The dielectric constant of graphene

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Electron-electron interactions

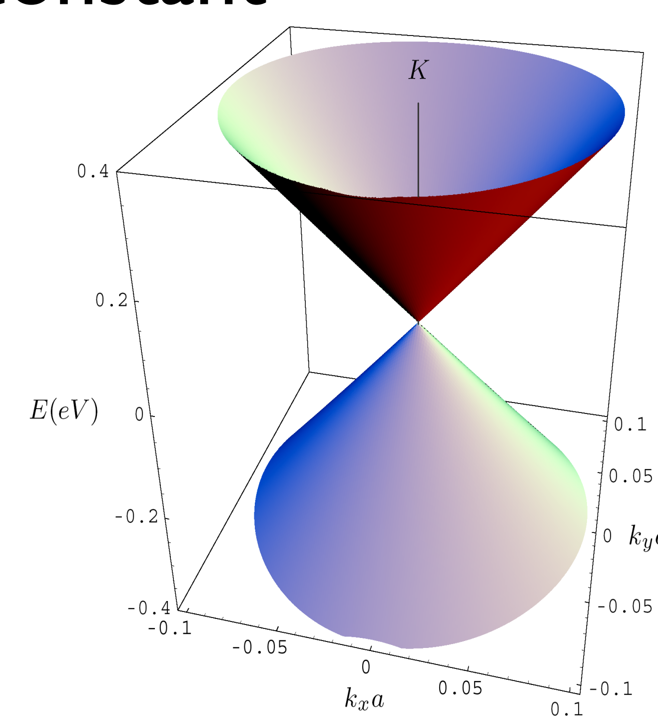
$$\frac{\text{Coulomb energy}}{\text{Kinetic energy}} = \text{strength of interactions}$$

$$E_C = e^2 n^{1/2} / \epsilon_0$$

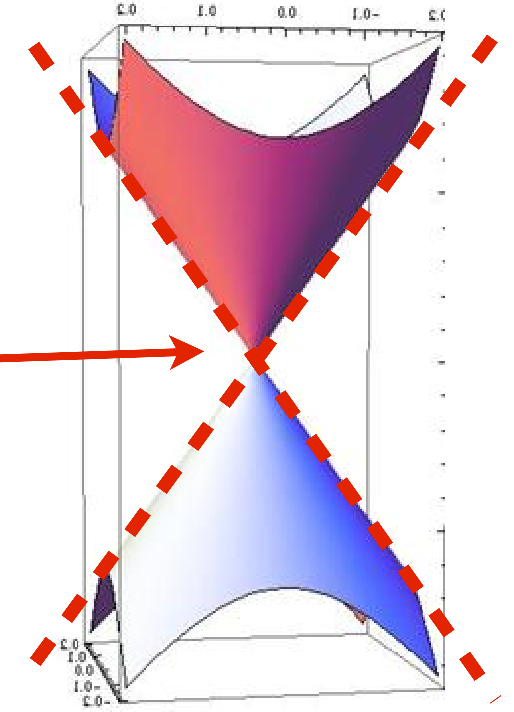
$$\alpha = \frac{E_C}{E_G} = \frac{e^2}{\epsilon_0 \hbar v_F}$$

Dimensionless fine structure constant

$$E_G \approx \hbar v_F n^{1/2}$$



Electron-electron interactions

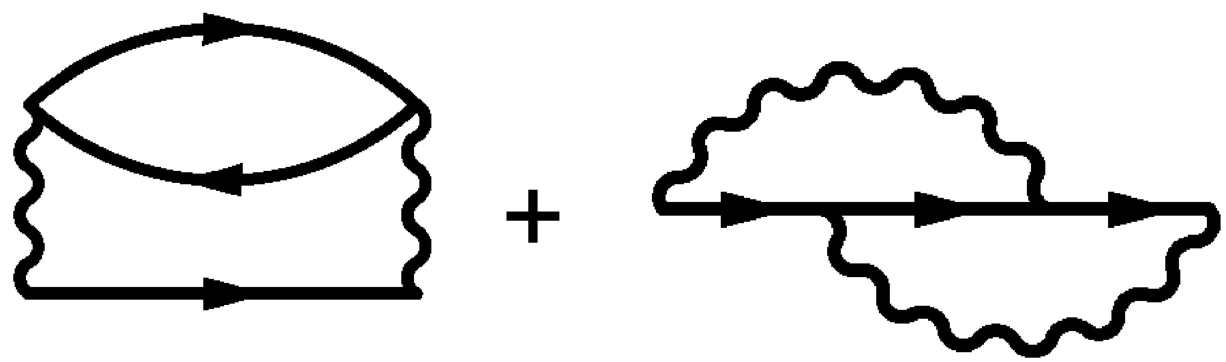


Velocity renormalization

Self-energy

$$v = v_0 \left[1 + \left(\frac{\alpha}{4} + a_1 \alpha^2 + \ldots \right) \ln \left(\frac{\Lambda}{q} \right) \right]$$

Hartree-Fock

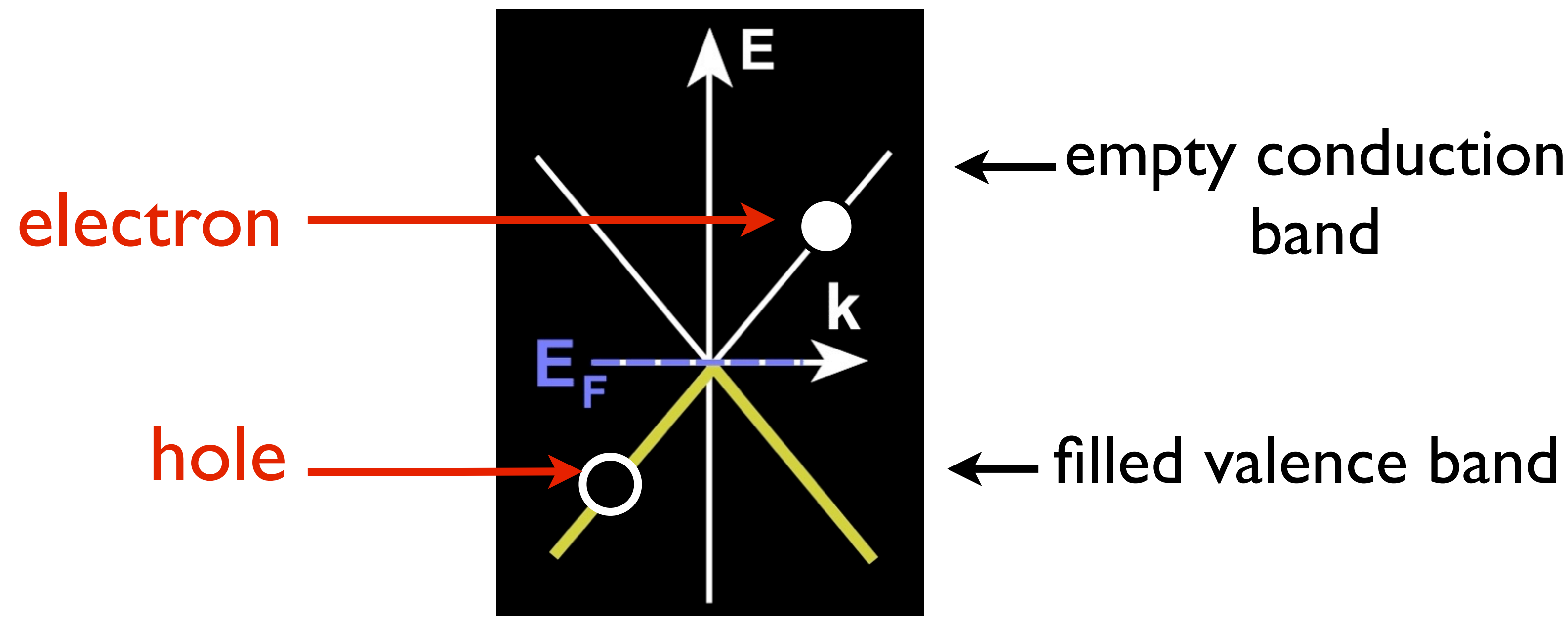


Gonzalez et al.
Nucl. Phys. B 424, 595 (94)

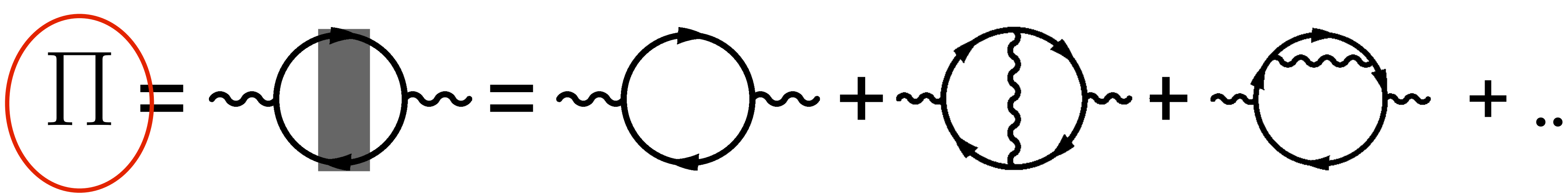
Mishchenko, PRL 98, 216801 (06)

Logarithmic singularity in the infrared! (no resumation)

Physical idea: Vacuum polarization



Polarization bubble



Creation and anihilation of particle-hole pairs!

When α=2.2 strong vacuum polarization effects can screen out interactions among quasiparticles!

Physical idea:

??

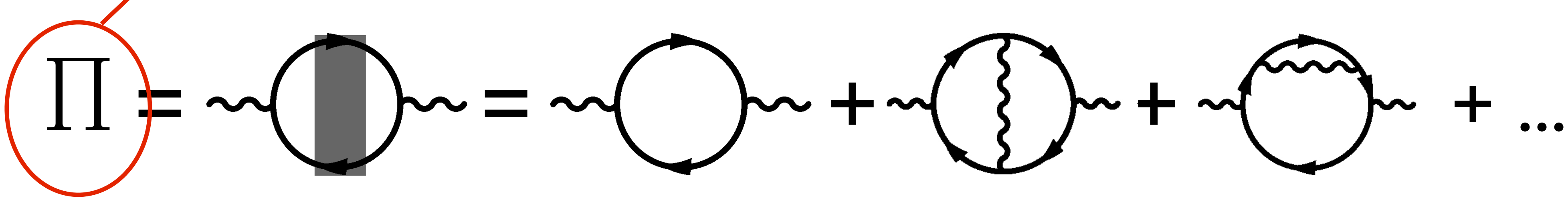
$$\alpha^{\star}(\mathbf{k},\omega) = \frac{e^2}{\hbar v\, \epsilon(\mathbf{k},\omega)} \ll 1$$

Dressed fine structure constant

$$\epsilon(\mathbf{k},\omega) = 1 - V(k)\Pi(\mathbf{k},\omega)$$

Dielectric function

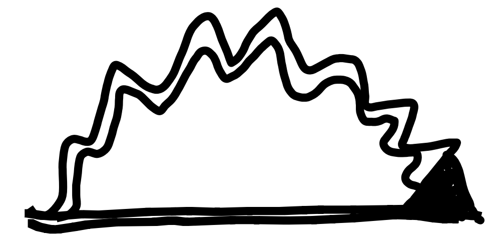
Polarization bubble



Creation and annihilation of particle-hole pairs

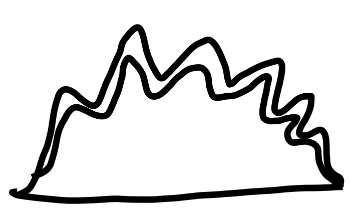
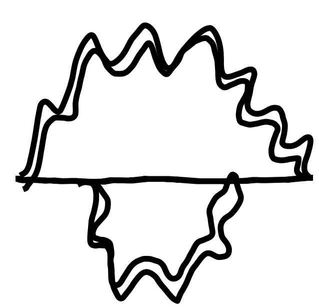
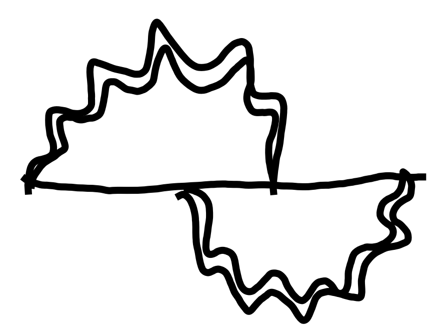
Self-energy

$$\Sigma^\star(q) = \sum_k V(k) G(k+q) \Gamma(k, k+q, q)$$



$$V^*(k) = \frac{V(k)}{1 - V(k)\Pi(k)} = \frac{V(k)}{\varepsilon(k)}$$

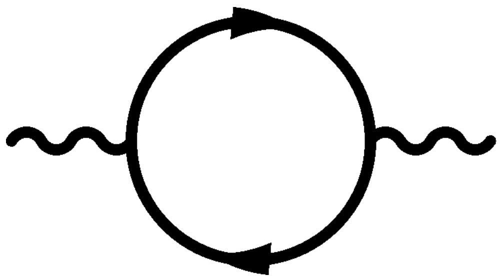
Expansion in the dressed interaction: $\alpha^\star(\mathbf{k}, \omega) = \dfrac{e^2}{\hbar v \, \epsilon(\mathbf{k}, \omega)}$

$\Sigma^\star(q)$ =  +  +  + $O[(\alpha^\star)^3]$

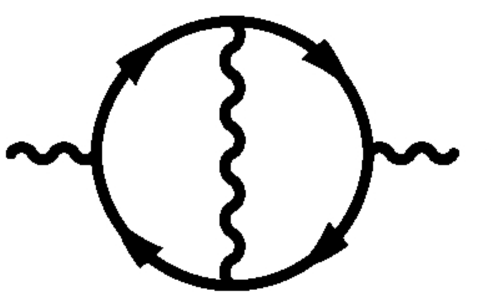
Is the dressed fine structure constant a controlled expansion parameter?

Freestanding graphene

$$\alpha = \frac{e^2}{\hbar v} \approx 2.2$$



$$\frac{2\pi e^2}{|\mathbf{q}|}\Pi^{(1)}(\mathbf{q}) = -\frac{\pi}{2}\alpha$$



$$\frac{2\pi e^2}{|\mathbf{q}|}\Pi^{(2)}(\mathbf{q}) = -0.53\alpha^2$$

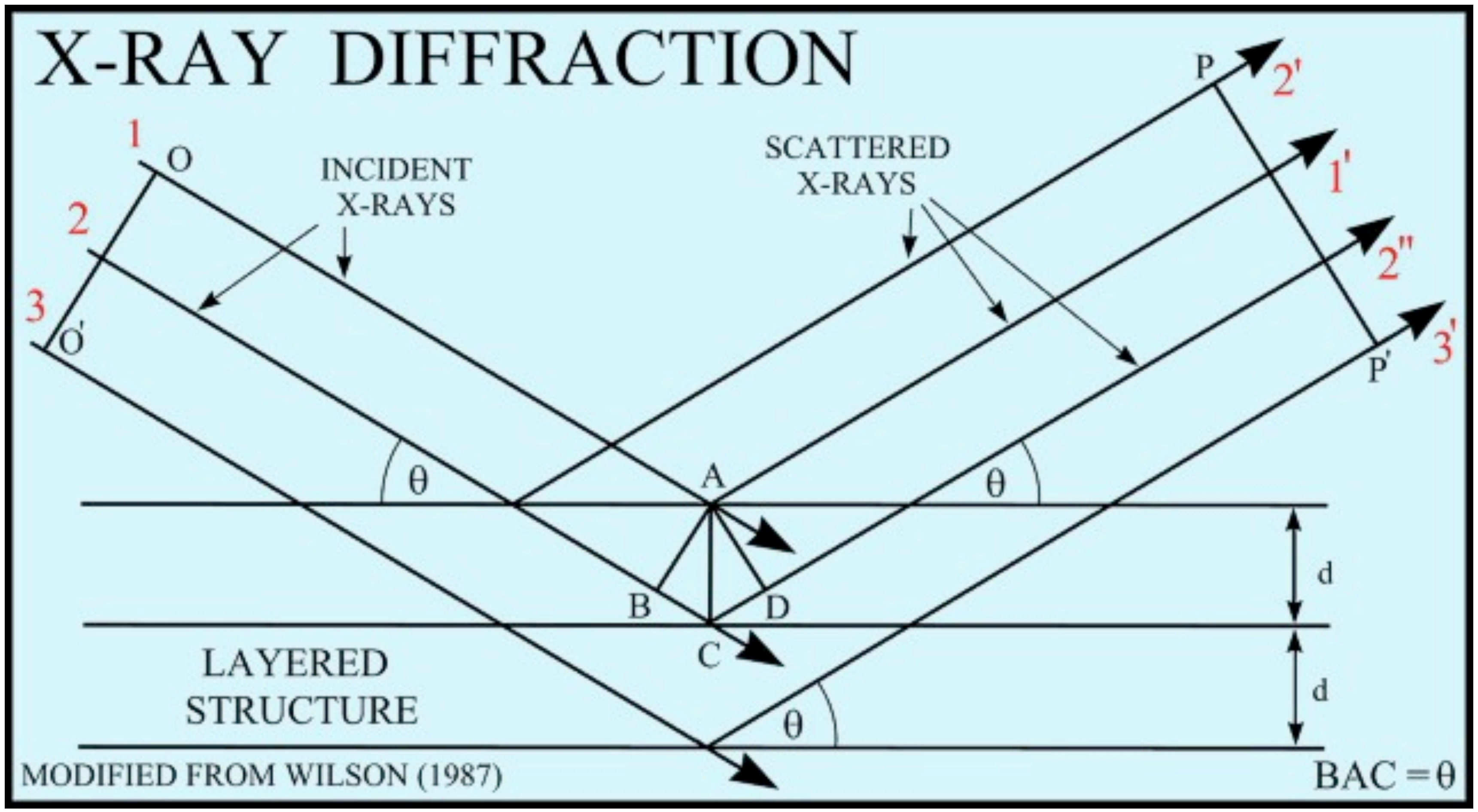
Dielectric constant $\mathcal{E} = 1 + \frac{\pi}{2}\alpha + 0.53\ \alpha^2 + O(\alpha^3).$

3.5 2.5

Perturbation theory brakes down!

V. N. Kotov, B. Uchoa, A. H. Castro Neto, PRB 78, 035119 (2008)

What about the experiments?

