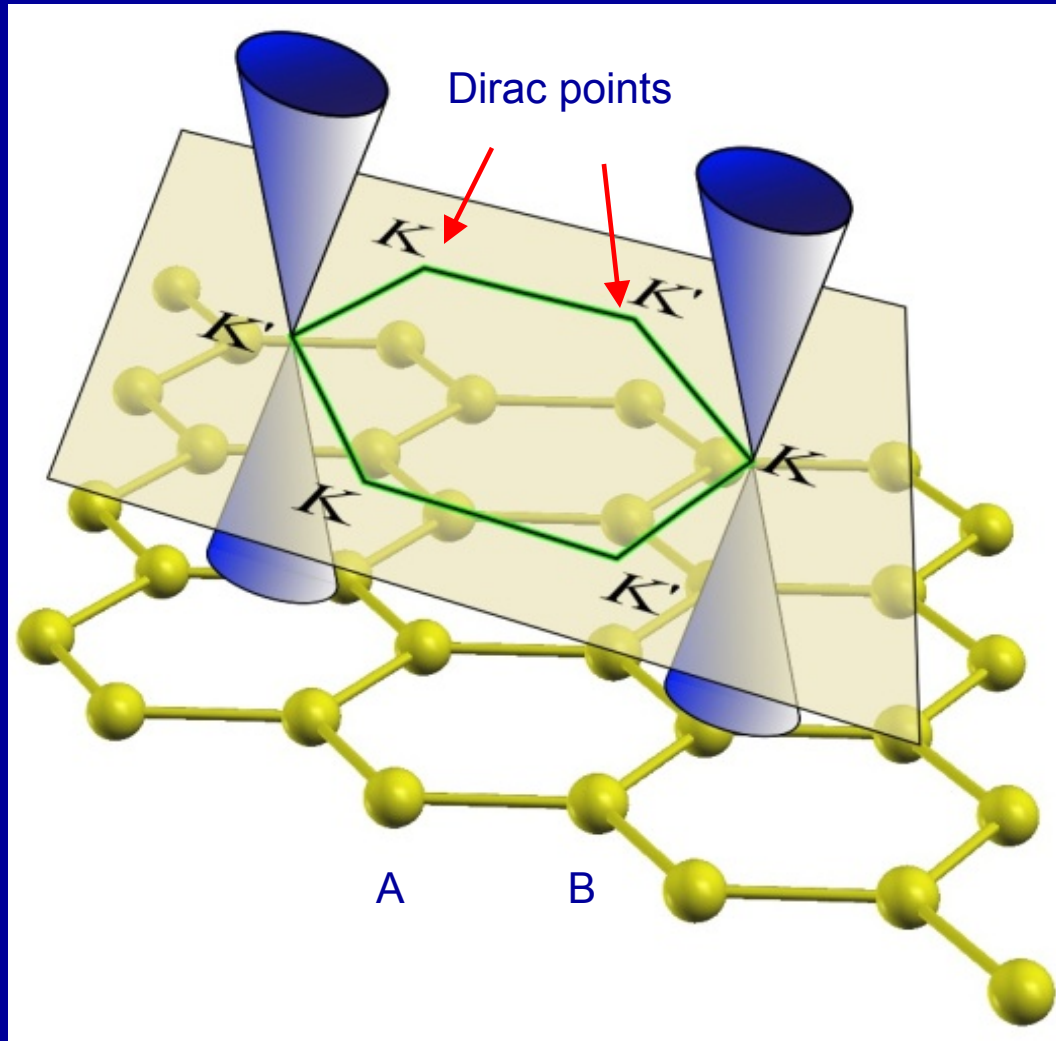


# Spin-orbit coupling in graphene

Jaroslav Fabian

University of Regensburg

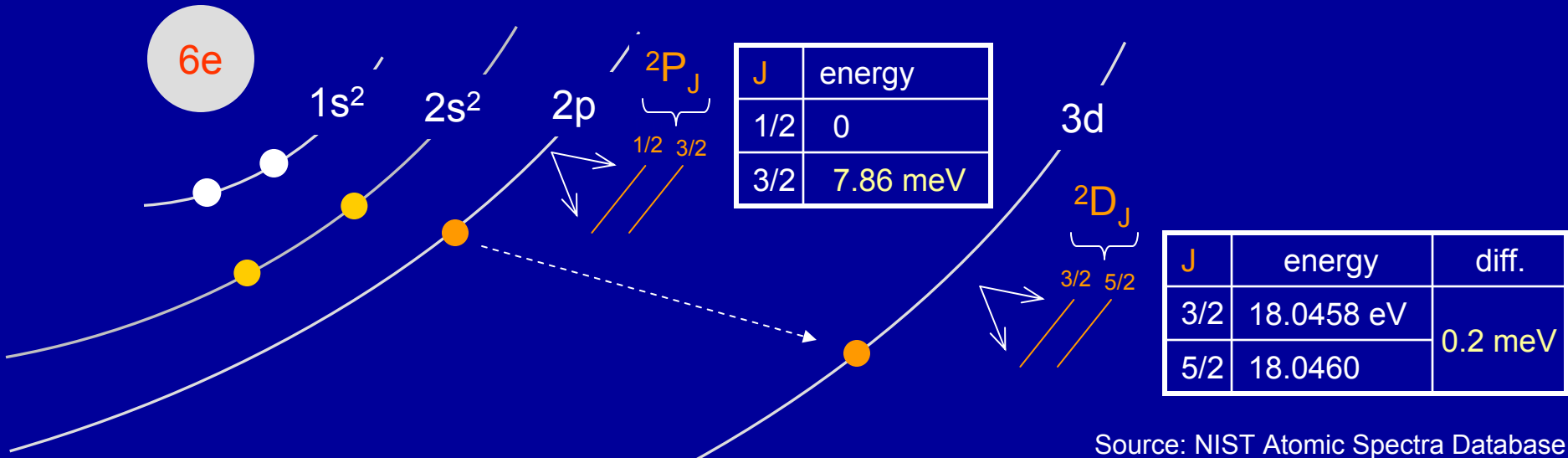
# graphene 101



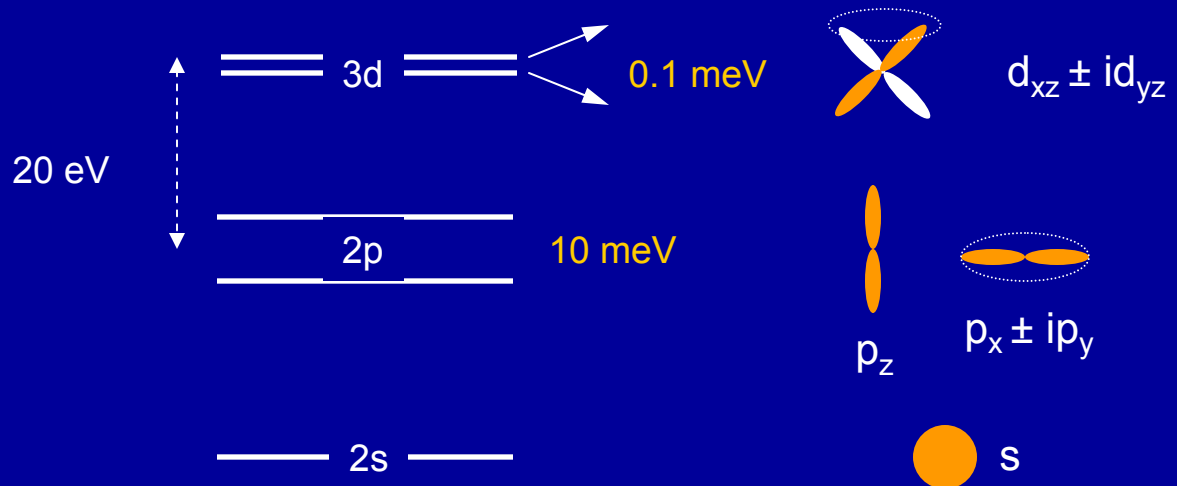
How conic are the cones?

How does spin-orbit coupling modify the band structure topology?

# :spin-orbit splitting in the carbon ion $2s^2 2p$ :



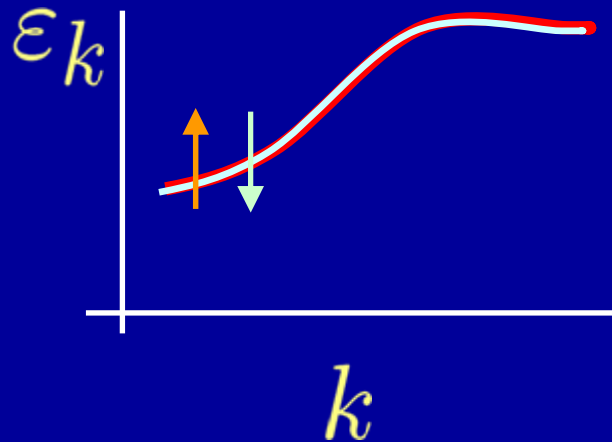
Source: NIST Atomic Spectra Database



# going to solids: intrinsic and extrinsic SOC

solids with space inversion symmetry

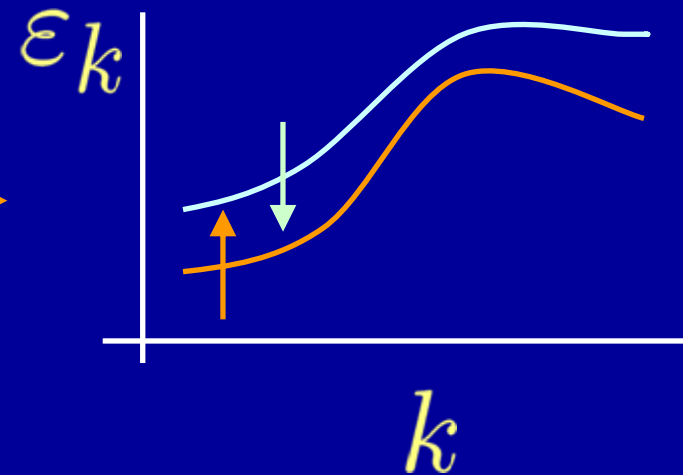
graphene, bilayer, ABC trilayer, graphite, ...  
silicon, aluminum, ...



