Chirality & Correlations in Graphene

KITP Workshop - 01.07

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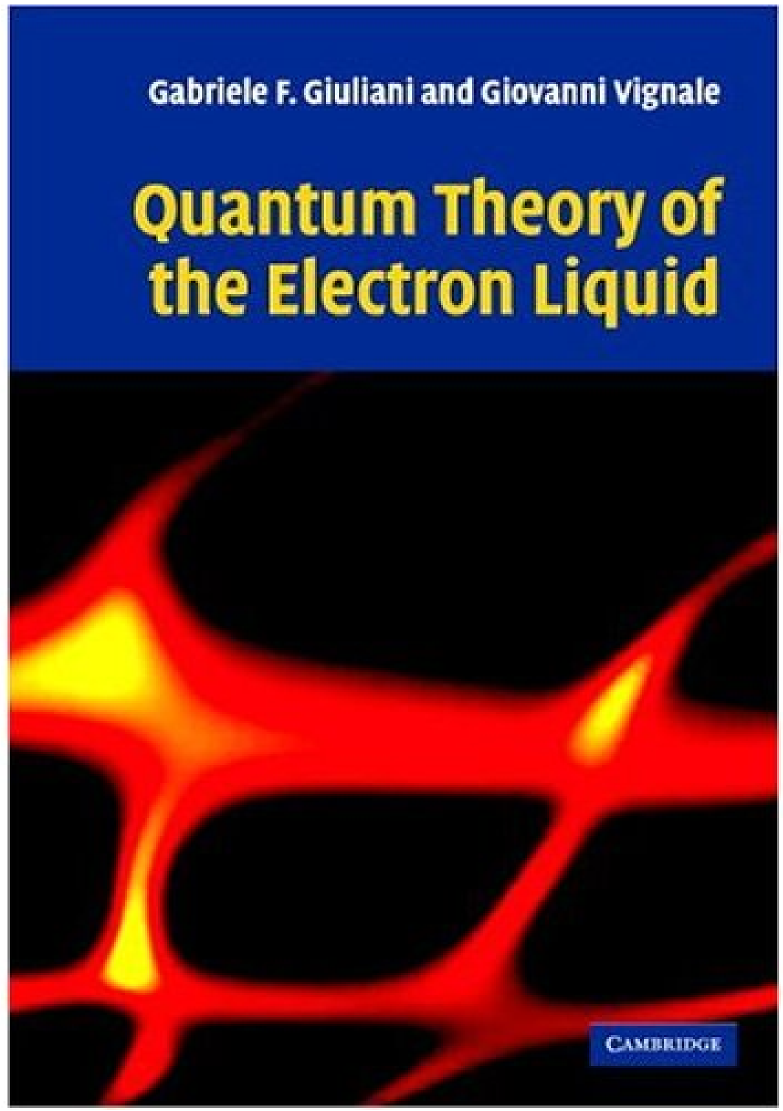
Electron Gas Theory



The MDF model for Graphene

$v = 2 \pi e^2/q$

$$\hat{\mathcal{H}} = v \sum_{\mathbf{k}} \hat{\psi}^\dagger_{\mathbf{k}} (\sigma_1 k_1 + \sigma_2 k_2) \hat{\psi}_{\mathbf{k}} + \frac{1}{2S} \sum_{\mathbf{q}\neq 0} v_q (\hat{n}_{\mathbf{q}} \hat{n}_{-\mathbf{q}} - \hat{N})$$

Wikipedia on Helicity vs. Chirality

Helicity (particle physics)

From Wikipedia, the free encyclopedia

In particle physics, **helicity** is the projection of the angular momentum to the direction of motion:

Because the angular momentum with respect to an axis has discrete values, helicity is discrete, too. For spin-1/2 particles such as the electron, the helicity can either be positive - the particle is then "right-handed" - or negative - the particle is then "left-handed".

For massless (or extremely light) spin-1/2 particles, helicity is equivalent to the operator of chirality multiplied by h/2

