

Spontaneously Ordered Electronic States in BLG and SLG

Leonid Levitov (MIT)

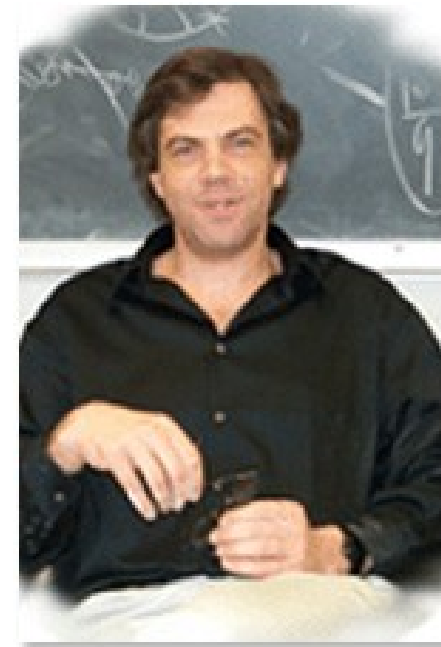
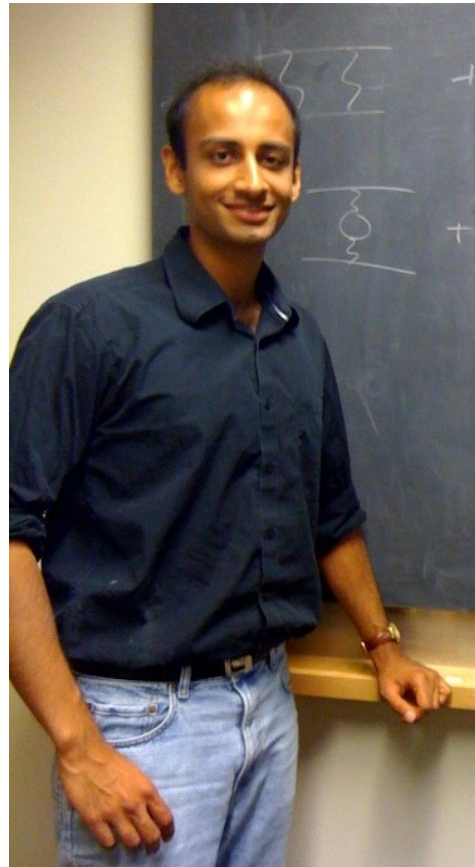
Fundamental Aspects of Graphene and Other
Carbon Allotropes
KITP UCSB 01/09/2012

Collaboration

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New ordered states in SLG and BLG

- Weak interactions in undoped SLG (low DOS) **both a blessing and a curse**: robustness vs. functionality
- Strengthen the effects of interaction: **use weakly dispersing states, $E_{\text{kinetic}} < E_{\text{potential}}$**
- ➡ (i) BLG at charge neutrality: excitonic insulator, spontaneous Hall effect at $B=0$ [charge QHE, spin QHE or valley QHE], nematic order
- ➡ (ii) SLG doped to saddle point: chiral d-wave superconductivity (broken time-reversal symmetry)
- (iii) alter electronic states using external fields (QHE, FQHE)
- ➡ ● Ways to experimentally distinguish different ordered states in BLG

Theory:

- Min, Borghi, Polini & MacDonald, Pseudospin magnetism in graphene. Phys. Rev. B 77, 041407 (2008).
- Nandkishore & Levitov, Dynamical screening and excitonic instability in bilayer graphene, Phys. Rev. Lett. 104, 156803 (2009)
- Nandkishore & Levitov, Flavor symmetry and competing orders in bilayer graphene. arXiv:1002.1966v1001 (2010).
- Zhang, Min, Polini, & MacDonald, Spontaneous inversion symmetry breaking in graphene bilayers. Phys. Rev. B 81, 041402 (R) (2010).
- Nandkishore & Levitov, Quantum Anomalous Hall State in Bilayer Graphene, Phys Rev B 82, 115124 (2010)
- Vafek & Yang, Many-body instability of Coulomb interacting bilayer graphene: Renormalization group approach. Phys. Rev. B 81, 041401 (2010).
- Lemonik, Aleiner, Toke & Falko, Spontaneous symmetry breaking and Lifshitz transition in bilayer graphene. Phys. Rev. B 82, 201408 (2010).
- Zhang, Jung, Fiete, Niu & MacDonald, Spontaneous quantum Hall states in chirally stacked few-layer graphene systems. Phys. Rev. Lett. 106, 156801 (2011).
- Kharitonov, Canted antiferromagnetic phase of the $\nu=0$ quantum Hall state in bilayer graphene. preprint, arXiv:1105.5386v1101 (2011).

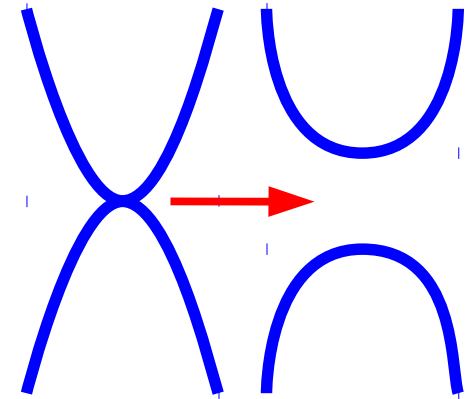
Experiment:

- Martin, Feldman, Weitz, Allen & Yacoby, Local Compressibility Measurements of Correlated States in Suspended Bilayer Graphene. Phys. Rev. Lett. 105, 256806 (2010).
- Weitz, Allen, Feldman, Martin, & Yacoby, Broken-symmetry states in doubly gated suspended bilayer graphene. Science 330, 812-816 (2010).
- Freitag, Trbovic, Weiss & Schonenberger, Spontaneously gapped ground state in suspended bilayer graphene. arXiv:1104.3816vs (2011)
- Zhao, Cadden-Zimansky, Jiang, & Kim, Symmetry Breaking in the Zero-Energy Landau Level in Bilayer Graphene. Phys. Rev. Lett. 104, 066801 (2010).
- Feldman, Martin & Yacoby, Broken-symmetry states and divergent resistance in suspended bilayer graphene. Nat. Phys. 5, 889-893 (2009).
- Bao, W. et al. Magnetoconductance oscillations and evidence for fractional quantum Hall states in suspended bilayer and trilayer graphene Phys. Rev. Lett. 105, 246601 (2010).
- Velasco, Jing, Bao, Lee, Kratz, Aji, Bockrath, Lau, Varma, Zhang, Jung & MacDonald, Transport Spectroscopy of Symmetry-Broken Insulating States in Bilayer Graphene, arXiv:1108.1609
- Mayorov, Elias, Mucha-Kruczynski, Gorbachev, Tudorovskiy, Zhukov, Morozov, Katsnelson, Falko, Geim, Novoselov, Interaction-Driven Spectrum Reconstruction in Bilayer Graphene, Science 333, 860 (2011)

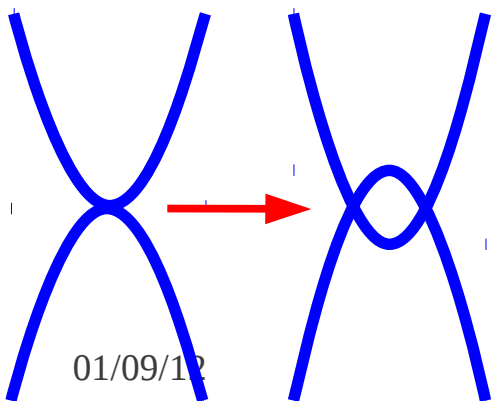
Spontaneous ordering in BLG at DP

- Particle-hole pairing instability
- BCS-like exciton condensate, no superfluidity, phase locking
- Gapped spectrum $\Delta = \pm \Delta_0$
- Another candidate state: “nematic” order, gapless spectrum, broken rotational symmetry