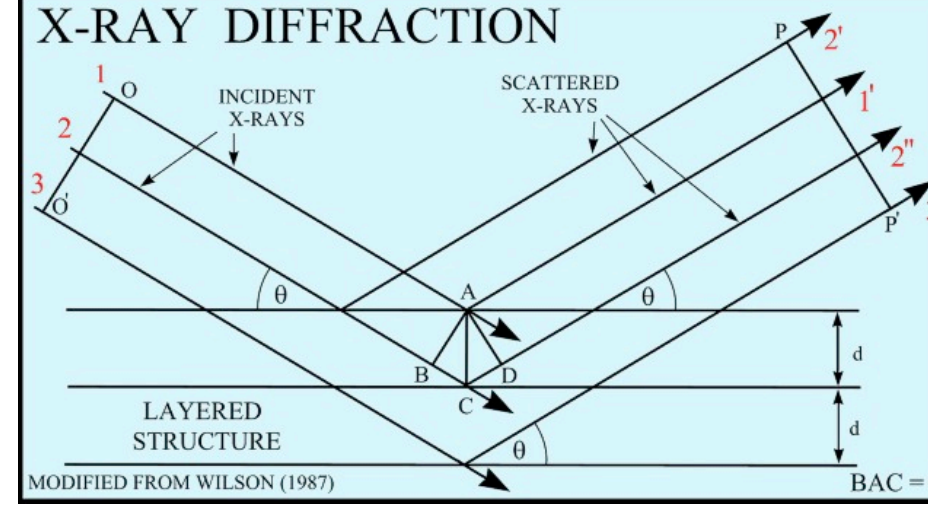


$$\chi(t, \mathbf{r}, \mathbf{r}') = \langle T[\hat{n}(t, \mathbf{r})\hat{n}(0, \mathbf{r}')]\rangle$$

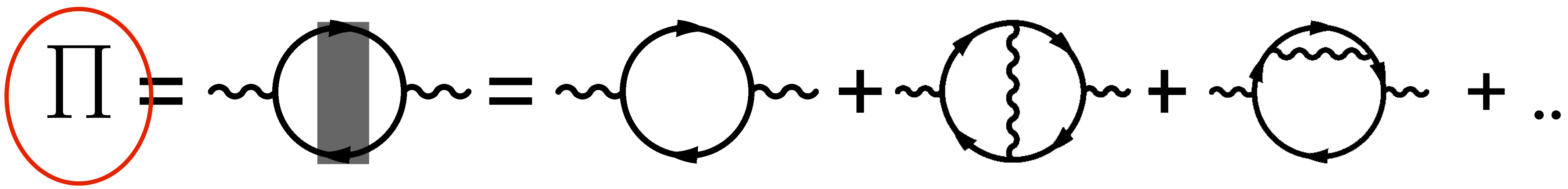
Density-density correlation function

Inelastic X-ray diffraction

$$\chi(t,\mathbf{r},\mathbf{r}') = \langle T[\hat{n}(t,\mathbf{r})\hat{n}(0,\mathbf{r}')]\rangle$$

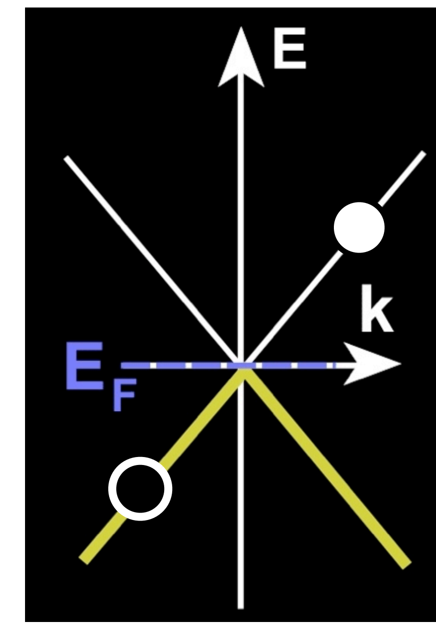


Polarization bubble



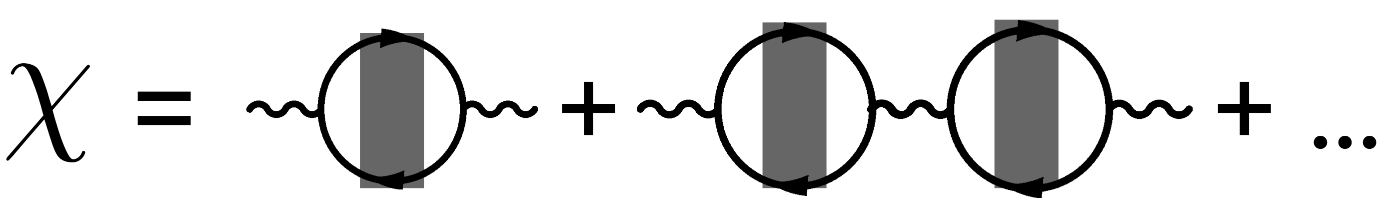
Building block of the response function!

Creation and anihilation of particle-hole pairs



Particle-hole pair

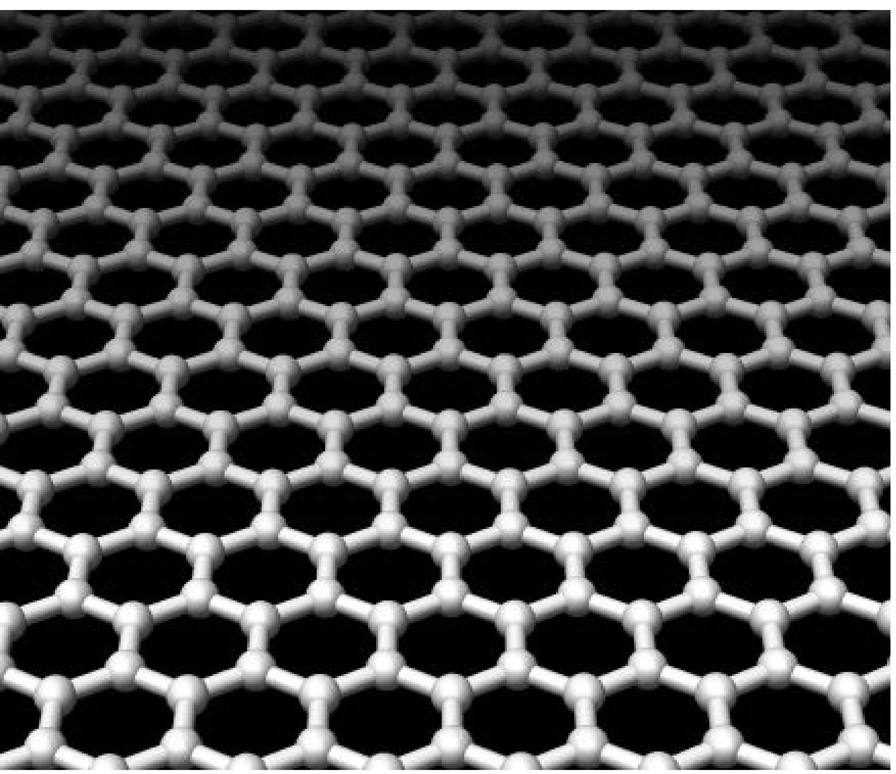
The response function

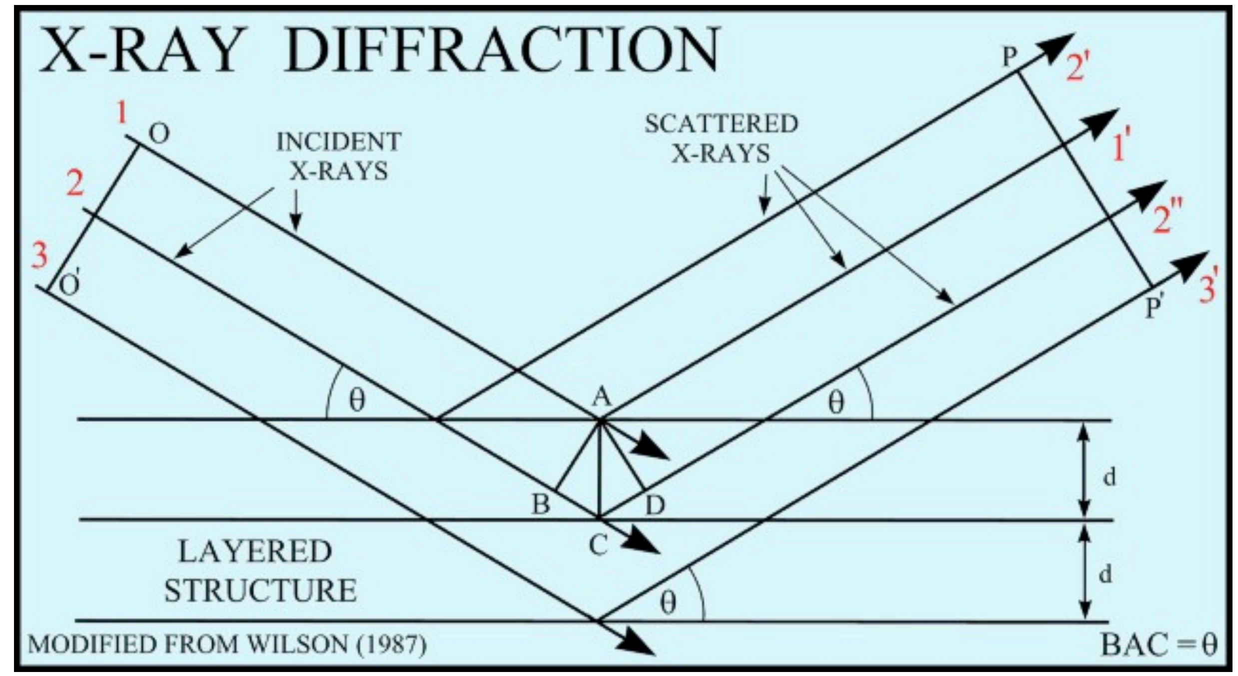


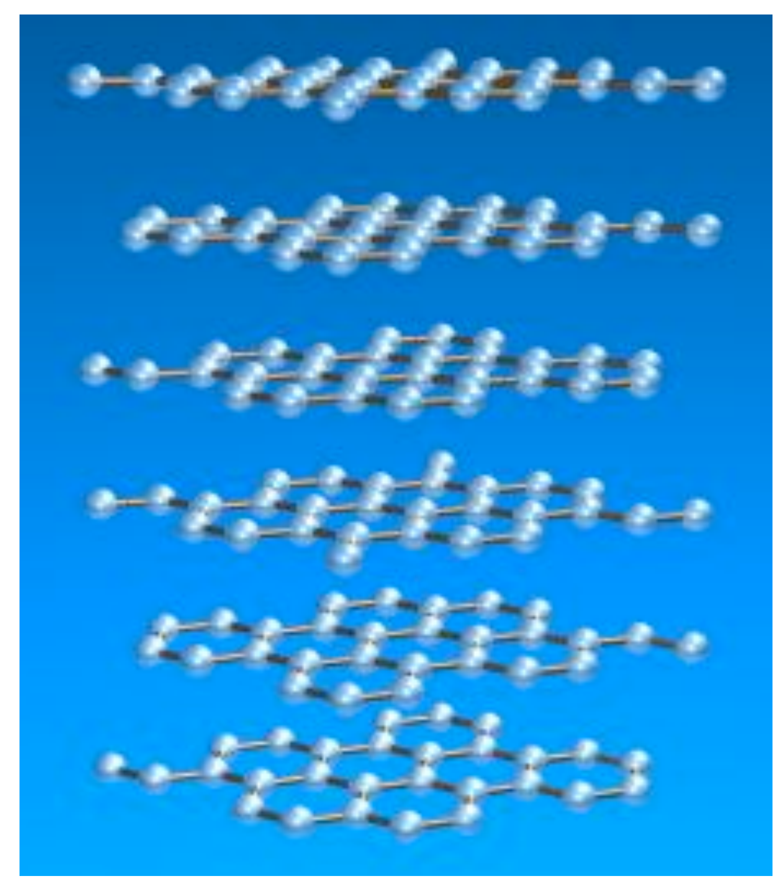
$$\chi = \ldots = \frac{\Pi}{1 - V^{\star}\Pi}$$

Problem: One cannot scatter X-rays in a single layer!

????



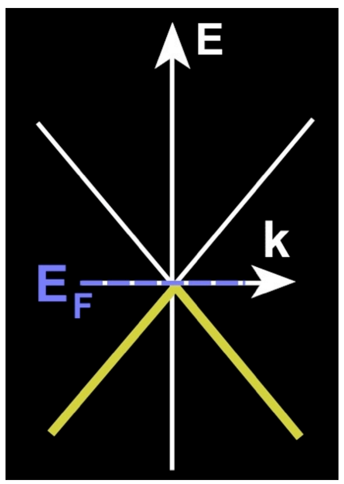


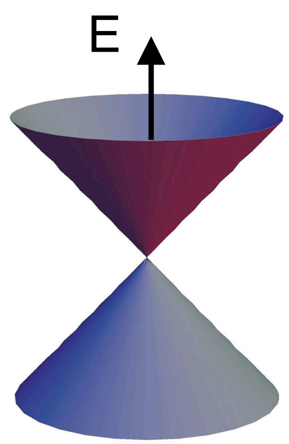


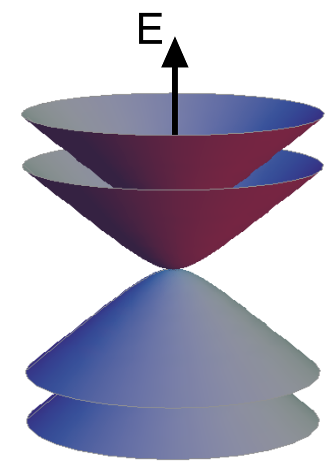
Graphene ← ? Graphite

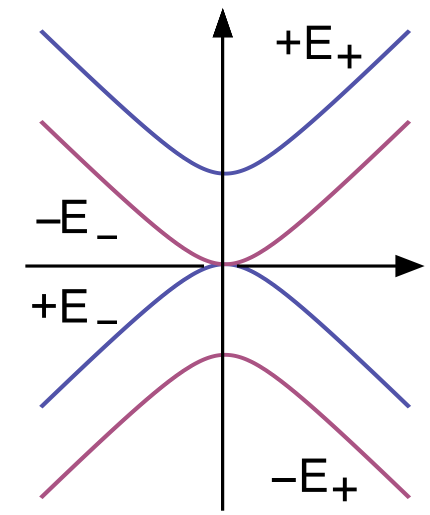
$$V(q, z=0) = \frac{e^2}{q}$$ 2D

3D $$V(q, k_z) = \frac{e^2}{q^2 + k_z^2}$$



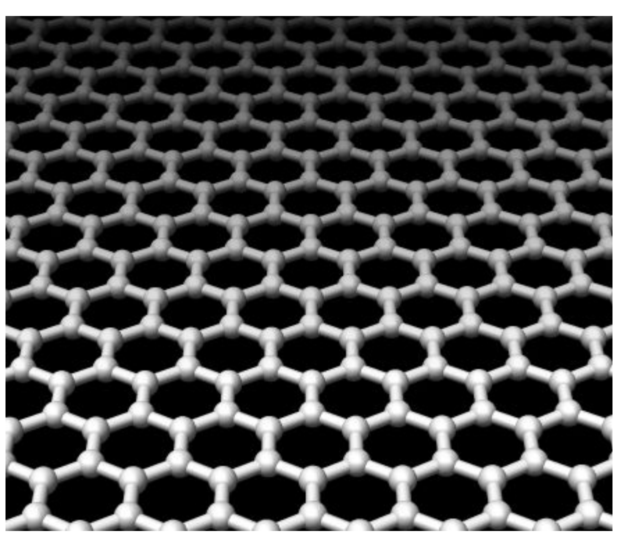


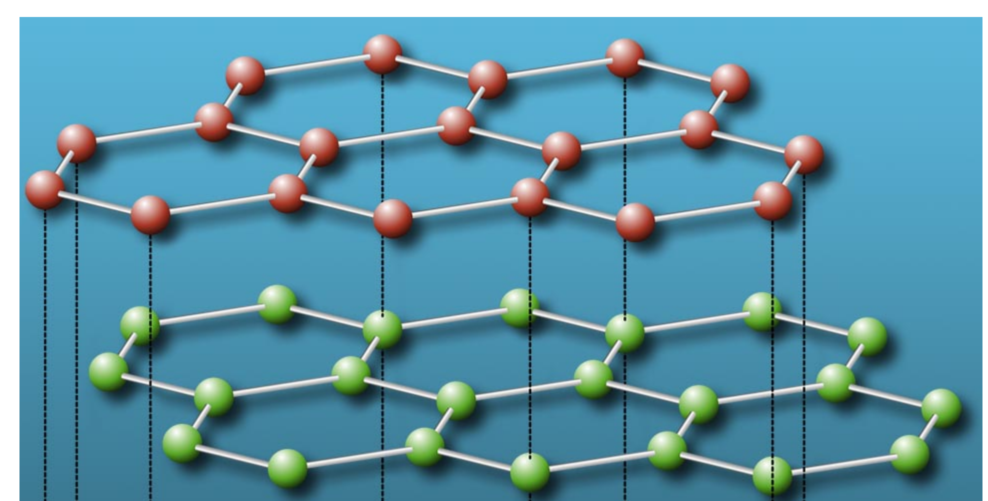




linear spectrum

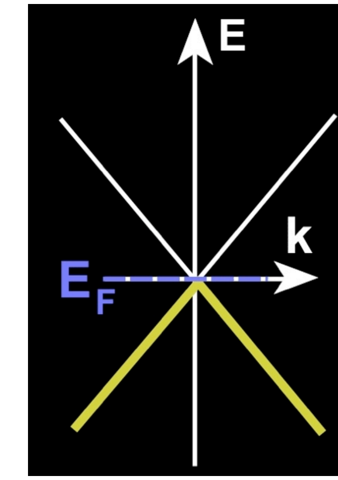
hyperbolic spectrum:
electronic hopping between layers

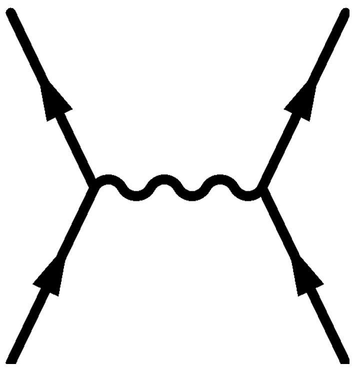




Argument:

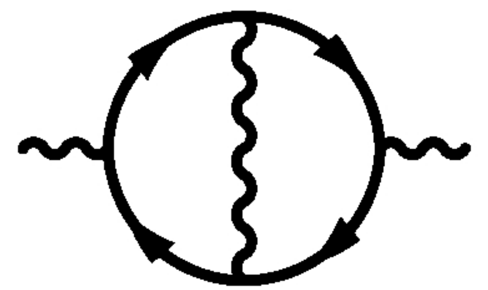
For $t_\perp = 0$ the fermion propagator is kz independent





$$\frac{1}{q^2 + k_z^2}$$

Coulomb 3D



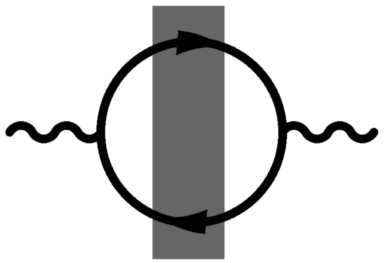
$$\int_{-\infty}^{\infty} \mathrm{d}k_z \frac{1}{q^2 + k_z^2} = \frac{1}{q}$$

Coulomb 2D

The polarizability of graphite and graphene are identical in higher order of perturbation theory!

