Symmetry and quantum transport in disordered graphene

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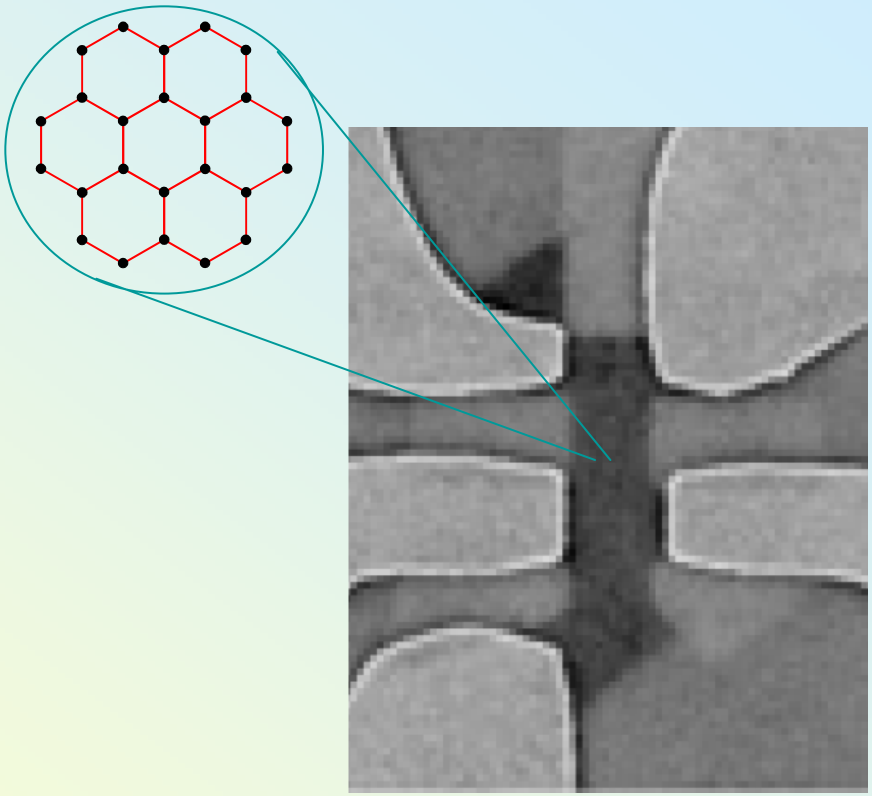


A.Geim and K.Novoselov
Nature Mat. 6, 183 (2007)

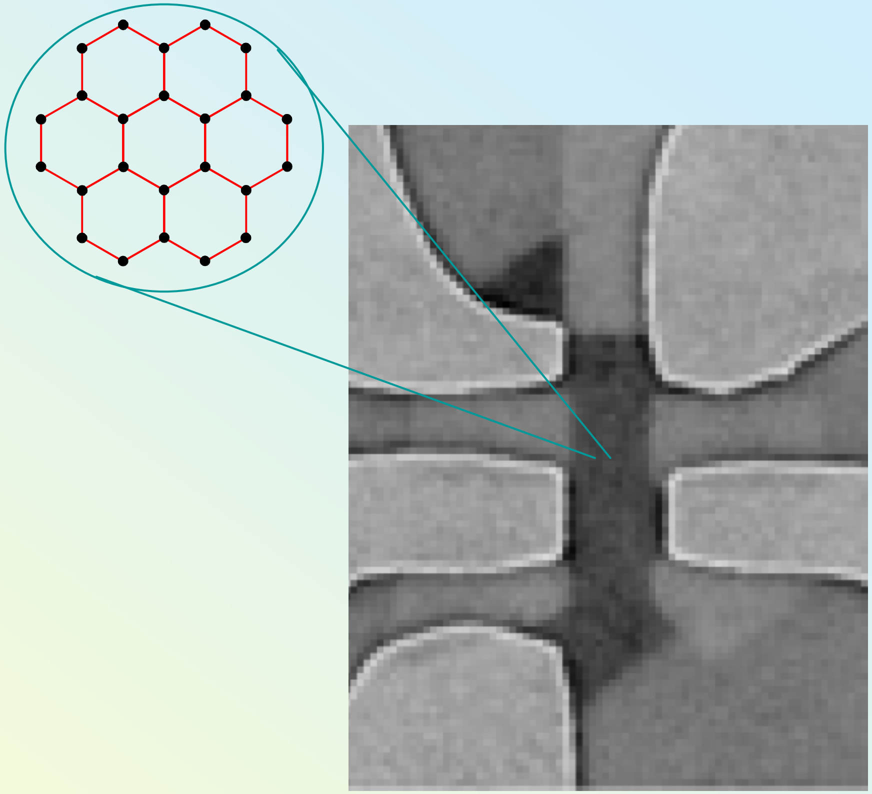
Symmetry and quantum transport in disordered graphene

List of properties of electrons in graphene.

Weak localisation *vs* anti-localisation: qualitative discussion.

Formal WL analysis:
effect of different types of disorder,
specifically - adatoms.

Universal conductance fluctuations
and correlation function thermometry.



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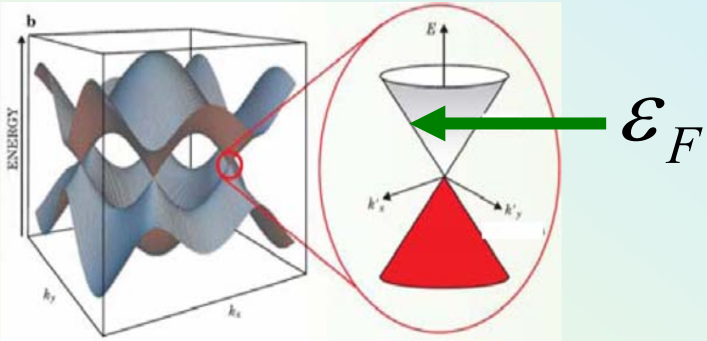
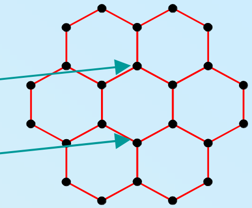
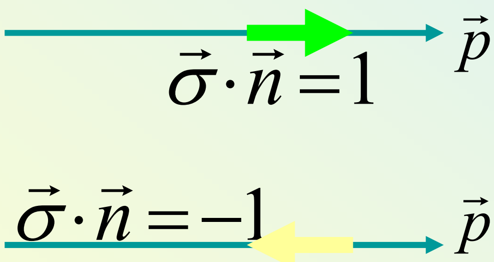


$$\hat{H} = v\vec{p}\cdot\vec{\sigma}$$

$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$



'chiral' electrons: sublattice 'isospin' $\vec{\sigma}$ is linked to the direction of the electron momentum

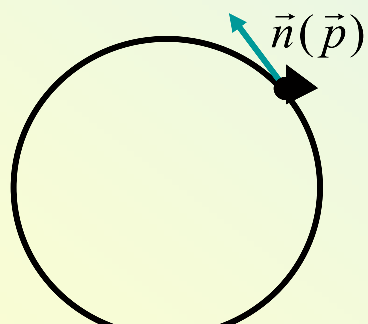


$$\vec{p} = (p\cos\vartheta, p\sin\vartheta)$$

$$\psi_{\vec{p}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i\vartheta} \end{pmatrix}$$

Berry phase

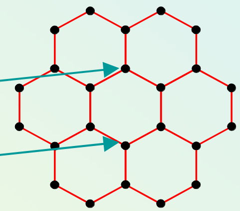
$$\pi = i\int_0^{2\pi} d\vartheta\, \psi^+ \frac{d}{d\vartheta}\psi$$

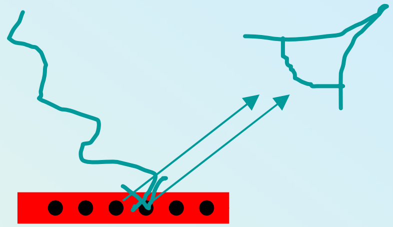


$$\psi \to e^{2\pi\frac{i}{2}\sigma_3}\psi = e^{i\pi\sigma_3}\psi$$

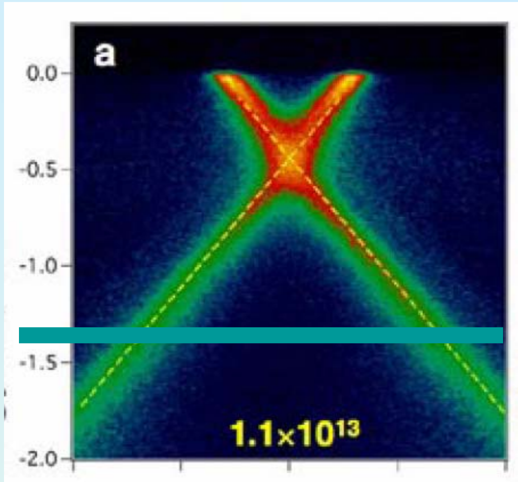
‘Chirality’ of electrons observed using ARPES of graphene

$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$





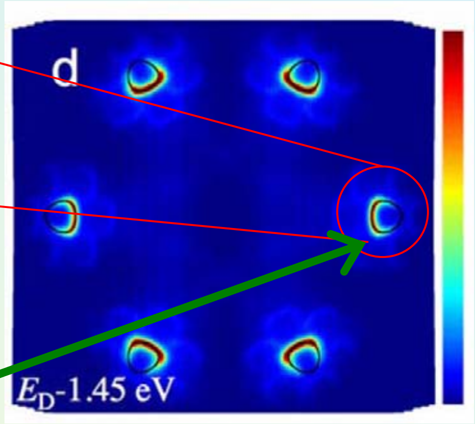
$$\varepsilon = -vp$$



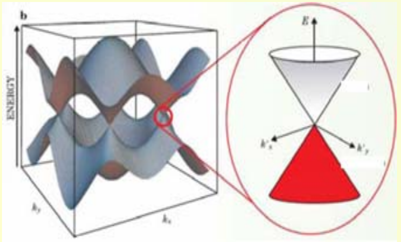
$$I_{ARPES} \sim |\varphi_A + \varphi_B|^2$$

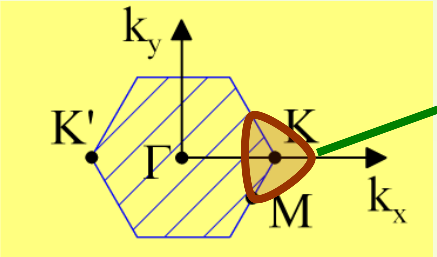
$$\vec{k}_{\parallel} = \vec{G} \pm \vec{K} + \vec{p}$$

$$\sim \sin^2\left(\frac{\vec{k}\cdot\vec{R}_{BA}}{2} + \frac{\vartheta}{2}\right)$$



Mucha-Kruczynski, Tsyplyatyev, Grishin, McCann, VF, Boswick, Rotenberg - PRB 77, 195403 (2008)





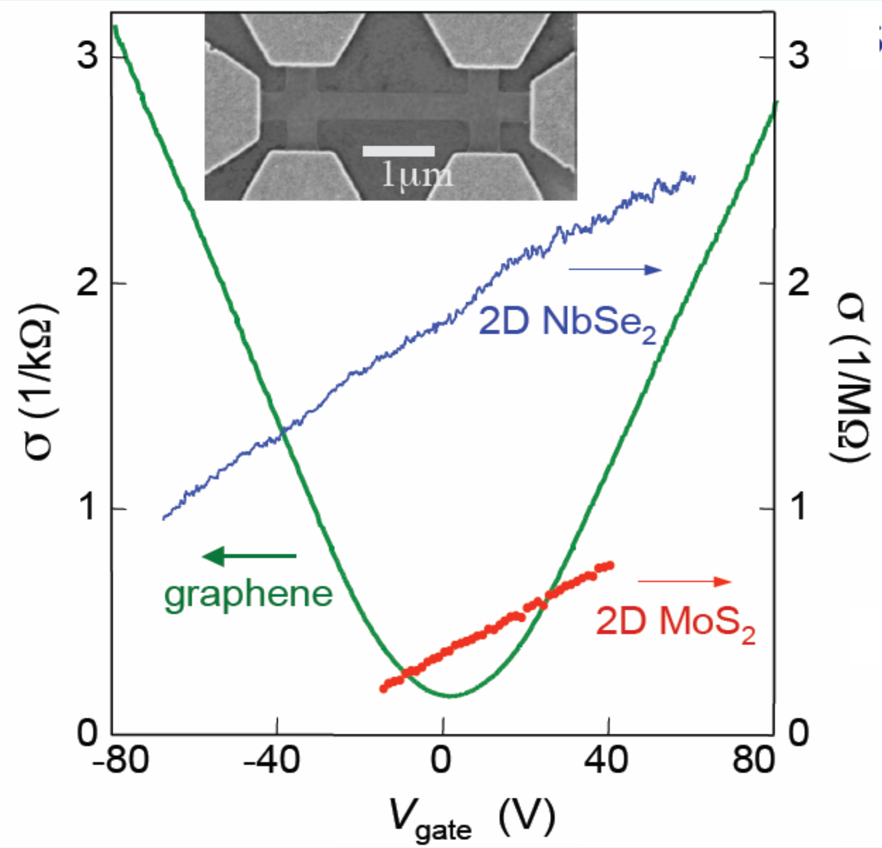
ARPES of heavily doped graphene synthesized on silicon carbide

Bostwick *et al* - Nature Physics, 3, 36 (2007)

Role of scattering from remote charges for graphene conduction in GraFETs

$$\sigma_{cl} = \frac{4e^2}{h}\frac{n_e}{n_i}F\left(\frac{e^2}{\hbar v}\right)$$

due to screening of potential of charges by electrons in graphene



Nomura and MacDonald - PRL 96, 256602 (2006)
Cheianov and VF - PRL 97, 226801 (2006)
Nomura and MacDonald - PRL 98, 076602 (2007)
Hwang, Adam, Das Sarma - PRL 98, 186806 (2007)

Electronic properties of graphene.

