Jamming in Dense Suspensions

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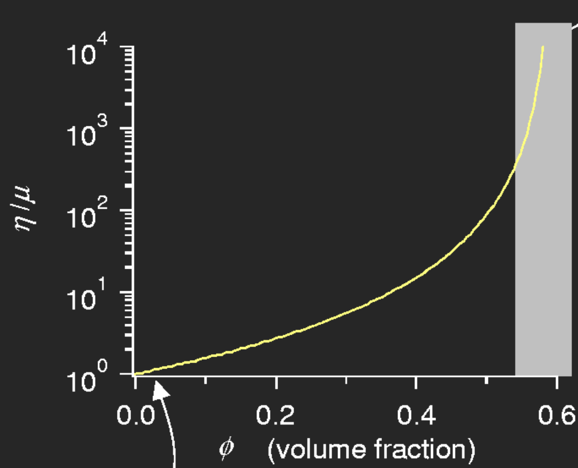
KITP Program on Granular Materials – 31 March 2005

Rheology of concentrated suspensions



$$\eta(\phi) = \mu\left(1 + \frac{5}{2}\phi + \ldots\right)$$

(not published in 1905)

- All existing data looks like this (roughly)
- Viscosity diverges at high volume fractions (<random close packing?)
- Universal behavior of viscosity?
  - Interactions – hard spheres
    – difficult to achieve
    – sensitive to short range repulsive interactions
  - Shear rate – Peclet number

$$\mathrm{Pe} = \frac{\dot{\gamma}}{D/a^2}$$

- System eventually jams
  - At what volume fraction? ϕ_{RCP}?
  - Depends on shear rate (Pe)?

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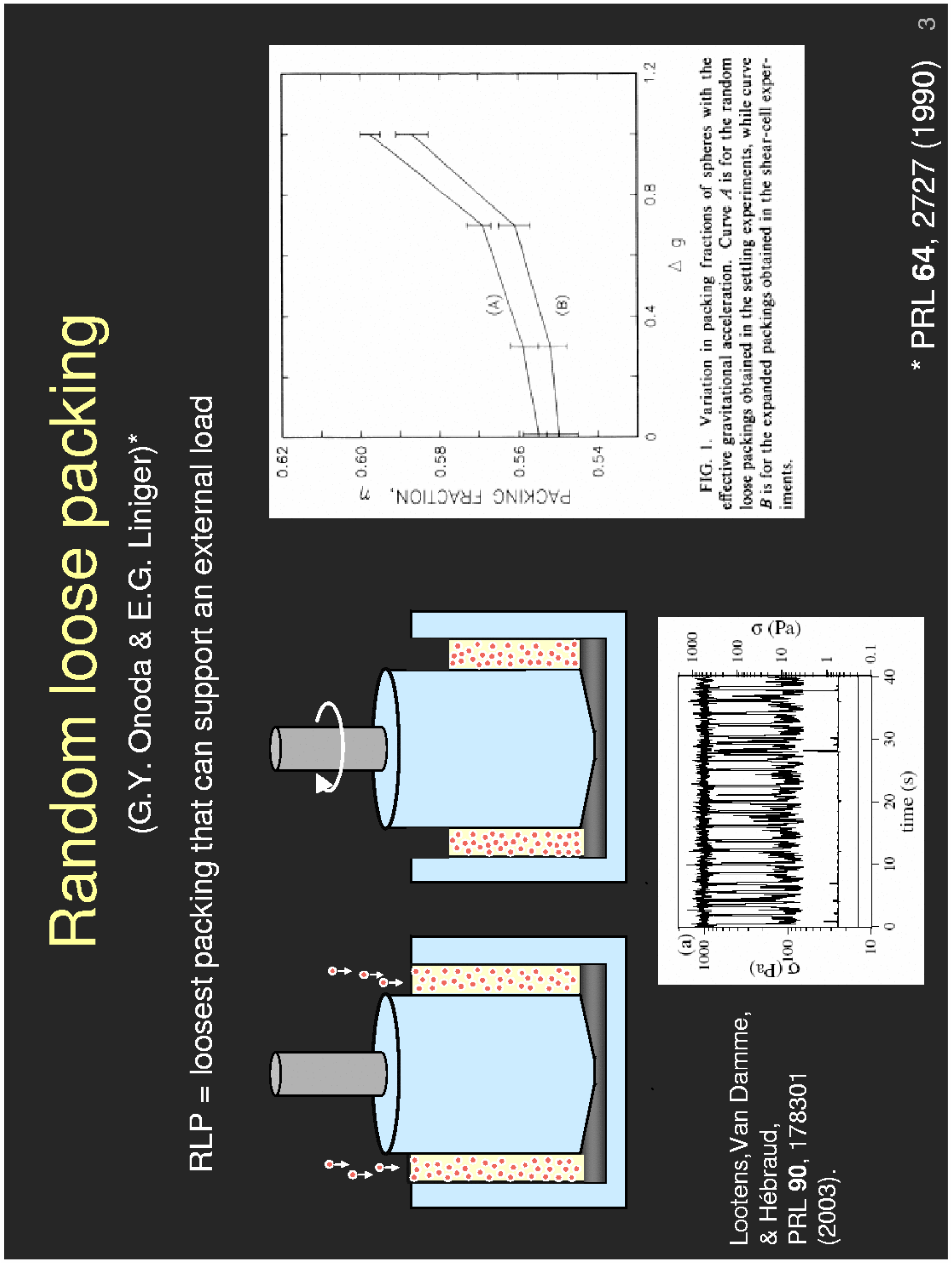
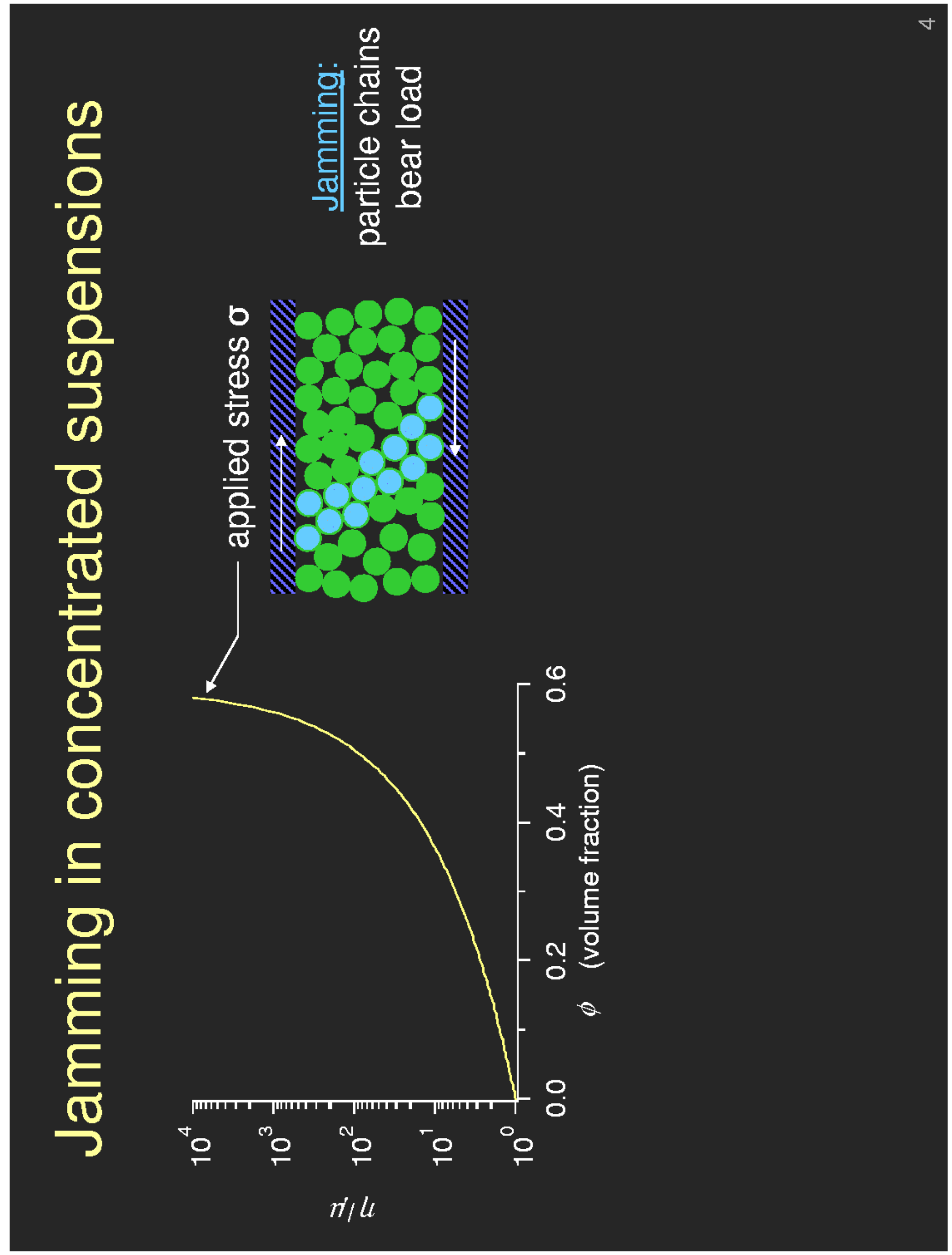
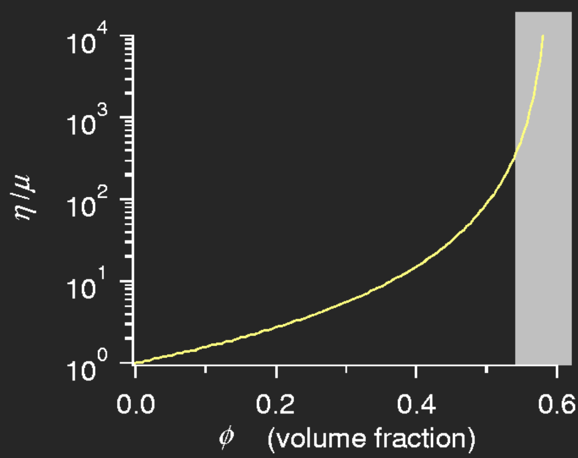


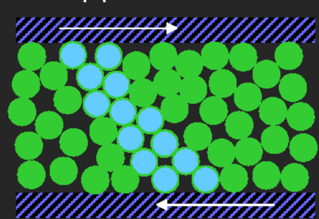
Random loose packing
(G.Y. Onoda & E.G. Liniger)*
RLP = loosest packing that can support an external load
PACKING FRACTION, η
Δ g
(A)
(B)
FIG. 1. Variation in packing fractions of spheres with the effective gravitational acceleration. Curve A is for the random loose packings obtained in the settling experiments, while curve B is for the expanded packings obtained in the shear-cell experiments.
σ (Pa)
time (s)
Lootens, Van Damme, & Hébraud, PRL 90, 178301 (2003).
* PRL 64, 2727 (1990)
3



Jamming in concentrated suspensions
applied stress σ
Jamming: particle chains bear load
η/η'
10^4
10^3
10^2
10^1
10^0
0.0
0.2
0.4
0.6
φ (volume fraction)
4

Jamming in concentrated suspensions





- Jamming usually studied by measuring viscosity divergence
  - Usually measured at constant applied shear rate

    $\Rightarrow$ Jamming is impossible under these conditions

    $$\sigma = \eta\dot{\gamma}$$

  - This approach *infers* jamming – does not measure it
- Jamming studies should be performed at constant stress

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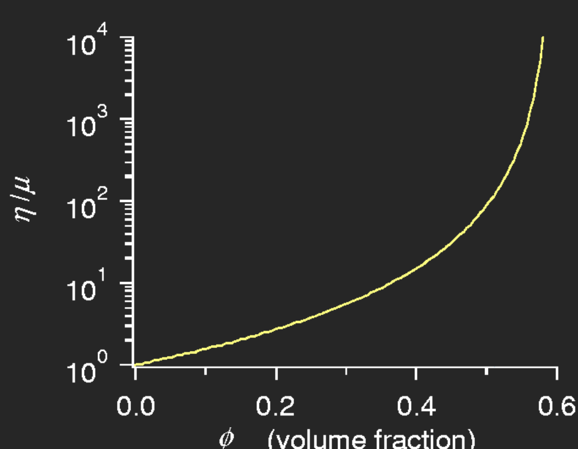
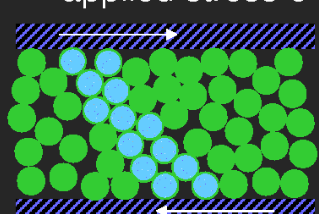
Jamming in concentrated suspensions





Why study jamming in *non-colloidal* suspensions?

- Shear dominates diffusion: $\dot{\gamma} >> D/a^2$

  diffusion frustrates jamming

- Can use *large particles* to *see inside bulk* $\Rightarrow$ *microstructure*
  - requires *matching refractive indices* of particles and suspending liquid
  - Can minimize effects of gravity by *matching density* of particles and suspending liquid

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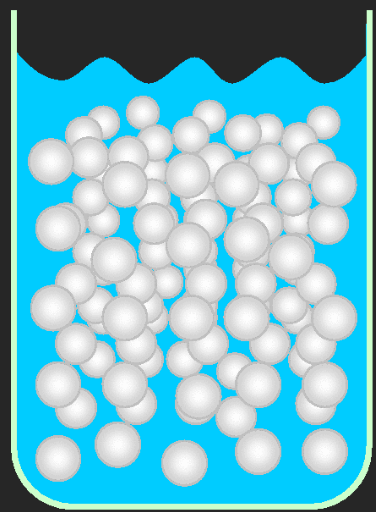
Our suspensions



Spheres
- PMMA
- diameter ≈ 210 – 250 µm
  - *no Brownian motion* (too big)
  - *polydisperse* to avoid ordering
- concentrated → $0.5 < \varphi < 0.6$

Liquid
- Newtonian rheology
- refractive index matched* to PMMA
- density matched* to PMMA

* Leighton formulation

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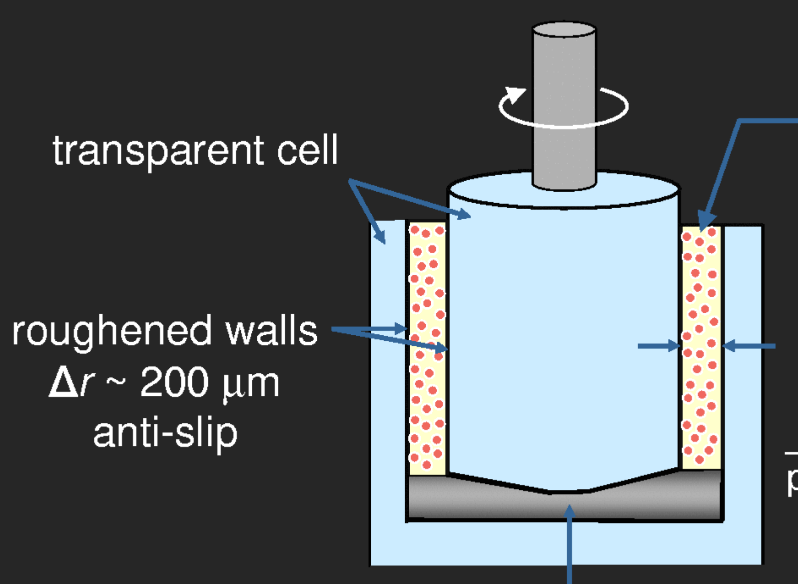
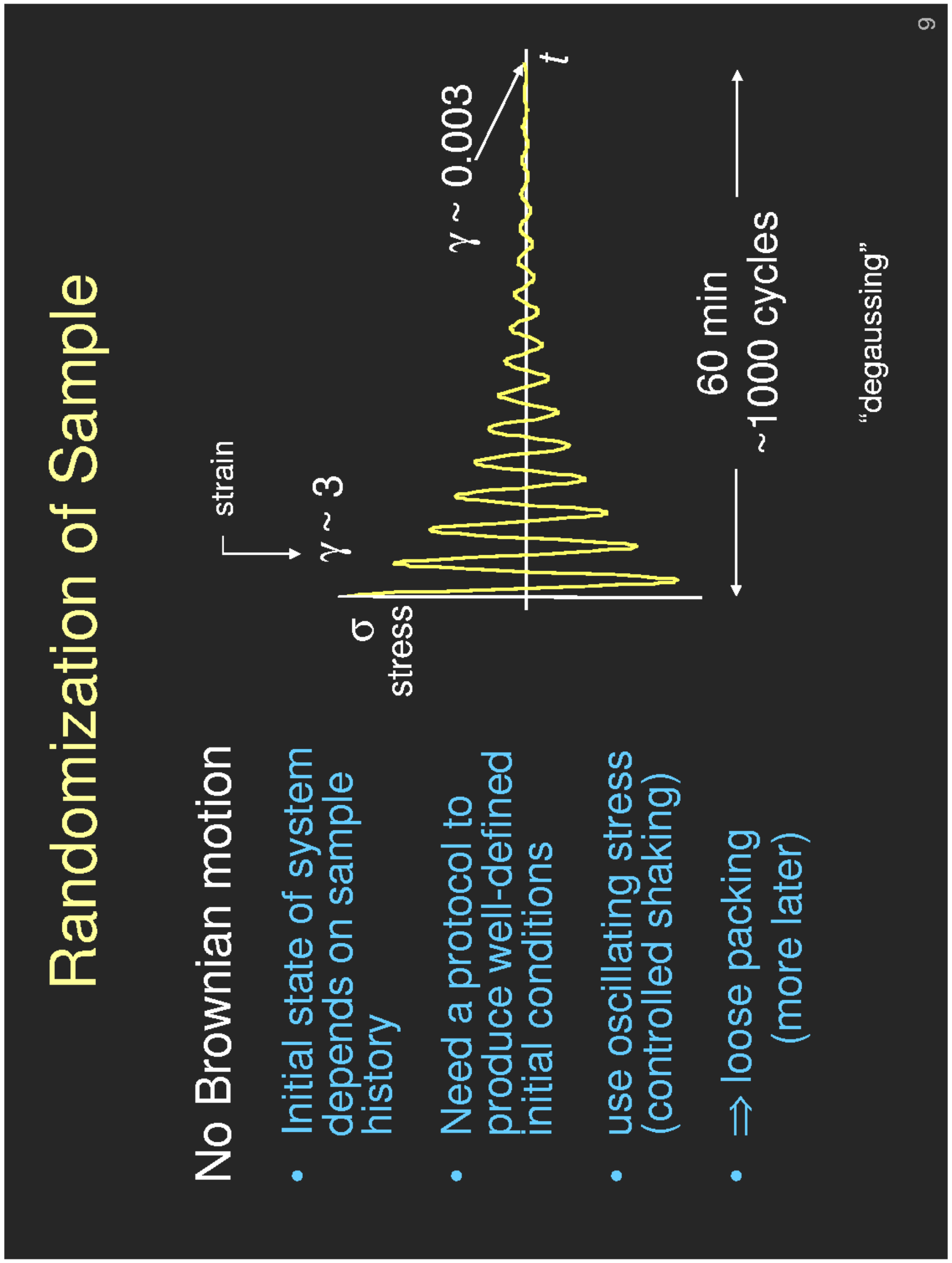
Couette cell for rheology

