Irradiated Graphene as a Topological Insulator

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I. Introduction: Graphene as a topological insulator
II. Circularly polarized microwaves and Floquet spectrum
III. Edge states and transport
IV. Evanescent penetration: “Superdiffusive” transport
V. Summary

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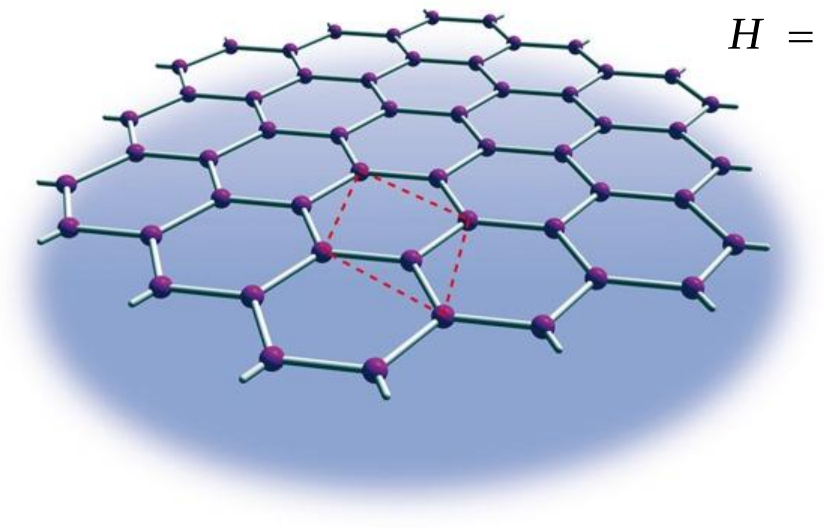
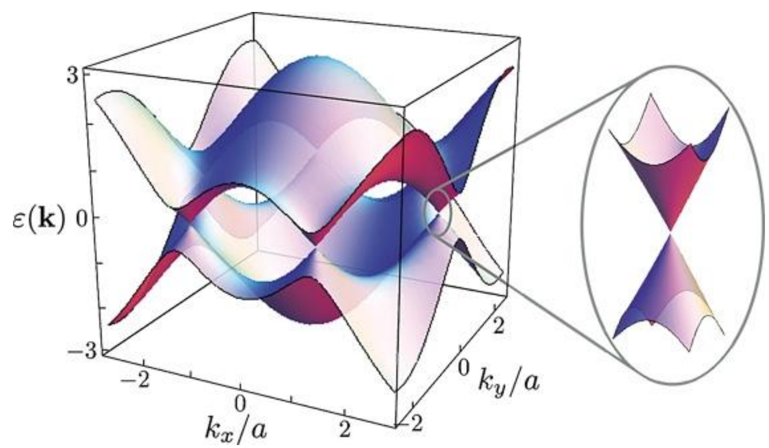
I. Introduction: Gapping the Graphene Spectrum



Simplest tight-binding model for p_z orbitals:

$$H = -t \sum_{n_1 n_2 = n.n.} \left| n_1 \right\rangle \left\langle n_2 \right| \Rightarrow H_0 = v_F \vec{\psi}^+ (\tau_z k_x \sigma_x + k_y \sigma_y) \vec{\psi}$$

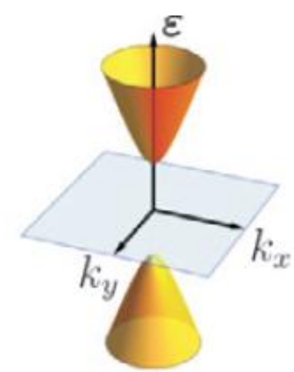


Dirac points

Kane-Mele model: Add

□1

$$H_{nnn} = it_2 \sum_{(n_1 n_2) = n.n.n.} \upsilon_{n_1 n_2} \left| n_1 \right\rangle \left\langle n_2 \right| \Rightarrow H_{SO} = \Delta_{SO} \vec{\psi}^+ \tau_z \sigma_z \vec{\psi}$$

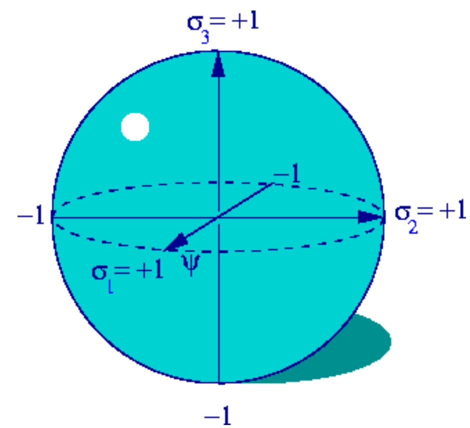


Gapped

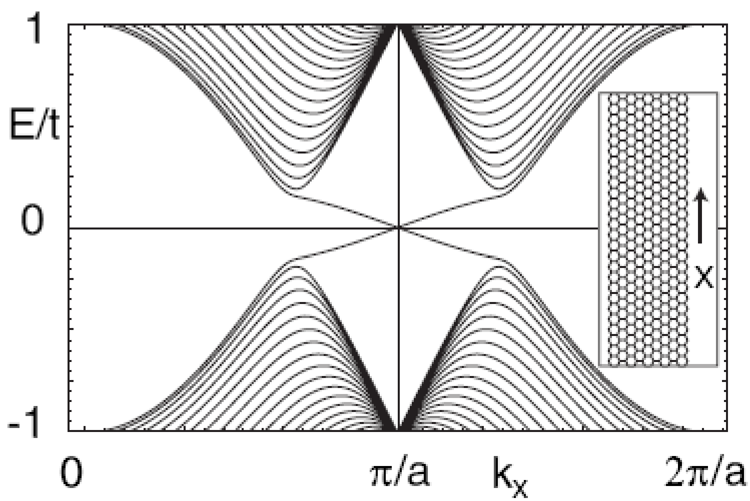
Simple model:

$$H = \begin{pmatrix} \pm m \mp ak^2 & v_F\left(k_x - ik_y\right) \\ v_F\left(k_x + ik_y\right) & \mp m \pm ak^2 \end{pmatrix} \equiv h(\vec{k})\begin{pmatrix} n_z(k) & n_x(\vec{k}) - in_y(\vec{k}) \\ n_x(\vec{k}) + in_y(\vec{k}) & -n_z(k) \end{pmatrix}$$

n wraps around
Bloch sphere once
□ Chern number □ 1



…so expect edge states!



Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele

PRL **95**, 226801 (2005)

But: in practice gap appears to be unobservably small

$$\Delta_{SO} \sim 5 \text{ mK}$$

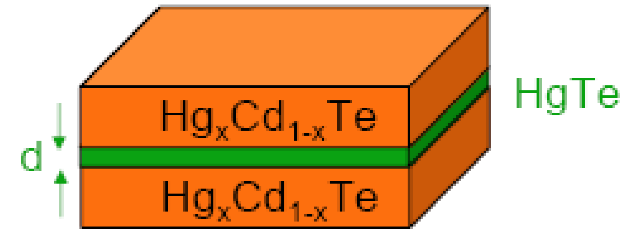
Min, et al. ’06, Yao et al. ‘06

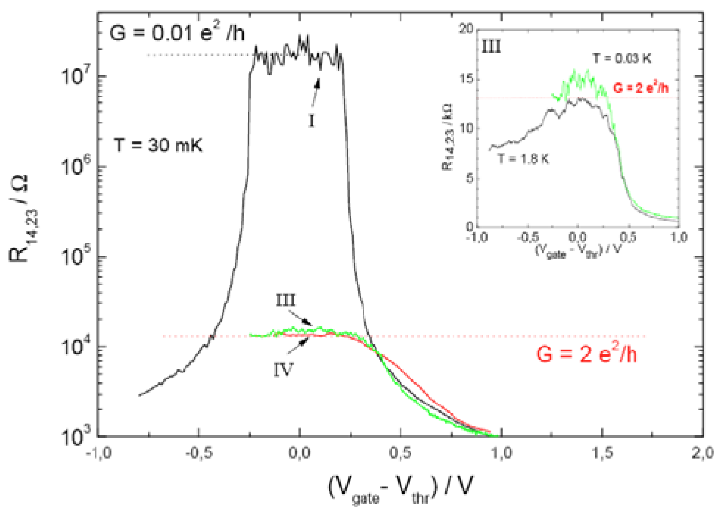
Analogous behavior observed in other materials:

HgTe/CdTe Heterostructure (Bernevig, Hughes, Zhang, Science 06)

HgTe has inverted bandstructure at Γ
2D Quantum well exhibits QSH phase

$2\,\Delta_{so} \sim$ 200 K for d ~ 70 Å.

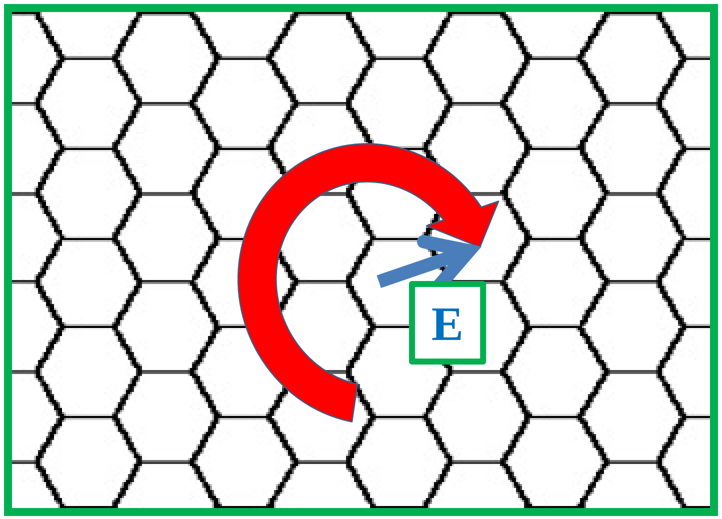




M. Konig et al., Science (2007)

Quantized Hall behavior
without magnetic field!

II. Non-zero Chern Numbers from Microwaves



Oka and Aoki, PRB (2009):

Consider electronic states in a rotating, spatially uniform electric field

Floquet theorem: If $H(t+T)=H(t)$, then □$(t) = e^{-i\varepsilon_{□} t}\, u(t)$ with u periodic in time. $\varepsilon_{□}$ are **quasienergies**.

-□/2 □ $\varepsilon_{□}$ □ □/2

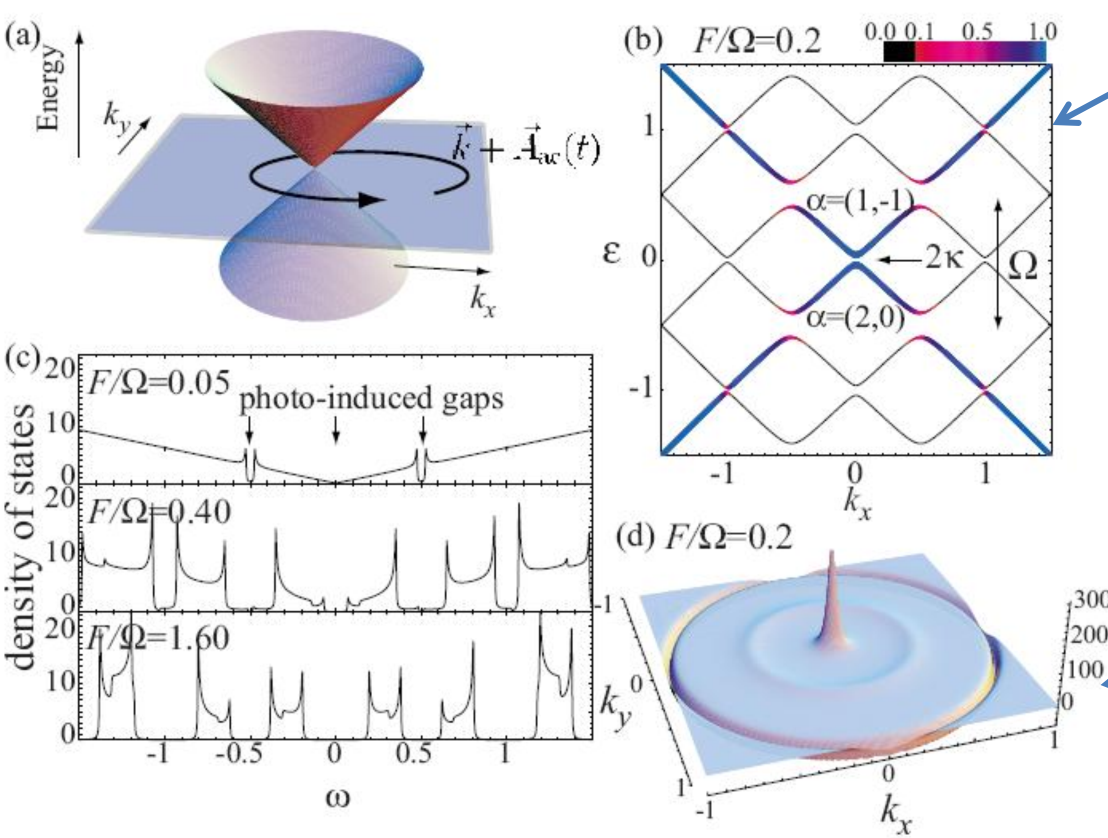
$$\vec{A}(t) = A_0(\cos\omega t, \sin\omega t) \Rightarrow H_0(\vec{k}) \rightarrow H_0(\vec{k} + \vec{A}(t))$$

