Ordered Loop Current States in Bilayer Graphene

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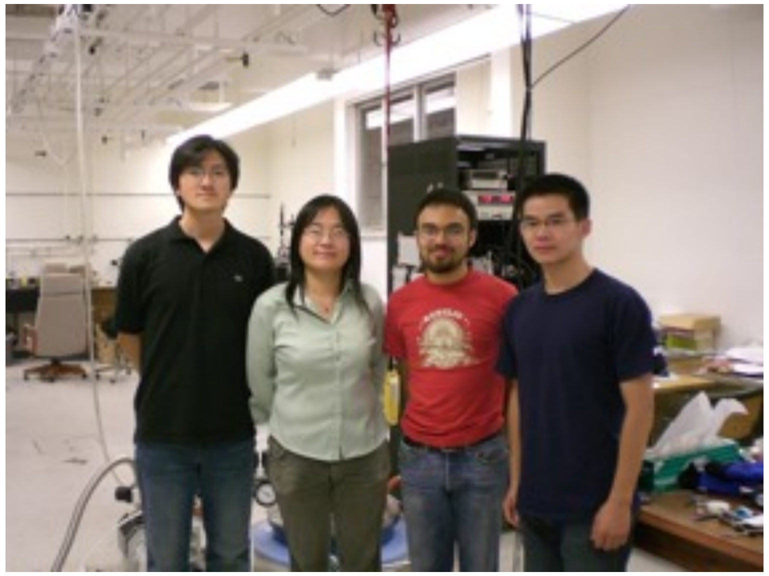
Acknowledgements

Chandra Varma

Lijun Zhu

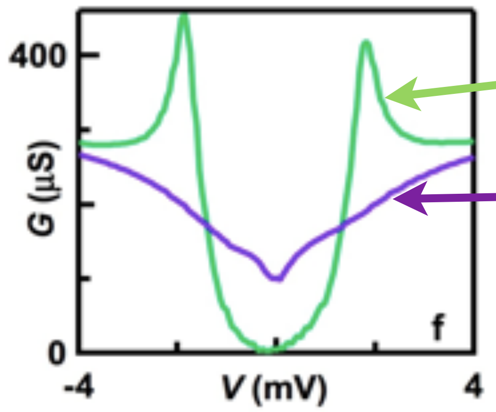
C. N. Lau Group



LZ, VA, CMV arXiv:1202.0821

The Observation

1) Insulating state at charge neutrality measured in two terminal transport measurement

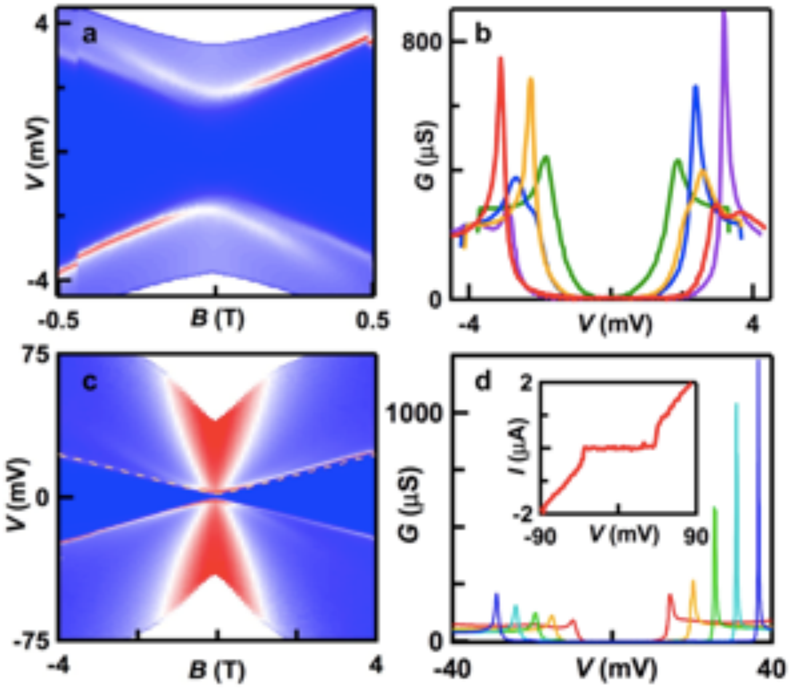


E=0, $G_{min} < 0.5\ \mu S$

E=-15 mV/nm

J.Velasco et al., Nature Nanotechology 2012

2) Evolution of the gap with magnetic field

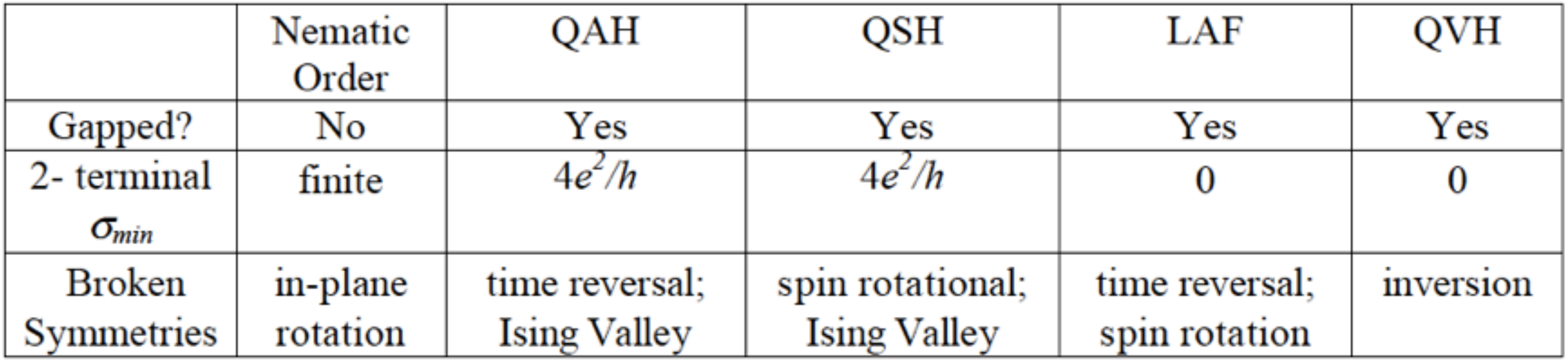


$$E_{gap} = \Delta_0 + \sqrt{a^2B^2 + \Delta_0^2}$$

1 meV

5.5 meV/T

The Puzzle



	Nematic Order	QAH	QSH	LAF	QVH
Gapped?	No	Yes	Yes	Yes	Yes
2- terminal σ_{min}	finite	$4e^2/h$	$4e^2/h$	0	0
Broken Symmetries	in-plane rotation	time reversal; Ising Valley	spin rotational; Ising Valley	time reversal; spin rotation	inversion

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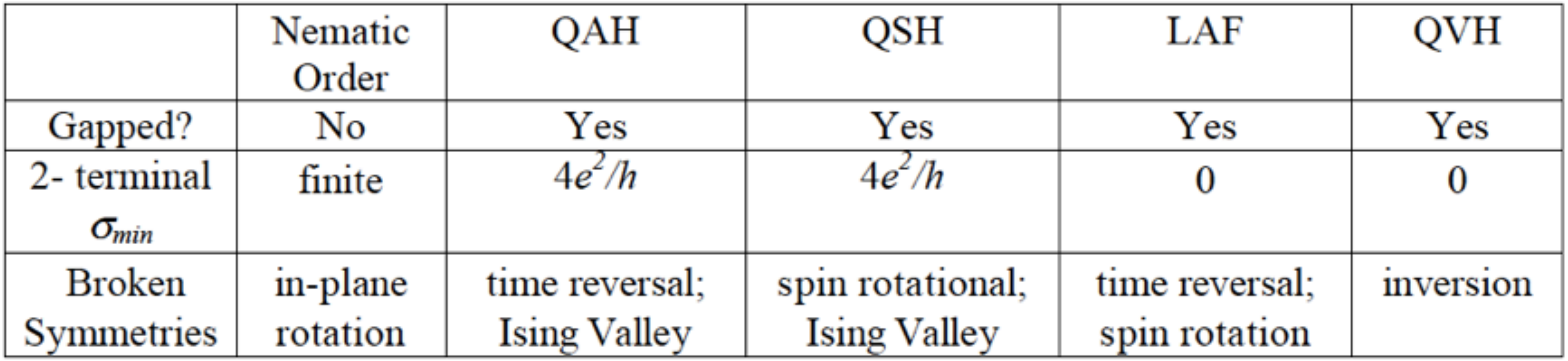

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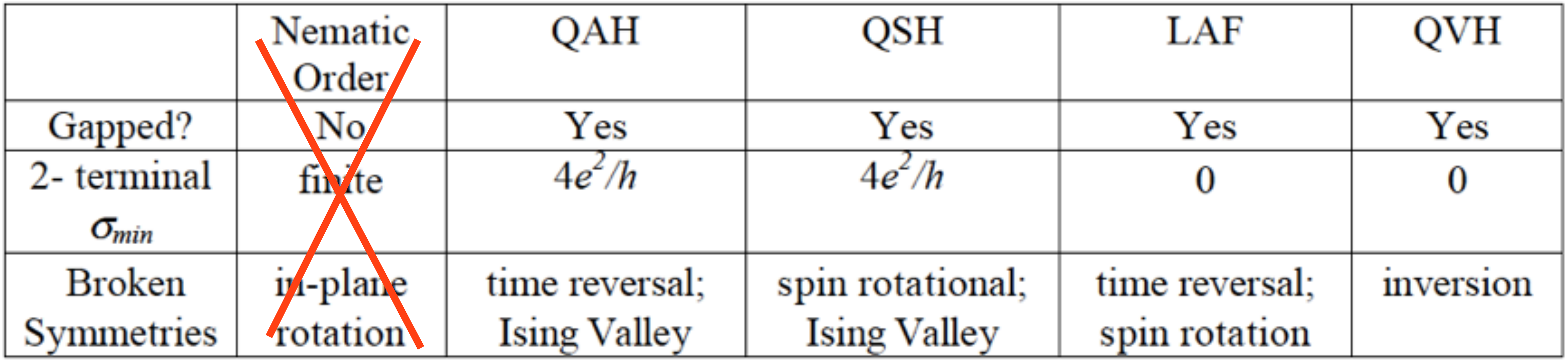

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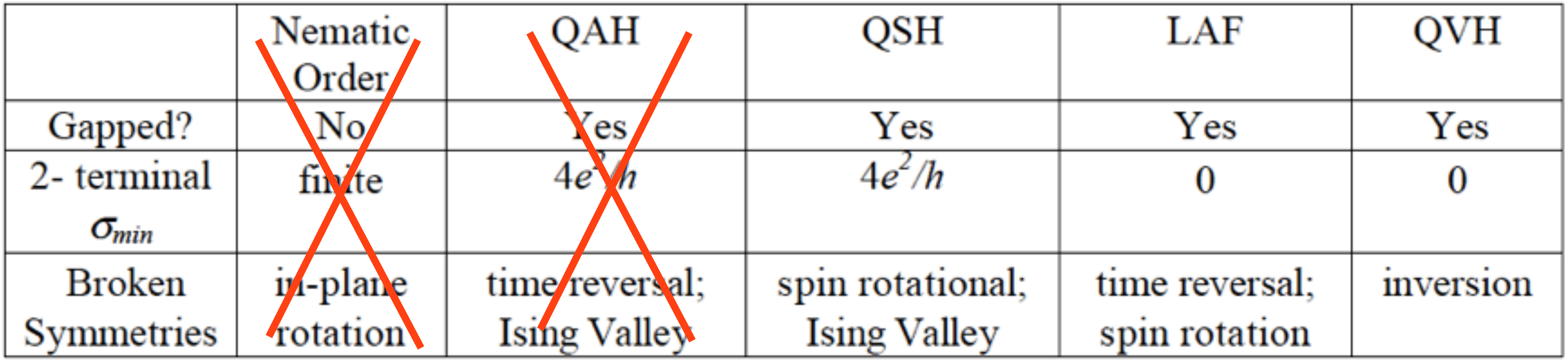

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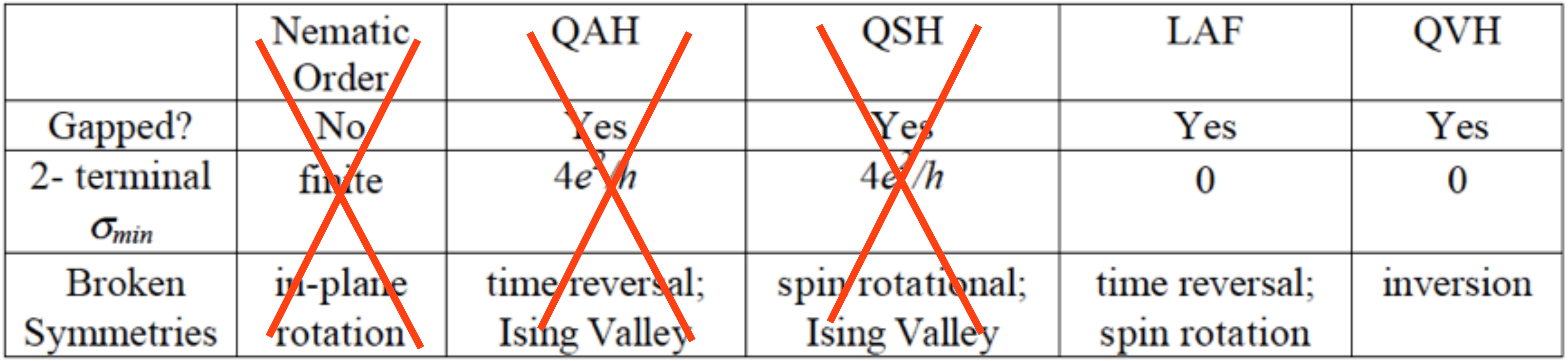

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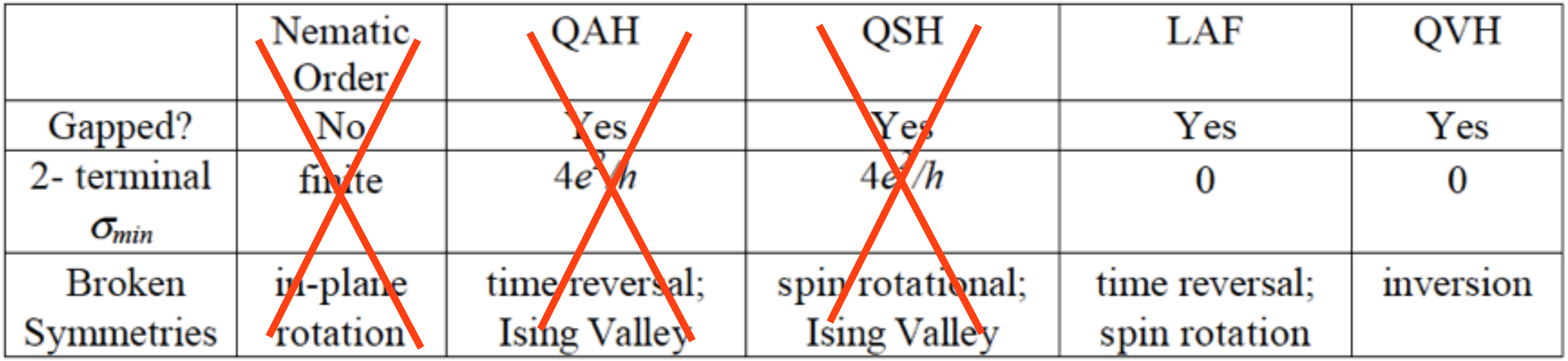

