Electron interactions and fractional quantum Hall effect in graphene in a strong magnetic field

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Collaborators: Roderich Moessner, Benoît Douçot, Nicolas Regnault

Overview

- Interactions and SU(4) (spin $\times$ valley) symmetry
  - symmetry breaking terms in $n \neq 0$ (and $n = 0$)
  - interactions and pseudopotentials

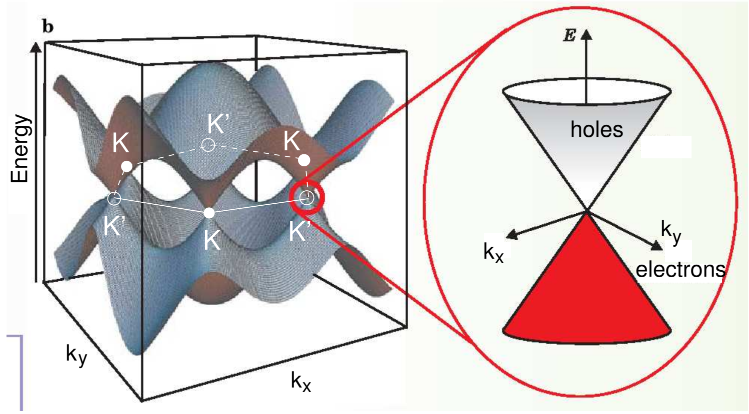
$\Rightarrow$ MOG, R. Moessner, and B. Douçot,
Phys. Rev. B **74**, 161407 (2006)

- SU(4) fractional quantum Hall effect
  - SU(4) generalisation of Halperin's wave function
  - comparison with numerical results (exact diagonalisation)

$\Rightarrow$ MOG and N. Regnault, *work in progress*

Graphene in a magnetic field

- Bandstructure near the points K and K' (valley $\alpha = \pm$):



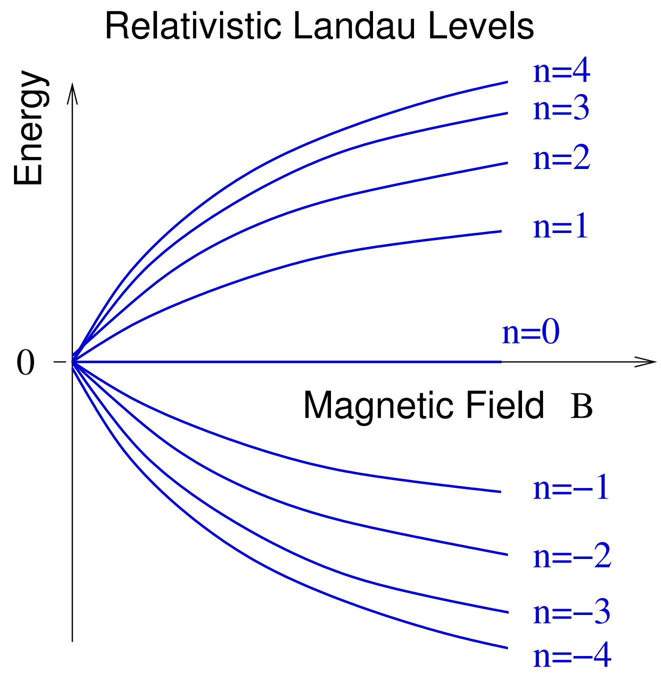
$$\mathcal{H}^{\pm}(\boldsymbol{\kappa}) = \frac{3}{2}ta \begin{pmatrix} 0 & \kappa_1 \mp i\kappa_2 \\ \kappa_1 \pm i\kappa_2 & 0 \end{pmatrix}$$

$\boldsymbol{\kappa}$: measured from K and K' points

- Energy dispersion with magnetic field (degenerate in valley index α):

$$\epsilon_n = \pm\hbar\frac{v_F}{l_B}\sqrt{|n|} \propto \sqrt{B|n|}$$

(Relativistic LLs)



Length and energy scales in graphene

Length scales

- Distance between neighbouring carbon atoms: $a = 0.14$nm
- Magnetic length: $l_B = 26$nm$/\sqrt{B[\mathrm{T}]}$
- Lattice effects (anisotropies, etc.): $a/l_B \sim 0.005\sqrt{B[\mathrm{T}]}$
- Larmor radius: $R_L = \sqrt{n} l_B$

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- Band width: $t = 2.7$eV
- LL 'spacing': $\sqrt{2}\hbar v_F/l_B = 3ta/\sqrt{2}l_B \sim 35\sqrt{B[\mathrm{T}]}$meV

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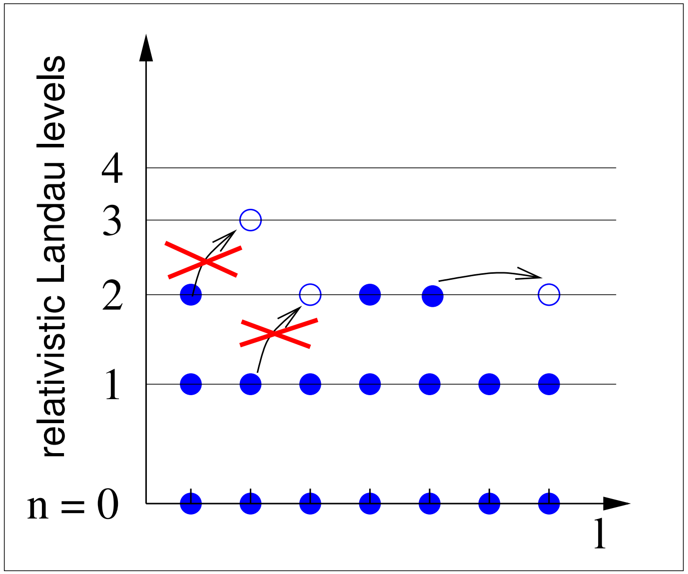
- Zeeman splitting: $\Delta_z = g\mu_B B \sim 0.1B[\mathrm{T}]\text{meV}$
- Interaction energy: $e^2/\epsilon l_B \sim 2.4...12\sqrt{B[\mathrm{T}]}\text{meV}$

