

Electronic properties of corrugated graphene.

F. Guinea

Instituto de Ciencia
de Materiales de Madrid

Consejo Superior de Investigaciones Científicas

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KITP Graphene week.
Apr. 13-17, 2009

- Midgap states in graphene quantum dots (with A. Akhmerov)
- Effective electric fields (with F. von Oppen, E. Mariani)
- Effective and real magnetic fields in suspended graphene samples (with M. Fogler, M. I. Katsnelson, G. León, E. Prada, P. San José, A. K. Geim)

Localized states at edges

M. Fujita, K. Wakabayashi, K. Nakada, and K. Kusakabe,
Journ. Phys. Soc. Jap. **65**, 1920 (1996)

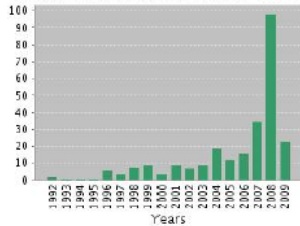
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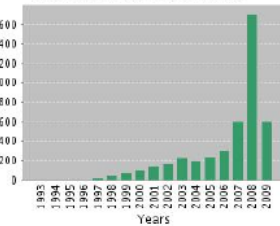
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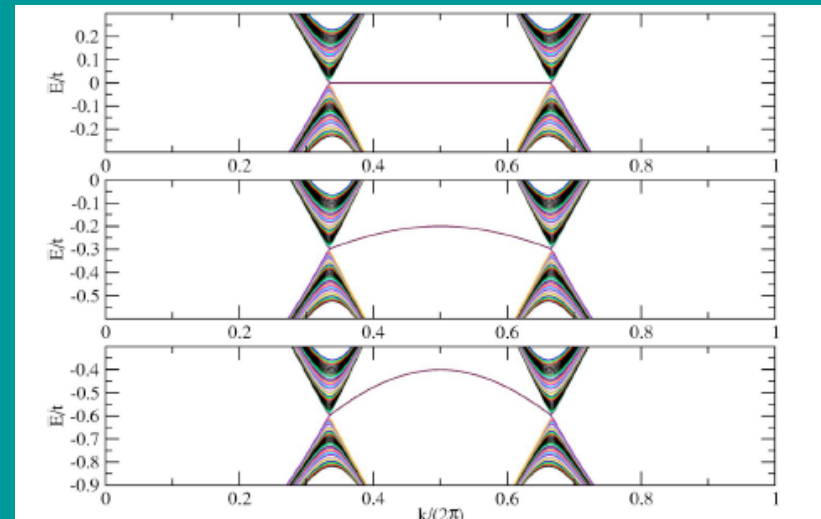
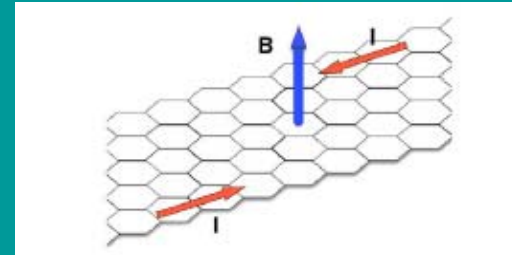
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N. M. R. Peres, F. G., and A. H. Castro Neto, Phys. Rev. B **73**, 125411 (2006)

Boundary conditions at edges

A. R. Akhmerov and C. W. J. Beenakker, Phys. Rev. B **77**, 085423 (2008)

See also:

M. V. Berry and R. J. Mondragon, Proc. R. Soc. London Ser. A **412**, 53 (1987),

E. McCann and V. I. Fal'ko, J. Phys.: Condens. Matter **16**, 2371, (2004),

L. Brey and H. A. Fertig Phys. Rev. B **73**, 235411 (2006)

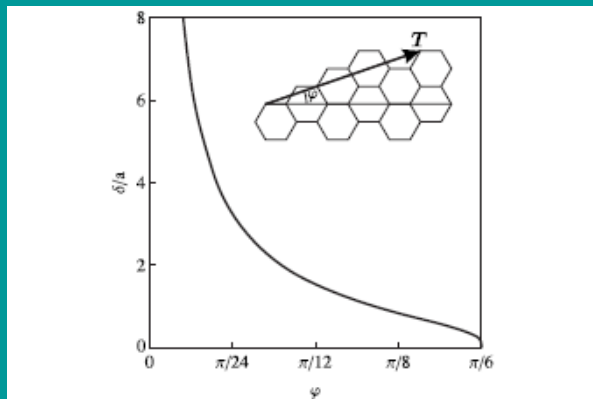


FIG. 2. Dependence on the orientation φ of the distance δ from the boundary within which the zigzag-type boundary condition breaks down. The curve is calculated from formula (3.14) valid in the limit $|T| \gg a$ of large periods. The boundary condition becomes precise upon approaching the zigzag orientation $\varphi = \pi/6$.

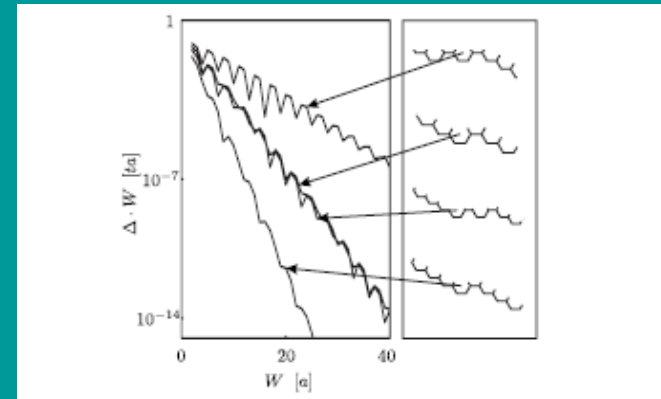


FIG. 7. Dependence of the band gap Δ of zigzaglike nanoribbons on the width W . The curves in the left panel are calculated numerically from the tight-binding equations. The right panel shows the structure of the boundary, repeated periodically along both edges.

From A. R. Akhmerov and C. W. J. Beenakker, Phys. Rev. B **77**, 085423 (2008)

Experiments

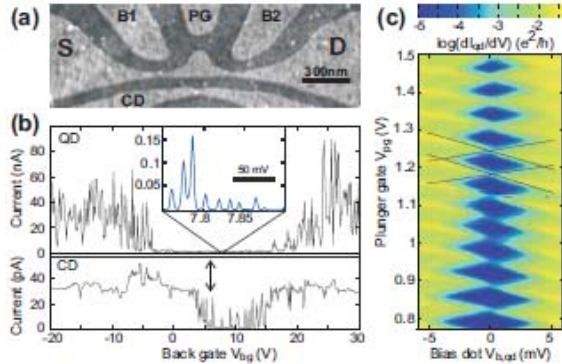


FIG. 1: (Color online) Nanostructured graphene quantum dot device with nanoribbon and characteristic transport measurements. (a) Scanning force micrograph of the measured devices. The central island is connected to source (S) and drain (D) contacts by two constrictions. The diameter of the dot is 200 nm and the constrictions are 35 nm wide. The graphene nanoribbon acts as charge detector (CD). Three lateral gates B1, B2 and PG are used to tune the devices. (b) Back gate characteristics of the quantum dot (QD) device (upper panel) and the charge detector (lower panel), shown in (a). Both measurements were performed at a source-drain (bias) voltage of $V_{b,qd} = V_{b,cd} = 500 \mu\text{V}$ and at 1.7 K. Inset shows Coulomb blockade resonances observed inside the transport gap as a function of the back gate voltage over a range of 150 mV. (c) Coulomb blockade diamonds in differential conductance (logarithmic scale) recorded as function of the plunger gate and bias voltage with fixed back gate voltage $V_{bg} = 2 \text{ V}$. The charging energy is estimated to be $E_c \approx 4.3 \text{ meV}$.

