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Graphene

an Electronic Crystalline Membrane

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Introduction

- Crystalline membranes – solid-like structures with *2D* character – are very common in our world, in biophysics and condensed matter.
- Graphene is the ultimate crystalline membrane:
  - Can be used as a toy model, as it is simple enough for simulations.
- In addition, structure is a prominent part of quality control and design of graphene applications, as well as in transport properties.
- However,
  - **Experiments** show that **graphene possesses intrinsic ripples,** of sizes 100-300 Å.
  - The **theory** of crystalline membranes predicts a **scale invariant cascade of corrugations**, for a suspended, tension-less membrane.

Nelson, Piran, Weinberg (eds.), "statistical mechanics of membranes and surfaces", (2004).
Novoselov et al. *Science, 306, 666 (2004) ; PNAS, 102, 10451 (2005).*

Introduction

- ***What is the origin of the difference?***
  - I argue that the origin is an interplay between the free electrons on the surface, and the elastic energy of the lattice.
  - Indeed, **Graphane** shows reduced curvature (Elias et al., Science (2009)).
- The same microscopic intrinsic source leads to two disorder phenomena:
  - Ripples – topographic disorder
  - Charge inhomogeneity - electron hole puddles.

Outline

- Crystalline membranes.

DG, Phys. Rev. E **80**, 041117 (2009)

- Graphene as an electronic crystalline membrane.
  - Connecting structure and the creation of electron hole puddles.

DG, Phys. Rev. B **80**, 161406 (2009)

  - Stress induced by doping.
    - Spontaneous buckling?
    - An asymmetry in the transport properties of hole and electron doped graphene.

DG, Phys. Rev. B **79**, 113411 (2009)

- Summary

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Fl e of crystalline membranes

Out of plane mode

$$F\left[\vec{u},\mathbf{h}\right]=\frac{1}{2}\int d^2\vec{x}\left[k\left(\nabla^2\mathbf{h}\right)^2+l\left(u_{ii}\right)^2+2mu_{ij}u_{ij}\right]$$

In-plane modes

Bending (curvature) energy

Strain energy

Deformation tensor:

$$u_{ij}\circ\frac{1}{2}\left(\partial_i\vec{u}_j+\partial_j\vec{u}_i\right)+\frac{1}{2}\partial_i\mathbf{h}\times\partial_j\mathbf{h}+\frac{1}{2}\partial_i u_k\partial_j u_k$$

The "size" of the 3rd dimension in graphene

- A 3D material is fully characterized by:
  - 2 elastic constants:

    μ and λ,

    or alternatively

    Young's modulus K_0 and Poisson ratio ν.
- The three constants in 2D elasticity are not independent!
  - can be expressed in terms of the 3D young modulus, 3D Poisson ratio, and lateral size ***h:***

$$h = \sqrt{12\left(1-\nu^2\right)\frac{\kappa}{K_0}}$$

- Thus, the "elastic thickness" of graphene is less 1Å
- *Smaller than the lattice size!*
- Graphene, in the continuum limit, is the ultimate membrane!

Flat phase of crystalline membranes

$$F\left[\vec{u},\mathbf{h}\right]=\frac{1}{2}\int d^2\vec{x}\left[k\left(\nabla^2\mathbf{h}\right)^2+\lambda\left(u_{ii}\right)^2+2\mu u_{ij}u_{ij}\right]$$

$$F_{eff}\left[\mathbf{h}\right]=\frac{1}{2}\int d^2\vec{x}\left[k\left(\nabla^2\mathbf{h}\right)^2+K_0\left(\frac{1}{2}\mathrm{P}_{\mathrm{ij}}^{\mathrm{T}}\partial_i\mathbf{h}\partial_j\mathbf{h}\right)^2\right]$$

$$K_0=\frac{4\mu(\mu+\lambda)}{2}$$

Harmonic part

Anharmonic term

tensor:

$$u_{ij}\ \text{o}\ \frac{1}{2}(\quad)\quad\frac{1}{2}\partial_i\mathbf{h}\times\partial_j\mathbf{h}$$

Remarks

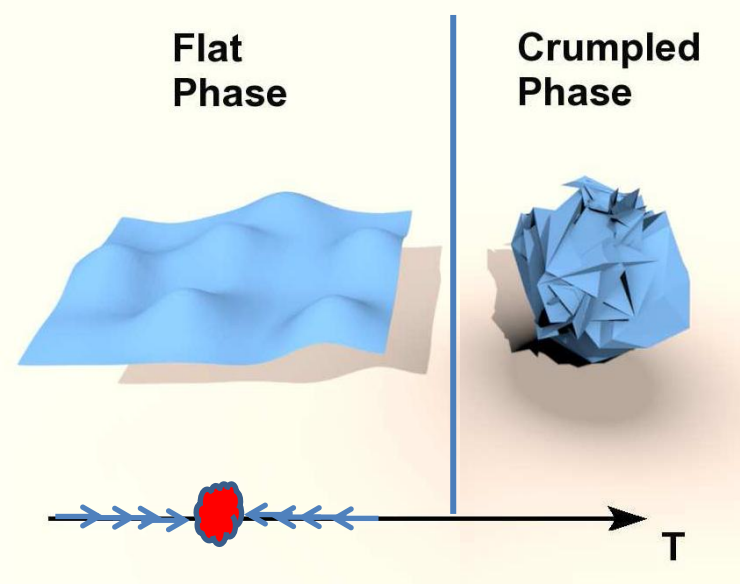
- The harmonic approximation leads to unstable flat phase.

$$\vec{n} = \frac{\hat{z} - \vec{\nabla} h}{\sqrt{1 + \left|\vec{\nabla} h\right|^2}}$$

$$G(q) = 1 - \left\langle |n|^2 \right\rangle \sim \text{å} \left\langle |\vec{q}\mathbf{h}|^2 \right\rangle \sim \text{ò}_{L^{-1}} \frac{d^2\vec{q}}{(2p)^2} \frac{q^2}{kq^4} \sim \ln L$$

Remarks

- The harmonic approximation leads to unstable flat phase.
- Stable flat phase is due to anharmonic coupling between bending and stretching mode.
- An indication of the non-trivial, attractive, fixed point in the RG analysis of the problem.
- Confirmed using ε-expansion.



Remarks

- The harmonic approximation leads to unstable flat phase.
- Stable flat phase is an indication of a non-trivial fixed point.

$$\left\langle \left|\mathbf{h}\right|^2 \right\rangle = \int_{L^{-1}} \frac{d^D\vec{q}}{(2p)^D} \frac{k_B T}{k_R(q) q^4} \sim L^{2z} \quad 0 < z < 1$$

- Normal-normal correlation function:

$$G(q) = \left\langle \left|n\right|^2 \right\rangle = \sum \left\langle \left|\vec{q}\mathbf{h}\right|^2 \right\rangle \sim \int_{L^{-1}} \frac{d^2\vec{q}}{(2p)^2} \frac{q^2}{k_R(q) q^4} \sim L^{2z-2} \xrightarrow[L\to\infty]{} 0$$

- An asymptotically flat surface, with scale invariant structure of softly diverging corrugations.
- Computing the anomalous exponents has been a huge challenge.

