Talk for Graphene Week of the KITP Program on
Low Dimensional Electron System

Effects of Interaction and Disorder for Graphene Quantum Hall States

Donna Sheng (CSUN and KITP)

In collaboration with:

Hao Wang (CSUN)
L. Sheng (Nanjing Univ. China)
Z. Y. Weng (Tsinghua Univ. China)
F. D. M. Haldane (Princeton) and L. Balents (UCSB)

NSF and DOE support

Outline:

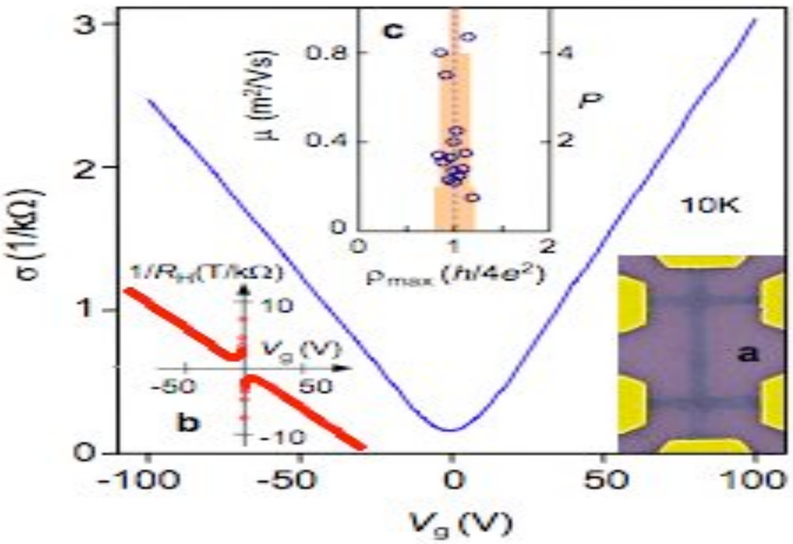
IQHE in graphene with disorder:
quantum phase transitions and phase diagram
edge states, and topological insulator
(QHE without B)

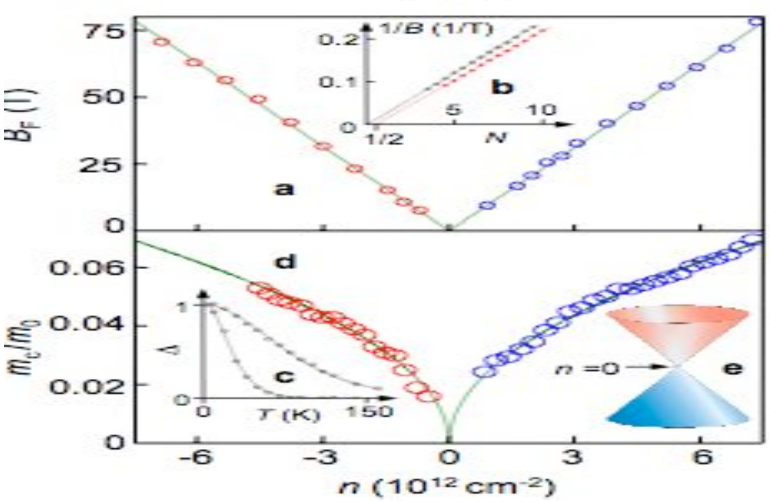
Effect of Interaction: Pseudospin ferromagnet and
odd integer QHE state for interacting
electrons in graphene

Disorder effect and comparison of
mobility gap at $\nu=1$ and $\nu=3$ odd IQHE
Bilayer Graphene (without tunneling)

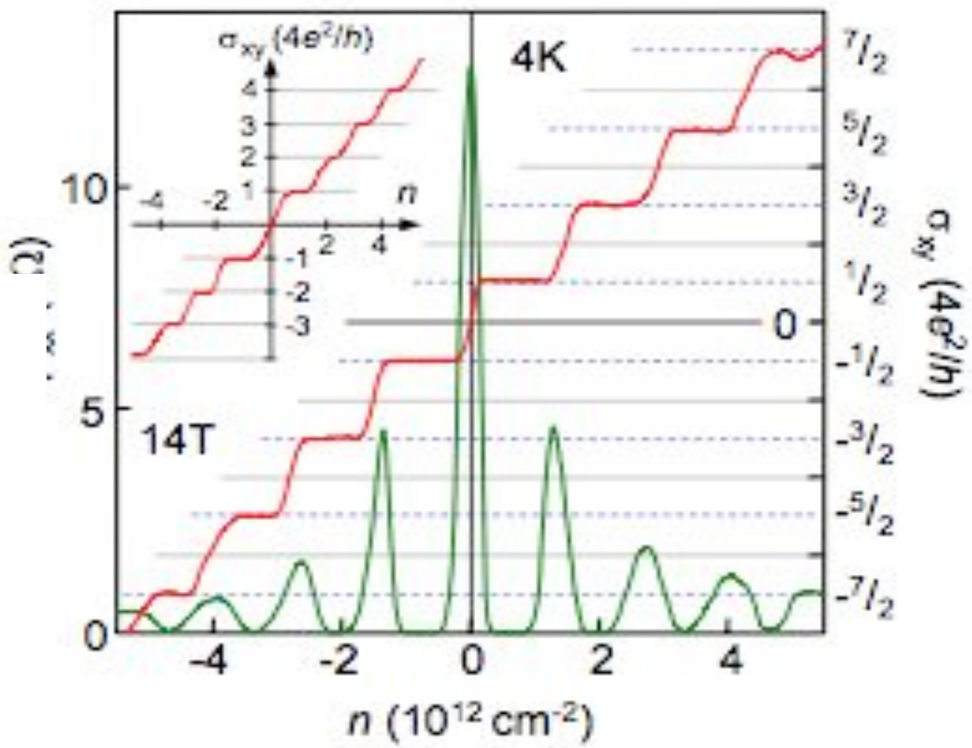
Prediction of symmetry broken states in higher
(n > or = 2) Dirac LLs

Experimental realization of graphene monolayer and discovery of "Half Integer" QHE





Novoselov et al (2005)



"Half-Integer" Quantized Quantum Hall Effect

Zhang et al (2005)

Theory:

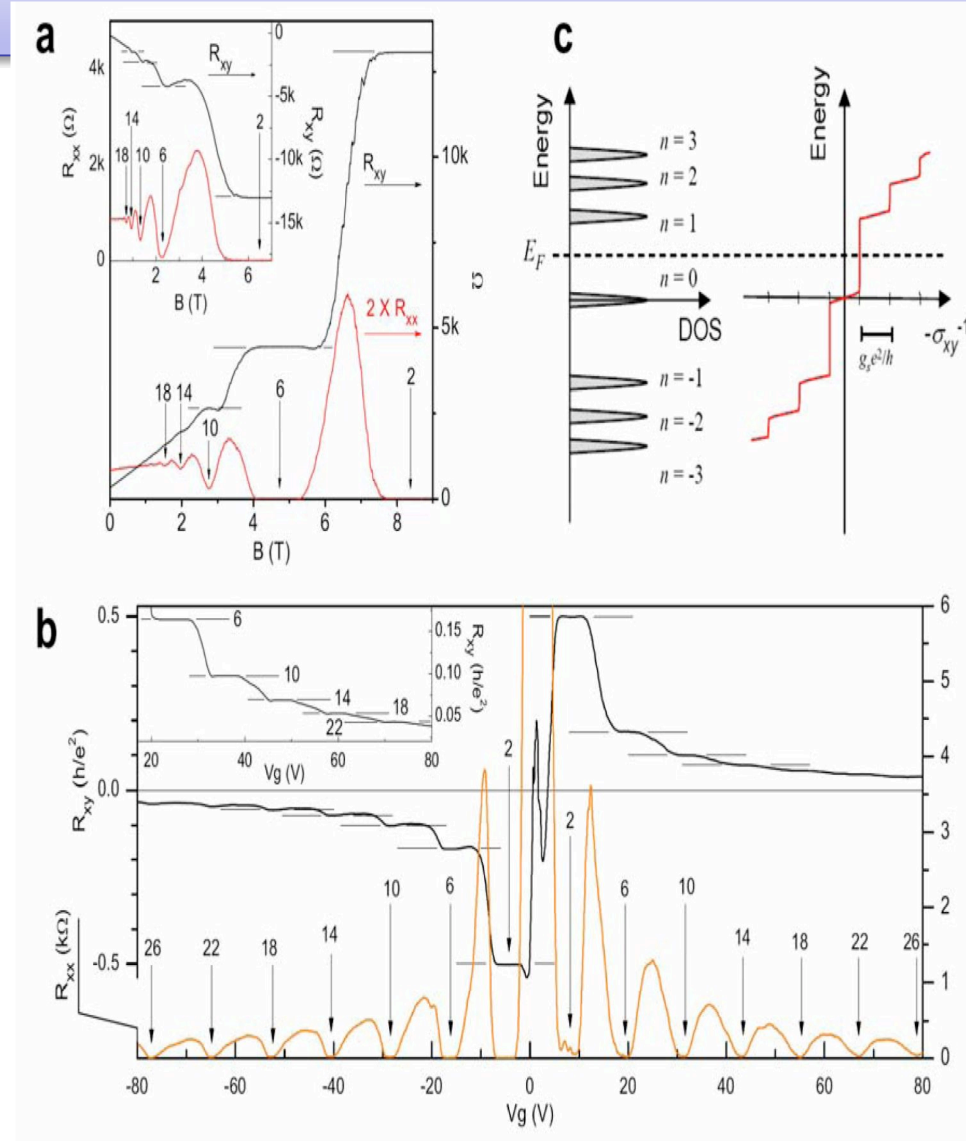
The "half-integer" quantized IQHE: (-3/2, 1/2, 1/2, 3/2)4e²/h
4= valley * spin degeneracy

Dirac nature, Berry phase shift

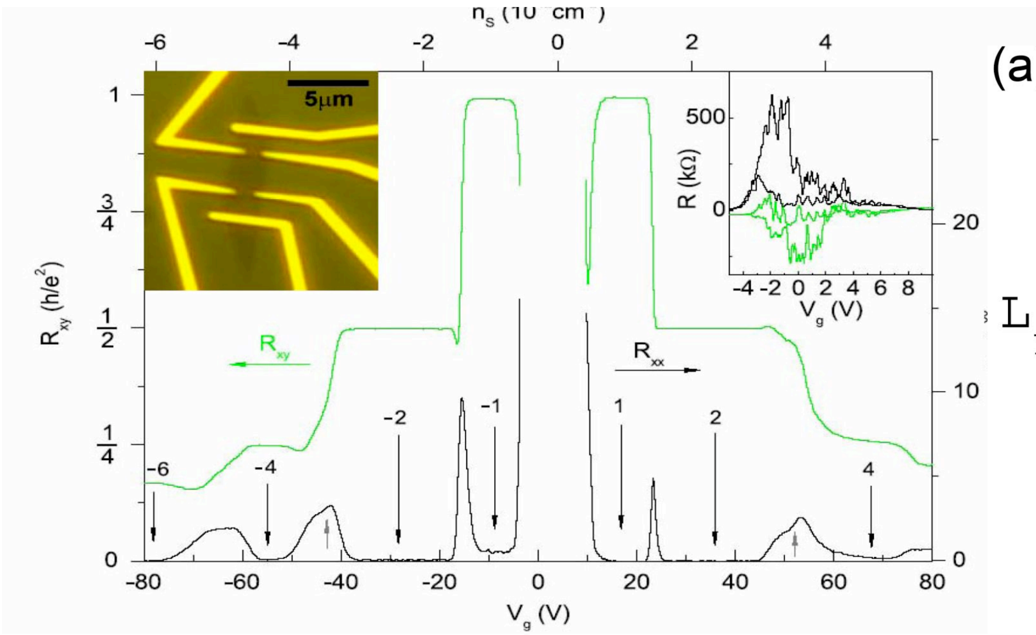
Continuous model for Dirac fermions
Gusynin et al. Peres et al.
McCann & Falko

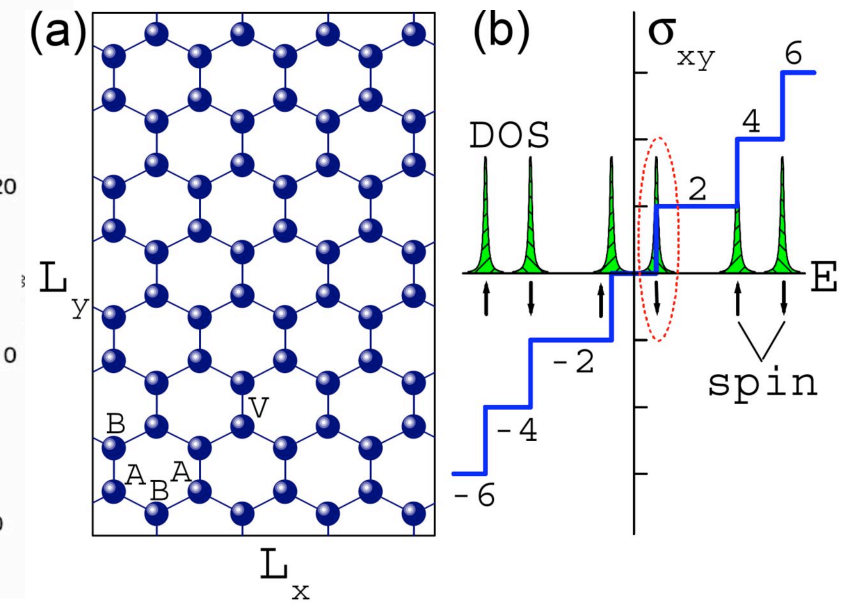
Zheng and Ando

What is the full picture from band model of electrons in honeycomb lattice ?



Experiment discovers ν=1 IQHE and "ν=0" insulating phase



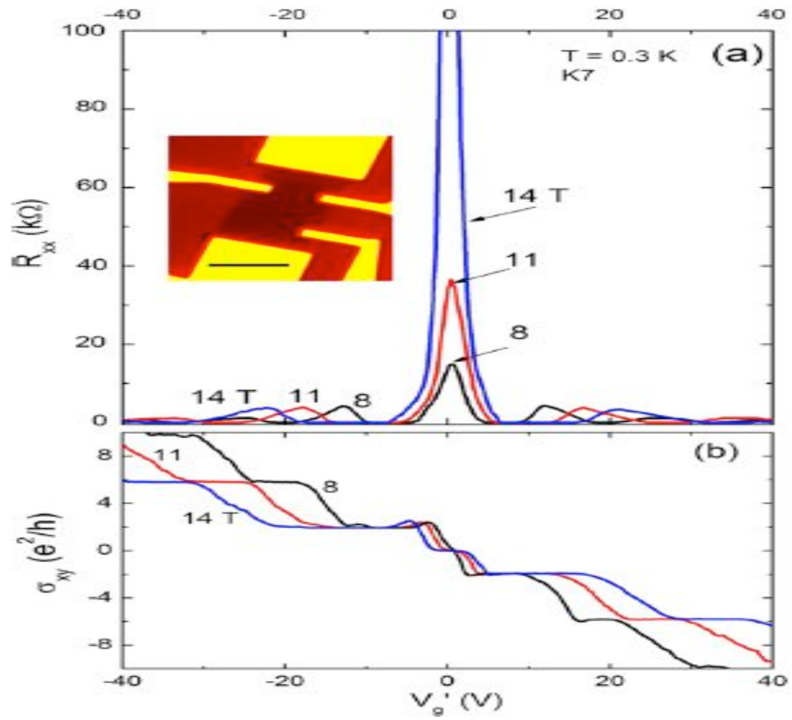


Y. Zhang et. al., PRL 2006

FIG 1. (color online) R_{xx} and R_{xy} measured in the device shown in the left inset, as a function of V_g at $B = 45$ T and $T = 1.4$ K. $-R_{xy}$ is plotted for $V_g > 0$. The numbers with the vertical arrows indicate the corresponding filling factor ν. Gray arrows indicate developing QH states at $\nu = \pm 3$. n_s is the sheet carrier density derived from the geometrical consideration. Right inset: R_{xx} (dark solid lines) and R_{xy} (light solid lines) for another device measured at $B = 30$ T and $T = 1.4$ K in the region close to the Dirac point. Two sets of R_{xx} and R_{xy} are taken at different time under the same condition. Left inset: an optical microscope image of a graphene device used in this experiment.

Interaction has to be taken into account to explain the ν=1 IQHE---Pseudospin Ferromagnet?

Insulating at Dirac point (n=0)?



