Graphene Nanomechanical Quantum Hall Magnetometry

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Graphene Nanomechanical Resonators for Thermodynamic Measurements in the Quantum Hall Regime

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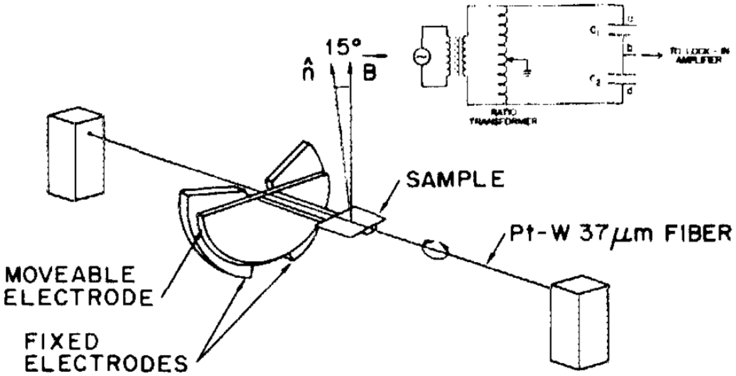
Thermodynamic Measurements in the QH Regime: probe of bulk properties and many-body effects

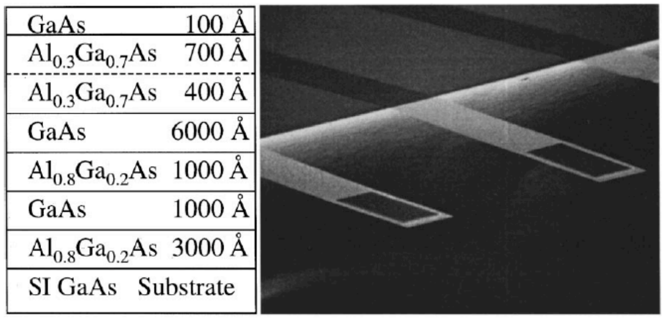
Magnetization

DC torque magnetometry



Eisenstein et al, PRL (1985)

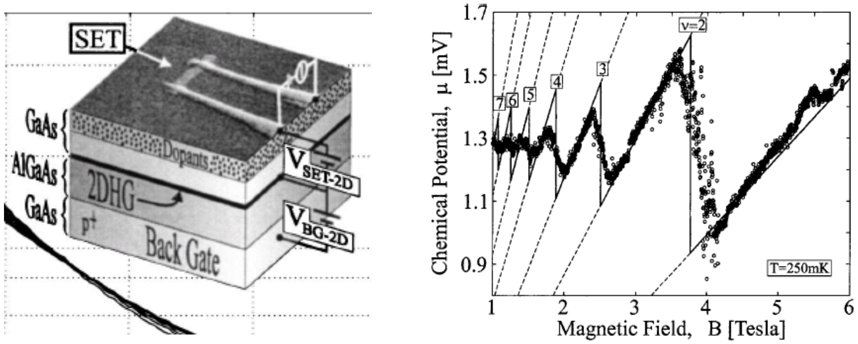
Oscillator torsional magnetometry



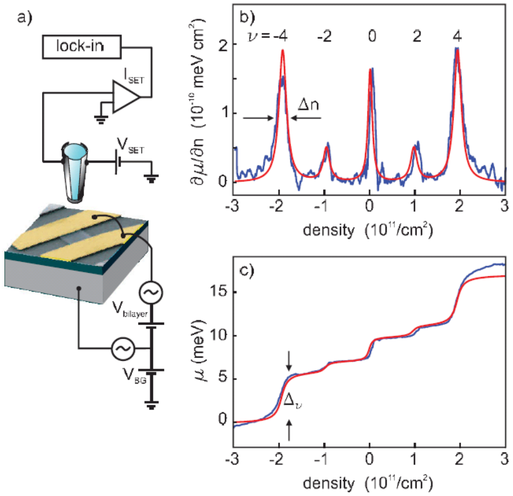
GaAs	100 Å
$Al_{0.3}Ga_{0.7}As$	700 Å
$Al_{0.3}Ga_{0.7}As$	400 Å
GaAs	6000 Å
$Al_{0.8}Ga_{0.2}As$	1000 Å
GaAs	1000 Å
$Al_{0.8}Ga_{0.2}As$	3000 Å
SI GaAs	Substrate

Harris et al, PRL (2001)

Chemical potential and compressibility



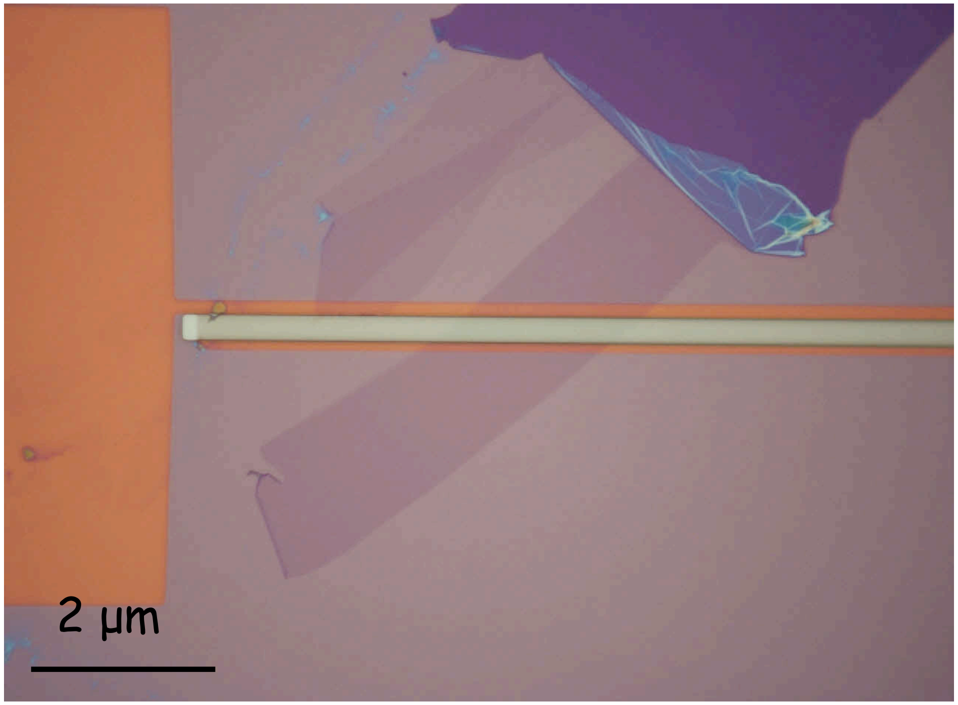
Ilani et al, PRL (2000)

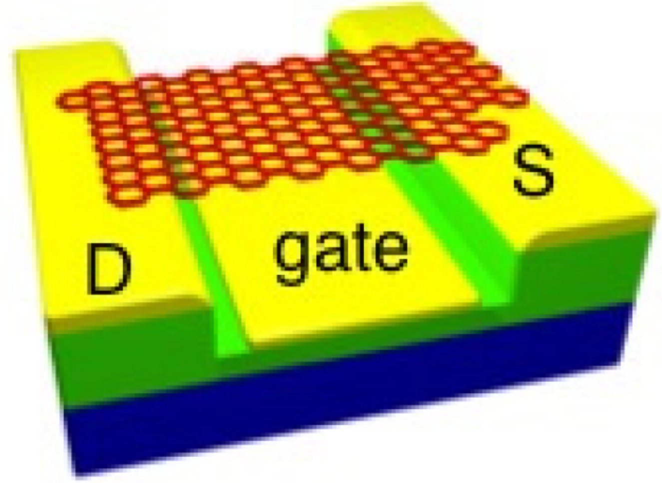


In suspended graphene:
Martin et al, PRL (2010)

