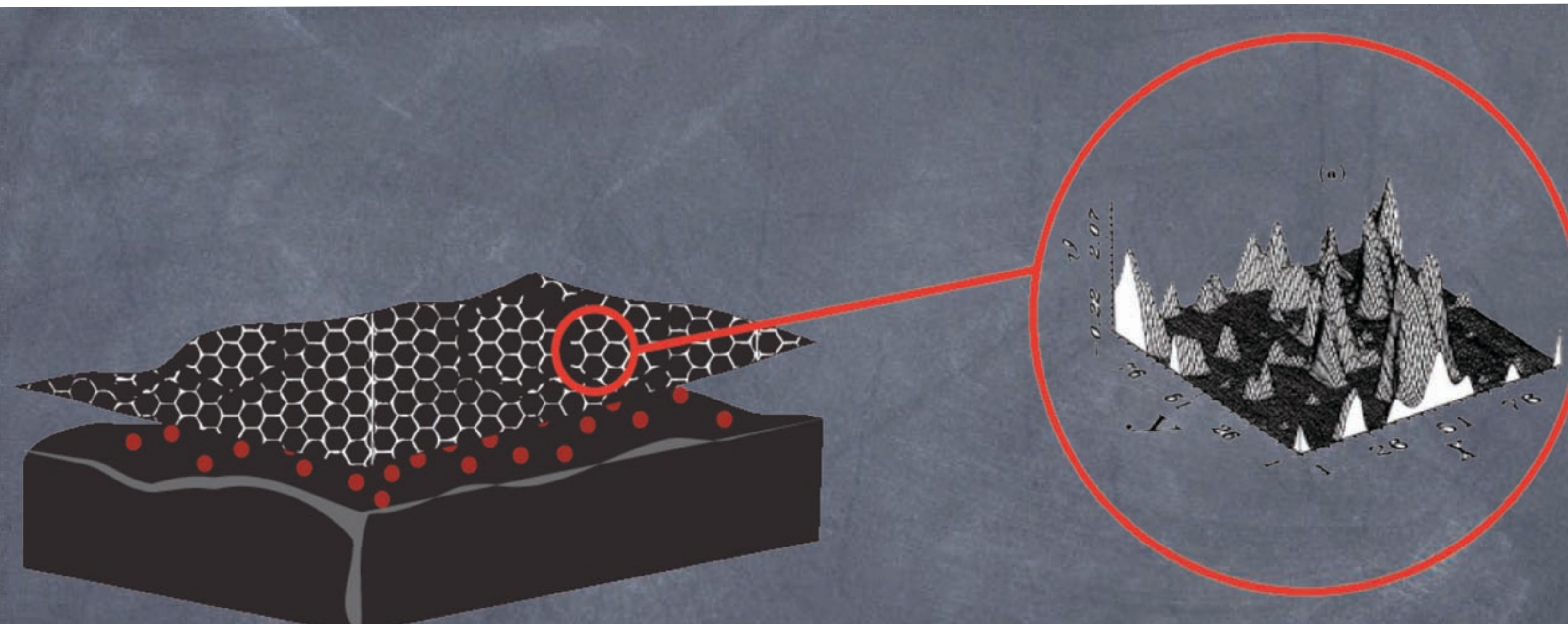
A self-consistent theory for graphene transport

Shaffique Adam



Collaborators: Sankar Das Sarma, Piet Brouwer, Euyheon Hwang, Michael Fuhrer, Enrico Rossi, Ellen Williams, Philip Kim, Victor Galitski, Masa Ishigami, Jian-Hao Chen, Sungjae Cho, and Chaun Jang.

Schematic

1. Introduction
   - Graphene transport mysteries
   - Need for a hirarchy of approximations
   - Sketch of self-consistent theory: discussion of ansatz and its predictions

2. Characterizing the Dirac Point
   - What the Dirac point really looks like
   - Comparison of self-consistent theory and energy functional minimization results

3. Quantum to classical crossover

4. Effective medium theory

5. Comparison with experiments

Introduction to graphene transport mysteries

Hole carriers Electron carriers

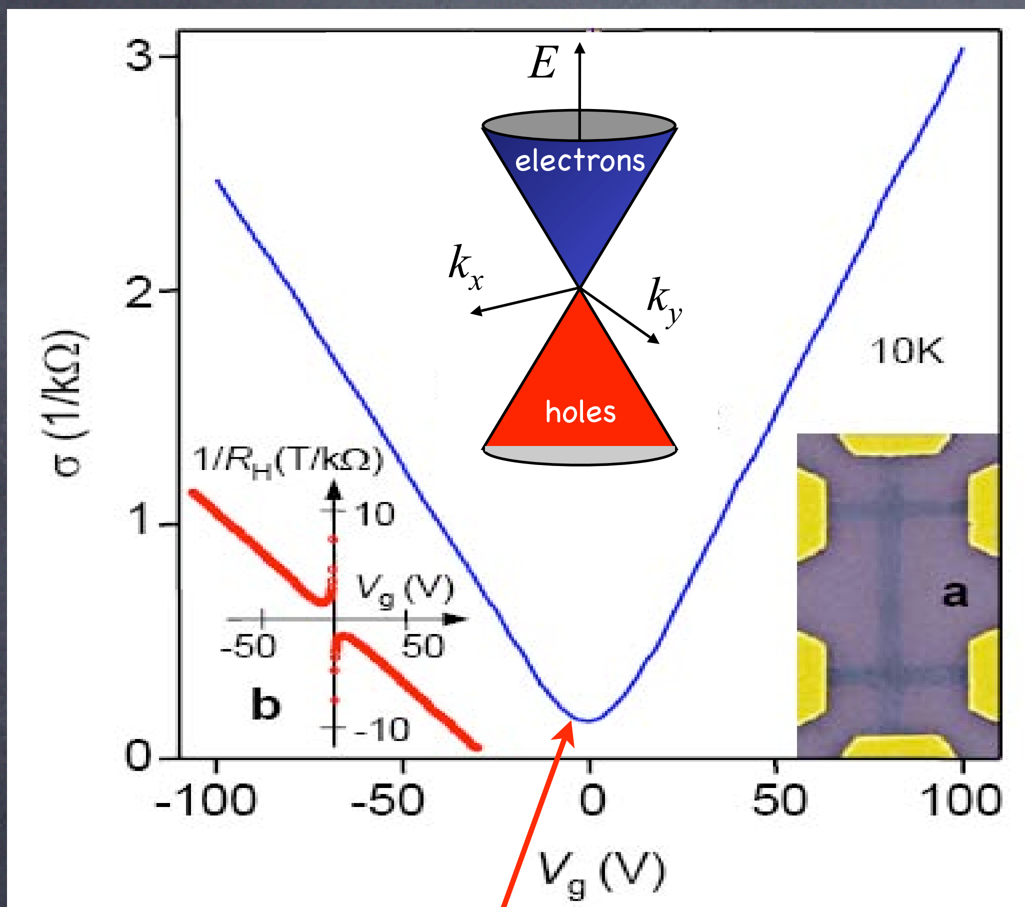
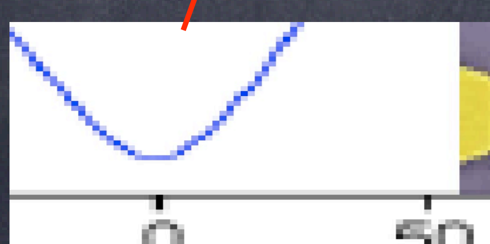
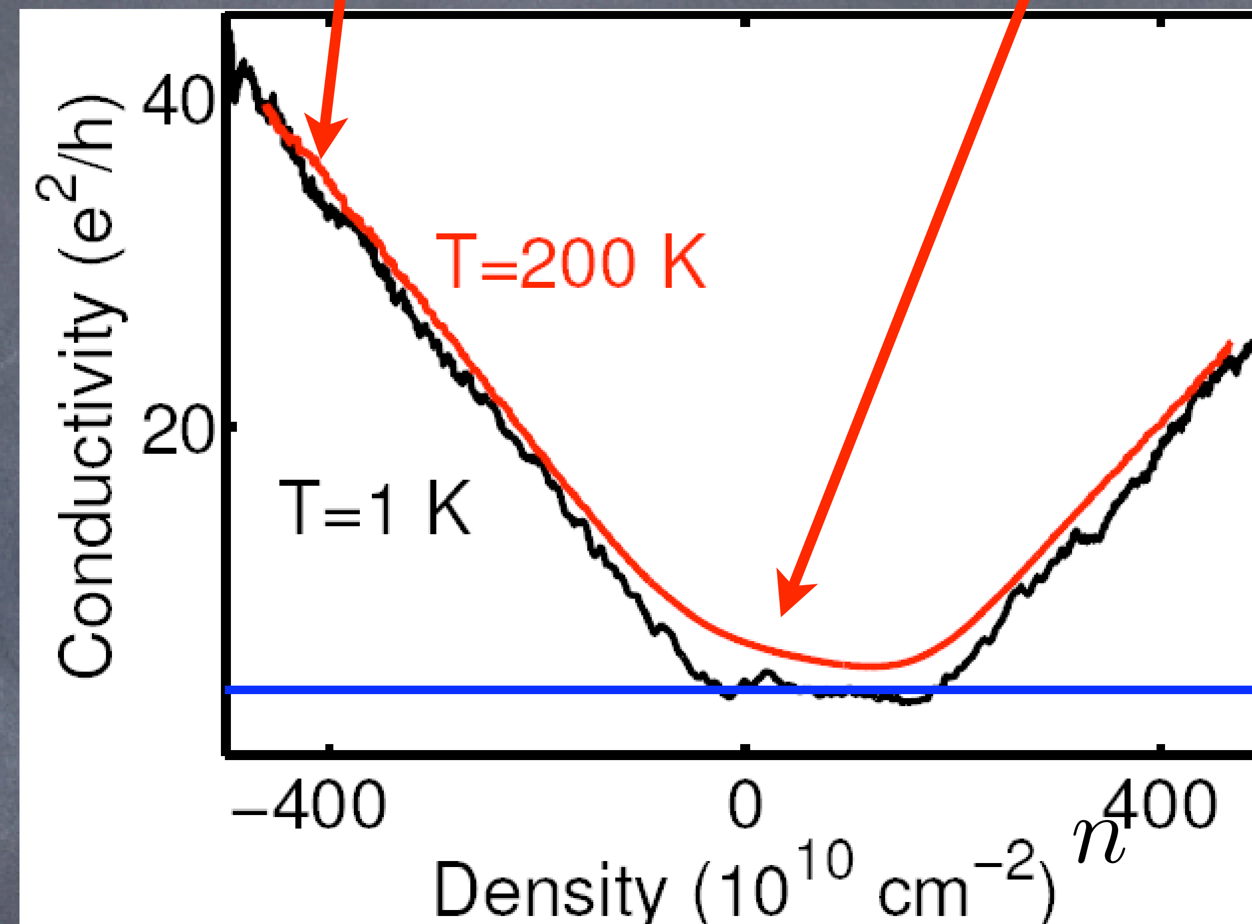


Figure from Novoselov et al. (2005)



$V_g \propto n$

High Density Low Density



n

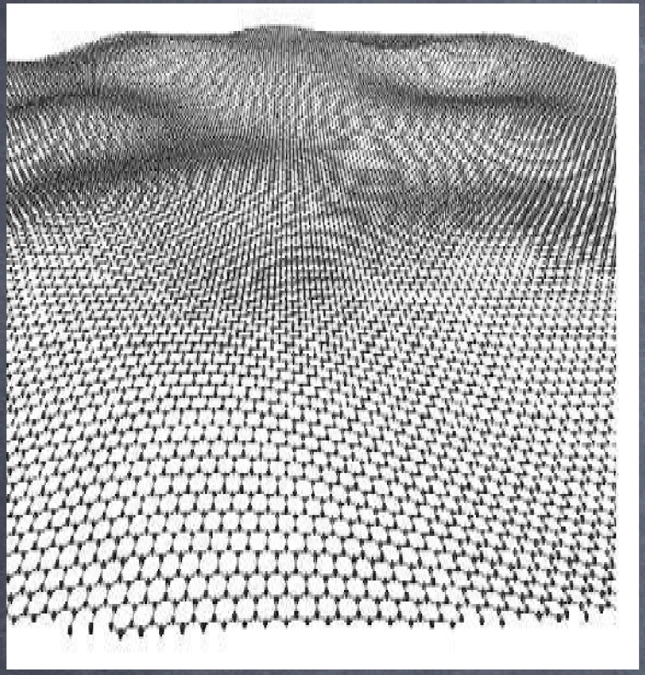
$\frac{4e^2}{h}$

Fuhrer group (unpublished) 2006

- Constant (and high) mobility over a wide range of density. Dominant scattering mechanism?
- Minimum conductivity plateau ?
- Mechanism for conductivity without carriers?

What could be going on?

Graphene



- Honeycomb lattice: Dirac cone with trigonal warping,
- Disorder: missing atoms, ripples, edges, impurities (random or correlated)
- Interactions: screening, exchange, correlation, velocity/disorder renormalization
- Phonons
- Localization: quantum interference
- Temperature
- ...

Exact solution is impossible -> reasonable hierarchy of approximations

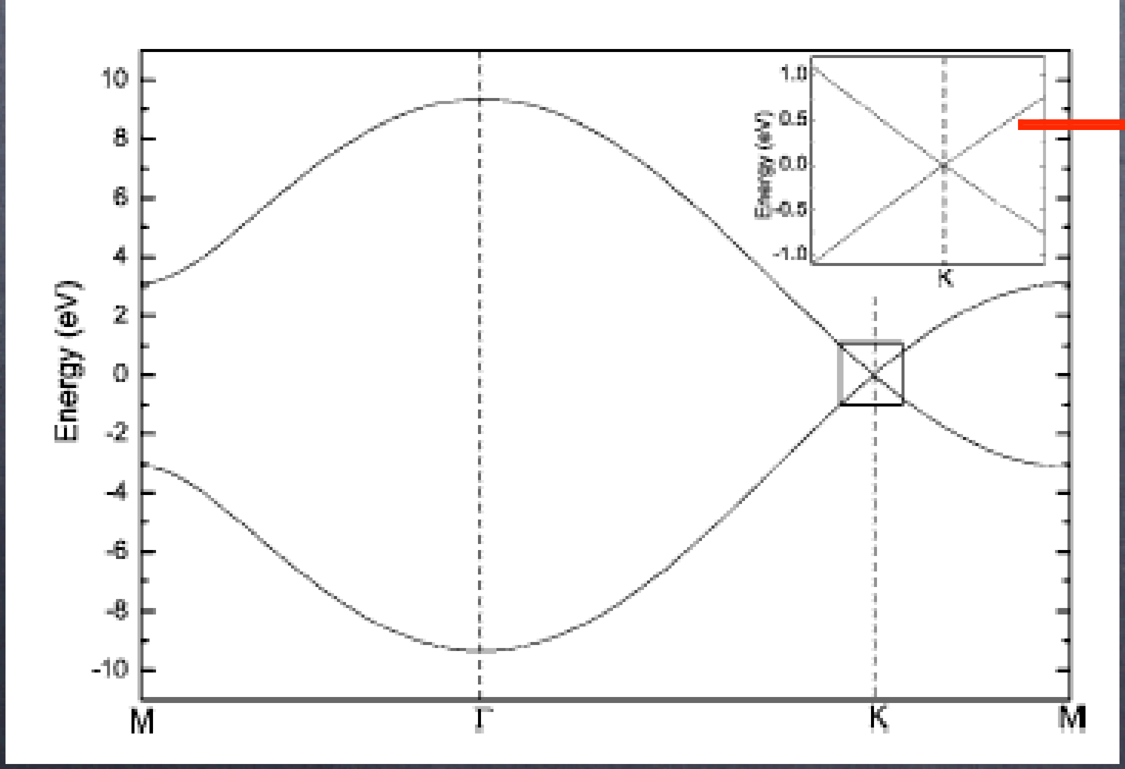
Any small parameters?

- For transport, we can use a low energy effective theory i.e. Dirac Hamiltonian. Corrections, e.g. band non-linearities set in at close to breakdown current!

$$E_w \sim \frac{\hbar v_{\mathrm{F}}}{a_0} \sim 3\ \mathrm{eV}$$

$$E_b \sim \hbar v_{\mathrm{F}} k_{\mathrm{max}} \sim \hbar v_{\mathrm{F}} \sqrt{7.2\ \times 10^{10} \mathrm{cm}^{-2}/V \times 100\ V} \sim 0.3\ eV$$

B. Partoens* and F. M. Peeters†
PHYSICAL REVIEW B 74, 075404 (2006)



$E_F = 0.3$ eV
$n \sim 10^{13}\ \mathrm{cm}^{-2}$
$V_g \sim 100$ V

Any small parameters?

