Geometric potential (in few-layer graphene)
a new twist on an old idea

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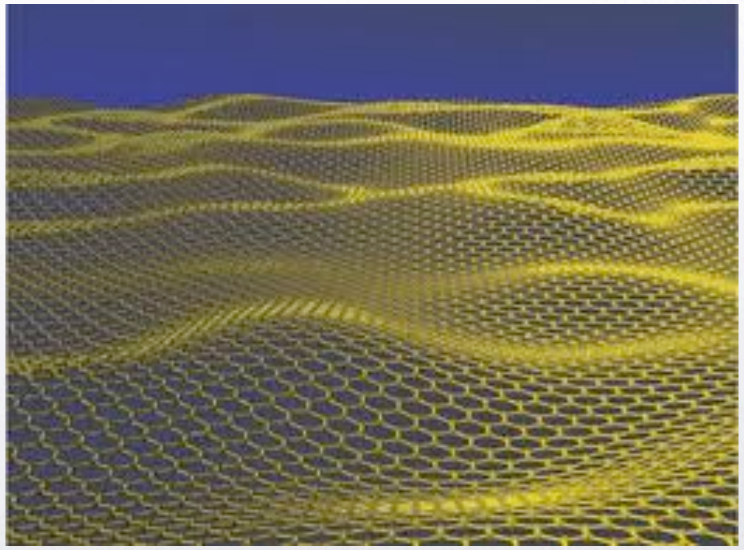
Outline

Quantum particle on a curved surface

Geometric potential: rediscovered time-and-again

Graphene: Is this the solution?



Conclusions

Quantum particle on a curved surface

ANNALS OF PHYSICS: **63**, 586–591 (1971)

Quantum Mechanics with Constraints

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Received September 9, 1970

$$-\frac{\hbar^2}{2M}U = -\frac{\hbar^2}{8M}\left\{2\sum_i \frac{1}{R_i^2} - \left(\sum_i \frac{1}{R_i}\right)^2\right\}.$$

“ Let us now turn to Quantum mechanics and consider the behaviour of a particle constrained to move on a surface $\Phi(x, y, z) = 0$. Since such systems do not exist, this is not a physical problem, and we are at complete liberty to invent any mathematical formalism we like, without the risk of being reputed by experiment. ”

Quantum particle on a curved surface

PHYSICAL REVIEW A VOLUME 23, NUMBER 4 APRIL 1981

Quantum mechanics of a constrained particle

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(Received 26 August 1980)

The motion of a particle rigidly bounded to a surface is discussed, considering the Schrödinger equation of a free particle constrained to move, by the action of an external potential, in an infinitely thin sheet of the ordinary three-dimensional space. Contrary to what seems to be the general belief expressed in the literature, this limiting process gives a perfectly well-defined result, provided that we take some simple precautions in the definition of the potentials and wave functions. It can then be shown that the wave function splits into two parts: the normal part, which contains the infinite energies required by the uncertainty principle, and a tangent part which contains "surface potentials" depending both on the Gaussian and mean curvatures. An immediate consequence of these results is the existence of different quantum mechanical properties for two isometric surfaces, as can be seen from the bound state which appears along the edge of a folded (but not stretched) plane. The fact that this surface potential is not a

$$V_G(q_1, q_2) = -\frac{\hbar^2}{8m}(\kappa_1 - \kappa_2)^2$$

$\kappa_1(q_1, q_2)$ and $\kappa_2(q_1, q_2)$ are local curvatures at point (q_1, q_2).

Geometric Potential: discovered and rediscovered...

Single bent waveguide: Goldstone and Jaffe, PRB 45; Carini et al., PRB 46 (1992).
Curved 2D electron gases: M. Pepper et al., J. Phys. (1994).
Spin-orbit curvature effect: M.V. Entin and L.I. Magarill, PRB 64 (2001).
Curved 2D electron gases: A. Lorke et al., Superlattices 33 (2003).
Constrained quantum particle in an em field: G. Ferrari et al., PRL 100 (2008).
Signatures of geometric potential in Y-junctions: G. Cuoghi et al., PRB 79 (2009).
Geometric potential in photonic crystals, PRL 104 (2010).