$$[H_0(\vec{k} + \vec{A}(t)) - i\partial_t]u = \varepsilon_\alpha u \quad \Rightarrow \quad \varepsilon_\alpha \equiv \varepsilon_\alpha(\vec{k})$$

Floquet spectrum has a band structure

Quasienergy spectrum is gapped at k=0!

$$\psi \propto e^{-i\varepsilon_\alpha t}\begin{pmatrix} 1 \\ (\varepsilon_\alpha / E_0)e^{i\omega t} \end{pmatrix} \qquad \varepsilon_{\alpha,\pm} = \tfrac{1}{2}\left(\omega \pm \sqrt{4A_0^2 + \omega^2}\right) \Rightarrow \Delta = \sqrt{4A_0^2 + \omega^2}$$



Spectrum in extended zone scheme

□ = □
2□ = □
F/□ = A_0

Berry curvature

Berry Curvature

dc Hall Conductivity

$$\sigma_{xy}(\boldsymbol{A}_{\mathrm{ac}}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_{\alpha} f_{\alpha}(\boldsymbol{k}) [\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_{\alpha}(\boldsymbol{k})]_z$$

Occupation

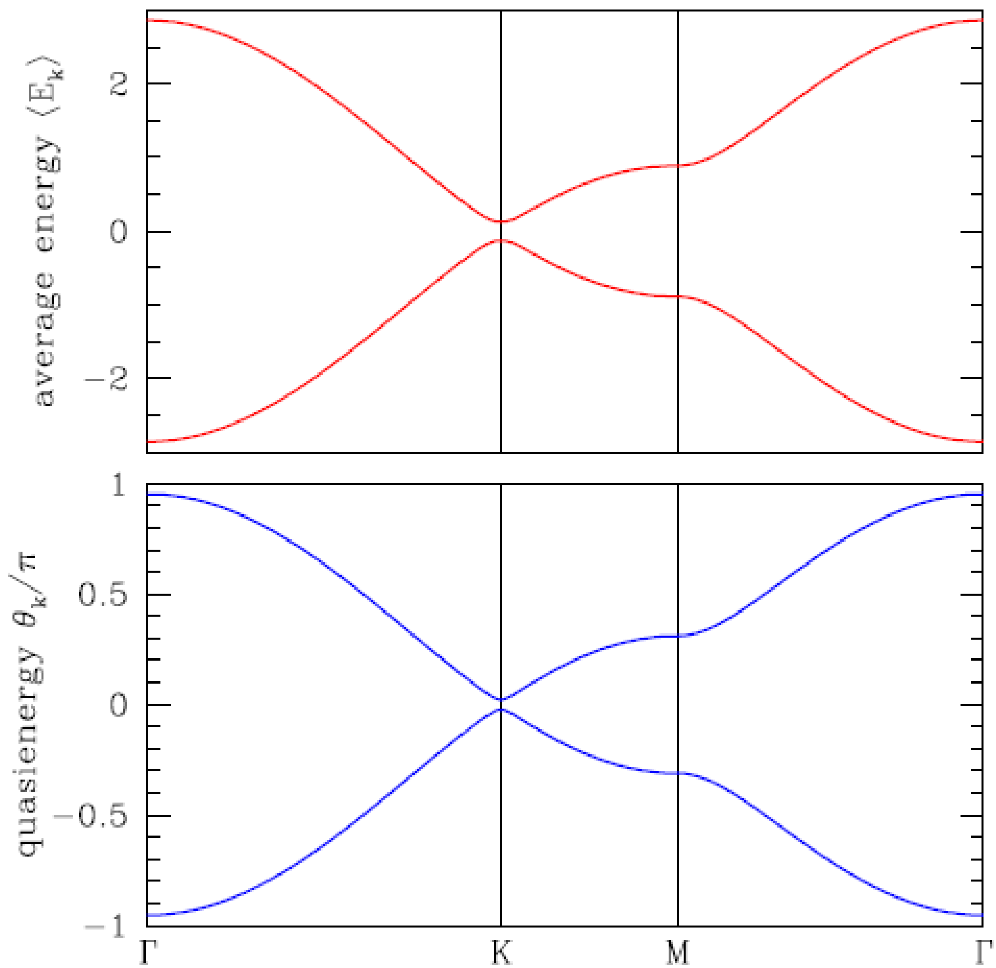
Berry curvature

$$\boldsymbol{\mathcal{A}}_{\alpha}(\boldsymbol{k}) \equiv -i \langle\langle \Phi_{\alpha}(\boldsymbol{k}) | \nabla_{\boldsymbol{k}} | \Phi_{\alpha}(\boldsymbol{k}) \rangle\rangle \,, \quad \psi_{\alpha}(t) = e^{-i\varepsilon_{\alpha} t} \Phi_{\alpha}(t)$$

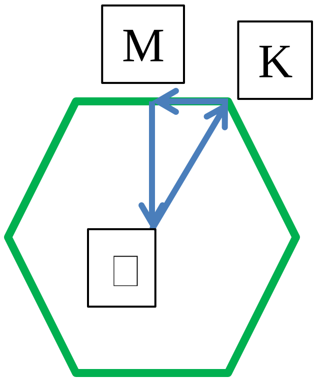
Time-averaged matrix element

- Hall conductance proportional to Chern number, if one band fully occupied
- So obtain something like quantized Hall conductance!
- Must deal with occupation factors. Can we use Fermi-Dirac with quasienergies instead of real energies?

