Higgs phases and zero-energy states in graphene

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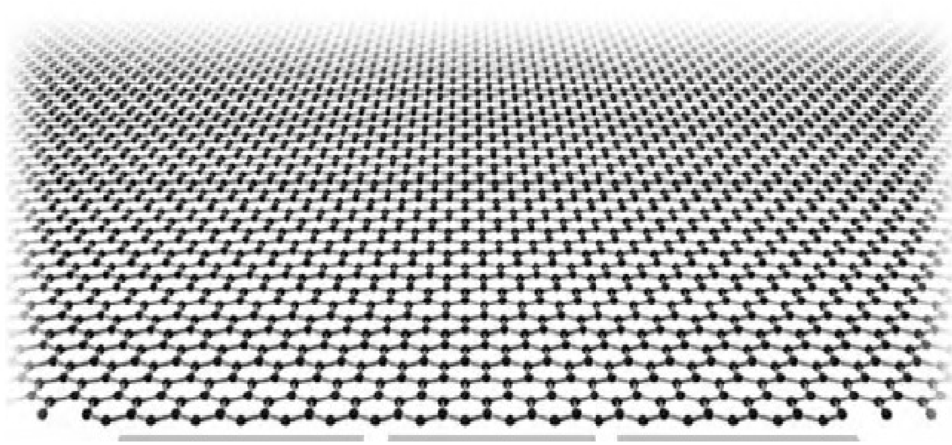
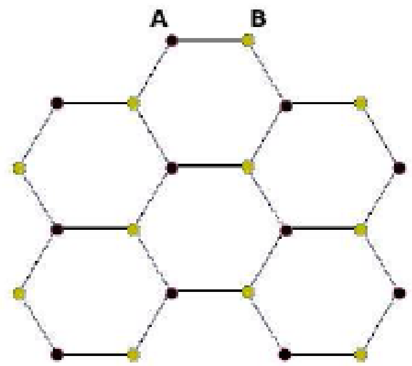


Graphene: 2D carbon (1s2, 2s2, 2p2)



Two triangular sublattices: A and B; one electron per site (half filling)

Tight-binding model (t = 2.5 eV):

$$H_0 = -t \sum_{\vec{A},i,\sigma=\pm 1} u^\dagger_\sigma(\vec{A}) v_\sigma(\vec{A}+\vec{b}_i) + H.c.$$

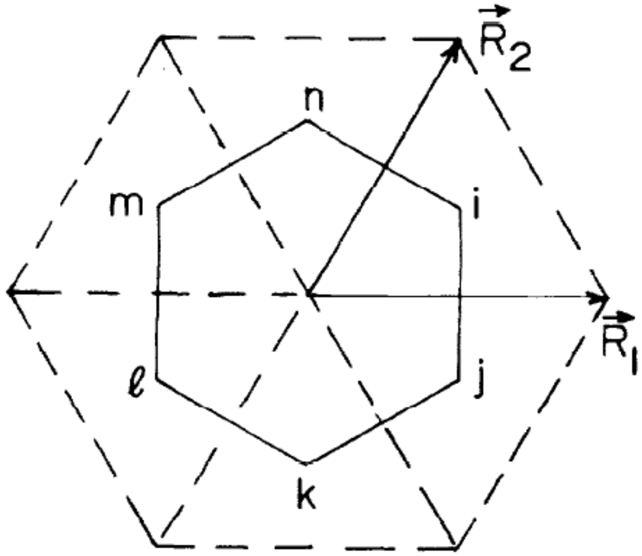
(Wallace, PR , 1947)

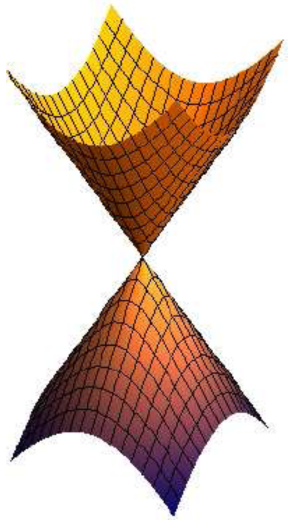
$$E(\vec{k}) = \pm t \left| \textstyle\sum_i \exp[\vec{k} \cdot \vec{b}_i] \right|$$