Interaction model – densities

- Electrons in a single rel. LL at $\nu \neq 2n+1$ (no spin):

$$H = \frac{1}{2}\sum_{\mathbf{q}} V(q)\rho^n(-\mathbf{q})\rho^n(\mathbf{q})$$



with Coulomb interaction

$$V(q) = \frac{2\pi e^2}{\epsilon q}$$

- Projected densities $\rho^n(\mathbf{q}) = \sum_{\alpha,\alpha'} F_n^{\alpha\alpha'}(\mathbf{q})\bar{\rho}^{\alpha\alpha'}(\mathbf{q})$:

$$\bar{\rho}^{\alpha\alpha'}(\mathbf{q}) = \sum_{m,m'} \langle m|e^{-i[\mathbf{q}+(\alpha-\alpha')\mathbf{K}]\cdot\mathbf{R}}|m'\rangle c^{\dagger}_{n,m,\alpha}c_{n,m',\alpha'}$$

$\mathbf{R}$: guiding center operator

Interaction model (II)

- Graphene form factors $l_B \equiv 1$:

$$
\begin{aligned}
F_n^{++}(\mathbf{q}) &= \frac{1}{2}\left[L_{|n|}\left(\frac{|\mathbf{q}|^2}{2}\right) + L_{|n|-1}\left(\frac{|\mathbf{q}|^2}{2}\right)\right] e^{-|\mathbf{q}|^2/4} = F_n^{--}(\mathbf{q}) \equiv \mathcal{F}_n(\mathbf{q}) \\
F_n^{+-}(\mathbf{q}) &= \left(\frac{-i(q + q^* - K - K^*)}{2\sqrt{2|n|}}\right) L^1_{|n|-1}\left(\frac{|\mathbf{q}-\mathbf{K}|^2}{2}\right) e^{-|\mathbf{q}-\mathbf{K}|^2/4} \\
F_n^{-+}(\mathbf{q}) &= \left[F_n^{+-}(-\mathbf{q})\right]^*
\end{aligned}
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$$

- Model: $H = \frac{1}{2}\sum_{\alpha_1,\ldots,\alpha_4}\sum_{\mathbf{q}} v_n^{\alpha_1,\ldots,\alpha_4}(\mathbf{q})\bar{\rho}^{\alpha_1\alpha_3}(-\mathbf{q})\bar{\rho}^{\alpha_2\alpha_4}(\mathbf{q})$

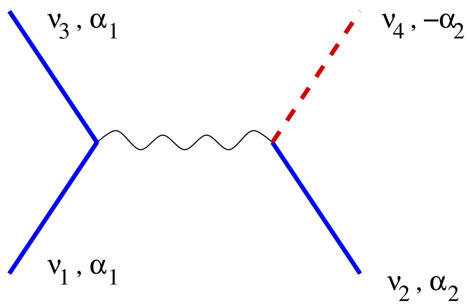
  with interaction vertex:

$$
v_n^{\alpha_1,\ldots,\alpha_4}(\mathbf{q}) = \frac{2\pi e^2}{\epsilon|\mathbf{q}|} F_n^{\alpha_1\alpha_3}(-\mathbf{q}) F_n^{\alpha_2\alpha_4}(\mathbf{q}),
$$

$\Rightarrow$ No SU(2) valley symmetry so far !!

Scattering Processes in $n \neq 0$

- Terms of the form $F_n^{\alpha,\alpha}(\mp\mathbf{q})F_n^{\alpha',-\alpha'}(\pm\mathbf{q})$:
  exp. suppressed $\sim \exp(-|\mathbf{K}|/8)$



- Umklapp terms $F_n^{\alpha,-\alpha}(-\mathbf{q})F_n^{\alpha,-\alpha}(\mathbf{q})$:
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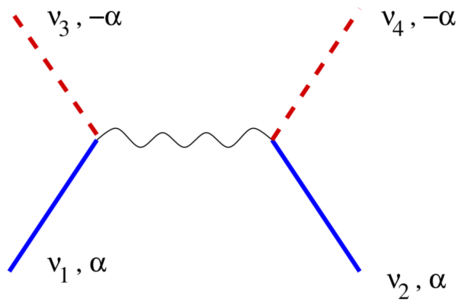

