Rheology and linear response of sheared granular flows

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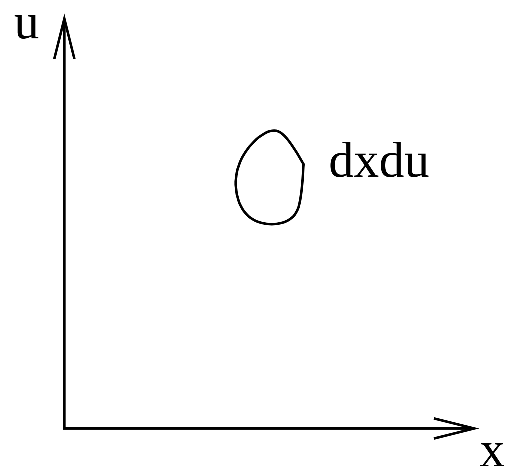
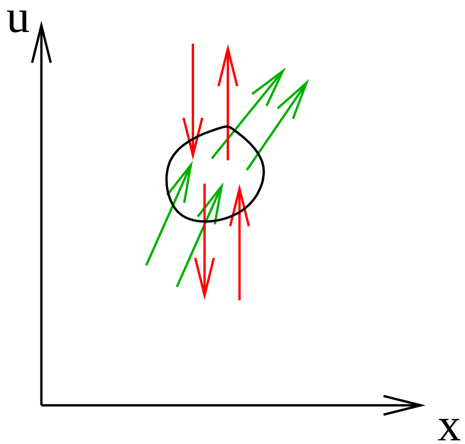
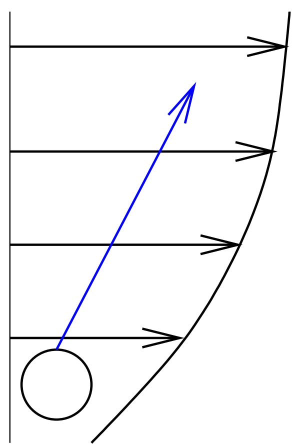
Kinetic theory — elastic hard spheres

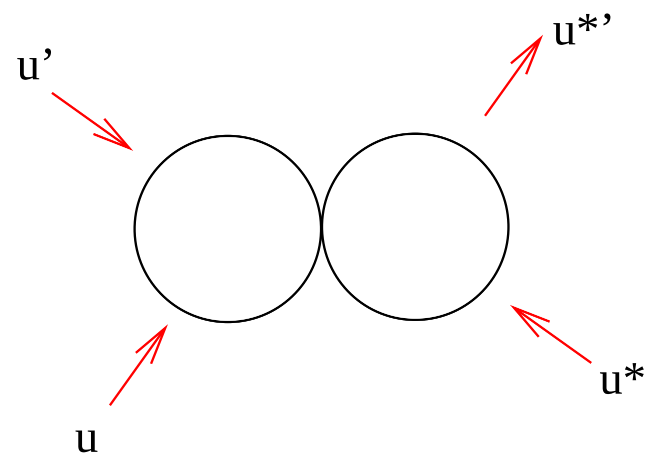
- Velocity distribution $f(\mathbf{x}, \mathbf{u})d\mathbf{x}d\mathbf{u}$.
- Fluctuating velocity $\mathbf{c} = \mathbf{u} - \mathbf{U}$



Boltzmann eq $\frac{\partial(\rho f)}{\partial t} + \frac{\partial(\rho c_i f)}{\partial x_i} + \frac{\partial(\rho a_i f)}{\partial c_i} - \frac{\partial U_i}{\partial x_j}\frac{\partial(\rho c_j f)}{\partial c_i} = \frac{\partial_c(\rho f)}{\partial t}$







Collision integral — molecular chaos approximation.

Boltzmann equation: $\frac{\partial(\rho f)}{\partial t} + \frac{\partial(\rho c_i f)}{\partial x_i} - \frac{\partial U_i}{\partial x_j}\frac{\partial(\rho c_j f)}{\partial x_i} = \frac{\partial_c(\rho f)}{\partial t}$

Equilibrium (no gradients)

$$\frac{\partial_c f}{\partial t} = 0$$

Solution — Maxwell-Boltzmann distribution

$$f = (2\pi T)^{-3/2} \exp\left(-mu^2/2T\right)$$

Non-equilibrium — Chapman-Enskog procedure:

$$\frac{\partial(\rho f)}{\partial t} + \frac{\partial(\rho c_i f)}{\partial x_i} - \frac{\partial U_i}{\partial x_j}\frac{\partial(\rho c_j f)}{\partial c_i} = \frac{\partial_c(\rho f)}{\partial t}$$

$$\frac{T^{1/2}\rho f}{L} \qquad G_{xy}\rho f \qquad \frac{T^{1/2}\rho(f - f_{eq})}{\lambda}$$

Asymptotic expansion in parameter $\epsilon = (\lambda/L)$; $f = f_0 + \epsilon f_1 + \ldots$

Leading order $\frac{\partial_c(\rho f)}{\partial t} = 0 \to f = f_{MB}$.

First correction

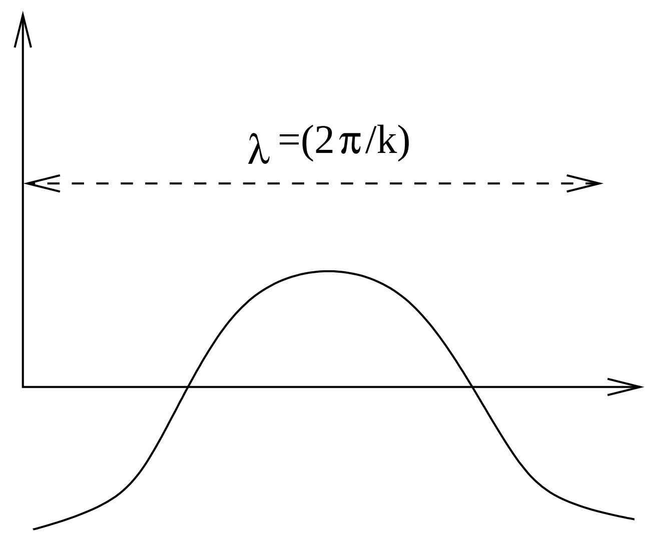
$$\frac{\partial(\rho f_0)}{\partial t} + \frac{\partial(\rho c_i f_0)}{\partial x_i} - \frac{\partial U_i}{\partial x_j}\frac{\partial(\rho c_j f_0)}{\partial c_i} = \frac{\partial_c(\rho f_1)}{\partial t}$$

Moments of Boltzmann equation

- 'Slow' Mass, Momentum & Energy, conserved in collisions.
- Other 'fast' moments decay over time scales $\sim$ collision time.

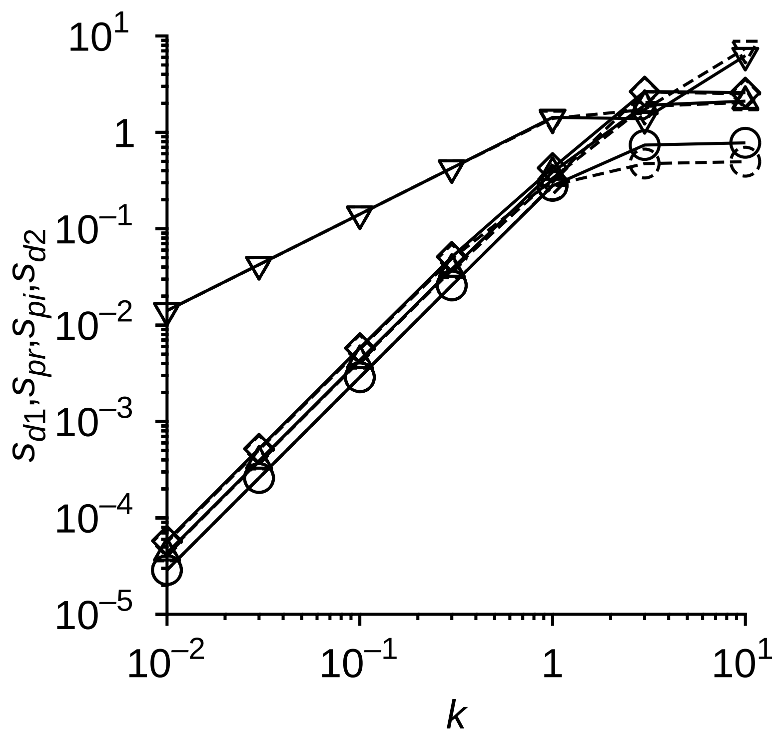
Linear response

- $f(\mathbf{c}) = f_0(\mathbf{c}) + \tilde{f}(\mathbf{c})e^{(st+\imath kx)}$
- Linearised Boltzmann equation
  $$\left[s + \imath k c_x - G_{ij}\frac{\partial c_i}{\partial c_j}\right]\tilde{f} = L[\tilde{f}]$$
- $\tilde{f}(\mathbf{c}) = \sum_{i=1}^{N} A_i\psi_i(\mathbf{c})$
- $(sI_{ij} + \imath k X_{ij} - G_{ij} - L_{ij})A_j = 0$



Hydrodynamic modes for elastic system

- Number of eigenvalues depends on number of basis functions chosen.
- For $k \to 0$,
  Transverse momenta $s_t = -(\mu/\rho)k^2$.
  Energy $s_e = -D_T k^2$.
  Mass & longitudinal mom.
  $s_l = \pm \imath k \sqrt{p_\rho} - \rho^{-1}(\mu_b + 4\mu/3)k^2$.
- All other modes with negative eigenvalues, indicating that other transients decay.