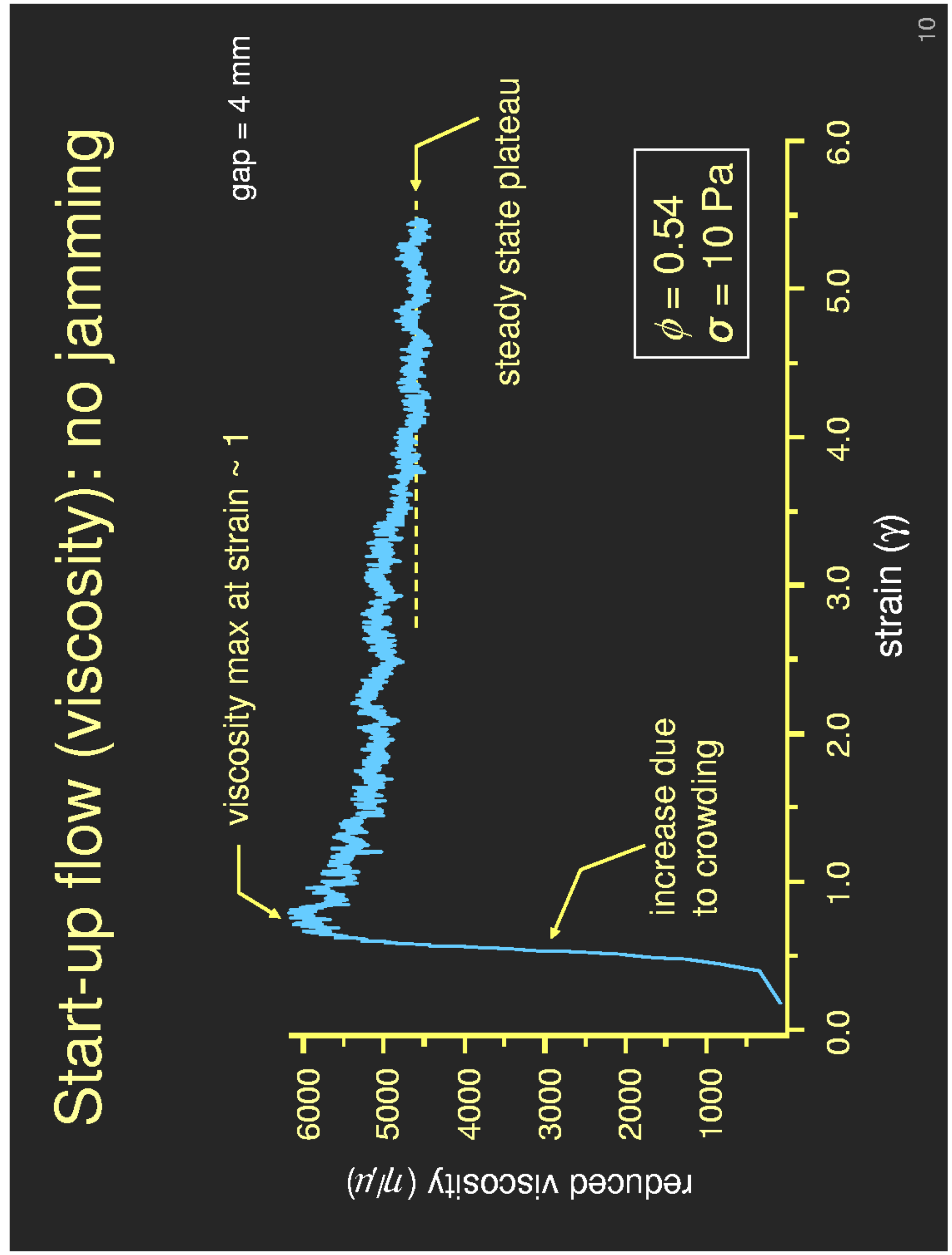
controlled stress $\eta = \frac{\sigma}{\dot{\gamma}}$ ← apply stress (σ), ← measure shear rate ($\dot{\gamma}$)



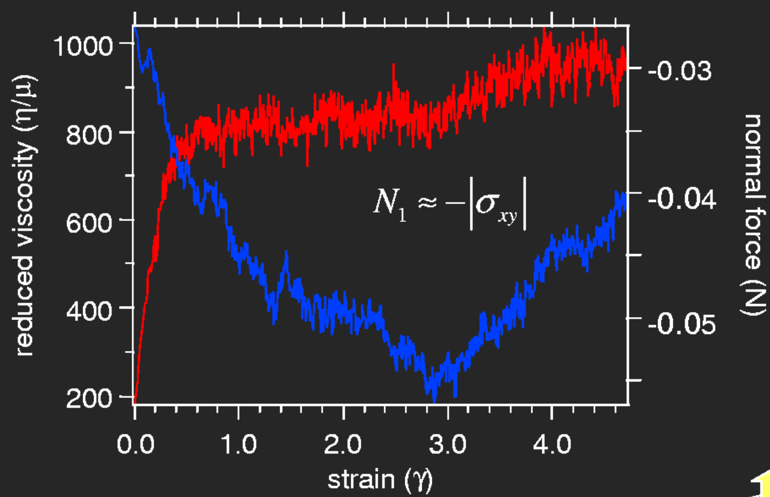
$\frac{\text{cell gap}}{\text{particle diameter}} \approx 10-100$

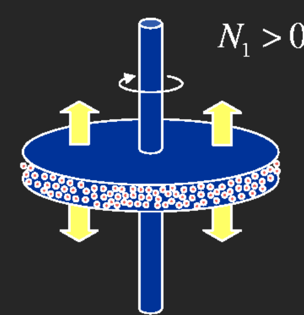
8


Randomization of Sample
No Brownian motion
• Initial state of system depends on sample history
• Need a protocol to produce well-defined initial conditions
• use oscillating stress (controlled shaking)
• ⇒ loose packing (more later)
strain
γ ~ 3
σ
stress
γ ~ 0.003
t
60 min
~1000 cycles
“degaussing”
9


Start-up flow (viscosity): no jamming
gap = 4 mm
viscosity max at strain ~ 1
steady state plateau
increase due to crowding
φ = 0.54
σ = 10 Pa
reduced viscosity (η/μ)
6000
5000
4000
3000
2000
1000
0.0
1.0
2.0
3.0
4.0
5.0
6.0
strain (γ)
10

First normal stress difference: no jamming





$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \quad ; \quad N_1 = \sigma_{xx} - \sigma_{yy}$$

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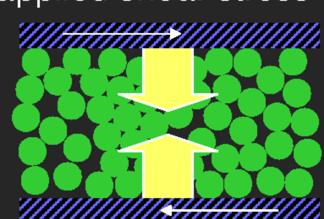
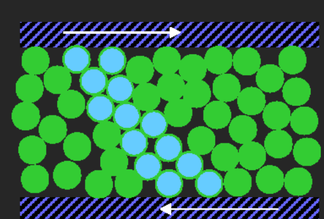
Negative first normal stress difference

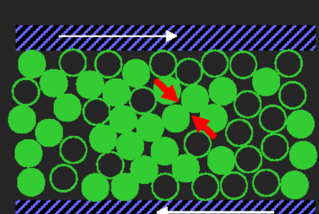
applied shear stress σ



Negative first normal stress difference $N_1 = \sigma_{xx} - \sigma_{yy}$ pulls plates together



Expect jamming would tend to push plates apart



Lubrication forces can suck plates together

(no Brownian motion)

(Laun, Brady, Morris, …)