The sum is complex => two equations for two variables for zero energy

=> Dirac points (no Fermi surface)

Brillouin zone:





Two inequivalent (Dirac) points at :

+K and -K

Dirac fermion:

$$\Psi_\sigma^\dagger(\vec{x},\tau) = T\sum_{\omega_n}\int^\Lambda \frac{d\vec{q}}{(2\pi a)^2} e^{i\omega_n\tau + i\vec{q}\cdot\vec{x}}(u_\sigma^\dagger(\vec{K}+\vec{q},\omega_n), v_\sigma^\dagger(\vec{K}+\vec{q},\omega_n), u_\sigma^\dagger(-\vec{K}+\vec{q},\omega_n), v_\sigma^\dagger(-\vec{K}+\vec{q},\omega_n))$$

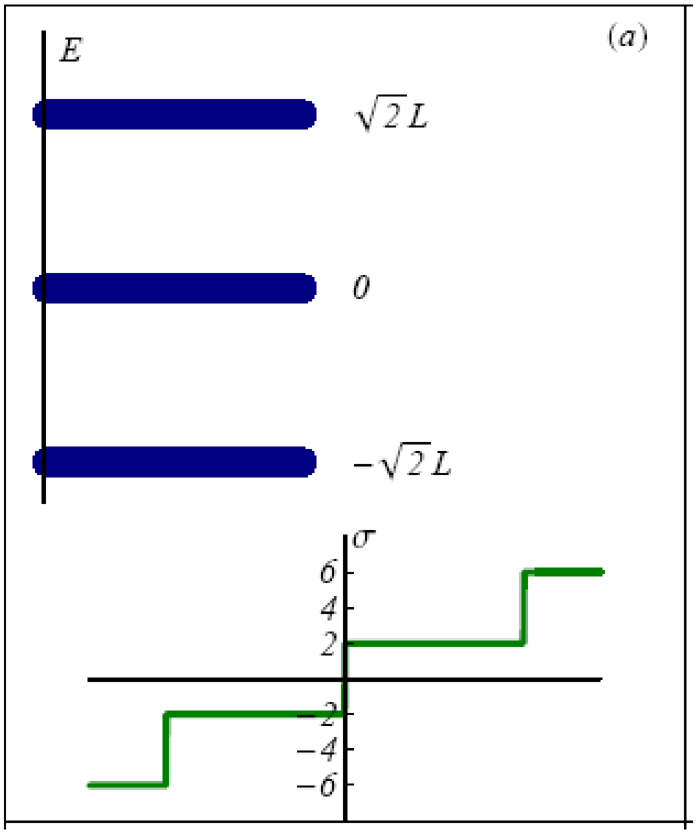
"Low - energy" Hamiltonian: $H_0 = i\gamma_0\gamma_i(-i\partial_i - A_i)$ **i=1,2**

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \qquad \nu,\, \mu = 0, 1, 2$$

(v = c/300 = 1, in our units)

Experiment: how do we detect Dirac fermions?

Quantum Hall effect (for example): $L = \sqrt{\hbar v_F^2 |eB|/c}$



Landau levels: each is

2 (spin) x 2 (Dirac) x eB (Area)/hc

degenerate => quantization in steps of

four !