Ong's group 2007, 2008
Abanin et al. (2007-2008)
Shimshoni, Fertig, Pai
Li Sheng, DNS (2009)
Edge states, spin-orbit, interaction

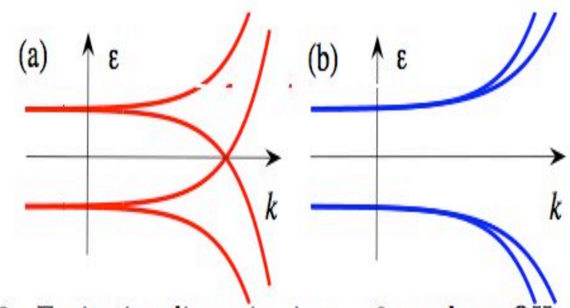
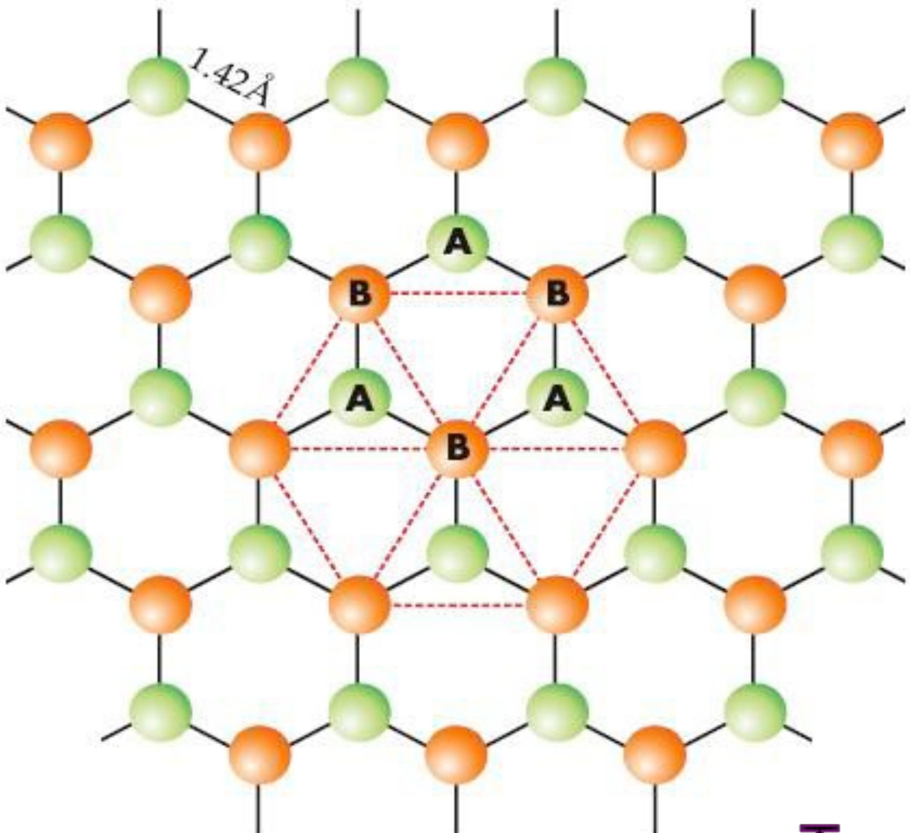
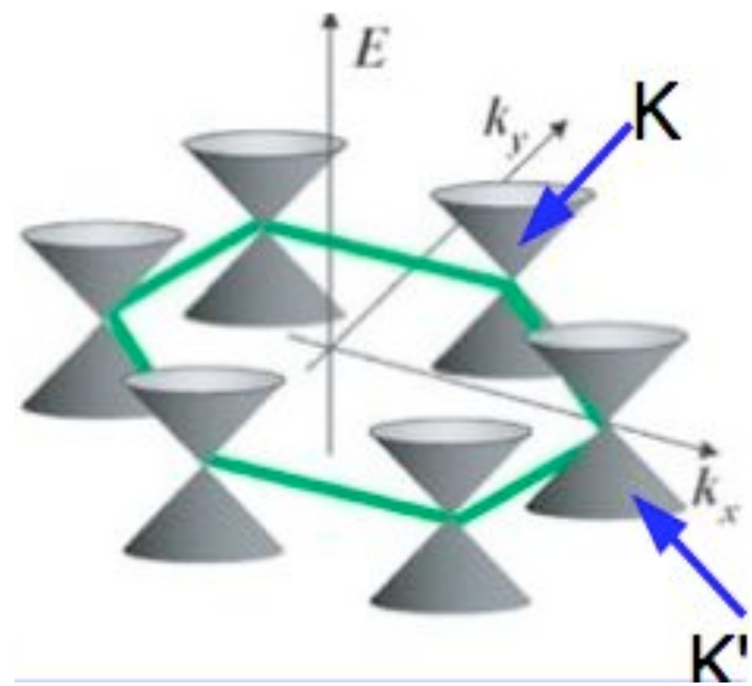


FIG. 2: Excitation dispersion in $\nu = 0$ graphene QH state for a system with and without gapless chiral edge modes, (a) and (b) respectively. Case (a) is realized in spin-polarized $\nu = 0$ state [4], while case (b) occurs when symmetry is incompatible with gapless modes, for example, in valley-polarized $\nu = 0$ state conjectured in Ref. [15]. In the latter a gap opens at branch crossing due to valley mixing at the sample boundary.

SO and interaction can open gap
(topological insulator without Z_2 symmetry)

Electrons on Honeycomb lattice





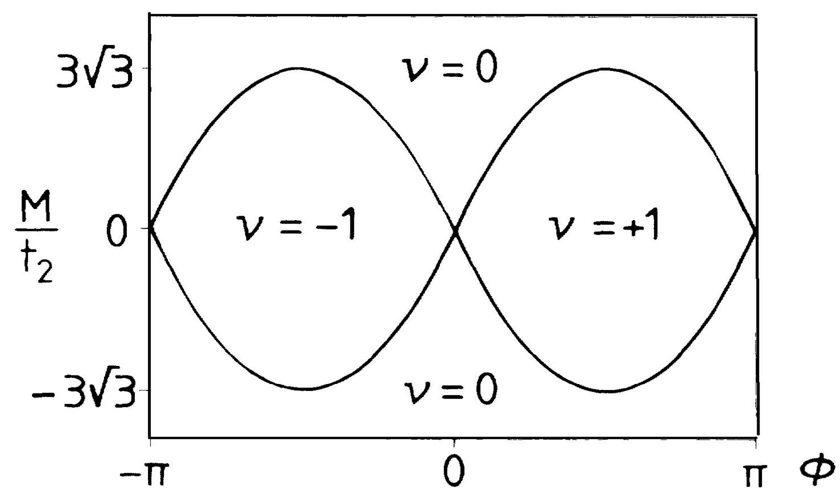
Two sublattices A and B, zero energy gap semimetal

Linear spectrum near two Dirac points: Valleys K and K'

Haldane's model for IQHE without magnetic field (Phys. Rev. Lett. 1988)

$$H(\mathbf{k}) = 2t_2\cos\phi\left(\sum_i \cos(\mathbf{k}\cdot\mathbf{b}_i)\right)\mathbf{I} + t_1\left(\sum_i [\cos(\mathbf{k}\cdot\mathbf{a}_i)\sigma^1 + \sin(\mathbf{k}\cdot\mathbf{a}_i)\sigma^2]\right) + \left[M - 2t_2\sin\phi\left(\sum_i \sin(\mathbf{k}\cdot\mathbf{b}_i)\right)\right]\sigma^3$$

t_2 is complex



IQHE does not require LLs,
It can occur if the time-reversal symmetry is broken

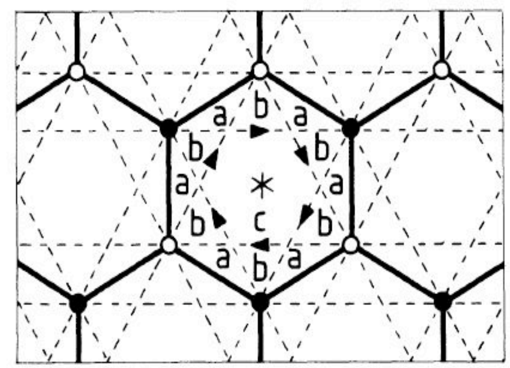
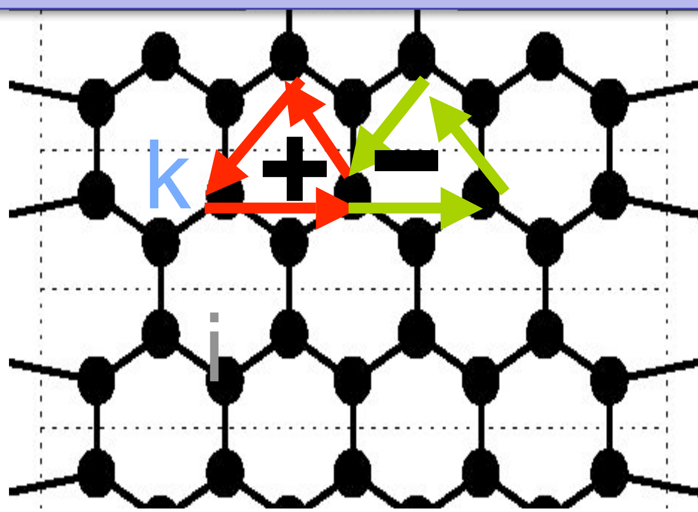


FIG. 1. The honeycomb-net model ("2D graphite") showing nearest-neighbor bonds (solid lines) and second-neighbor bonds (dashed lines). Open and solid points, respectively, mark the *A* and *B* sublattice sites. The Wigner-Seitz unit cell is conveniently centered on the point of sixfold rotation symmetry (marked "*") and is then bounded by the hexagon of nearest-neighbor bonds. Arrows on second-neighbor bonds mark the directions of positive phase hopping in the state with broken time-reversal invariance.

Topological order in SOC band insulator and SHE---honeycomb lattice model



other models:
B. A. Bernevig and S. C. Zhang,
X. L. Qi, Y. S. Wu and S. C. Zhang
J. Moore, Sinitsyn et al, P. A. Lee

spin species, + & - IQHE

role of Rashba V_R

$$H = -t\sum_{\langle ij\rangle} c_i^\dagger c_j + \frac{2i}{\sqrt{3}} V_{\rm SO} \sum_{\langle\langle ij\rangle\rangle} c_i^\dagger \hat{\boldsymbol{\sigma}} \cdot (\mathbf{d}_{kj} \times \mathbf{d}_{ik}) c_j$$

$$+ \; iV_{\rm R} \sum_{\langle ij\rangle} c_i^\dagger \hat{\mathbf{z}} \cdot (\hat{\boldsymbol{\sigma}} \times \mathbf{d}_{ij}) c_j + \sum_i \epsilon_i c_i^\dagger c_i \;,$$

F. D. M. Haldane 1988, Kane and Mele 2004, 2005

L. Sheng, D. N. Sheng, C. S. Ting & F.D.M.Haldane 2005, 2006

Chern number

Sheng et al, 2006 (PRL)

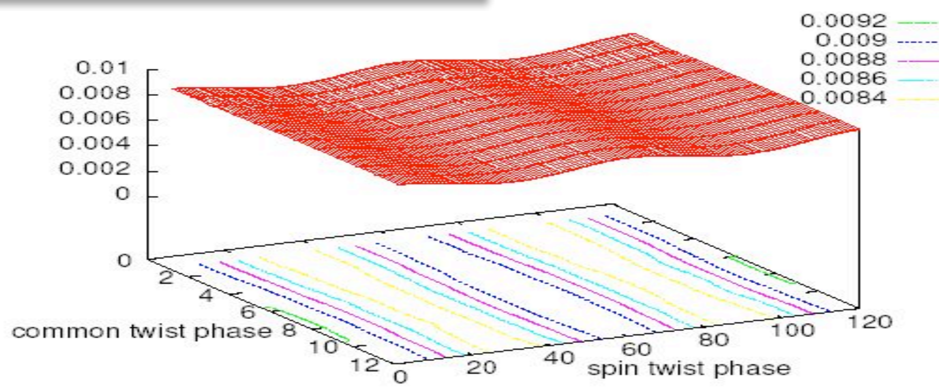
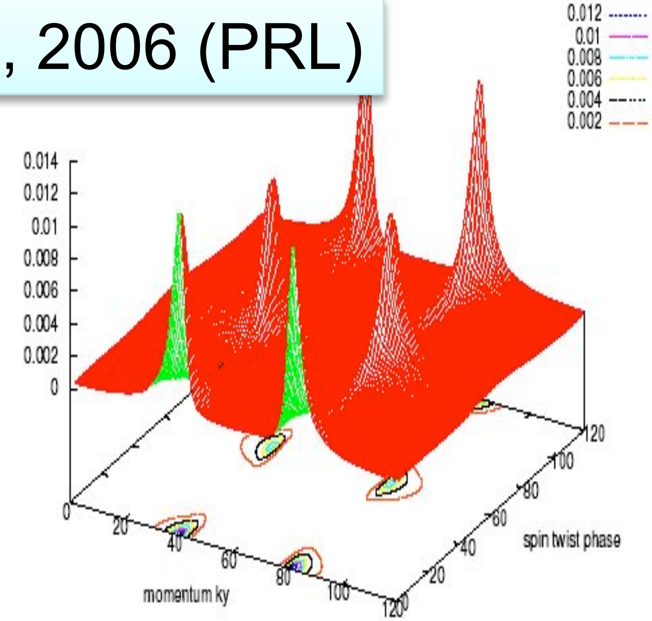


FIG. 1: Solid angle Ω_j as a function of two boundary phases (θ_x^s, θ_y^c), each θ unit cell is meshed into $N_{mesh} = 120 \times 12$ points for a pure system with $N_x = N_y = 60 \times 60$ at $V_{so} = 0.1$ and $V_R = 0.1$. Thus $\sum_{j=1}^{N_{mesh}} \Omega_j = 4\pi$, and $C_{sc} = 2$.



