heat flux expressions
- 3-D Finite Element model reveals some radial dependence of T, η fields
- Novel NMR technique developed to provide phase-resolved measurements of $P(z,v_z)$ in a 3-D granular bed
- Time-averaged PDFs and temperature distributions mask several interesting phenomena
- When combined with a 1-D dynamic model, such data can be expected to provide a more sensitive test of existing constitutive equations