(Gusynin and Sharapov, PRL, 2005)

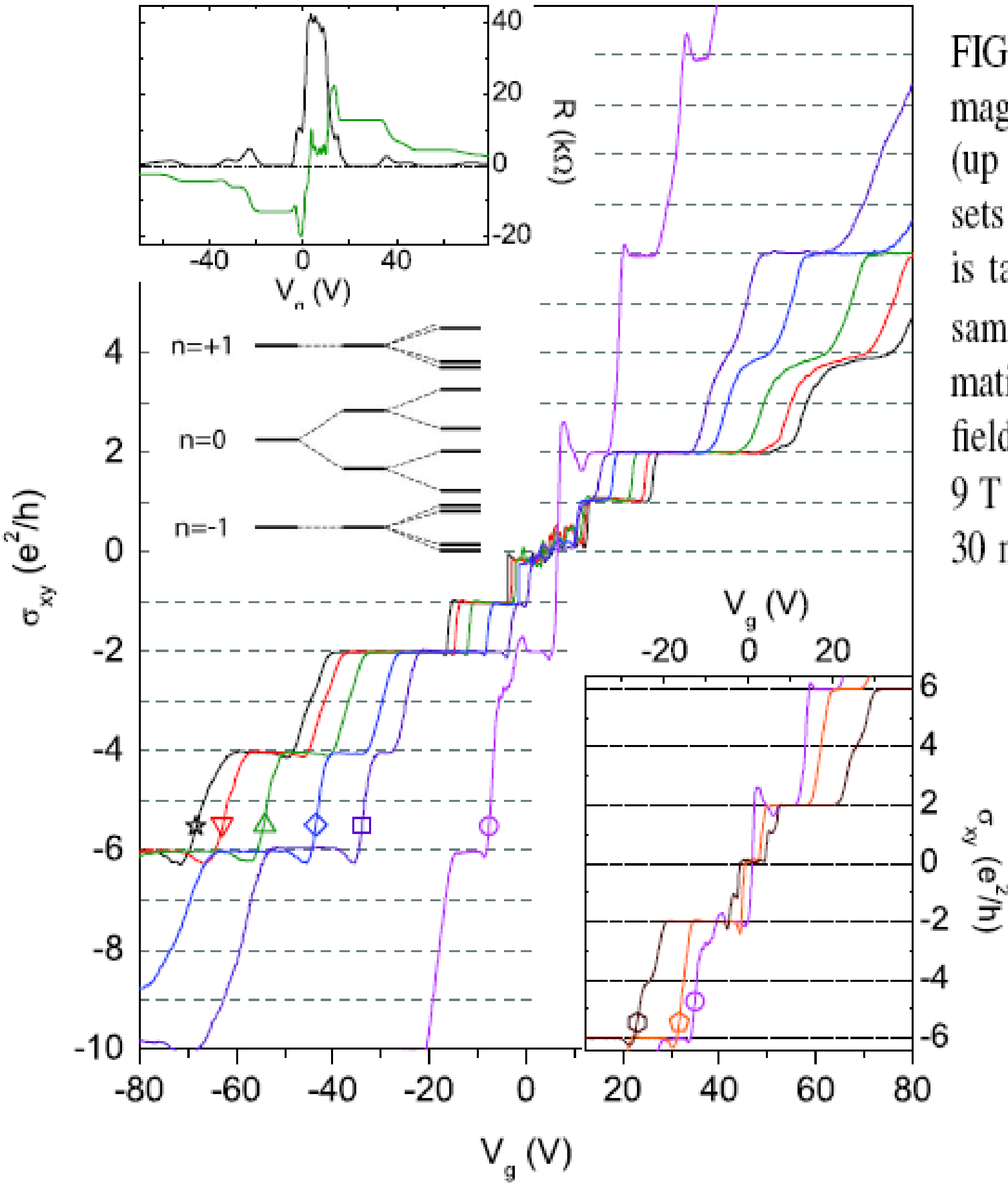


FIG. 2 (color online). σ_{xy}, as a function of V_g at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$ T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xx} and R_{xy} for the same device measured at $B = 25$ T. Left lower inset: a schematic drawing of the LLs in low (left) and high (right) magnetic field. Right inset: detailed σ_{xy} data near the Dirac point for $B = 9$ T (circle), 11.5 T (pentagon), and 17.5 T (hexagon) at $T = 30$ mK.