degeneracy at  $k$

solids without space inversion symmetry

graphene in transverse electric field ... *extrinsic*  
graphene on a substrate ... *extrinsic*  
ABA trilayer ... GaAs

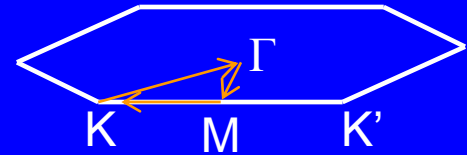


no degeneracy  
(only Kramers left)

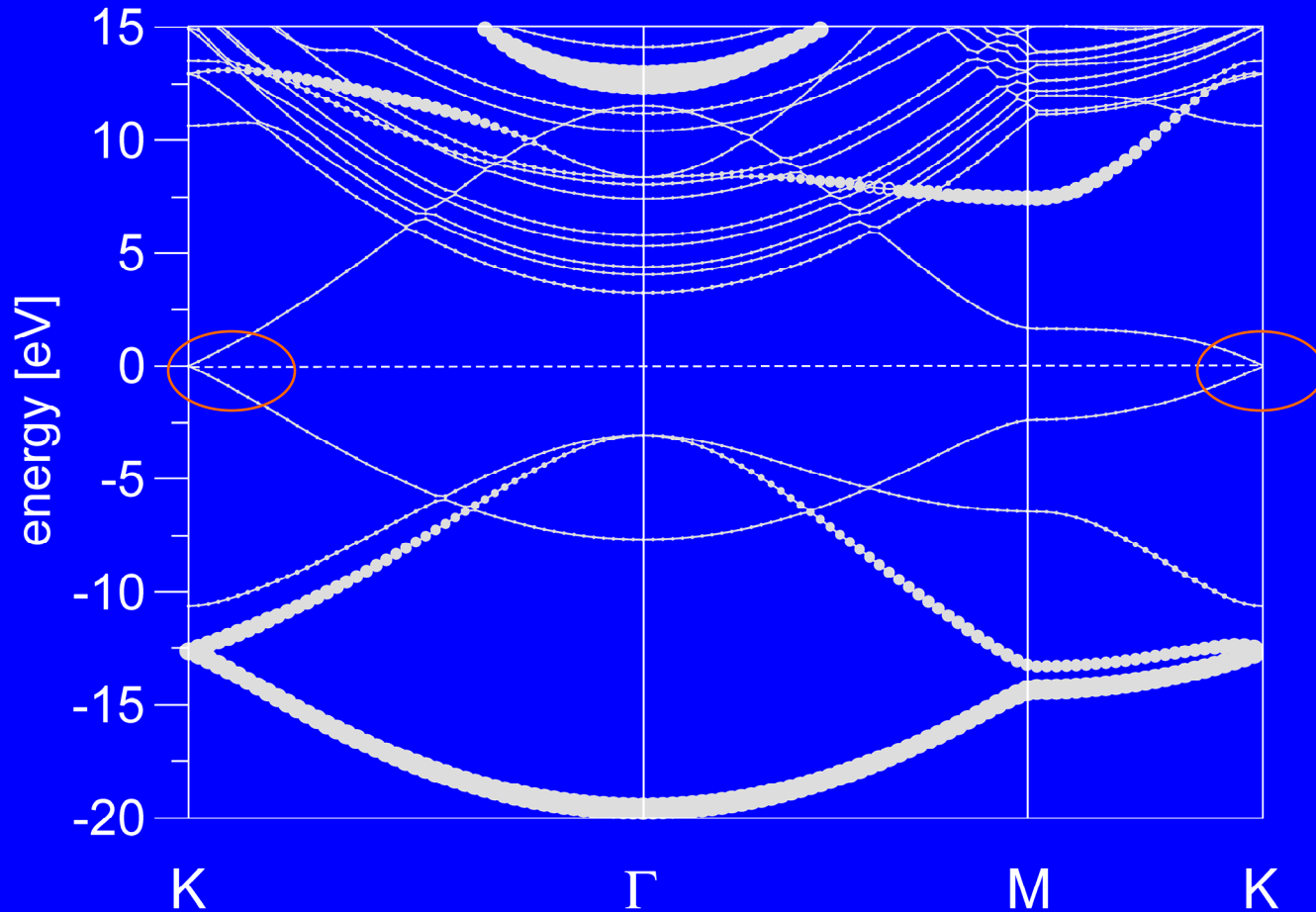
What are the functional forms (spectrum) of the spin-orbit field?

