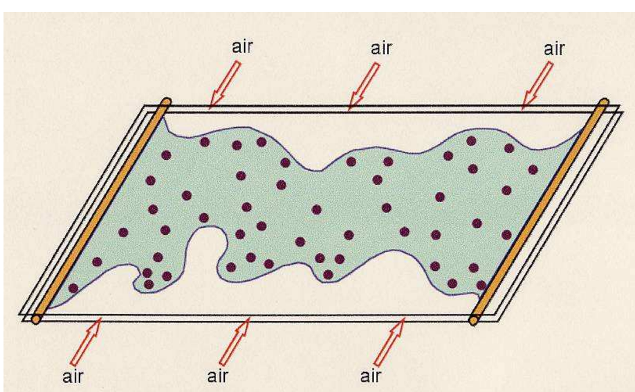
2005/5/2 at KITP

Sweeping Pattern by drying process of water-powder mixture

Takuya Iwashita
Yumino Hayase (Hokkaido)
Ryo Yamamoto
Hiizu Nakanishi
Kyushu University

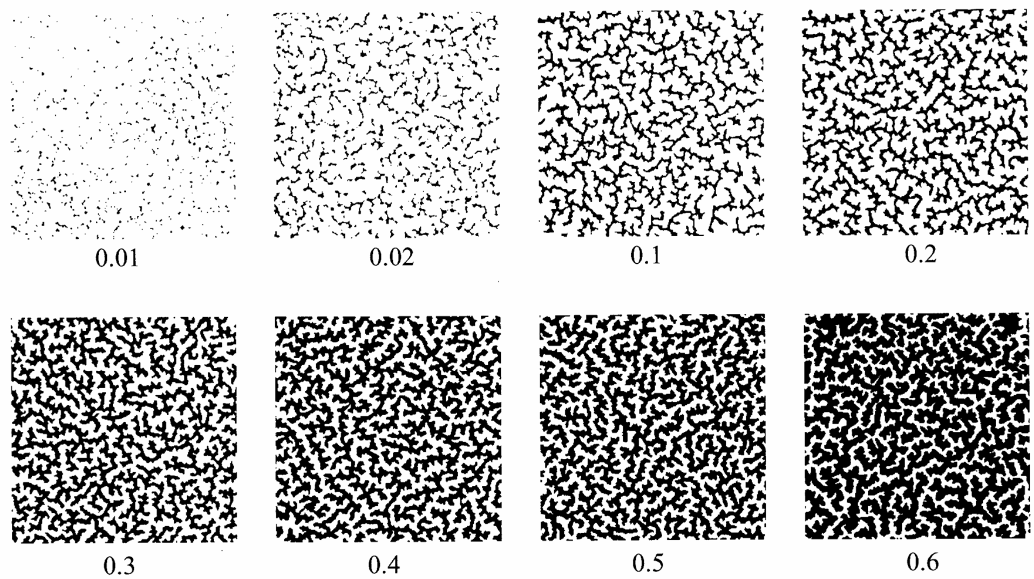
Drying Experiment of Water-Cornstarch Mixture

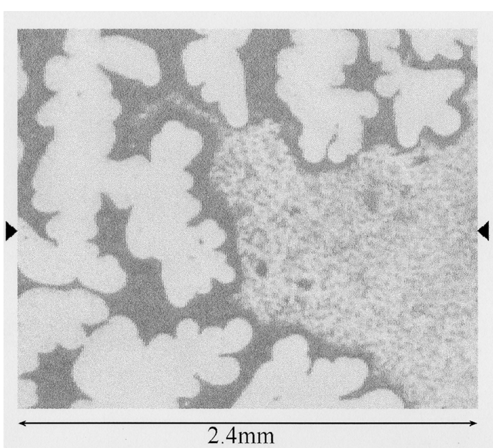
Yamazaki and Mizuguchi, JPSJ 69 (2000) 2387.

