Single-parameter pumping in graphene

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Phys. Rev. B **80**, 245414 (2009)

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Outline

- Graphene: chirality, evanescent modes, bipolarity...
- Adiabatic quantum pumping
- Non-adiabatic pumping
- Conclusions

Chirality

Chirality

Chiral

Chirality

versus

Chiral

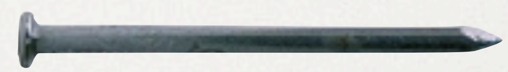
Non-chiral

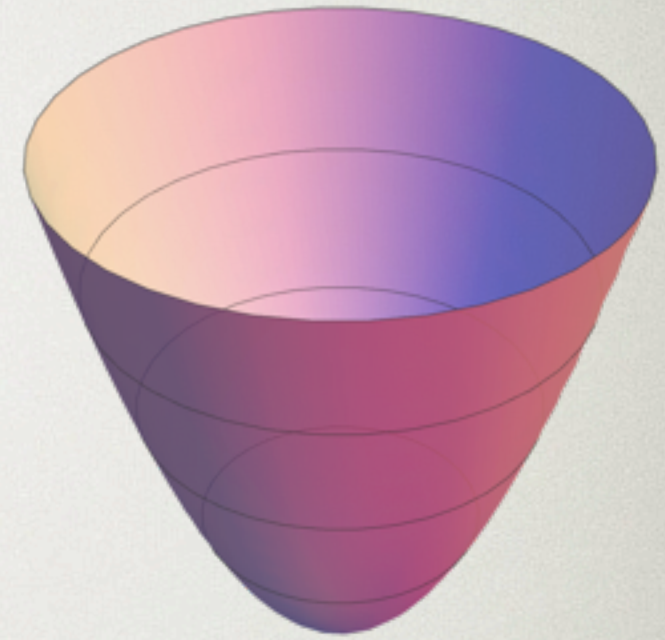
Chiral Electrons

Chiral Electrons

- 2D electron gas

$$H = \frac{k^2}{2m^*}$$





  - Spin is independent of k

Chiral Electrons

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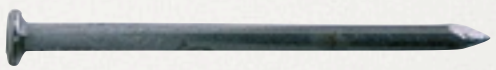
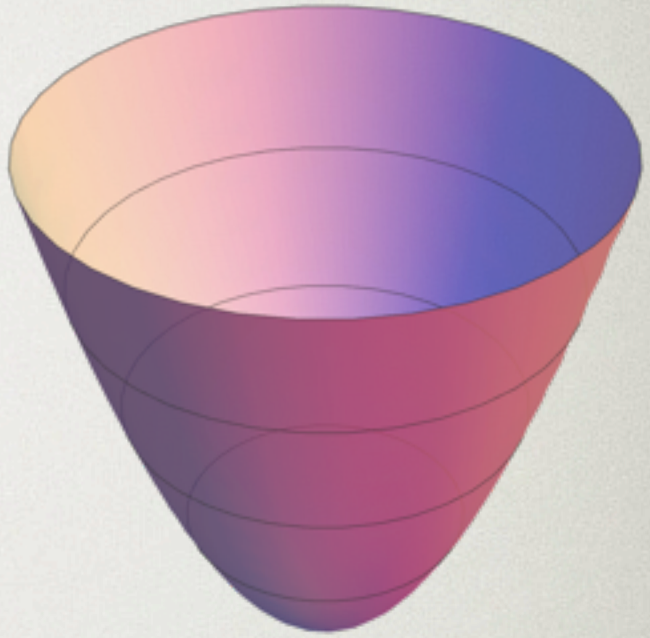
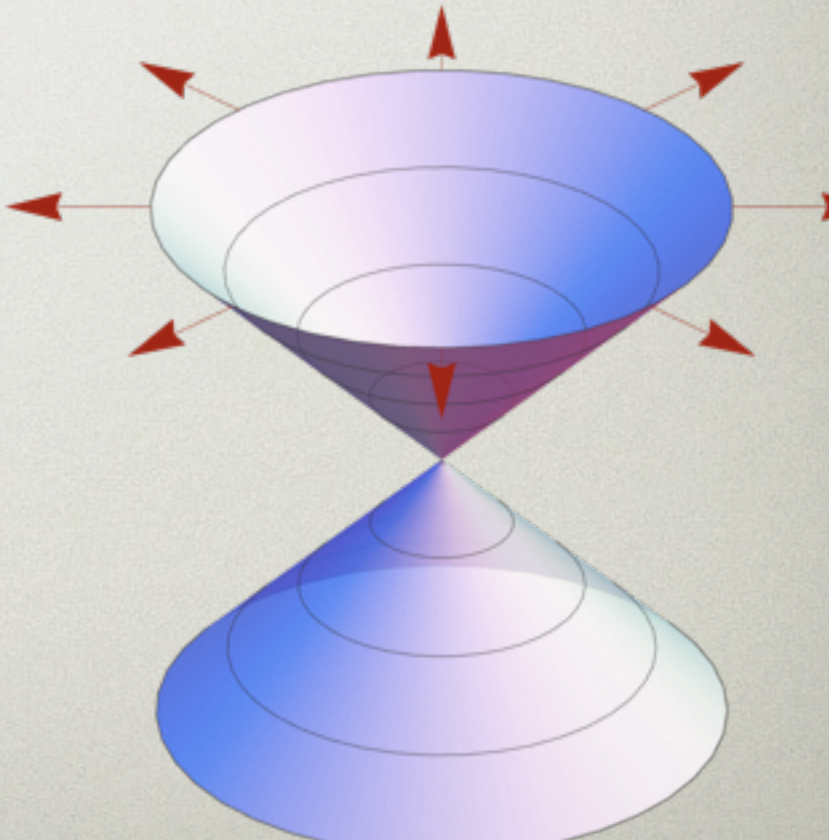




  - Spin is independent of k

- Graphene

$$H = v_F \boldsymbol{k} \cdot \boldsymbol{\sigma}$$



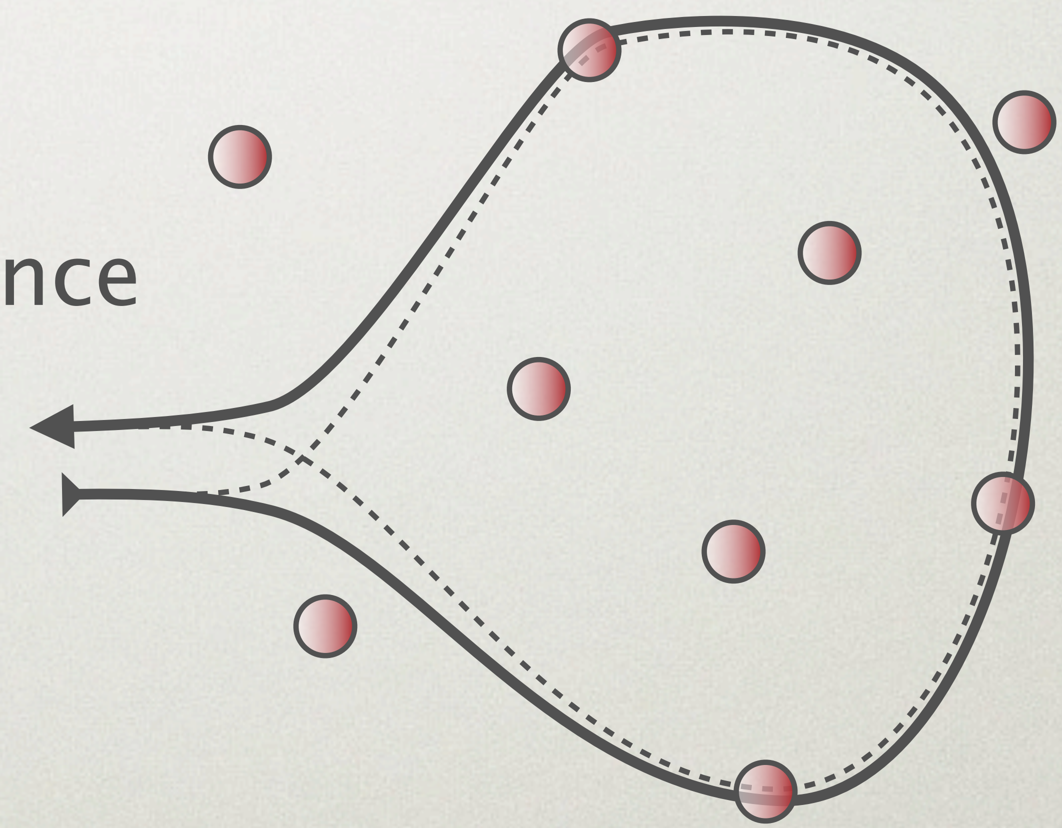
  - Eigenstates are chiral

Chirality and Transport

- Chirality tends to delocalize electrons
  - Weak antilocalization

Destructive interference

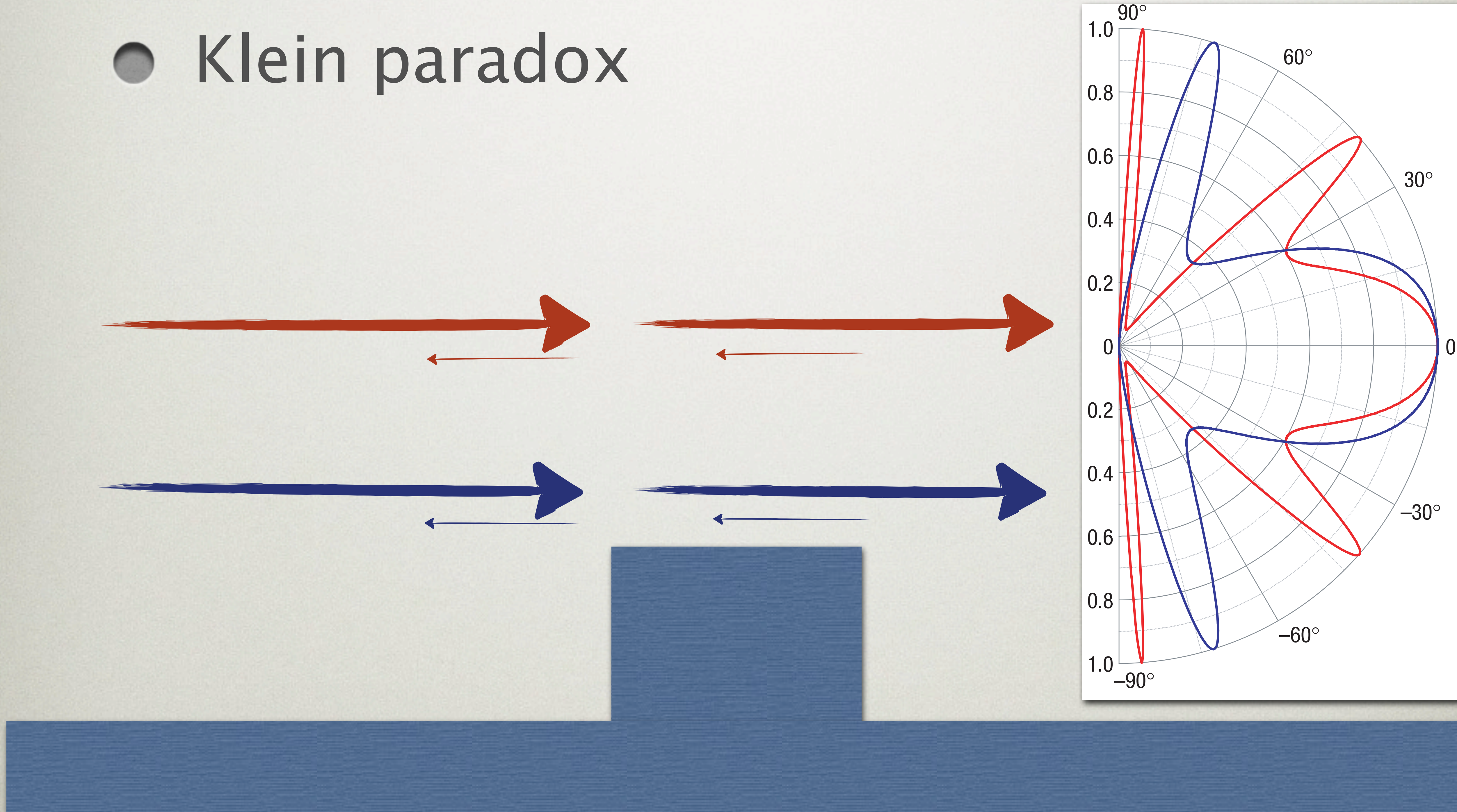
$$e^{\frac{i}{2}2\pi\sigma_z} = -\sigma_0$$



