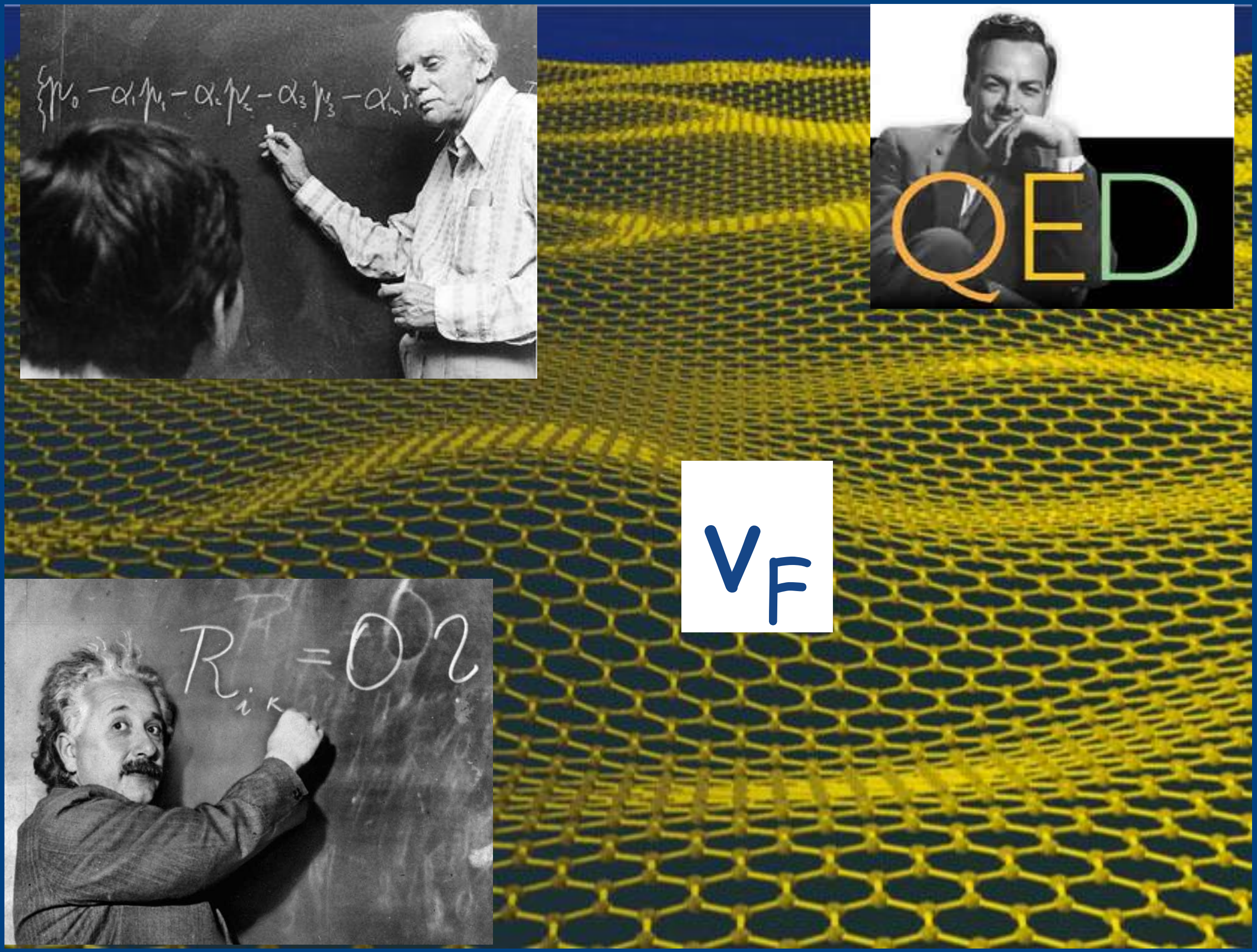
KAVLI 2012



Curved graphene revisited

María A. H. Vozmediano
Instituto de Ciencia de Materiales de Madrid
CSIC

Collaborators

A. Cano
E. V. Castro
J. González,
F. Guinea
M. I. Katsnelson,
M. P. López-Sancho,
J. L. Mañes,
T. Stauber
M. Sturla

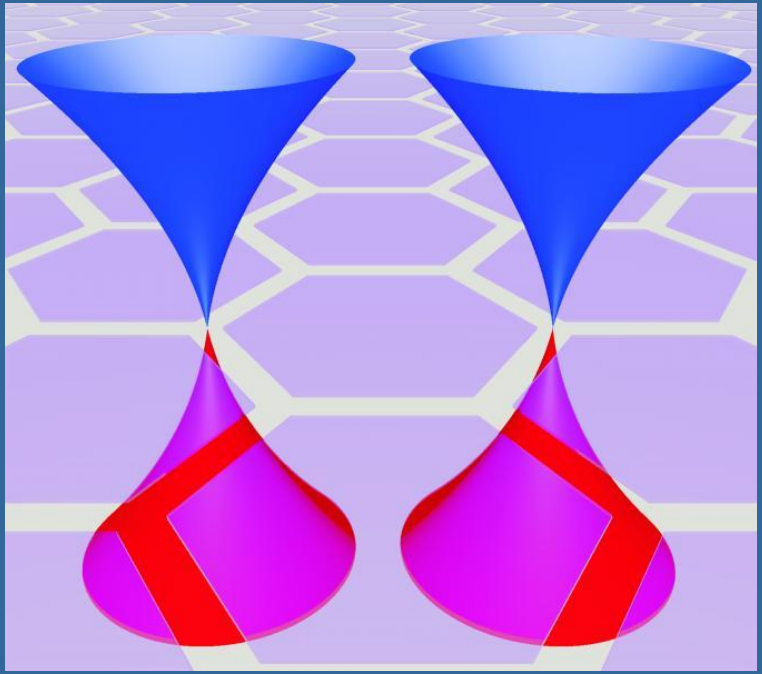
This work:

F. de Juan,
M. Sturla

F. de Juan, A. Cortijo,
A. G. Grushin, B. Valenzuela

Summary

- Fermi velocity is the only parameter in the continuum model.
- Energy dependence: disorder and interactions.
- Space dependence: ripples, strain.
- Reconciling the GR and TB approaches.
- Homework for the experimentalists.



Artistic view of Dirac cones from Manchester's group

Summary of graphene features

- The electronic properties described by 2D massless spinors.

Not a tight-binding feature: C_3 symmetry+ low energy.

- Spinor structure given by the two sublattices A and B

$$H = v_F \int d^2x \, \bar{\Psi}(x) \sigma^{\mu} \partial_{\mu} \Psi(x)$$

- They come in **two flavors** associated to the **two Fermi points** (related by time reversal symmetry)

- Real spin did not play much a role until the recent advent of the topological insulators

The Fermi velocity is the only free parameter in the continuum model. All observable quantities depend on it.