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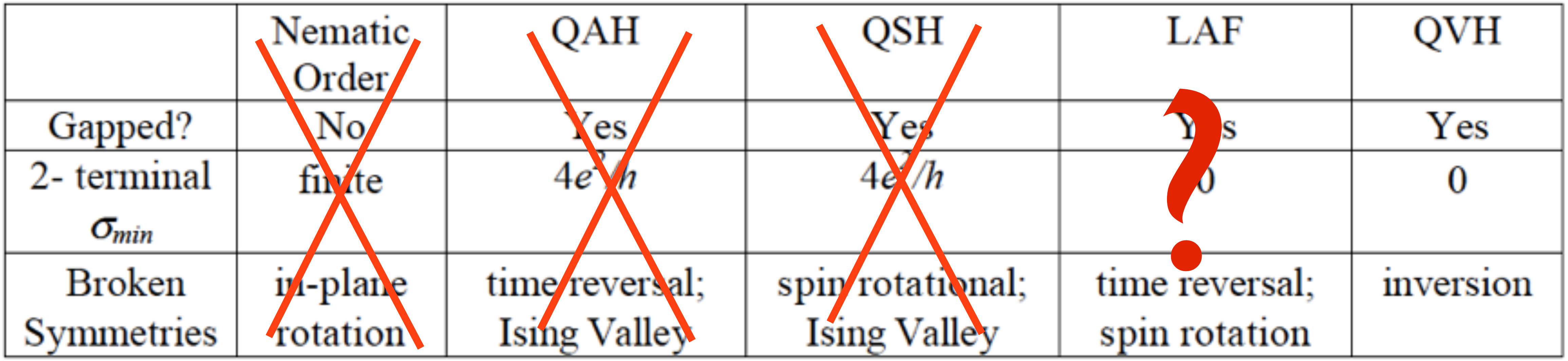

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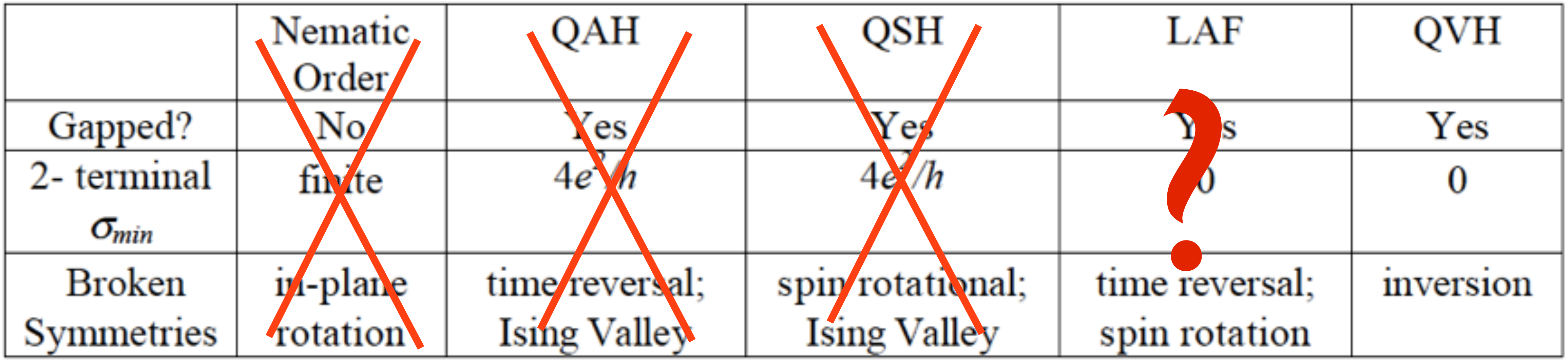

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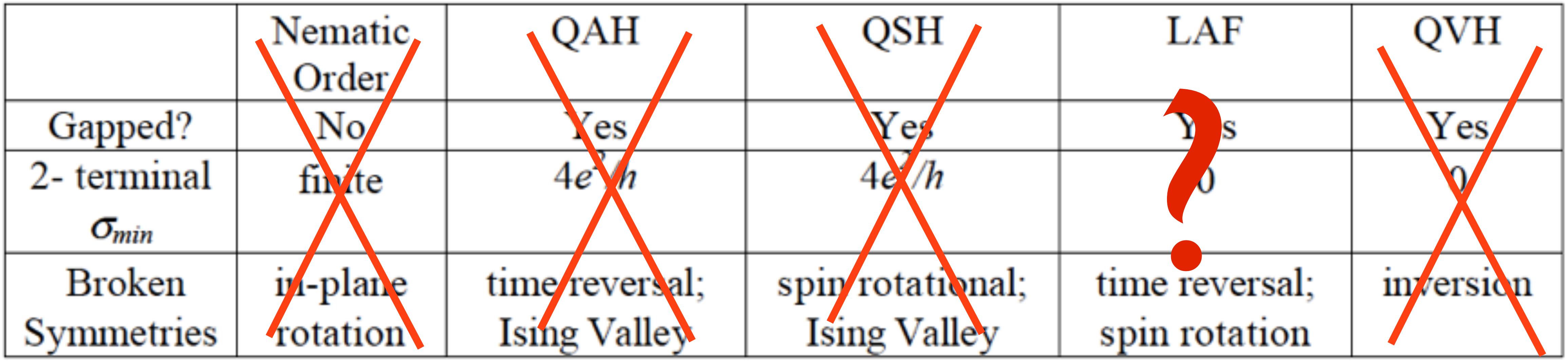

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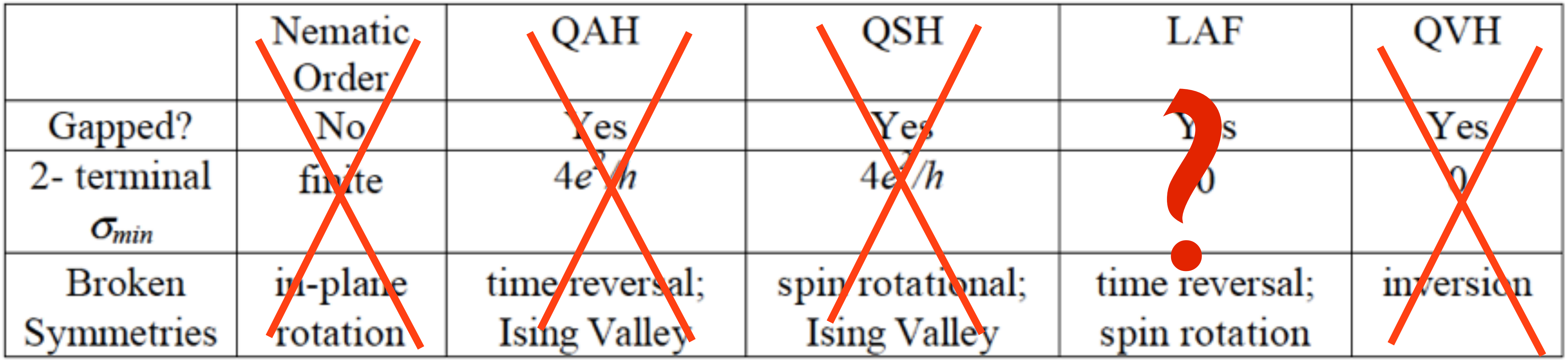

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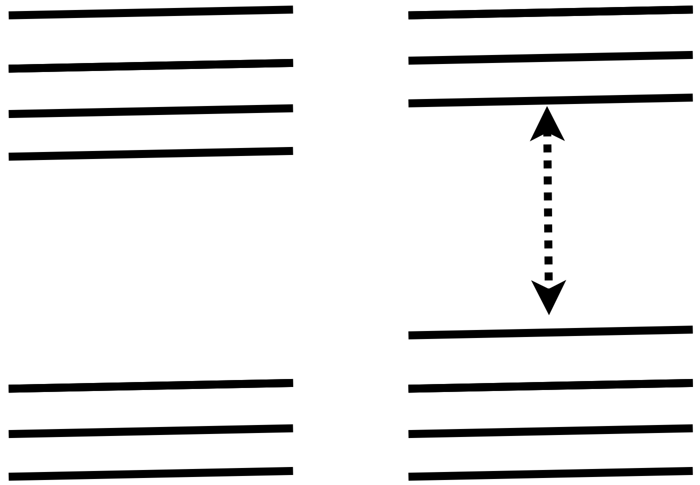
Whats going on ?

Layered Antiferromagnet



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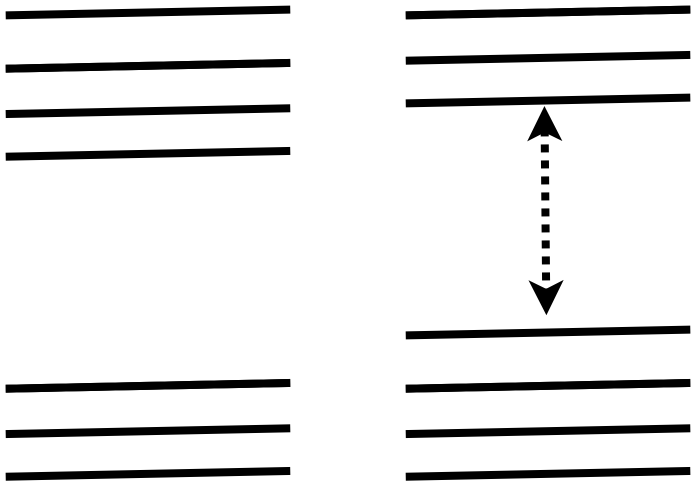

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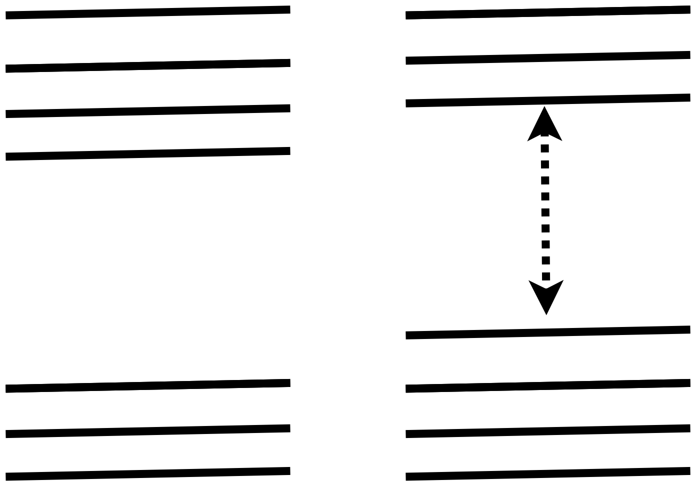

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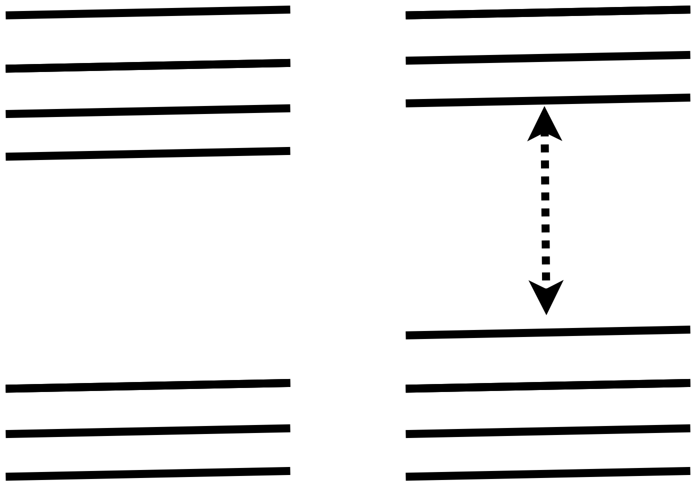

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The gap can evolve with magnetic field due to interaction

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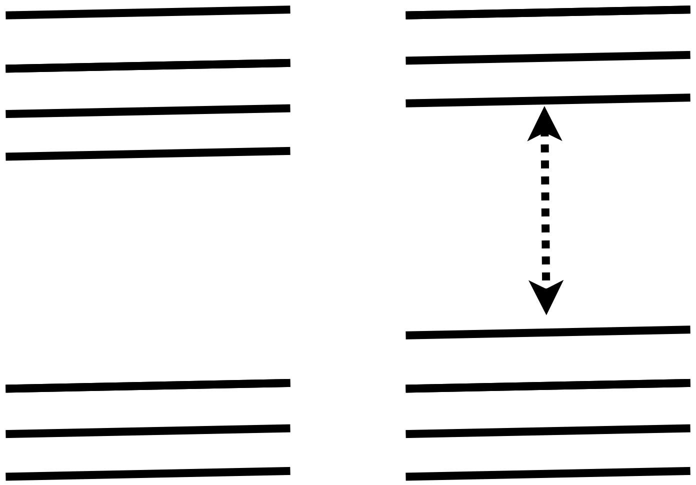