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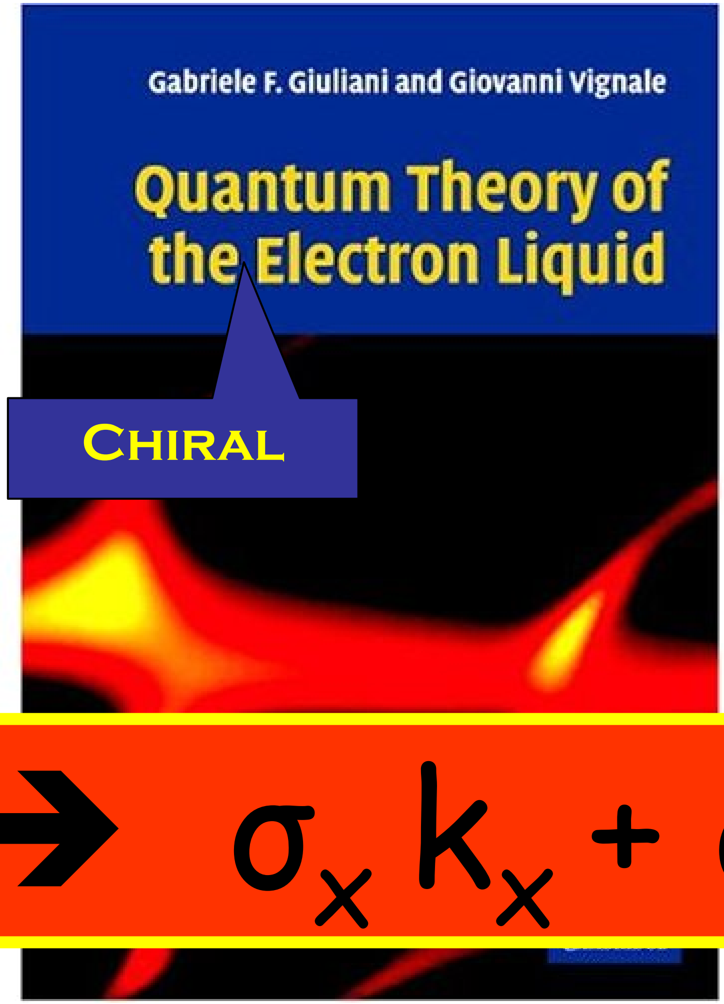
Electron Gas Theory



$$k^2 \rightarrow \sigma_x k_x + \sigma_y k_y$$

The MDF model for Graphene

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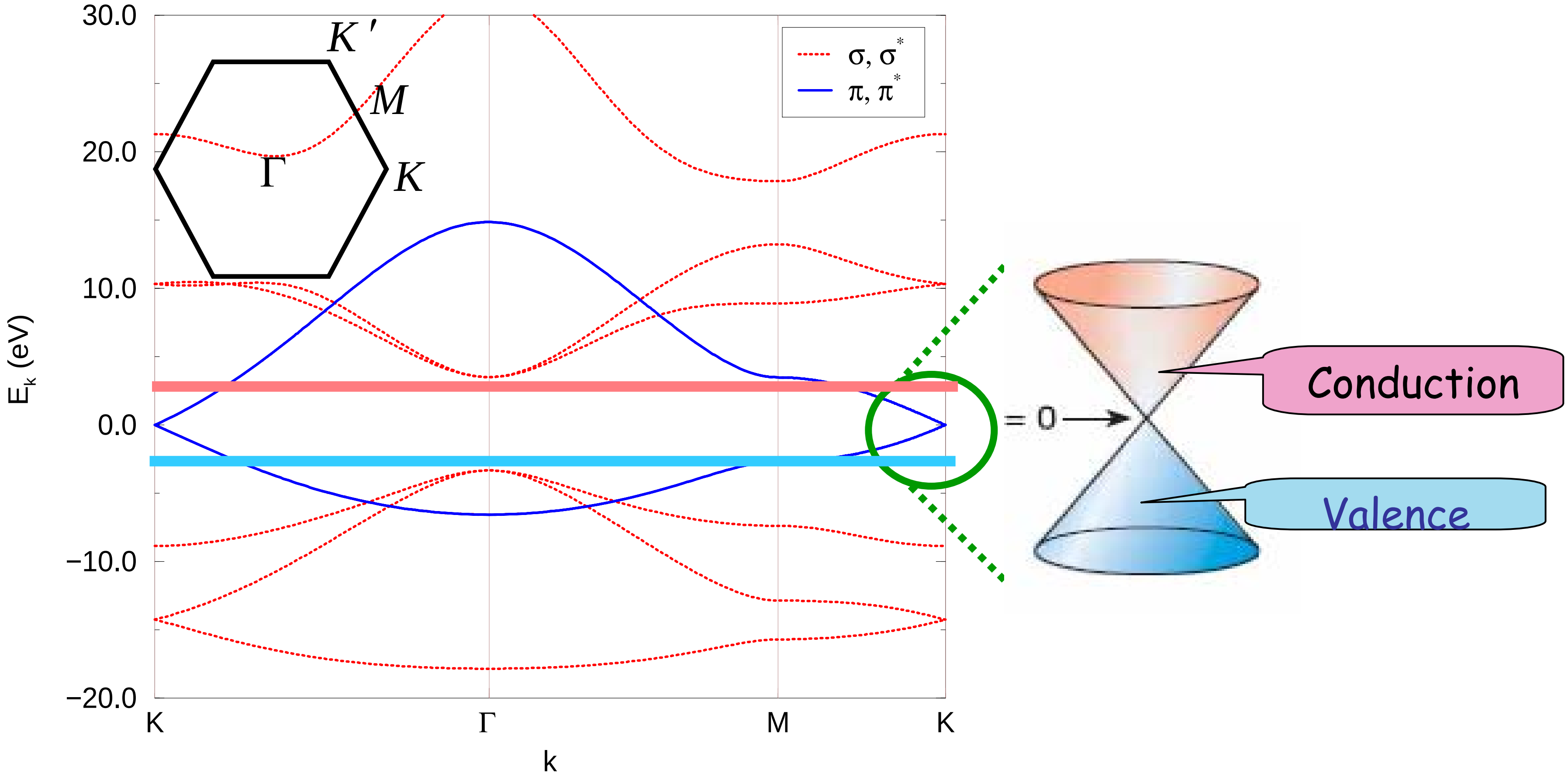
… plus uv cutoff