Calculation of Transport coefficients (dilute):

$$\begin{aligned}\sigma_{xy} &= -\rho\langle u_x u_y\rangle \\ &= -\rho\int d\mathbf{u} f_1(\mathbf{u}) u_x u_y \\ &= \eta G_{xy}\end{aligned}$$

Beyond molecular chaos — incorporate correlated collisions.

Two dimensions $\sigma_{xy} = \eta G_{xy} + \eta' G_{xy} \log(G_{xy})$

Three dimensions $\sigma_{xy} = \eta G_{xy} + \eta' G_{xy} |G_{xy}|^{1/2}$

Green-Kubo formula (shear viscosity):

$$\eta = \frac{\beta}{V} \lim_{k \to 0} \int_0^\infty dt \langle \sigma_{xy}(k,t) \sigma_{xy}(-k,0) \rangle$$

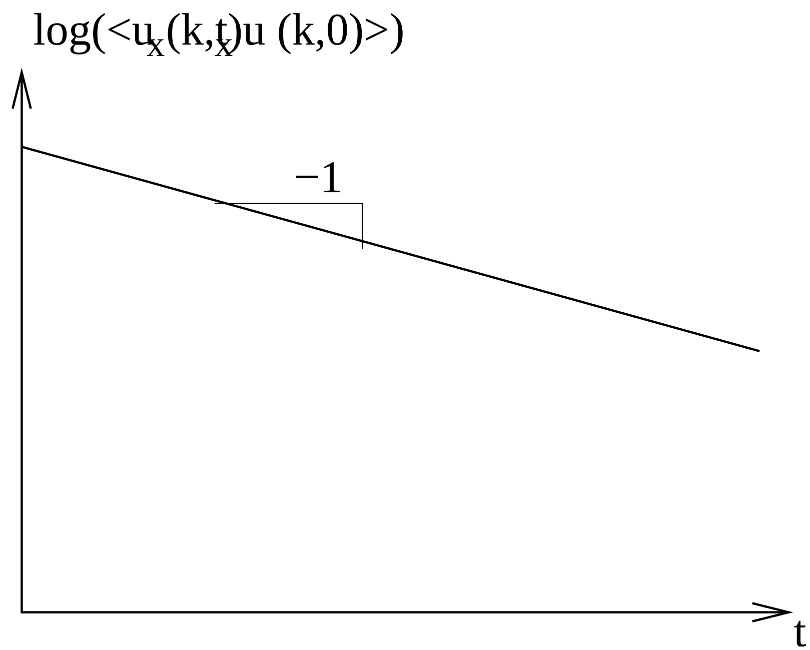
Microscopic stress:

$$\sigma_{xy}(\mathbf{k}) = \int_{\mathbf{k}'} u_x(\mathbf{k}-\mathbf{k}') u_y(\mathbf{k}')$$

Velocity fluctuations:

$$\partial_t u_x(\mathbf{k}) = -\eta k^2 u_x(\mathbf{k})$$

$$u_x(\mathbf{k},t) = \exp(-\eta k^2) u_x(\mathbf{k},0)$$

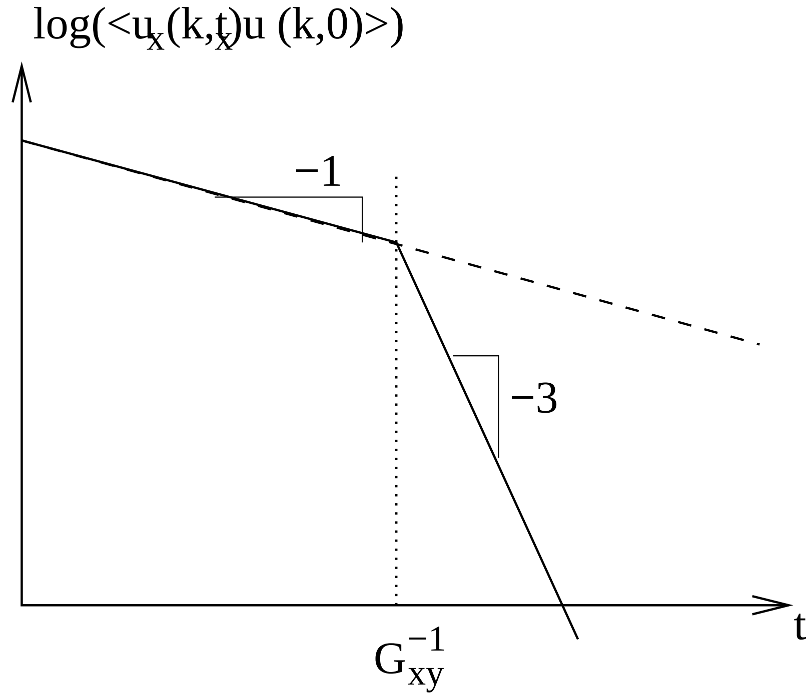


Viscosity

$$\eta = \frac{\beta}{V} \int d\mathbf{k}' \int_0^\infty dt \langle u_x(\mathbf{k}',t) u_x(-\mathbf{k}',0) \rangle \langle u_y(-\mathbf{k}',t) u_y(\mathbf{k}',0) \rangle$$

Time correlation — long time tail:

$$\int d\mathbf{k}\langle u_x(\mathbf{k},t)u_x(-\mathbf{k},0)\rangle$$
$$\sim \int d\mathbf{k}\exp\left(-\eta k^2 t\right)$$
$$\sim t^{-d/2}$$



Sheared system:

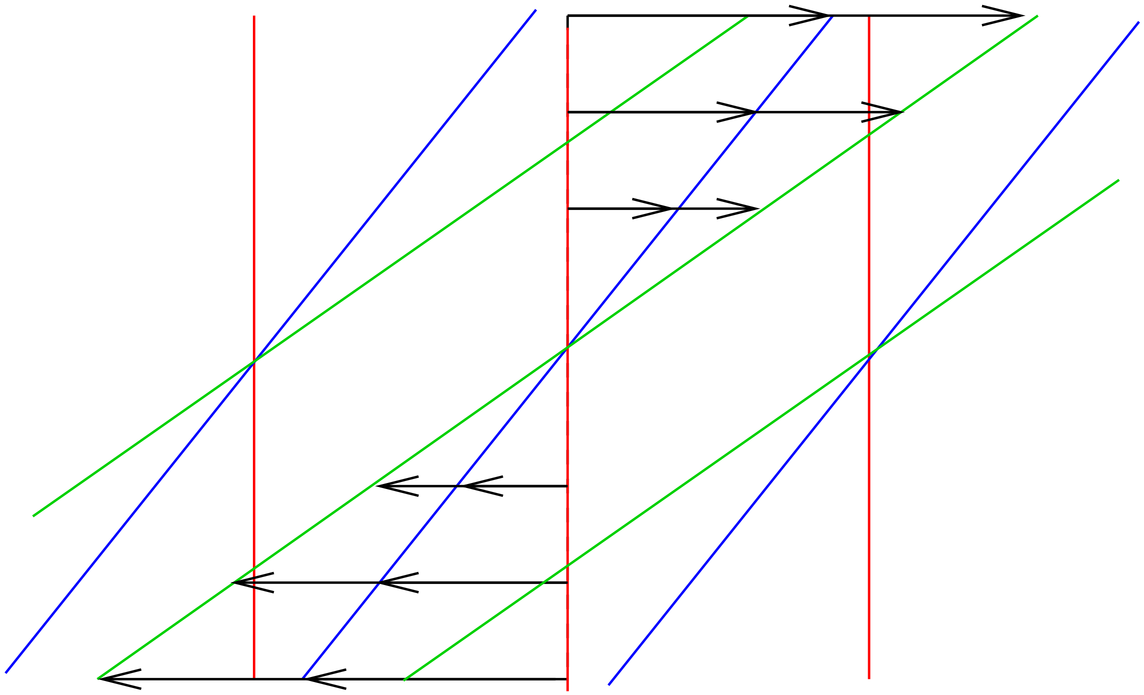
$$(\partial_t + G_{xy}k_x\frac{\partial}{\partial k_y})u_x = -\eta k^2 u_x$$

$$u_x(t) = u_x(0)\exp\left[-Dt\left(k^2 - G_{xy}tk_xk_y + \frac{1}{3}G_{xy}^2t^2k_x^2\right)\right]$$

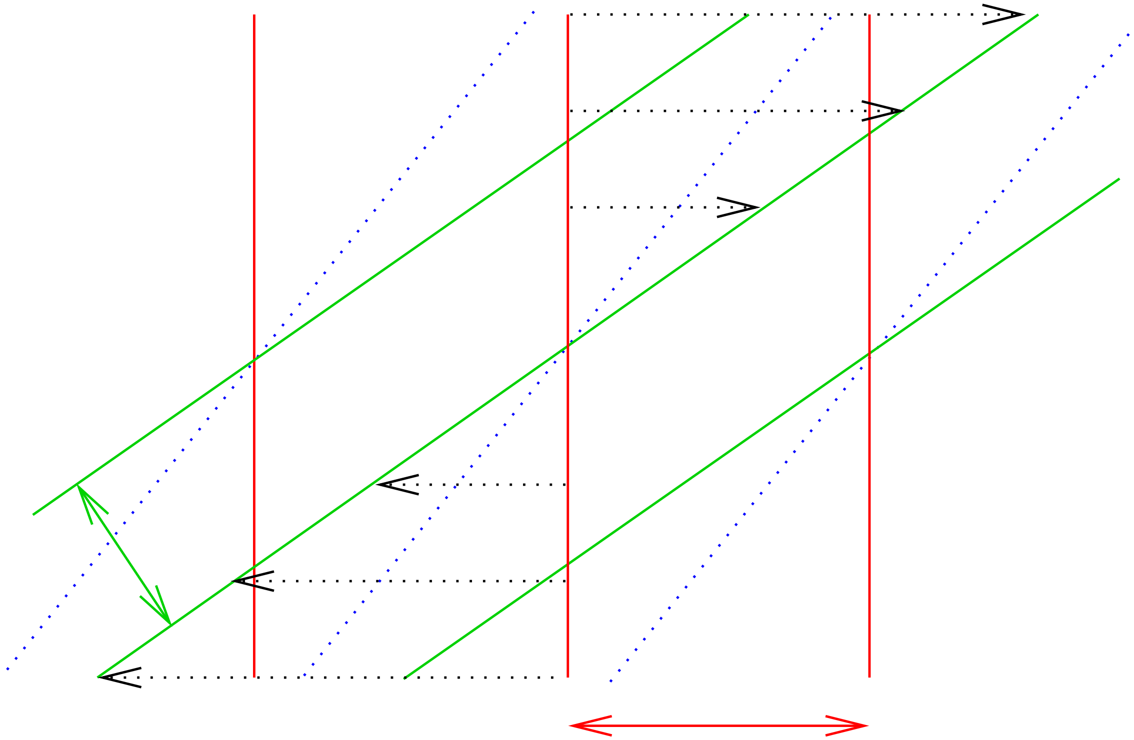
$$u_x(t) \sim \exp\left(-1/3G_{xy}^2k_x^2t^3\right)$$

.