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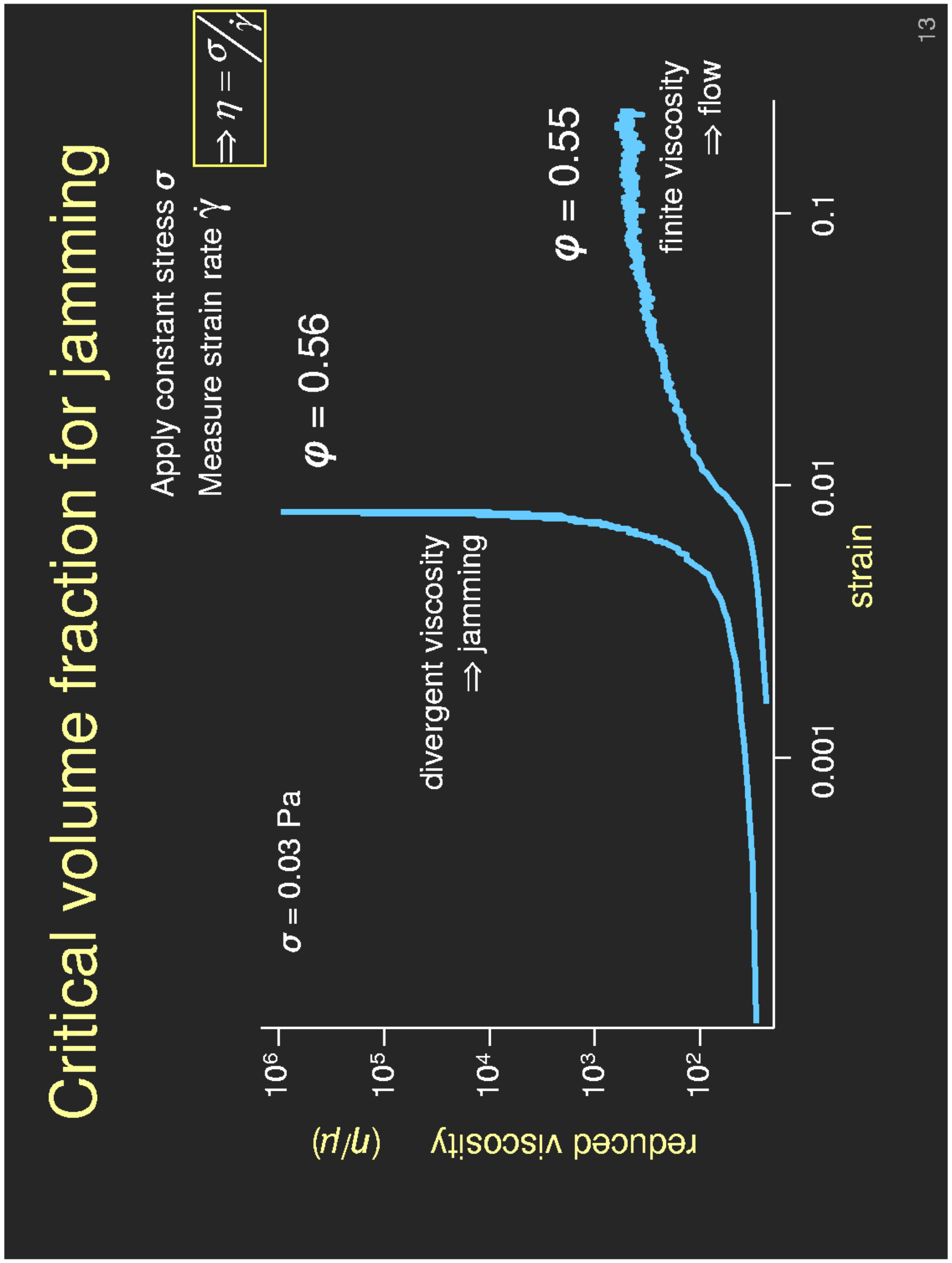
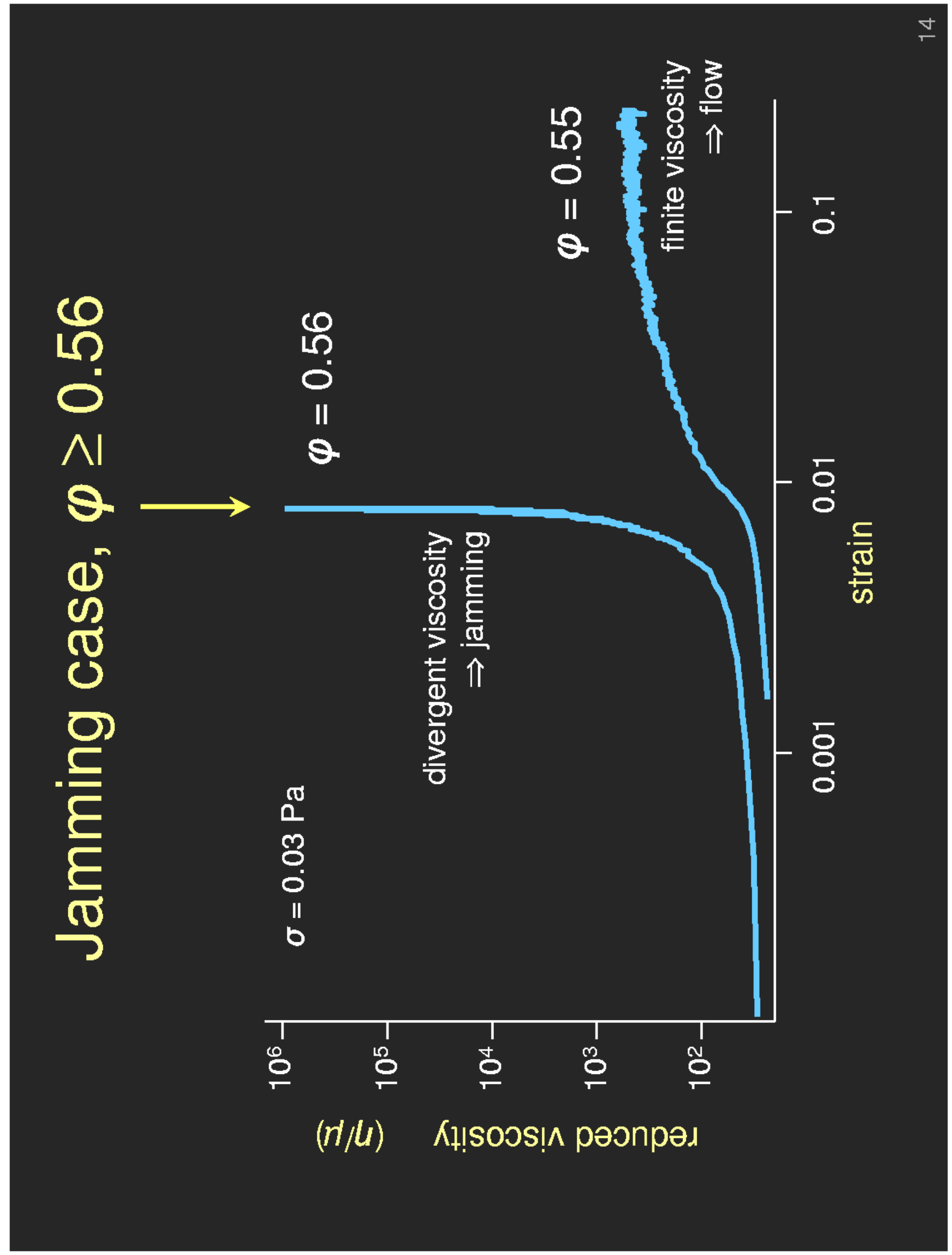
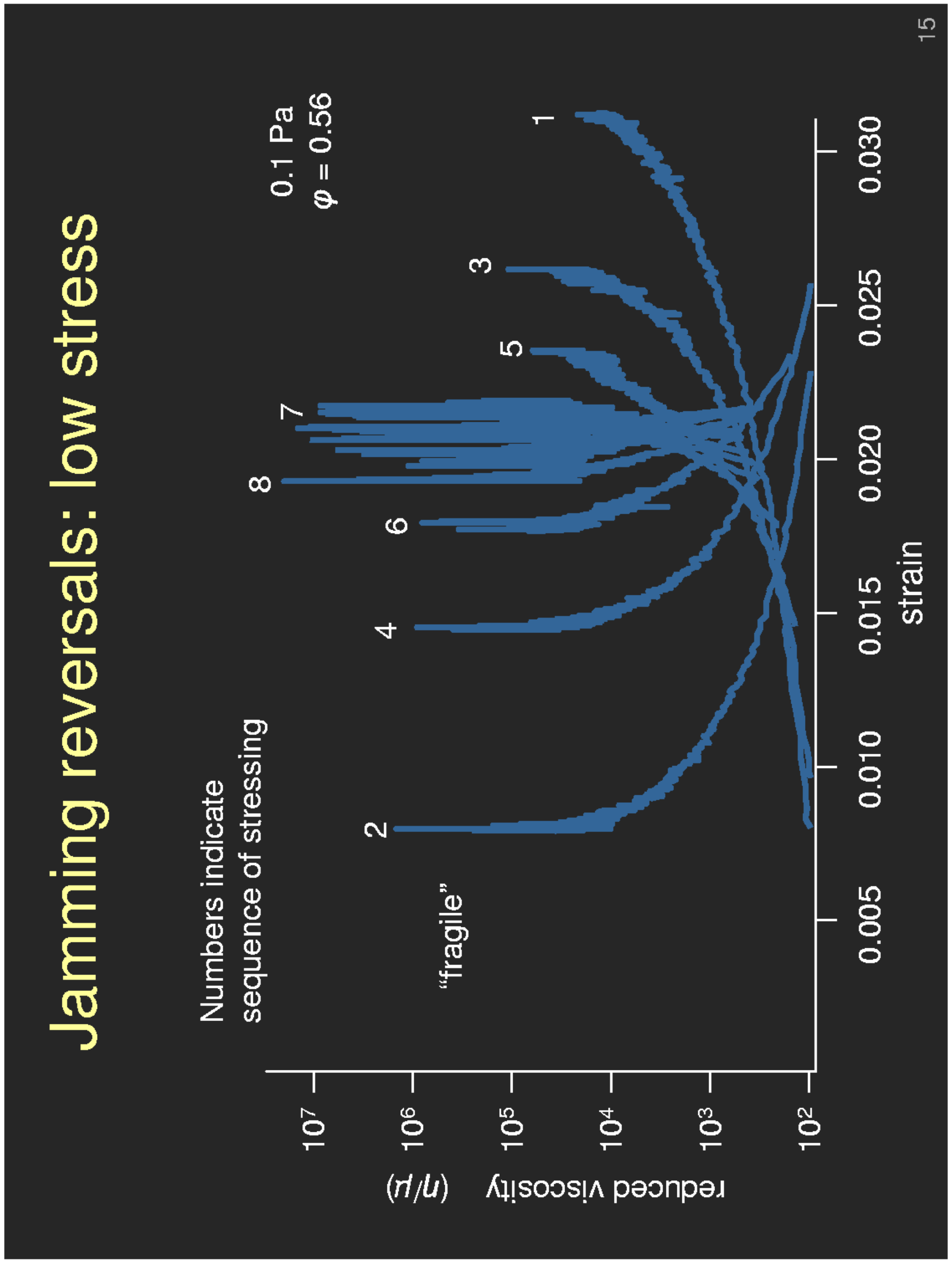
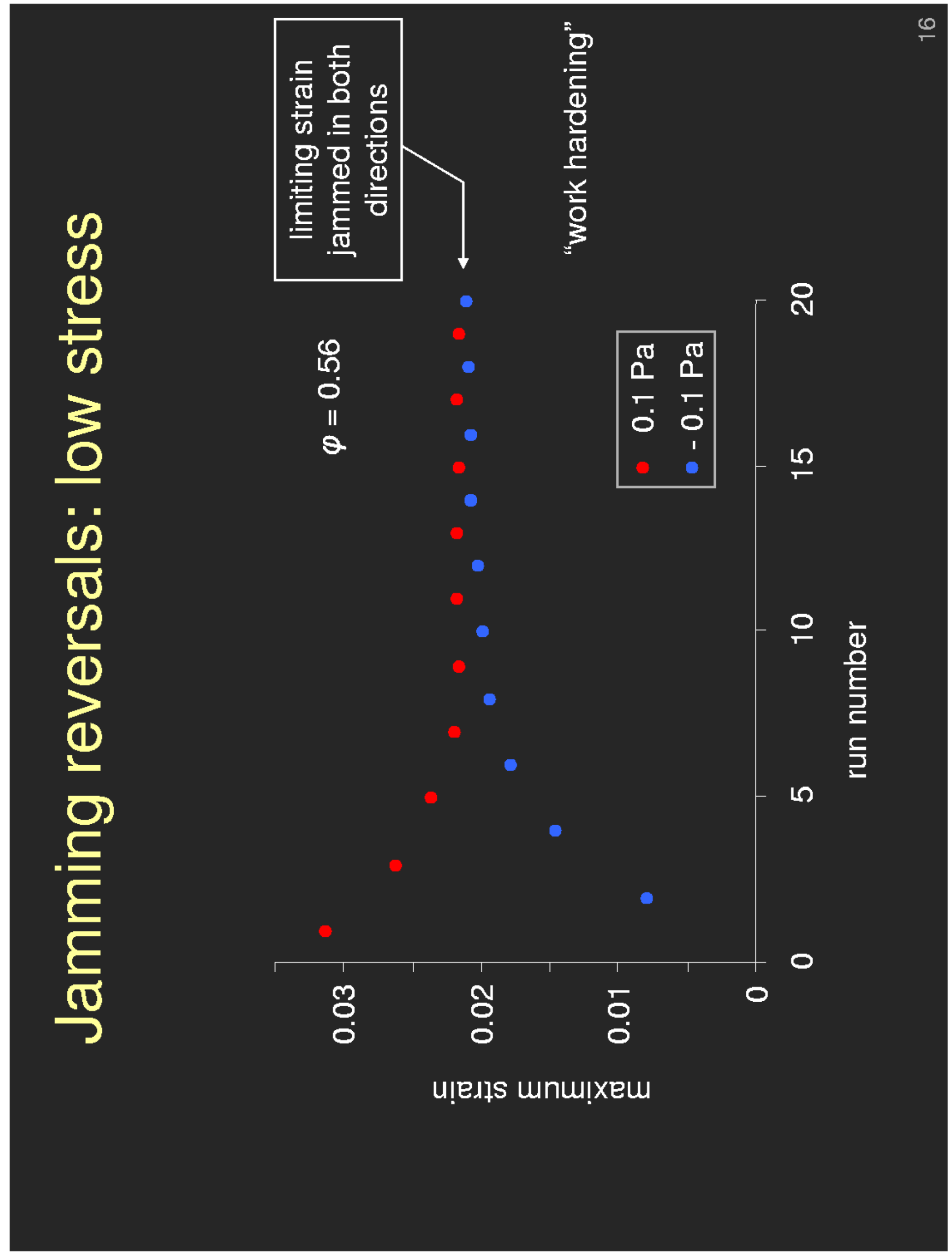
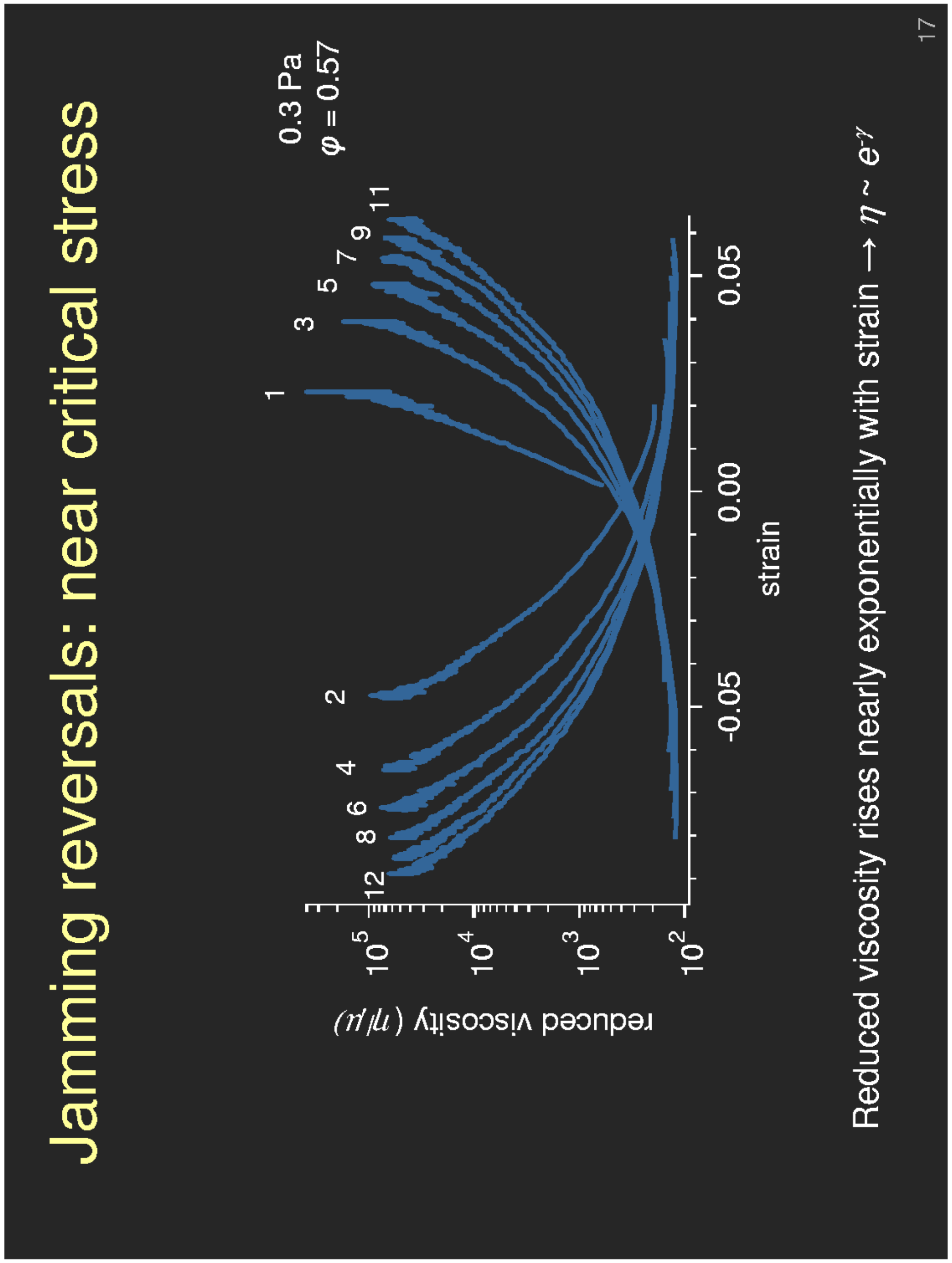
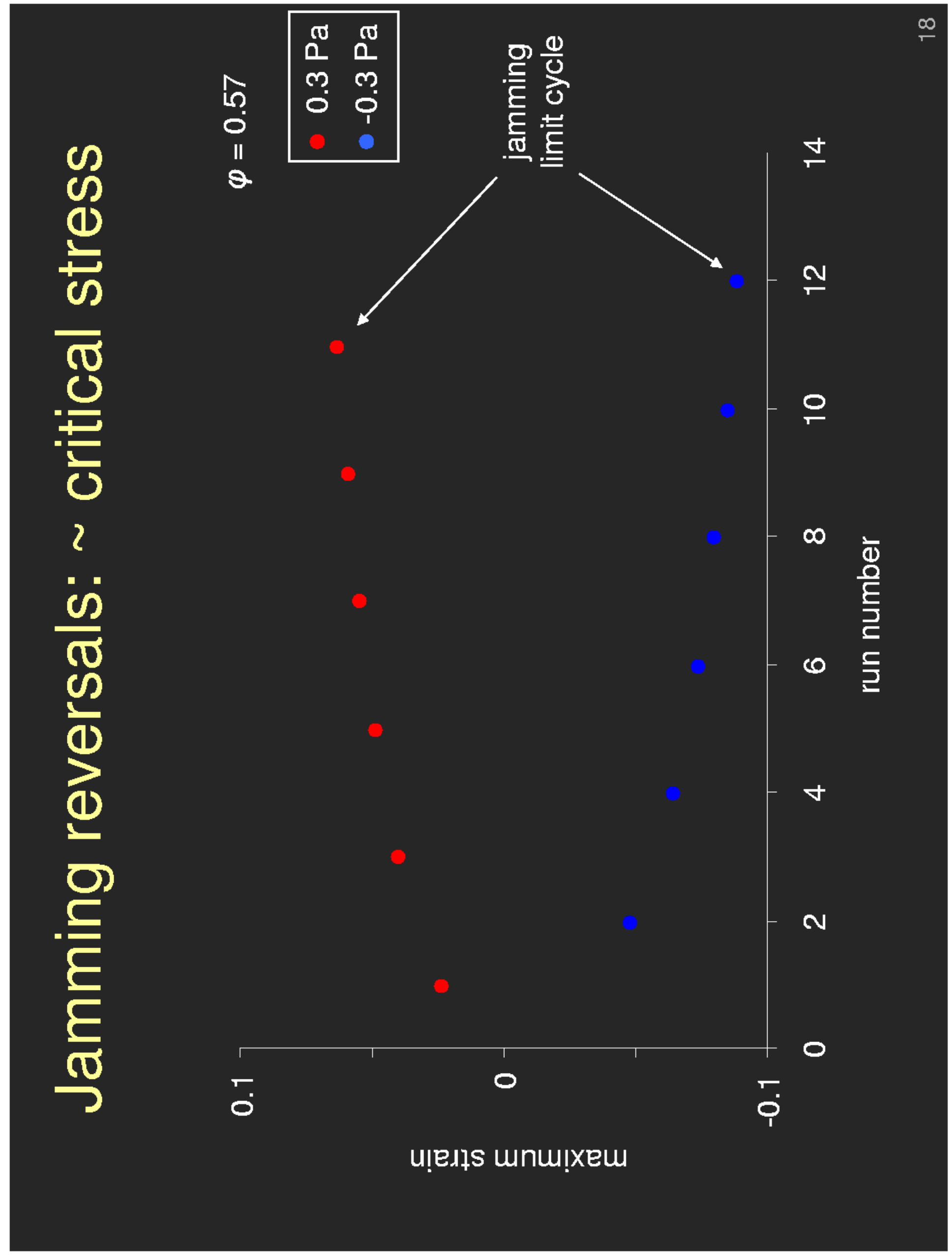

Critical volume fraction for jamming
Apply constant stress σ
Measure strain rate $\dot{\gamma}$
$\Rightarrow \eta = \sigma / \dot{\gamma}$
σ = 0.03 Pa
φ = 0.56
divergent viscosity
⇒ jamming
φ = 0.55
finite viscosity
⇒ flow
reduced viscosity (η/μ)
10^6
10^5
10^4
10^3
10^2
0.001
0.01
0.1
strain
13

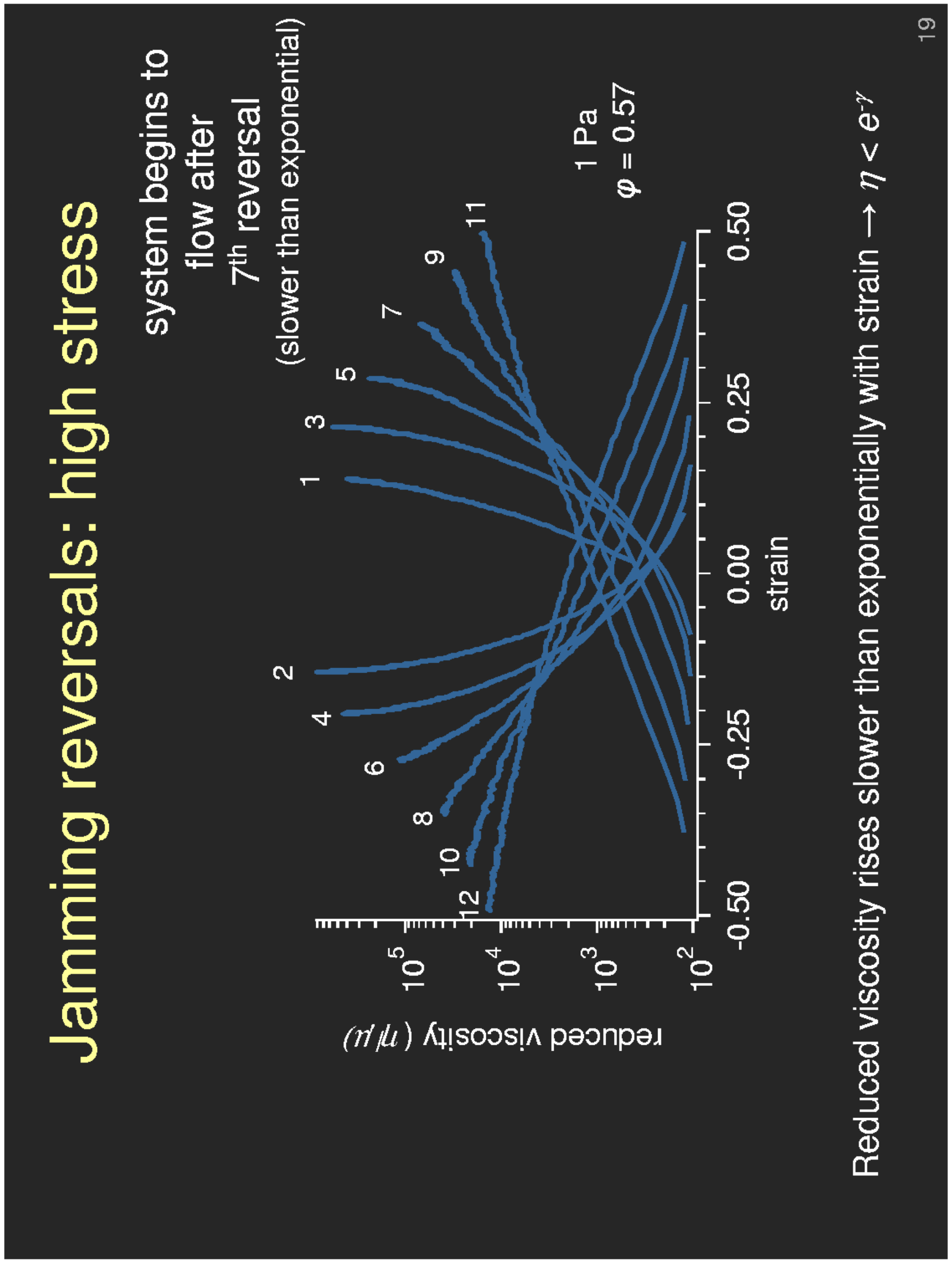

Jamming case, φ ≥ 0.56
σ = 0.03 Pa
φ = 0.56
divergent viscosity
⇒ jamming
φ = 0.55
finite viscosity
⇒ flow
reduced viscosity (η/μ)
10^6
10^5
10^4
10^3
10^2
0.001
0.01
0.1
strain
14

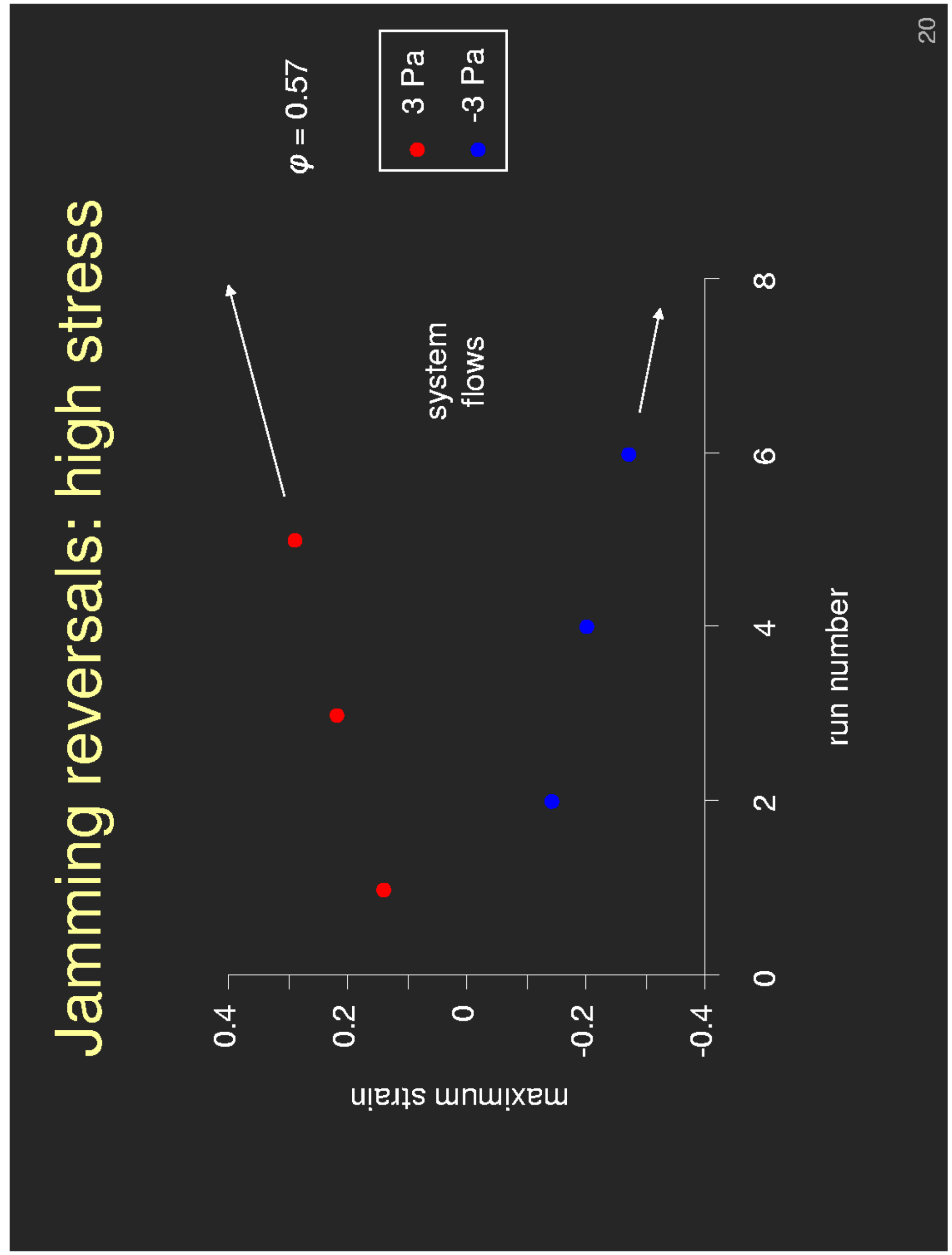

Jamming reversals: low stress
Numbers indicate
sequence of stressing
"fragile"
0.1 Pa
φ = 0.56
1
2
3
4
5
6
7
8
reduced viscosity (η/μ)
10^2
10^3
10^4
10^5
10^6
10^7
0.005
0.010
0.015
0.020
0.025
0.030
strain
15


