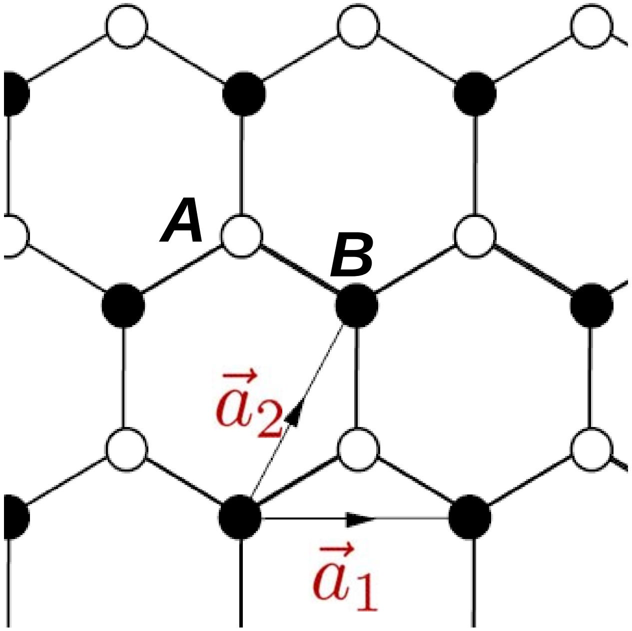
Transport properties of graphene.

1. Effect of disorder on transport in graphene.
   I.L. Aleiner and K.B. Efetov, PRL, 97, 236801, (2006)

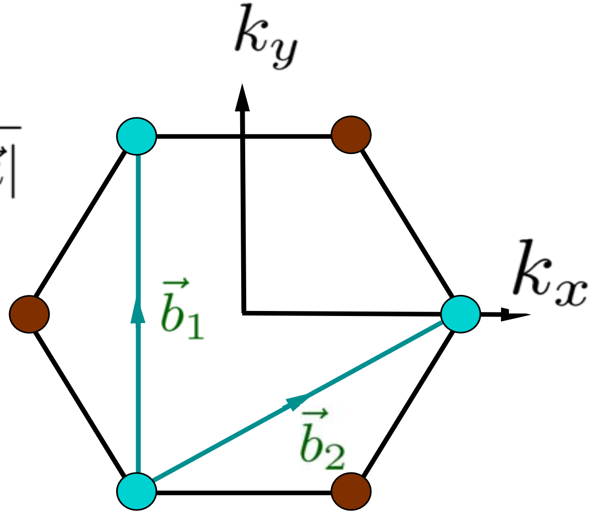
2. Quantum dots in graphene.
P.G. Silvestrov and K.B. Efetov, PRL, 98, 016802 (2007)

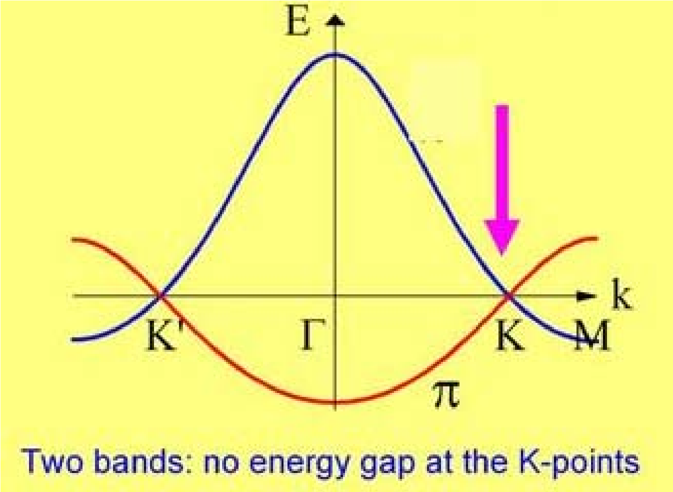
Lattice



Inverse lattice.

$$|\vec{b}| = \frac{4\pi}{\sqrt{3}|\vec{a}|}$$





Hamiltonian H_0 for free electrons

$$H_0 = -iv\vec{\Sigma}\vec{\nabla}$$

v-velocity

$$\Sigma_{x,y} = 1^{K,K'} \otimes \tau_{x,y}^{A,B},$$

$\tau_{x,y,z}$ -Pauli matrices

A,B-sublattice space, K,K' -valley space

Full Hamiltonian (no spin): $H = H_0 + \hat{V}$

The external potential $\hat{V}$ is generally a 4x4 matrix.
It is proportional to unity matrix only if it varies slowly on the lattice period.