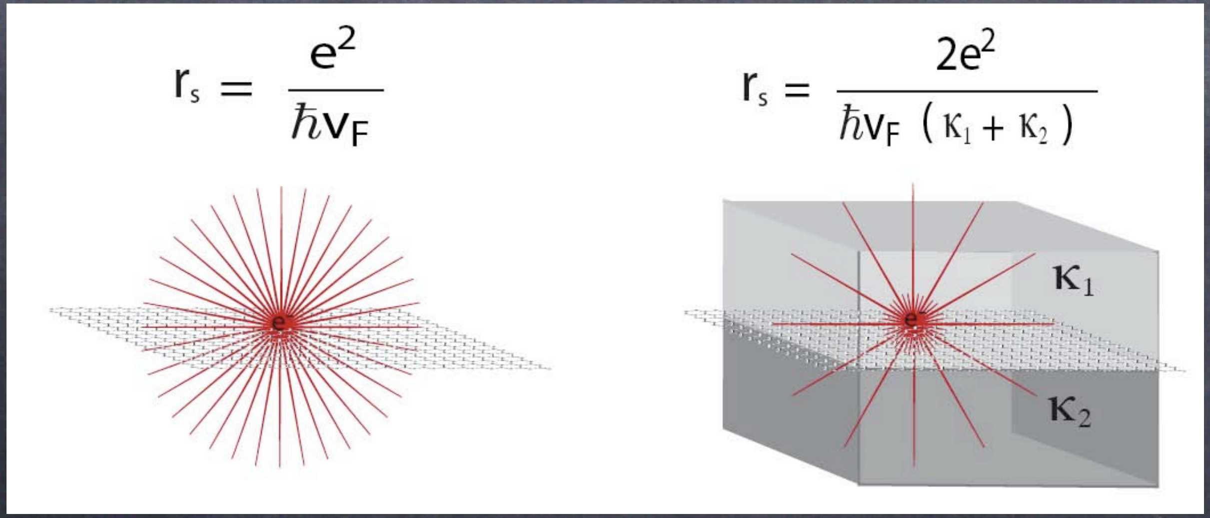
- Physical limit is weakly interacting electrons, where graphene is a Fermi Liquid

- For massless Dirac Fermions, interaction strength is given by ratio of potential to kinetic energy

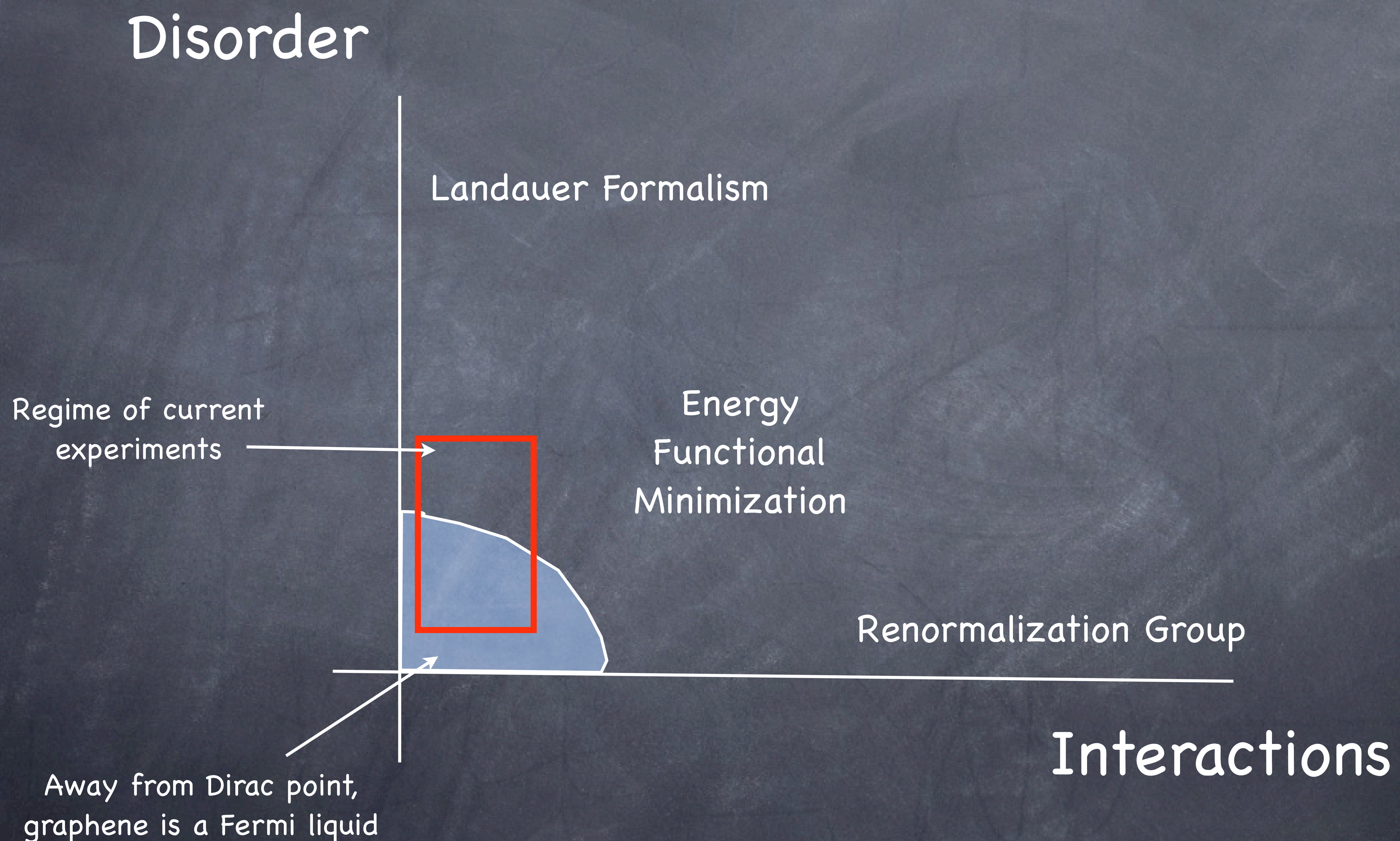
$$\alpha = \frac{P.E}{K.E} = \left[\frac{e^2 k_\mathrm{F}}{\kappa}\right] / (\hbar v_\mathrm{F} k_\mathrm{F}) \sim \frac{1}{137}\frac{300}{\kappa} \lesssim \frac{2}{\kappa}$$

$$\alpha_{SiO_2} \approx 0.8$$

- Interactions are tuned NOT by carrier density, but by dielectric environment!



Any small parameters?



Hierarchy of approximations

Honeycomb lattice ($sp_2 + p_z$) -> Tight binding gives low energy "Dirac Hamiltonian"

Interactions -> "Bubble diagrams" give RPA screening

Disorder -> "Ladder diagrams" give semi-classical Boltzmann transport

Density inhomogeneity (at Dirac point) -> "Self-consistent Approximation"

Mean field theory -> Conductivity a function of effective carrier density

Hierarchy of approximations

Honeycomb lattice (sp_2 + p_z) -> Tight binding gives low energy "Dirac Hamiltonian"

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Density inhomogeneity (at Dirac point) -> "Self-consistent Approximation"

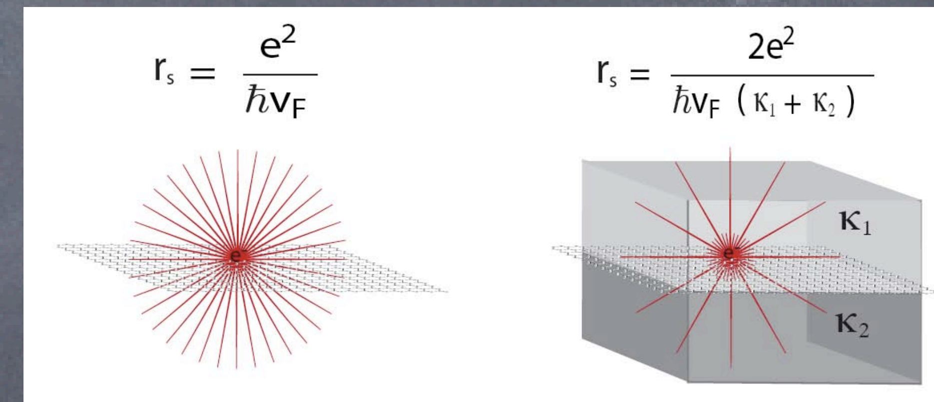
Mean field theory -> Conductivity a function of effective carrier density

A note about high density

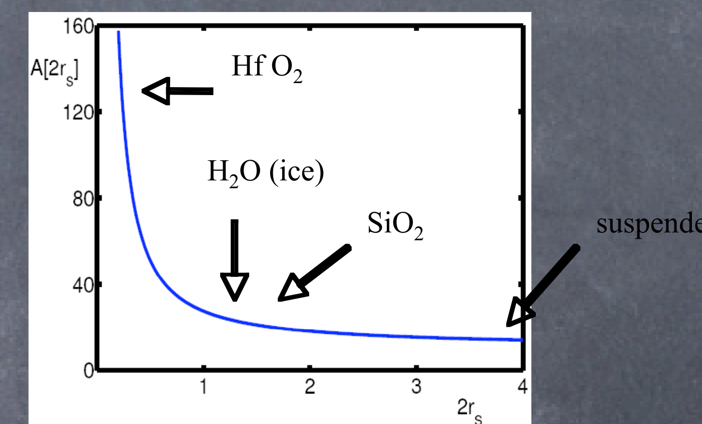
1. The linear in density conductivity is due to screened Coulomb impurities. Dielectric screening increases mobility.

$$\sigma = A[2r_{\rm s}]\frac{n}{n_{\rm imp}} = \frac{e^2}{h}\frac{2}{F_1[x]}\frac{n}{n_{\rm imp}}$$

$$\frac{F_1(x)}{x^2} = \frac{\pi}{4} + 3x - \frac{3x^2\pi}{2} + x(3x^2-2)\frac{\arccos[1/x]}{\sqrt{x^2-1}}.$$



$A[2r_s]$



2. Short-range (i.e. delta-correlated scatterers) give a density independent conductivity. Dielectric screening decreases conductivity.

$$\sigma^s = B[2r_s]\frac{1}{K_0} = \frac{4\pi e^2}{h}\frac{(\hbar v_{\rm F})^2}{n_{\rm imp}u^2}\frac{1}{F_2(x)}$$

$$F_2(x) = \frac{\pi}{2} - \frac{16x}{3} + 40x^3 + 6\pi x^2 - 20\pi x^4 + 8x^2(5x^3-4x)\frac{\arccos[1/x]}{\sqrt{x^2-1}}.$$

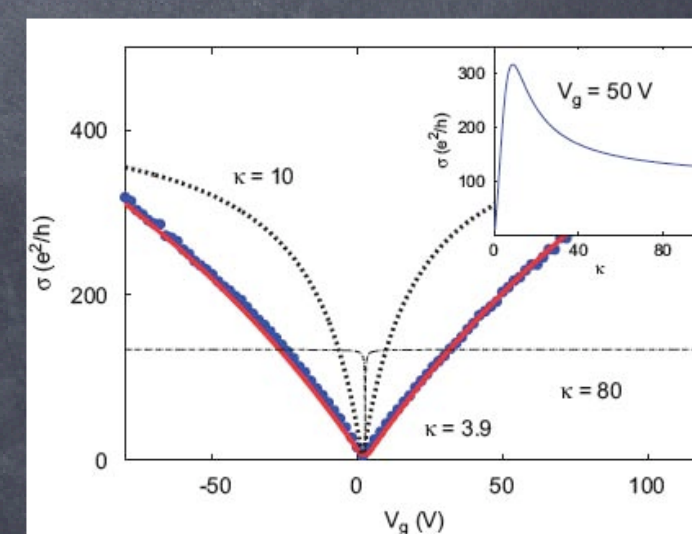


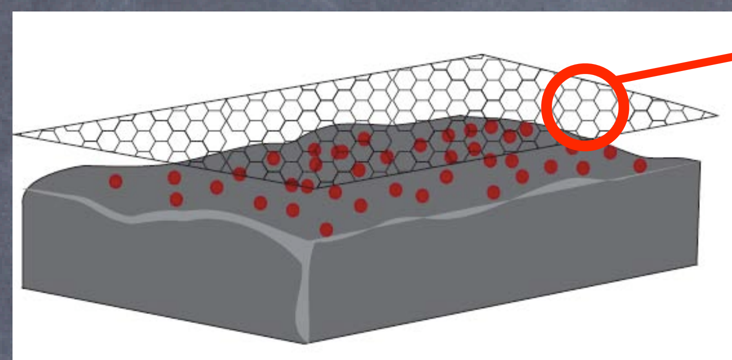
Figure from Adam, Hwang and Das Sarma, Physica E (2008)

Low Density: self-consistent ansatz

A self-consistent theory for graphene transport

Shaffique Adam†, E. H. Hwang, V. M. Galitski, and S. Das Sarma

PNAS | November 20, 2007 | vol. 104 | no. 47 | 18393



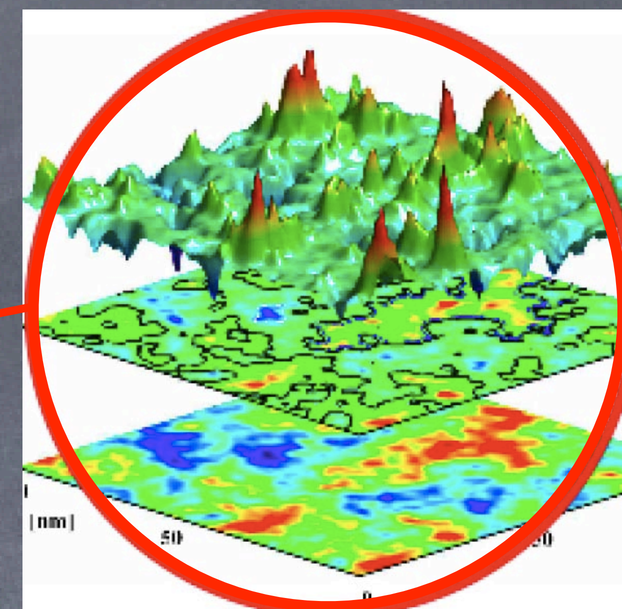


Figure from Rossi, Adam and Das Sarma (2008)

System breaks up into electron and hole puddles -- conductivity is given by the Drude-Boltzmann conductivity of a homogenous system with an effective carrier density n*, where n* is calculated using a self-consistent "Fermi-Thomas" condition: $E_F^2 = \langle V_D^2 \rangle$. Here V_D is the RPA-screened disorder potential of charged impurities and $\langle \cdots \rangle$ denotes disorder averaging

Assumptions underlying the self-consistent ansatz

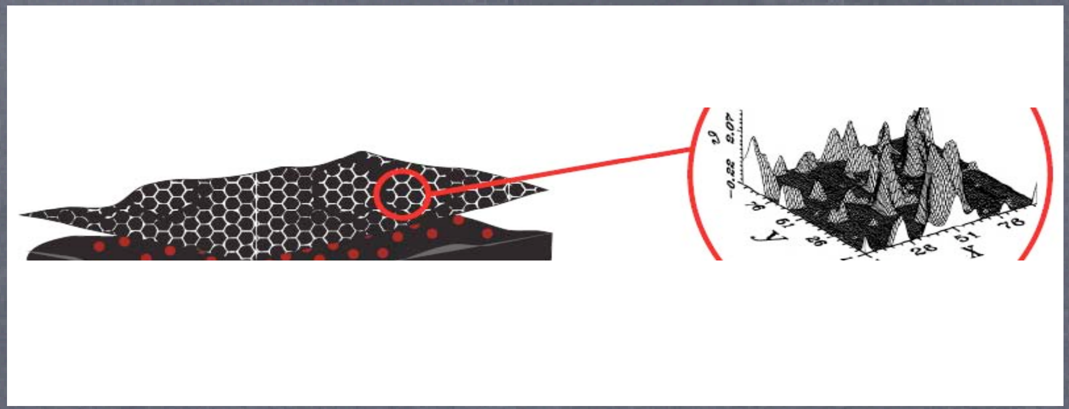
Q1. Does the self-consistent procedure give meaningful results for the "statistical properties" of the inhomogeneous system? i.e. How accurate is n^*, ξ ?

Q2. When can we map this highly inhomogeneous electron/hole puddle system into a homogeneous medium?

Shaffique Adam[1], Piet W. Brouwer[2], and S. Das Sarma

Q3. What is the conductivity of this effective medium in terms of the "statistical properties" of the inhomogeneous system? i.e Is it really that $\sigma_{\min} = \sigma[n^*]$?

Why do we need a self-consistent theory?



Imagine increasing charged impurity density n_{imp}. This increases the potential fluctuations which increases the induced carrier density. But an increased carrier density screens more effectively which decreases the potential fluctuations and decreases the induced density.

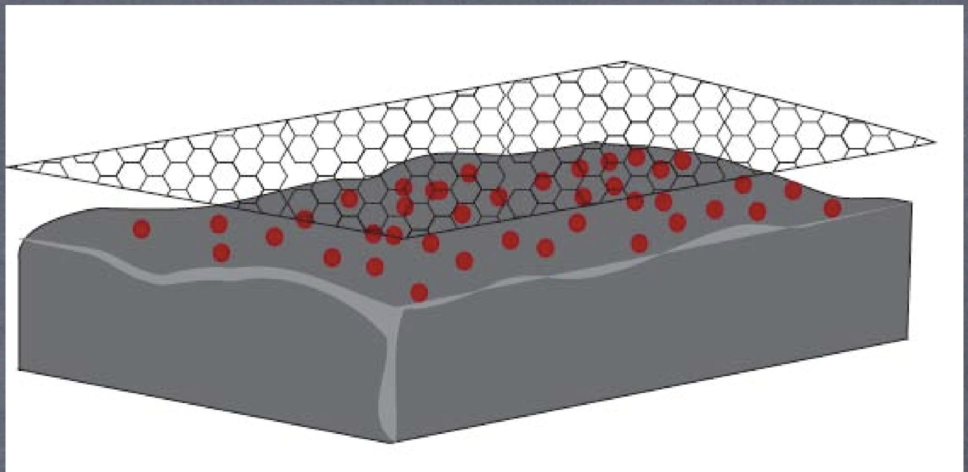
Self-consistent approximation [1]

Calculating n*, the effective carrier density

$$E_{\mathrm{F}}^2 = \langle V_D^2 \rangle$$

g(n) ⟷ n* ⟷ f(n)

$$\pi \hbar^2 v_{\mathrm{F}}^2 n$$



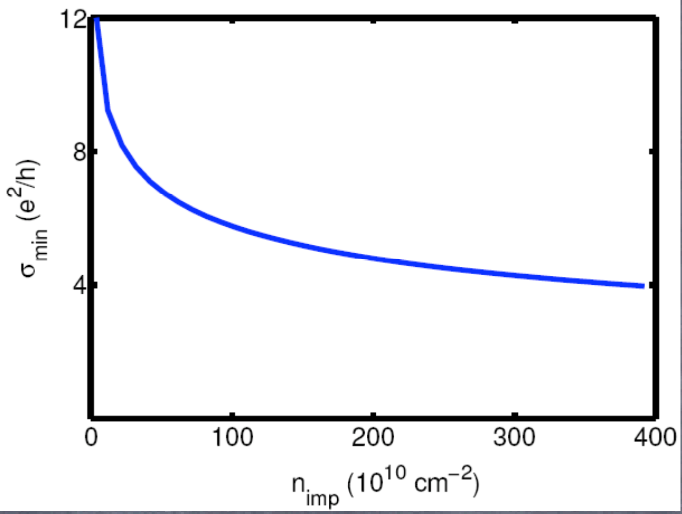
Computing $\langle V_D^2 \rangle$: statistics problem of averaging over uncorrelated disorder

Predictions of the theory [1]

I. As one increases disorder, minimum conductivity is determined by the competition between the increased puddle carrier density (that increases conductivity) and increased scattering (that decreases conductivity).

$$\sigma_{\min}^{\mathrm{SCA}} = \frac{2e^2}{h}\,\frac{n^*}{n_{\mathrm{imp}}}\,\frac{1}{F_1(2r_s)}$$

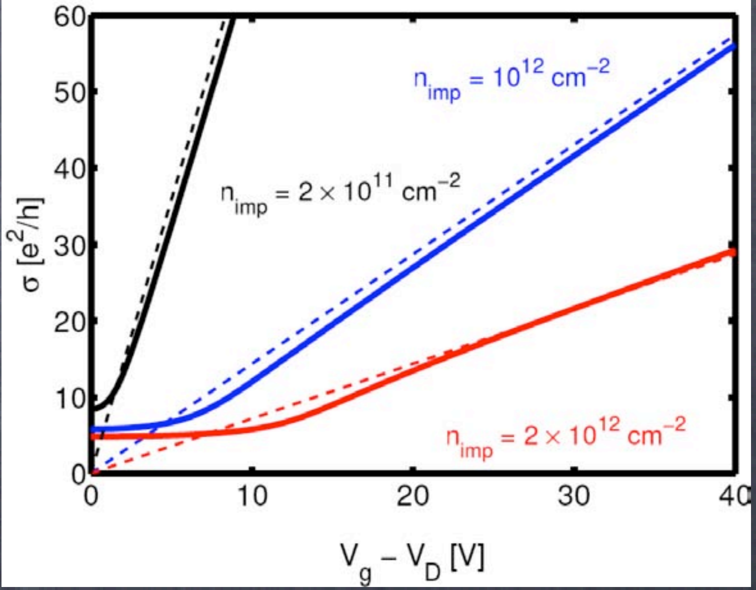
$$\frac{n^*}{n_{\mathrm{imp}}} = 2r_s^2 C_0^{\mathrm{RPA}}(r_s,\, a = 4d\sqrt{\pi n^*})$$



$$C_0^{\mathrm{RPA}}(r_s, a) = -1 + \frac{4E_1(a)}{(2+\pi r_s)^2} + \frac{2e^{-a r_s}}{1+2r_s} + (1+2r_s a)e^{2r_s a}(E_1[2r_s a] - E_1[a(1+2r_s)]),$$

II. Minimum conductivity is a plateau (not a point): the plateau width is determined by the residual density n*.

$$\frac{n^*}{n_{\mathrm{imp}}} = 2r_s^2 C_0^{\mathrm{RPA}}(r_s,\, a = 4d\sqrt{\pi n^*})$$



III. Dirac point offset is determined by (disorder averaged) first moment of the screened Coulomb potential.

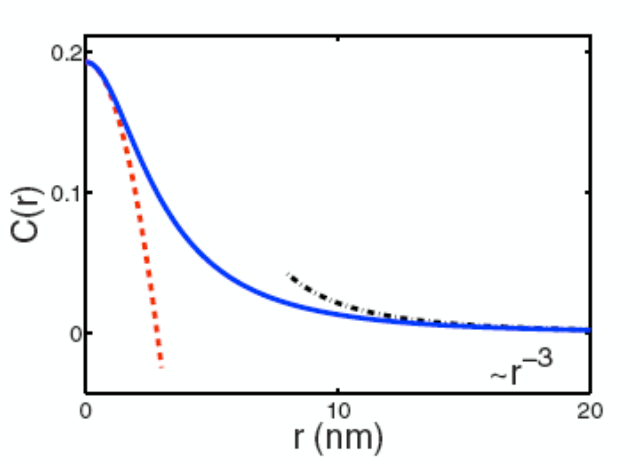
$$\bar{n} = \frac{n_{\mathrm{imp}}^2}{4n^*},$$

Predictions of the theory [2]

IV a. Screened Coulomb potential gives disorder potential correlation function with a power law tail.