Crystalline Membranes – Partial History

- **80's – understanding the problem:**
  1. D. R. Nelson and L. Peliti, J. Phys. France **48,** 1085 (1987).
  2. J. A. Aronovitz and T. C. Lubensky, Phys. Rev. Lett. **60**, 2634 (1988).
  3. M. Kardar and D. R. Nelson, Phys. Rev. A **38,** 966 (1988)**.**
  4. F. David and E. Guitter, Europhys. Lett. **5,** 709 (1988)**.**
  5. E. Guitter, F. David, S. Leibler, and L. Peliti, J. Phys. France **50** 1787 (1989)
  6. J. Aronovitz, L. Golubovic, and T. Lubensky, J. Phys. France **50,** 609 (1989)
- **90's – understanding the solution – theory, simulations and experiment:**
  1. P. Le Doussal and L. Radzihovsky, Phys. Rev. Lett. **69**, 1209 (1992)
  2. Z. Zhang, H. T. Davis, and D. M. Kroll, Phys. Rev. E **48,** 651(R) (1993).
  3. L. Radzihovsky and J. Toner, Phys. Rev. Lett. **75**, 4752 (1995)
  4. L. Radzihovsky and J. Toner, Phys. Rev. E **57**, 1832 (1998)**.**
  5. C. Schmidt, K. Svoboda, N. Lei, I. Petsche, L. Berman, C., Safinya, and G. Grest, Science **259,** 952 (1993)**.**
  6. Z. Zhang, H. T. Davis, and D. M. Kroll, Phys. Rev. E **53,** 1422 (1996).
  7. Mark J. Bowick, Simon M. Catterall, Marco Falcioni, Gudmar Thorleifsson, and Konstantinos N. Anagnostopoulos, J. Phys. I 6, 1321 (1996)
  8. M. Falcioni, M. J. Bowick, E. Guitter, and G. Thorleifsson, Europhys. Lett. **38**, 67 (1997)**.**
- **"Graphene induced" revival:**
  1. J.-P. Kownacki and D. Mouhanna, Phys. Rev. E **79**, 040101(R) (2009)
  2. J. H. Los, M. I. Katsnelson, O. V. Yazyev, K. V. Zakharchenko, and A. Fasolino, Phys. Rev. B **80**, 121405(R) (2009).
  3. D. Gazit, Phys. Rev. **E 80**, 041117 (2009).
  4. Zakharchenko, K. V.; Roldán, R.; Fasolino, A.; Katsnelson, M. I., Phys. Rev. B **82**, 125435.
  5. Essafi, K.; Kownacki, J. -P.; Mouhanna, D. arXiv: 1011.6173
  6. Hasselmann, N.; Braghin, F. L., arXiv: 1012.0313.

Self Consistent Screening Approximation

$$F\left[\vec{u},\mathbf{h}\right]=\frac{1}{2}\int d^{D}\vec{x}\left[k\left(\nabla^{2}\mathbf{h}\right)^{2}+\lambda\left(u_{ii}\right)^{2}+2\mu u_{ij}u_{ij}\right]$$

In general dimensionality (d-world, D-membrane, $d_C=d-D$):

$$F_{eff}\left[\mathbf{h}\right]=\frac{1}{2}\int\frac{d^{D}\vec{q}}{\left(2\pi\right)^{D}}\left\{kq^{4}\left|\mathbf{h}_{\vec{q}}\right|^{2}+\right.$$

$$\left.\frac{1}{2}\int\frac{d^{D}\vec{k}}{\left(2\pi\right)^{D}}\frac{d^{D}\vec{k}'}{\left(2\pi\right)^{D}}\frac{R^{(D)}\left(\vec{q},\vec{k},\vec{k}'\right)}{4\left(d-D\right)}\mathbf{h}_{\vec{k}}\times\mathbf{h}_{\vec{q}-\vec{k}}\mathbf{h}_{\vec{k}'}\times\mathbf{h}_{-\vec{q}-\vec{k}'}\right\}$$

$k_i k_j k'_l k'_m \tilde{r}_{ij,lm}(q)$

=0 for D=2

$$\tilde{r}=2\mu\tilde{N}\left(\vec{q}\right)+2b\mu\tilde{M}\left(\vec{q}\right)$$

$$b=\frac{\mu\left(2\mu+D\lambda\right)}{2\mu+\lambda}$$

$$N_{ij,lm}\left(\vec{q}\right)=\frac{1}{D-1}\mathrm{P}^{\mathrm{T}}_{ij}\left(\vec{q}\right)\mathrm{P}^{\mathrm{T}}_{lm}\left(\vec{q}\right)$$

$$M_{ij,lm}\left(\vec{q}\right)=\frac{1}{2}\left(\mathrm{P}^{\mathrm{T}}_{il}\left(\vec{q}\right)\mathrm{P}^{\mathrm{T}}_{jm}\left(\vec{q}\right)+\mathrm{P}^{\mathrm{T}}_{im}\left(\vec{q}\right)\mathrm{P}^{\mathrm{T}}_{jl}\left(\vec{q}\right)\right)-N_{ij,lm}\left(\vec{q}\right)$$

Self Consistent Screening Approximation

A diagrammatic expansion:

1. Every interaction introduces a $1/d_C$ factor.
2. Since the h propagator:

$$\left\langle \mathbf{h}\left(\vec{q}\right)\mathbf{h}\left(-\vec{q}\right)\right\rangle = \mathbf{I}_{d_C' d_C} k_B T G\left(q\right)$$