Topological stability of the Fermi points

J. Mañes , F. Guinea, MAHV
PRB07

Graphene and multilayer with ABC stacking: Fermi points protected by discrete TI invariance

$$H(K + k)$$

H defines a map from the circle |k|=R to 2x2 Hamiltonians $H = \vec{h} \cdot \vec{\sigma}$

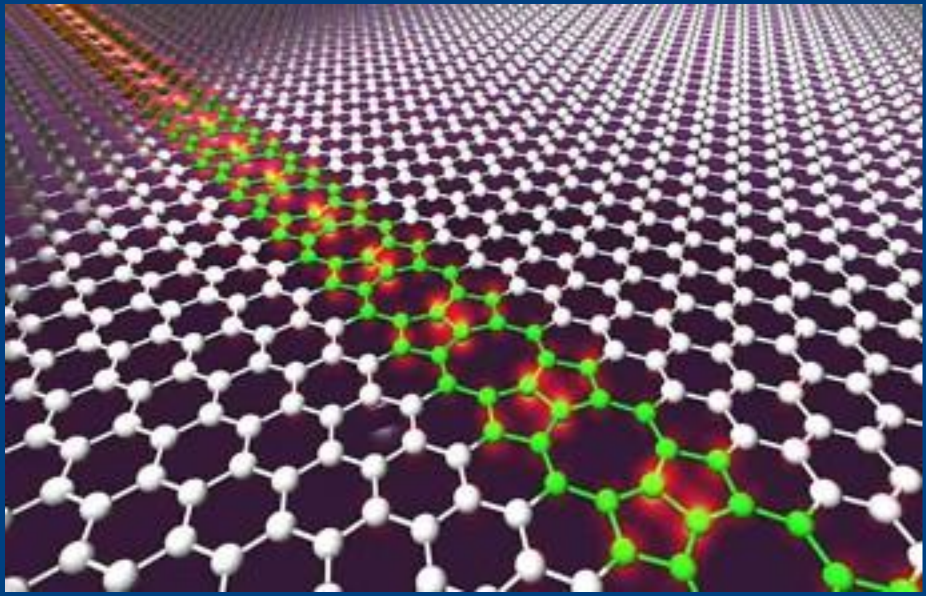
Fermi points are zeroes of the determinant. A perturbation will open a gap if the loop

$$k = Re^{i\theta} \rightarrow (h_x, h_y, h_z) = R(\cos\theta, \sin\theta, 0).$$

is contractible in the space of hamiltonians with non-vanishing determinant. If TI holds this space is homeomorphic to R^2-{0}.

$$\pi_1(R^2 - \{0\}) = \pi_1(S^1) = Z$$

Fermi points robust to lattice deformations and interactions preserving TI.



Coulomb interactions: Graphene vs QED

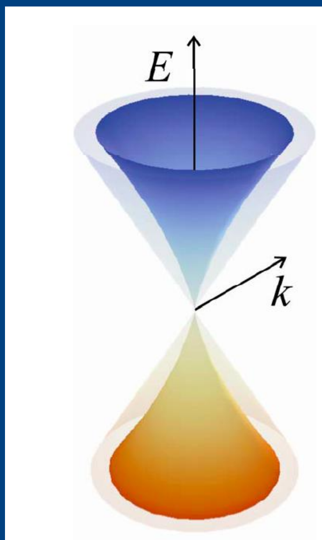
$$L = \int d^2r \; \mathrm{dt} \; \bar{\psi}(\mathbf{r},\mathrm{t})\gamma^{\mu}(\partial_{\mu} - ieA_{\mu})\psi(\mathbf{r},t)$$

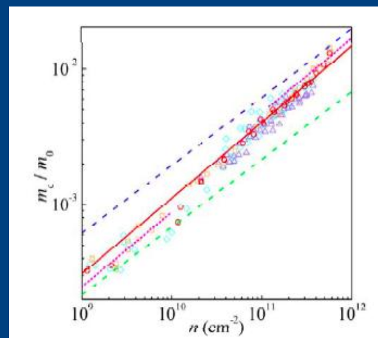
Non-relativistic QED (2+1)?

$$j^{\mu} \sim (\bar{\psi}\gamma^{0}\psi \;, \mathrm{v}\,\bar{\psi}\vec{\gamma}\psi)$$

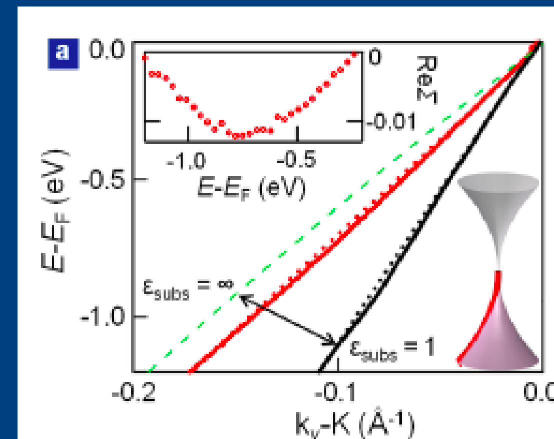
The Fermi velocity

It plays the same role as the effective mass in usual 2DEG





From cyclotron mass- Suspended. Clean. (Elias et al Nat. Phys. 2011)



From ARPES. Epitaxial (Lanzara's group PNAS 2011)

Energy dependent!

Coulomb interactions make it grow at lower energies.
Disorder does the opposite.
If you see it constant as decreasing energies it intrinsically grows.

Also space dependent?

Graphene as a bridge between high and low energy physics

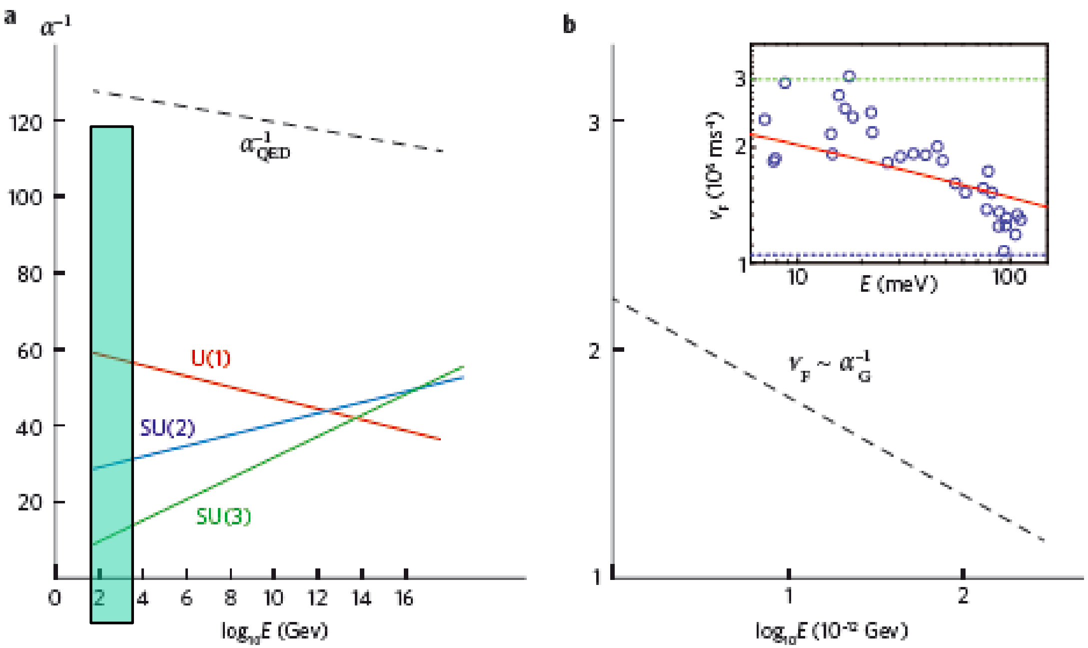
María A. H. Vozmediano
Instituto de Ciencia de Materiales de Madrid
Consejo Superior de Investigaciones Científicas

nature physics | news & views

GRAPHENE

The running of the constants

Maria A. H. Vozmediano



$$\alpha_{QED} = \frac{e^2}{4\pi\hbar c} \qquad \alpha_G = \frac{e^2}{4\pi\hbar v_F}$$