Chern numbers in tight-binding calculations



$$\theta_k = \varepsilon_\alpha\left(\vec{k}\right)T$$

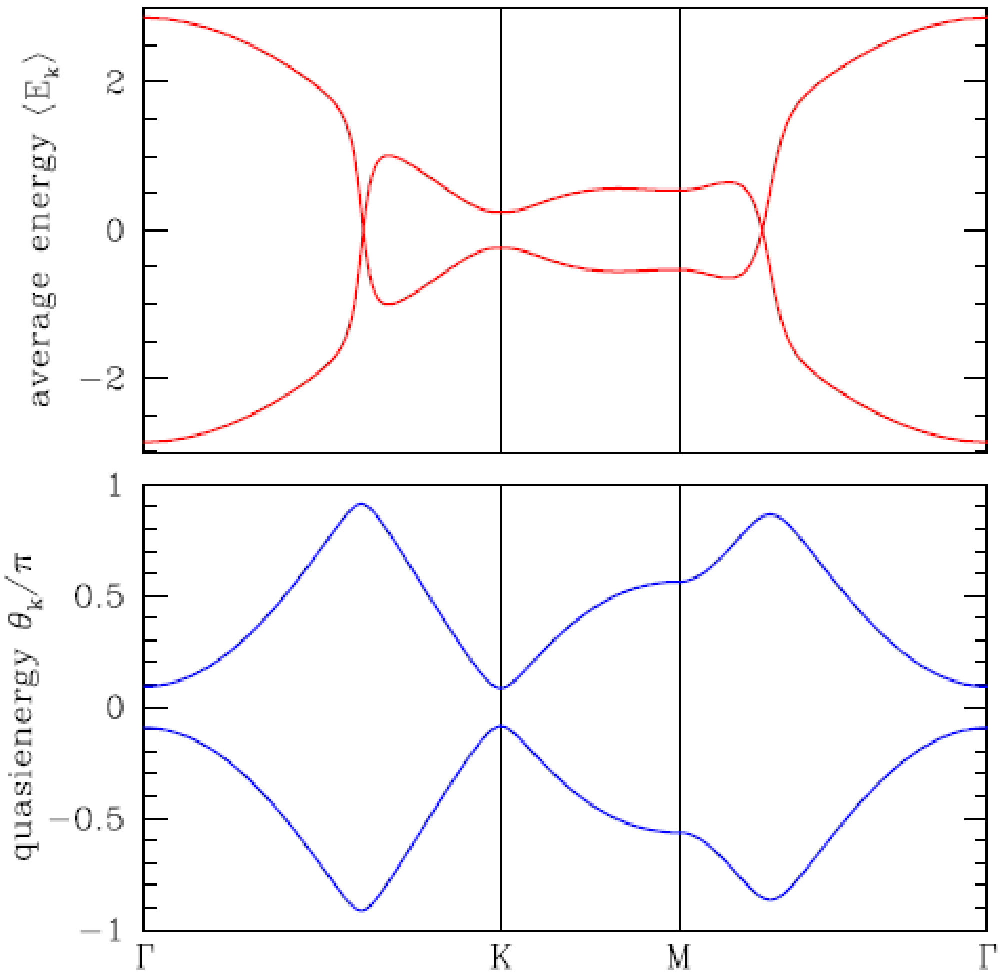


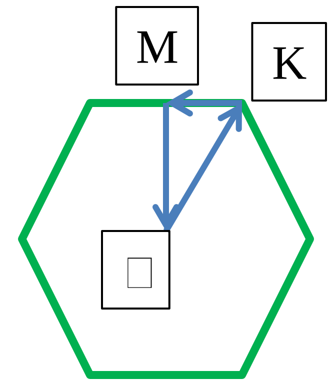
$$\omega = 6t$$

$$A_0 a = 0.75$$

Chern number for lower band = +1

$$\theta_k = \varepsilon_\alpha\left(\vec{k}\right)T$$





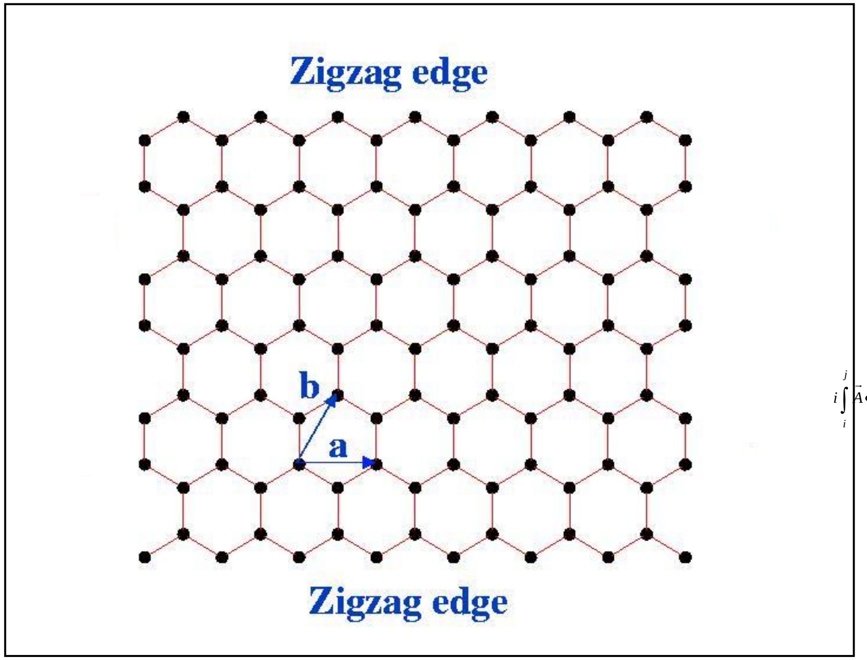
$$\omega = 3t$$

$$A_0 a = 0.75$$

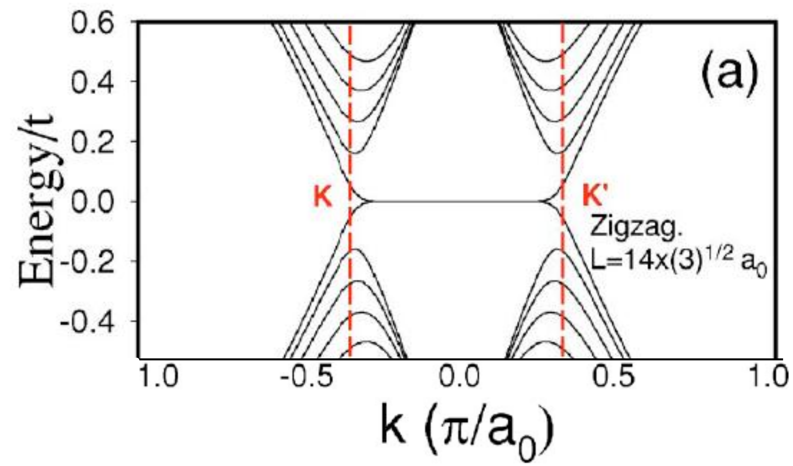
Chern number for lower band = +3

III. Edge States and Transport

Even without microwaves, graphene ribbons support edge states. But not current-carrying.