# :graphene band structure:

character of the bands

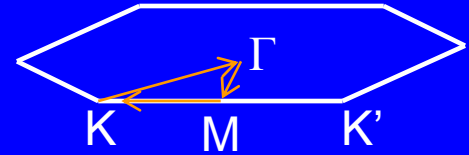


s character

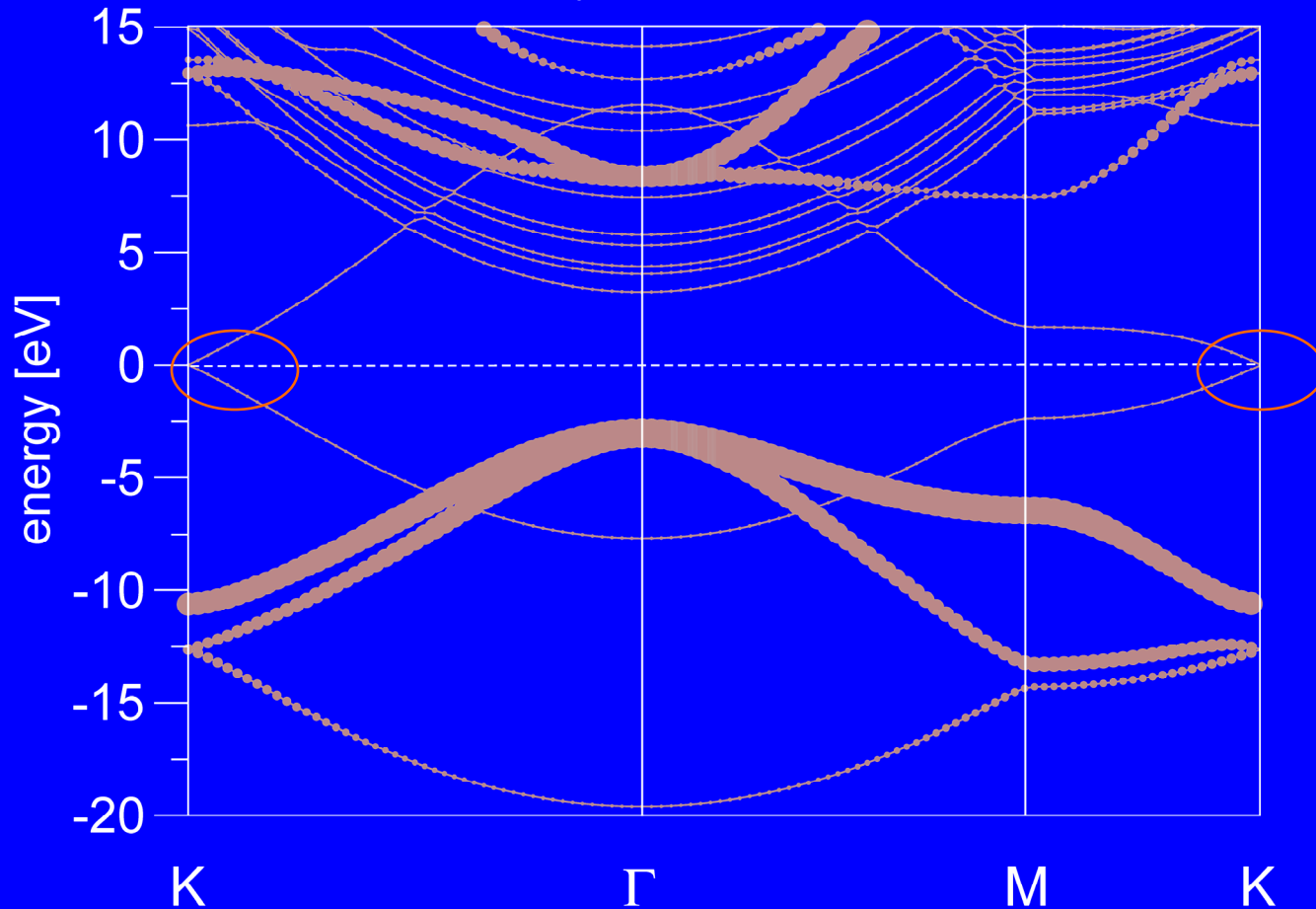


# :graphene band structure:

character of the bands

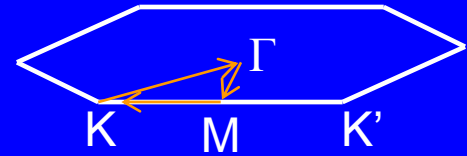


$p_{x,y}$  character

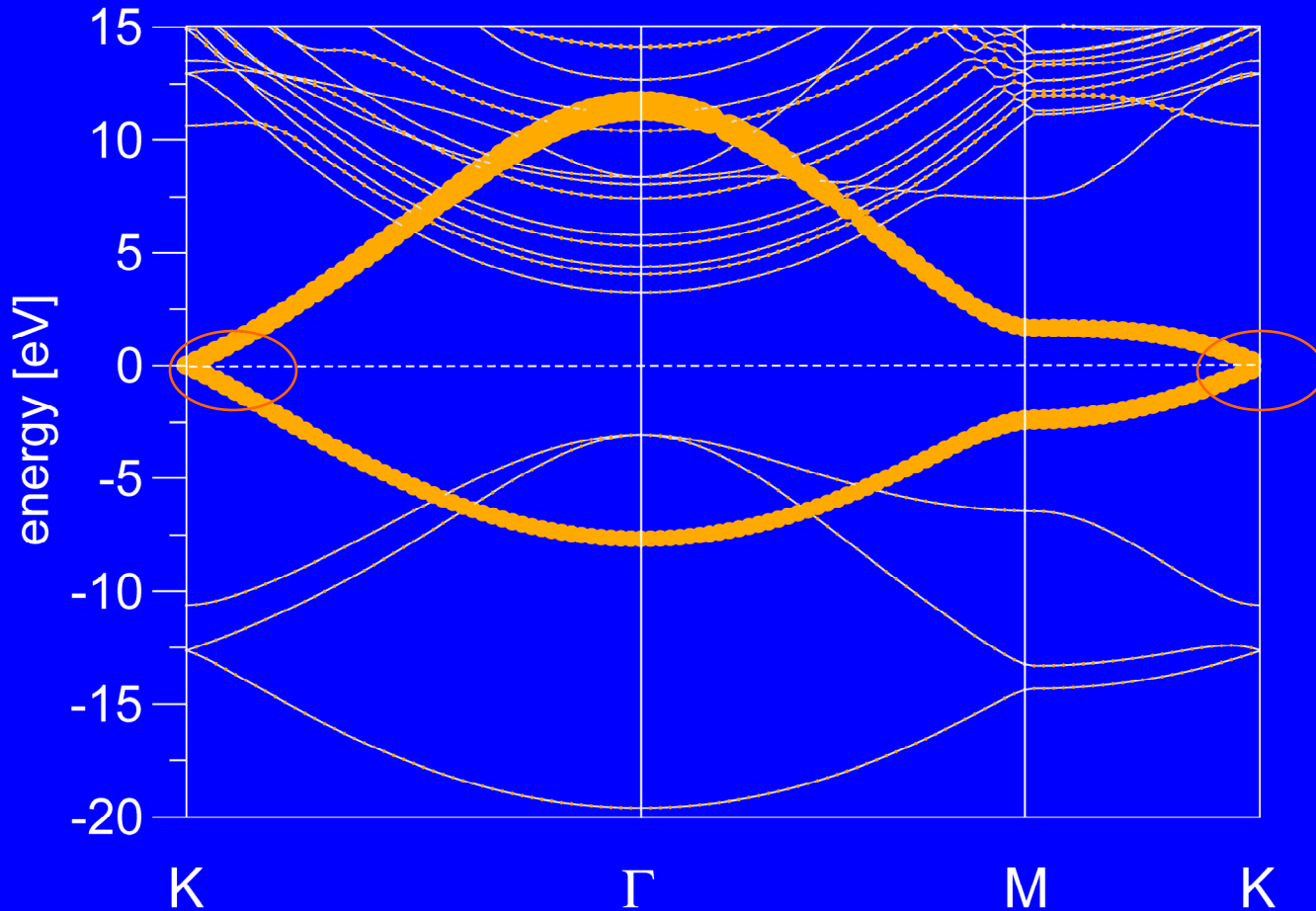


# :graphene band structure:

character of the bands

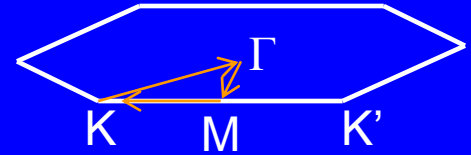


$p_z$  character

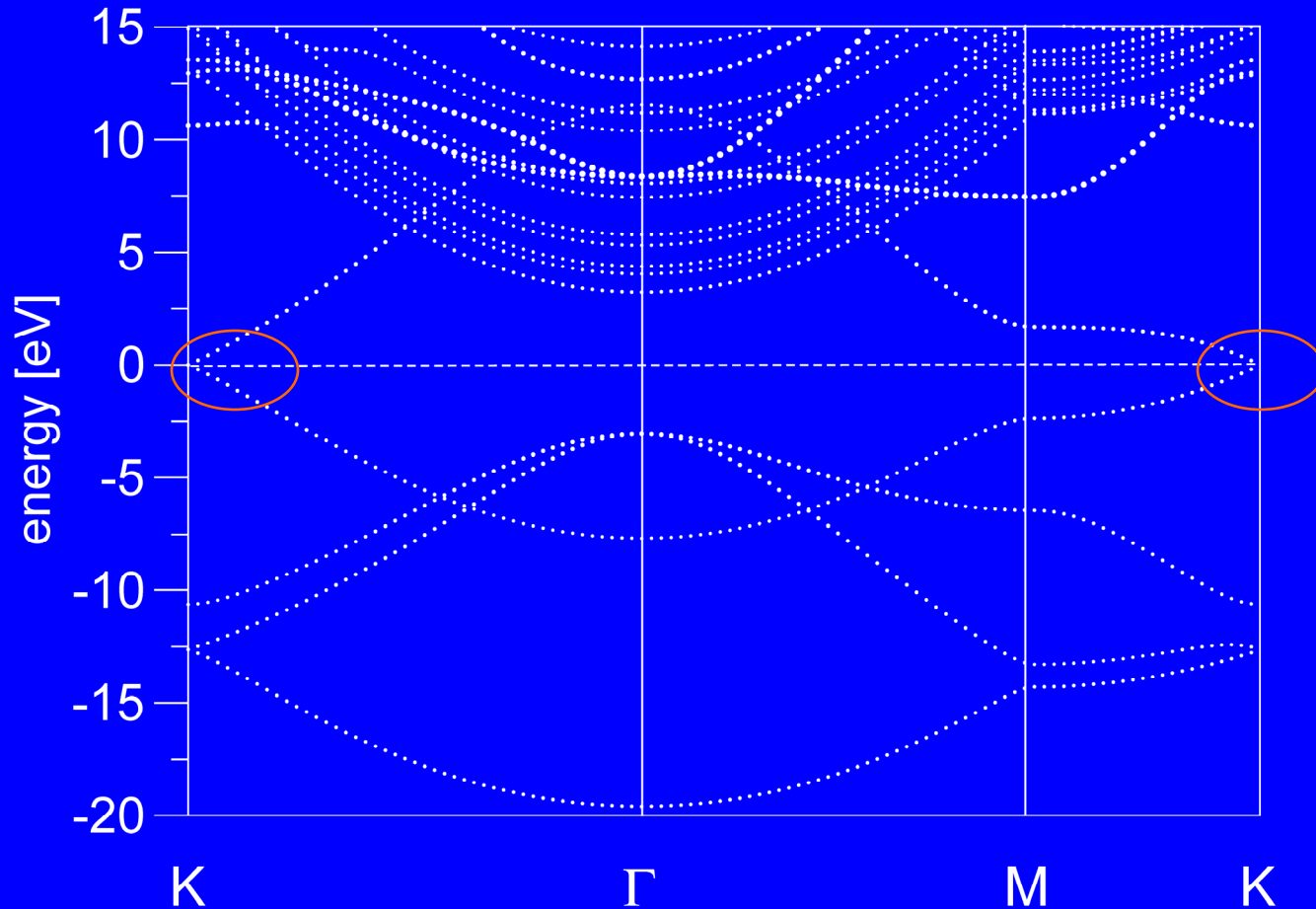


# :graphene band structure:

character of the bands



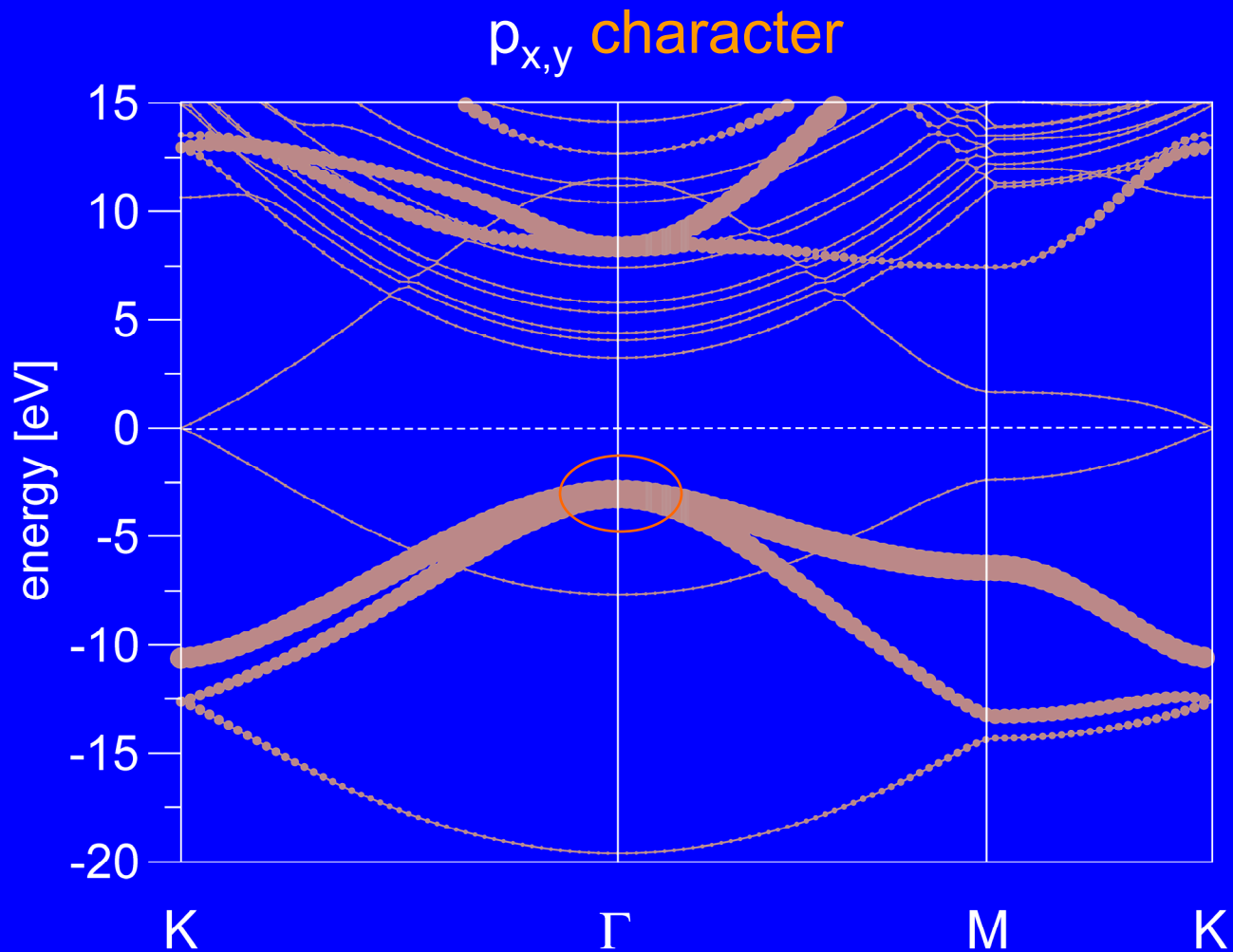
d character





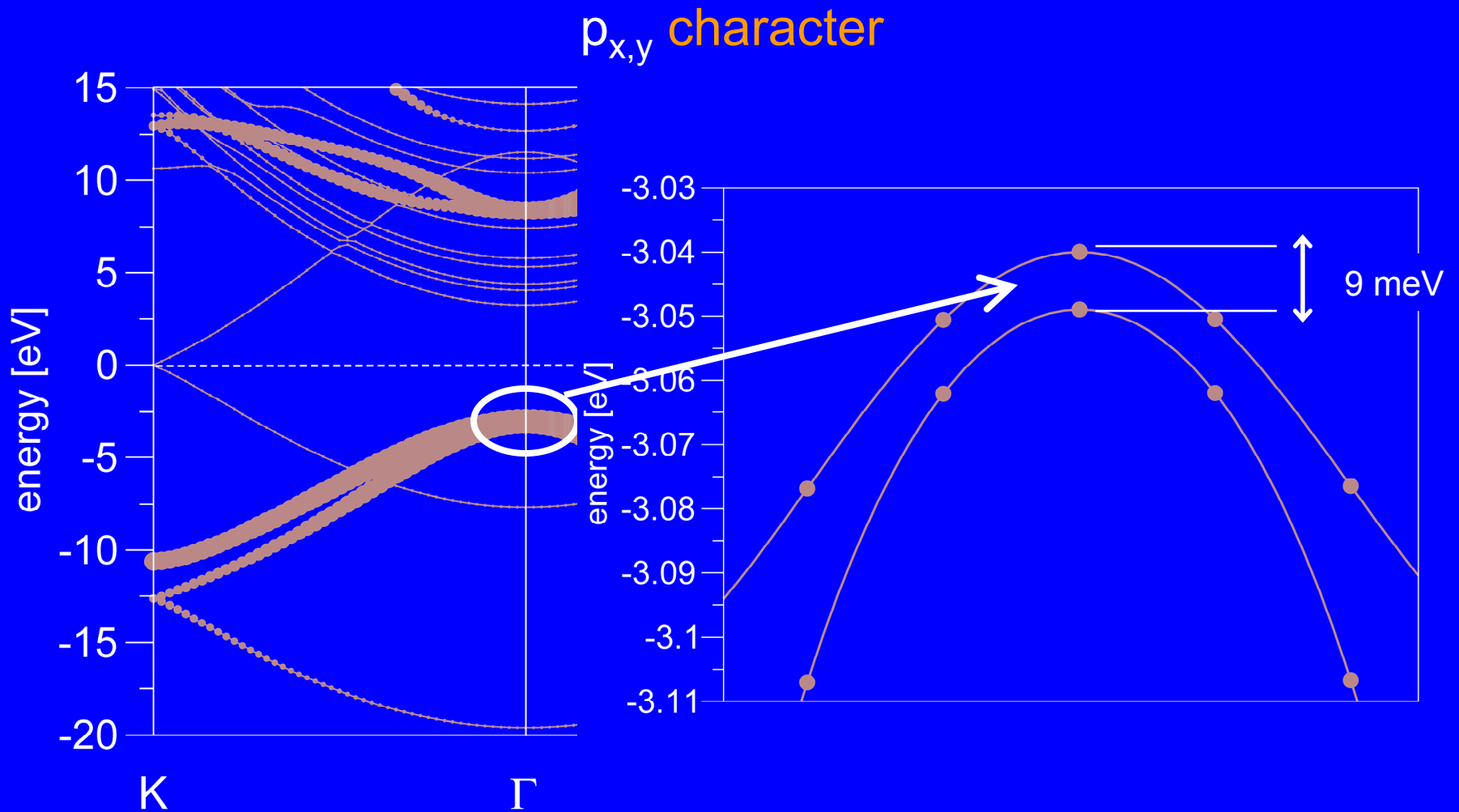
# :graphene band structure:

Spin-orbit coupling at  $\Gamma$ : 9 meV