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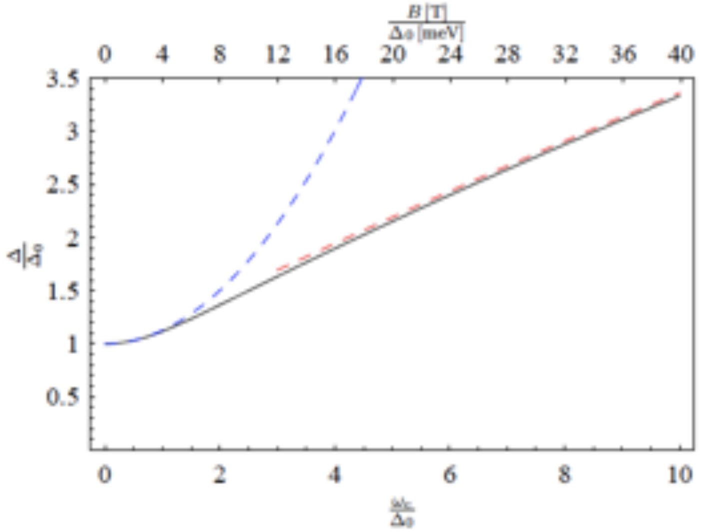
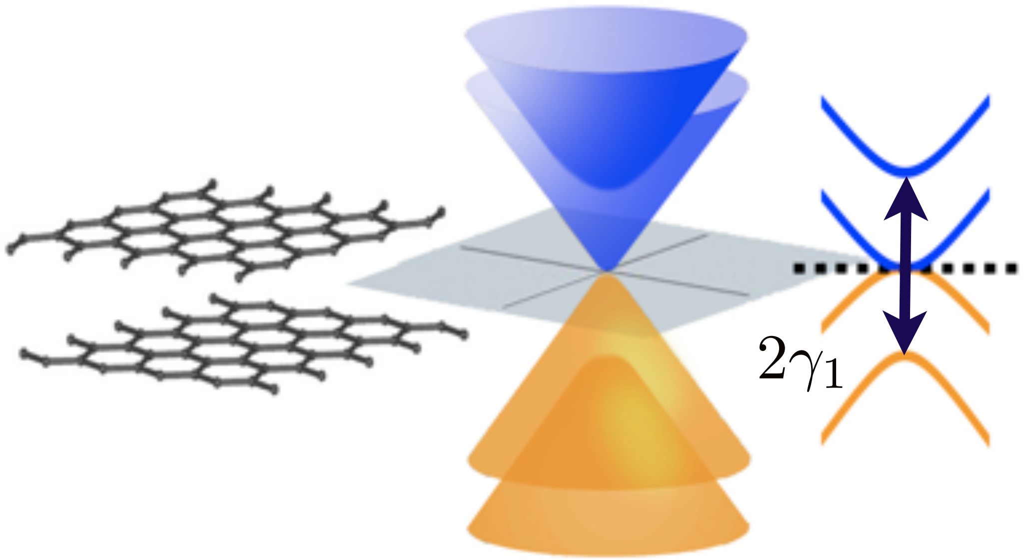


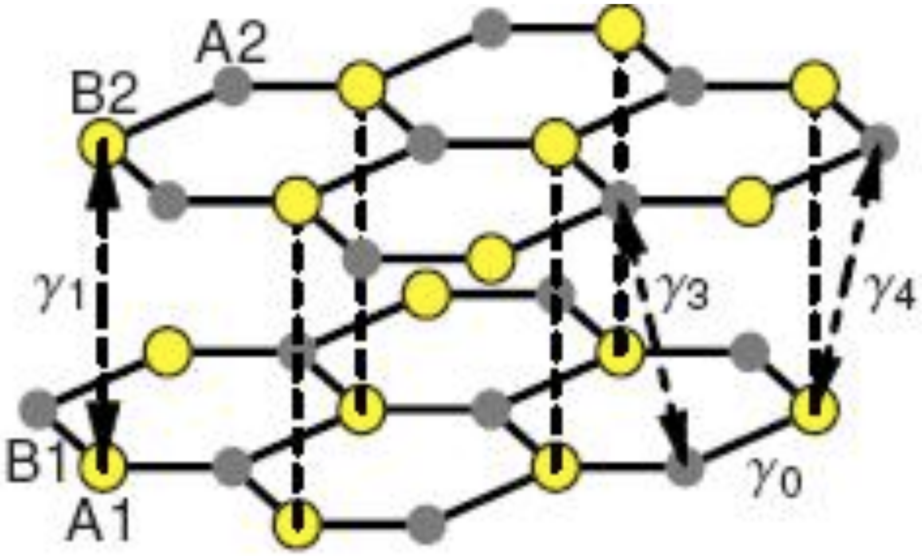
a is underestimated by a factor of 4 at weak fields

Throckmorton, Vafek arXiv:1101.2076

Similar to previous result by M. Kharitonov arXiv:1109.1553

Band Structure





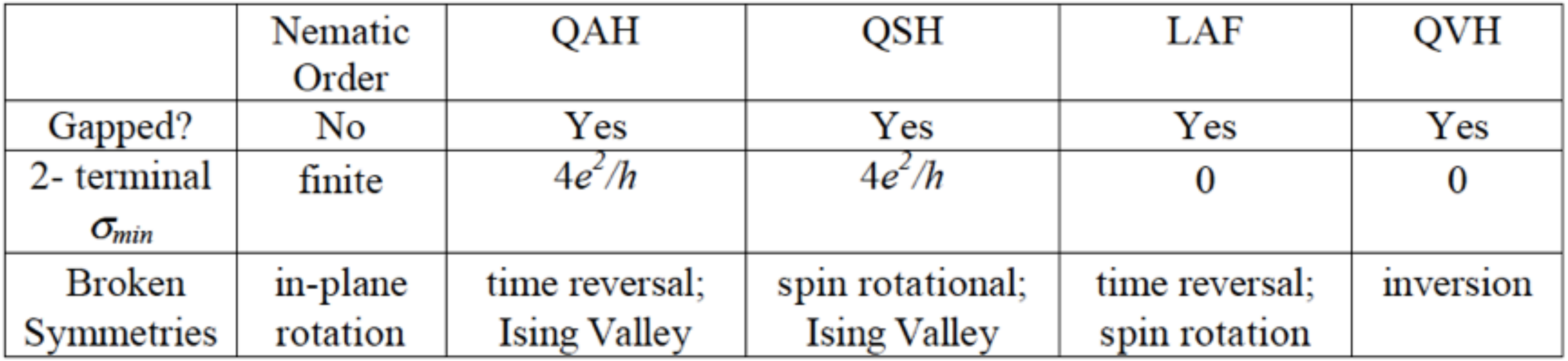
E. McCann & V.I. Falko, PRL 2006,
Y. Zhang et al., Nature 2010

Quadratic dispersion at the nodes

Unstable to infinitesimal interactions and perturbations

Need high quality samples and devices to access the intrinsic ground state

Suggested Broken symmetry states



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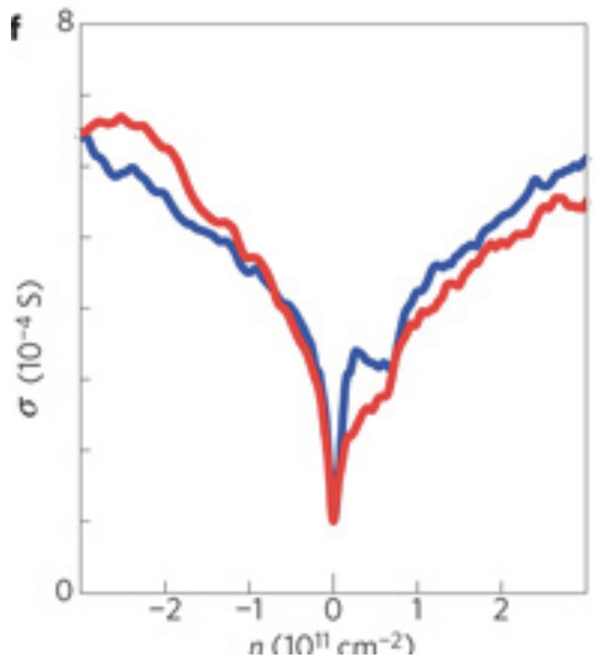
O. Vafek et al., PRB 2010,
Y. Lemonik, et al., PRB, 2010,
R. Nandkishore & L. Levitov, PRB 2010,
F. Zhang, PRL 2011,
J. Jung et al., PRL 2011,
....
.....
(apologies to those not listed)

None of these account for the data in J. Velasco et al., Nature nano 2012

Experimental Precedent (?)



Feldman et al., Nature Physics 2009

Single gated sample

Mobilities estimated $10^5 \text{cm}^2/\text{Vs}$

Gapped state ?

Number to remember: $e^2/h = 4 \times 10^{-5}$ S or $h/e^2 = 25\text{k}\Omega$

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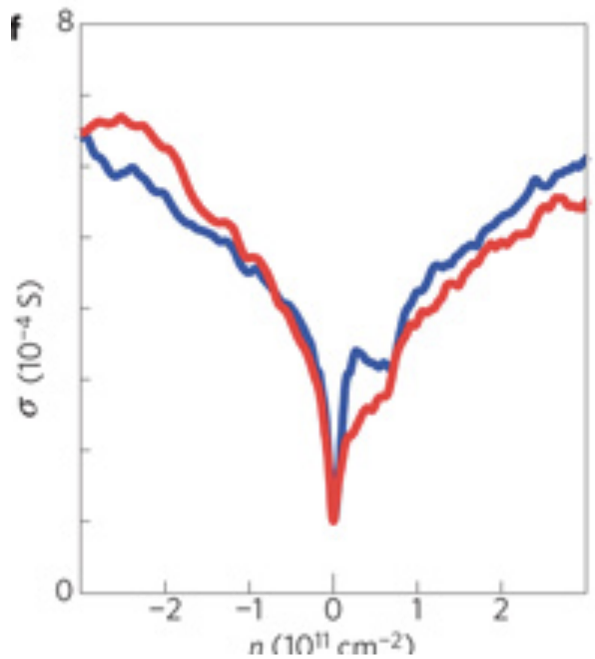

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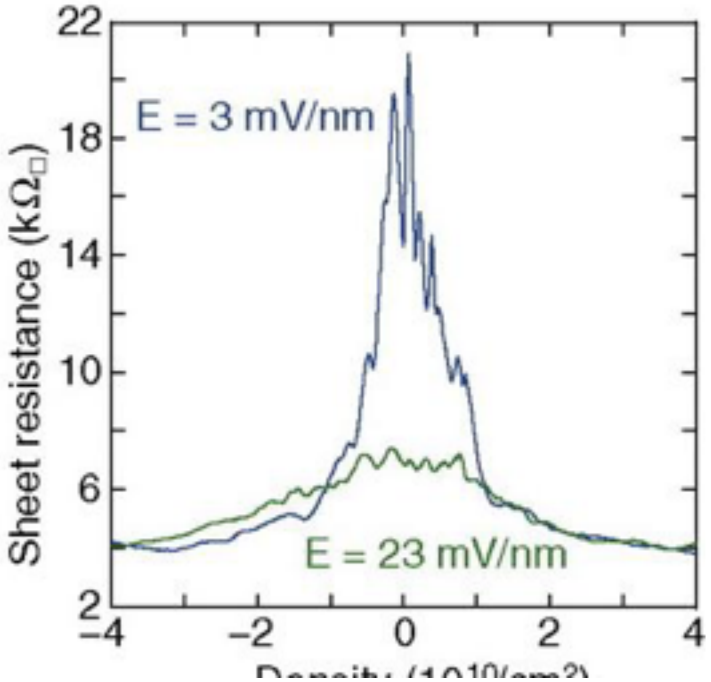

Weitz et al., Science 2010

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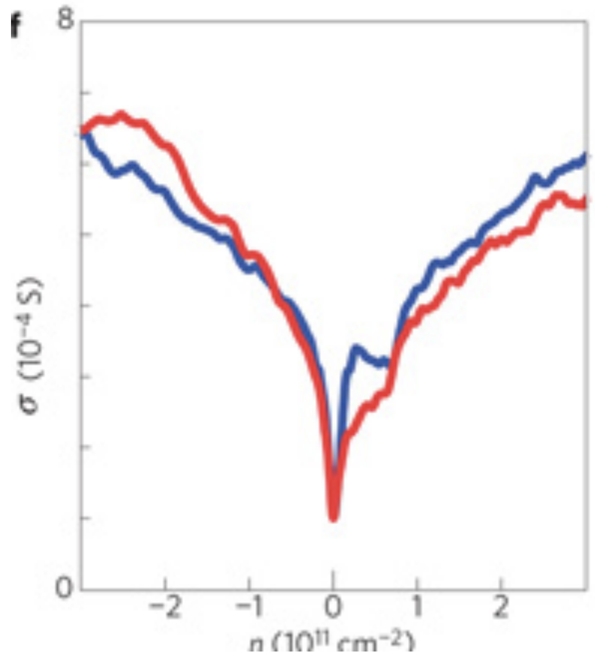

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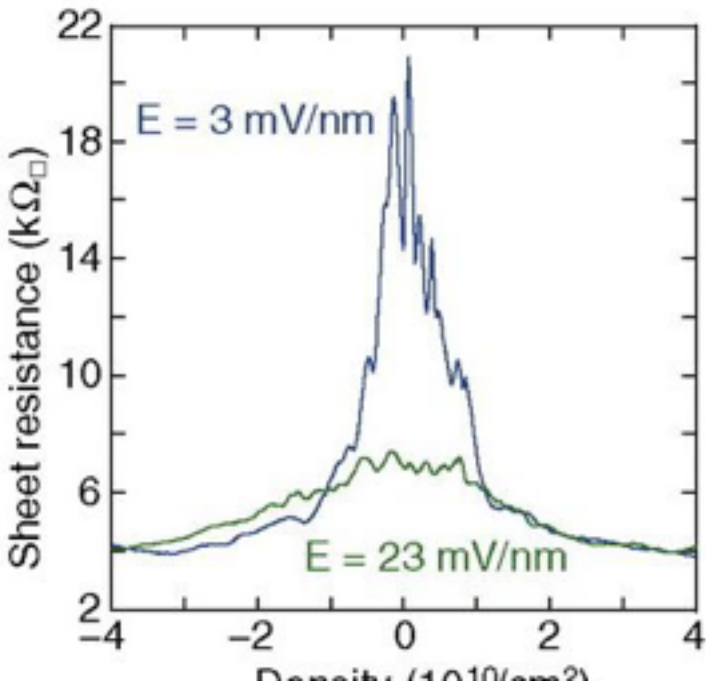

Weitz et al., Science 2010

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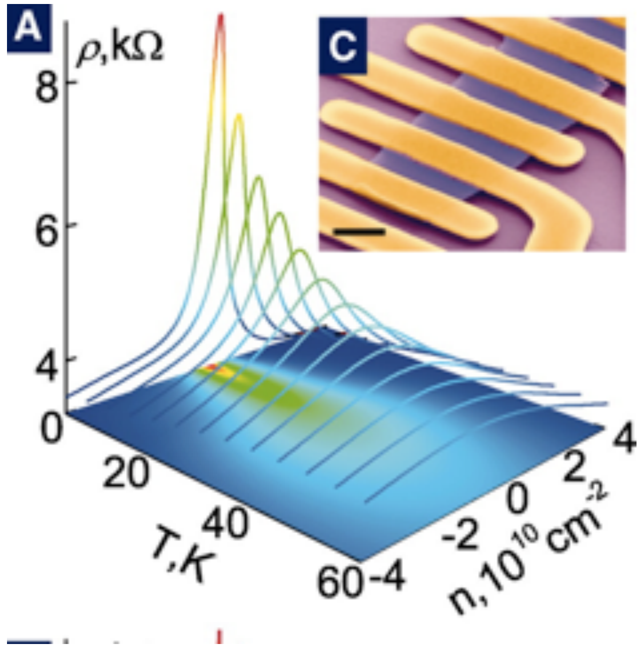