Two problems:

1. $\hat{V}$ is disorder potential (full matrix)
2. $\hat{V}$ is a barrier across the graphene strip (unity matrix).

New results for 1).

a) Most general structure of $\hat{V}$ is identified
(using most general symmetries).

b) Going beyond the self-consistent Born approximation (SCBA)
(derivation and solution of RG equations).

c) derivation of a supermatrix non-linear σ-model
(ultimate localization but a complicated crossover).

Symmetries of the model:

1. Time reversal (TR) symmetry (exact)

$$\varphi^T(r) = ((\varphi_A, \varphi_B)_{AB}; (\varphi_B^*, -\varphi_A^*)_{AB})_{K,K'}$$

$$\Downarrow$$

$$\varphi^*(r) = \hat{z}\varphi(r), \quad \hat{z} = \tau_y^{A,B} \otimes \tau_y^{K,K'} \quad \Rightarrow \quad \boxed{H = \hat{z} H^T \hat{z}}$$

Disorder may break all the symmetries except the time reversal one!

The most general form:

$$\hat{V}(r) = u_0(r) + \sum_{\{m,i\}=\{x,y,z\}} \hat{G}_{m,i} u_{m,i}(r), \qquad \hat{G}_{m,i} = \tau_m^{K,K'} \otimes \tau_i^{A,B}$$

$u_0(r), u_m(r)$ are real independent random functions

However, averaging must restore rotation, reflection, translation and C_{6v} symmetries!

$$\begin{aligned}
\langle \hat{V}(\mathbf{r}_1) \otimes \hat{V}(\mathbf{r}_2) \rangle = &\ \delta(|\mathbf{r}_1 - \mathbf{r}_2|)\ \gamma_0\ \mathbb{1} \otimes \mathbb{1} \\
&+ \delta(|\mathbf{r}_1 - \mathbf{r}_2|)\ \gamma_{\parallel}\ \hat{G}_{z,z} \otimes \hat{G}_{z,z} \\
&+ \delta(|\mathbf{r}_1 - \mathbf{r}_2|)\ \gamma_{\perp} \sum_{j=x,y} \hat{G}_{z,j} \otimes \hat{G}_{z,j} \\
&+ \delta(|\mathbf{r}_1 - \mathbf{r}_2|)\ \beta_{\parallel} \sum_{m=x,y} \hat{G}_{m,z} \otimes \hat{G}_{m,z} \\
&+ \delta(|\mathbf{r}_1 - \mathbf{r}_2|)\ \beta_{\perp} \sum_{j,m=x,y} \hat{G}_{m,j} \otimes \hat{G}_{m,j}
\end{aligned}$$

Five independent real constants for the disorder.

γ_0 is due to long range impurities (with respect to the lattice period)

β -intervalley scatering

Impurities conserving the chirality.

$$\hat{V}(\mathbf{r}) = -\hat{\Sigma}_z \hat{V}(\mathbf{r}) \hat{\Sigma}_z \Longrightarrow \gamma_0 = \gamma_{||} = \beta_{||} = 0$$

Consequence: One delocalized state exist at E=0!
Minimum metallic conductivity at the center.

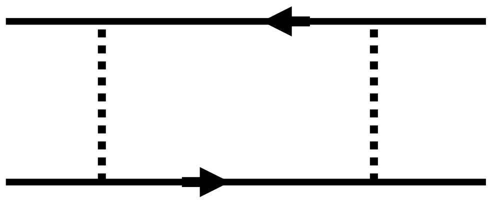
(See review: Altland, Simons, Zirnbauer, Phys. Rep. (2002)

However: no reason for this symmetry!