# :graphene band structure:

spin-orbit coupling at  $\Gamma$ :  $\sim 10$  meV, as in C atom



# :spin-orbit coupling in graphene at K:

## band-structure topologies in a transverse E-field

M. Gmitra, S. Konschuh, C. Ertler, C. Ambrosch-Draxl, and J. Fabian, Phys. Rev. B 80, 235431 (2009)

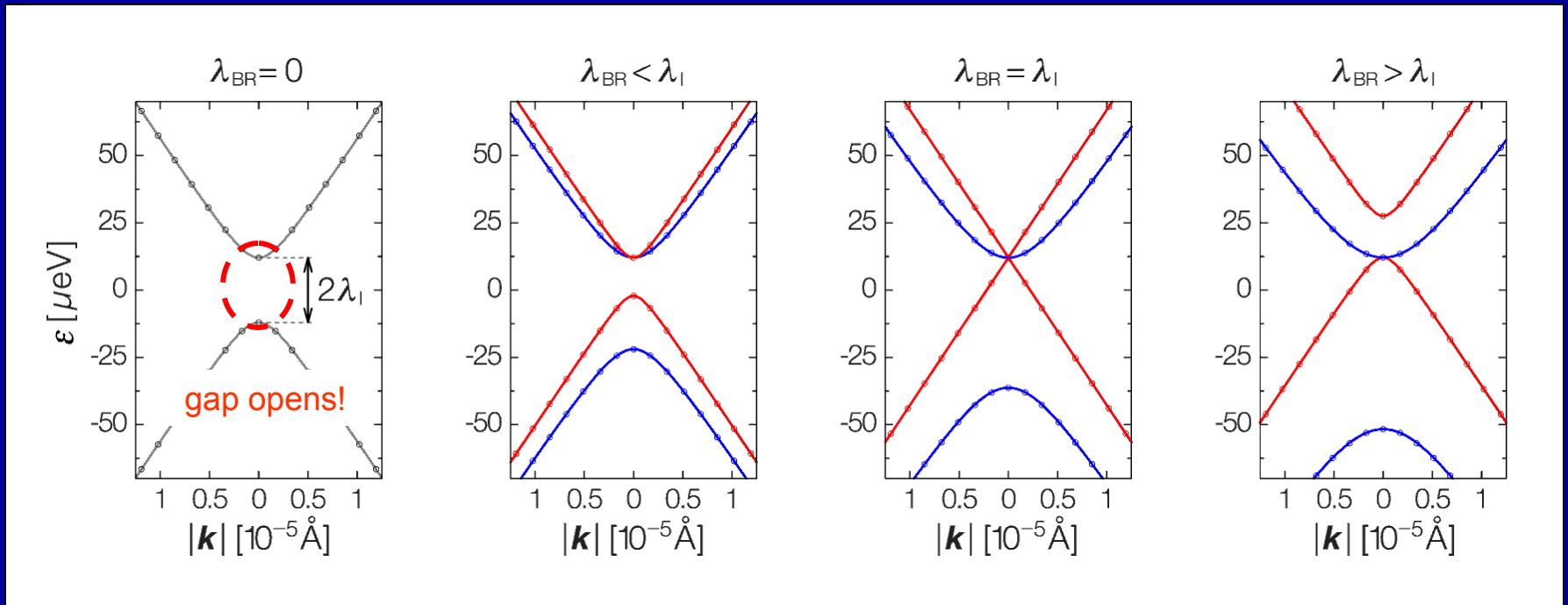
S. Konschuh, M. Gmitra, and J. Fabian, Phys. Rev. B 82, 245412 (2010)

Slonczewski, McClure & Yafet, Kane and Mele

$$H_I = \lambda_I \kappa \sigma_z s_z$$

$$H_{BR} = \lambda_{BR} (\kappa \sigma_x s_y - \sigma_y s_x)$$

$$E_{\mu\nu} = \mu \lambda_{BR} + \nu \sqrt{(\hbar v_F k)^2 + (\lambda_{BR} - \mu \lambda_I)^2}$$