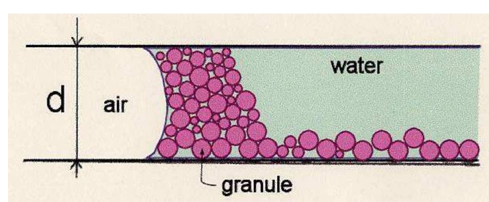


Drying Patterns

2.7. Dependence of the pattern on ϕ_g $(d = 30\mu m)$

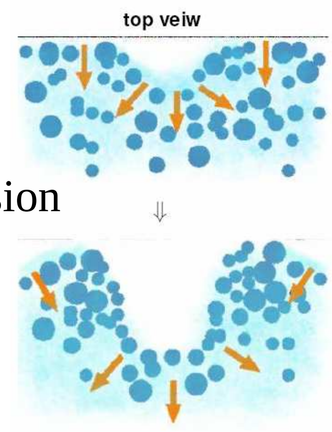






Mechanism of pattern formation

- Volume shrink by evaporation
- Retreat of interface
- Sweeping of granules by interface tension
- Resistance against interface motion by accumulated granules
- Instability of flat interface



- Pinning of large granules by glass plates

Modeling for Sweeping Interface

- Phase Field Model
- Invasion Percolation Model
- Boundary Dynamics

Other Interface Dynamics and its Instability

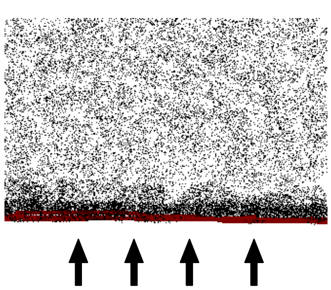
- Viscous Fingering ▯
  Saffman-Taylor Instability

- Crystal Growth ▯
  Mullins-Sekerka Instability

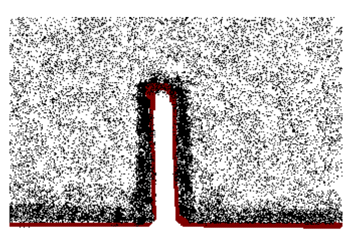
Sweeping Instability

Small material diffusion

⇒ Accumulation along interface



Stuck Interface



sticking out 1-d path

Phase Field Model of Sweeping Dynamics

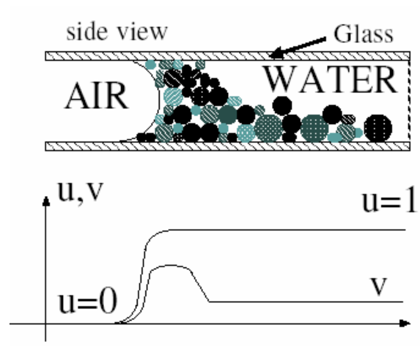
u : phase field, v : granular density

$$\frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v))$$
$$\frac{\partial v}{\partial t} = -\nabla \cdot \boldsymbol{J}, \quad \boldsymbol{J} = (A(v)\nabla u)v - D(u)\nabla v$$

$u(\mathbf{r},t)$: represents the interface

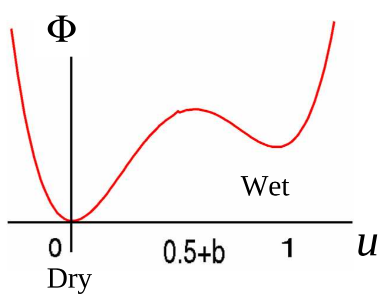
$u=1$: wet
$u=0$: dry

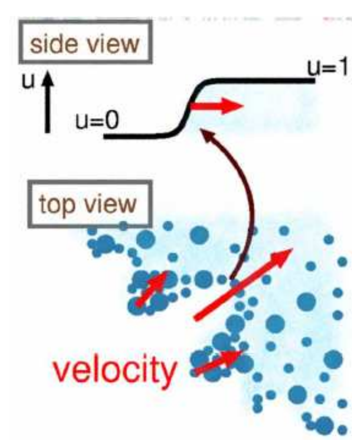
$v(\mathbf{r},t)$: represents granular distribution
a conserved quantity