Renormalize

Downwards   Upwards E->0

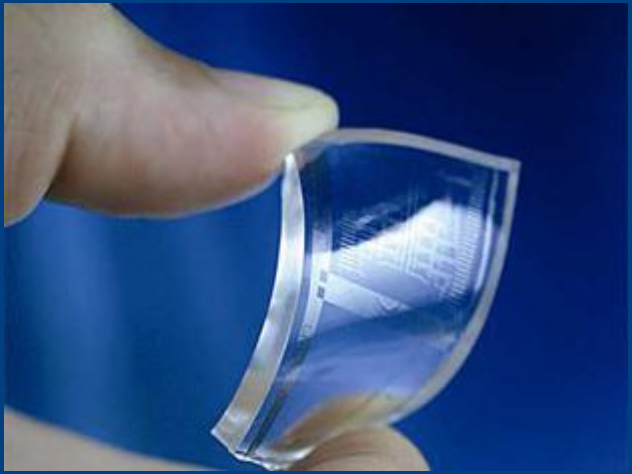
$$\alpha^{-1}_{QED}(1GeV) \approx 137; \quad \alpha^{-1}_{QED}(100GeV) \approx 128$$

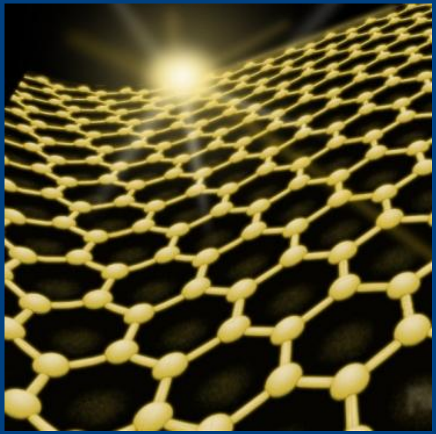
$$v_F(0.2meV) \approx 2.3 \times 10^6 m/s; \quad v_F(100meV) \approx 1.4 \times 10^6 m/s$$

Infrared stable. QED is probably "trivial" free fixed point. Graphene runs to α_{QED}.

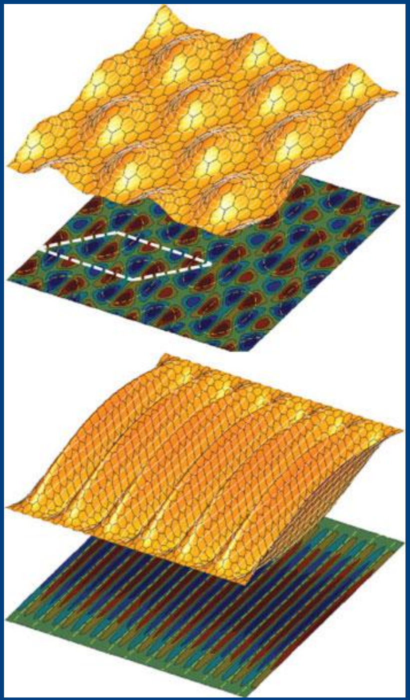
Renormalization is not just numerical correction!

Curved and strained graphene

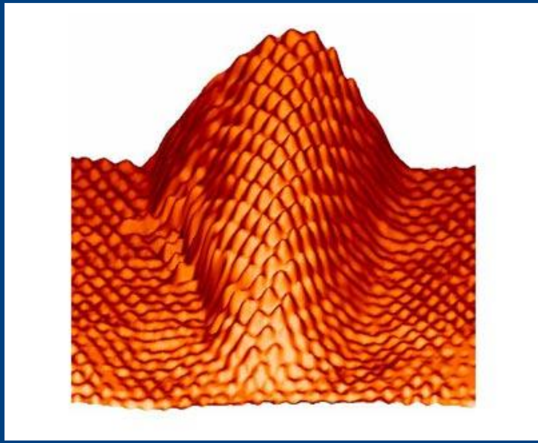




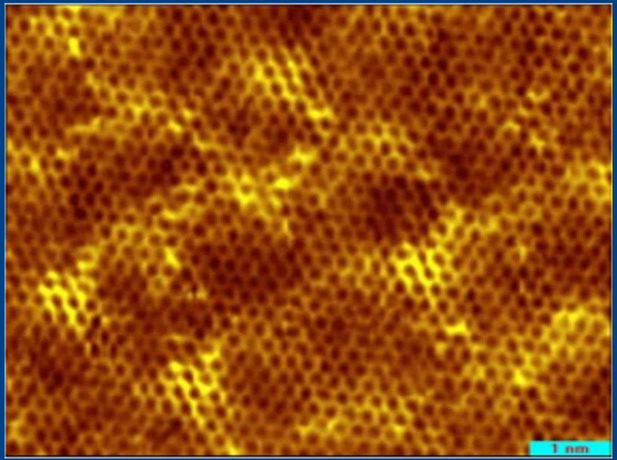
Artistic



Low, Guinea,
Katsnelson 2011

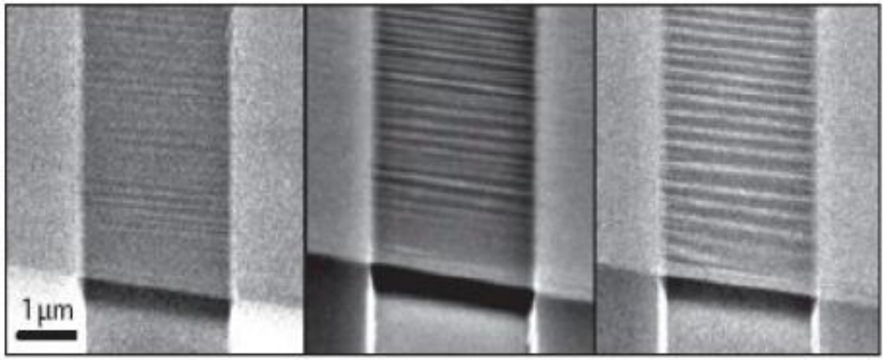


Graphene wrinkle,
Sun et al, Nanotec. (09)