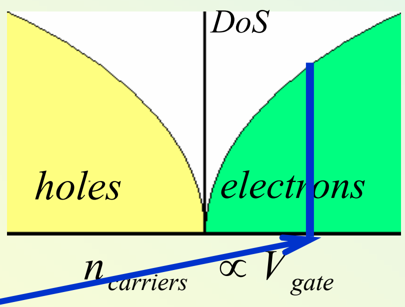
Weak localisation *vs* anti-localisation: qualitative discussion.

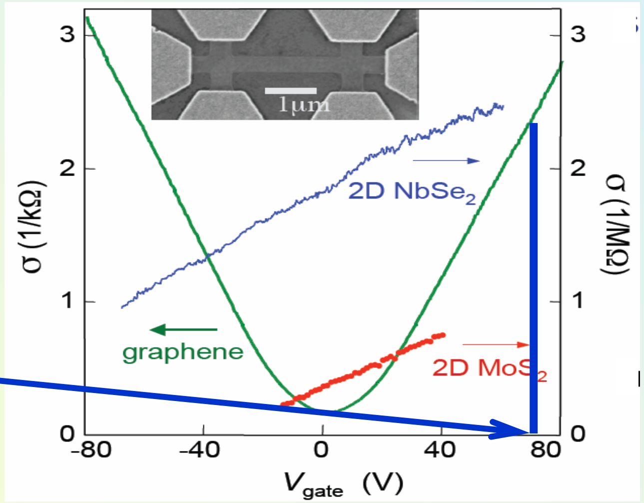
Formal WL analysis: effect of different types of disorder.

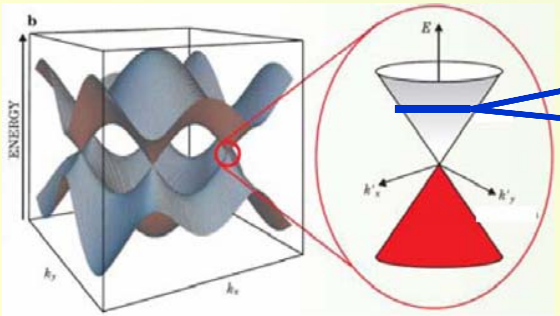
Universal conductance fluctuations.

Metallic (high-density) regime

$p_F l >> 1$ and $\delta n_e << n_e$

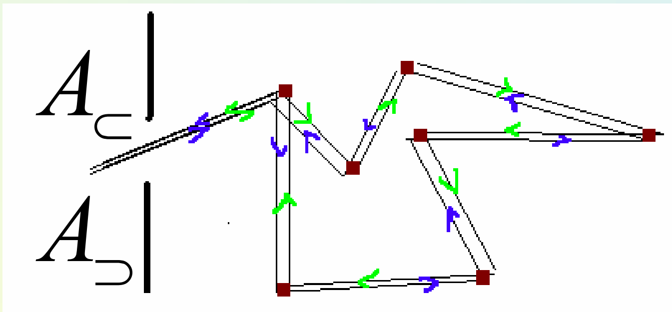






Interference correction: weak localisation effect...

$$w \sim |A_{\subset} + A_{\supset}|^2 = |A_{\subset}|^2 + |A_{\supset}|^2 + [A_{\subset}^* A_{\supset} + A_{\subset} A_{\supset}^*]$$



$$e^{i\varphi_{\supset}} = e^{i\varphi_{\subset}} \qquad A_{\supset} = A_{\subset}$$

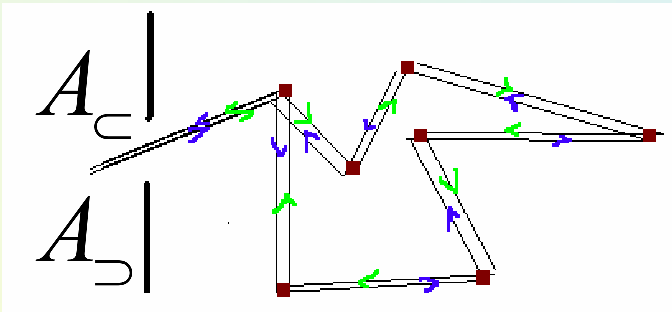
$$A_{\subset}^* A_{\supset} = |A_{\subset}|^2 > 0$$

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h} \ln\left(\tau_{\varphi} / \tau\right)$$

WL = enhanced backscattering in time-reversal-symmetric systems

Interference correction: weak localisation effect...

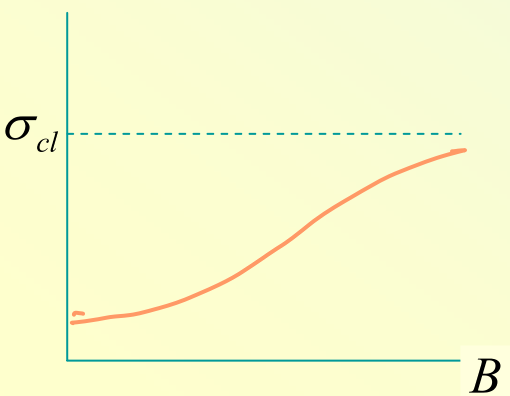
$$w \sim |A_\subset + A_\supset|^2 = |A_\subset|^2 + |A_\supset|^2 + [A_\subset^* A_\supset + A_\subset A_\supset^*]$$



$$e^{i\delta} A_\supset^{(0)} \neq e^{-i\delta} A_\subset^{(0)}$$

Random (path-dependent) phase factor due to a magnetic field

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h} \ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$



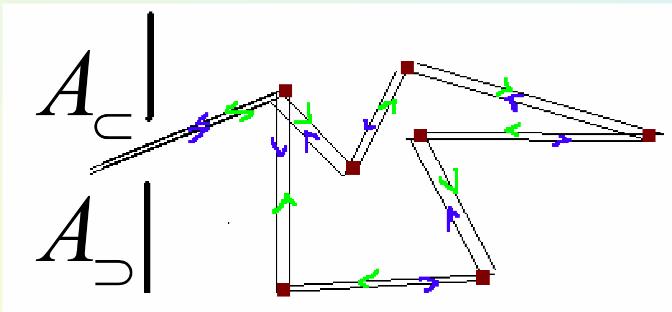
Broken time-reversal symmetry, *e.g.*, due to a magnetic field ***B*** suppresses the weak localisation effect

↓

WL magnetoresistance

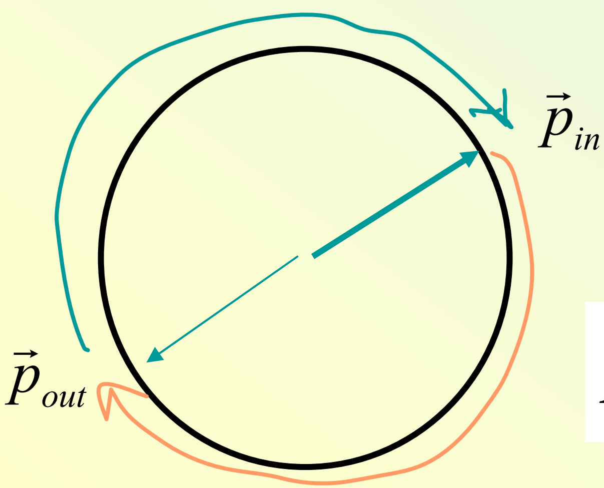
... but ...

$$w \sim |A_{\subset} + A_{\supset}|^2 = |A_{\subset}|^2 + |A_{\supset}|^2 + [A_{\subset}^* A_{\supset} + A_{\subset} A_{\supset}^*]$$



WL = enhanced backscattering
for non-chiral electrons in
time-reversal-symmetric systems

WAL = suppressed backscattering
for Berry phase π electrons

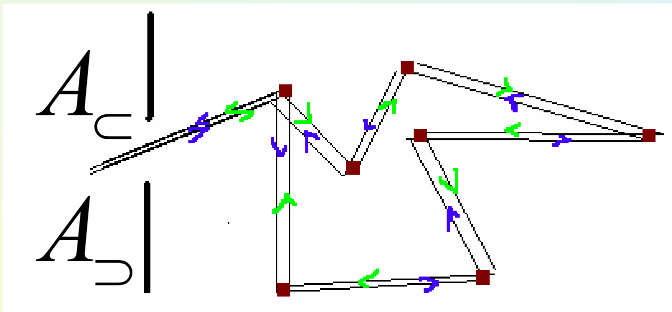


chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)}\psi_{in}$

$$A_{\subset} A_{\supset}^* = e^{-i2\pi(\sigma_z/2)} |A_{\subset}|^2 = -|A_{\subset}|^2 < 0$$

... but ...

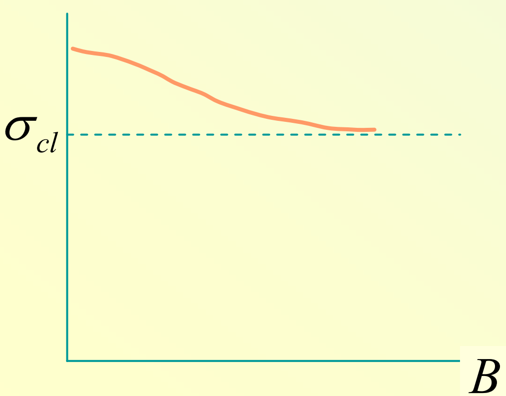
$$w \sim |A_\subset + A_\supset|^2 = |A_\subset|^2 + |A_\supset|^2 + [A_\subset^* A_\supset + A_\subset A_\supset^*]$$



WL = enhanced backscattering
for non-chiral electrons in
time-reversal-symmetric systems

$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi h} \ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$

WAL = suppressed backscattering
for Berry phase π electrons



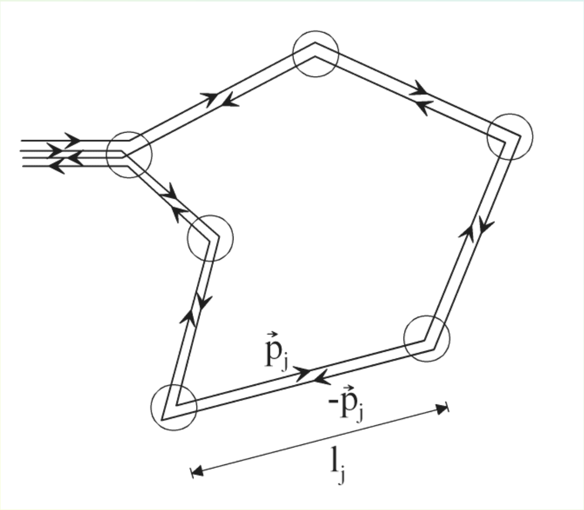
chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)}\psi_{in}$

$$A_\subset A_\supset^* = e^{-i2\pi(\sigma_z/2)} |A_\subset|^2 = -|A_\subset|^2 < 0$$

Suzuura, Ando - PRL 89, 266603 (2002)

... however ...