‘Turning’ of wave vector due to shear:

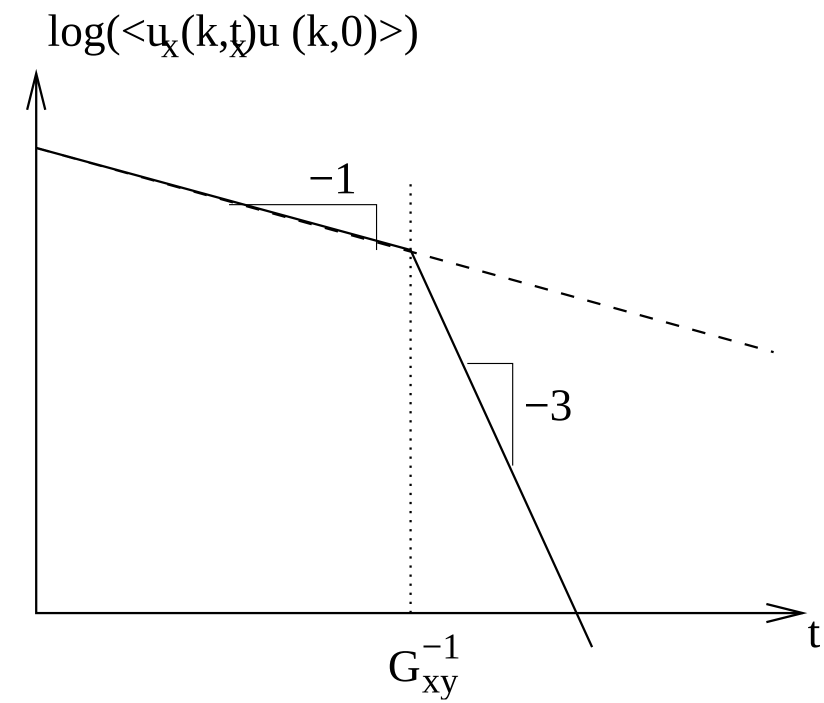


‘Turning’ of wave vector due to shear:



Green-Kubo relation:

$$\eta = \frac{\beta}{V} \int d\mathbf{k}' \int_0^{G_{xy}^{-1}} dt \;\; t^{-d/2}$$



Two dimensions:

$$\eta = \eta_0 + \eta_1 \log\left(G_{xy}\right)$$

Three dimensions:

$$\eta = \eta_0 + \eta_1 |G_{xy}|^{1/2}$$

Beyond the Boltzmann equation:

One particle distribution

$f_\alpha(\mathbf{x}_\alpha, \mathbf{u}_\alpha)$.

Two-particle distribution:

$f_{\alpha\beta}(\mathbf{x}_\alpha, \mathbf{u}_\alpha, \mathbf{x}_\beta, \mathbf{u}_\beta)$

Molecular chaos truncation:

$f_{\alpha\beta} = f_\alpha f_\beta$.

Ring kinetic truncation:

$f_{\alpha\beta} = f_\alpha f_\beta (1 + g_{\alpha\beta})$.

$$f_{\alpha\beta\gamma} = f_\alpha f_\beta f_\gamma (1 + g_{\alpha\beta} + g_{\alpha\gamma} + g_{\beta\gamma})$$

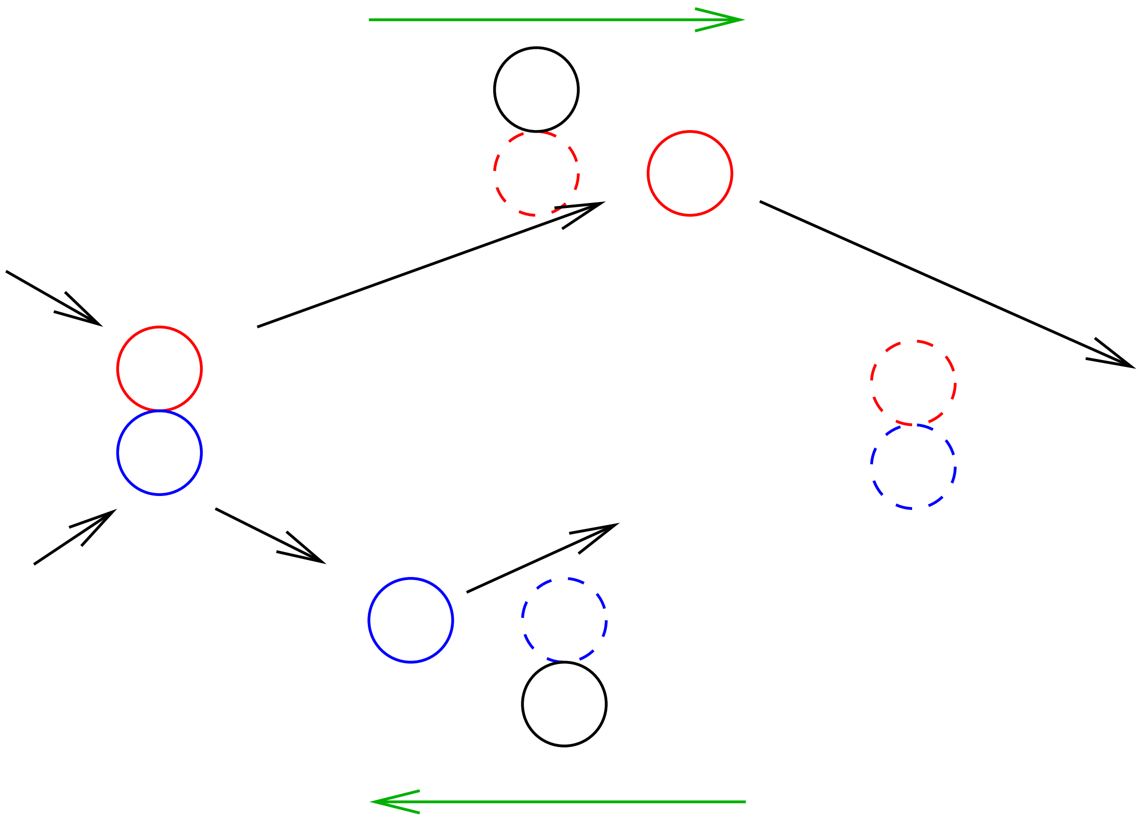
Single particle distribution

$$\frac{\partial (c_{\alpha i} f_\alpha)}{\partial x_{\alpha i}} - G_{ij} c_{\alpha j} \frac{\partial f_\alpha}{\partial x_{\alpha i}} = \frac{\partial_c f_\alpha}{\partial t}$$

Ring kinetic equation:

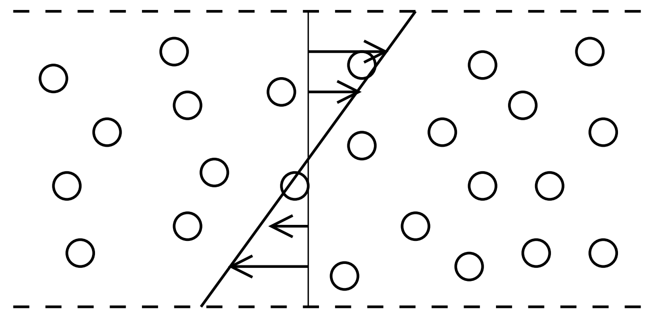
$$\partial_t f_{\alpha\beta} - G_{ij} x_{\alpha\beta j} \frac{\partial f_{\alpha\beta}}{\partial x_i} + c_{\alpha\beta i} \frac{\partial f_{\alpha\beta}}{\partial x_i} - G_{ij} \left(c_{\alpha j} \frac{\partial f_{\alpha\beta}}{\partial c_{\alpha i}} + c_{\beta j} \frac{\partial f_{\alpha\beta}}{\partial c_{\beta i}} \right) = \frac{\partial_c f_{\alpha\beta}}{\partial t}$$

Propagator in ring kinetic equation:



Steady homogeneous shear flow of inelastic particles:

$$-G_{ij}\frac{\partial(\rho c_j f)}{\partial c_i} = \frac{\partial_c(\rho f)}{\partial t}$$



Nearly elastic collisions:

$e_n \ll 1 \rightarrow$ Dissipation $\ll$ Particle energy

Expand in $\varepsilon_n = (1 - e_n)^{1/2}$.