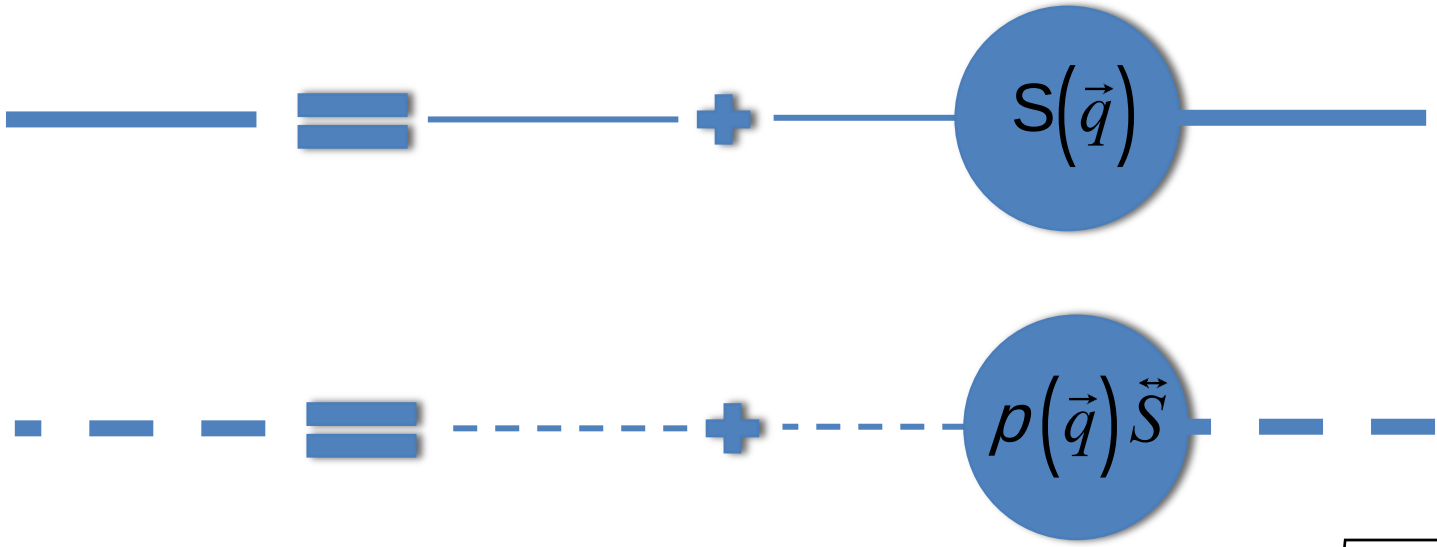
   Every loop introduces a d_C factor.

A $1/d_C$ expansion is called for.

Order of expansion: $n = Number\ of\ interactions - Number\ of\ loops$

Radzihovsky, Le-Doussal, PRL **69,** 1209 (1992);
DG, PRE **80**, 041117 (2009)

Self Consistent Screening Approximation



The long-wavelength approximation: $q << q_T = \sqrt{\frac{K_0 k_B T}{k^2}}$

$$K_0(\vec{q}) \underset{q\to 0}{\sim} q^{h_u} \qquad k(q) \underset{q\to 0}{\sim} S(q) \sim q^{-h}$$

A general result:

$$h_u = 4 - D - 2h$$

Self Consistent Screening Approximation

A Dyson equation for the interaction: $\vec{r}_R = \vec{r} - p(q)\,\vec{r} \times \vec{S} \times \vec{r}_R$

$$m_R(\vec{q}) = \frac{m}{1+2mp(\vec{q})}$$

$$b_R(\vec{q}) = \frac{b}{1+(D+1)bp(\vec{q})}$$

$D \neq 1,2$

$$\Rightarrow \quad p(\vec{q}) \sim q^{-h_u} \quad \Rightarrow \quad \lim_{q\to 0} \frac{I(q)}{m(q)} = -\frac{2}{D+1}$$

A universal negative Poisson's Ratio for $D \neq 1,2$

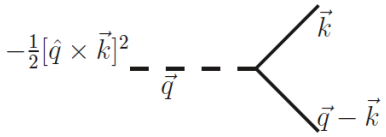
D=2: simulations give $\nu = -0.32(1)$,
extrapolation of SCSA $\nu = -1/3$.

Falcioni *et al*, Europhys. Lett. **38,** 67 (1997)

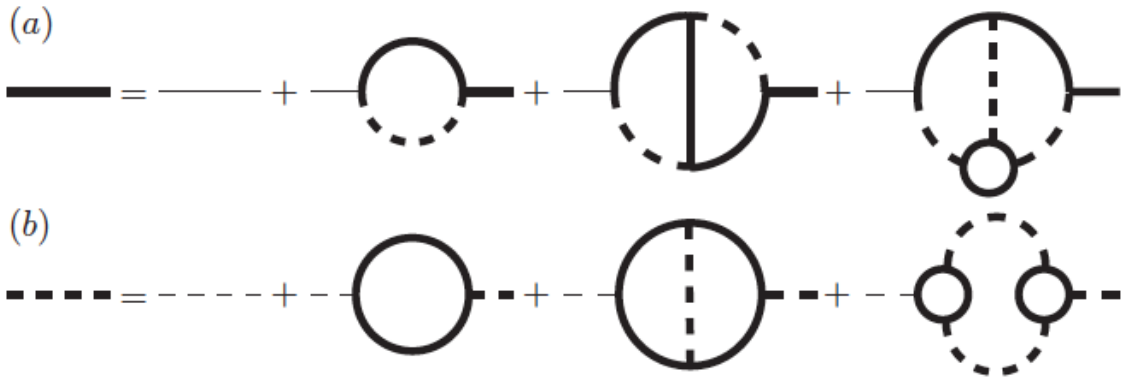
SCSA Results for physical membranes

h propagator

Interaction propagator



Interaction vertex



SCSA Results for physical membranes

1st order

- A one loop expansion.
- Recovers all other expansions.

$$z = 0.590...$$

$$\lim_{q\to 0} \frac{\sqrt{k_B T K_R(q)}}{q k_R(q)} = 3.351...$$

$$\lim_{q\to 0} \frac{I(q)}{m(q)}_{D^1 1,2} = -\frac{2}{D+1}$$

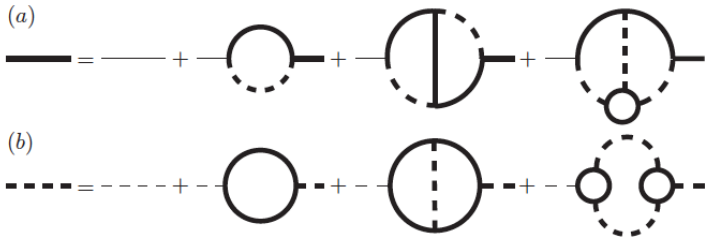
2nd order

- Gives a result ***iff*** regarded as a $1/d_C$ expansion.