$i\int_i^j \vec{A}\bullet d\vec{\ell}$

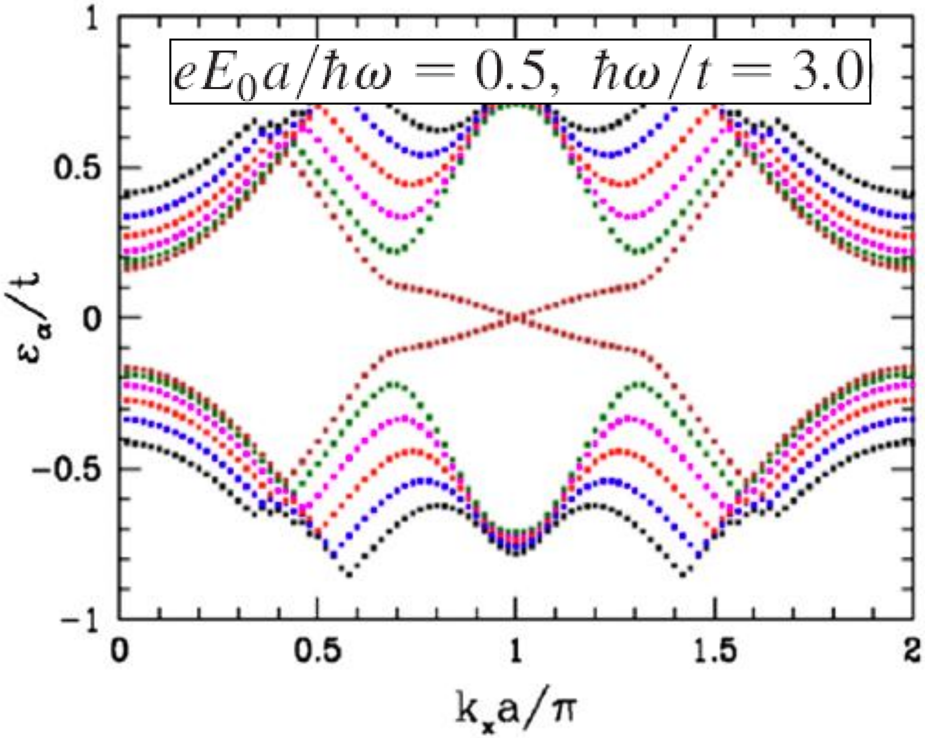


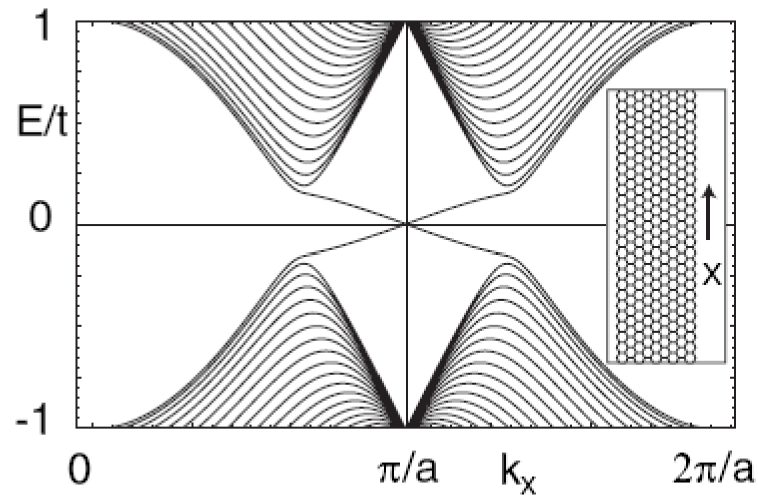
To compute (Floquet) spectrum with microwaves,

$$t_{ij} \rightarrow t \exp\left(i\int_i^j \vec{A}\bullet d\vec{\ell} \right)$$

Expand $H_F = H_0(t) - i\partial_t$ in Fourier harmonics in *x* and *t*, sites in *y*

□ Floquet spectrum (quasienergies) in extended zone scheme



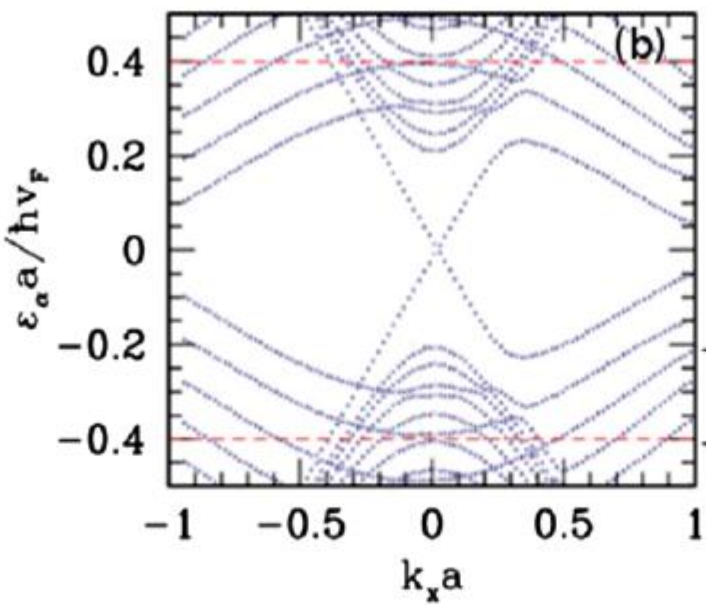


Current carrying states emerge…   …just as in TI spectrum.