Interface Motion

$$\frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v))$$





Interface Motion

$$\frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u - 0.5 - b(v))$$

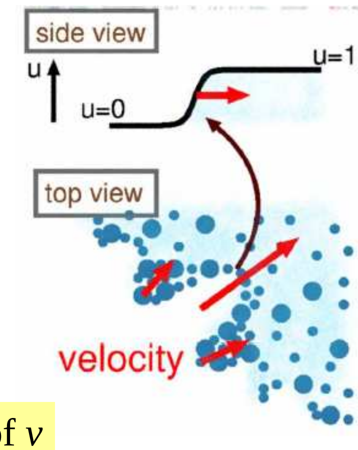
Steady propagation when b is constant.

$$u(x,t) = \frac{1}{2}\left[1 - \tanh\left(\frac{x - \sqrt{2}bt}{2\sqrt{2}}\right)\right]$$

Speed is prop. to b.

Granules resist interface motion.

⇨ $b(v)$: a decreasing function of v



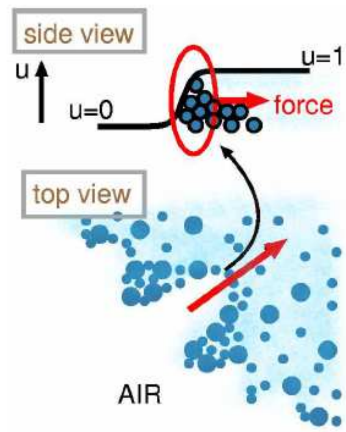
Granule Motion

$$\frac{\partial v}{\partial t} = -\vec{\nabla} \cdot \boldsymbol{J}, \qquad \boldsymbol{J} = (A(v)\vec{\nabla} u)v - D(u)\vec{\nabla} v$$

Conserved quantity

⇩

Eq. continuity for flux $\boldsymbol{J}$



Granule Motion

$$\frac{\partial v}{\partial t} = -\vec{\nabla} \cdot \boldsymbol{J}, \qquad \boldsymbol{J} = (A(v)\vec{\nabla} u)v - D(u)\vec{\nabla} v$$

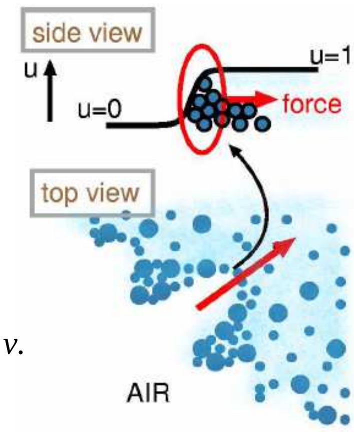
Flux driven by the surface tension of air-water interface

proportional to $\vec{\nabla} u$

Blocking effect

A should be a decreasing function of *v*.

$$A(v) = \frac{A_0}{v+1}$$

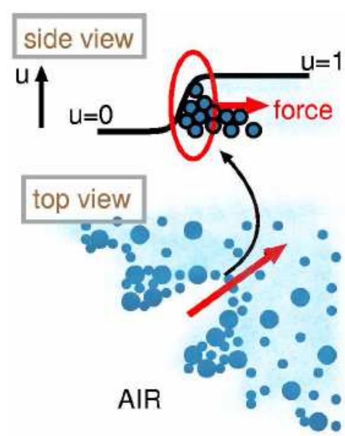


Granule Motion

$$\frac{\partial v}{\partial t} = -\vec{\nabla} \cdot \boldsymbol{J}, \qquad \boldsymbol{J} = (A(v)\vec{\nabla} u)v - D(u)\vec{\nabla} v$$