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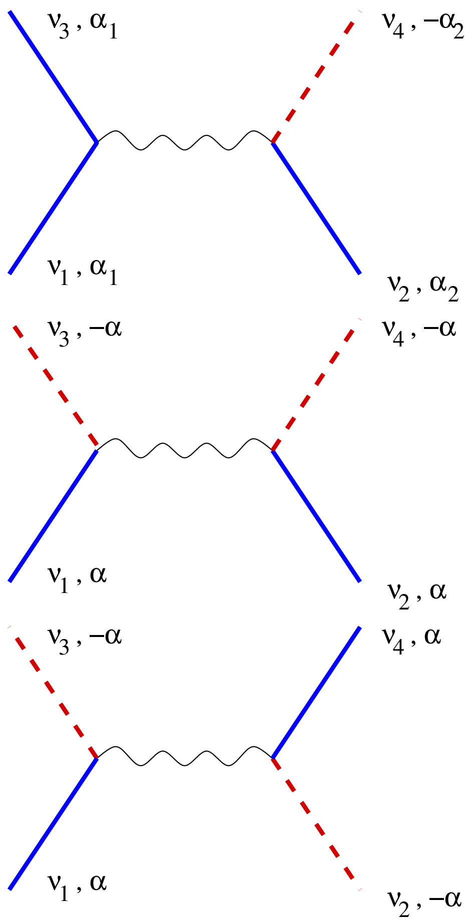
- Backscattering terms
  $F_n^{\alpha,-\alpha}(-\mathbf{q})F_n^{-\alpha,\alpha}(\mathbf{q})$:
  alg. small $\sim 1/|\mathbf{K}| \sim a/l_B$



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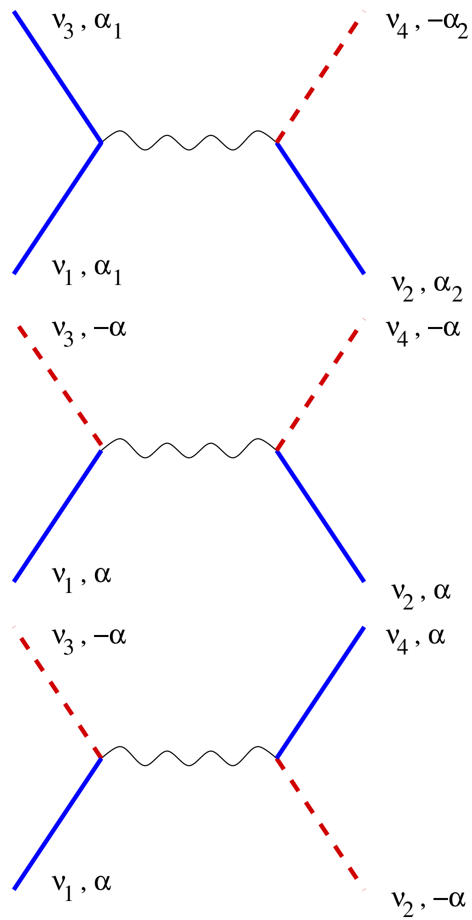

$$\Rightarrow H^n_{SU(2)} = \frac{1}{2}\sum_{\alpha,\alpha'}\sum_{\mathbf{q}} \frac{2\pi e^2}{\epsilon|\mathbf{q}|}\left[\mathcal{F}_n(q)\right]^2 \bar{\rho}^{\alpha,\alpha}(-\mathbf{q})\bar{\rho}^{\alpha',\alpha'}(\mathbf{q}) + \mathcal{O}(a/l_B)$$

SU(2) Model

- SU(2) Interaction Hamiltonian:

$$H^n_{SU(2)} = \frac{1}{2}\sum_{\mathbf{q}} v^G_n(q)\bar{\rho}(-\mathbf{q})\bar{\rho}(\mathbf{q})$$

with total projected density $\bar{\rho}(\mathbf{q}) = \bar{\rho}^{++}(\mathbf{q}) + \bar{\rho}^{--}(\mathbf{q})$
and effective interaction potential for graphene:

$$v^G_{n\neq 0}(q) = \frac{\pi e^2}{\epsilon q} e^{-q^2/2}\left[L_{|n|}\left(\frac{q^2}{2}\right) + L_{|n|-1}\left(\frac{q^2}{2}\right)\right]^2, \quad v^G_0(q) = \frac{2\pi e^2}{\epsilon q} e^{-q^2/2}$$

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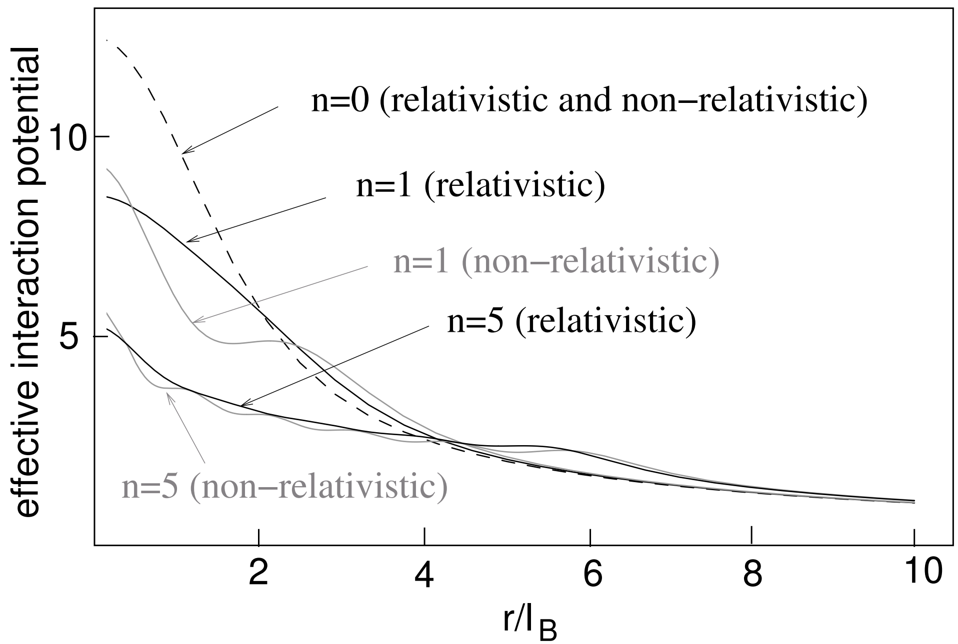
- Magnetic translation algebra for projected densities:

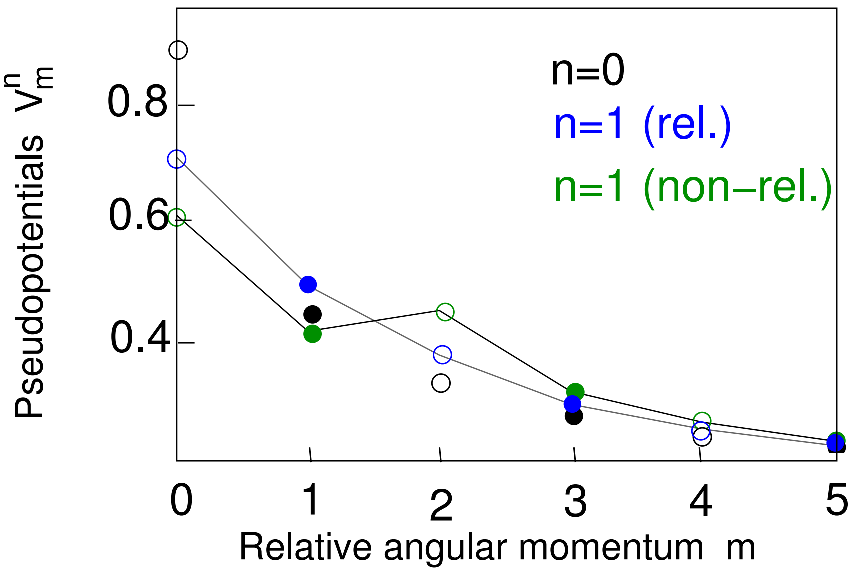
$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{q}')] = 2i\sin\left(\frac{\mathbf{q}\wedge\mathbf{q}'}{2}\right)\bar{\rho}(\mathbf{q}+\mathbf{q}')$$

$\Rightarrow$ Same structure as for non-relativistic QH systems, but

- SU(4) symmetry (spin $\times$ valley)
- modified interaction potential

Effective SU(2) interaction potentials – FQHE





- Largest difference between rel. and non-rel. case in $n = 1$
- Similar behaviour of rel. interaction in $n = 0$ and $n = 1$:
  - "valley-polarised" FQHE states most stable in $n = 1$
  - absence of non-rel. $n = 1$ physics: Pfaffian at $\nu = 5/2$?

SU(2) symmetry-breaking terms of $\mathcal{O}(a/l_B)$

- Backscattering terms in $n \neq 0$:

$$H_{bs} = \frac{1}{2}\sum_{\alpha}\sum_{\mathbf{q}} v_n^{\alpha,-\alpha}(\mathbf{q})\bar{\rho}^{\alpha,-\alpha}(-\mathbf{q})\bar{\rho}^{-\alpha,\alpha}(\mathbf{q})$$

with interaction $v_n^{+-}(\mathbf{q}) = v_n^{-+}(-\mathbf{q})$ peaked at $\mathbf{q} = \pm\mathbf{K}$:

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- Electrostatics in $n = 0$:

  charge distributed homogeneously on both sublattices

  $\Rightarrow$ "easy-plane anisotropy" SU(2)$\rightarrow$U(1)

  BUT : other possible symmetries (e.g. for on-site U)

  Alicea and Fisher, PRB 74, 075422 (2006); Herbut, cond-mat/0610249

Towards a FQHE in graphene

- Chern-Simons approach

$\Rightarrow$ Relativistic composite fermions (CF) ?

Peres, Guinea, Castro Neto, PRB 73, 125411 (2006)

Khveshchenko, cond-mat/0607174

- Exact diagonalisation with SU(2) chirality symmetry only (fully spin-polarised)

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  Töke et al., PRB 74, 235417 (2006)

- CF wave functions with SU(4) symmetry at $\nu = p/(2sp+1)$

  Töke and Jain, cond-mat/0701026

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$\Rightarrow$ SU(4) generalisation of Halperin's wave functions at "new fractions"

MOG and N. Regnault, work in progress

SU(K) generalisation of Halperin's wave functions

- Laughlin's wave function (apart from Gaussian factor):

$$\phi_m^L = \prod_{k<l}^{N} (z_k - z_l)^m, \qquad m \;\; \text{odd}$$

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- Halperin's generalisation for SU(2) spin:

$$\phi^H_{m_1,m_2,n} = \prod_{k_1<l_1}^{N_1} \left(z^{(1)}_{k_1} - z^{(1)}_{l_1}\right)^{m_1} \prod_{k_2<l_2}^{N_2} \left(z^{(2)}_{k_2} - z^{(2)}_{l_2}\right)^{m_2} \prod_{k_1,k_2}^{N_1,N_2} \left(z^{(1)}_{k_1} - z^{(2)}_{k_2}\right)^{n}$$