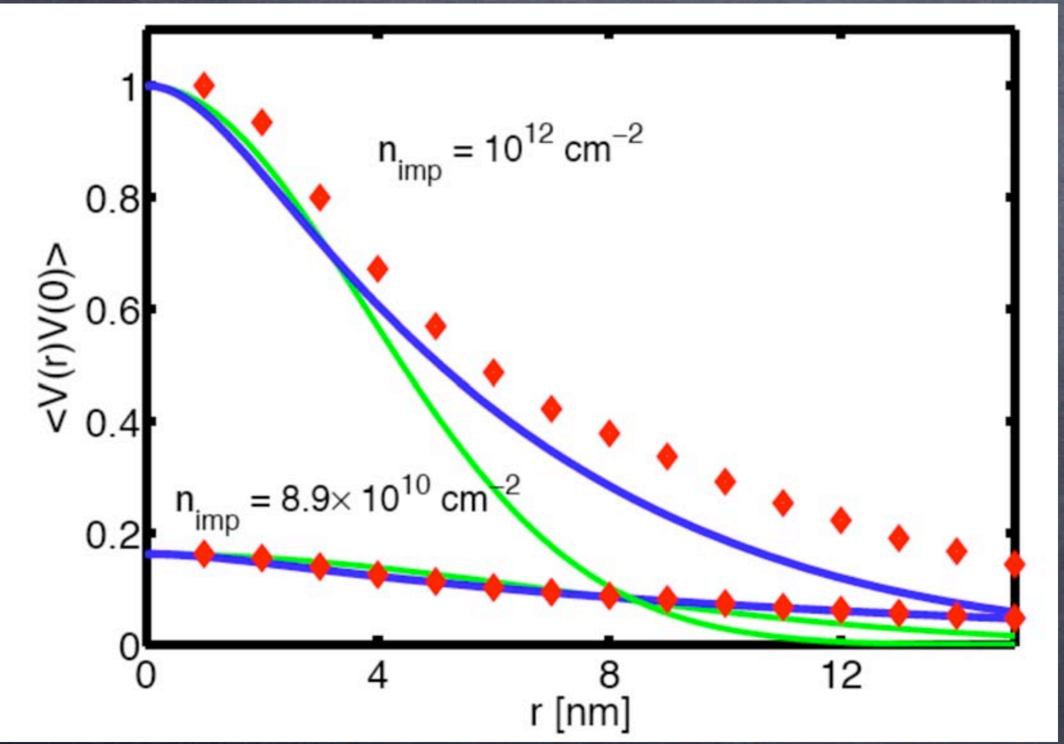
$$C(r) = C_0(2q_s d) + \sum_{m>0} (-1)^m (q_s r)^{2m} \frac{(2m-1)!!}{2^m m!} C_m(2q_s d),$$

$$C_m(x) = [1/(2m)!](\partial_x^{2m+1})[x e^x E_1(x)].$$



$$C(r) \sim 2|1 - q_s d|(q_s r)^{-3}.$$

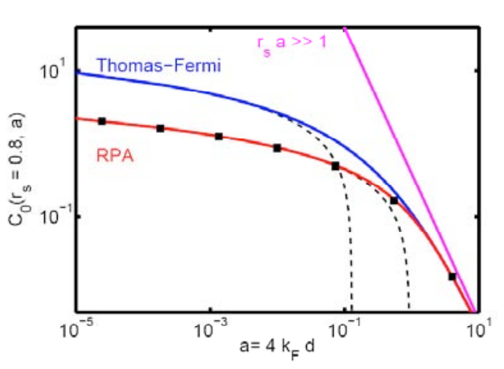
IV b. A Gaussian approximation for the correlation function captures many of the salient features.



$$\langle V(r)V(0)\rangle \approx \frac{K_0 \gamma^2}{2\pi\xi^2} \exp\left[\frac{-r^2}{2\xi^2}\right].$$

$$K_0 = \frac{1}{4r_s^2}\left(\frac{D_0}{C_0}\right)^2,$$

$$\xi = \frac{1}{\sqrt{n_{\mathrm{imp}}}} \frac{D_0}{4\pi r_s^2} \frac{1}{(C_0)^{3/2}},$$



$C_0(d^2 n^*, r_s)$ and $D_0(d^2 n^*, r_s)$ are analytic functions reported in Adam et al. PRL (2008)

Schematic

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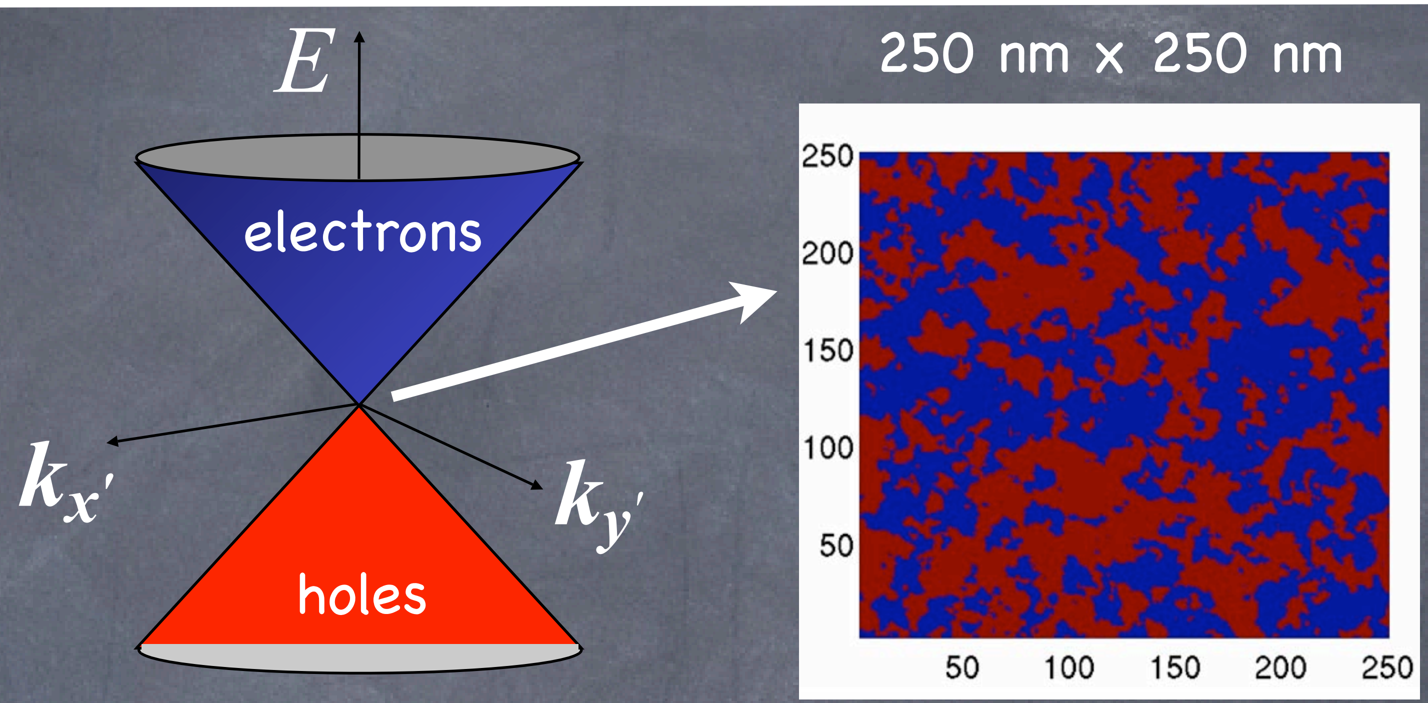
Characterizing the Dirac Point

How do you characterize the Dirac point in the presence of disorder?

Dirac cone
(momentum space)



Real space inhomogeneity -> puddles of electrons and holes

- Screened potential correlation $\langle v(r)v(0)\rangle$

  e.g. Full width at half maximum of $\langle v(r)v(0)\rangle$ is related to the correlation length ξ

- Distribution function P[n] (histogram of carrier density)

  e.g. width of P[n] gives root mean square carrier density: n_{rms}

Statistical properties of Dirac Point

Q1. Does the self-consistent procedure give meaningful results for the "statistical properties" of the inhomogeneous system?

- Does the self-consistency capture the right physics?
- What about many body effects: exchange, correlation?

Recall that in self-consistent procedure:

$$n^* = \frac{1}{\pi(\hbar v_{\mathrm{F}})^2} \langle V(0)V(0) \rangle$$

Once n* is determined, the full distribution $P[V(r_1)\, V(r_2)\, \ldots\, V(r_n)]$ can be computed. In particular, characterizing all higher moments or correlation functions becomes a matter of quadrature e.g.

$$\begin{aligned} \langle V(r)V(0)\rangle &= n_{\mathrm{imp}} \int d\mathbf{q} [\phi(q,n^*)]^2 e^{i\mathbf{q}\cdot\mathbf{r}} \\ &\approx \frac{n_{\mathrm{imp}}(\hbar v_{\mathrm{F}})^2 K_0[r_s, d\sqrt{n^*}]}{2\pi(\xi[r_s, d\sqrt{n^*})^2} \exp\left[\frac{-n_{\mathrm{imp}} r^2}{2(\xi[r_s, d\sqrt{n^*}])^2}\right] \end{aligned}$$

$$n_{\mathrm{rms}} = \sqrt{\langle V^4\rangle}/[\pi(\hbar v_{\mathrm{F}})^2]$$

$$n_{\mathrm{rms}} \approx n^* \sqrt{3 + (n_{\mathrm{imp}}\xi^2)^{-1}}$$

(From our perspective, potential correlation is a "second moment" of screened disorder potential, whereas n_{rms} is a "fourth moment")

PRL **101,** 166803 (2008) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2008

Ground State of Graphene in the Presence of Random Charged Impurities

Enrico Rossi and S. Das Sarma

PHYSICAL REVIEW B **78**, 115426 (2008)

Density functional theory of graphene sheets

Marco Polini,[1,*] Andrea Tomadin,[1] Reza Asgari,[2] and A. H. MacDonald[3]

Local Density Approximation or "Poor man's DFT" for graphene

$$E[n] = E_{kin}[n(\mathbf{r})] + E_H[n(\mathbf{r})] + E_{exch}[n(\mathbf{r})] - \int_A \mathbf{V_D} n(\mathbf{r}) d^2r - \mu \int_A n(\mathbf{r}) d^2r \qquad \frac{\delta E}{\delta n} = 0$$

Then average over 500-1000 ensembles

How does the ground state obtained by "Energy Functional Minimization" compare with the self-consistent Ansatz?

What does the Dirac point really look like?

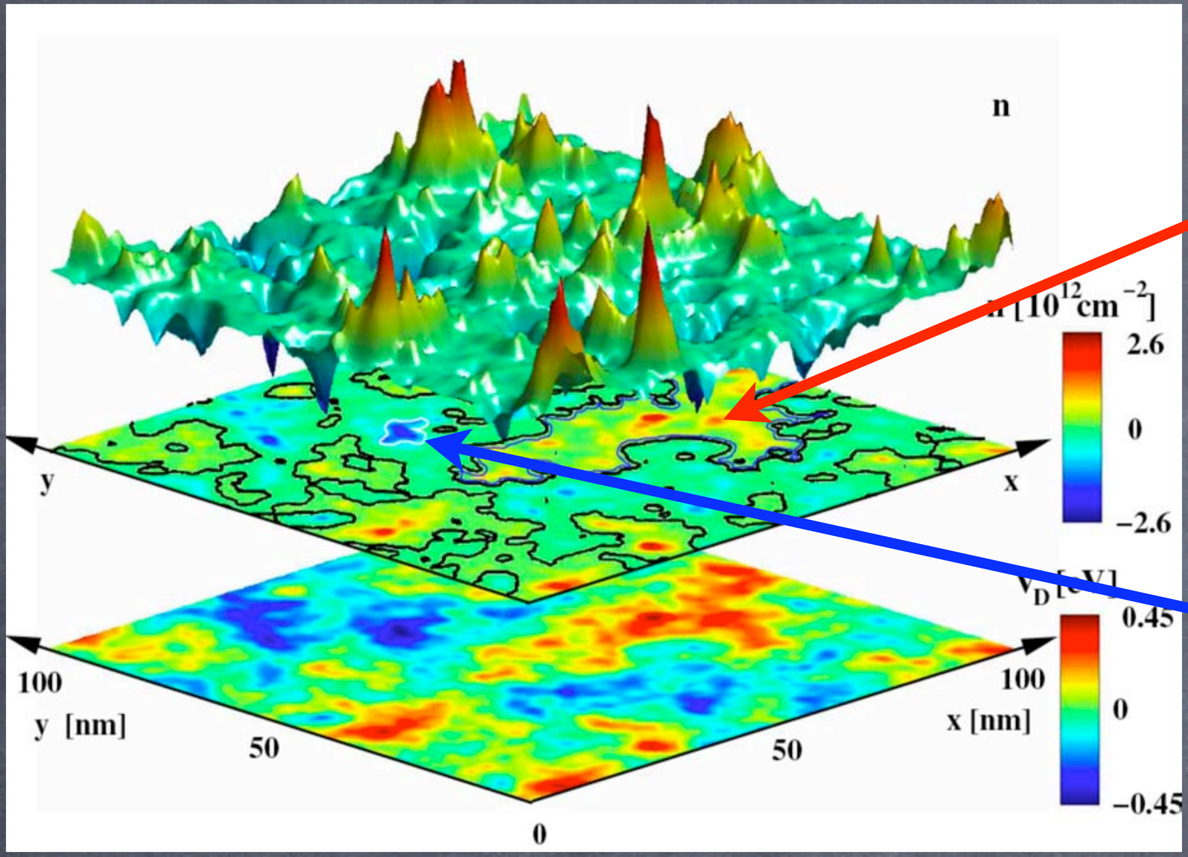


Figure from Rossi, Adam and Das Sarma (2008)

Large puddles

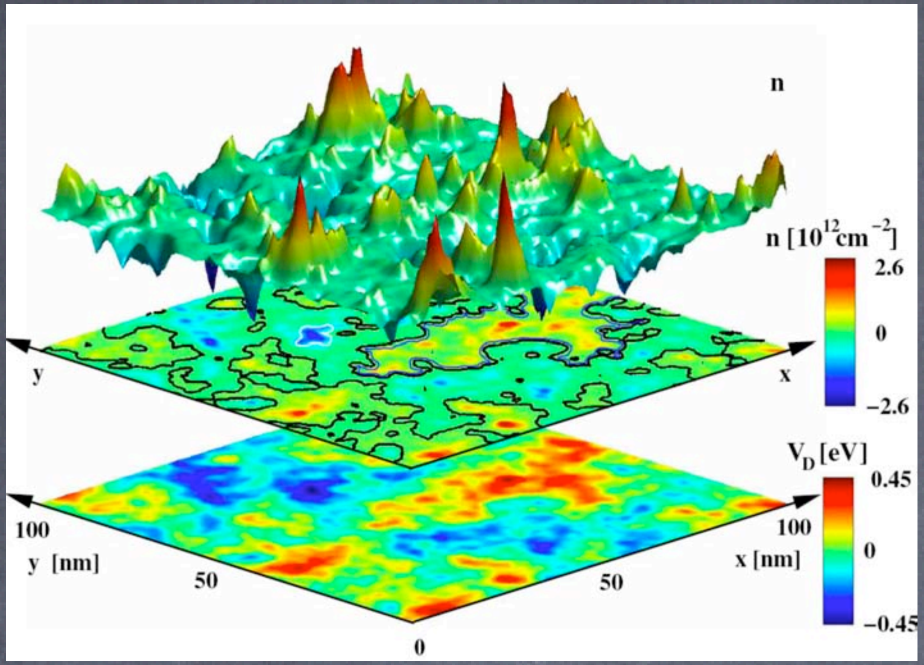
$$N_e \sim n_{\mathrm{rms}} L^2 \sim 500$$