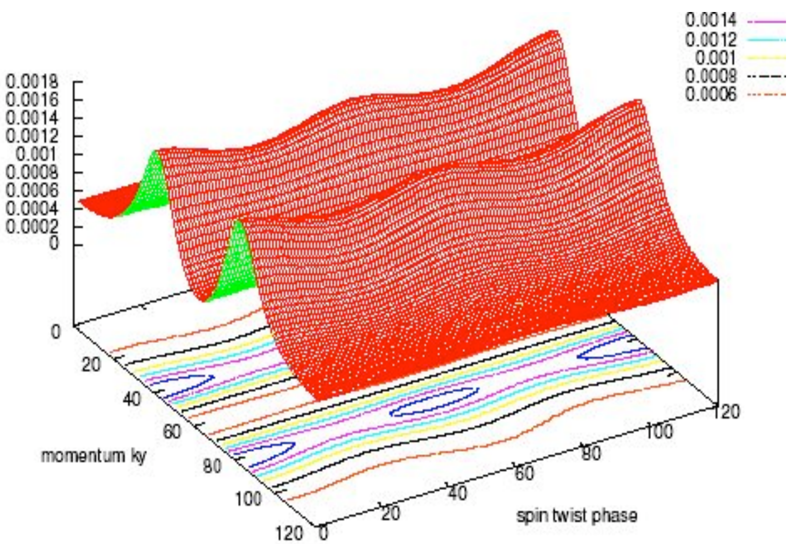
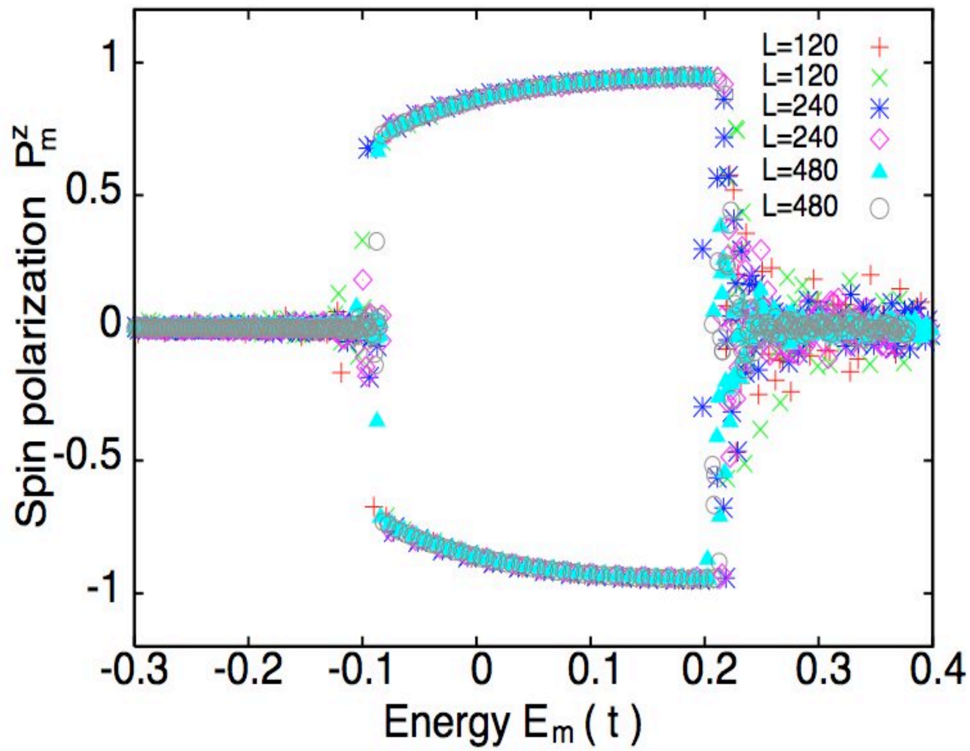
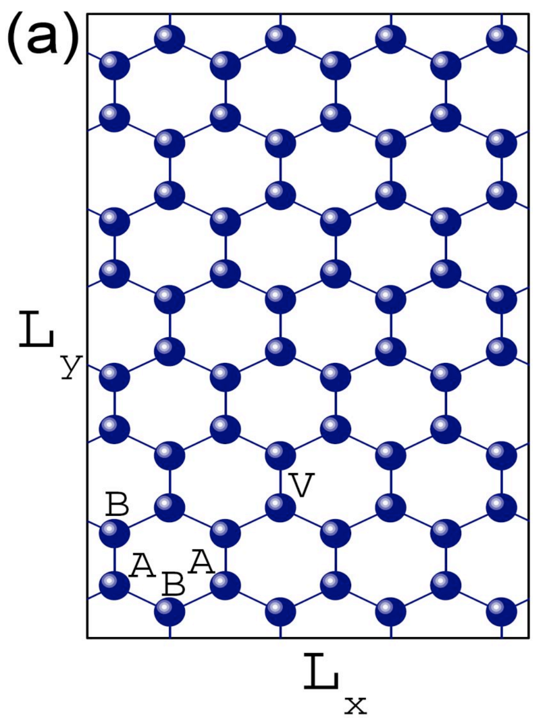
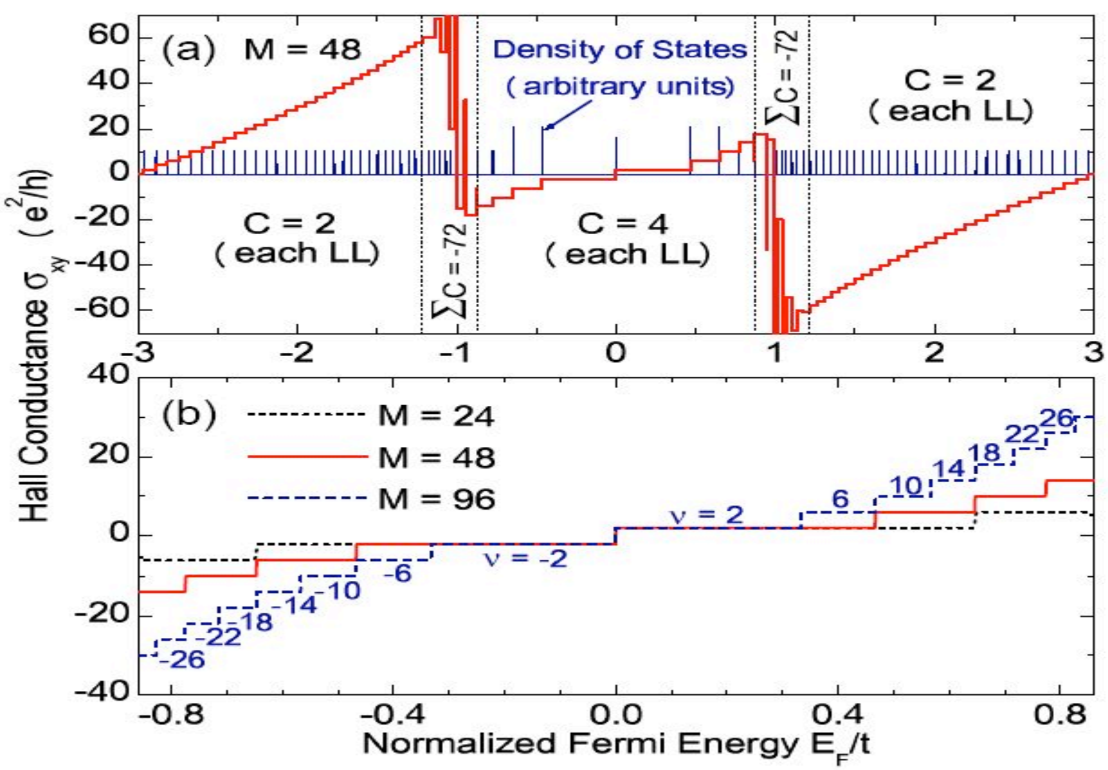


FIG. 2: Solid angle Ω_j as a function of spin twist and momentum k_y. Each $2\pi \times 2\pi$ unit cell is meshed into $N_{mesh} = 120 \times 120$ points for a pure system with $N_x = 60$ at $V_{so} = 0.1$ and $V_R = 0.1$. Thus $\sum_{j=1}^{N_{mesh}} \Omega_j = 4\pi$, and $C_{sc} = 2$.



Tight-binding model study of LLs and IQHE
Three regions of IQHE in the energy band





$$H = -t\sum C_{iB}^{+} C_{jA} e^{iA_{ij}} + h.c + \sum w_i C_i^{+} C_i$$

$\Phi = 2\pi/M$

Effect of disorder and phase diagram: PRB 73 (2006)

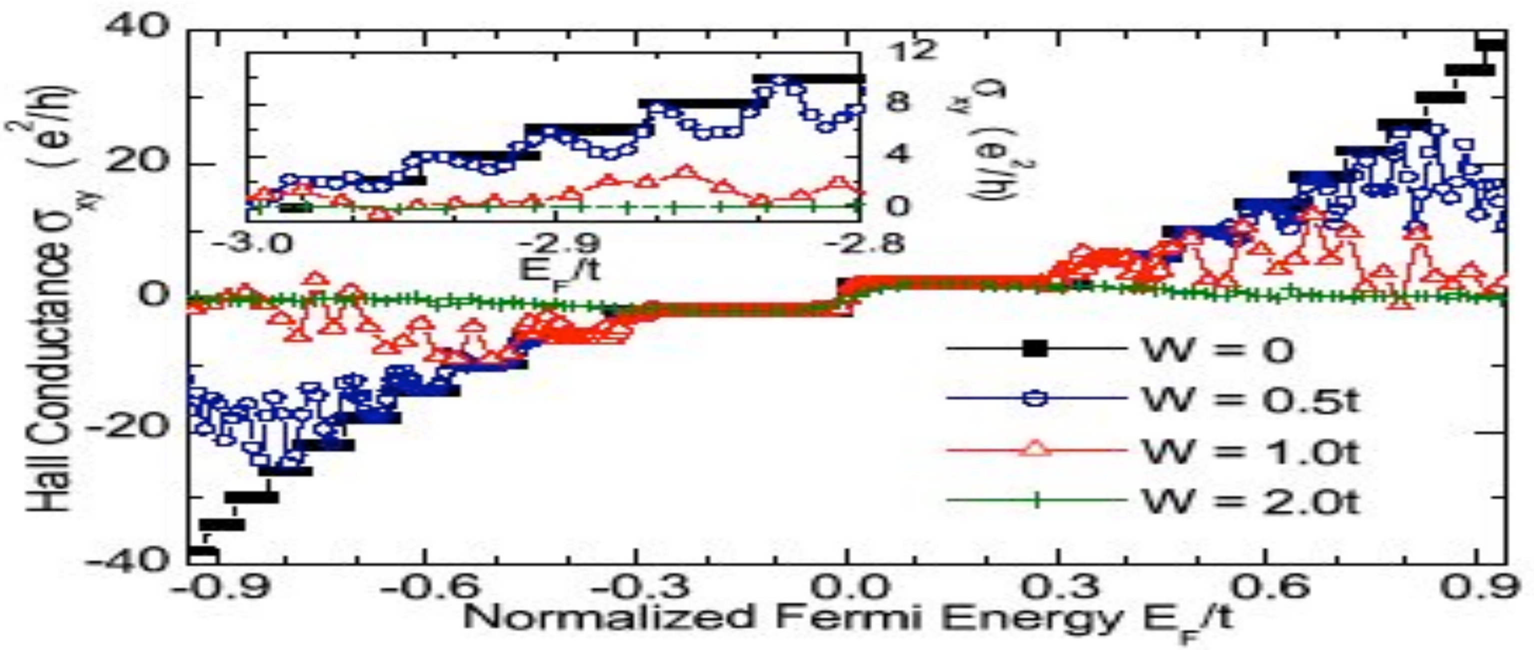


FIG. 2: Unconventional Hall conductance as a function of electron Fermi energy near the band center for four different disorder strengths each averaged over 200 disorder configurations. Inset: conventional Hall conductance near the lower band edge. Here $M = 96$ and the sample size is 96×48.

Similar work: Hasegawa & Kohmoto et al

Phase diagram

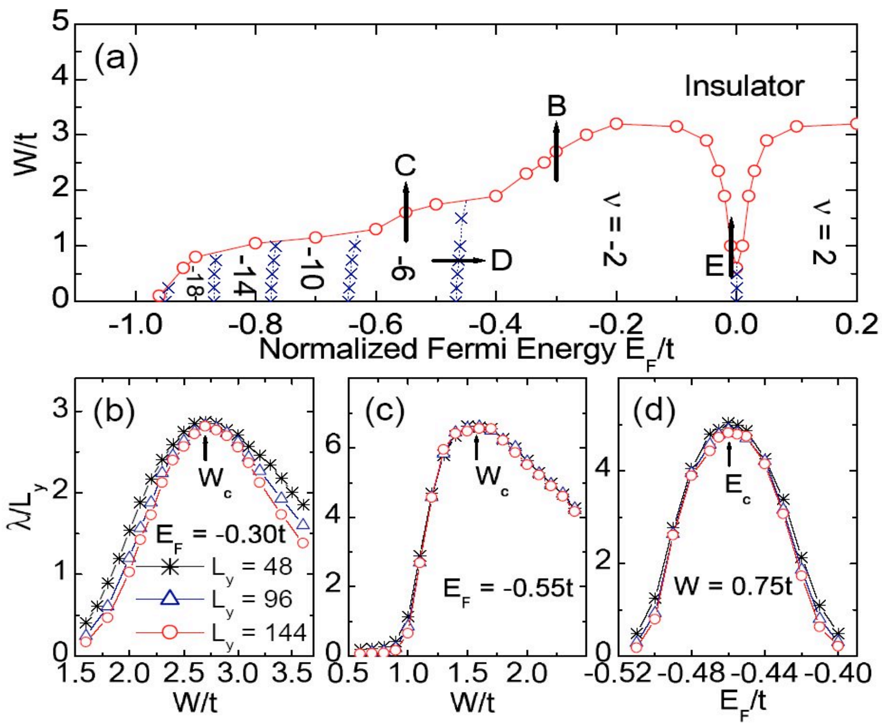


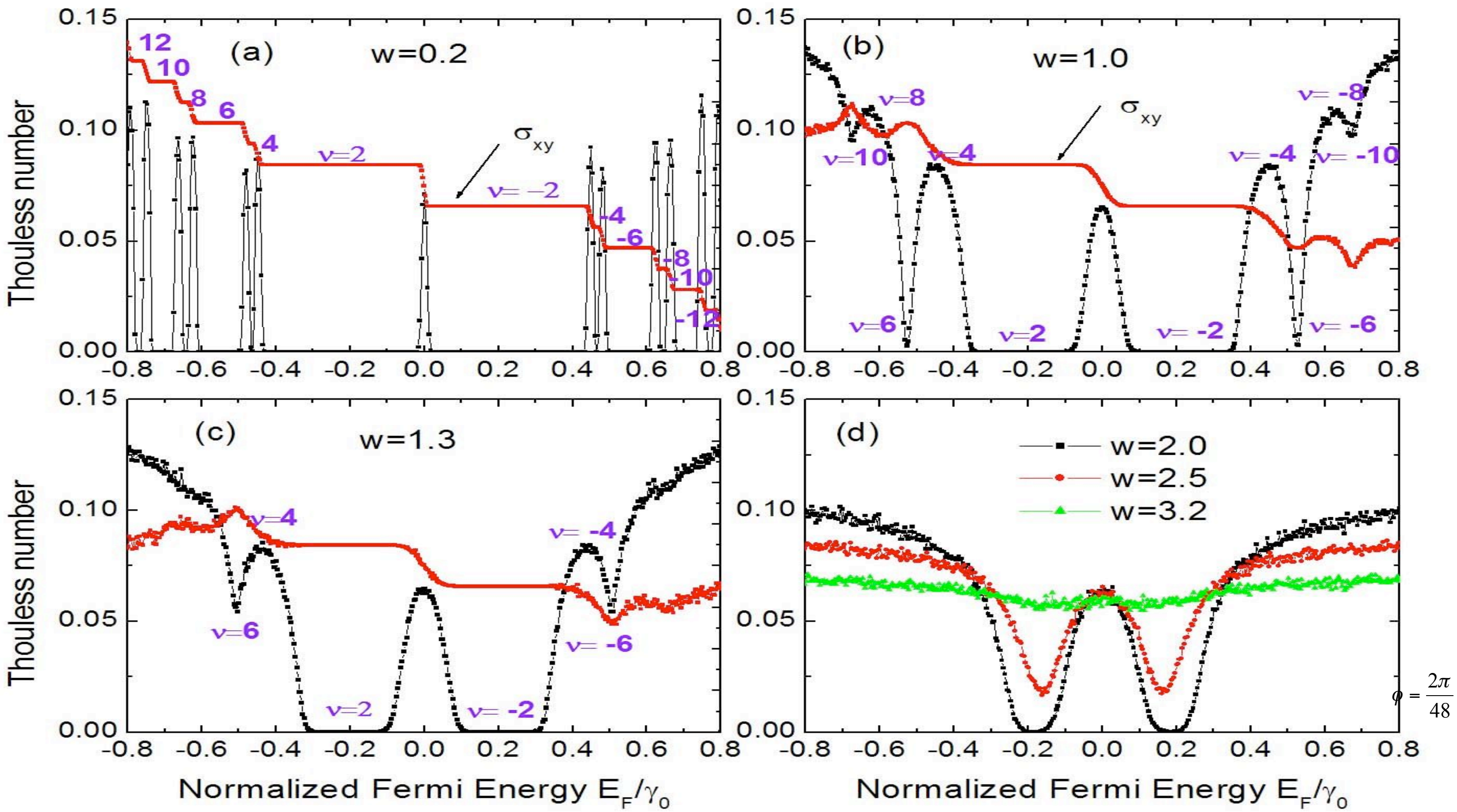
FIG. 3: (a) Phase diagram for the unconventional QHE regime in graphene at $M = 48$, which is symmetric abou
$E_F = 0$. (b) to (d): Normalized localization lengths calc
lated for three bar widths $L_y = 48$, 96 and 144, as the pha
boundary is crossed by the paths indicated by the arrows
C and D in (a), respectively.

Prediction: Direct IQHE to insulator transition

Similar phase diagram for bilayer QHE system

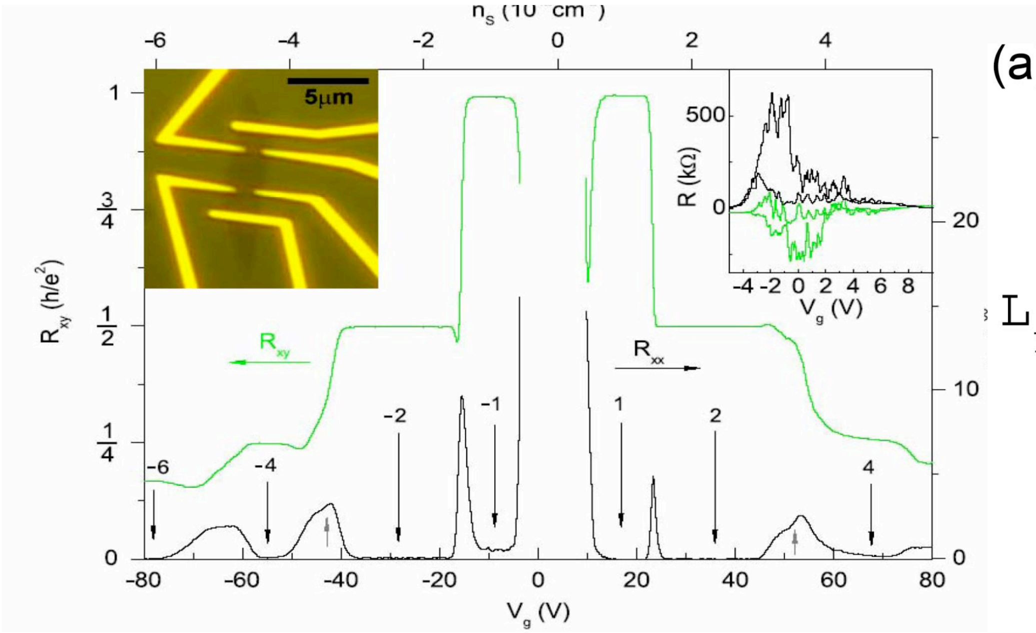
IQHE vs. metallic phase if No intervally scattering

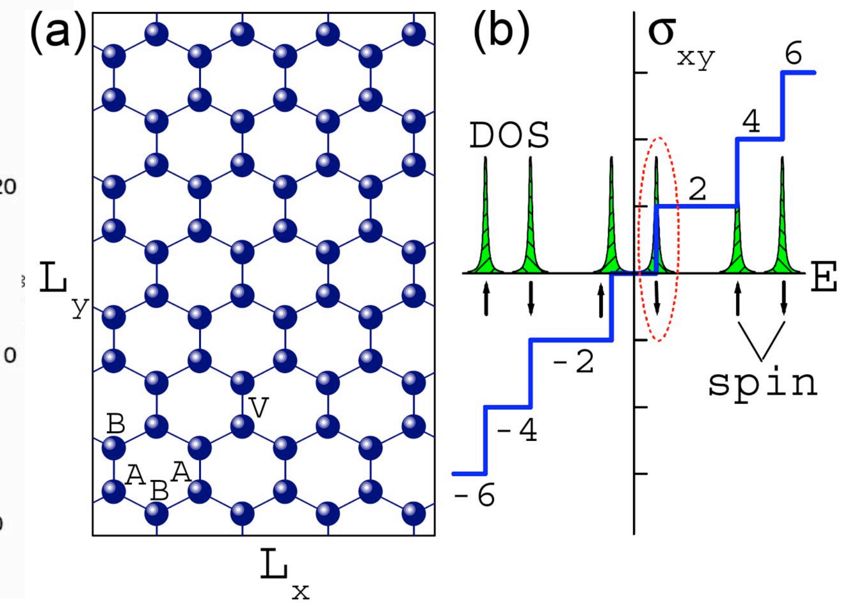
IQHE in bilayer Graphene



Calculated Thouless number and Hall conductivity for different disorder strengths

Experiment discovers ν=1 IQHE and "ν=0" insulating phase





Y. Zhang et. al., PRL 2006

FIG 1. (color online) R_{xx} and R_{xy} measured in the device shown in the left inset, as a function of V_g at $B = 45$ T and $T = 1.4$ K. $-R_{xy}$ is plotted for $V_g > 0$. The numbers with the vertical arrows indicate the corresponding filling factor ν. Gray arrows indicate developing QH states at $\nu = \pm 3$. n_s is the sheet carrier density derived from the geometrical consideration. Right inset: R_{xx} (dark solid lines) and R_{xy} (light solid lines) for another device measured at $B = 30$ T and $T = 1.4$ K in the region close to the Dirac point. Two sets of R_{xx} and R_{xy} are taken at different time under the same condition. Left inset: an optical microscope image of a graphene device used in this experiment.