$$z = 0.605...$$

$$\lim_{q\to 0} \frac{\sqrt{k_B T K_R(q)}}{q k_R(q)} = 3.573...$$

$$\lim_{q\to 0} \frac{I(q)}{m(q)}_{D^1 1,2} = -\frac{2}{D+1}$$



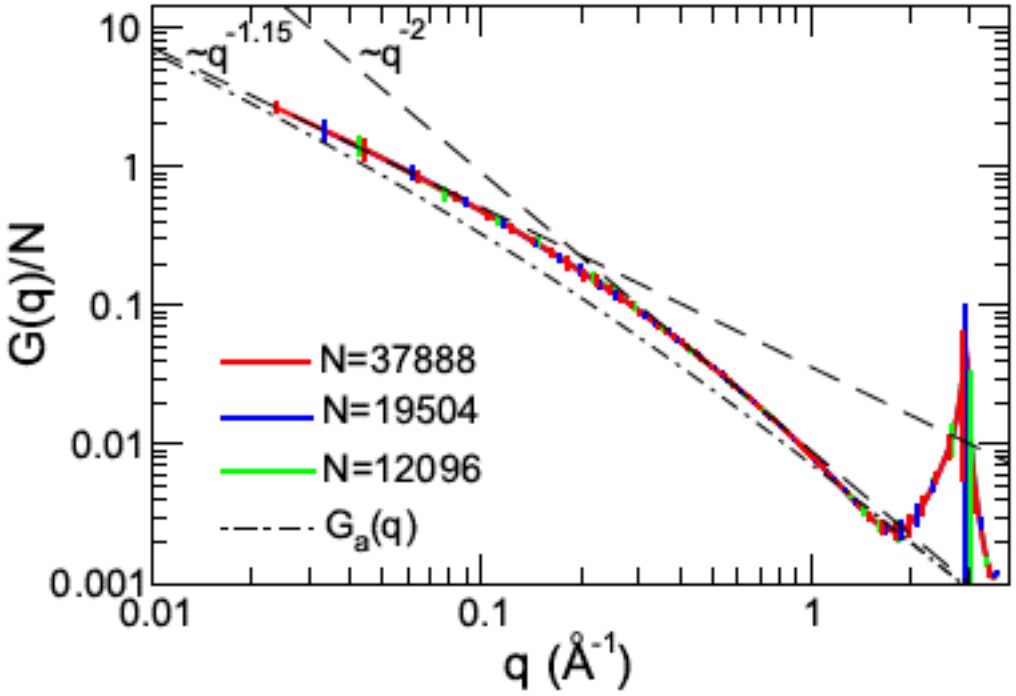
Crystalline membranes and graphene

- Going to second order has not changed the general results:
  - Scale free structure at wavelengths much longer than the thermal wavelength ≈1.2 nm in graphene.
- Numerical solution of the SCSA equation to first order leads to the same conclusion.

Zakharchenko, Roldán, Fasolino, Katsnelson, Phys. Rev. B **82**, 125435.

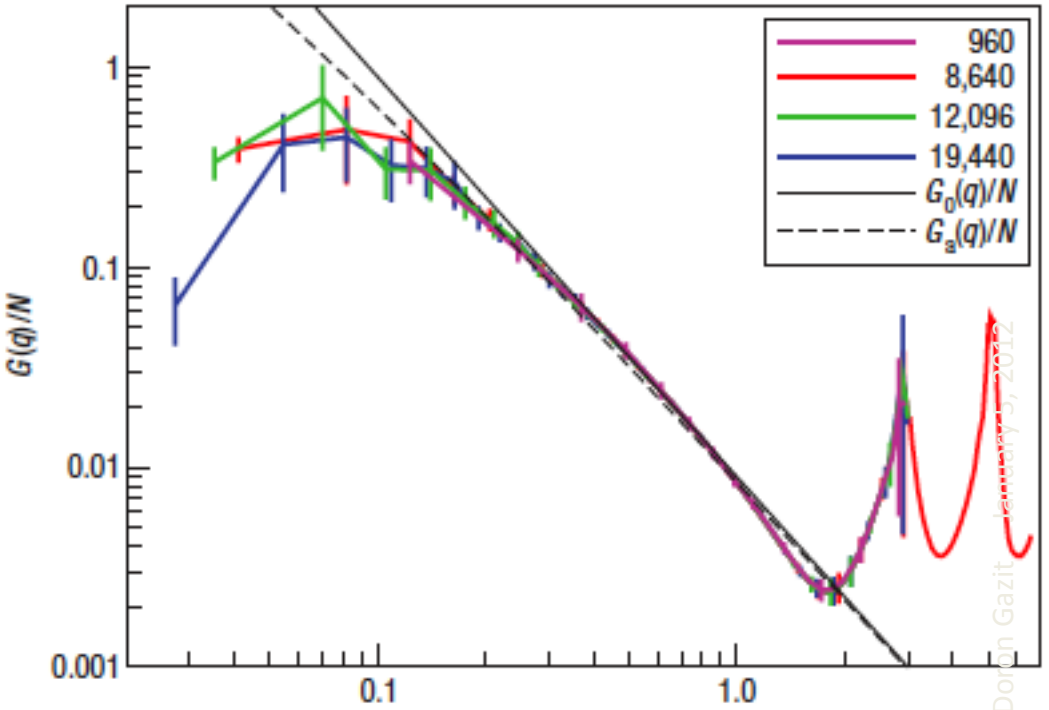
Atomistic simulations of graphene

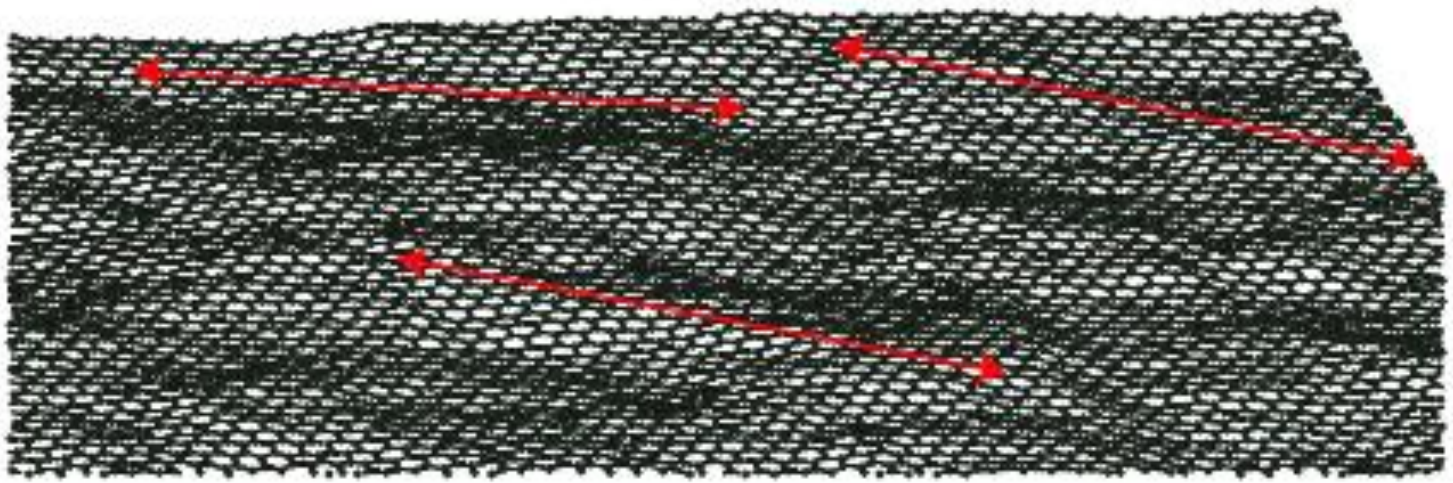
- Los et al. [PRB(R) (2009)] have used carbon-carbon potential to calculate ***normal-normal correlation***.
- They found a behavior consistent with the theory of crystalline membranes.
- **No sign of ripples**.