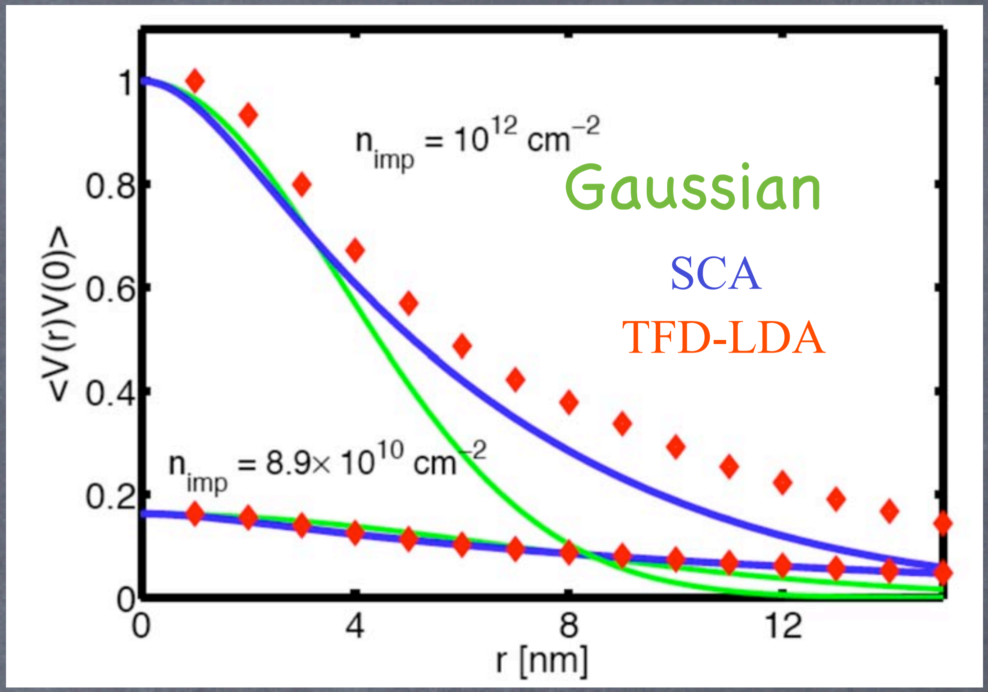
Small puddles

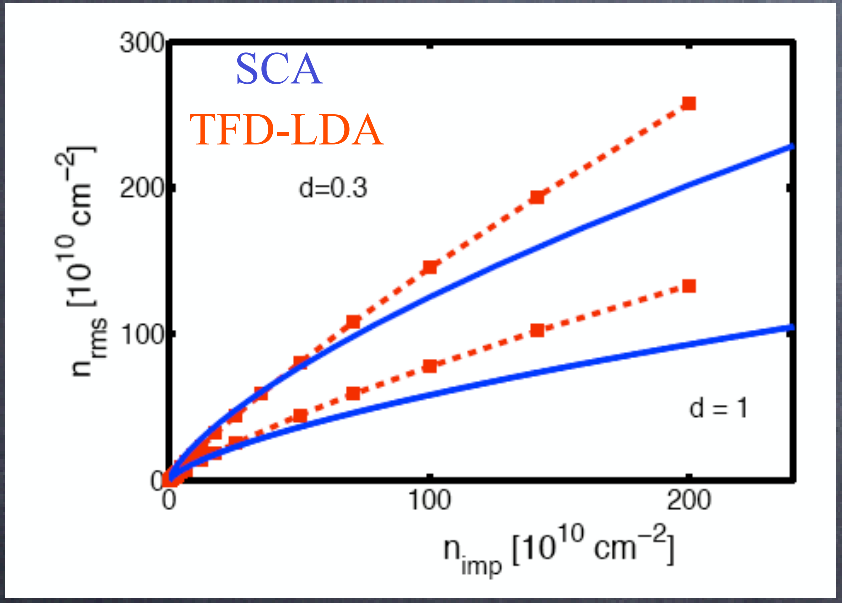
$$N_e \sim n_{\mathrm{max}} \xi^2 \sim 2$$

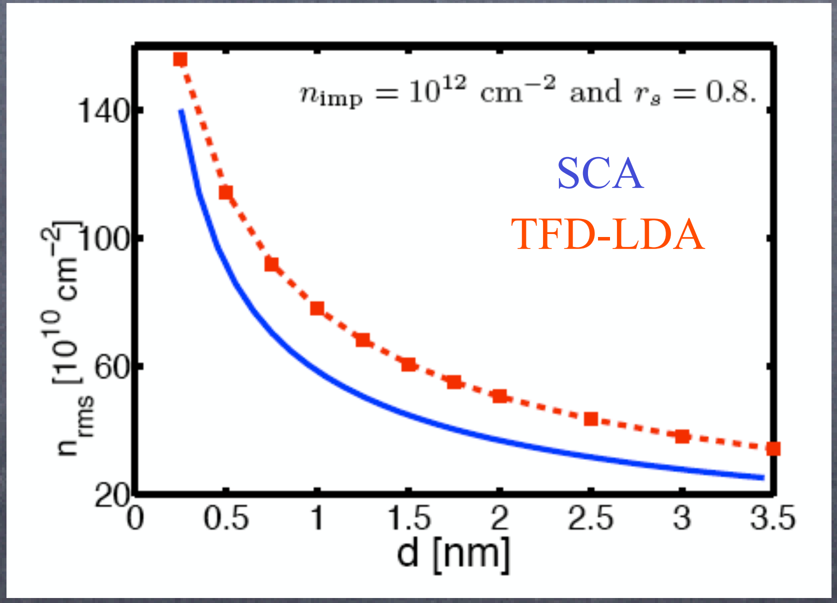
How does this compare with predictions of the self-consistent theory?

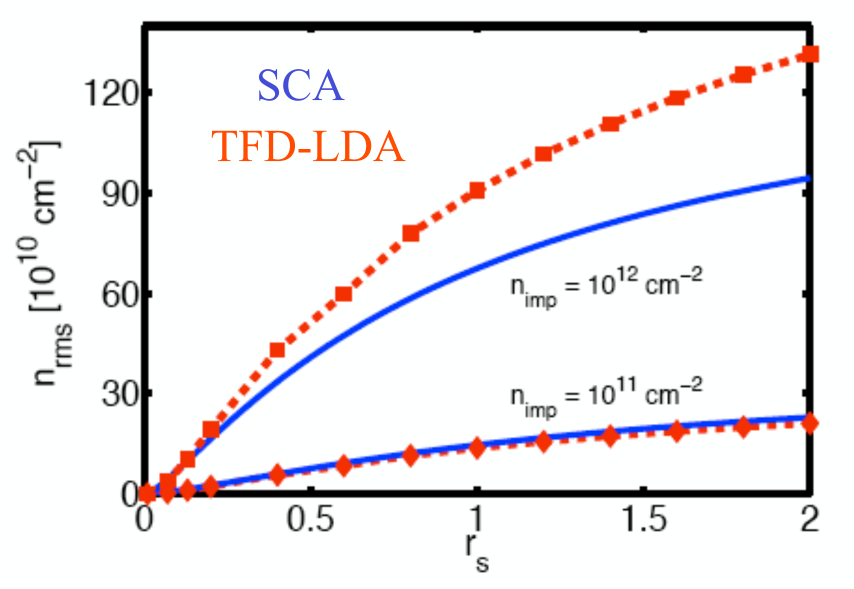
Comparison with self-consistent theory









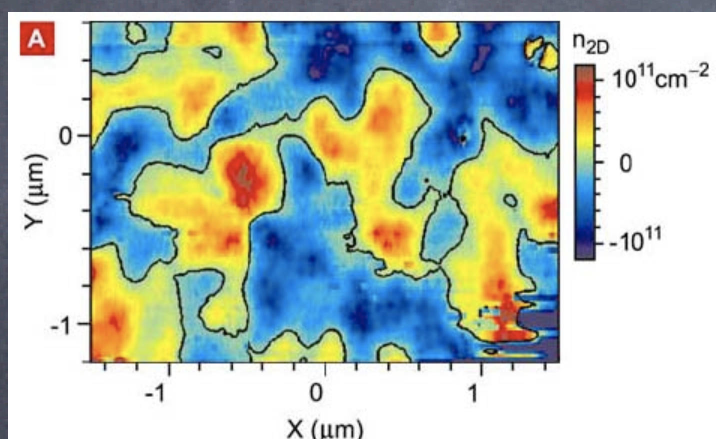


Two approaches agree over a wide range of parameters

Self-consistent procedure captures most of the physics

Results: Characterizing the inhomogeneity

Our theory was verified quantitatively by recent experiments...



Observation of electron–hole puddles in graphene using a scanning single-electron transistor

J. MARTIN[1], N. AKERMAN[1], G. ULBRICHT[2], T. LOHMANN[2], J. H. SMET[2], K. VON KLITZING[2] AND A. YACOBY[1,3]*

nature physics | VOL 4 | FEBRUARY 2008

arXiv:0705.2180

Origin of Spatial Charge Inhomogeneity in Graphene

Yuanbo Zhang[1*§], Victor W. Brar[1,2*], Caglar Girit[1,2], Alex Zettl[1,2], Michael F. Crommie[1,2§]

arXiv:0902.4793

we report a new technique of Dirac point mapping that we have used to determine the origin of charge inhomogeneities in graphene. We find that fluctuations in graphene charge density are not caused by topographical corrugations, but rather by charge-donating impurities below the graphene. These impurities induce

Similar results from Arizona and Riverside groups.

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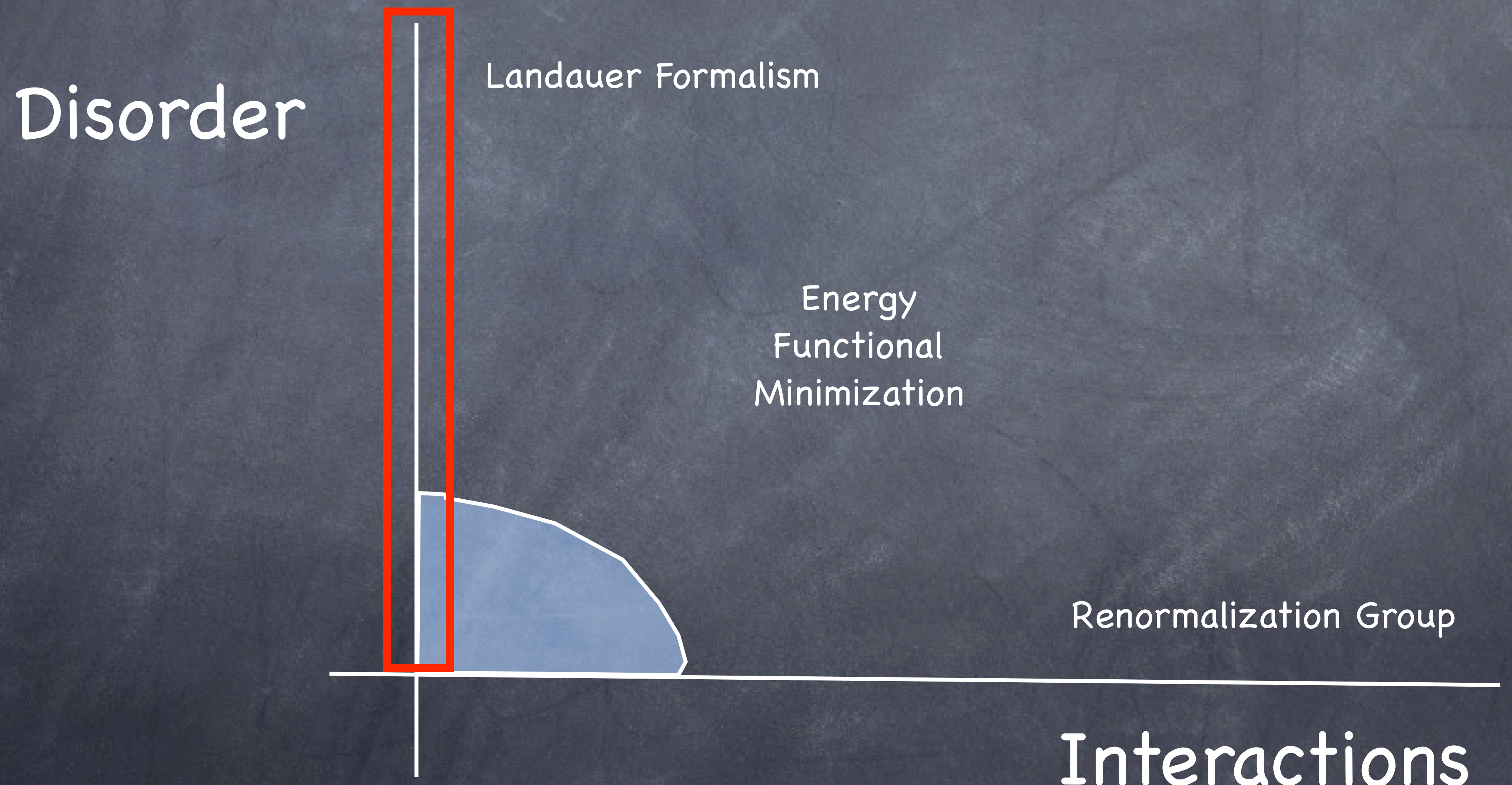
Q2. When can we map this highly inhomogeneous electron/hole puddle system into a homogeneous medium?

- Discuss the case of non-interacting electrons by solving the fully quantum mechanical problem.



Fully quantum solution [1]

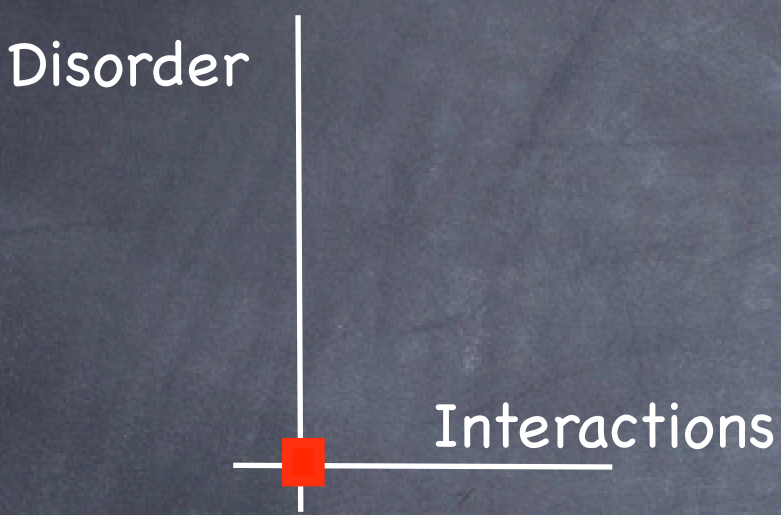
First lets look at the clean limit (no disorder): a problem in quantum mechanics

Zitterbewegung, chirality, and minimal conductivity in graphene

M.I. Katsnelson[a]

Eur. Phys. J. B **51**, 157–160 (2006)
DOI: 10.1140/epjb/e2006-00203-1

THE EUROPEAN PHYSICAL JOURNAL B

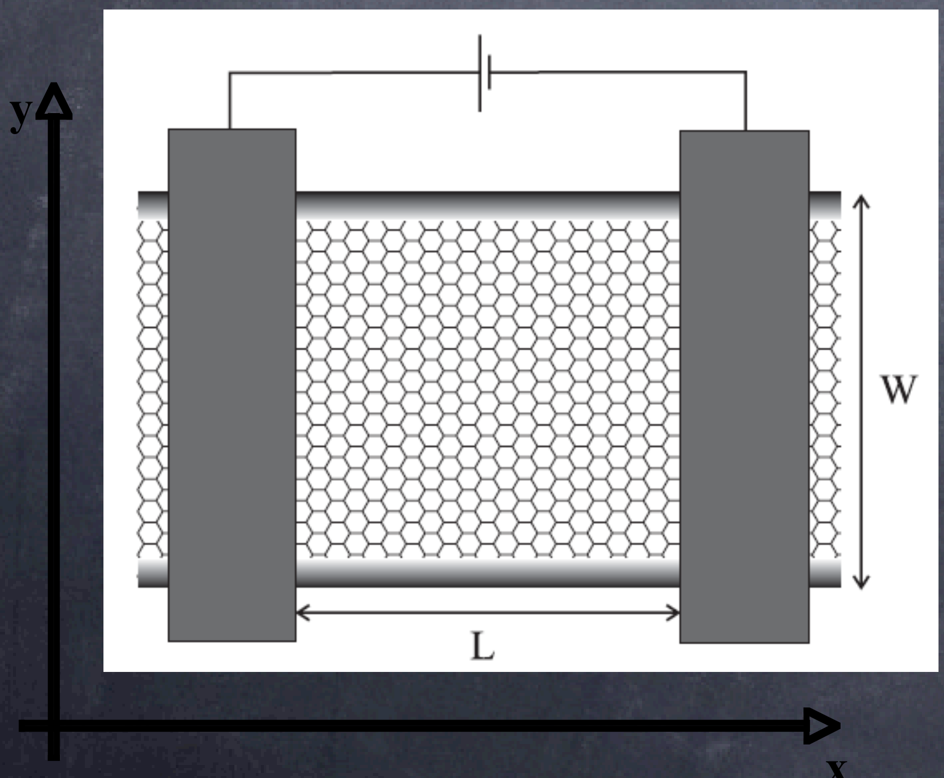


PRL **96**, 246802 (2006) PHYSICAL REVIEW LETTERS week ending 23 JUNE 2006

Sub-Poissonian Shot Noise in Graphene

J. Tworzydło,[1] B. Trauzettel,[2] M. Titov,[3] A. Rycerz,[2,4] and C. W. J. Beenakker[2]

$$T_n = \frac{1}{\cosh^2 Lq_n + (q_n/k_\infty)^2 \sinh^2 Lq_n} \rightarrow \frac{1}{\cosh^2[\pi(n+1/2)L/W]}$$



$$G = \frac{4e^2}{h} \sum_{n=0}^{N-1} T_n \rightarrow \frac{4e^2}{\pi h} \frac{W}{L} \text{ for } W \gg L$$

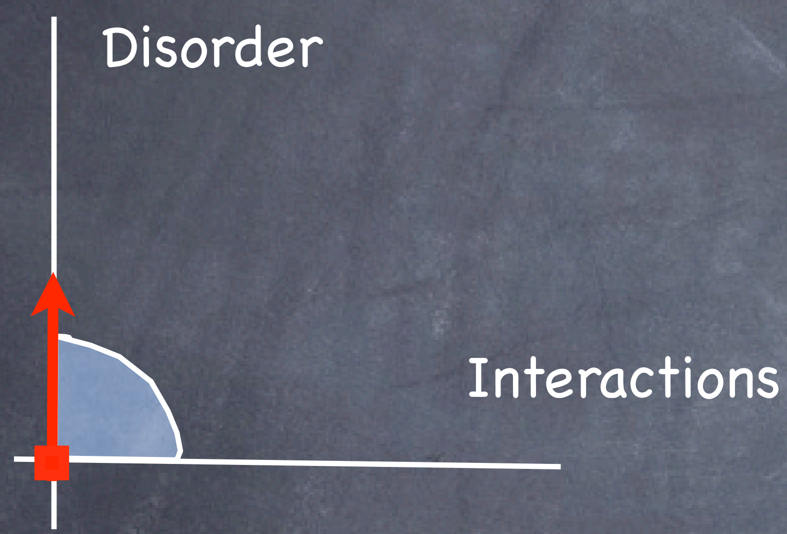
$$\sigma \equiv G \times L/W$$

$$\sigma_{\min} = \frac{4e^2}{\pi h}$$

Same result as "static" Kubo formalism T-> 0

Fully quantum solution [2]

- Consider non-interacting model but fully quantum mechanical
- Electron interference gives anti-localization (symplectic symmetry)



$$H = -i\hbar v_{\mathrm{F}} \left[\sigma_x \partial_x + \sigma_y \partial_y\right]$$

$$H = \sigma_y H^* \sigma_y$$

VOLUME 89, NUMBER 26 PHYSICAL REVIEW LETTERS 23 DECEMBER 2002

Crossover from Symplectic to Orthogonal Class in a Two-Dimensional Honeycomb Lattice

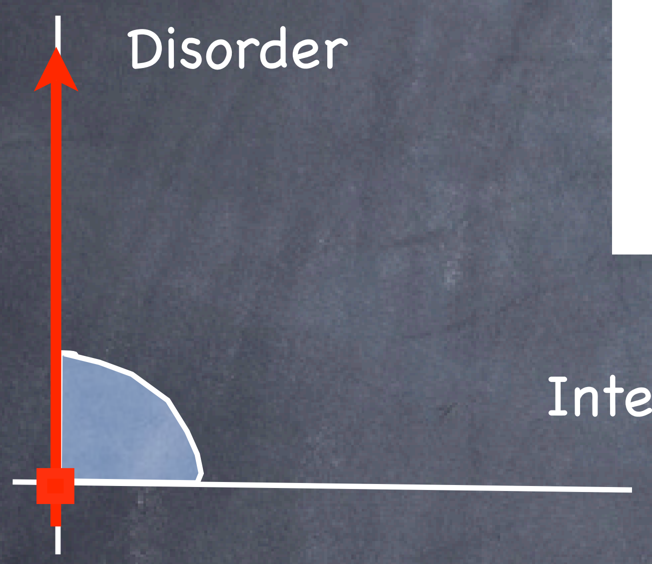
Hidekatsu Suzuura* and Tsuneya Ando*

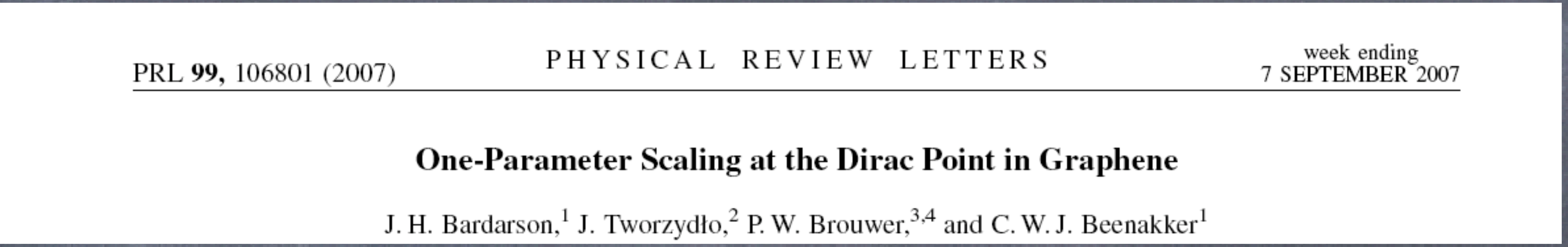
$$\Gamma_{k_\alpha k_\beta}(q) = \frac{nu^2}{2S} e^{i(\varphi_{k_\alpha} - \varphi_{k_\beta})} \frac{1}{(v_F \tau q)^2},$$

Due to quantum interference, disorder increases the conductivity

Fully quantum solution [3]

- So long as disorder is smooth, quantum interference does not localize graphene electrons even for strong disorder. No Anderson localization in graphene!