Interaction has to be taken into account to explain the ν=1 IQHE---Pseudospin Ferromagnet?

Interaction and pseudospin state:Pseudospin (or real Spin) Ferromagnet

Theoretical works

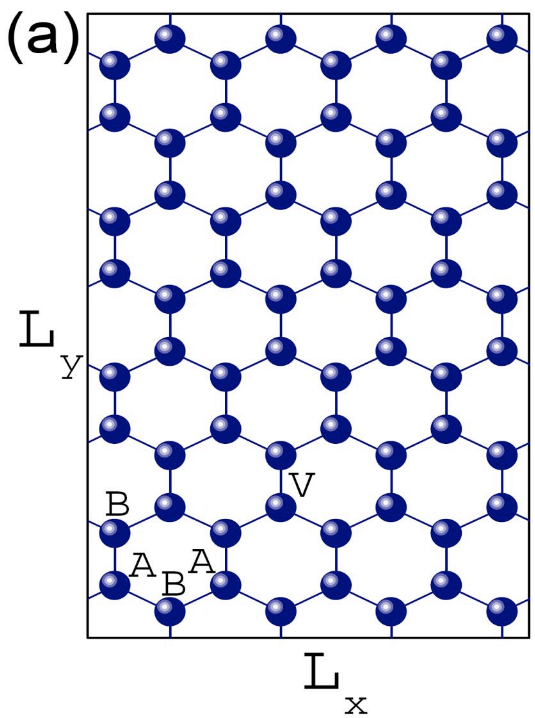
Nomura & MacDonald (2006)
Stoner criteria for pseudospin FM

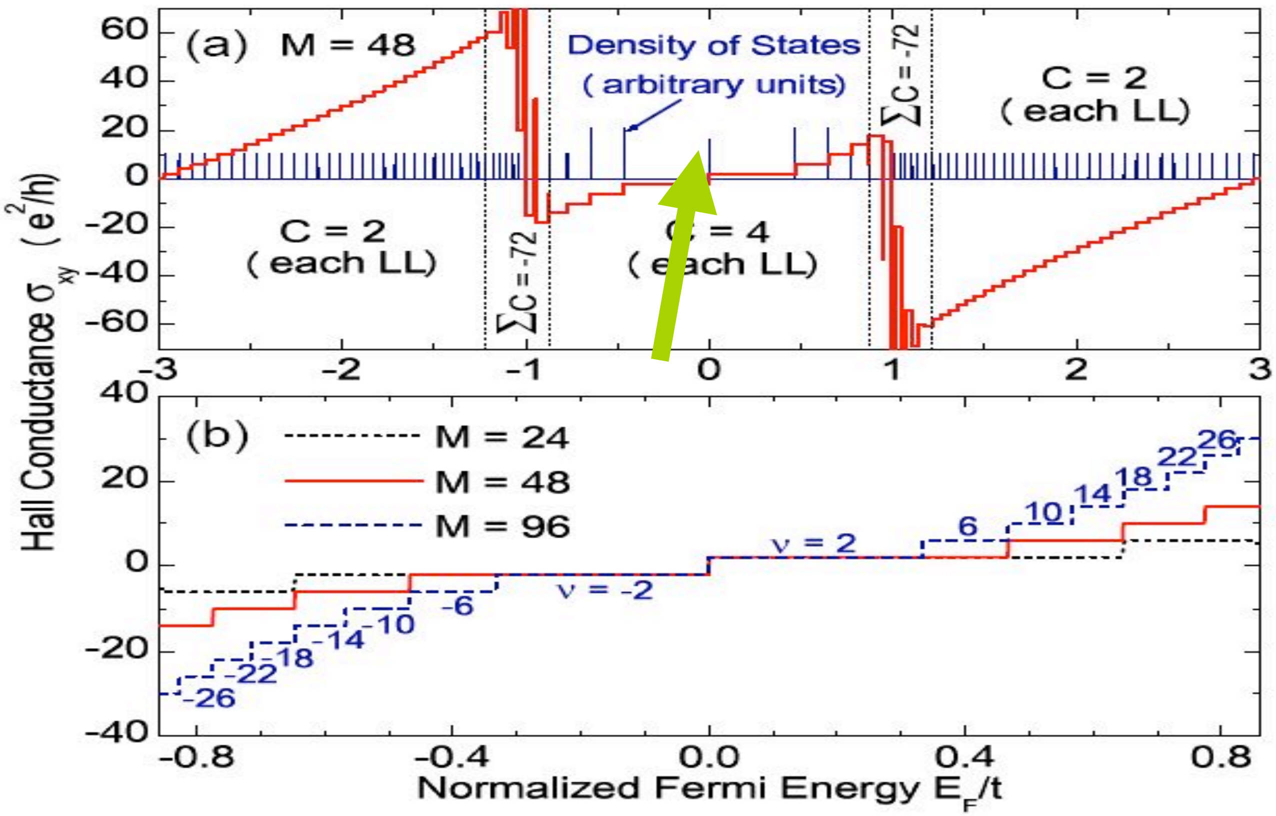
Alicea & Fisher (2006)
lattice effect is relevant

SU(4) (2006-2007)
Yang et al., Gusynin et al.
Toke & Jain, Goerbig et al.

Haldane's Pseudo-Potential gives rise to incompressible state, SU(4) invariant, Algebraic correction (a/l) to SU(4) may also be important

Exact diagonalization using lattice model





100X100 Lattice sites

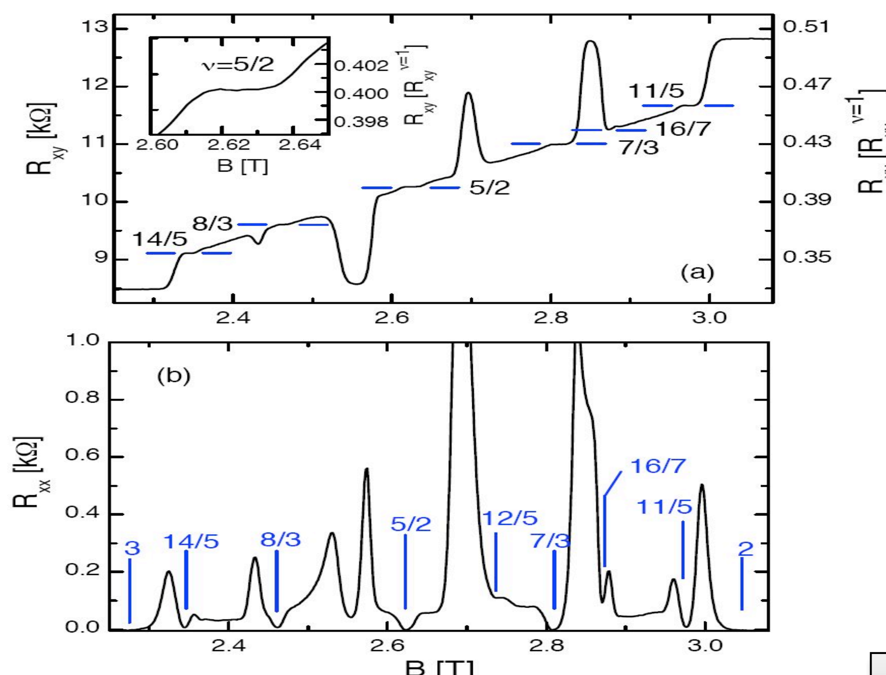
$$H = \sum C_{iB}^{+} C_{jA} + h.c + \sum w_i C_i^{+} C_i$$

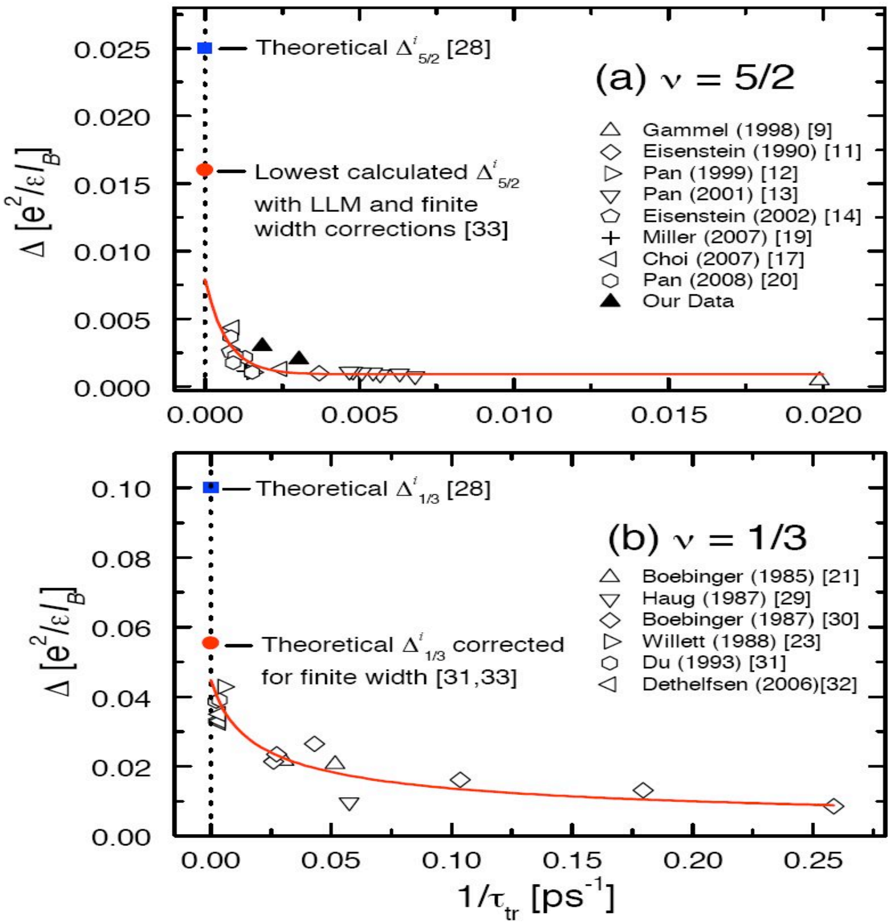
Only keep states inside the top-Landau level, large lattice size and keep a degeneracy of Ns=24

Importance of Disorder and Mobility Gap in FQHE

- Disorder effect in FQHE (developing a topological method to identify Mobility Gap, which is directly measured in experiments)

Experimental Observations





Dean et al. (2008)

Comparison of the Calculated Mobility Gap With Experimental Data for 1/3 FQHE

collaboration with X. Wan, K. Yang et at. (2003-2005)

Pseudospin Ferromagnet and Graphene Odd IQHE

Sheng et al. (2007)

MOBILIT

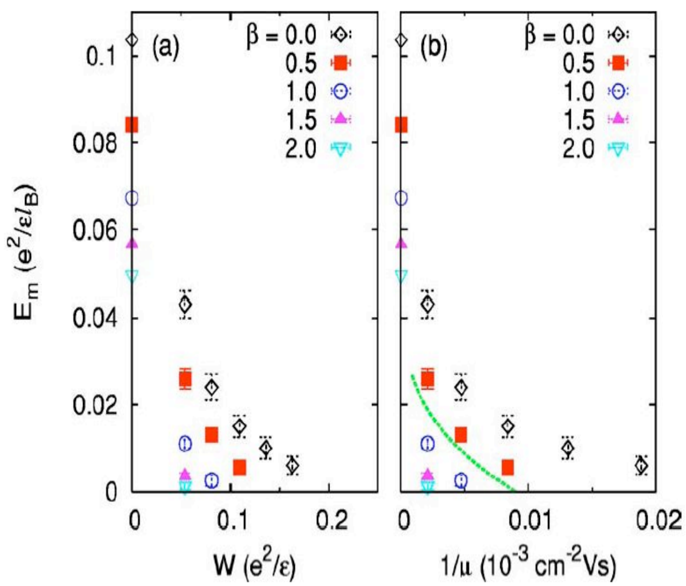
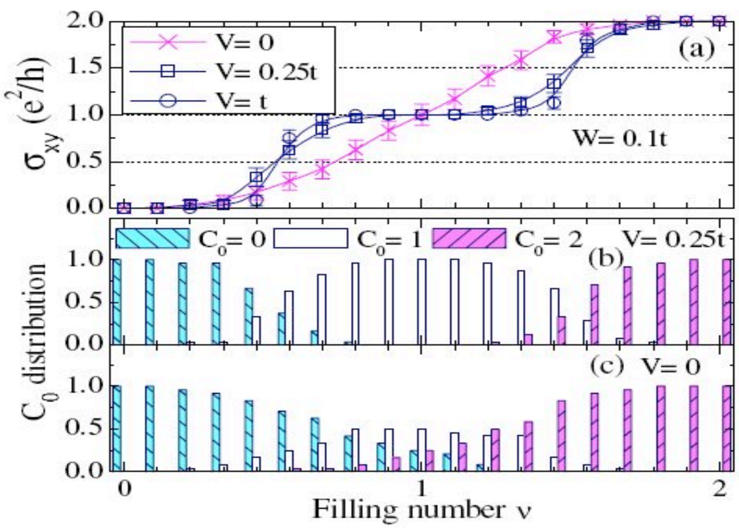
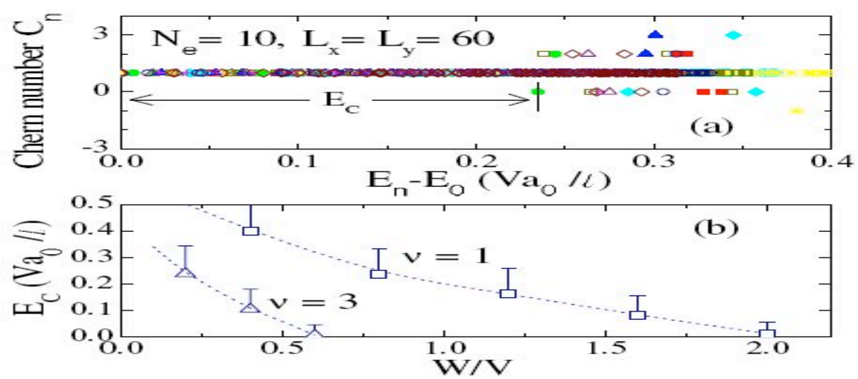


FIG. 7. (Color online) (a) Mobility gap E_m as a functi
for various layer thickness β. E_m is extrapolated from sys
N_e=4–8 electrons to the limit $1/N_e \to 0$. (b) Dependence o
inverse mobility $1/\mu^0$ for various β. The dashed line is c
from a fit to experimental data (taken from Ref. 4). Here, w
empirical mobility-density relation as well as a mobility-
relation in the Born approximation $\mu^0 = e\hbar^3/(m^{*2}W^2)$. T