J. Güttinger, C. Stampfer, S. Hellmüller, F. Molitor, T. Ihn and K. Ensslin, *Appl. Phys. Lett.* **93**, 212102 (2008), see also:

C. Stampfer, J. Güttinger, F. Molitor, D. Graf, T. Ihn, and K. Ensslin, *Appl. Phys. Lett.*, **92**, 012102 (2008).

C. Stampfer, E. Schurtenberger, F. Molitor, J. Güttinger, T. Ihn, and K. Ensslin, *Nano Lett.* **8**, 2378 (2008).

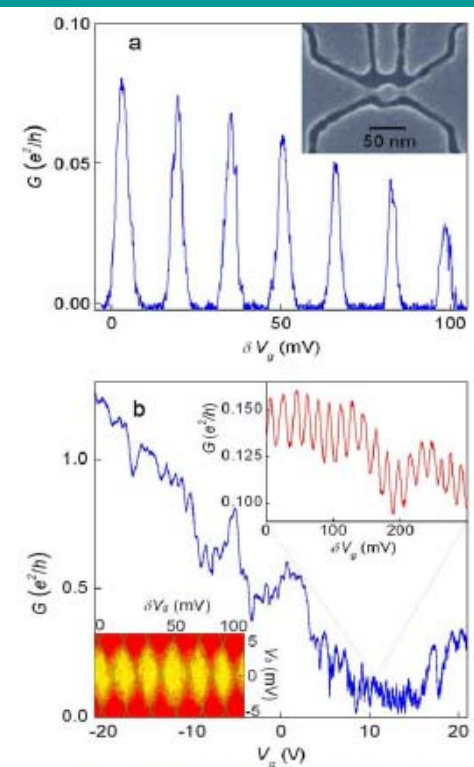
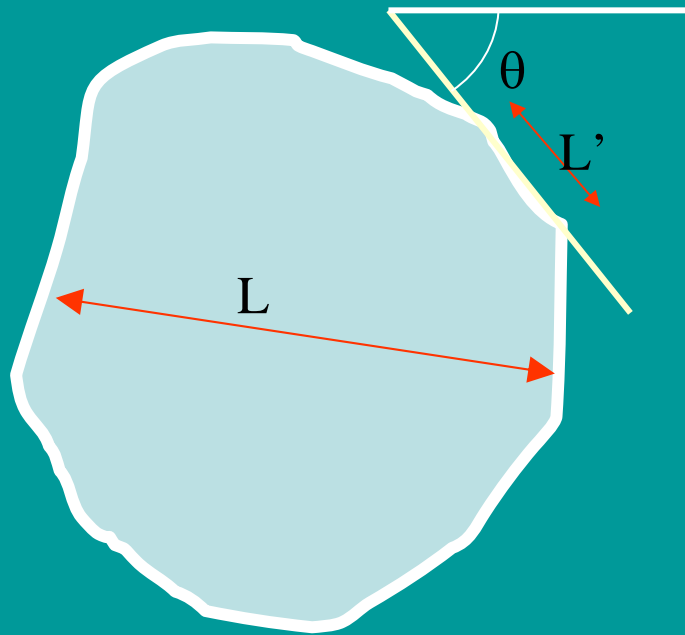


Figure 1. Graphene-based single-electron transistor. a, Conductance G of a device with the central island of 250nm in diameter and distant side gates [SI] as function of V_g in the vicinity of +15V; $T=0.3\text{K}$. The inset shows one of our smaller devices to illustrate the high resolution of our electron-beam lithography that allows features down to 10 nm. Dark areas in the scanning electron micrograph are gaps in the PMMA mask so that graphene is removed from these areas by plasma etching. In this case, a 30-nm QD is connected to contact regions through narrow constrictions and there are 4 side gates. b, Conductance of the same device as in Fig. 1a over a wide range of V_g ($T=4\text{K}$). Upper inset: Zooming into the low- G region reveals hundreds of CB oscillations. The lower inset shows Coulomb diamonds: differential conductance $G_{diff} = dI/dV$ as a function of V_g (around +10V) and bias V_b (yellow-to-red scale corresponds to G_{diff} varying from zero to $0.3e^2/h$; note that our color diagrams often appear smudged if printed in gray).

L. A. Ponomarenko, F. Schedin, M. I. Katsnelson, R. Yang, E. H. Hill, K. S. Novoselov, A. K. Geim, *Science* **320**, 356 (2008)

Ballistic quantum dots

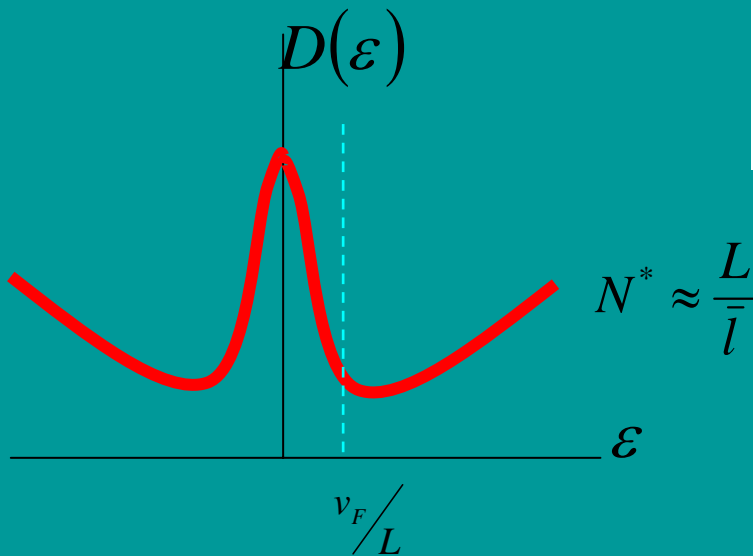


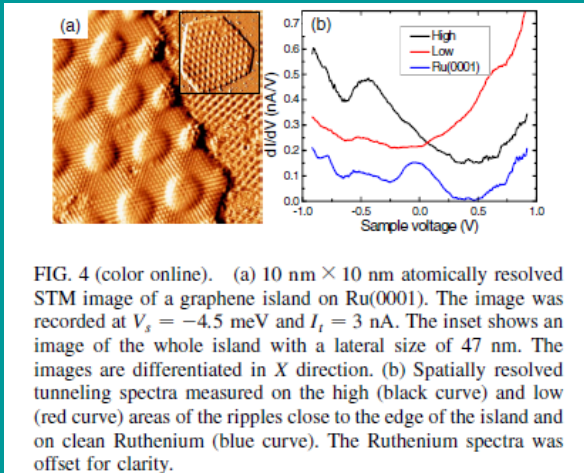
$$\Psi_i(r_{\parallel}, r_{\perp}) \propto e^{ik_i r_{\parallel}} e^{-\lambda_i r_{\perp}}$$

$$\varepsilon_i \approx \frac{v_F}{L} e^{-\lambda_i L'}$$

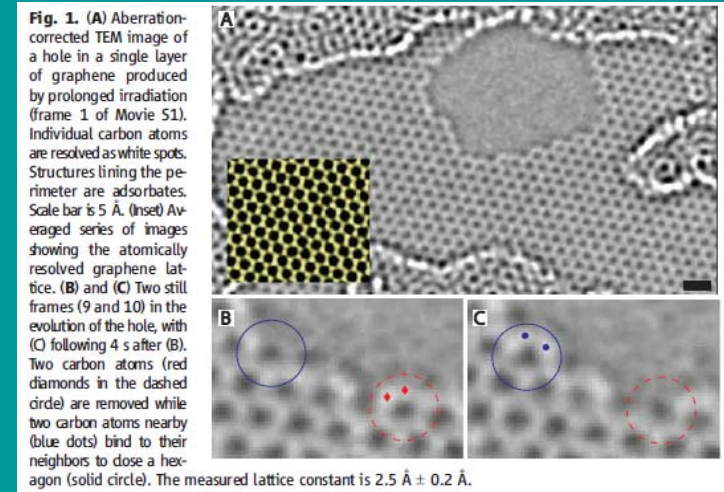
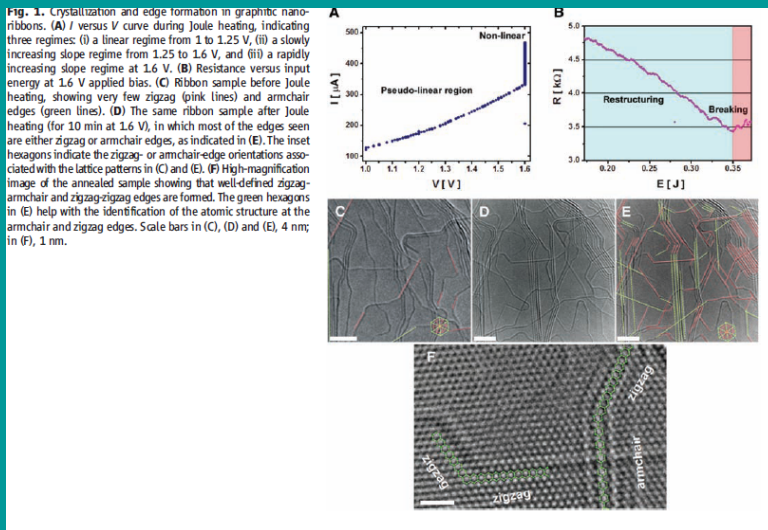
$$\lambda_i \approx c(\theta) \frac{2\pi}{L'}, \quad c(\theta) \frac{4\pi}{L'}, \dots, \frac{1}{l(\theta)}$$