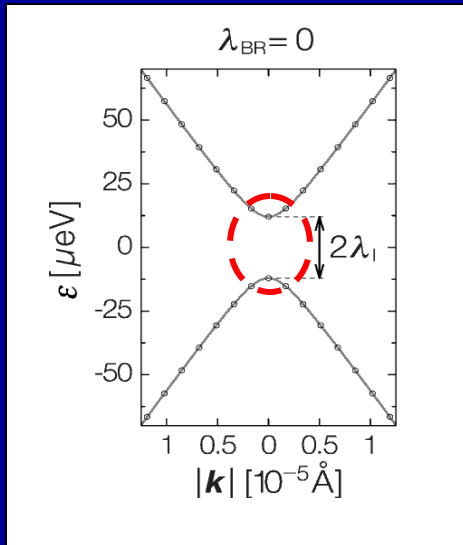
0

transverse E-field

- LAPW Wien 2k
- self-consistent transverse electric field
- analytical description within 5% for  $\pm 200$  meV

# :spin-orbit coupling in graphene at K: the intrinsic gap puzzle

M. Gmitra, S. Konschuh, C. Ertler, C. Ambrosch-Draxl, and J. Fabian, Phys. Rev. B 80, 235431 (2009)



- our calculated gap is **24 micro eV**
- previous calculations:

*200 micro eV Kane and Mele, PRL 95, 226801 (2005)*

*1 micro eV Min et al, PRB 74, 165310 (2006)*

*1 micro eV Yao et al, PRB 75, 041401(R) (2007)*

*50 micro eV Boettger and Trickey, PRB 75, 121402(R) (2007)*

*25 micro eV (Jülich, S. Bihlmaier, private communication)*

*24.5 micro eV S. Abdelouahed et al, Phys. Rev. B 82, 125424 (2010)*

?

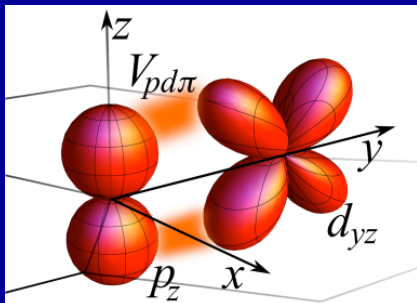
TB  
PP

all e

**$\pi$ -d mediated SOC**

J. C. Slonczewski, PhD Thesis, Rutgers University, 1955

J. W. McClure and Y. Yafet, Proceedings of the Fifth Conference on Carbon, (Pergamon, 1962), Vol. 1, p 22.

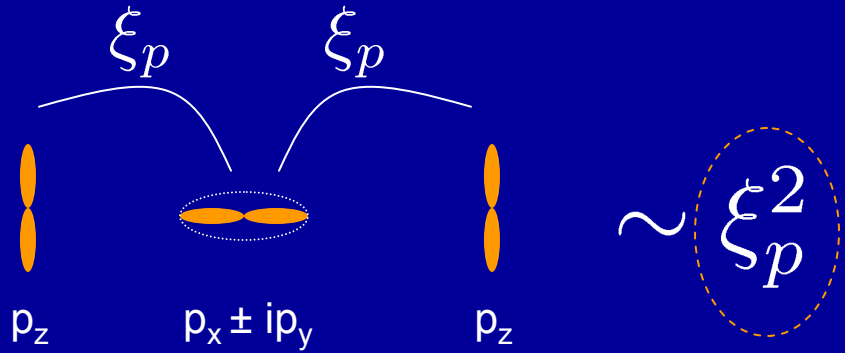
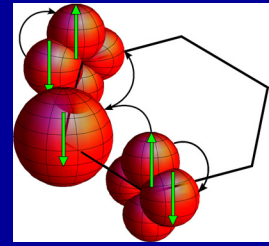


$$|p_z^A\rangle + i\gamma(\pm|d_{xz}^B\rangle \pm i|d_{yz}^B\rangle)$$

S. Konschuh, M. Gmitra, and J. Fabian, Phys. Rev. B 82, 245412 (2010)

# $\sigma$ - $\pi$ mediated SOC

2<sup>nd</sup> order effect

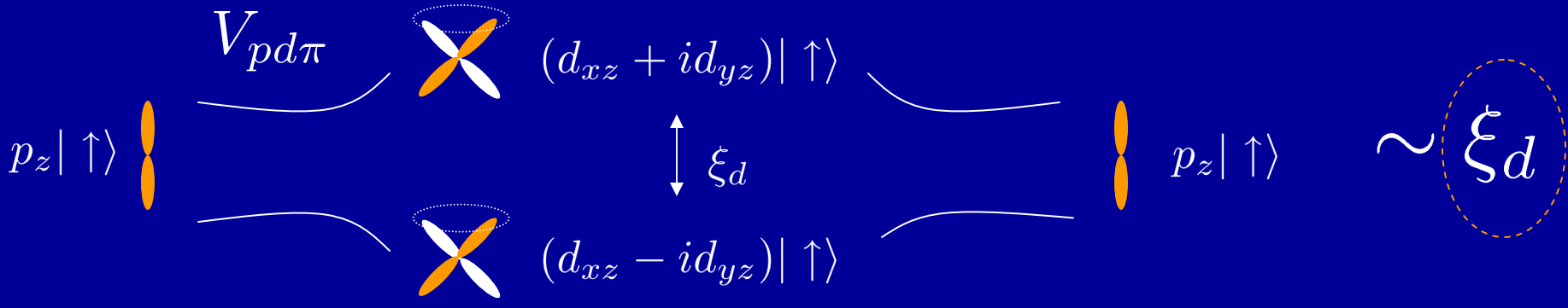
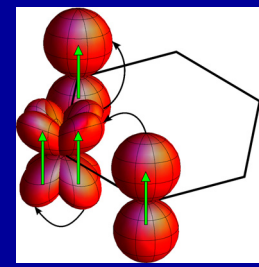


$$\langle p_z | \mathbf{L} | p_z \rangle = 0$$

$$\langle p_z | L_y | p_x \rangle \neq 0$$

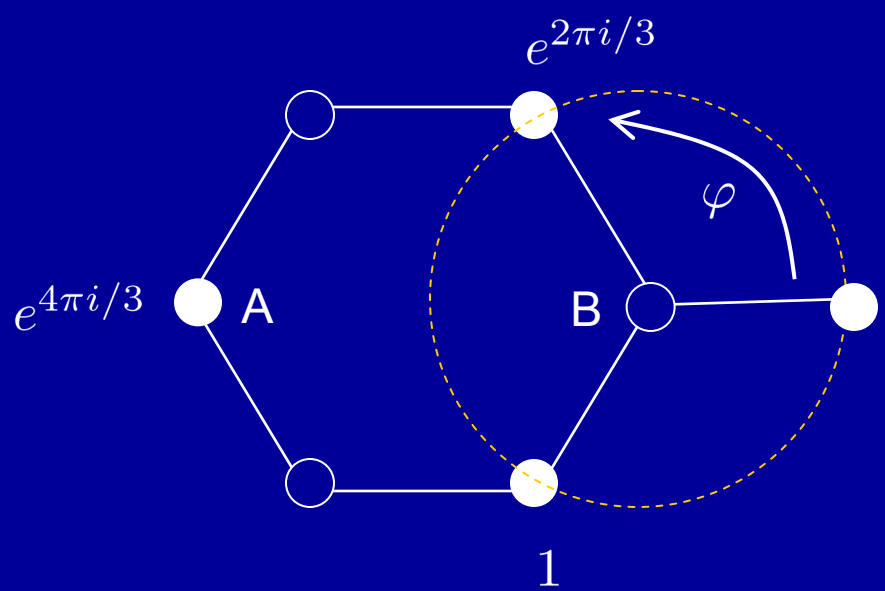
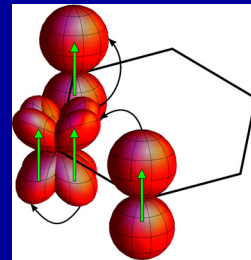
# $\pi$ -d mediated SOC

1<sup>st</sup> order effect



# why d orbitals?

symmetry at the K point



$$f(r)ze^{-i\varphi}$$

$$|p_z\rangle_A + |d_{xz} - id_{yz}\rangle_B$$



rotation about B by 120

$$[|p_z\rangle_A + |d_{xz} - id_{yz}\rangle_B] e^{2\pi i/3}$$

$$[|p_z\rangle_B + |d_{xz} + id_{yz}\rangle_A] |\uparrow\rangle$$

$$[|p_z\rangle_A + |d_{xz} - id_{yz}\rangle_A] |\downarrow\rangle$$

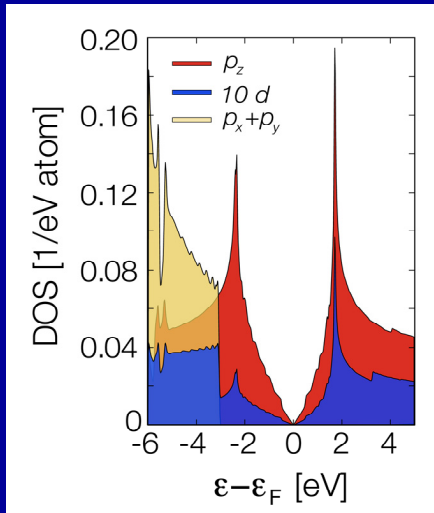
$$[|p_z\rangle_B + |d_{xz} + id_{yz}\rangle_A] |\downarrow\rangle$$

$$[|p_z\rangle_A + |d_{xz} - id_{yz}\rangle_A] |\uparrow\rangle$$

$2\lambda_I$

# :spin-orbit coupling in graphene: the intrinsic gap puzzle

M. Gmitra, S. Konschuh, C. Ertler, C. Ambrosch-Draxl, and J. Fabian, Phys. Rev. B 80, 235431 (2009)  
S. Konschuh, M. Gmitra, and J. Fabian, Phys. Rev. B 82, 245412 (2010)