(Y. Zhang, PRL, 2006)

Symmetries: exact and emergent

1) **Lorentz**

(microscopically, only Z2 (A <-> B) x Z2 (K <-> -K) = D2, dihedral group)

2) **Chiral** : $SU(2)$ = $\{\gamma_{35}, \gamma_3, \gamma_5\}$, $\gamma_{35} = i\gamma_3\gamma_5$

Generators commute with the Dirac Hamiltonian (in 2D). Only two are emergent!

3) **Time-reversal** (exact) : $I_t = U_t K, \quad U_t = i\gamma_1\gamma_5$

(+ K <-> - K and complex conjugation)

(IH, Juricic, Roy, PRB, 2009)

4) Particle-hole (supersymmetry): anticommute with Dirac Hamiltonian

$$\vec{M} = (\gamma_0, i\gamma_0\gamma_3, i\gamma_0\gamma_5) \,, \qquad \tilde{M} = i\gamma_1\gamma_2$$

=> $$\psi_{-E} = M\,\psi_E$$

and so map zero-energy states, when they exist, into each other!

The zero-energy subspace is invariant under both symmetry (commuting) and supersymmetry (anticommuting) operators.

"Masses" = supersymmetries

1) Broken chiral symmetry, preserved time reversal

$$H_0 \rightarrow H_0 + \vec{m} \cdot \vec{M}$$

2) Broken time reversal symmetry, preserved chiral

$$H_0 \rightarrow H_0 + \tilde{m}\tilde{M}$$

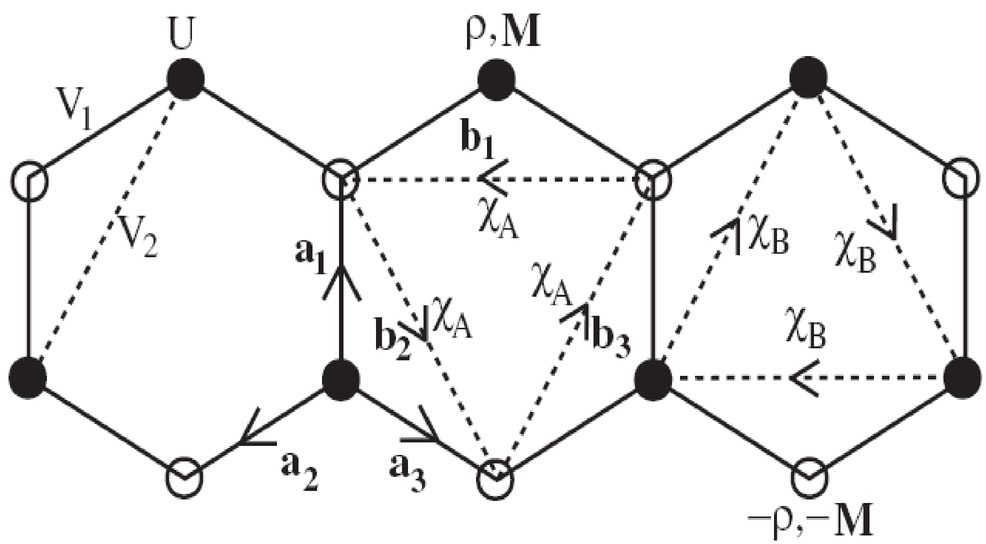
In either case the spectrum becomes gapped:

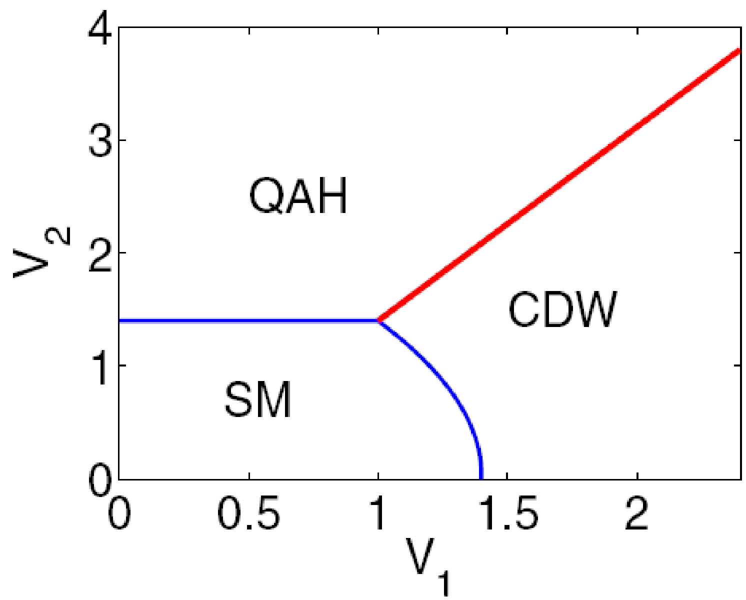
$$\varepsilon_{\pm}(\boldsymbol{p}) = \pm\sqrt{|\boldsymbol{p}|^2 + |\Delta_0|^2}, \quad \Delta_0 = m, \ \tilde{m}$$

On lattice?

1) m γ_0 staggered density, or Neel (with spin); preserves translations (Semenoff, PRL, 1984)

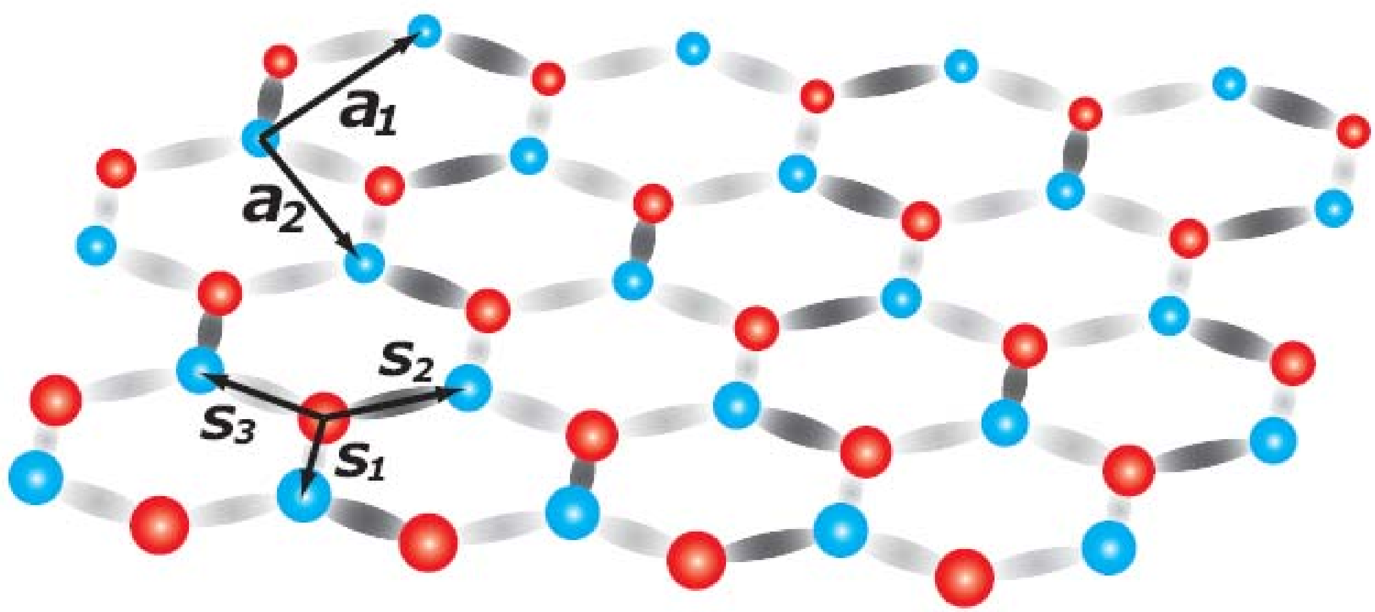
2) $\tilde{m}\tilde{M}$ circulating currents (Haldane, PRL, 1988)