Experimental observation: only for light!
No experimental observation for fermions.

Slowly varying potential for fermions

Large at saddle points, $\kappa_1\kappa_2 < 0$ and small on bumps.

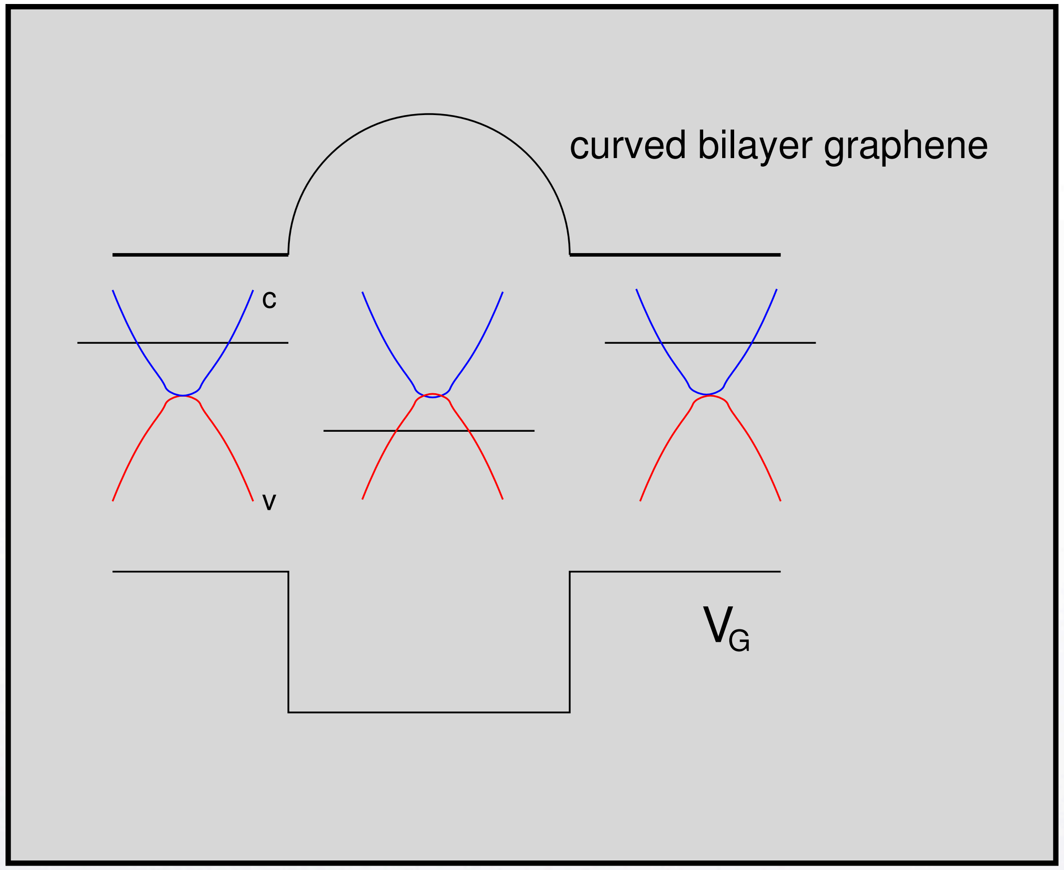
"Smooth" potential will locally lower the Fermi energy.

Gapless material: if $V_G \sim E_F$ the Fermi energy will shift from the conduction to the valance band.

Continuum model requires $\kappa a \ll 1$ where a is the lattice constant.