X. Wu, X. Li, Z. Song, C. Berger and W. A. de Heer, Phys. Rev. Lett. **98** 136801 (2007)

Chirality and Transport

- Chirality tends to delocalize electrons
    - Klein paradox



M. Katsnelson,K. Novoselov and A. Geim. Nature Physics **2** 620--625 (2006)

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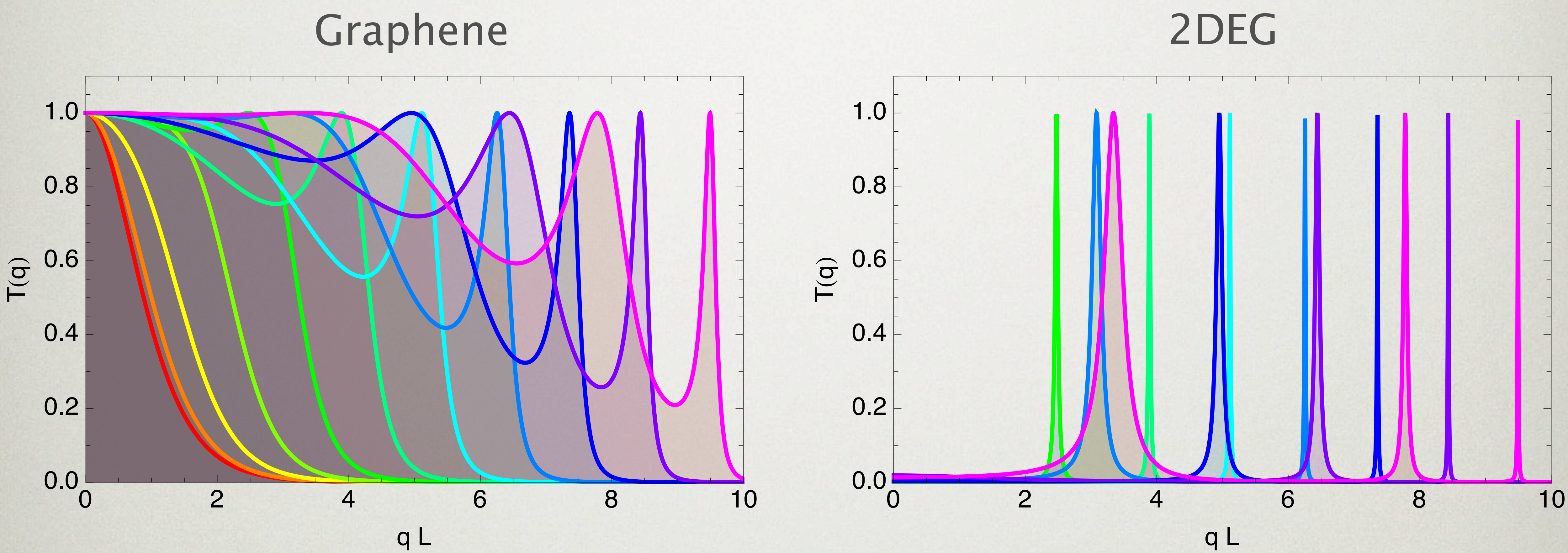
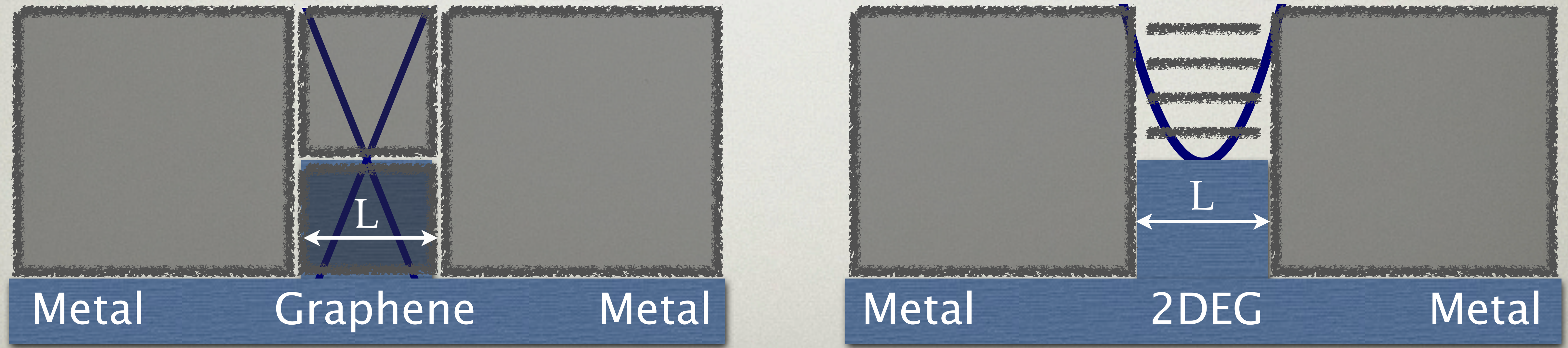


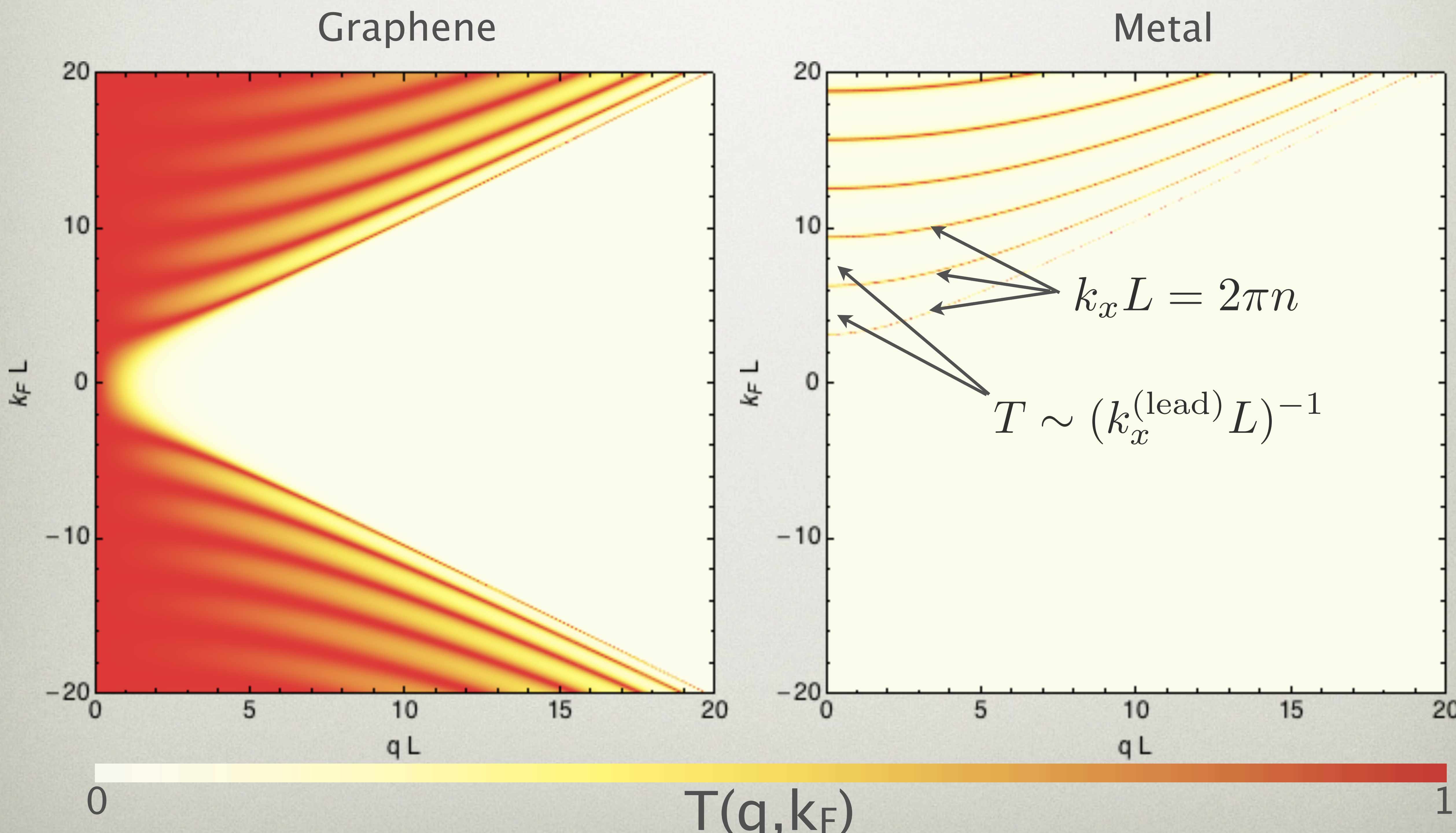


Open system

Closed system

Chirality and Transport

- Chirality tends to delocalize electrons



Motivation of our work

We are interested in how
exotic properties of
graphene
influence electron transport

Motivation of our work

How to drive a d.c. current?

Motivation of our work

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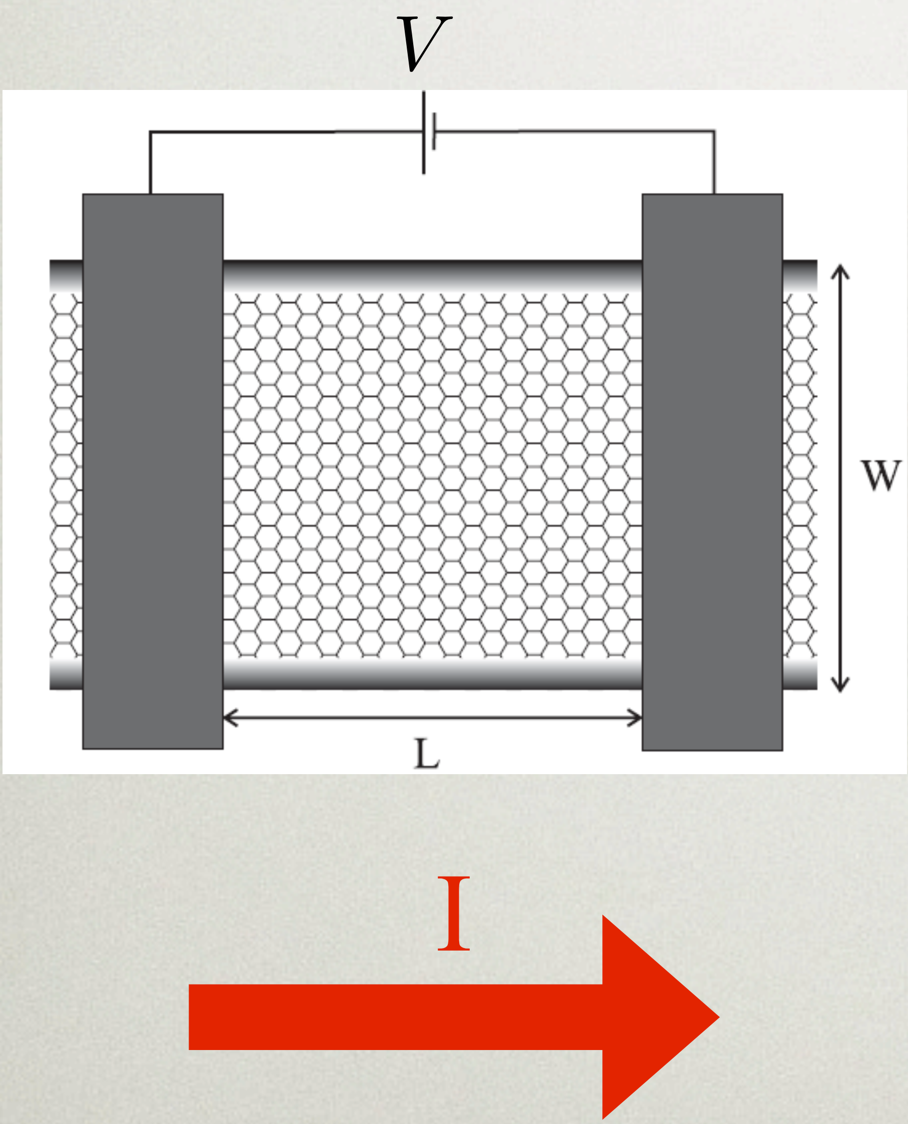
Normally...