Graphene Bands



$K_c \sim 0.5\ (\Gamma - M) = \pi/3a \sim 1/a$

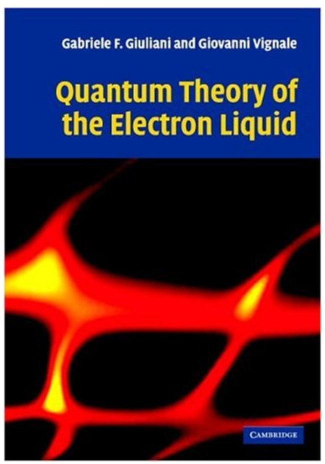
Chiral Electron Gas Properties

- $\chi/\chi_0 < 1$ — Perez et al. PRB (2005)
- $\kappa/\kappa_0 < 1$
- $v/v_0 > 1$ — Katnelson cond-mat/0606611

Coupling Constant Integration

$$E_{\rm int} = \frac{N}{2}\int_0^1 d\lambda \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, v_q \left[S^{(\lambda)}(q) - 1\right]$$

Static Structure Factor

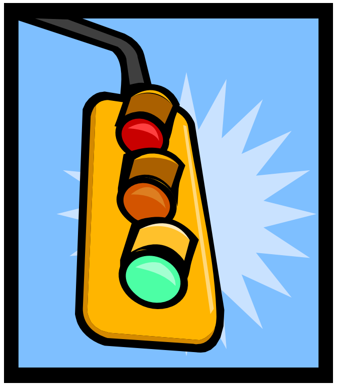


Fluctuation-Dissipation Theorem

$$S^{(\lambda)}(q) = -\frac{1}{\pi n}\int_0^{+\infty} d\Omega\, \chi^{(\lambda)}_{\rho\rho}(\mathbf{q}, i\Omega)$$

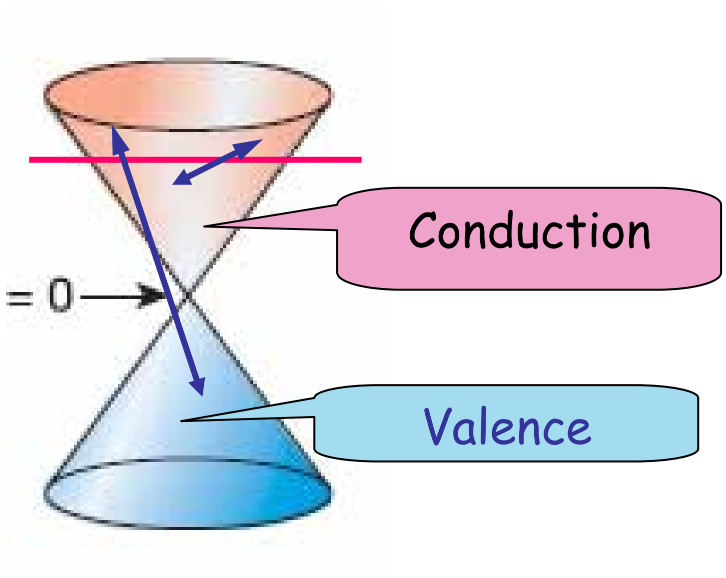
RPA Theory

$$\chi_{\rho\rho}^{(\lambda)}(\mathbf{q}, i\Omega) = \frac{\chi^{(0)}(\mathbf{q}, i\Omega)}{1 - \lambda v_q \chi^{(0)}(\mathbf{q}, i\Omega)}$$



Why RPA?