Mayorov et al. Science 2011

Single gated sample

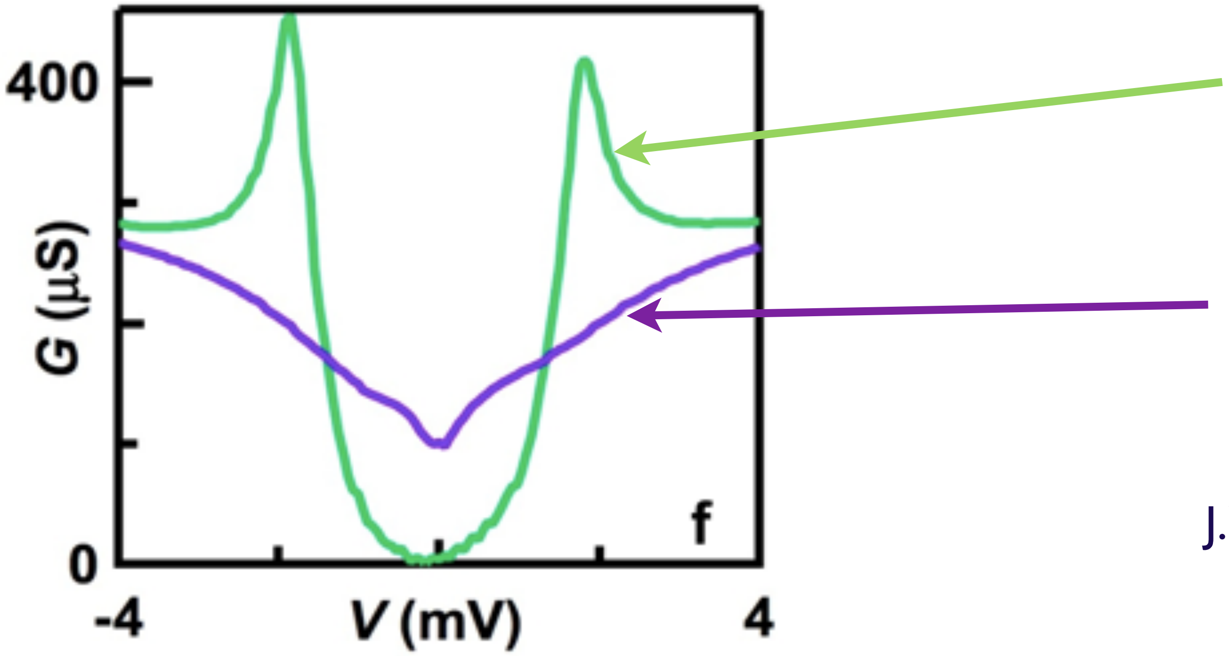
Mobilities estimated from onset of quantum oscillations $10^6 cm^2/Vs$

No gap

Data consistent with nematic state

Number to remember: $e^2/h = 4 \times 10^{-5}$ S or $h/e^2 = 25k\Omega$

Experimental Data



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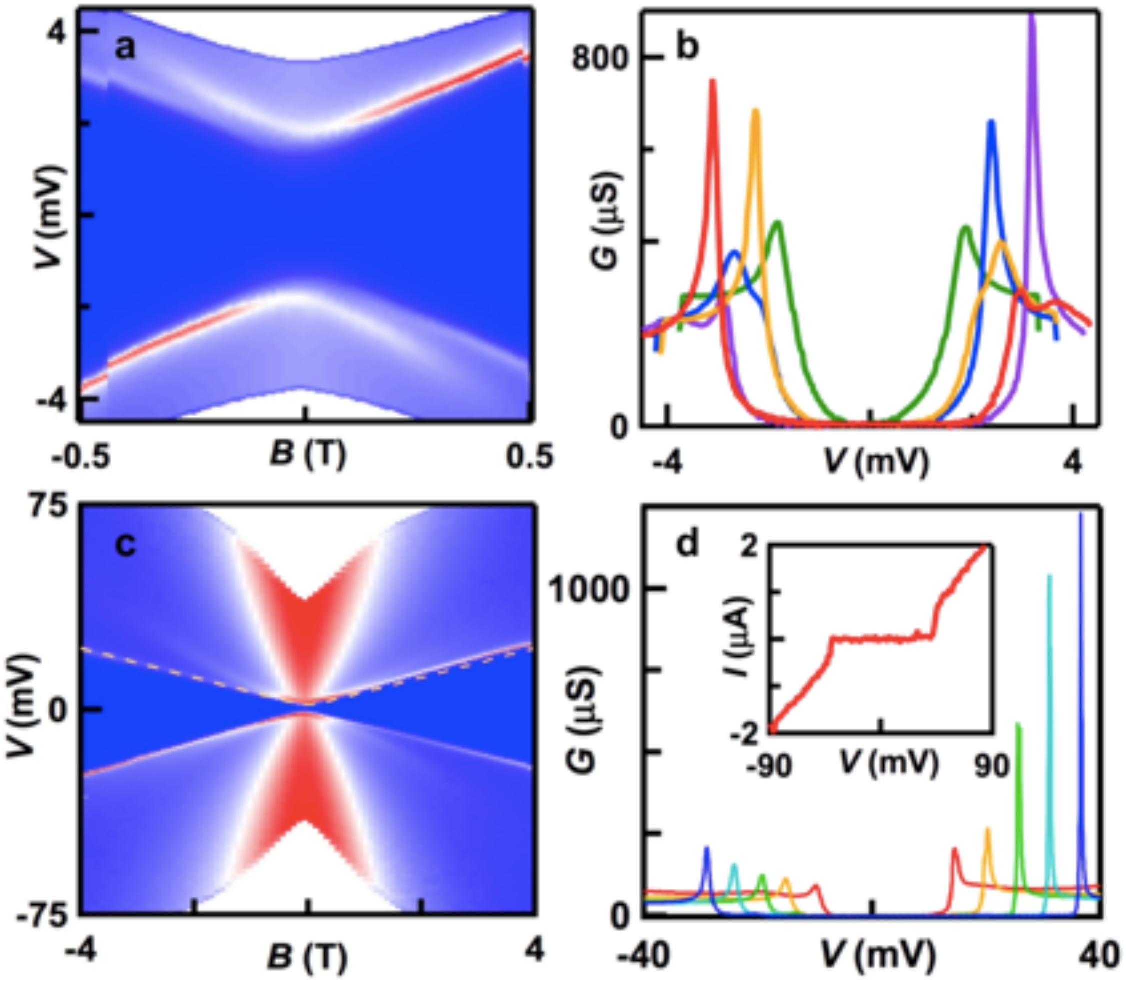

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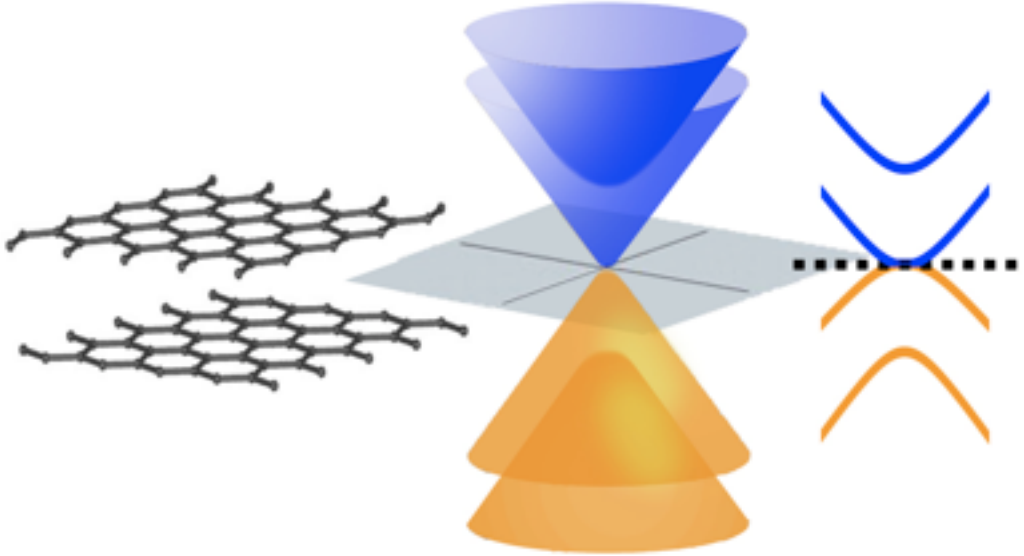
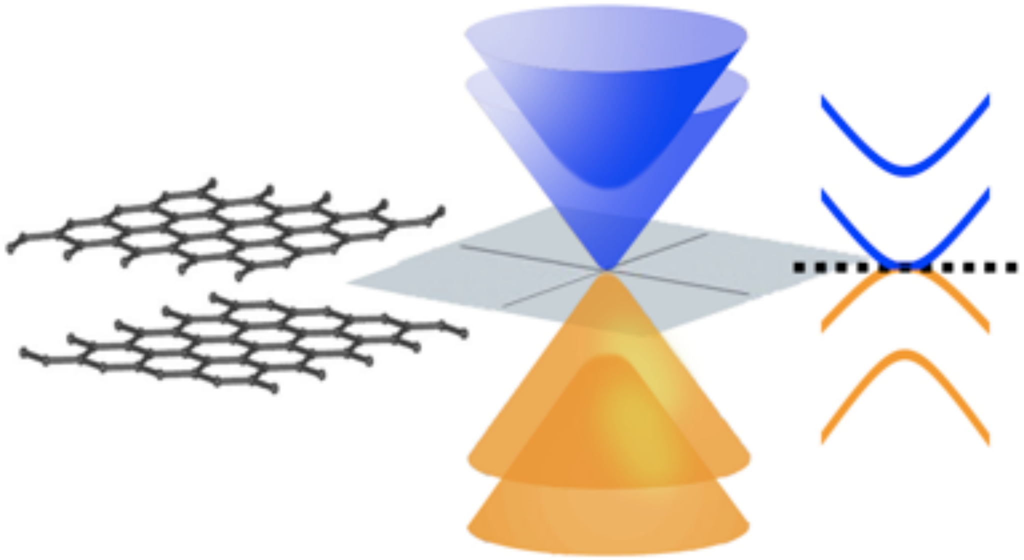
3) The single particle gap should evolve with magnetic field

Effective theories



Effective theories

effectively ignore stacked atoms and concentrate on the two low energy bands. (Weak coupling)



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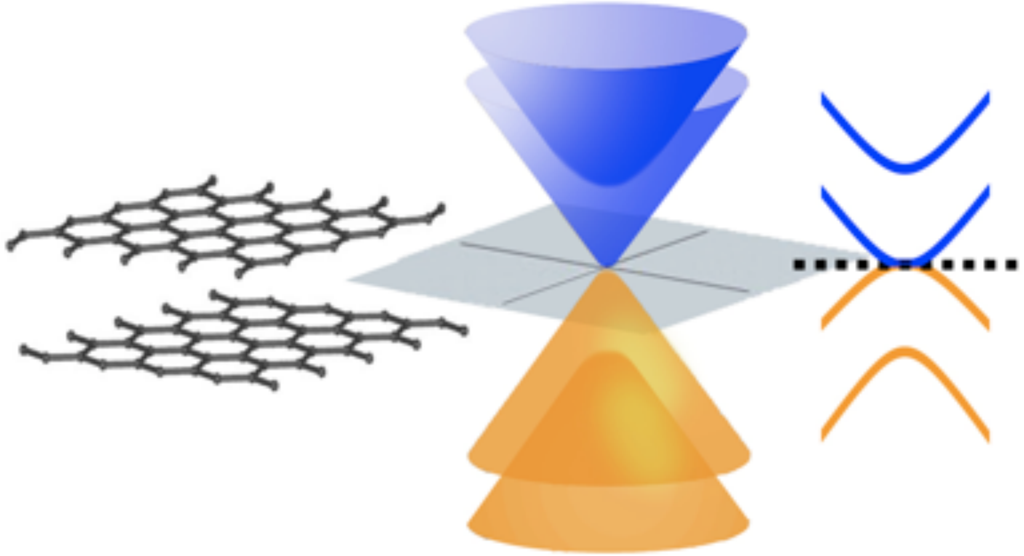

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Two valleys, two spins, two layers

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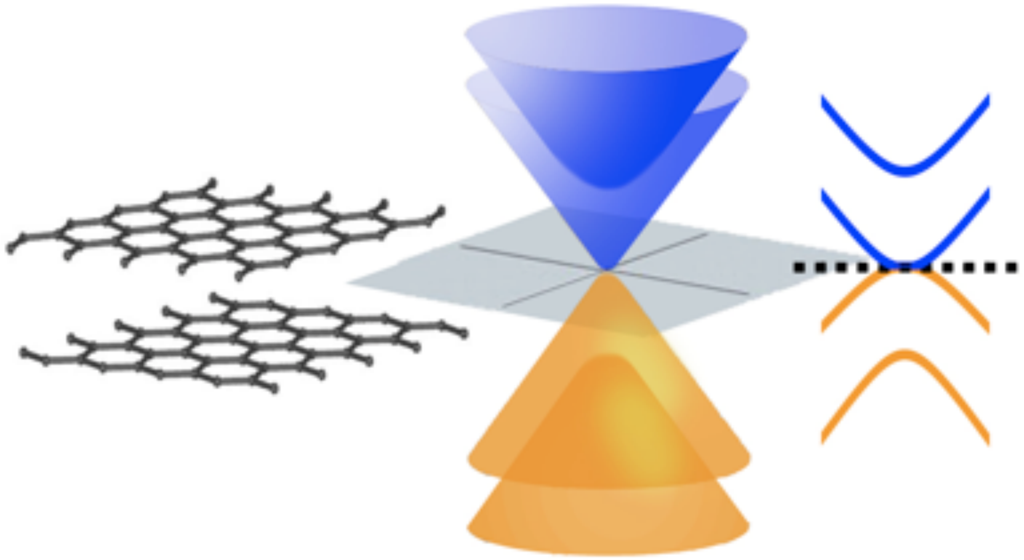

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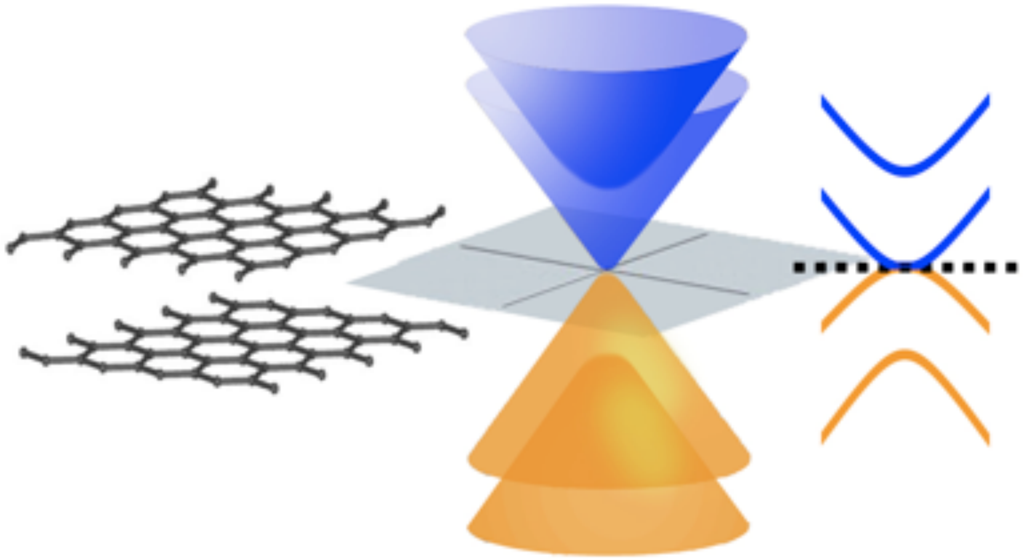