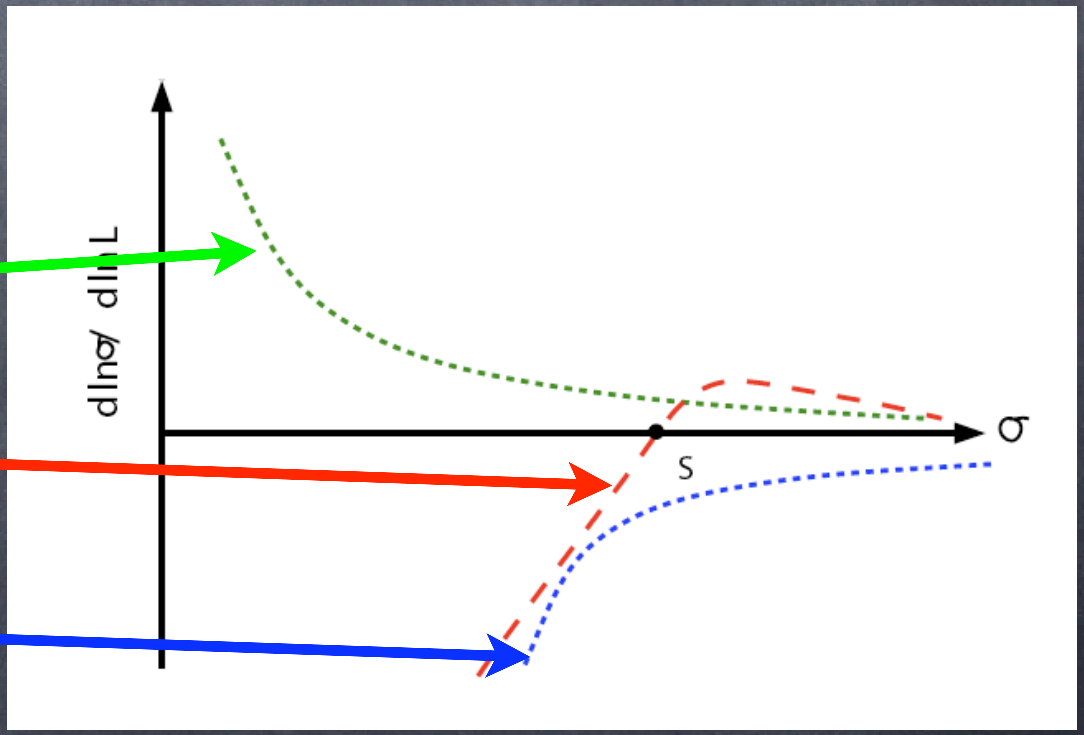



PRL **99**, 106801 (2007) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2007

One-Parameter Scaling at the Dirac Point in Graphene

J. H. Bardarson,[1] J. Tworzydło,[2] P. W. Brouwer,[3,4] and C. W. J. Beenakker[1]

Also: Nomura, Koshino, Ryu PRL (2007)
San Jose et al. PRB (2007)



Graphene

Spin-orbit
(Weak anti-localization)

Orthogonal
(Weak localization)

Ballistic to diffusive crossover [1]

$$\sigma = \left[W \frac{dR}{dL} \right]^{-1}, \quad R = 1/G.$$

$\pi n \xi^2 = 0,\ 0.25,\ \text{and}\ 1$

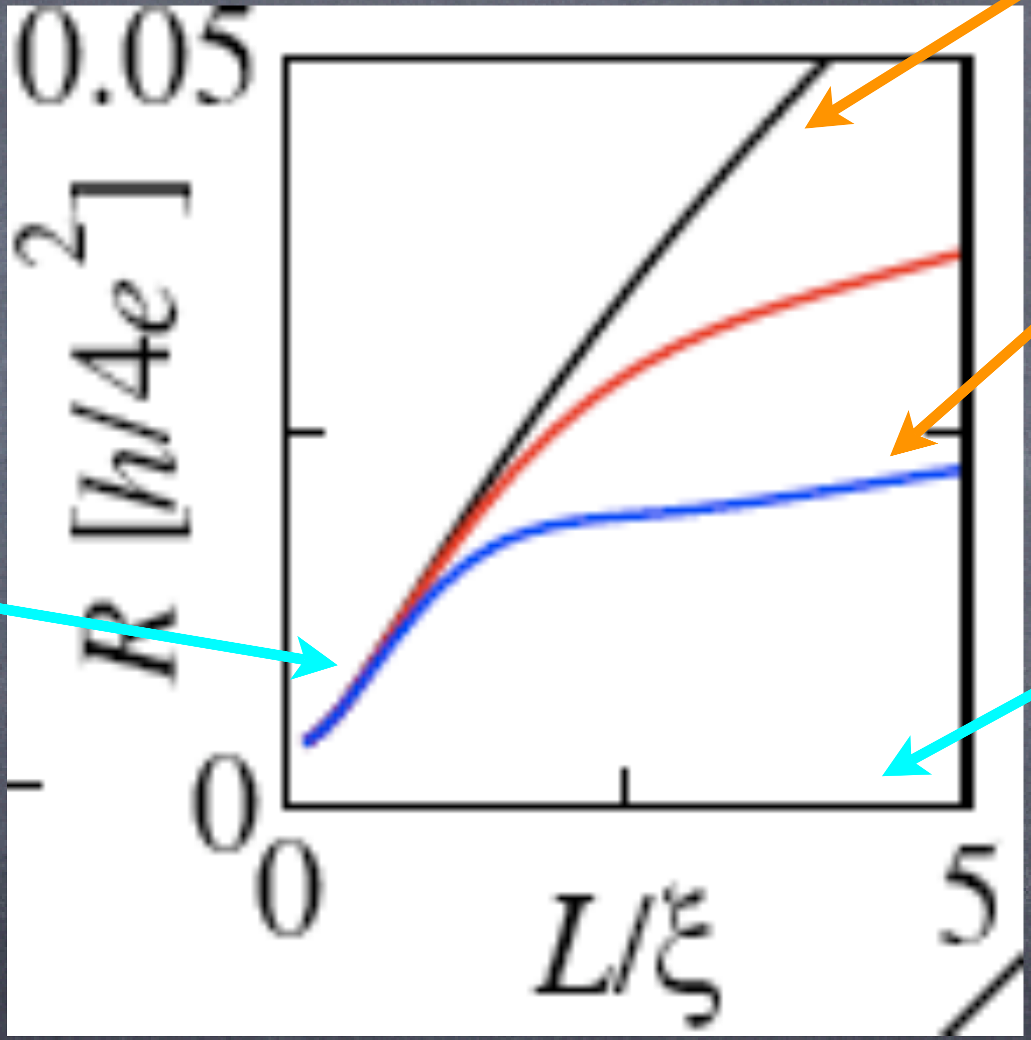
Dirac point

Away from Dirac point

Ballistic regime, close to universal value

$$\frac{4e^2}{\pi h}$$

$$L \lesssim \xi$$



Diffusive regime, subject of our study

$$L \gg \xi$$

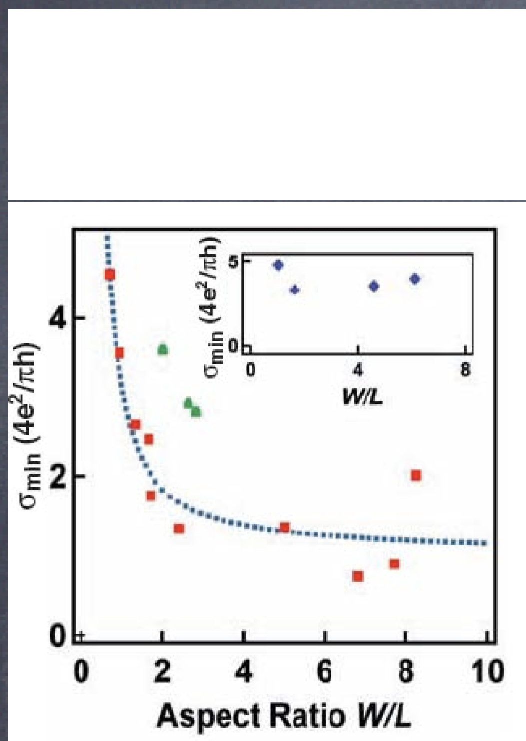
Figure from Adam, Brouwer and Das Sarma, arXiv 0811.0609

L System size

ξ Disorder correlation length

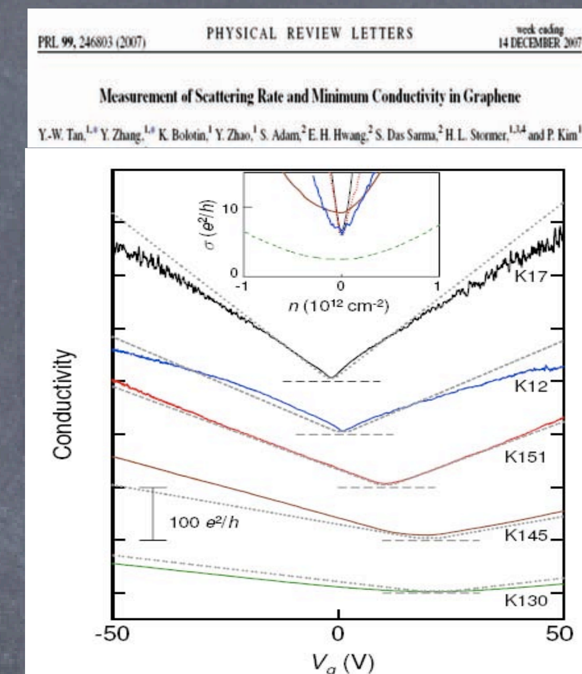
Ballistic to diffusive crossover [2]

Ballistic transport



- Universal
- Landauer formalism

Diffusive transport



- Non-Universal
- Boltzmann formalism

See also: Lewenkopf, Mucciolo and Castro Neto PRB (2008)

A note about Weak Anti-localization

- Boltzmann and quantum theory give opposite predictions. WAL implies cleaner samples will have lower conductivity, Boltzmann predicts higher conductivity!

- How do we deal with a finite system size?

(b) Take $L \longrightarrow \infty$ limit and then extrapolate back to origin.

$$\sigma' = \lim_{L \to \infty}[\sigma(L) - \pi^{-1}\ln(L/\xi)]$$

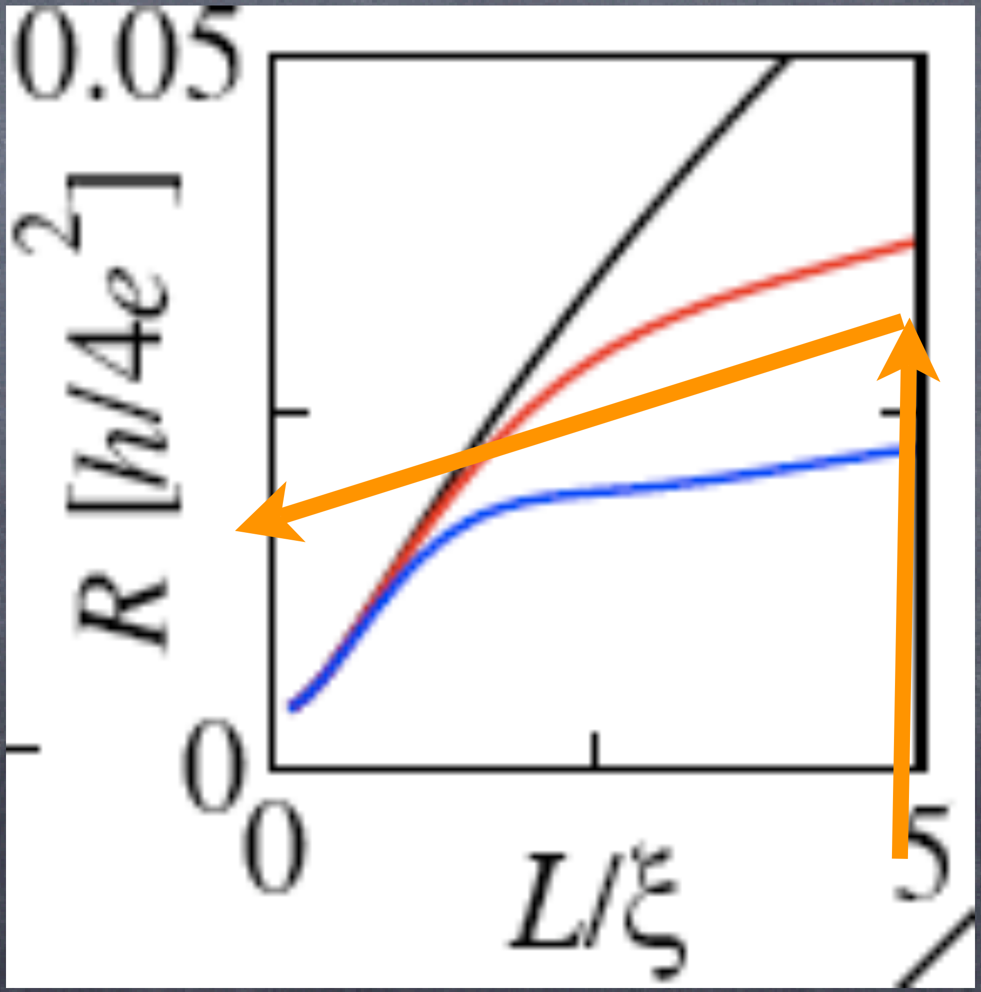


Figure from Adam, Brouwer and Das Sarma, arXiv 0811.0609

(a) Pick some large value such that we are always in the diffusive regime e.g. $L/\xi = 50$

$$\sigma(L = 50\xi)$$

Quantum to semi-classical crossover [1]

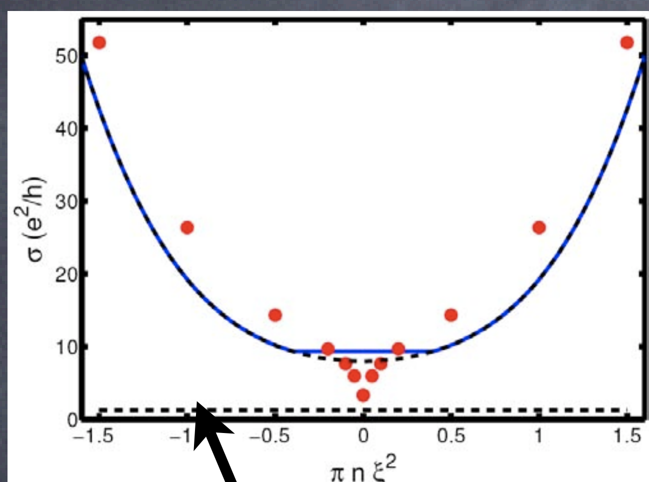
Away from Dirac point, transport is classical

Comparison of Landauer (data points) and Boltzmann (solid lines) at high density for different values of impurity concentration

Quantum to semi-classical crossover [2]

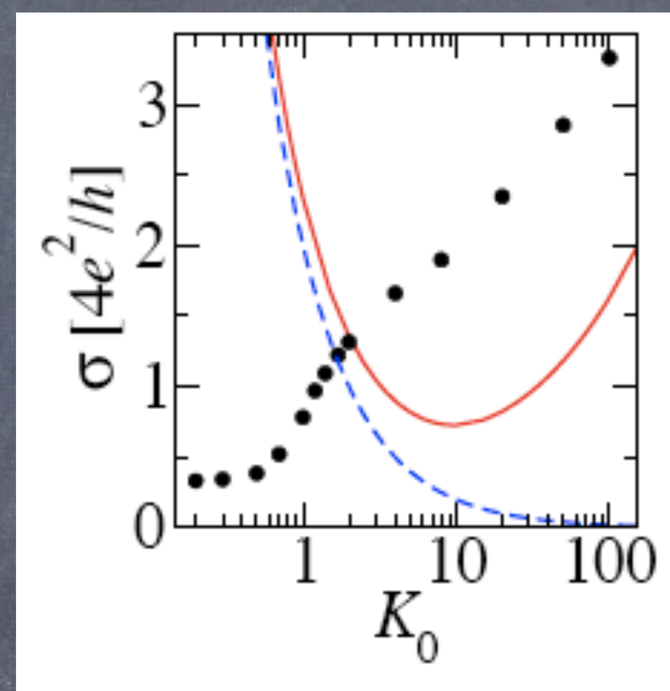
At the Dirac point, transport is "quantum" for low impurity concentration, and consistent with "self-consistent" theory at large impurity concentration

