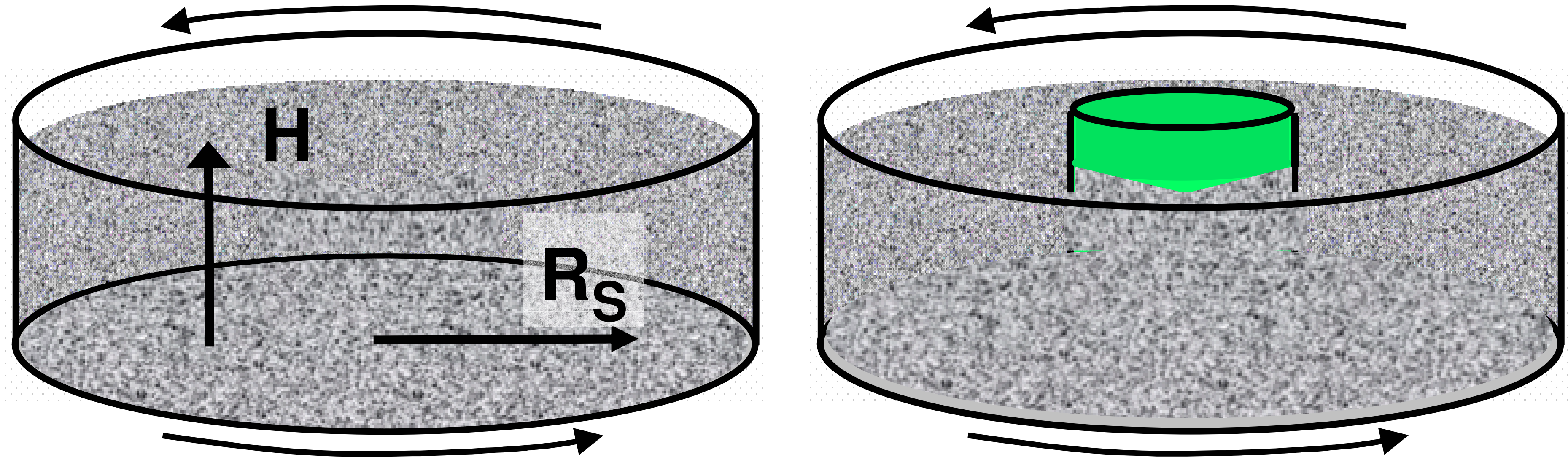
Robust Slow Granular Flows

Denis Fenistein, Rene Mikkelsen, Jan-Willem van de Meent, Paul Umbanhowar & MvH

Split-bottomed geometry



Shallow layers: Universality

Deep layers: Precession / Localization

3D: Velocity and Density

Model(s)

Wide & Universal Shear Zones

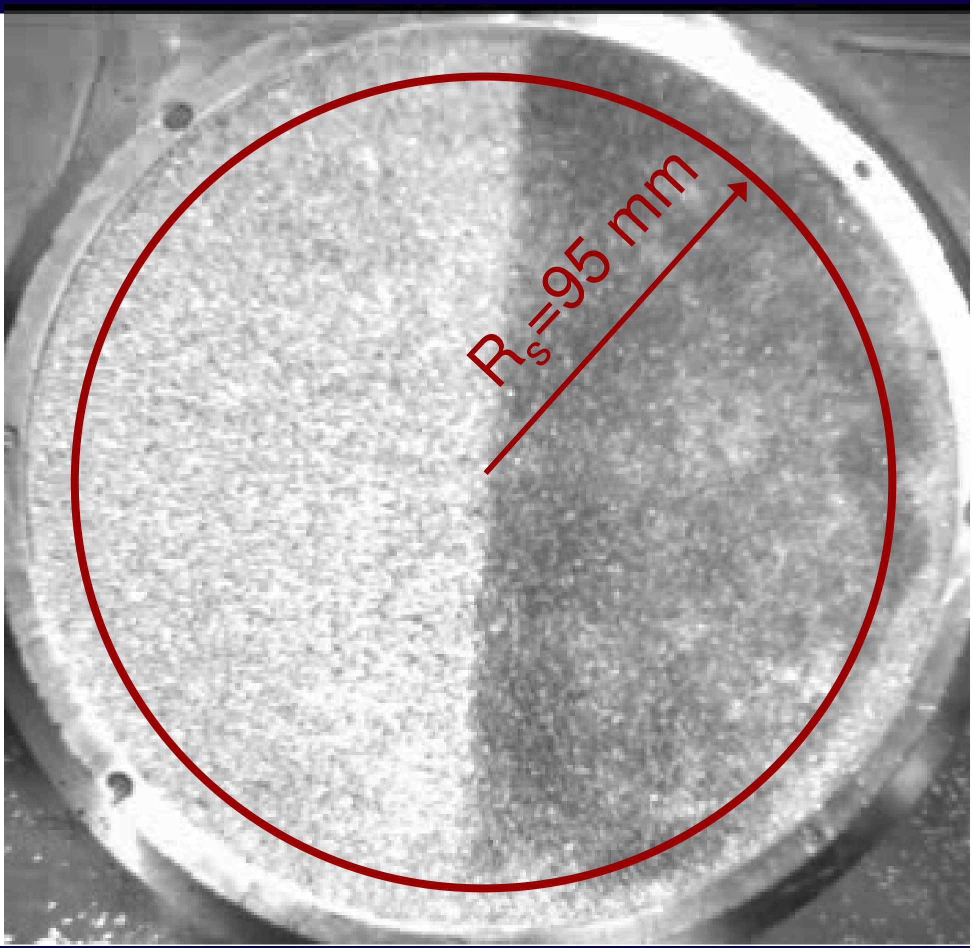
Velocity profiles

Rate independent

Short transients

Azimuthal

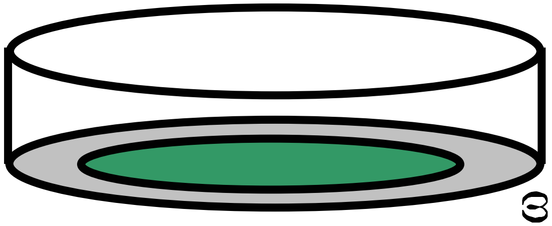
$\omega(r)$: nondimensional

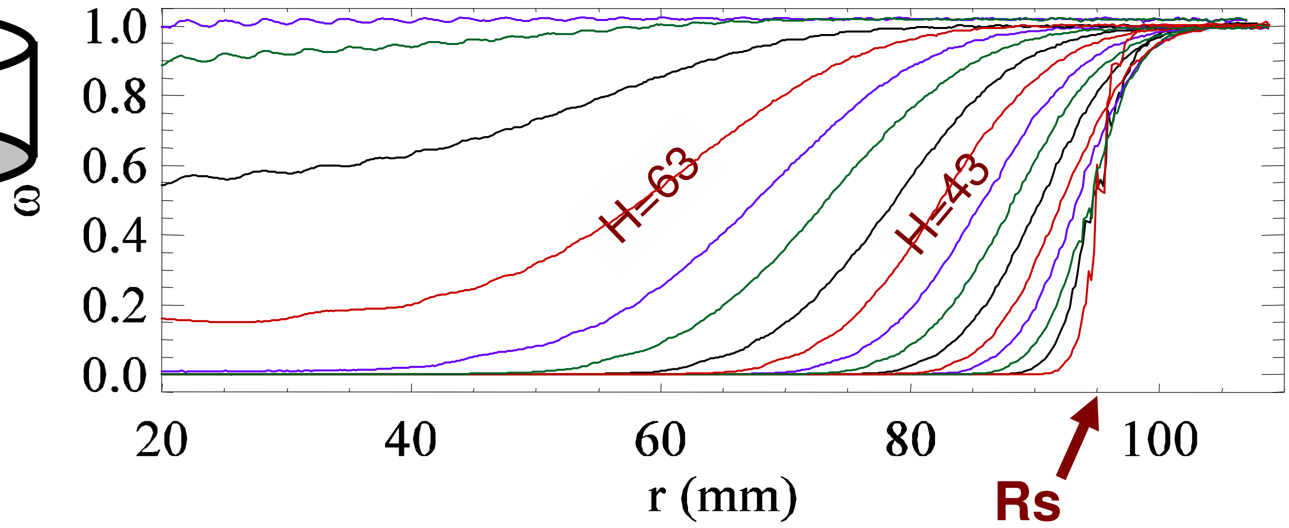


Denis Fenistein, Rene Mikkelsen, Jan-Willem van de Meent, Henri Bontebal & MvH

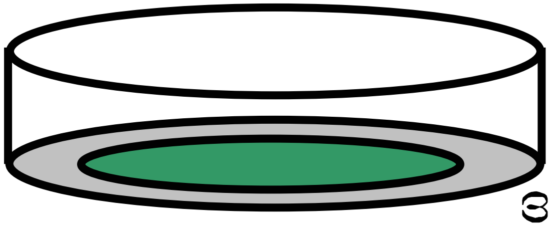
Nature **425**, 256 (2003); PRL **92**, 094301 (2004)