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Long Range leads to nematic instability within one loop RG

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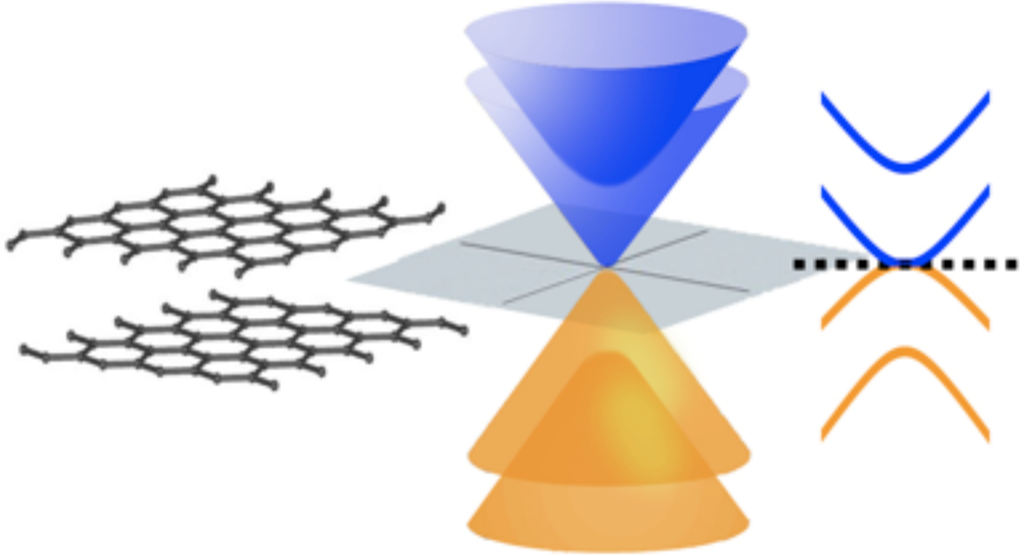

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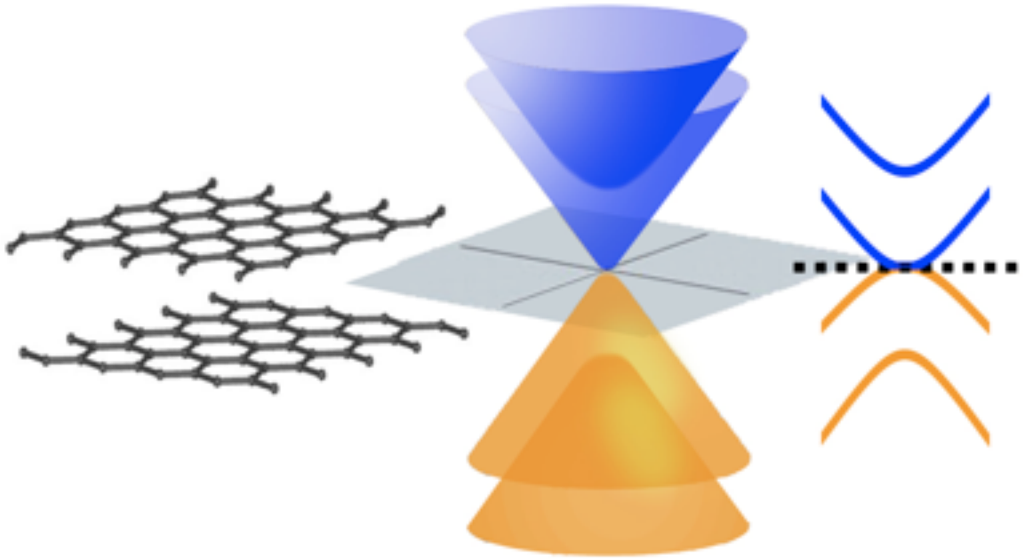

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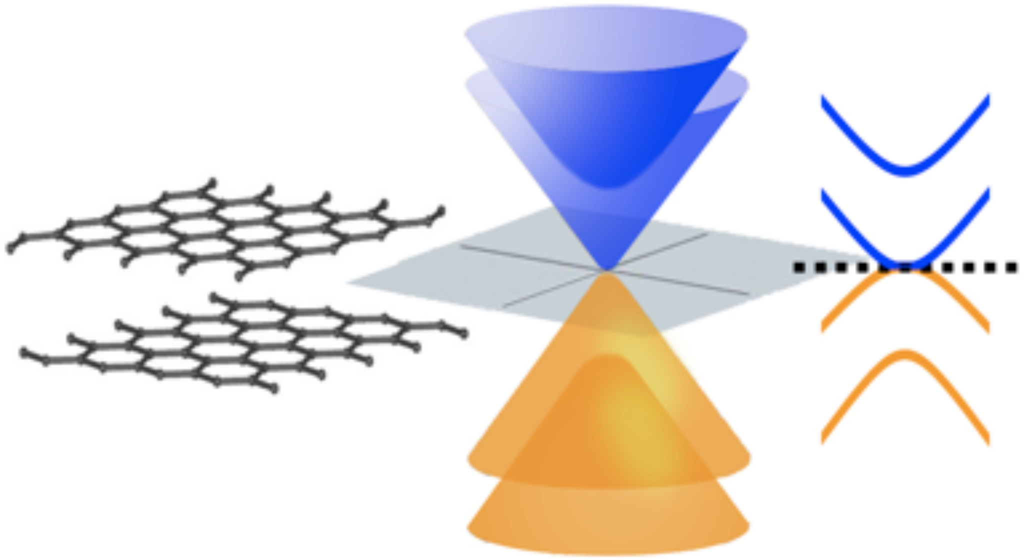

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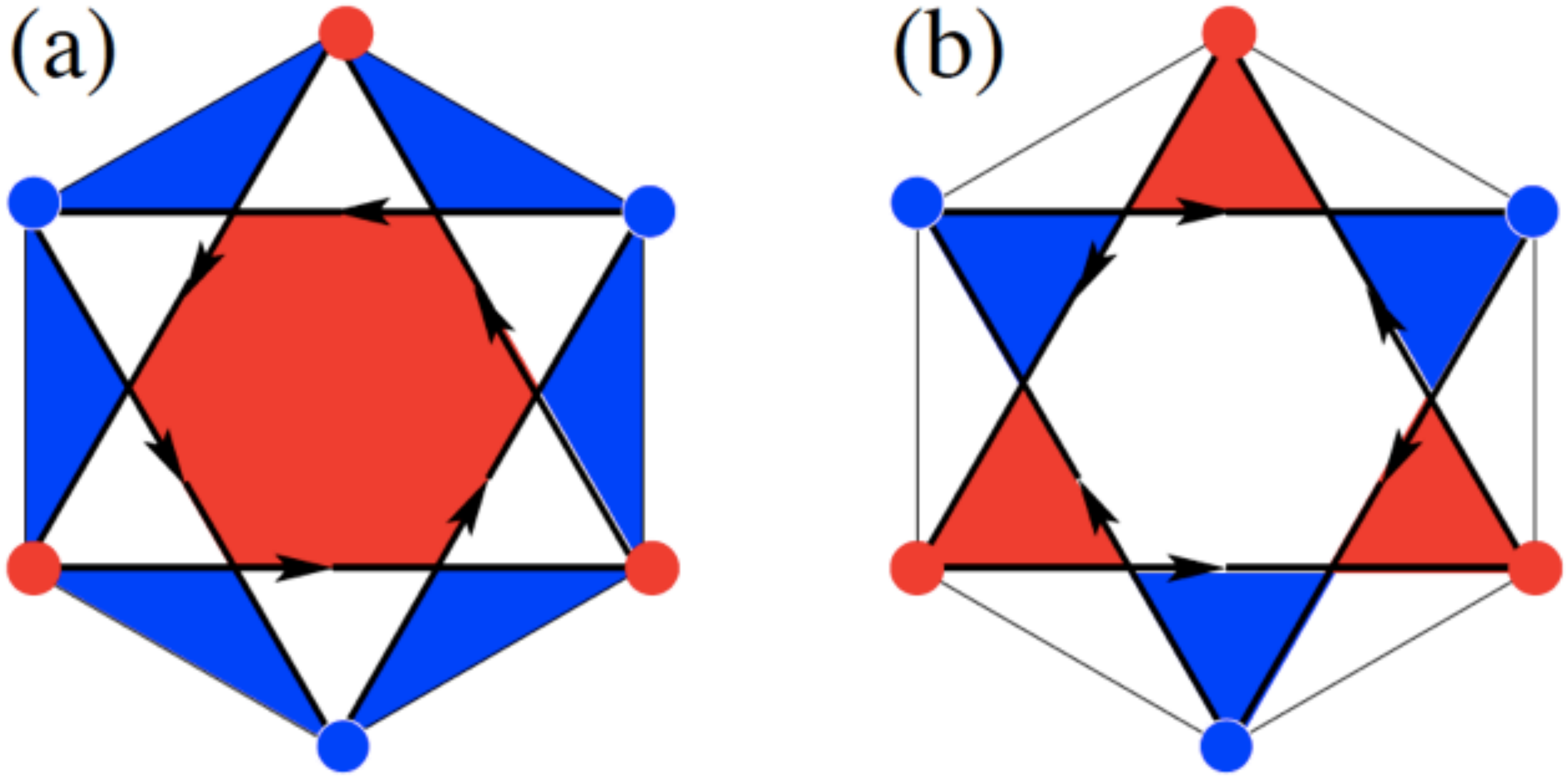
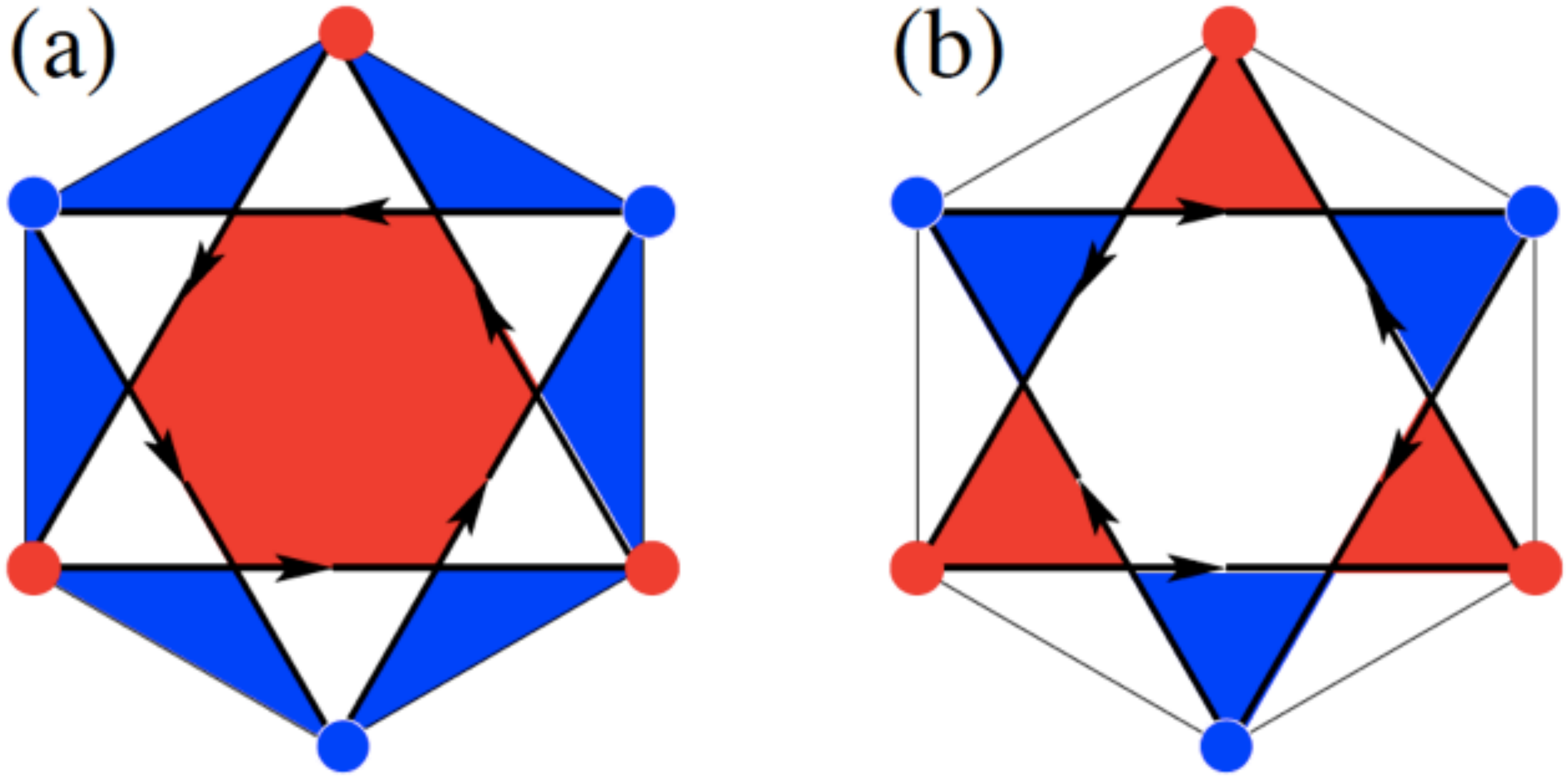
Suggest one has to go beyond the weak coupling theories
Include the gapped bands as well

Flux Phases in single layer



(a) Haldane Phase with currents in the same direction in each sublattice

Flux Phases in single layer



(a) Haldane Phase with currents in the same direction in each sublattice

(b) Pattern of fluxes with opposite currents in the two sublattice

Model

$$\begin{aligned} H &= H_1 + H_2 + H_{12} + H_{int} \\ H_l &= t \sum_{i,\delta_\nu} \left(a^\dagger_{li} b_{li+\delta_\nu} \right) + t_1 \sum_{l,\tilde{\delta}_\mu} \left(a^\dagger_{li} a_{li+\tilde{\delta}_\mu} \right) + h.c. \\ H_{12} &= t_\perp \sum_i a^\dagger_{1i} b_{2i} + h.c. \\ H_{int} &= V_{nn} \sum_{l,i,\delta_\nu} n_{li} n_{li+\delta_\nu} + V_{nnn} \sum_{l,i,\tilde{\delta}_\nu} n_{li} n_{li+\tilde{\delta}_\nu} \end{aligned}$$

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Haldane phase results from next nearest neighbor interactions.