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- SU(K) generalisation:

$$\psi^{SU(K)}_{m_1,\dots,m_K;n_{ij}} = \prod_{j=1}^{K} \prod_{k_j<l_j}^{N_j} \left(z^{(j)}_{k_j} - z^{(j)}_{l_j}\right)^{m_j} \prod_{i<j}^{K} \prod_{k_i,k_j}^{N_i,N_j} \left(z^{(i)}_{k_i} - z^{(j)}_{k_j}\right)^{n_{ij}}$$

Symmetries of SU(4) states (I)

- Symmetric exponent matrix $M = (n_{ij})$, $n_{ji} = n_{ij}$; $n_{jj} \equiv m_j$
- Filling factors :

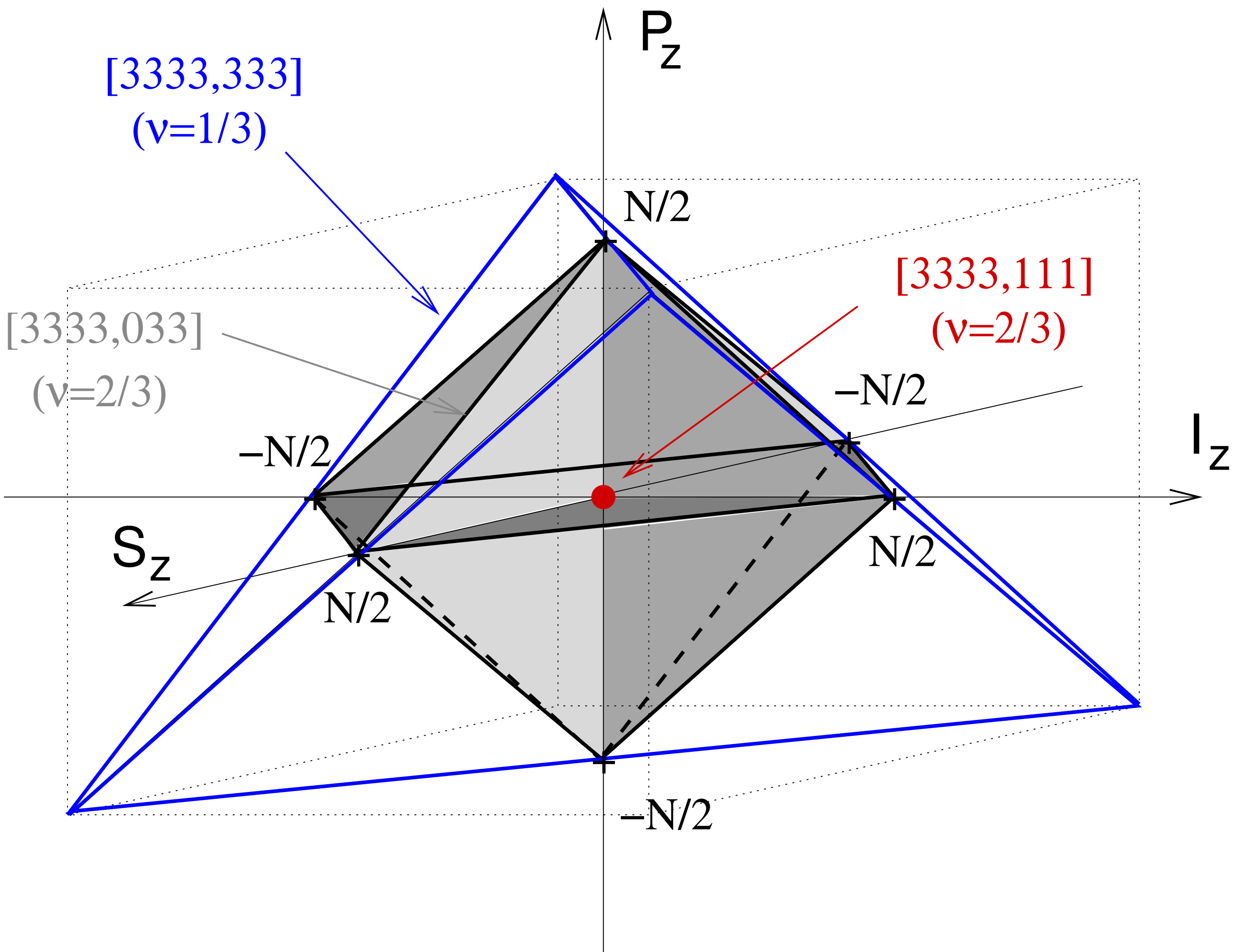
$$\begin{pmatrix} \nu_1 \\ \vdots \\ \nu_4 \end{pmatrix} = M_4^{-1} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$[m_1 m_2 m_3 m_4, n_e n_+ n_-]$	r	ν_T	$\frac{S_z}{N/2}$	$\frac{I_z}{N/2}$	$\frac{P_z}{N/2}$
$[3333, 111]$	4	$2/3$	0	0	0
$[3333, 033]$	2	$2/3$	–	0	–
$[3555, 222]$	4	$2/5$	$1/3$	$1/3$	$1/3$
$[3333, 233]$	2	$2/5$	–	0	–
$[3535, 222]$	4	$8/19$	0	$1/2$	0
$[5555, 222]$	4	$4/11$	0	0	0
$[3737, 233]$	3	$4/11$	–	$1/2$	–
$[3535, 235]$	2	$4/11$	–	$1/2$	–
$[3333, 333]$	1	$1/3$	–	–	–