Argument:

Leading term (order N)



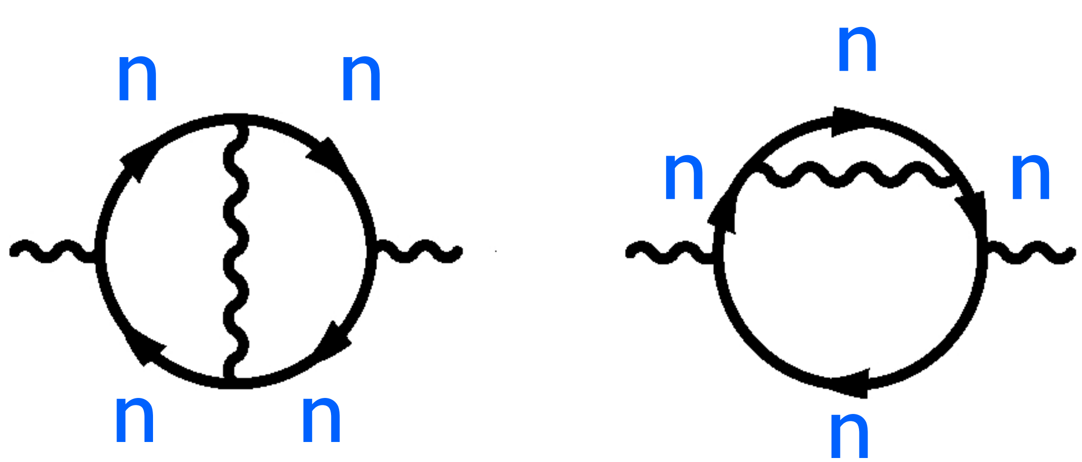
$$= \int_{|\mathbf{q}_{P_1}| \ll |\mathbf{q}_{P_2}| \ll \ldots \ll |\mathbf{q}_{P_N}|}^{\Lambda} \mathrm{d}\mathbf{q}_1 \ldots \mathrm{d}\mathbf{q}_N \times f(\mathbf{q}_1, \ldots \mathbf{q}_N, \omega_1, \ldots \omega_N)$$

$$t_\perp \ll \max(q, \omega) \ll |\mathbf{q}_{P_1}| \ll |\mathbf{q}_{P_2}| \ll \ldots \ll |\mathbf{q}_{P_N}|$$

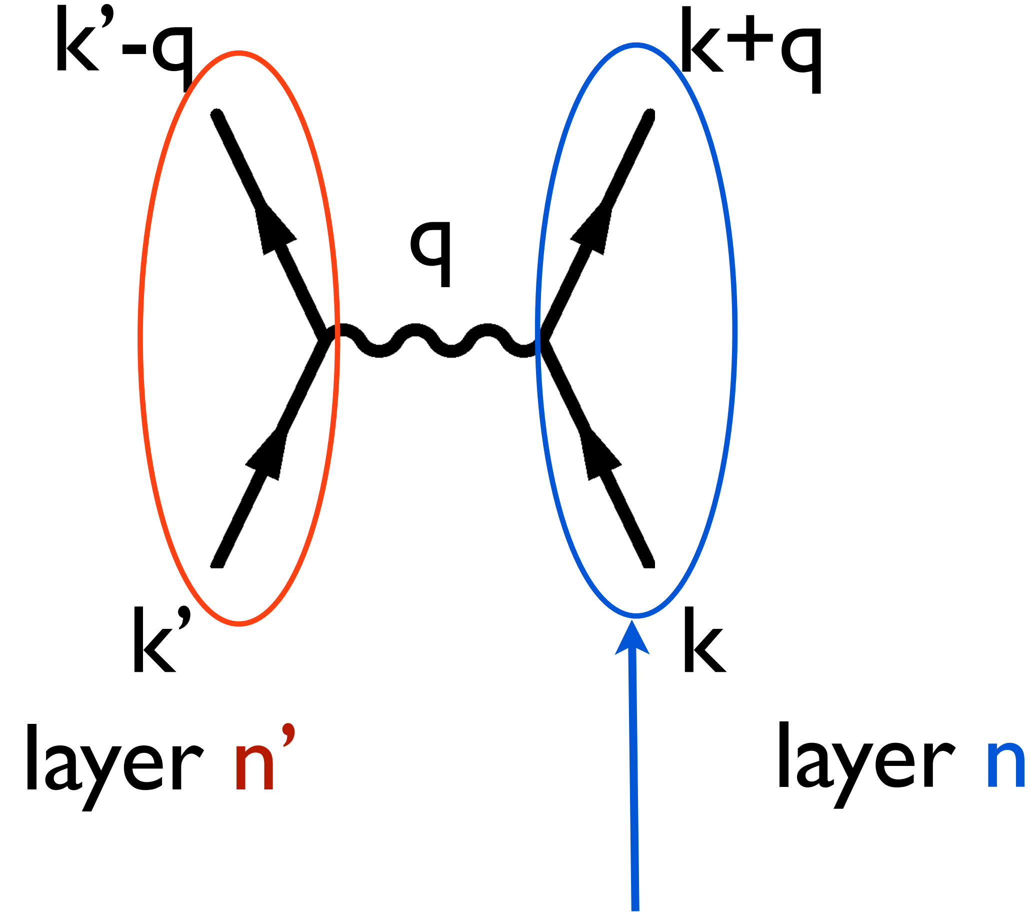
Infrared cut-off!

$t_\perp$ gives subleading corrections to the polarization when $\max(\omega, \hbar v k) \gg t_\perp$!

Physical Argument:

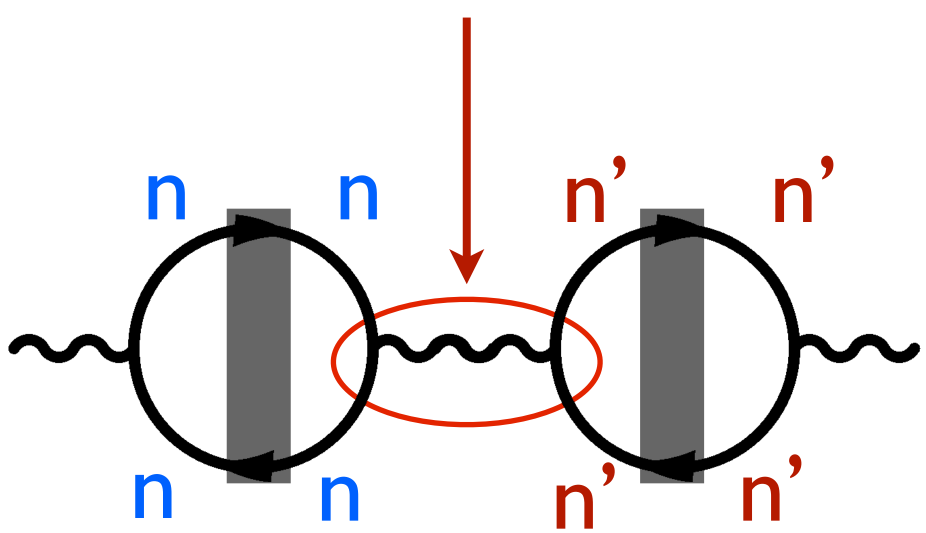


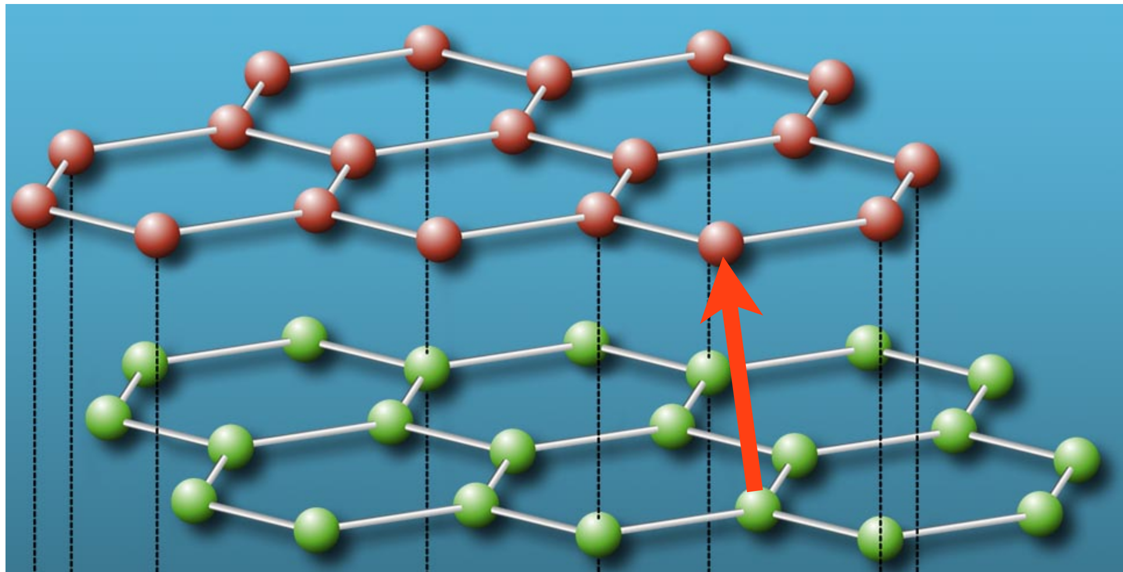
Intra-layer interaction

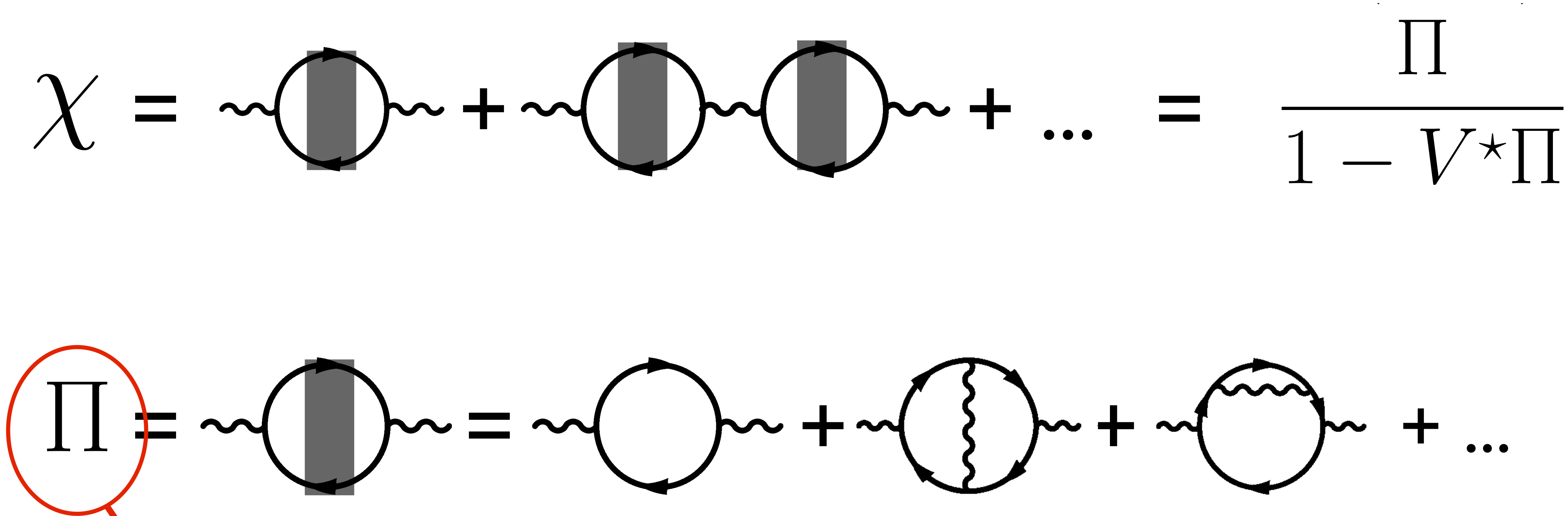


At energy scales much larger than $t_\perp \approx 0.4\,\mathrm{eV}$

Coulomb coupling between different layers







$$\boxed{\Pi_{3D}(\mathbf{k},\omega) = \frac{1}{d}\Pi_{2D}(\mathbf{k},\omega)}$$

Distance between layers

for $\max(\omega, \hbar v k) \gg t_{\perp} \approx 0.4\,\mathrm{eV}$

The polarizability of graphite and graphene are approximately the same!

Charge susceptibility (of a single freestanding graphene sheet)

$$\chi(\mathbf{k},\omega) = \frac{\chi_{3D}(\mathbf{k},-\mathbf{k},\omega)\cdot d}{1 - V(k)\left[1 - F(\mathbf{k})\right]\chi_{3D}(\mathbf{k},-\mathbf{k},\omega)\cdot d}$$

$\chi(\mathbf{k},\omega)$ → Graphene!

$V(k)$ → $\frac{2\pi e^2}{q}$

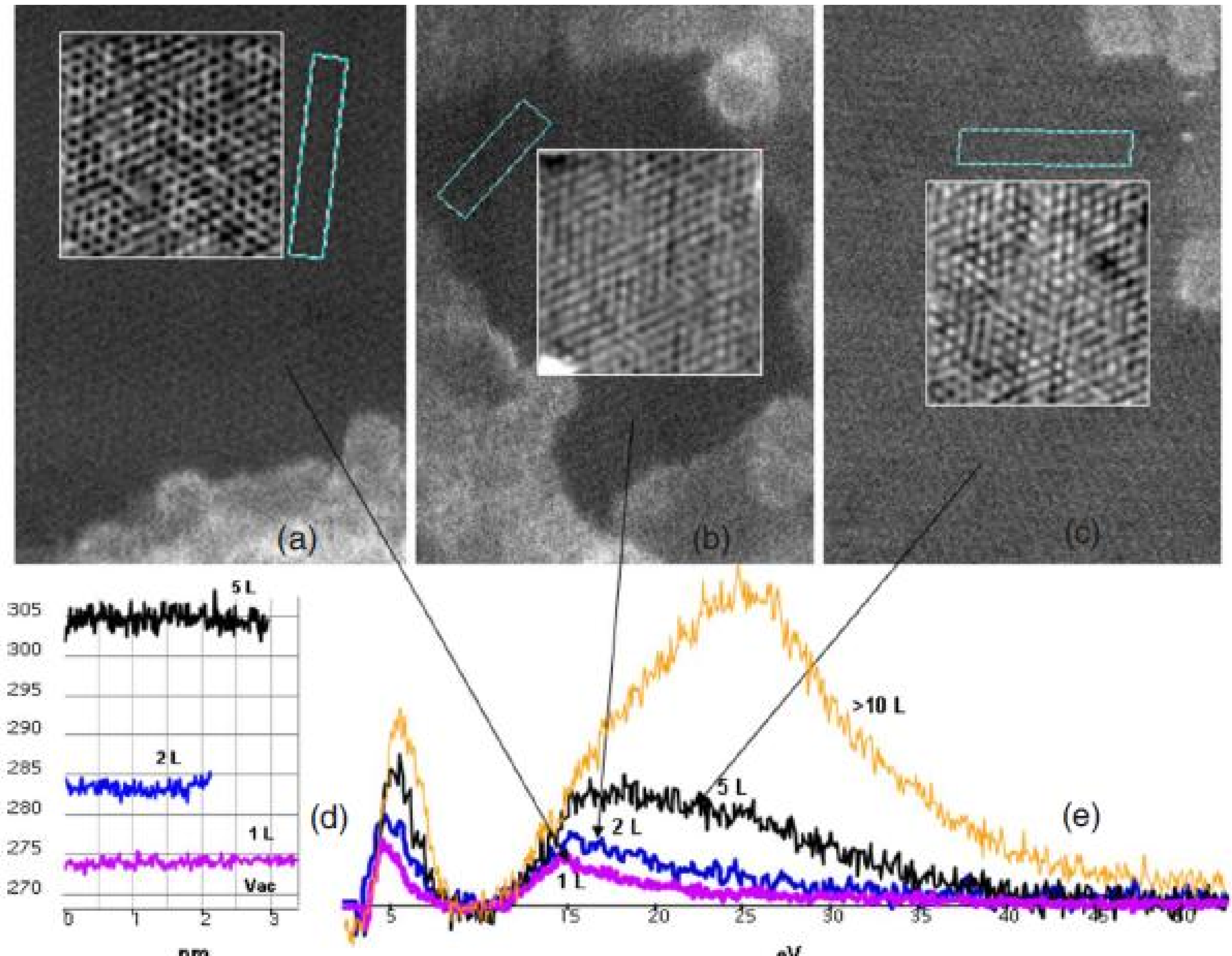
$\chi_{3D}(\mathbf{k},-\mathbf{k},\omega)$ → data

$F(\mathbf{k})$ →

$$F(\mathbf{k}) = \frac{\sinh(qd)}{\cosh(qd) - \cos(k_z d)}$$

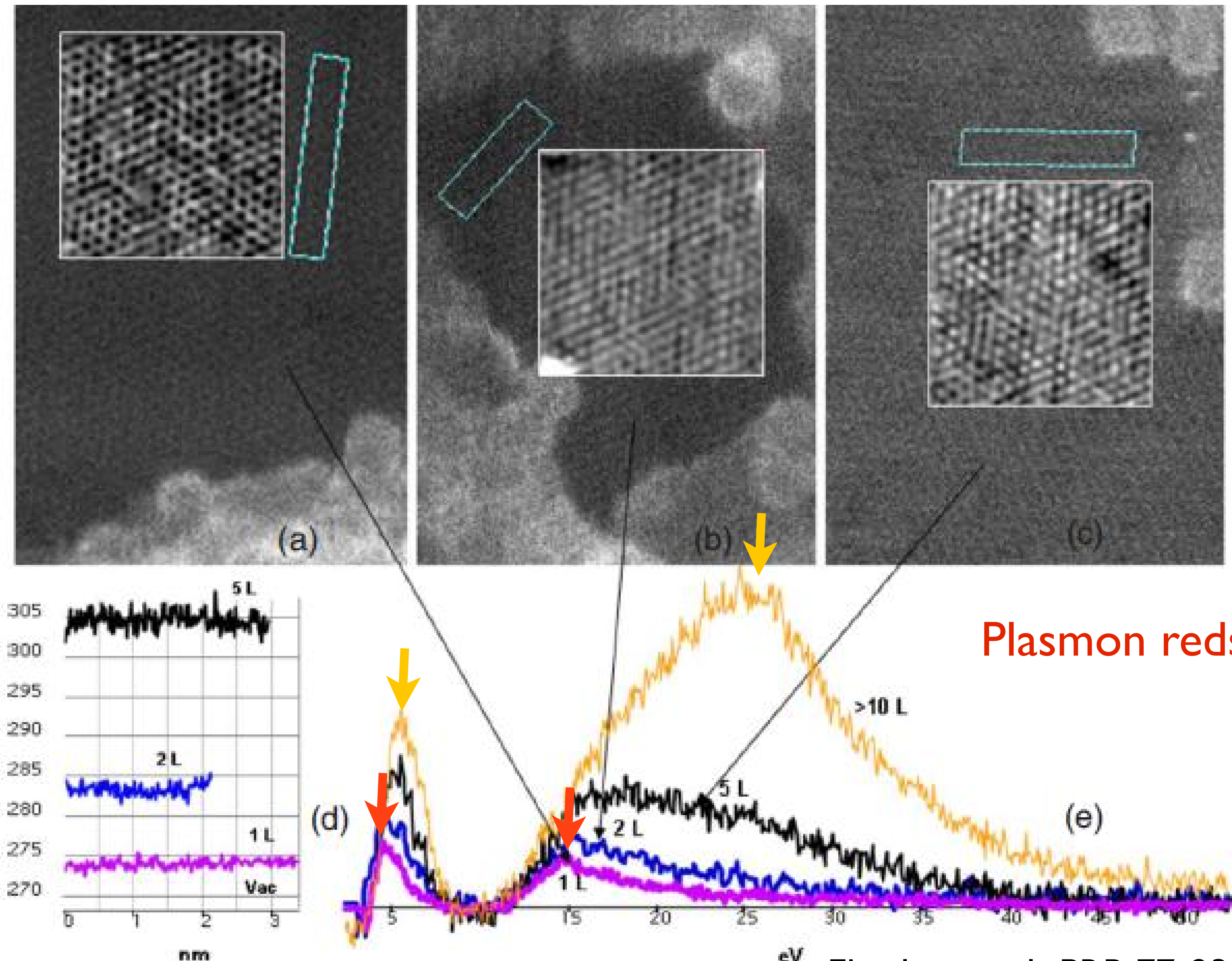
Graphite structure factor
(infinite number of layers)