Min et al 2008;
Nandkishore, LL 2010
Zhang et al 2010



Vafeek, Yang 2010; Lemonik et al 2010

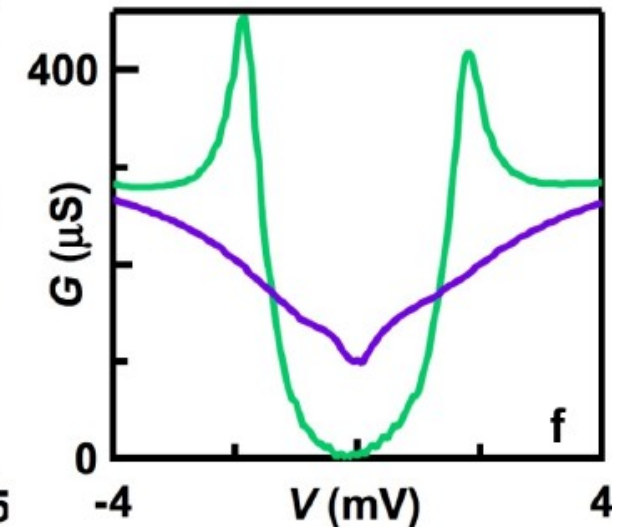
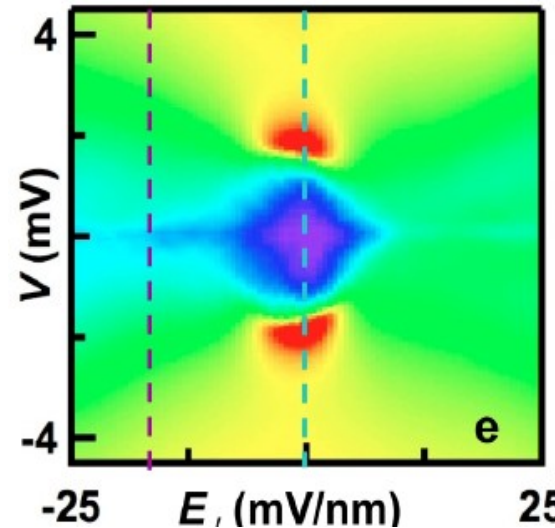
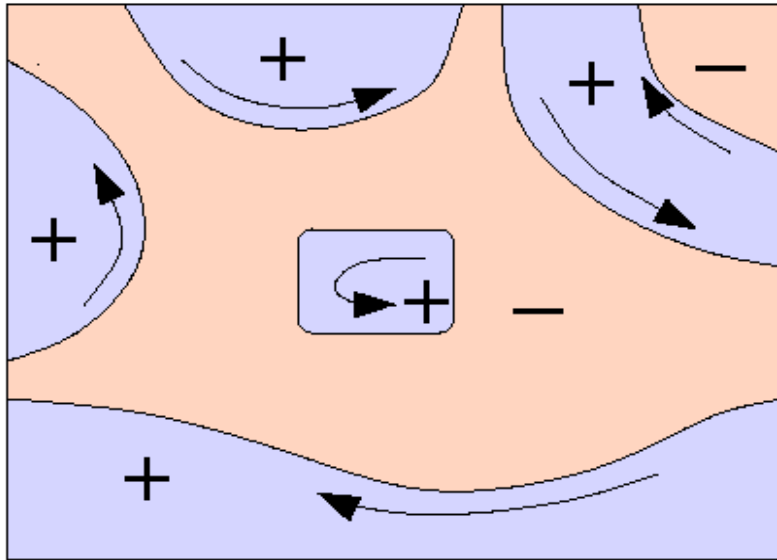


$$H_{nema} = \begin{pmatrix} 0 & \frac{p_-^2}{2m} + \Delta \\ \frac{p_+^2}{2m} + \Delta & 0 \end{pmatrix}$$

$$H_{gapped} = \begin{pmatrix} \Delta & \frac{p_-^2}{2m} \\ \frac{p_+^2}{2m} & -\Delta \end{pmatrix}$$

Spontaneous gap opening in BLG

Nandkishore & LL, PRL 104, 156803 (2010), PRB 82, 115124 (2010)



- 'Which-layer' symmetry breaking Velasco et al arXiv:1108.1609
- Domains of + and - polarization
- Charge, valley or spin polarized current along domain boundaries, QHE, VQHE, SQHE, etc
- SU(4) symmetry and the variety of possible states
- Time reversal symmetry breaking at $B=E=0$: Anomalous Quantum Hall state, quantized σ_{xy}
- Experiment (Yacoby, Lau and Geim groups)

Large variety of possible states

$$H_K = \begin{pmatrix} \Delta_K & p_-^2/2m \\ p_+^2/2m & -\Delta_K \end{pmatrix} \quad H_{K'} = \begin{pmatrix} \Delta_{K'} & p_+^2/2m \\ p_-^2/2m & -\Delta_{K'} \end{pmatrix}$$

$$\Delta_{K,\sigma} = \pm \Delta_{K',\sigma} = \pm \Delta_{K,-\sigma} = \pm \Delta_{K',-\sigma} \quad p_{\pm} = p_1 \pm i p_2$$

- Four-fold spin/valley degeneracy
- Many gapped states: valley “antiferromagnet”, ferromagnetic, ferrimagnetic, ferroelectric, etc (Min et al 2008, Nandkishore & LL 2010, Zhang et al 2010)
- Degeneracy on a mean field level: instability threshold **the same for all states**: short-range interaction, screened long-range interaction models
- SU(4) symmetry?

Opposite chirality of two valleys conceals SU(4) symmetry, made manifest by performing unitary transformation

$$H_0 = \frac{p_+^2}{2m} \tilde{\tau}_- + \frac{p_-^2}{2m} \tilde{\tau}_+,$$

Approximate SU(4) symmetry (weakly broken by trigonal warping and capacitor energy)

$$H = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger H_0 \psi_{\mathbf{p}} + \frac{1}{2} \sum_{\mathbf{q}} V_+(q) \rho_{\mathbf{q}} \rho_{-\mathbf{q}} + V_- \lambda_{\mathbf{q}} \lambda_{-\mathbf{q}},$$

Strategy: Diagonalize SU(4) invariant Hamiltonian and incorporate anisotropies perturbatively

Mean field description of gapped states

$$H = \frac{p_+^2 \tau_- + p_-^2 \tau_+}{2m} + \Delta \tau_3 Q;$$

Classification into manifolds (4,0), (3,1), (2,2) and distinction between symmetry protected and accidental degeneracies

$$\sigma_{xy} = (M_{>} - M_{<}) \frac{e^2}{h},$$

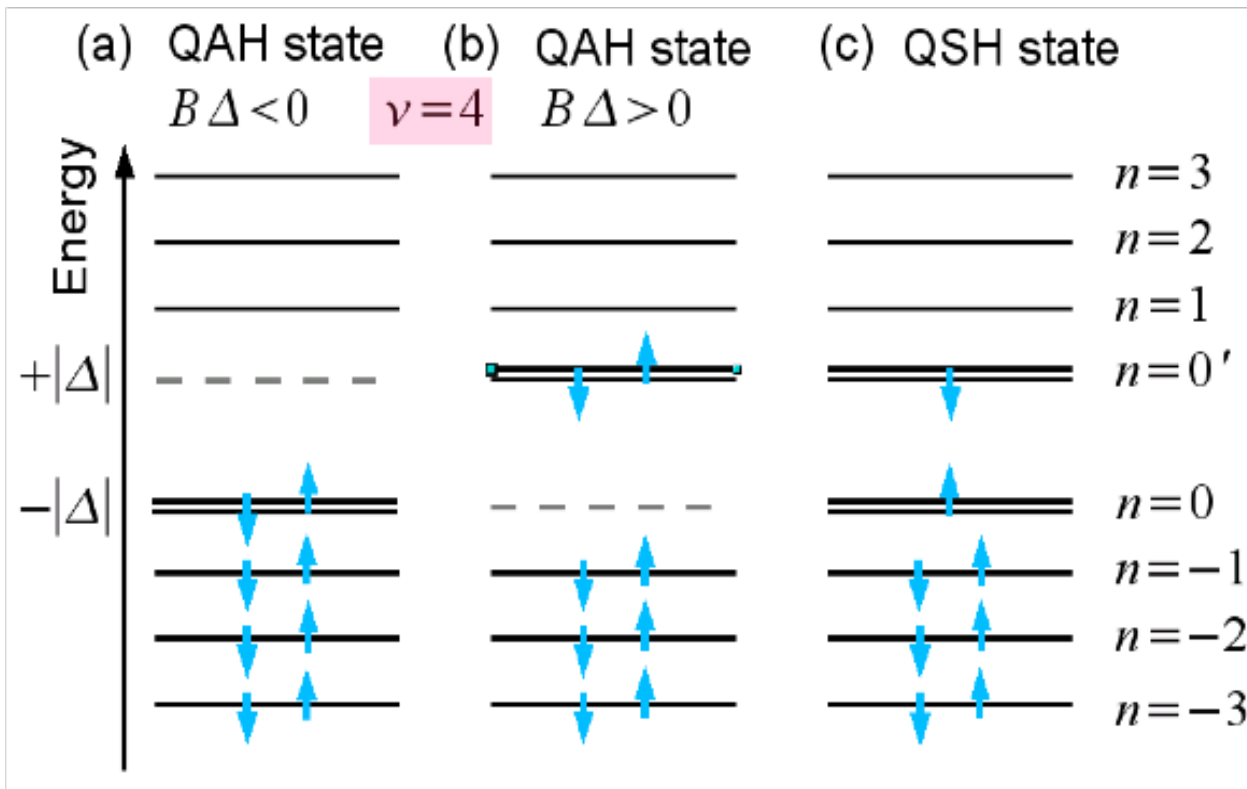
(4,0) and (3,1) states feature QHE, “anomalous QHE”, $B=0$