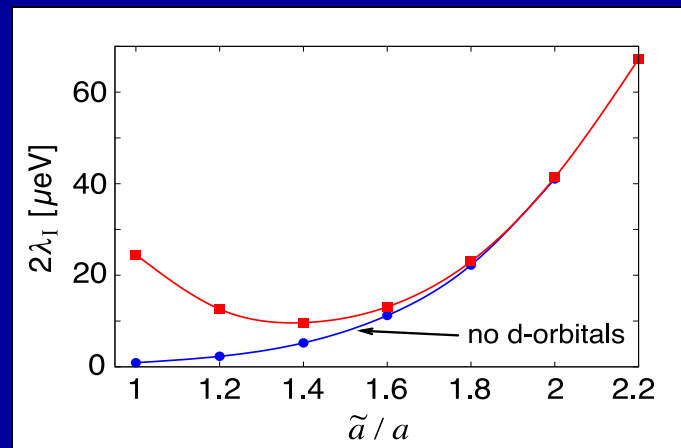


The intrinsic gap is 96% from d and higher orbitals!!!

$$2\lambda_I = \frac{4(\epsilon_p - \epsilon_s)}{9V_{sp\sigma}^2} \xi_p^2 + \frac{9V_{pd\pi}^2}{(\epsilon_d - \epsilon_p)^2} \xi_d$$

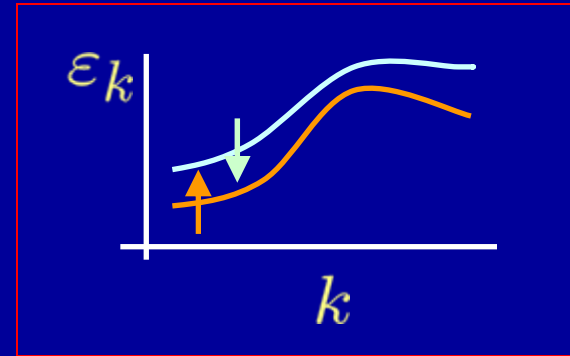
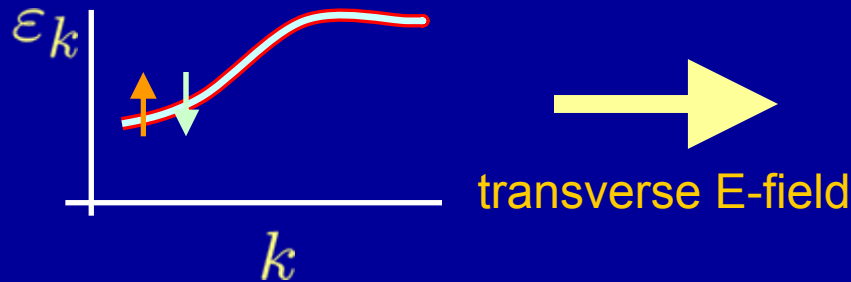
$\sigma$ - $\pi$  SOC: 1  $\mu\text{eV}$  irrelevant       $\pi$ -d SOC: 23  $\mu\text{eV}$  dominates

intrinsic gap versus strain  
also confirms the picture:



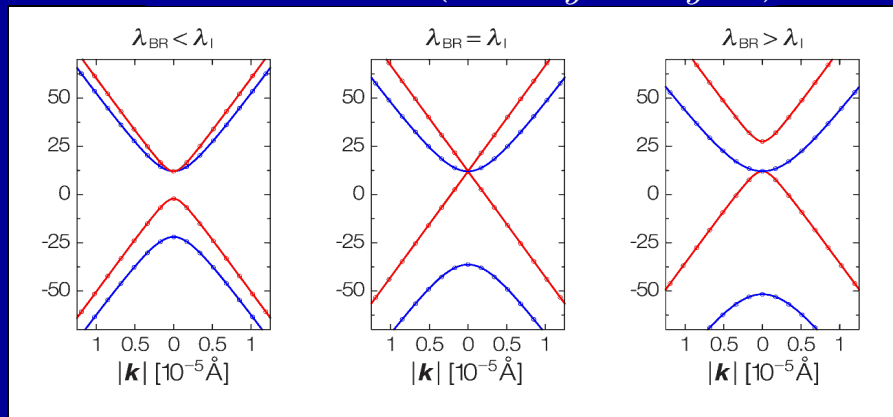
# :spin-orbit coupling in graphene: extrinsic (Bychkov-Rashba) coupling

M. Gmitra, S. Konschuh, C. Ertler, C. Ambrosch-Draxl, and J. Fabian, Phys. Rev. B 80, 235431 (2009)  
S. Konschuh, M. Gmitra, and J. Fabian, Phys. Rev. B 82, 245412 (2010)



$$H_I = \lambda_I \kappa \sigma_z s_z$$

$$H_{BR} = \lambda_{BR} (\kappa \sigma_x s_y - \sigma_y s_x)$$



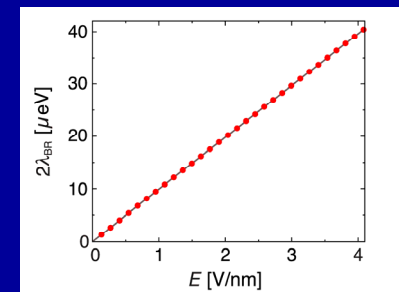
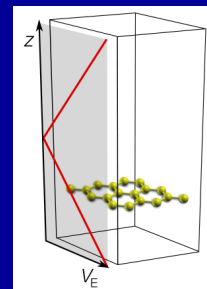
$\sigma$ - $\pi$  mediated Stark effect + SOC

$$\lambda_{BR} = \frac{eEz_{sp}}{3V_{sp\sigma}} \xi_p + \sqrt{3} \frac{eEz_{pd}}{(\epsilon_d - \epsilon_p)} \frac{3V_{pd\pi}}{(\epsilon_d - \epsilon_p)} \xi_d$$

$\sigma$ - $\pi$  SOC  
dominates

$\pi$ -d SOC  
irrelevant

Min et al, PRB 74, 165310 (2006)

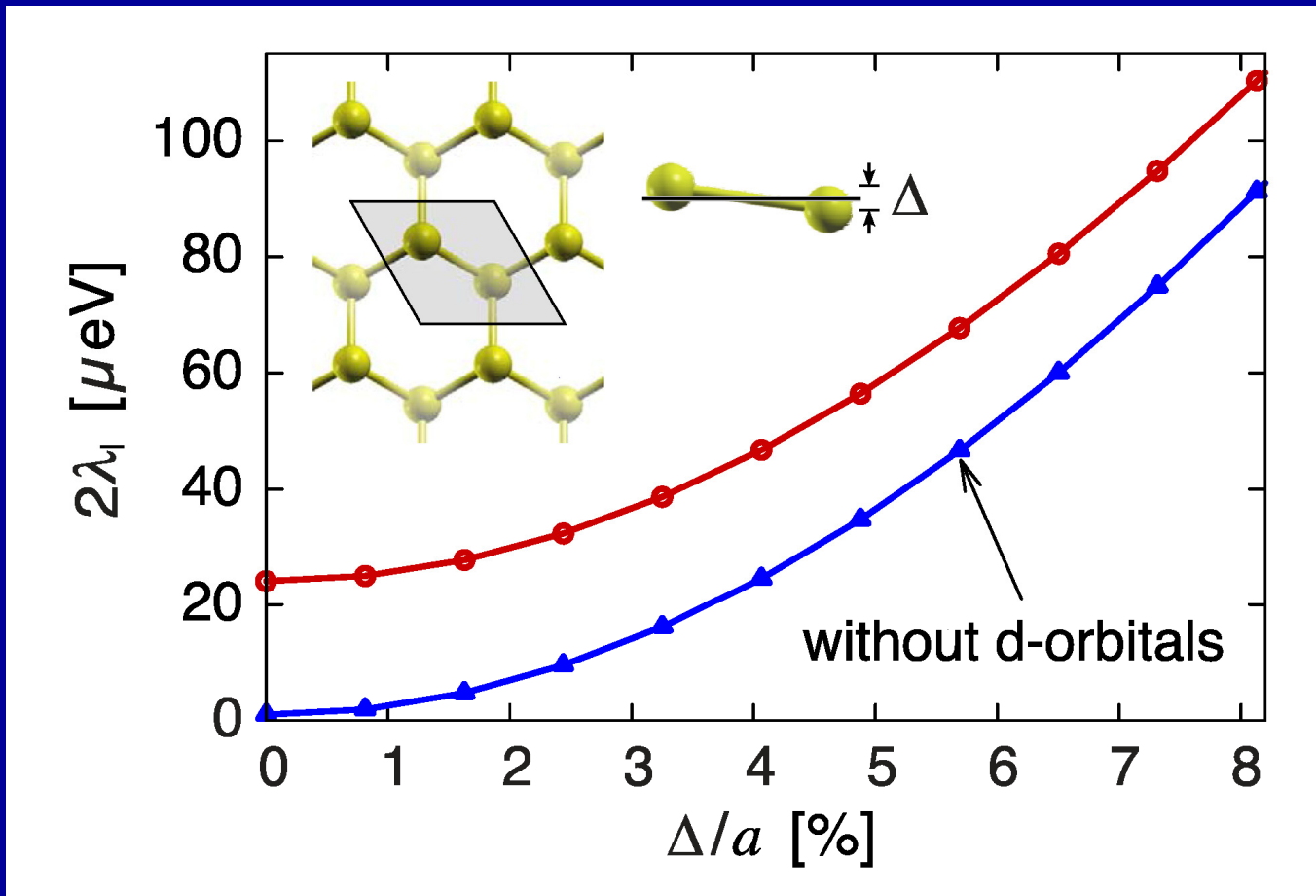


10  $\mu$ eV per V/nm



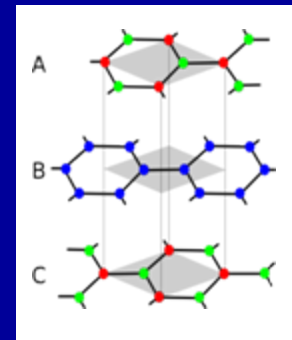
# :buckling of graphene: sigma-pi coupling

M. Gmitra, S. Konschuh, C. Ertler, C. Ambrosch-Draxl, and J. Fabian, Phys. Rev. B 80, 235431 (2009)



# :outline:

- bilayer, trilayer, and graphite



Order-of-magnitude increase SOC due to interlayer hopping?