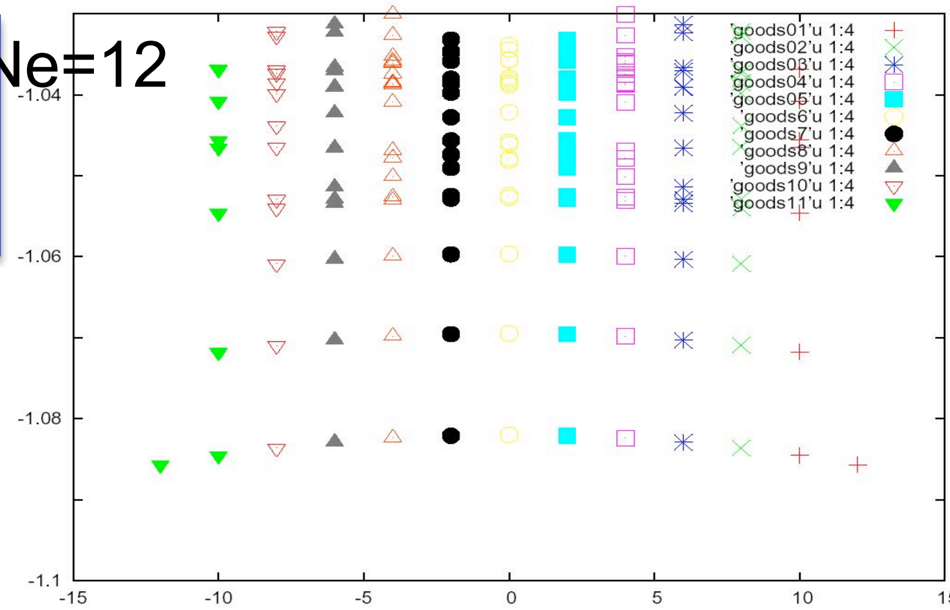




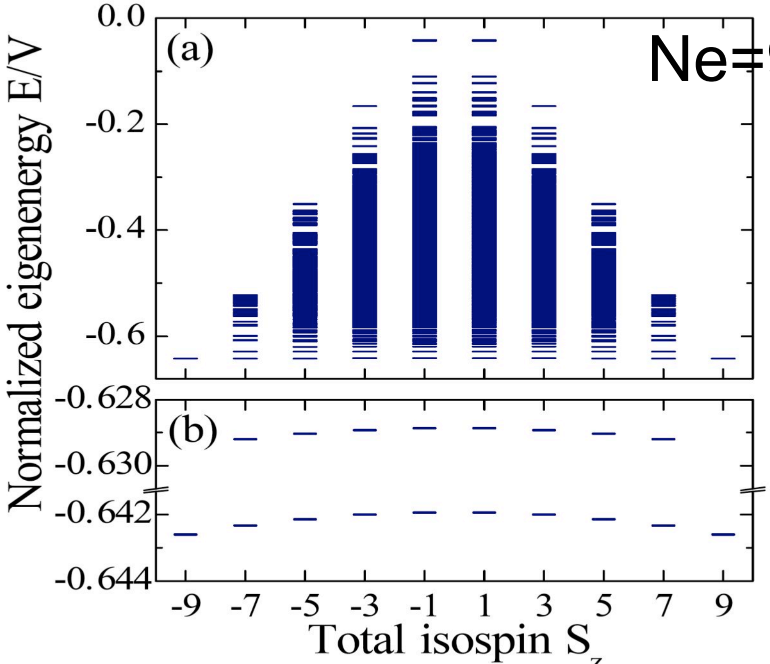
5 (color online). (a) Calculated Chern numbers of 6
y eigenstates as a function of $E_n - E_0$ with E_n t
energy at $\nu = 1$, for $V = 0.5t$, $W = 0.8V$, and 10 r
ler configurations. (b) Critical energy E_C for filling
$\nu = 1$ (squares) and $\nu = 3$ (triangles) as functi
alized disorder strength W/V, where the error ba
ean deviation of E_C due to disorder sampling.

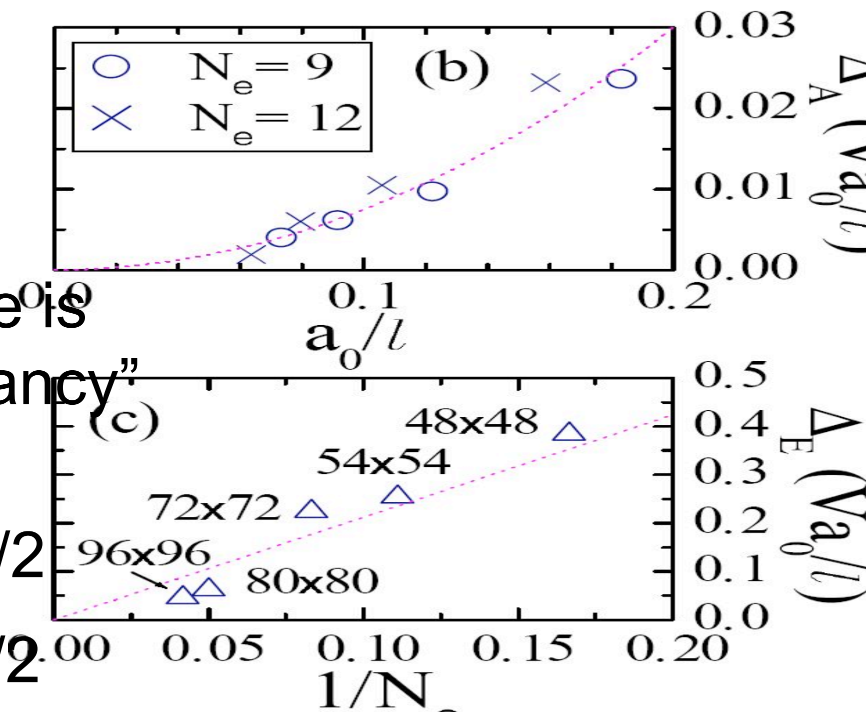
Energy spectrum for pure system

Ne=12

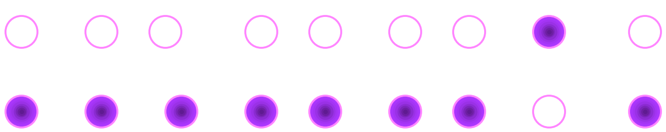


Ne=9





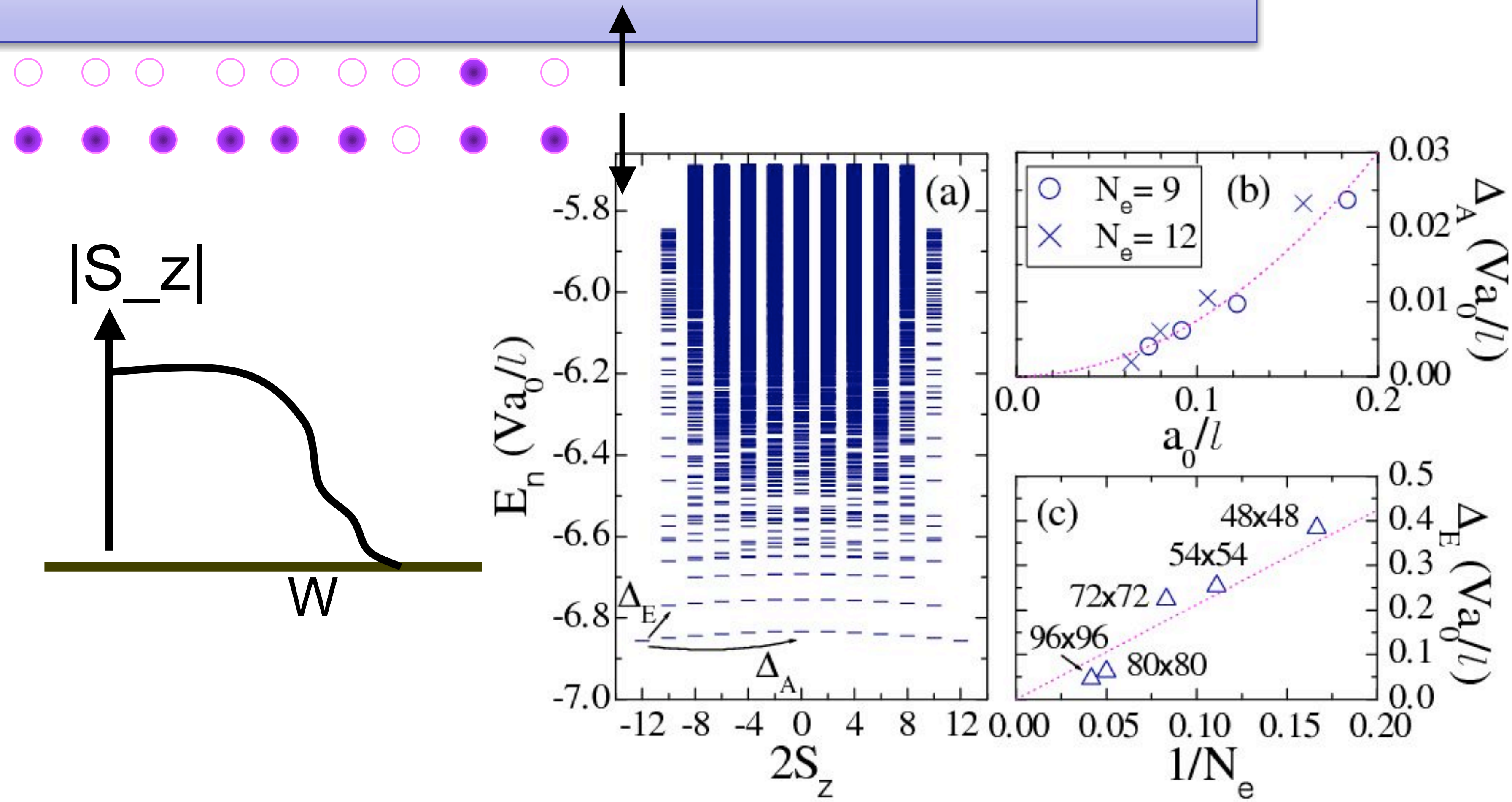
In each pseudo-spin sector, there is a FM state with “no double occupancy” and phase coherence



Sz=1/2

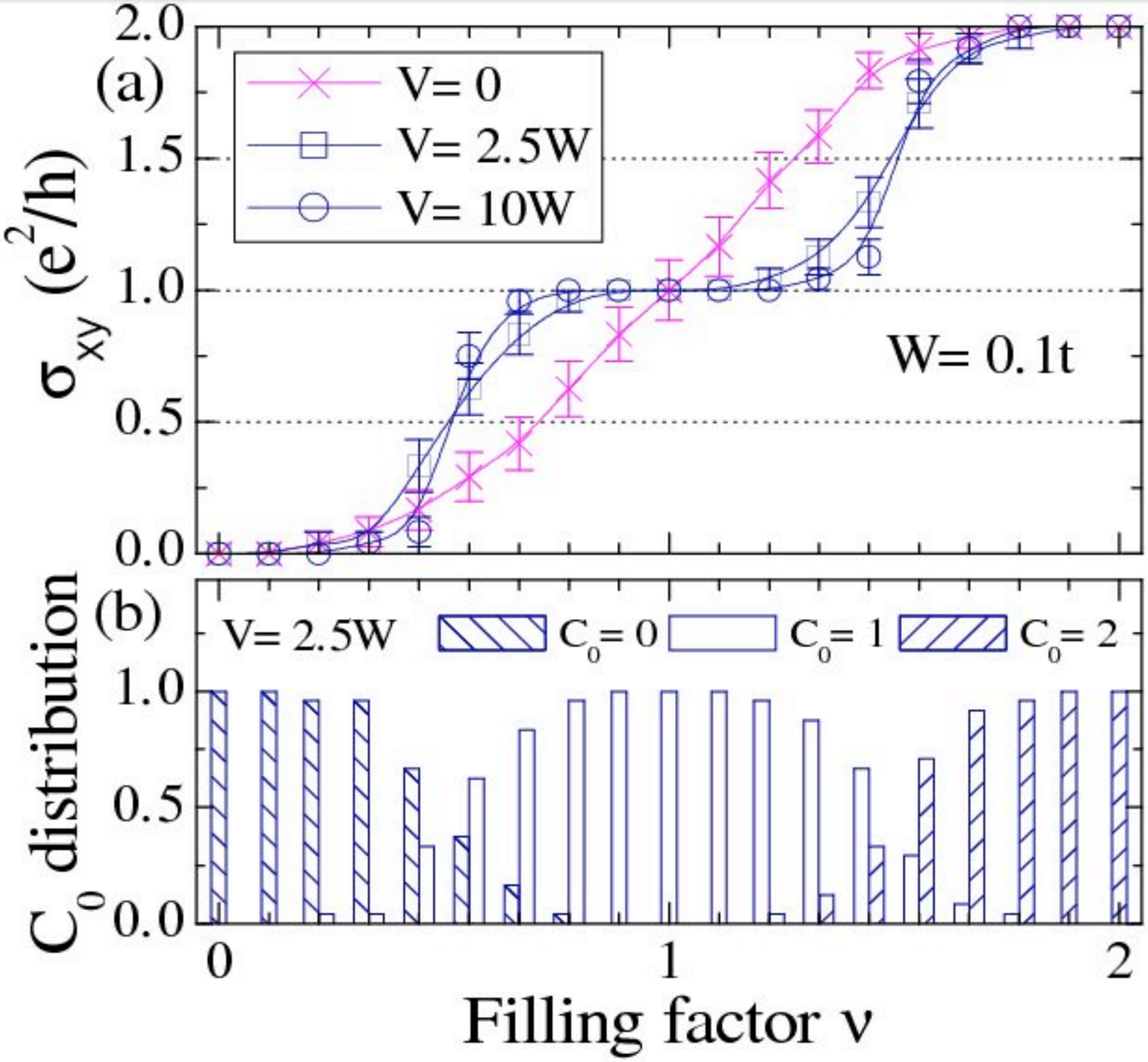
Sz=-1/2

the excitation energy gap (with double occupation), mobility gap



the excitation gap scales with 1/Ne, possibly extrapolates to zero at large Ne limit

Directly look at the transport property instead of "gaps"



Chern number "IS" Hall conductance

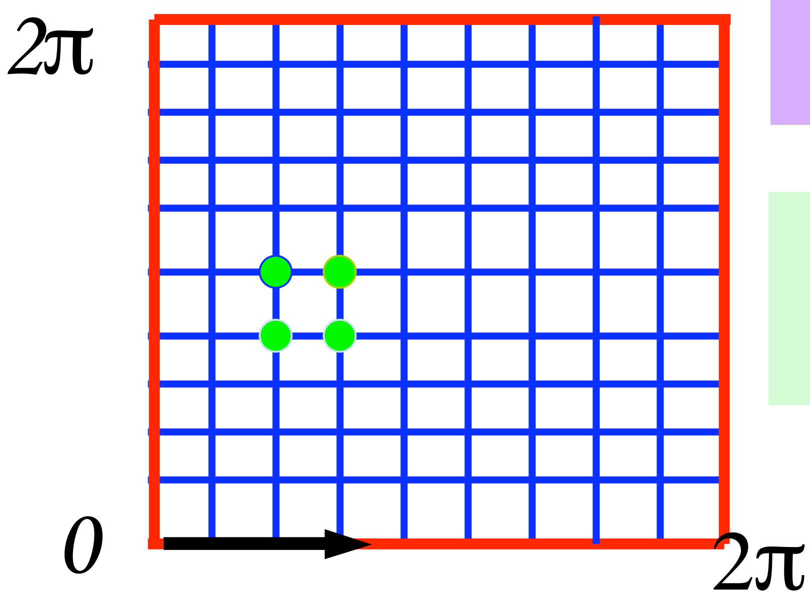
D. J. Thouless et al 1982, J. E. Avron et al. 1883

$$\sigma_{xy} = \frac{ie^2h}{2\pi} \sum_{n>0} \frac{<0|p_x|n><n|p_y|0> - c.c}{(E_n - E_0)^2}$$

$$C = \frac{i}{4\pi} \oint d\theta_j \{ \langle \psi | \frac{\partial \psi}{\partial \theta_j} \rangle - \langle \frac{\partial \psi}{\partial \theta_j} | \psi \rangle \} \qquad \sigma_{xy} = C\frac{e^2}{h}$$