$$D(\varepsilon) = \sum_i \delta(\varepsilon - \varepsilon_i) \approx \begin{cases} \frac{1}{\varepsilon} & \frac{v_F}{L} e^{-L/\bar{l}} \leq \varepsilon \leq \frac{v_F}{L} \\ \frac{\varepsilon L^2}{v_F^2} & \frac{v_F}{L} \leq \varepsilon \end{cases}$$





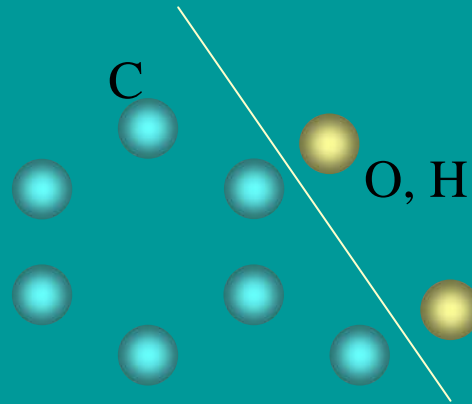
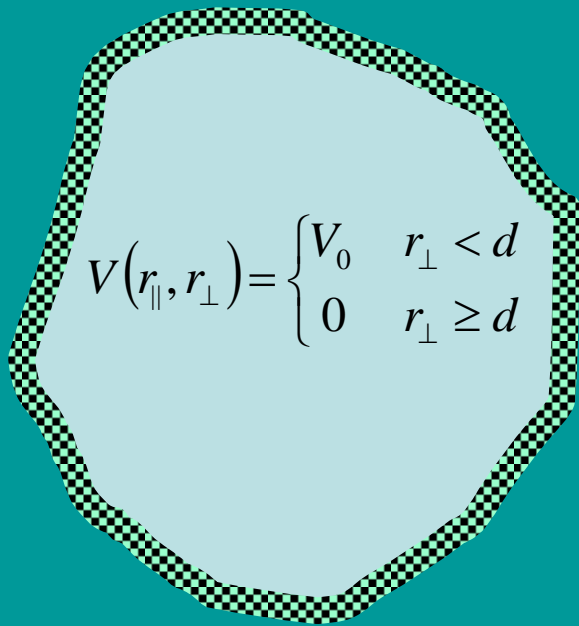
A. L. Vázquez de Parga, F. Calleja, B. Borca, M. C. G. Passeggi, Jr., J. J. Hinarejos, F. G., and R. Miranda, Phys. Rev. Lett. **100**, 056807 (2008)



X. Jia, M. Hofmann, V. Meunier, B. G. Sumpter, J. Campos-Delgado, J. M. Romo-Herrera, H. Son, Y.-P. Hsieh, A. Reina, J. Kong, M. Terrones, M. S. Dresselhaus, Science **323**, 1701 (2009)

Ç. Ö. Girit, J. C. Meyer, R. Erni, M. D. Rossell, C. Kisielowski, L. Yang, C.-H. Park, M. F. Crommie, M. L. Cohen, S. G. Louie, A. Zettl, Science **323**, 1705 (2009)

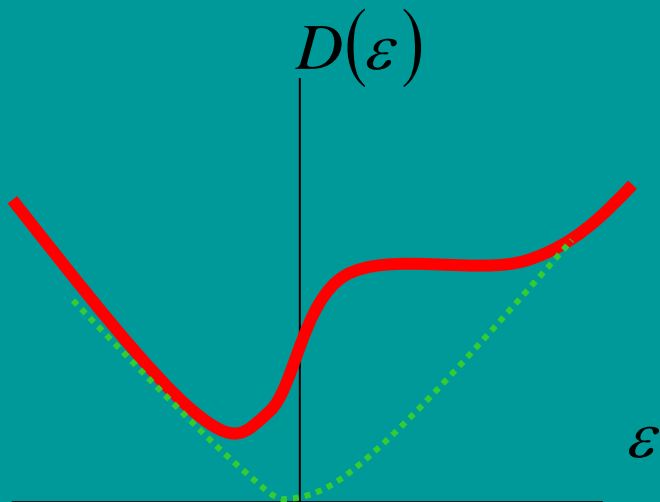
Ballistic quantum dots. Edge potentials



$$\varepsilon_i = \frac{v_F}{L} e^{-\lambda_i L} + V_0 \lambda_i d + t' a^2 \lambda_i^2$$

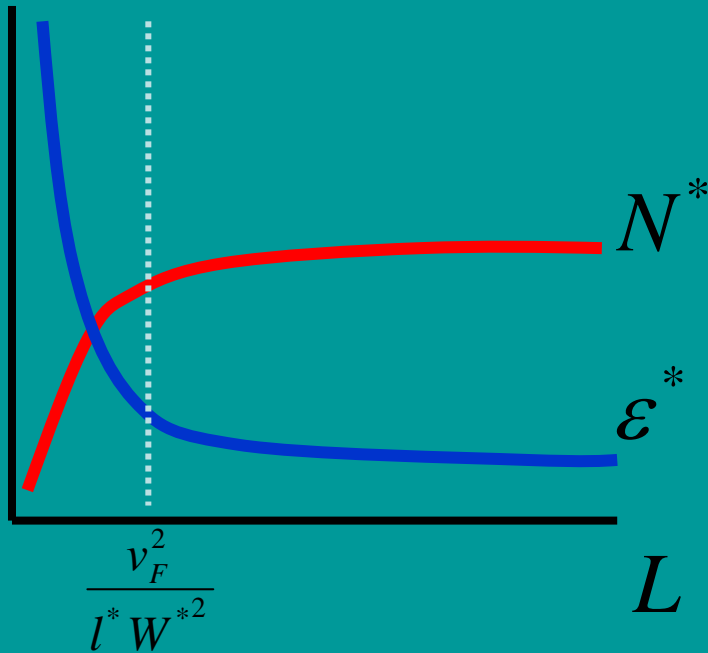
$$W^* = \text{Max}(V_0, t')$$

$$l^* = \text{Max}(\bar{l}, d)$$



$$D(\varepsilon) \approx \begin{cases} \frac{L}{l^*} \frac{1}{W^*} + \frac{\varepsilon L^2}{v_F^2} & 0 \leq \varepsilon \leq W^* \\ \frac{\varepsilon L^2}{v_F^2} & W^* \leq \varepsilon \end{cases}$$