S. Raghu et al., PRL 2008

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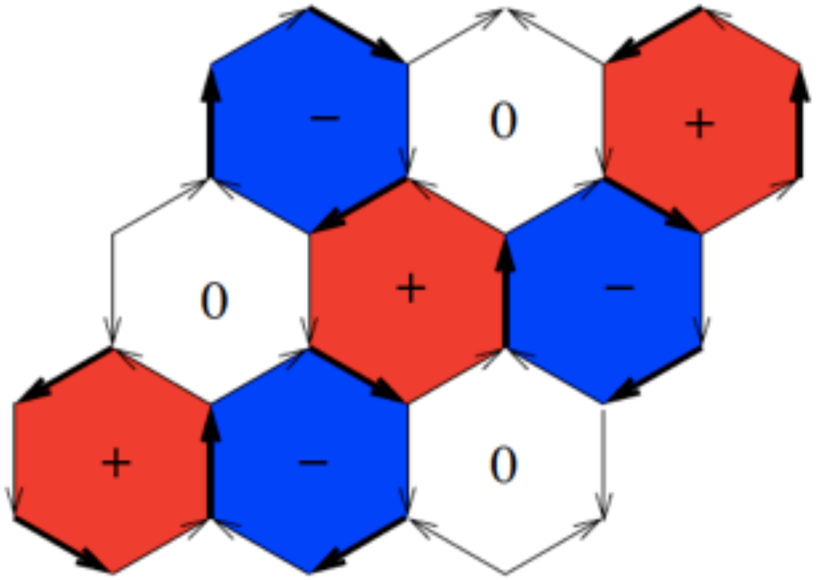

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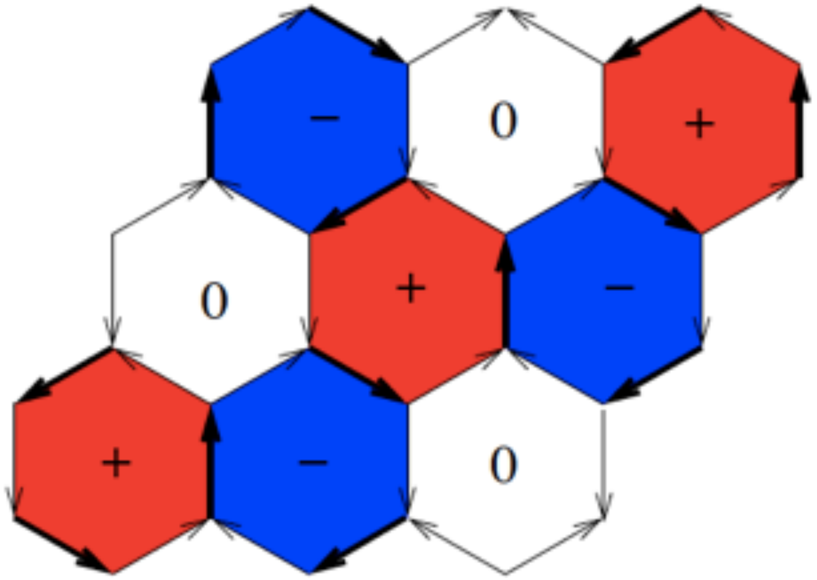

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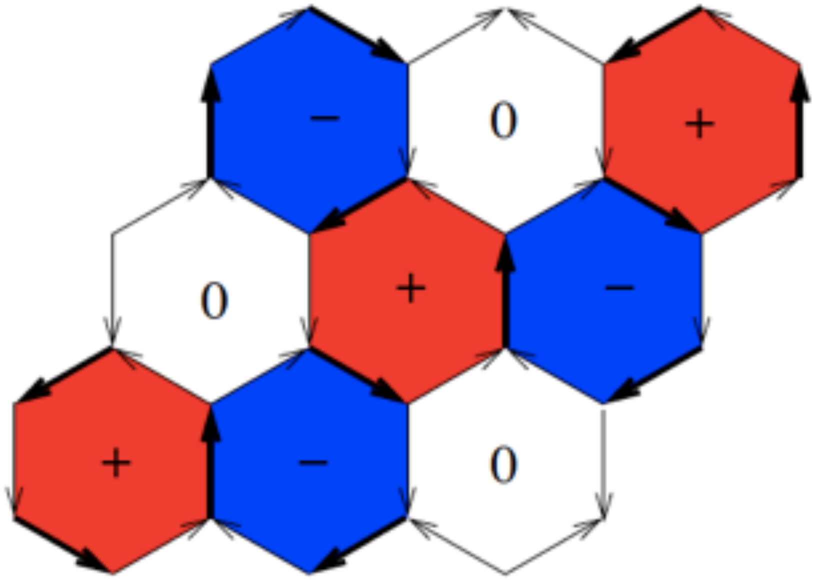

They do not take advantage of the singularities afforded by the density of state

Need much larger interaction (of order the band width)

Possible ground states

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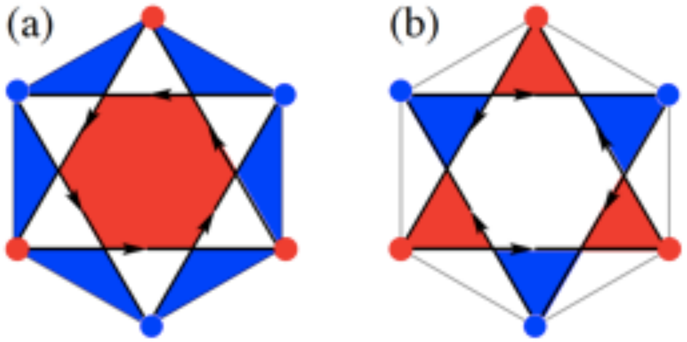
Even and odd parity (across layers) combination of the flux patterns



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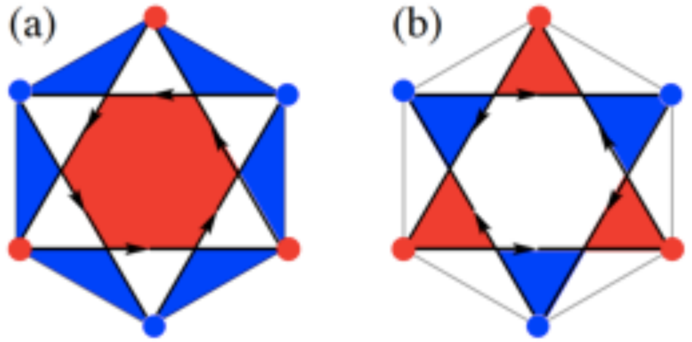

Combinations of (b) energetically disfavored

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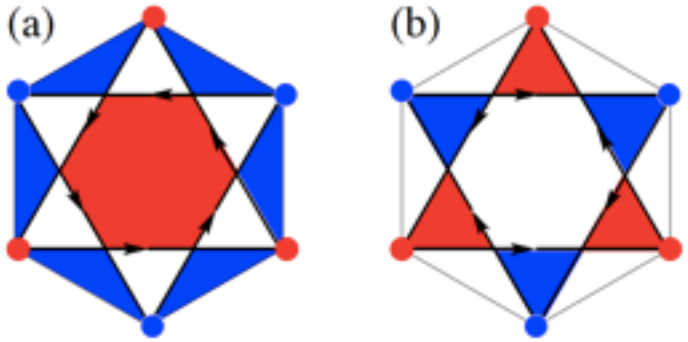
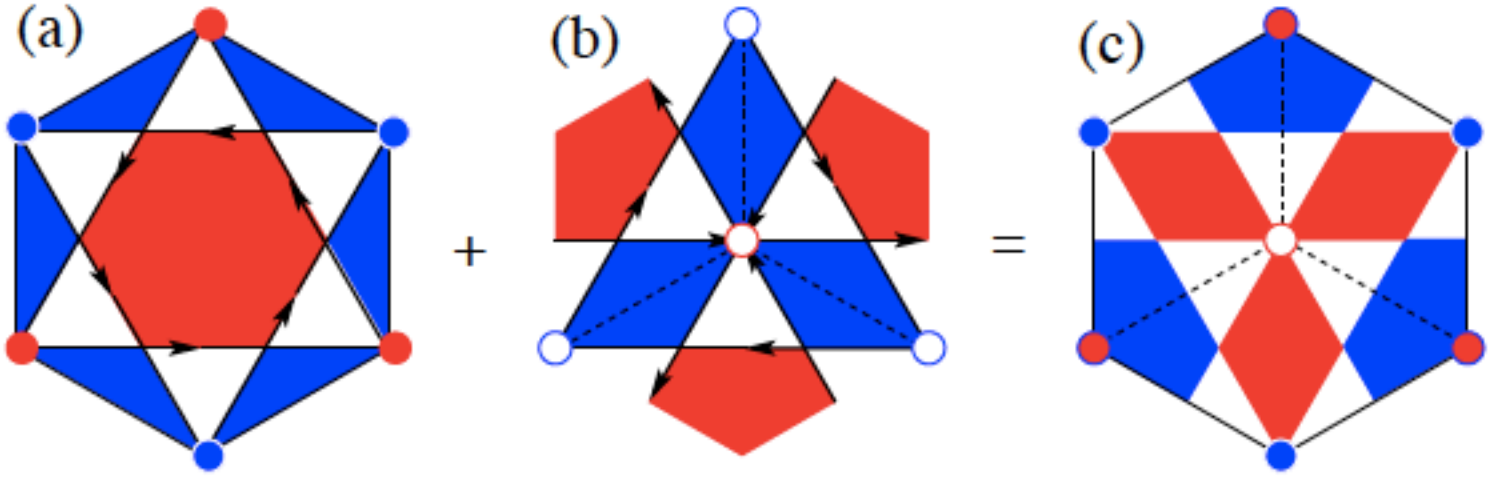


Combinations of (b) energetically disfavored

Even parity of (a) is the Quantum Anomalous Hall state (QAH).

Note that the relative flux in the two sublattice need not be equal. All fluxes on only the unstacked sublattice is state obtained when the gapped bands are ignored



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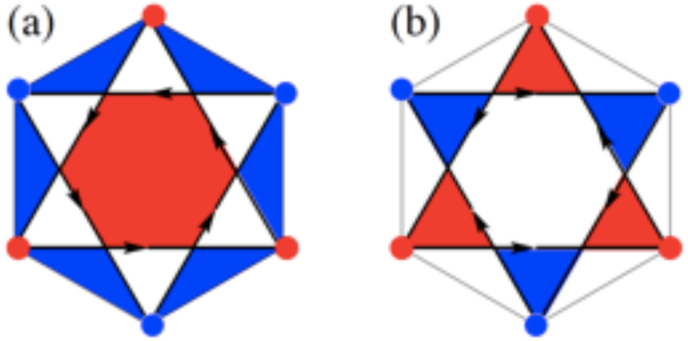

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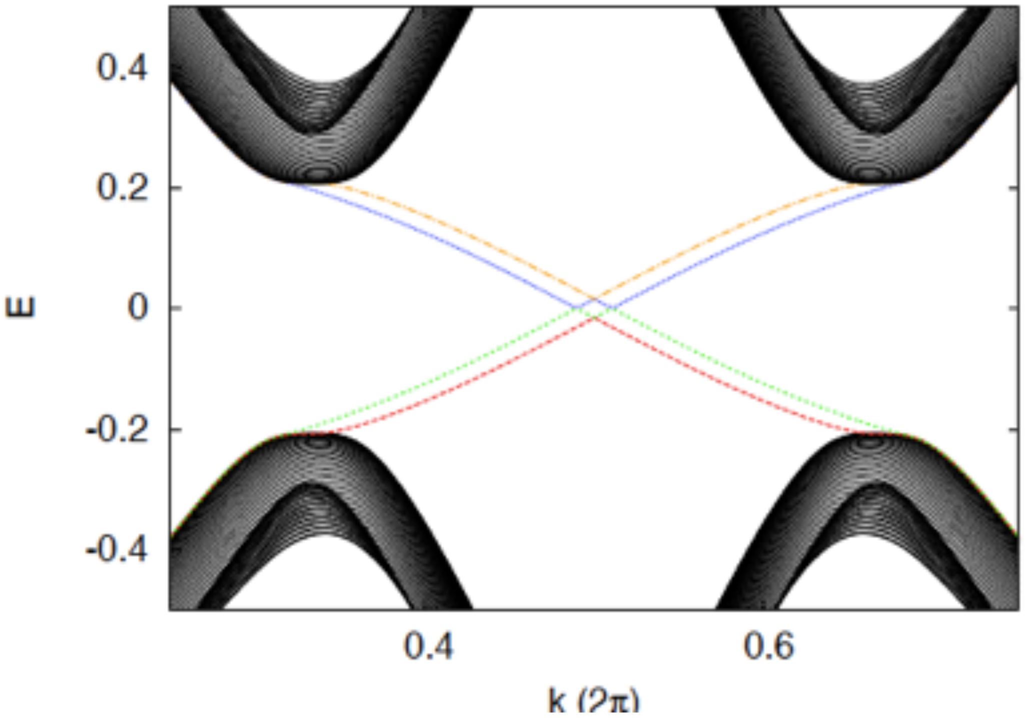