$$C = \sum_{\varepsilon_m < \varepsilon_F} C^{(m)}$$

C is for many-body

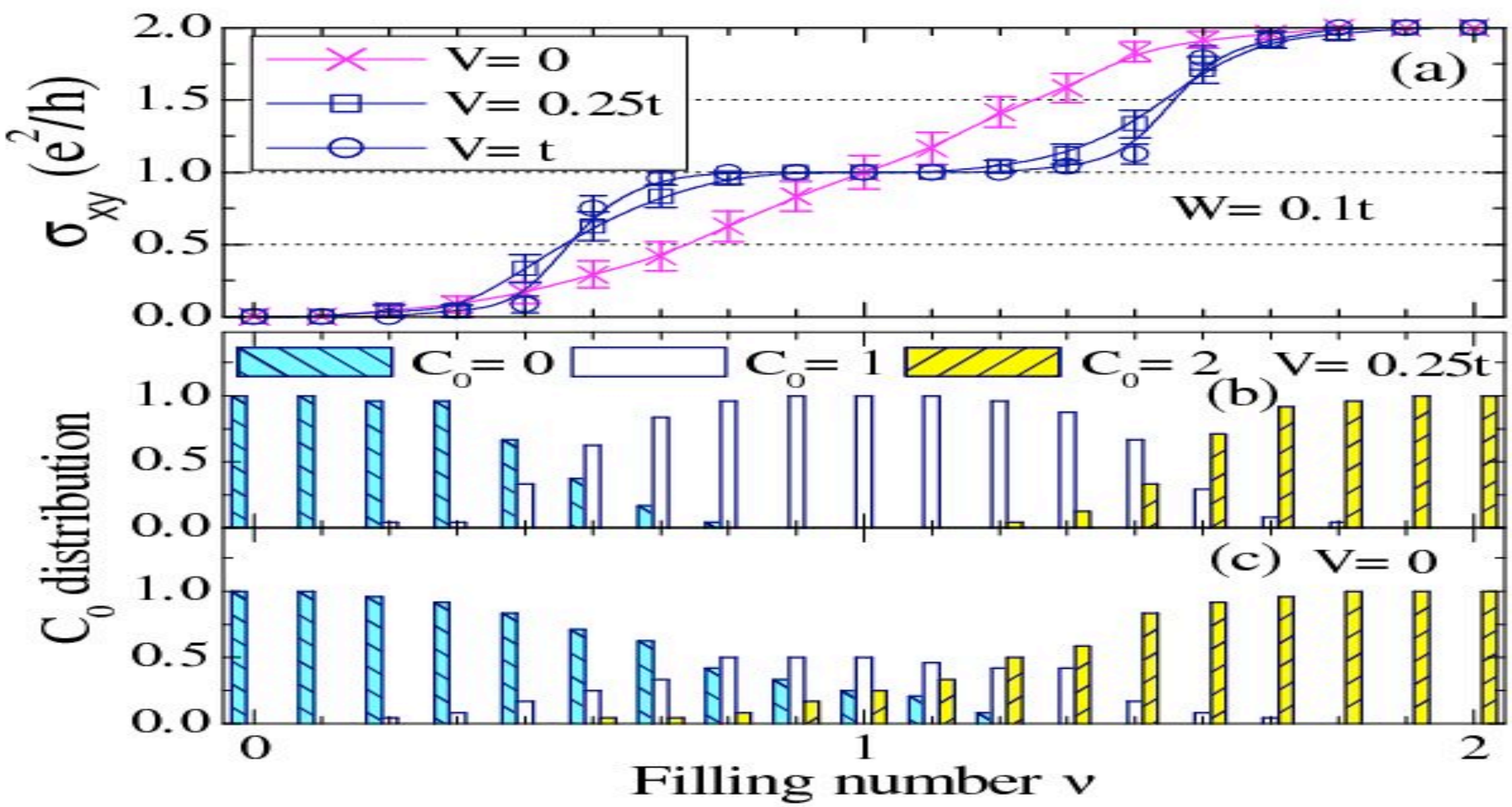


(θ_x, θ_y) **boundary phase**

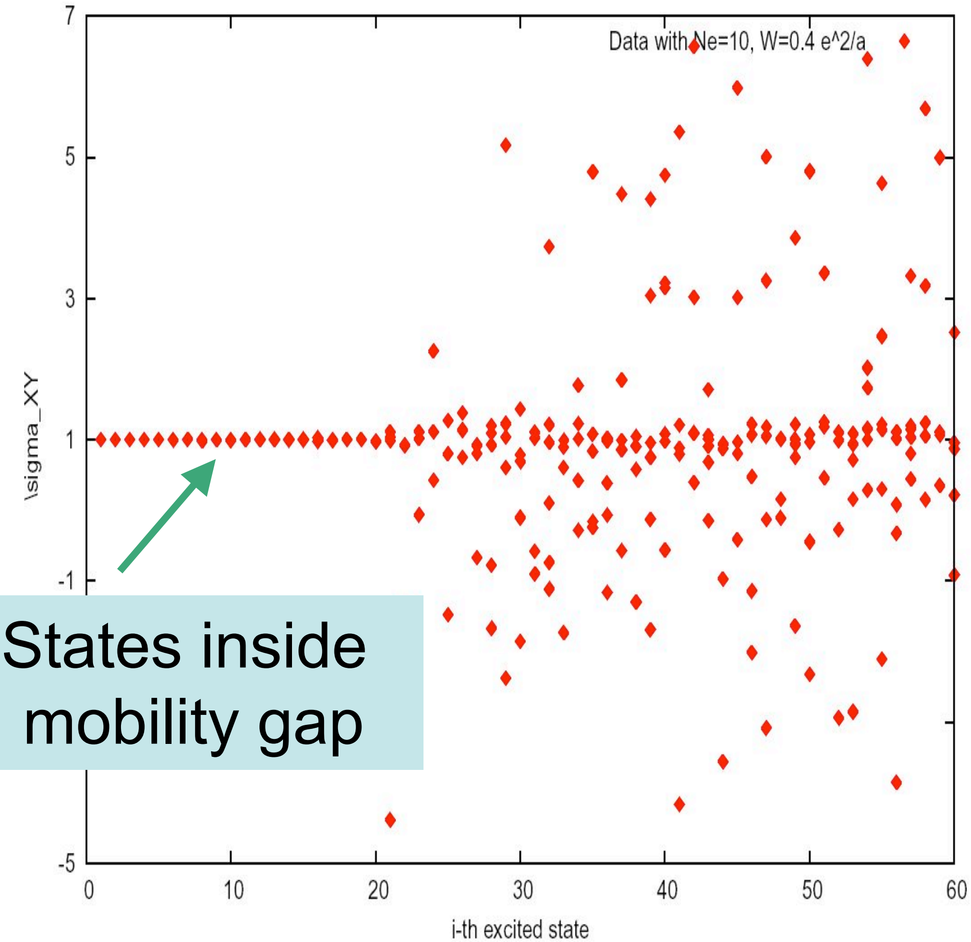
D.N. Sheng, Xin, Yang., PRL 2003;
Sheng, Balents, Wang
Xin et al. PRB (FQHE)

Just get $\psi(\theta)$ at all nodes of mesh of 100-1000 points, overlap of $\psi(\theta)$ at nearest points

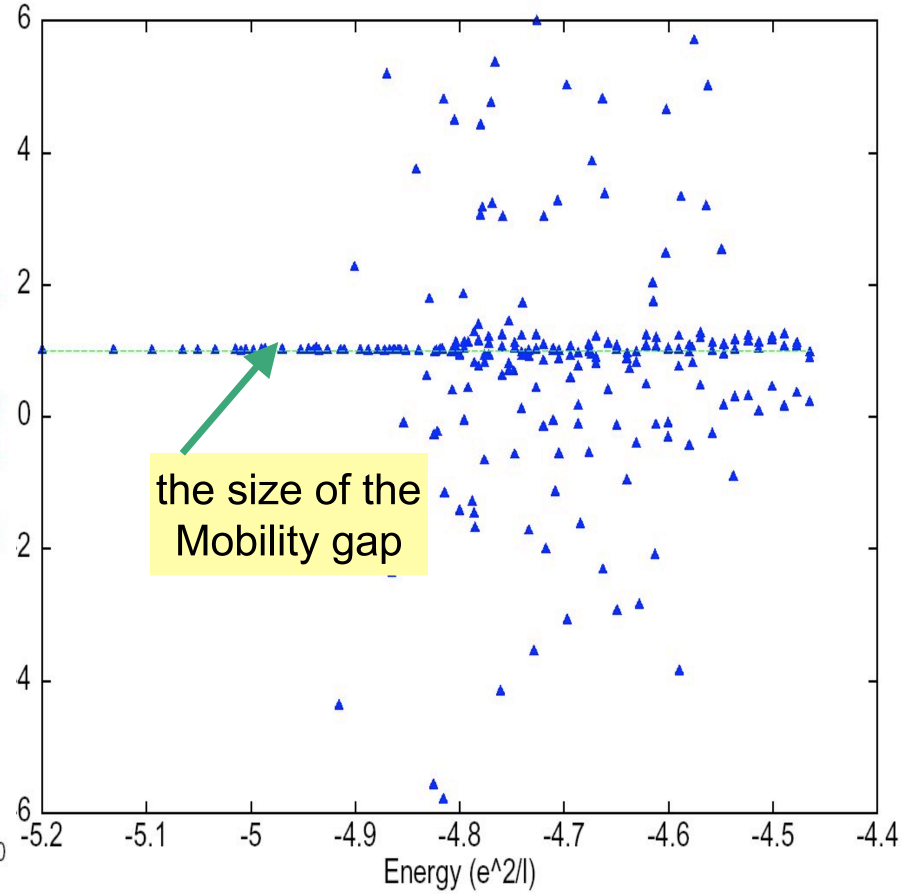
The destruction of odd IQHE is due to the mixing of various Chern numbers



The importance of mobility gap (activation gap of experiment): from direct Hall conductance and Chern number calculations



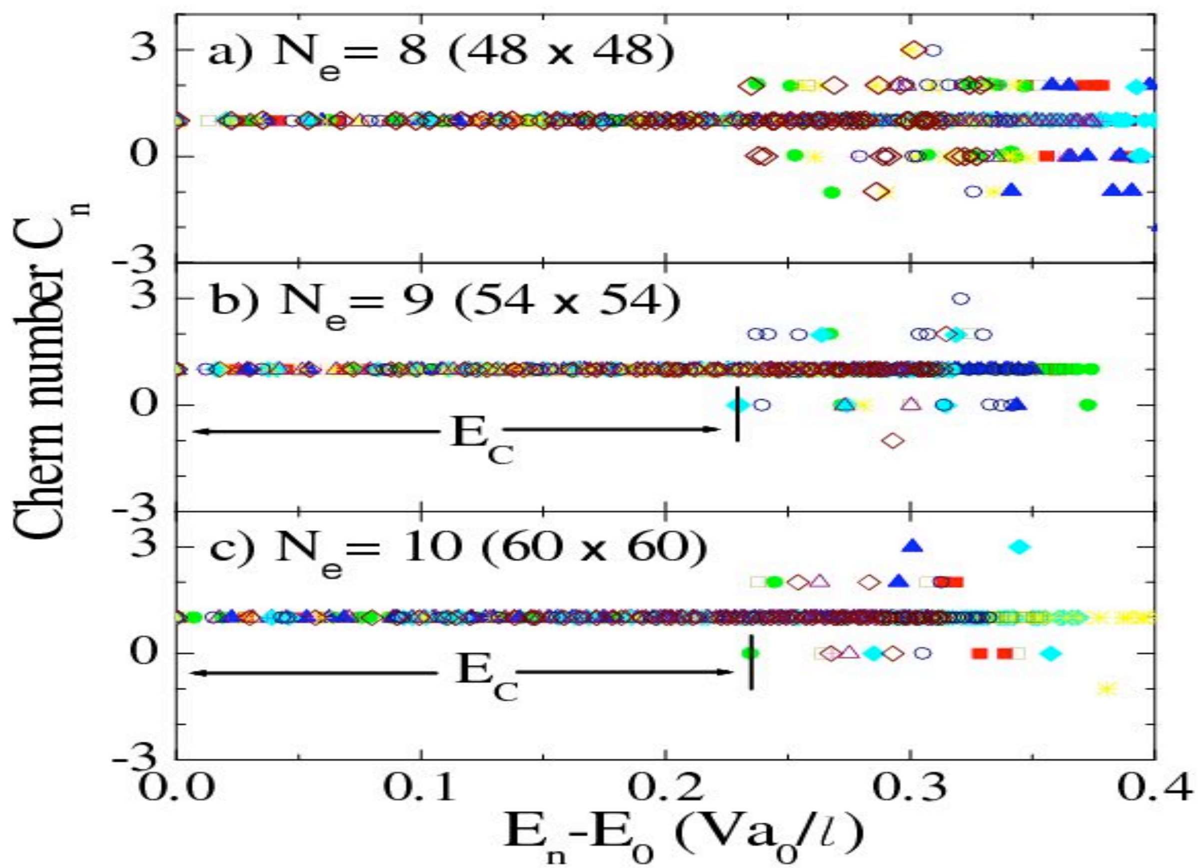
States inside mobility gap



the size of the Mobility gap

finite size scaling confirms a finite transport gap at large size limit (more data are coming)

Fluctuation of Chern numbers determine a mobility edge



e^2/

εl

Mobility gap for odd IQHE states

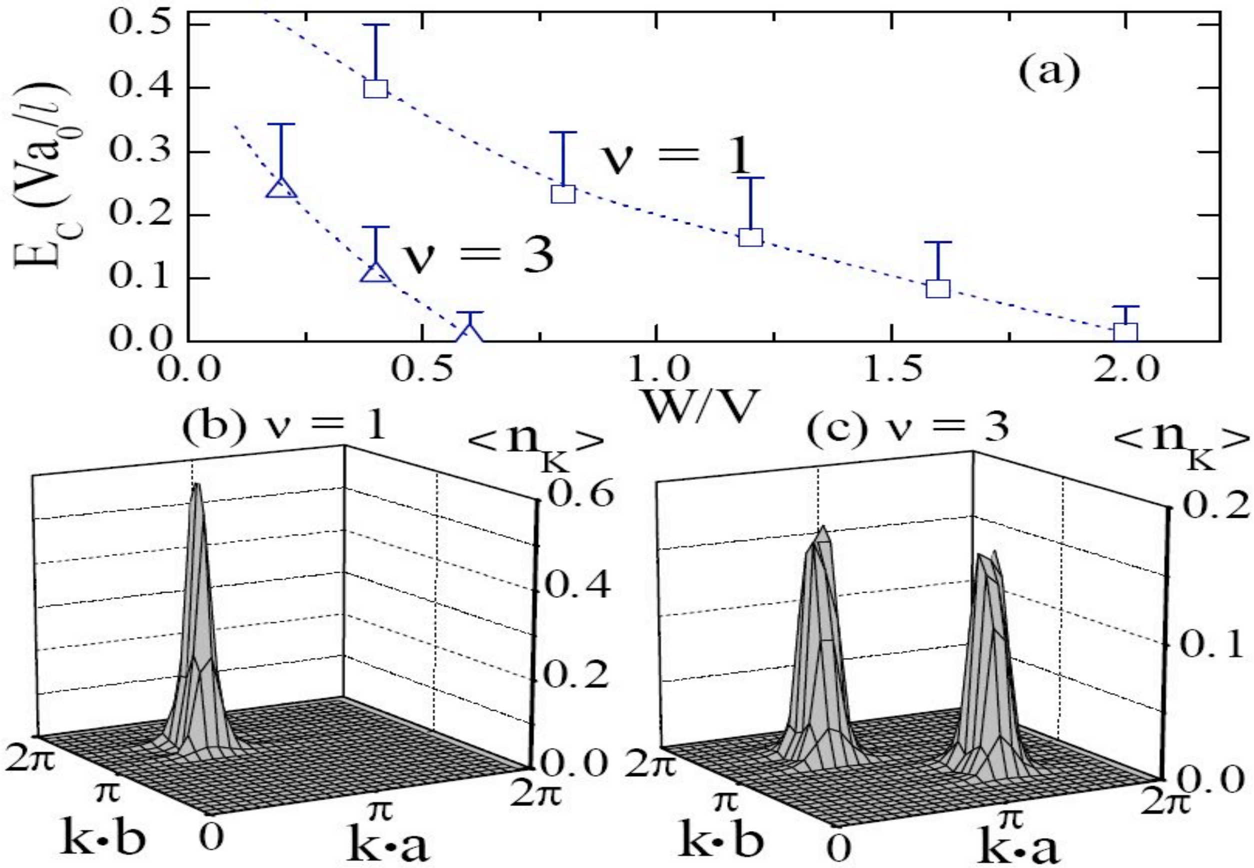
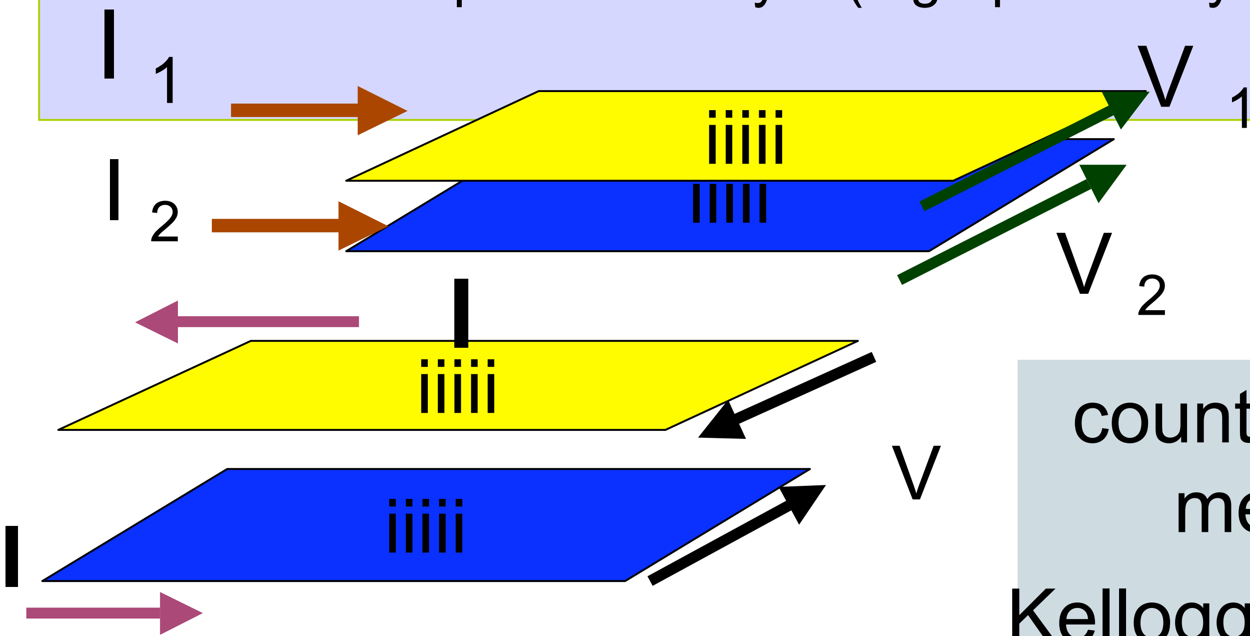


FIG. 5: (color figure online) (a) The critical energy E_C for filling numbers $\nu = 1$ (squares) and $\nu = 3$ (triangles) as a function of normalized disorder strength W/V, where the error bars are the mean deviation of E_C due to disorder sampling. (b) and (c) are the electron number distribution in the wavevector **k** space for $\nu = 1$ and $\nu = 3$, respectively, where $L_x = L_y = 54$ and $M = 3 \times 54$, $V = 0.5t$, and $W = 0.01t$.