Jamming reversals: low stress
φ = 0.56
limiting strain
jammed in both
directions
"work hardening"
0.1 Pa
- 0.1 Pa
maximum strain
0
0.01
0.02
0.03
run number
0
5
10
15
20
16

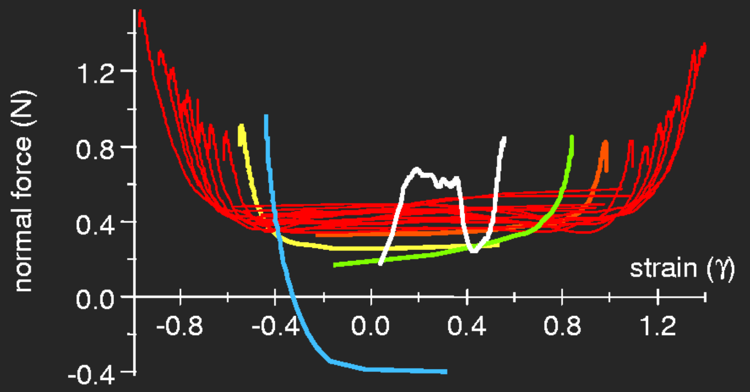

Jamming reversals: near critical stress
0.3 Pa
φ = 0.57
reduced viscosity (η/μ)
strain
Reduced viscosity rises nearly exponentially with strain → η ~ e^γ
17

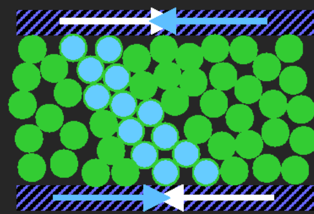

Jamming reversals: ~ critical stress
φ = 0.57
0.3 Pa
-0.3 Pa
jamming
limit cycle
maximum strain
run number
18

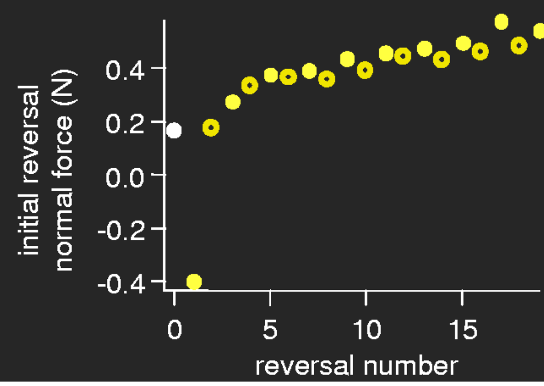

Jamming reversals: high stress
system begins to flow after 7th reversal (slower than exponential)
1 Pa
φ = 0.57
reduced viscosity (η/μ)
10^5
10^4
10^3
10^2
strain
-0.50
-0.25
0.00
0.25
0.50
1
2
3
4
5
6
7
8
9
10
11
12
Reduced viscosity rises slower than exponentially with strain → η < e^γ
19


Jamming reversals: high stress
φ = 0.57
3 Pa
-3 Pa
system flows
maximum strain
0.4
0.2
0
-0.2
-0.4
run number
0
2
4
6
8
20

Jamming normal stress







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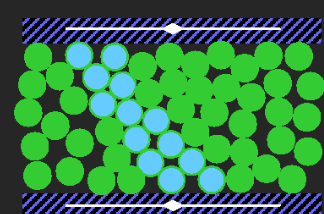
Jamming both ways

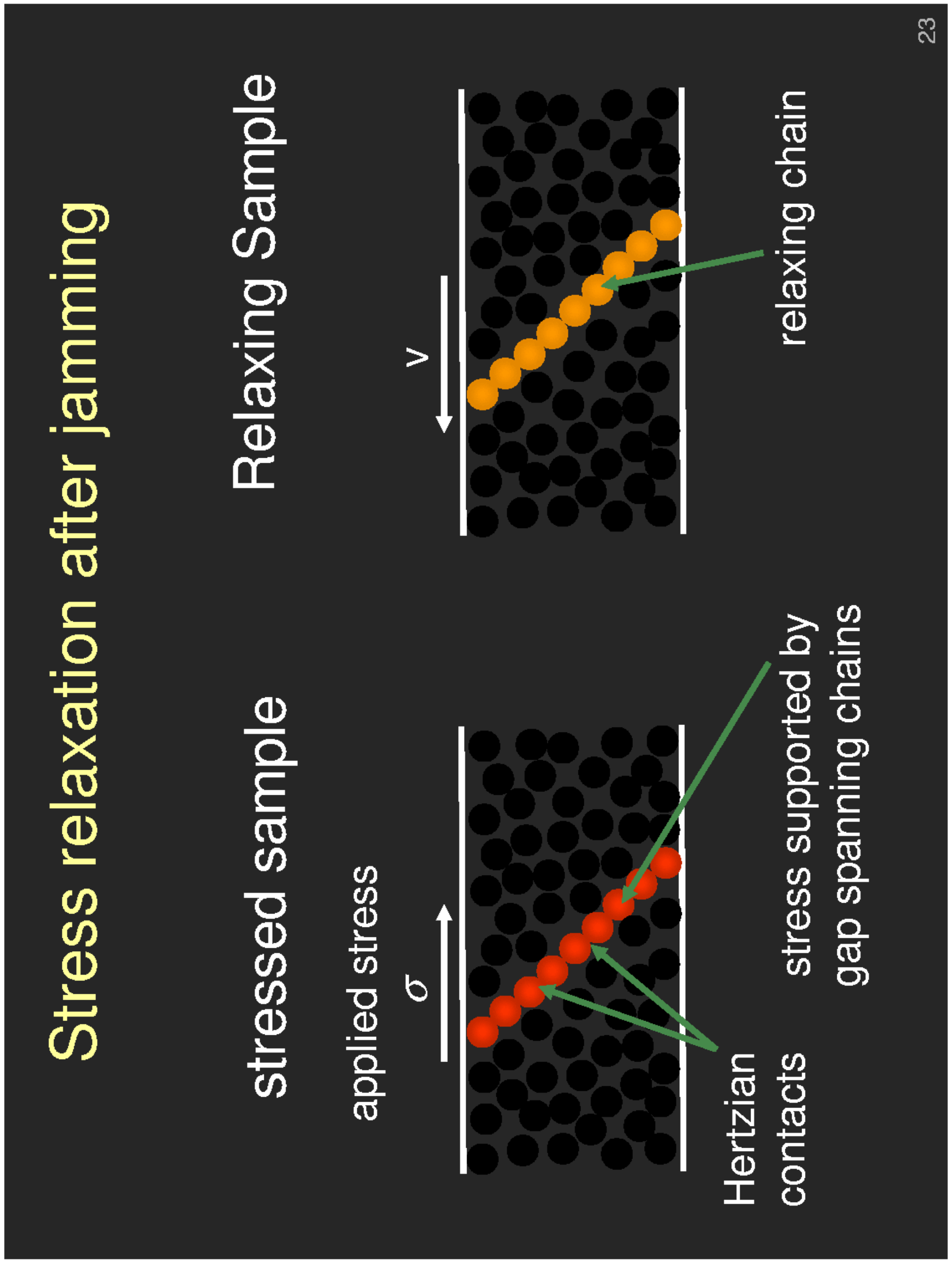


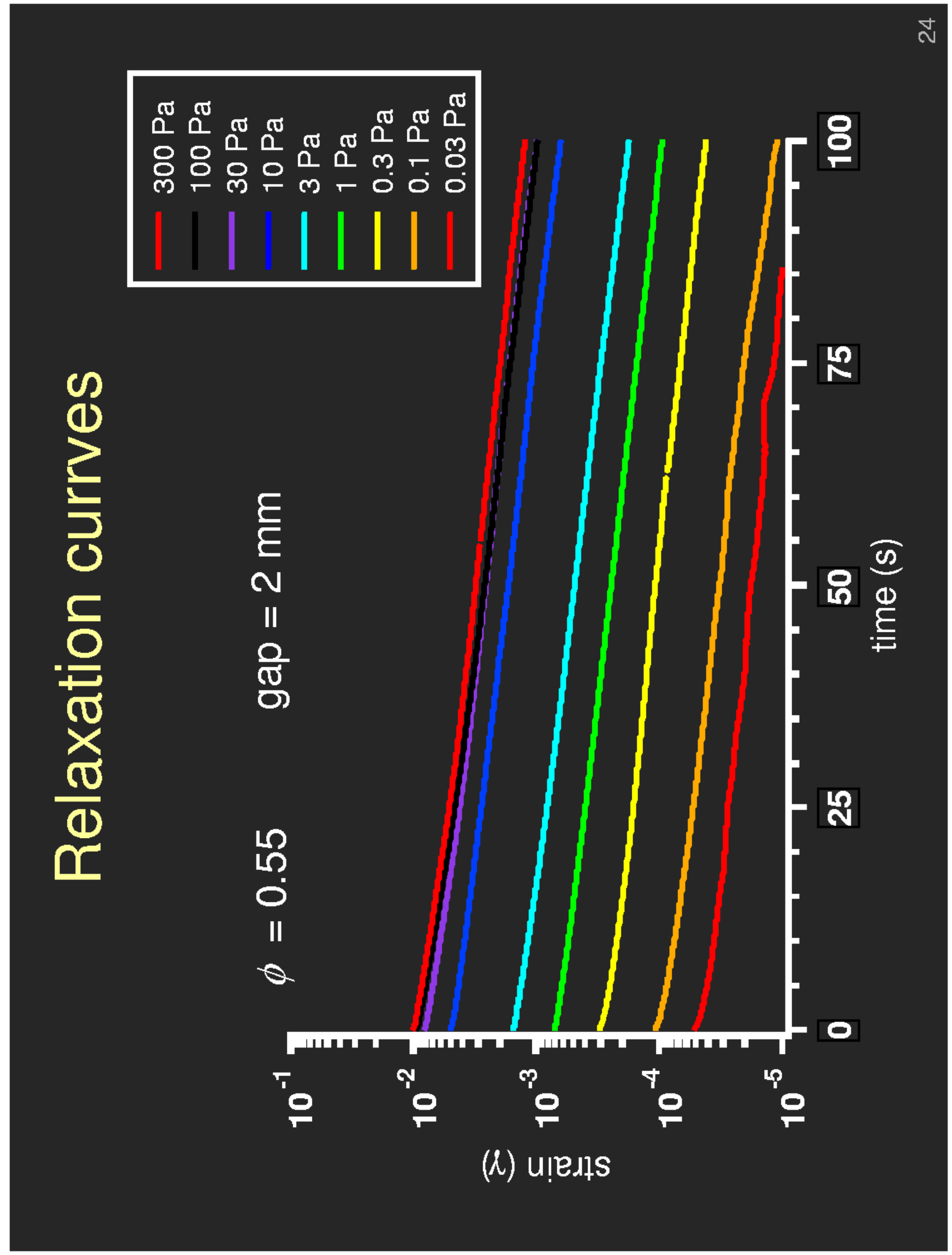
particles touch with
Hertzian contacts

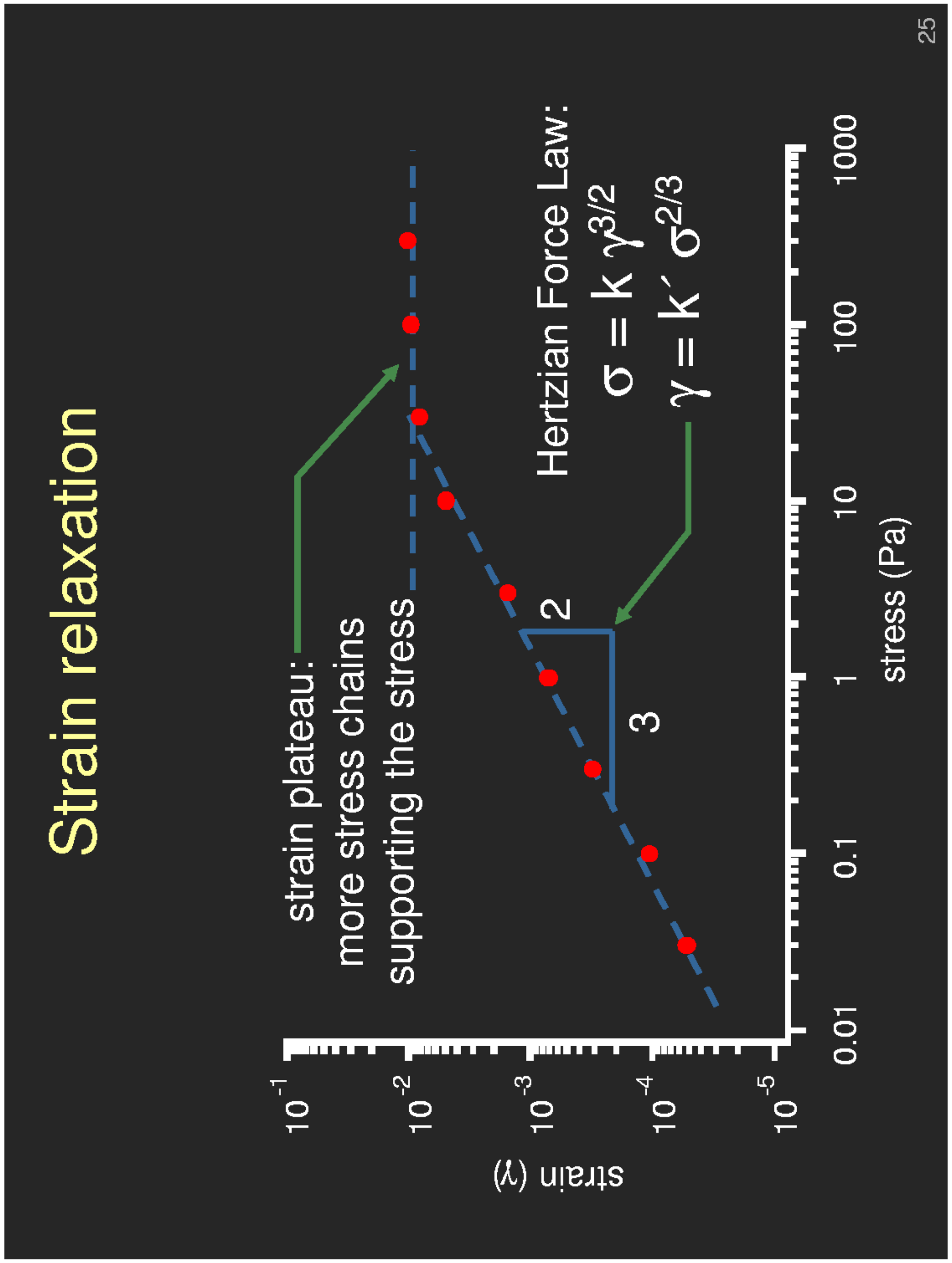
Jamming produced by *flow* reversal,
not by quenching from high *T*