Motivation of our work

Normally...



Strip of graphene contacted by two electrodes

A voltage source creates a voltage bias between the contacts , that drives a current through the strip

Motivation of our work

But there is another way...

Quantum Pumping

Quantum Pumping

An electron pump is a device that generates a d.c. current between two electrodes that are kept at the same bias

Quantum Pumping

An electron pump is a device that generates a d.c. current between two electrodes that are kept at the same bias

Electrons are transferred between the reservoirs by externally varying the scattering properties of the pumping region over time

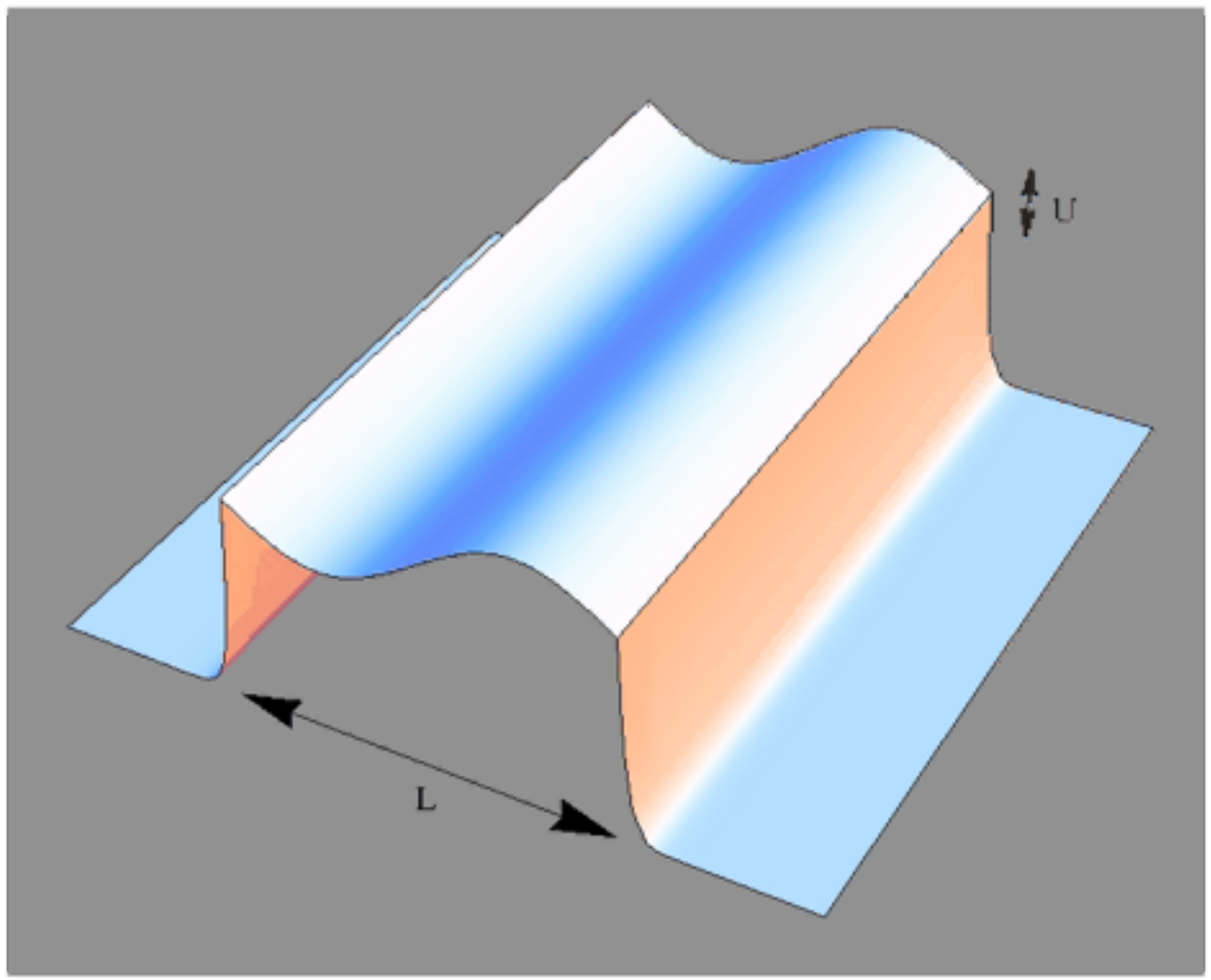
Quantum Pumping

- How does chirality impact the response of electrons under local driving?

Level spacing

E_L

$$E_L^N = \frac{\hbar^2}{2m^* L^2}$$
$$E_L^G = \frac{\hbar v_F}{L}$$



A quantum pump

Frequency ω
Amplitude U

Pumping Regimes

- Adiabatic limit $\omega \ll E_L$
  - Weak driving $U \ll E_L$
  - Strong driving $U \gg E_L$
- Non-adiabatic limit $\omega \gtrsim E_L$
  - Weak driving $U \ll \omega$
  - Strong driving $U \gg \omega$

Adiabatic Pumping

Quantum pumping in graphene

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Department of Physics, Lancaster University, Lancaster LA1 4YB, United Kingdom
(Received 27 August 2009; revised manuscript received 3 November 2009; published 10 December 2009)

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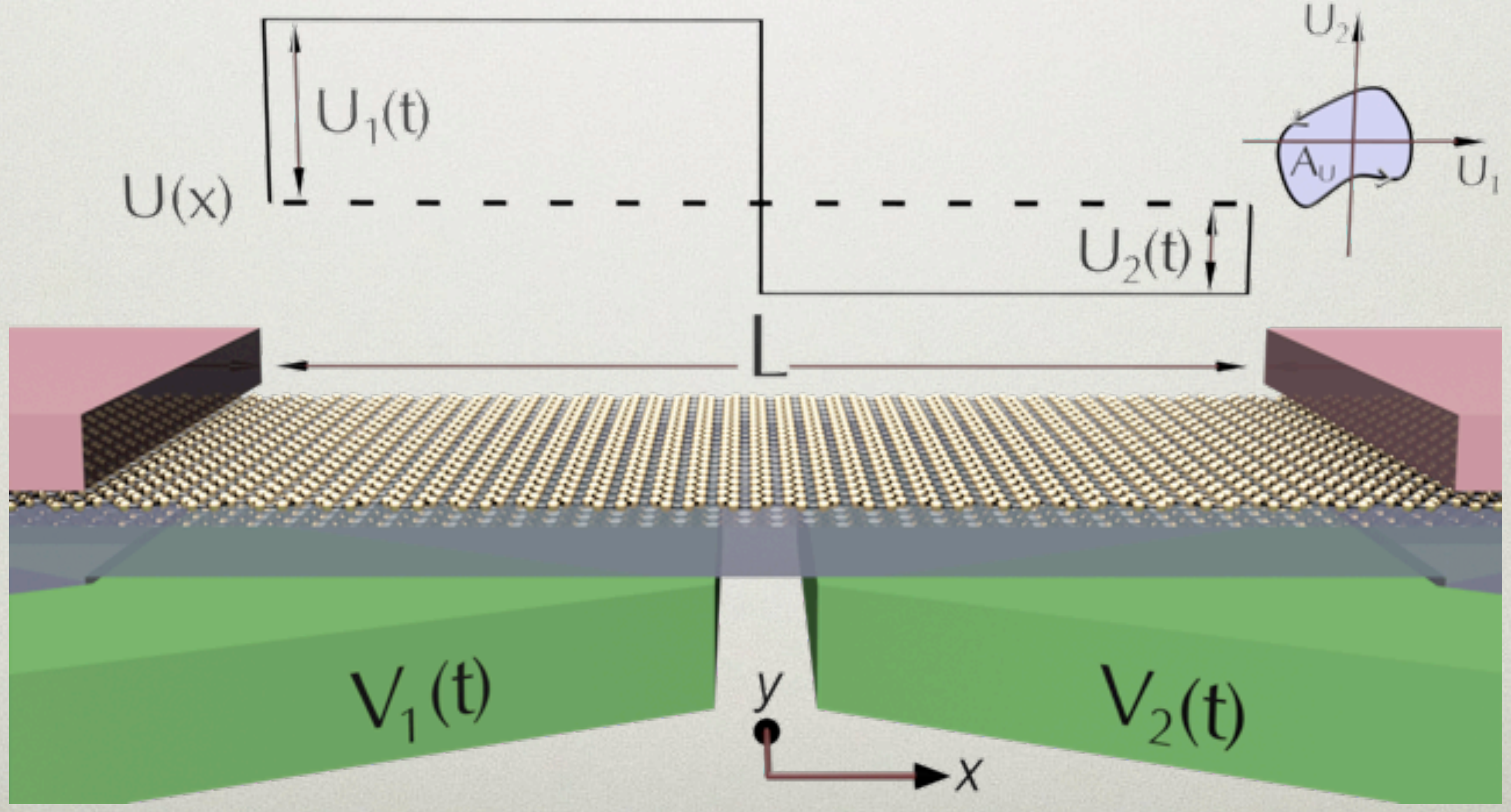


Adiabatic Pumping

- Geometric formulation of pumping

PHYSICAL REVIEW B VOLUME 58, NUMBER 16 15 OCTOBER 1998-II

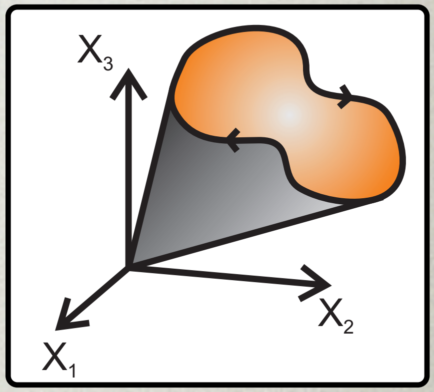
Scattering approach to parametric pumping

P. W. Brouwer

Assuming:

- ✦ Phase coherent system
- ✦ Negligible interactions
- ✦ Zero temperature
- ✦ Adiabatic driving at frequency $\omega \ll 1/\tau_D$ ← dwell time

the charge pumped between two reservoirs is:



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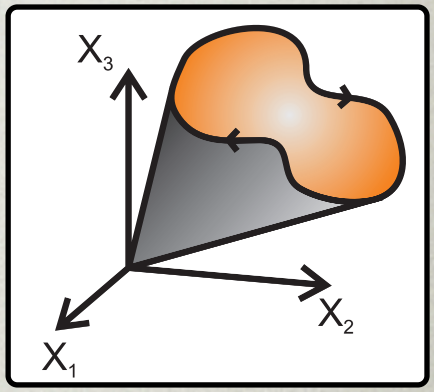
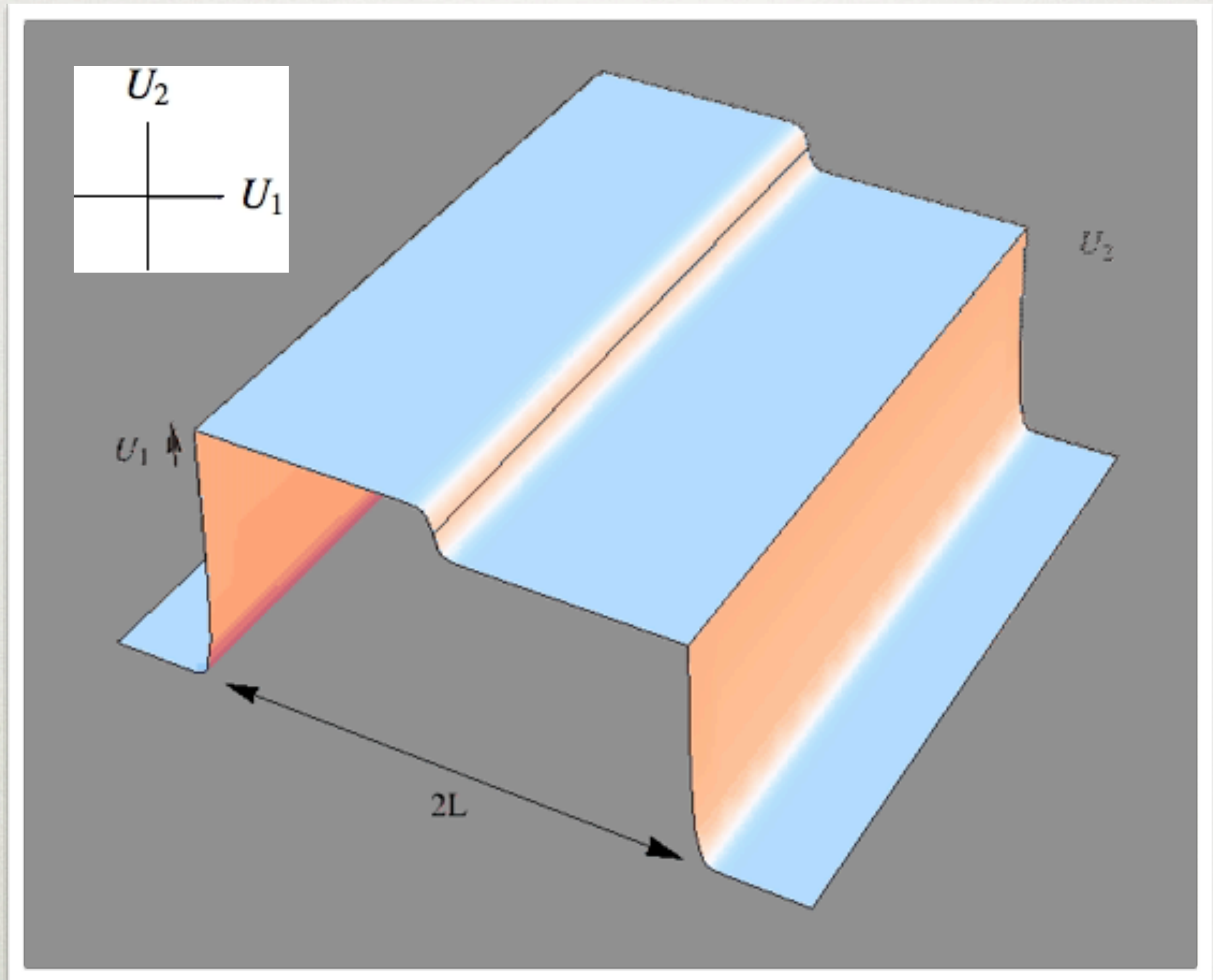


the charge pumped between two reservoirs is:

$$Q(m) = \frac{e}{\pi} \int_A dX_1 dX_2 \sum_{\beta} \sum_{\alpha \in m} \mathrm{Im} \frac{\partial S^*_{\alpha\beta}}{\partial X_1} \frac{\partial S_{\alpha\beta}}{\partial X_2}$$

Adiabatic Pumping

- Minimal setup: two parameter pumping

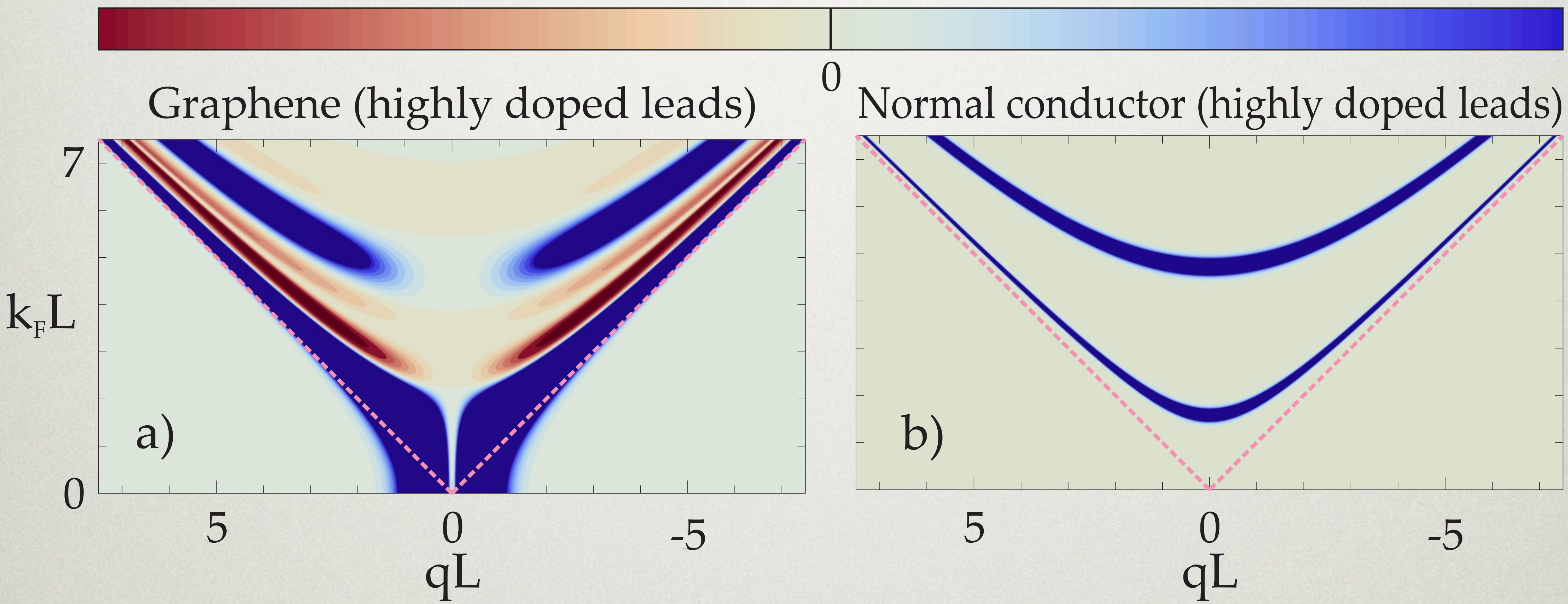


$$Q_{L\to R} = \frac{e}{\pi}\mathrm{Im}\iint dU_1 dU_2 \, \langle \partial_{U_1} s_L | \partial_{U_2} s_L \rangle$$

Adiabatic pumping

- Adiabatic pumping response

$$\chi_{uq} = \frac{Q^{q}_{L\to R}}{\pi\,(U/E_L)^2}\frac{1}{N_p}$$

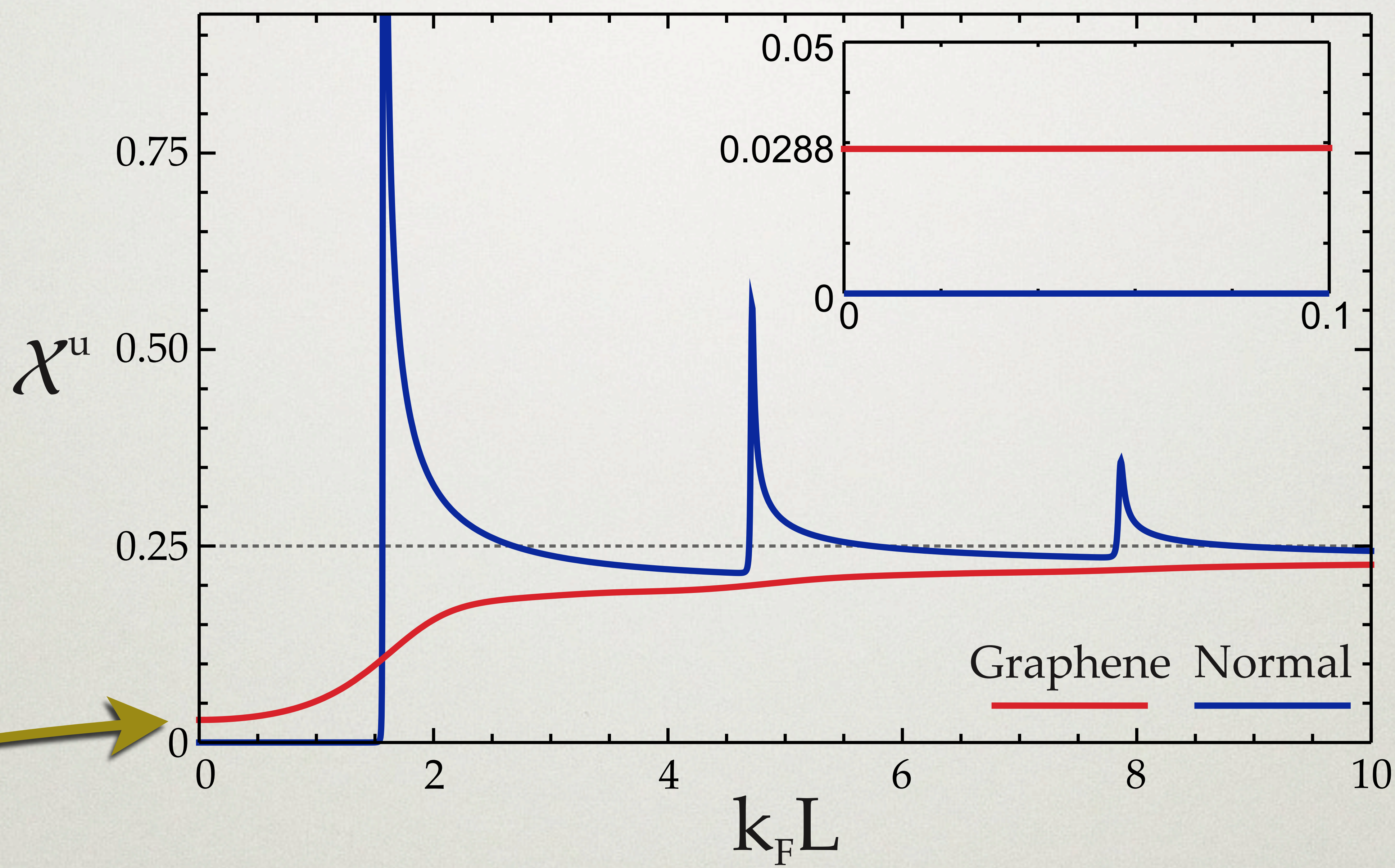


- Chirality enables pumping of evanescent modes close to q=0

E. Prada, P. San-Jose and H. Schomerus, Phys. Rev. B **80**, 245414 (2009).

Adiabatic pumping

- Total response: summing over modes



Universal response
(W≫L)

$$\int_0^\infty dq \frac{\sinh^2(q)\,[2q\cosh(2q)-\sinh(2q)]}{\pi q^3 \cosh^4(2q)} = 0.0288$$

Adiabatic Pumping

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- Chirality enables efficient pumping of evanescent modes in graphene

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- Not so in conventional pumps

Adiabatic Pumping

- Chirality enables efficient pumping of evanescent modes in graphene
- Not so in conventional pumps
- Dirac point pumping is universal for weak driving in W>>L pumps – close analogy to the minimal conductivity of ballistic graphene.