Nandkishore, LL 2010 Vafeek, Yang 2010

Near-degeneracy and selection:

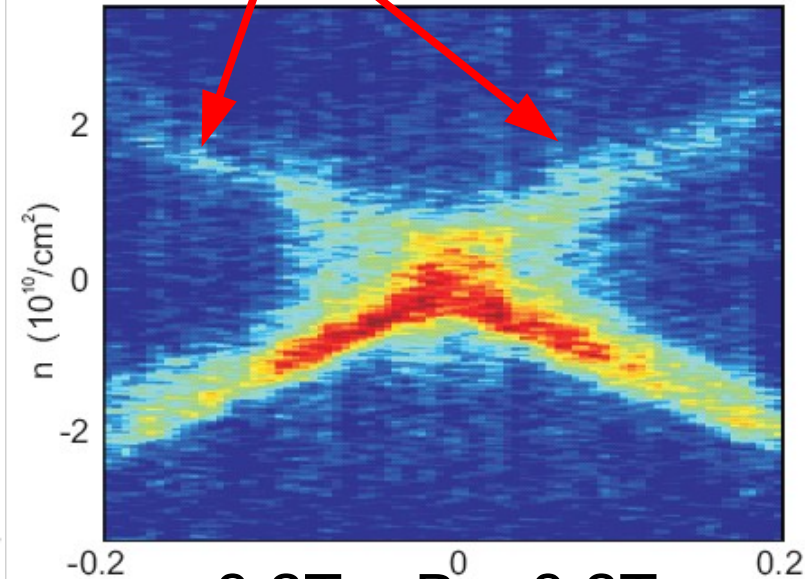
Quantum fluctuations favor (4,0) state;
thermal fluctuations favor (2,2) state

The QAH state stabilized by a B field



$$n = \pm 4 \times eB/h$$

inverse compressibility $d\mu/dn$



$$-0.2\text{T} < B_0 < 0.2\text{T}$$

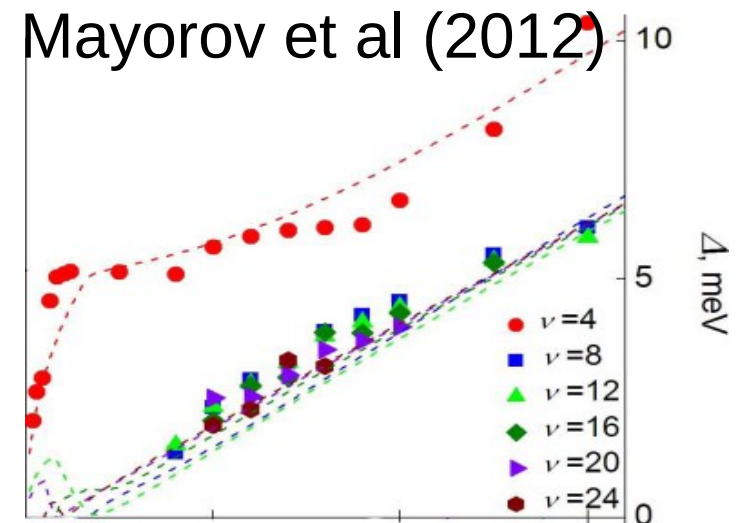
Martin et al (2010)

Mayorov et al (2012)

Theory: the QAH state favored at small nonzero B and $\nu = +4, -4$

Experiment: Incompressible $\nu = +4, -4$ states observed at **very low B**

Consistent with the QAH state



Transport experiments compatible with the QAH state (but indecisive)

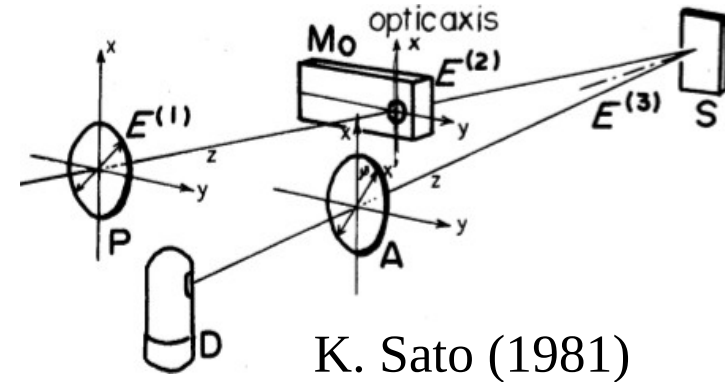
- Incompressible regions at low B , $\nu=4$ (if field induced), $\nu=+4$ and $\nu=-4$ (if intrinsic); no such feature at higher filling factor (unlike nematic or other states)
- Incompressible (bulk gap)+finite two-probe conductivity; distinguishes QAH state from (2,2) state but not from nematic state or trigonal warping
- Phase transition at zero ν , finite B to (2,2) QHFM state (likewise)
- Phase transition at finite E to trivial insulator (Ising universality class)

The QAH state not yet observed

EXPERIMENTAL SIGNATURES

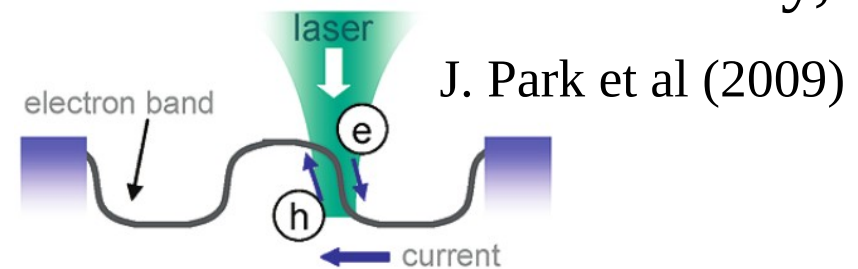
- 1) Direct test: measurement of QHE at $B=0$; requires four-probe measurement on suspended BLG at low T
- 2) TRS breaking via violation of Onsager symmetry $B, -B$ in a four-probe measurement

3) Optically detect TRS breaking: contactless measurement of σ_{xy} by polar Kerr effect (not Faraday effect)



Nandkishore & LL, PRL 107, 097402 (2011)

4) Scanning photocurrent imaging: domains with different chirality, p-n droplets, edge states