(Raghu et al, PRL, 2008, generic phase diagram IH, PRL, 2006)

3) $m_1 \; i\gamma_0\gamma_3 \; + \; m_2 \; i\gamma_0\gamma_5$ Kekule hopping pattern

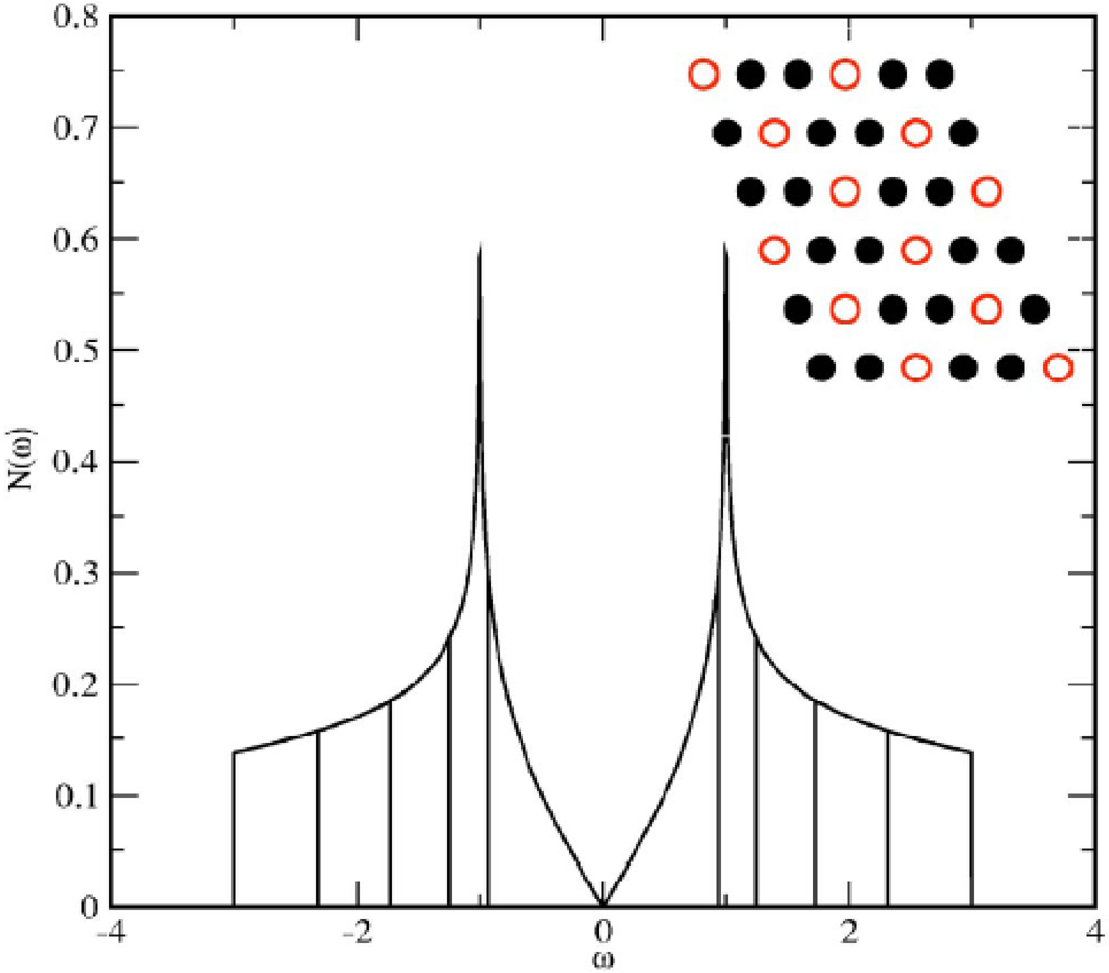


(Hou, Chamon, Mudry, PRL, 2007)