Moreover, all weak coupling constants grow in the process of the renormalization!

One should go beyond SCBA.

Scattering amplitudes in perturbation theory.



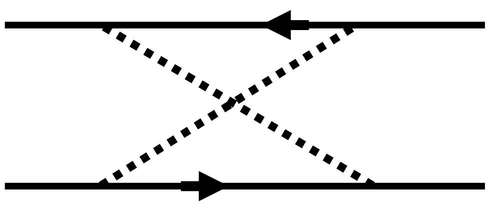
SCBA

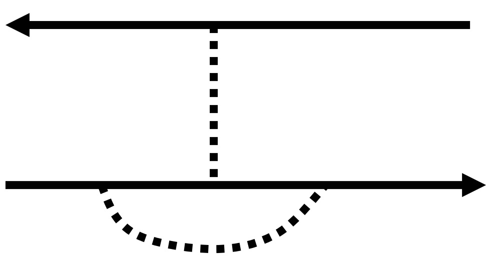
$$G_\varepsilon(p) = \frac{1}{\varepsilon - p\Sigma + i\delta}$$

$$\int G_\varepsilon(p) G_\varepsilon(p) d^2 p \propto \ln(J/\varepsilon)$$

Logarithmic contributions to the amplitudes!

However, other logarithmic contributions exist!





SCBA fails: RG treatment.

Physical quantities as integrals over supervectors ψ (averaging has been performed)

$$\langle \ldots \rangle = \int \ldots \exp\left(-L[\psi]\right) D\psi, \qquad L[\psi] = L_0[\psi] + L_{\mathrm{int}}[\psi]$$

$$L_0[\psi] = i \int \bar{\psi} \left[\varepsilon - \hat{\mathbb{H}}_0 - \hat{\mathbf{\Lambda}} \left(\frac{\omega}{2} + i0\right)\right] \psi d\mathbf{r},$$

$$L_{int}[\psi] = \frac{1}{2} \int \left[\gamma_0 \left(\bar{\psi}\psi\right)^2 + \Gamma_m^i \left(\bar{\psi}\hat{\mathbb{G}}_{m,i}\psi\right)^2\right]$$

$$\Gamma_z^z = \gamma_z, \ \Gamma_z^{\{x,y\}} = \gamma_\perp, \ \Gamma_{\{x,y\}}^z = \beta_z, \ \Gamma_{\{x,y\}}^{\{x,y\}} = \beta_\perp$$

ψ has 16 components for the density of states

RG treatment: $\psi = \psi_0 + \tilde{\psi}$

Fast (ψ_0) Slow ($\tilde{\psi}$)

Integration over the fast field.

Reproducing the form of the Lagrangian: RG equations

Assumption: $\gamma_0 \gtrsim \gamma_{||,\perp}, \beta_{||,\perp}$

New coupling constants:

$$\begin{aligned} g_{||} &= \gamma_{||} + 2\gamma_{\perp}; \qquad & g_{\perp} = \beta_{||} + 2\beta_{\perp} \\ \delta g_{||} &= \gamma_{||} - \gamma_{\perp}; \qquad & \delta g_{\perp} = \beta_{||} - \beta_{\perp} \end{aligned}$$

Equations:

$$2\pi v \partial_t v = -\left(\gamma_0 + g_{||} + 2g_{\perp}\right);$$

$$9\pi v^2 \partial_t \gamma_0 \approx 2\left(g_{||}^2 + 2g_{\perp}^2\right);$$

$$9\pi v^2 \partial_t g_{||} \approx -8g_{||}^2 - 20 g_{||} g_{\perp} + 14 g_{\perp}^2;$$

$$9\pi v^2 \partial_t g_{\perp} \approx 4 g_{||} g_{\perp} - 18 g_{\perp}^2.$$

$$\pi v^2 \partial_t \delta g_{||,\perp} \approx -3\gamma_0 \delta g_{||,\perp};$$