Ballistic quantum dots. Localized states



There are approximately N^* states localized at the edges, within a range ϵ^* of the Dirac point

$$\epsilon^* \approx \text{Min} \left(W^*, \frac{v_F^2}{L l^* W^*} \right)$$

$$N^* \approx \text{Min} \left(\frac{L}{l^*}, \frac{v_F^2}{l^{*2} W^{*2}} \right)$$

$$N_{\text{ribbon}}^* \approx \text{Min} \left(\frac{L}{l^*}, \frac{v_F^2}{l^{*2} W^{*2}} \frac{L}{W_{\text{ribbon}}} \right)$$

$$L \approx 1 \mu m$$

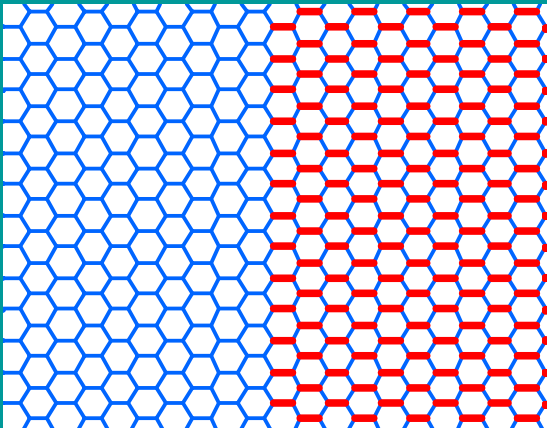
$$l^* \approx 0.2 nm$$

$$W^* \approx 0.3 eV$$

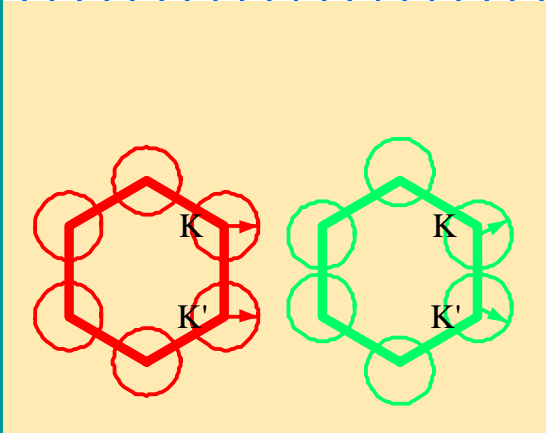
$$N^* \approx 100$$

$$\epsilon^* \approx 6 meV$$

Effective gauge fields

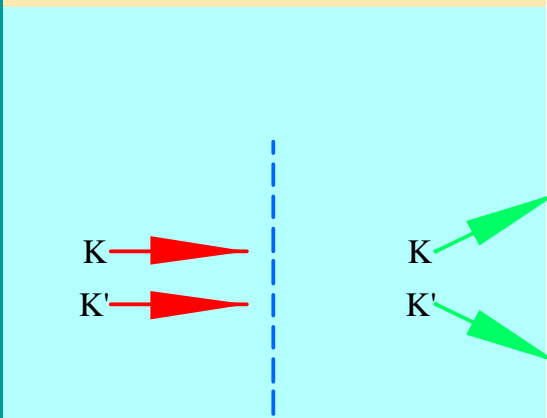


$$H \equiv \begin{pmatrix} 0 & t_1 e^{i\vec{k}_1 \vec{a}_1} + t_2 e^{i\vec{k}_2 \vec{a}_2} + t_3 e^{i\vec{k}_3 \vec{a}_3} \\ t_1 e^{-i\vec{k}_1 \vec{a}_1} + t_2 e^{-i\vec{k}_2 \vec{a}_2} + t_3 e^{-i\vec{k}_3 \vec{a}_3} & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{3\bar{t}a}{2}(k_x + ik_y) + \Delta t \\ \frac{3\bar{t}a}{2}(k_x + ik_y) + \Delta t & 0 \end{pmatrix}$$

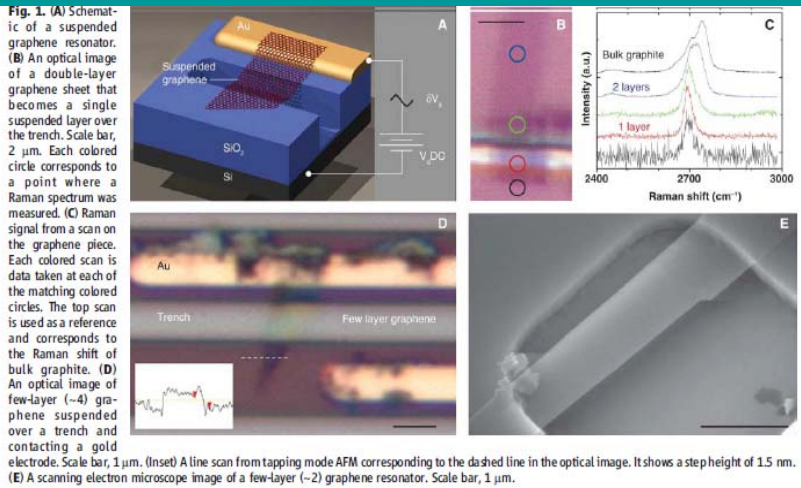


A modulation of the hoppings leads to a term which modifies the momentum: an effective gauge field.

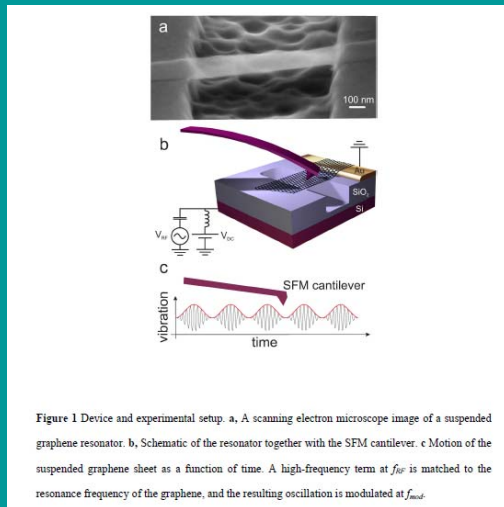
The induced “magnetic” fields have opposite sign at the two corners of the Brillouin Zone.



These terms are forbidden by symmetry in clean graphene.



J. Scott Bunch, A. M. van der Zande, Scott S. Verbridge, I. W. Frank, D. M. Tanenbaum, J. M. Parpia, H. G. Craighead, P. L. McEuen, *Science* **315**, 490 (2007)



D. García-Sánchez, A. M. van der Zande, A. San Paulo, B. Lassagne, P. L. McEuen, A. Bachtold, *Nano Lett.* **8**, 1399 (2008)

$$H_K \equiv \begin{pmatrix} V \sum_i u_{ii} & v_F (-i\partial_x - \partial_y - A_x + iA_y) \\ v_F (-i\partial_x + \partial_y - A_x - iA_y) & V \sum_i u_{ii} \end{pmatrix}$$

$$A_x = \frac{\beta}{2a} (u_{xx} - u_{yy}) \quad A_y = \frac{\beta}{2a} 2u_{xy}$$

$$V \approx 10 - 20 \text{ eV} \quad \beta = \frac{\partial \log(t)}{\partial \log(a)} \approx 2 - 3$$

H. Suzuura and T. Ando, *Phys. Rev. B* **65**, 235412 (2002)
J. L. Mañes, *Phys. Rev. B* **76**, 045430 (2007)

$$\vec{E} = -V \nabla \left(\sum_i u_{ii} \right) \pm \frac{\partial \vec{A}}{\partial t}$$

$$|\vec{E}_{\vec{q}}| \approx \left(V |\vec{q}|^2 \times \frac{|\vec{q}|}{k_{FT}} \pm \frac{N\beta}{2a} |\vec{q}| \omega_{\vec{q}} \right) |\vec{B}_{\vec{q}}|$$

$$\omega_{\vec{q}} = c |\vec{q}| \approx \frac{c}{L}$$

screening

$$\frac{E_{\text{gauge}}}{E_{\text{scalar}}} \approx N\beta \frac{c a^{-1}}{V / k_{FT} L} \approx N\beta \frac{\theta_{\text{Debye}}}{V} k_{FT} L \gg 1$$

Energy dissipation

$$D = v_F^2 \frac{\tau \tau_V}{\tau + \tau_V}$$

$$\sigma_{DC} = \nu D$$

$$\frac{1}{\tau_V} \approx \frac{1}{\tau_{V0}} + c \frac{T^2}{E_F}$$

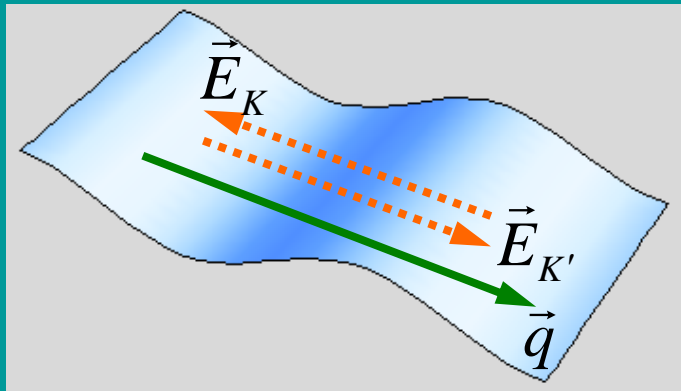
$$\frac{dW}{dt} = \vec{j} \vec{E} = \omega_{\vec{q}} \Gamma_{\vec{q}}$$