NOW...

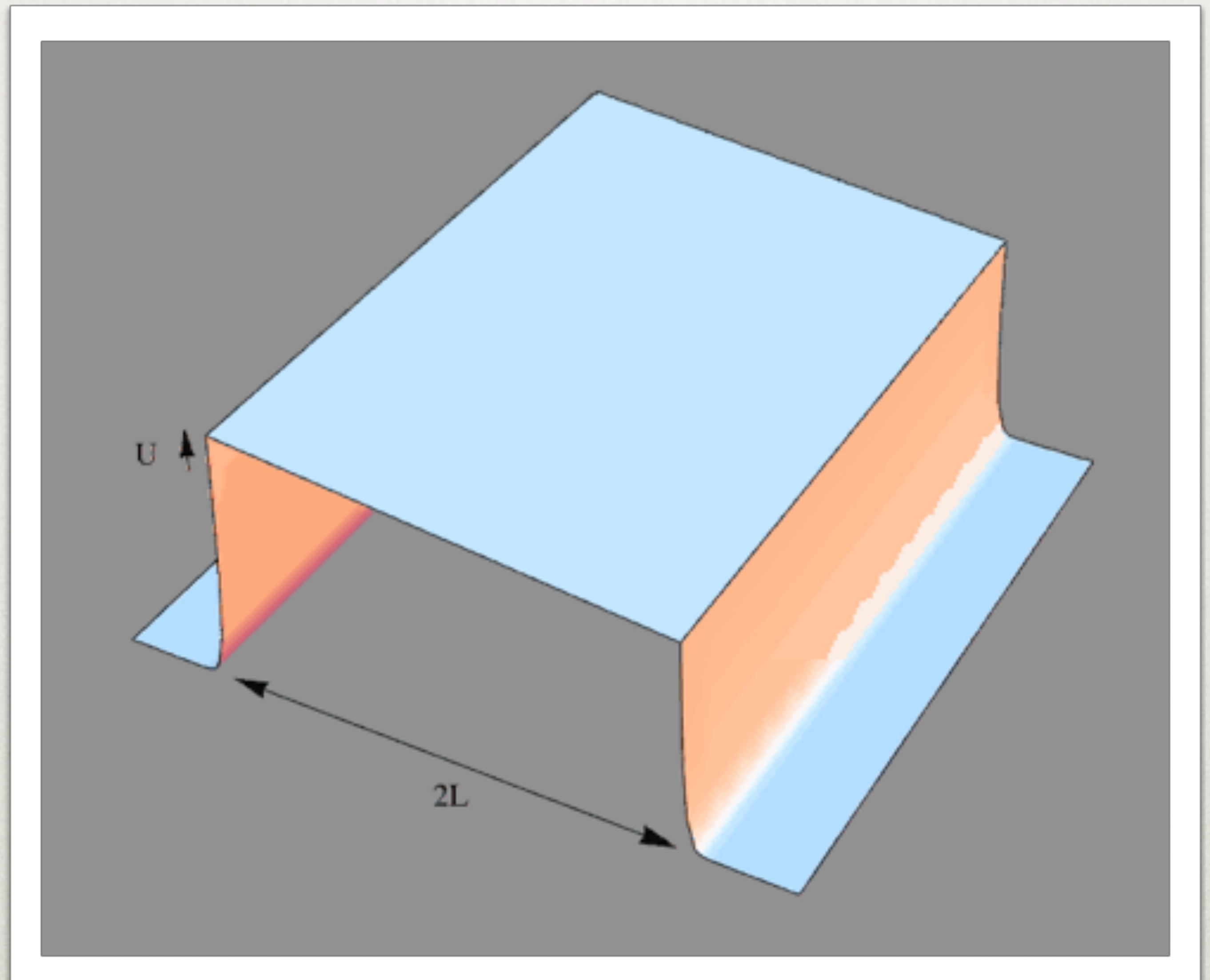
NON-ADIABATIC PUMPING

$$\omega \gtrsim E_L$$

Non-Adiabatic Pumping

- Minimal pumping requirements:

single-parameter driving + left-right asymmetry



Minimal non-adiabatic pump

M. Wagner and F. Sols, Phys. Rev. Lett. 83, 4377 (1999).
S. Kohler, J. Lehmann and P. Hänggi, Physics Reports **406** 379 - 443 (2005)

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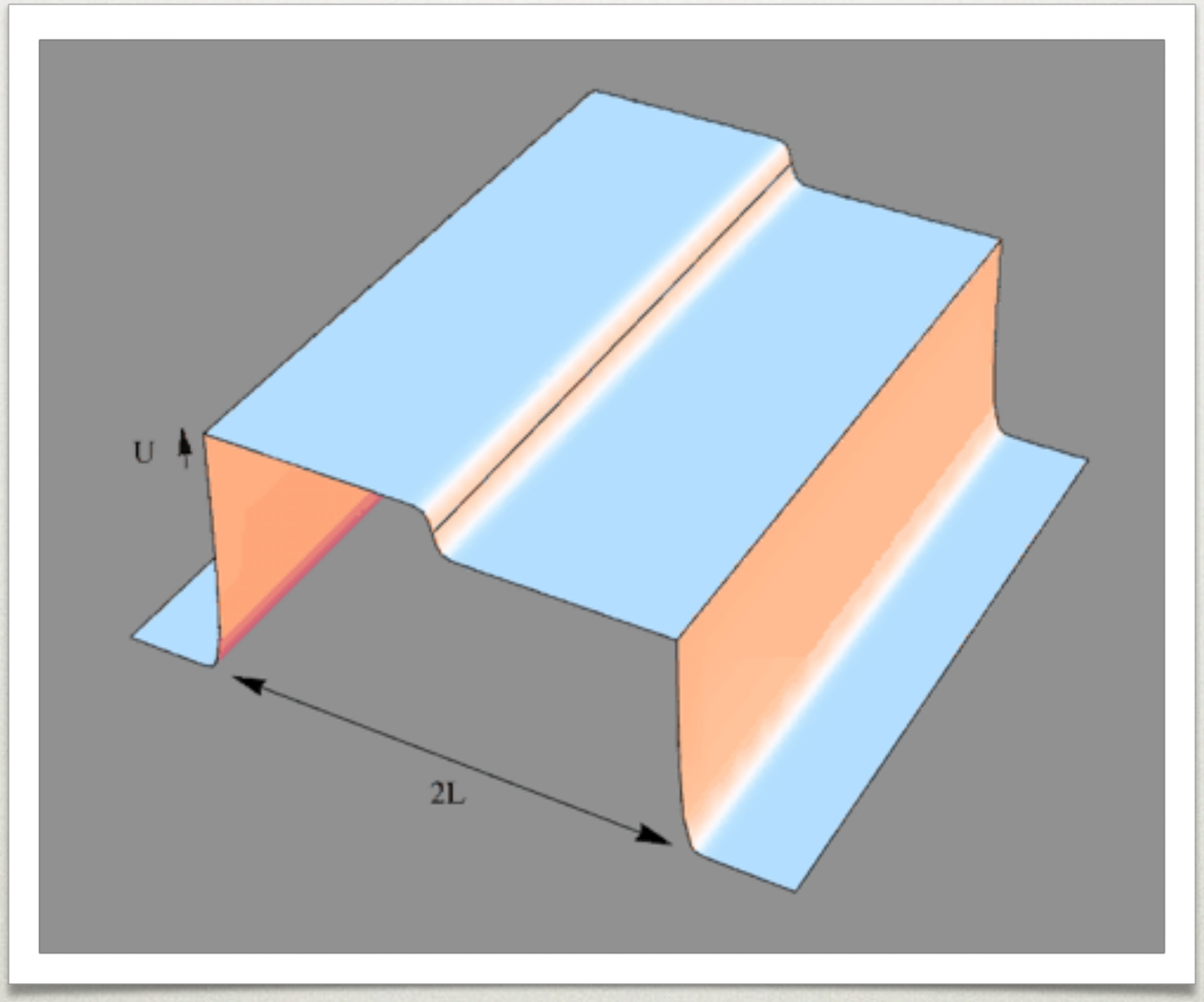

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- Possible due to **spatiotemporal symmetry breaking**
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- **Floquet theory**

Floquet theory

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- **Floquet theory**: turns a periodic, time dependent problem into a static one

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  - Floquet theorem for the stationary limit

$$\text{If } i\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle \text{ and } H(t) = H(t+T), \text{ then}$$
$$|\Psi(t)\rangle = e^{-i\epsilon t}|\phi(t)\rangle, \text{ with } |\phi(t+T)\rangle = |\phi(t)\rangle$$

Floquet theory

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  - Then $|\phi(t)\rangle = \sum_n e^{in\omega t}|\phi_n\rangle$, with $\omega = 2\pi/T$

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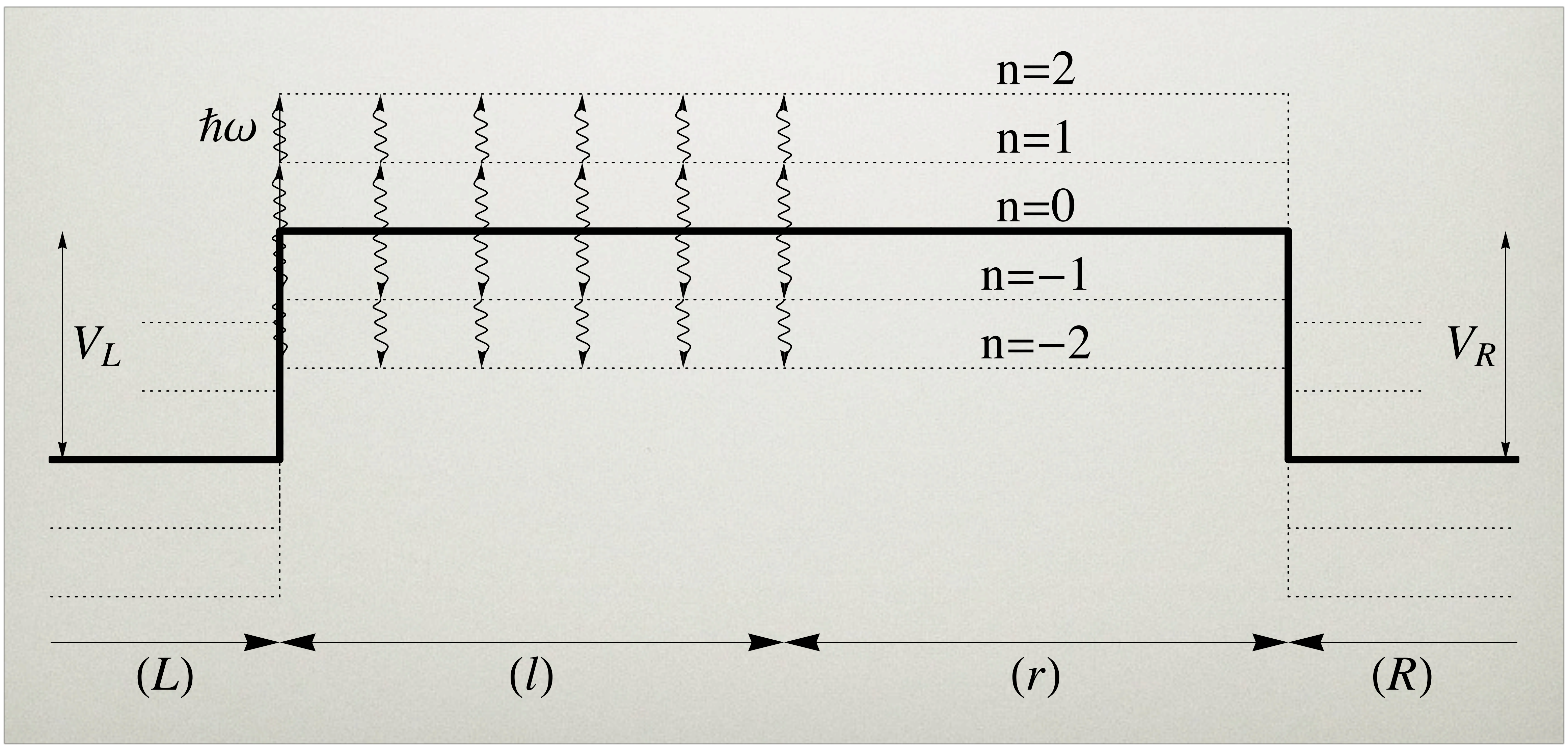
- Then $|\phi(t)\rangle = \sum_n e^{in\omega t}|\phi_n\rangle$, with $\omega = 2\pi/T$
- If $H(t) = H^{(0)} + \cos(\omega t)U$

$$\left(H^{(0)} + n\hbar\omega\right)|\phi_n\rangle + \frac{1}{2}U\left(|\phi_{n+1}\rangle + |\phi_{n-1}\rangle\right) = \epsilon|\phi_n\rangle$$