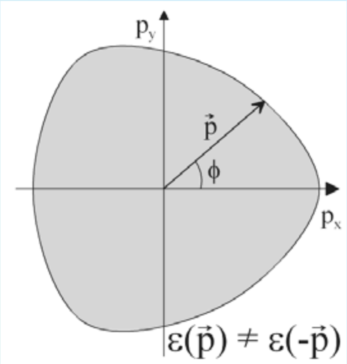
weak trigonal warping leads to a random phase difference, δ for long paths.



$$\hat{H} = v\vec{\sigma}\cdot\vec{p} - \mu\left((p_x^2 - p_y^2)\sigma_x - 2p_x p_y \sigma_y\right) + \hat{I}u(\vec{r}) + \hat{V}(\vec{r})$$

$$e^{i\delta} A_{\supset}^{(0)} \neq e^{-i\delta} A_{\subset}^{(0)}$$

$$\delta = \sum_j \left[\varepsilon(\vec{p}_j) - \varepsilon(-\vec{p}_j)\right] l_j / \hbar v_F$$

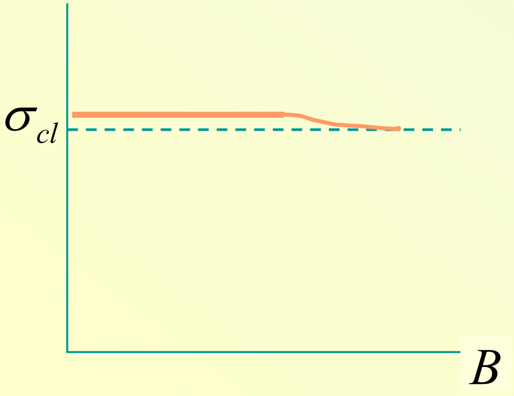


$$\sigma = \sigma_{cl} + \frac{e^2}{2\pi h}\ln\left(\min[\tau_\varphi, \tau_B]/\tau\right)$$

Some types of disorder (*e.g.*, lattice deformation) lead to a similar effect.

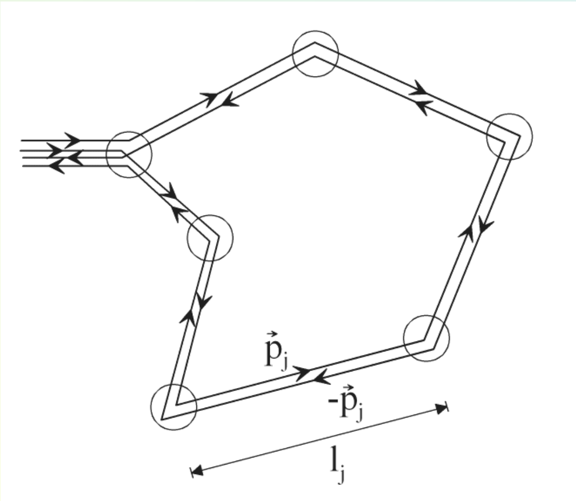
chiral electrons $\psi_{out} = e^{-i\phi(\sigma_z/2)}\psi_{in}$

$$A_{\subset}A_{\supset}^{*} = e^{-i2\pi(\sigma_z/2)} |A_{\subset}|^2 = -|A_{\subset}|^2 < 0$$



McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)
for bilayers: Kechedzhi, McCann, VF, Altshuler – PRL 98, 176806 (2007)

... and, finally, ...

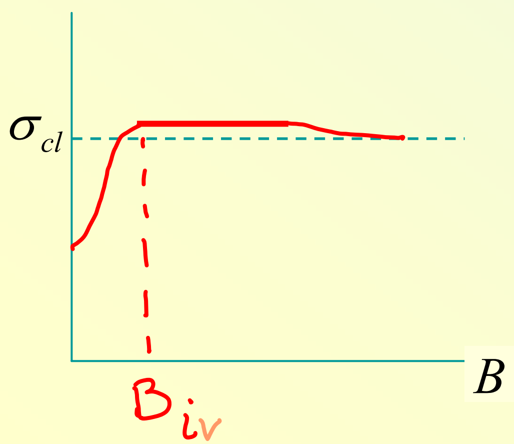


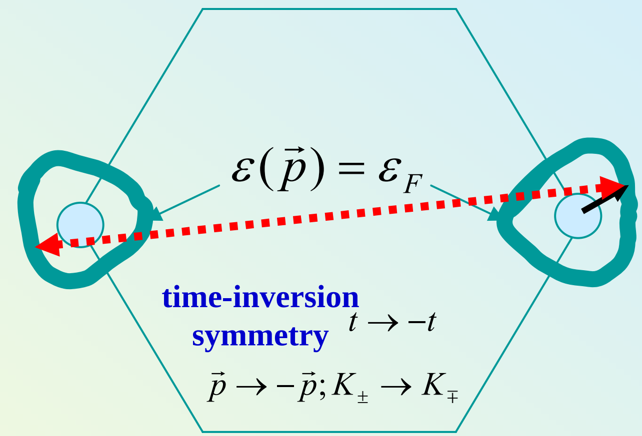
$$A_{\supset}^{K_\pm} = A_{\subset}^{K_\mp}$$

Inter-valley scattering restores the WL behaviour typical for electrons in time-inversion symmetric systems

$$\hat{H} = \pm v\vec{\sigma}\cdot\vec{p} - \mu\left((p_x^2 - p_y^2)\sigma_x - 2p_x p_y \sigma_y\right) + \hat{I}u(\vec{r}) + \hat{V}(\vec{r})$$

$$\sigma = \sigma_{cl} - \frac{e^2}{2\pi h}\ln\left(\min[\tau_\varphi, \tau_B]/\tau_{iv}\right)$$





McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)
for bilayers: Kechedzhi, McCann, VF, Altshuler – PRL 98, 176806 (2007)

Electronic properties of graphene.

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Weak localisation and universal conductance fluctuations.

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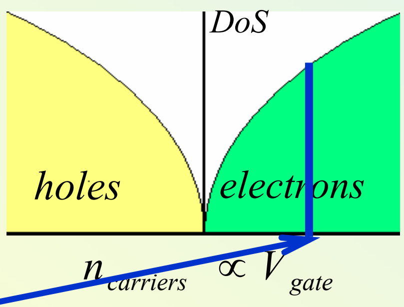
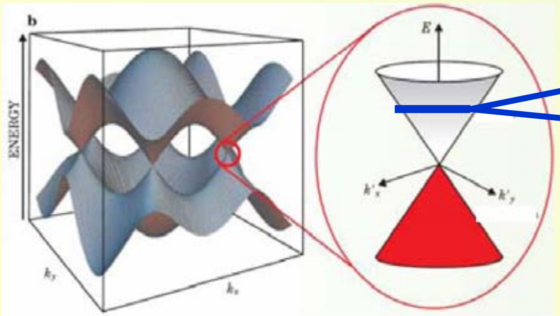
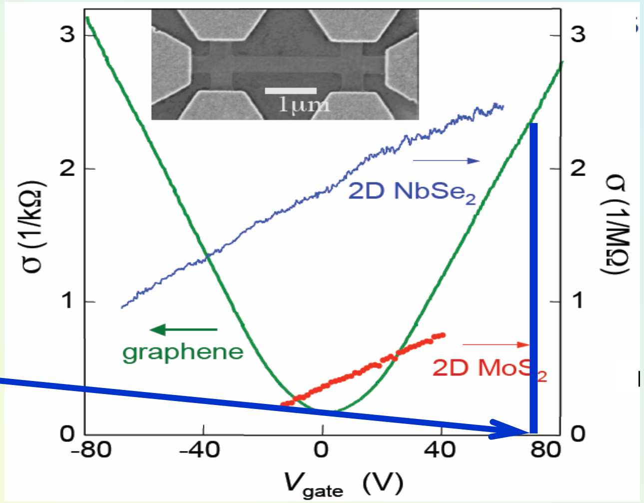
$p_F l >> 1$ and $\delta n_e << n_e$







Dirac electrons in disordered graphene.

smooth potential; intra-valley scatterers

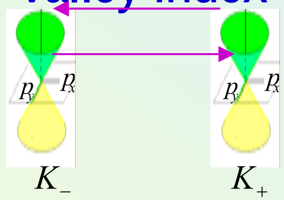
$$\hat{H} = \begin{pmatrix} v\vec{\sigma}\cdot\vec{p} & 0 \\ 0 & -v\vec{\sigma}\cdot\vec{p} \end{pmatrix} + \begin{pmatrix} \hat{V}_{KK} & \hat{V}_{iv} \\ \hat{V}_{iv}^{+} & \hat{V}_{K'K'} \end{pmatrix}$$

intervalley-scattering disorder

sublattice index 'isospin'

$$\begin{pmatrix} \varphi_{A,K} \\ \varphi_{B,K} \\ \varphi_{B,K'} \\ \varphi_{A,K'} \end{pmatrix}$$

valley index

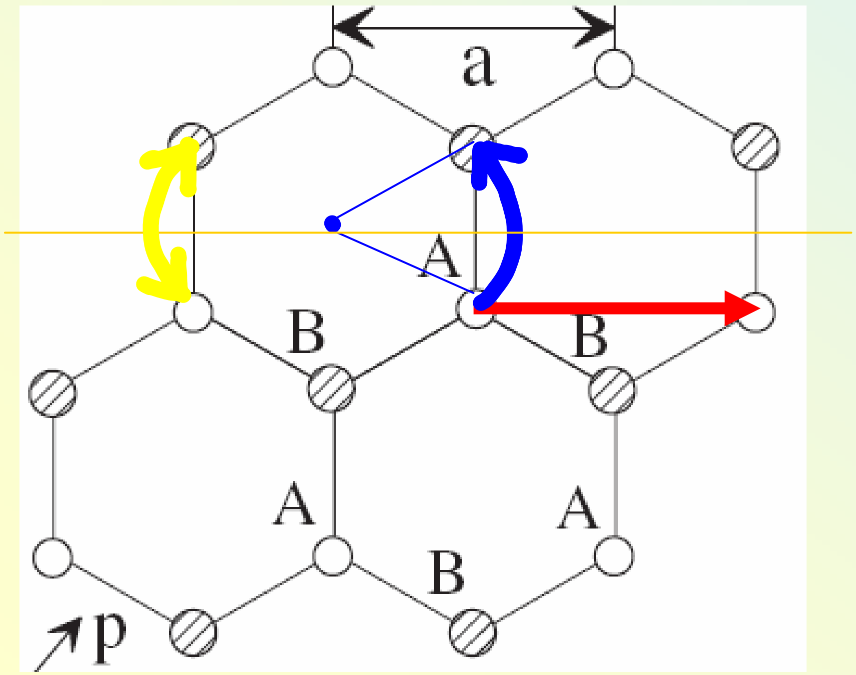


4-dimensional representation of the symmetry group of the honeycomb lattice

$$G\{C_{6v} \otimes T\}$$

$$\begin{matrix} A,K \\ B,K \\ B,K' \\ A,K' \end{matrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix};\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix};\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix};\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Generating elements: $T_{A\to A}, C_{\frac{\pi}{3}}, S_x$



Translation $T_{A\to A}$

$$\begin{pmatrix} e^{i\frac{4\pi}{3}} & & & \\ & e^{i\frac{4\pi}{3}} & & \\ & & e^{-i\frac{4\pi}{3}} & \\ & & & e^{-i\frac{4\pi}{3}} \end{pmatrix}$$

Rotation $C_{\frac{\pi}{3}}$

$A \longleftrightarrow B$

$K \longleftrightarrow K'$

$$\begin{pmatrix} & & e^{i\frac{2\pi}{3}} & \\ & & & e^{-i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} & & & \\ & e^{-i\frac{2\pi}{3}} & & \end{pmatrix}$$

Mirror reflection S_x

$A \longleftrightarrow B$

$$\begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$$

Symmetry operations and transformations of matrices

Generators of the group $G\{T, C_{6v}\}$: $T_{A\to A}, C_{\frac{\pi}{3}}, S_x$

$$\Phi(C_{\pi/3}\vec{r}) = \hat{U}(C_{\pi/3})\Phi(\vec{r})$$

$$\Phi(S_x\vec{r}) = \hat{U}(S_x)\Phi(\vec{r})$$

$$\Phi(T_1\vec{r}) = \hat{U}(T_1)\Phi(\vec{r})$$

$$\Phi = \begin{pmatrix} \mathrm{A,K} \\ \mathrm{B,K} \\ \mathrm{B,K'} \\ \mathrm{A,K'} \end{pmatrix}$$