Valley drag:

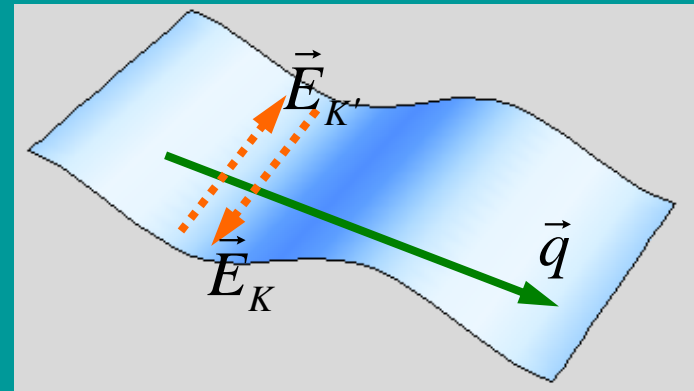
K. Flensberg, B. Y.-K. Hu, A.-P. Jauho, J. M. Kinaret, Phys. Rev. B **52**, 14761 (1995)

A. Kamenev, Y. Oreg, Phys. Rev. B **52**, 7516 (1995)

W.-K. Tse, Y.-K. Yu, and S. Das Sarma, Phys. Rev. B **76**, 081401 (2008)



Longitudinal polarization



Transverse polarization

Conductivity

See: B. I. Halperin, Patrick A. Lee, and N. Read, Phys. Rev. **47**, 7312 (1993)

$$\frac{\partial n_K}{\partial t} + \nabla \cdot \vec{j}_K = -\frac{1}{\tau_V} (n_K - n_{K'}) \quad \text{Continuity}$$

$$\frac{\partial n_{K'}}{\partial t} + \nabla \cdot \vec{j}_{K'} = -\frac{1}{\tau_V} (n_{K'} - n_K)$$

Ohm's law

$$\left(\frac{1}{\tau} + \frac{1}{\tau_V} \right) \vec{j}_K - \frac{1}{\tau_V} \vec{j}_{K'} = -\frac{v_F^2}{2} \nabla n_K + \frac{e^2 v v_F^2}{2} \vec{E}_K$$

$$-\frac{1}{\tau_V} \vec{j}_K + \left(\frac{1}{\tau} + \frac{1}{\tau_V} \right) \vec{j}_{K'} = -\frac{v_F^2}{2} \nabla n_{K'} + \frac{e^2 v v_F^2}{2} \vec{E}_{K'}$$

$$\sigma_{\parallel}(\vec{q}, \omega) = \sigma_{DC} \frac{\left[\omega^2 + \frac{2}{\tau_V} \left(D|\vec{q}|^2 + \frac{2}{\tau_V} \right) \right]}{\omega^2 + \left(D|\vec{q}|^2 + \frac{2}{\tau_V} \right)^2}$$

Longitudinal polarization

$$\sigma_{\perp}(\vec{q}, \omega) = \sigma_{DC}$$

Transverse polarization

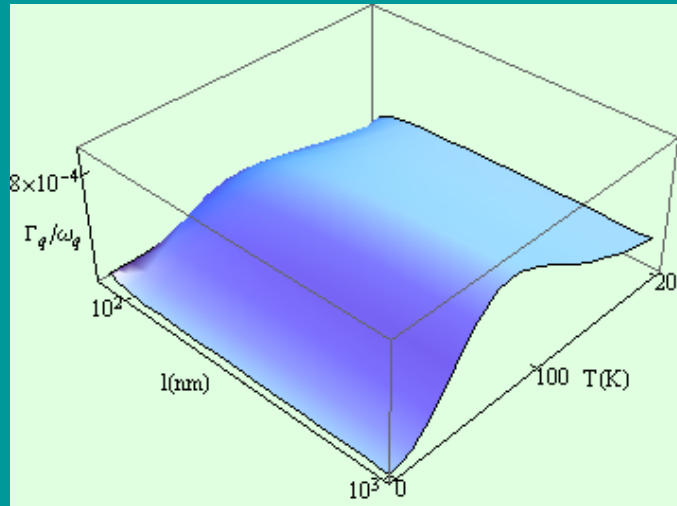
Ballistic limit:

$$D|\vec{q}|^2 \leftrightarrow v_F |\vec{q}|$$
$$l \leftrightarrow |\vec{q}|^{-1}$$

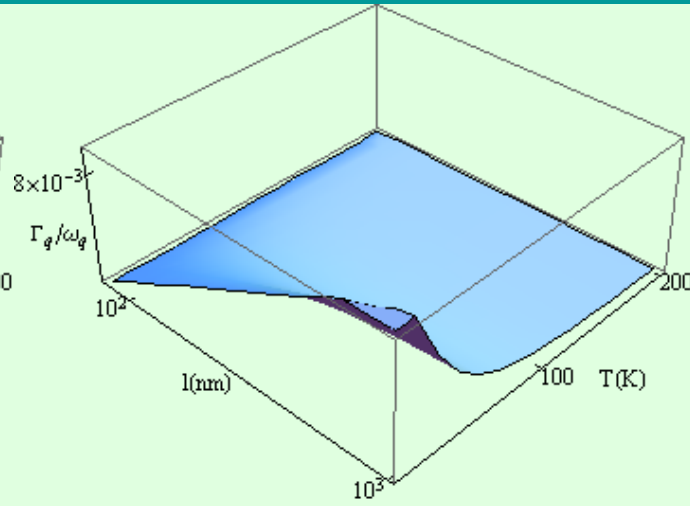
$$n = 10^{12} \text{ cm}^{-2}$$

$$\tau_V^{-1} = \frac{1}{10\tau} + \frac{T^2}{E_F}$$

$$L = q^{-1} = 1 \mu\text{m}$$

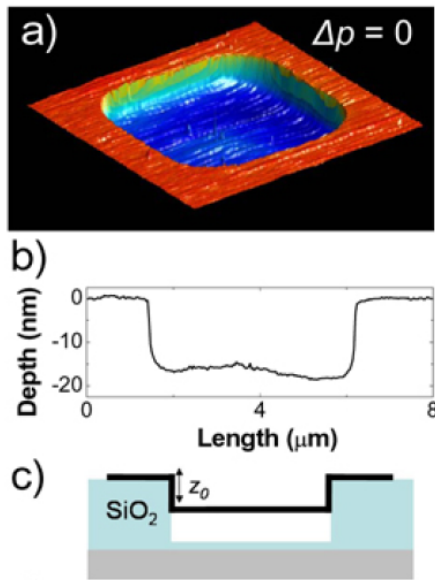


Longitudinal polarization



Transverse polarization

Semiclassical analysis
Valid for multilayered systems



J. S. Bunch, S. S. Verbridge, J. S. Alden, A. M. van der Zande, J. M. Parpia, H. G. Craighead, and P. L. McEuen, *Nano Lett.* **8**, 2458 (2008)

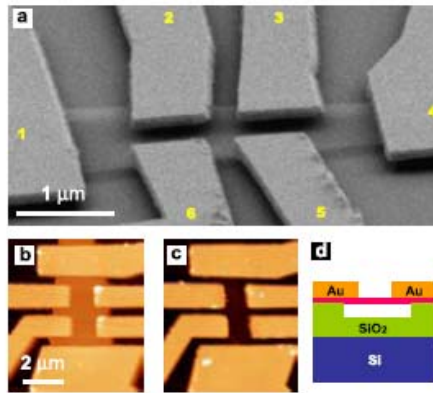
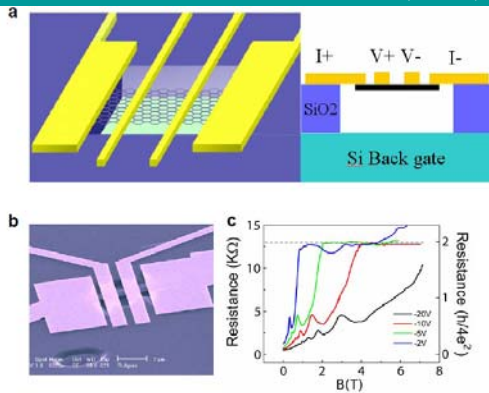


FIG. 1: (a) SEM image of a typical suspended six-probe graphene device taken at 15° with respect to the sample plane. (b) AFM image of the suspended device #1 before the measurements. (c) AFM image of the device #1 after the measurements with graphene removed by a short oxygen plasma etch (same z scale). (d) Device schematic, side-view. Degenerately doped silicon gate (blue), partly etched SiO_2 (green), suspended single-layer graphene (pink) and Au/Cr electrodes (orange).