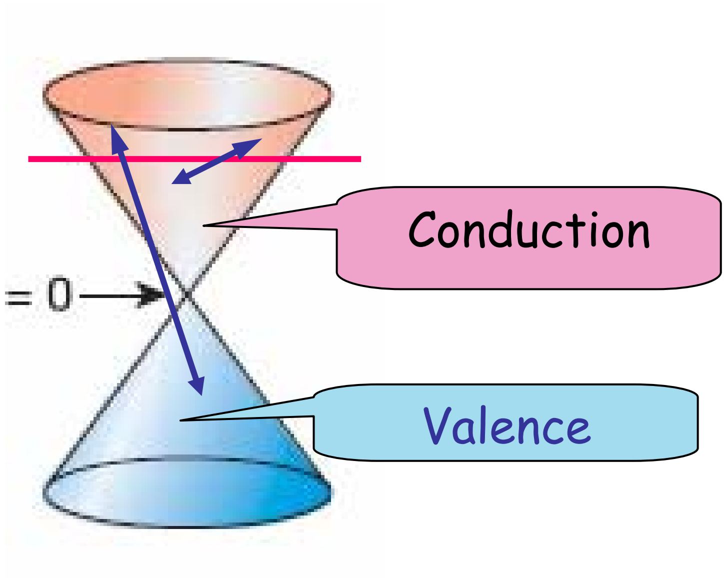
Chiral `Lindhard' Function



Shung PRB (1986)
Wang cond-mat/0605498
Wunch cond-mat/0610630
Hwang cond-mat/0610562

$$\chi^{MDF}(\mathbf{q}, i\Omega) = -\frac{q^2}{16\sqrt{\Omega^2 + v^2 q^2}} - \frac{\varepsilon_{\mathrm{F}}}{2\pi v^2} + \frac{q^2}{8\pi\sqrt{\Omega^2 + v^2 q^2}} \Re e \left[\sin^{-1}\left(\frac{2\varepsilon_{\mathrm{F}} + i\Omega}{vq}\right) + \left(\frac{2\varepsilon_{\mathrm{F}} + i\Omega}{vq}\right) \sqrt{1 - \left(\frac{2\varepsilon_{\mathrm{F}} + i\Omega}{vq}\right)^2} \right]$$

Chiral `Lindhard' Function



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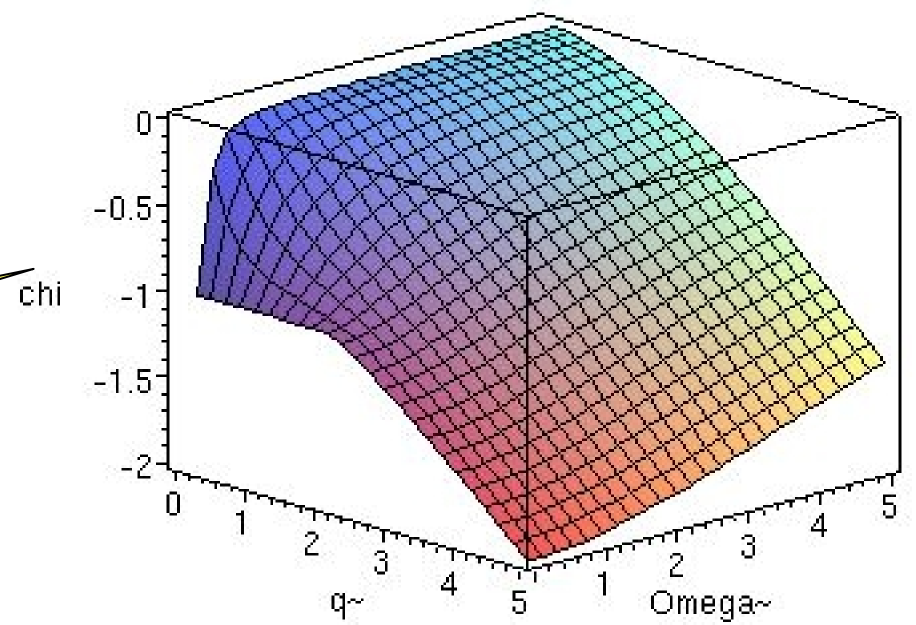
$\chi^0(q,i\Omega)$