- How should ripples look?

Los, Fasolino, Katsnelson, Nature Materials **6**, 858 [2007].





Physical membranes:

Method	ζ	*References*
1st order SCSA	0.590…	Radzihovsky, Le-Doussal, PRL, **69,** 1209 (1992)

Crystalline membranes and graphene

- Going to second order has not changed the general results:
  - Scale free structure at wavelengths much longer than the thermal wavelength ≈1.2 nm in graphene.
- Numerical solution of the SCSA equation to first order leads to the same conclusion.
- No signature of ripples using state of the art atomic carbon-carbon interaction in QMC or MD simulations.
- Existence of intrinsic ripples in graphene implies that:

Graphene is not a regular crystalline membrane.

Outline

- Crystalline membranes.

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  - Stress induced by doping.
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DG, Phys. Rev. B **79**, 113411 (2009)

- Summary

What about the π electrons?

An interplay between charge inhomogeneities and corrugations

- In the presence of corrugations, the surface area changes:

$$\delta S \sim a^2 u_{ii}$$

- This changes the ion density, and thus the electron density.
- As a result, the electron's chemical potential is locally changed
  ➔ an effective induced electric field:

$$V(r) = D u_{ii} \quad ; D = 10 - 30\text{eV}$$

Suzuura, Ando PRB, 65, 235412 (2002)
Katsnelson, Perreira, Castro-Neto, Guinea, Geim, Kim, Fogler, Horovitz, ….

The total Classical free energy

Charge inhomogeneity

Electron-electron interaction

$$F[u,h,\delta n] = E_{ee}[\delta n] + \frac{1}{2}\int d^2\vec{x}\kappa(\Delta h)^2 + \frac{1}{2}\int d^2\vec{x}\left[2\mu u_{ij}^2 + \lambda u_{ii}^2 + 2Du_{ii}\delta n\right].$$

Elastic energy

Coupling between structure and charge inhomogeneity

$$E_{ee} = \frac{e^2}{2\varepsilon}\int\int d^2x d^2y \frac{\delta n(\vec{x})\,\delta n(\vec{y})}{|\vec{x}-\vec{y}|} = \frac{1}{2\varepsilon}\int \frac{d^2q}{(2\pi)^2}\frac{|\delta n(\vec{q})|^2}{q}$$

Intrinsic ripples in graphene

$$F_{eff}\left[h\right]=\frac{1}{2}\int\frac{d^2\vec{q}}{\left(2p\right)^2}\left[k\left(q^2\mathbf{h}\right)^2+K\left(q\right)\underbrace{\mathrm{F}\left(h\left(\vec{x}\right)\right)=\frac{1}{2}\mathrm{P}_{ij}^{\mathrm{T}}\P_i h\P_j h}\mathrm{F}^2\left(q\right)\right]$$

$$K\left(q\right)=K_0\frac{1-\frac{2\mu+\lambda}{\mu+\lambda}\frac{q}{q_0}}{1-\frac{q}{q_0}}\qquad\xi_0=\frac{2\pi}{q_0}\equiv\frac{D^2\frac{E}{e^2}}{2\mu+\lambda}$$

- The screening factor was estimated using RPA calculations

V. N. Kotov, B. Uchoa, and A. H. Castro Neto, Phys. Rev. B **78**, 035119 (2008)

- For 2D materials, this means inherent competition between bending and stretching.
- Ripples compensate this competition.

Intrinsic ripples in graphene

The long wavelength limit is not changed, however a new scale comes into play due to the free electrons on the membrane.

$(q)]$

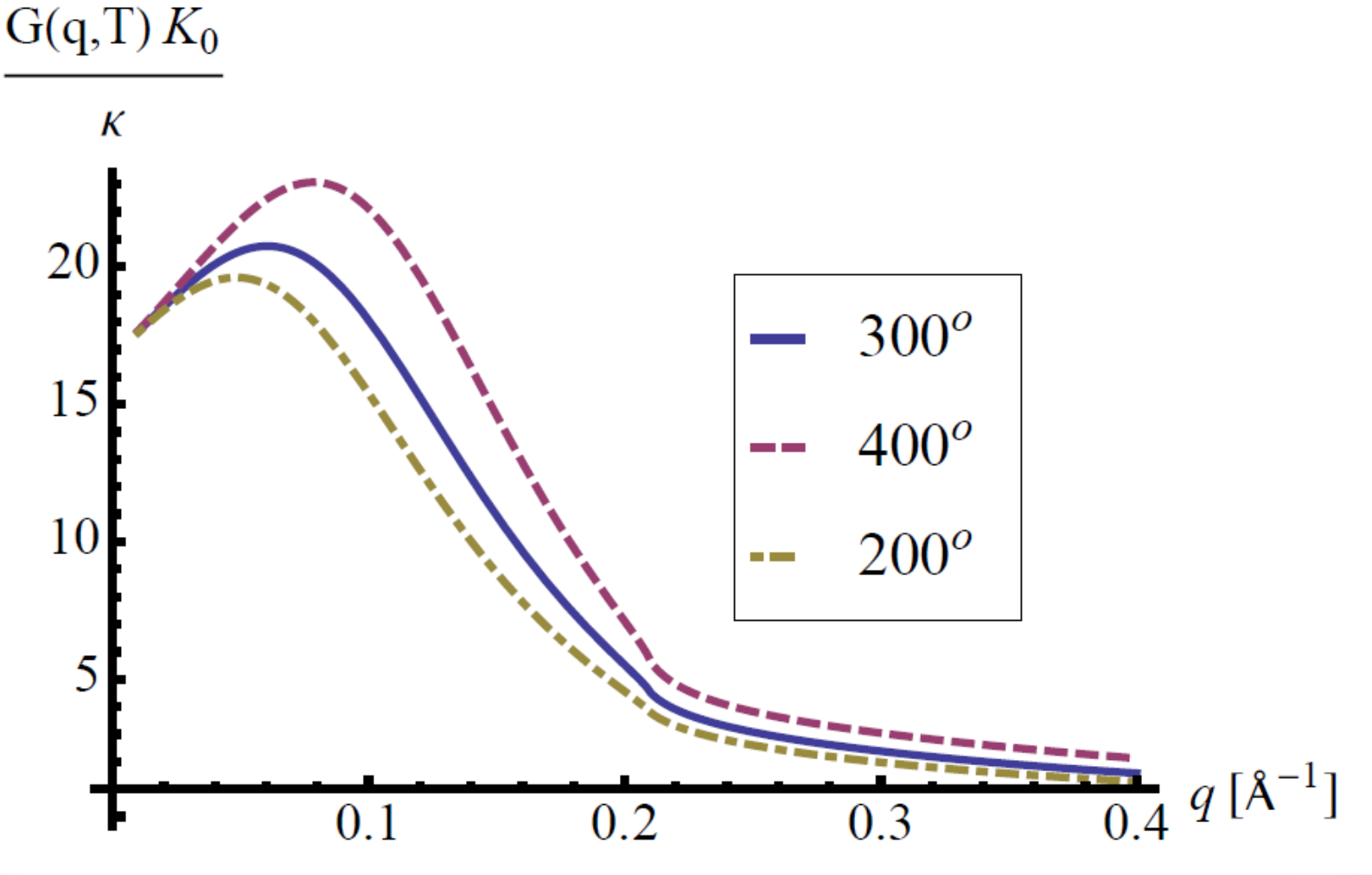
Need to solve:

$$\begin{cases} y(q) = 1 + \left(\frac{q_T}{q}\right)^2 \Sigma(q) \\ z(q) = 1 + \frac{1}{2}\left(\frac{q_T}{q}\right)^2 \Psi(q) \end{cases}$$

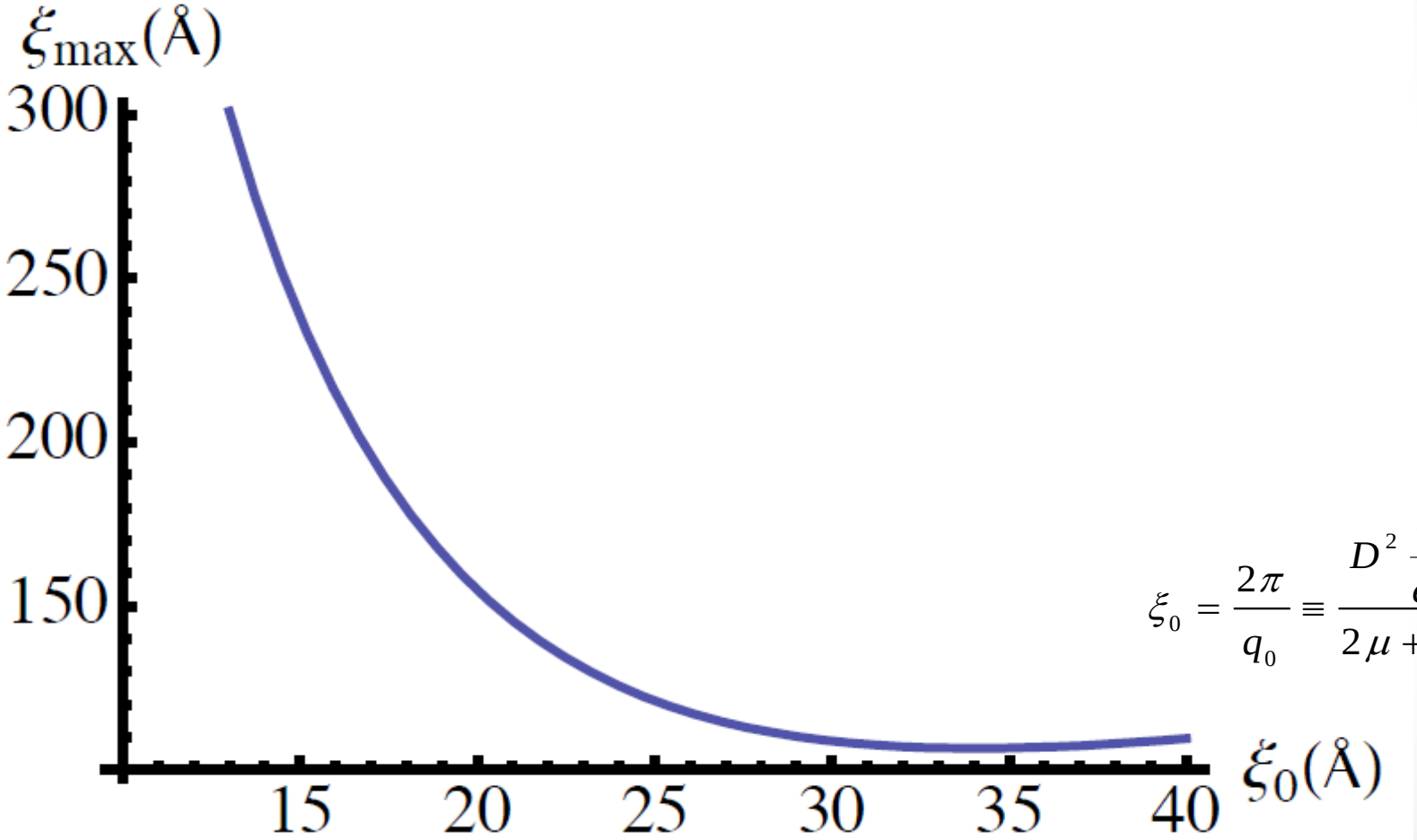
around $q \approx q_0$.

$$\Sigma(q) = \mathcal{P}\int \frac{d^2\vec{k}}{(2\pi)^2} \frac{K(qk)}{K_0} \frac{|\hat{q}\times\hat{k}|^4}{|\hat{q}-\vec{k}|^4}.$$

Normal-Normal Correlation Function



Characteristic Size of Ripples

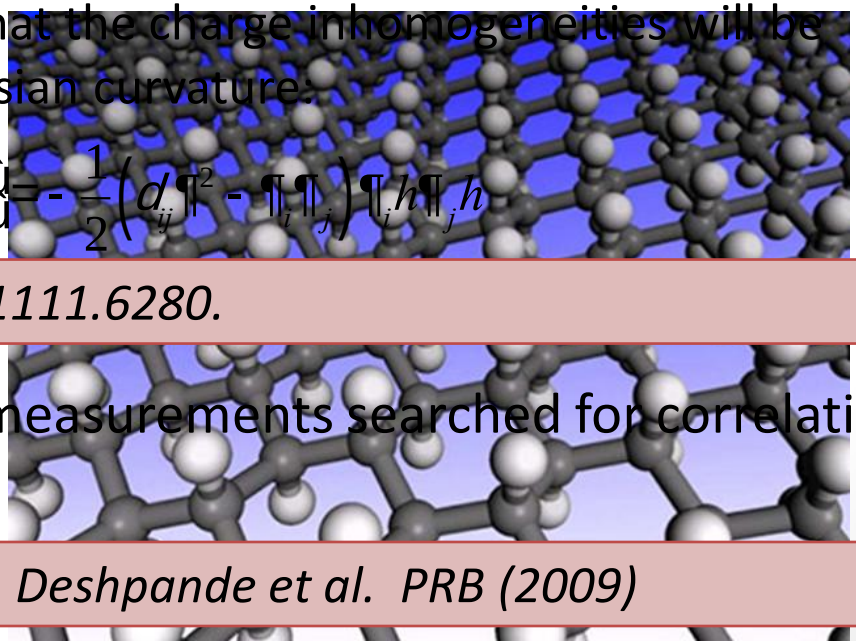


$$\xi_0 = \frac{2\pi}{q_0} \equiv \frac{D^2 \frac{E}{e^2}}{2\mu + \lambda},$$

Characteristics of Ripples

- Wavelength: 100-300Å.
  - Recovers measurements of Meyer et al. (2007).
- Amplitude: 5-20 Å.
- Correlation between charge inhomogeneities and ripples:
  - Circumstantial evidence: Graphane's structure is consistent with no ripples!
  - This model indicates that the charge inhomogeneities will be correlated to the Gaussian curvature:

$$d\eta \mu S\left[h(x)\right] = -\frac{1}{2}\left(d_{ij}\nabla^2 - \nabla_i\nabla_j\right)\nabla_i h \nabla_j h$$



Marco Gibertini et al., arXiv:1111.6280.