Absence of intervalley scattering is not stable

Solution of the RG equations:

$$v(\varepsilon) = \left(\frac{\gamma_0}{\pi} \ln \frac{|\tilde{\varepsilon}|}{\varepsilon_0} \right)^{1/2}, \quad \gamma_0(\varepsilon) = \gamma_0 + \mathcal{O}\left(\frac{g_{\|}(\varepsilon)}{\gamma_0} \right)$$

$$g_{\|}(\varepsilon) \approx g_{\perp}(\varepsilon) \approx \frac{9\gamma_0}{14 \ln \left[t^* / \ln |\tilde{\varepsilon}|/\varepsilon_0 \right]},$$

$$\varepsilon_0 \approx J \exp\left(-\pi v_0^2 / \gamma_0 \right)$$

$$\tilde{\varepsilon} = \max(\varepsilon, \varepsilon_0)$$

ε_0 -is the energy at which the 1-loop approximation breaks down

All coupling constants are important. No chance for moving to chiral disorder!

Physical quantities with the "ultraviolet" logarithmic renormalization

Diffusion coefficient:

$$D(\epsilon) = \frac{v^2 \tau_{tr}(\epsilon)}{2}; \qquad \frac{1}{\tau_{tr}(\epsilon)} = \frac{\pi \gamma_0 \nu(\epsilon)}{4}$$

Density of states:

$$\nu(\epsilon) = \frac{|\tilde{\varepsilon}|}{\pi \hbar^2 v^2(\epsilon)}$$

Conductivity:

$$\sigma(\epsilon) = 2e^2 \nu(\epsilon) D(\epsilon) = \frac{4e^2}{\pi^2 \hbar} \ln\left(\frac{|\tilde{\varepsilon}|}{\varepsilon_0}\right)$$

However, this is not the end of the story: localization effects!

Non-linear supermatrix σ-model for describing localization effects

General scheme: following *"Supersymmetry in disorder and chaos"*, K.B. Efetov, Cambridge University Press, (1997)

$$\langle O \rangle = \int O(\hat{Q}) \exp\left(-F\left[\hat{Q}\right]\right) \mathcal{D}\hat{Q}$$

For any correlation function O

$$1 = \int \exp\left(-F\left[\hat{Q}\right]\right) \mathcal{D}\hat{Q}$$

Due to supersymmetry

For graphene: Q are 16x16 supermatrices (unity in the sublattice space), $Q^2 = 1$

Free energy functional F

$$F = \frac{\pi\hbar\nu(\epsilon)}{16} Str \int \Big\{ D(\epsilon)\left(\nabla\hat{Q}\right)^2 + 2i\omega\hat{\Lambda}\hat{Q}$$

$$- \frac{\pi\nu(\epsilon)g_{\|}(\epsilon)}{4}\left[\hat{\rho}_z, \hat{Q}\right]^2 - \frac{\pi\nu(\epsilon)g_{\perp}(\epsilon)}{4}\left(\left[\hat{\rho}_x, \hat{Q}\right]^2 + \left[\hat{\rho}_y, \hat{Q}\right]^2\right)\Big\} d\mathbf{r};$$

$$\hat{\rho}_j = \tau_j^{KK'} \otimes \mathbb{1}^{eh} \otimes \mathbb{1}^{AR} \otimes \mathbb{1}^{g}; \quad \hat{\Lambda} = \mathbb{1}^{KK'} \otimes \mathbb{1}^{eh} \otimes \tau_z^{AR} \otimes \mathbb{1}^{g};$$

All the constants should be taken from the solution of the RG equations!