Is edge state structure robust with respect to boundary conditions?

Continuum model with "infinite mass boundary conditions"
(valley preserving)

$$\psi_A(x, y=0) = \psi_B(x, y=0) \qquad \psi_A(x, y=W) = -\psi_B(x, y=W)$$



K

So can consider transport via edge states in ribbon geometry.
But what are their occupations?

Since leads have no microwaves, can define transmission matrix connecting transverse modes among various leads

$T_{pq}(E, E+E_n) =$ Probability of transmission from mode p to q absorbing n photons

Current out of mode p

$$I_p = 4\pi e \int_{-\infty}^{\infty} dE \left\{ f_p(E) - \sum_{n,q} T_{pq}(E+E_n, E) f_q(E) \right\}$$

Current out of left lead

$$I^{(L)} = 4\pi e \int_{-\infty}^{\infty} dE \left\{ \sum_p f_p^L(E) - \sum_{n,q,p} T_{pq}^{LL}(E+E_n, E) f_q^L(E) - \sum_{n,p,q} T_{pq}^{RL}(E+E_n, E) f_q^R(E) \right\}$$

Current conservation + inversion symmetry

$$I^{(L)} = 4\pi e \int dE \sum_{p,q,n} T_{qp}^{LR}(E, E+E_n)[f_q^L(E) - f_q^R(E)] \Rightarrow G = 4\pi e^2 \sum_{p,q,n} T_{qp}^{LR}(E_F, E_F+E_n)$$

- Zero temperature conductance
- No charge pumping

Computing Transmission Probability

- Discrete time, intervals □t
- Lead states analytically known, Floquet eigenvalues

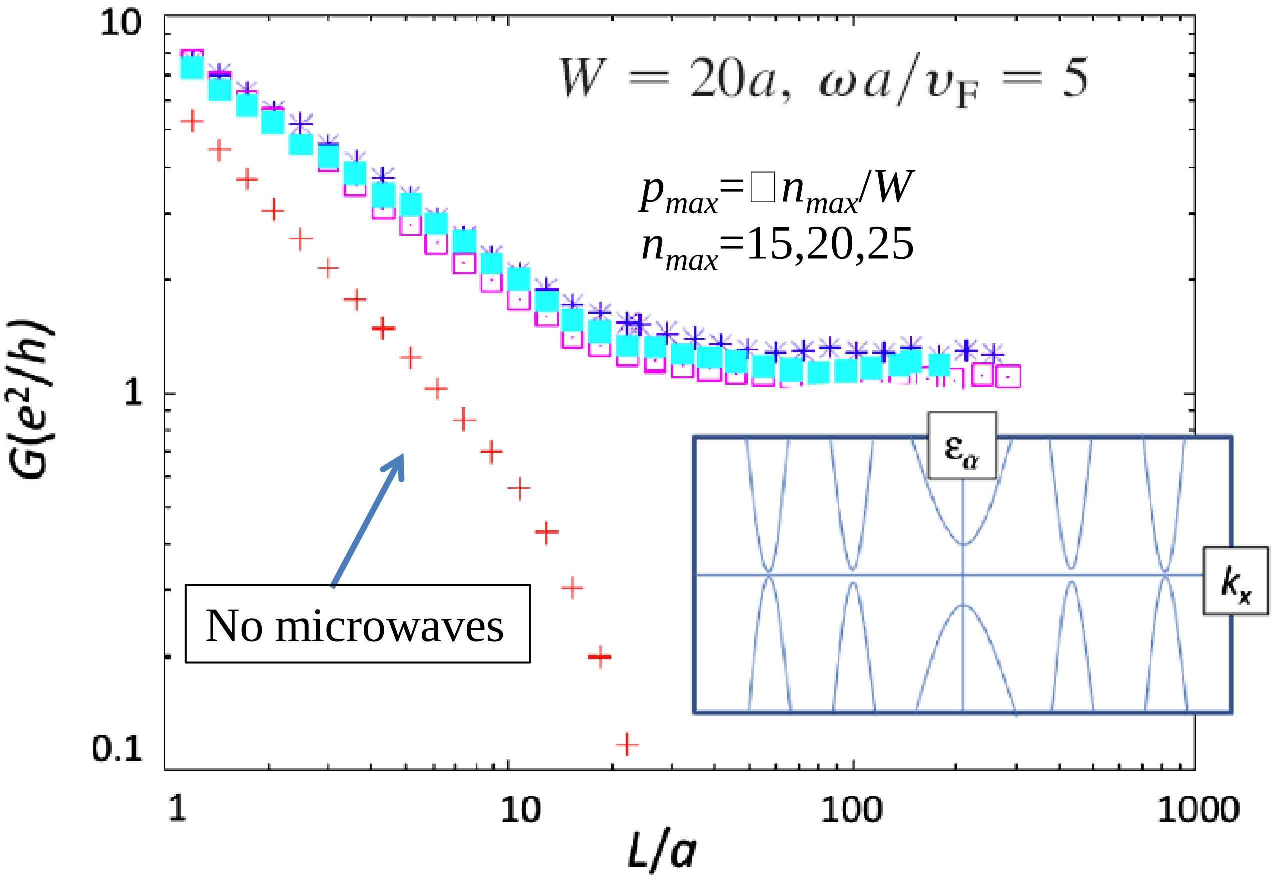
$$\varepsilon_{\alpha} = \varepsilon_{p,n}(k_x) = \pm v_F \sqrt{k_x^2 + k_p^2} + E_n; \quad E_n = \sin(n\omega\Delta t)/\Delta t$$

- Match this to states in scattering region with same Floquet eigenvalue. These are state of the form □$(y,t)\exp(ik_x x)$ which satisfy

$$[(-i\Delta_t - \varepsilon_{\alpha})\sigma_x + i\hat{p}_y\sigma_z]\Phi(y,t) = k_x\Phi(y,t)$$