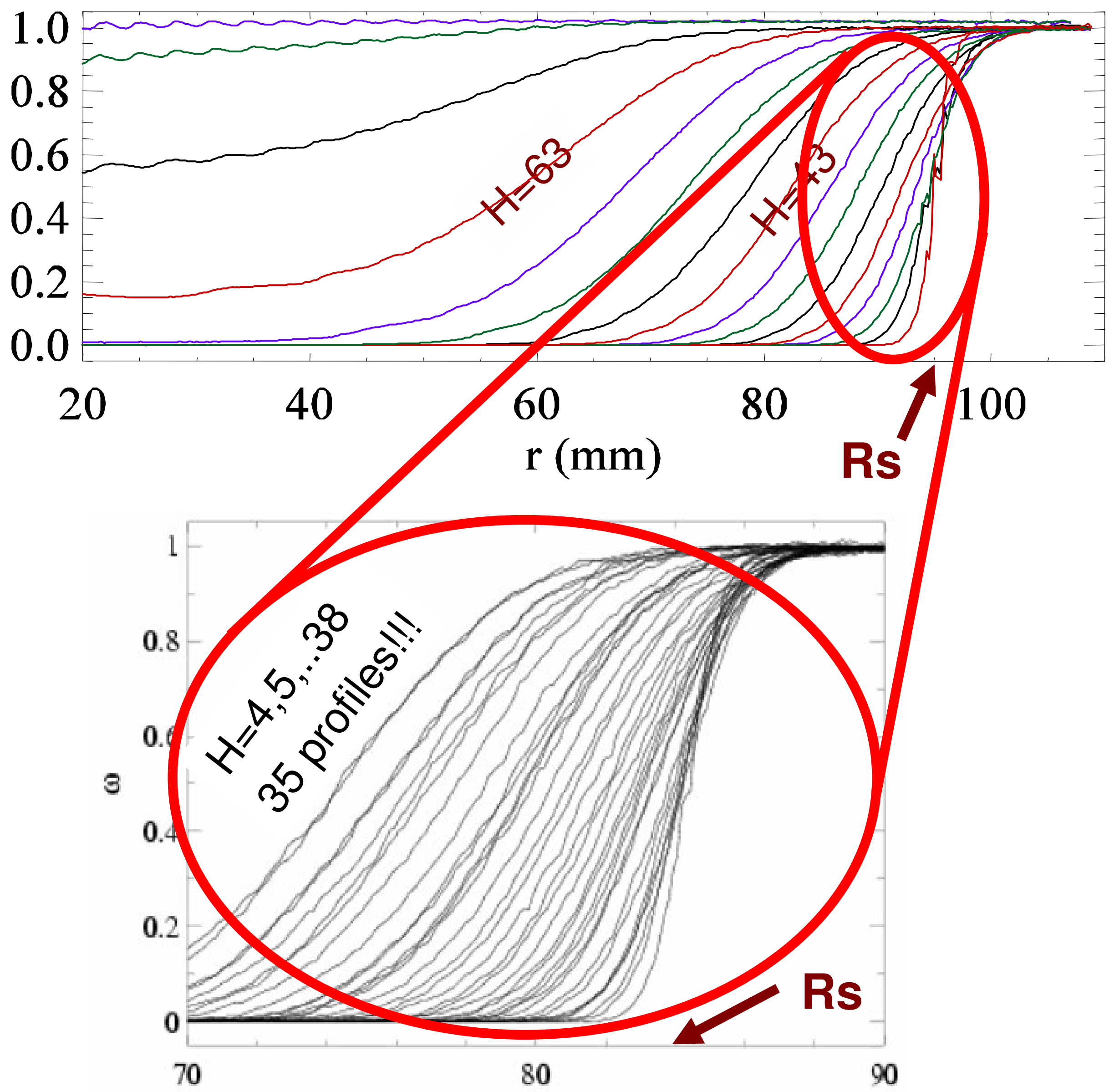
Surface Velocity Profiles



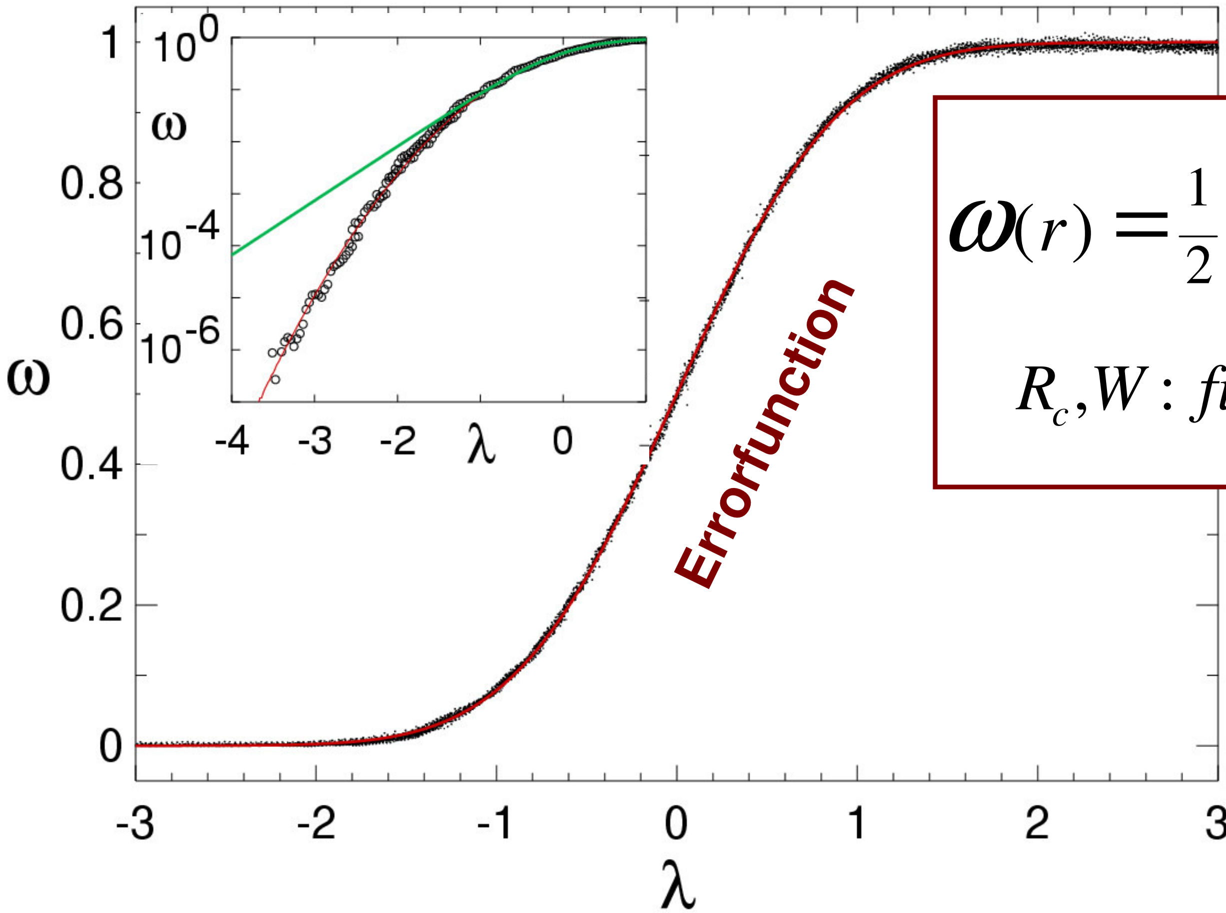


Shallow Layers





Shallow Layers

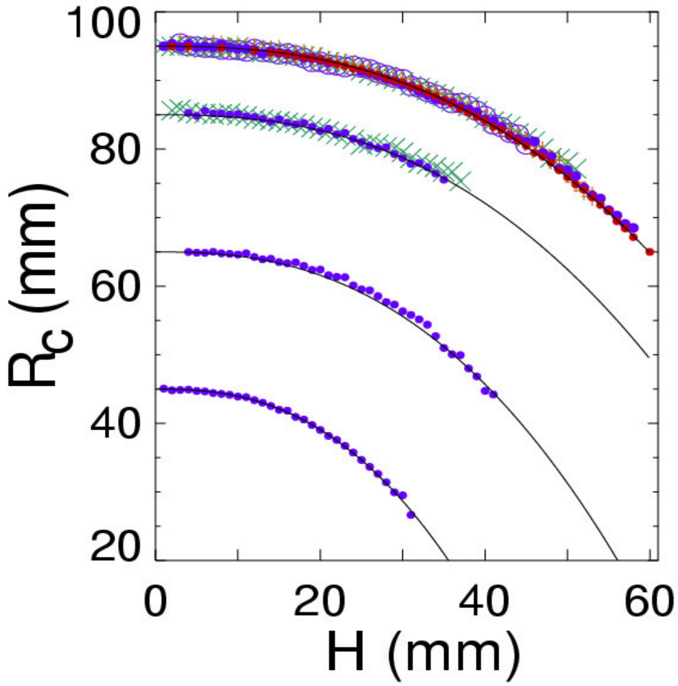


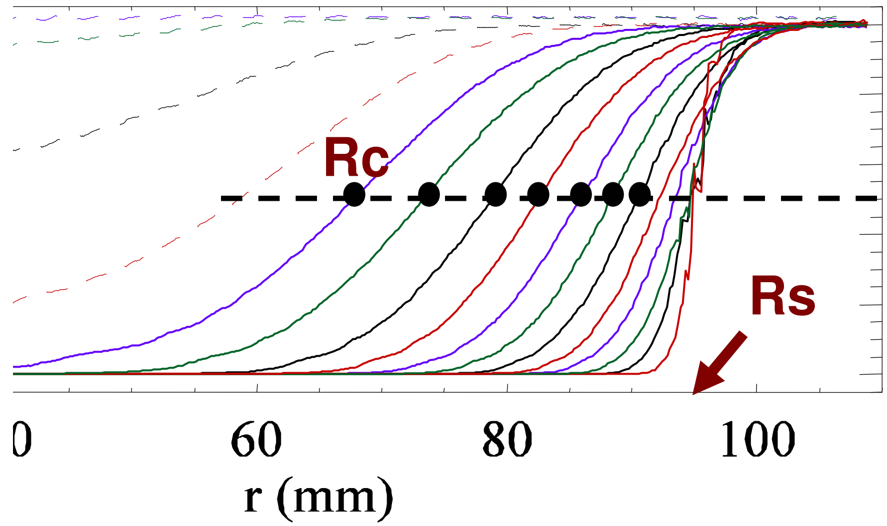
$$\omega(r) = \frac{1}{2} + \frac{1}{2} erf\left(\frac{r - R_c}{W}\right)$$