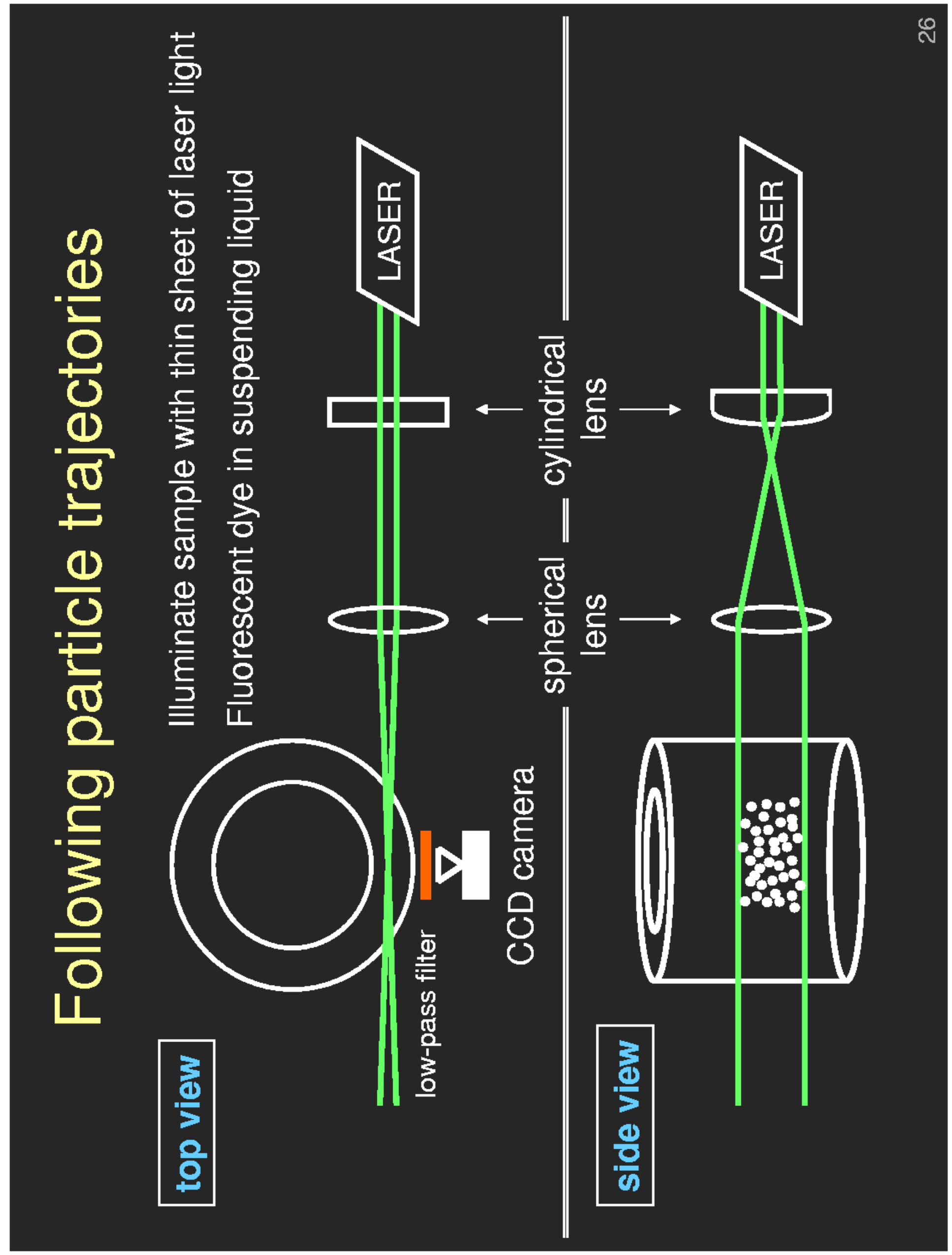
The system jams in both directions

22

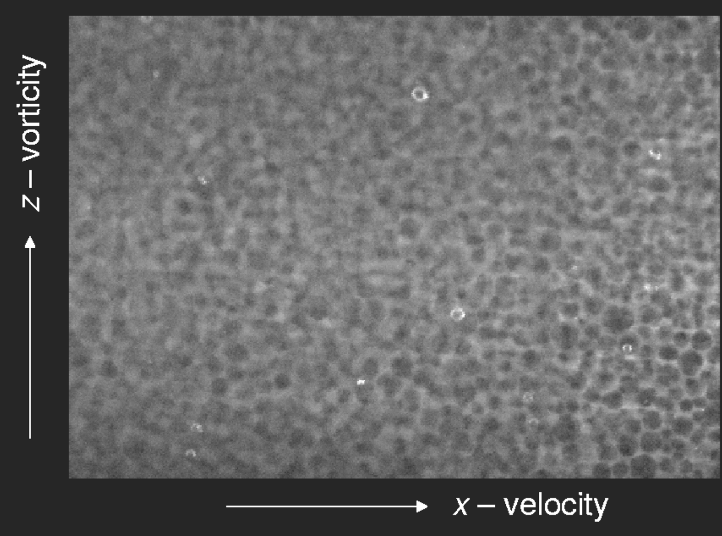

Stress relaxation after jamming
stressed sample
Relaxing Sample
applied stress
σ
v
Hertzian contacts
stress supported by gap spanning chains
relaxing chain
23

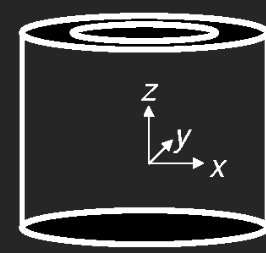

Relaxation curves
ϕ = 0.55
gap = 2 mm
300 Pa
100 Pa
30 Pa
10 Pa
3 Pa
1 Pa
0.3 Pa
0.1 Pa
0.03 Pa
strain (γ)
10^-1
10^-2
10^-3
10^-4
10^-5
time (s)
0
25
50
75
100
24


Strain relaxation
strain plateau:
more stress chains
supporting the stress
Hertzian Force Law:
σ = k γ^3/2
γ = k´ σ^2/3
2
3
strain (γ)
10^-1
10^-2
10^-3
10^-4
10^-5
0.01
0.1
1
10
100
1000
stress (Pa)
25


Following particle trajectories
Illuminate sample with thin sheet of laser light
Fluorescent dye in suspending liquid
top view
LASER
low-pass filter
CCD camera
spherical
lens
cylindrical
lens
side view
LASER
26

2-d image in velocity-vorticity plane

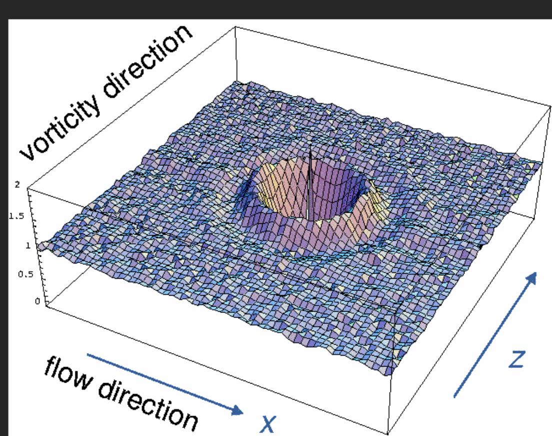


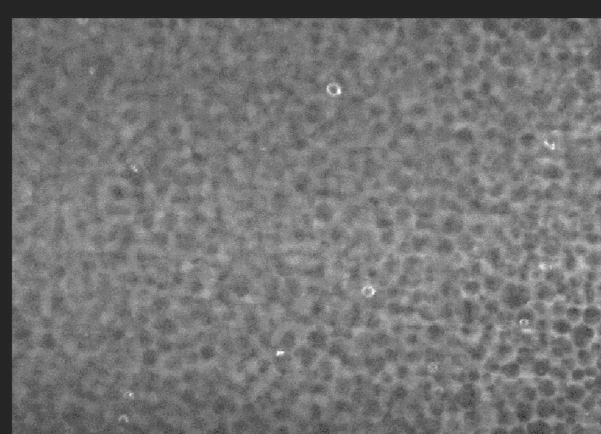


x – velocity direction
y – gradient direction
z – vorticity direction

Pair correlation functions g(r)

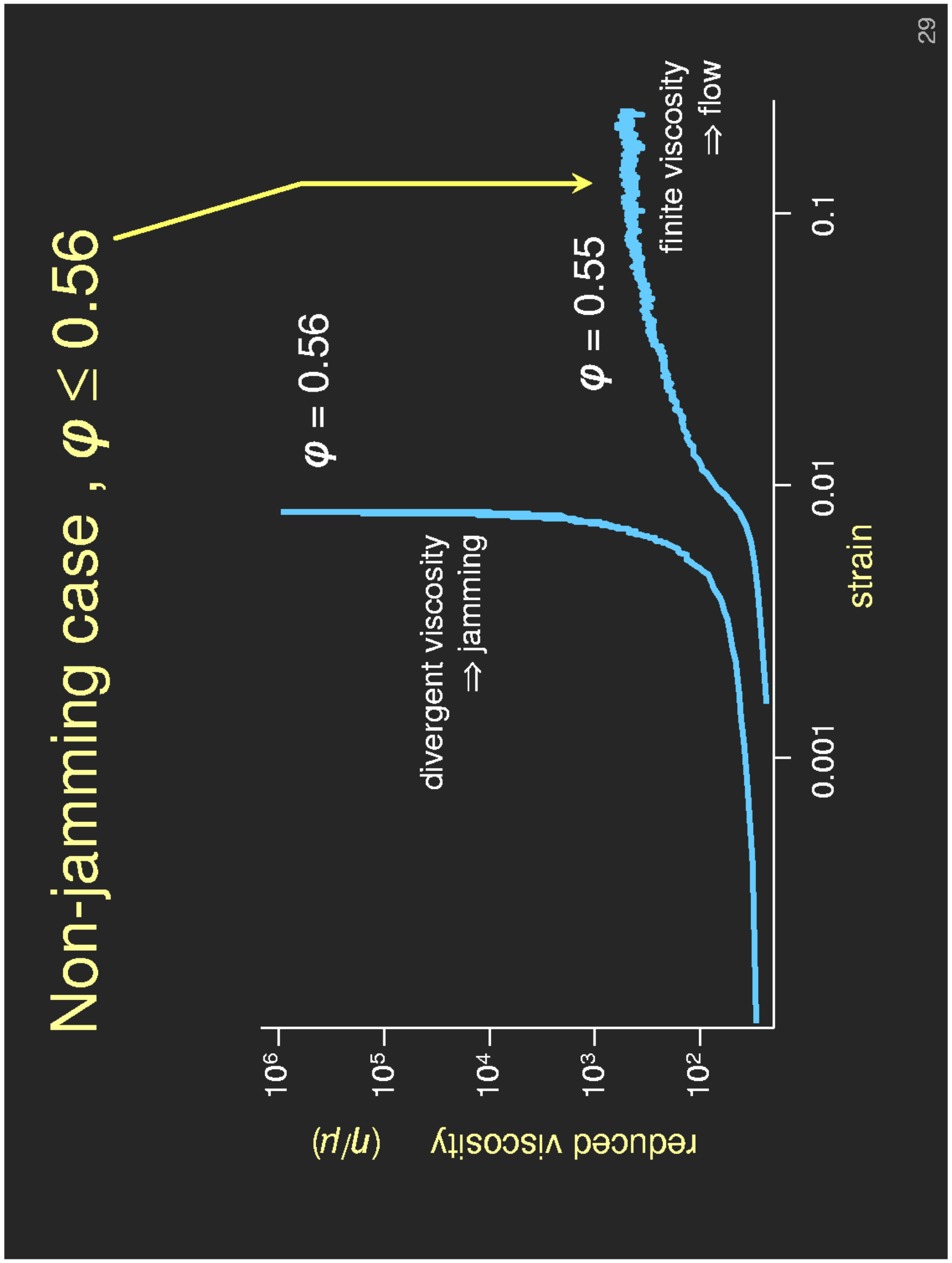
jammed state - no flow

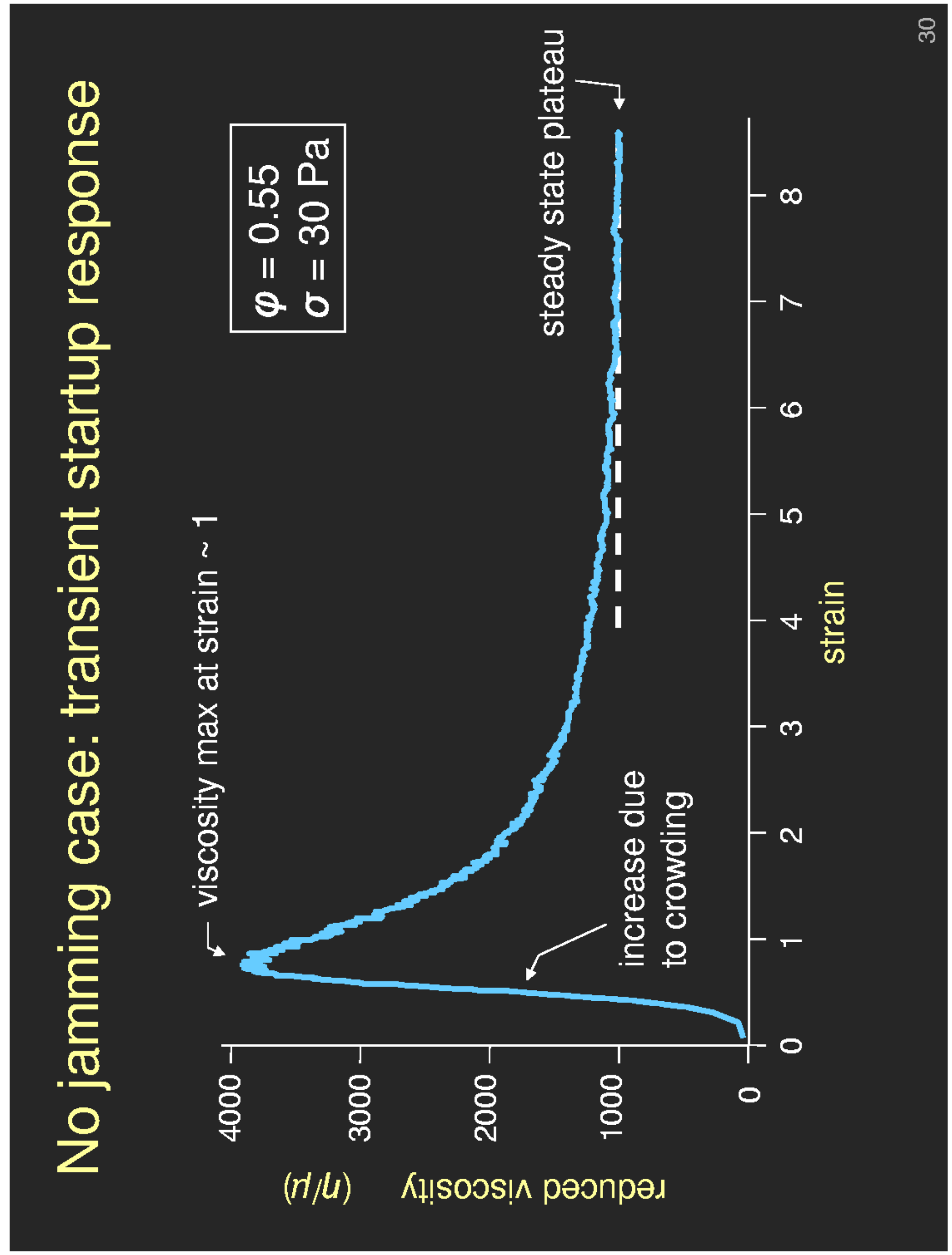




$$g(\vec{r}) = \frac{\langle n(\vec{r}_0)n(\vec{r}_0+\vec{r})\rangle}{\langle n(\vec{r}_0)\rangle^2}$$

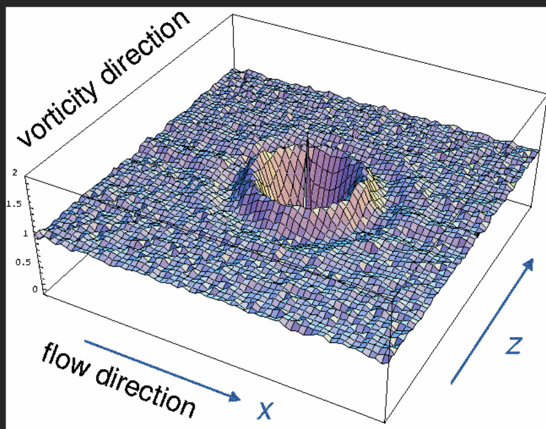
random particle positions


Non-jamming case , φ ≤ 0.56
φ = 0.56
φ = 0.55
divergent viscosity
⇒ jamming
finite viscosity
⇒ flow
reduced viscosity (η/μ)
10^6
10^5
10^4
10^3
10^2
0.001
0.01
0.1
strain
29

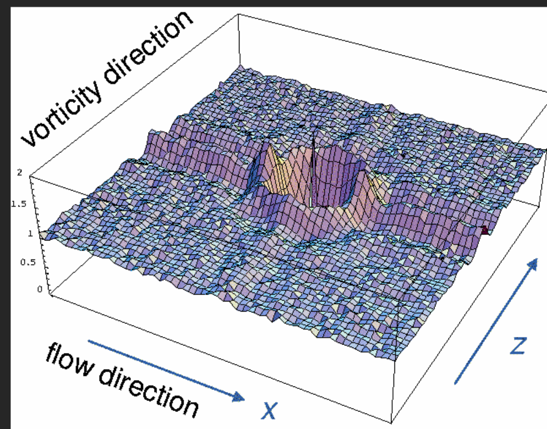

No jamming case: transient startup response
φ = 0.55
σ = 30 Pa
viscosity max at strain ~ 1
increase due
to crowding
steady state plateau
reduced viscosity (η/μ)
4000
3000
2000
1000
0
0
1
2
3
4
5
6
7
8
strain
30

Pair correlation functions *g*(*r*)

no flow



flow – steady state



Note structure in flow direction:
strings or layers?

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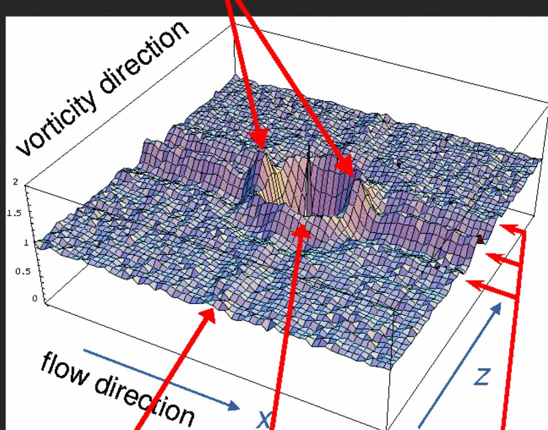
Pair correlation functions *g*(*r*)

peaks in flow direction

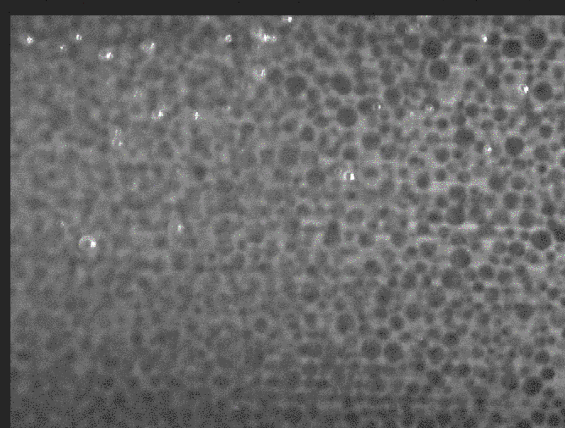


weak registry in vorticity plane

nearest neighbor ring

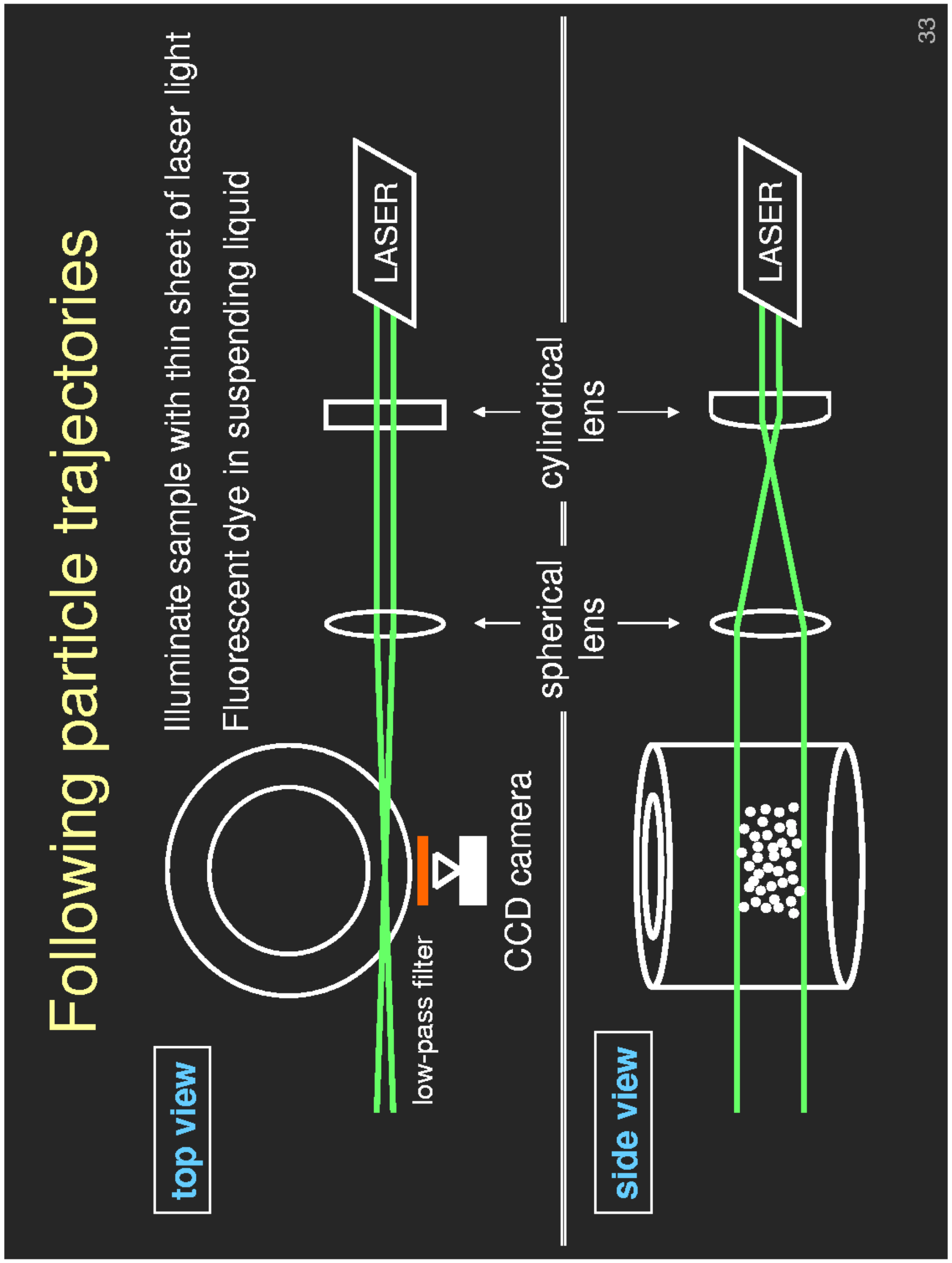
maxima along flow direction indicates particles line up in the flow direction