4-components wave-functions arrange a *4D* irreducible representations of the lattice symmetry group.

$$\hat{X} \to U[\hat{X}] = \hat{U}^{+}\hat{X}\hat{U}$$

The *16D* space of matrices $\hat{\mathrm{X}}$ can be separated into irreducible representations of the symmetry group G

Examples of convenient 4x4 matrices

sublattice 'isospin' matrices:

$$\Sigma_x = \begin{bmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{bmatrix} \quad \Sigma_y = \begin{bmatrix} \sigma_y & 0 \\ 0 & -\sigma_y \end{bmatrix} \quad \Sigma_z = \begin{bmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Sigma_{s_1}, \Sigma_{s_2}] = 2i\varepsilon^{s_1 s_2 s_3} \Sigma_{s_3}$$

valley 'pseudospin' matrices:

$$\Lambda_x = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix} \quad \Lambda_y = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix} \quad \Lambda_z = \begin{bmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Lambda_{l_1}, \Lambda_{l_2}] = 2i\varepsilon^{l_1 l_2 l_3} \Lambda_{l_3}$$

$$[\Sigma_s, \Lambda_l] = 0$$

$$\Phi = \begin{pmatrix} \mathrm{A,K} \\ \mathrm{B,K} \\ \mathrm{B,K'} \\ \mathrm{A,K'} \end{pmatrix}$$

Irreducible matrix representation of $G\{\ T, C_{6v}\}$

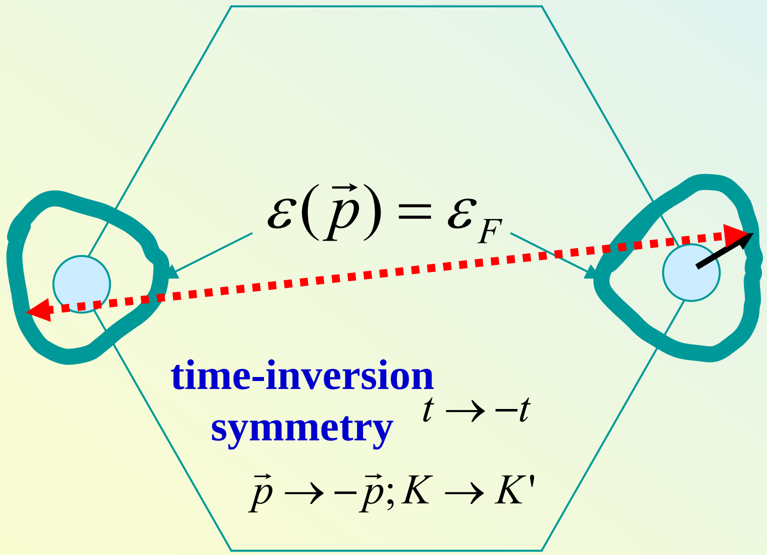
four 1D-representations
four 2D-representations
one 4D-representation

$\Sigma_{(x,y)}\ \Lambda_{(x,y)}$

	$C_{\pi/3}$	s_x	T	
I	1	1	1	A_1
Σ_z	1	-1	1	A_2
$\Lambda_z\Sigma_z$	-1	-1	1	B_1
Λ_z	-1	1	1	B_2

	$C_{\pi/3}$	s_x	T	
$\begin{bmatrix}\Sigma_x\\ \Sigma_y\end{bmatrix}$	$\begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	E_1
$\begin{bmatrix}\Lambda_z\Sigma_x\\ \Lambda_z\Sigma_y\end{bmatrix}$	$\begin{pmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	E_2
$\begin{bmatrix}\Lambda_x\\ \Lambda_y\end{bmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}$	
$\begin{bmatrix}\Lambda_x\Sigma_z\\ \Lambda_y\Sigma_z\end{bmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\begin{pmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}$	

Time-reversal



$$\Phi = \begin{pmatrix} \mathrm{A,K} \\ \mathrm{B,K} \\ \mathrm{B,K'} \\ \mathrm{A,K'} \end{pmatrix}$$

Time inversion of Σ, Λ matrices:

I,	invariant
$\vec{\Sigma}$, $\vec{\Lambda}$	invert signs
$\vec{\Sigma} \otimes \vec{\Lambda}$	invariant

Full basis of symmetry-classified 4x4 matrices

sublattice 'isospin' matrices:

$$\Sigma_x = \begin{bmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{bmatrix} \quad \Sigma_y = \begin{bmatrix} \sigma_y & 0 \\ 0 & -\sigma_y \end{bmatrix} \quad \Sigma_z = \begin{bmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Sigma_{s_1}, \Sigma_{s_2}] = 2i\varepsilon^{s_1 s_2 s_3}\Sigma_{s_3}$$

valley 'pseudospin' matrices:

$$\Lambda_x = \begin{bmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{bmatrix} \quad \Lambda_y = \begin{bmatrix} 0 & -i\sigma_z \\ i\sigma_z & 0 \end{bmatrix} \quad \Lambda_z = \begin{bmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{bmatrix}$$

SU_2 Lie algebra with:

$$[\Lambda_{l_1}, \Lambda_{l_2}] = 2i\varepsilon^{l_1 l_2 l_3}\Lambda_{l_3}$$

$$[\Sigma_s, \Lambda_l] = 0$$

$$t \to -t$$

16 generators of group U4

I,	symmetric
$\vec{\Sigma}, \ \vec{\Lambda}$	invert sign
$\vec{\Sigma} \otimes \vec{\Lambda}$	symmetric

$$\hat{H} = \pm v\vec{\sigma}\cdot\vec{p} - \mu\left((p_x^2 - p_y^2)\sigma_x - 2p_x p_y \sigma_y\right) + \hat{V}(\vec{r})$$

Dirac term ↕ warping term

$$\hat{H} = v\vec{\Sigma}\cdot\vec{p} - \mu\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Lambda_z\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Sigma_x + \hat{I}u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\Sigma_s\Lambda_l$$

general form of time-inversion-symmetric disorder

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler - PRL 97, 146805 (2006)

Microscopic origin of various disorder terms

$$u(r)\hat{I} \qquad \langle uu \rangle \sim \alpha_0$$

Comes from potential of charged impurities in the substrate, deposits on its surface (water-ice) and doping molecules screened by electrons in graphene.

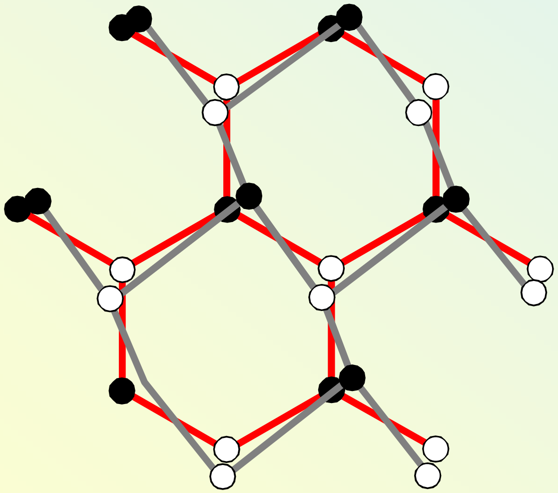
Unavoidable in graphene-based field-effect transistors (GraFETs) in Si/SiO_2 substrate.

Geim and Novoselov - Nature Materials 6, 183 (2007)
Jang, Adam, Chen, Williams, Das Sarma, Fuhrer
PRL 101, 146805 (2008)

Nomura and MacDonald - PRL 96, 256602 (2006)
Cheianov and VF - PRL 97, 226801 (2006)
Nomura and MacDonald - PRL 98, 076602 (2007)
Hwang, Adam, Das Sarma - PRL 98, 186806 (2007)

Lattice deformation – bond disorder

$$V = u_{sz}(r)\Sigma_{s(x,y)}\Lambda_z$$
$$= \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp$$



$$\vec{d} = \vec{d}_A - \vec{d}_B$$
$$\vec{u}_\perp \sim \vec{l}_z \times \vec{d}$$

Fictitious 'magnetic field': $b_{fict} = \mathrm{rot}\,\vec{u}_\perp \neq 0$

Foster, Ludwig - PRB 73, 155104 (2006)
Morpurgo, Guinea - PRL 97, 196804 (2006)

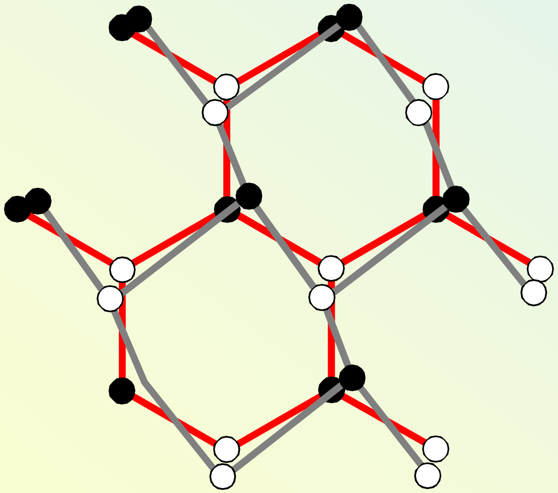
$$\hat{H} = \vec{\Sigma} \cdot \vec{p} + \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp$$
$$= \vec{\Sigma} \cdot (\vec{p} + \Lambda_z \vec{u}_\perp)$$

Suppresses the interference of electrons in one valley, similarly to the warping effect in the band structure.

Lattice deformation – bond disorder

$$\hat{V} = \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp$$

$$\frac{1}{2}\left\langle \vec{u}_\perp^{\,2} \right\rangle = \gamma_\perp$$



$$\vec{d} = \vec{d}_A - \vec{d}_B$$

$$\vec{u}_\perp \sim \vec{l}_z \times \vec{d}$$

$$K \to K'$$

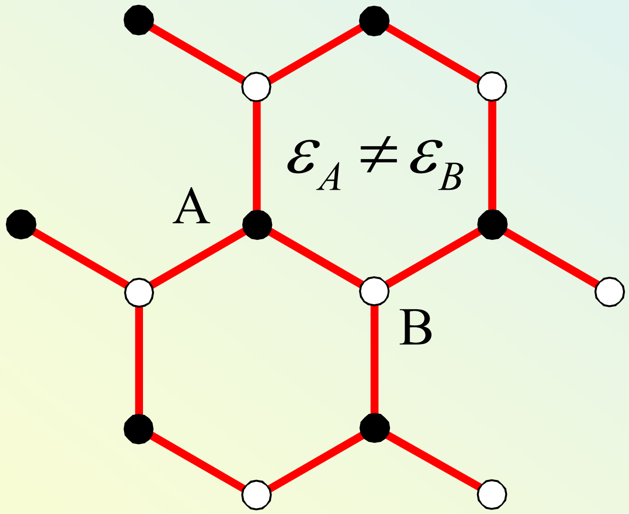
$$b_{fict} \to -b_{fict}$$

Foster, Ludwig - PRB 73, 155104 (2006)
Morpurgo, Guinea - PRL 97, 196804 (2006)

$$\hat{H} = \vec{\Sigma} \cdot \vec{p} + \Lambda_z \vec{\Sigma} \cdot \vec{u}_\perp = \vec{\Sigma} \cdot (\vec{p} + \Lambda_z \vec{u}_\perp)$$

The phase coherence of two electrons propagating in different valleys is not affected (real time-reversal symmetry is preserved).

intra-valley AB disorder



$$\hat{V} = \Lambda_z \Sigma_z u_z(\vec{r})$$

$$u_z \sim \varepsilon_A - \varepsilon_B$$

different energy on A and B sites opens a gap and thus suppresses chiralty of electrons.