Leading order $\dfrac{\partial_c(\rho f_0)}{\partial t} = 0 \rightarrow f = f_{MB}$.

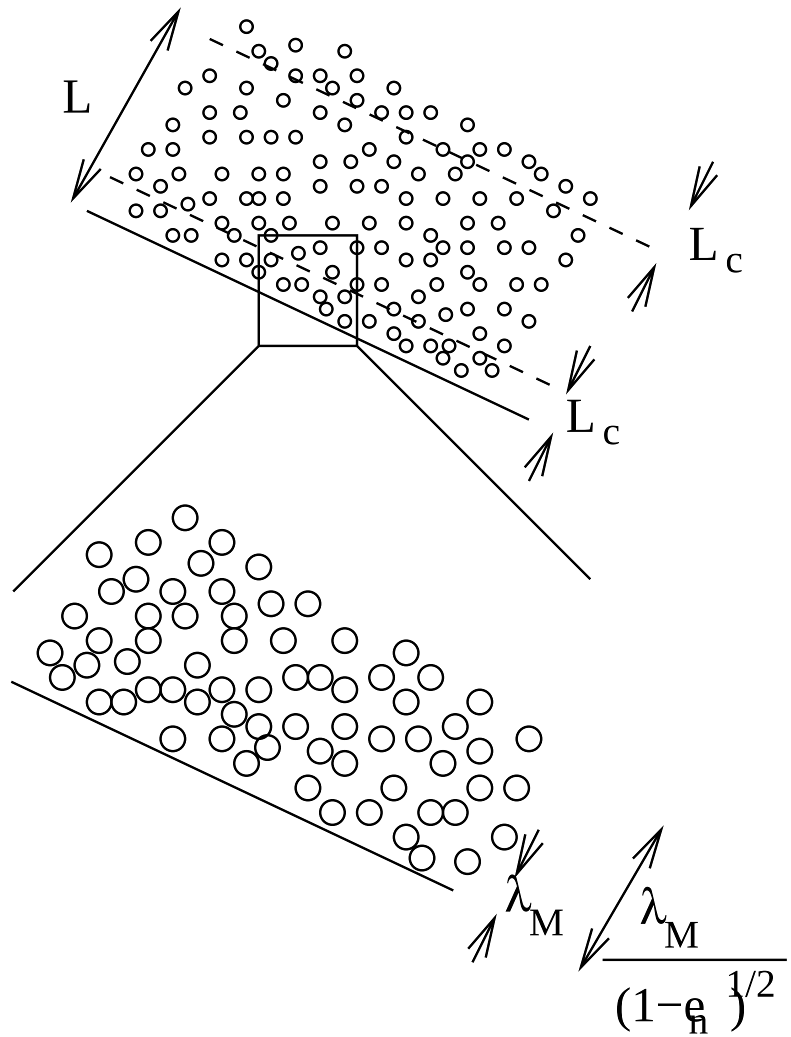
Rate of energy production $\sim \mu G_{xy}^2 \sim (T^{1/2}/d^2) G_{xy}^2$.

Rate of energy dissipation $\sim \rho^2 T^{3/2}(1 - e_n^2)^{1/2}$.

$\rightarrow G_{xy} \sim (1 - e_n^2)^{1/2} T^{1/2} \sim \varepsilon_n T^{1/2}$.

Hydrodynamic modes for smooth inelastic spheres

- Energy *not conserved.*
- Source of energy.
- Rate of conduction $(\lambda_M T^{1/2}/L^2)$.
- Rate of dissipation $((1-e)T^{1/2}/\lambda_M)$.
- Conduction length $L_c = (\lambda_M/(1-e)^{1/2}$.
- Energy conserved $L \ll L_c$.
- *Adiabatic approx.* $L \gg L_c$. Local balance between source and dissipation.



Smooth nearly elastic particles $\qquad O(1) \qquad O(\varepsilon_n) \qquad O(\varepsilon_n^2)$

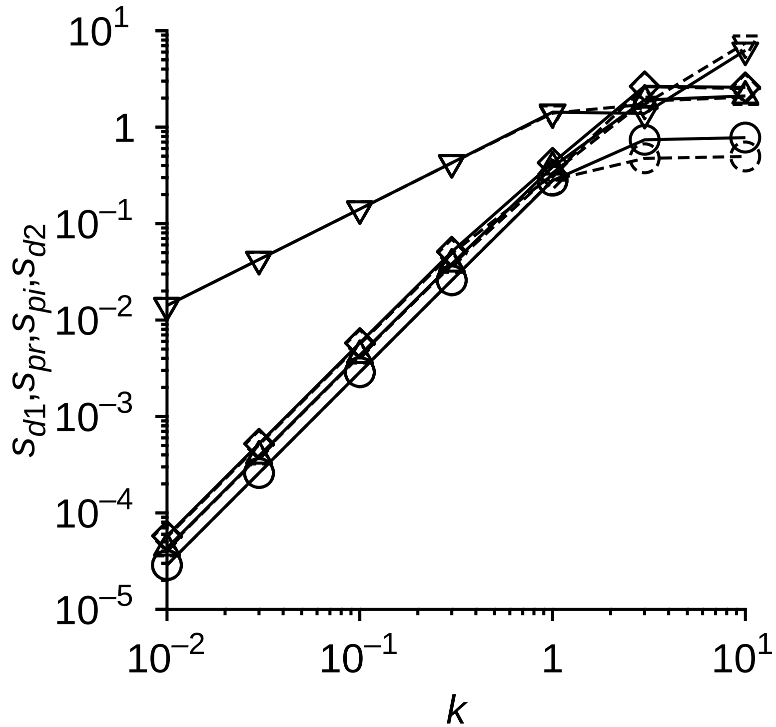
$$
\begin{aligned}
\sigma_{ij} \quad = \quad & -p(\phi, S_{ij}, G_{ii})\delta_{ij} + 2\mu(\phi, S_{ij}G_{ii})S_{ij} + \mu_b(\phi, S_{ij}, G_{ii})\delta_{ij}G_{kk} \\
& + (\mathcal{A}(\phi)(S_{ik}S_{kj} - (\delta_{ij}/3)S_{kl}S_{lk}) + \mathcal{B}(\phi)\delta_{ij}G_{kk}^2 + \mathcal{C}(\phi)S_{ij}G_{kk}) \\
& + \mathcal{D}(\phi)(S_{ik}A_{kj} + S_{jk}A_{ki}) + \mathcal{F}(\phi)(A_{ik}A_{kj} - (\delta_{ij}/3)A_{kl}A_{lk}) \\
& - \frac{\mathcal{D}(\phi)}{2}\left(\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_i}\right) - \frac{2\delta_{ij}}{3}\frac{\partial}{\partial x_k}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_k}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
p \quad &= \quad \rho T(1 + (4 - 2\epsilon^2)\phi\chi(\phi)) \\
\mu(\phi) \quad &= \quad \frac{5T^{1/2}}{16\sqrt{\pi}\chi(\phi)}\left(1 + \frac{8\phi\chi(\phi)}{5}\right)^2 + \frac{48\phi^2\chi(\phi)T^{1/2}}{5\pi^{3/2}} \\
\mu_b(\phi) \quad &= \quad \frac{16\phi^2\chi T^{1/2}}{\pi^{3/2}}
\end{aligned}
$$

Coefficients $\mathcal{A}$ - $\mathcal{G}$ identical to Burnett expansion for $\varepsilon_n \to 0$.

Linear response — $L \ll L_c$

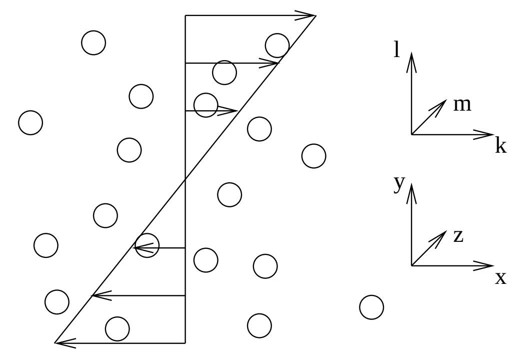
- Number of eigenvalues depends on number of basis functions chosen.
- For $k \to 0$,
  Transverse momenta $s_t = -(\mu/\rho)k^2$.
  Energy $s_e = -D_T k^2$.
  Mass & longitudinal mom. $s_l = \pm \imath k\sqrt{p_\rho} - \rho^{-1}(\mu_b + 4\mu/3)k^2$.
- All other modes with negative eigenvalues, indicating that other transients decay.