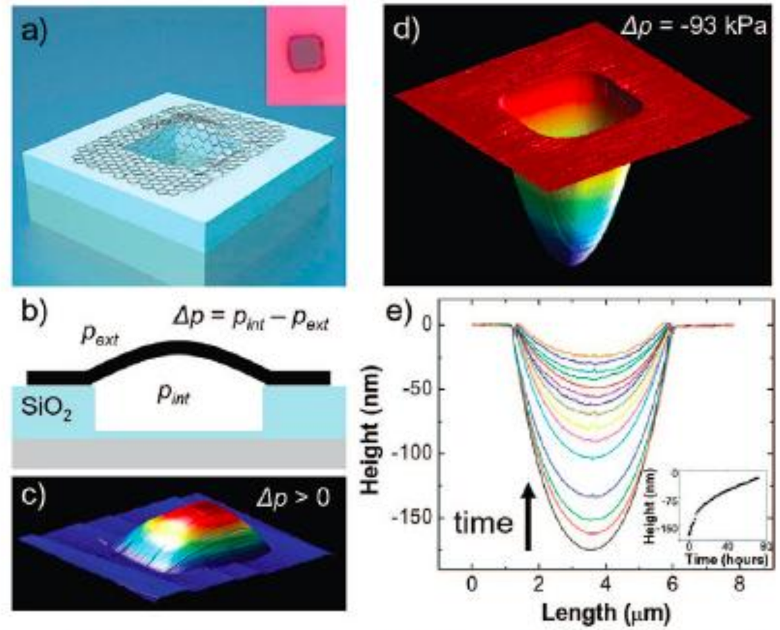


Atomically resolved STM image of a monolayer of graphene on SiC(IIT).

Controlling strain



"**Controlled ripple texturing** of suspended graphene and ultrathin graphite membranes" Bao *et al.* , Nat. Nanotech. **4** 562 (2009)

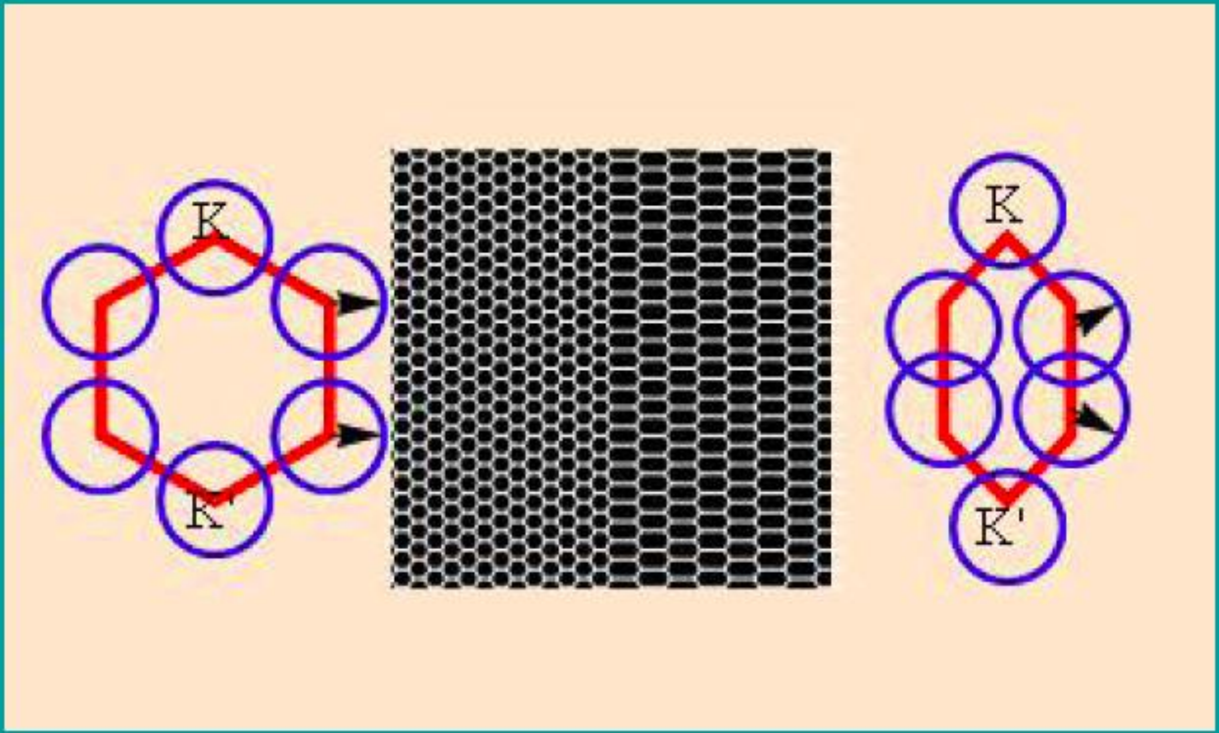


"Impermeable atomic membranes from graphene sheets" Scott Bunch *et al.*, Nanolett. **8**, 2458 (2008)

- "**Introducing Nonuniform Strain** to Graphene Using Dielectric Nanopillars" (cond-mat/1106.1507 Tomori *et al.*)
- "Graphene bubbles with **controllable curvature**" (Cond-mat/1108.1701, Manchester group)
- "Topological properties of artificial graphene assembled by atom manipulation", where they produced **atomically engineered strains**. (Manoharan group, APS 2011)
- And the list goes on...

Effective gauge fields (from tight binding)