Transport for bi-layer (2 graphene layer) system



counter-flow current measurement,

Kellogg et al. PRL 2004

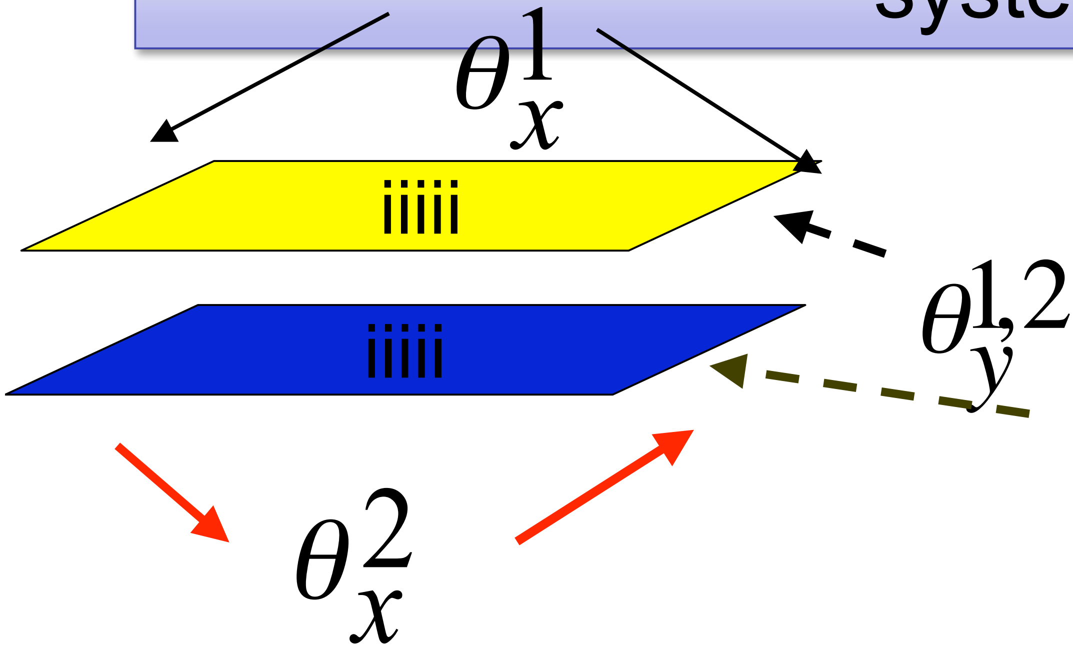
$$I_1 = \sigma^{11} V_1 + \sigma^{12} V_2$$

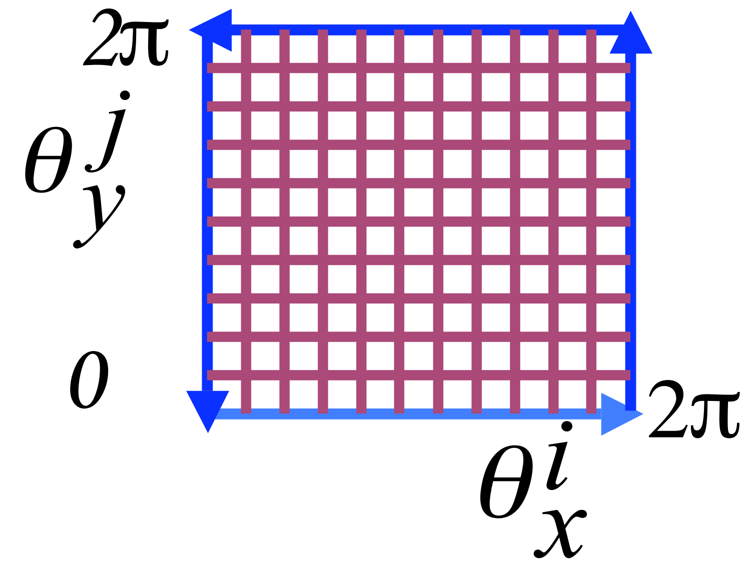
$$I_2 = \sigma^{21} V_1 + \sigma^{22} V_2$$

2*2 Hall conductance

symmetric and anti-symmetric (pseudo-spin) transport channels

Chern number matrix in two-component systems





$$\sigma_{xy} = \frac{ie^2 h}{2\pi} \sum_{n>0} \frac{<0|p_x|n><n|p_y|0> - c.c}{(E_n - E_0)^2}$$

$$C = \frac{i}{4\pi} \oint d\theta_j \{ \langle \psi | \frac{\partial \psi}{\partial \theta_j} \rangle - \langle \frac{\partial \psi}{\partial \theta_j} | \psi \rangle \} \qquad \sigma_{xy} = C \frac{e^2}{h}$$

Charge twist $\theta_i^1 = \theta_i^2$

Sheng, Balents, Wang 2003

Two single layer graphenes without tunneling
Counter flowing superfluid state (measurement highly desired)

- The pseudospin degeneracy=4

Feromagnetism has smaller stiffness,
but mobility gap remains rob

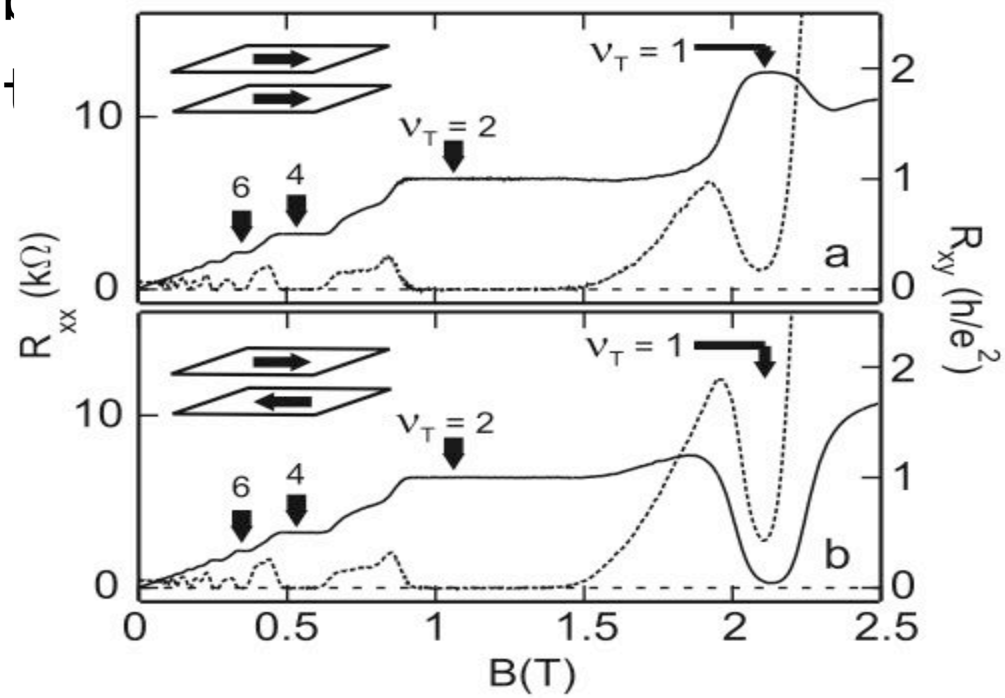
If d < magnetic lengt

Quantized Hall
$\sigma_{xy}=e^2/h$

Quantized Hall Drag e^2/h

Vanish $\rho_{xy}=\rho_{xx}=0$ (T=0) in counter flow

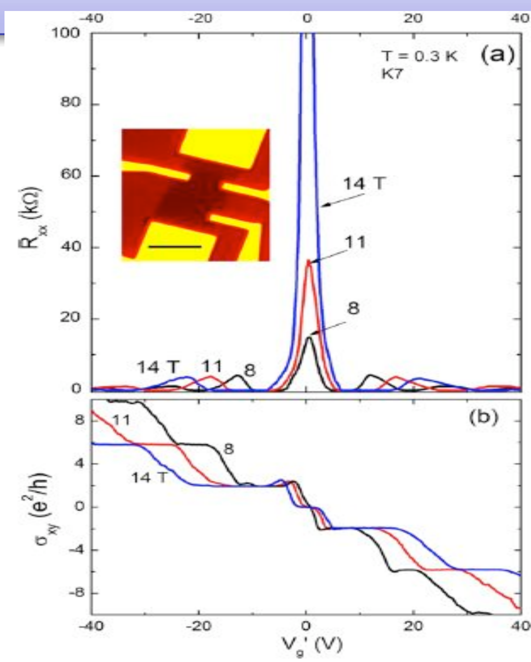
Similar physics to bilayer ?



Kellogg et al. (2004)

Eva Andrei group: decoupled graphene

Insulating at Dirac point (n=0)? Open question!



Ong's group 2007, 2008
Abanin et al. (2007-2008)
Shimshoni, Fertig, Pai
Li Sheng, DNS (2009)
Edge states, spin-orbit,
interaction

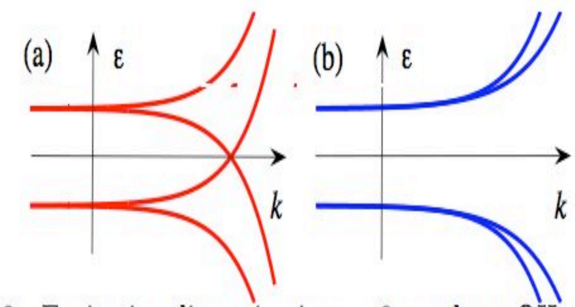


FIG. 2: Excitation dispersion in $\nu = 0$ graphene QH stat
a system with and without gapless chiral edge modes, (a)
(b) respectively. Case (a) is realized in spin-polarized ν
state [4], while case (b) occurs when symmetry is incompa

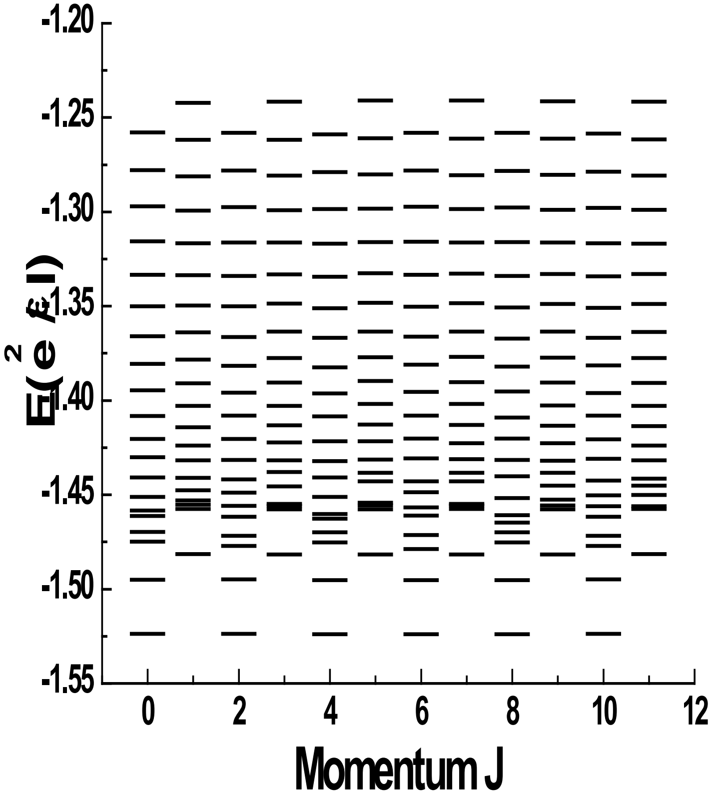
Corbino disk measurement of σ_{xx} relation to $1/\rho_{xx}$

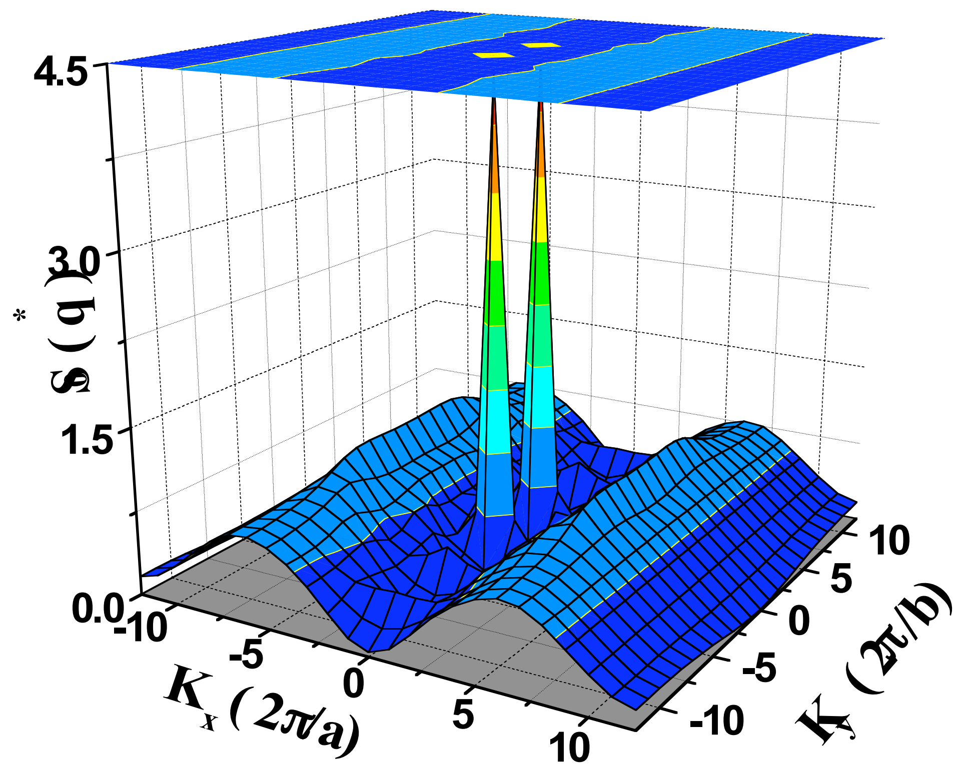
Backward scattering can bring $\sigma_{xy}^{up/down}$ to zero

SO and interaction can open gap (topological insulator without Z_2 symmetry)

Weak B, metallic ?