Linear response

Infinite sheared granular material

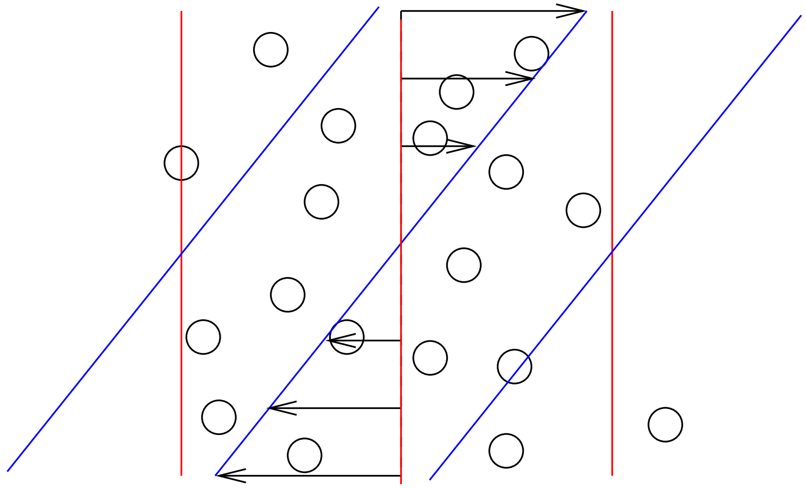
- Mean flow $\bar{u}_x = \bar{G}y$, $\bar{u}_y = 0$, $\bar{u}_z = 0$.
- Small dissipation $\epsilon = (1 - e_n)^{1/2} \ll 1$.
- Macroscopic length $L \gg L_c$.
- Mass conservation
  $\partial_t \rho + \nabla.(\rho \mathbf{u}) = 0$.
- Momentum conservation
  $\rho(\partial_t \mathbf{u} + \mathbf{u}.\nabla \mathbf{u}) = \nabla.\sigma$.
- Perturbations



$$\begin{pmatrix} \rho(\mathbf{x}, t) \\ \mathbf{u}(\mathbf{x}, t) \end{pmatrix} = \begin{pmatrix} \tilde{\rho}(t) \\ \tilde{\mathbf{u}}(t) \end{pmatrix} \exp\left(\imath k x + \imath l y + \imath m z\right)$$

Linear response

- Infinite shear flow — not homogeneous.
- Time dependent wave vector $k = k_0, l = l_0 - k_0\bar{G}t, m = m_0$.
- 'Linear' response equations

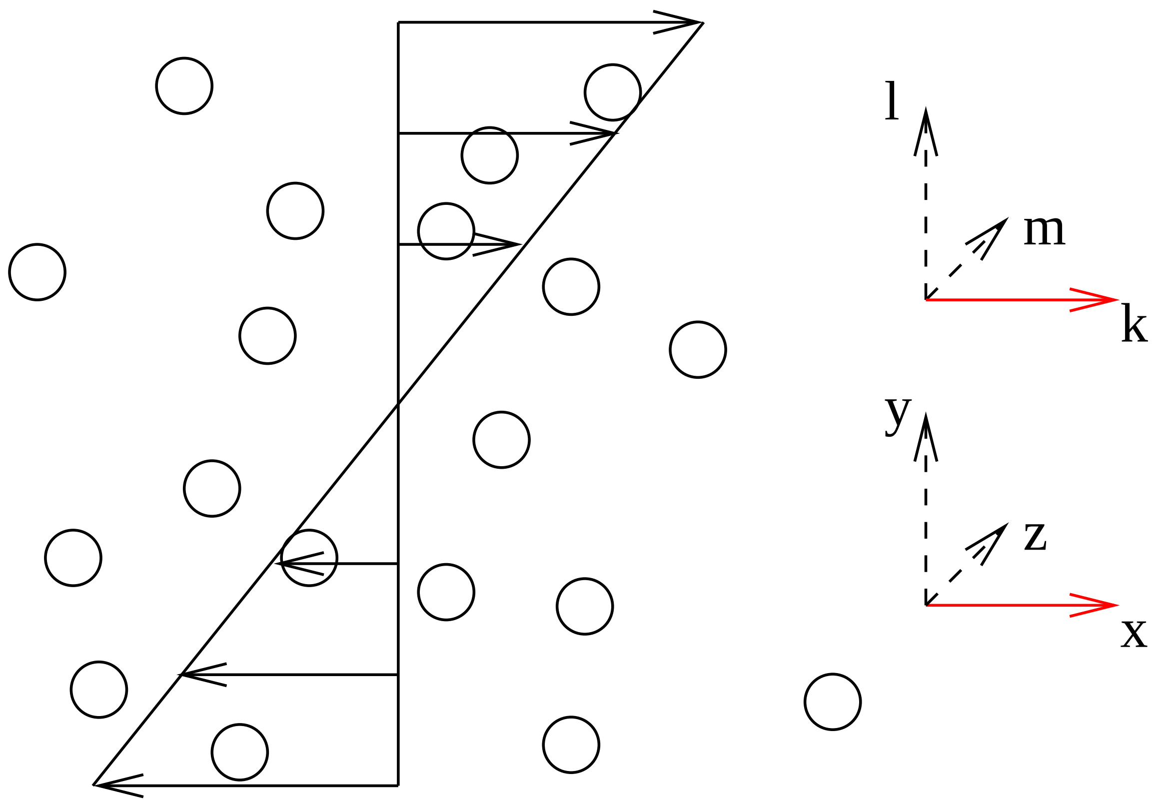


$$\partial_t \begin{pmatrix} \tilde{\rho}(t) \\ \tilde{\mathbf{u}}(t) \end{pmatrix} + (\mathcal{L}_0 + t\mathcal{L}_1 + t^2\mathcal{L}_2) \begin{pmatrix} \tilde{\rho}(t) \\ \tilde{\mathbf{u}}(t) \end{pmatrix} = 0$$

$$\begin{pmatrix} \tilde{\rho}(t) \\ \tilde{\mathbf{u}}(t) \end{pmatrix} = \exp\left(-t\mathcal{L}_0 - (t^2/2)\mathcal{L}_1 - (t^3/3)\mathcal{L}_2\right) \begin{pmatrix} \tilde{\rho}(0) \\ \tilde{\mathbf{u}}(0) \end{pmatrix}$$

For $k_0 = 0$, $\mathcal{L}_1 = 0$, $\mathcal{L}_2 = 0$.

Linear response — flow plane



Linear response — flow plane transverse mode

Perturbations to $\tilde{u}_z$:

$$\tilde{u}_z(t) = \tilde{u}_z(0)\exp\left(s_{0z}t + (s_{1z}t^2/2) + (s_{2z}t^3/3)\right)$$

$$\begin{aligned}
s_{0z} &= -\frac{(\bar{\mu} + \bar{\mathcal{E}}\bar{G}^2/8)}{\bar{\rho}}(k_0^2 + l_0^2) \\
s_{1z} &= \left(\frac{\bar{\mathcal{A}}\bar{G}^2 k_0^2}{2} + \frac{2\bar{G}k_0 l_0}{\bar{\rho}}(\bar{\mu} + (\bar{\mathcal{E}}\bar{G}^2/8))\right) \\
s_{2z} &= -\frac{\bar{G}^2 k_0^2}{\bar{\rho}}(\bar{\mu} + (\bar{\mathcal{E}}\bar{G}^2/8))
\end{aligned}$$

For $t \ll \bar{G}^{-1}$, $\tilde{u}_z \sim \exp(-\bar{\mu}k_0^2 t)$.

For $t \gg \bar{G}^{-1}$, $\tilde{u}_z \sim \exp(-\bar{\mu}\bar{G}^2 k_0^2 t^3)$.

Linear response — flow plane

Short time $t \ll \bar{G}^{-1}$:

$$\begin{pmatrix} \tilde{\rho}(t) \\ \tilde{u}_x(t) \\ \tilde{u}_y(t) \end{pmatrix} = \exp\left(s_{\rho xy}\right) \begin{pmatrix} \tilde{\rho}(0) \\ \tilde{u}_x(0) \\ \tilde{u}_y(0) \end{pmatrix}$$

where

$$s_{\rho xy}^3 = -\bar{G}^2 k_0^2 \left(\bar{\mu}_\rho + \frac{\bar{G}^2 \bar{\mathcal{E}}}{8}\right) + k_0 l_0 \bar{G} \left(\bar{p}_\rho - \frac{\bar{G}^2}{4}(\bar{\mathcal{A}}_\rho + 2\bar{\mathcal{C}}_\rho)\right)$$

- Three solutions — two propagating, one diffusive.
- For $l_0 = 0$, $s_{\rho xy} \propto -(-1, (-1)^{1/3}, (-1)^{2/3})\bar{G}^{2/3} k_0^{2/3} \bar{\mu}_\rho^{1/3}$.
- For $l_0 \neq 0$, $s_{\rho xy} \propto (-1, (-1)^{1/3}, (-1)^{2/3}) k_0^{1/3} l_0^{1/3} \bar{p}_\rho^{1/2}$

Linear response — flow plane

$$s_{\rho xy}\tilde{\rho} + \bar{\rho}\imath k_0\tilde{u}_x + \bar{\rho}\imath l_0\tilde{u}_y = 0$$