Regularization

$E=E-E(\varepsilon_F=0)$

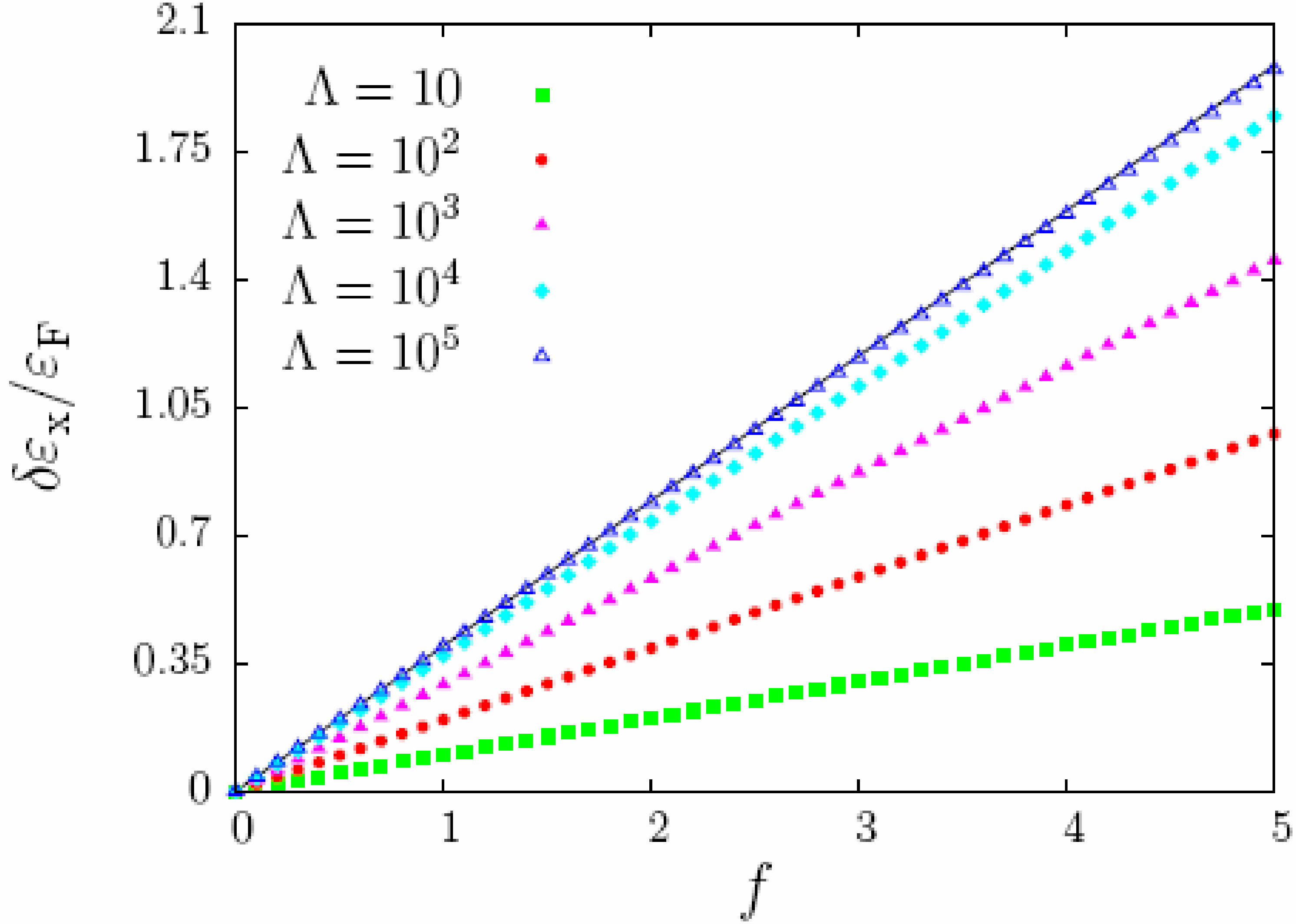
$$\delta\varepsilon_{\rm x} = -\frac{1}{2\pi n}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\, v_q \int_0^{+\infty} d\Omega\, \left[\chi^{(0)}(\mathbf{q}, i\Omega, \varepsilon_{\rm F}) - \chi^{(0)}(\mathbf{q}, i\Omega, \varepsilon_{\rm F}=0)\right]$$