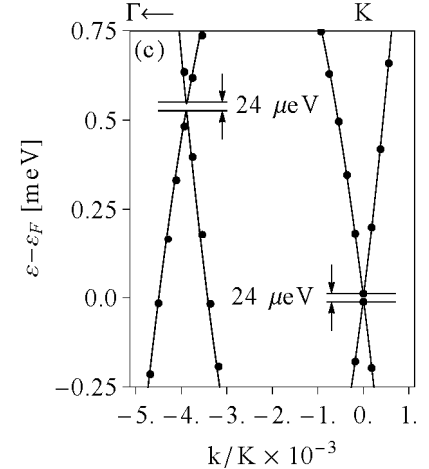
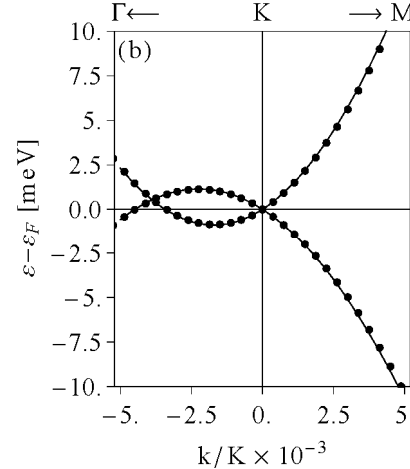
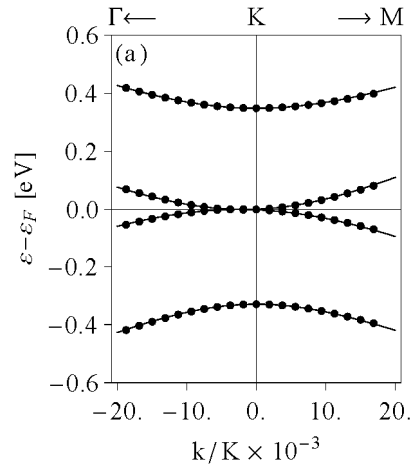
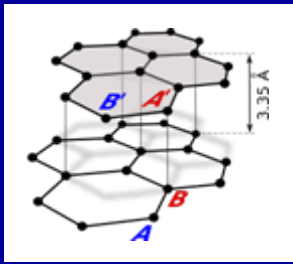
F. Guinea, New J. Phys. 12, 083063 (2010)

E. McCann and M. Koshino, Phys. Rev. B 81, 241409(R) (2010)

# :AB bilayer:

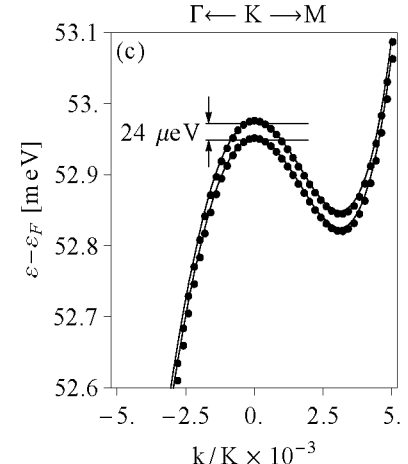
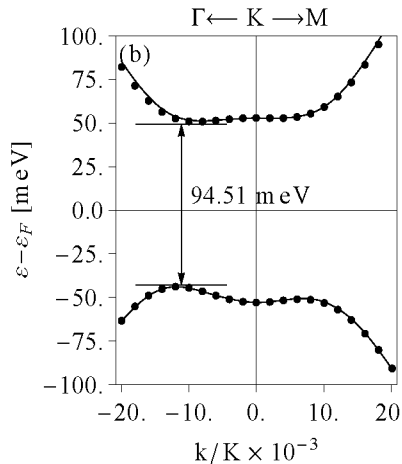
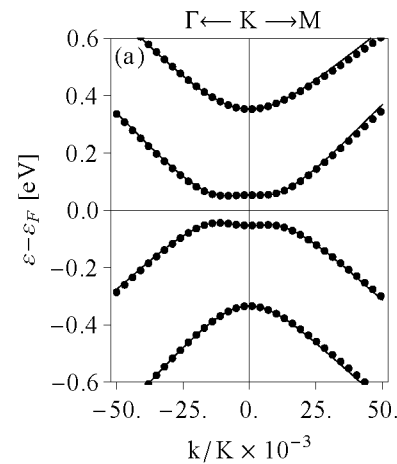
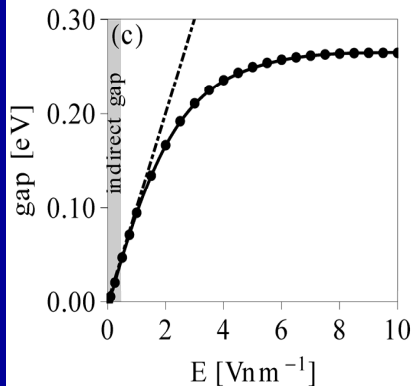
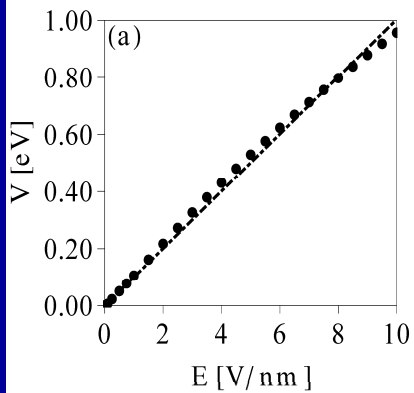
Wien2k and tight binding

$E=0$



$E=1$  V/nm

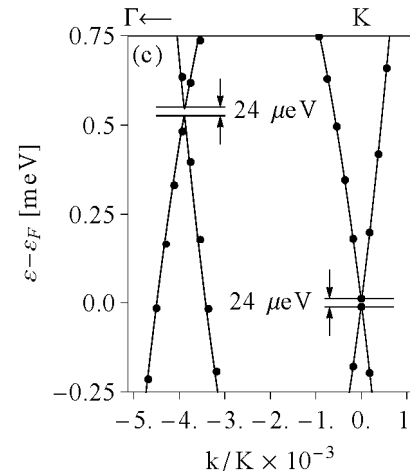
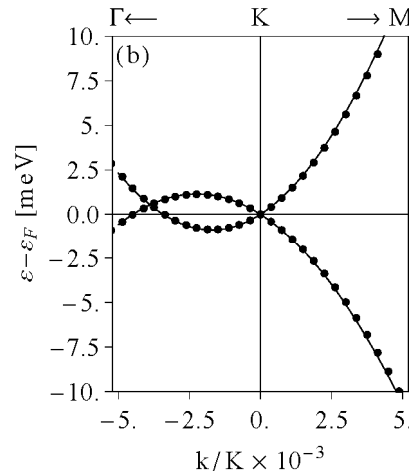
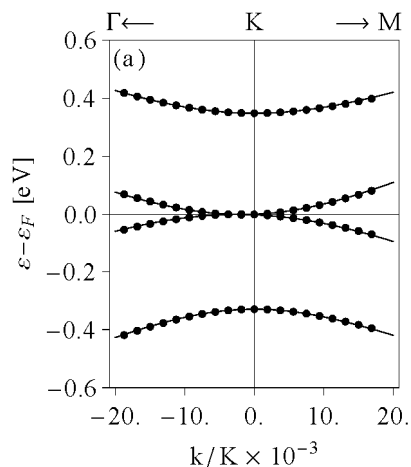
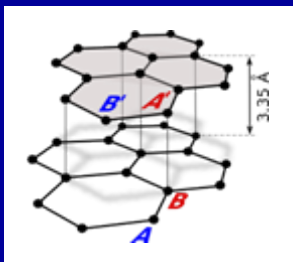
Extrinsic splitting given by intrinsic value!!!