Testing conversion with EELS data



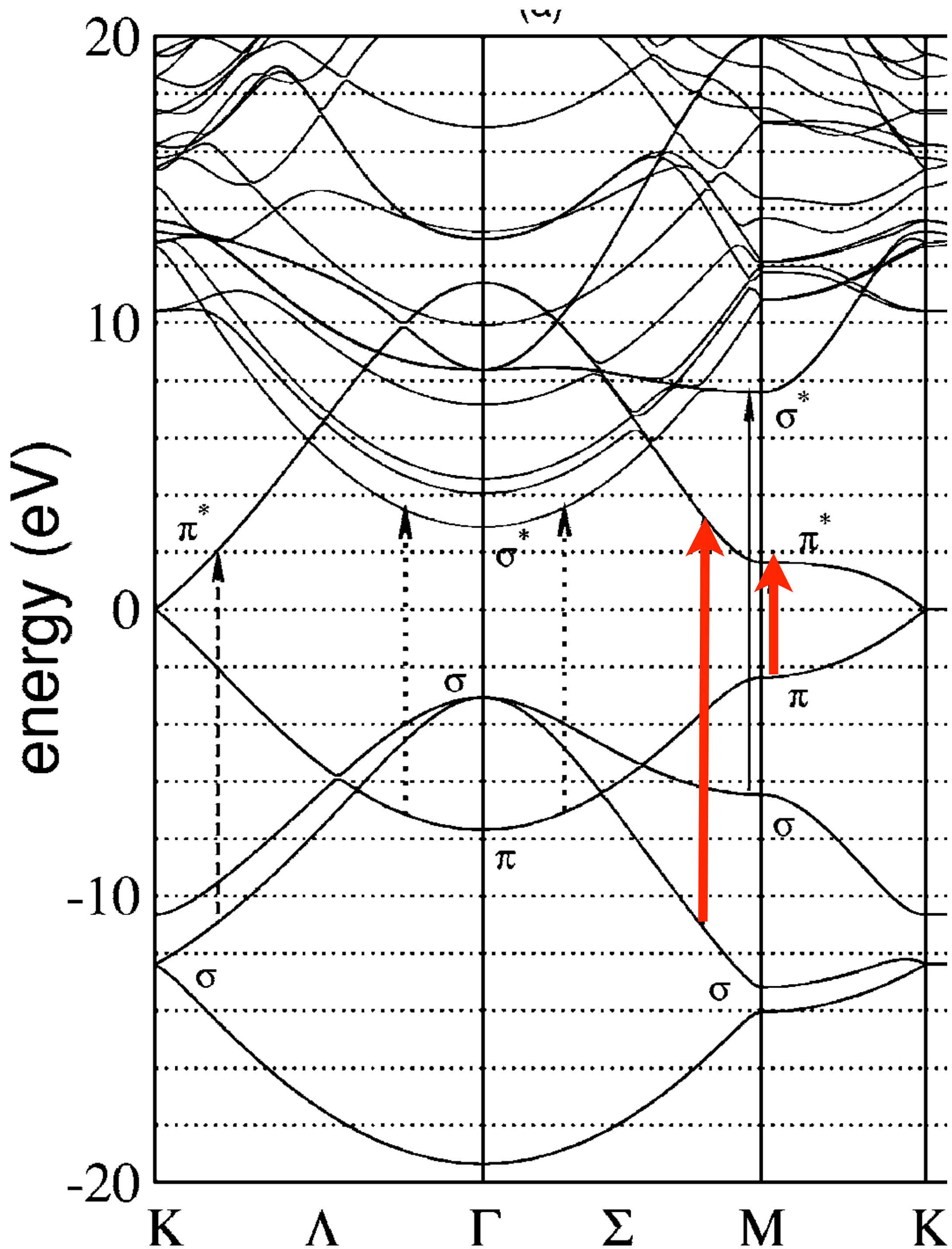
Eberlein et al., PRB 77, 233406 (07)

Testing conversion with EELS data



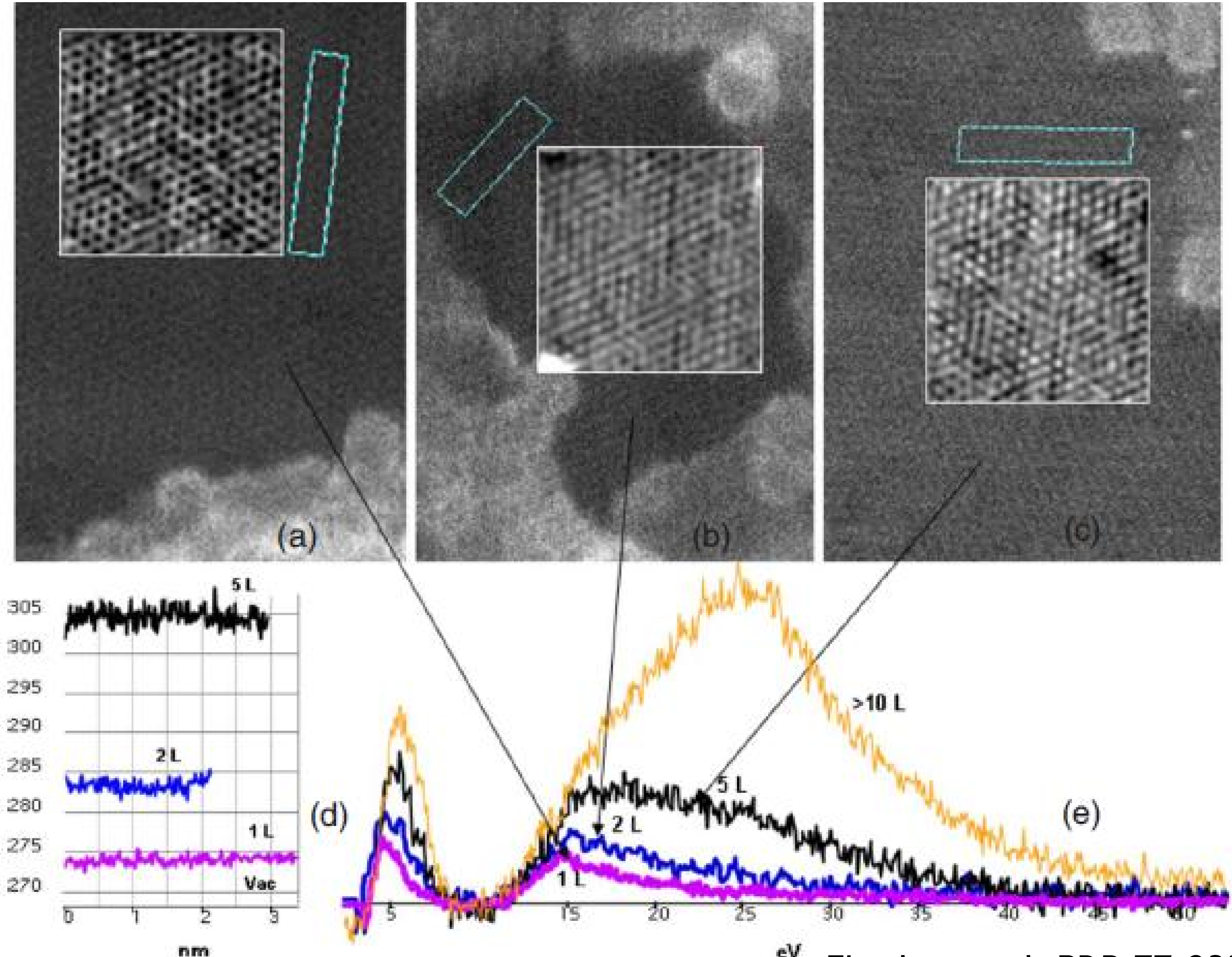
Plasmon redshift!

Eberlein et al., PRB 77, 233406 (07)



high energy plasmons
with 7 and 30 eV

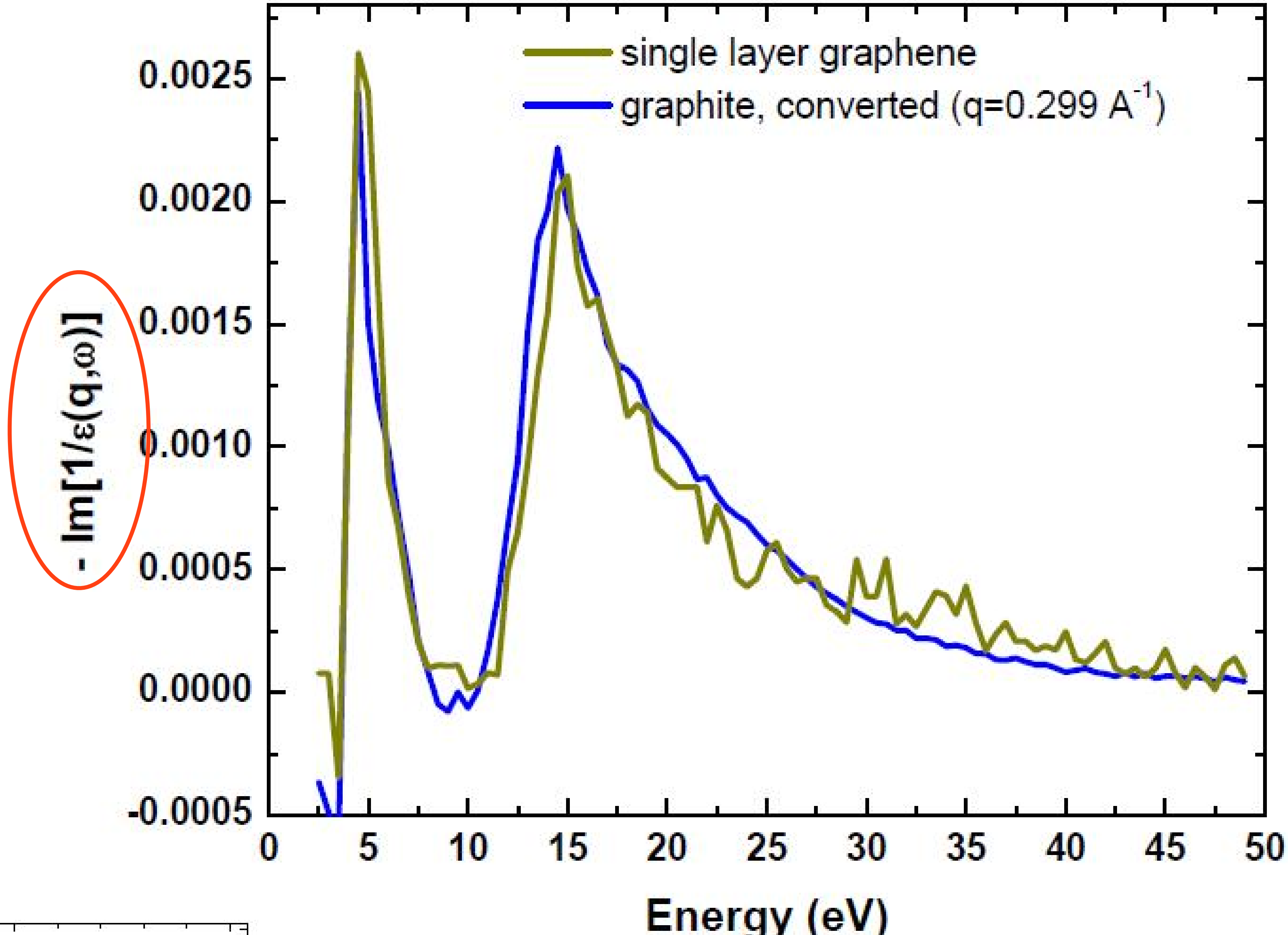
Testing conversion with EELS data



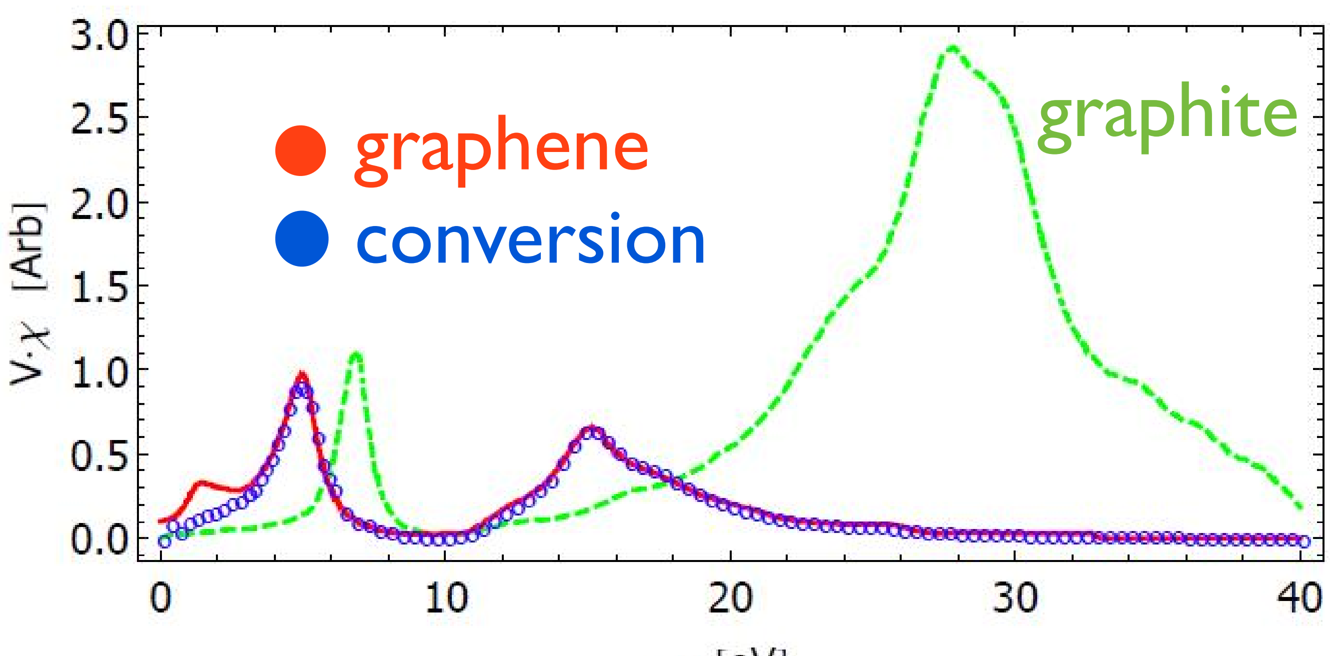
Eberlein et al., PRB 77, 233406 (07)

Testing conversion with EELS data

Response function for freestanding graphene!



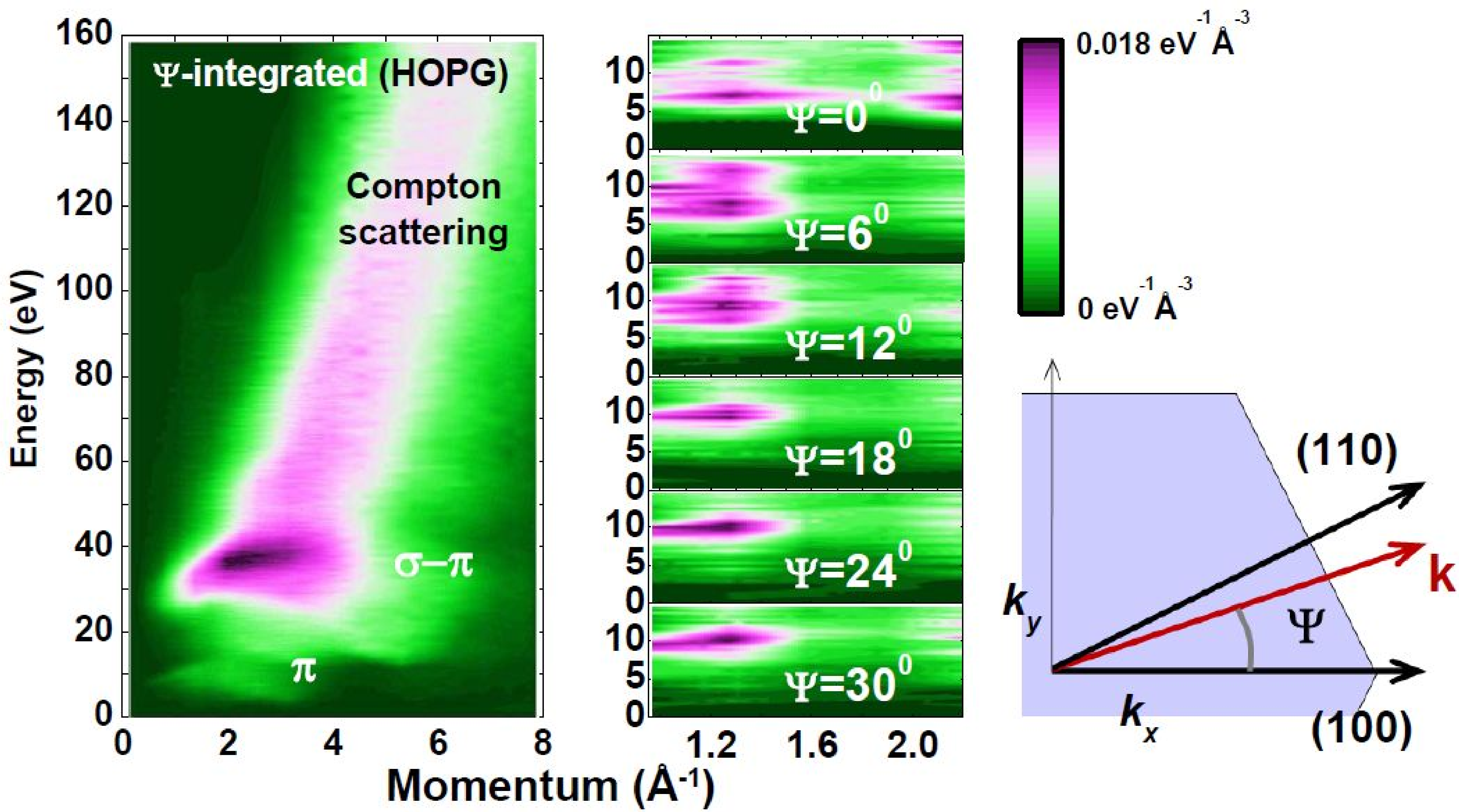
Ab initio



Eberlein et al., PRB 77, 233406 (07)