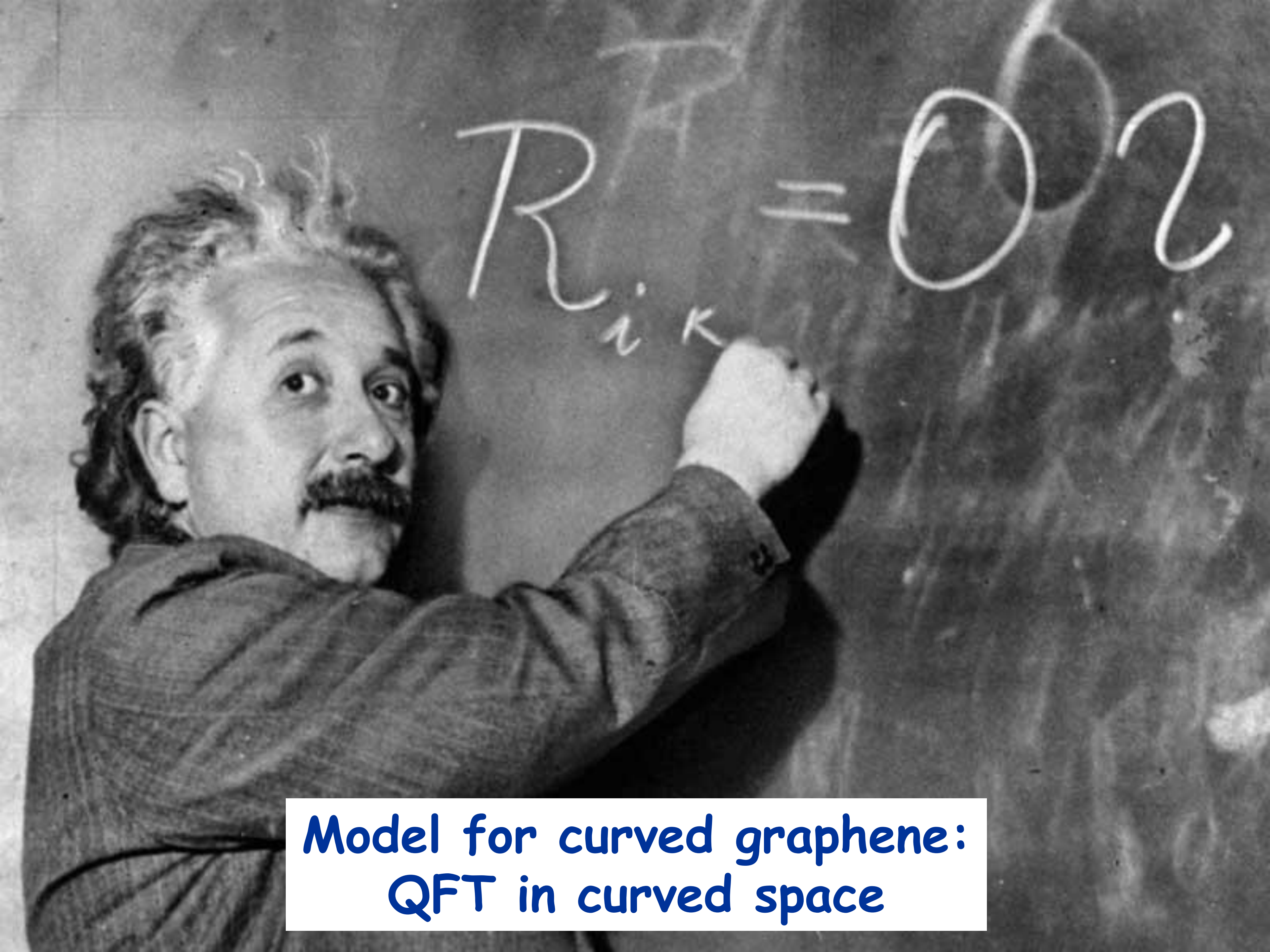
$$H \equiv \begin{pmatrix} 0 & t_1 e^{i\vec{k}_1\vec{a}_1} + t_2 e^{i\vec{k}_2\vec{a}_2} + t_3 e^{i\vec{k}_3\vec{a}_3} \\ t_1 e^{-i\vec{k}_1\vec{a}_1} + t_2 e^{-i\vec{k}_2\vec{a}_2} + t_3 e^{-i\vec{k}_3\vec{a}_3} & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{3\bar{t}a}{2}\left(k_x + ik_y\right) + \Delta t \\ \frac{3\bar{t}a}{2}\left(k_x + ik_y\right) + \Delta t & 0 \end{pmatrix}$$



Courtesy of F. Guinea

A modulation of the hoppings leads to a term which modifies the momentum: an effective gauge field.

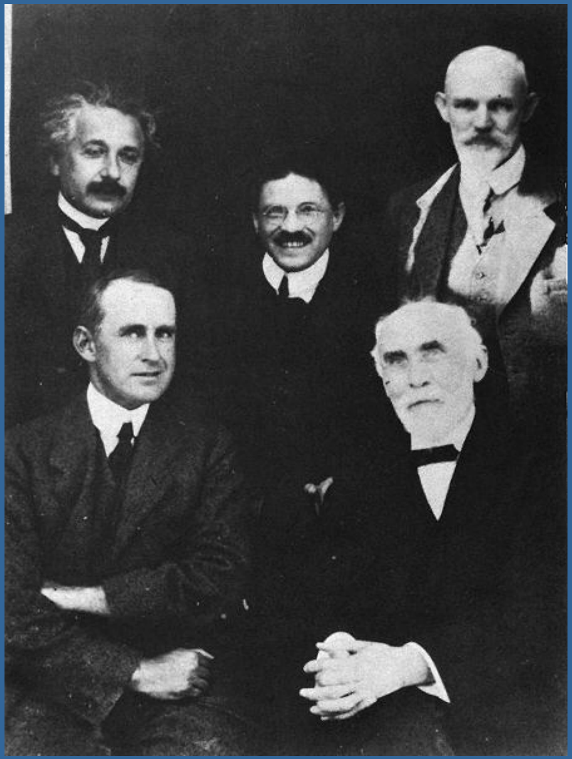
The induced "magnetic" fields have opposite sign at the two corners of the Brillouin Zone.



Model for curved graphene: QFT in curved space

Dirac in curved space

We can include curvature effects by coupling the Dirac equation to a curved space



$$\gamma^a e_a^\mu \left(\partial_\mu - \Omega_\mu(x)\right)\psi = E\Psi$$

Need a metric and a "tetrad".

$$e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu}$$

Generate r-dependent Dirac matrices and an effective "gauge" field.

$$\Omega_\mu = \frac{1}{4}\gamma^a \gamma^b e_{a;\mu}^\nu e_{b\nu}$$

Effects of the curvature

1. The curved gamma matrices:

$$\gamma^{\mu}(r) = \gamma^{a} e_{a}^{\mu}(r)$$

$$H = i v_F \left(\sigma^1(r)\partial_1 + \sigma^2(r)\partial_2 \right) \equiv i(v_1(r)\sigma^1\partial_1 + v_2(r)\sigma^2\partial_2)$$

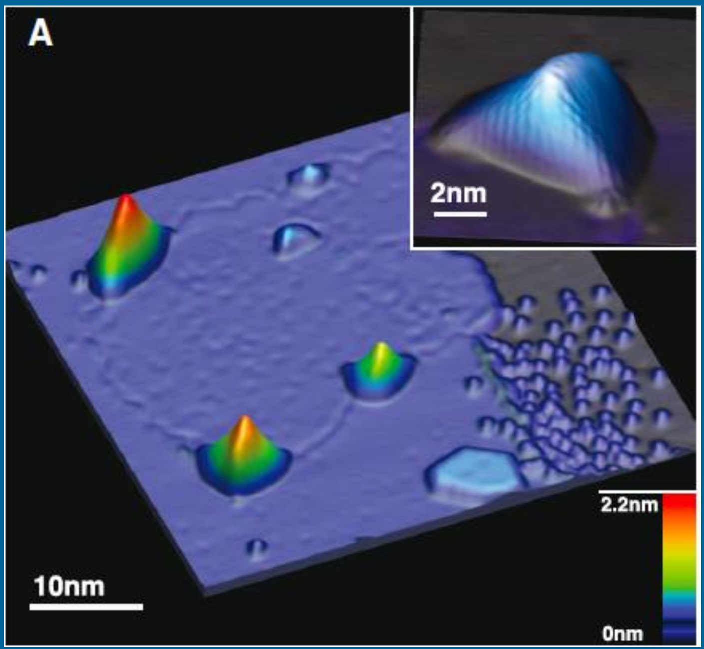
Can be seen as a position-dependent Fermi velocity

2. The spin connection:

- It can be seen as an effective gauge field. It is constructed with derivatives of the metric and depends on the γ matrix representation (Dirac point).
- It has different signs at the two Fermi points (time reversal symmetry preserved).