- previous theories and measurements searched for correlations in the mean curvature.

Castro-Neto, Kim, EPL (2008); Deshpande et al. PRB (2009)

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Doped Graphene

- Until now, we neglected the dynamics of the electrons.
- In other words, the Dirac action should be taken into account:

$$S = -\sum_{n=1}^{2} \int_0^b dt \int d^2\vec{x}\, \bar{\Psi}_n\left(t,\vec{x}\right)$$

$$\left\{ g^0 \partial_0 + i\left(Du_{ii} + dV\right) + v_f \vec{g}\times\left(\vec{\nabla} + ig_5\vec{A}\right)\right\} \Psi_n\left(t,\vec{x}\right)$$

Deformation Energy

Chemical potential

Pseudo-Magnetic gauge fields

$$\vec{A}\left(\vec{r}\right) = \frac{g_2}{v_f}\begin{pmatrix} 2u_{xy} \\ u_{xx} - u_{yy} \end{pmatrix} \qquad g_2 \sim 1\text{-}\ 3\ \text{eV}$$

π electrons effect on the surface structure

- The surface structure is determined by the elastic free energy:
  - A low *q* expansion of the free energy.
  - The contribution of the σ electrons – elasticity.
- At low *q*, the π electrons are well described by the Dirac picture.
- All we have to do is to evaluate the π electrons contribution to the free-energy.
- The acoustic velocity is much smaller than the Fermi velocity:

$$v_{ph} \approx 2 \cdot 10^{6} \frac{\text{cm}}{\text{sec}} << v_{f} \approx 10^{8} \frac{\text{cm}}{\text{sec}}$$

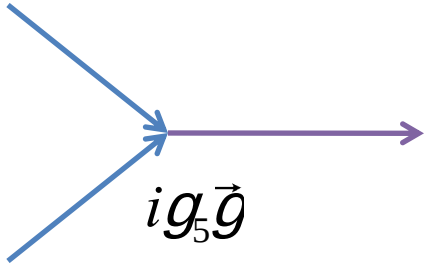
- Thus – one can integrate out the degrees of freedom of the electrons (Average the effect of the free electrons).

Feynman Diagrams for π-Electrons

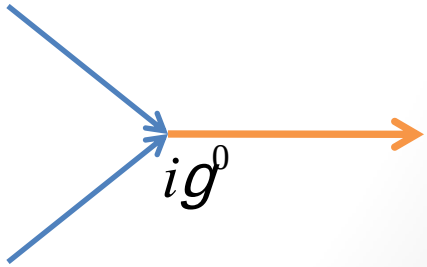
- Fermion propagator
- No Vector/Electro-chemical potential propagator.

$$\frac{i}{g_m k^m}$$

- Vector pot./Fermion vertex



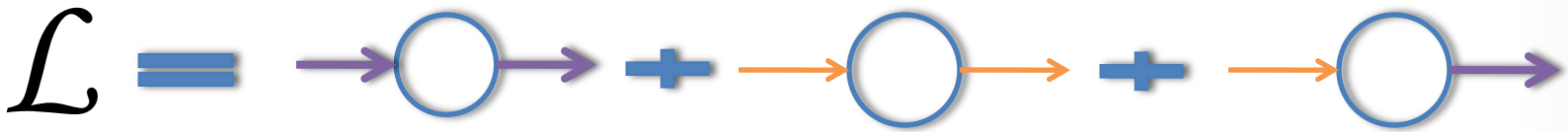
- Electro-chem. Pot/Fermion vertex



Integrating out the π-electrons

- The resulting (perturbative) Lagrangian is pure gauge.

$$\mathcal{L} = \frac{1}{2} A_i \mathrm{P}^{A}_{ij} A_j + \frac{1}{2} \mathrm{P}^{V} V^2$$



“Structure” Polarization operators

$$P^{V}\left(q_0=0;\vec{q}\right)=\frac{1}{b}\sum_{s=1}^{2}\sum_{w_f}\int\frac{d^2\vec{k}}{(2p)^2}\mathrm{Tr}\left[\frac{g^a k_a}{k^2}g^0\frac{g^b(k+q)_b}{(k+q)^2}g^0\right]$$

$$P_A^{ij}\left(q_0=0;\vec{q}\right)=\frac{1}{b}\sum_{s=1}^{2}\sum_{w_f}\int\frac{d^2\vec{k}}{(2p)^2}\mathrm{Tr}\left[\frac{g^a k_a}{k^2}g_5g^i\frac{g^b(k+q)_b}{(k+q)^2}g_5g^j\right]$$