"Catalysis" of order: magnetic and otherwise

Strong interactions are needed for the gap because there are very few states near the Fermi level:

$$U_c/t \approx 5.5$$



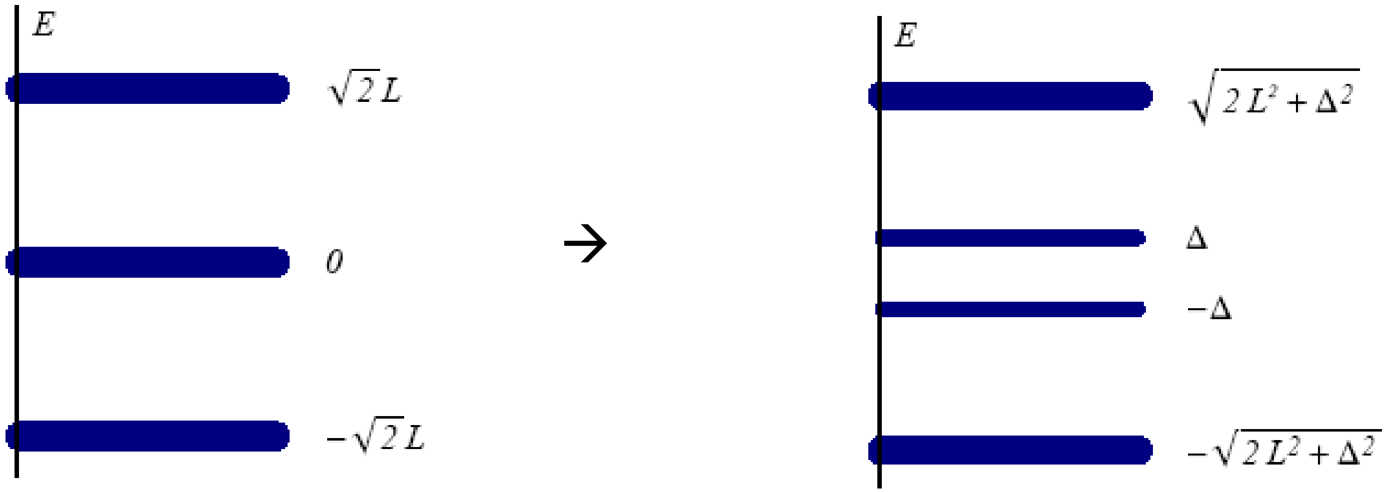
Density of states is linear near Dirac point:

$$N(\omega) \propto \omega^{(2-z)/z}$$

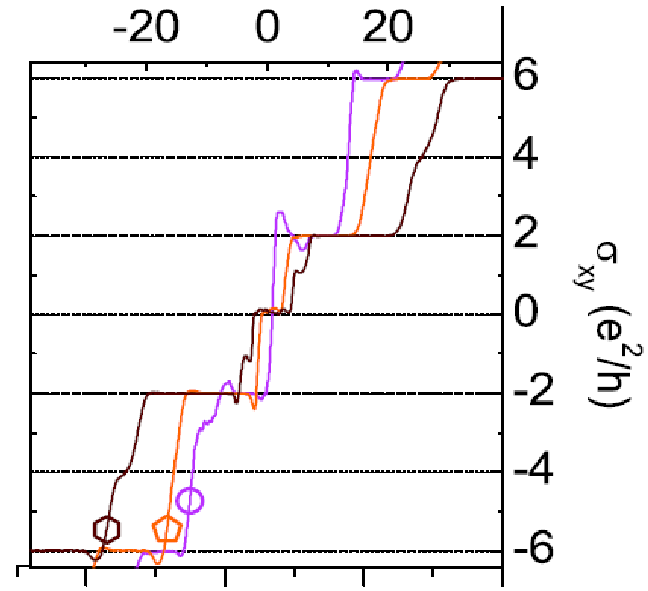
and $z = 1$

In the magnetic field: Landau quantization => DOS infinite

=> critical interaction infinitesimal
(Gusynin, Miransky, Shovkovy, PRL, 1994)



Zero-energy level is split, others only shifted
=> quantum Hall effect at new filling factor at
zero!