Data points: Landauer

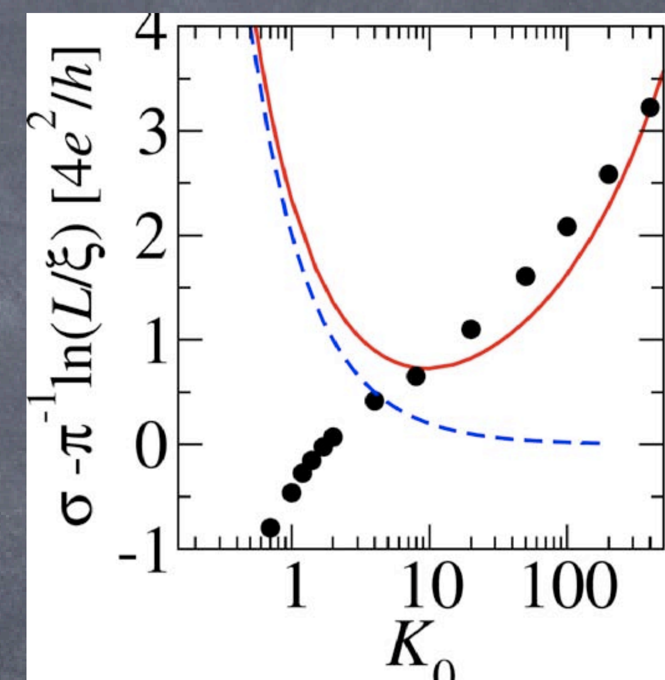


Ballistic universal value

$\sigma(L = 50\xi)$



$\sigma' = \lim_{L\to\infty}[\sigma(L) - \pi^{-1}\ln(L/\xi)]$



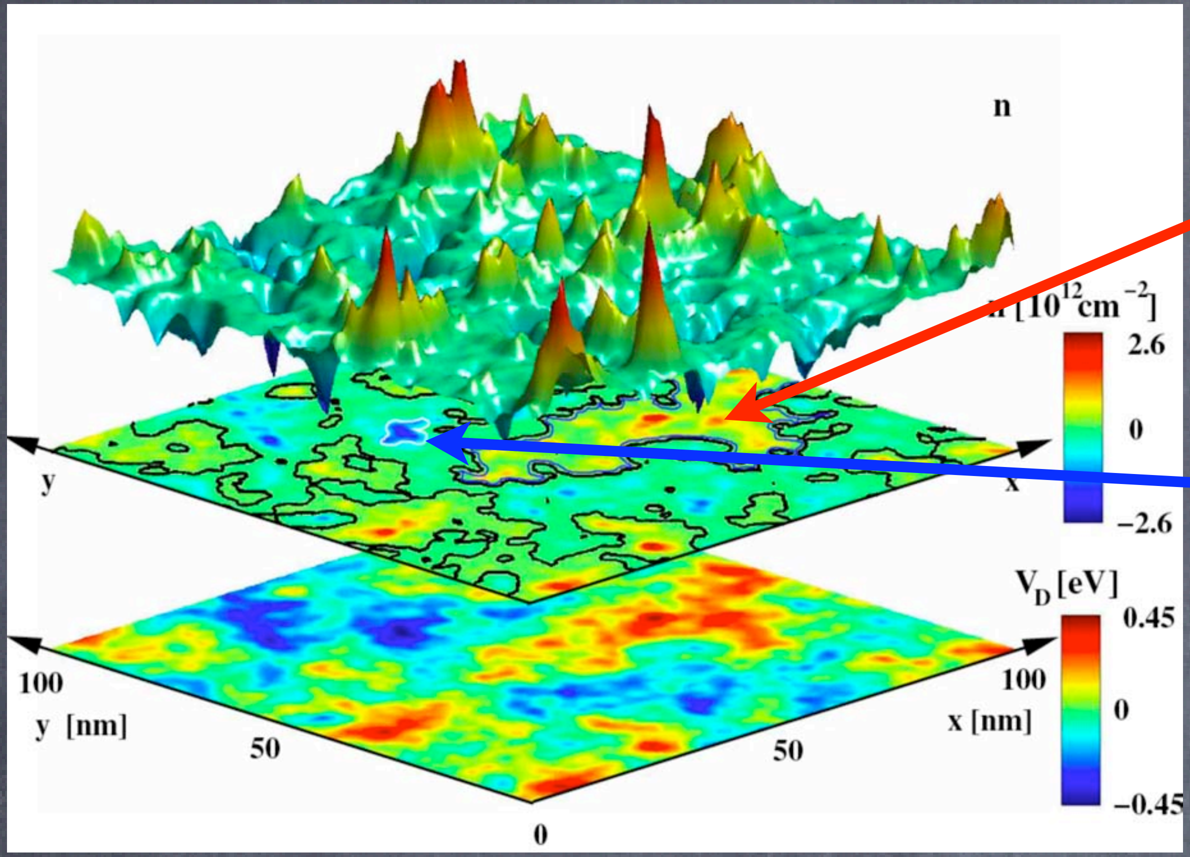
Intuitive picture:

$$\frac{K_0}{2\pi} = \pi\xi^2 n^*$$ roughly corresponds to number of electrons per puddle N_e

If $N_e \gtrsim 1.5$ then transport is semiclassical and mapping to homogeneous system works. If $N_e \lesssim 1.5$ then transport is quantum.

Relation to experiments?

Figure from Rossi, Adam and Das Sarma (2008)



Large puddles

$$N_e \sim n_{\rm rms} L^2 \sim 500$$

Small puddles

$$N_e \sim n_{\max} \xi^2 \sim 2$$

For realistic graphene, both the long-range correlation of the Coulomb potential, and the effect of exchange both work in the favor of the semiclassical regime!

$$\frac{K_0}{2\pi} \approx \xi^2 \pi \frac{n_{\rm rms}}{\sqrt{3}}$$

For typical graphene

$$n_{\rm imp} = 2 \times 10^{12} cm^{-2}, r_s = 0.8, d = 1 \text{ nm}$$
$$\longrightarrow \xi \approx 5 \text{ nm and } K_0 \approx 2$$

For very clean graphene

$$n_{\rm imp} = 10^{11} cm^{-2}, r_s = 0.8, d = 1 \text{ nm}$$
$$\longrightarrow \xi \approx 10 \text{ nm and } K_0 \approx 1$$

For suspended graphene

$$n_{\rm imp} = 10^{10} cm^{-2}, r_s = 2, d = 0.5 \text{ nm}$$
$$\longrightarrow \xi \approx 10 \text{ nm and } K_0 \approx 0.3$$

Schematic

1. Introduction
   - Graphene transport mysteries
   - Need for a hirarchy of approximations
   - Sketch of self-consistent theory: discussion of ansatz and its predictions

2. Characterizing the Dirac Point
   - What the Dirac point really looks like
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3. Quantum to classical crossover

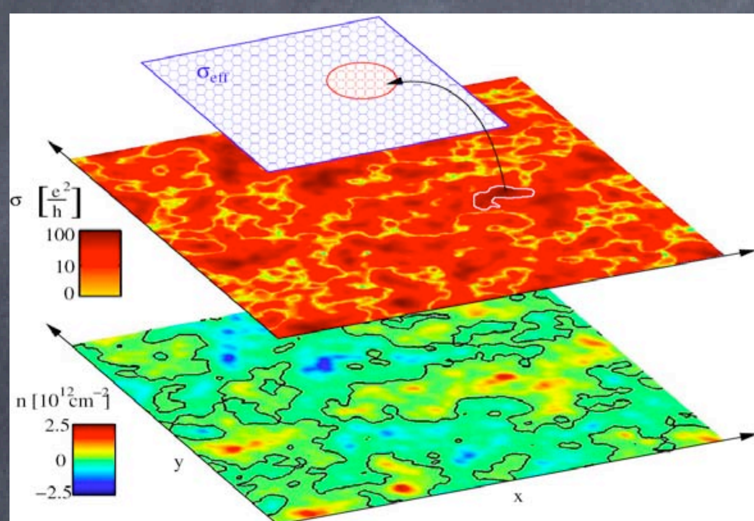
4. Effective medium theory

5. Comparison with experiments

Effective Medium Theory [1]

Q3. What is the conductivity of this effective medium in terms of the "statistical properties" of the inhomogeneous system? i.e Is it really that $\sigma_{\min} = \sigma[n^*]$?

Answer this question using Effective Medium Theory



[Bruggeman, Ann. Physik (1935)]
[Landauer, J. Appl. Phys. (1952)]
[Rossi, Adam and Das Sarma, arXiv:0809.1425]
[Fogler, arXiv:0810.1755]

$$\int dn \frac{\sigma_0 \frac{|n|}{n_{imp}} - \sigma_{\text{eff}}}{\sigma_0 \frac{|n|}{n_{imp}} + \sigma_{\text{eff}}} P(n) = 0$$

P[n] from TFD-LDA or SCA

[Rossi and Das Sarma, PRL (2008)]
[Adam, Hwang, Galitski and Das Sarma, PNAS (2007)]

Bulk conductivity from Boltzmann theory with charged impurities

[Ando J. Phys. Soc. Jpn. (2006)]
[Nomura and MacDonald, PRL (2006), PRL (2007)]
[Cheianov and Falko, PRL (2006)]
[Hwang, Adam and Das Sarma, PRL (2007)]
[Adam, Hwang, Galitski and Das Sarma, PNAS (2007)]

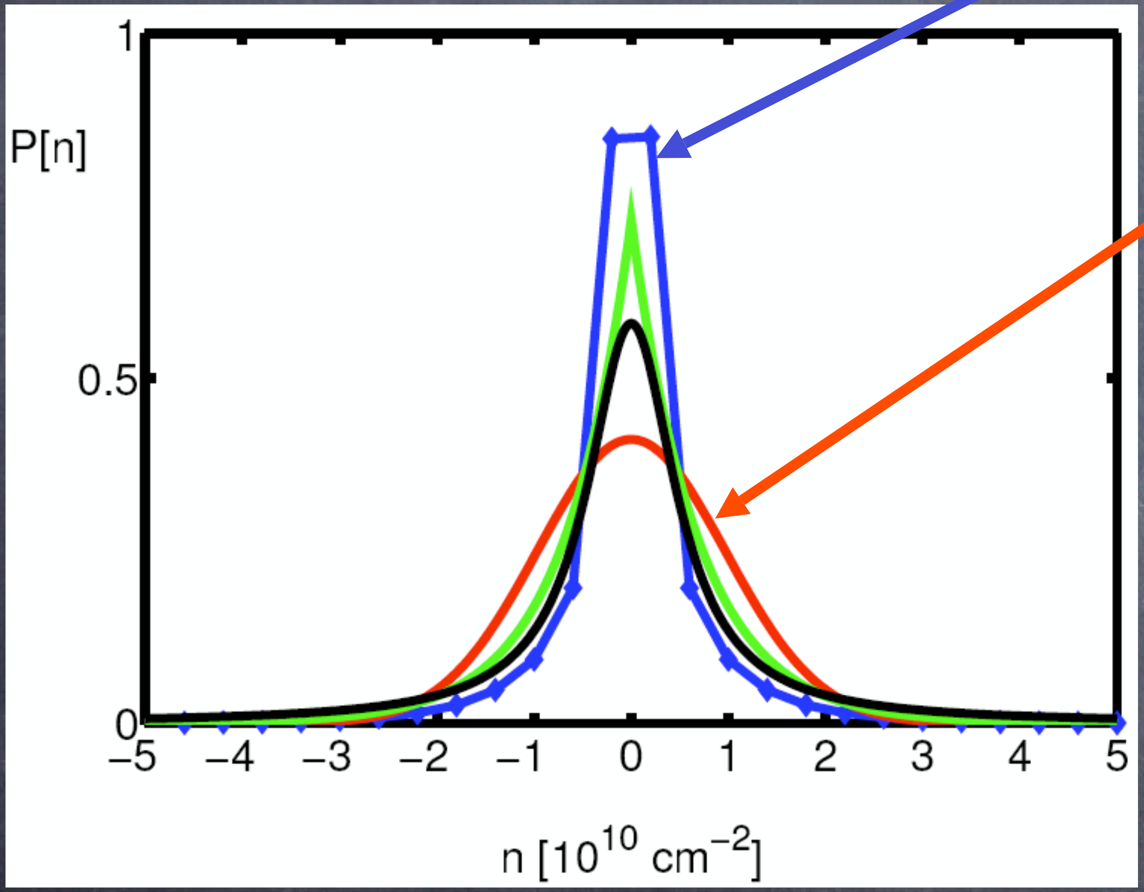
Range of validity:

1. Interface resistance is negligible
   [Cheianov and Falko,PRB (2006)]
   [Fogler, Novikov, Glazman, Shklovskii, PRB (2008)]
2. Several electrons per puddle

$$l \ll \left[\frac{\nabla\sigma(\mathbf{r})}{\sigma(\mathbf{r})}\right]^{-1}$$

Effective Medium Theory [2]

$$\int dn \frac{\sigma_0 \frac{|n|}{n_{imp}} - \sigma_{\rm eff}}{\sigma_0 \frac{|n|}{n_{imp}} + \sigma_{\rm eff}} P(n) = 0$$



Example of TDF-LDA numerical data with given $n_{\rm rms}$

Gaussian distribution with the same $n_{\rm rms}$

Parameters fully specified by normalization and $n_{\rm rms}$

$$x = \frac{\sigma_{\rm EMT}}{2\sqrt{2}\ n_{\rm rms}/(n_{\rm imp}F_1(2r_s))}$$

$$\frac{\sqrt{\pi}}{2} = e^{-x^2}(\pi x {\rm Erf[x]} + {\rm xE_i[x^2]})$$

If P[n] is Gaussian with the same $n_{\rm rms}$ then

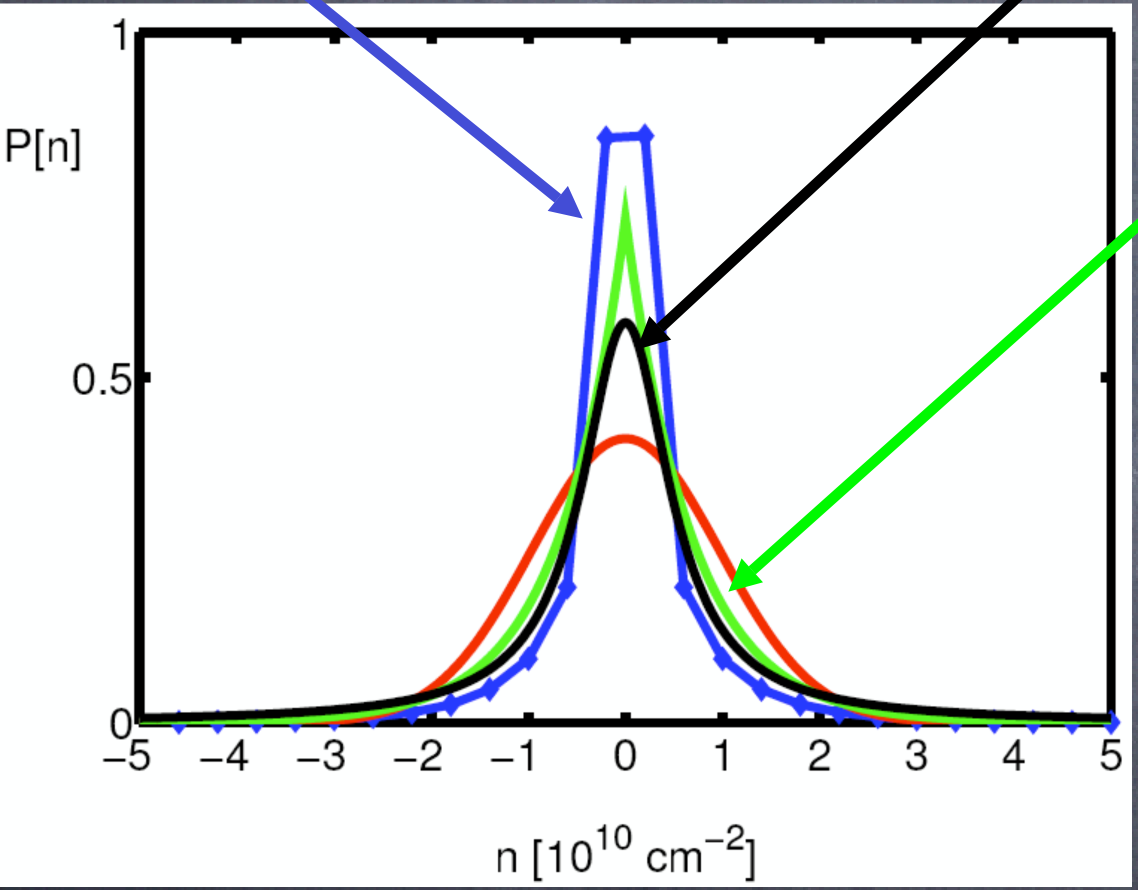
$$\sigma_{\rm min}^{\rm EMT} \approx 0.9925\ \sigma_{\rm min}^{\rm SCA}$$

Recall: $\sigma_{\rm min}^{\rm SCA} = \frac{2e^2}{h}\frac{n^*}{n_{\rm imp}}\frac{1}{F_1(2r_s)}$ $\quad n_{\rm rms} \approx \sqrt{3}n^*$

Effective Medium Theory [3]

$$\int dn \frac{\sigma_0 \frac{|n|}{n_{imp}} - \sigma_{\text{eff}}}{\sigma_0 \frac{|n|}{n_{imp}} + \sigma_{\text{eff}}} P(n) = 0$$

Example of TDF-LDA numerical data with given n_{rms}



If $P[n]$ is Lorentzian with the width given by $\frac{n_{\text{rms}}}{\sqrt{3}}$ then

$$\sigma_{\text{min}}^{\text{EMT}} = \sigma_{\text{min}}^{\text{SCA}}$$

Recently proposed by Fogler, that for $r_s \rightarrow 0$

$$P[n] = (1/\sqrt{2}\ n_{\text{rms}}) \exp[-\sqrt{2}|n|/n_{\text{rms}}]$$

Again parameters fully specified by normalization and n_{rms}

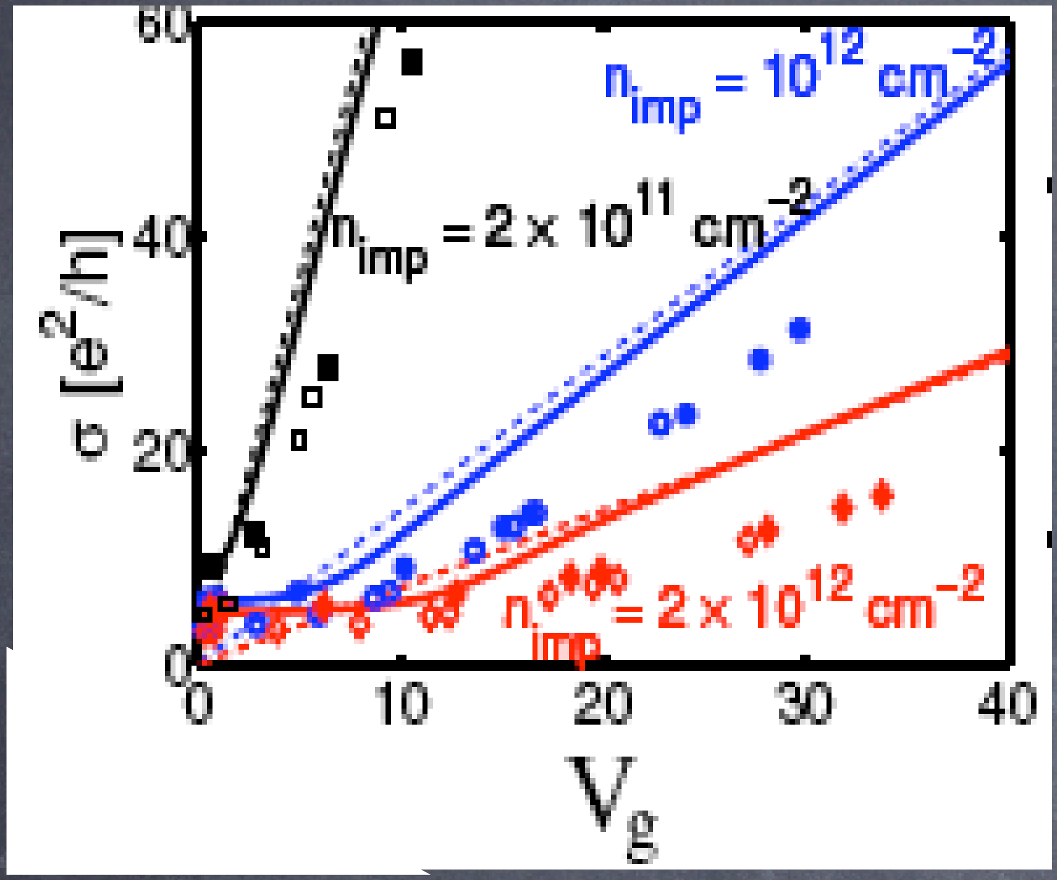
$$y = \frac{\sigma_{\text{EMT}}}{\sqrt{2}\ n_{\text{rms}}/(n_{\text{imp}} F_1(2r_s))}$$