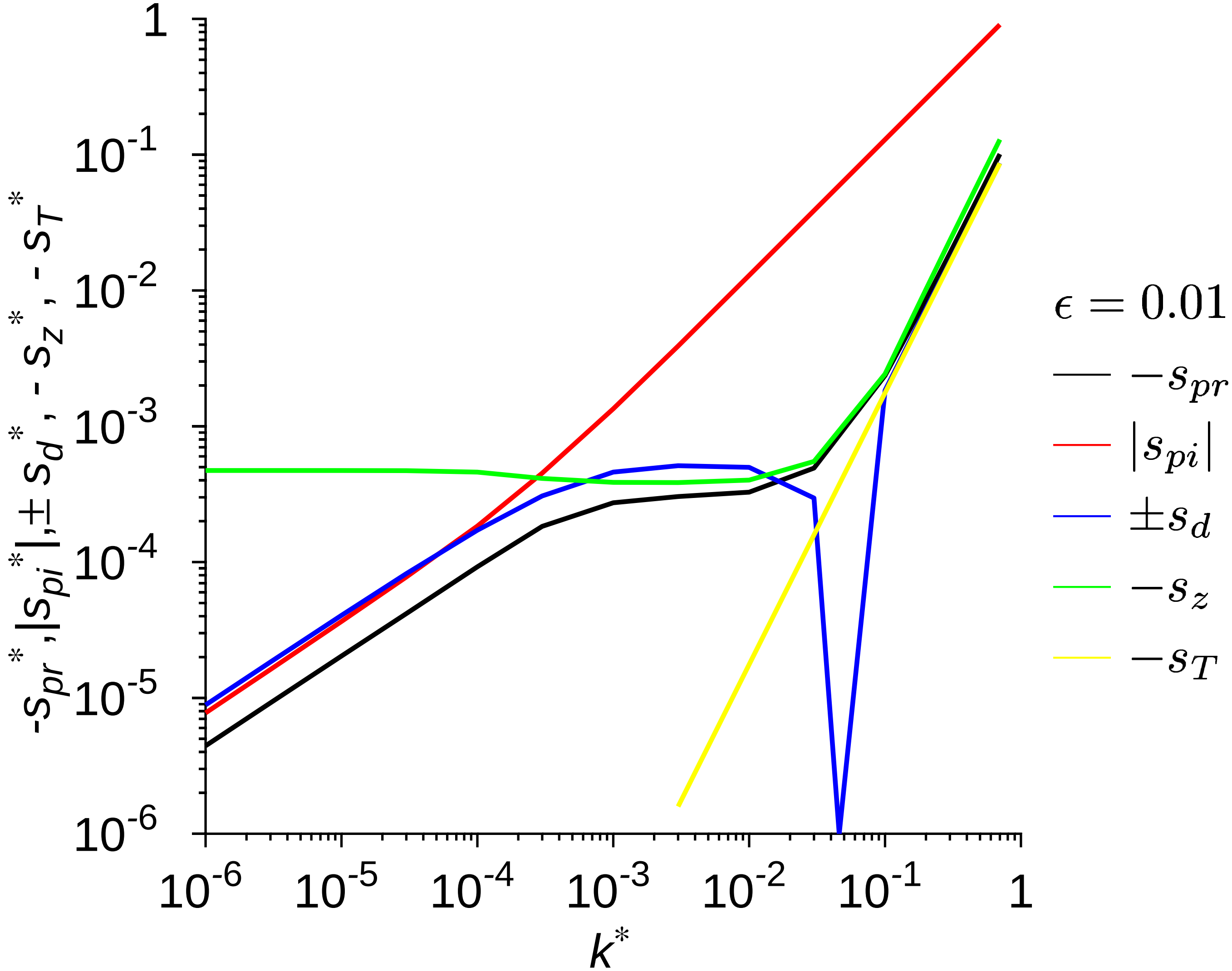
$$\bar{\rho}(s_{\rho xy}\tilde{u}_x + \bar{G}\tilde{u}_y) = 0$$

$$\bar{\rho}s_{\rho xy}\tilde{u}_y - (\imath\bar{G}k_0\bar{\mu}_\rho\tilde{\rho} + \imath l_0\bar{p}_\rho)\tilde{\rho} = 0$$

Summary — Flow direction:

		$k \ll \epsilon$	$k \gg \epsilon$
Propagating	s_{pr}	$-k^{2/3}$	$-k^2$
	s_{pi}	$\pm k^{2/3}$	$\pm k$
Diffusive	s_d	$+k^{2/3}$	$-k^2$
Transverse	s_z	$-k^2$	$-k^2$
Energy	s_T	$-k^0$	$-k^2$

Linear response — flow plane



Linear response — flow plane

Long time $t \gg \bar{G}^{-1}$:

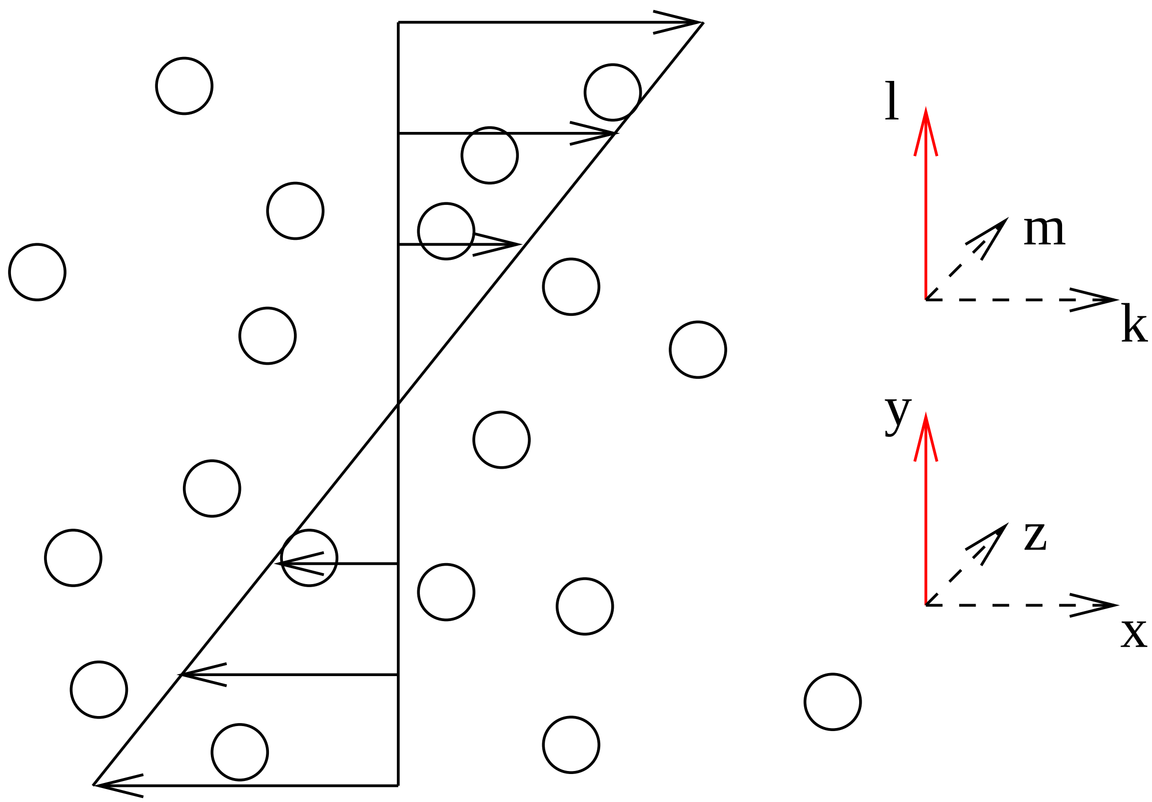
$$\begin{pmatrix} \tilde{u}_x(t) \\ \tilde{u}_y(t) \end{pmatrix} = \exp\left(-s_{xy}t^3/3\right) \begin{pmatrix} \tilde{u}_x \\ \tilde{u}_y \end{pmatrix}$$

$$\begin{aligned} s_{xy} \quad = \quad & -\frac{\bar{G}^2 k_0^2}{\bar{\rho}} \left(\frac{5\bar{\mu}}{3} + \frac{\bar{\mu}_b}{2} + \frac{\bar{p}\bar{R}}{\bar{G}} \right. \\ & \left. \pm \left(\frac{1}{9}\left(\bar{\mu} - \frac{3\bar{\mu}_b}{2}\right)^2 + \frac{4\bar{\mu}\bar{p}\bar{R}}{3\bar{G}} + \frac{\bar{\mu}_b\bar{p}\bar{R}}{\bar{G}} + \frac{\bar{p}^2\bar{R}^2}{\bar{G}^2} \right)^{1/2} \right) \end{aligned}$$

$s_{xy1} = (-2k_0^2\bar{G}\bar{p}\bar{R}/\bar{\rho})$

$s_{xy2} = (-\bar{G}^2 k_0^2 \bar{\mu}/\bar{\rho})$.

Linear response — gradient direction



Linear response — gradient direction

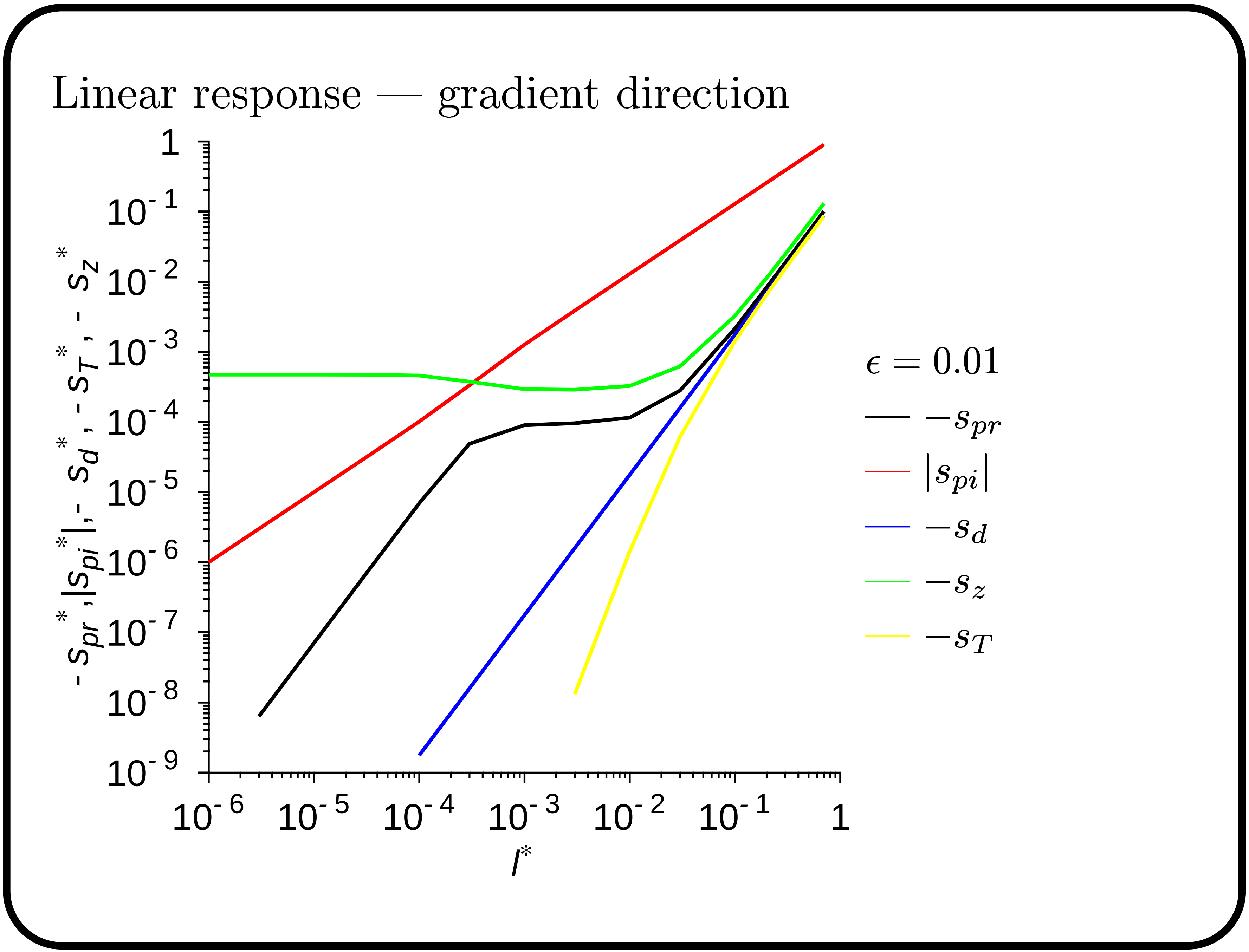
- Diffusive mode correct to $O(\epsilon^3)$