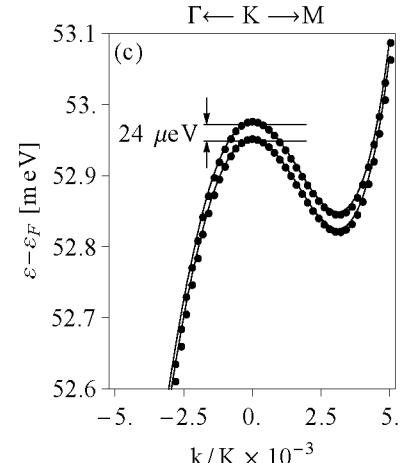
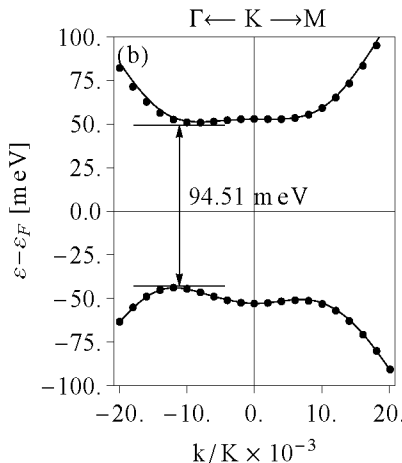
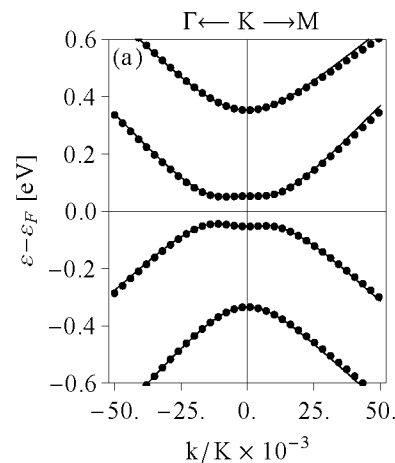
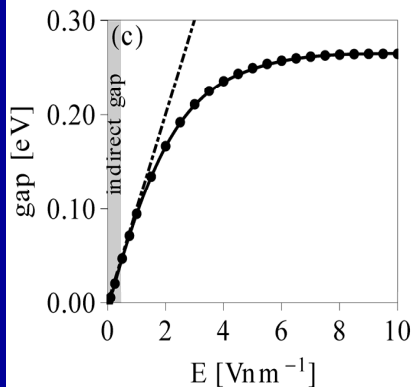
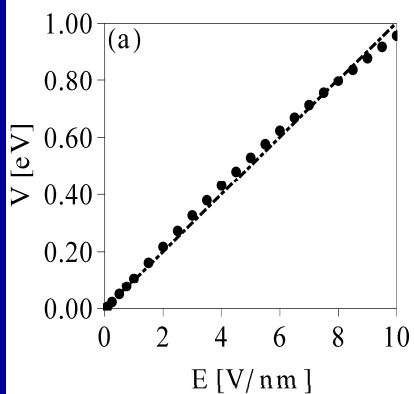
# :AB bilayer:

Wien2k and tight binding

$E=0$



$E=1$  V/nm



Extrinsic splitting given by intrinsic value!!!

# :Hamiltonian for AB bilayer:

S. Konschuh, M. Gmitra, D. Kochan, and J. Fabian,  
*Theory of spin-orbit coupling in bilayer graphene*, arXiv: 1111.7223

Intrinsic: F. Guinea, *New J. Phys.* 12, 083063 (2010)

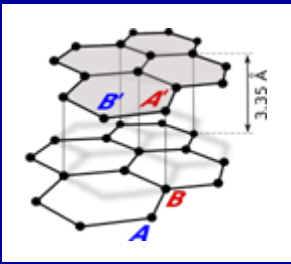
$$H_{\text{SOC}} = H_{\text{I}} + H_{\text{BR}} + H_{\text{inter}} + H_{\text{el}}$$

$$H_{\text{I}} = \frac{1}{2} [(\Delta_{\text{I}} - \Delta'_{\text{I}})\sigma_0 + (\lambda_{\text{I}} + \lambda'_{\text{I}})\sigma_z] \mu_0 \tau s_z \\ + \frac{1}{2} [(\Delta_{\text{I}} + \Delta'_{\text{I}})\sigma_0 + (\lambda_{\text{I}} - \lambda'_{\text{I}})\sigma_z] \mu_z \tau s_z,$$

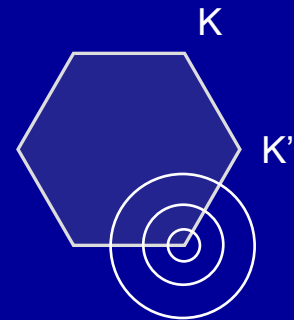
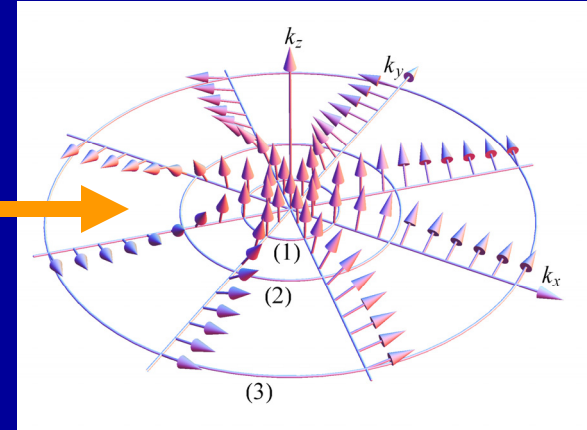
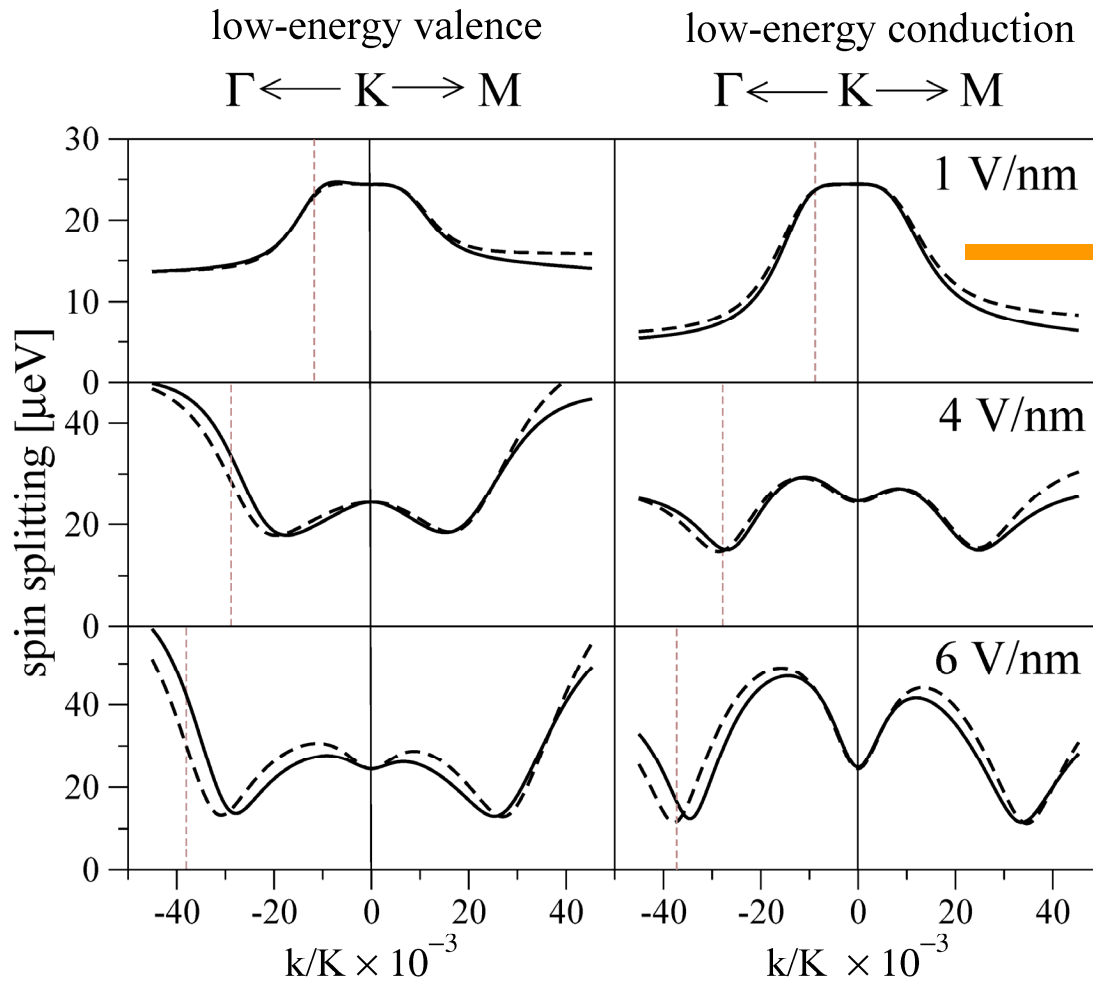
$$H_{\text{BR}} = \frac{1}{2} (\lambda_{0-} \mu_0 + \lambda_{0+} \mu_z) (\tau \sigma_x s_y - \sigma_y s_x),$$

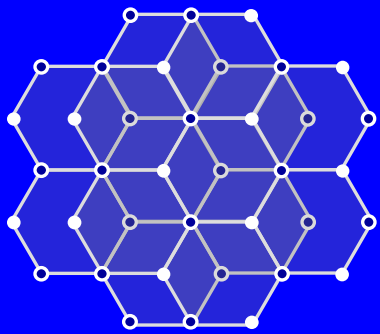
$$H_{\text{inter}} = -\frac{1}{2} (\lambda_{4+} \sigma_z + \lambda_{4-} \sigma_0) (\tau \mu_x s_y + \mu_y s_x),$$

$$H_{\text{el}} = \frac{\lambda_1}{2} s_z (\mu_x \sigma_x - \mu_y \tau \sigma_y) \\ + \frac{\lambda_3}{4} [\mu_x (\tau \sigma_x s_y + \sigma_y s_x) + \mu_y (\tau \sigma_y s_y - \sigma_x s_x)].$$



# :spin splittings in AB bilayer:





$c=3.355$  (6.71) Å

# :graphite: Wien2k