$$w_f=\frac{2p}{b}\left(n+1/2\right)\qquad k^m=\left(w_f,\vec{k}\right)$$

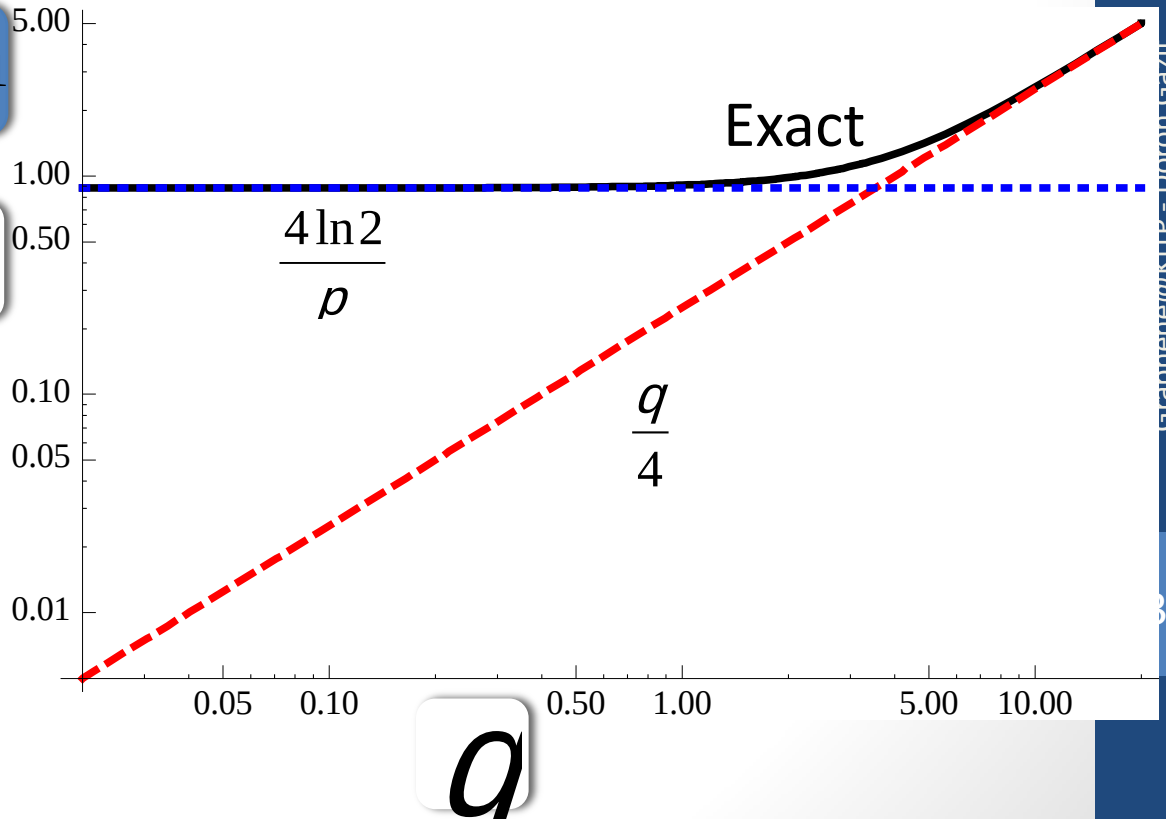
Electro-chemical "Structure" Polarization Operator

$$\mathcal{P}\left(\hbar v_f q, \alpha\right) = \frac{4}{\pi}\int_0^1 dx \ln\left[2\cosh\left(\frac{\alpha q}{2}\sqrt{x(1-x)}\right)\right] = \begin{cases} q \ll 1 & \dfrac{4\ln 2}{\pi} \\ q \gg 1 & \dfrac{q}{4} \end{cases}$$

$$\hbar v_f \alpha q \approx 15\left(\frac{T}{300^\circ K}\right)^{-1}\left(\frac{I}{100 A}\right)^{-1} \gg 1$$

$$P_{ij}^{A}(\vec{q}) = \frac{\hbar v_f q}{4}\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right)$$

$$P^{V}(\vec{q}) = \frac{q}{4\hbar v_f}$$



Elastic Free-Energy

$$F[\vec{u},h]=\frac{1}{2}\int\frac{d^2\vec{q}}{(2\pi)^2}\left[\kappa q^4|h|^2+\lambda(q)(u_{ii})^2+2\mu(q)(u_{ij})^2-\frac{\hbar v_f q}{4}\left|\vec{A}\times\hat{q}\right|^2\right]$$

$$\lambda(q)=\lambda+\frac{D^2 q}{4\hbar v_f}$$

$$\mu(q)=\mu+\frac{g_2^2 q}{8\hbar v_f}$$

- q dependence of elastic constants.
- Preferred directions.

P. San-Jose, J. González, and F. Guinea, Phys. Rev. Lett. 106, 045502 (2011).

Effect of an External Electro-chemical potential

$$F\left[\vec{u},h\right]=\frac{1}{2}\int\frac{d^2\vec{q}}{(2\pi)^2}\left[\kappa q^4|h|^2+\lambda\left(u_{ii}\right)^2+2\mu\left(u_{ij}\right)^2+2D\left(u_{ii}\right)_{-q}P^{V}\left(q;b\right)dV_q\right]$$

- For constant external potential:

$$dF\left[\vec{u},h\right]=t\int d^2\vec{x}\,u_{ii}\left(\vec{x}\right)$$

$$t=\frac{4D\ln 2\times k_BT}{\pi\left(\hbar v_f\right)^2}dV$$

Chemical potential induces additional stress!

DG, Phys. Rev. B **79**, 113411 (2009)

Spontaneous Buckling of hole doped graphene

- Take the harmonic approximation:

$$F[h] = \frac{1}{2}\int d^2\vec{x}\left[k\left(\nabla^2 h\right)^2 + t\left(\vec{\nabla}h\right)^2\right]$$

$$0 \overset{!}{=} \frac{\delta F}{\delta h} = \kappa q^4 h + \tau q^2 h \Rightarrow \begin{cases} \tau < 0 & q^* = \sqrt{\frac{|\tau|}{\kappa}} \\ \tau \geq 0 & q^* = 0 \end{cases}$$

negative stress (hole-doping) leads to buckling!

$$\xi = 2\pi\sqrt{\frac{\pi\left(\hbar v_f\right)^2 \beta\kappa}{4D\ln 2\cdot\delta V}} = 144\text{A}\cdot\left(\frac{T}{300°\text{K}}\right)^{-1/2}\cdot\left(\frac{D}{20\text{eV}}\right)^{-1/2}\cdot\left(\frac{\delta V}{100\text{meV}}\right)^{-1/2}$$

- When using the electric field effect: $\varepsilon_f \sim \hbar v_f \sqrt{\pi n}$

$$x \sim 270\text{A}\left(\frac{|V_g|}{10\text{V}}\right)^{1/4}$$

Rippling side-effect: Additional Resistivity

- The disorder phenomena: ripples and charge inhomogeneities are a source for deviation from ballistic conductance.

$$r \approx \frac{\hbar}{e^2}\left(\frac{T}{k}\cdot\frac{x}{a}\right)^2 \approx 200 - 600\Omega\left(\frac{T}{300^\circ K}\right)\cdot\left(\frac{|n|}{10^{12}}\right)^{-1/2}$$

- The different structural response of hole- and electron-doped graphene creates an inherent asymmetry!

Katsnelson and Geim, Phil. Trans. R. Soc. A 366, 195 (2008).

Bolotin et al. 2008

Nevertheless, this large asymmetry is unusual and presently unresolved and, together with the observed n dependence, may point to a scattering behavior in suspended graphene that is more complex than simple electron-phonon interaction.

Summary

- Graphene is an example of a new class of materials:
  electronic crystalline membranes,
  in which a strong interplay exists between the structure and the free electrons in the membrane.
- The π-electrons induce:
  - Ripples due to competition between electron-electron interaction and electron-phonon interaction.
  - Additional stress in the presence of a chemical potential.
    - Can lead to buckling in the case of hole doping.
- Outlook:
  - Phase diagram in the presence of impurities and other defects.
  - Effect on transport.