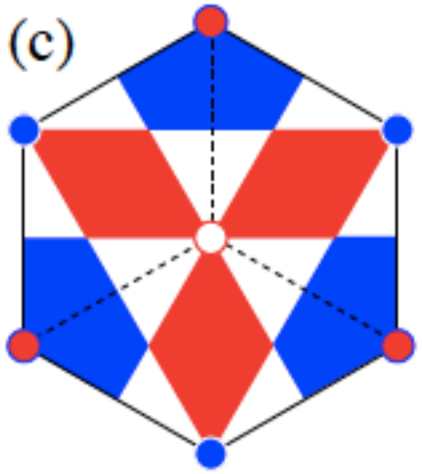

ME (Magneto electric) State

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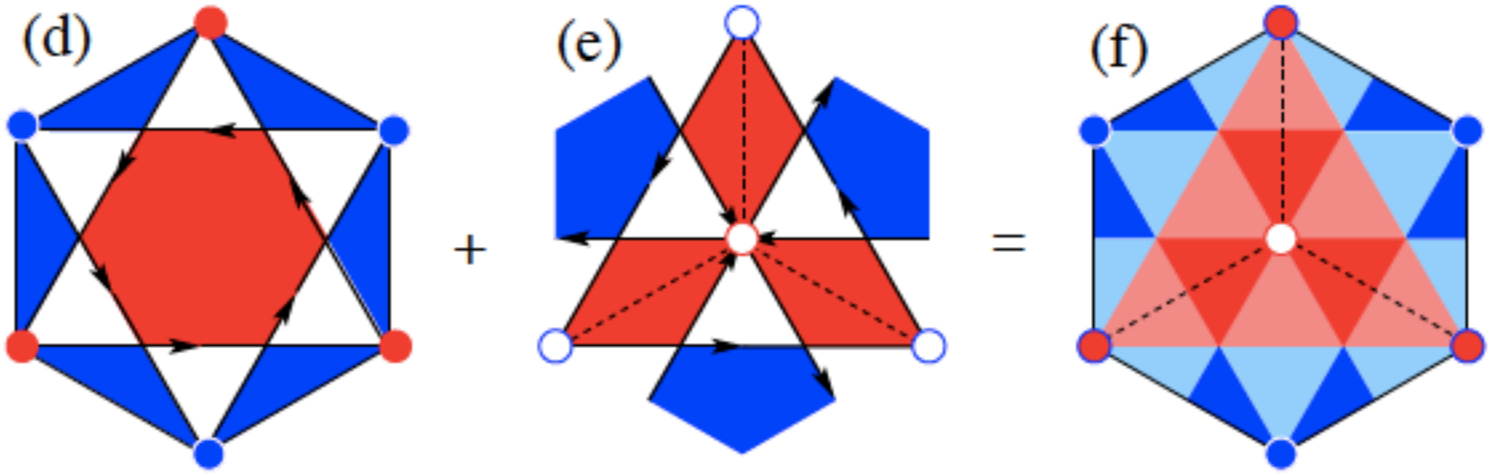
Odd parity combination is the ME state



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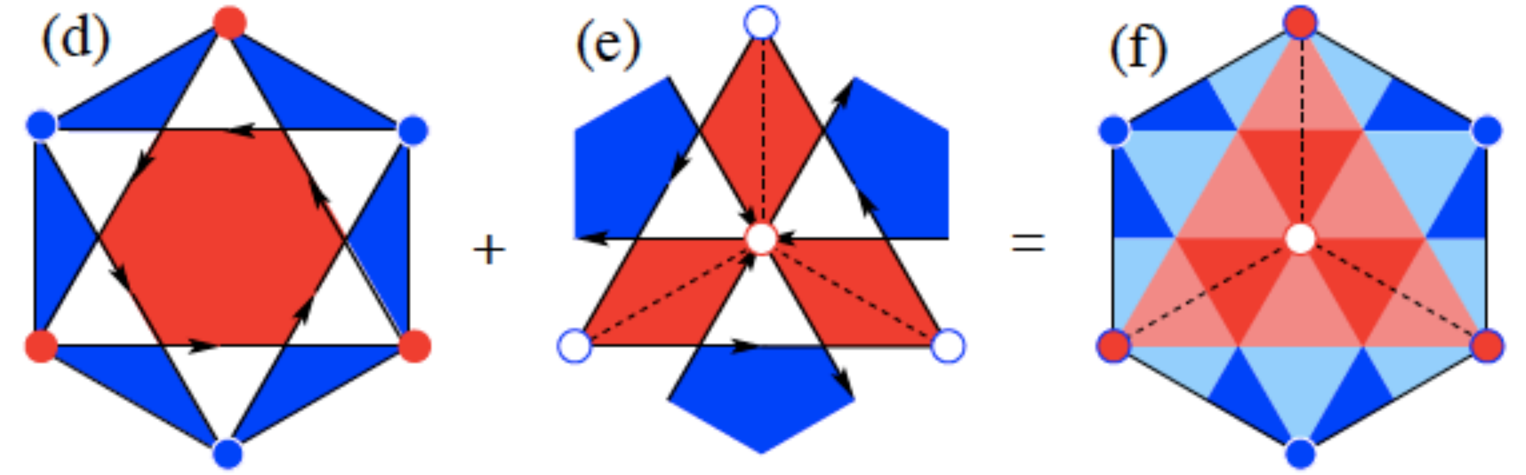
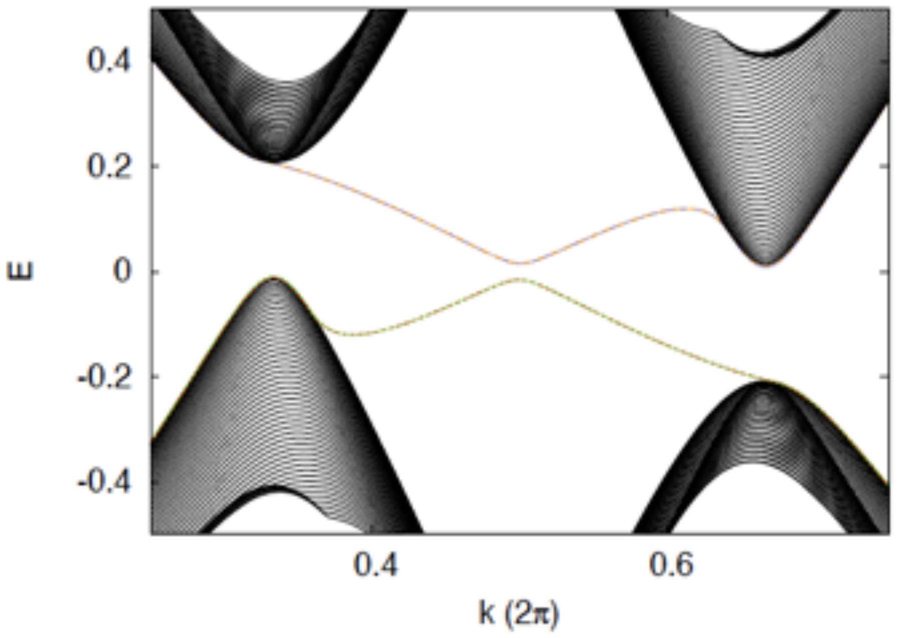


Indirect gap opens up for finite order parameter strength



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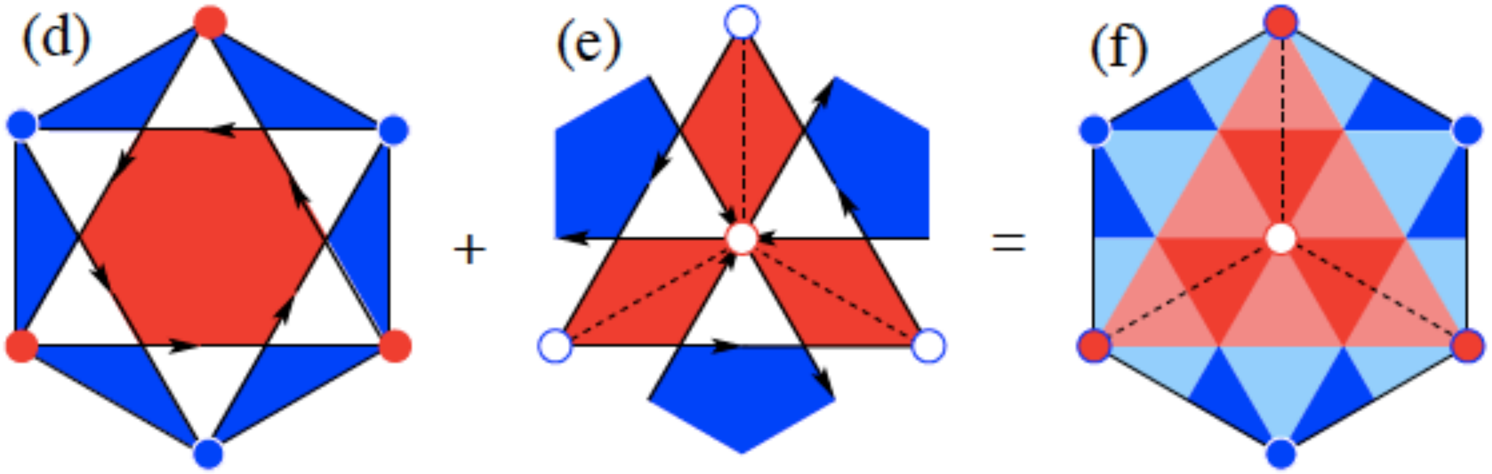

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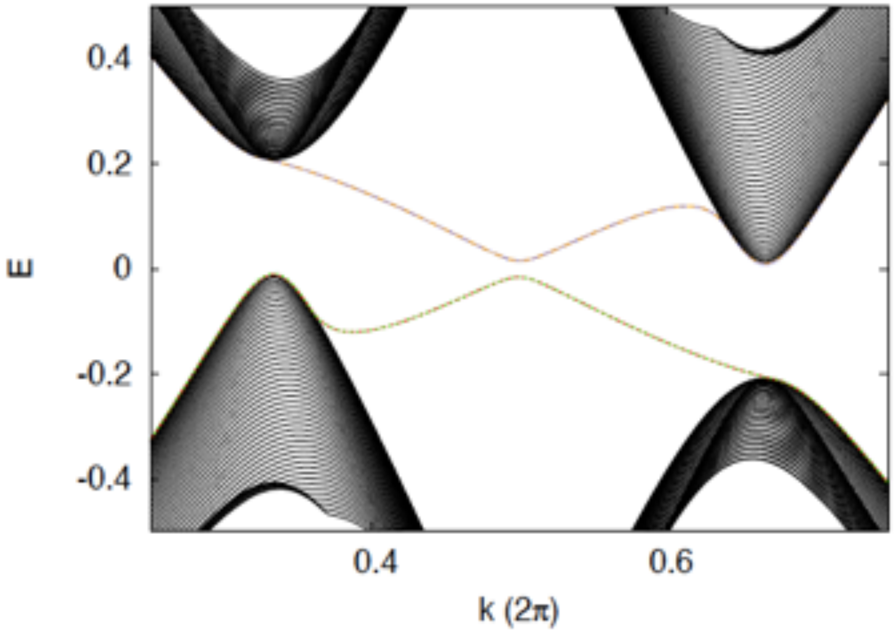
No topologically protected edge modes



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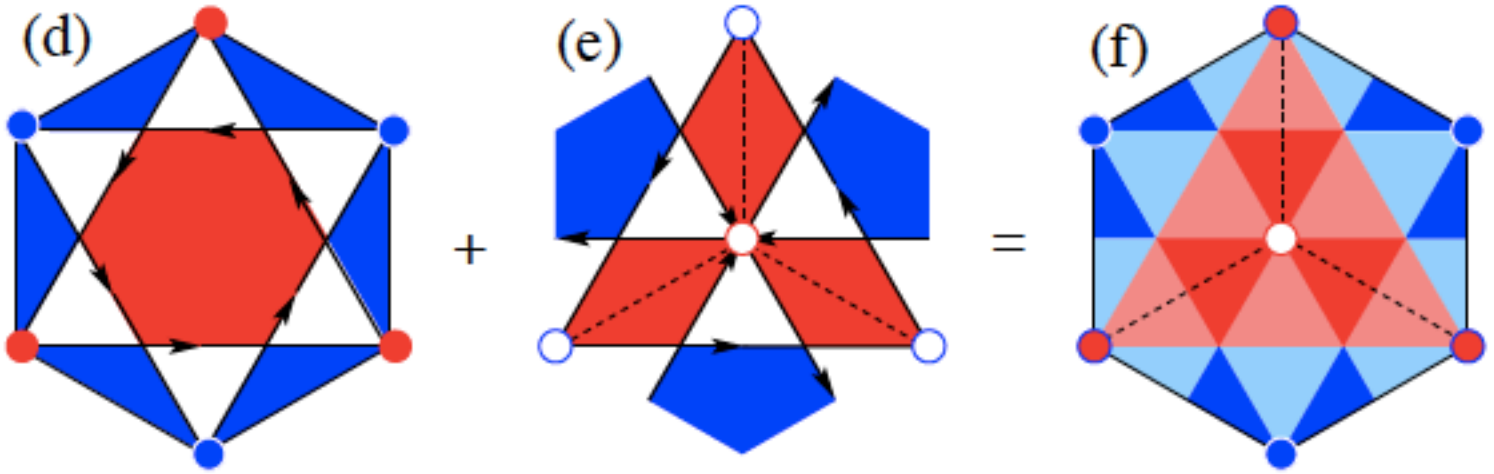

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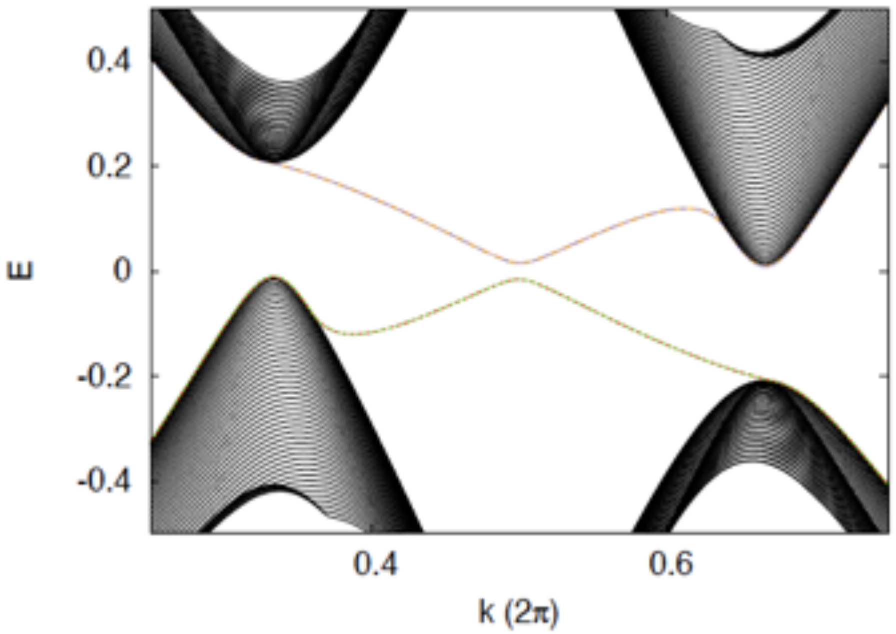
Stable state among the flux phases for next nearest neighbor interaction of order 0.05 t