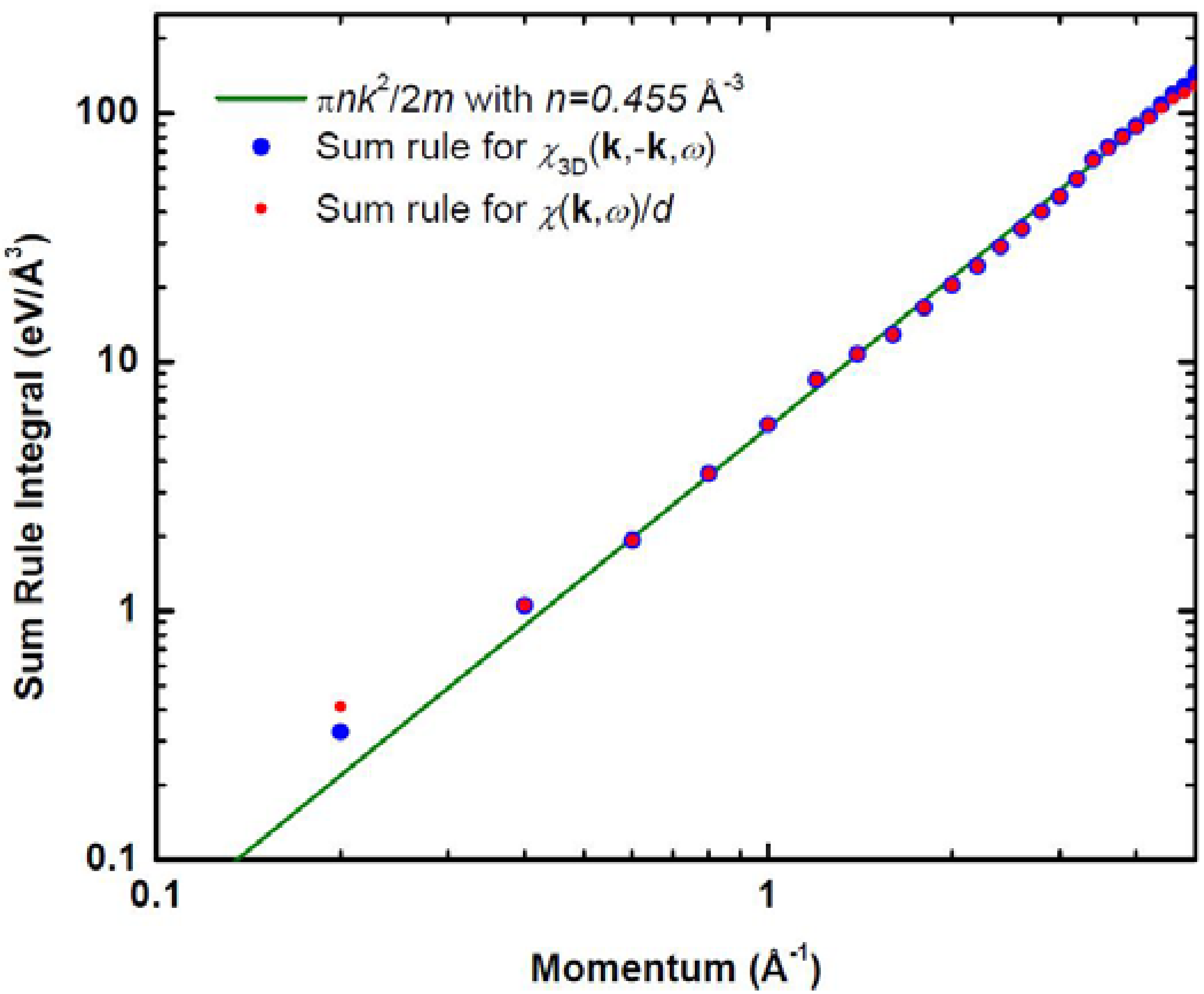
The Effective Fine-Structure Constant of Freestanding Graphene Measured in Graphite

James P. Reed,[1] Bruno Uchoa,[1] Young Il Joe,[1] Yu Gan,[1] Diego Casa,[2]
Eduardo Fradkin,[1] Peter Abbamonte[1]*

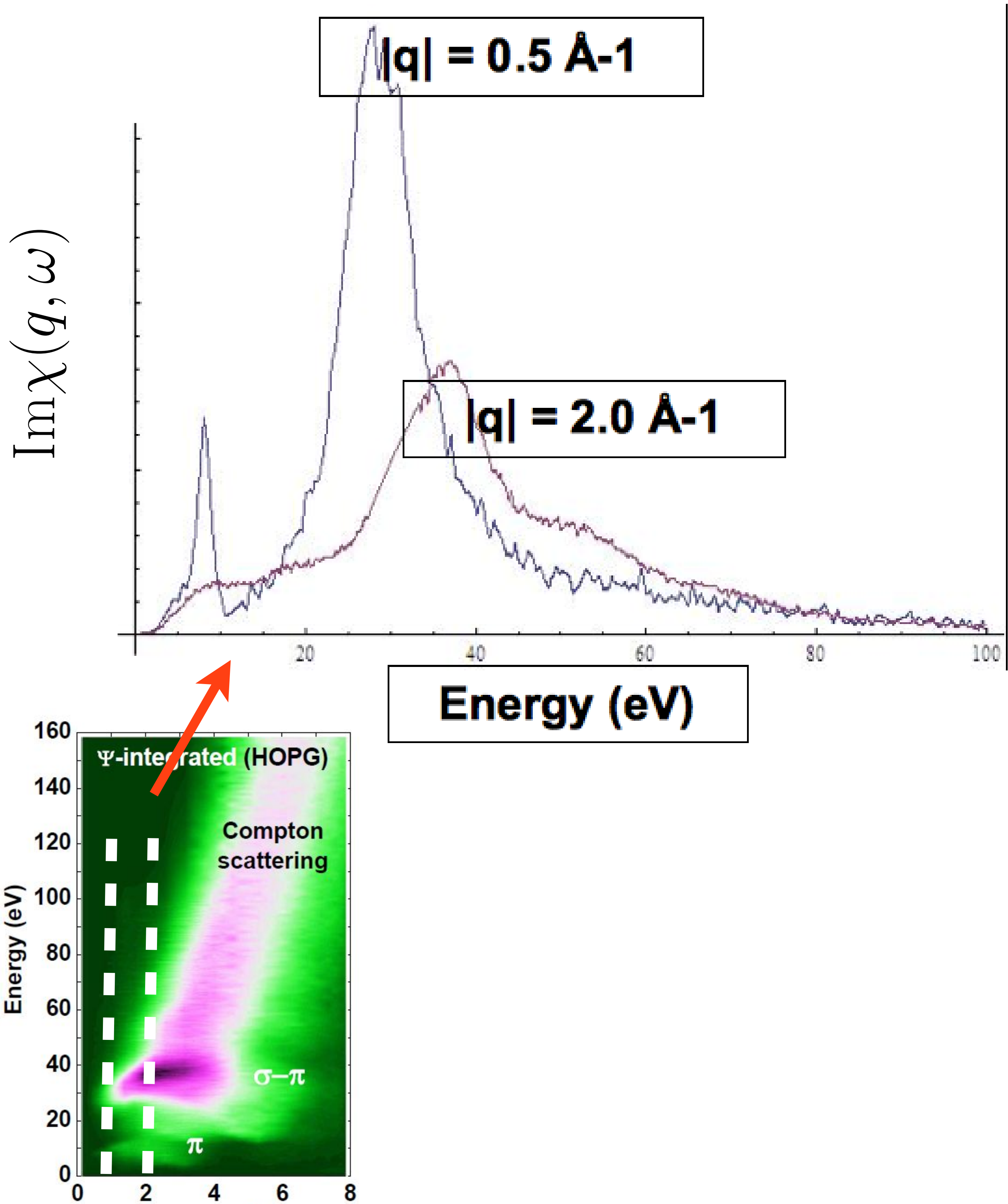


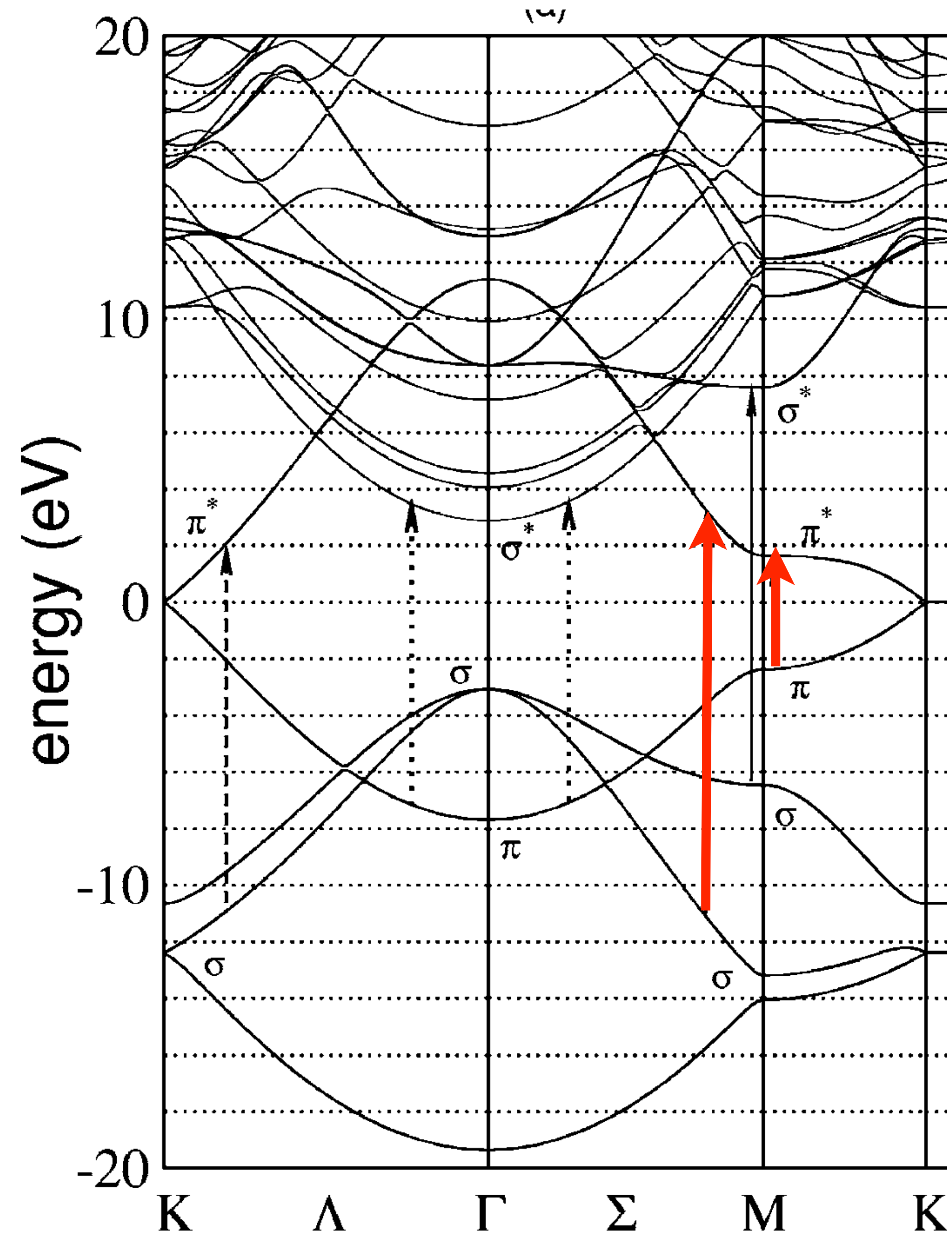
Calibration of the data: f-sum rule

$$\int_{-\infty}^{\infty} \mathrm{d}\omega\, \omega \mathrm{Im}\Pi^{(1)}(k,\omega) = \pi \frac{N_e k^2}{m}$$



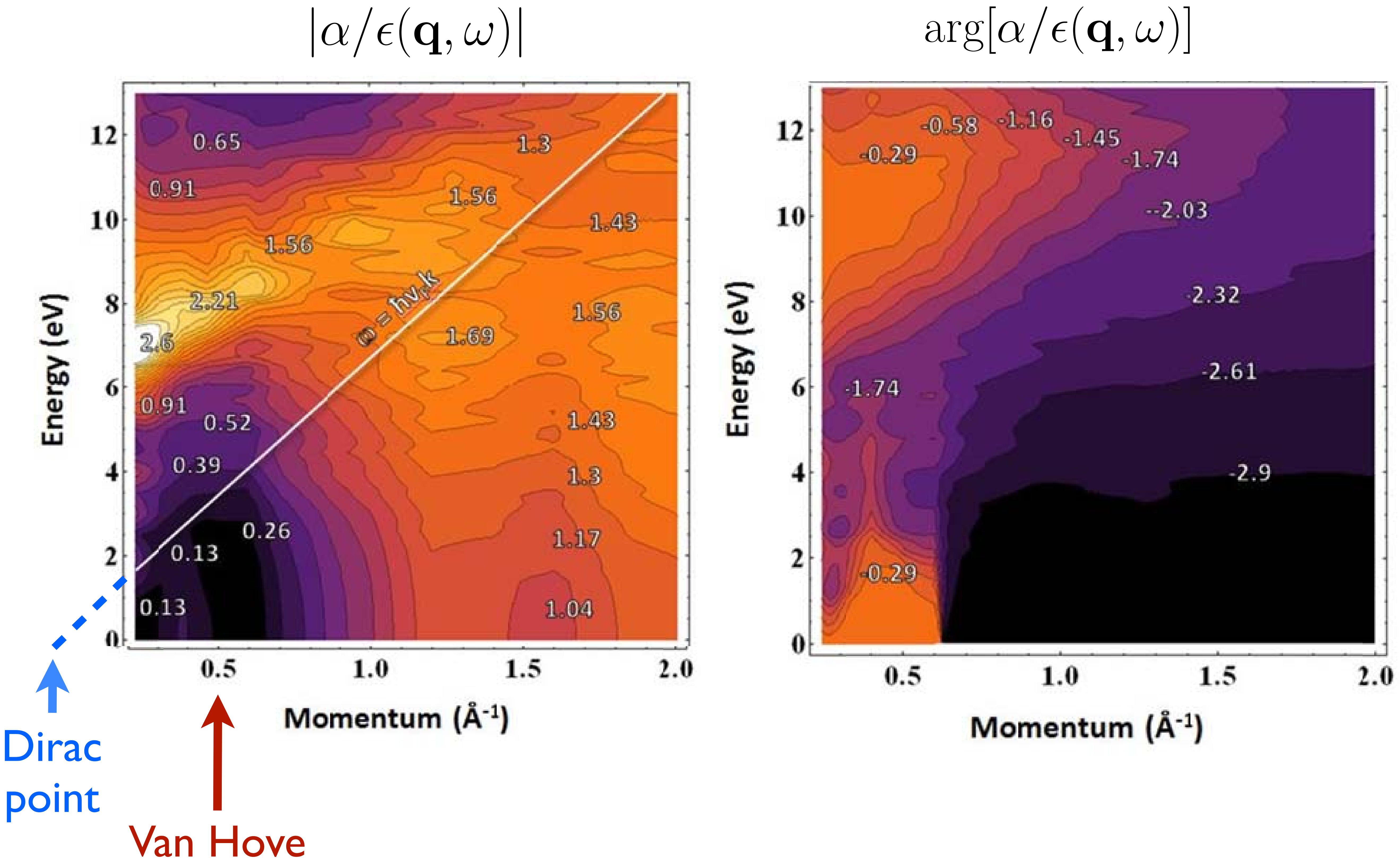
X-ray data in graphite





high energy plasmons
with 7 and 30 eV

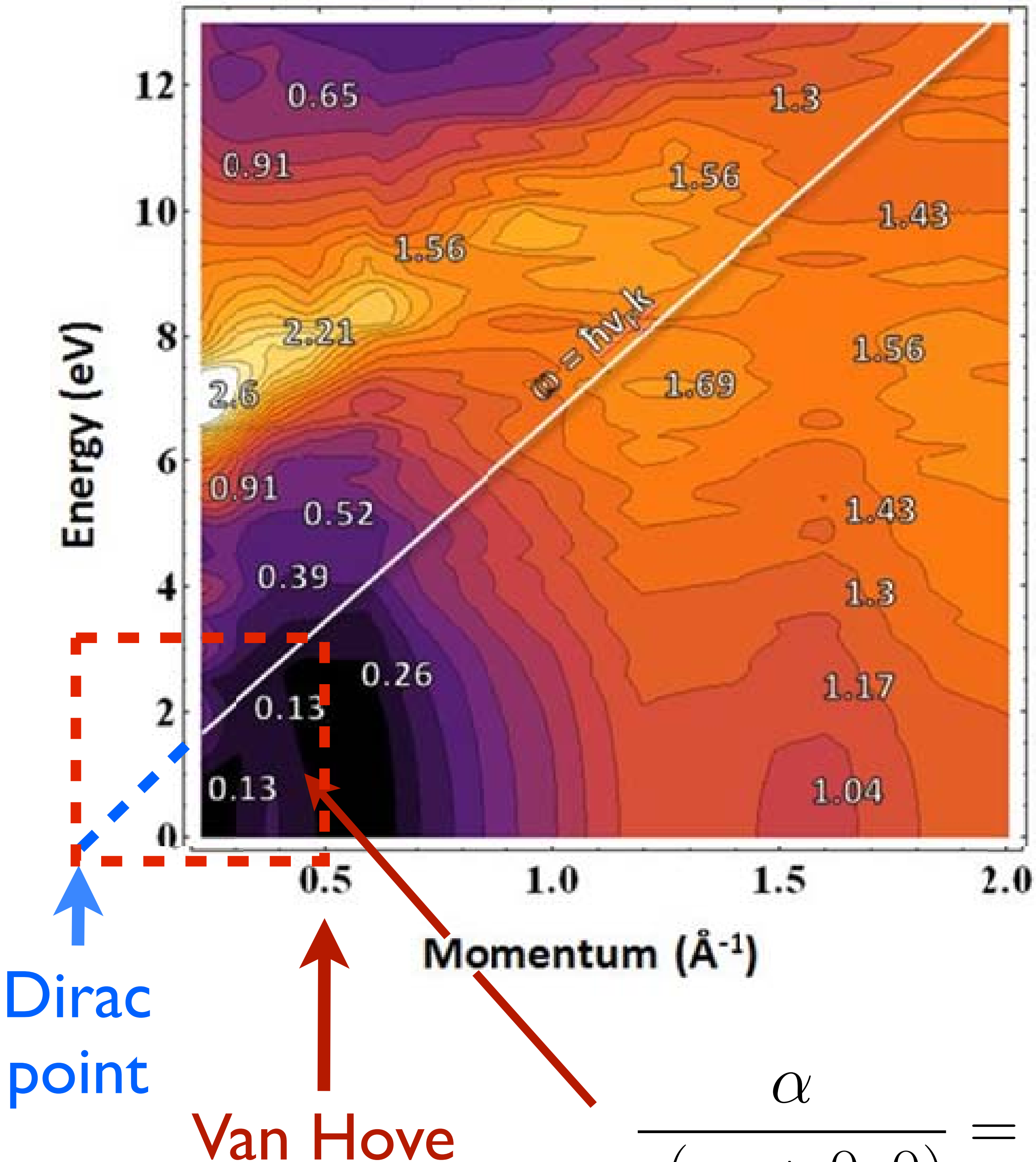
The effective fine structure constant

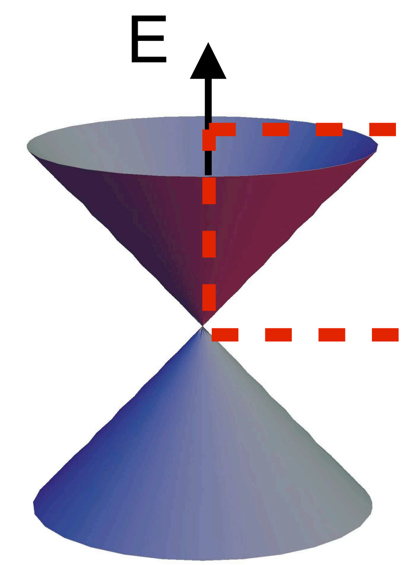


Reed et al., Science 330, 805 (2010)