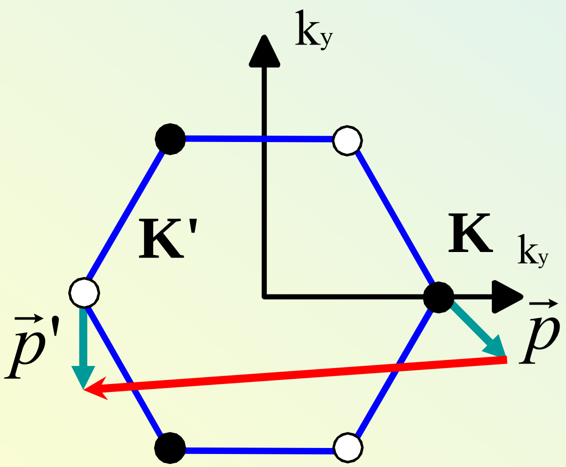
$$\langle u_z^2 \rangle = \gamma_z$$

Intra-valley disorder $\Lambda_z \Sigma_s u_s$ suppresses the interference of electrons in one valley (however, does not affect coherence of a pair of electrons in the opposite valleys)

$$\tau_z^{-1} \equiv \gamma_F(\gamma_z + 2\gamma_\perp)$$

Inter-valley disorder

Deformations similar to those produced by the phonons in the corners of the Brillouin zone. Can be induced by deposits on graphene sheet, points of mechanical contact with the substrate, atomic defects, and sample edges.



$$u_{sl}(r)\Sigma_s\Lambda_l$$

$$s = x, y; \quad l = x, y$$

$$\left\langle u_{sl}^2 \right\rangle = \beta_\perp$$

$$u_{zl}(r)\Sigma_z\Lambda_l$$

$$l = x, y$$

$$\left\langle u_{zl}^2 \right\rangle = \beta_z$$

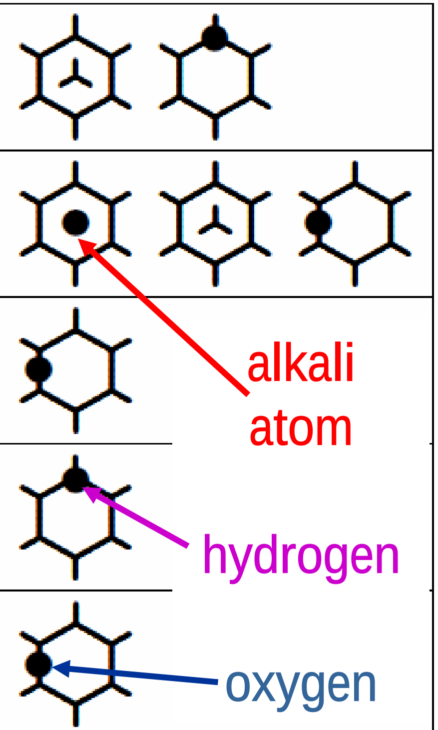
intervalley scattering rate

$$\tau_{iv}^{-1} \sim \gamma_F (4\beta_\perp + 2\beta_z)$$

Symmetry breaking by adatoms

McCann and VF, Phys. Rev. B 71, 085415 (2005)

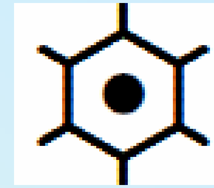
$s\Sigma_z\Lambda_z, \qquad s=\pm 1$	
$\Sigma_z(\mathbf{\Lambda}\cdot\mathbf{v}), \qquad \mathbf{v}\in\mathbb{V}_3$	
$\Lambda_z(\mathbf{\Sigma}\cdot\mathbf{v}), \qquad \mathbf{v}\in\mathbb{V}_3$	
$(\mathbf{\Lambda})_\alpha u_{\alpha\beta}(\mathbf{\Sigma})_\beta, \qquad u\in\mathbb{U}_6$	
$(\mathbf{v}\times\mathbf{\Sigma})(\mathbf{u}\times\mathbf{\Lambda}), \quad \mathbf{u},\mathbf{v}\in\mathbb{V}_3$	



$$\mathbb{V}_3 = \left\{(1,0);\; -\frac{1}{2}\left(1,-\sqrt{3}\right);\; -\frac{1}{2}\left(1,\sqrt{3}\right)\right\}$$

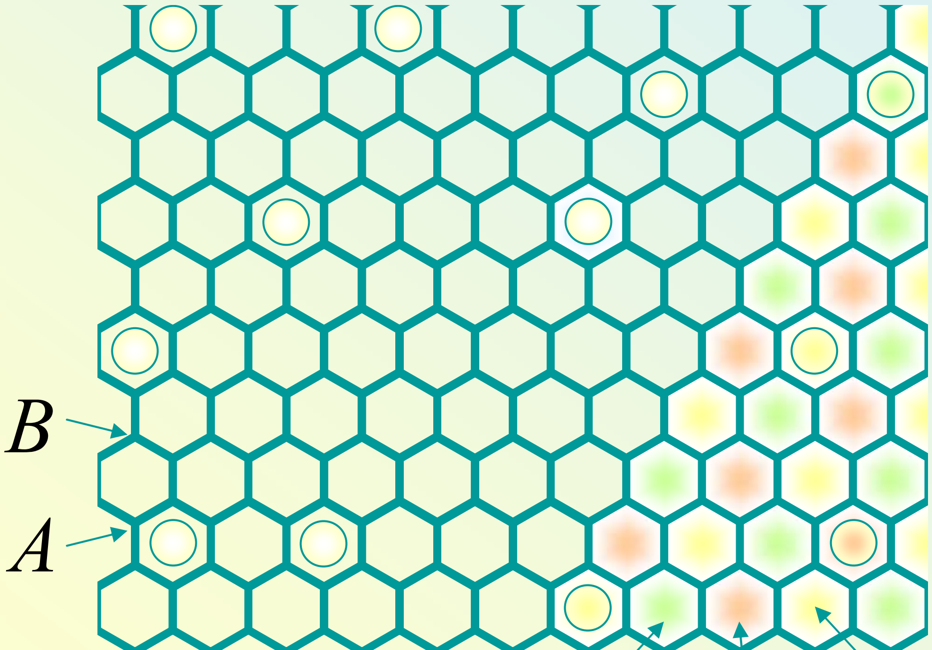
$$\mathbb{U}_6 = \{\, g_1^n g_2^n \mid n=1,\ldots,6\}, \qquad g_1 = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} \qquad g_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Adatom in the middle of hexagon:
AB symmetric but scatters between valleys



$$\alpha, \beta_z \neq 0$$

$$\gamma_z, \gamma_\perp, \beta_\perp = 0$$



$$V_{\odot} = [\hat{I}u + u_{iv} v_l \Lambda_l \Sigma_z] \delta(\vec{r} - \vec{r}_i)$$

$l = x$ **or** y

$$\left\{ (1,0);\ -\frac{1}{2}\left(1, -\sqrt{3}\right);\ -\frac{1}{2}\left(1, \sqrt{3}\right) \right\} \supset \mathbf{v}$$

McCann and VF, Phys. Rev. B 71, 085415 (2005)

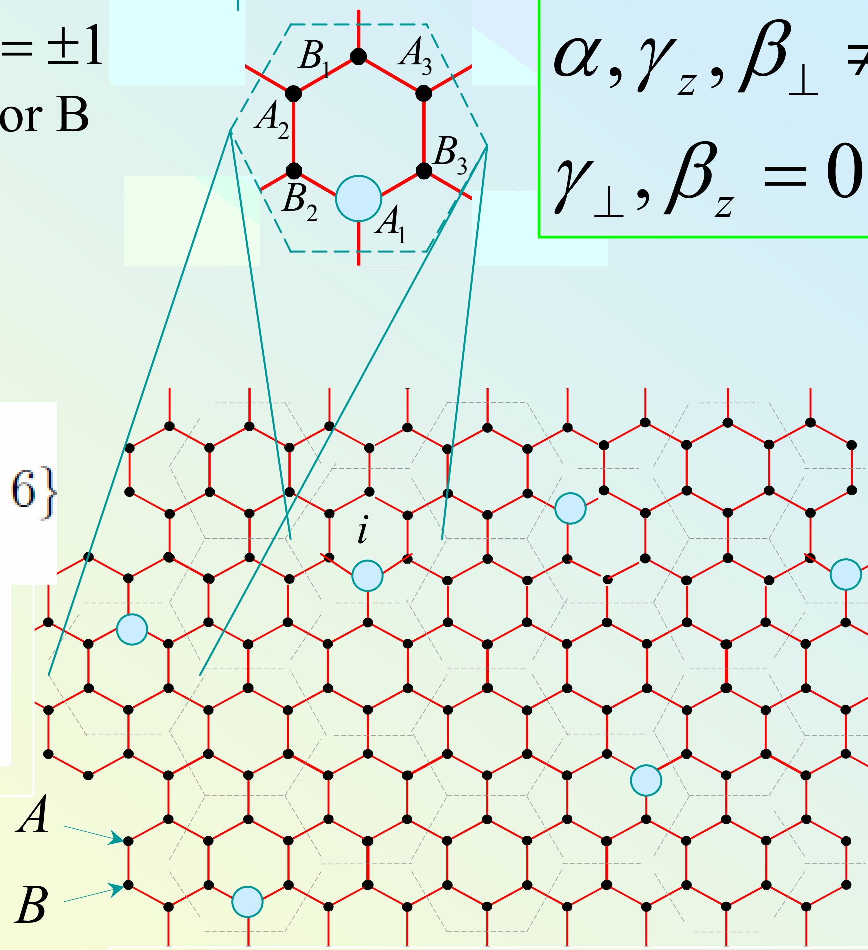
On-site adatom ('hydrogen')

$$V = [u\hat{I} + u_z m_i \Lambda_z \Sigma_z + u_{iv} w_i^{ls} \Lambda_l \Sigma_s]\,\delta(\vec{r}-\vec{r}_i)$$

$m = \pm 1$
A or B

$l, s = x \text{ or } y$

$$\alpha, \gamma_z, \beta_\perp \neq 0$$
$$\gamma_\perp, \beta_z = 0$$

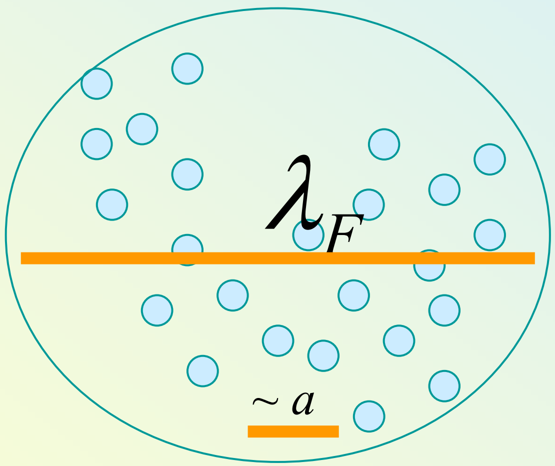


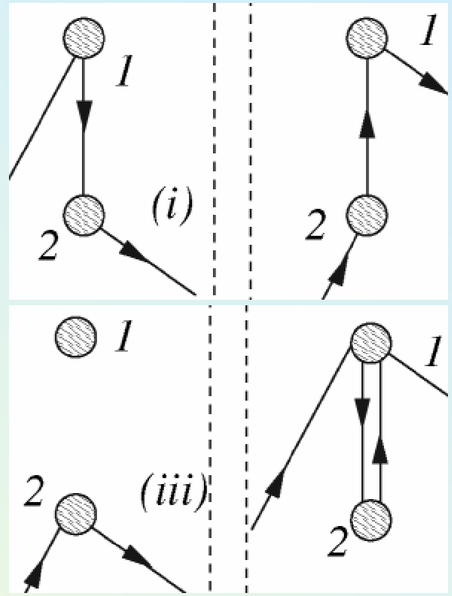
$$w \in \mathbb{U}_6 = \{\, g_1^n g_2^n \mid n = 1, \ldots, 6 \}$$

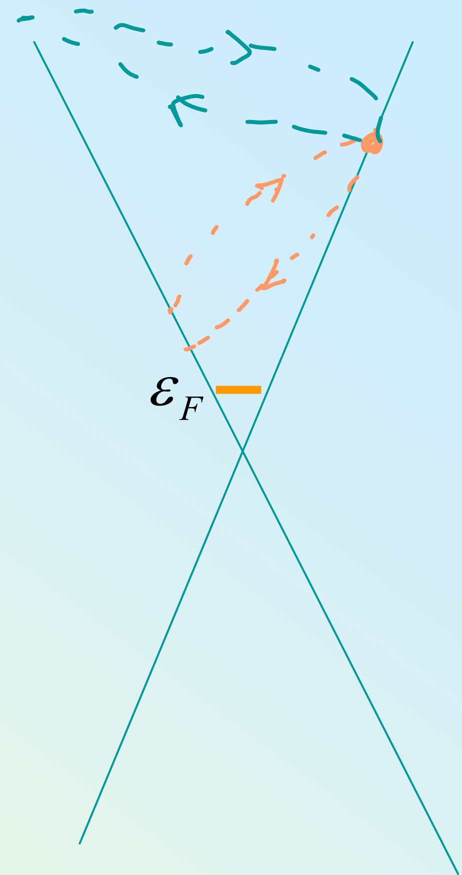
$$g_1 = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$$

$$g_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

McCann and VF, Phys. Rev. B 71, 085415 (2005)