$$
\begin{aligned}
s_d &= \frac{8\bar{\mu}\bar{p}_\rho - 8\bar{p}\bar{\mu}_\rho + 2\bar{G}^2\bar{\mu}_\rho(\bar{\mathcal{A}} + 2\bar{\mathcal{C}}) - 2\bar{G}^2\bar{\mu}(\bar{\mathcal{A}}_\rho + 2\bar{\mathcal{C}}_\rho)}{-4\bar{\rho}\bar{p}_\rho - 8\bar{p} + 2\bar{G}^2(\bar{\mathcal{A}} + 2\bar{\mathcal{C}}) + \bar{\rho}\bar{G}^2(\bar{\mathcal{A}}_\rho + 2\bar{\mathcal{C}}_\rho)} l^2 \\
&\approx \frac{2(\bar{p}\bar{\mu}_\rho - \bar{\mu}\bar{p}_\rho)}{2\bar{p} + \bar{\rho}\bar{p}_\rho} l^2
\end{aligned}
$$

Qualitative difference — $(\bar{p}\bar{\mu}_\rho - \bar{\mu}\bar{p}_\rho) = 0$ at low and high density.

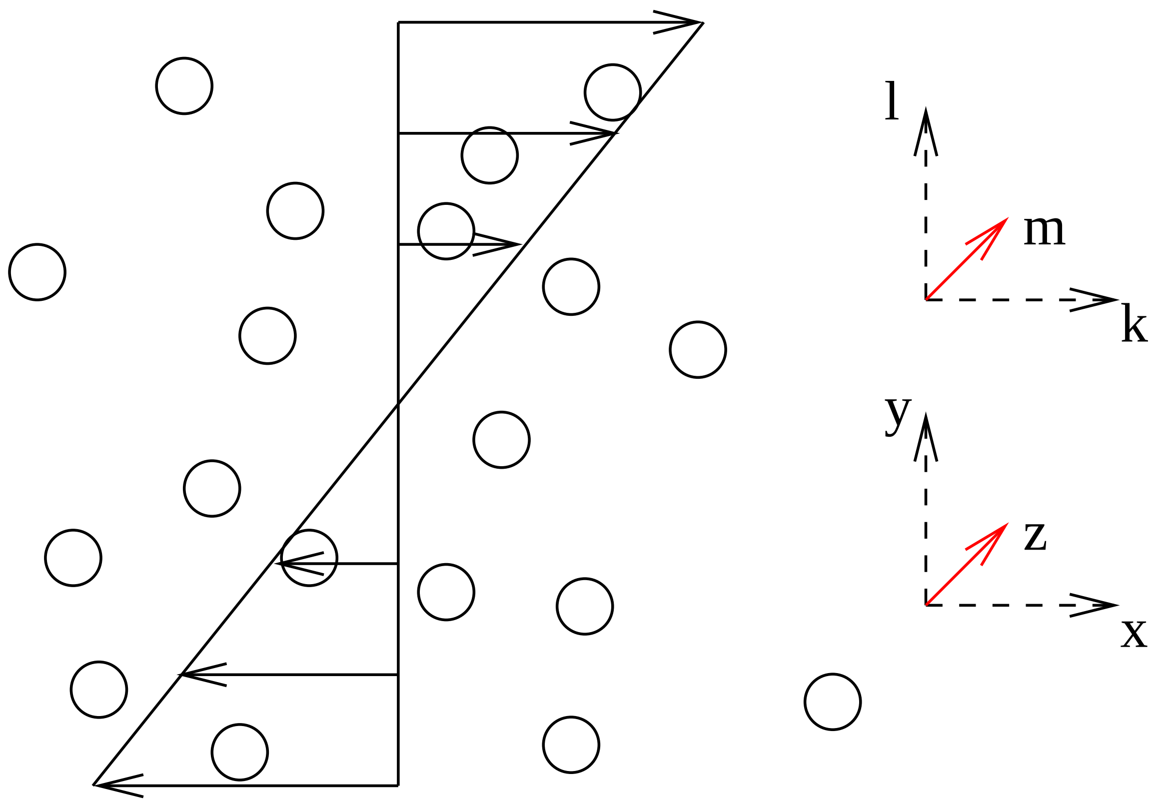
- Propagating modes

$$
\begin{aligned}
s_{pi} &= \pm\imath l\sqrt{\bar{p}_\rho + (2\bar{p}/\bar{\rho})} \\
&\quad -l^2\left(\frac{\bar{p}_\rho(\bar{G}(4\bar{\mu} + 3\bar{\mu}_b) + 6\bar{p}\bar{R}) + 6\bar{G}\bar{\mu}_\rho\bar{p}}{6\bar{G}(2\bar{p} + \bar{\rho}\bar{p}_\rho)} + \frac{5\bar{\mu}}{3\bar{\rho}} + \frac{\bar{\mu}_b}{2\bar{\rho}} + \frac{\bar{p}\bar{R}}{\bar{G}\bar{\rho}}\right)
\end{aligned}
$$



Linear response — gradient direction
1
10-1
10-2
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-6
10-5
10-4
10-3
10-2
10-1
1
l*
-s_pr*, |s_pi*|, -s_d*, -s_T*, -s_z*
ϵ = 0.01
−s_pr
|s_pi|
−s_d
−s_z
−s_T

Linear response — vorticity direction



Linear response — vorticity direction

Decoupling $\rho - z$ and $x - y$.

- $$s_{\rho z} = \pm \imath m \sqrt{\bar{p}_\rho - (\bar{\mathcal{C}}_\rho \bar{G}^2/2)} - \frac{m^2}{2\bar{\rho}} \left(\frac{4\bar{\mu}}{3} + \bar{\mu}_b + \frac{2\bar{p}\bar{R}}{\bar{G}} \right)$$

  For $\phi \ll 1$, $\bar{p}_\rho < 0 \rightarrow$ unstable. For $\phi \rightarrow \phi_c$, $\bar{p}_\rho > 0 \rightarrow$ stable.

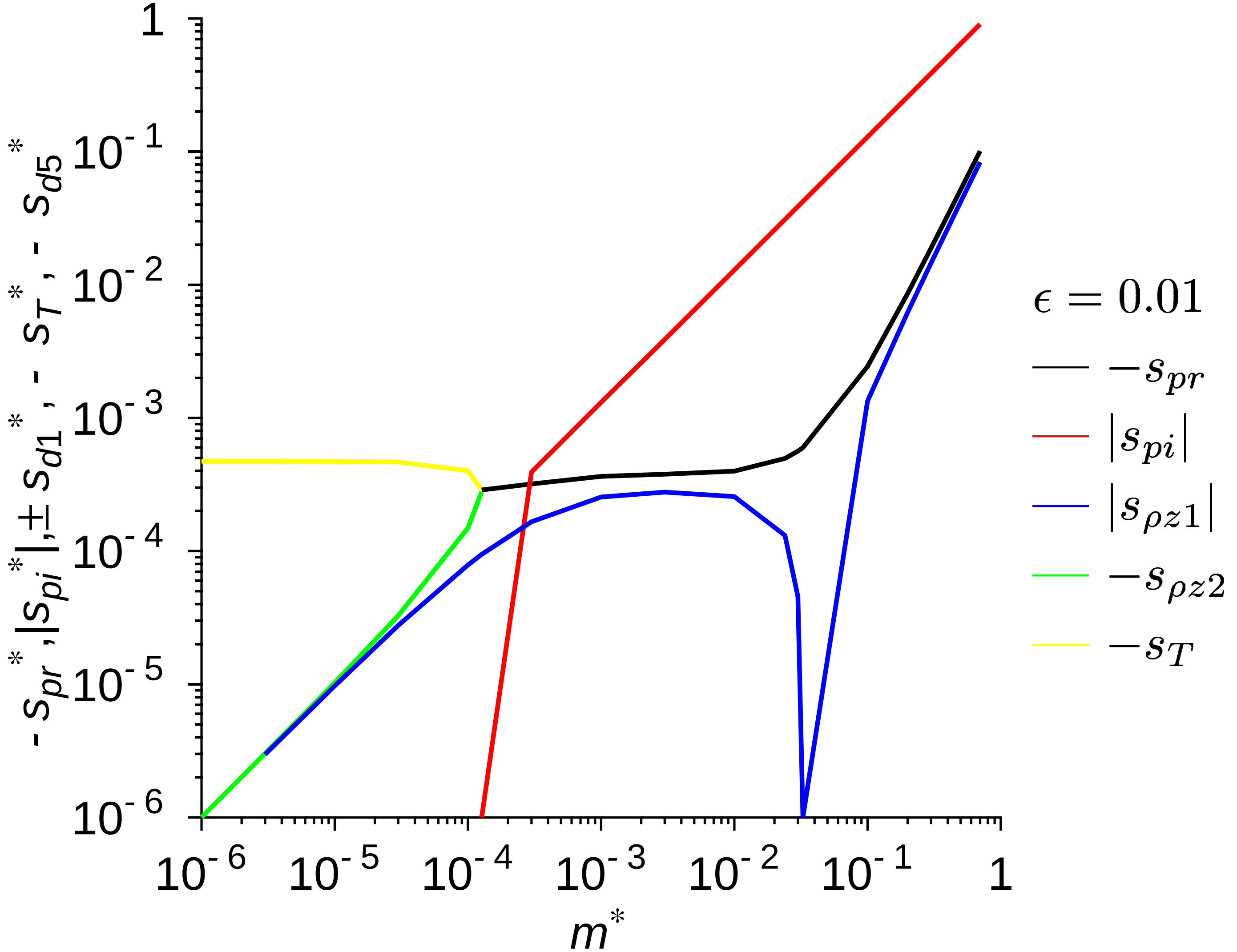
- $$s_{xy} = \pm m \sqrt{\frac{\bar{\mathcal{A}}\bar{G}}{4\bar{\rho}}} - m^2 \frac{\bar{\mu}}{\bar{\rho}}$$