Physical reality of the fictitious gauge fields

Observing fictitious gauge fields



Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§

30 JULY 2010 VOL 329 **SCIENCE** www.sciencemag.org

$$A_x = \frac{\beta}{a}\left(u_{xx} - u_{yy}\right)$$

$$A_y = \frac{2\beta}{a} u_{xy}$$

$$\beta = \frac{\partial \log(t)}{\partial \log(a)} \approx 2$$

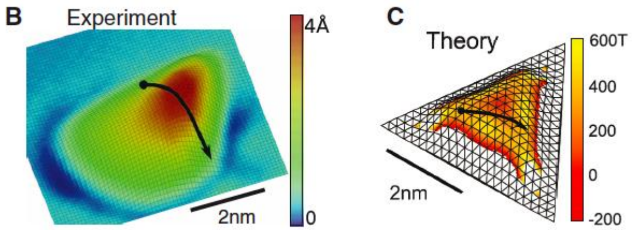


Fig. 3. (**A**) Experimental topographic line scan and experimentally determined B_s profile over the tip trajectory shown by black line in (B). (**B**) STM topography of graphene nanobubble. (**C**) Topography of theoretically simulated graphene nanobubble with calculated B_s color map. (**D**) Simulated topographic line scan and B_s profiles extracted from line shown in (C).

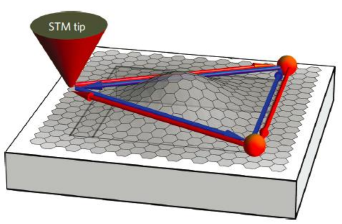
Need strong deformations giving rise to strong-uniform fictitious fields

nature physics

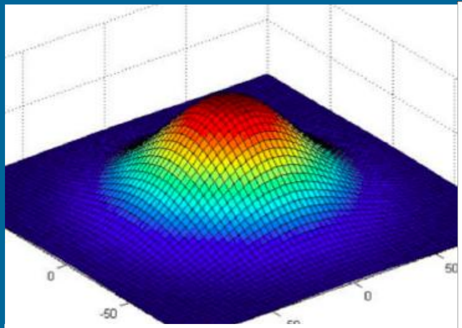
ARTICLES

PUBLISHED ONLINE: 10 JULY 2011 | DOI: 10.1038/NPHYS2034

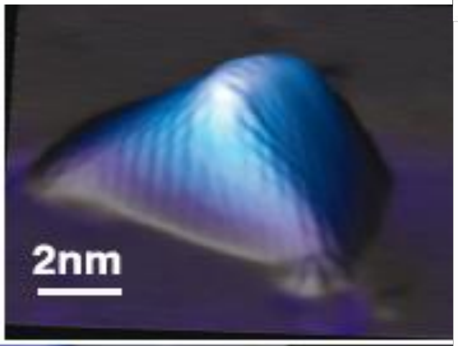
Aharonov–Bohm interferences from local deformations in graphene



Fernando de Juan[1], Alberto Cortijo[2], María A. H. Vozmediano[3]* and Andrés Cano[4]

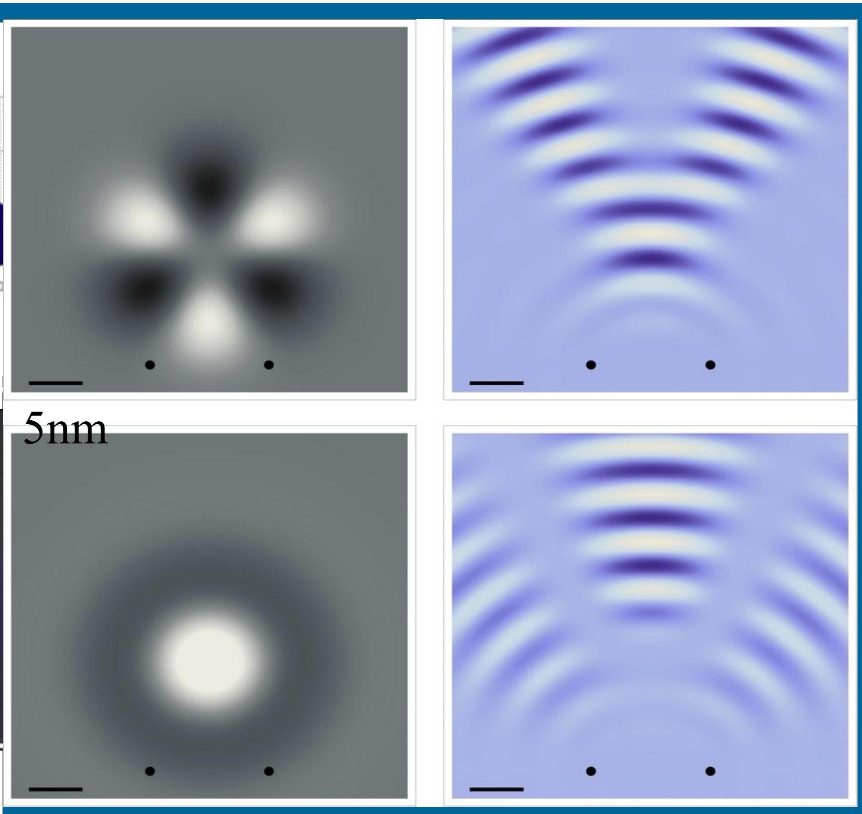


$$z(r) = A \exp(-r^2/b^2)$$



$$u_r = u_0 \exp(-r^2/2\sigma^2) r^2 \sin 3\theta$$

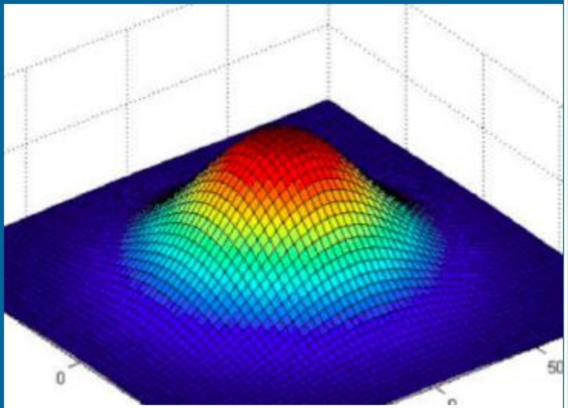
$$u_\theta = u_0 \exp(-r^2/2\sigma^2) r^2 \cos 3\theta$$