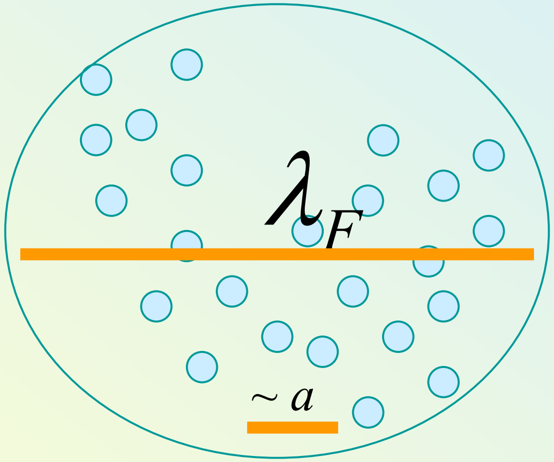


Aleiner, Efetov - PRL 97, 236801 (2006)

Many adatoms: renormalisation of disorder parameters

$$\Gamma(v/a) \Rightarrow \Gamma(\varepsilon') \Rightarrow .... \Rightarrow \Gamma(\varepsilon_F)$$

$$\Gamma = \{\alpha, \gamma_z, \gamma_\perp, \beta_z, \beta_\perp\}$$



Aleiner, Efetov - PRL 97, 236801 (2006)
Ostrovsky, Gornyi, Mirlin, PRB 74, 235443 (2006)
Foster, Aleiner, PRB 77, 195413 (2008)

$$
\begin{aligned}
\dot{\alpha}_0 &= 2\alpha_0(\alpha_0 + \beta_\perp + \gamma_\perp + \beta_z + \gamma_z) + \beta_\perp\beta_z + 2\gamma_\perp\gamma_z\,, \\
\dot{\beta}_\perp &= 4(\alpha_0\beta_z + \beta_\perp\gamma_\perp + \beta_z\gamma_z), \\
\dot{\beta}_z &= 2(\alpha_0\beta_\perp - \beta_z\alpha_0 + \beta_\perp\gamma_z + \beta_z\gamma_z), \\
\dot{\gamma}_\perp &= 4\alpha_0\gamma_z + \beta_\perp^2 + \beta_z^2, \\
\dot{\gamma}_z &= 2\gamma_z(-\alpha_0 - \beta_\perp + \beta_z + \gamma_\perp - \gamma_z) + 2\alpha_0\gamma_\perp + \beta_\perp\beta_z, \\
\dot{\varepsilon} &= \varepsilon(1 + \alpha_0 + \beta_\perp + \gamma_\perp + \beta_z + \gamma_z),
\end{aligned}
$$

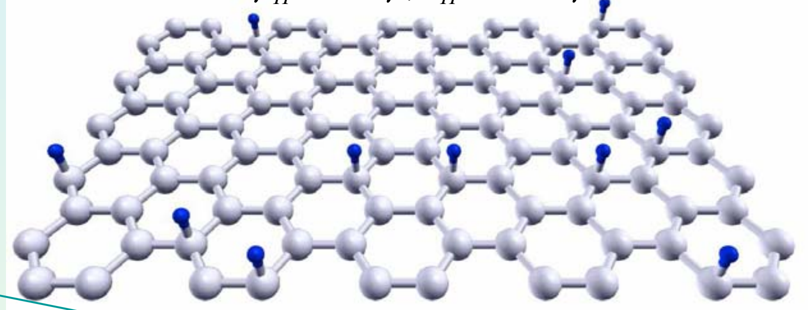
Adsorbate - induced disorder in graphene

$$\mathcal{H} = -\gamma \sum_{\langle l,m \rangle} c_l^\dagger c_m + \sum_n \mathcal{T}_n,$$

$$\mathcal{H}_n = \varepsilon_i d_n^\dagger d_n + \gamma_i (c_{\alpha_n}^\dagger d_n + c_{\alpha_n} d_n^\dagger)$$

$$\mathcal{T} = t_0(\varepsilon) c_\alpha^\dagger c_\alpha, \qquad t_0(\varepsilon) = \frac{\gamma_i^2}{\varepsilon - \varepsilon_i - \gamma_i^2 g_0(\varepsilon)}$$

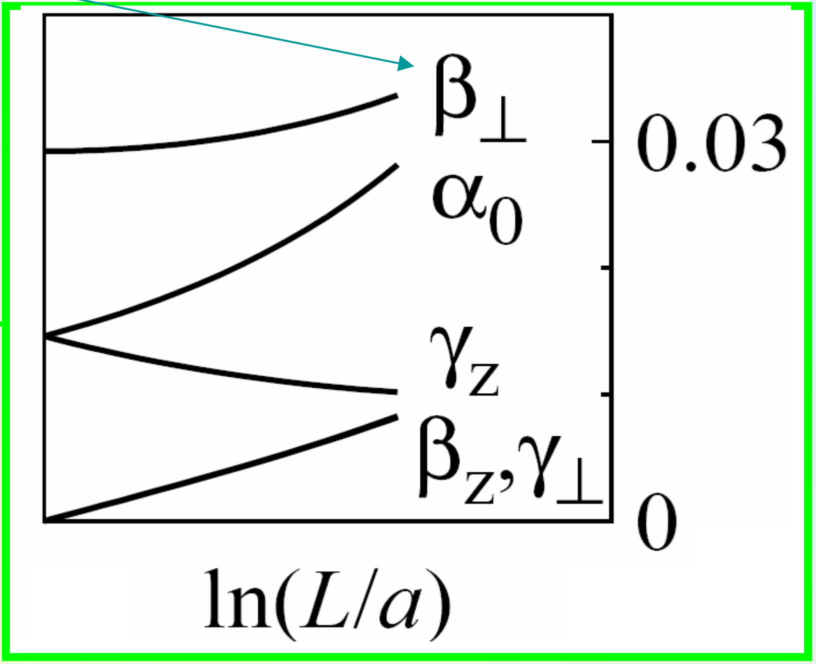
$\gamma_H \sim 2.2\gamma; \varepsilon_H \sim 0.66\gamma$



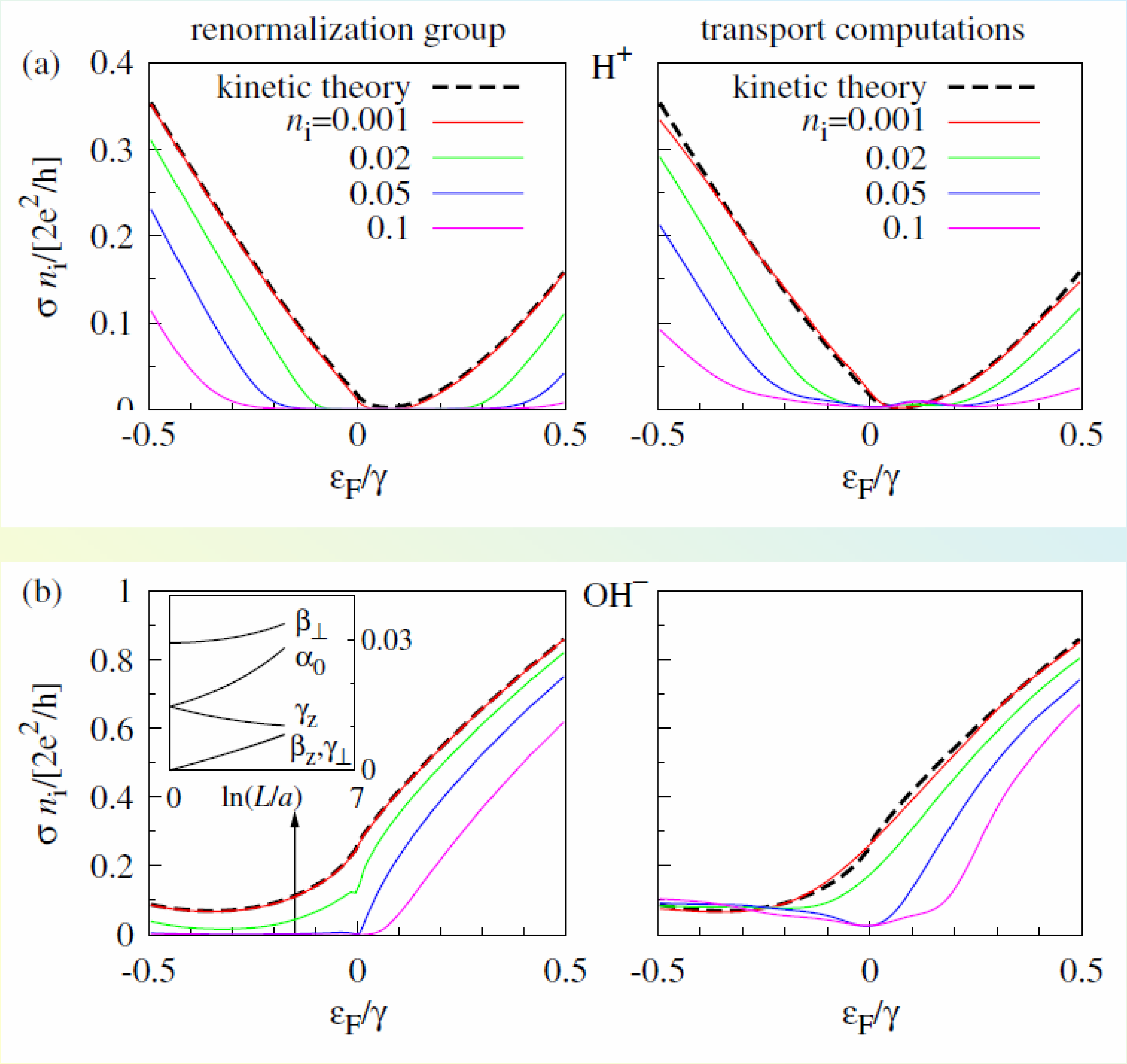
Strong inter-valley scattering

$$\alpha_0 = \gamma_z = \beta_\perp/2 = \frac{A_c n_i |t_0(\varepsilon_F)|^2}{2\pi \langle v_{\mathbf{k}} \rangle_{\varepsilon_F} / \langle v_{\mathbf{k}}^{-1} \rangle_{\varepsilon_F}}$$

$$\beta_z = \gamma_\perp = 0$$



Robinson, Schomerus, Oroszlany, VF
PRL 101, 196803 (2008)



Robinson, Schomerus, Oroszlany, VF - PRL 101, 196803 (2008)

Electronic properties of graphene.

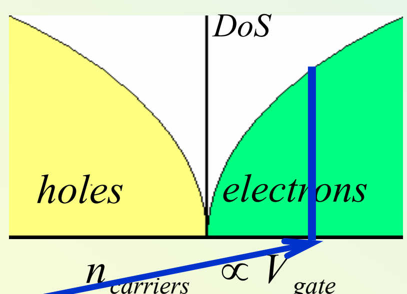
Weak localisation *vs* anti-localisation: qualitative discussion.

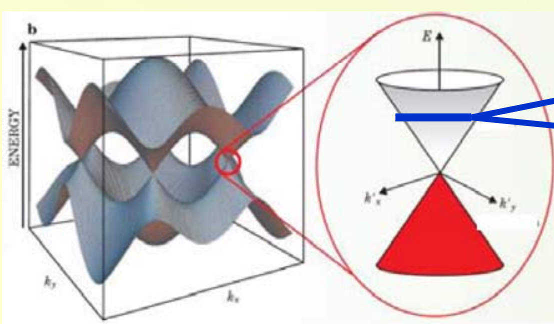
Formal WL analysis taking into account different types of disorder.

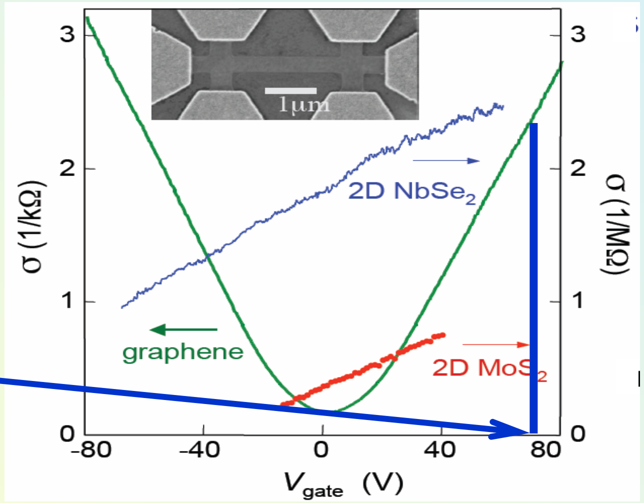
Universal conductance fluctuations.