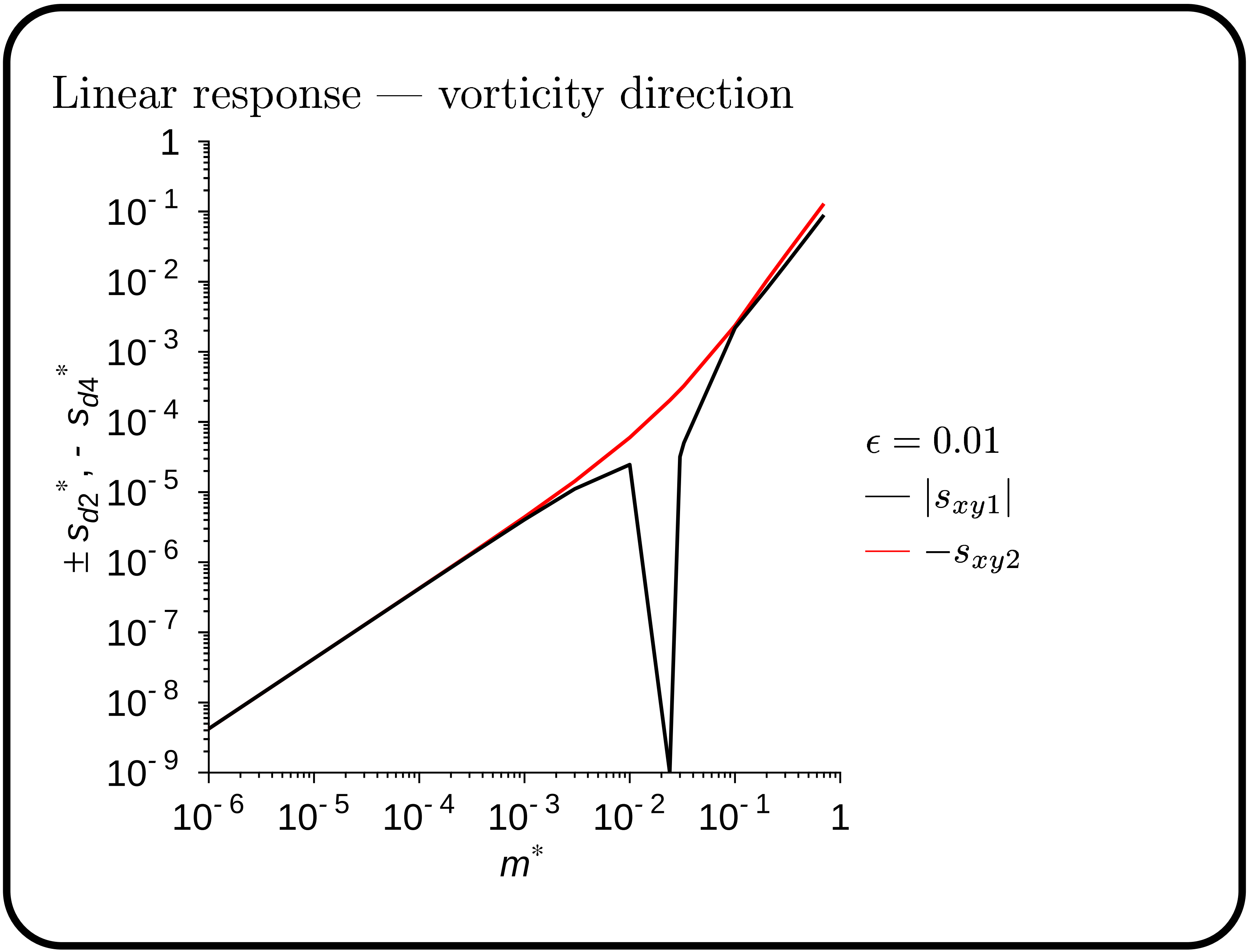
  One stable and one unstable mode.

Summary — vorticity direction

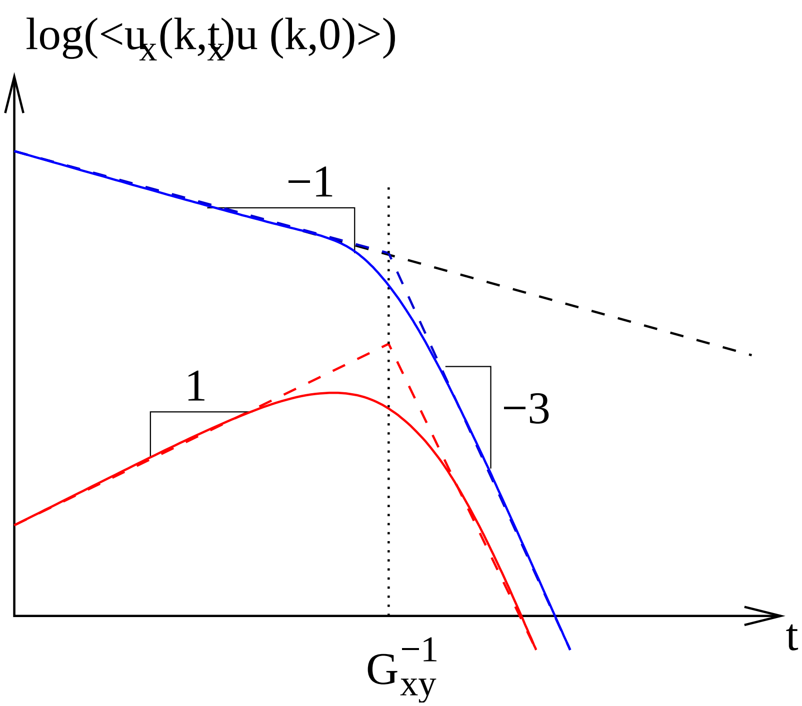
		$m \ll \epsilon$	$m \gg \epsilon$
Diffusive	$s_{\rho z}$	$+m$	$-m^2$
	$s_{\rho z}$	$-m$	$\pm\imath m$
Transverse	s_{xy}	$+m$	$-m^2$
	s_{xy}	$-m$	$-m^2$
Energy	s_T	$-m^0$	$-m^2$

Linear response — vorticity direction




Linear response — vorticity direction
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
$\pm s_{d2}^*$, - s_{d4}^*
10⁻⁶
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
m^*
$\epsilon = 0.01$
$|s_{xy1}|$
$-s_{xy2}$

Time correlation functions:



- $k \gg \varepsilon$

$$\int_{\mathbf{k}} \langle u_x(\mathbf{k},t) u_x(-\mathbf{k},0)\rangle \quad \sim \quad t^{-d/2}$$

$$\sigma_{xy} = \eta G_{xy} + \eta' G_{xy} \log(|G_{xy}|)$$

- $k \ll \varepsilon$

$$\int_{\mathbf{k}} \langle u_x(\mathbf{k},t) u_x(-\mathbf{k},0)\rangle$$

$$\sim \quad \int d\mathbf{k} \exp\left(-\eta k^{2/3} t\right)$$

$$\sim \quad t^{-3d/2}$$

$$\sigma_{xy} = \eta G_{xy} + \eta' G_{xy}^3 + \ldots$$

Conclusions

Linear response for shear flow:

- Perturbations grow at short times, decay at long times in the flow directions. Growth rate $\propto k^{2/3}, (kl)^{1/3}$ at short times, $\propto k^{2/3}$ at long times.
- Perturbations stable in gradient direction. Diffusive mode $s_d \propto -l^2$, propagating modes $\propto \pm \imath l - l^2$.
- Diffusive mode in gradient direction not adequately described by Navier-Stokes approximation.
- Perturbations in vorticity directions $\propto \pm m$ at low density, $\propto \pm \imath m - m^2$ at high density.
- Not adequately described by Navier-Stokes approximation.

Conclusions

- **Cautious conclusion:** transport coefficients do not diverge in two dimensions, regular in three dimensions.
- **However:** transport coefficients could be different from their microscopic values.