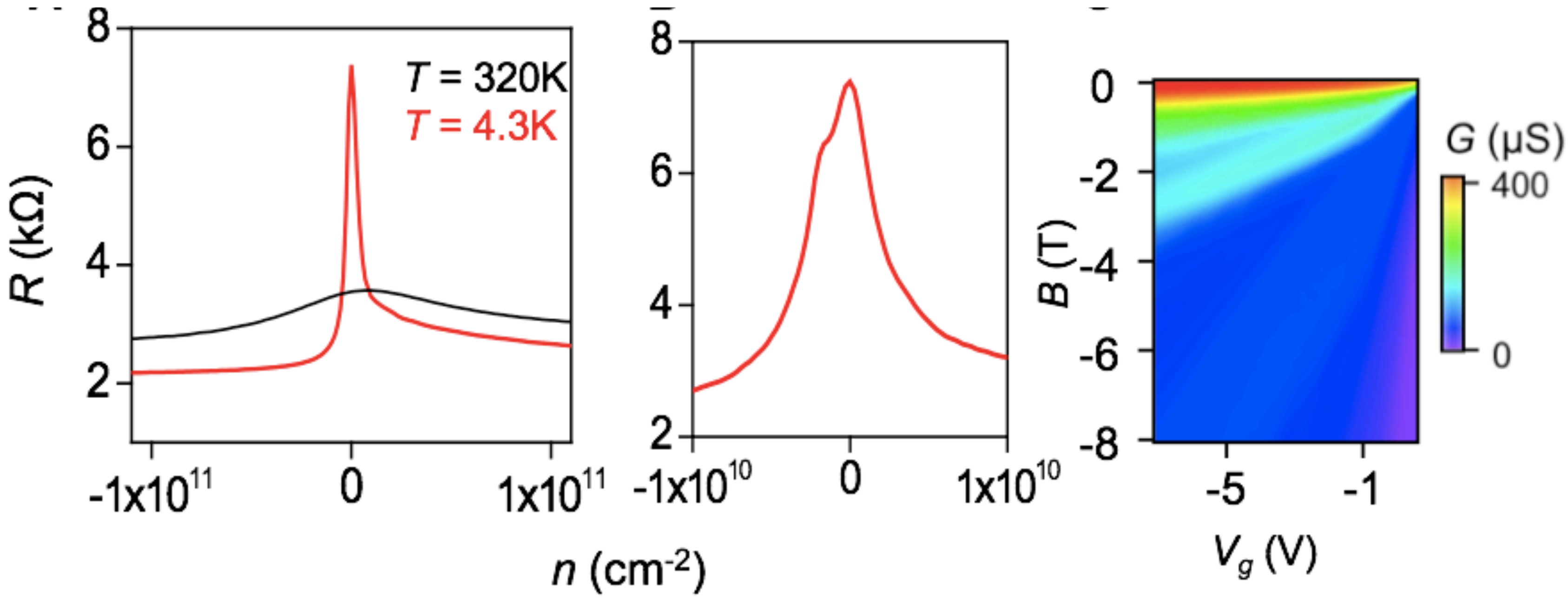
Ultraclean suspended graphene

Exfoliate directly onto patterned electrodes



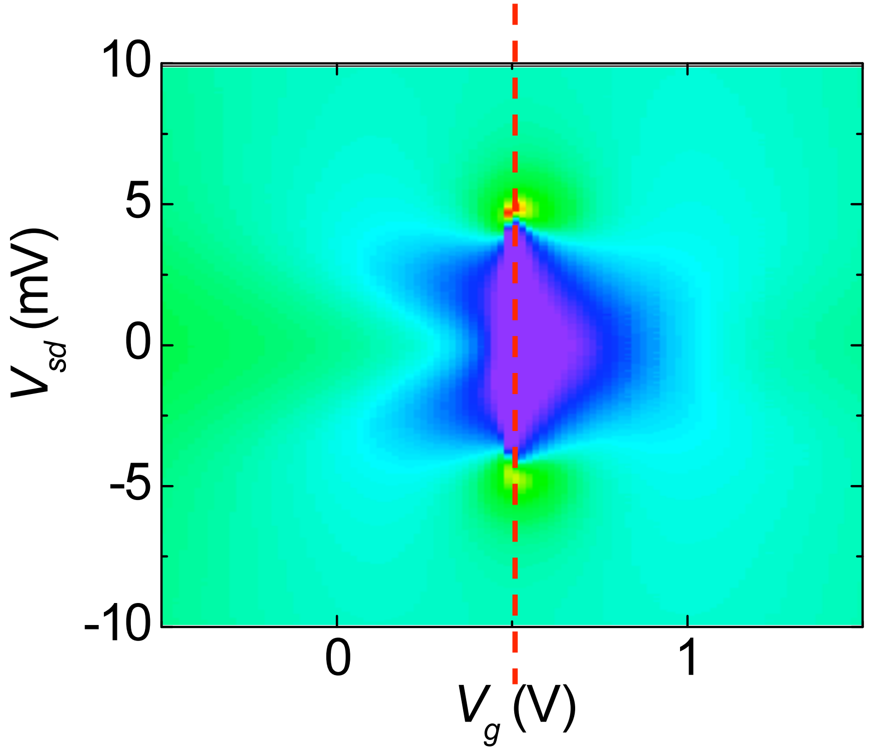


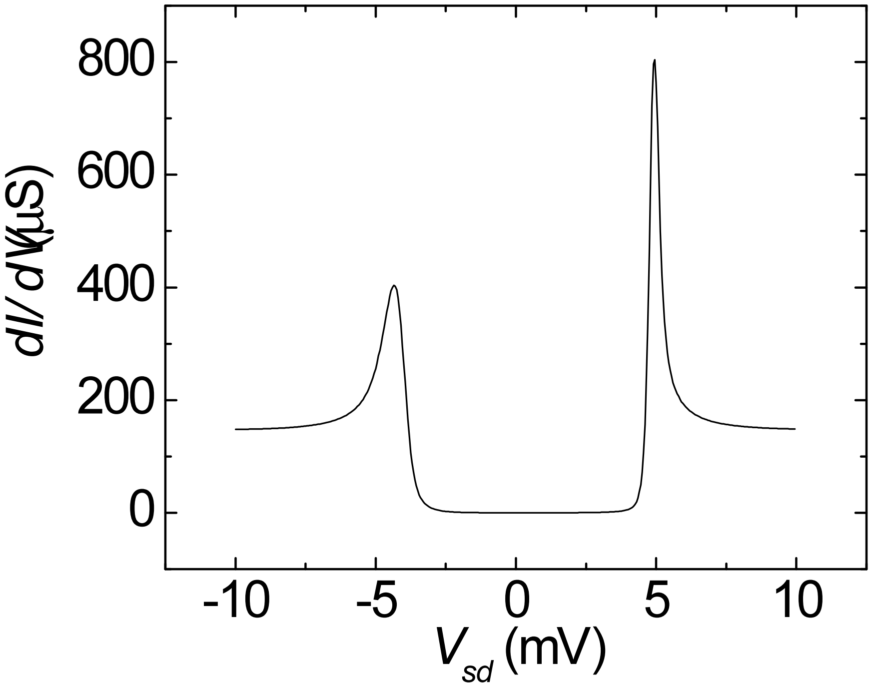
Sample characterization for QHE measurements



`	Charge inhomogeniety δn (cm^{-2})	Disorder potential ΔE (meV)
SiO_2 (many groups)	~10^{11}	~100
h-BN (Dean et al)	~4×10^{10}	~20
Suspended, under-etched (Bolotin et al, Du et al)	~10^{10}	~10
Suspended, resist-free (This work, Bao et al)	~2×10^{9}	~5-10

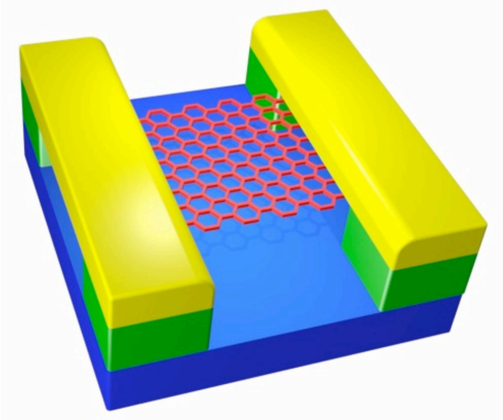
Energy gap in bilayer graphene

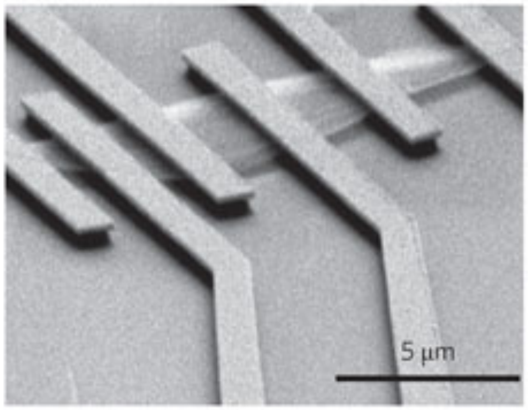


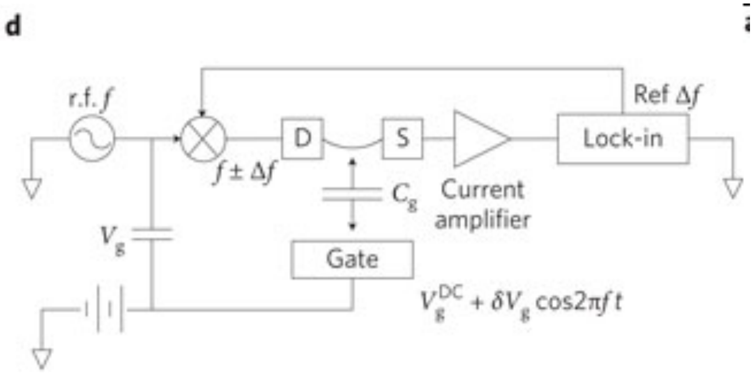


- Gap of ~4-5 meV in bilayer samples, from non-linear transport
- Temperature dependence fits a simply activated gap ~2meV
- No top gate: cannot rule out built-in electric field (unlikely)

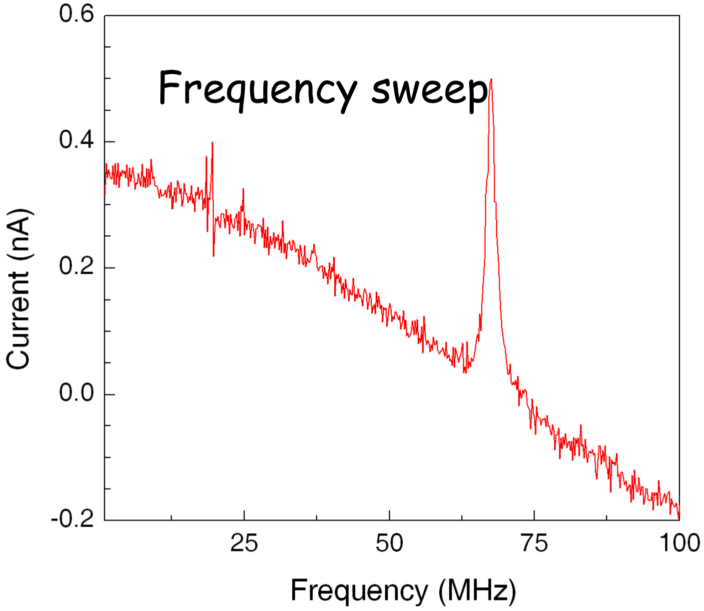
Graphene nanomechanical resonators – electrical mixing readout (old work)