Floquet Theory

- Static Hamiltonian of coupled sidebands

$$H = \sum_n \left(H^{(0)} + n\hbar\omega \right) |\phi_n\rangle\langle\phi_n| + \frac{1}{2} \sum_n U(x) \left(|\phi_{n+1}\rangle\langle\phi_n| + |\phi_n\rangle\langle\phi_{n+1}| \right)$$



Floquet Theory

- The time-averaged pumped current is:

$$\bar{I} = \frac{e}{h} \sum_{n=-\infty}^{\infty} \int d\epsilon \left[T^{(n)}_{L\rightarrow R}(\epsilon) - T^{(n)}_{R\rightarrow L}(\epsilon) \right] f(\epsilon)$$

in terms of the sideband-resolved transmissions $T^{(n)}$

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in terms of the sideband-resolved transmissions $T^{(n)}$

- At zero temperature

$$\frac{d\bar{I}}{dE_F} = \frac{e}{h} \sum_{n=-\infty}^{\infty} \left[T^{(n)}_{L\to R}(E_F) - T^{(n)}_{R\to L}(E_F) \right] = \frac{e}{h} \Delta T(E_F)$$

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Parameter Regime

Strong non-adiabatic pumping:
only one sideband

$$\hbar\omega \gg E_L$$

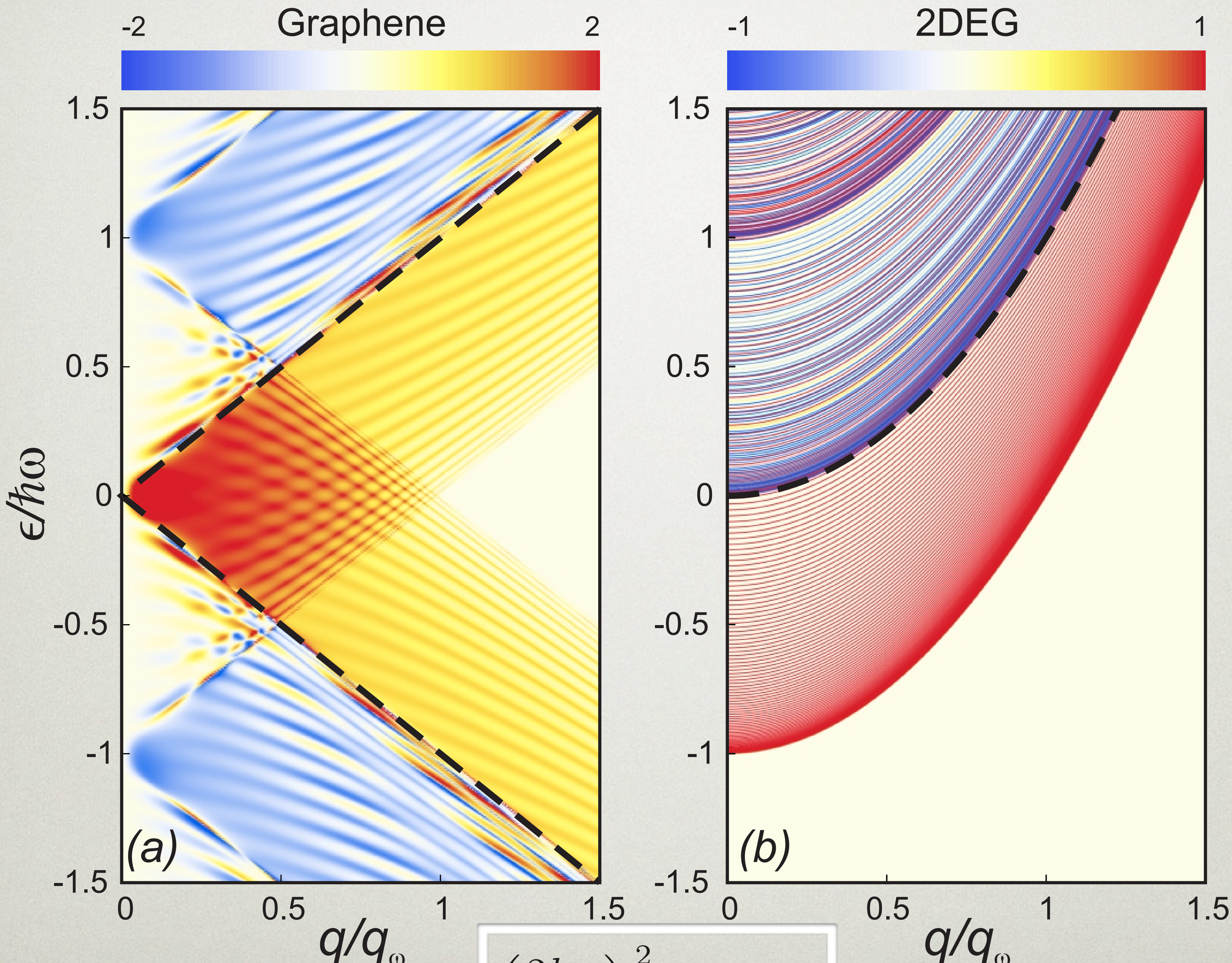
Weak driving regime:

$$U \ll \hbar\omega$$

Wider than long pumps:

$$W \gg L$$

Results

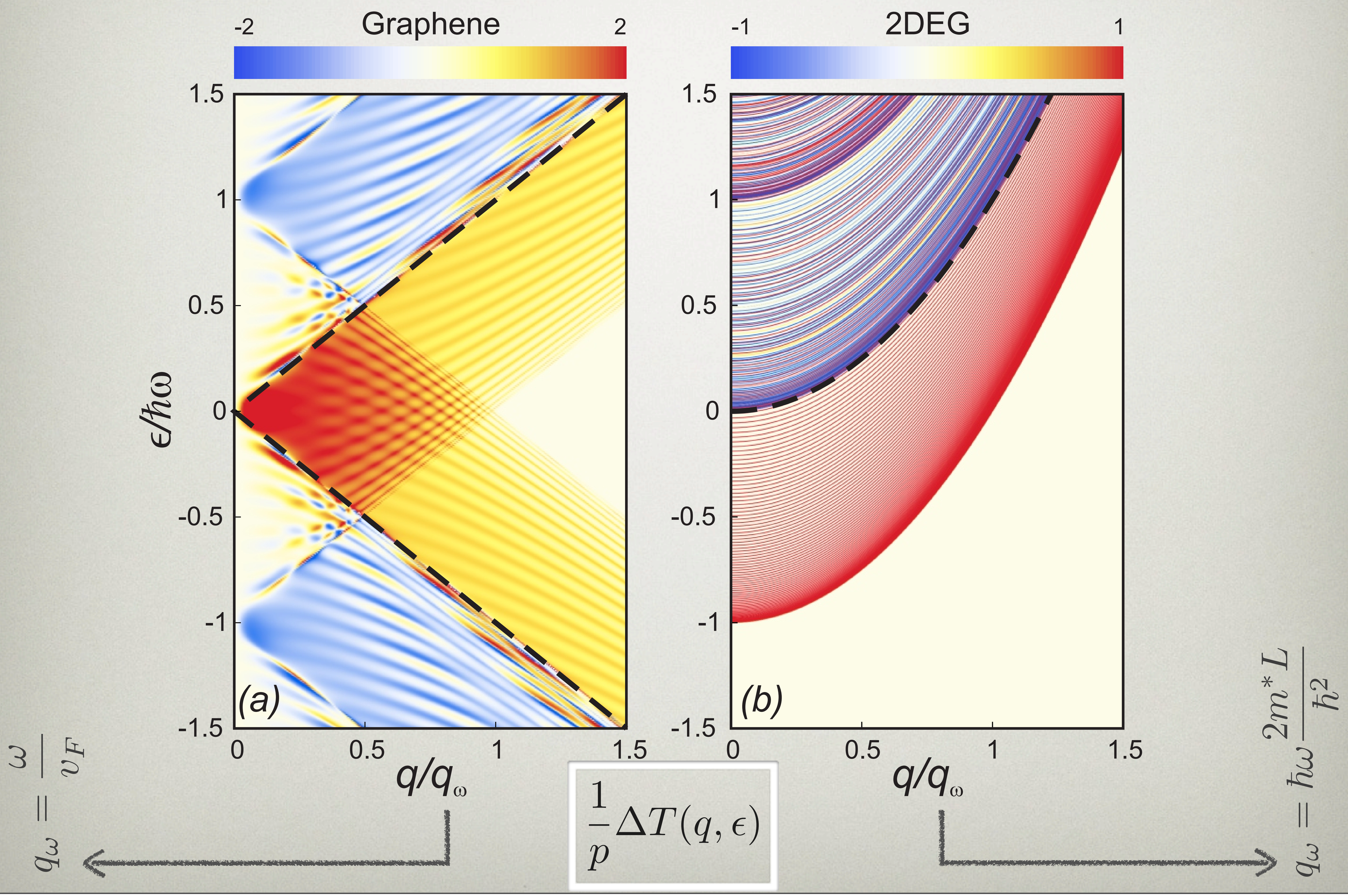


$$q_\omega = \frac{\omega}{v_F}$$

$$\left(\frac{2\hbar\omega}{U}\right)^2 \Delta T(q,\epsilon)$$

$$q_\omega = \hbar\omega\frac{2m^*L}{\hbar^2}$$

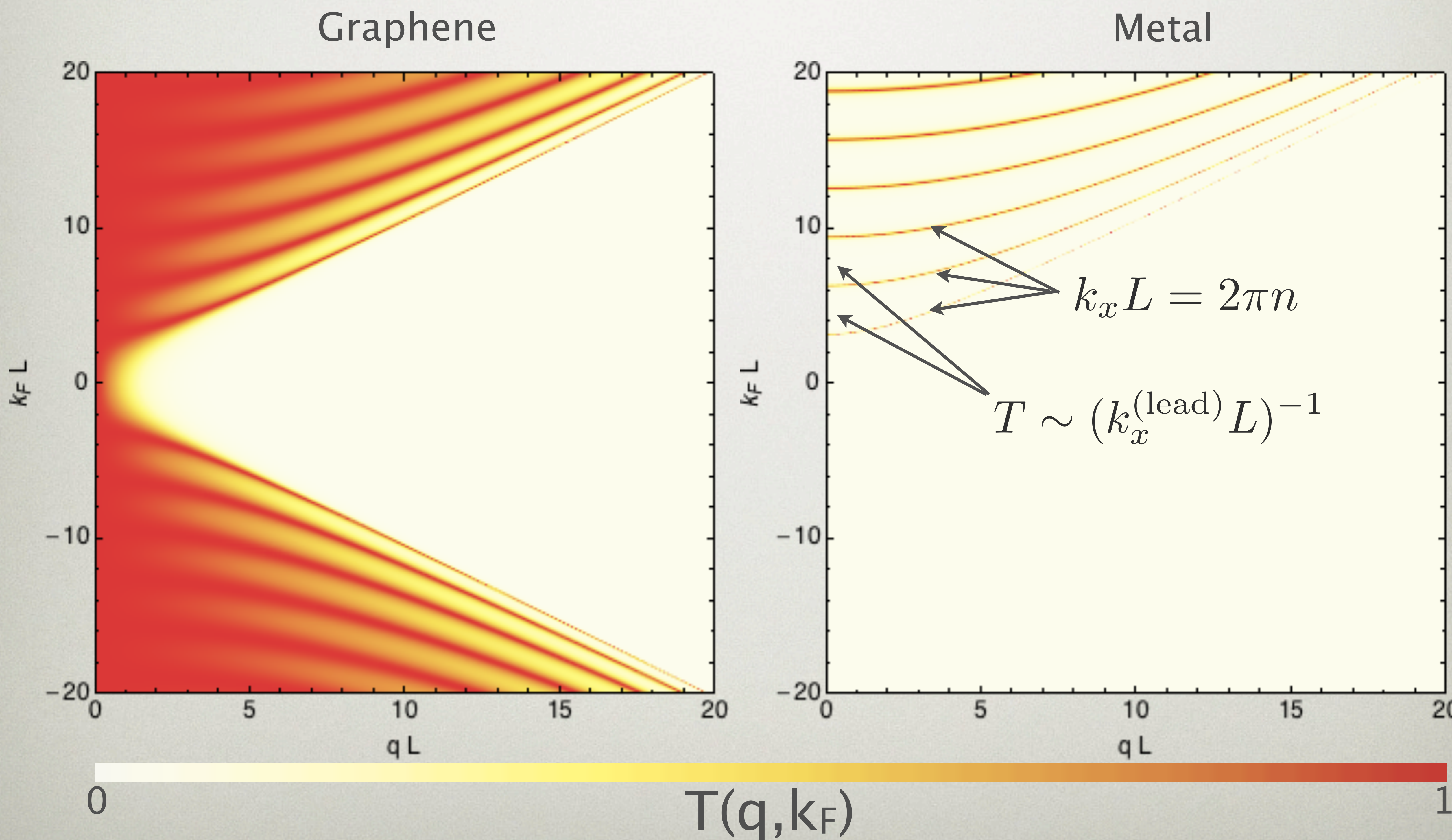
Results



Graphene
-2
2
2DEG
-1
1
ϵ/ħω
q/q_ω
(a)
(b)
$q_\omega = \frac{\omega}{v_F}$
$\frac{1}{p}\Delta T(q,\epsilon)$
$q_\omega = \hbar\omega\frac{2m^*L}{\hbar^2}$

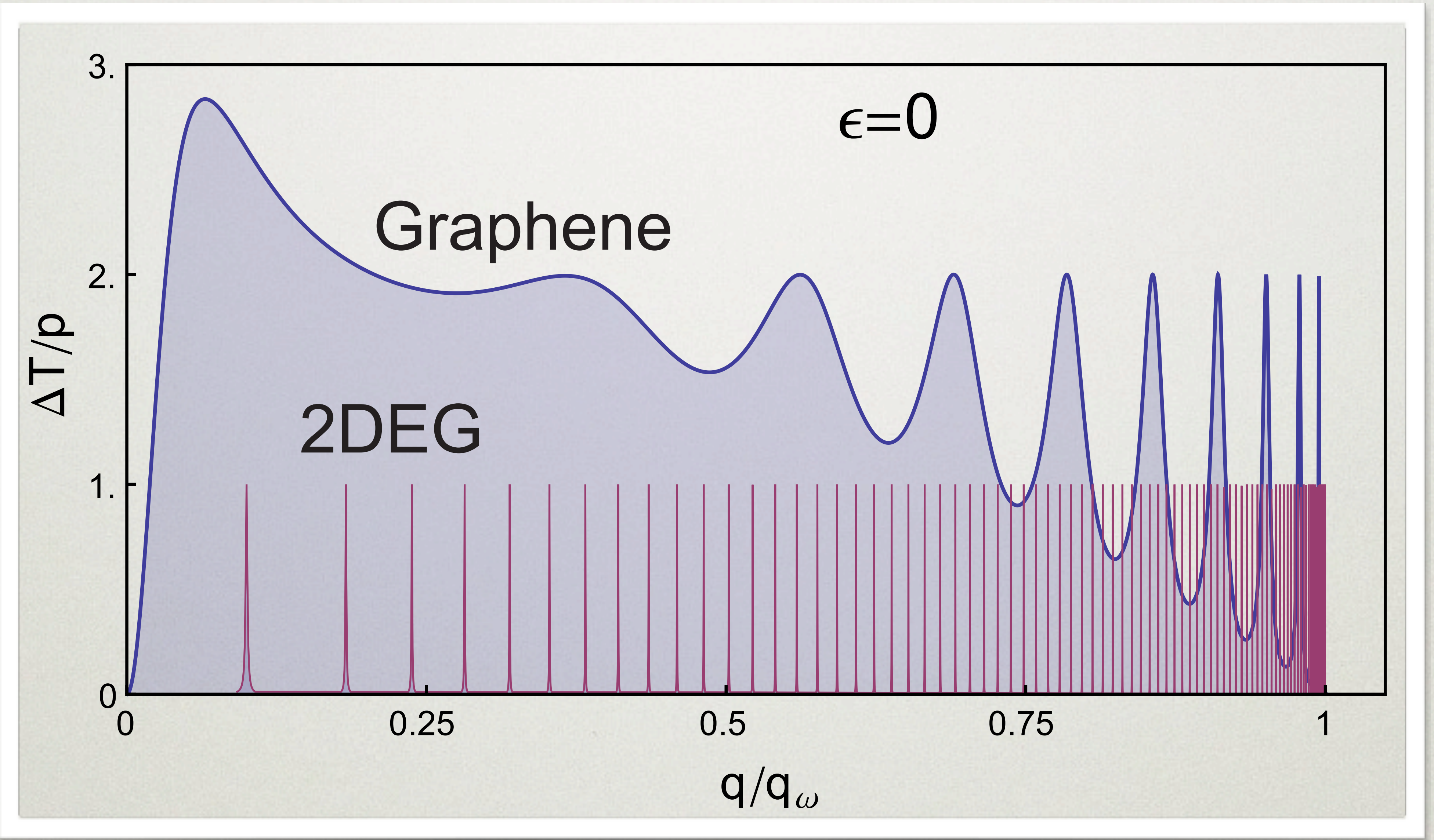
Remember...

- For voltage-driven transport we had:



Results

- Purely evanescent pumping response

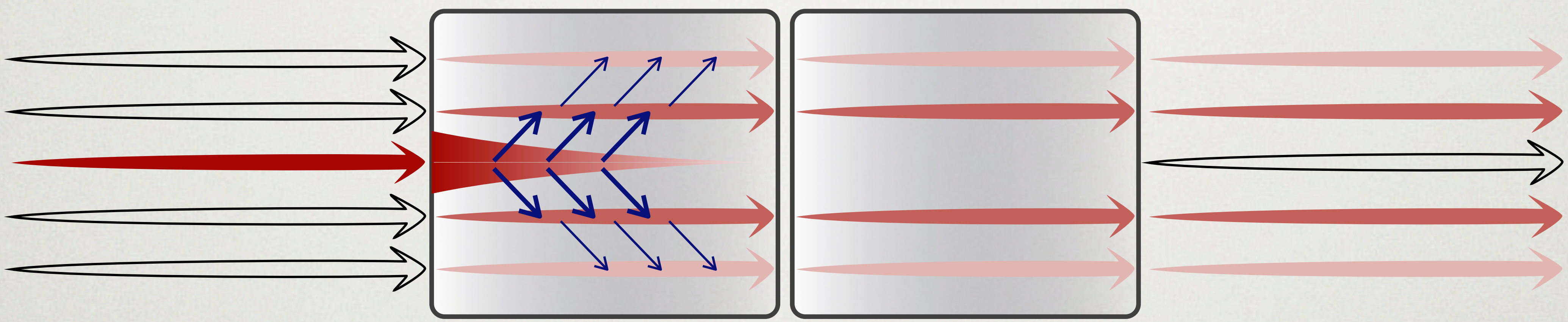


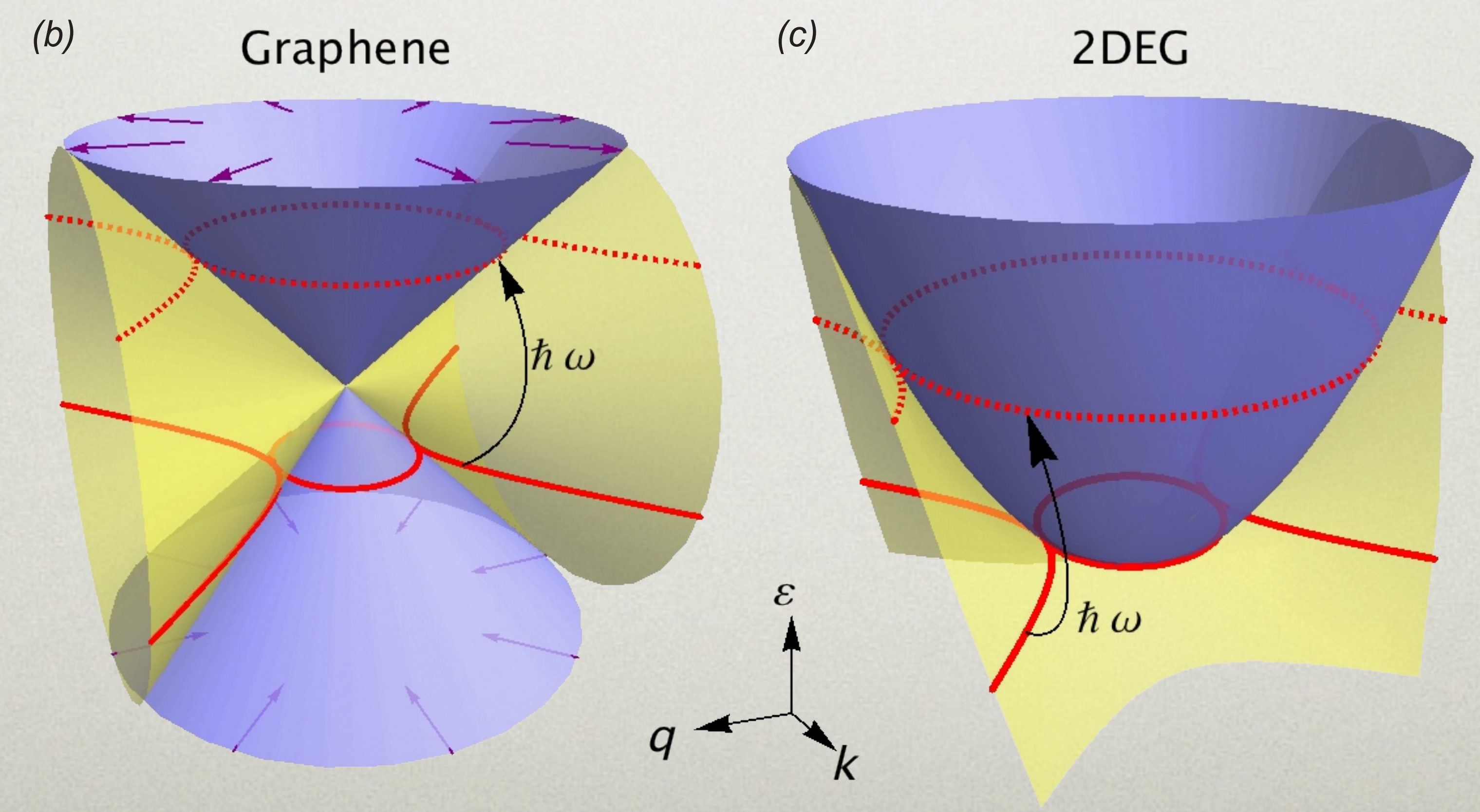
- Rectified! (does not change sign)