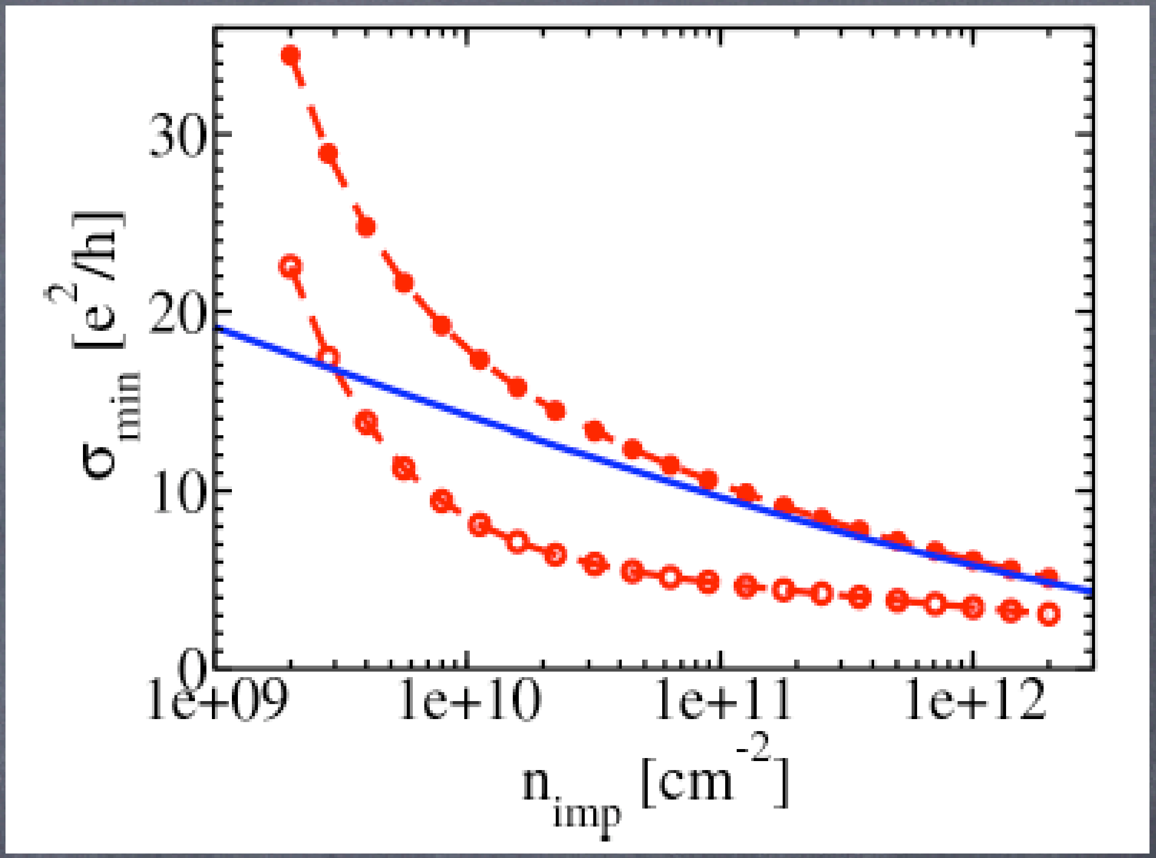
$$y e^y \Gamma[y] = 1/2$$

$$\sigma_{\text{EMT}} \approx 0.75\ \sigma_{\text{SCA}}$$

Recall: $\sigma_{\text{min}}^{\text{SCA}} = \frac{2e^2}{h} \frac{n^*}{n_{\text{imp}}} \frac{1}{F_1(2r_s)}$ $\quad n_{\text{rms}} \approx \sqrt{3} n^*$

Comparison of EMT with self-consistent theory





- EMT results shows that conductivity calculated using self-consistent Ansatz ($\sigma_{\min} = \sigma[n^*]$) and numerical P[n] using Thomas-Fermi-Dirac local density approximation agree for $n_{imp} > 10^{10}\ cm^{-2}$. Adding corrections to SCA (e.g. Gaussian approximation for P[n]) gives agreement down to very low imp. concentration ~ $10^9\ cm^{-2}$

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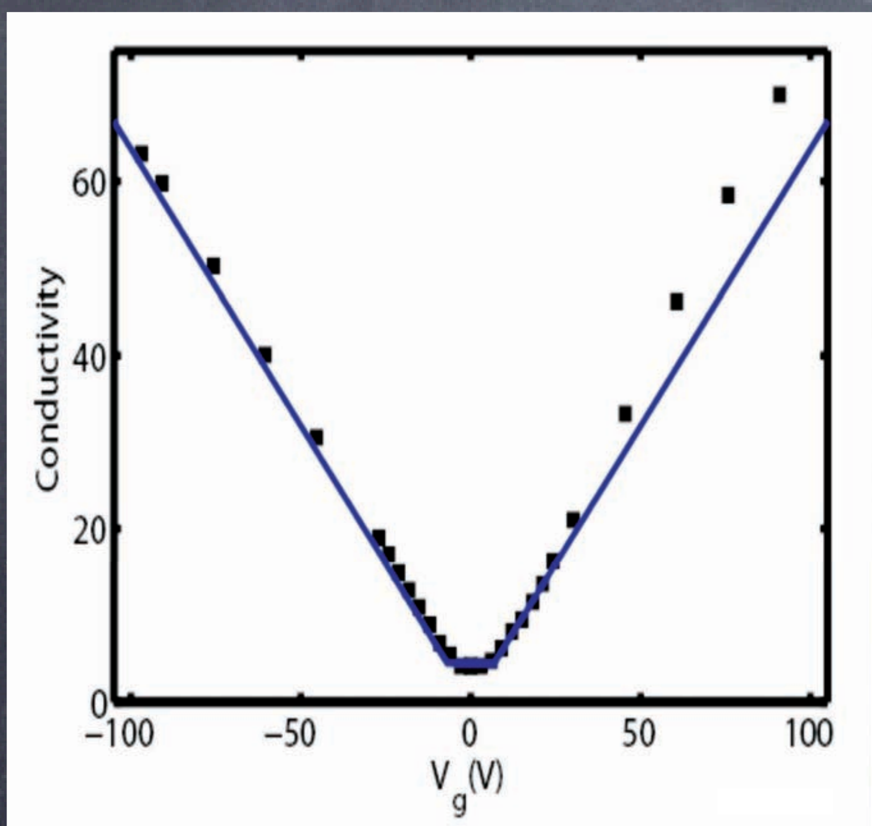
Manchester Experiments

Magnetoresistance

Novoselov et al. Nature (2005)
Schedin et al. Nature Materials (2007)

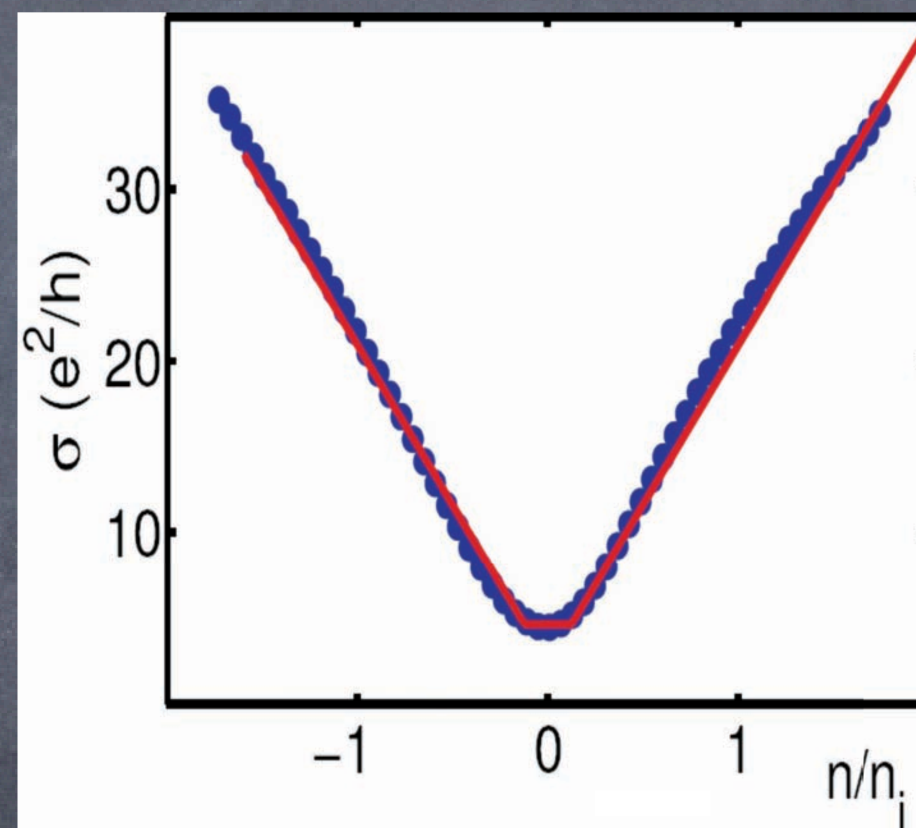
Sample 1

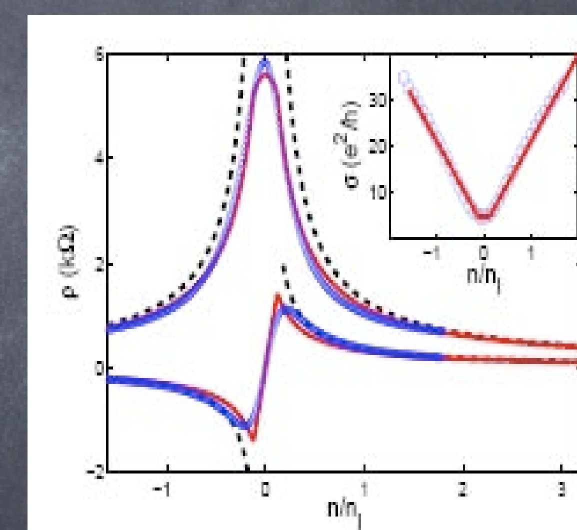
$n_{\mathrm{imp}} = 230 \times 10^{10}\ \mathrm{cm}^{-2}$



Sample 2

$n_{\mathrm{imp}} = 175 \times 10^{10}\ \mathrm{cm}^{-2}$





$$\sigma_{xx}^{(c)} = \sigma_{yy}^{(c)} = \frac{\sigma_0^{(c)}}{1 + \left(\sigma_0^{(c)} R_H^{(c)} B\right)^2}, \qquad \sigma_{xy}^{(c)} = -\sigma_{yx}^{(c)} = -\frac{\left[\sigma_0^{(c)}\right]^2 R_H^{(c)} B}{1 + \left(\sigma_0^{(c)} R_H^{(c)} B\right)^2},$$

Columbia Experiments [1]

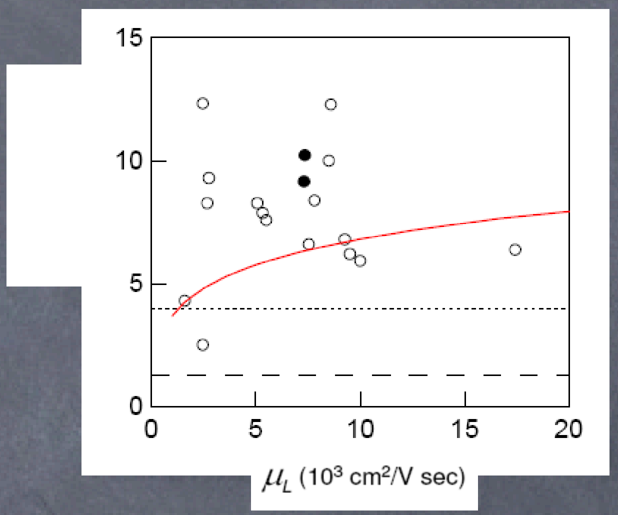
Samples showing an order of magnitude variation in mobility

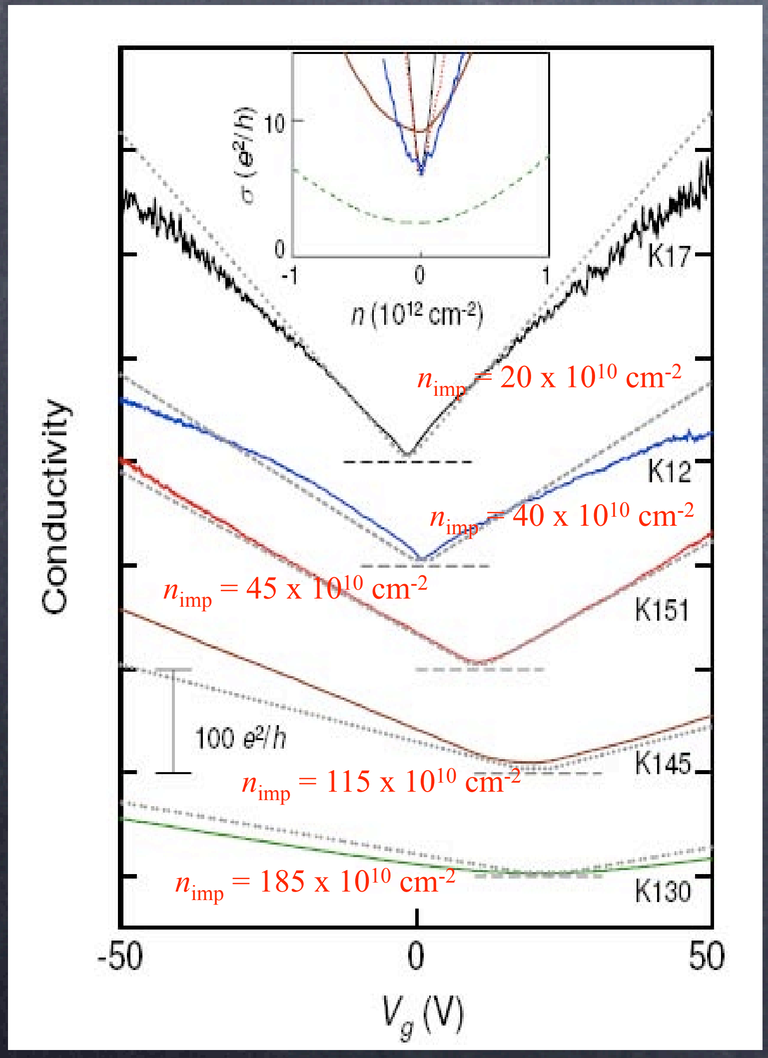
No fit parameter


PRL 99, 246803 (2007) PHYSICAL REVIEW LETTERS week ending 14 DECEMBER 2007

Measurement of Scattering Rate and Minimum Conductivity in Graphene

Y.-W. Tan,[1,*] Y. Zhang,[1,*] K. Bolotin,[1] Y. Zhao,[1] S. Adam,[2] E. H. Hwang,[2] S. Das Sarma,[2] H. L. Stormer,[1,3,4] and P. Kim[1]



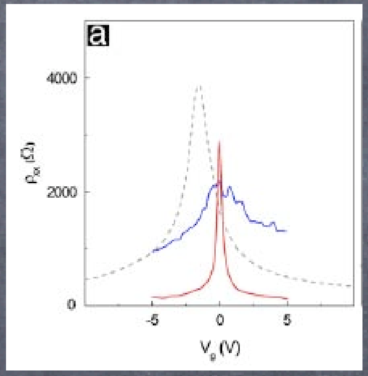


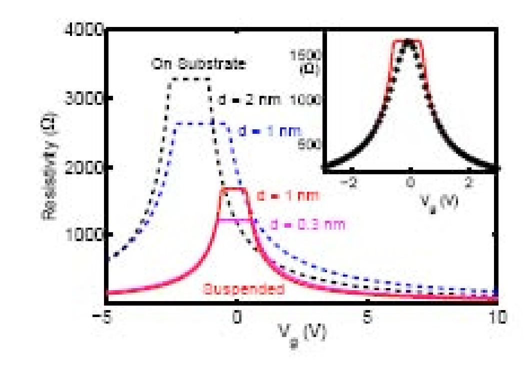
Clean Sample
- High mobility
- Small Vg offset
- Narrow plateau