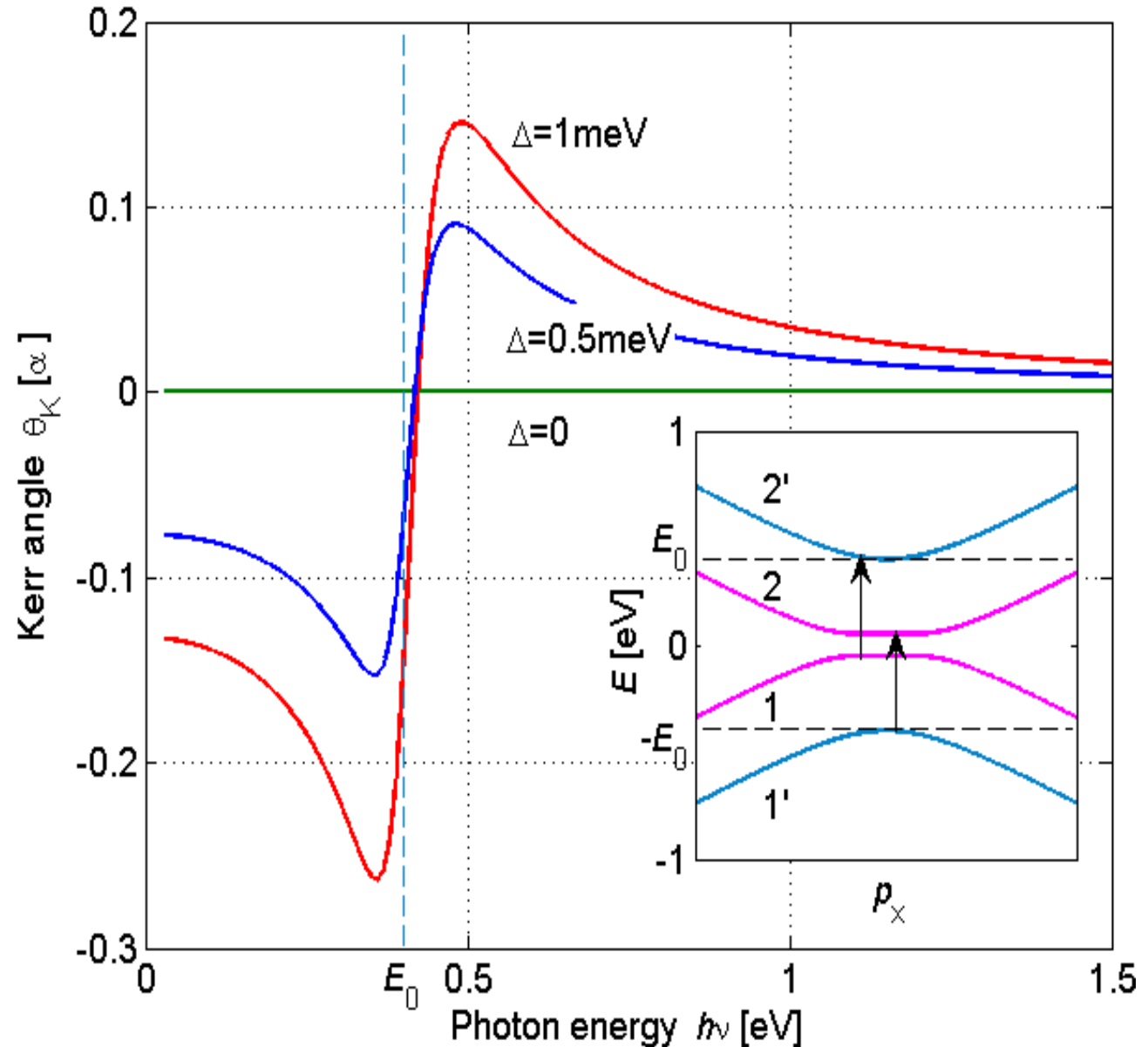
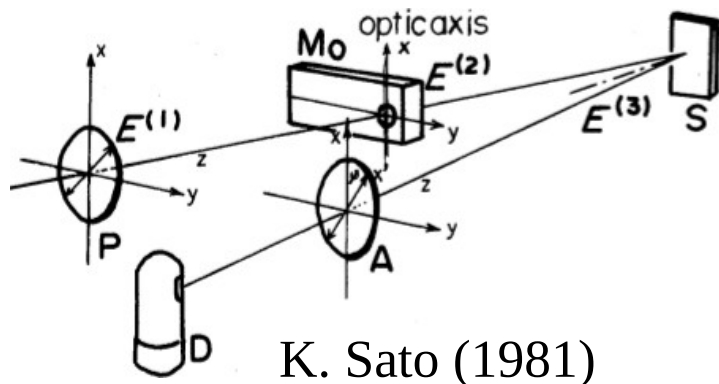
Song & LL, arXiv:1112.5654 (2011)

5) Tunneling probes and local capacitance probes: local gap, filling factor, compressibility

Kerr effect: optical detection of TRS breaking, contactless measurement of σ_{xy}

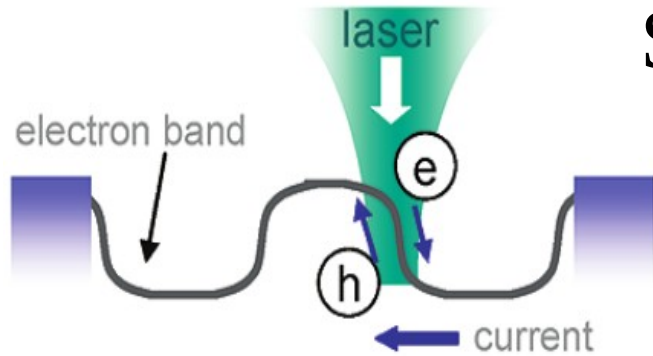
Large polar Kerr effect in TRS-broken states: interband transitions sensitive to low-energy physics at Dirac point

Nandkishore & LL
PRL 107, 097402
(2011)



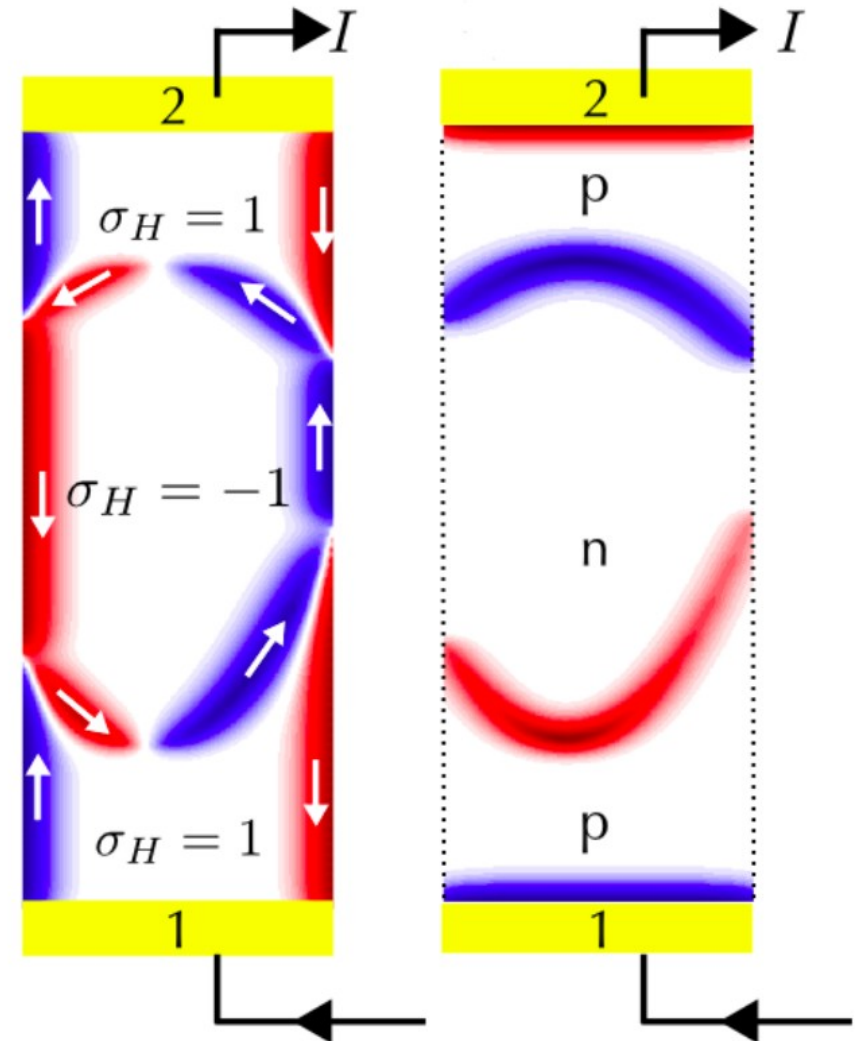
Scanning photocurrent (PC) imaging

Song & LL, arXiv:1112.5654 (2011)



$$j_{local} = (a \nabla n + b \hat{z} \times \nabla n) J_{laser}$$

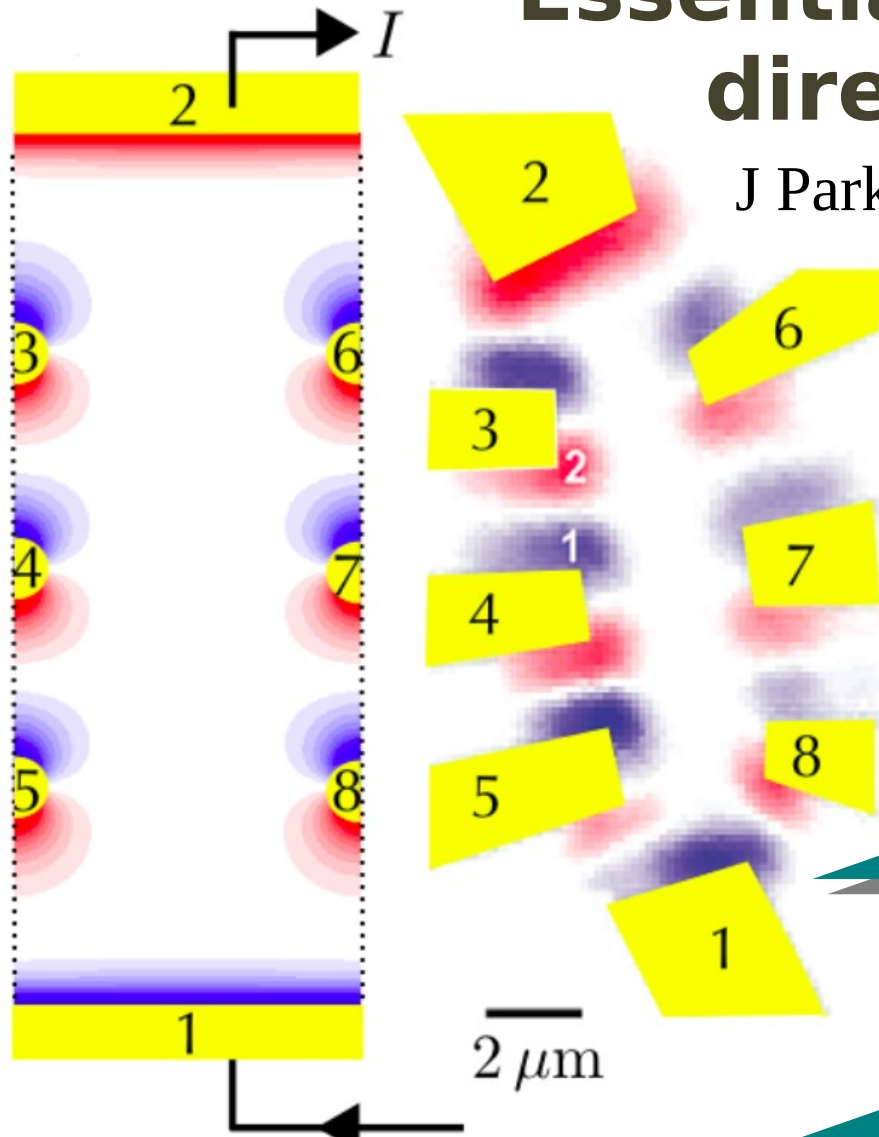
- Unpolarized light generates PC at interfaces, inhomogeneities, edges
- PC can image domains of opposite chirality, p-n boundaries, etc
- How are local properties manifested in system-wide PC?