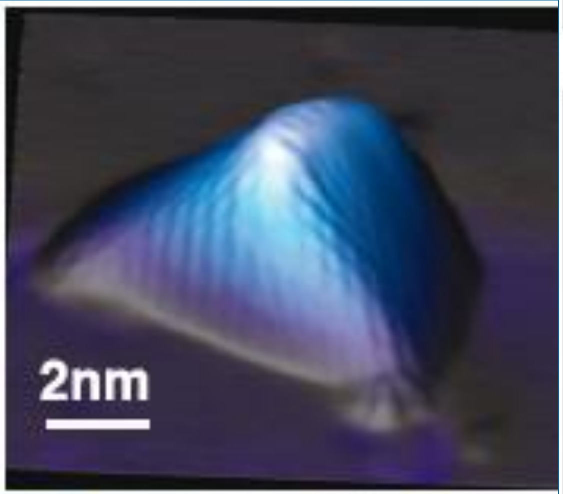
Magnetic field

Interference pattern

Profiles of fictitious magnetic fields

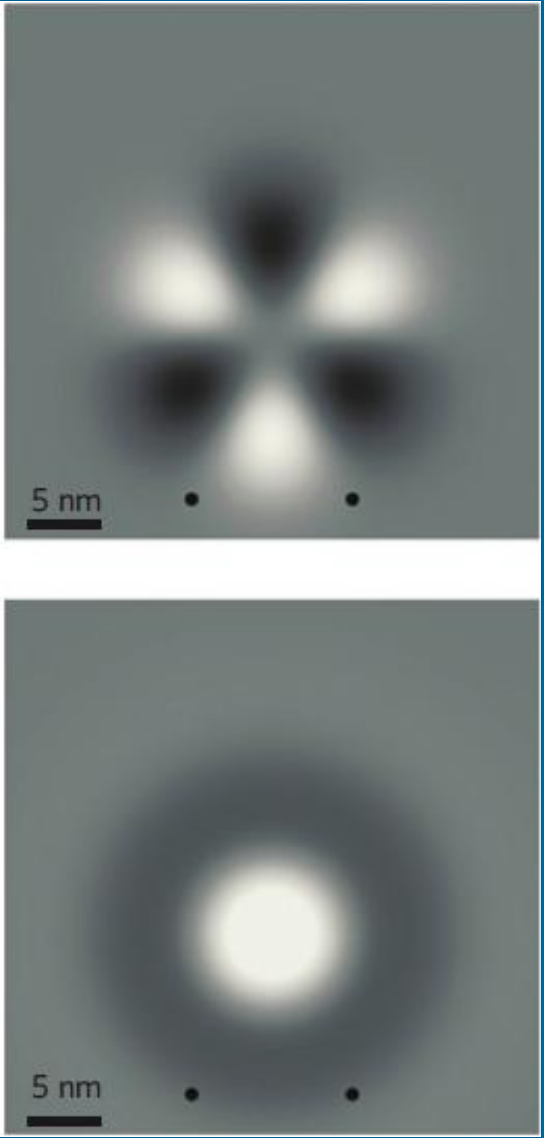


$$z(r) = A\ \exp\left(-r^2/b^2\right)$$



$$u_r = u_0 \exp\left(-r^2/2\sigma^2\right) r^2 \sin 3\theta$$

$$u_\theta = u_0 \exp\left(-r^2/2\sigma^2\right) r^2 \cos 3\theta$$



TB-Magnetic field

Covariant

$$B_z = -\frac{1}{r}\partial_r (rA_\theta) = \frac{1}{4r}\frac{\alpha z'}{\left(1+\alpha z\right)^{3/2}}$$

Axial symmetry

TB-strain

$$A_x = \frac{\beta}{a}\left(u_{xx} - u_{yy}\right)$$

$$A_y = \frac{2\beta}{a} u_{xy}$$

Relating TB-elasticity and geometric

$$g^{ij} = \eta^{ij} + u^{ij}$$

Geometric	TB-elasticity
• Space dependent Fermi velocity	• ??
• Effective gauge fields with the symmetry of the deformation	• Effective gauge fields with different symmetry
• Dimensions: $A \sim \partial g \sim \partial u$	• $A \sim u$
• Need intrinsic curvature	• In-plane strain OK
• Material independent	• Material dependent $\beta = v_F \partial t / \partial a \approx 2 \text{ eV}$

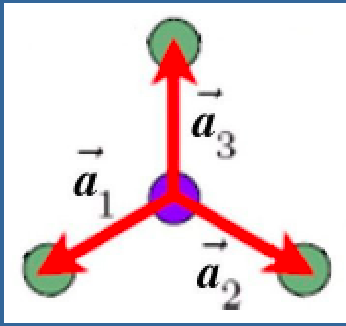
An important issue: is v(x) a physical effect? (observable)

Strain + symmetry

Build an effective H at low energy with C_3 symmetry

- What can we build with (σ^i, q^i, u^{ij}) ? $H_0 = v_F \vec{\sigma}.\vec{q}$

C_3 invariant tensor:



$$f^{ijk} = \frac{1}{a^3}\sum_{n=1}^{3} a_n^i a_n^j a_n^k$$

Terms compatible with C_3 symmetry:

Even # indices: contract with the flat metric

- $\sigma_i \partial_j$, the flat Hamiltonian
- $\sigma_i(\partial_j u_{kl})$ the geometric gauge field
- $u_{kl}\sigma_i\partial_j$ the space dependent Fermi velocity

Odd # indices: contract with f or ε_{ijl}

- $\sigma_i u_{jk}$ the trigonal gauge field
- $\sigma_i \partial_k \partial_j$ the trigonal warping term

Deformed graphene: TB beyond linear approximation

$$H = -\sum_{n=1}^{3}(t+\delta t_n)\begin{pmatrix} 0 & e^{-i(\vec{K}+\vec{q})\cdot\vec{d}_n} \\ e^{i(\vec{K}+\vec{q})\cdot\vec{d}_n} & 0 \end{pmatrix}$$

Expand in q:
Dirac fermions

Expand in δt_n:
Gauge fields

$$H_{trigonal} = f^{ijk}\sigma^i q^j q^k$$

Expand in both:

$$A^i = \frac{2}{3a}\epsilon^{ij} f^{jkl} u^{kl}$$

$$H_{v_F} = \sigma^i q^j u^{kl} f^{ijkl}$$

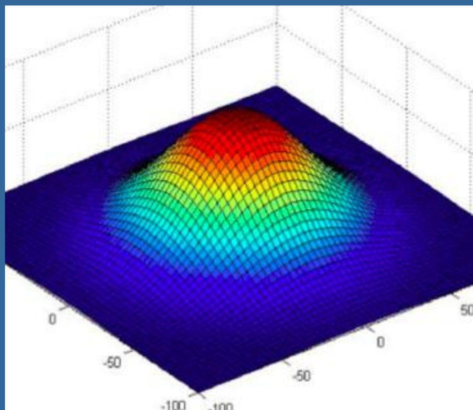
Related works

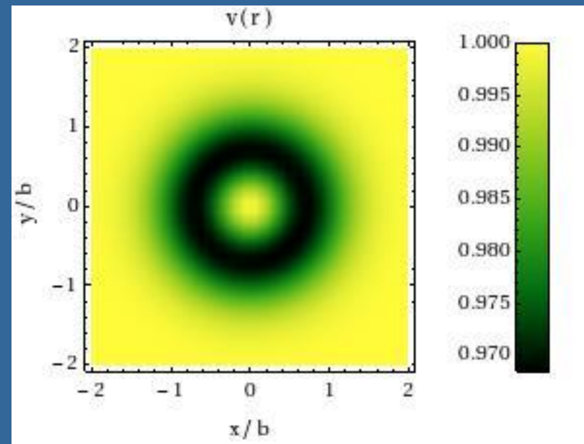
J. L. Manes 2007 *Phys. Rev. B* ***76 045430***
Winkler R and Zulicke U 2010 *Phys. Rev. B* ***82 245313***
T. L. Linnik arXiv1111.3924