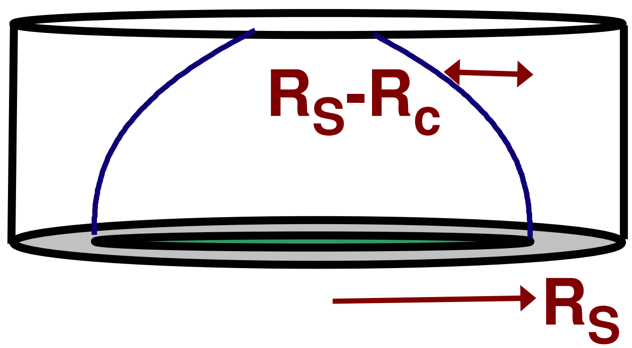
$R_c, W : functions(R_s, H, d, \ldots)$

Shallow Layers: R_c

independent of grain size & shape

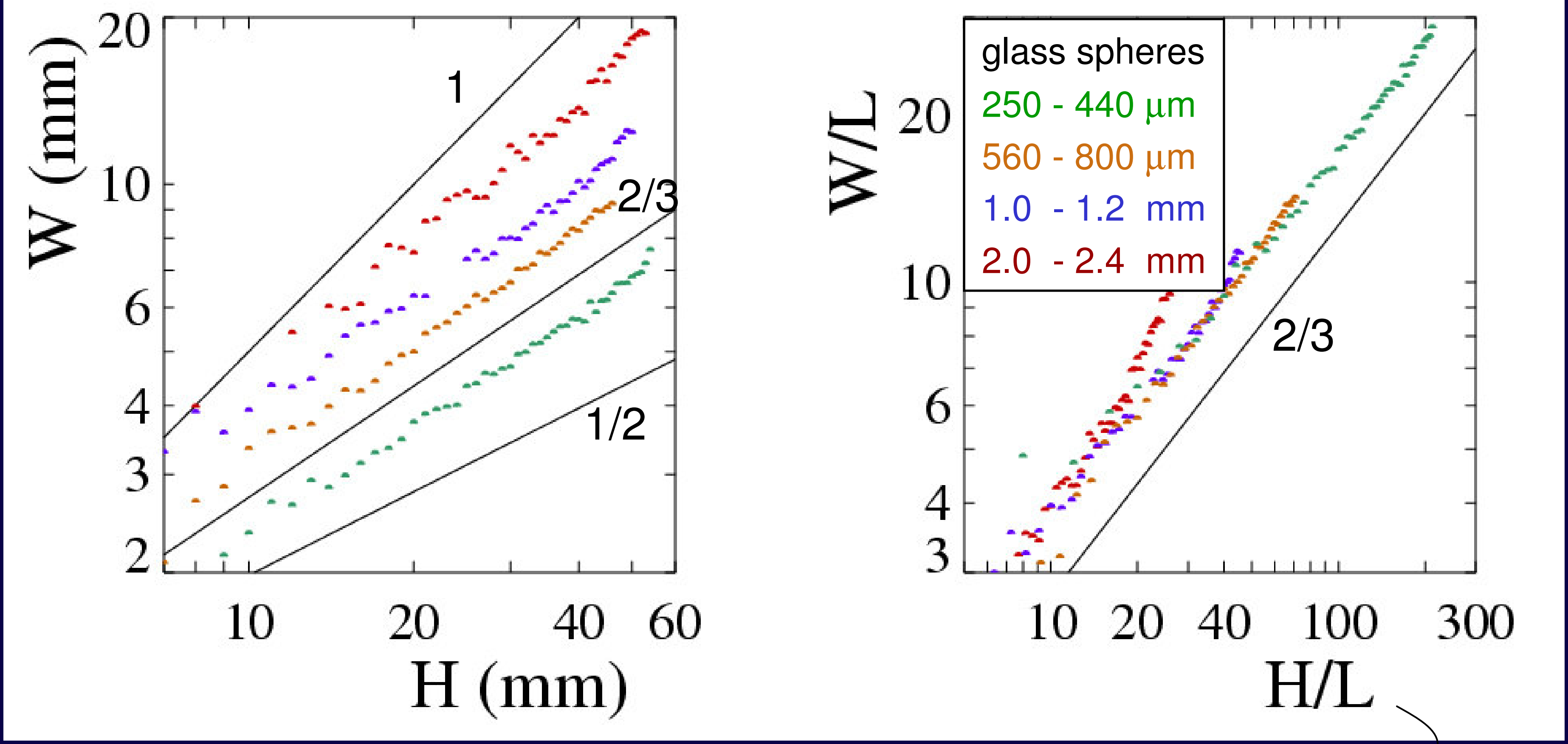






$$(R_s - R_c)/R_s = (H/R_s)^{5/2}$$

Shallow Layers: W



1.1 mm bronze: W 30% smaller

L: best fits, 0.25, 0.65, 1.1, 2.2 mm

$$W/L \sim (H/L)^{2/3} \quad , L \approx d$$

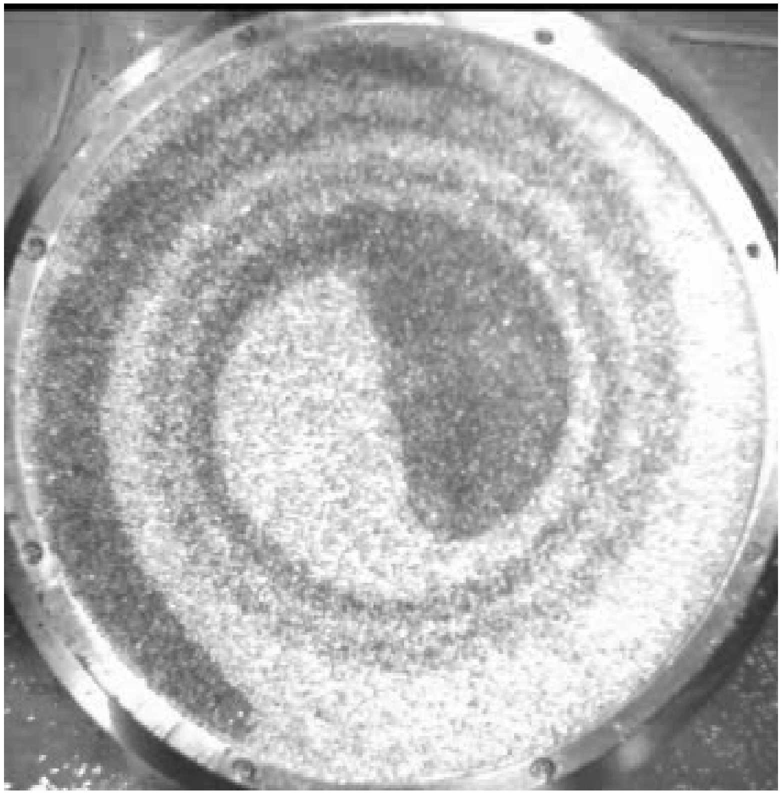
Shallow Layers

$$\omega(r)=1/2+1/2\ \mathrm{erf}[(r-R_c)/W] := \mathrm{nerf}\ [(r-R_c)/W]$$

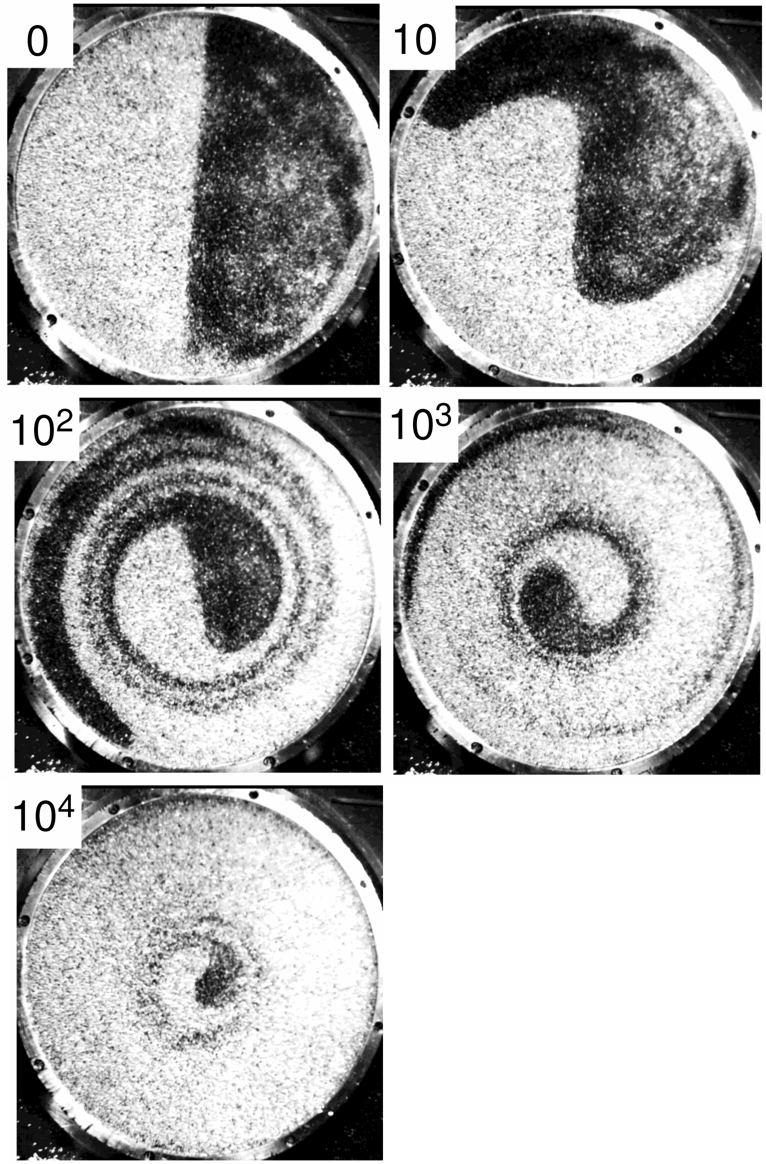
$(R_s-R_c)/R_s = (H/R_s)^{5/2}$: Independent particles