Exchange

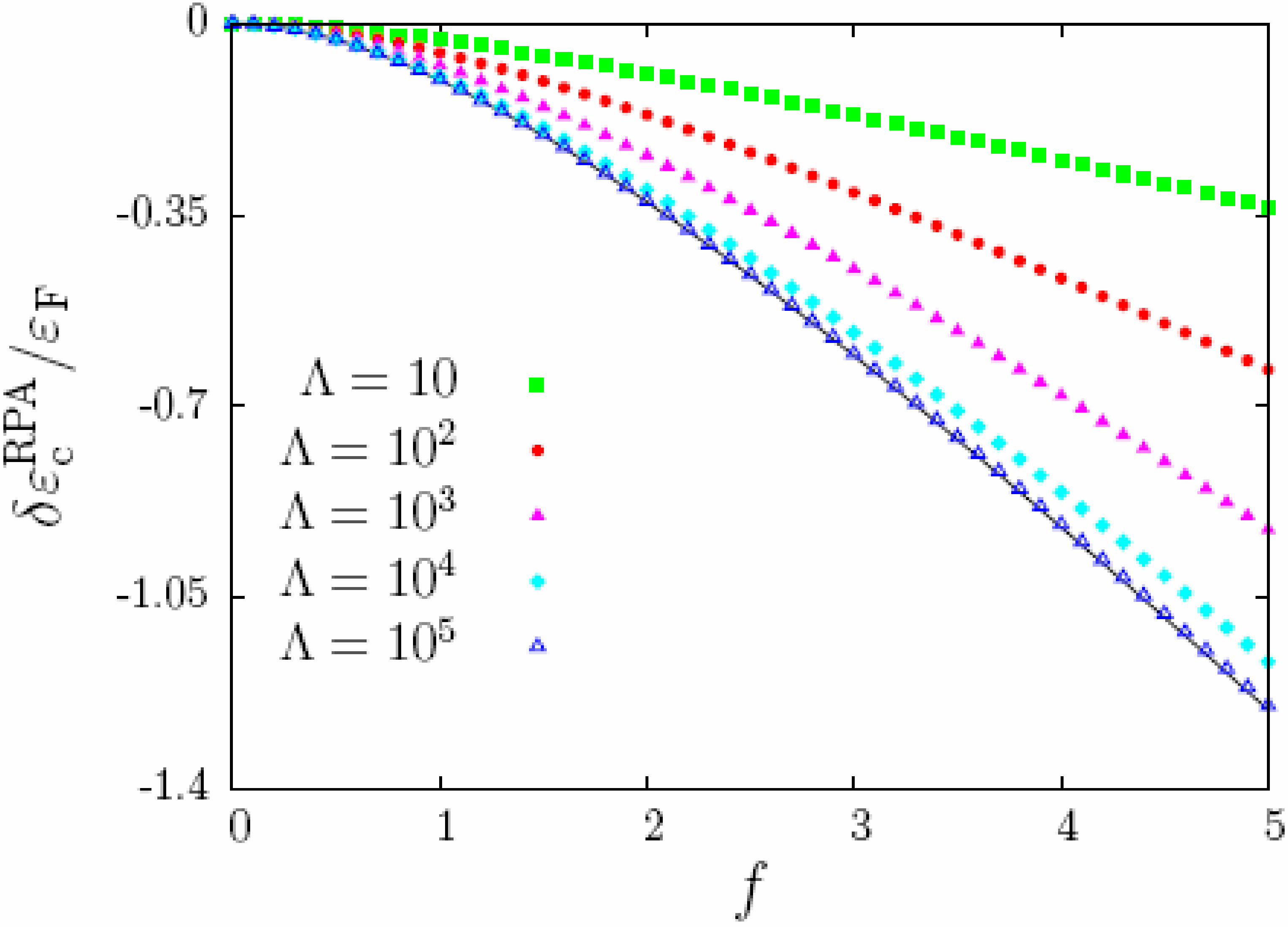
Correlation

$$\delta\varepsilon_{\rm c}^{\rm RPA} = \frac{1}{2\pi n}\int \frac{d^2\mathbf{q}}{(2\pi)^2} \int_0^{+\infty} d\Omega\, \left\{ v_q \delta\chi^{(0)}(\mathbf{q}, i\Omega) + \ln\left[\frac{1 - v_q\chi^{(0)}(\mathbf{q}, i\Omega, \varepsilon_{\rm F})}{1 - v_q\chi^{(0)}(\mathbf{q}, i\Omega, \varepsilon_{\rm F}=0)}\right]\right\}$$