The effective fine structure constant

$$|\alpha/\epsilon(\mathbf{q},\omega)|$$





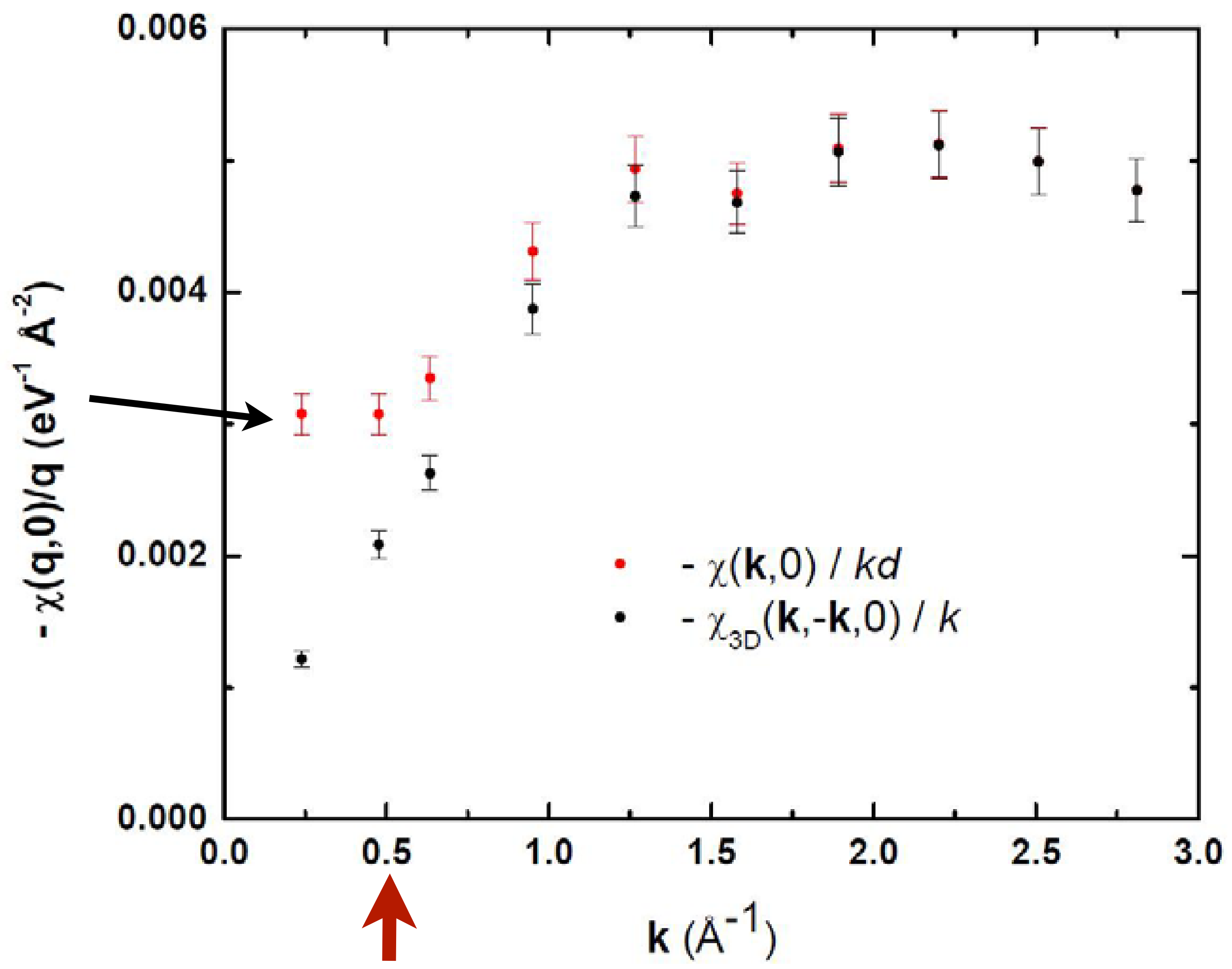
$$\frac{\alpha}{\epsilon(q \to 0, 0)} = 0.13 \approx 1/7$$

Dielectric screening!

The effective fine structure constant

$$\Pi(\mathbf{q} \to 0, 0) \propto q$$

Extrapolates linearly to zero!

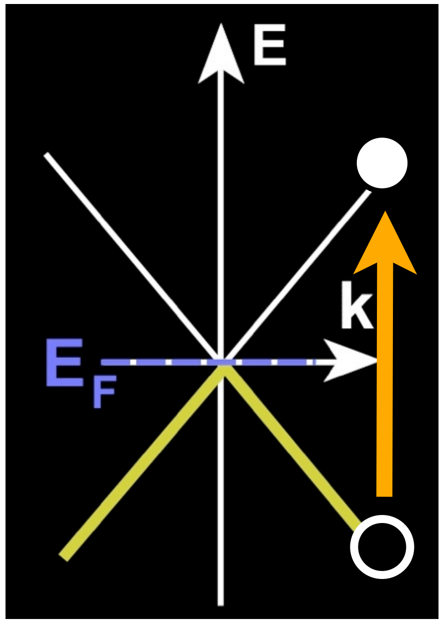


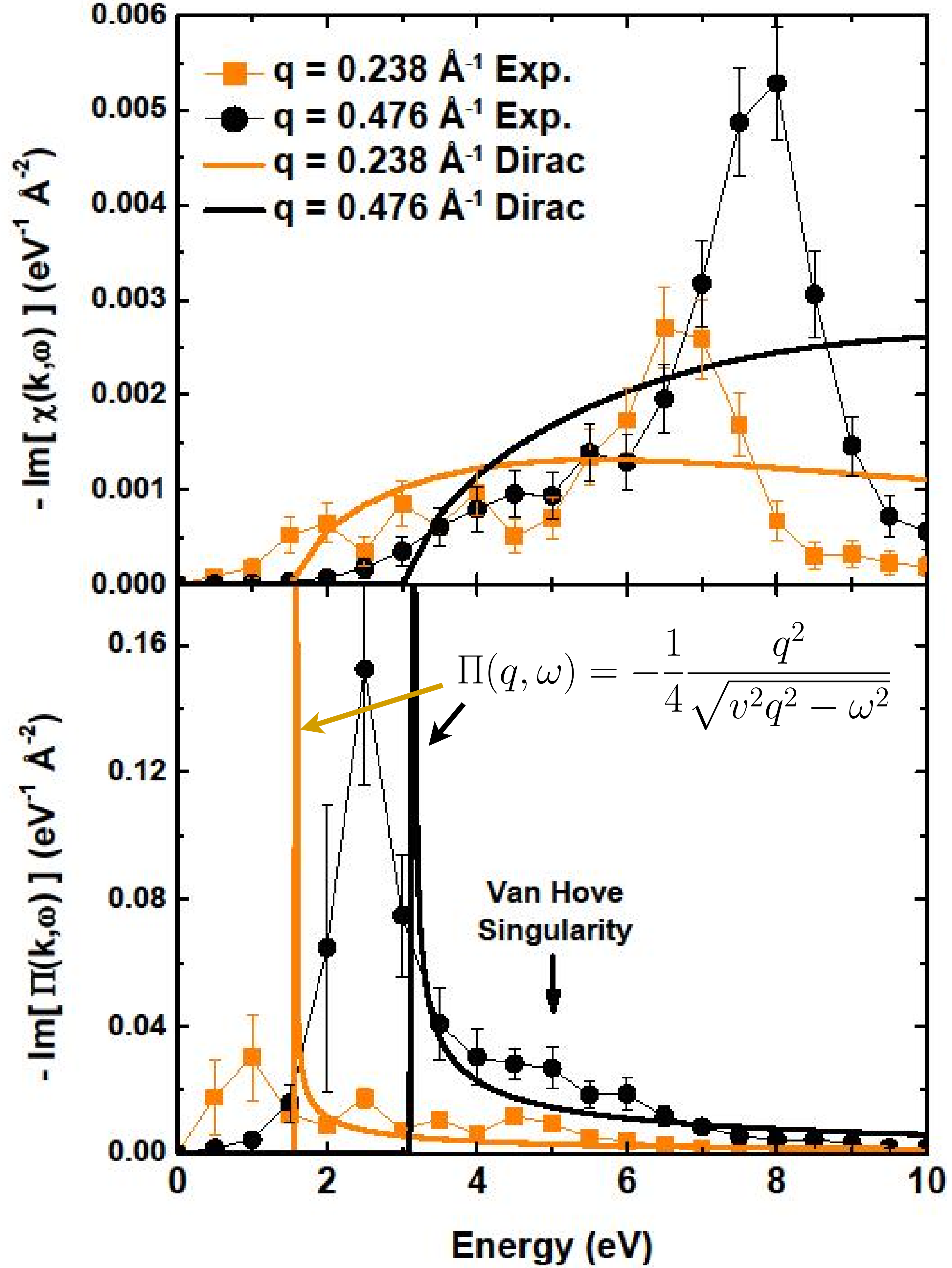
Van Hove

$$\epsilon(\mathbf{q}, \omega) = 1 - \frac{2\pi e^2}{q}\Pi(q, \omega)$$

$$\frac{\alpha}{\epsilon(q \to 0, 0)} = 0.13 \approx 1/7 \ !$$

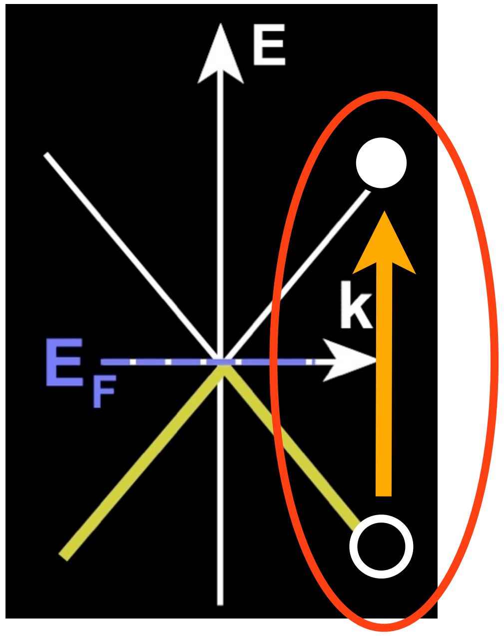
Polarization bubble (graphene)



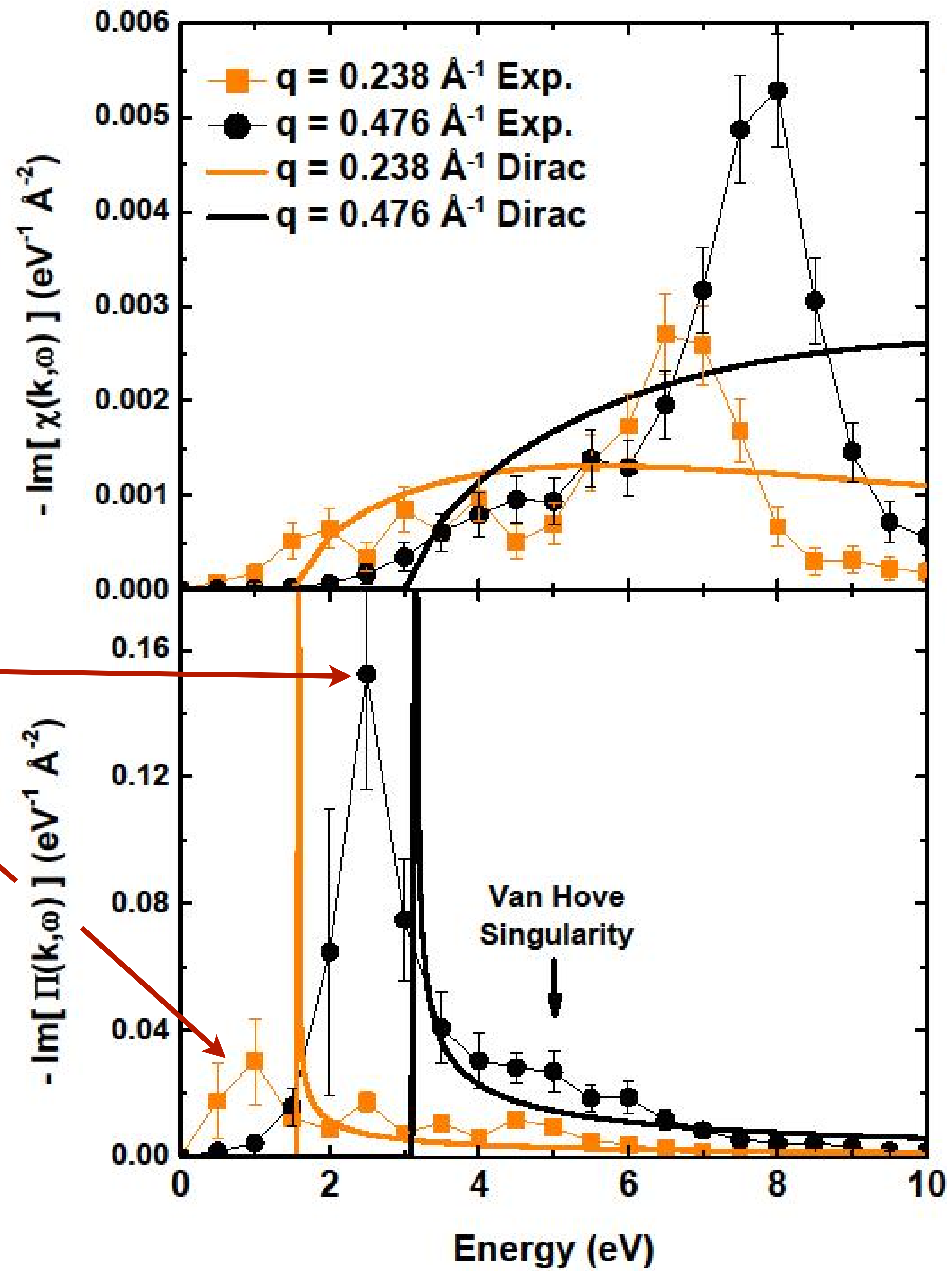


Optical absorption edge

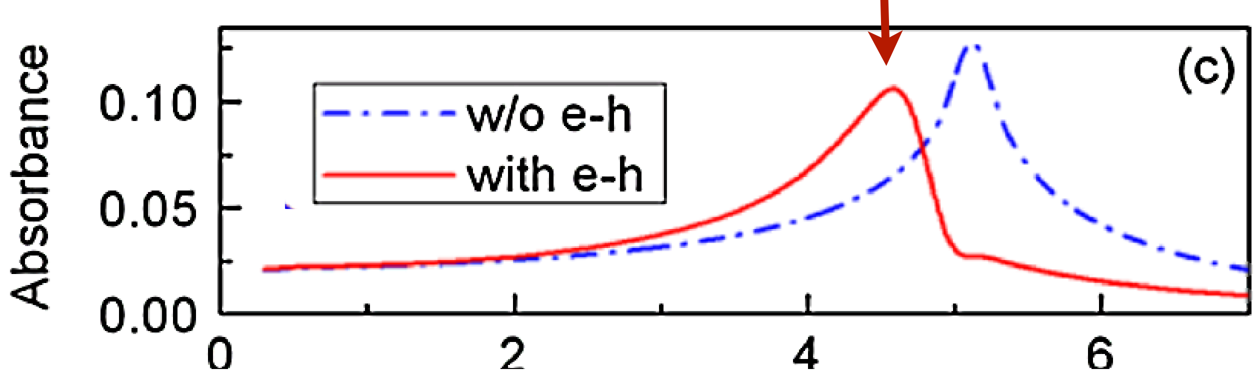
Polarization bubble (graphene)



Bound state!



Excitons!