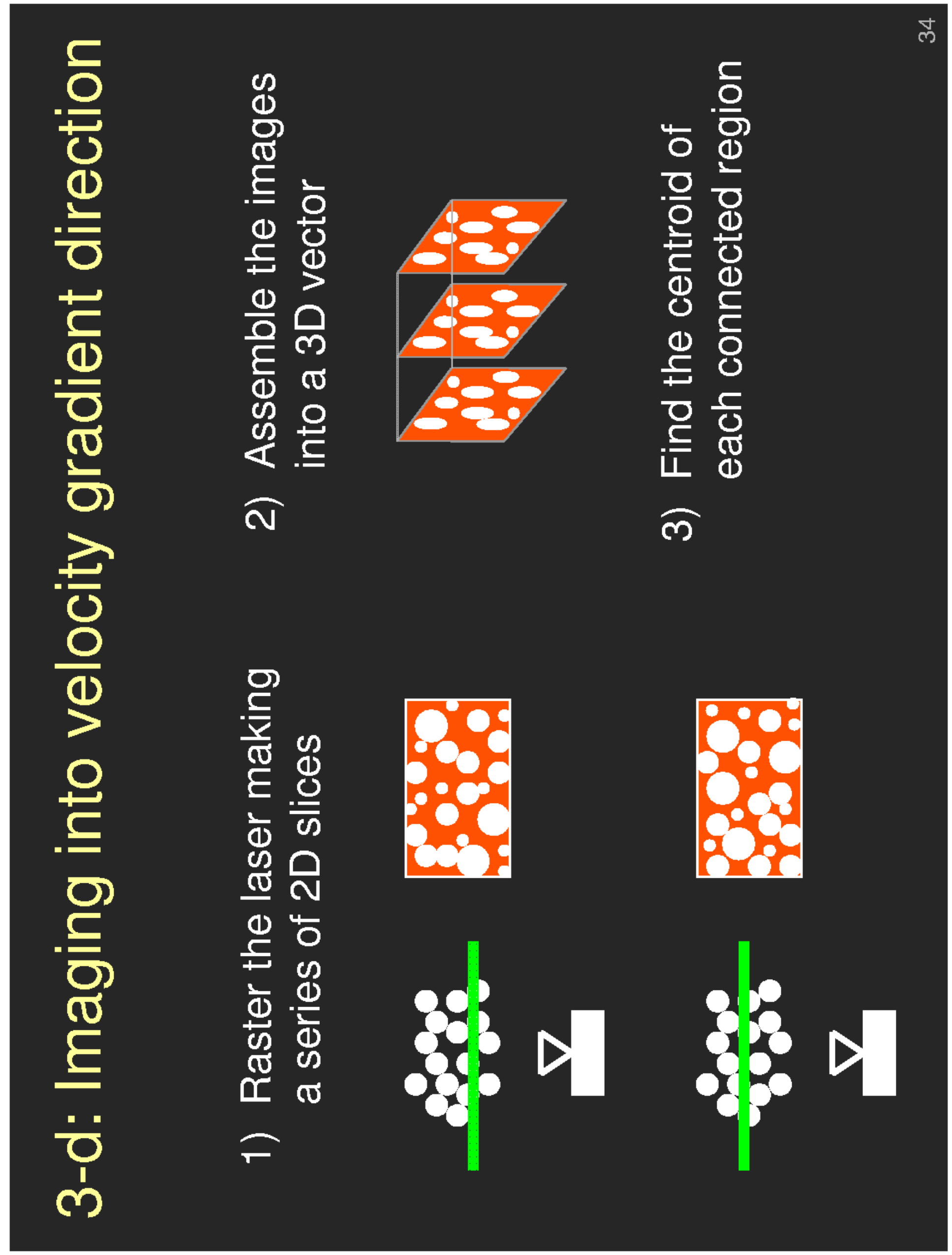
flow – steady state

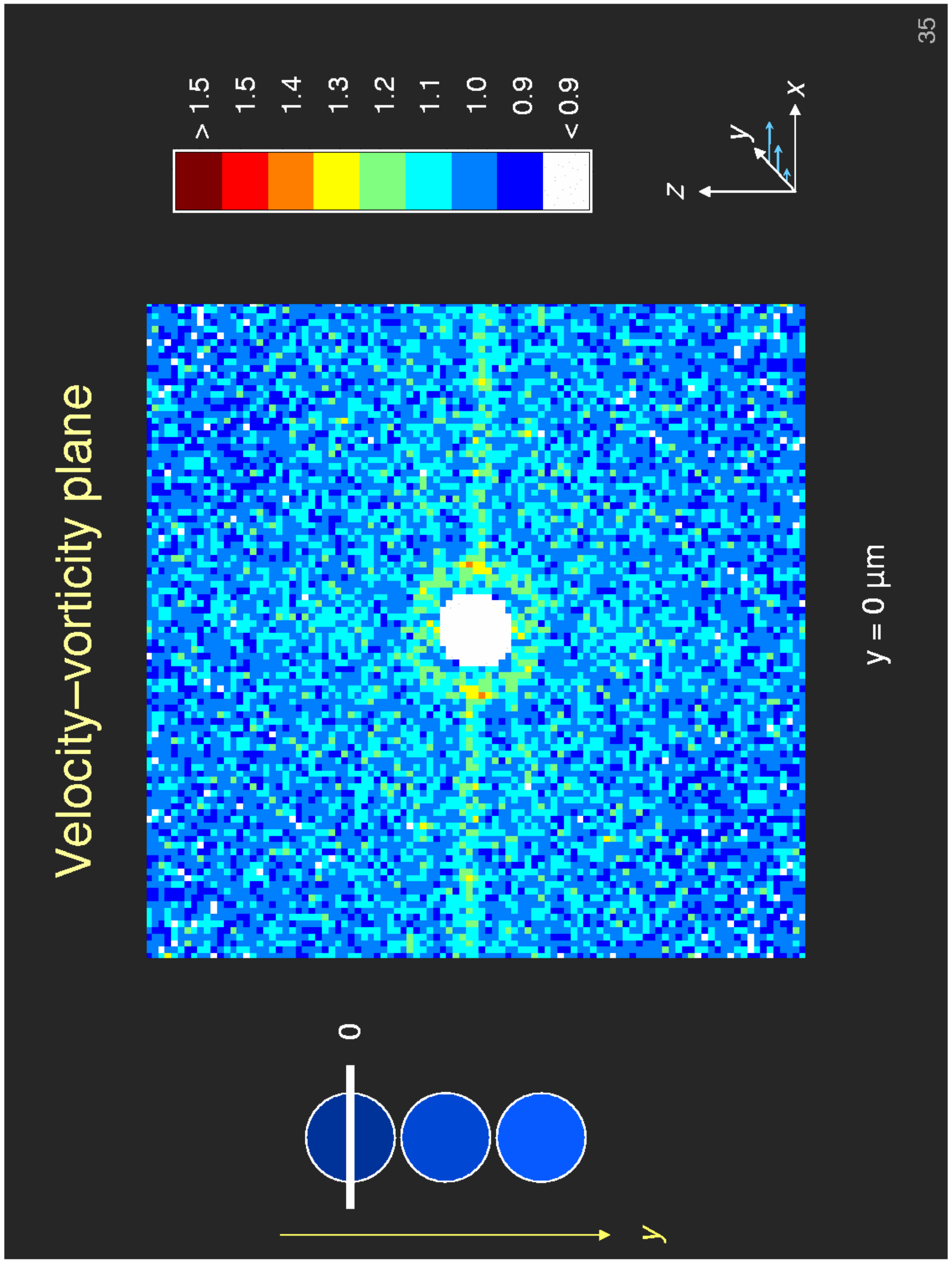


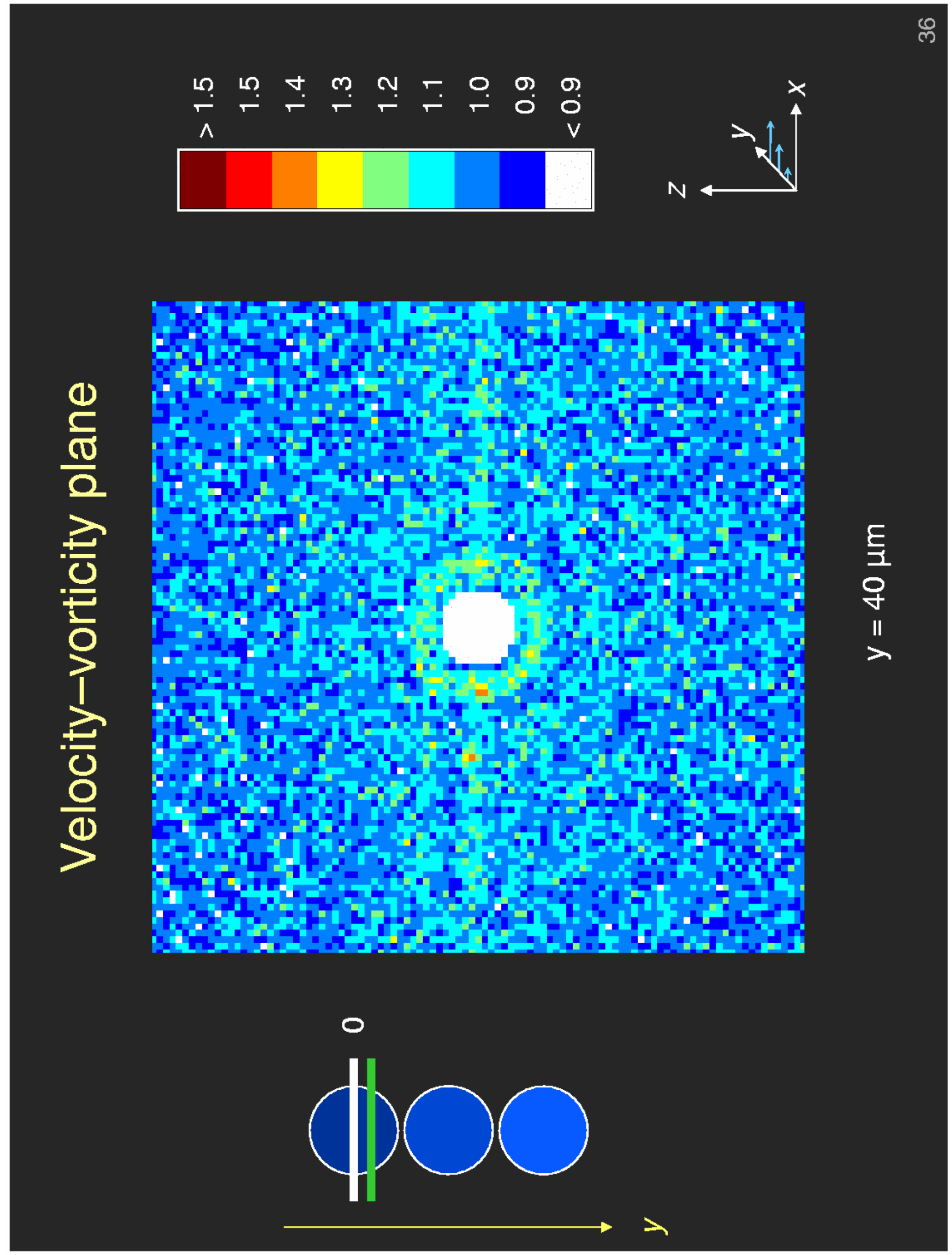
Structure in flow direction:
- strings
- layers?