$$D(\epsilon) = \frac{v^2(\epsilon)\tau_{tr}(\epsilon)}{2}; \quad \frac{1}{\tau_{tr}(\epsilon)} = \frac{\pi\gamma_0\nu(\epsilon)}{4}$$

$$v(\varepsilon) = \left(\frac{\gamma_0}{\pi}\ln\frac{|\tilde{\varepsilon}|}{\varepsilon_0}\right)^{1/2}, \quad \gamma_0(\varepsilon) = \gamma_0 + \mathcal{O}\left(\frac{g_{\|}(\varepsilon)}{\gamma_0}\right)$$

$$g_{\|}(\varepsilon) \approx g_{\perp}(\varepsilon) \approx \frac{9\gamma_0}{14\ln\left[t^*/\ln|\tilde{\varepsilon}|/\varepsilon_0\right]}, \qquad \tilde{\varepsilon} = max(\epsilon, \#\varepsilon_0)$$

1) If only diagonal disorder (γ_0) is present :

$$F = \frac{\pi\nu(\varepsilon)}{16} Str \int \left[D(\varepsilon)(\nabla Q)^2 + 2i\omega\Lambda Q \right] dr$$

$$\hat{Q} = \hat{C}\hat{Q}^T\hat{C}^T; \quad \hat{C} = i\tau_y^{KK'} \otimes 1\!\!1^{AR} \otimes \left(\tau_-^{eh} \otimes 1\!\!1^g - \tau_+^{eh} \otimes \tau_z^g\right)$$

The symmetry of Q corresponds to 2 replicas of the symplectic ensemble. ⇨ Antilocalization!

The symplectic symmetry was first noticed by Suzuura and Ando (2002)

However, the terms with ρ_i break the symmetry and the ensemble becomes orthogonal!

At large distances: $\hat{Q} = 1\!\!1_{KK'} \otimes \hat{Q}_o$

-is 8x8 supermatrix of the orthogonal symmetry: localization!

Perturbation theory: first order gives the weak localization correction.

$$\Delta\sigma_{WL} = \frac{e^2}{2\pi^2\hbar}\left[-\ln\frac{\tau_\phi}{\tau_{tr}} + 2\ln\frac{\tau_2}{\tau_{tr}} + \ln\frac{\tau_3}{\tau_{tr}}\right]$$

$$\frac{1}{\tau_2} = \frac{1}{\tau_\phi} + \frac{1}{\tau_\perp};\ \ \frac{1}{\tau_3} = \frac{1}{\tau_\phi} + \frac{1}{2\tau_{||}} + \frac{1}{\tau_\perp};$$

τ_ϕ -inelastic dephasing time

Agrees with McCann et al (2006)

Ultraviolet logarithmic renormalization

$$\frac{\tau_{tr}}{\tau_1} \approx \frac{\tau_{tr}}{\tau_2} \approx \frac{36}{7\ln[t^*/\ln|\tilde{\varepsilon}/\varepsilon_0|]}$$

Qualitative picture (from Aleiner and Efetov (2006)).