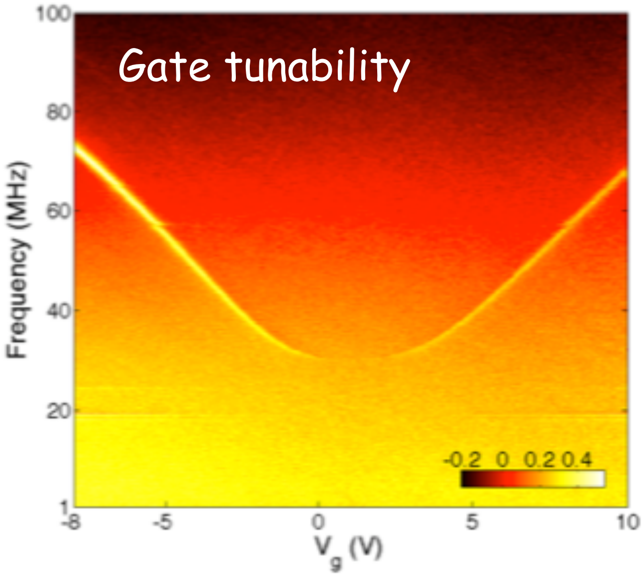




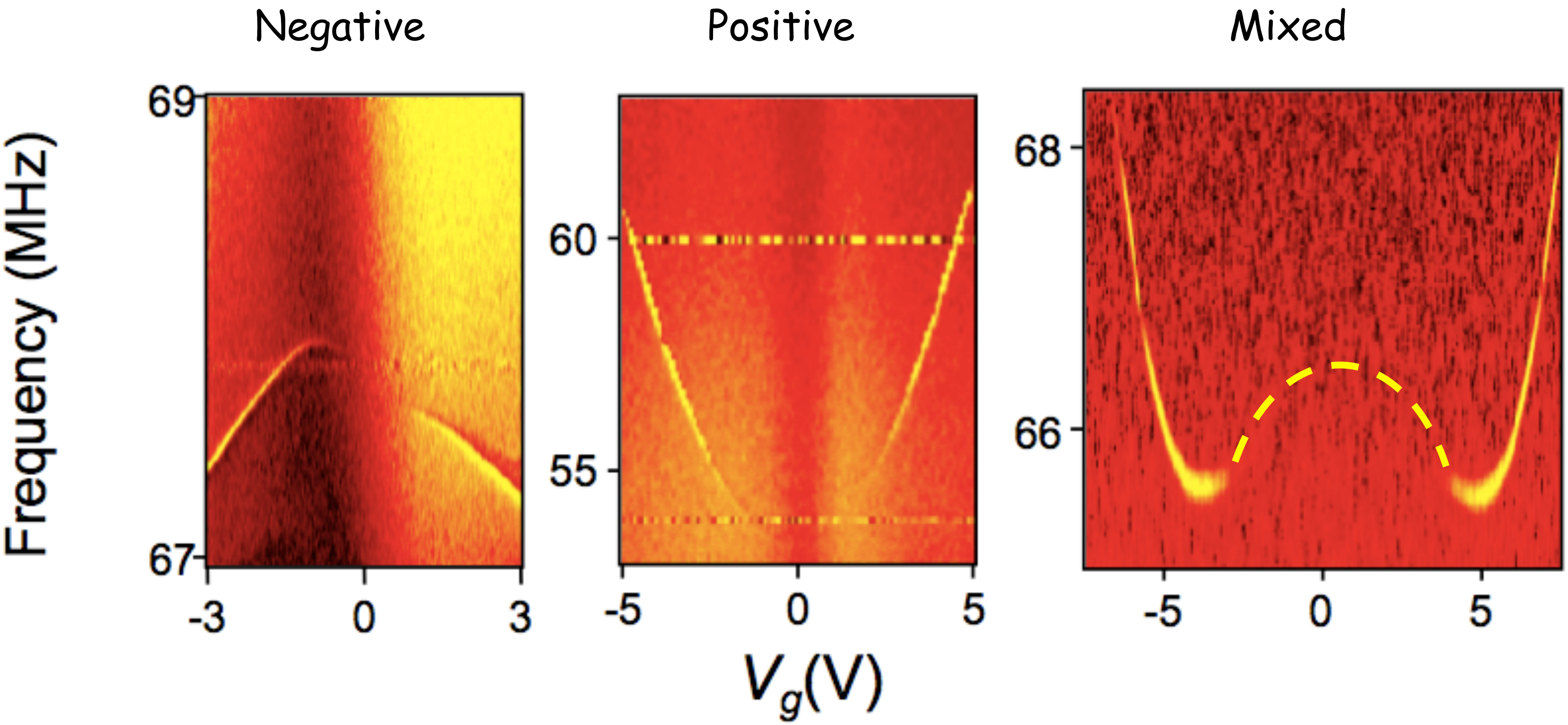


Changyao Chen, Sami Rosenblatt, Kirill I. Bolotin, William Kalb, Philip Kim, Ioannis Kymissis, Horst L. Stormer, Tony F. Heinz & JH, *Nature Nanotechnology* (2009).





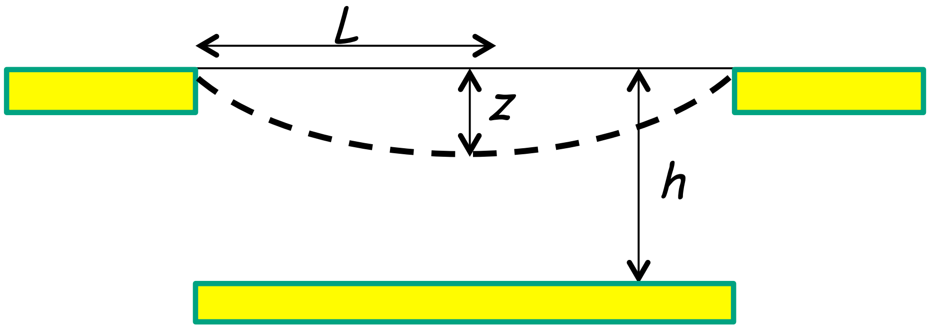
Gate tunability



See also: Deshmukh, Nanotechnology 2010

Mechanical model to explain dispersion

<u>1D string model</u>



$$U_{es} = -\frac{1}{2}V_g^2\left[C_0 + C'z + \tfrac{1}{2}C''z^2\right]$$

$$U_{elastic} = \frac{16z^2}{3L}\left[\frac{T_0}{2} + \frac{2ESz^2}{3L^2}\right]$$

x-sectional area

Built-in tension

Young's modulus

<u>Equilibrium displacement:</u>

$$F_{static} = -\frac{\delta U_{total}}{\delta z} = 0 \text{ at } z = z_0$$

<u>Spring constant:</u>

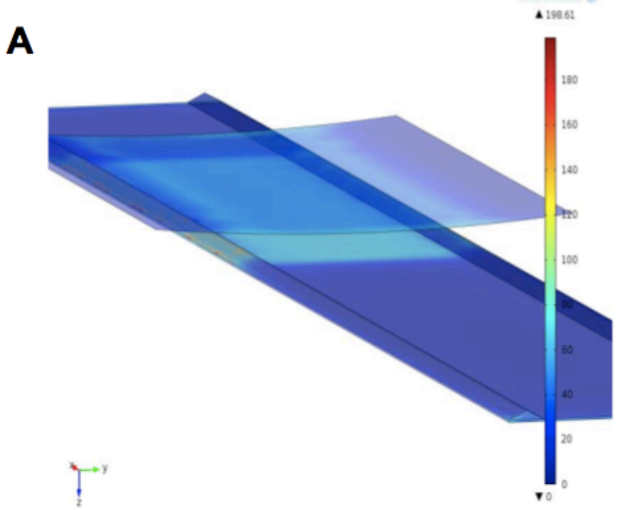
$$k = \frac{\partial^2 U}{\partial z^2} = \frac{16T_0}{3L} + \frac{128ES}{3L^3}z_0^2 - \frac{1}{2}C''V_g^2$$

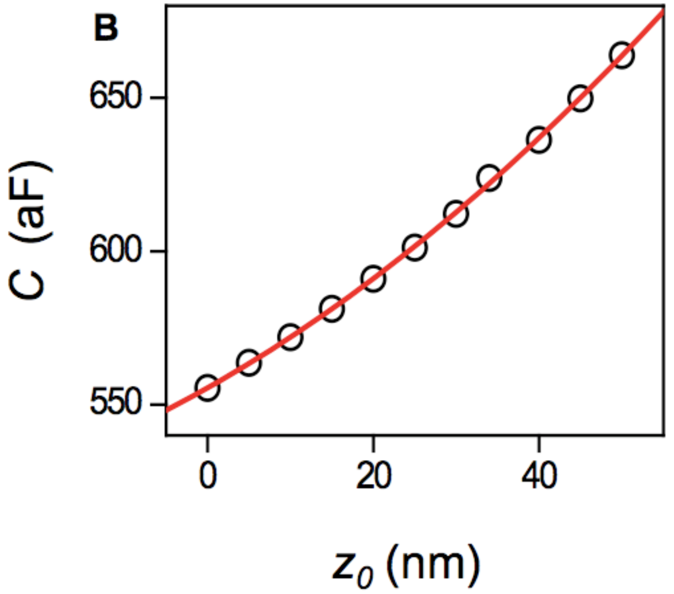
Stiffening with V_g

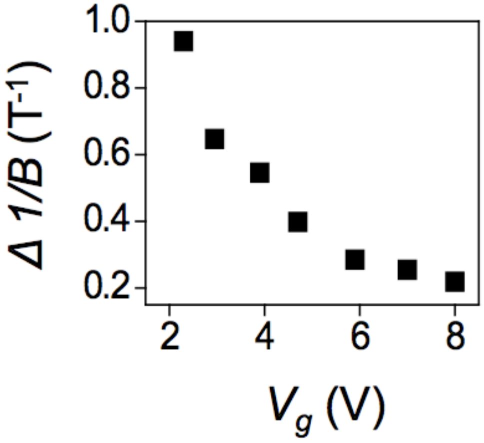
Softening with V_g

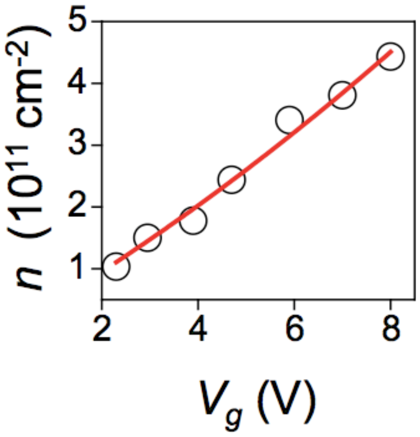
Mechanical model to explain dispersion

Calculate C_0, C', C'' using finite element model.