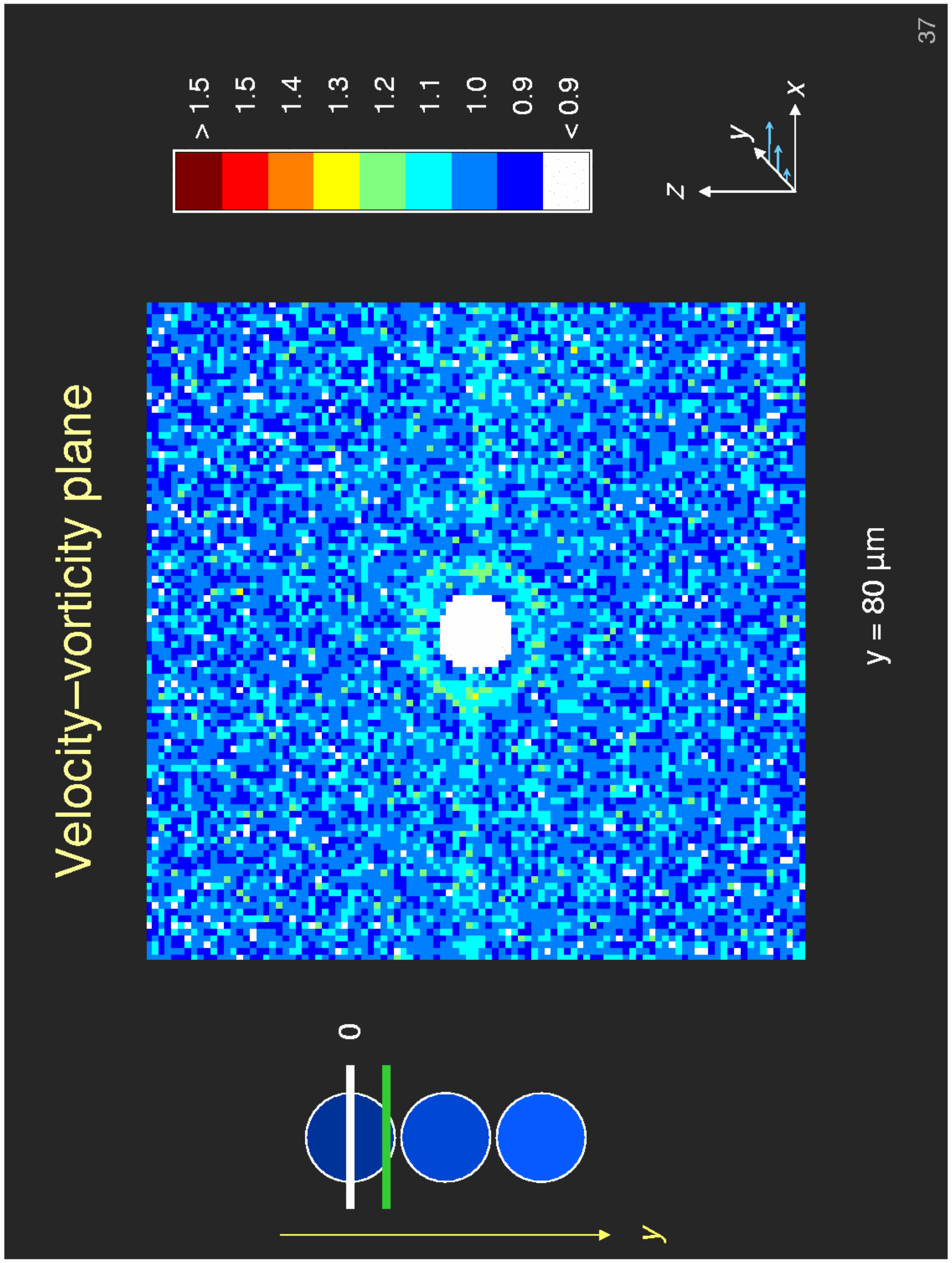
32

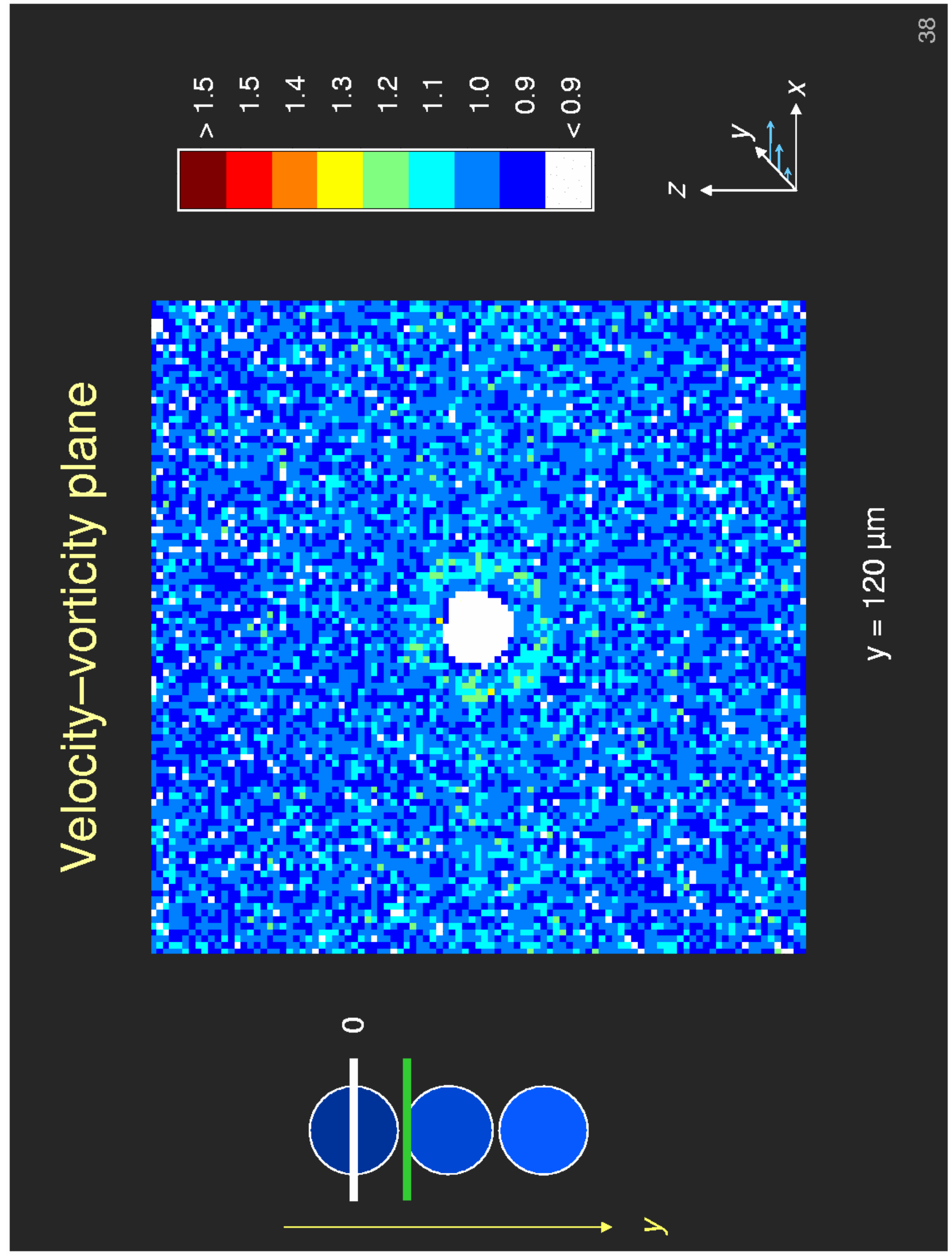

Following particle trajectories
top view
Illuminate sample with thin sheet of laser light
Fluorescent dye in suspending liquid
LASER
low-pass filter
CCD camera
spherical lens
cylindrical lens
side view
LASER
33

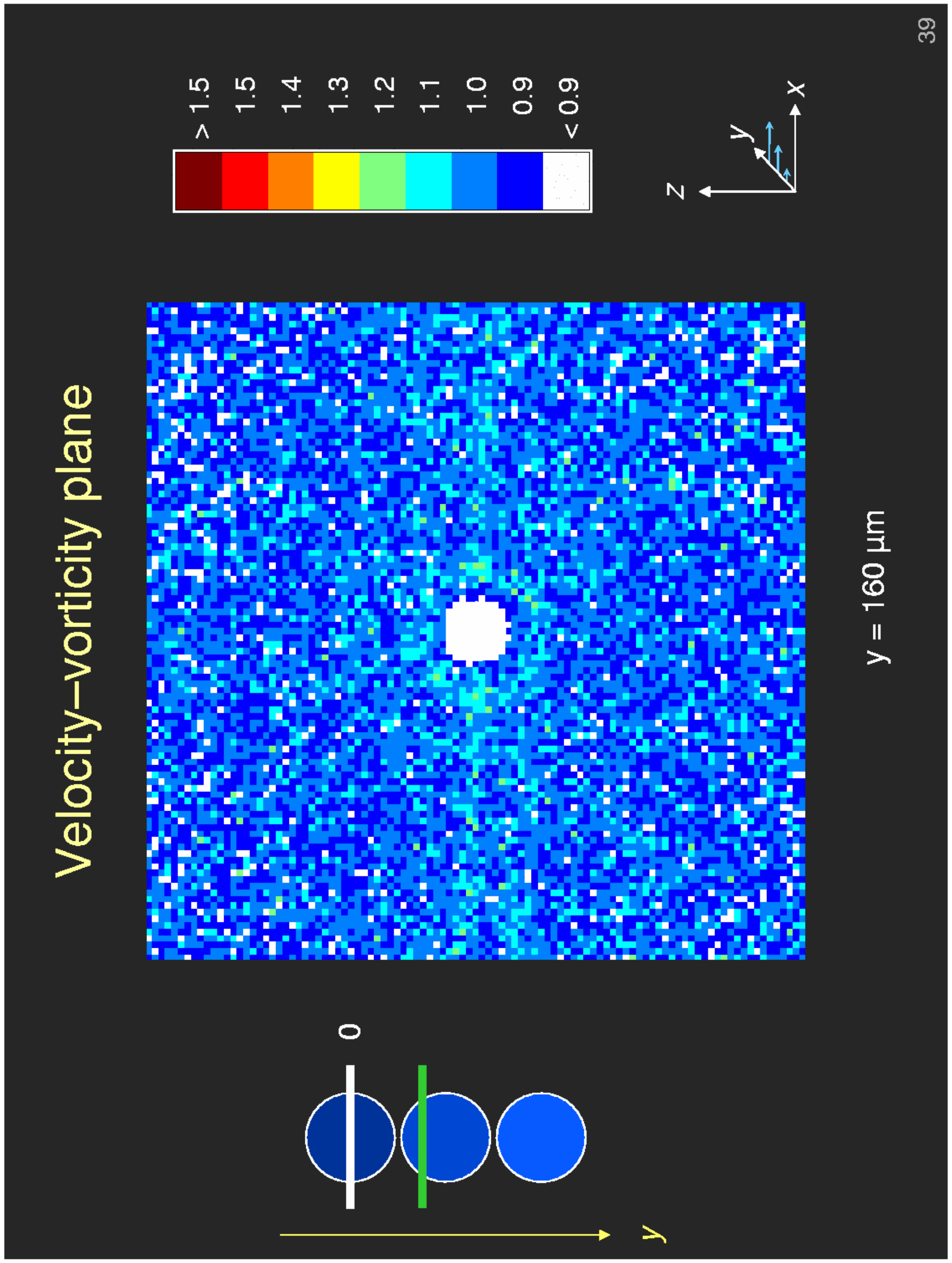

3-d: Imaging into velocity gradient direction
1) Raster the laser making a series of 2D slices
2) Assemble the images into a 3D vector
3) Find the centroid of each connected region
34