Metallic (high-density) regime

$p_F l >> 1$ and $\delta n_e << n_e$







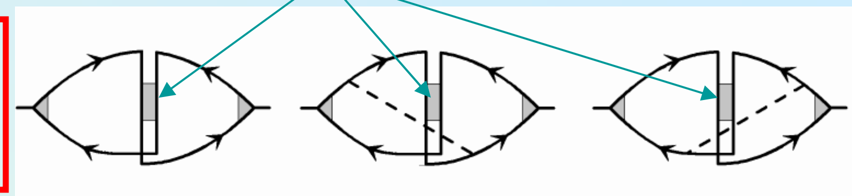
WL correction

SU_2^{Λ}

$$\delta\sigma \sim C_0^x + C_0^y + C_0^z - C_0^0$$

SU_2^{Σ}

$C_s^l(\vec{r},\vec{r}\,')$



Particle-particle correlation function ‘Cooperon’

$$\left(D\nabla_{\vec{r}}^2 - i\omega + \tau_{sl}^{-1}\right)C_s^l(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$$

τ_{sl}^{-1} relaxation rate of the corresponding ‘Cooperon’

$$\hat{H} = v\vec{\Sigma}\cdot\vec{p} - \mu\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Lambda_z\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Sigma_x + \hat{I}\,u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\Sigma_s\Lambda_l$$

leading terms do not contain valley operators Λ , thus, they remain invariant with respect to valley transformations SU_2^{Λ}: this allows for all four Cooperons.

All types of symmetry breaking disorder

$$\hat{H} = v\vec{\Sigma}\cdot\vec{p} - \mu\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Lambda_z\Sigma_x\left(\vec{\Sigma}\cdot\vec{p}\right)\Sigma_x + \hat{I}\,u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\Sigma_s\Lambda_l$$

Trigonal warping

$$\tau_w^{-1}$$

inter-valley + intra-valley disorder

$$\tau_{iv}^{-1} + \tau_z^{-1}$$

inter-valley disorder

$$\tau_{iv}^{-1} \propto \beta$$

same valley | inter-valley

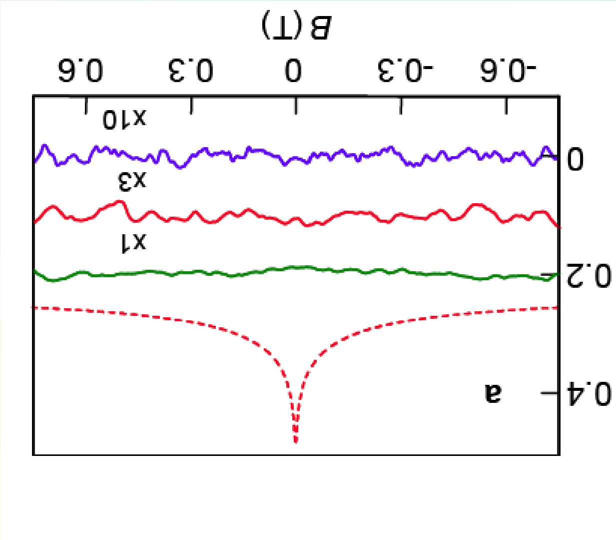
$$\delta\sigma \sim C_0^x + C_0^y + C_0^z - C_0^0$$

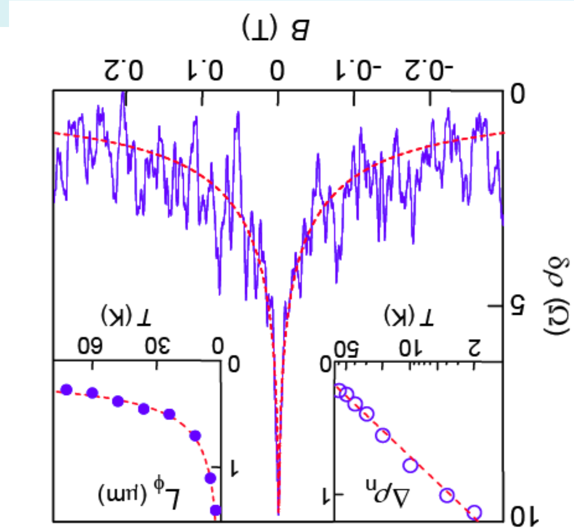
$$\tau_*^{-1} = \tau_w^{-1} + \tau_z^{-1} + \tau_{iv}^{-1}$$

The only surviving mode

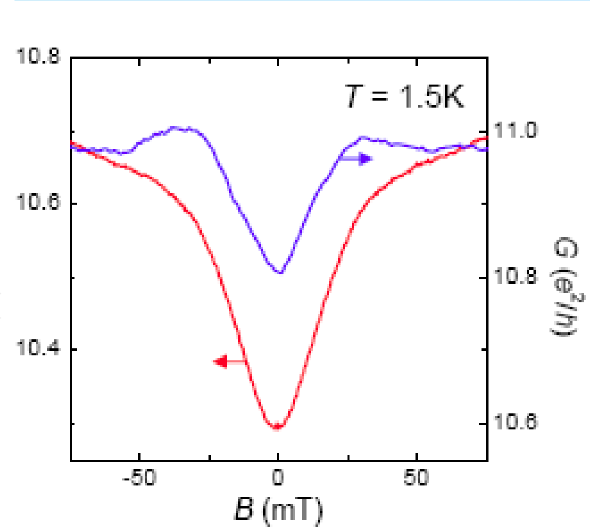
$$\tau_\varphi^{-1} << \tau_w^{-1},\ \ \tau_{iv}^{-1},\ \ \tau_z^{-1}$$

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler PRL 97, 146805 (2006)

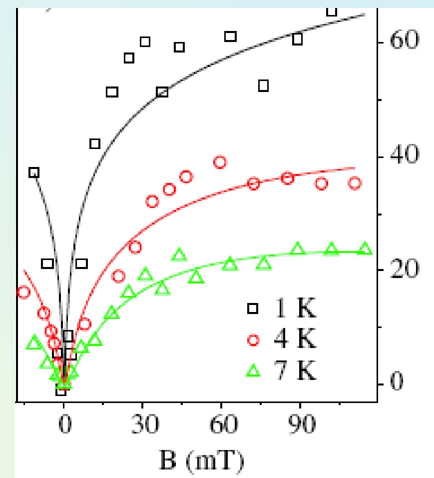




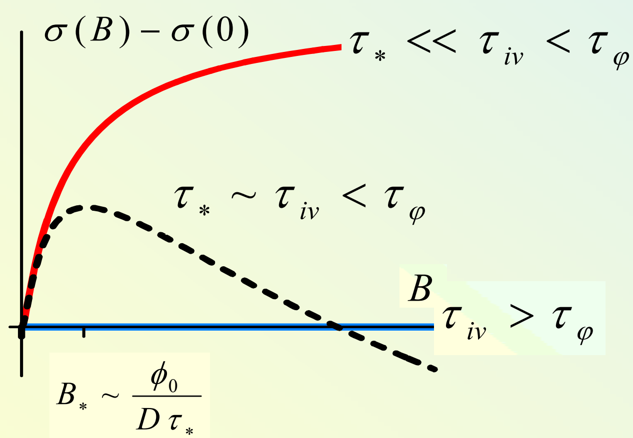
Morozov et al, PRL 97, 016801 (2006)



Heersche et al,
Nature 446, 56-59 (2007)



Tikhonenko *et al*
PRL 100, 056802 (2008)



$$\Delta\sigma \sim \frac{e^2}{\pi h}\left(F\left(\tfrac{B}{B_\varphi+2B_{iv}}\right)+2F\left(\tfrac{B}{B_\varphi+B_*}\right)-F\left(\tfrac{B}{B_\varphi}\right)\right)$$

$$F(z)=\ln z+\psi\left(\tfrac{1}{2}+z^{-1}\right)$$

McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler, PRL 97, 146805 (2006)

PRL **100,** 056802 (2008) PHYSICAL REVIEW LETTERS week ending 8 FEBRUARY 2008

Weak Localization in Graphene Flakes

F. V. Tikhonenko, D. W. Horsell, R. V. Gorbachev, and A. K. Savchenko
School of Physics, University of Exeter, Stocker Road, Exeter, EX4 4QL, United Kingdom

WL used to test ‘what type’ of disorder:

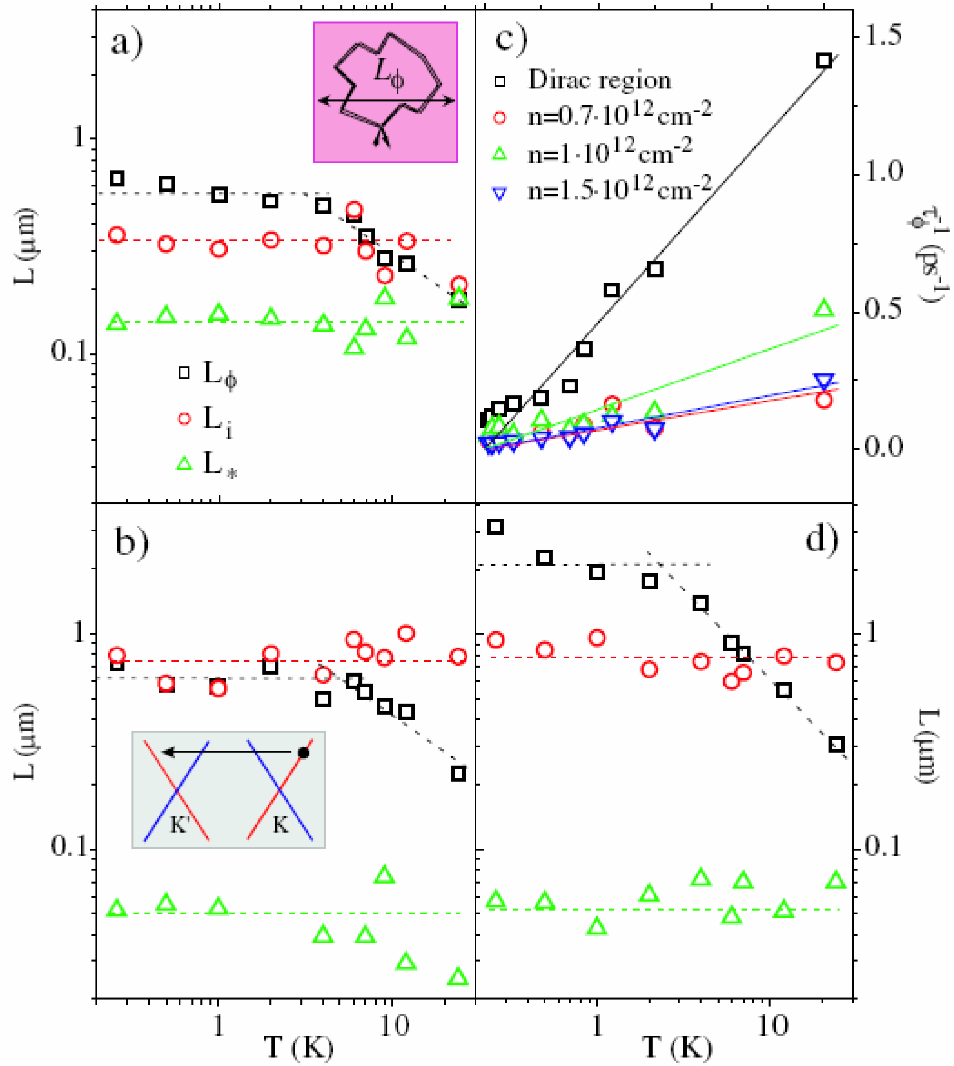
$$L_i >> L_* > l$$

(substrate charge seems to dominate in the resistance of GraFETs on Si/SiO_2).

$$L_i = \sqrt{\tau_{iv} D}$$

$$L_* = \sqrt{\tau_* D}$$

$$\tau_*^{-1} = \tau_w^{-1} + \tau_z^{-1} + \tau_{iv}^{-1}$$



Narrow ribbons of graphene feature strong inter-valley scattering due to edges and thus shows robust WL magnetoresistance.