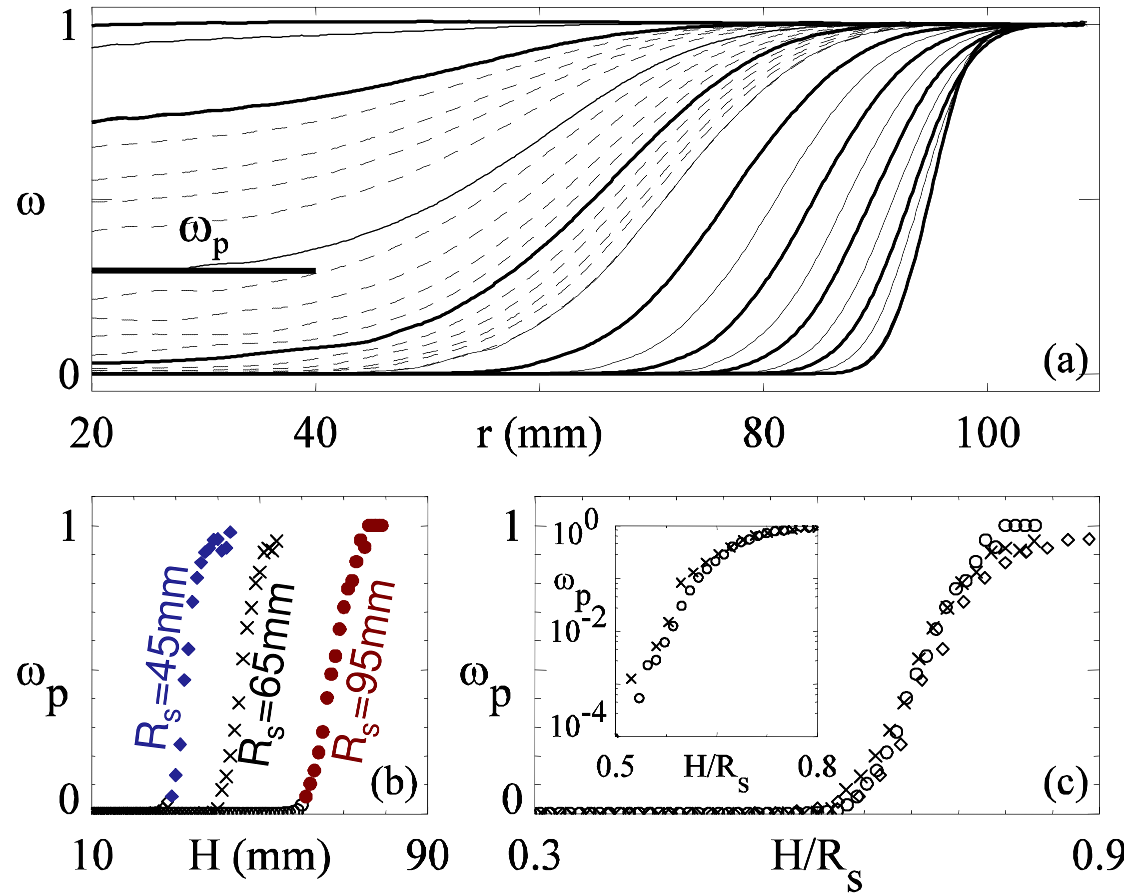
$W/d \sim (H/d)^{2/3}$: Independent R_s

Deep layers: Core Precession

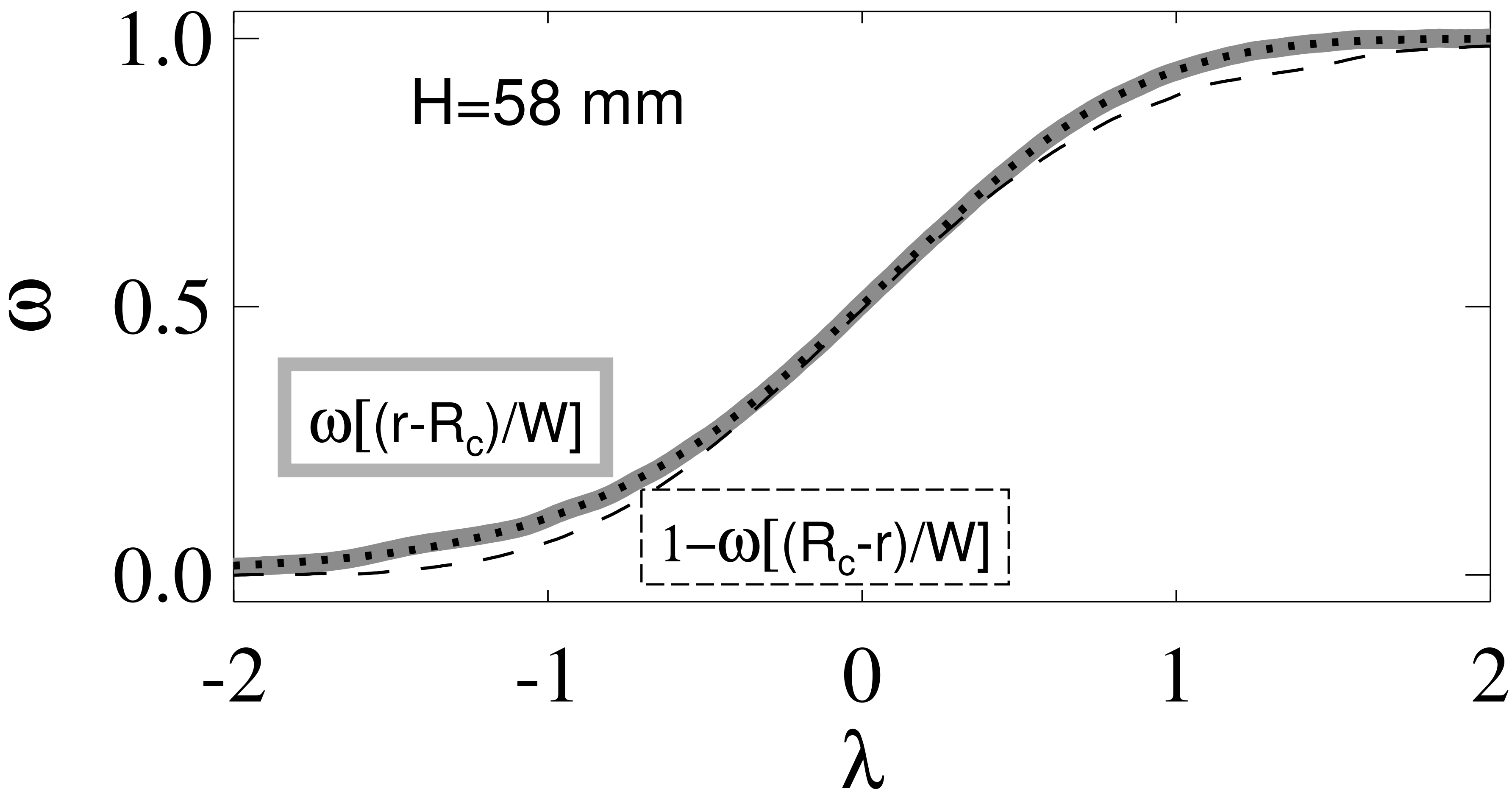


Deep layers: Core Precession

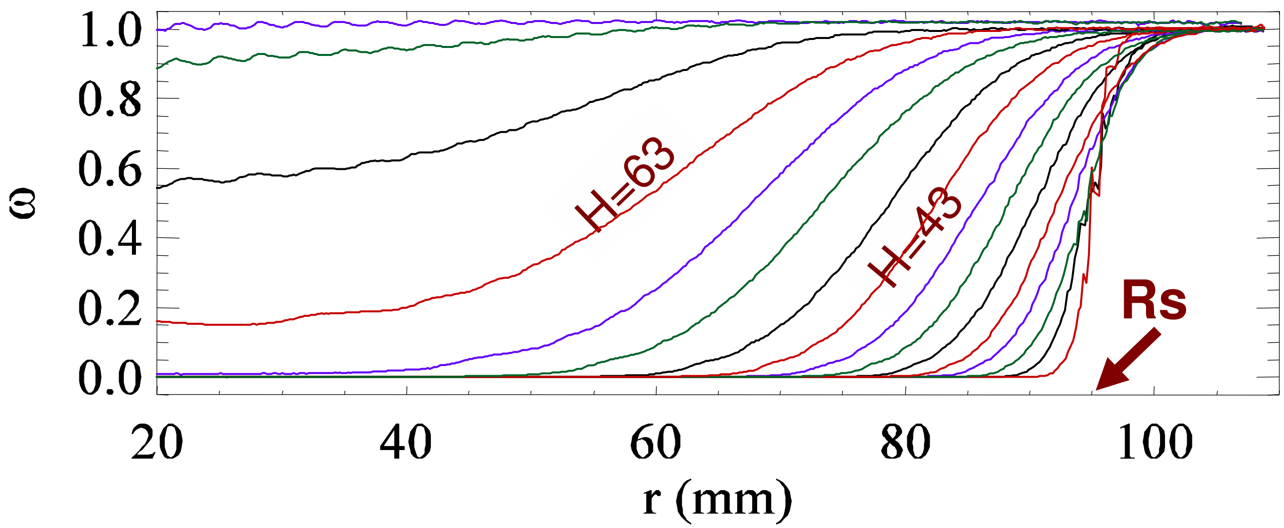




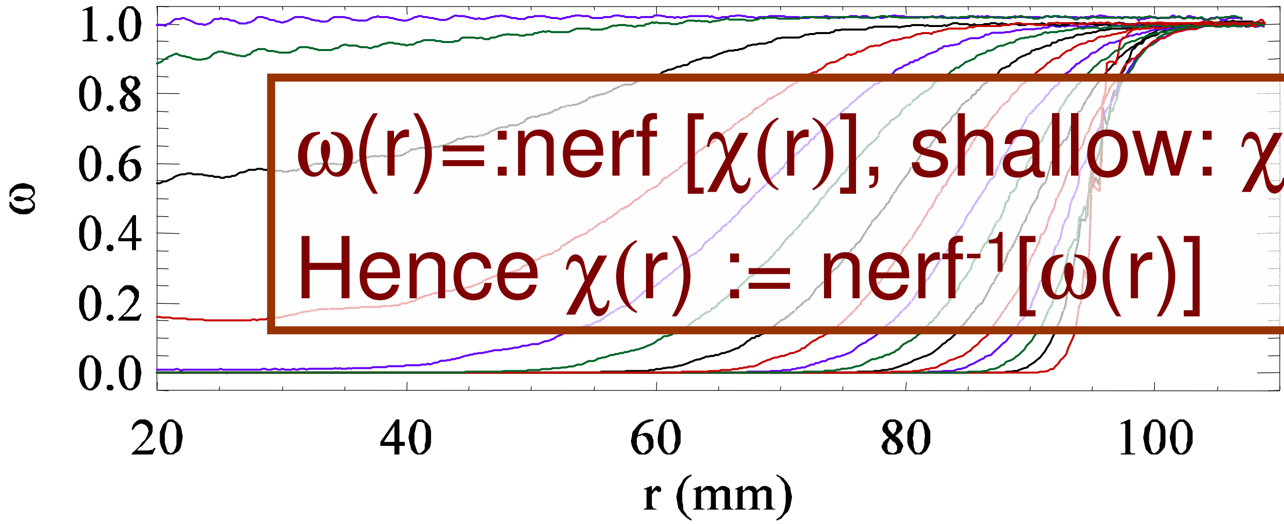
Deep layers: Asymmetry



Precession, Asymmetry



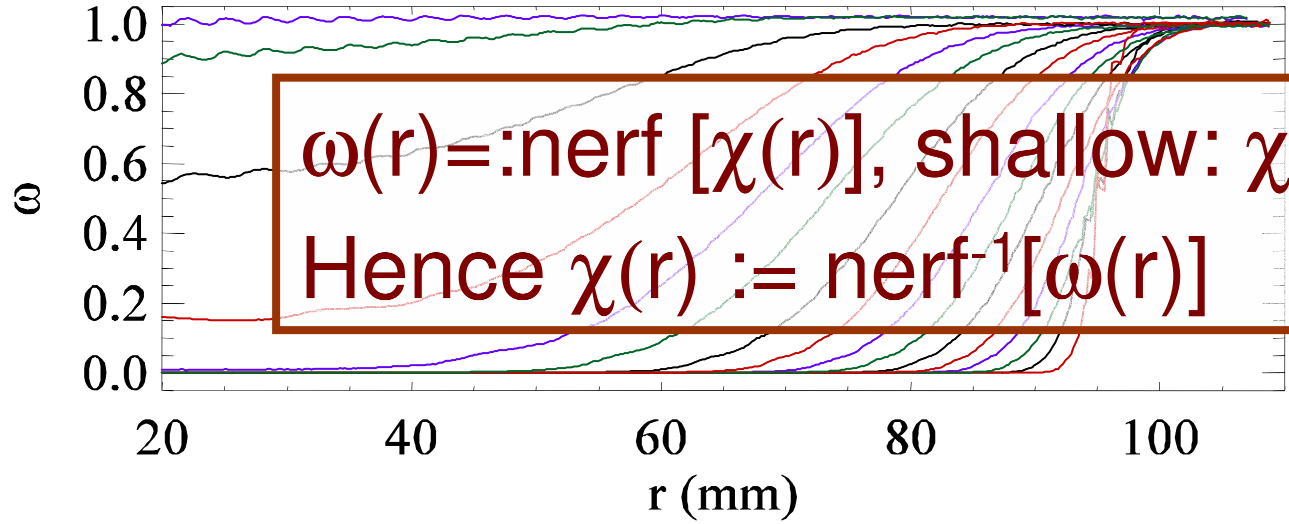
Precession, Asymmetry



$\omega(r) =: \mathrm{nerf}\,[\chi(r)]$, shallow: $\chi(r) = (r - R_c)/W$.

Hence $\chi(r) := \mathrm{nerf}^{-1}[\omega(r)]$

Precession, Asymmetry



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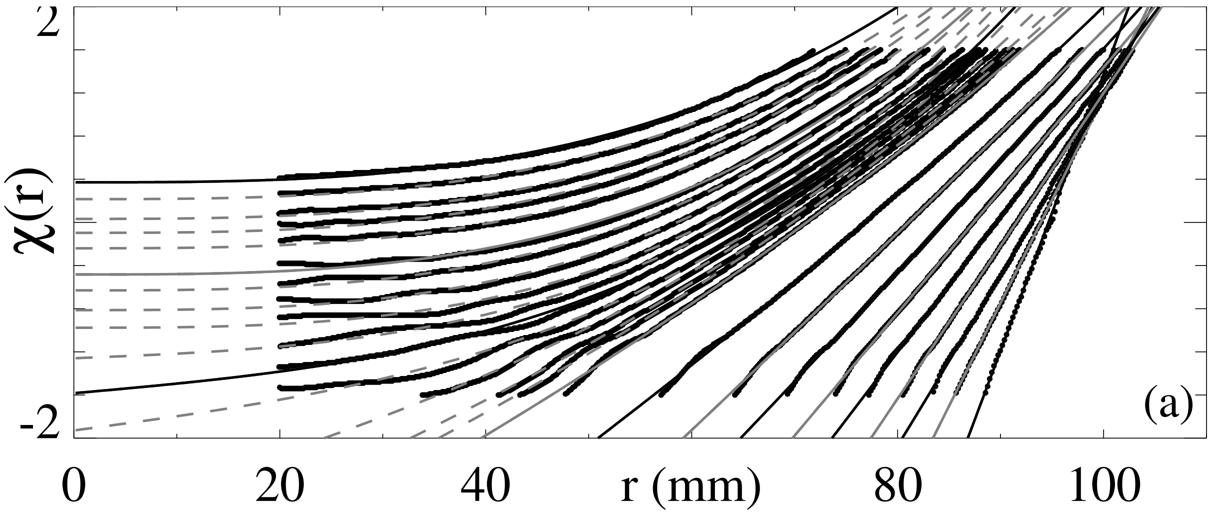
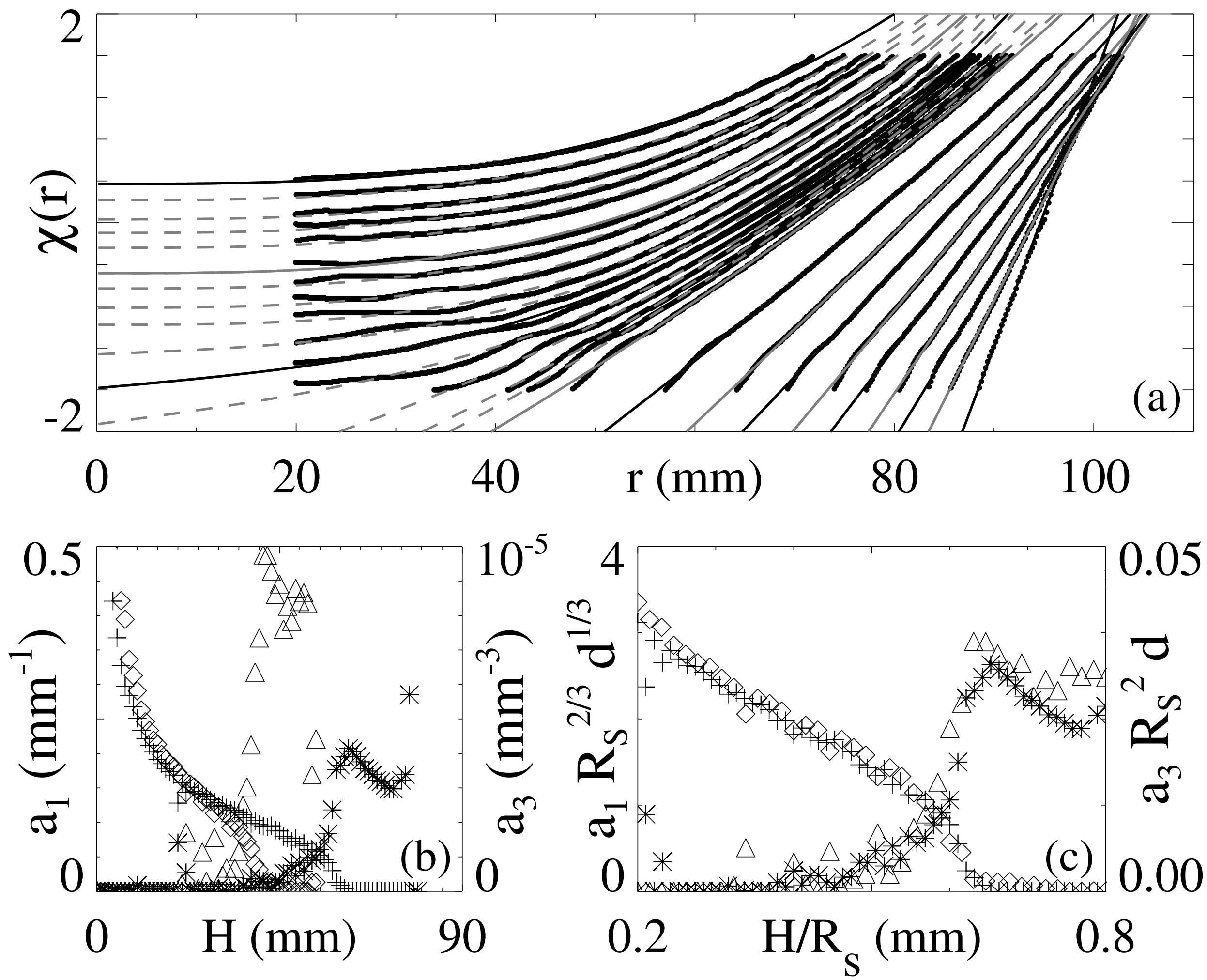


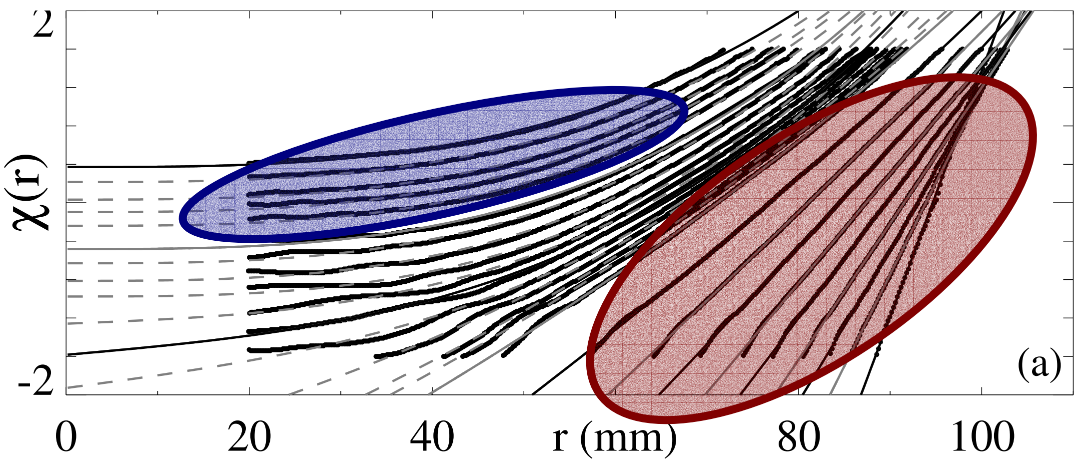
Fit: $\chi(r) = a_0 + a_1\, r + a_3\, r^3$