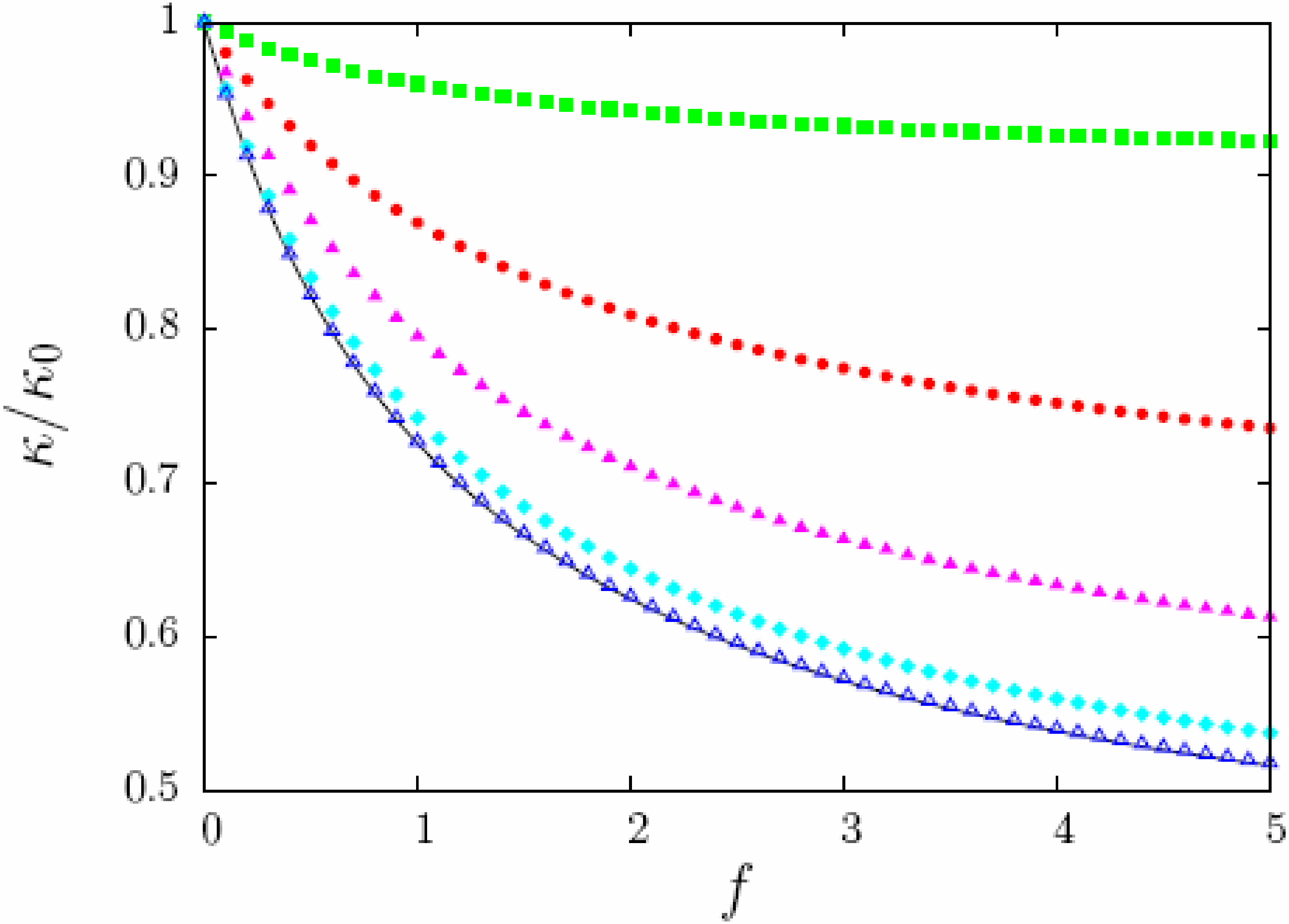
Exchange Energy



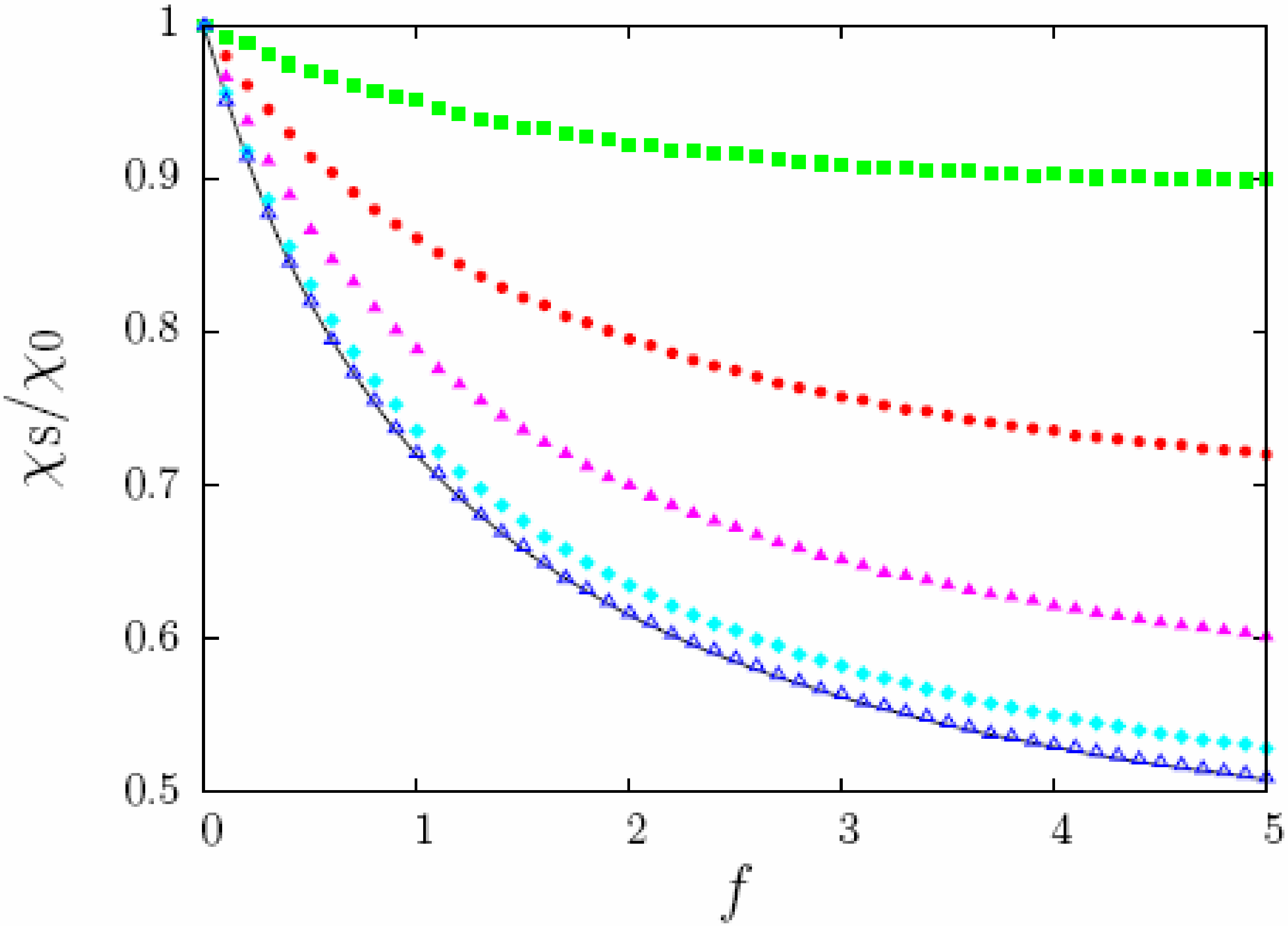
Correlation Energy



Compressibility



Spin-susceptibility



Alternate Form for X-energy

Ordinary

$$\delta\varepsilon_{\mathrm{x}} = -\frac{1}{2nS^2}\sum_{s,s'}\sum_{\mathbf{k},\mathbf{k}'} V_{s,s'}(\mathbf{k},\mathbf{k}')\,\delta n_{\mathbf{k}s}\delta n_{\mathbf{k}'s'}$$

$$+ \frac{1}{nS}\sum_{\mathbf{k},s}\Sigma^{(0)}_{\mathbf{k},s}\delta n_{\mathbf{k}s}$$

Extra-ordinary

Renormalized Velocity

$$\Sigma^{(0)}_{\mathbf{k},s} = -\frac{1}{S}\sum_{\mathbf{k}',s'} V_{ss'}(\mathbf{k},\mathbf{k}')n^{(0)}_{s'}(\mathbf{k}')$$

Contribution to Band Energy

$$V_{s,s'}(\mathbf{k},\mathbf{k}') = \frac{2\pi e^2}{|\mathbf{k}-\mathbf{k}'|}\left[\frac{1+ss'\cos(\theta_{\mathbf{k},\mathbf{k}'})}{2}\right]$$

$$\Rightarrow \quad v \to v\left[1+\frac{f}{4g}\ln(\Lambda)\right]$$

Katnelson cond-mat/0606611

$$\frac{v_p}{v_0} = 1 + \left[\frac{g}{4} - g^2\left(\frac{5}{6} - \ln 2\right) + O(g^3)\right]\ln(\mathcal{K}/p).$$

Mischenko cond-mat/0612651

Chiral Electron Gas Theory

The Cast

Chiral Electron Gas Properties

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  Perez et al. PRB (2005)

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- $v/v_0 > 1$

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