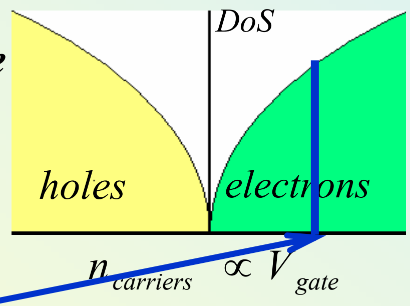
McCann, Kechedzhi, VF, Suzuura, Ando, Altshuler, PRL 97, 146805 (2006)

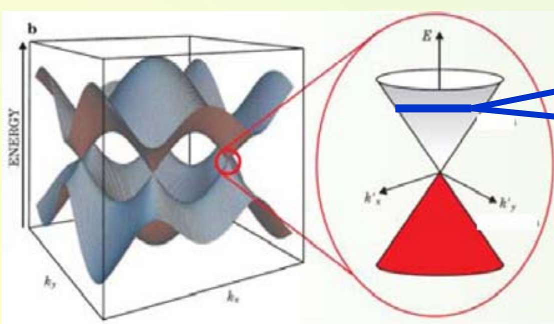
Symmetry and quantum transport in disordered graphene

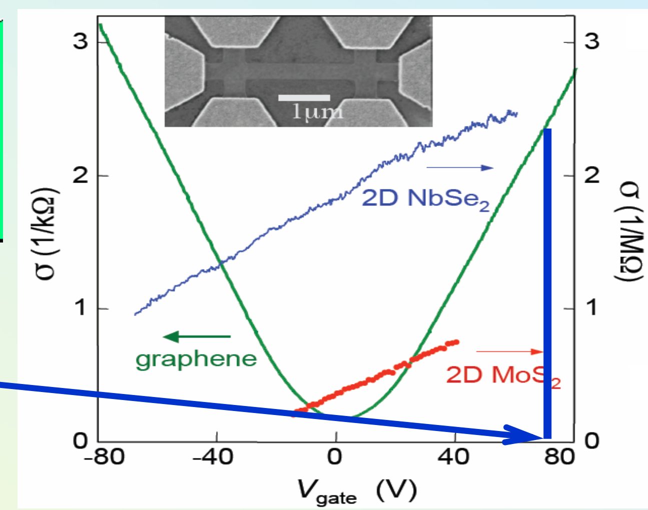
QT is strongly effected both by graphene band structure and the type of disorder in the sample.

Measurements of weak localisation can be used as a tool to grade the efficiency of different types of disorder in material.

$p_F l >> 1$ and $\delta n_e << n_e$

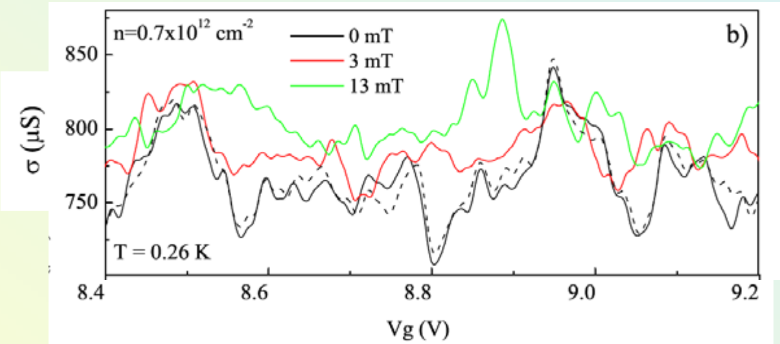




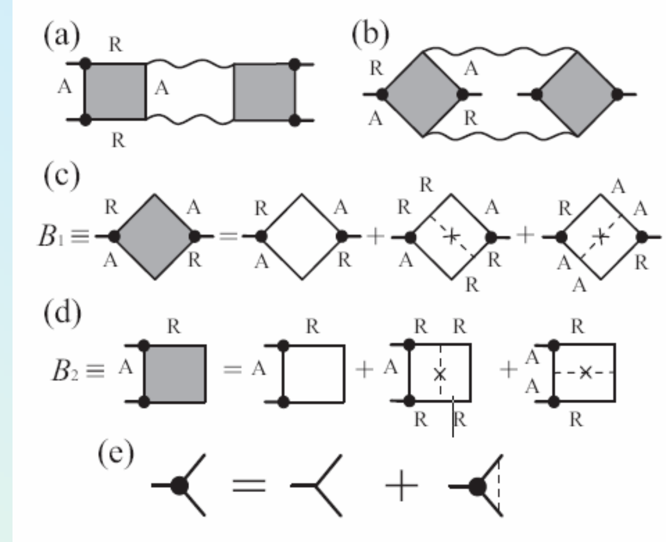


Universal conductance fluctuations and correlation function thermometry of graphene.

UCF in graphene



Tikhonenko et.al. PRL (2008)



inter-valley

same valley

$$\langle \delta \mathcal{G}^2 \rangle \sim \left|D_{KK}\right|^2 + \left|D_{KK'}\right|^2 + \left|D_{KK(valley-anti-symm)}\right|^2 + \left|D_{KK(valley-symm)}\right|^2$$

$$\tau_*^{-1} = \tau_w^{-1} + \tau_z^{-1} + \tau_{iv}^{-1}$$

$$\tau_{iv}^{-1}$$

$$\tau_\varphi^{-1} << \tau_w^{-1},\ \tau_{iv}^{-1},\ \tau_z^{-1}$$

Diffusion pole: $\left(D\nabla_{\vec{r}}^2 - i\omega + \tau_{sl}^{-1}\right) D_s^l(\vec{r},\vec{r}') = \delta(\vec{r}-\vec{r}')$

Wide (2D) graphene sheet, 1<**α**<4

$$\langle \delta \mathcal{G}^2 \rangle = \alpha \frac{3\zeta(3)}{2\pi^3} \frac{L_y}{L_x} \left(\frac{2e^2}{h}\right)^2$$

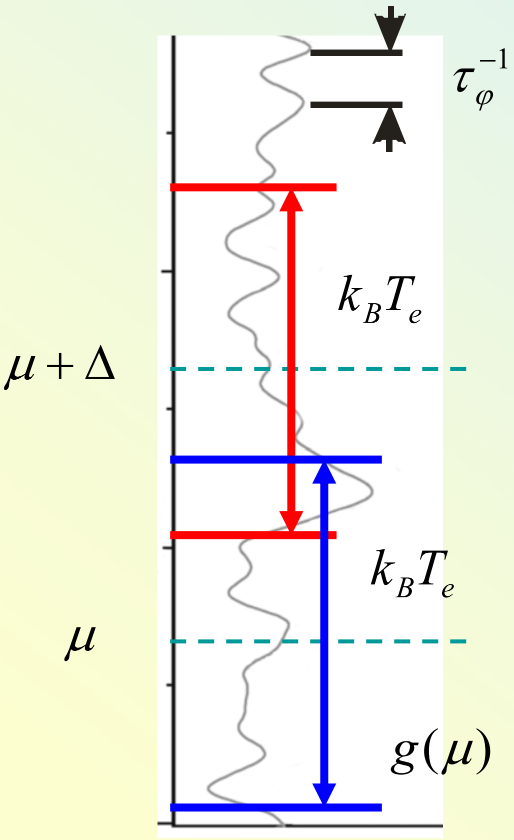
Kechedzhi, Kashuba, VF
PRB 77, 193403 (2008)

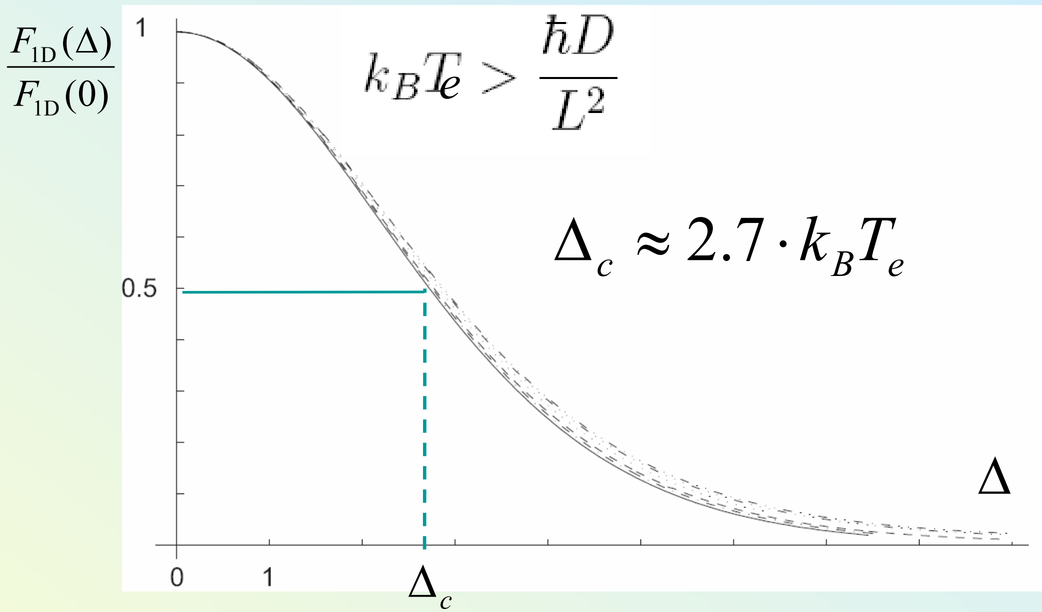
Narrow ribbon, **α**=1

$$\langle \delta \mathcal{G}^2 \rangle = \frac{1}{15} \left(\frac{2e^2}{h}\right)^2$$

Correlation thermometry function of graphene ribbons

$$F(\Delta) \equiv \left\langle \delta\, g(\mu)\delta\, g(\mu+\Delta)\right\rangle$$



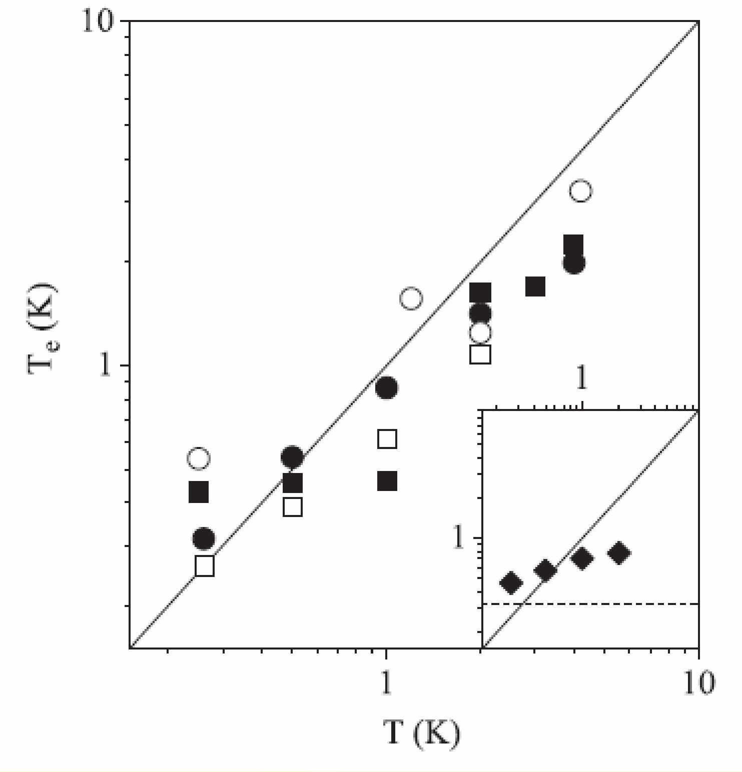


$$k_B T_e \gg \frac{\hbar}{\tau_\varphi} \gg \frac{\hbar D}{L^2}$$

$$F_{1D}(\Delta) \approx 2\pi\left(\frac{\hbar D}{L^2}\right)\frac{L_\varphi}{L}\int d\varepsilon\; f'(\varepsilon,\mu) f'(\varepsilon,\mu+\Delta)$$

Correlation thermometry function of graphene ribbons

Width at half maximum of the correlation function of UCF



Kechedzhi, Horsell, Tikhonenko, Savchenko, Gorbachev, Lerner, VF - PRL 102, 066801 (2009)

Sample	L	W	L_φ
F2	3.8	1.8	3.8
B1	3.7	0.3	2.7

$$k_B T > \frac{\hbar D}{L^2}$$

Electron temperature from the correlation function $\approx$ bath temperature (in low current measurements)

Correlation function of UCF can be used to measure temperature of electrons in graphene ribbons

Summary: Quantum transport in disordered graphene

a) QT is strongly effected both by graphene band structure and the type of disorder in the sample.

a) Measurements of weak localisation enable one to grade the efficiency of different types of disorder in material.

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c) Universal conductance fluctuations can be used for the correlation function thermometry of graphene.