IH, PRB 2006, PRB 2007, IH and Roy, PRB 2008

But which order is catalyzed?

The Hamiltonian is : $$H[A^0] = i\gamma_0\gamma_i(p_i - A_i^0)$$

so that, $$H^2[A^0] = (p_i - A_i^0)^2 + \tilde{M}\epsilon_{ij}\partial_i A_j^0$$

where, $\tilde{M} = i\gamma_1\gamma_2$ **is TRS breaking mass.**

so all zero energy states have the same eigenvalue (+1 or -1) of $\tilde{M}$ **for a uniform magnetic field.**

For a non-uniform magnetic field:

$$H[A^0,0] = e^{-\chi(\vec{x})\tilde{M}} H[0,0] e^{-\chi(\vec{x})\tilde{M}},$$

where $A_i^0 = \epsilon_{ij}\partial_j\chi.$ (In Coulomb gauge, $\partial_i A_i^0 = 0$.)

Zero-energy states with and without magnetic field are simply related:

$$\Phi_{0,n}[A^0](\vec{x}) \propto e^{\chi(\vec{x})\tilde{M}} \Phi_{0,n}[0](\vec{x})$$

since at large distance, for a localized flux F

$$\chi(\vec{x}) = F \ln|\vec{x}|$$

normalizability requires them to be -1 eigenstates of $\tilde{M}$!!

Since, however,

$$Tr_0 \vec{M} = 0$$

half of zero-energy states have +1, and half -1 eigenvalue of $\vec{M}$

For any anticommuting traceless operator, such as $\vec{M}$

$$\langle \vec{M} \rangle = \frac{1}{2}\left[\sum_{n,\mathrm{occup}} - \sum_{n,\mathrm{empty}} \right] \Phi^{\dagger}_{0,n}(\vec{x}) \vec{M} \Phi_{0,n}(\vec{x}),$$

(IH, PRL, 2007)

So:

TRS broken explicitly => CS broken spontaneously

Is the opposite also true?

Consider the non-Abelian potential: $H = i\gamma_0\gamma_i(p_i - A_i)$

$$A_i = A_i^3\gamma_3 + A_i^5\gamma_5 + A_i^{35}\gamma_{35}$$

which manifestly breaks CS, but preserves TRS. Since,

$$\gamma_{35} = \sigma_z \otimes I_2$$

A_i^{35} **for example, represents a variation in the position of the Dirac point, induced by height variations or strain.**

$\tilde{M}$ **still anticommutes with** H **but also with** I_t **so** $Tr_0\tilde{M} = 0$

CS broken explicitly => TRS broken spontaneously

(IH, PRB, 2008)

Pseudo-magnetic catalysis:

Flux of non-abelian pseudo-magnetic field

⇓

Subspace of zero-energy states (Atiyah-Singer, Aharonov-Casher)

⇓

Equally split by TRS breaking mass

⇓

With next-nearest neighbor repulsion, TRS spontaneously broken

In a non-uniform pseudo-magnetic field (bulge):

$$\langle \Psi^\dagger(\vec{x})\tilde{M}\Psi(\vec{x})\rangle \quad \propto \quad B^{35}(r)$$

local TRS breaking!

In sum:

1) Dirac Hamiltonian in 2D has plenty of (emergent) symmetry

2) Chiral symmetry + Time reversal
=> variety of Mott ("Higgs") insulators

3) Interactions need to be strong for Higgs, but

4) Ubiquitous zero-energy states => catalyze Higgs
(QHE at filling factors zero and one in uniform magnetic field)

5) Non-abelian flux catalyzes time-reversal symmetry breaking! (IH, PHYSICAL REVIEW B 78, 205433 (2008)