F. de Juan, M. Sturla, MAHV, work in progress.

See also Pereira on Thursday

A working example





F. de Juan, A. Cortijo, MAHV PRB76, 165409 (2007).

$$z(r) = A \exp(-r^2/b^2)$$

$$v_r(r) = \frac{1}{\sqrt{1 + z'(r)^2}}$$

$$1 + z'(r)^2 \equiv \alpha f(r),$$

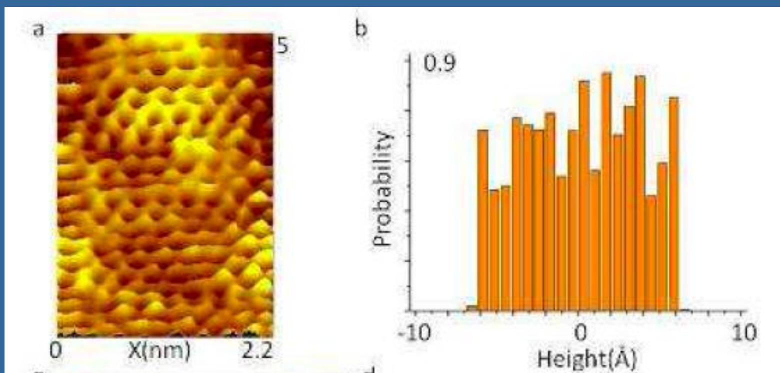
$$\alpha = \left(\frac{A}{b}\right)^2 \approx 10^{-2}, \quad f(r) = \frac{4}{b^2} r^2 \exp(-2r^2/b^2)$$

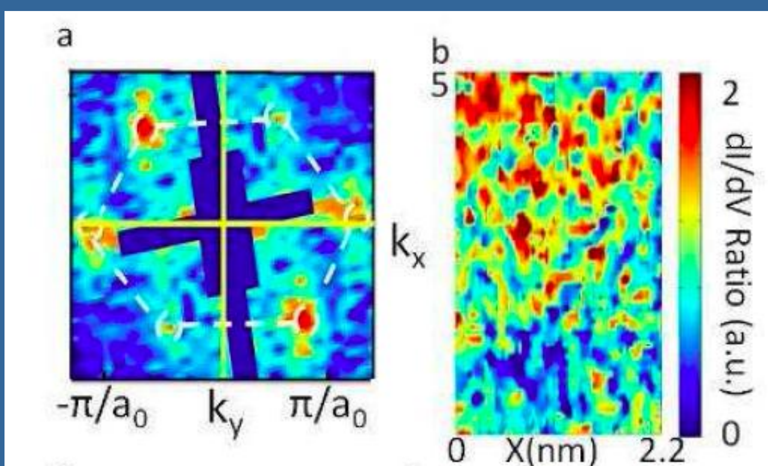
α measures the departure from flat space $(h/L)^2$. The effects are of order α.

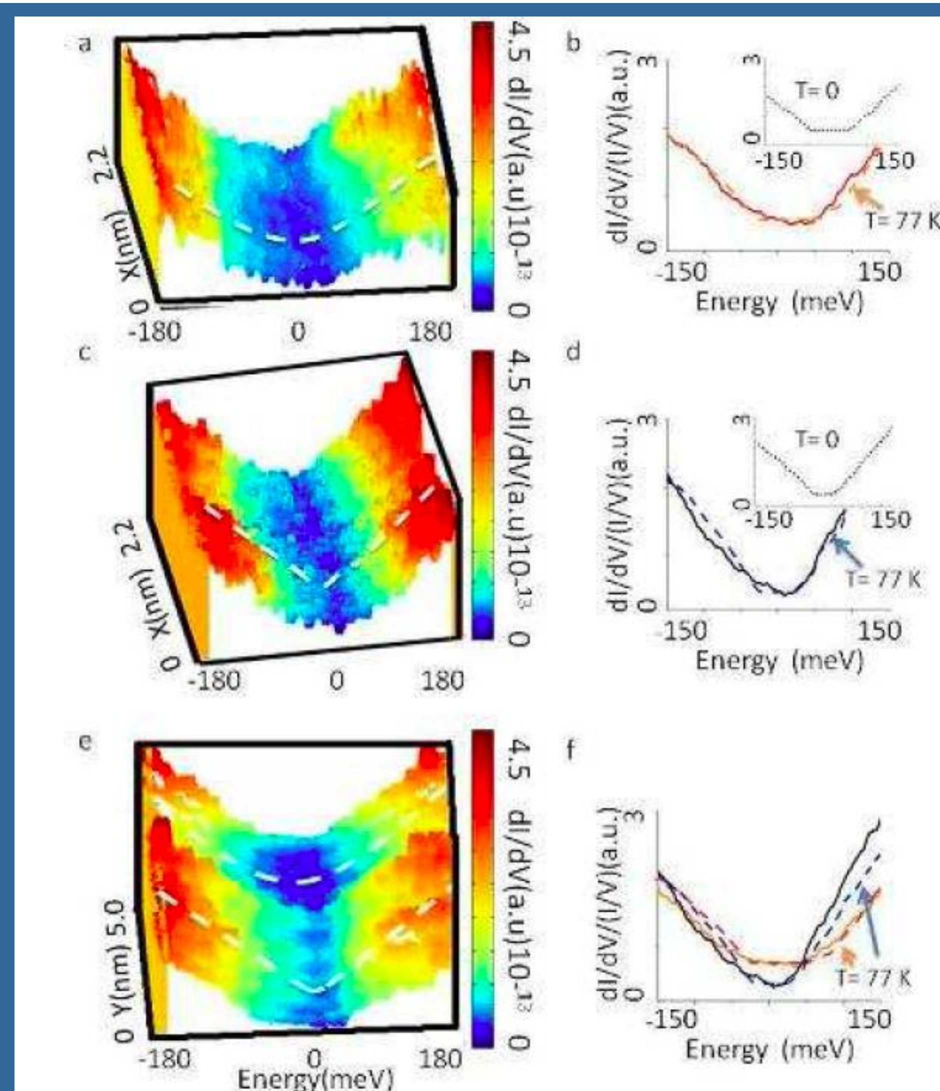
Notice:
A distribution of ripples will give rise to a landscape of v_F.

Relating morphology and electronics

Evidence for Strain-Induced Local Conductance Modulations in Single-Layer Graphene on SiO2M. L. Teague, A. P. Lai, J. Velasco, C. R. Hughes, A. D. Beyer, M. W. Bockrath, C. N. Lau, and N.-C. Yeh, Nano Letters 2009 9 (7), 2542-2546







Correlation of the tunneling spectrum with strain tensor ->

Conclusion

Pay attention to the Fermi velocity!

(It is not the constant that it looks like)

Some tasks for the experimentalists

(May 2010)

- Establish the existence (or not) of a gap in the clean, neutral system. ?
- Measure the variations (or not) of the Fermi velocity with energy at low energies. √
- Measure the variations (or not) of the Fermi velocity with the curvature in rippled samples.

(January 2012)

- Establish the correlation (or not) of morphology and electronics (see Lau).
- Revise the experiments where the value of v_F is a crucial quantity. Notice that it varies with the energy **and with location on the sample.**