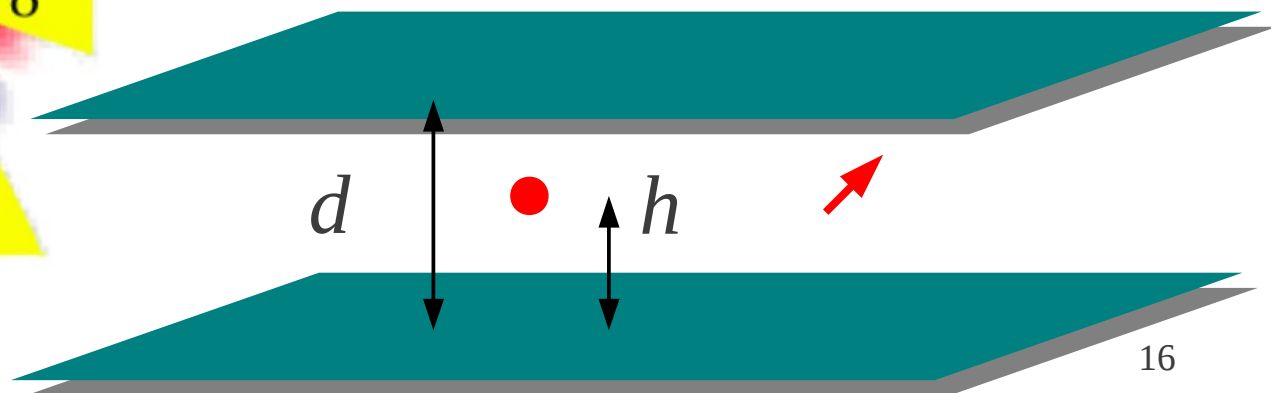
System-wide (global) PC in gapless materials: imaging local properties

Essential nonlocality and directional effect

J Park et al (2009), J Song & LL (2011)



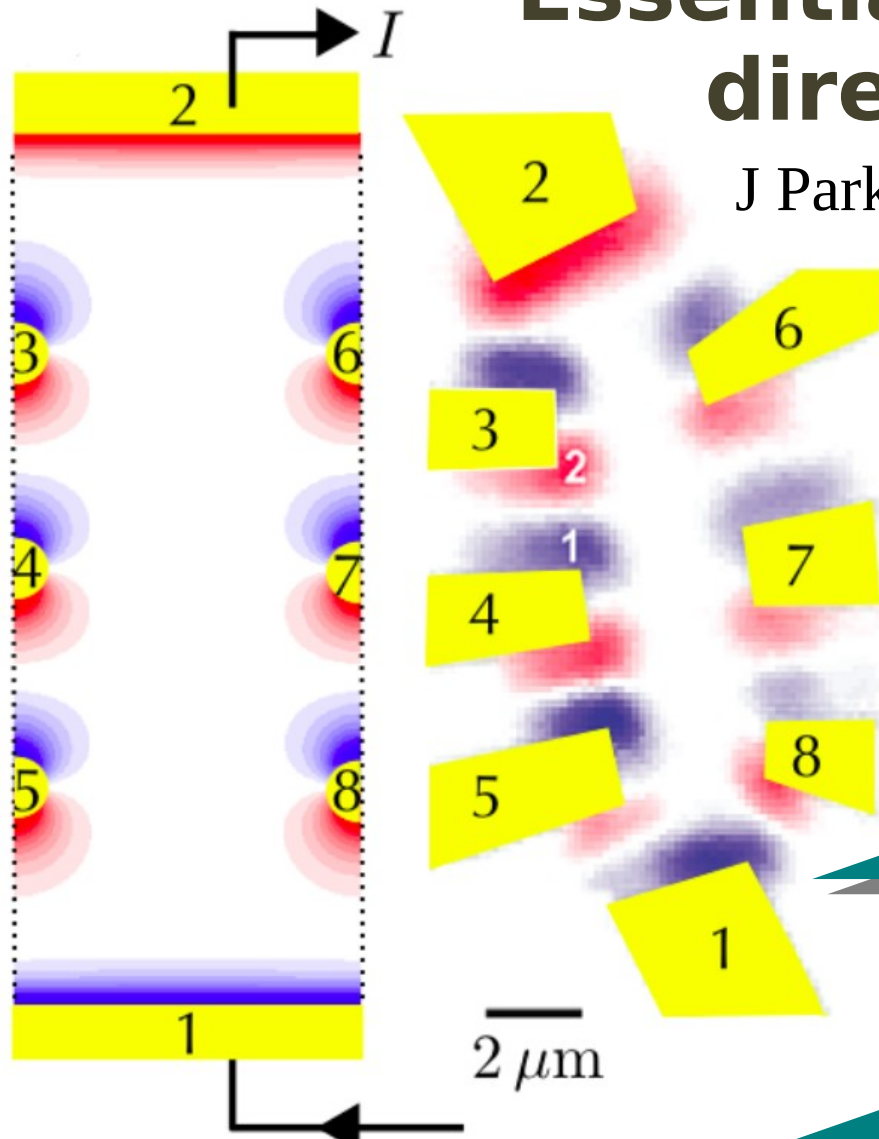
Electrostatic analogy
 h dependence?



System-wide (global) PC in gapless materials: imaging local properties

Essential nonlocality and directional effect

J Park et al (2009), J Song & LL (2011)