K. I. Bolotin, K. J. Sikes, Z. Jiang, G. Fudenberg, J. Hone, P. Kim, and H. L. Stormer, *Solid St. Commun.* **146**, 351 (2008)



X. Du, I. Skachko, A. Barker, E. Y. Andrei, *Nature Nanotechnology* **3**, 491 (2008)

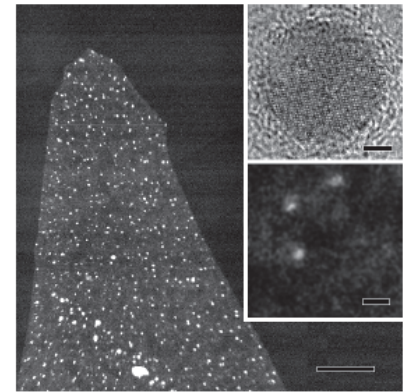
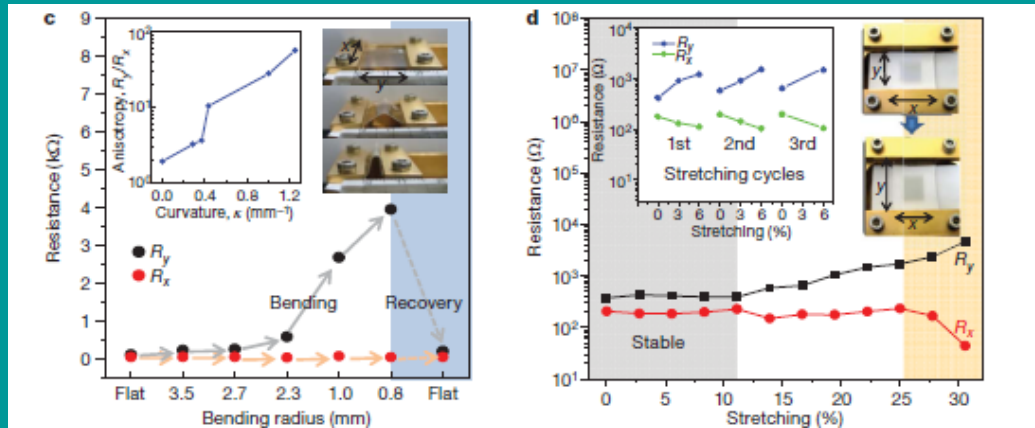
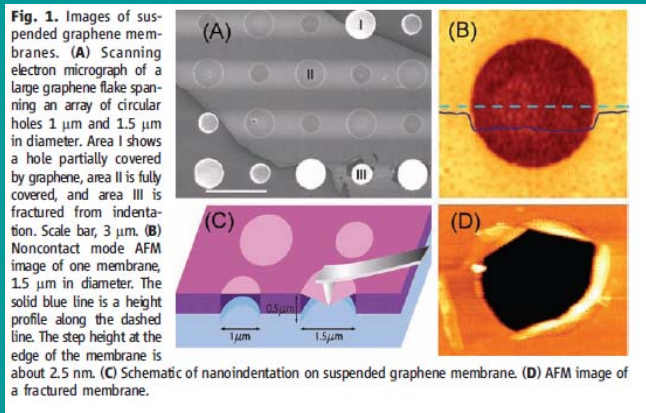


Figure 4: HAADF micrograph of a section of a graphene membrane that fractured during annealing. The graphene crystal is supported from one side only. White dots are copper nanoparticles. Scale bar: $1\ \mu\text{m}$. Top inset: high resolution bright field STEM micrograph of such a Cu particle (\varnothing 8.0 nm; scale bar: 2 nm). Low inset: HAADF image of individual atoms on graphene; scale bar: 2 Å.

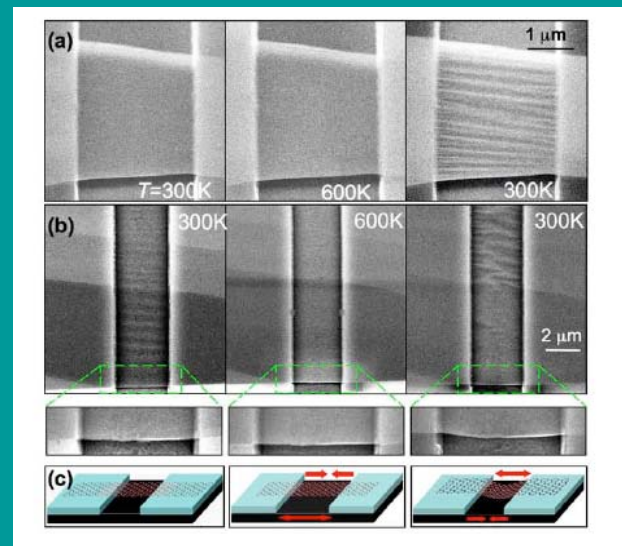
T. J. Booth, P. Blake, R. R. Nair, D. Jiang, E. W. Hill, U. Bangert, A. Bleloch, M. Gass, K. S. Novoselov, M. I. Katsnelson, and A. K. Geim, *Nano Lett.* **8**, 2442 (2008)



K. S. Kim, Y. Zhao, H. Jang, S. Y. Lee, J. M. Kim, K. S. Kim, J. H. Ahn, P. Kim, J.-Y. Choi, B. H. Hong, *Nature* **457**, 706 (2008)



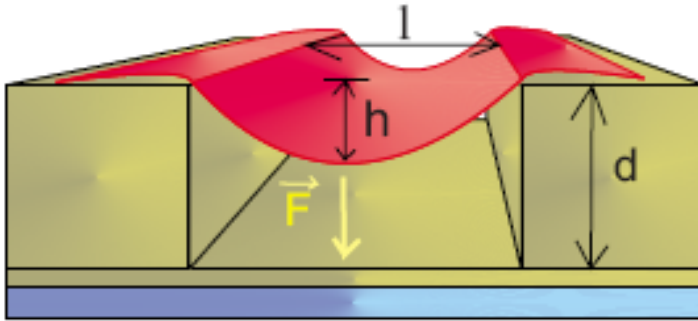
C. Lee, X. Wei, J. W. Kysar, J. Hone, *Science* **321**, 385 (2008)



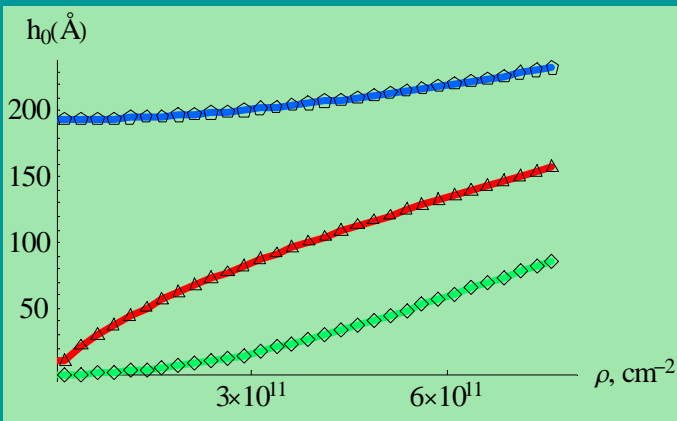
W. Bao, F. Miao, Z. Chen, H. Zhang, W. Jang, C. Dames, C. N. Lau, arXiv:0903.0414