Evanescent mode mechanism

- Evanescent modes are pumped only in one direction

$T_{LR} > 0$





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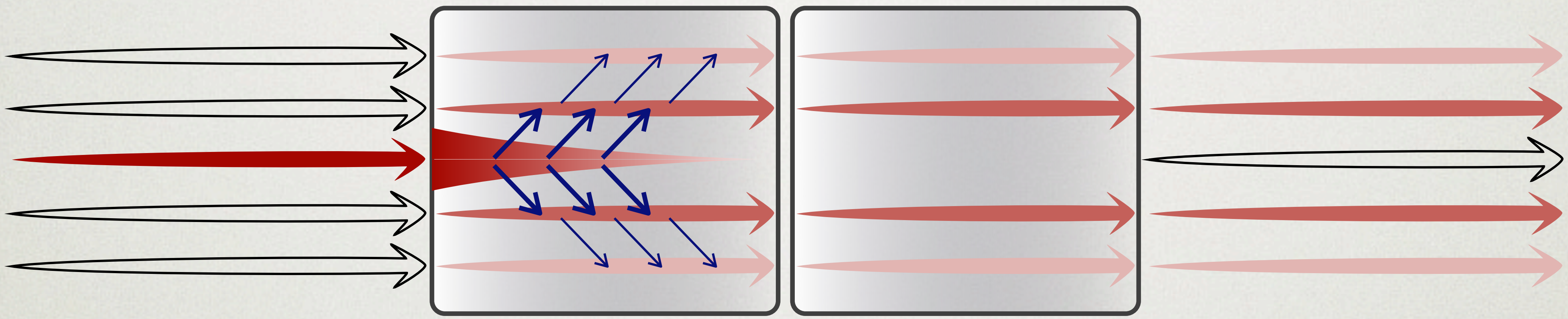
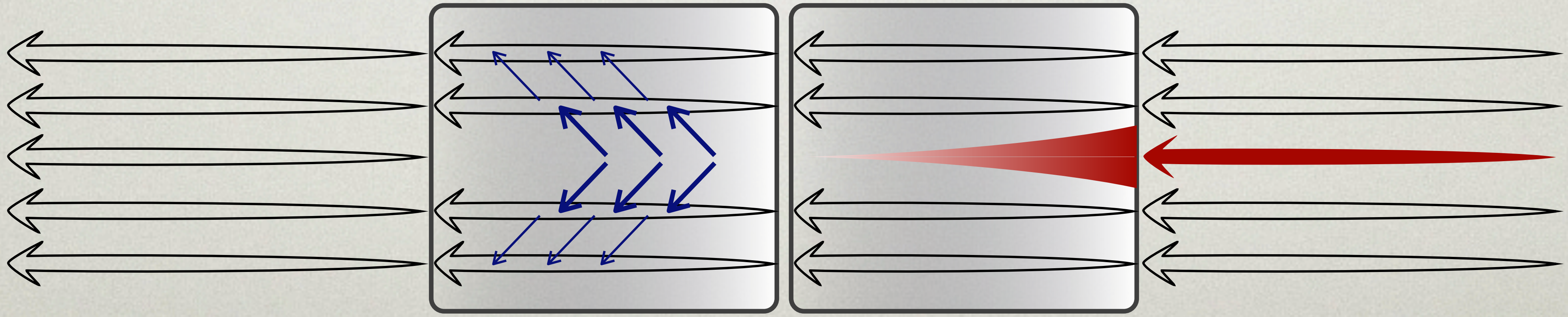
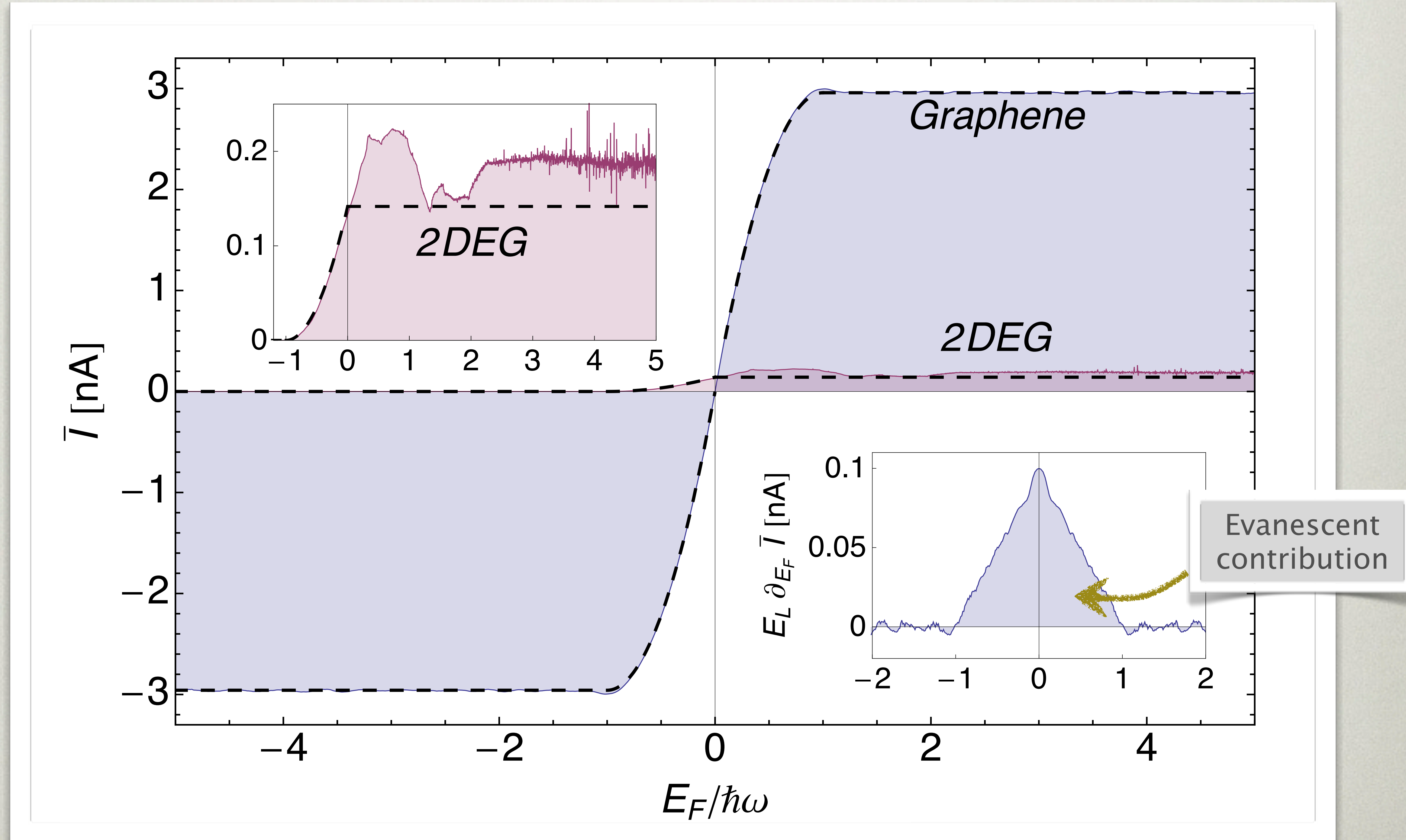


$$T_{RL} \approx 0$$



Total pumped current

- Chirality-enhanced evanescent pumping



Some Numbers

Relative pump performance:

$$k_F^\infty = 12nm^{-1}$$
$$m^* = 0.067m_e$$

Independent of L, W, ω, U

Some Numbers

Relative pump performance:

$$\nu \equiv \frac{\langle I_G \rangle^{\max}}{\langle I_N \rangle^{\max}} = \frac{\sigma_G^{\max}}{\sigma_N^{\max}} = \frac{\hbar k_F^{(\infty)}}{m^* v_F} \approx 20.9$$

$$k_F^{\infty} = 12 nm^{-1}$$
$$m^* = 0.067 m_e$$

Independent of L, W, ω, U

Some Numbers

Length and energy scales

$L = 1\mu m$

$$E_L^G = \frac{\hbar v_F}{L} \approx 0.66 meV$$

$W/L = 4$

$$E_L^N = \frac{\hbar^2}{2m^* L^2} \approx 0.57 \mu eV$$

$U = 200 \mu eV$

$$\hbar\omega = 10 meV (\approx 2.4 \mathrm{THz})$$

$I_{max}^G \approx 15 nA$

$L = 5\mu m$

$$E_L^G = \frac{\hbar v_F}{L} \approx 0.13 meV$$

$W/L = 4$

$$E_L^N = \frac{\hbar^2}{2m^* L^2} \approx 0.02 \mu eV$$

$U = 40 \mu eV$

$$\hbar\omega = 2 meV (\approx 500 \mathrm{GHz})$$

$I_{max}^G \approx 3 nA$

Conclusions

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- In a 2DEG only isolated resonances contribute

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- Resonant conditions are required in N systems
- Non-adiabatic driving pumps any evanescent mode that can be excited to propagating
- There is a whole range of evanescent modes in graphene available in graphene - efficiency
- In a 2DEG only isolated resonances contribute
- All such modes are rectified (driven in the direction dictated by spatial asymmetry)

Analytical Results

$$\bar{I} = \frac{ge}{h}\left(\frac{U}{2\hbar\omega}\right)^2 W \int_{-\infty}^{E_F} d\epsilon \int_{-\infty}^{\infty} dq \ \frac{\Delta T}{p}$$

$$\bar{I}_G \approx \frac{e}{\hbar}\frac{(U/2)^2}{E_W^G} \times \begin{cases} \left(2 - \frac{|E_F|}{\hbar\omega}\right)\frac{E_F}{\hbar\omega}, & |E_F| < \hbar\omega \\ \pm 1, & |E_F| > \hbar\omega \end{cases}$$

$$\bar{I}_N \approx \frac{e}{\hbar}\frac{(U/2)^2}{2k_F^{(\infty)} W E_W^N} \times \begin{cases} 0, & E_F < -\hbar\omega \\ \left(1 + \frac{E_F}{\hbar\omega}\right)^2, & -\hbar\omega < E_F < 0 \\ 1, & E_F > 0 \end{cases}$$

Analytical Results

- Weak driving limit:

$$\bar{I} = \frac{ge}{h}\left(\frac{U}{2\hbar\omega}\right)^2 W \int_{-\infty}^{E_F} d\epsilon \int_{-\infty}^{\infty} dq \, \frac{\Delta T}{p}$$

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Analytical Results

- Weak driving limit:

$$\bar{I} = \frac{ge}{h}\left(\frac{U}{2\hbar\omega}\right)^2 W \int_{-\infty}^{E_F} d\epsilon \int_{-\infty}^{\infty} dq\, \frac{\Delta T}{p}$$

- Semiclassical approximation

$$\bar{I}_G \approx \frac{e}{\hbar}\frac{(U/2)^2}{E_W^G} \times \begin{cases} \left(2 - \frac{|E_F|}{\hbar\omega}\right)\frac{E_F}{\hbar\omega}, & |E_F| < \hbar\omega \\ \pm 1, & |E_F| > \hbar\omega \end{cases}$$

$$\bar{I}_N \approx \frac{e}{\hbar}\frac{(U/2)^2}{2k_F^{(\infty)} W E_W^N} \times \begin{cases} 0, & E_F < -\hbar\omega \\ \left(1 + \frac{E_F}{\hbar\omega}\right)^2, & -\hbar\omega < E_F < 0 \\ 1, & E_F > 0 \end{cases}$$