Electrostatic analogy

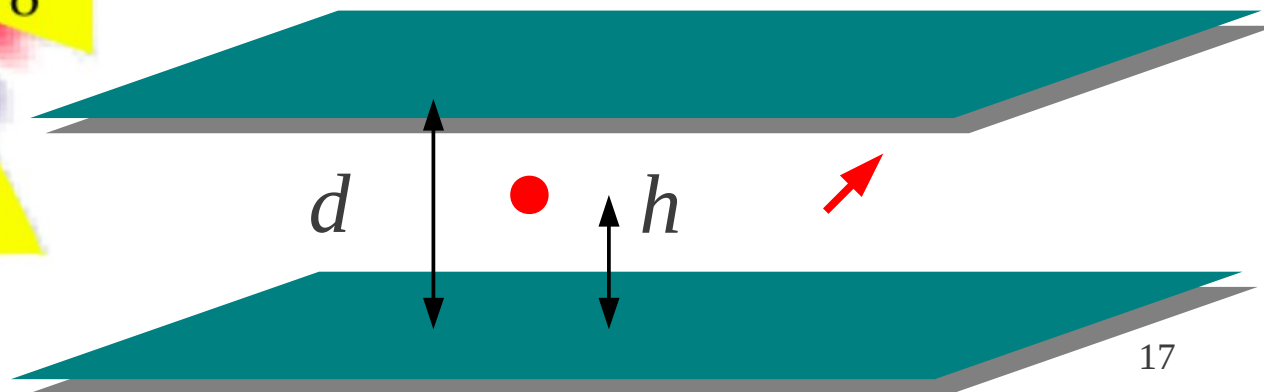
$$q' = -\frac{h}{d}q$$

$$q'' = -\frac{d-h}{d}q$$

$$q'_{dipole} = -p_z/d$$

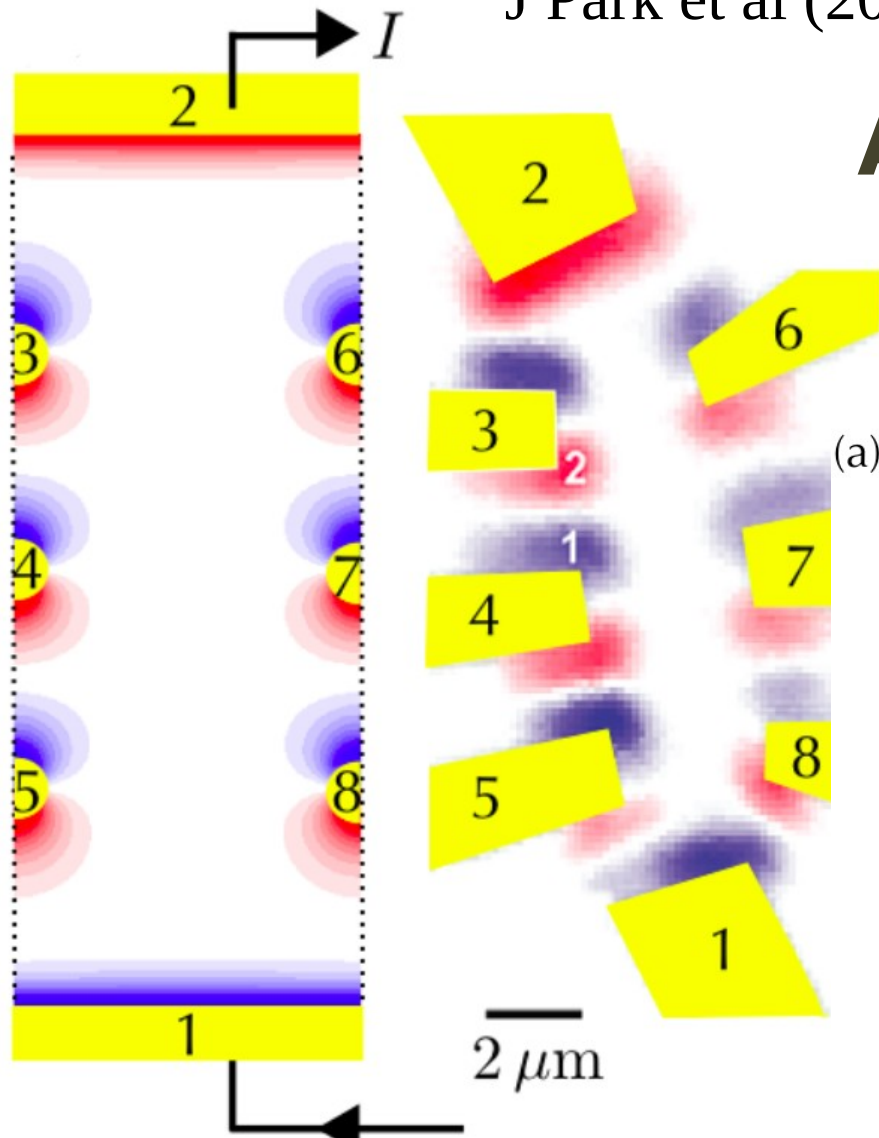
$$q''_{dipole} = p_z/d$$

h -independent!

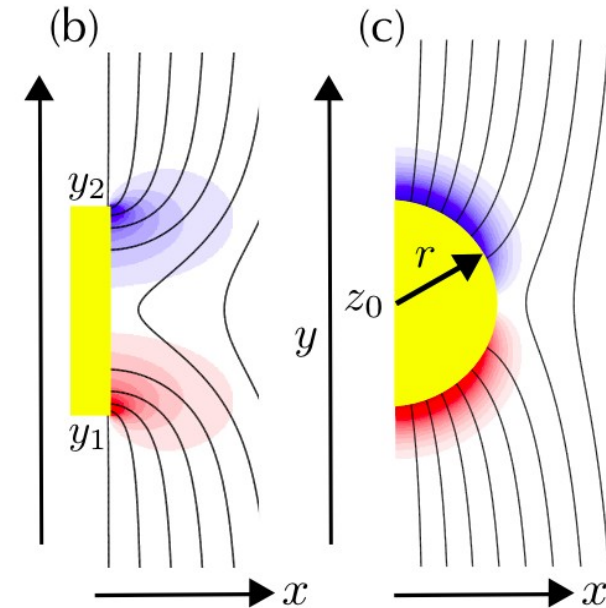
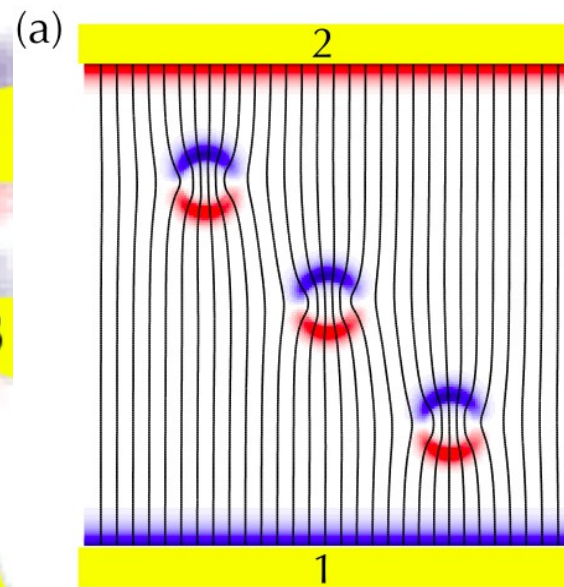


Nonlocality and directional effect “Shockley-Ramo theory”

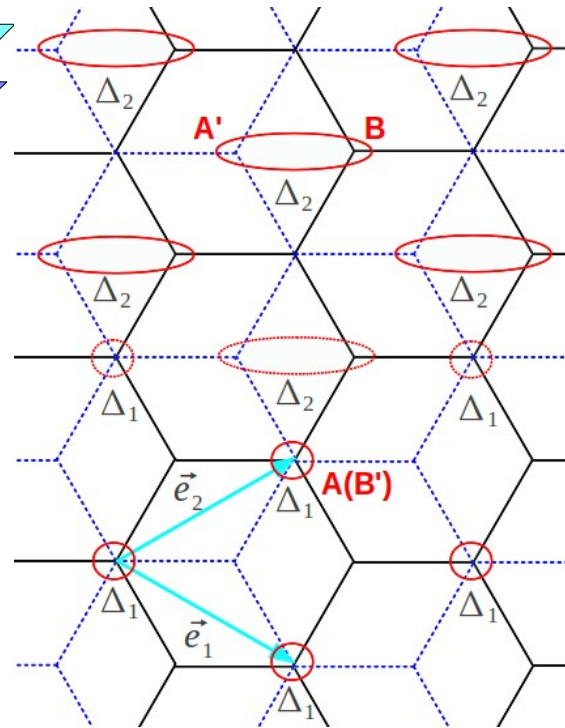
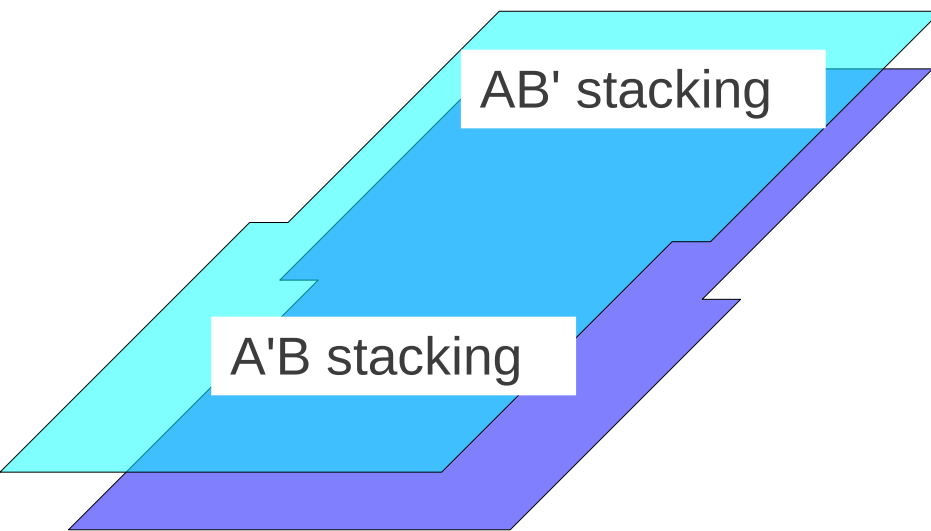
J Park et al (2009), J Song & LL (2011)