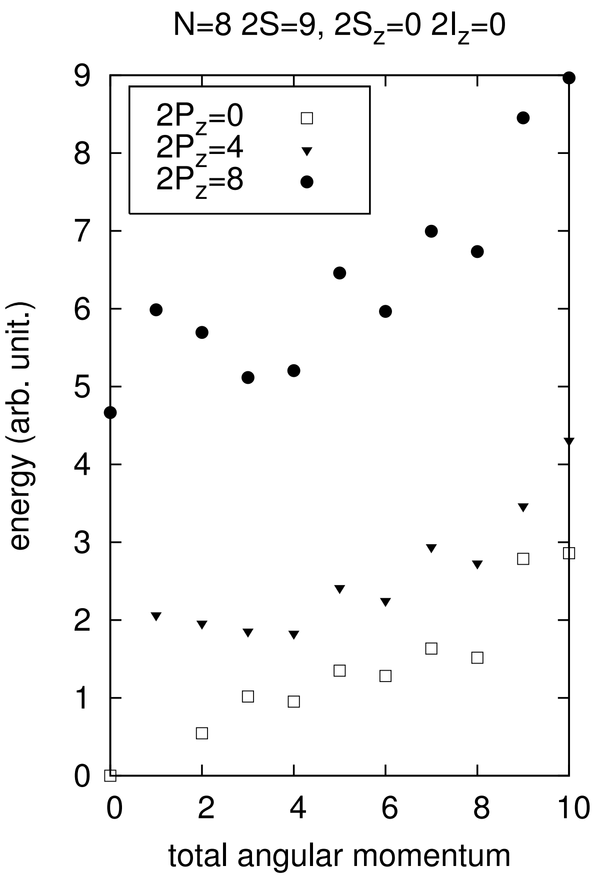
- Matrix rank r of M_4: residual symmetries
  - $r = 1$: SU(4) ferromagnet (at $\nu = 1/m$)
  - $r = 2$: e.g. SU(2)×SU(2) ferromagnet ($[3333, 033]$)
  - $r = 3$: e.g. SU(2) ferromagnet only at K (or K')
  - $r = 4$: invertible matrix (e.g. $[3333, 111]$)

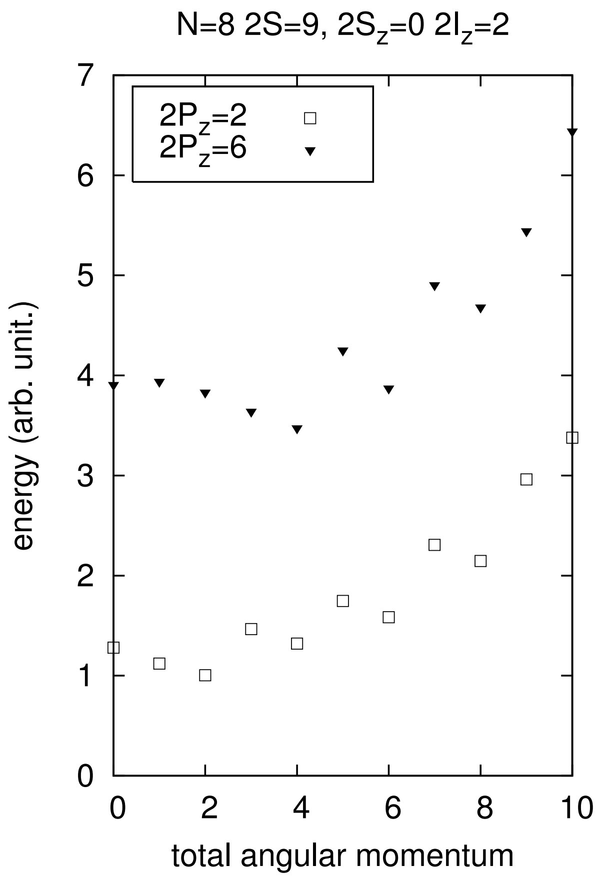
Symmetries of SU(4) states (II)

Good quantum numbers: "z-components" of SU(4) spin,
$S_z \sim \tau_z \otimes 1, I_z \sim 1 \otimes \tau_z, P_z \sim \tau_z \otimes \tau_z$