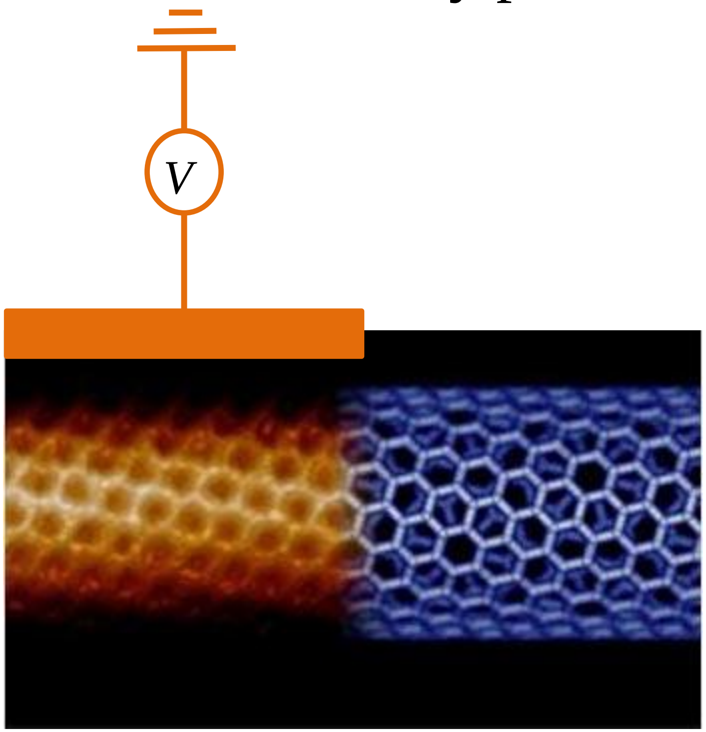
- Real space continuum model, infinite mass BC, use H for K valley
- Note operator is non-Hermitian. k_x may be imaginary □ evanescent waves
- For given state (p,n=0) on left, match at x=0 and x=L to find transmission amplitude $t_{pq}^{LR}(\varepsilon, \varepsilon + E_n)$
- Finally $T=|t|^2$.

Results



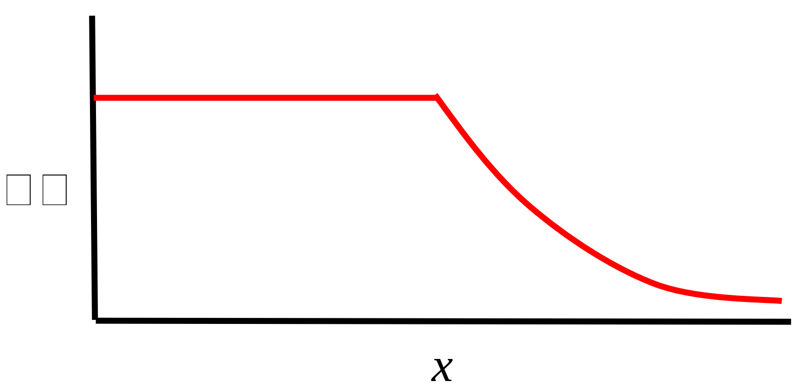
- Microwaves always increase conductance
- Edge state transport for large L
- Power law, superdiffusive behavior!

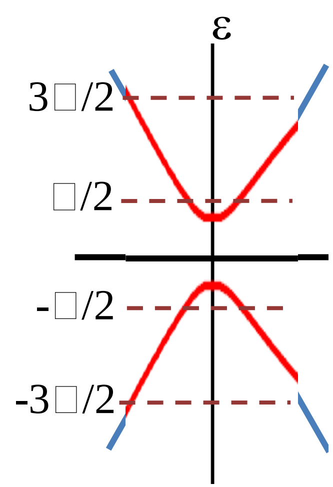
IV. Why power law behavior for $L < W$?



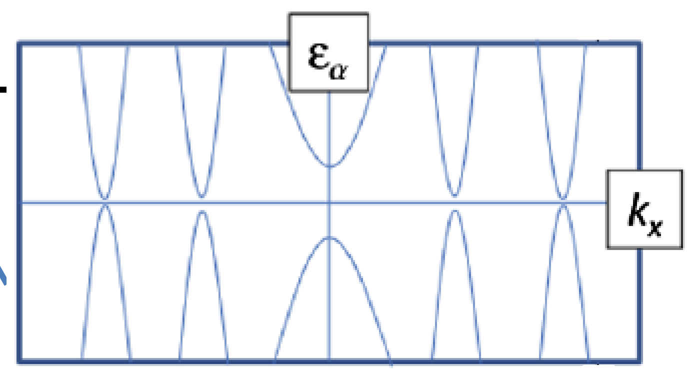
Simpler question:

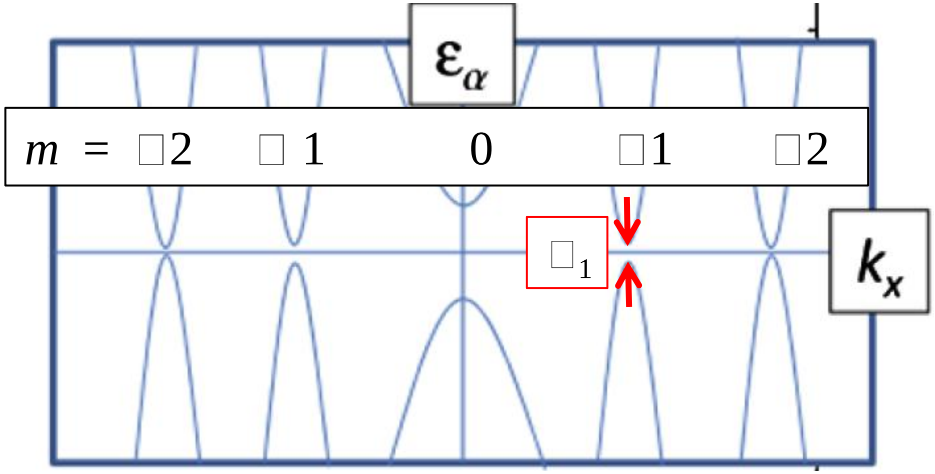
How does density decay in a nanotube geometry where half is held fixed at high density and half undoped but subject to microwaves?