Precession, Asymmetry

Fit: $\chi(r) = a_0 + a_1 r + a_3 r^3$



Local vs Global shearmodes



$\omega(r) = \mathrm{nerf}\,[(a_0 + a_1 r)] = \mathrm{nerf}\,[(r - R_c)/W]$

$\omega(r) = \mathrm{nerf}\,[(a_0 + a_3 r^3)]$

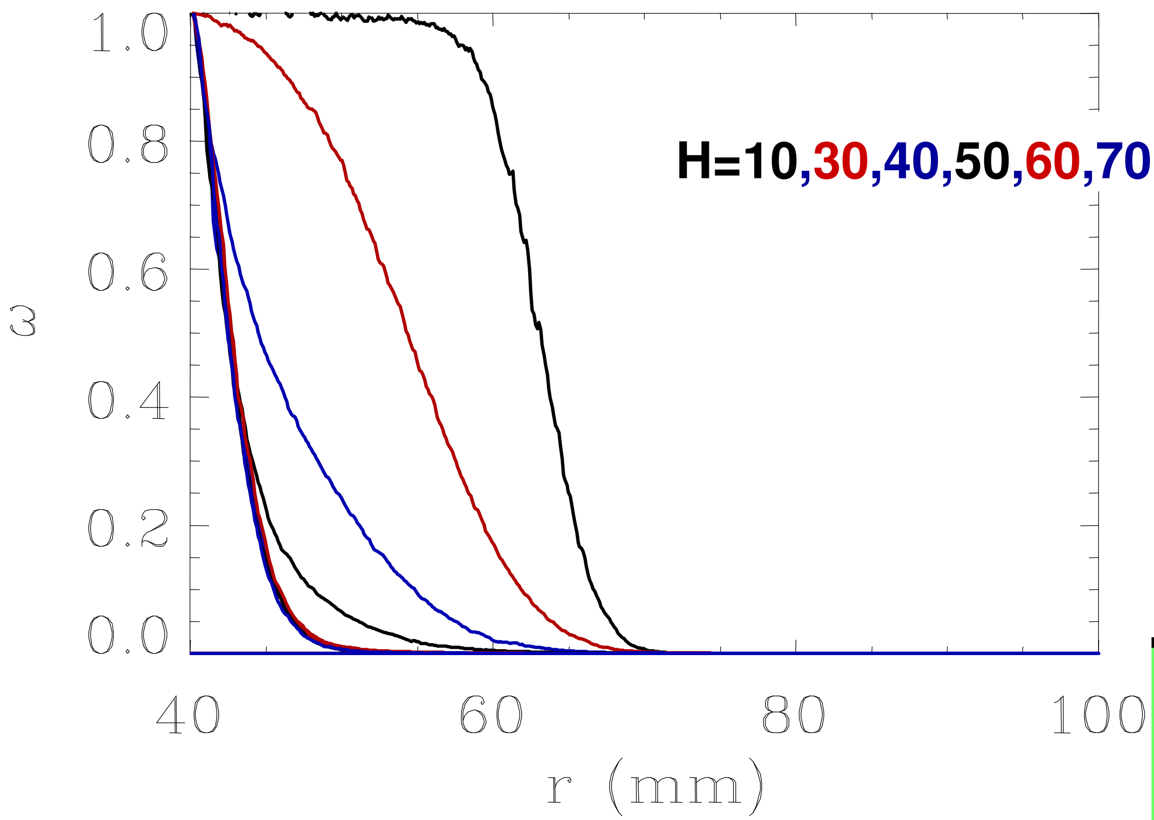
Deep Layers

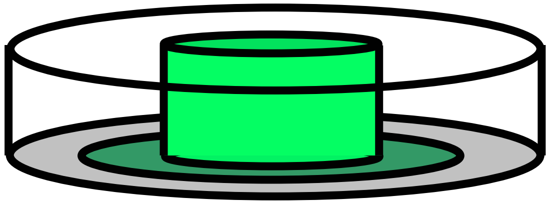
$\omega(r)=\text{nerf}[(a_0+ a_1 +a_3 r^3)]$