Exact diagonalisation for $\nu = 2/3$, $[3333, 111]$





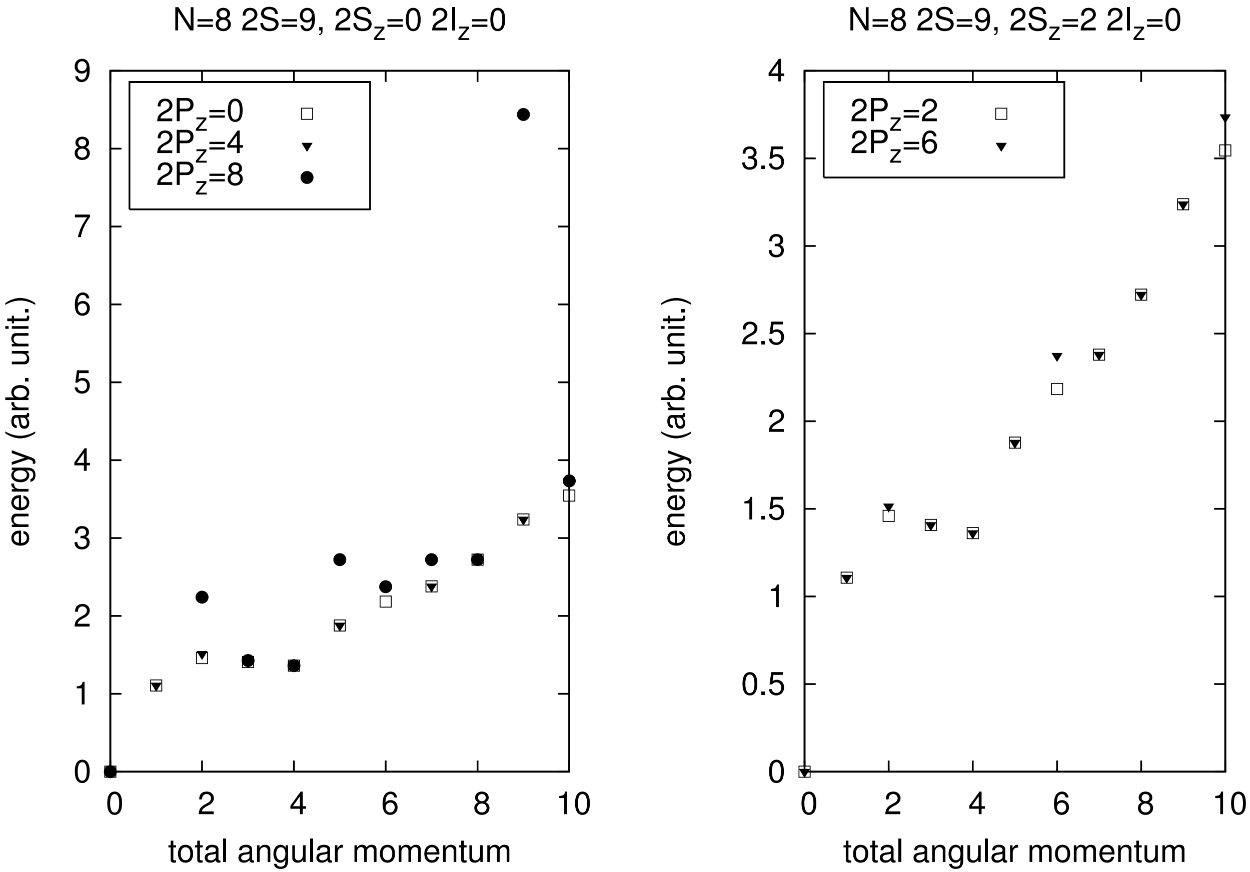
$N = 8$, $2S = 9$

calculations for model potential:

$$V_1^{\text{intra}} > 0, \quad V_{m>1}^{\text{intra}} = 0, \quad V_0^{\text{inter}} > 0, \quad V_{m>0}^{\text{inter}} = 0$$

$[3333, 111]$ is non-degenerate ground state with $S_z = I_z = P_z = 0$

Exact diagonalisation for $\nu = 2/3$, $[3333, 033]$



$N = 8$, $2S = 9$

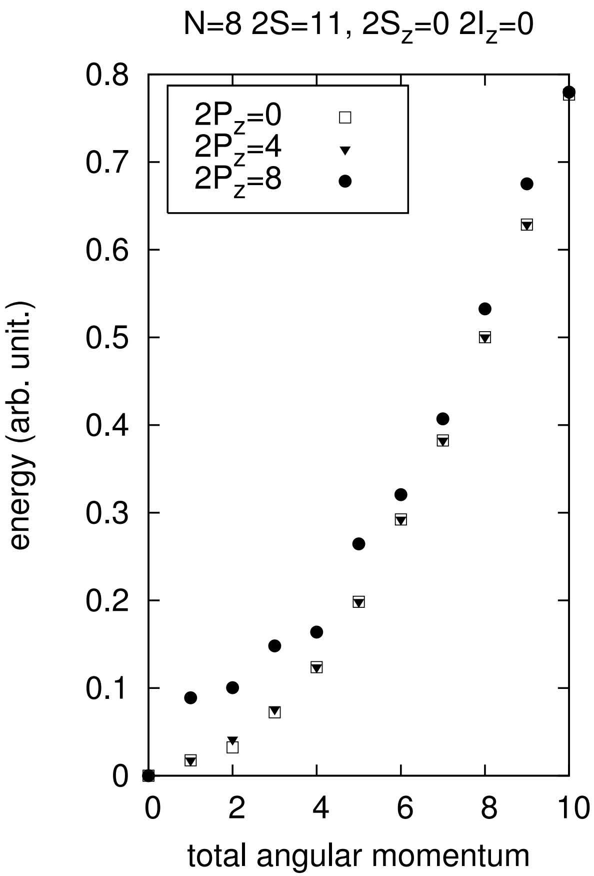
calculations for model potential

MC calculations for Coulomb: higher in energy than $[3333, 111]$

$[3333, 033]$ is ground state with SU(2)$\times$SU(2) symmetry

($S_z = 0$, $I_z = 0$, or $P_z = 0$)