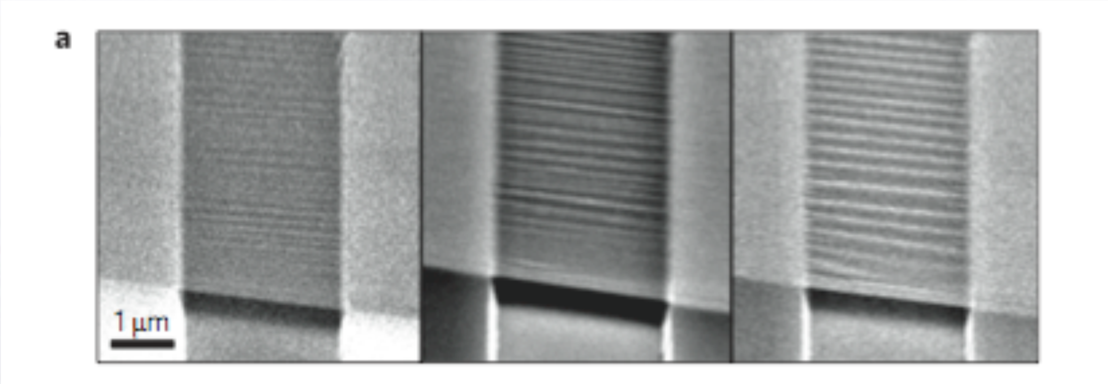
Two dimensional surface requires $\kappa(3a) \ll 1$.

Curved bilayer graphene with massive particles



E_F supression: $\kappa^{-1} \sim k_F^{-1} \sim$ 10-30 nm

Well-defined quantum wire.
PN junction without gating.
Create ripples using suspended (bilayer) graphene



Phys. Rev. B **80**, 153405 (2009)

Lau et al., Nat. Nano. **4**, 562 (2009)

Graphene: monolayer, bilayer, few-layer

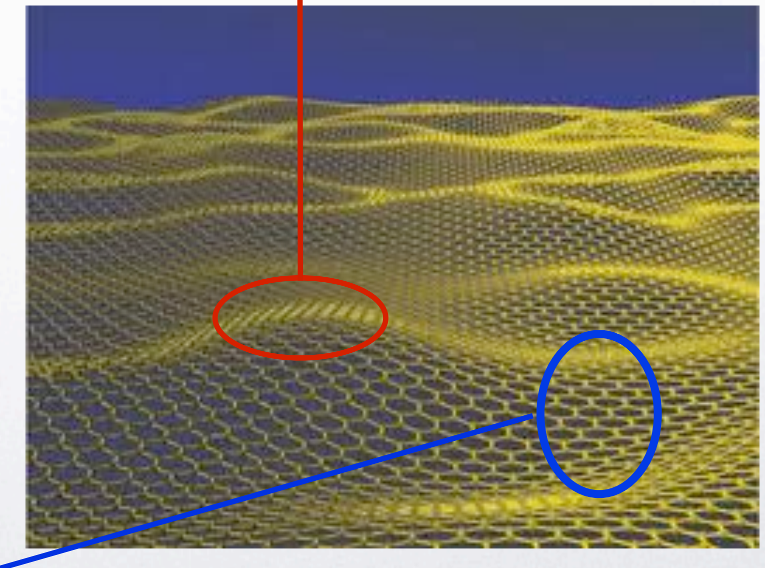
Elasticity model:
scalar potential for mono/bilayer graphene.

Proportional to Gaussian curvature: same for isometric surfaces.

Geometric potential: NOT the same for isometric surfaces.

Is there a lattice analog of the geometric potential result?

$$\kappa_1(q_1, q_2)\kappa_2(q_1, q_2) > 0$$



$$\kappa_1(q_1, q_2)\kappa_2(q_1, q_2) < 0$$

Conclusions and Outlook

- Novel properties of a particle on a curved surface.
- Graphene may provide an experimental realization of the geometric potential for fermions.

Thank you!