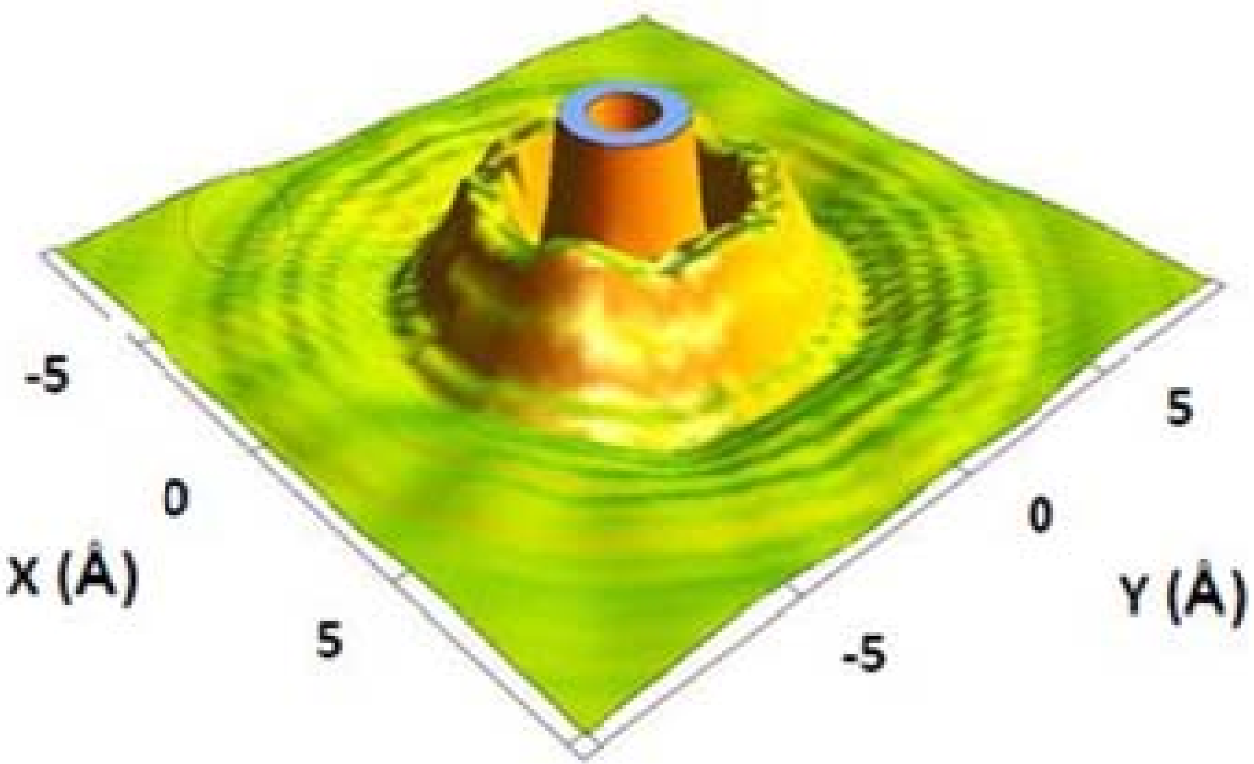


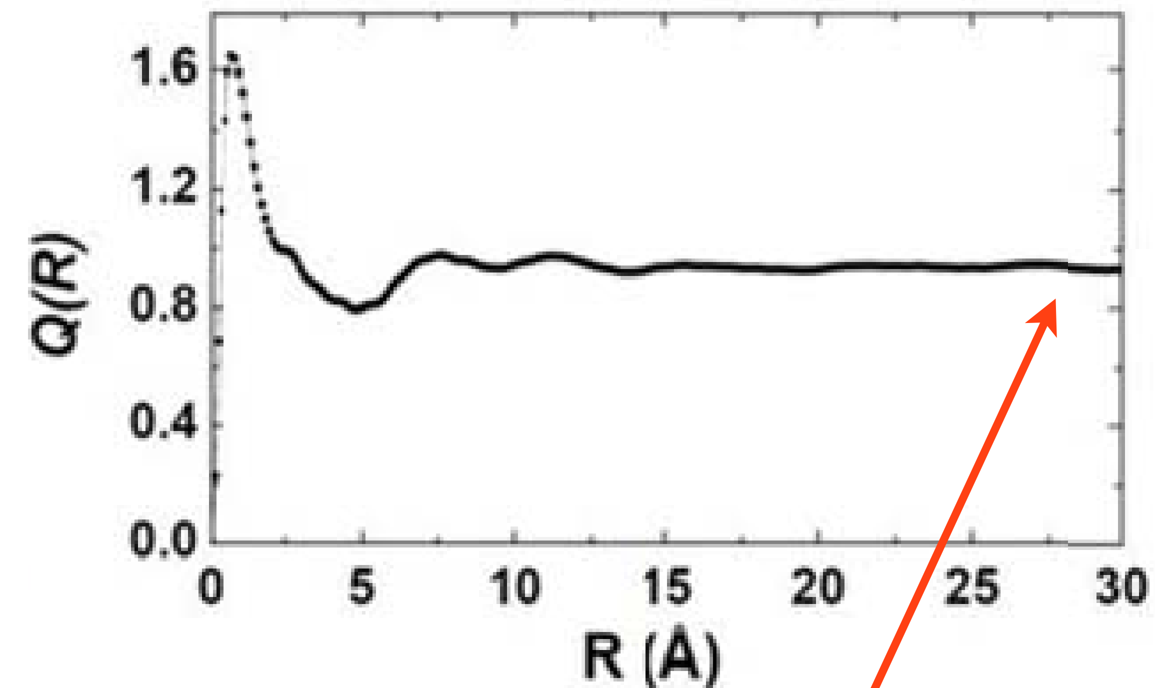
Yang et al. PRL **103**, 186802 (2009)

Test charge + cloud (x-ray data)

Q - 0.924 Q = 0.076 Q

$$Q(R) = \int_0^R \mathrm{d}^2 r \, \rho(\mathbf{r})$$





Induced charge density ρ(r)

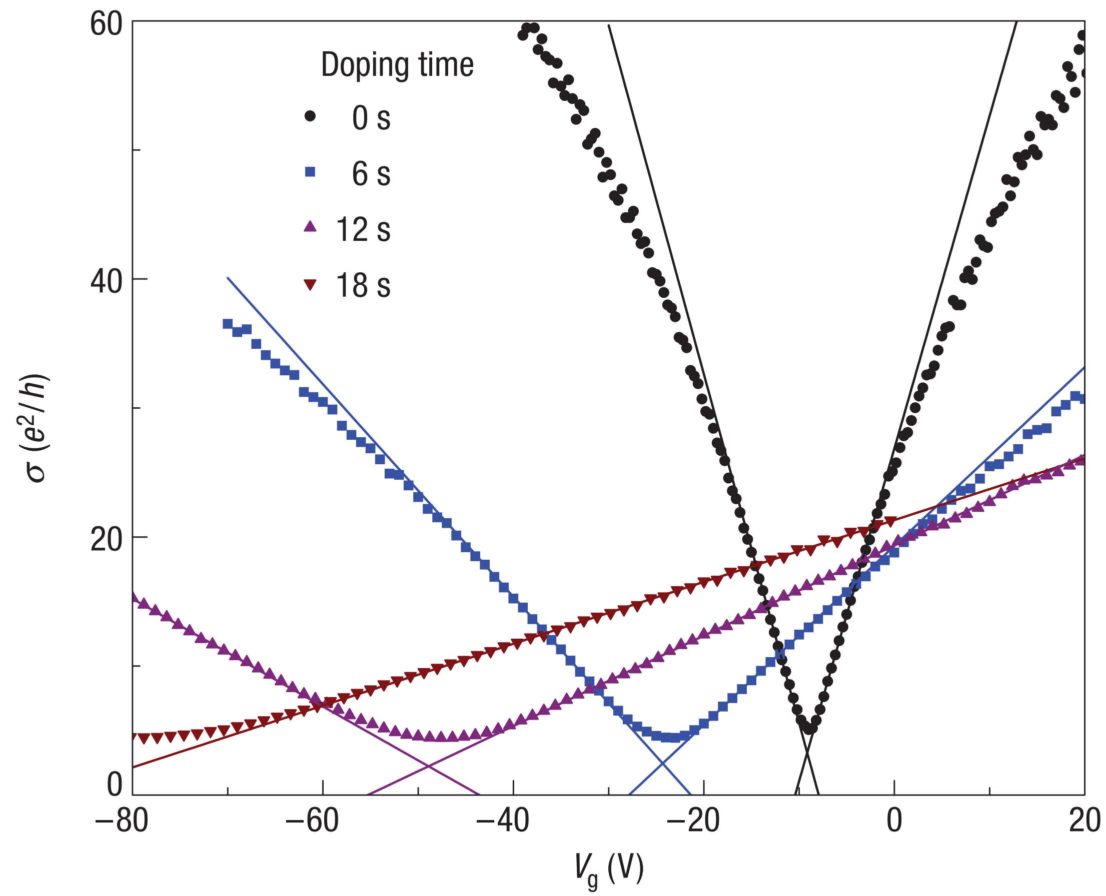
$$Q(\infty) = (0.924 \pm 0.046)e$$

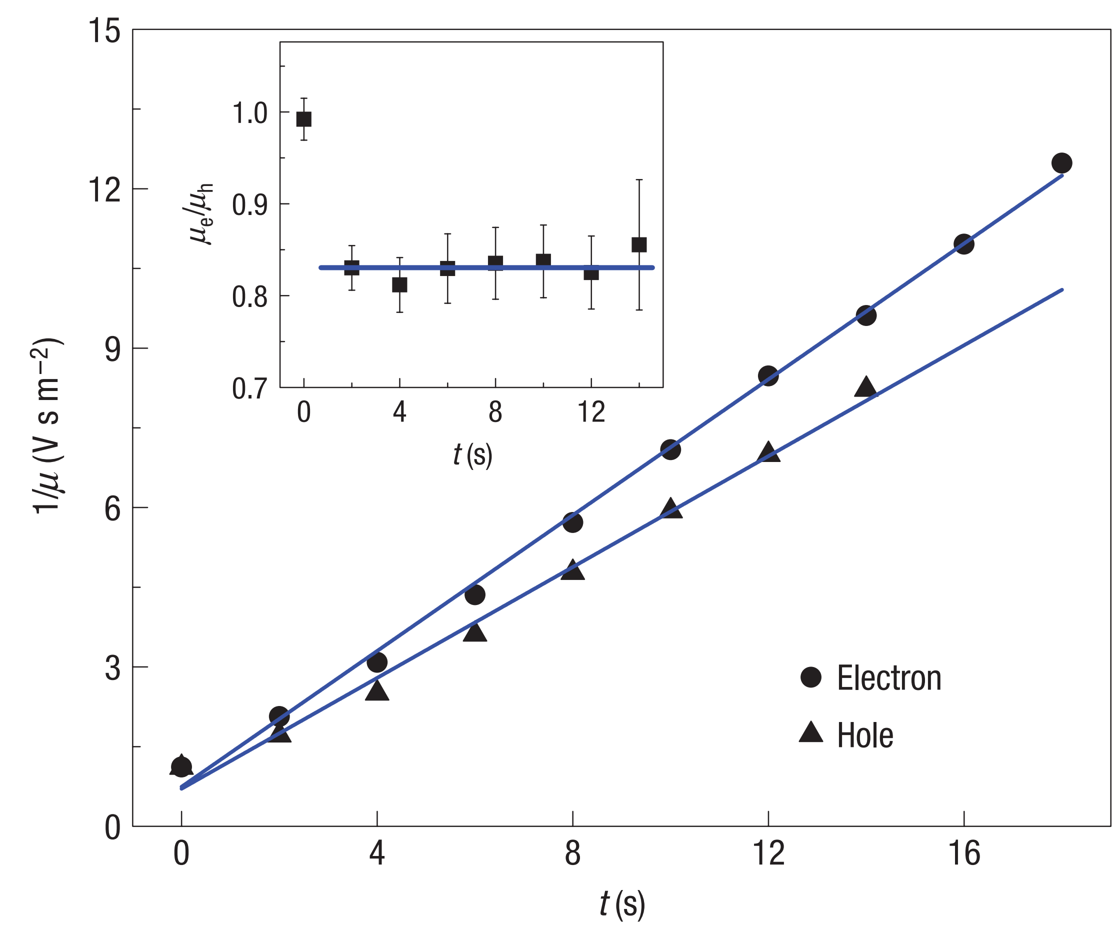
Coulomb charges are almost completely screened!

Charged-impurity scattering in graphene

J.-H. CHEN[1,2,3*], C. JANG[1,2,3*], S. ADAM[2,3,4], M. S. FUHRER[1,2,3], E. D. WILLIAMS[1,2,3,5,6]
AND M. ISHIGAMI[2,3†‡]

inverse mobility





Adsorption of K atoms in graphene

Significant mobility change with concentration!

PRL 102, 206603 (2009) PHYSICAL REVIEW LETTERS week ending 22 MAY 2009

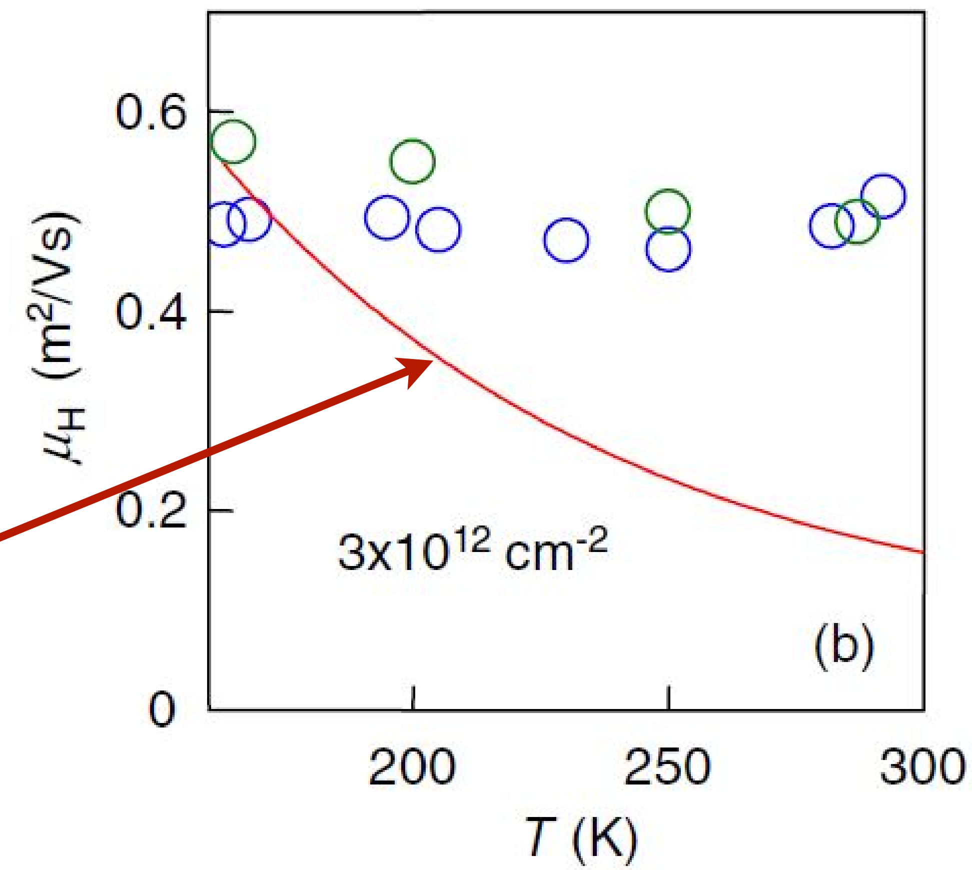
Effect of a High-κ Environment on Charge Carrier Mobility in Graphene

L. A. Ponomarenko,[1] R. Yang,[1] T. M. Mohiuddin,[1] M. I. Katsnelson,[2] K. S. Novoselov,[1] S. V. Morozov,[1,3] A. A. Zhukov,[1] F. Schedin,[1] E. W. Hill,[1] and A. K. Geim[1]

$\kappa \approx 25$ at 300 K (ethanol)

○ $\mu_{\mathrm{FE}} = \sigma / ne$

○ $\mu_H = \rho_{xy} / \rho_{xx} B$



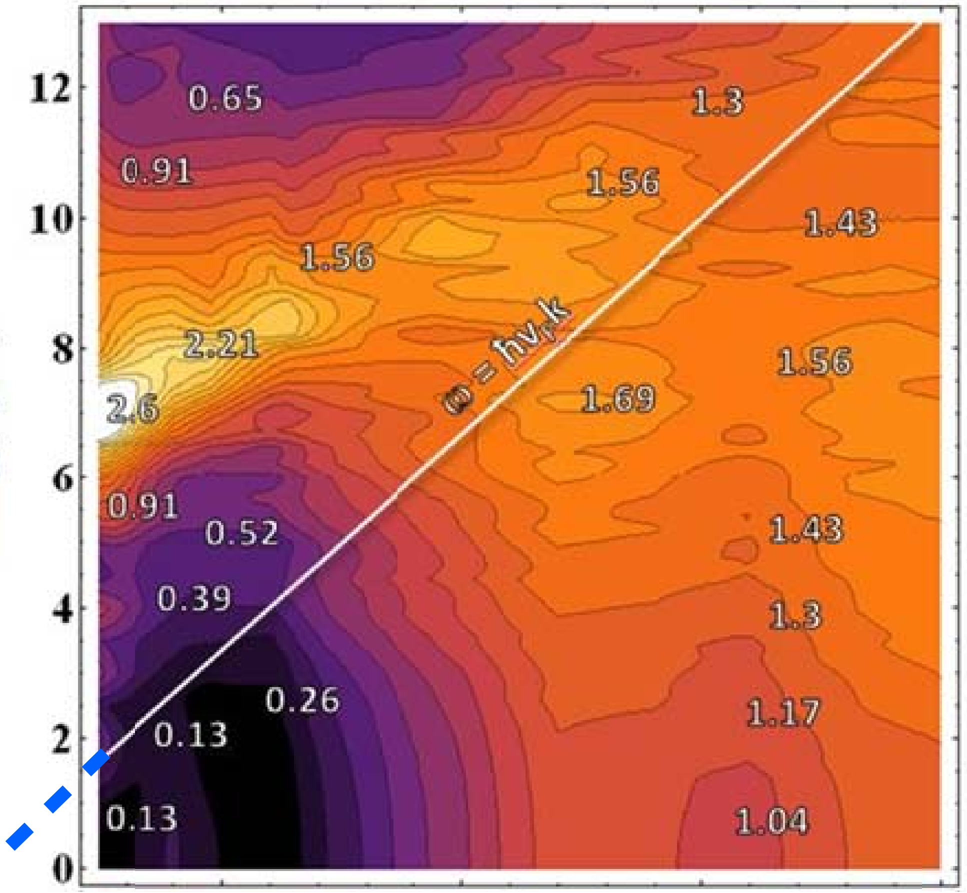
Mobility limited by Coulomb scatterers

Mobility is nearly insensitive to high-dielectric substrates!