Ballistic transport in suspended graphene

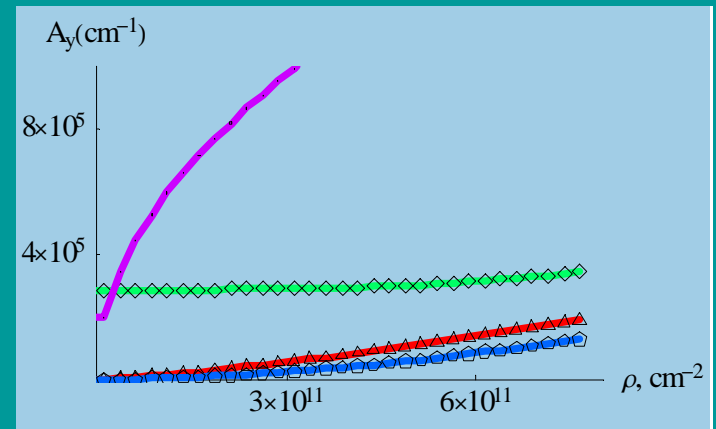
M. M. Fogler, F. G., M. I. Katsnelson, Phys. Rev. Lett. **101**, 226804 (2008)



- The graphene layer is deformed by the applied electric field, slack, ...
- Stresses lead to effective gauge potentials



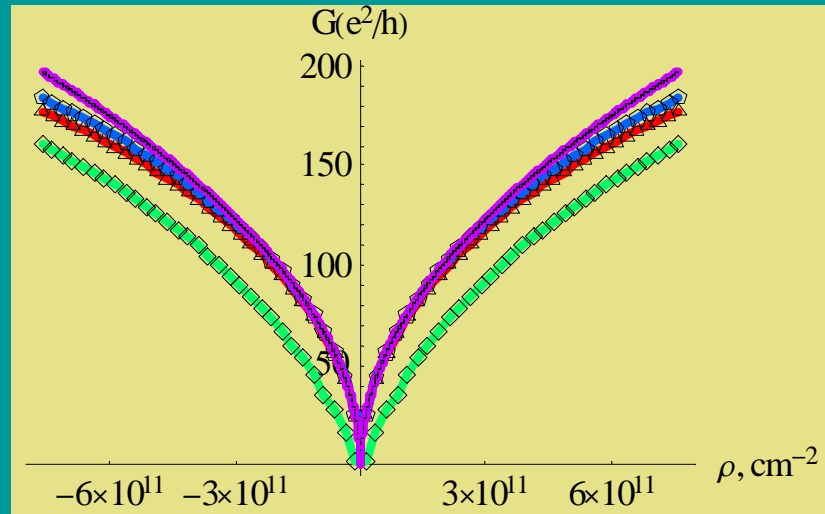
Maximum height as function of carrier density for different values of the slack



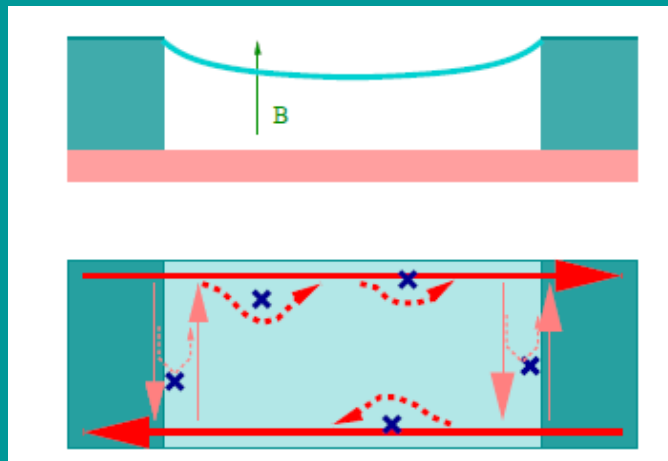
Vector potential inside the suspended region as function of carrier density for different values of the slack

Ballistic transport in suspended graphene

17



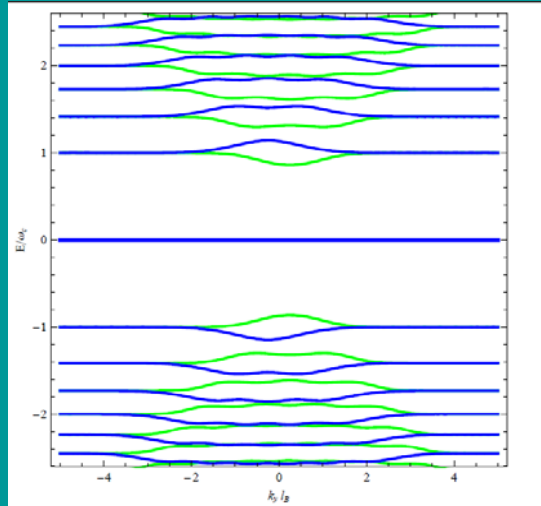
Transmission through a deformed graphene sheet as function of density for different values of the slack



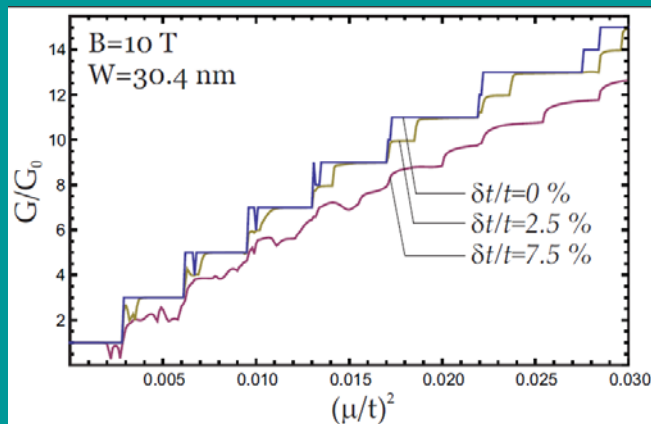
Hall currents in a magnetic field

Effects of a real magnetic field

G. León, E. Prada, P. San José, F. G., unpublished



Landau levels as function of the momentum parallel to the boundary



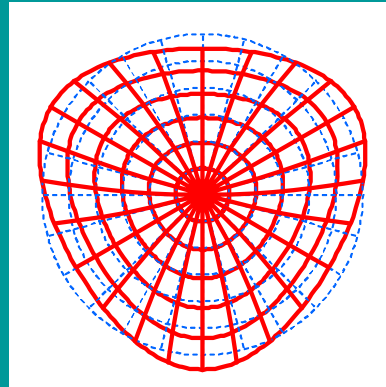
Integer Quantum Hall steps in a graphene ribbon where part of it is under a constant stress

Effective magnetic fields in strained graphene

A. K. Geim, M. I. Katsnelson, F. G., unpublished

$$\begin{aligned}
 u_r(r, \theta) &= Ar^2 \sin(3\theta) \\
 u_\theta(r, \theta) &= Ar^2 \cos(3\theta) \\
 \sigma_{rr}(r, \theta) &= 4\mu Ar \sin(3\theta) \\
 \sigma_{\theta\theta}(r, \theta) &= -4\mu Ar \sin(3\theta) \\
 \sigma_{r\theta}(r, \theta) &= 4\mu Ar \cos(3\theta)
 \end{aligned}$$

$$\begin{aligned}
 u_x(x, y) &= 2Axy \\
 u_y(x, y) &= A(x^2 - y^2) \\
 \sigma_{xx}(x, y) &= 4\mu Ay \\
 \sigma_{yy}(x, y) &= -4\mu Ay \\
 \sigma_{xy}(x, y) &= 4\mu Ax
 \end{aligned}$$