Diffusion within interface for numerical stability

$$D(u) = Du(1-u)$$



Simulation of Phase Model

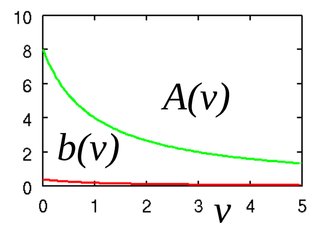
Phase Field $$\frac{\partial u}{\partial t} = \nabla^2 u + u(1-u)(u-0.5-b(v))$$

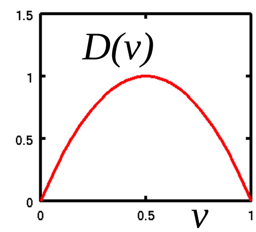
Gran. Density $$\frac{\partial v}{\partial t} = -\nabla \cdot \boldsymbol{J}, \qquad \boldsymbol{J} = (A(v)\nabla u)v - D(u)\nabla v$$

$b(v) = \frac{b_0}{v+1}$; $b_0 = 0.4$,

$A(v) = \frac{A_0}{v+1}$; $A_0 = 8.0$,

$D(v) = D_0 u(1-u)$; $D_0 = 1.0$





Simulation Results

t=0

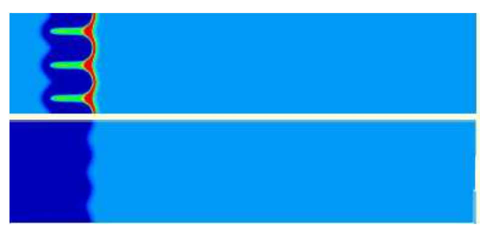
Initial wavy interface in u at $t=0$.

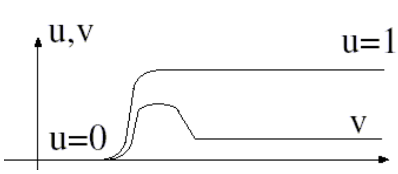
$v_0 = 0.5$ Initial granular density

system width W= 90

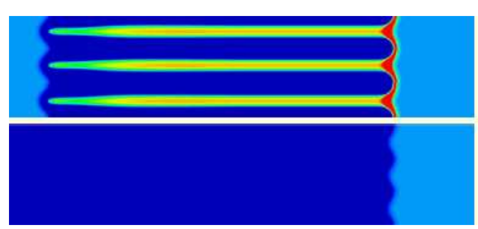
$\lambda = 30$

t=150



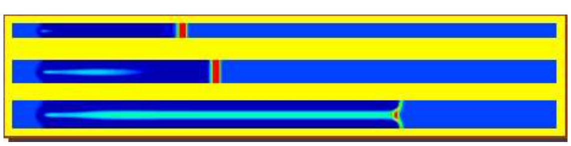


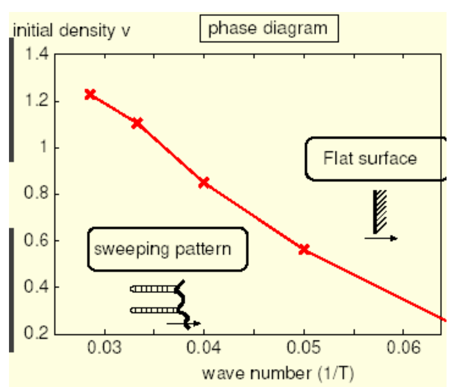
t=1800



v_0=0.5 λ = 15, 25, 30

v_0=1.0





Flat interface is stable

for higher initial granular density

for longer wave length of interface perturbation.

Problems of Phase Field Model

- Time consuming for simulations
- Local dynamics ←→ Experiment: global dyn.
- Pinning effect

evaporation from overall interface
interface advance at the weakest

Modeling of Sweeping Dynamics using Invasion Percolation

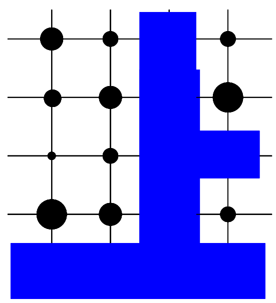
- Dynamics is analogous to Invasion Percolation
  --- Global dynamics
- Sweeping of granules
- Randomness and Pinning

Invasion Percolation

Invasion from the bottom

- The 0'th row is initially occupied.
- Strength of each lattice site is represented by a random number assigned to it in advance.