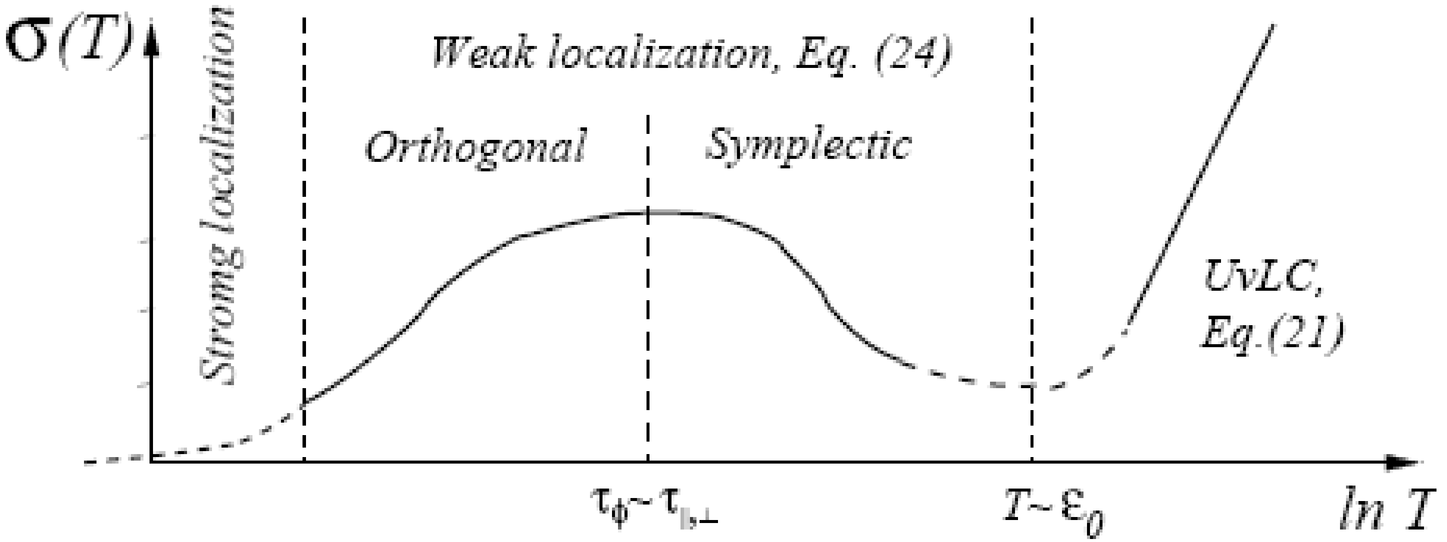


FIG. 2: Schematic dependence, $\sigma(T)$, fot the undoped graphene and for $\tau_\phi^{-1} \propto T$.

How to make a quantum dot in graphene?

Silvestrov and Efetov (2007)

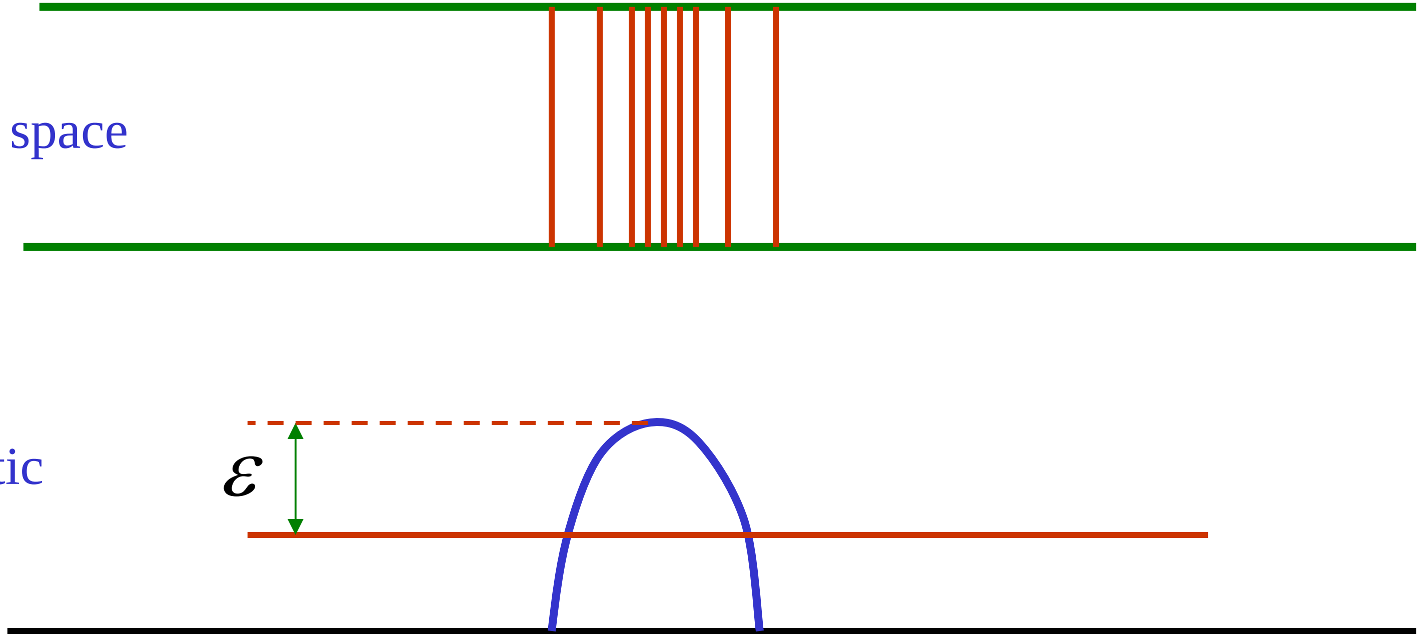
Peculiarity of graphene with respect to conventional 2D electron gases: One barrier is sufficient!

