Sweeping Pattern by drying process of water-powder mixture

Takuya Iwashita
Yumino Hayase (Hokkaido)
Ryo Yamamoto
Hiizu Nakanishi
Kyushu University

Drying Experiment of Water-Cornstarch Mixture

Drying Patterns

2.7. Dependence of the pattern on $\phi_g$

$(d = 30 \mu m)$

- 0.2
- 0.6
- 0.1
- 0.5
- 0.02
- 0.4
- 0.01
- 0.3

Diagram showing the relationship between water and granule layers with air flow.
Mechanism of pattern formation

- Volume shrink by evaporation
- Retreat of interface
- Sweeping of granules by interface tension
- Resistance against interface motion by accumulated granules
- Instability of flat interface
- Pinning of large granules by glass plates

Modeling for Sweeping Interface

- Phase Field Model
- Invasion Percolation Model
- Boundary Dynamics
Other Interface Dynamics and its Instability

- Viscous Fingering
  Saffman-Taylor Instability

- Crystal Growth
  Mullins-Sekerka Instability

Sweeping Instability

Small material diffusion

\[\rightarrow\] Accumulation along interface

Stuck Interface

sticking out 1-d path
Phase Field Model of Sweeping Dynamics

\[ \frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v)) \]

\[ \frac{\partial v}{\partial t} = -\nabla \cdot J, \quad J = (A(v)\nabla u)v - D(u)\nabla v \]

\( u(r,t) \): represents the interface

\( u=1 \): wet
\( u=0 \): dry

\( v(r,t) \): represents granular distribution

a conserved quantity

Interface Motion

\[ \frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v)) \]
**Interface Motion**

\[
\frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v))
\]

Steady propagation when \( b \) is constant.

\[
u(x, t) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{x - \sqrt{2}bt}{2\sqrt{2}} \right) \right]
\]

Speed is prop. to \( b \).

Granules resist interface motion.

- \( b(v) \): a decreasing function of \( v \)

**Granule Motion**

\[
\frac{\partial v}{\partial t} = -\nabla \cdot J, \quad J = (A(v)\nabla u)v - D(u)\nabla v
\]

Conserved quantity

Eq. continuity for flux \( J \)
**Granule Motion**

\[ \frac{\partial v}{\partial t} = -\nabla \cdot J, \quad J = (A(v)\nabla u)v - D(u)\nabla v \]

Flux driven by the surface tension of air-water interface

proportional to \( \nabla u \)

Blocking effect

\[ A(v) = \frac{A_0}{v + 1} \]

**Granule Motion**

\[ \frac{\partial v}{\partial t} = -\nabla \cdot J, \quad J = (A(v)\nabla u)v - D(u)\nabla v \]

Diffusion within interface for numerical stability

\[ D(u) = Du(1-u) \]
Simulation of Phase Model

Phase Field
\[ \frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v)) \]

Gran. Density
\[ \frac{\partial v}{\partial t} = -\nabla \cdot J, \quad J = (A(v) \nabla u)v - D(u) \nabla v \]

\[ b(v) = \frac{b_0}{v+1}; \quad b_0 = 0.4, \]

\[ A(v) = \frac{A_0}{v+1}; \quad A_0 = 8.0, \]

\[ D(v) = D_0 u(1-u); \quad D_0 = 1.0 \]

Simulation Results

Initial wavy interface in \( u \) at \( t=0 \)

\( v_0 = 0.5 \) \[ \text{Initial granular density} \]

system width \( W = 90 \)

\( \lambda = 30 \)

\( t=150 \)

\( t=1800 \)
\[ v_0 = 0.5 \quad \lambda = 15, 25, 30 \]

\[ v_0 = 1.0 \]

Flat interface is stable for higher initial granular density for longer wave length of interface perturbation.
Problems of Phase Field Model

• Time consuming for simulations

• Local dynamics ↔ Experiment: global dyn.

• Pinning effect evaporation from overall interface
  interface advance at the weakest
Modeling of Sweeping Dynamics using Invasion Percolation

- Dynamics is analogous to Invasion Percolation
  --- Global dynamics

- Sweeping of granules

- Randomness and Pinning

Invasion Percolation

Invasion from the bottom

- The 0'th row is initially occupied.
- Strength of each lattice site is represented by a random number assigned to it in advance.

1. Occupy the weakest unoccupied site adjacent to the occupied sites.
2. Repeat the occupation process.
Invasion Percolation Model for Sweeping Dynamics

- Random number at each site  □ granular quantity
- Invasion  □ drying process
- Site strength  □ resistance against drying
  ↓ granular quantity

Sweeping:
Redistribution of granules at drying site

Leftover:
Granules on the sites where accumulation is over a threshold are not redistributed.
→ pattern formation

Surface tension:
Site facing more dry sites is easier to dry.

Diagram:

- Granules on sites where accumulation is over a threshold
- Sweeping pattern formation
Parameters in Model

- **Initial Granular Distribution**
  - upper limit $g$
  - width $\Delta g$
- **Threshold** $g_M$
- **Strength of surface tension** $\gamma=1$

**Simulation results**

Experiments by Yamazaki
Summary

• We analyzed **Sweeping Dynamics** that produces complex patterns during the drying process of water-granule mixture.

• Interface instability in the sweeping dynamics is a close analog of Saffman-Taylor instability in viscous fingering and Mullins-Sekerka instability in crystal growth.

• We have tried three type of modeling:
  – Phase field model
  – Invasion percolation
  – Boundary dynamics