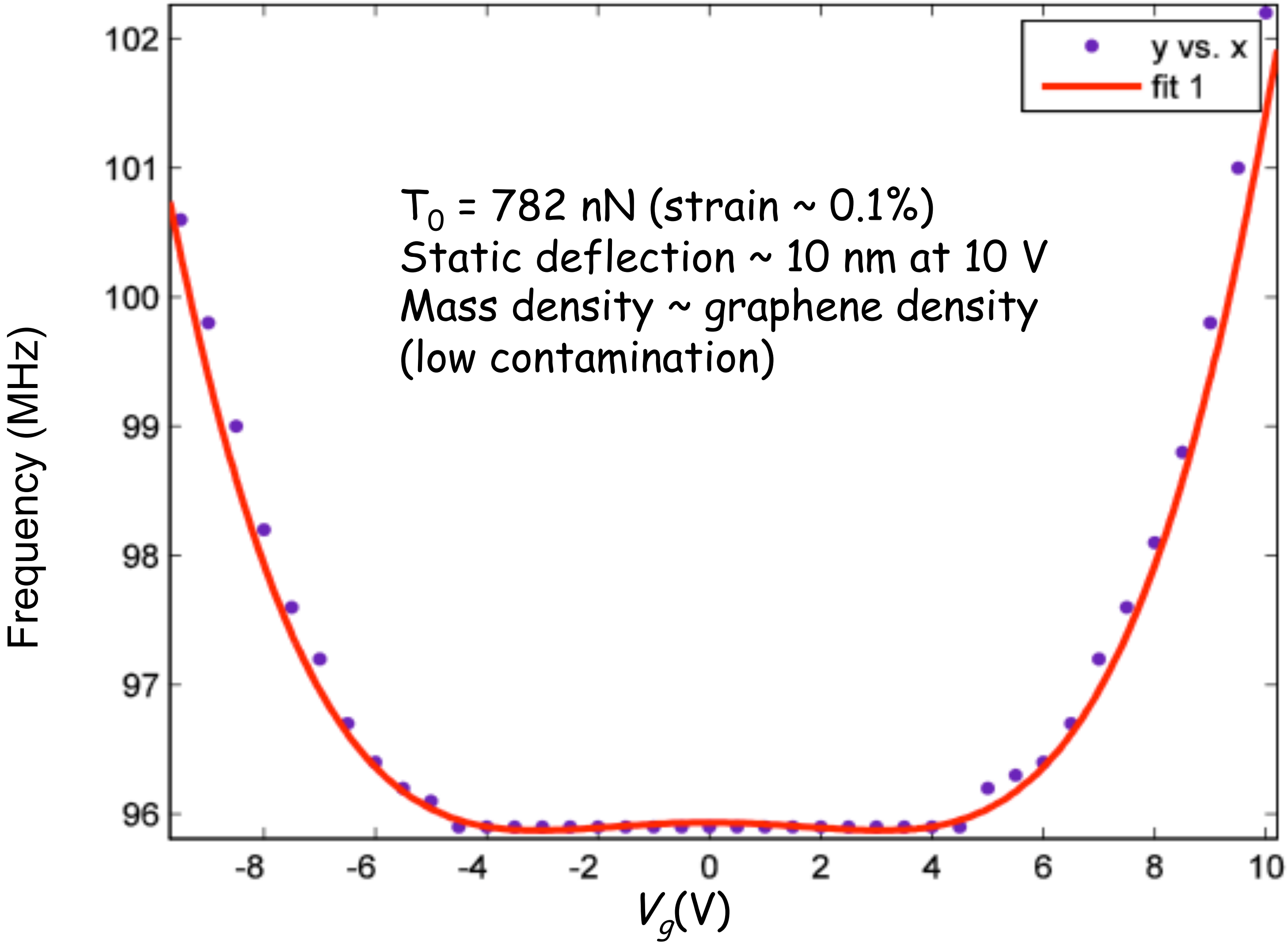




Compare to C_0 from SdH Oscillations

$$\Delta\left(\frac{1}{B}\right)=\frac{2e}{\hbar\pi n}$$

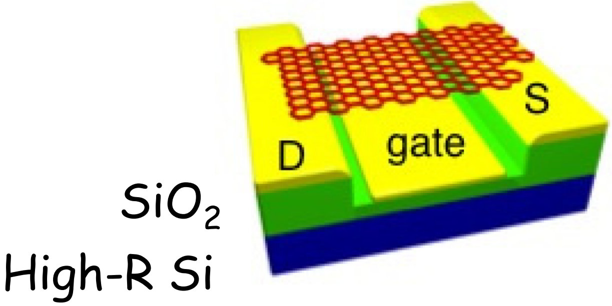
Mechanical model to explain dispersion

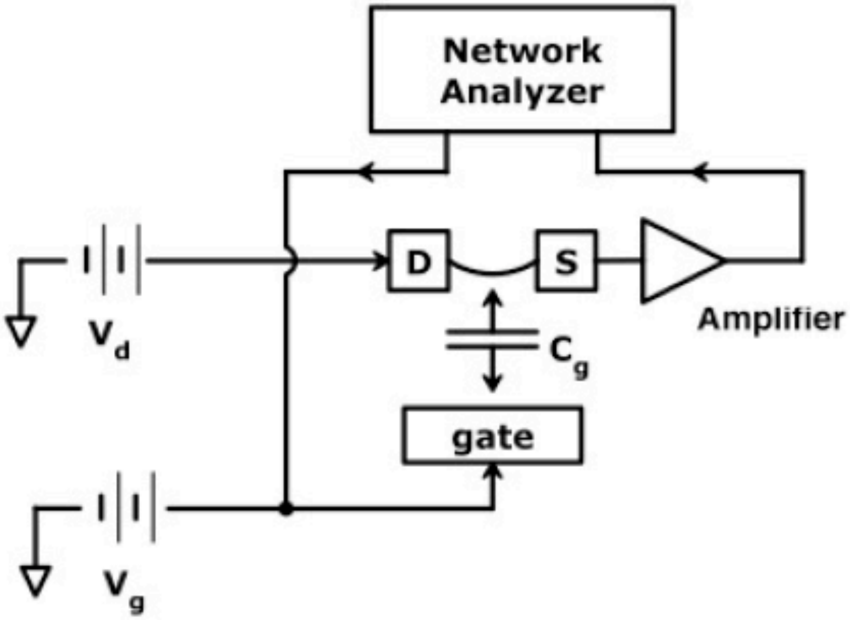


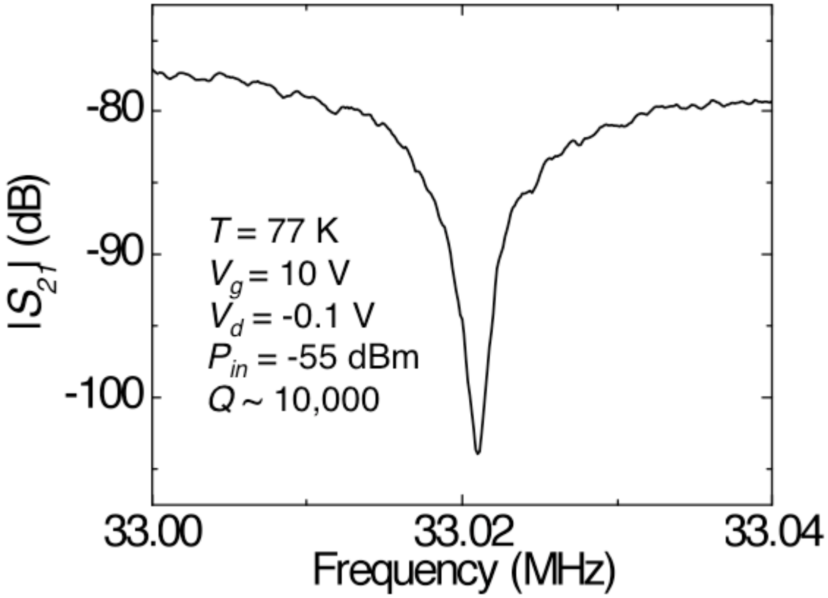
T_0 = 782 nN (strain ~ 0.1%)
Static deflection ~ 10 nm at 10 V
Mass density ~ graphene density
(low contamination)

Direct RF readout

The resonant channel transistor (RCT)







Xu, Chen, Deshpande, JH APL 2010

Working of the RCT

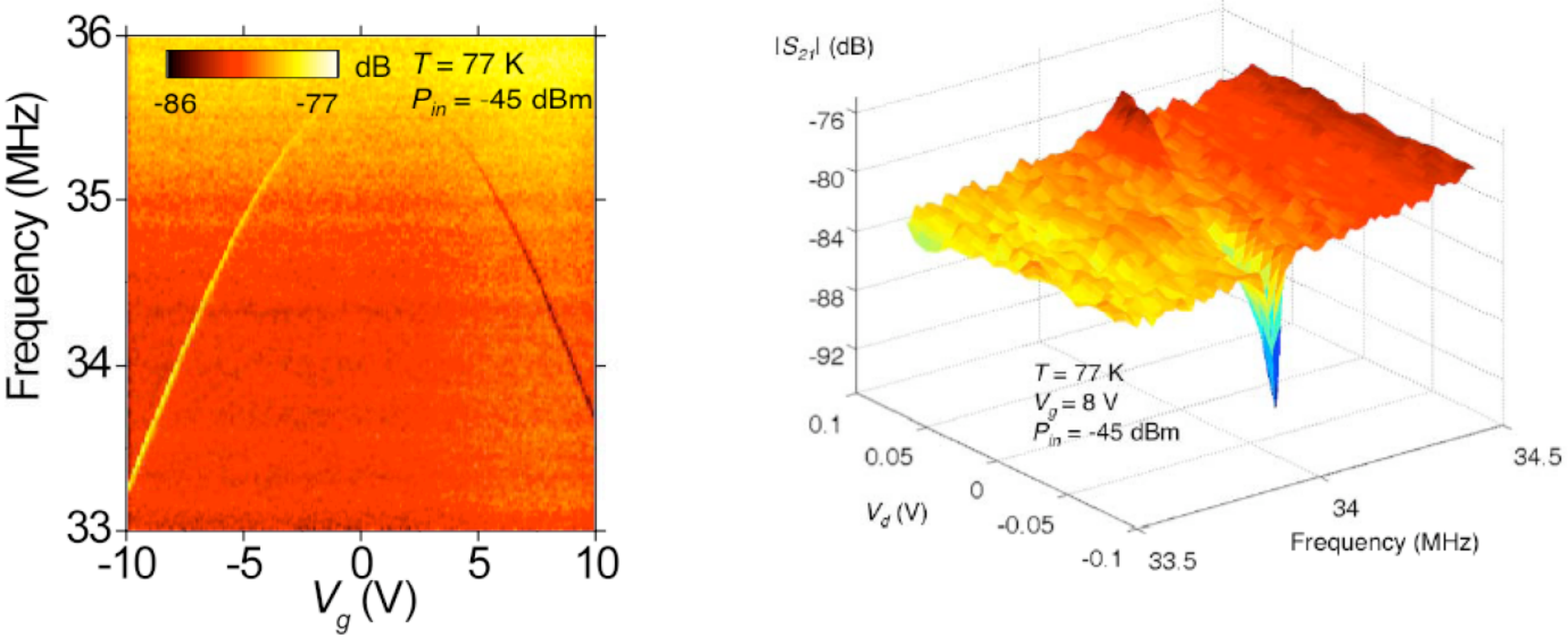
$$\tilde{I} = j\omega C_{tot}\tilde{V}_g \boxed{- j\omega\frac{\tilde{z}}{z_0}C_g V_g} + V_d\frac{dG}{dV_g}\tilde{V}_g \boxed{- V_d\frac{dG}{dV_g}\frac{\tilde{z}}{z_0}V_g}$$