Dirty Samples
- low mobility
- large Vg offset
- wide plateau

Suspended graphene

$$\rho = \frac{1}{\sigma}$$

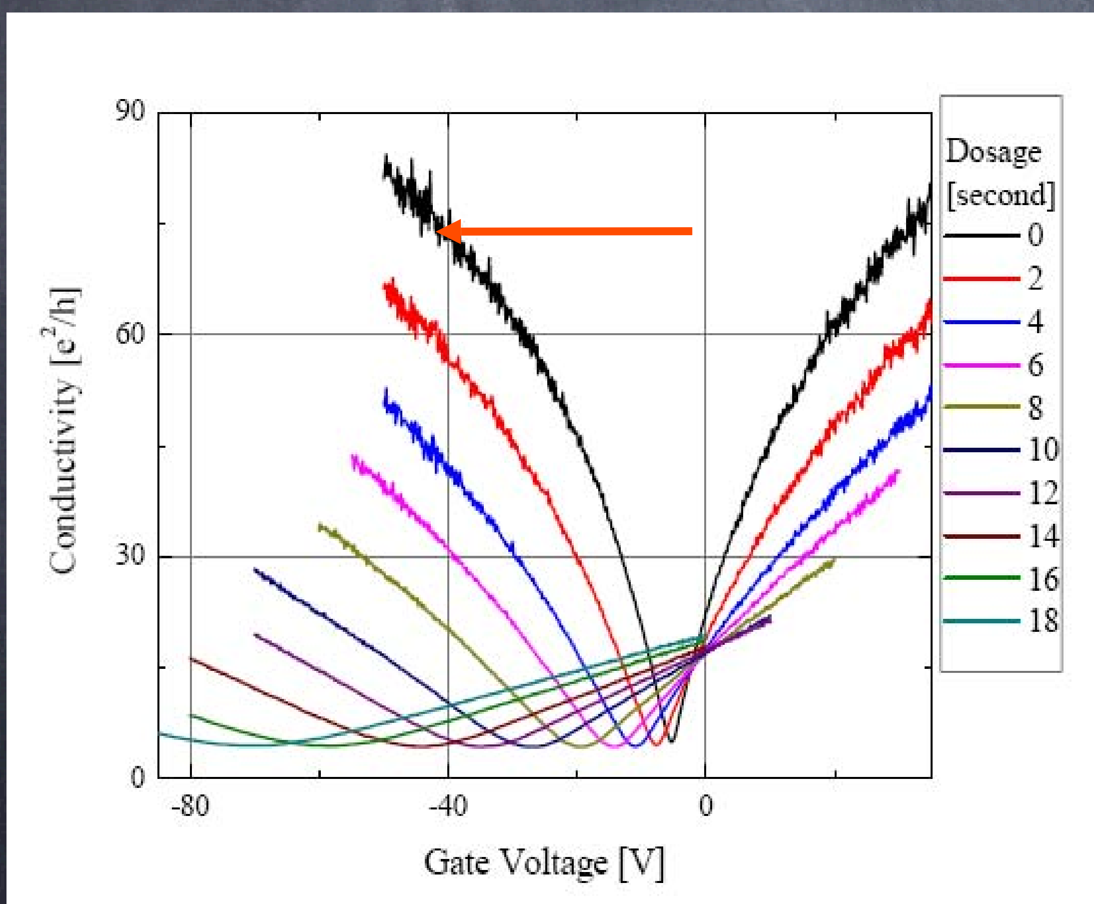




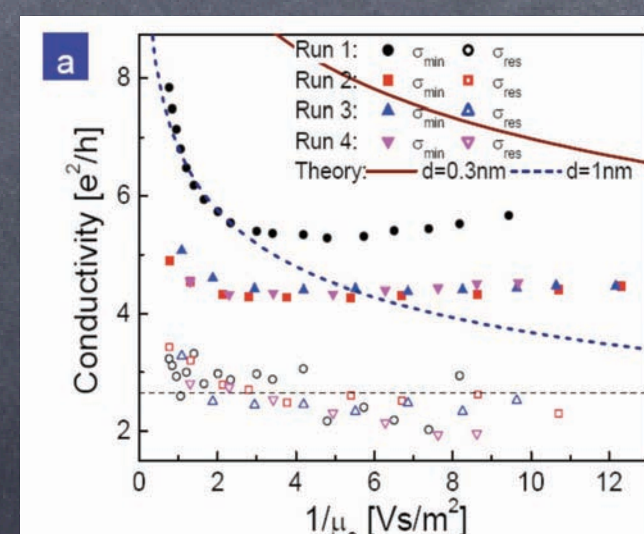
See also: Du et al. Nature Nanotechnology (2008)

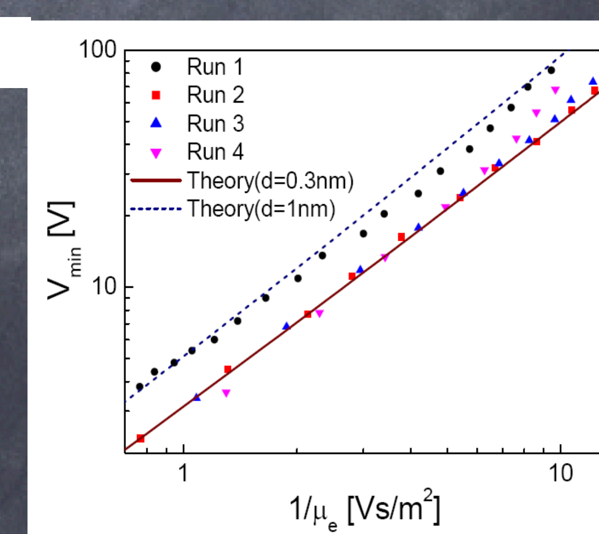
Adding charged impurities to graphene

Potassium Doping: Tuning the n_{imp} knob!

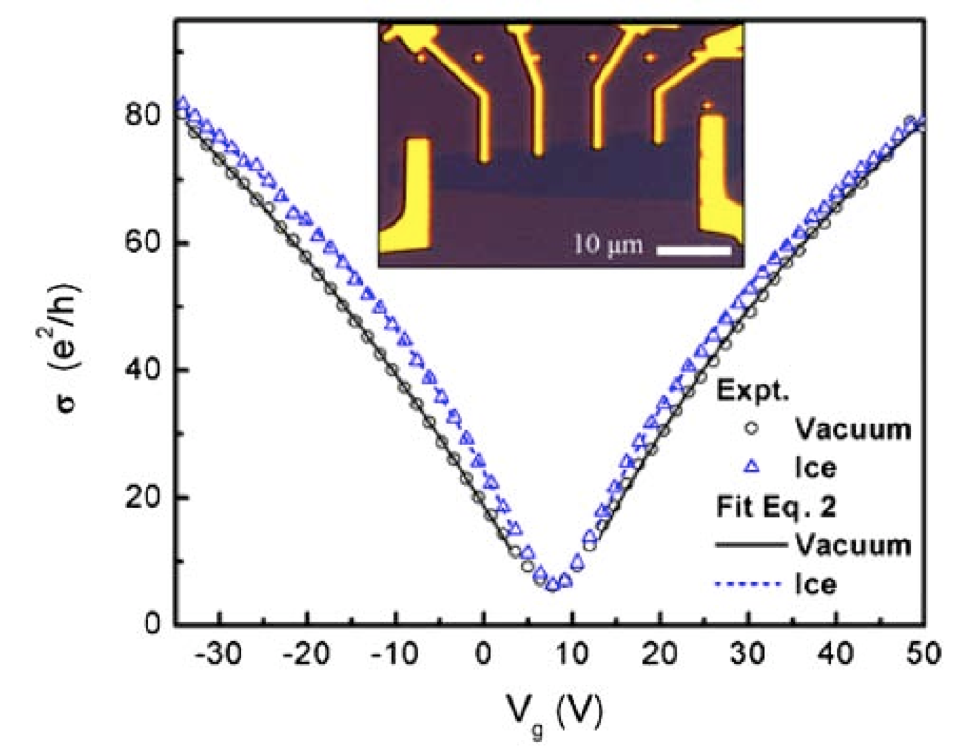


Potassium Doping
- Mobility decreases
- Sub-linearity vanishes
- Vg offset increases
- Plateau width increases
- Minimum conductivity decreases!





Dielectric Screening



$$\alpha = \frac{e^2}{\hbar v_F}$$

$$\alpha = \frac{2e^2}{\hbar v_F\,(\kappa_1 + \kappa_2)}$$

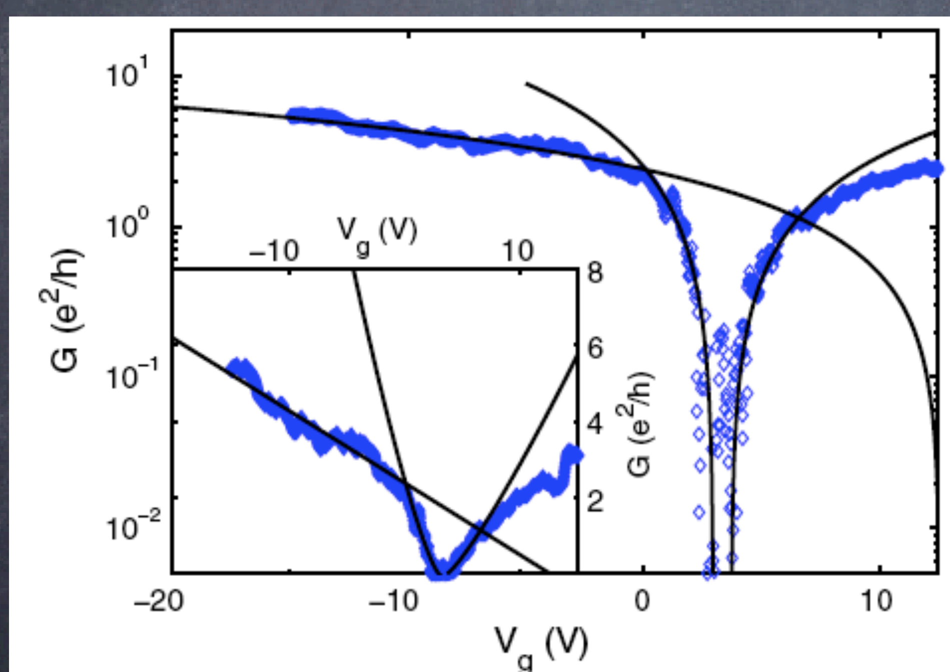
Graphene with gap: classical percolation [1]

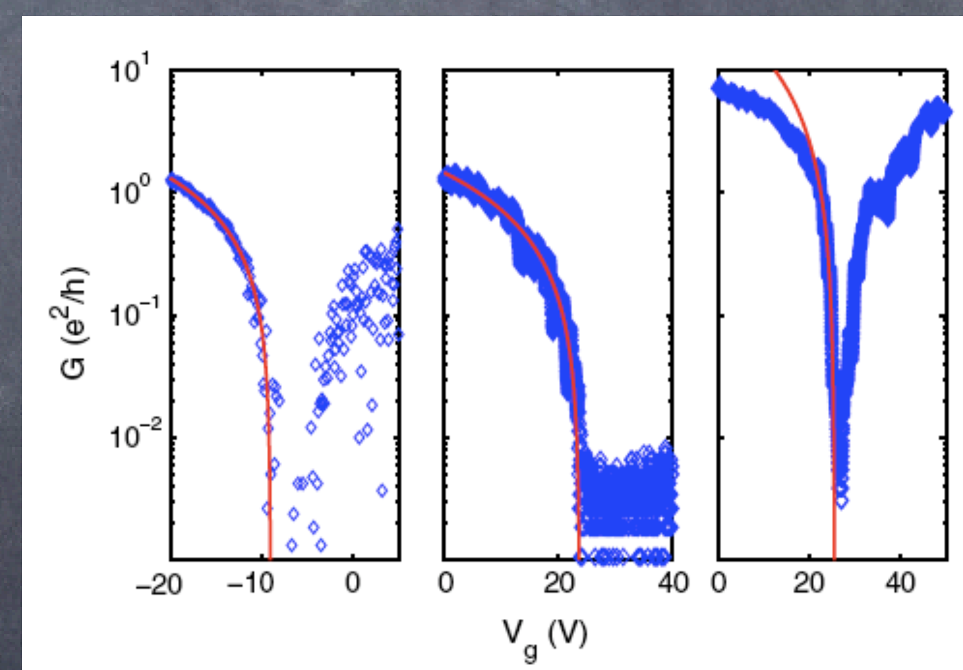
PRL **101,** 046404 (2008) PHYSICAL REVIEW LETTERS week ending 25 JULY 2008

Density Inhomogeneity Driven Percolation Metal-Insulator Transition and Dimensional Crossover in Graphene Nanoribbons

S. Adam,[1] S. Cho,[2] M. S. Fuhrer,[2] and S. Das Sarma[1,2]

Prediction: If p-n junction resistance increases, e.g. gap (in a non-quasi 1D nanoribbon), or magnetic field, or electric field (bilayer); then physics should become a classical percolation transition



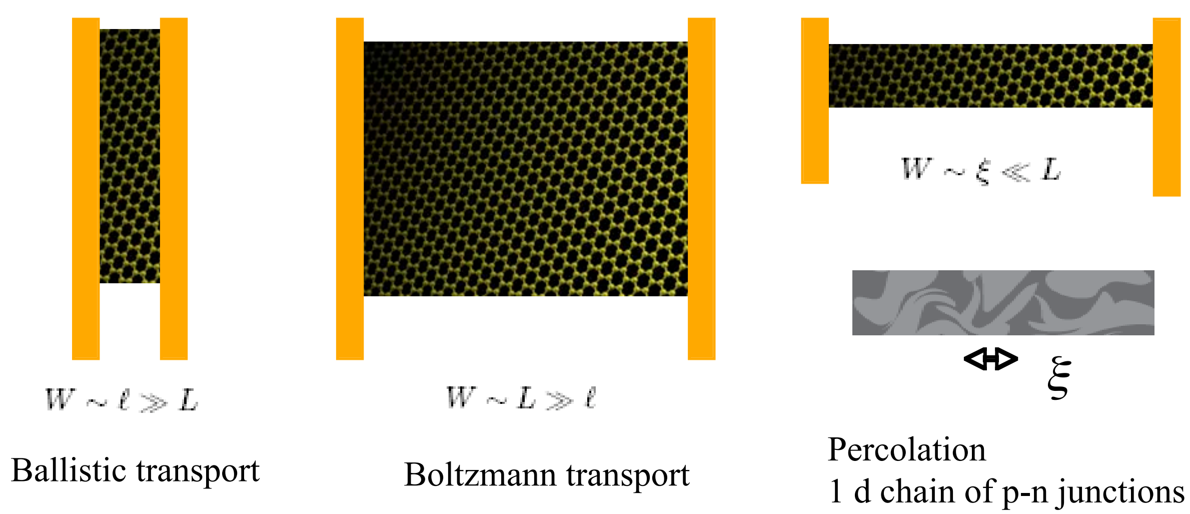


Same physics observed in analysis of Columbia Nanoribbon samples [Han et al. PRL (2007)]

Graphene with gap: classical percolation [2]

Nanoribbons have 4 different "gaps"

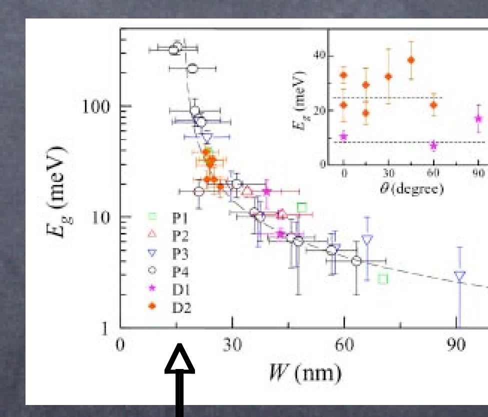
- Spectrum gap (i.e. in single particle spectrum)
- Transport gap (difference in n_c for electrons and holes)
- Non-linear transport gap (by tuning V_{sd})
- Activated gap as function of Temperature (~0.5 meV)



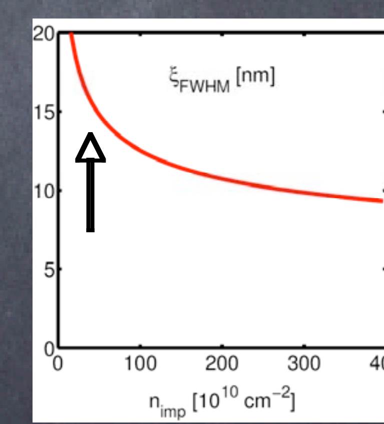
L	Length	ℓ	Mean free path
W	Width	ξ	Disorder correlation length

$$\langle V(r)V(0)\rangle \approx \frac{K_0\gamma^2}{2\pi\xi^2}\exp\left[\frac{-r^2}{2\xi^2}\right].$$

$$K_0 = \frac{1}{4r_s^2}\left(\frac{D_0}{C_0}\right)^2,$$
$$\xi = \frac{1}{\sqrt{n_{\mathrm{imp}}}}\frac{D_0}{4\pi r_s^2}\frac{1}{(C_0)^{3/2}},$$



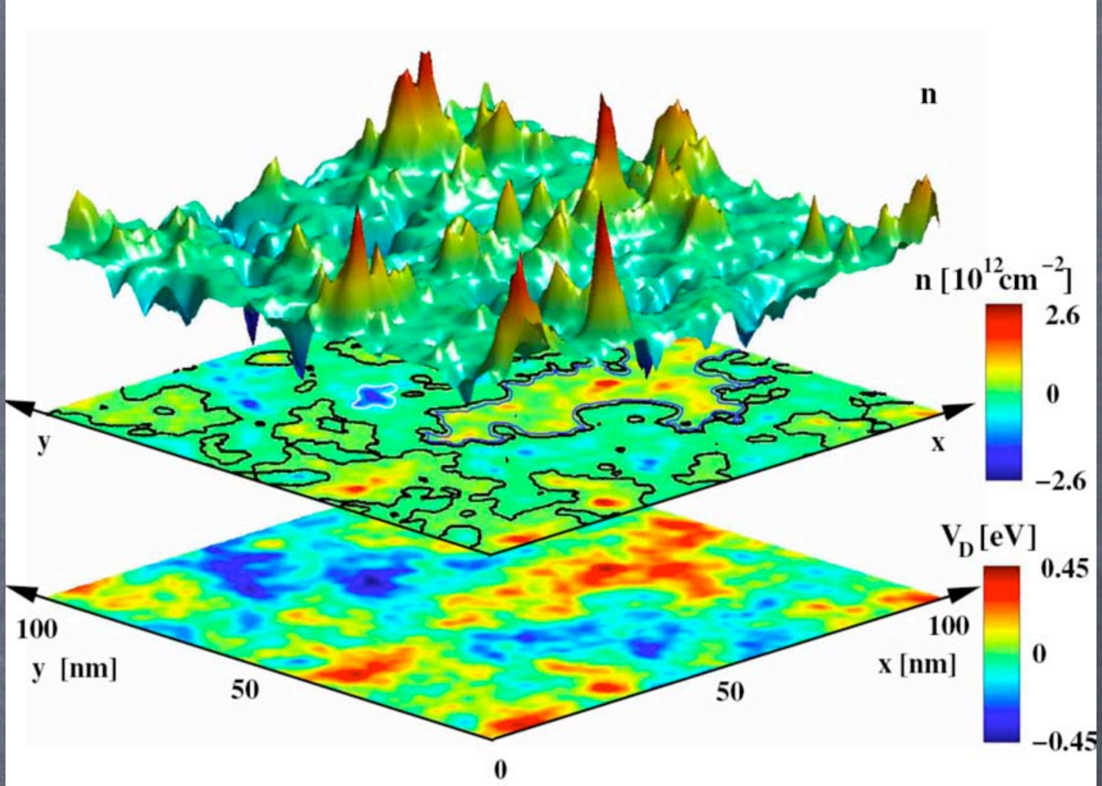
Han et al. PRL (2007)



Alternate explanations by: Sols, Guinea, and Castro Neto PRL, (2007); Martin and Blanter, arXiv:0705.0532

Concluding Remarks

- We understand the experimental observation of graphene "minimum conductivity" that arises from interplay of disorder and screening



- Several interesting bits of physics at play including Klein tunneling, unusual screening properties of graphene, etc.

- Employed Landauer formalism to understand quantum to classical crossover

- We have tested the assumptions of the self-consistent theory using other methods such as Energy functional minimization and an Effective Medium Theory.

For more details see: PNAS 104, 18392 (2007); as well as arXiv:0809.1425, arXiv:0811.0609 and arXiv:0812.1795 for the more recent work.

Back-up Slides