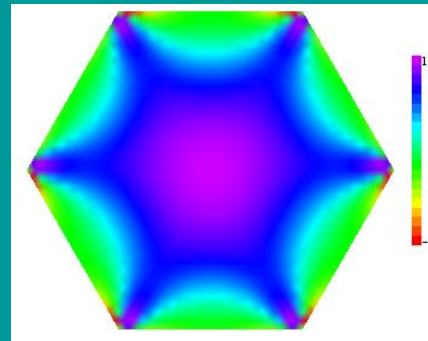
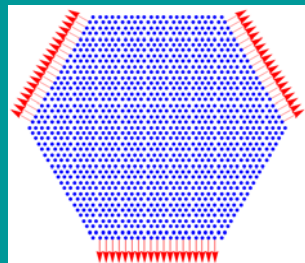
Shear deformation:
Constant effective
magnetic field

$$l_B \approx \sqrt{\frac{aR}{8\bar{u}\beta}}$$

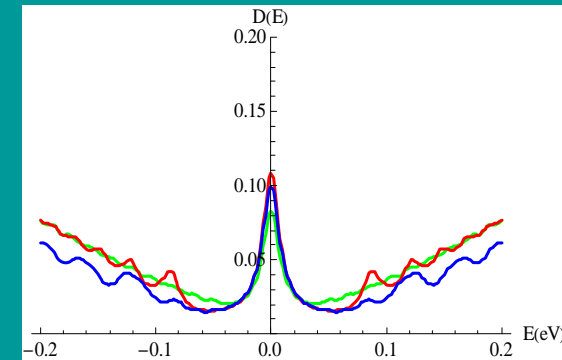
$$R \approx 1\mu m, \quad a \approx 0.1nm, \quad \beta \approx 3, \quad \bar{u} \approx 0.1$$

$$l_B \approx 7nm, \quad B \approx 10T$$

Applied forces



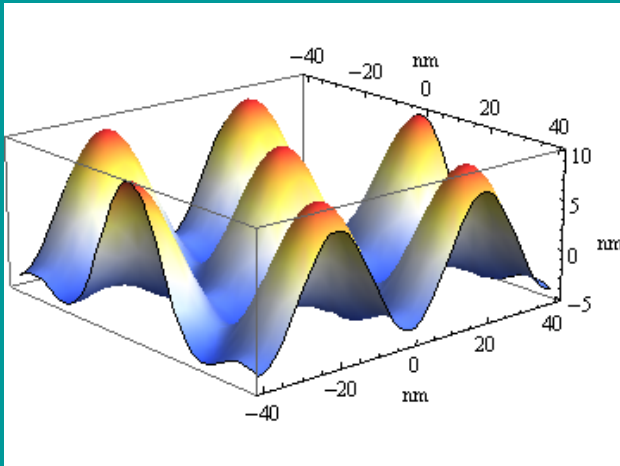
Effective magnetic field



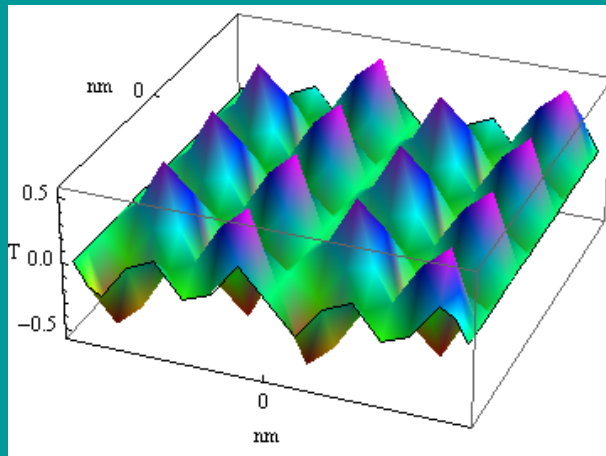
Total density of states

Dependence on
boundary
conditions:

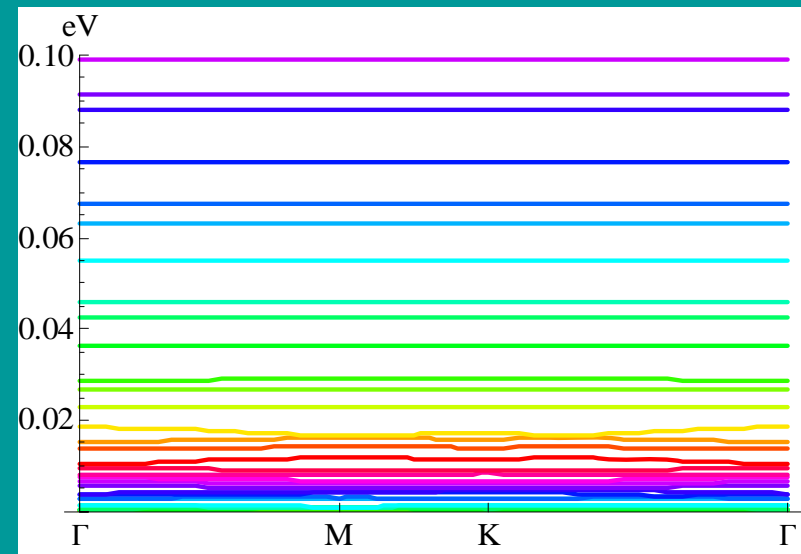
Strain superlattices



Height modulations



Effective magnetic field



Electronic bands

Wrinkles and mechanical instabilities in strained graphene

F. G., B. Horovitz, P. Le Doussal, arXiv:0811.4670, Solid St. Commun., in press

Wrinkling instability:

E.Cerda and L. Mahadevan, Phys. Rev. Lett. **90**, 074302 (2003)

T. A. Witten, Rev. Mod. Phys. **79**, 643 (2007)

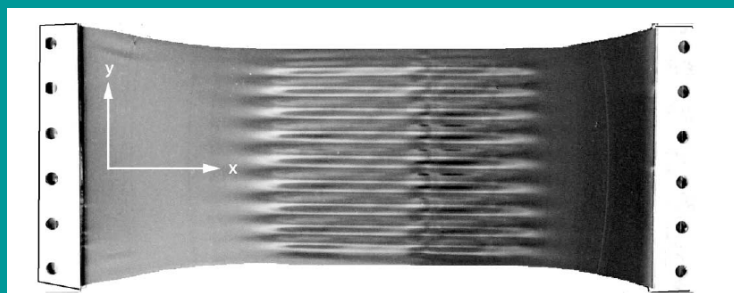
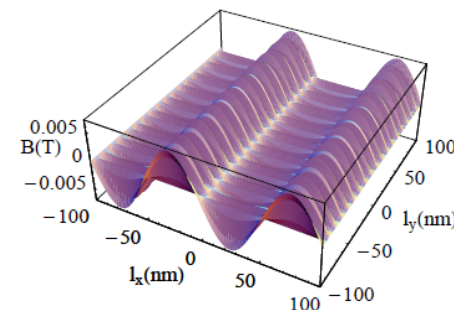
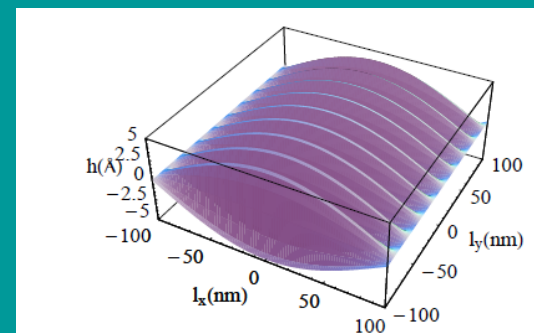
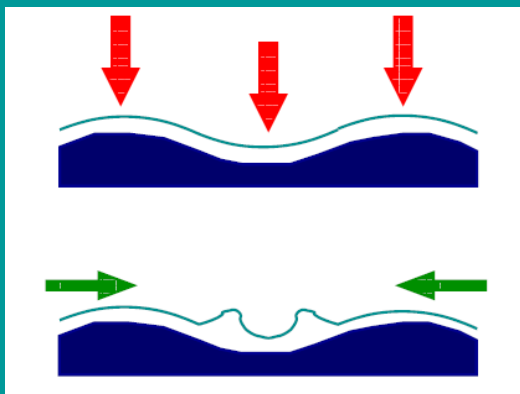


FIG. 1. Wrinkles in a polyethylene sheet of length $L \approx 25$ cm, width $W \approx 10$ cm, and thickness $t \approx 0.01$ cm under a uniaxial tensile strain $\gamma \approx 0.10$. (Figure courtesy of K. Ravi-Chandar)



E.Cerda and L. Mahadevan, Phys. Rev. Lett. **90**, 074302 (2003)



Possible instabilities in graphene on a substrate

- Localized states at edges can lead to observable consequences in graphene quantum dots
- Synthetic electric fields are induced in vibrating graphene. They give a simple physical picture of dissipation mechanisms.
- Synthetic magnetic fields interfere with real magnetic fields in suspended samples, modifying the properties in the Quantum Hall regime.