1. Occupy the weakest unccupied site adjacent to the occupied sites.
2. Repeat the occupation process.



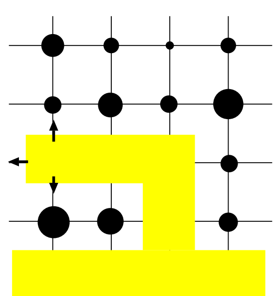
Invasion Percolation Model for Sweeping Dynamics

- Random number at each site □ granular quantity
- Invasion □ drying process
- Site strength □ resistance against drying
  ▯ granular quantity

Sweeping▯ Redistribution of granules at drying site

Leftover▯ Granules on the sites where accumulation is over a threshold are not redistributed.
→ pattern formation

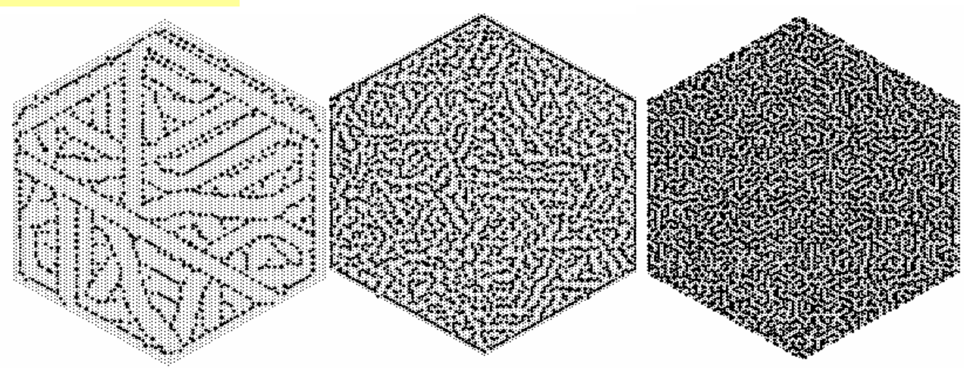
Surface tension ▯ Site facing more dry sites is easier to dry.



Parameters in Model

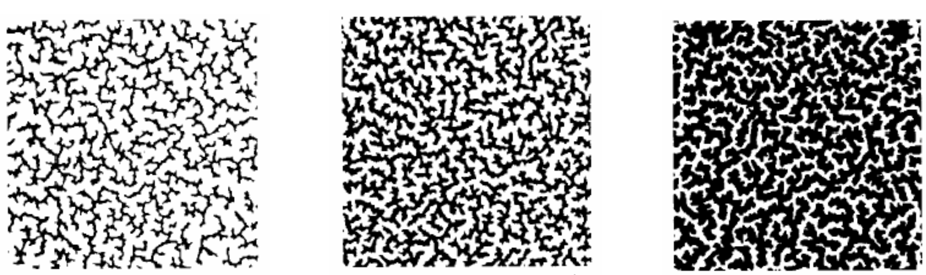
- Initial Granular Distribution
  - upper limit g
  - width Δg
- Threshold g_M
- Strength of surface tension $\gamma=1$

Simulation results



$g_M = 2.0,\ g/g_M = 0.2$ $g_M = 2.0,\ g/g_M = 0.5$ $g_M = 2.0,\ g/g_M = 0.8$

Experiments by Yamazaki



0.1 0.3 0.6

Summary

- We analyzed ***Sweeping Dynamics*** that produces complex patterns during the drying process of water-granule mixture.
- Interface instability in the sweeping dynamics is a close analog of Saffman-Taylor instability in viscous fingering and Mullins-Sekerka instability in crystal growth.
- We have tried three type of modeling:
  - Phase field model
  - Invasion percolation
  - Boundary dynamics