capacitive background

purely capacitive mechanical signal

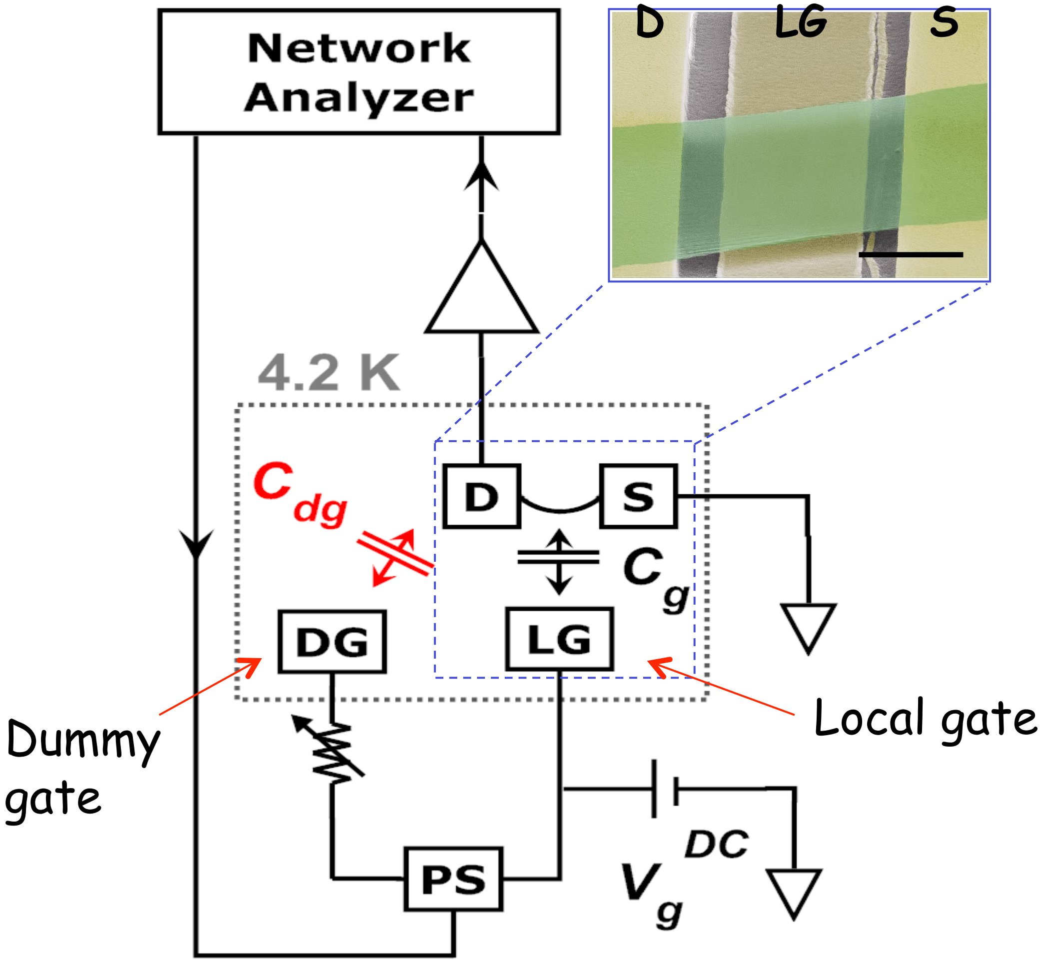
transistor rf background

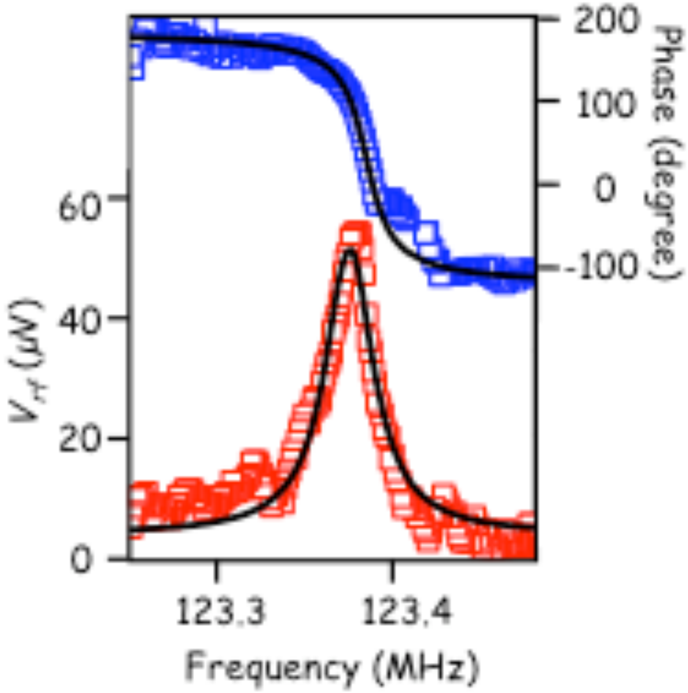
Transconductance-enhanced mechanical signal



Over two orders of magnitude faster than the mixing technique!

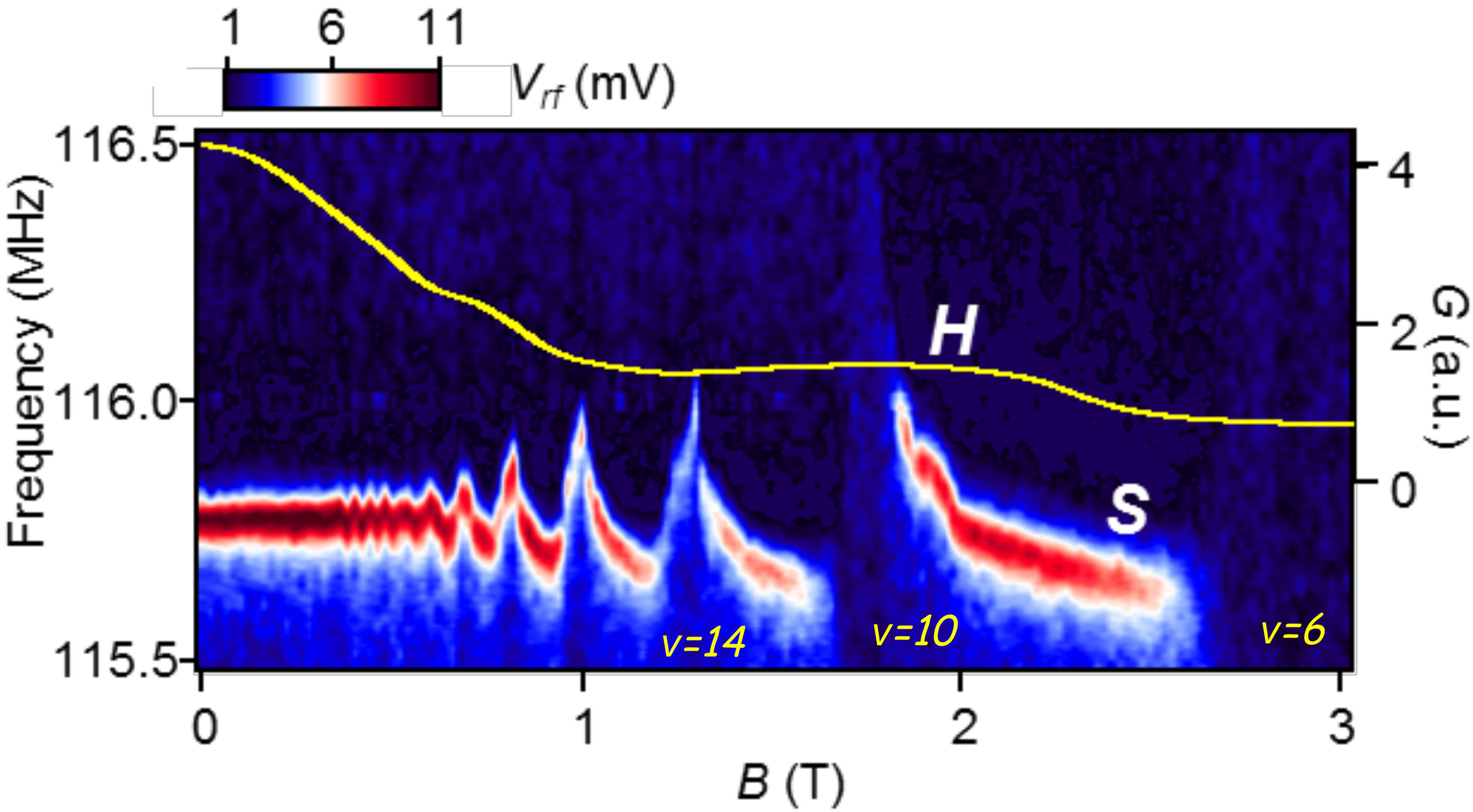
Purely Capacitive Readout





- Reduce background using balance bridge technique (based on work by Ekinci et al (2002) in Si NEMS)

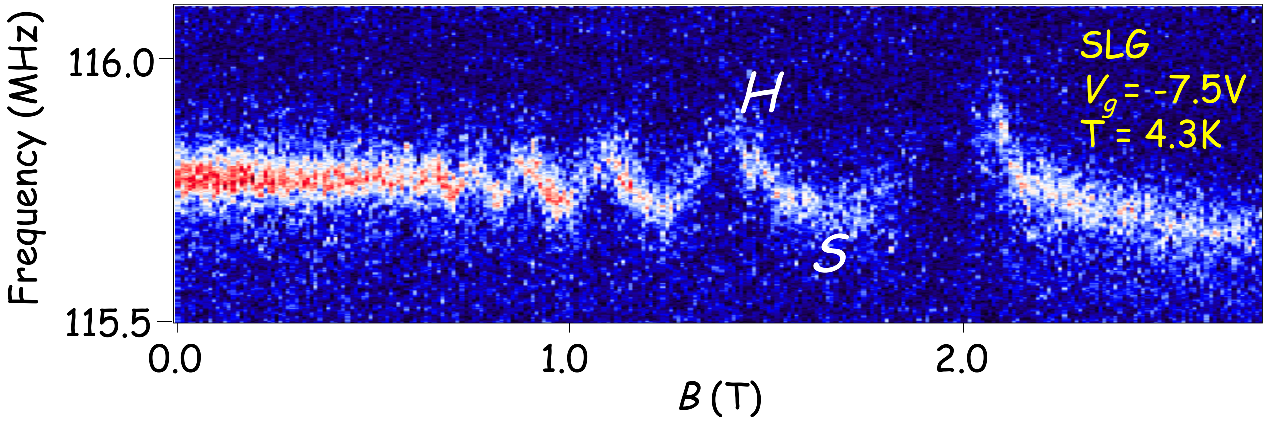
Graphene resonators in the quantum Hall regime