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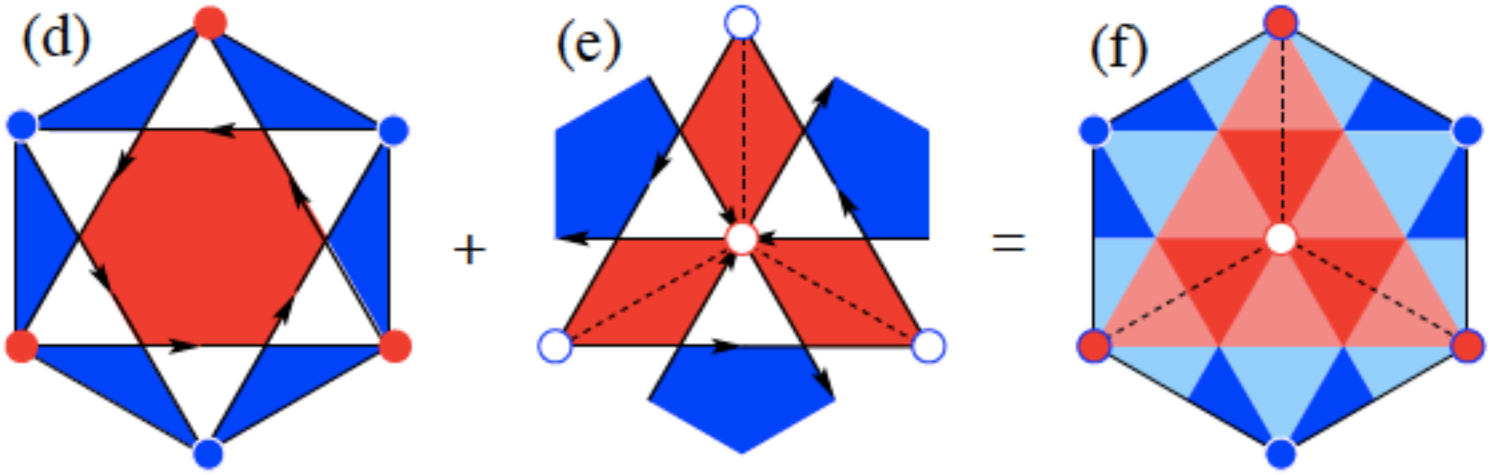

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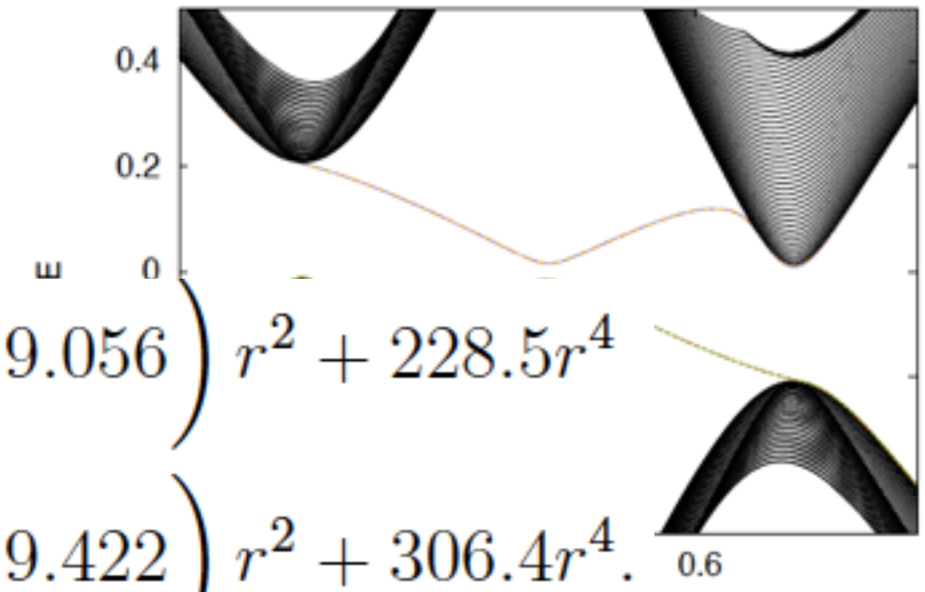

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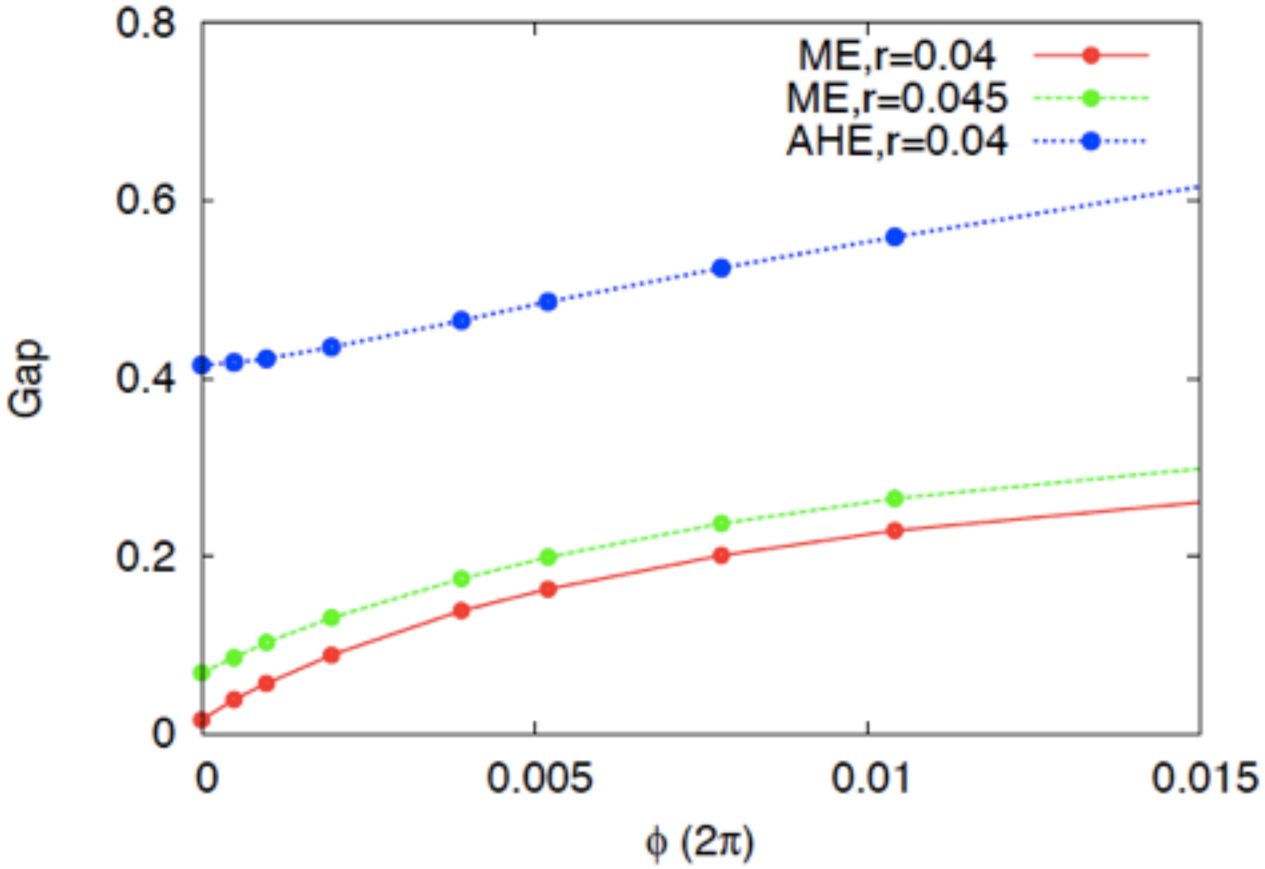
$$E(r)_{ME}/t = \left(\frac{1}{2V_{nnn}} - 9.056\right) r^2 + 228.5 r^4$$

$$E(r)_{AHE}/t = \left(\frac{1}{2V_{nnn}} - 9.422\right) r^2 + 306.4 r^4.$$

Magnetic Field Dependence



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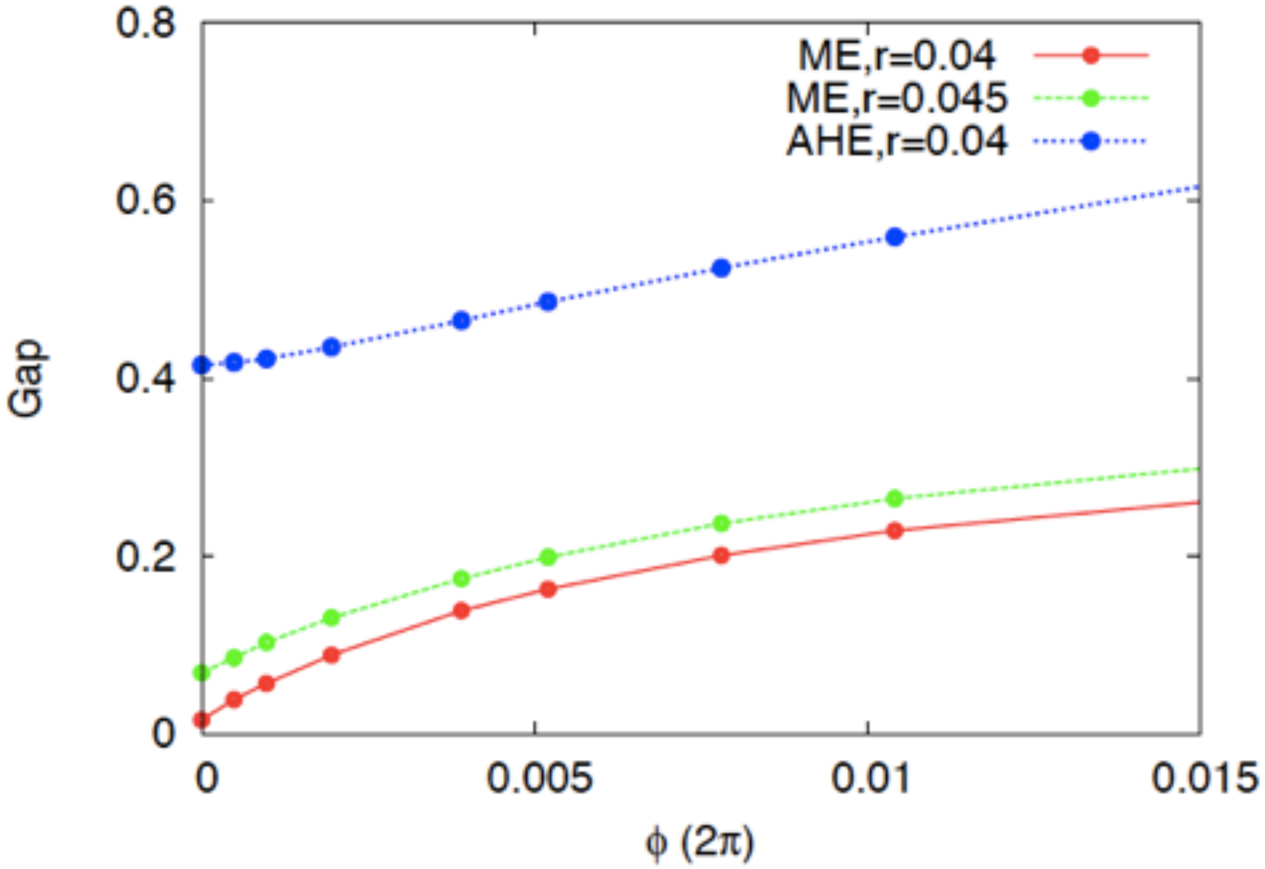

Magnetic field measured in flux per unit cell

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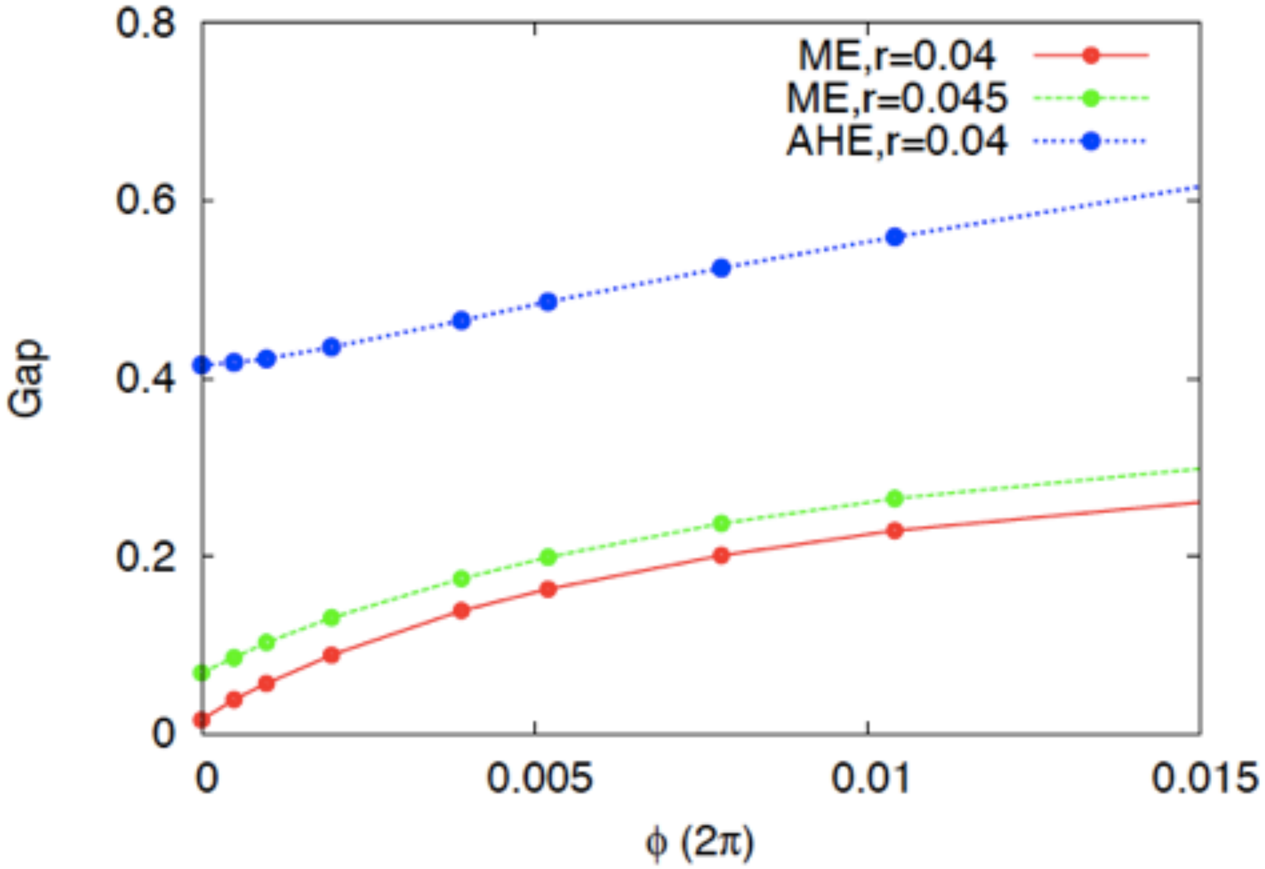

Magnetic field measured in flux per unit cell

The experimental fields not accessible yet in numerics

Conclusions

LZ, VA, CMV arXiv:1202.0821

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New candidate for the ground state of bilayer graphene

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Inclusion of gapped band can lead to states with smaller effective gap

LZ, VA, CMV arXiv:1202.0821