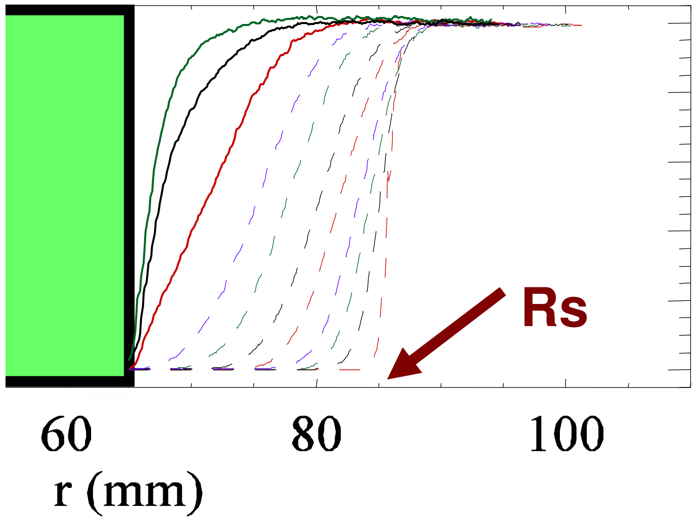
Shallow: $a_3=0$, Deep: $a_1=0$

Precession, Symmetry breaking

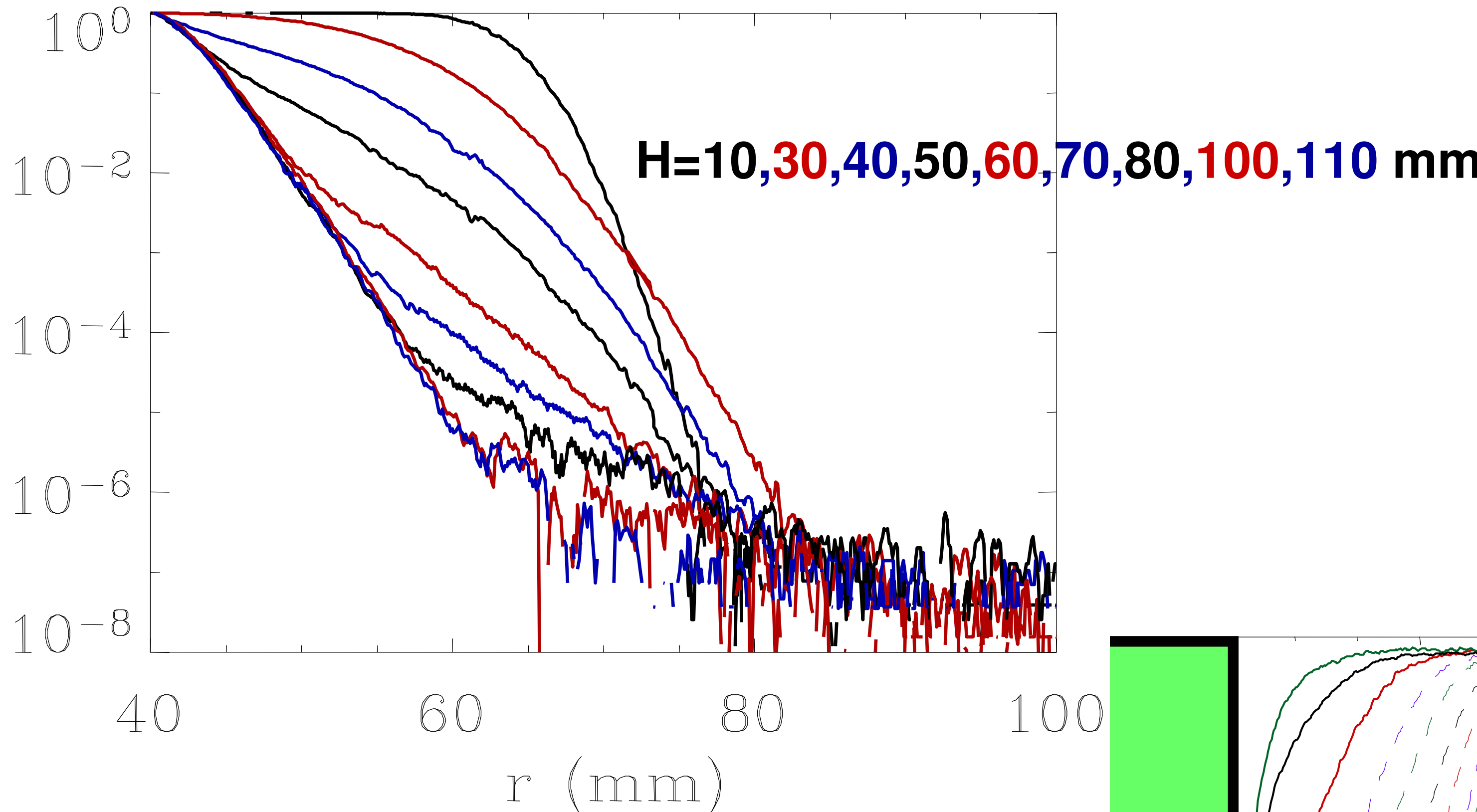
Wall-localized shearbands

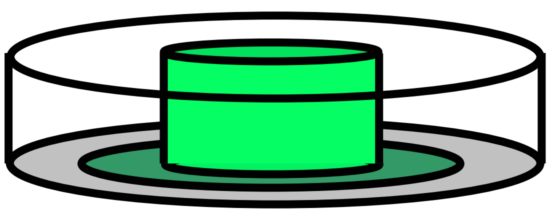


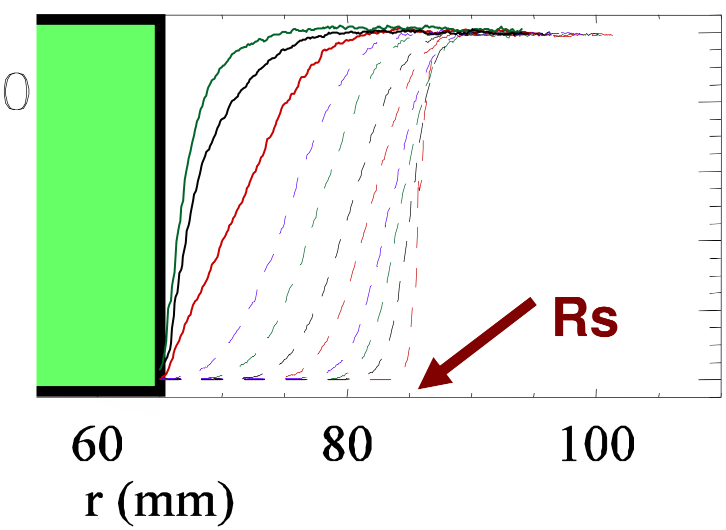




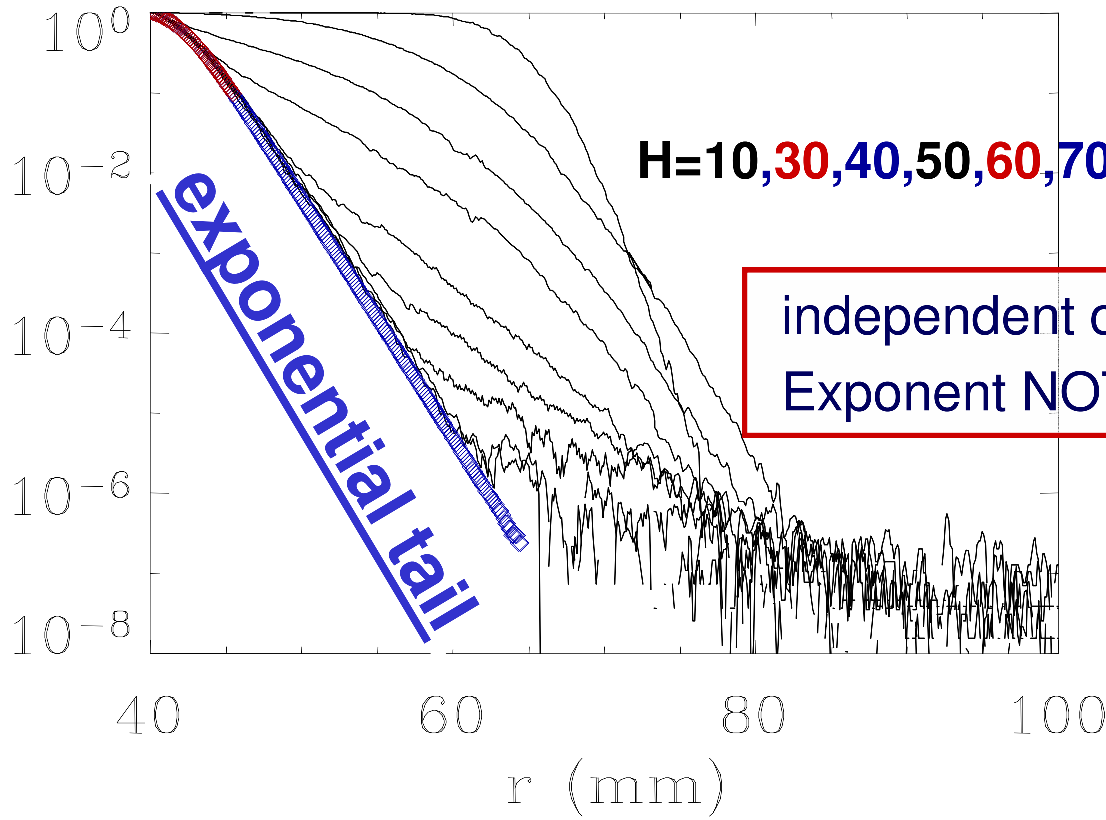
Wall-localized shearbands



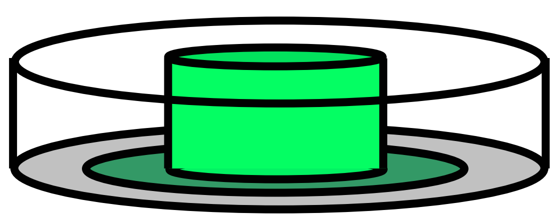


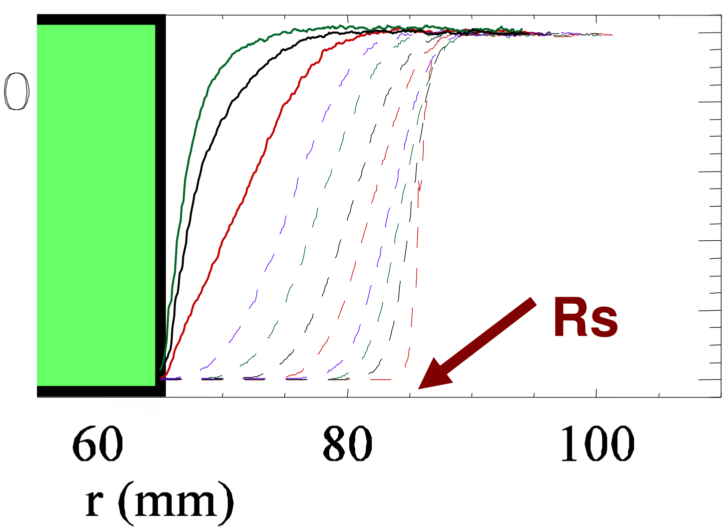


Wall-localized shearbands

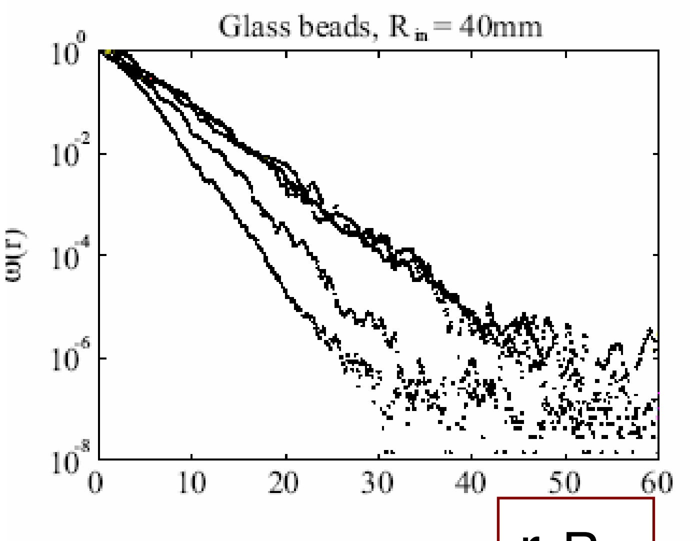


independent of grain shape, polydispersity
Exponent NOT~grain diameter

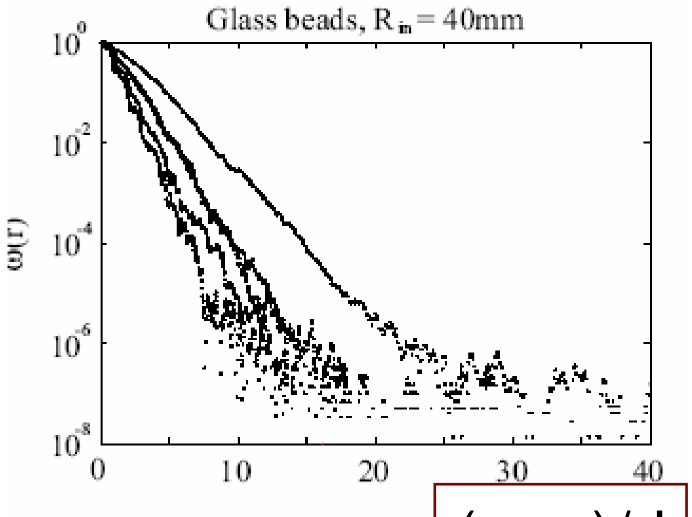




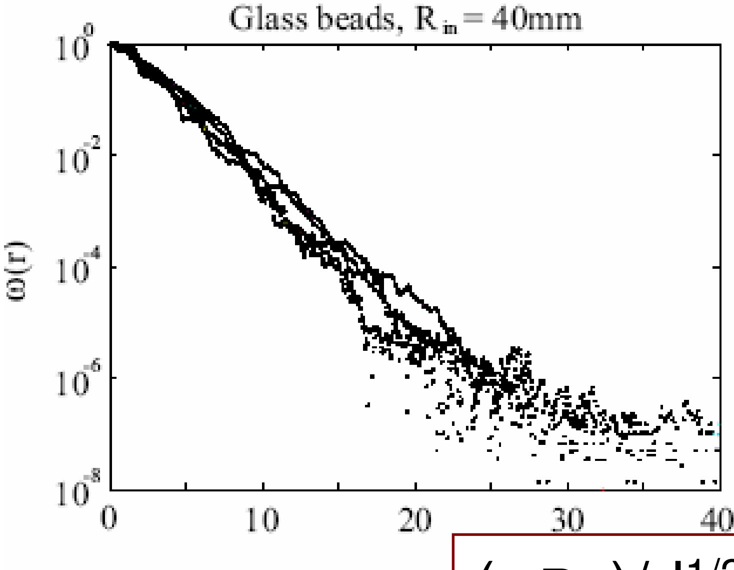
Wall-localized shearbands



r-R_{in}

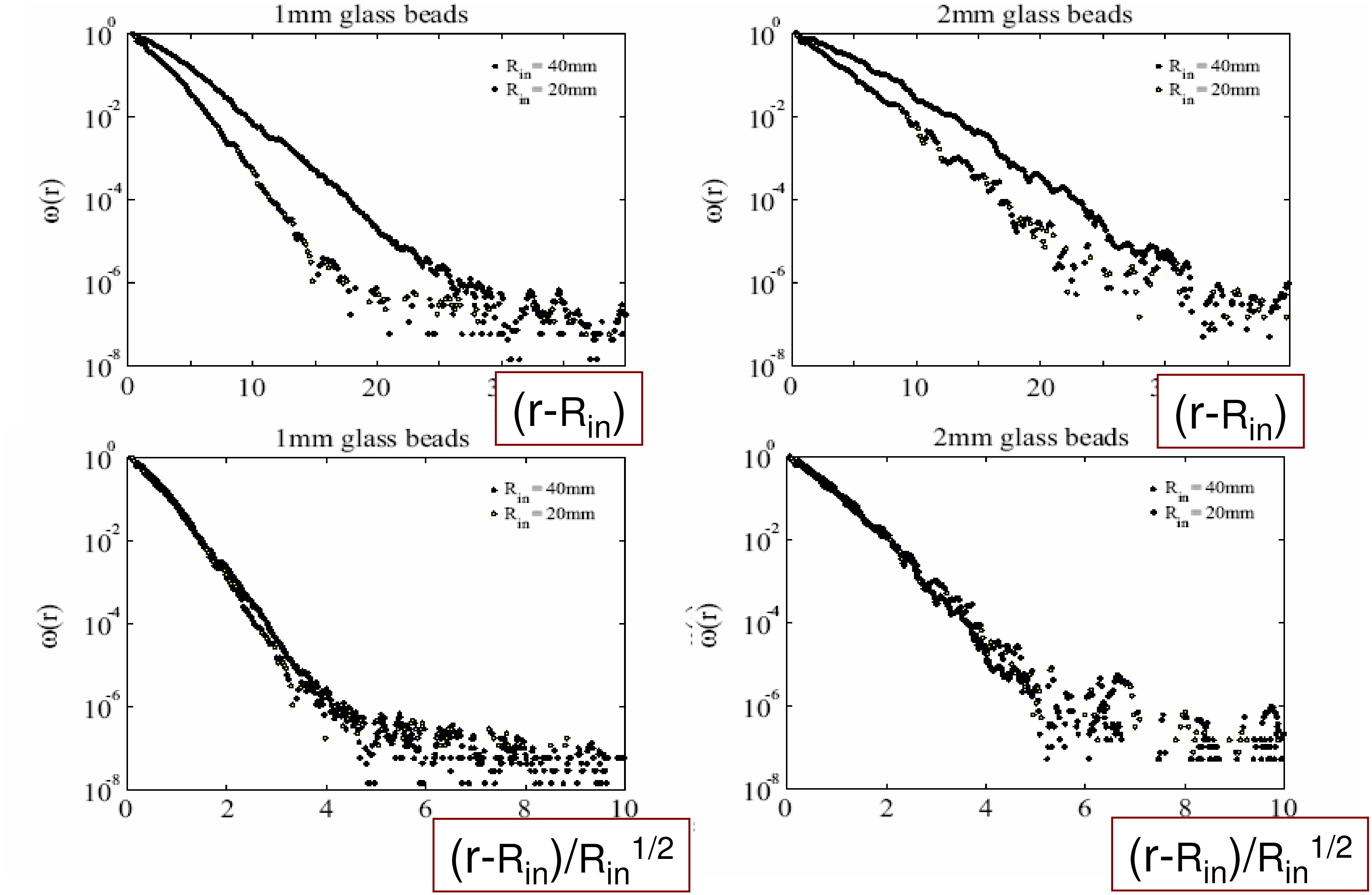


$(r-R_{in})/d$



$(r-R_{in})/d^{1/2}$

Wall-localized shearbands



Localized Shearbands

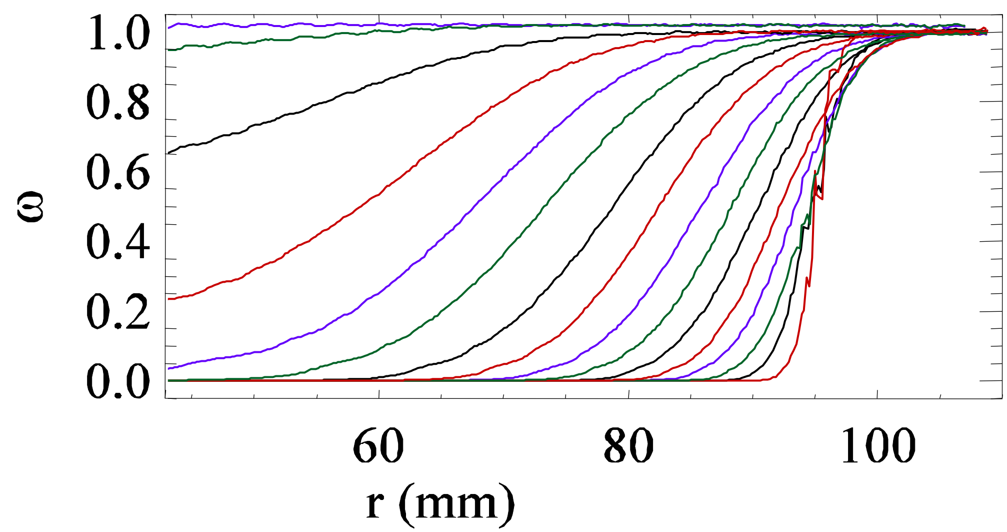
Exponential tail

Lengthscale: radius of curvature, grainsize

Surface Velocity Profiles

Shallow

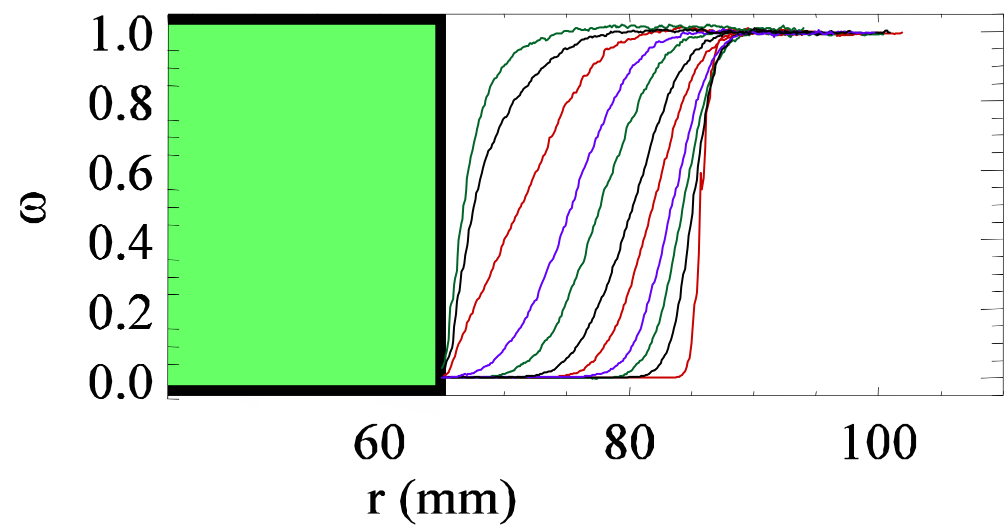