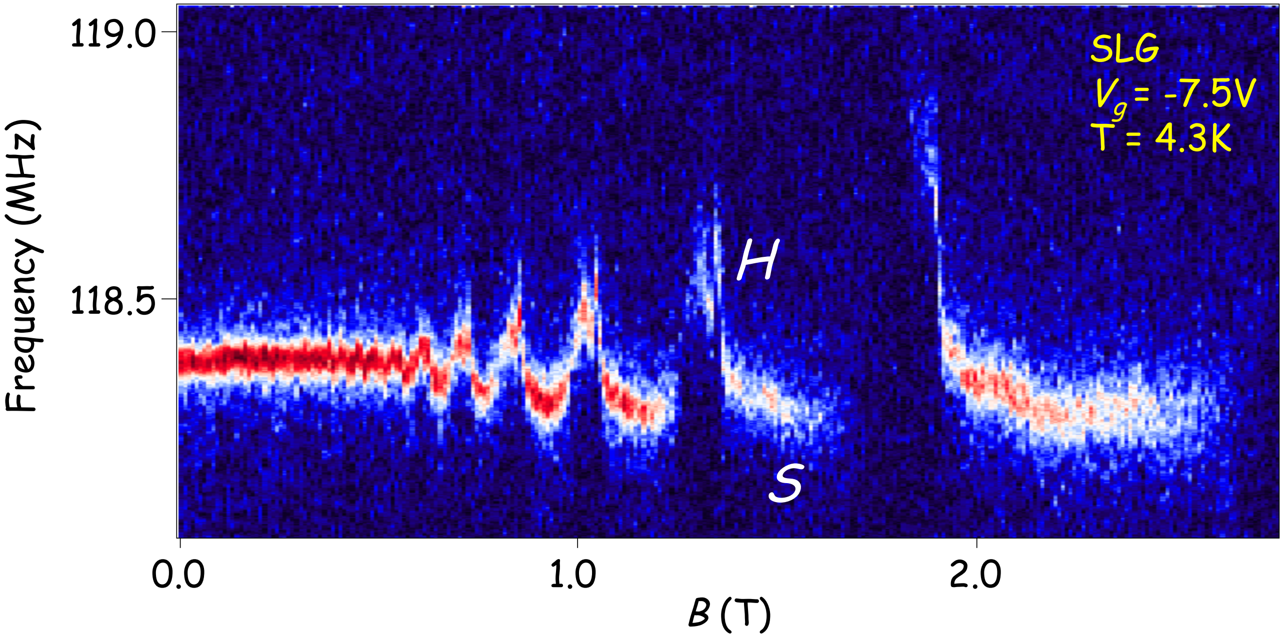
Mechanical response in the QH regime

- Hardening on QH plateaus (features 'H')
- Softening in partially filled LLs (features 'S')

Disorder dependence



- As fabricated
- $\Delta n \sim 10^{10}\text{cm}^{-2}$
- $\Delta E_F \sim 10$ meV

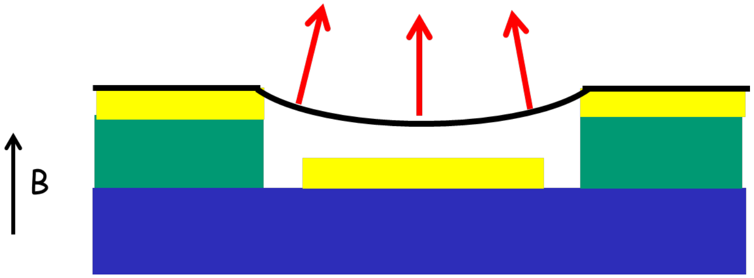


- After anneal
- $\Delta n \sim 4\text{x}10^{9}\text{cm}^{-2}$
- $\Delta E_F \sim 5$ meV

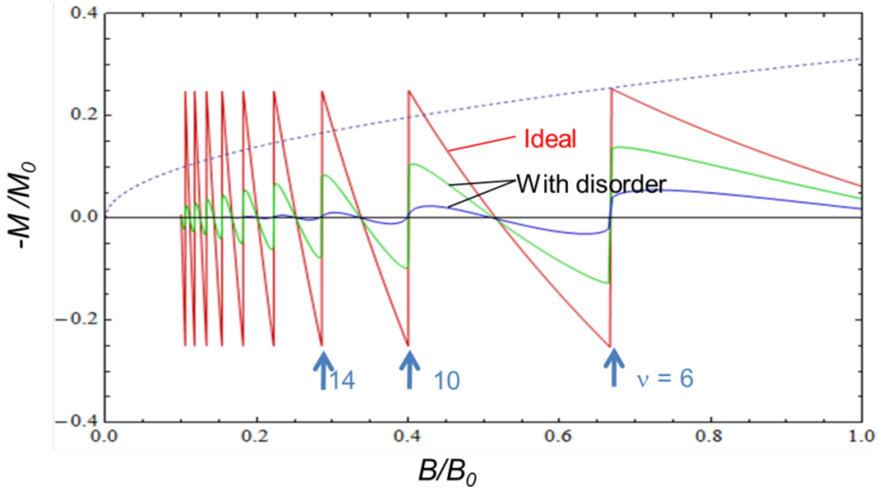
Note:

- Features 'H' become taller and spike-like with increasing *B* and decreasing disorder
- Fine-structure in feature 'H' for lower disorder

Is the response governed by torque?



MXB torque:
Static: tends to flatten (softening)
Dynamic: tends to stiffen



Calculated dHvA for graphene

$$U_{mag} = -\mathbf{M}\cdot\mathbf{B} \approx -MB\left(1-\frac{z_0^2}{2L^2}\right)$$

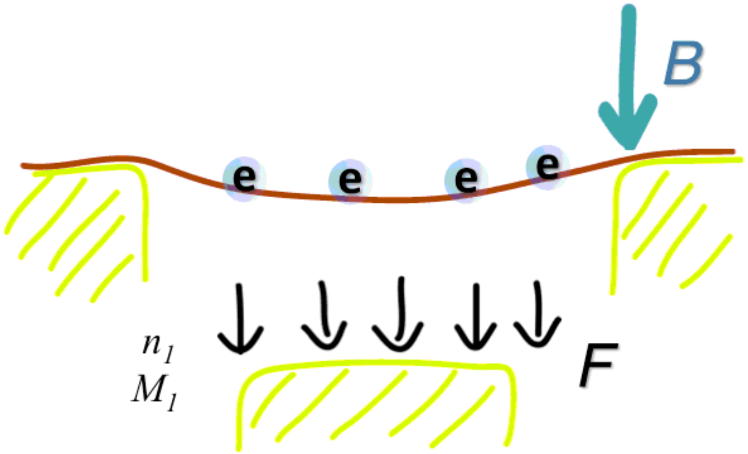
$$\Rightarrow \Delta k = \frac{d^2U_{mag}}{dz_0^2} \approx \frac{MB}{L^2}$$

$$\Delta f \approx f\left(\frac{\Delta k}{2k}\right) \approx 100\,Hz$$

- Compare with data Δf~*100 kHz* *i.e.* 3 orders of magnitude larger!
- Also, torque magnetometry does not yield sharp spikes
- Note: τ_{RC}~10ns precludes eddy current induced spikes

Electrostatic modulation of magnetization

$$\mathbf{F}_{mag} = \mathbf{M}\cdot\nabla\mathbf{B} + \nabla\mathbf{M}\cdot\mathbf{B}$$



Usually, M is constant… but in this case.

$$\frac{dM}{dz} = \frac{dM}{dn}\frac{dn}{dz}$$

Because of the proximity to the gate,
M changes with displacement.

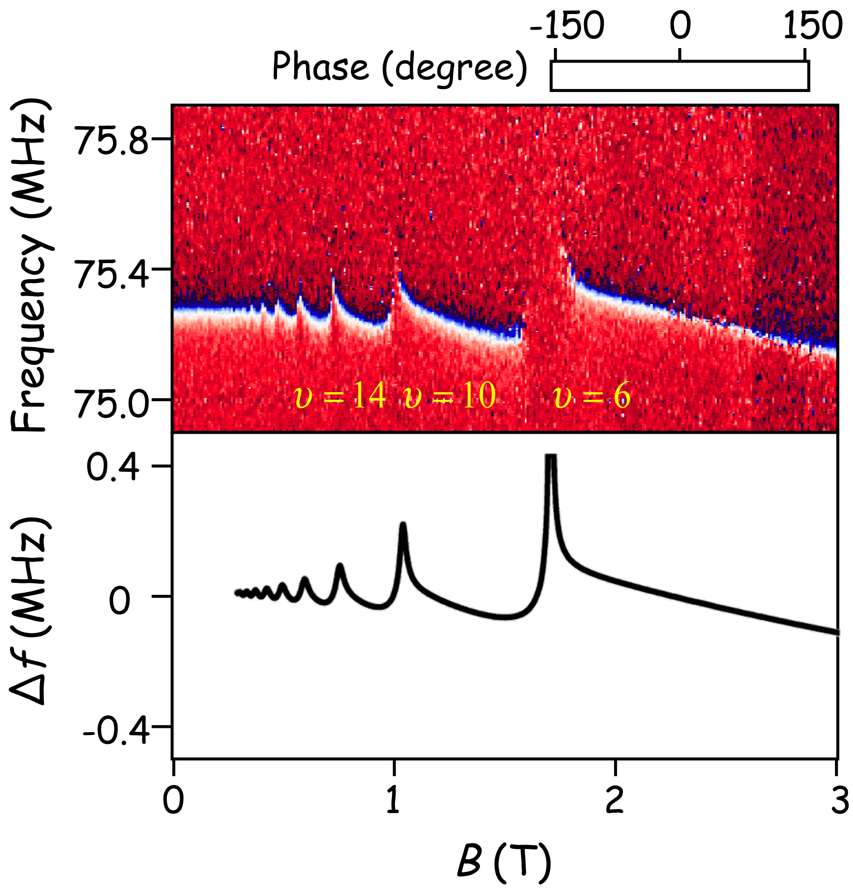
Softening 'S'

Spikes 'H'

$$\Delta f(F_{mag}) = -\frac{\lambda}{C_g V_g}\left(\frac{3}{2}U_{mag} + MB\right).$$

$$\Delta f(k_{mag}) = \frac{f}{2k}\left[\frac{C''}{C_g}\left(\frac{3}{2}U_{mag} + MB\right) + \left(\frac{C'}{C_g}\right)^2\left(\frac{3}{4}U_{mag} + MB - B^2\frac{dM}{dB}\right)\right]$$

Data vs. Model



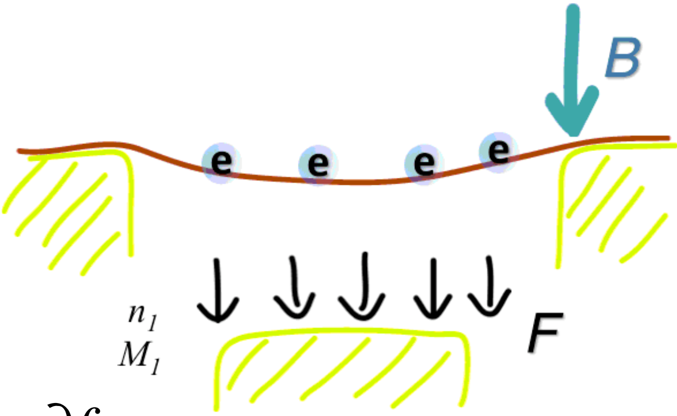
Calculation matches data to within a factor of a few and simulates all features

Only fitting parameter: disorder (~5 meV)

Simpler Interpretation*

1. Static contribution:

$$F_{mag} = -\frac{\partial U}{\partial z} = -\frac{\partial U(n)}{\partial n}\frac{\partial n}{\partial z} = -\mu V_g \frac{\partial C_g}{\partial z}$$



Obtain frequency shift directly from gate tunability:

$$\Delta f = F_{mag} \times \frac{1}{C_g' V_g}\lambda = \mu\lambda, \quad \text{where} \quad \lambda \equiv \frac{\partial f}{\partial V_g}$$