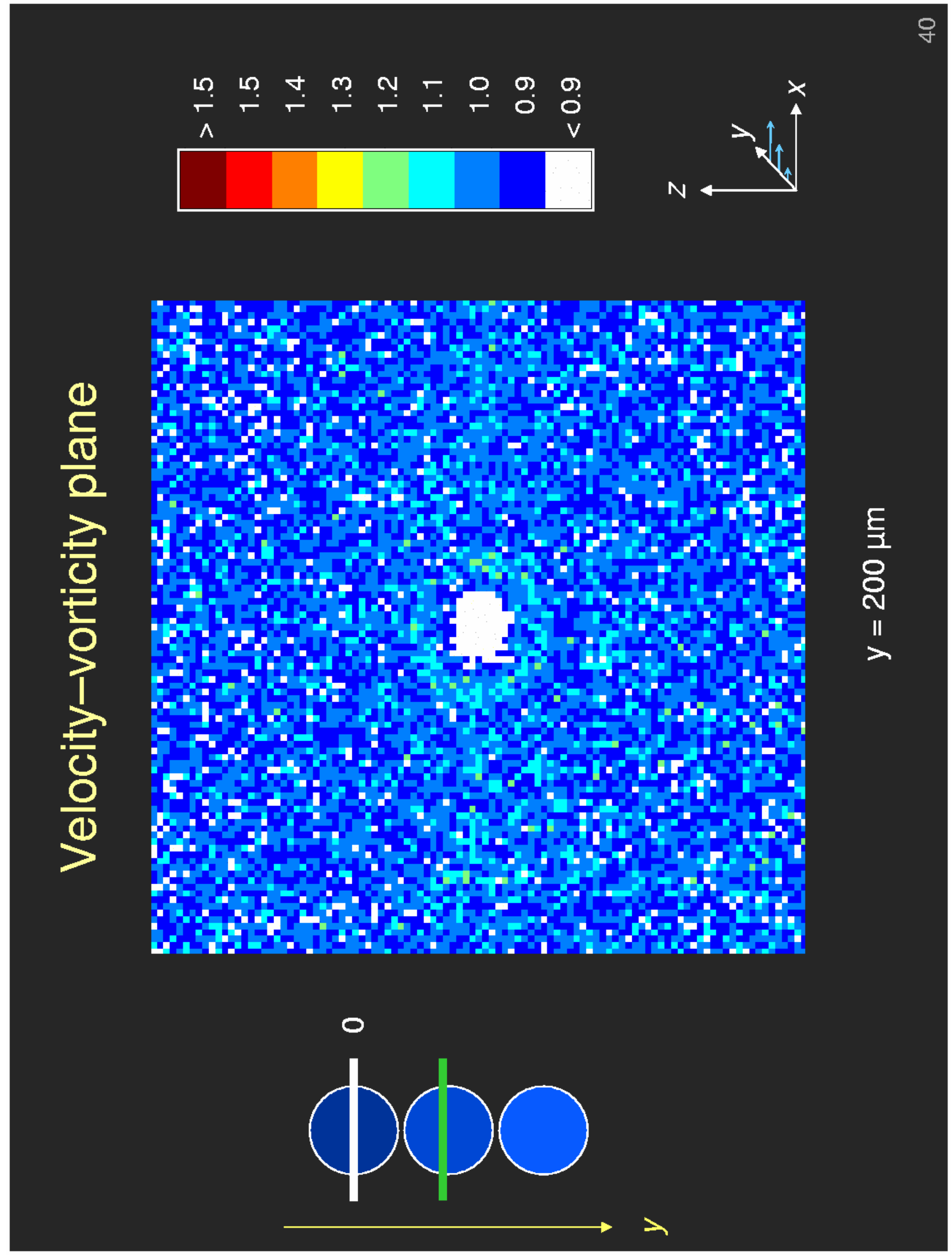

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 0 μm
0
y
35

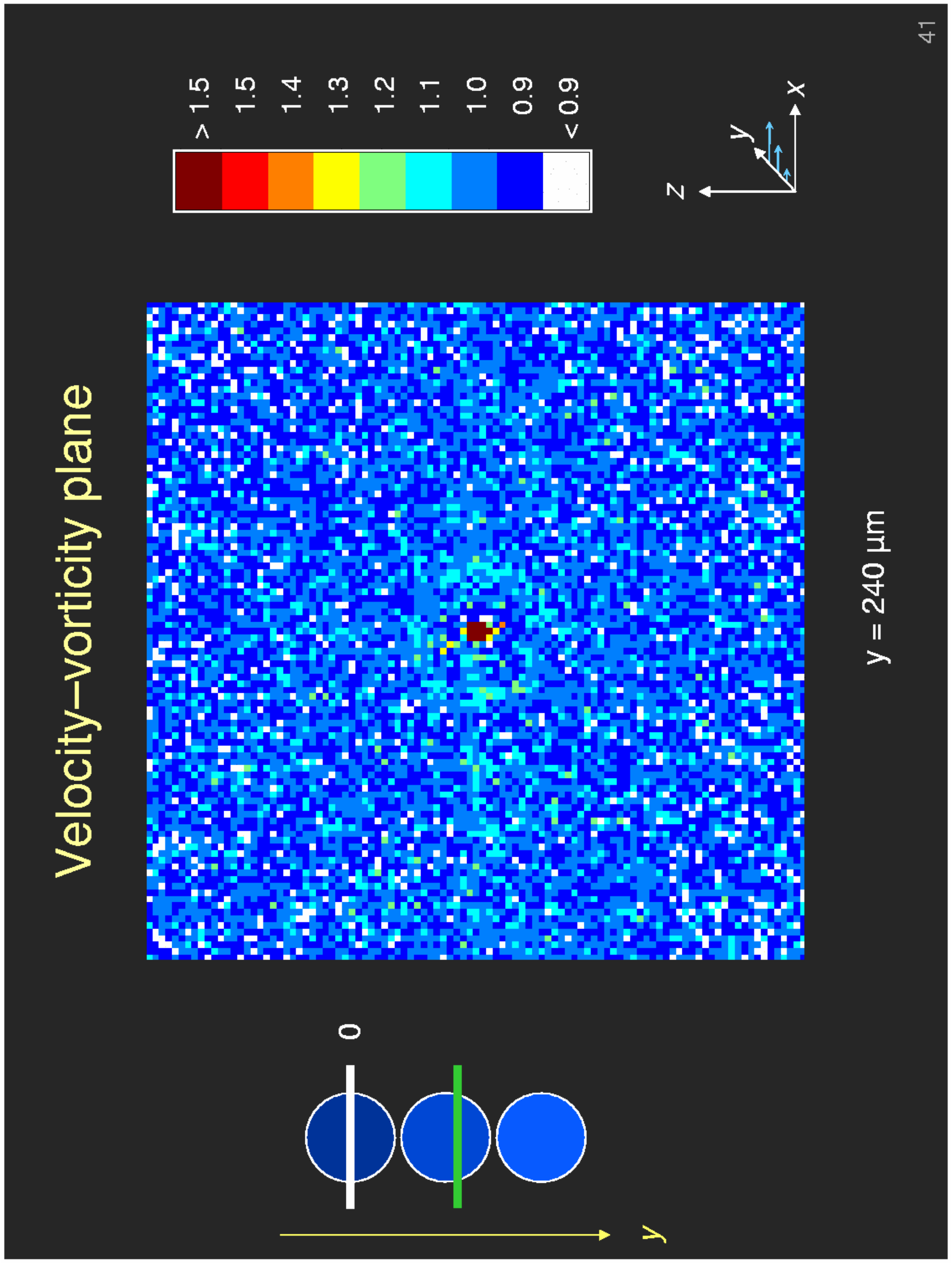

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 40 μm
0
y
36

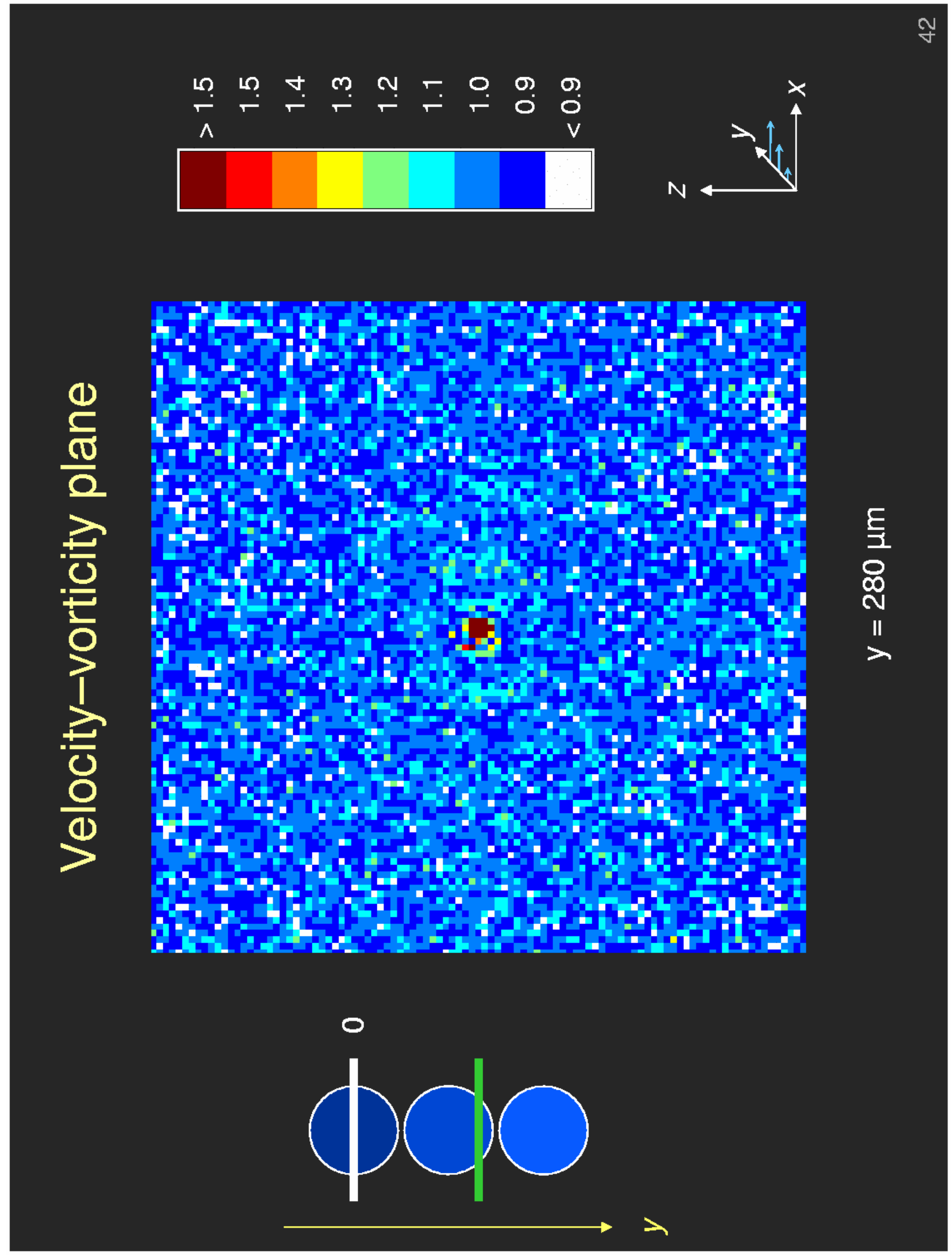

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 80 µm
0
y
37

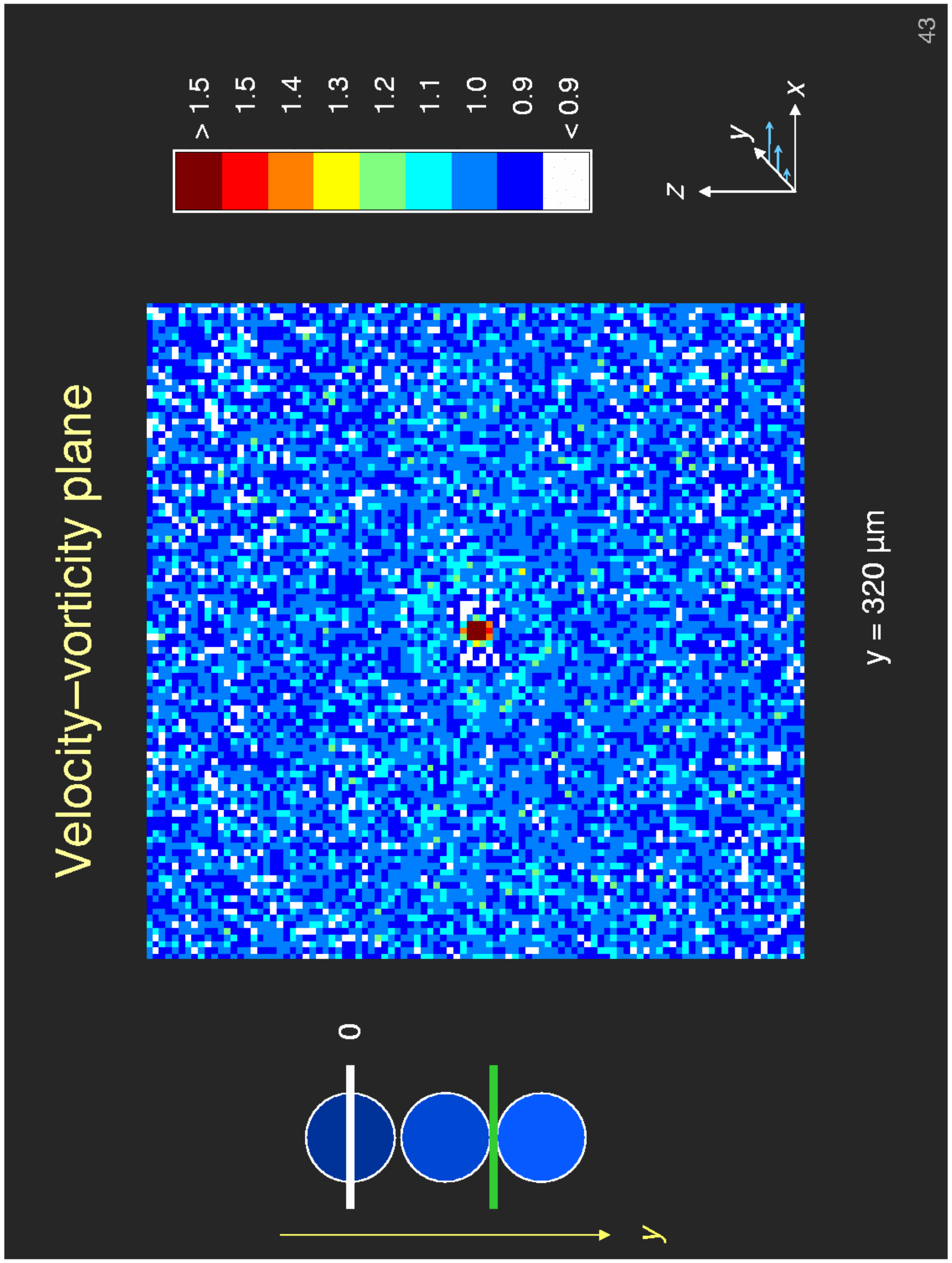

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 120 µm
0
y
38

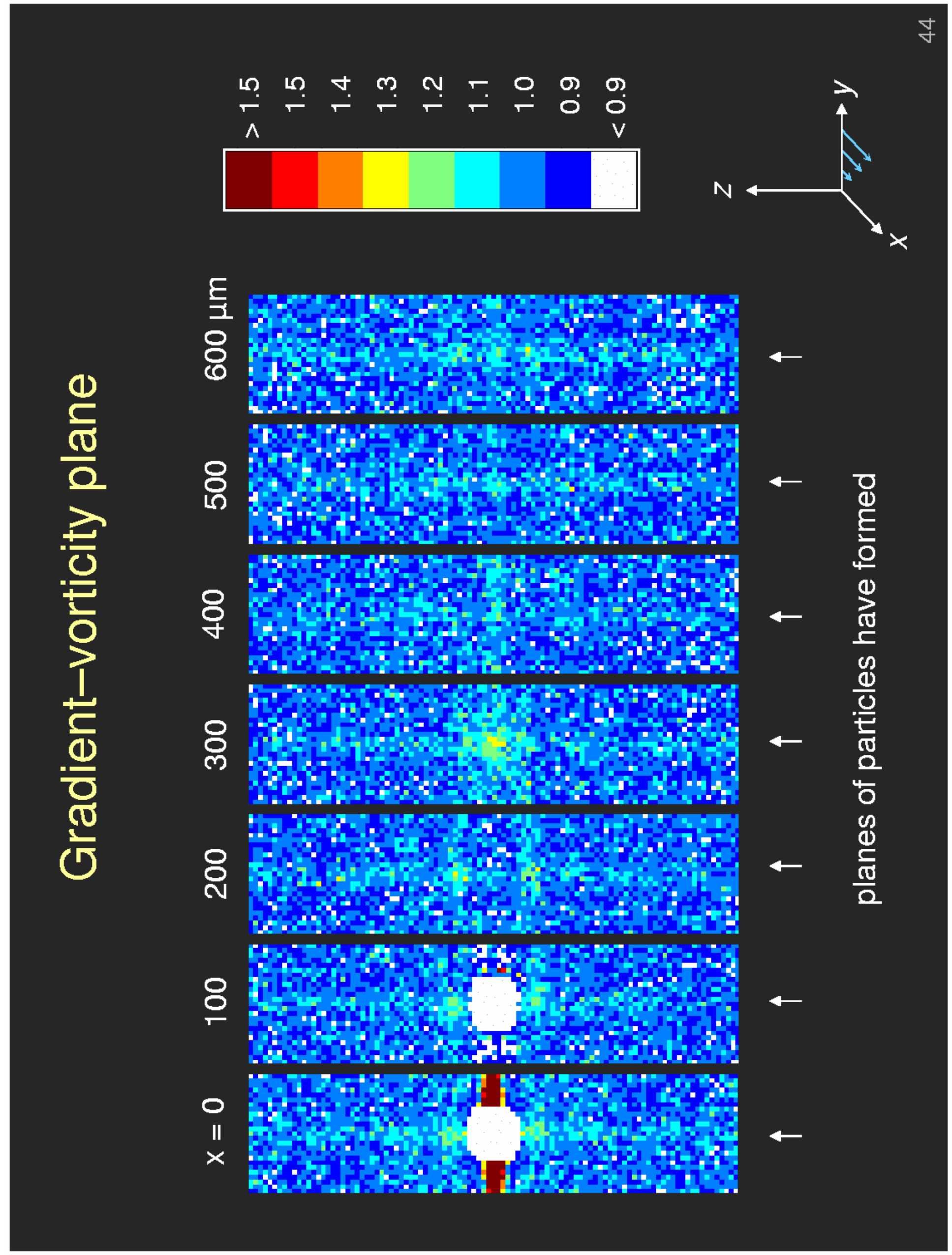

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 160 µm
0
y
39

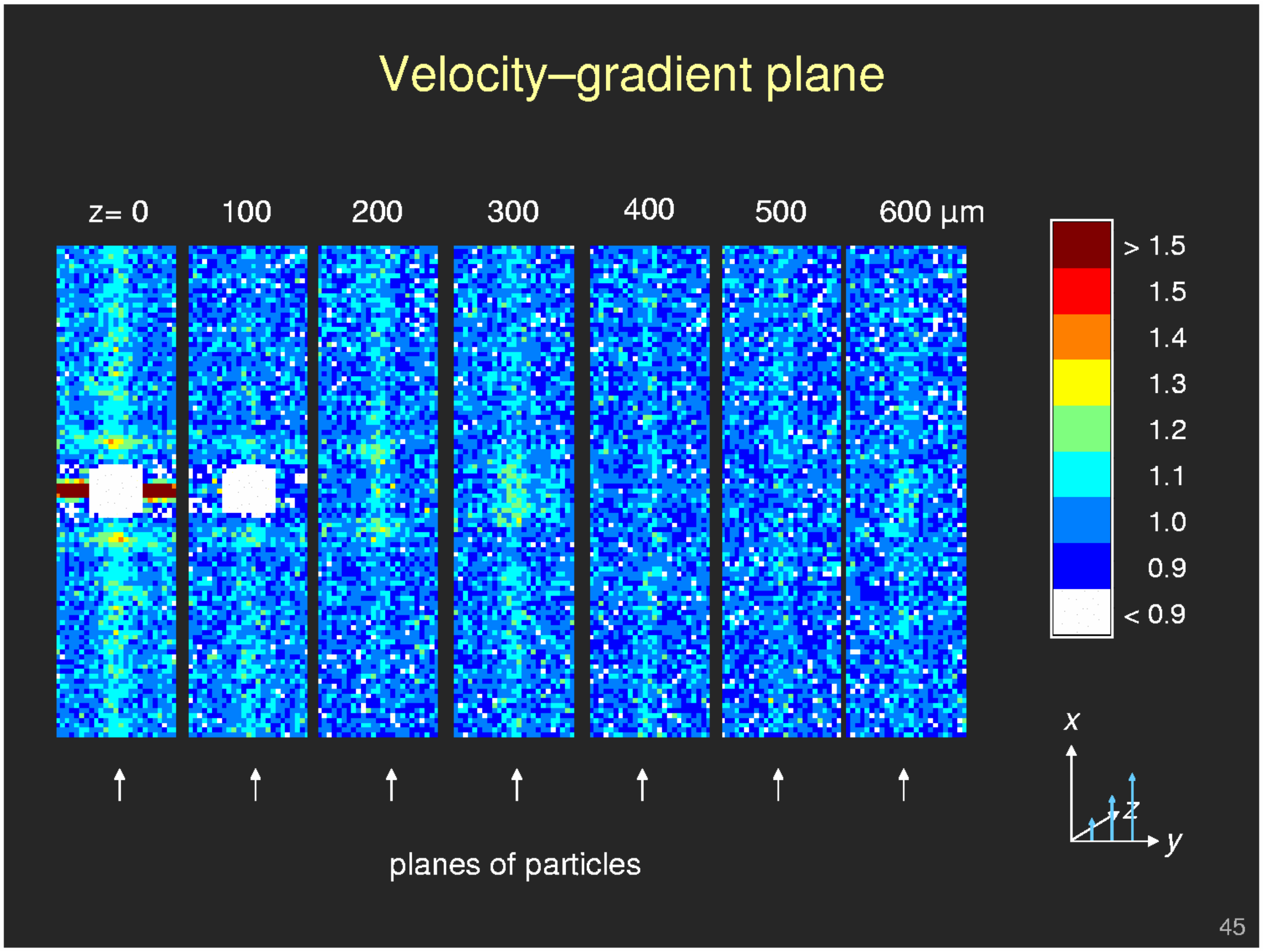

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 200 µm
0
y
40

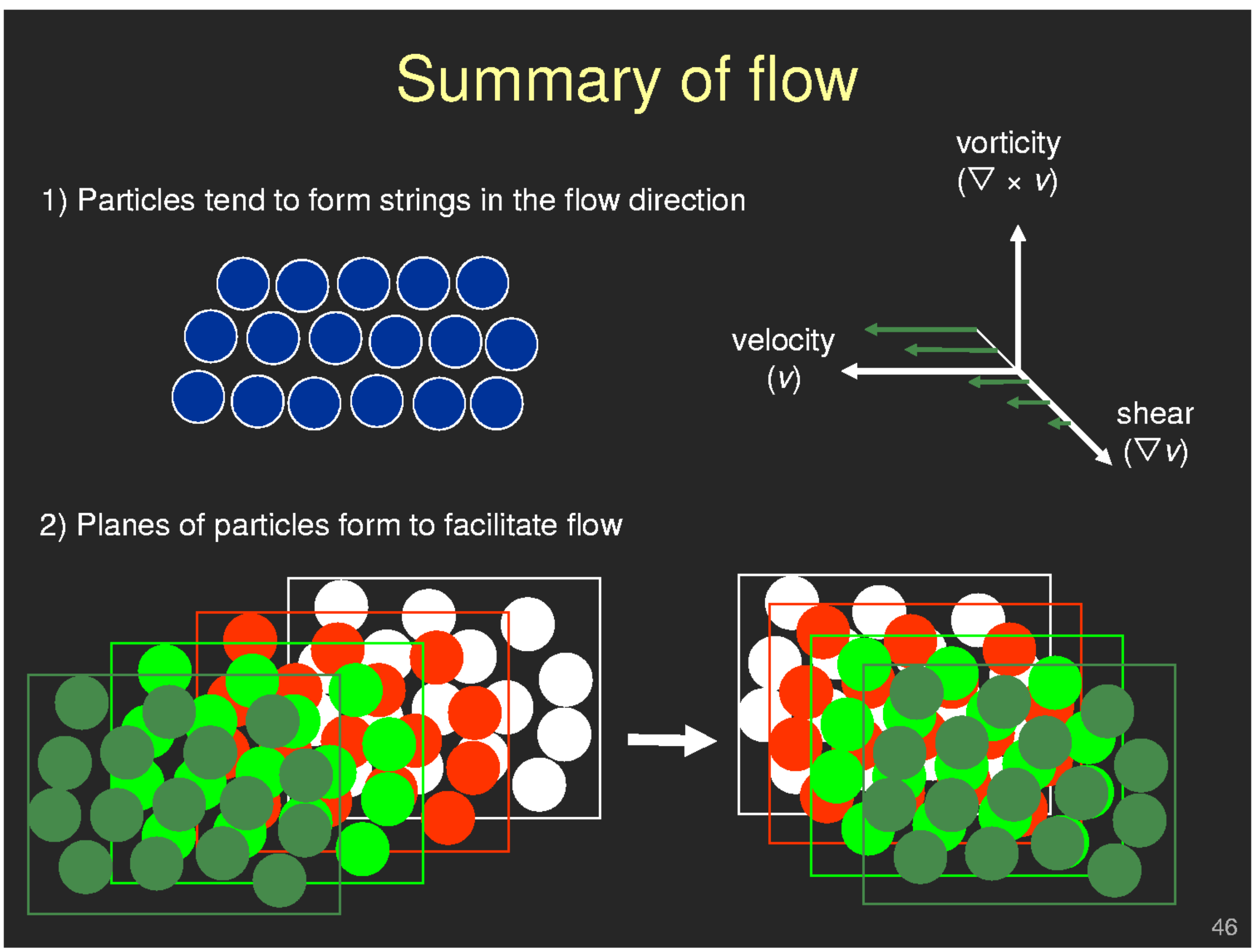

Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
z
y
x
y = 240 µm
0
y
41


Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
z
y
x
y = 280 µm
0
y
42


Velocity–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
y = 320 µm
0
y
43


Gradient–vorticity plane
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
y
z
x
x = 0
100
200
300
400
500
600 µm
planes of particles have formed
44


Velocity–gradient plane
z= 0
100
200
300
400
500
600 µm
> 1.5
1.5
1.4
1.3
1.2
1.1
1.0
0.9
< 0.9
x
y
z
planes of particles
45


Summary of flow
1) Particles tend to form strings in the flow direction
vorticity (∇ × v)
velocity (v)
shear (∇v)
2) Planes of particles form to facilitate flow
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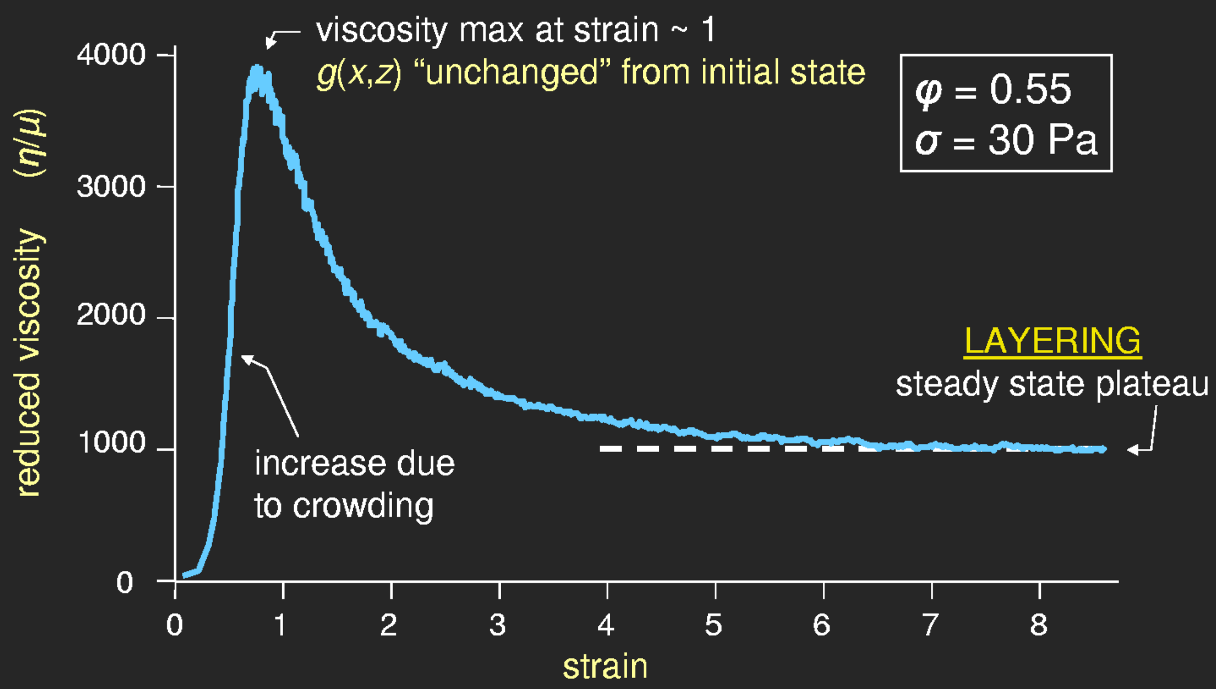
Layering → steady state viscosity



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Summary

- Non-colloidal suspensions jam above a (gap-dependent) volume fraction
- Suspensions jam both ways ("work hardening")
- Flowing dense suspensions organize into "strings" in the flow direction
- These "strings" are organized into sheets in the velocity-vorticity plane
- σ_c: Jamming occurs only below a critical stress

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Acknowledgements

- Kurt Knipmeyer
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- NSF MRSEC Program at UCSB

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