Errorfunction, universal center



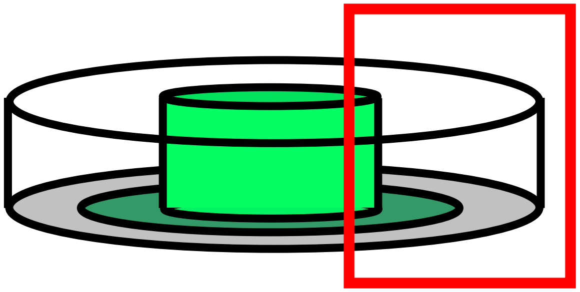
Deep

Precession, Global modes

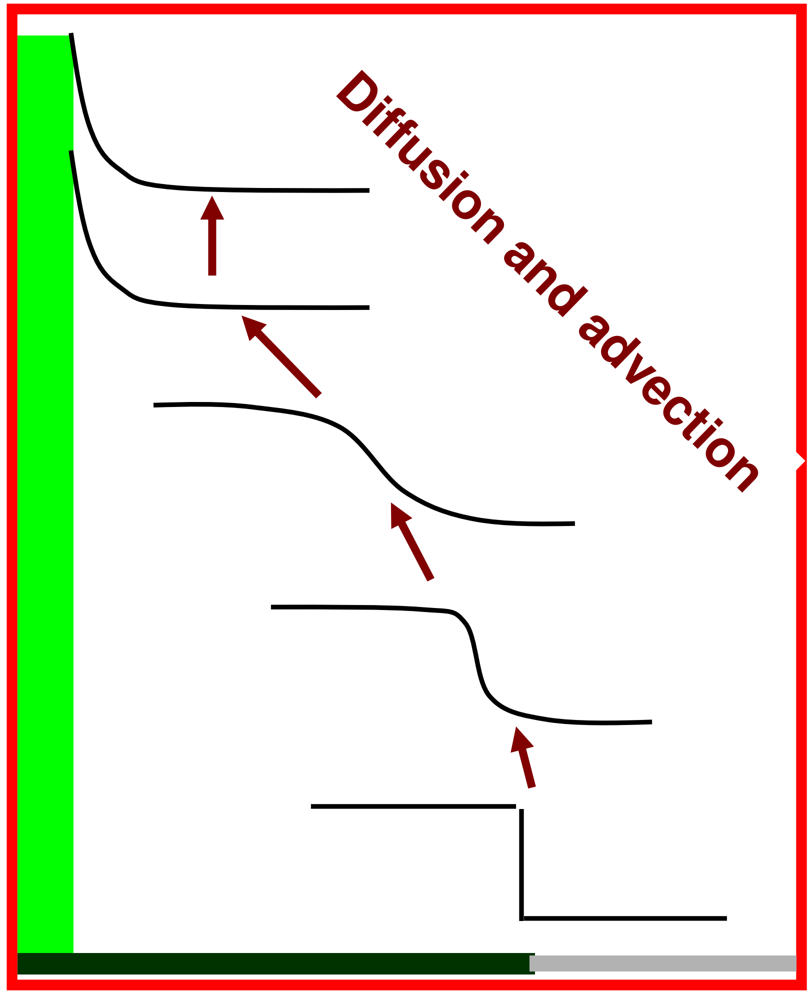
Wall localization, **Exponential** tail, Radius of Curvature



Reconciling two types of tails?



$$\partial_z \omega \sim \partial_r^2 \omega + \partial_r \omega$$



Stepfunction -> erf

Large height: z-indep

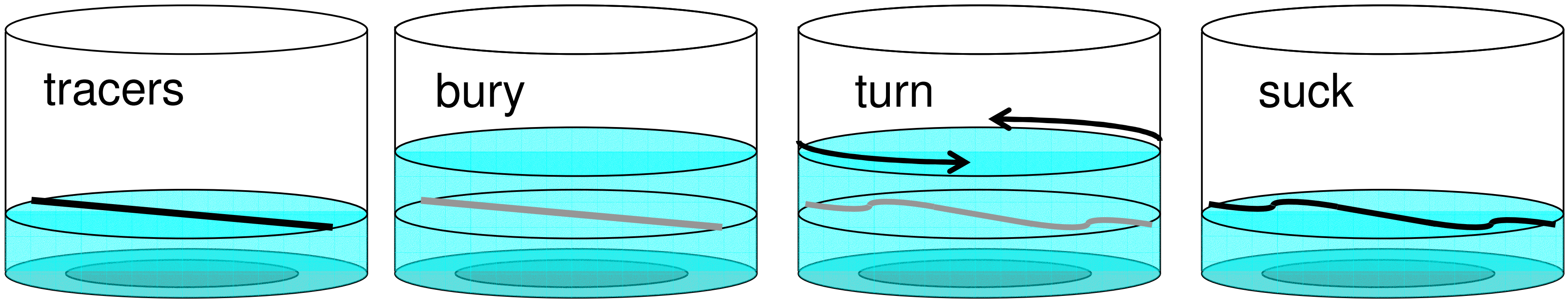
$$0 = \partial_r^2 \omega + \partial_r \omega$$

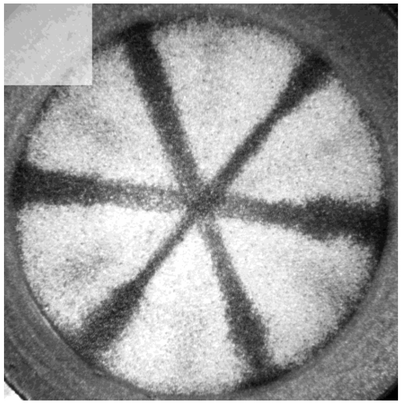
Exponential tail

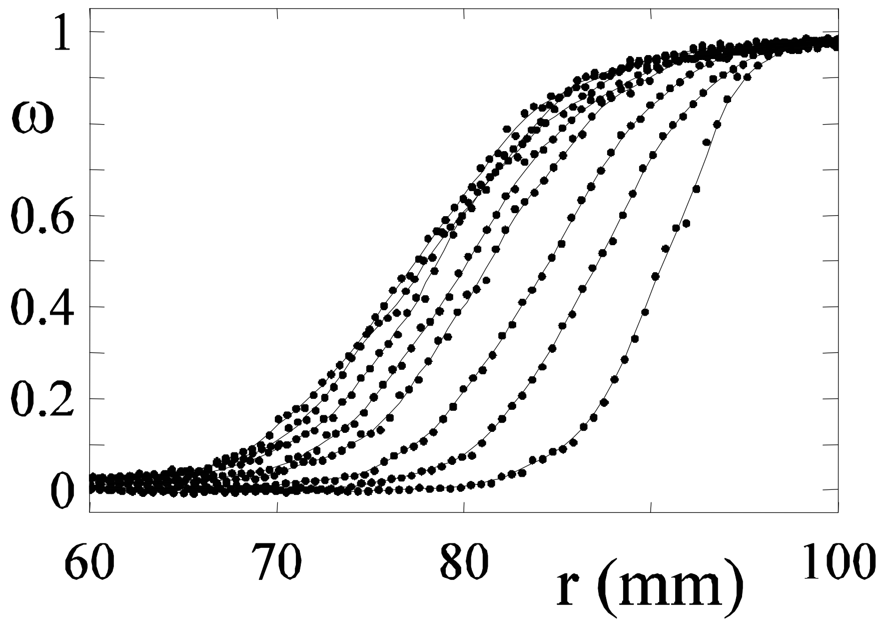
But width: ~ $H^{2/3}$

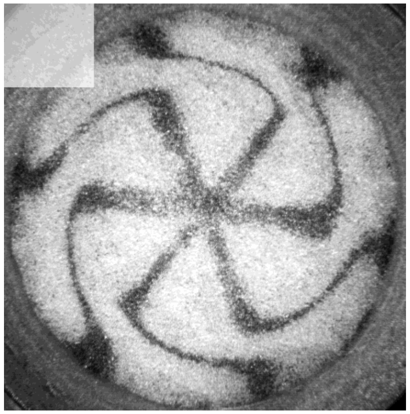
Location?

3D Profile: What happens inside?

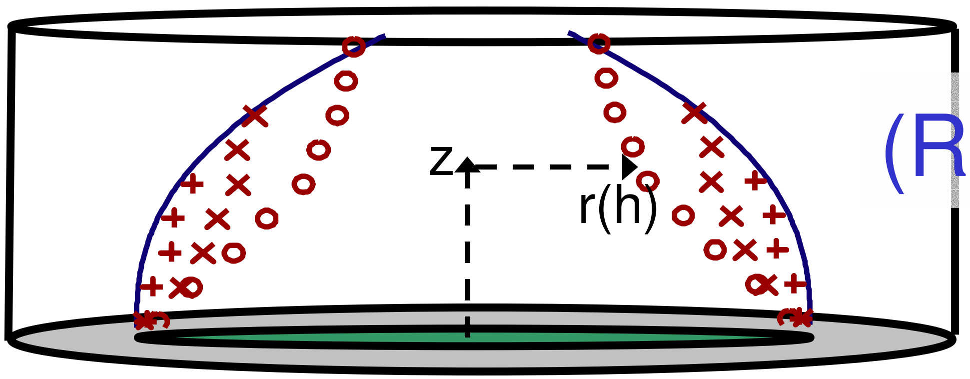








Model: Location in Bulk



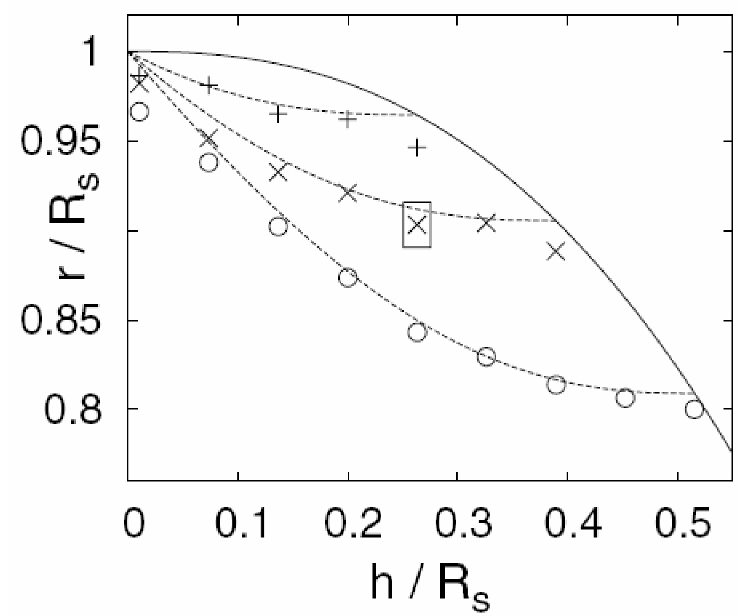
$(R_s - R_c)/R_s = (H/R_s)^{5/2}$

R_c in bulk: Scaling argument virtual bottom OK

$$R_c(R_s, H) = R_c(r, H - h).$$

α=5/2

$$h = H - r\left[1 - \frac{R_s}{r}\left[1 - (H/R_s)^\alpha\right]\right]^{1/\alpha}$$