Direct measure of chemical potential

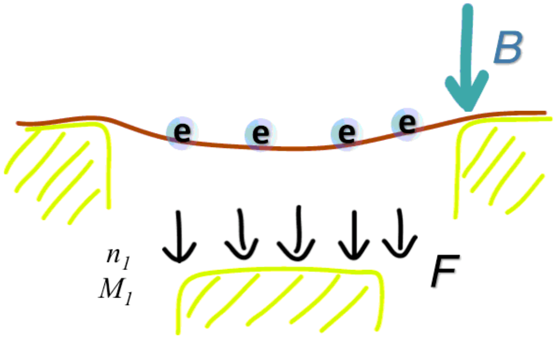
* Thanks to 'referee 2' for useful insights…

Ground state energy governed by electrostatics

1. Static contribution:

$$\Delta f = \mu\lambda$$

2. Dynamic contribution:

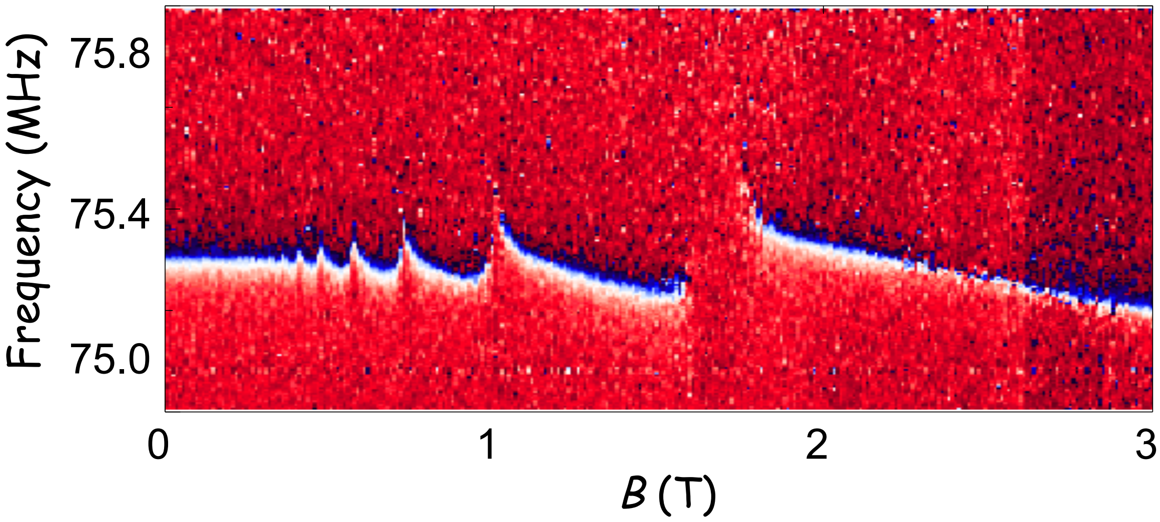


$$\Delta k_{mag} = -\frac{\partial F_{mag}}{\partial z} = \frac{\partial\mu}{\partial n}\left(V_g C_g'\right)^2 \Big/ eA$$

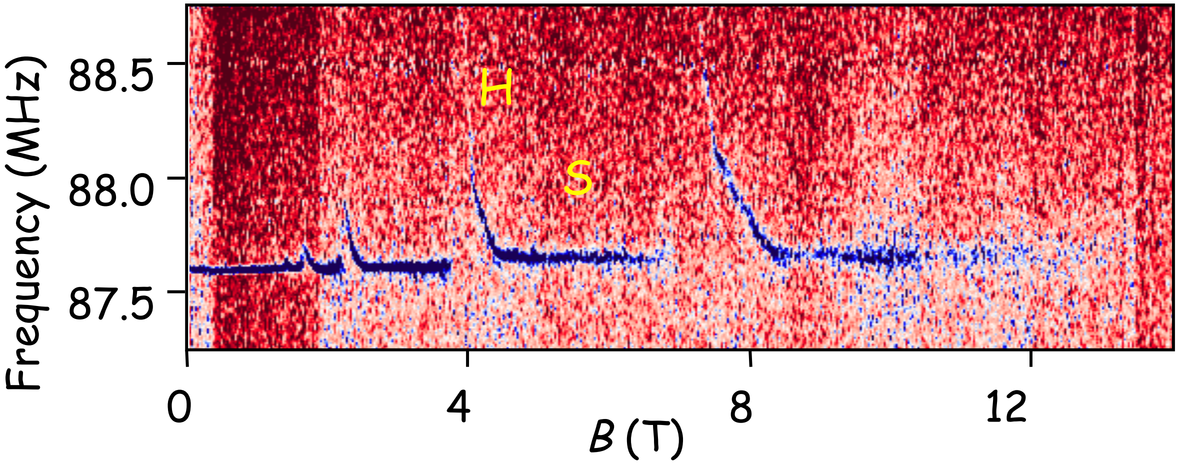
Spring stiffening is direct measure of compressibility dμ/dn

- Term 2 only appears on plateaus - dμ/dn small elsewhere.
- In devices with small built-in tension (large λ), feature 'S' measures chemical potential.
- Feature 'H' always measures compressibility, independent of tension

Samples with different built-in tension

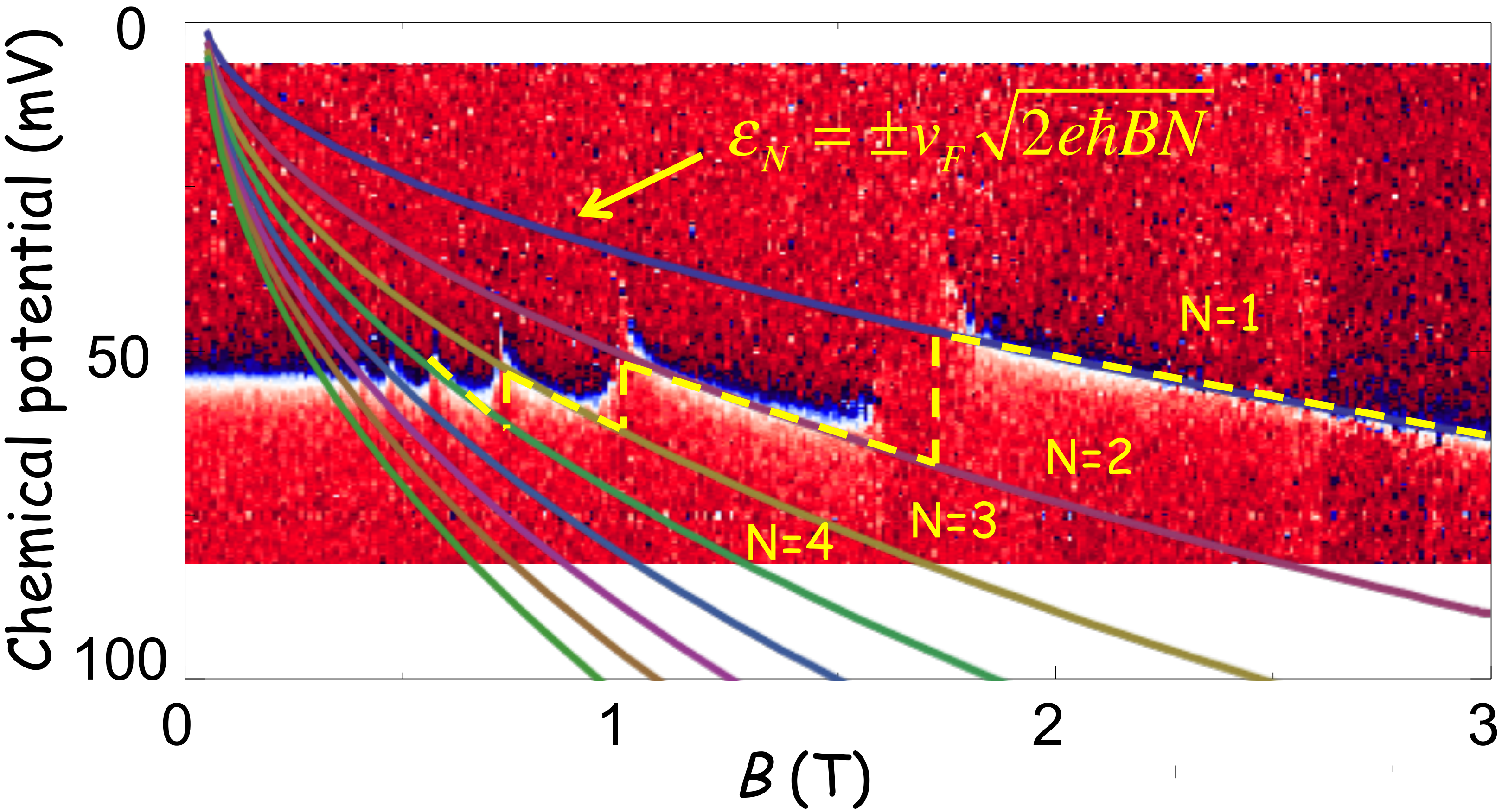


λ~10 MHz/V



λ~1 MHz/V

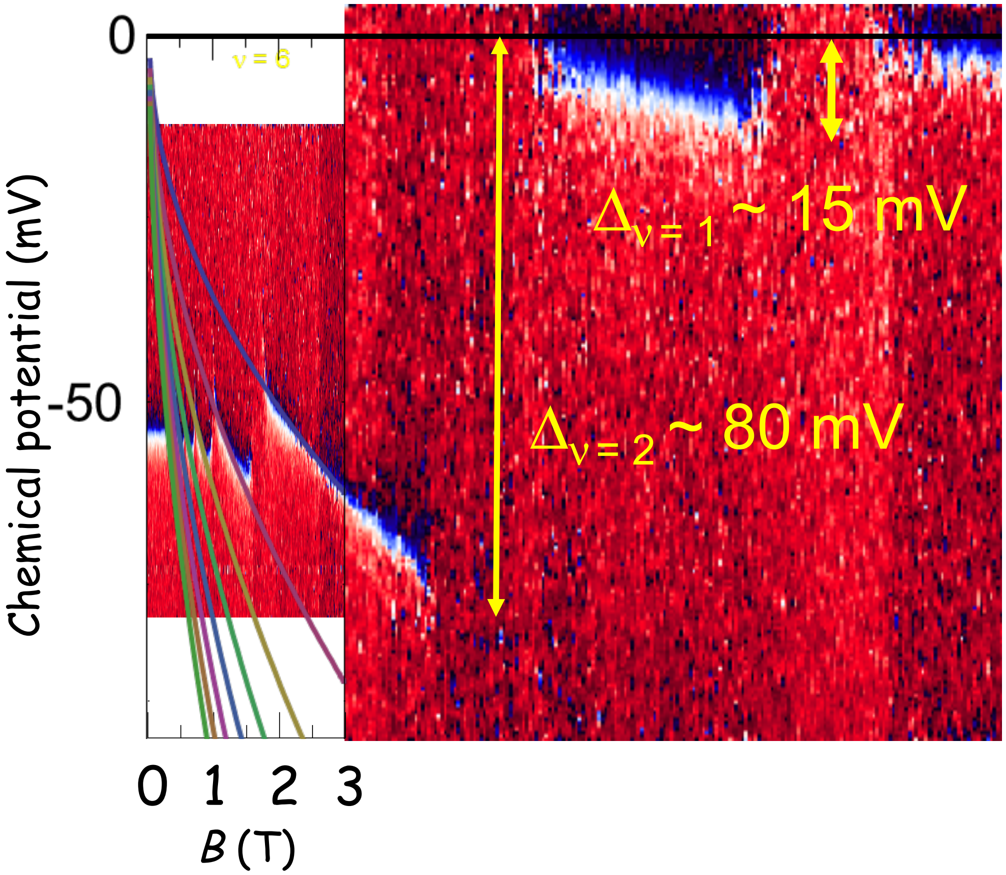
Overlay calculated chemical potential on data



- Chemical potential referenced to N=0 (which has μ =0)
- Calculated chemical potentials line up very well with feature 'S'
- Allows one to directly read-off quantum Hall gaps

Why is this useful?