Avoiding the reflectionless penetration (Klein paradox) through the barrier: making a strip with a barrier across it.

Transversal quantization removes the problem of the confinement!

Real space

Applied electrostatic potential



Sufficient to make the quantum dot!

Quasiclassical treatment complemented by numerics.

Conductance as a function of the energy (gate voltage)

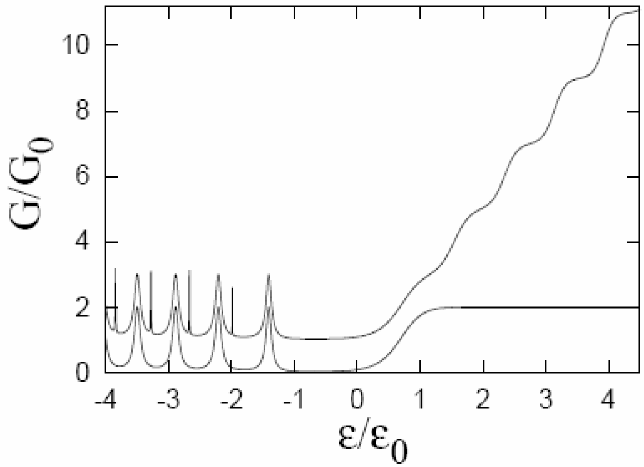


FIG. 1: Upper curve: Conductance of the graphene quantum dot as a function of the Fermi energy for the metallic armchair edges for $L = 4\xi$, Eq. (4). The lower curve: the contribution to the conductance from the transmission channels with $p_y = \pm\pi\hbar/L$. All calculations are carried out for the zero temperature, $T = 0$.

$$V = -(x/x_0)^2 U/2.$$

$$\xi = \left[\hbar c x_0^2/U\right]^{1/3}, \quad \varepsilon_0 = \hbar c/\xi.$$

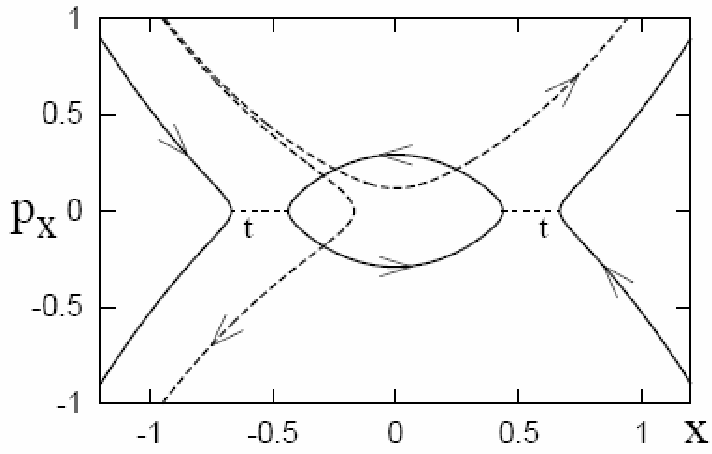


FIG. 2: Examples of trajectories described in the text drawn on the x, p_x plane (arbitrary units). Solid lines show the trajectories with $\varepsilon < 0$ either bouncing inside the barrier, or reflected by it from the left/right. Tunnelling events between the bounded and unbounded trajectories are shown schematically (t). Thick dashed lines show the trajectories with $\varepsilon > 0$ either transmitted for $|p_y| < \varepsilon/c$ (open channels) or reflected for $|p_y| > \varepsilon/c$ (closed channels).

Fabrication of quantum dots in graphene: essential detail for quantum computer! Experiments are being carried out!