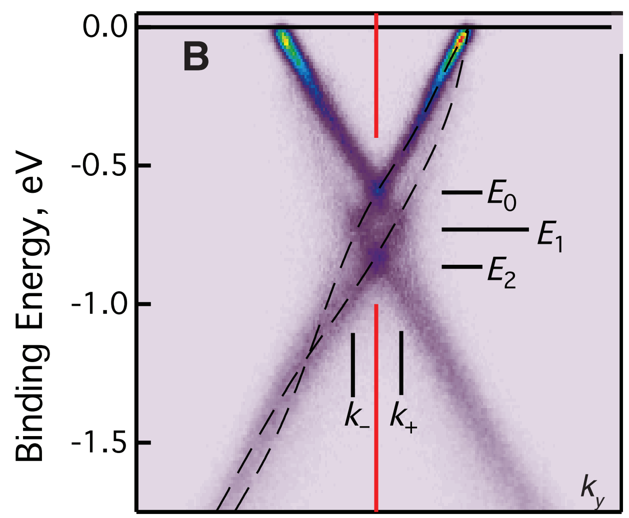
liquid crystal MLC6204 ($\kappa \approx 44$)

Summary of part I:

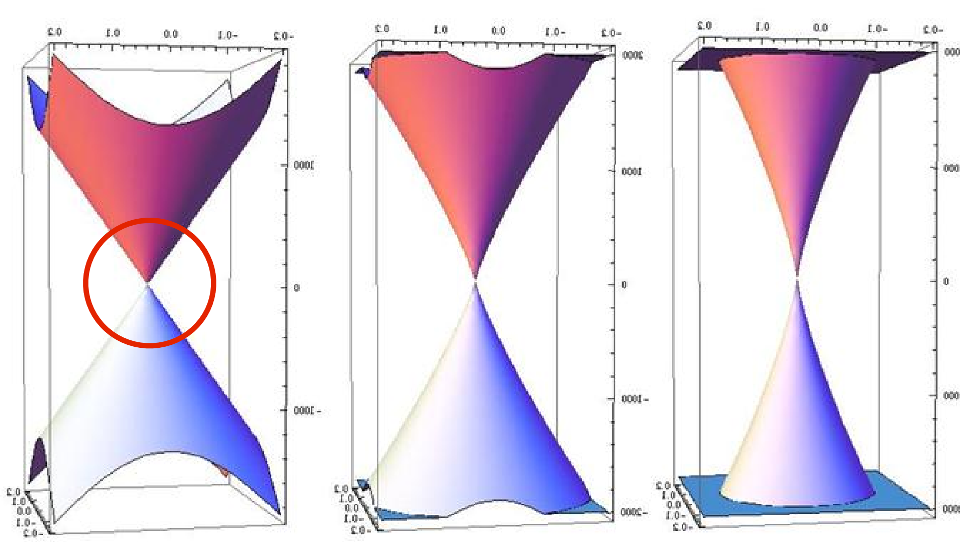
Excitonic effects make Dirac fermions more polarizable for $\omega < \hbar v q$



Dirac point



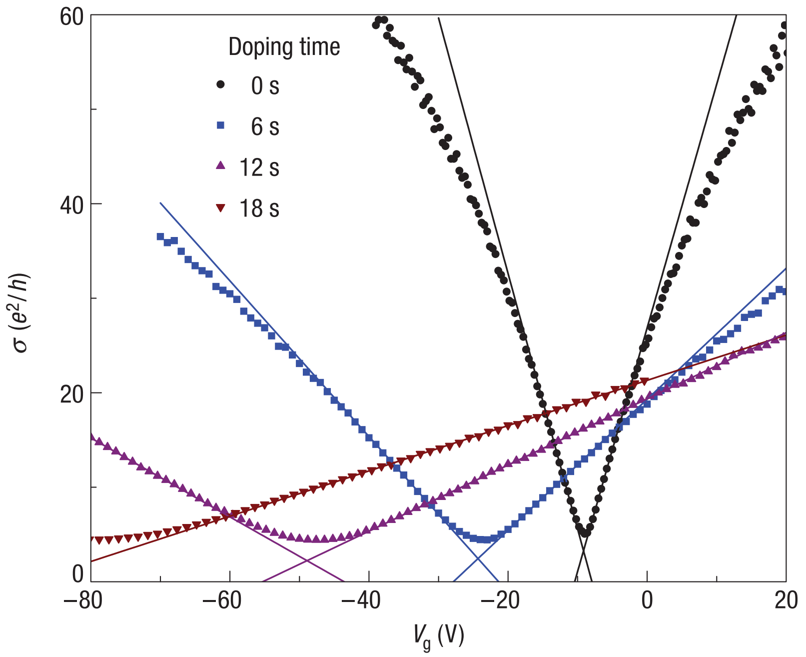
Plasmarons

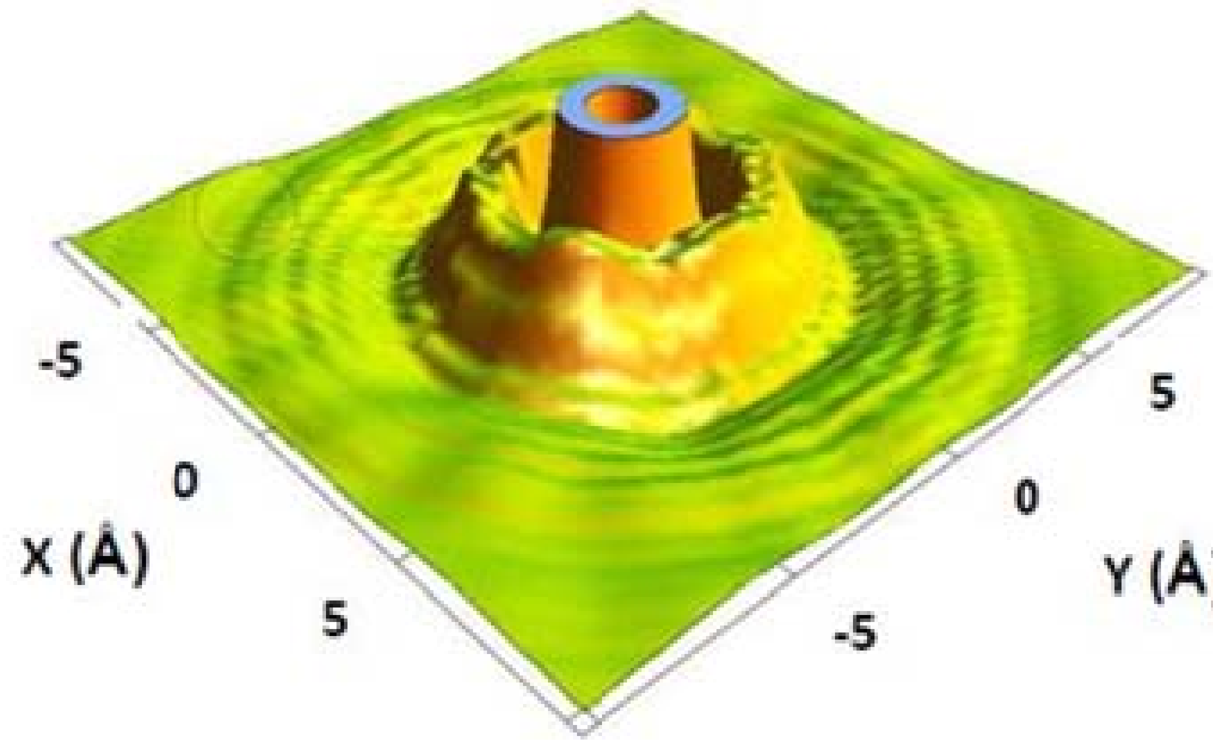


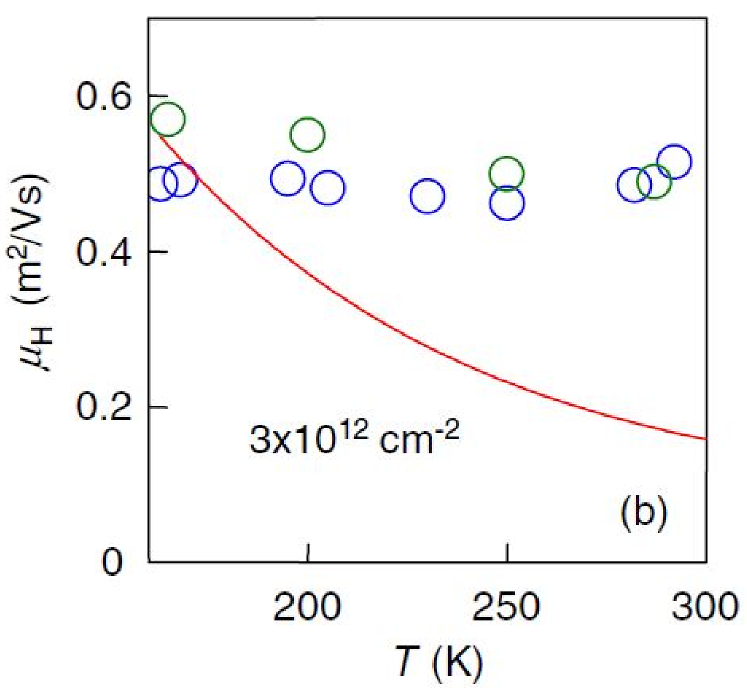
$$v \rightarrow v(1 + \frac{\alpha}{4}) \ln\left(\frac{\Lambda}{q}\right)$$

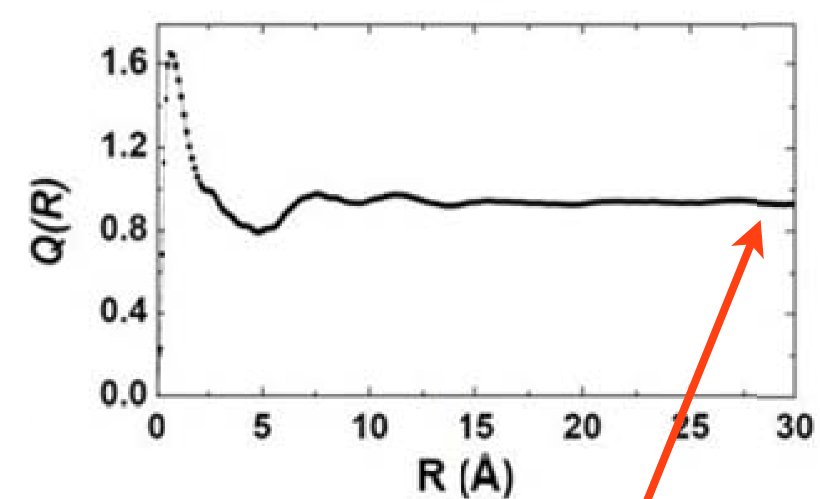
Velocity renormalization in half filled graphene (no plasmon) is hard to see!

Summary







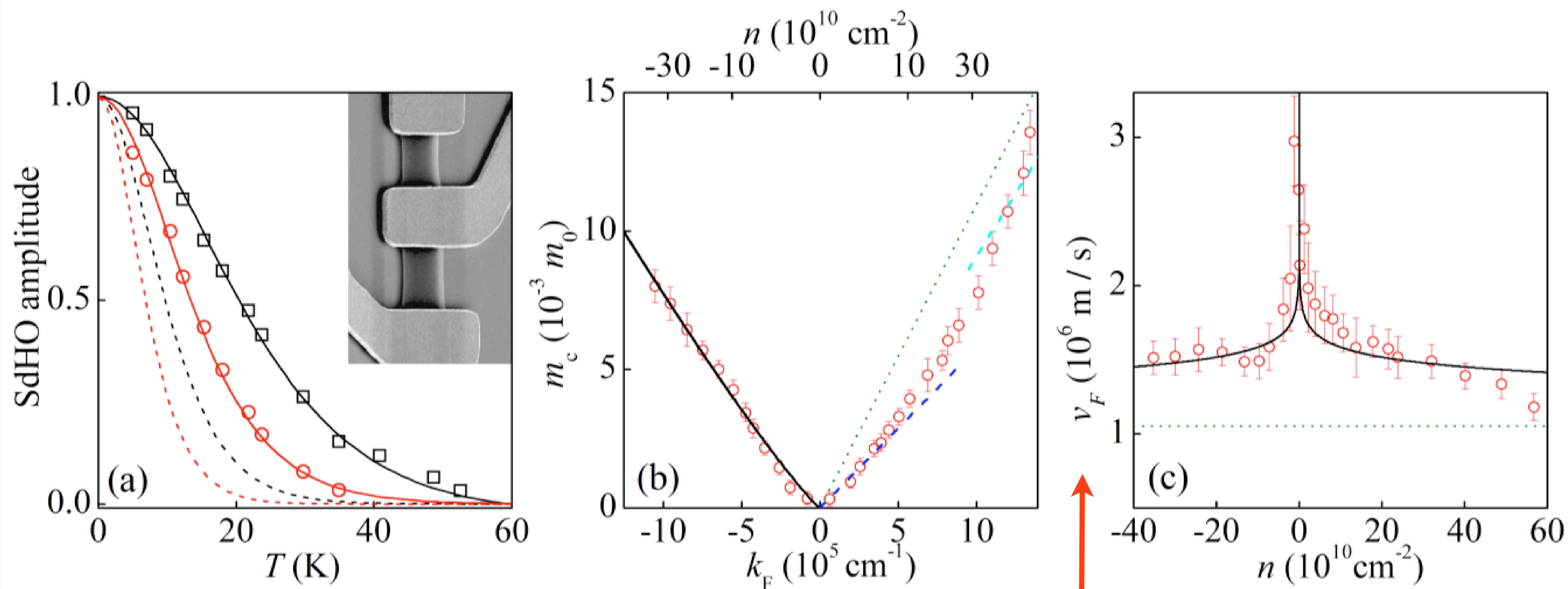


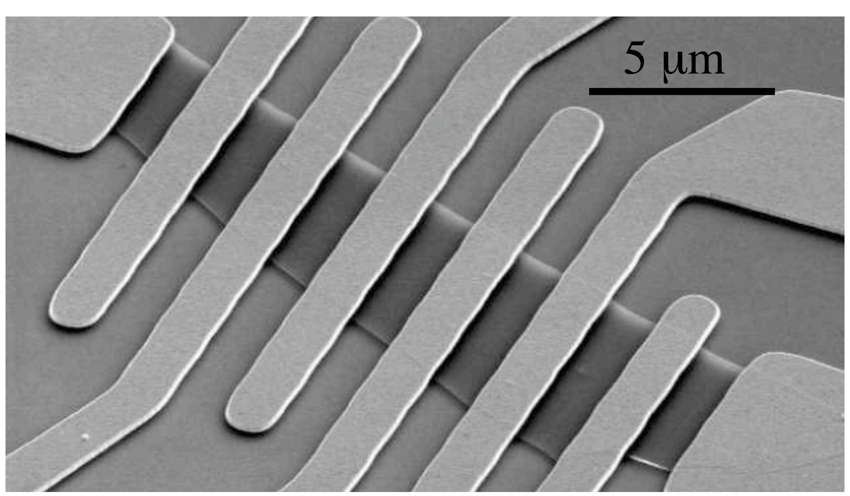
$$Q(\infty) = (0.924 \pm 0.046)e$$

Long range piece of the Coulomb interaction seems unlikely to limit the mobility of the samples (Coulomb impurities)!

Shubnikov-deHaas oscillations

fitting: $\alpha = 0.5$



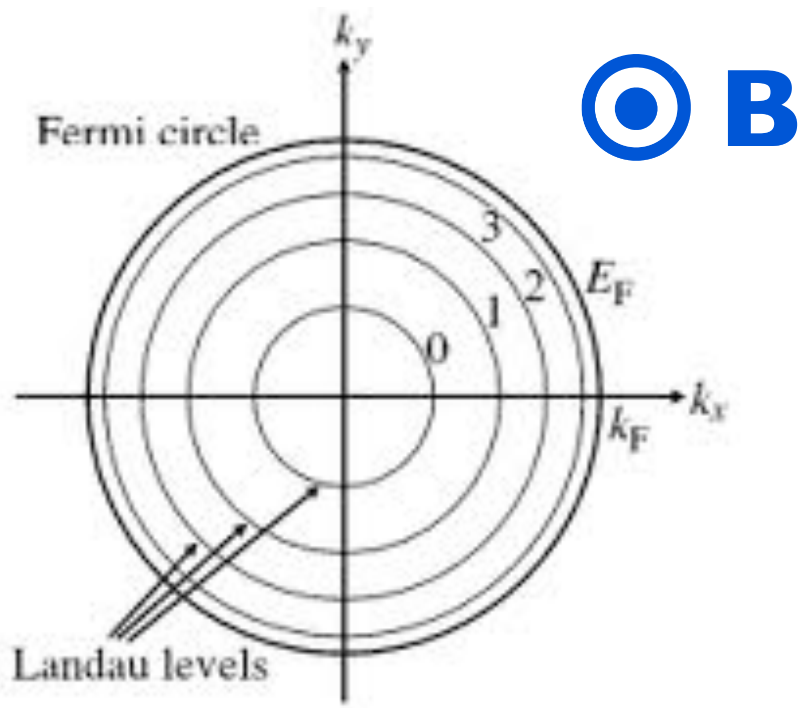


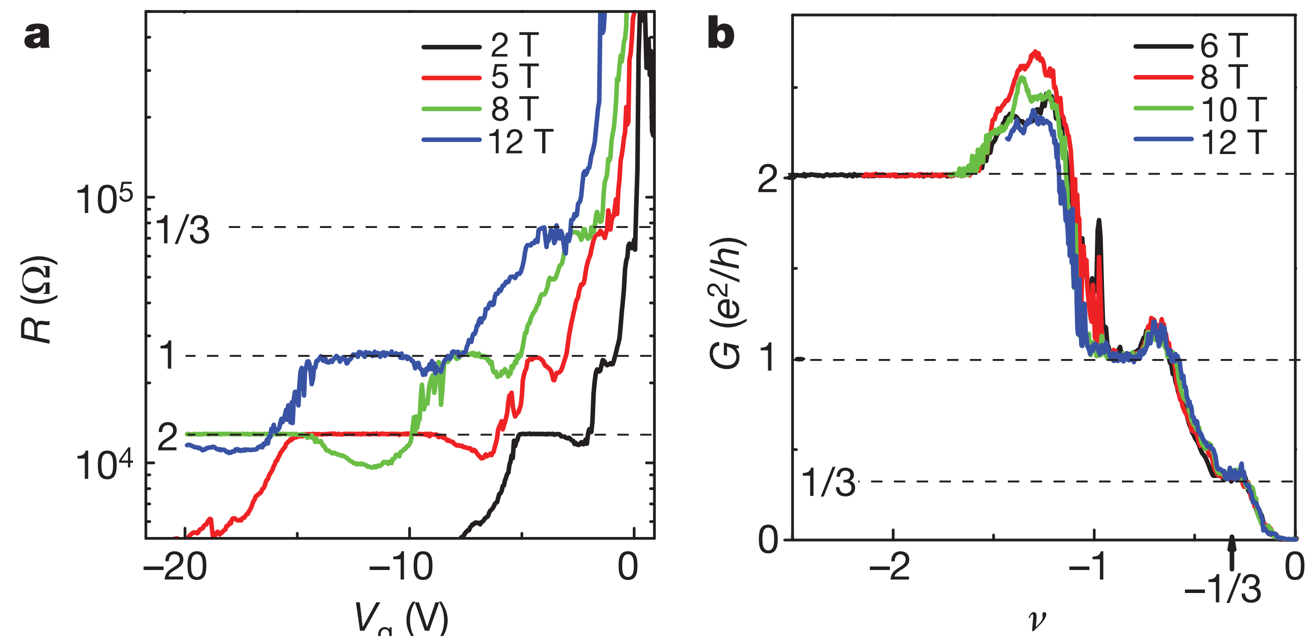
B=0.01 T

Elias et al., arXiv:1104.1396 (2011)

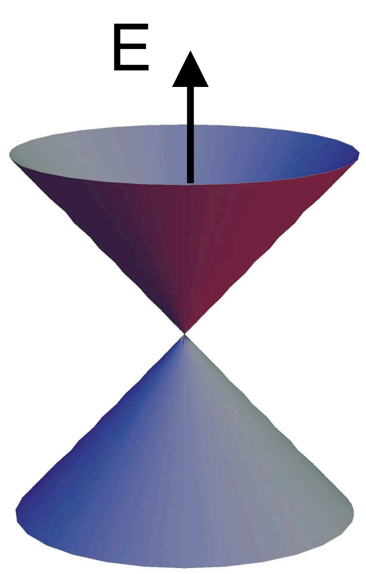
First observation of a Log!

Summary of part I:





At wavelengths longer than the cyclotronic wavelength graphene becomes strongly interacting again!



fractional quantum Hall effect