With microwaves,
□ < band width





- For each k_y, evanescent states at ε□=0
- Anticrossings of ε(k_x)+m□
- Decay length □$_m$ ~ 1/□$_m$
- Excess density due to occupation of these states

<u>To estimate gaps</u>: Define Floquet Green's function

$$\hat{G} = (z - \hat{H}_F)^{-1}$$

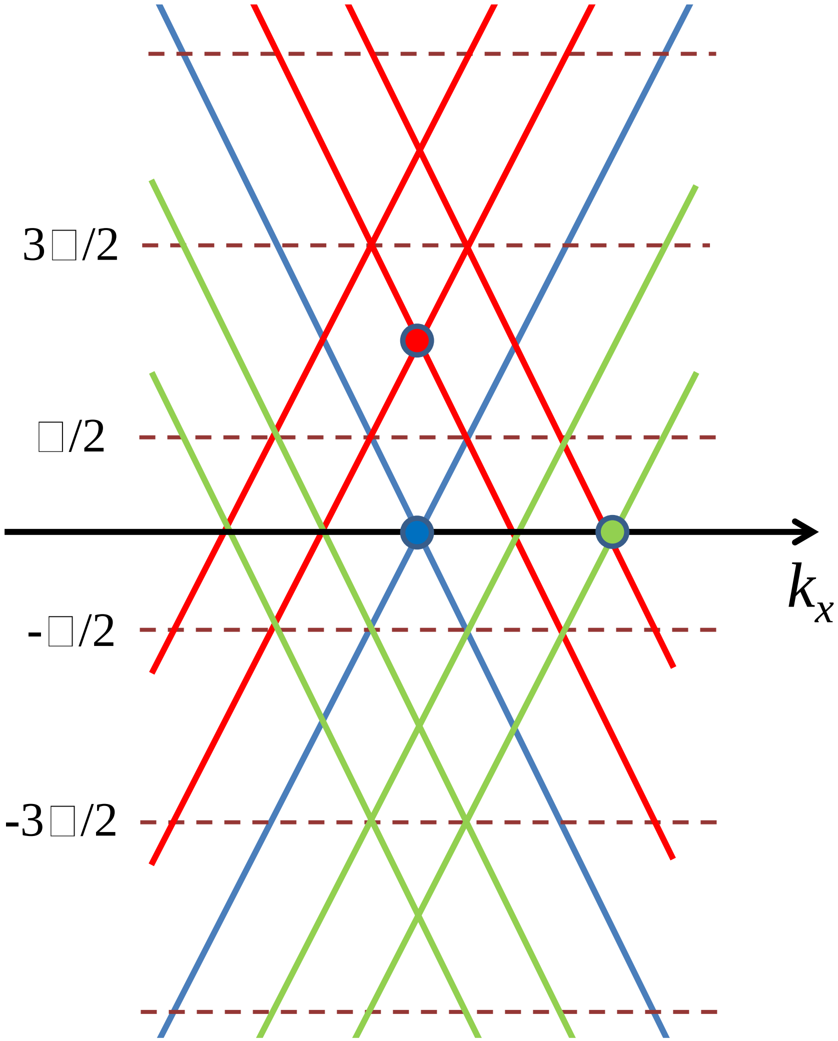
and treat microwaves perturbatively: $\hat{V} = A_0[e^{-i\omega t}\sigma_+ + e^{i\omega t}\sigma_-]$

$$\hat{G} = [z - \hat{H}_{F,0} - \Sigma(k_x, k_y, z)]^{-1}$$

$H_{F,0}$=v_F**k**□□-i□$_t$
□=Self-energy

k_x determined by $(z + m\omega)^2 = v_F^2(k_x^2 + k_y^2)$ with z=0.

$$\xi_m = 1/\Delta_m = 1/2\Sigma_m$$



Perturbation

$$\hat{V} = A_0[e^{-i\omega t}\sigma_+ + e^{i\omega t}\sigma_-]$$

admixes Floquet copy *m*□ with (*m*□1)□

□ □□ $(A_0)^{2m} / (m□)^{2m\text{-}1}$

Gap shrinks very rapidly with *m*
Decay length grows very rapidly with *m!*

$$\psi\left(k_x = \sqrt{V_0^2 / v_F^2 - k_y^2}, k_y\right) \rightarrow$$

$$\leftarrow \sum_m r_m \psi\left(k_x = -\sqrt{(V_0 + m\omega)^2 / v_F^2 - k_y^2}, k_y\right)$$

$$x < 0$$

$$\sum_m t_m \psi\left(k_x = i\sqrt{\Delta_m(k_y) / v_F^2}, k_y\right)$$

$$x > 0$$

- r's and t's linearly related by matching coeffs of $e^{□in□y}$ for both sublattices
- Include first order corrections to □ due to A_0
- Obtain recursion relations for t_{n+1} in terms of t_n and t_{n-1}

Large m $\Rightarrow t_m \sim \left(\dfrac{A_0}{\tau\omega}\right)^m$ (□ = constant of order unity)

Time-averaged density on irradiated side

$$\delta\rho \sim \sum_{m,k_y} \left|t_m\right|^2 e^{-\Delta_m x / v_F}$$

Note competition between increasing localization length and decreasing amplitude

Assume sum dominated by a single value of *n*

$$\delta\rho(x) \sim \exp\left\{2m_{\max}\ \ln\left(v_F A_0 / \omega\tau\right) - x/\xi_{m_{\max}}\right\}$$

$$m_{\max} \approx \frac{\ln(\omega x / v_F)}{-2\ln(A_0 v_F / \omega) + C} \quad \text{(for large } x\text{)}$$

$$\Rightarrow \delta\rho(x) \sim x^{-p}, \quad p = 2\ln(v_F A_0 / \omega\tau)/[\ln(v_F A_0/\omega) + const.]$$

+ ln(x) corrections

- Density falls off approximately as a power law with *x*
- Power is a weak function of □ and A_0
- Result of competition between rapidly increasing □$_{\mathrm{m}}$ and decreasing t_m with *m*

Summary

- Floquet spectrum of irradiated graphene has non-trivial topology similar to topological insulators
- Edge states, robust with respect to boundary conditions
- Can address question of occupation by including thermal reservoirs as leads
- Time averaged conductance increased by time-dependent potential
- For large *L* transport dominated by edge states
- For small *L* transport is dominated by evanescent states and is anomalous

Z. Gu, HAF, D. Arovas, A. Auerbach, PRL **107**, 216601 (2011)

Thank you!