Exact diagonalisation for $\nu = 2/3$, Coulomb



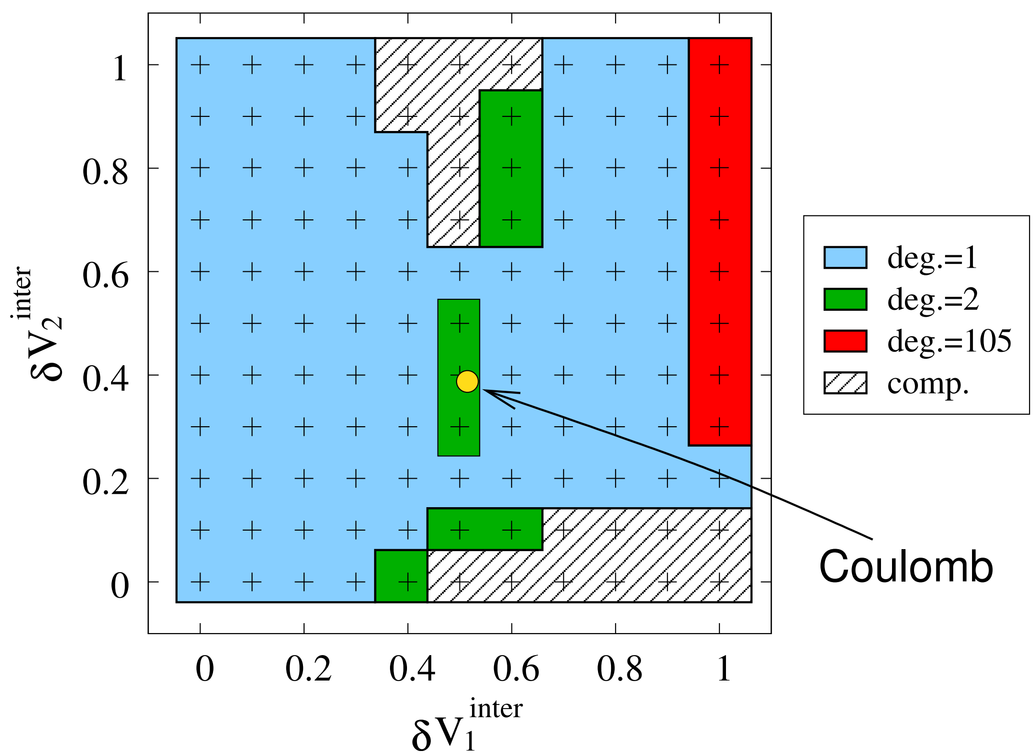
$N = 8, 2S = 11$

Goldstone mode ?

calculations for Coulomb potential

same ground state as in SU(2), orthogonal to $[3333, 111]$

Pseudopotential variation



Conclusions

- Electron interactions in graphene in the QH regime:
  - effective SU(4) interaction model
  - symmetry-breaking terms of order $a/l_B \sim 0.02$

- FQHE in graphene (what is different from GaAs):
  - graphene interaction potential
  - relativistic electrons but non-relativistic behaviour in lowest LL
  - SU(4) internal symmetry

$\Rightarrow$ possibly new fractions

Outlook: From SU(2) to SU(4) in a fixed LL (I)

- Theoretical limit of vanishing Zeeman splitting:
  valley SU(2) symmetry $\times$ spin SU(2) symmetry
- Generators of SU(2) symmetry: projected spin densities

$$\bar{S}^{\mu}(\mathbf{q}) = \bar{\rho}(\mathbf{q}) \otimes S^{\mu} = \frac{1}{2} \sum_{m,m;\sigma,\sigma'} \left\langle m \left| e^{-i\mathbf{q}\cdot\mathbf{R}} \right| m' \right\rangle c^{\dagger}_{m;\sigma} \tau^{\mu}_{\sigma,\sigma'} c_{m';\sigma'}$$

$$\begin{aligned}
[\bar{S}^{\mu}(\mathbf{q}), \bar{\rho}(\mathbf{q}')] &= 2i \sin\left(\frac{\mathbf{q}\wedge\mathbf{q}'}{2}\right) \bar{S}^{\mu}(\mathbf{q}+\mathbf{q}') \\
[\bar{S}^{\mu}(\mathbf{q}), \bar{S}^{\nu}(\mathbf{q}')] &= \frac{i}{2}\delta^{\mu\nu} \sin\left(\frac{\mathbf{q}\wedge\mathbf{q}'}{2}\right) \bar{\rho}(\mathbf{q}+\mathbf{q}') + i\epsilon^{\mu\nu\sigma} \cos\left(\frac{\mathbf{q}\wedge\mathbf{q}'}{2}\right) \bar{S}^{\sigma}(\mathbf{q}+\mathbf{q}')
\end{aligned}$$

- Additional SU(2) symmetry – SU(2)$\otimes$SU(2):

$$\bar{S}^{\mu}(\mathbf{q}) = \bar{\rho}(\mathbf{q}) \otimes (S^{\mu} \otimes \mathbb{1}) \qquad \bar{I}^{\nu}(\mathbf{q}) = \bar{\rho}(\mathbf{q}) \otimes (\mathbb{1} \otimes \bar{I}^{\nu})$$

Outlook: From SU(2) to SU(4) in a fixed LL (II)

- Other generators of SU(4) obtained from commutators:

$$[\bar{S}^{\mu}(\mathbf{q}), \bar{I}^{\nu}(\mathbf{q}')] = 2i \sin\left(\frac{\mathbf{q} \wedge \mathbf{q}'}{2}\right) \bar{\rho}(\mathbf{q}+\mathbf{q}') \otimes \left(S^{\mu} \otimes I^{\nu}\right),$$

$\Rightarrow$ SU(4) extension of magnetic translation algebra
[Ezawa, PRB 67, 125314 (2003); Tsitsishvili and Ezawa, PRB 70,125304 (2004)]

- Necessary spin-valley entanglement in model
  (c.f. spin-charge entanglement in SU(2) extension)

- SU(4) skyrmion physics at $\nu = 1$
  [Arovas et al., PRB 59, 13147 (1999); Ezawa, PRL 82, 3512 (1999)]