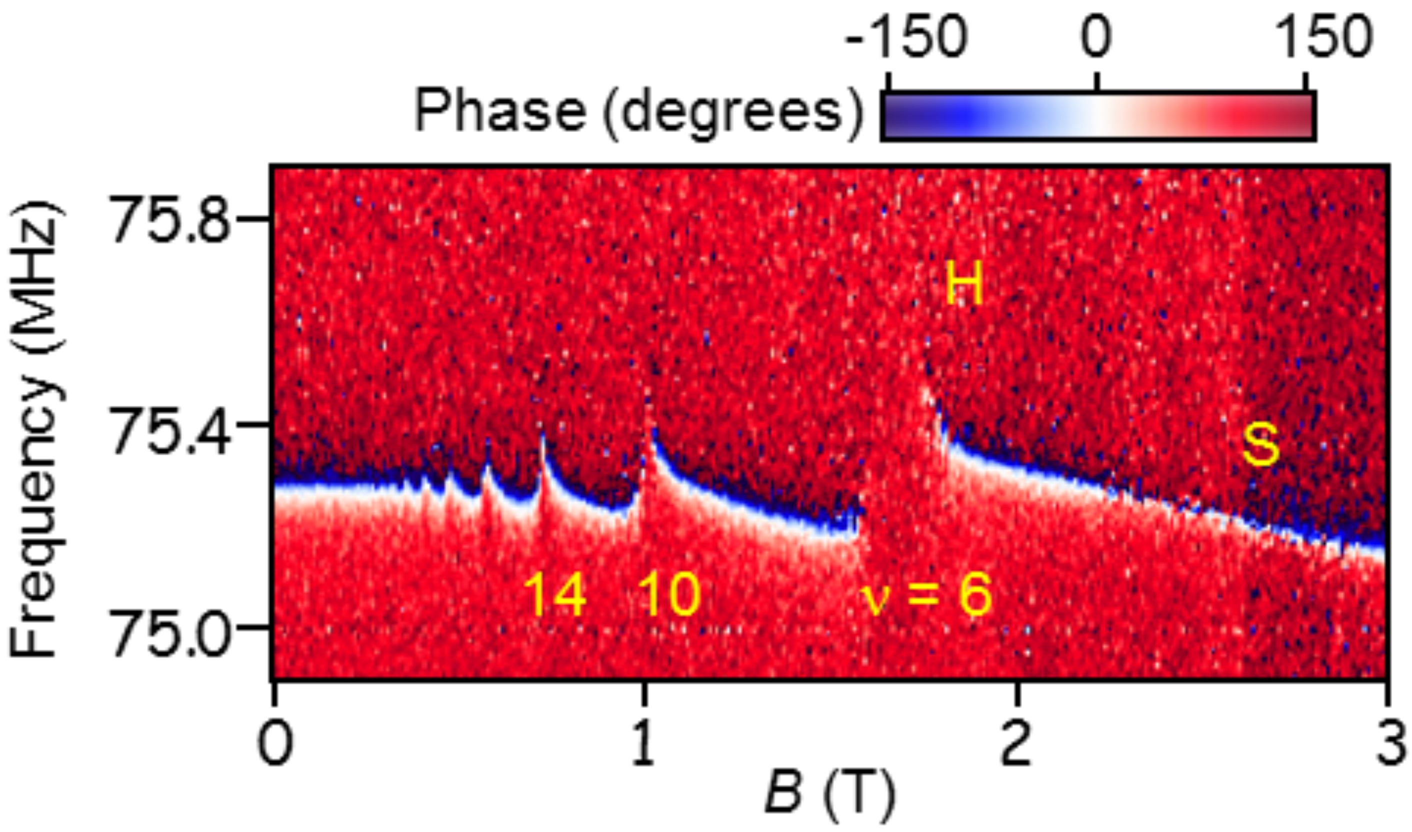
Allows one to measure gaps where single-particle picture does not predict any.



Note

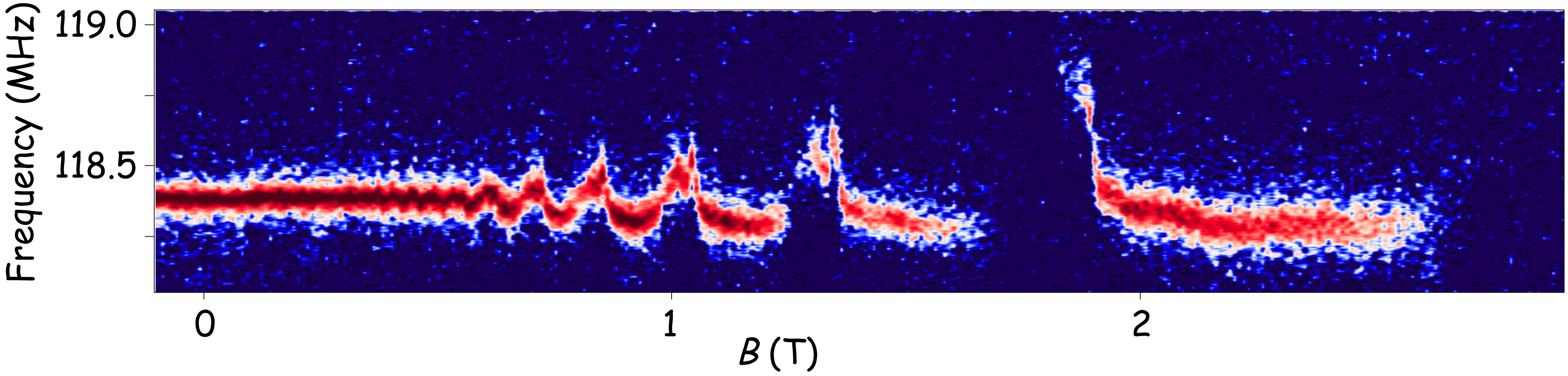
1. μ jumps to zero at ν=2, at 5T ν=2 gap is larger than room temperature.
2. Allows one to read-off ν=1 gap. Much larger than Zeeman gap (~1mV).

How about the hardening spikes 'H'?



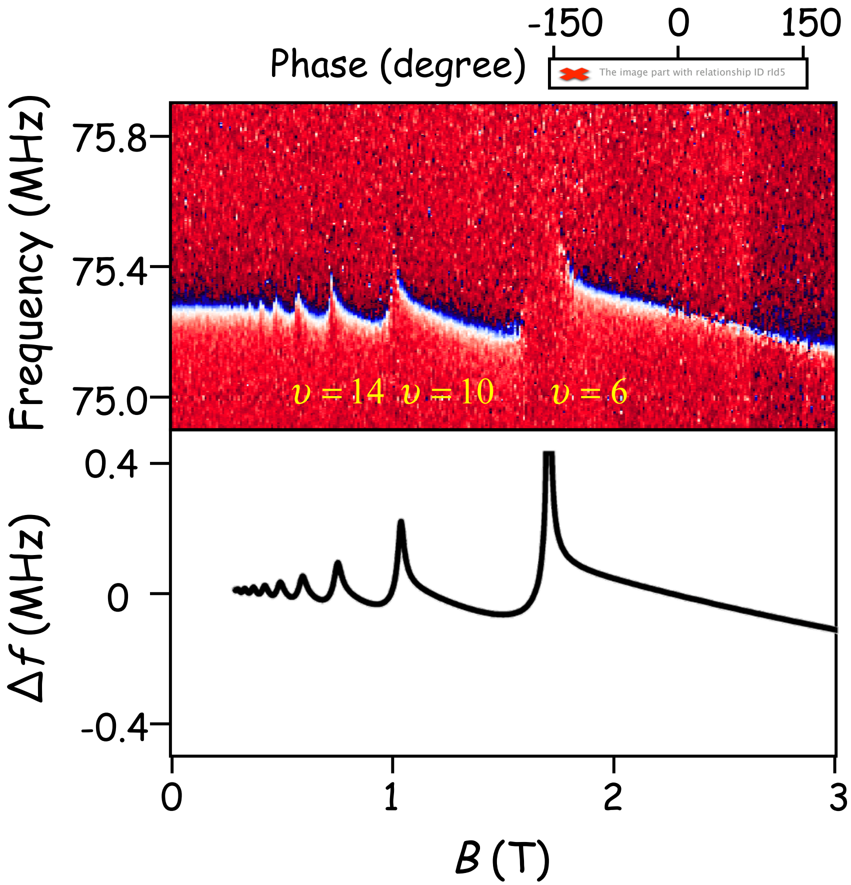
- Features 'H' allow one to estimate compressibility
- Of the order of 10^{-9} mVcm^2 for the largest gaps, in accordance with Martin et al

Fine structure



- Not broken symmetry states (don't appear at the right B)
- Domains – unlikely, since they appear in the cleaner samples
- Negative correction to compressibility, due to Hartree Fock terms
  - Needs further work
- Chemical potential contribution of edge (due to static term)
  - Appears because bulk charge is governed by C_Q and $dC_Q/dz = 0$, while edge charge is always governed by C_g

Magnetomety Applications?



- Moment sensitivity of 10^{-3} μ_B/e/(Hz)$^{1/2}$: competitive with the best magnetometers
- Flux sensitivity of 10^{-20} Wb/(Hz)$^{1/2}$: two orders of magnitude off from state-of-the-art flux sensors

Plenty of scope for improving signal and sensitivity

Acknowledgements

All the work: Vikram Deshpande, Changyao Chen

Fabrication help: Sasha Gondarenko

Funding: AFOSR MURI, NSF NRI

Very helpful discussions:
Nigel Cooper
Igor Aleiner
Allan MacDonald
Paco Guinea