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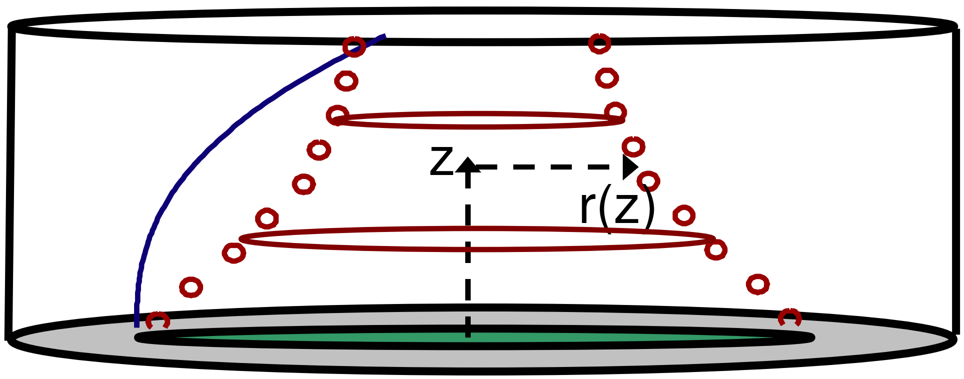
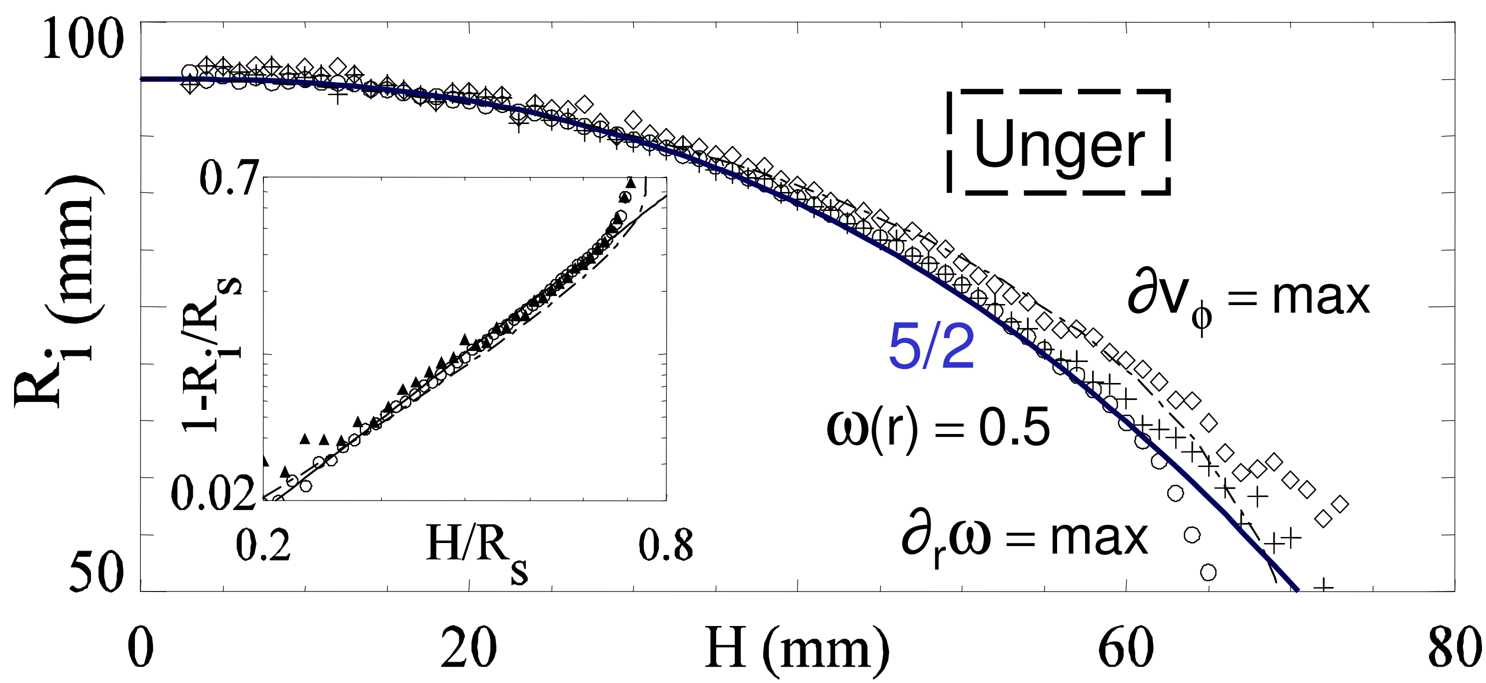
Model: R_c at surface



R_c : Torque minimalization

$$\int dz\, r^2(z)\sqrt{r'^2+1}\,(H-z)$$



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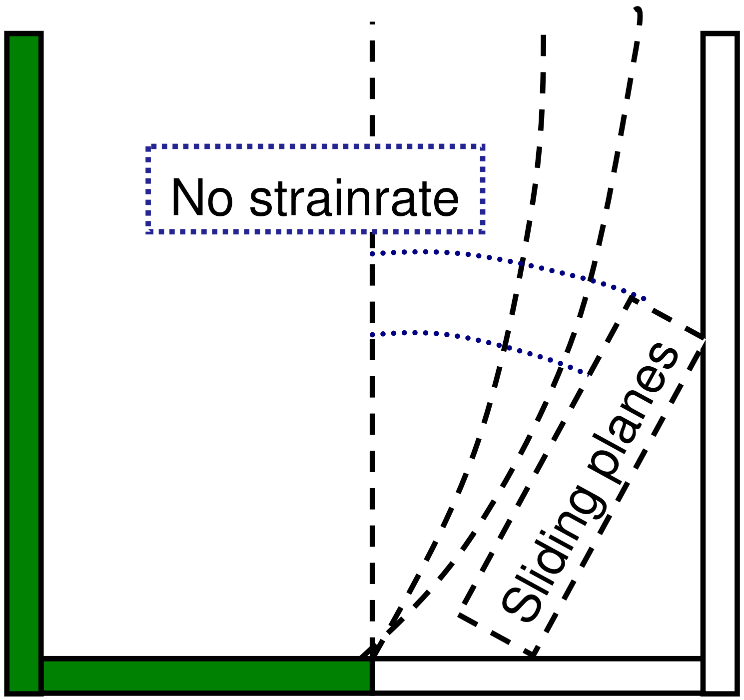
Mohr Coulomb: A Riddle

Vibrations: No strainrate = No shearstress
Quasistatic: shearstress constant along sliding planes of equal depth



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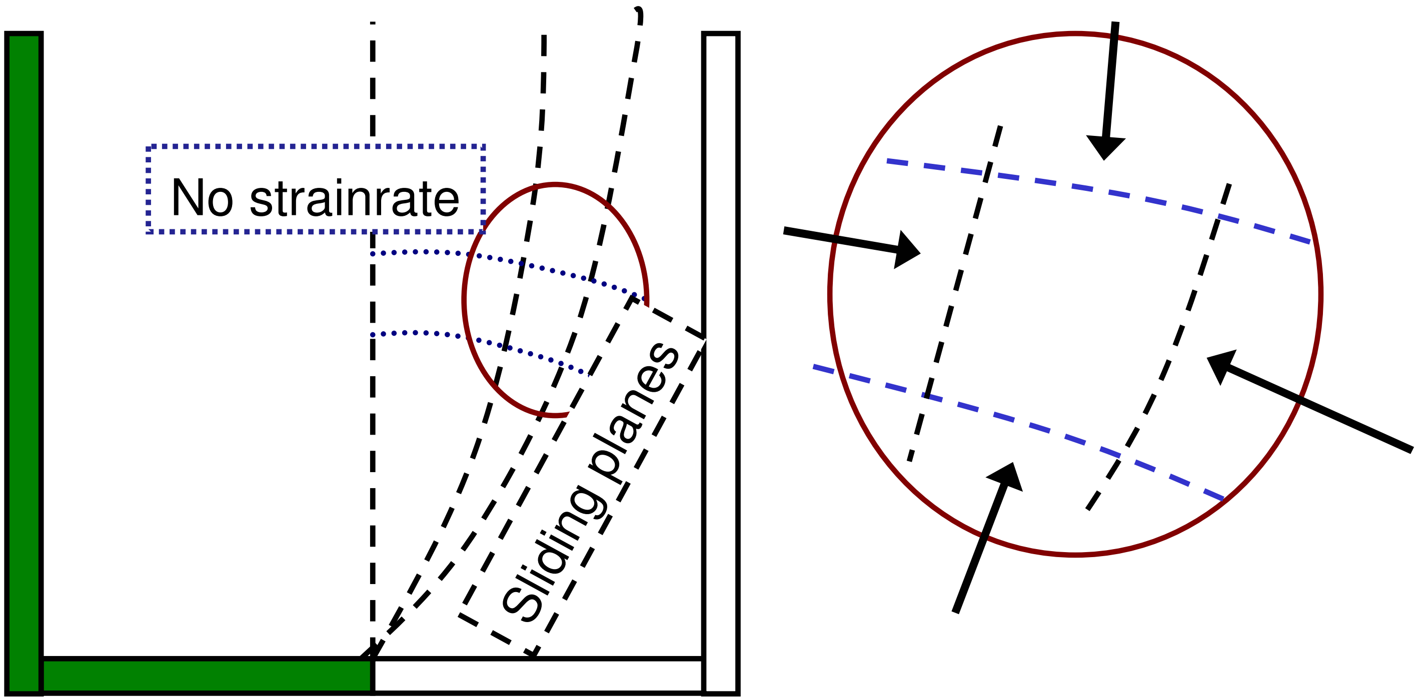
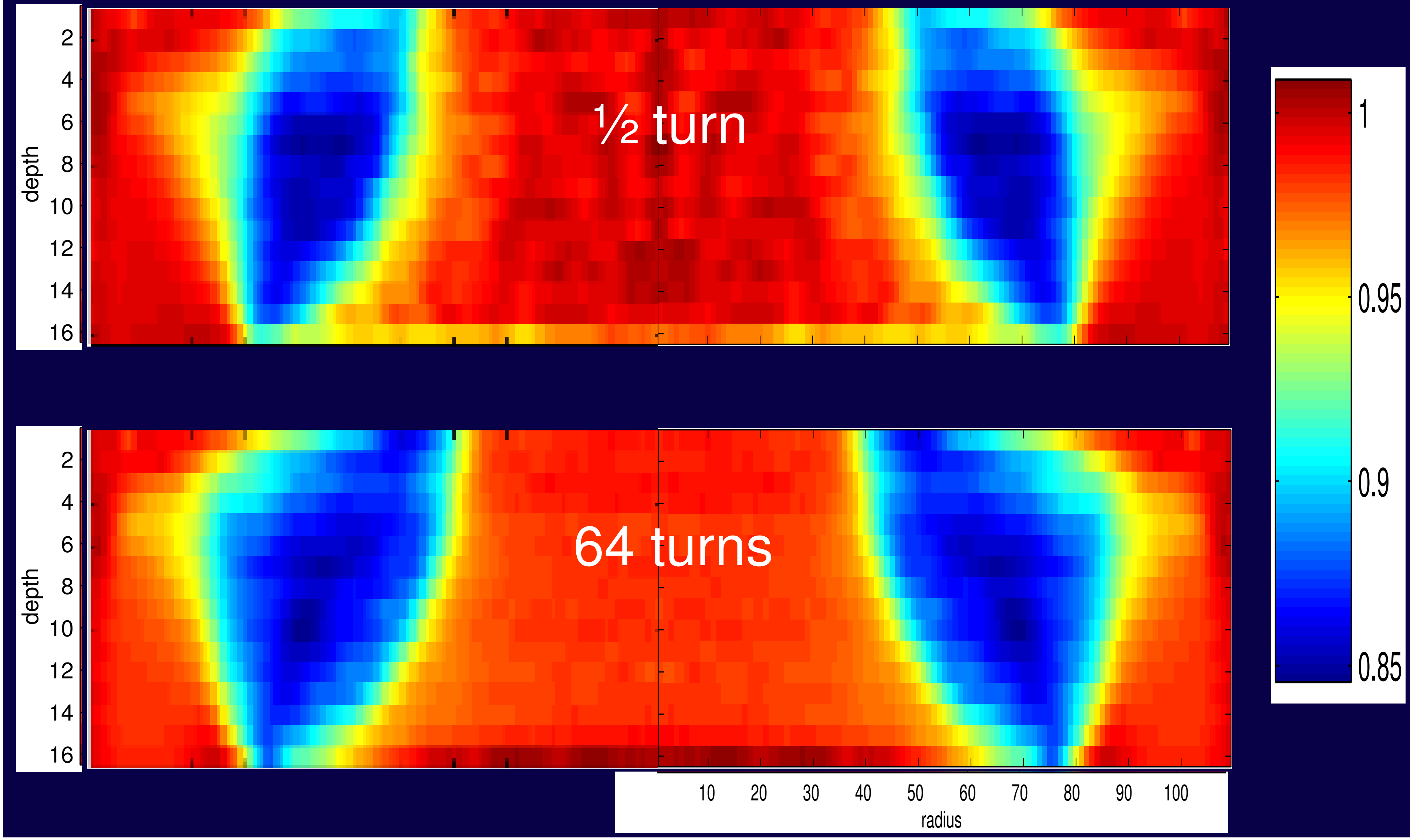


MRI: **density**

With Paul Umbanhowar (NWU)



Outlook

Split Bottom: wide range of flows

3 regimes: shallow, precession, localization

All hold surprises

Dilation

MD/Theory: stresses