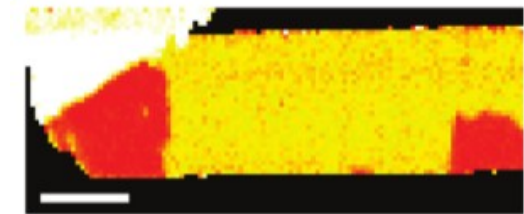
Angle-dependent global response, no position dependence



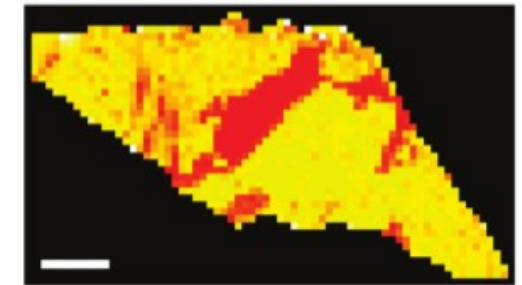
Tunneling heterojunctions in BLG: domains with different stacking order



ABA & ABC stacking in trilayer, Lui et al (2010)

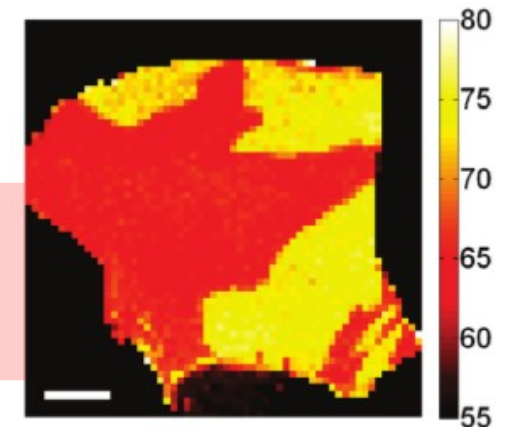


10 μm



Features:

- Tunneling transport (depends on orientation)
- New tunneling probe of ordered states
- Energy-dependent conductance, suppressed near DP (can mimic/obscure gapped state)

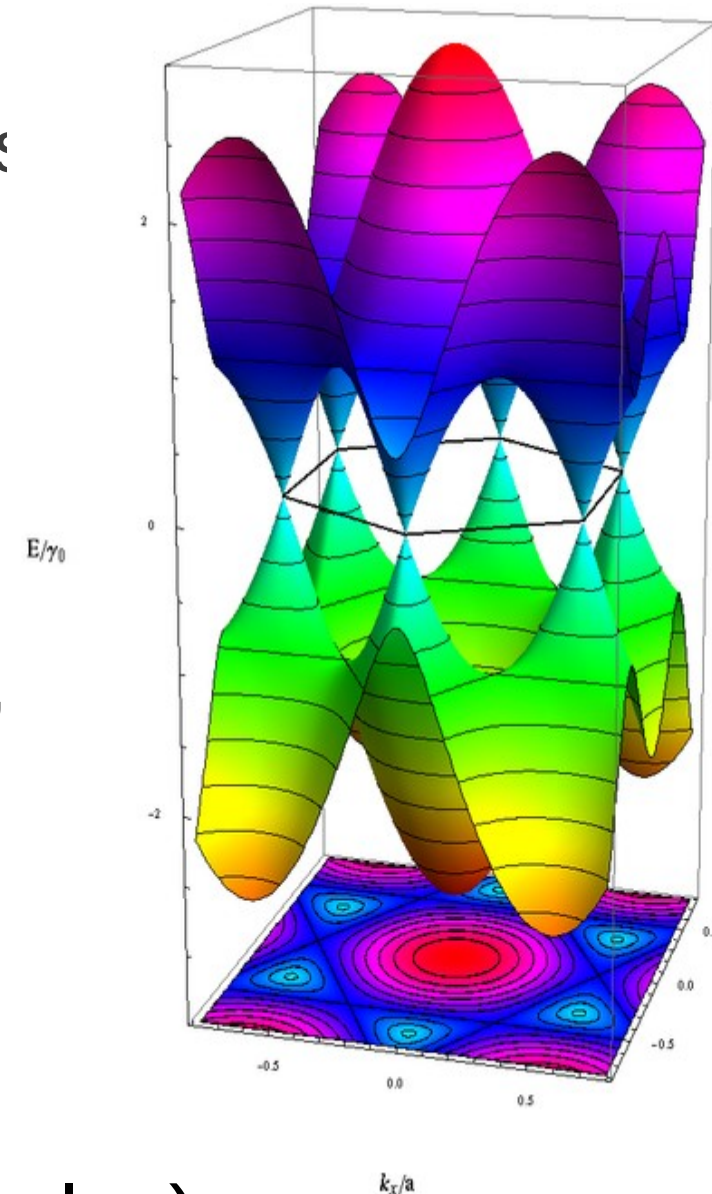


BLG summary

- Rich pattern of phases, $SU(4)$ classification
- Possibility of realizing QAH state at low T
- Inducing QAH state with B field
- **Experimental verdict:** QAH order plausible, but more work needed
- Additional experimental probes: optical Kerr effect, photocurrent imaging, tunneling

Electronic states in strongly doped graphene

- ◆ Quadratic dispersion near saddle points:
 $E = \pm t_0, -t_0$
- ◆ Logarithmic Van Hove singularity
- ◆ Hexagonal FS @ $n = 3/8, 5/8$
- ◆ Similar to square lattice @ $n = 1/2$
- ◆ Various competing orders: CDW, SDW, superconductivity, nematic order (Pomeranchuk instability)



High doping required ($\delta n = 1/8$)

Electrostatic gating challenging

Can be achieved chemically (Berkeley)

or with liquid dielectric gating (Columbia, Geneva)

Different scenarios

- Nesting and vH singularity enhance interaction effects
- d-wave pairing, Kohn-Luttinger framework (Gonzalez 2008)
- Pomeranchuk (nematic) order, mean field (Valenzuelo, Vozmediano 2008)
- SDW order, mean field
(Li arxiv:1103.2420, Makogon et al arxiv:1104.5334)
- Legitimate mean-field states: superconductor, metal, insulator
- Need renormalization group (RG) to compare these orders on equal footing

Attraction from repulsion

- Approach developed for square lattice

Schulz 1987, Dzyaloshinskii 1987, Furukawa, Rice, Salmhofer 1998, LeHur, Rice 2009

- RG treats all potential instabilities on equal footing
- Progressively integrate out high energy states, examine flow of couplings
- Marginal with log corrections
- Three sources of log divergences: DOS, BCS, nesting
- Pairing interaction induced by spin fluctuations
- New scenario for competition of SDW and SC

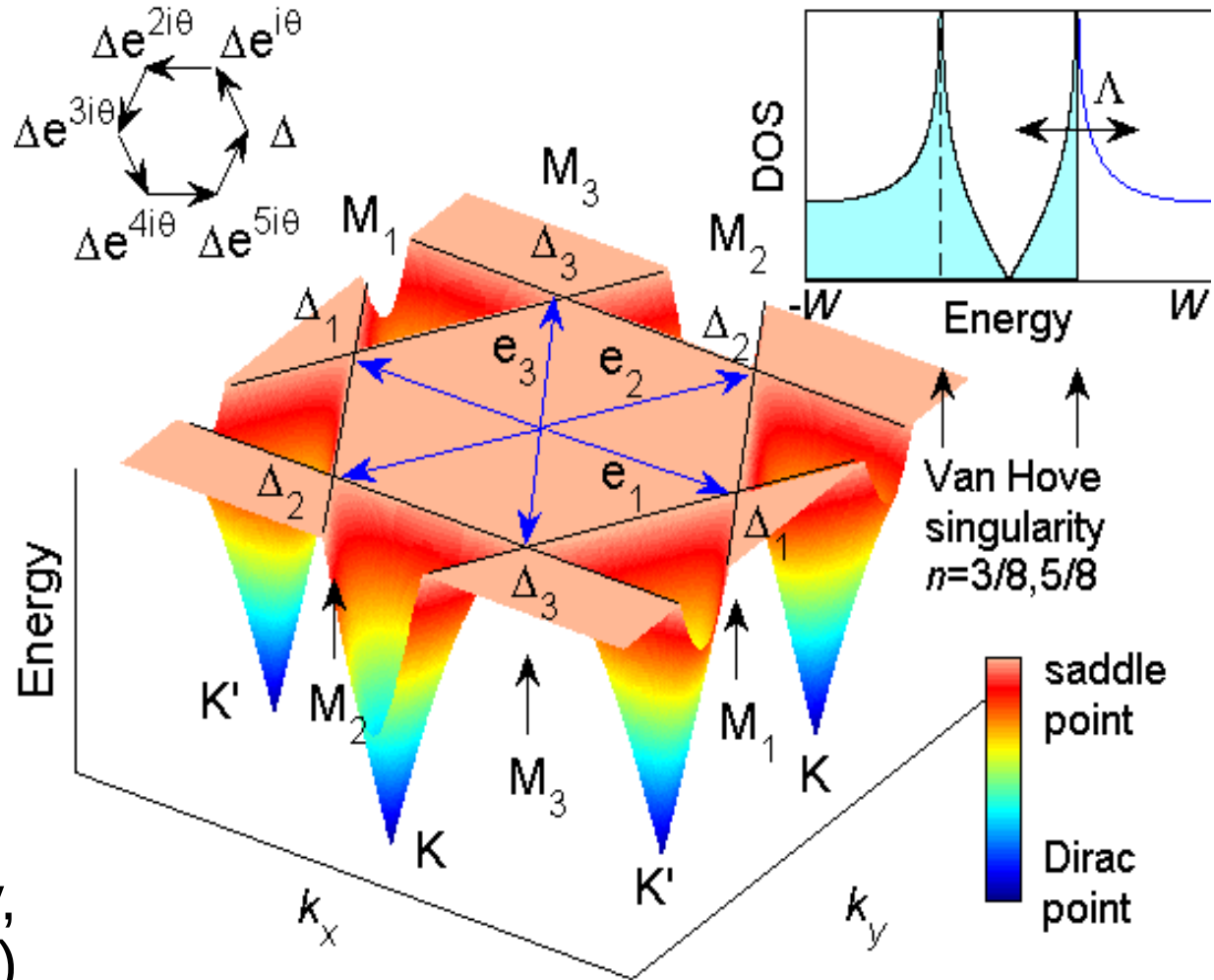
$$L = \sum_{\alpha=1}^3 \Psi_{\alpha}^{\dagger} (\partial_t - \epsilon_k + \mu) \Psi_{\alpha} - H_{two-particle}$$

Low energy description: three inequivalent patches

$$L = \sum_{\alpha=1}^3 \psi_{\alpha}^{\dagger} (\partial_t - \epsilon_k + \mu) \psi_{\alpha}$$

$-H_{\text{two-particle}}$

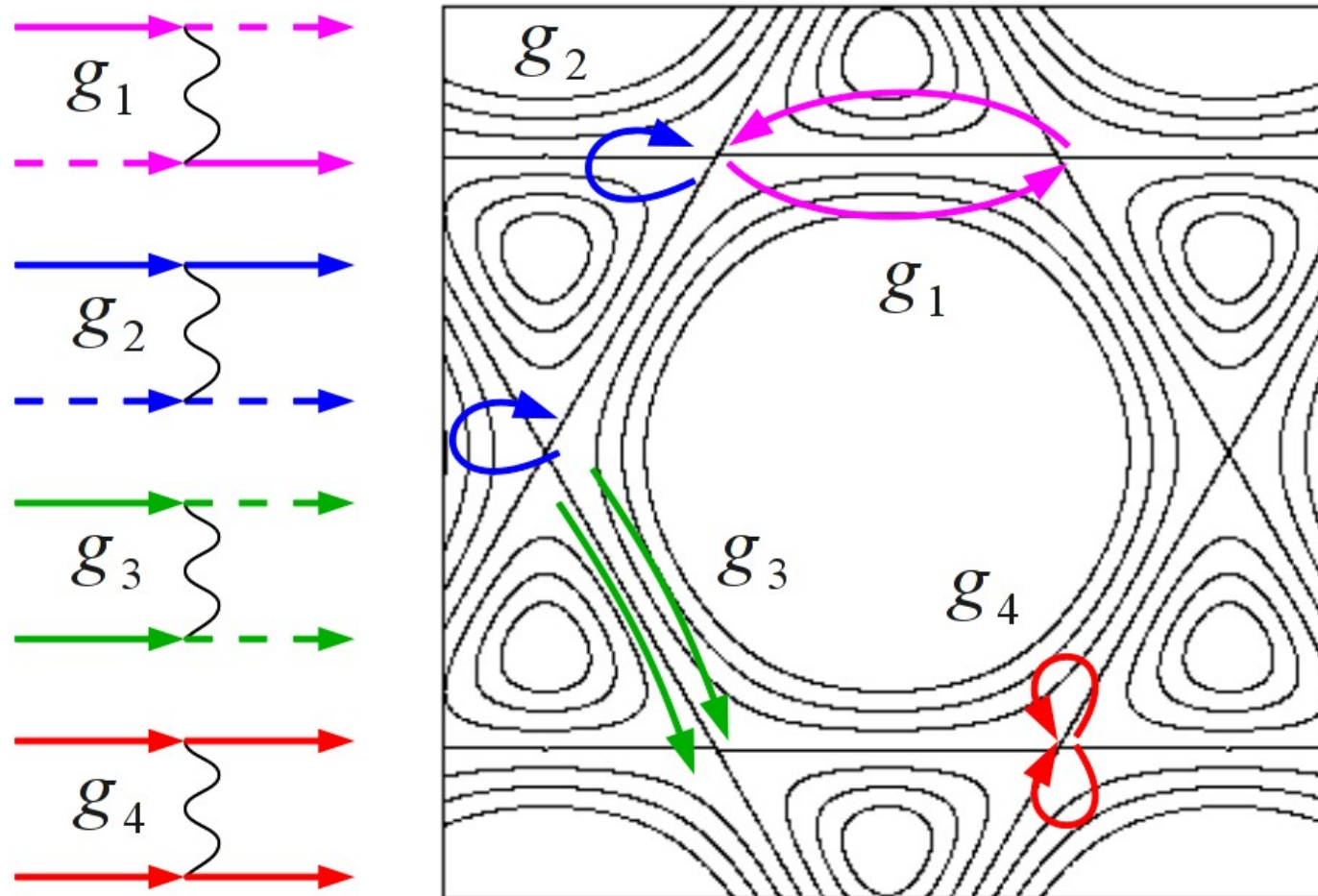
- four interactions
- (i) marginal at tree level
- (ii) log's



Nandkishore, Chubukov, Levitov, Nat Phys (2012)

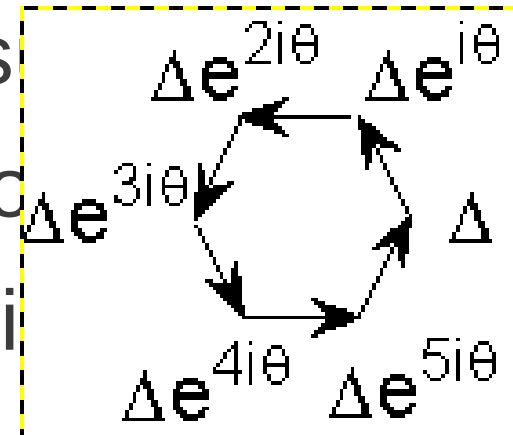
Two-particle inter-patch scattering processes

$$H_{two-particle} = \sum_{\alpha, \beta=1}^3 \frac{g_1}{2} \Psi_{\alpha}^{\dagger} \Psi_{\beta}^{\dagger} \Psi_{\alpha} \Psi_{\beta} + \frac{g_2}{2} \Psi_{\alpha}^{\dagger} \Psi_{\beta}^{\dagger} \Psi_{\beta} \Psi_{\alpha} + \frac{g_3}{2} \Psi_{\alpha}^{\dagger} \Psi_{\alpha}^{\dagger} \Psi_{\beta} \Psi_{\beta} + \sum_{\alpha=1}^3 \frac{g_4}{2} \Psi_{\alpha}^{\dagger} \Psi_{\alpha}^{\dagger} \Psi_{\alpha} \Psi_{\alpha}$$



Chiral superconductivity from repulsive interactions

- Pairing gap winds around the Fermi surface
- Induced by (weak) repulsive interactions
- d-wave pairing wins over s-wave pairing
- d+id state: *time reversal symmetry broken*
- Once a candidate for high T_c , long abandoned
- Rich phenomenology, as for p+ip states in ^3He films, SrRuO, FQHE $\nu=5/2$ (Volovik 1988, Laughlin 1998, Fu, Kane 2008, SC Zhang 2009):
 - (i) nonzero Chern class (“charge QHE” at $B=0$);
 - (ii) spin and thermal QHE; edge charge current in B field
 - (iv) Majorana states @ vortices and boundaries



Summary: chiral SC in highly doped SLG

- Interaction driven instability in graphene doped at saddle points
- Weak repulsive interaction stabilizes chiral superconducting state $d+id$ or $d-id$
- Enhanced T_c
- Topological superconductor with broken TRS
- Zoo of interesting phenomena
- Graphene easily combined with other materials into hybrid structures and heterostructures: pathway to applications of chiral superconductivity