Symmetry broken states (stripes and bubbles) in n=3 and n=2 Dirac LLs

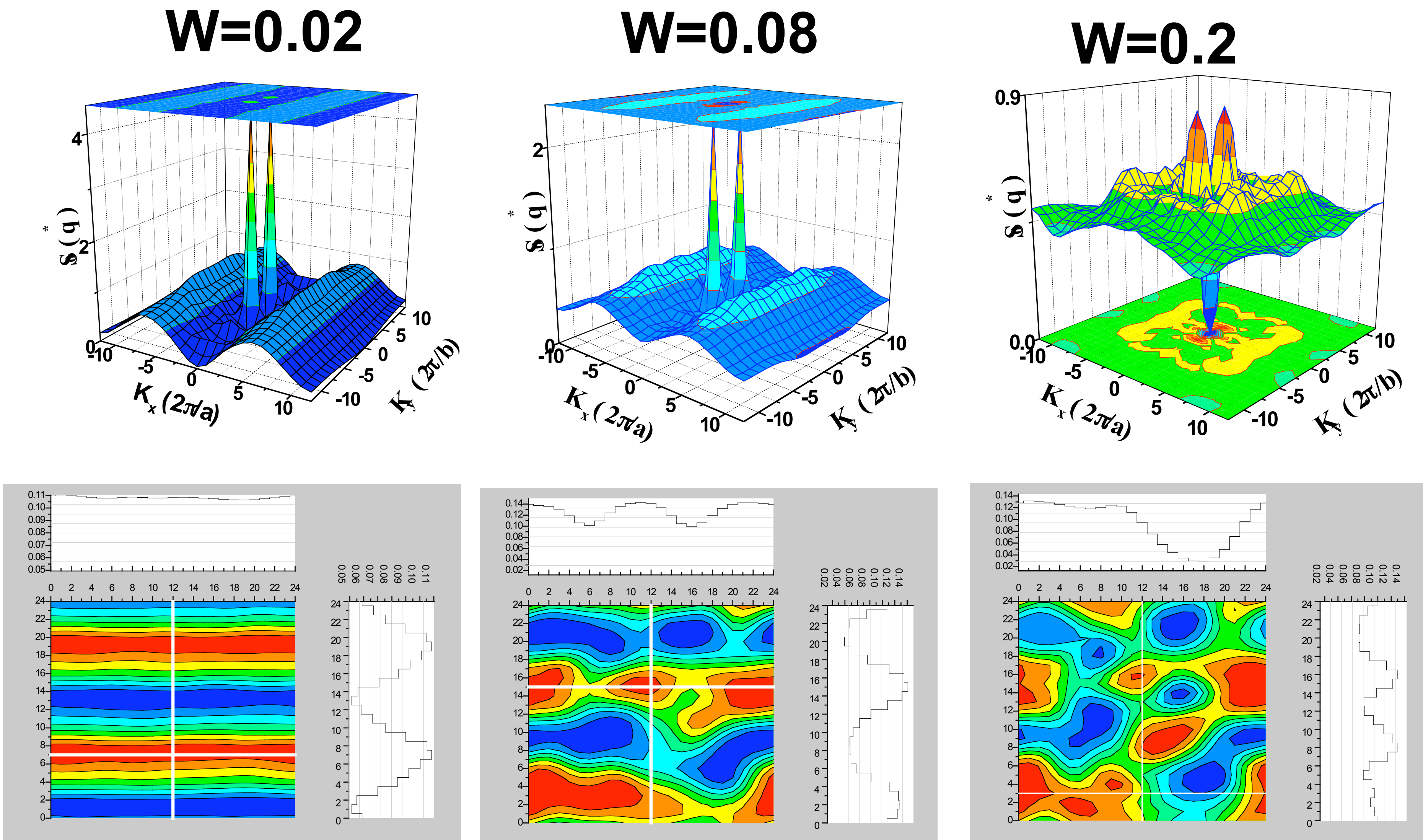


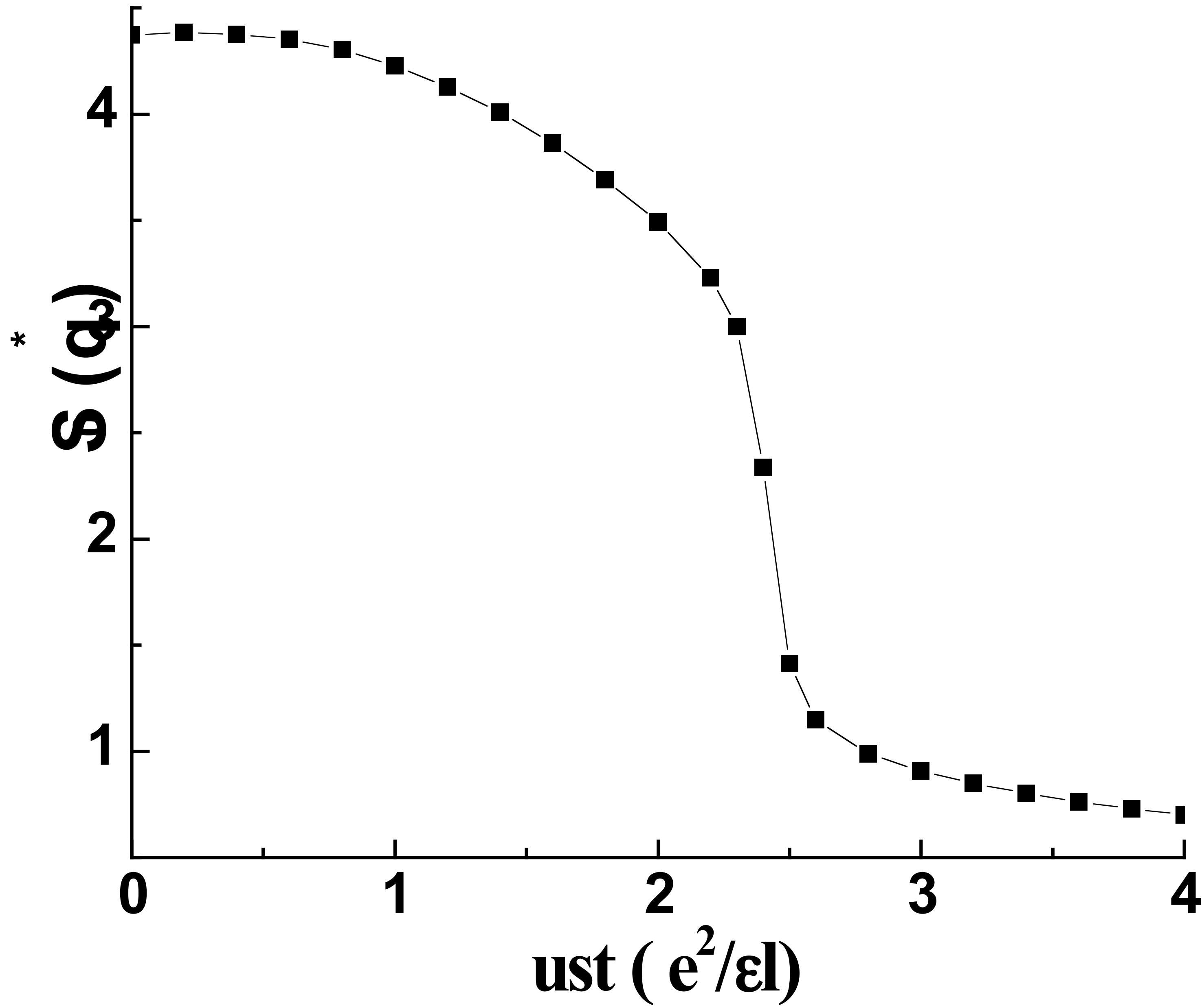


n=3(LL=2&3) Graphene, 12/24 systerm,
a/b=0.74,q^*=(0,±0.595)

Hartree-Folk:
Zhang & Joglekar (2007)

Disorder-Caused Phase Transition

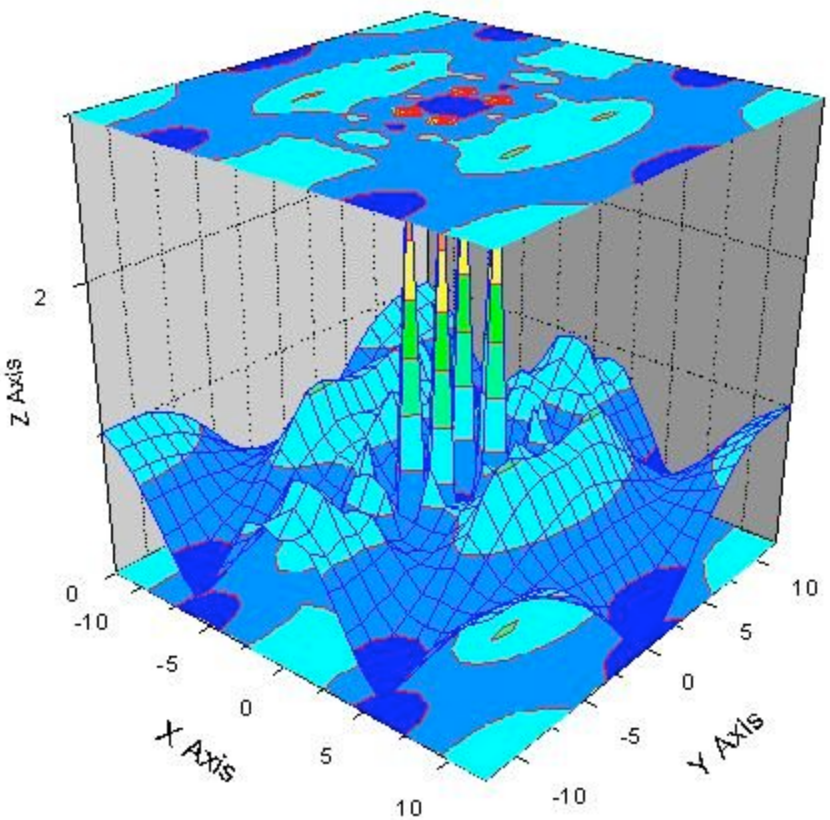




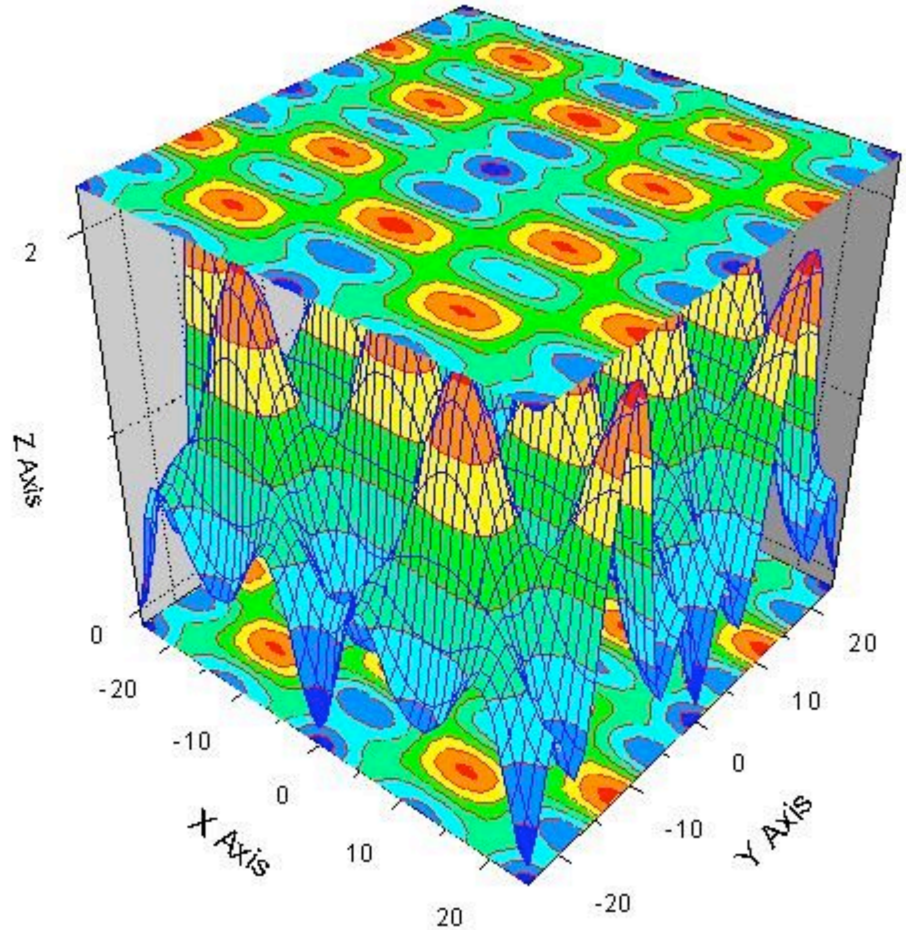
S (q*)
4
2
1
0
1
2
3
4
ust (e²/εl)

8/24 system, LL**2+3**, **a/b=0.65**, bubble phase

Structure factor:S0(q)

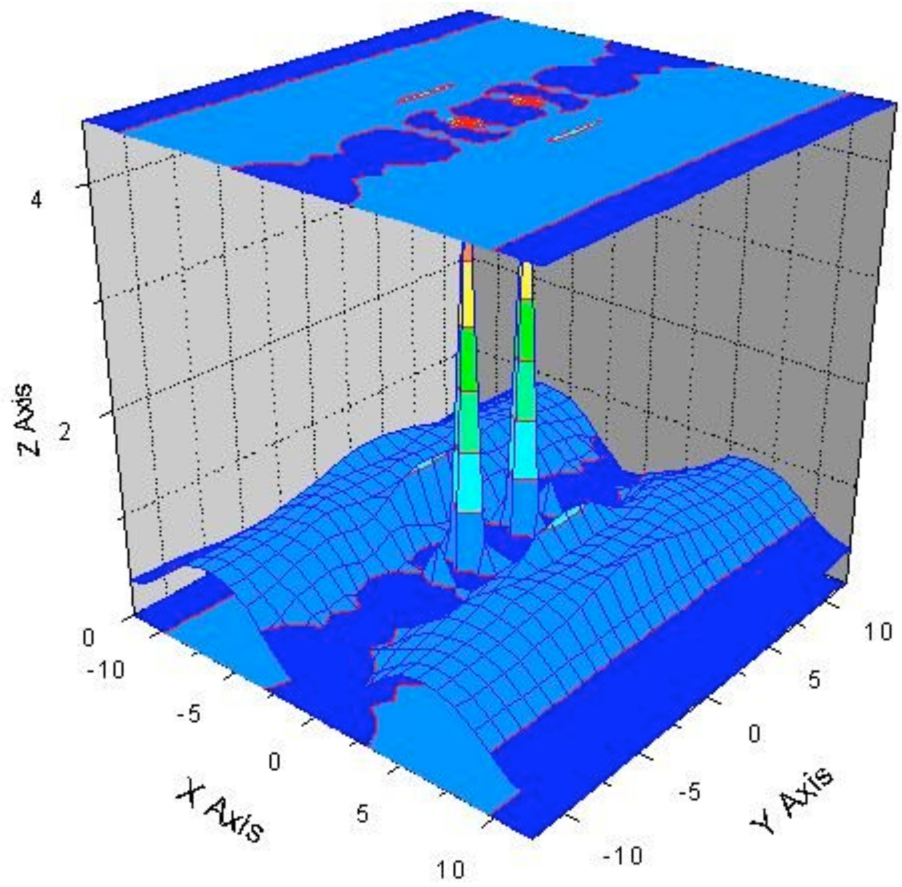


Correlation function:G(r)

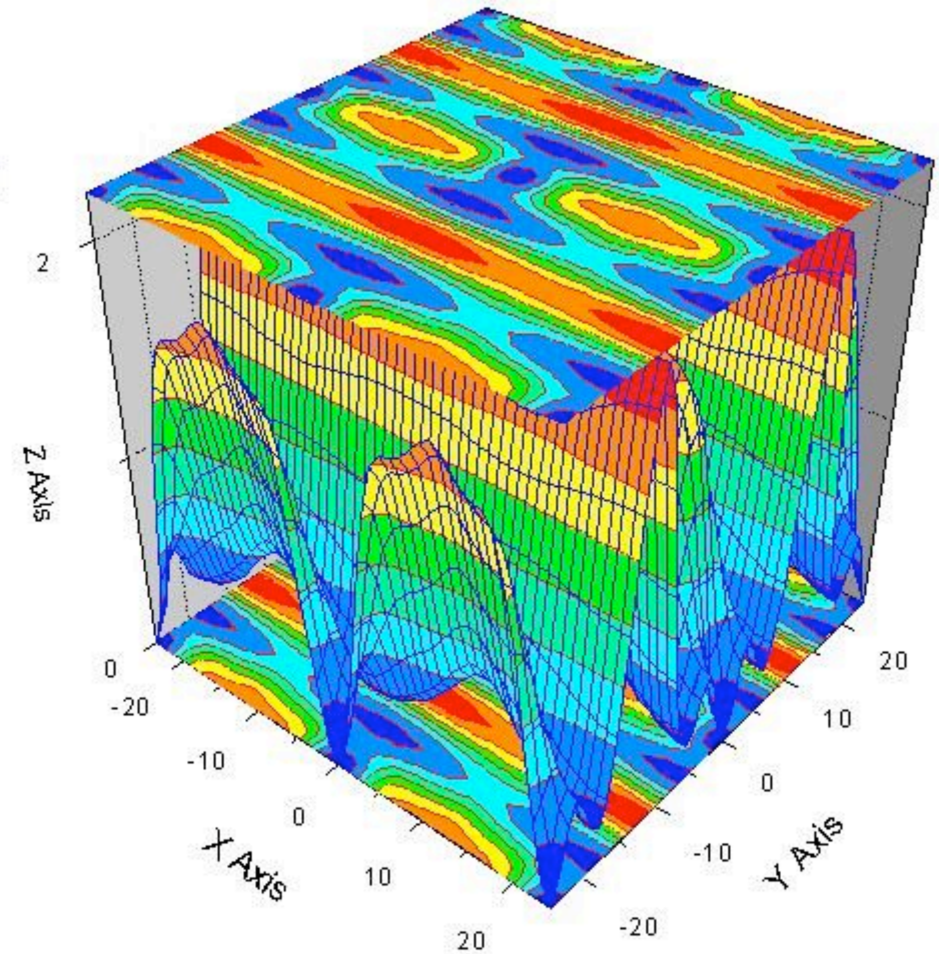


8/24 system, LL**2+3**, **a/b=0.86**, stripe phase

Structure factor:S0(q)



Correlation function:G(r)



Summary:

The Dirac QHE shows interesting phase diagram, explains how conventional QHE can be connected to Dirac ones. Extended levels merge together to separate insulating (or metallic phase) from Dirac IQHE

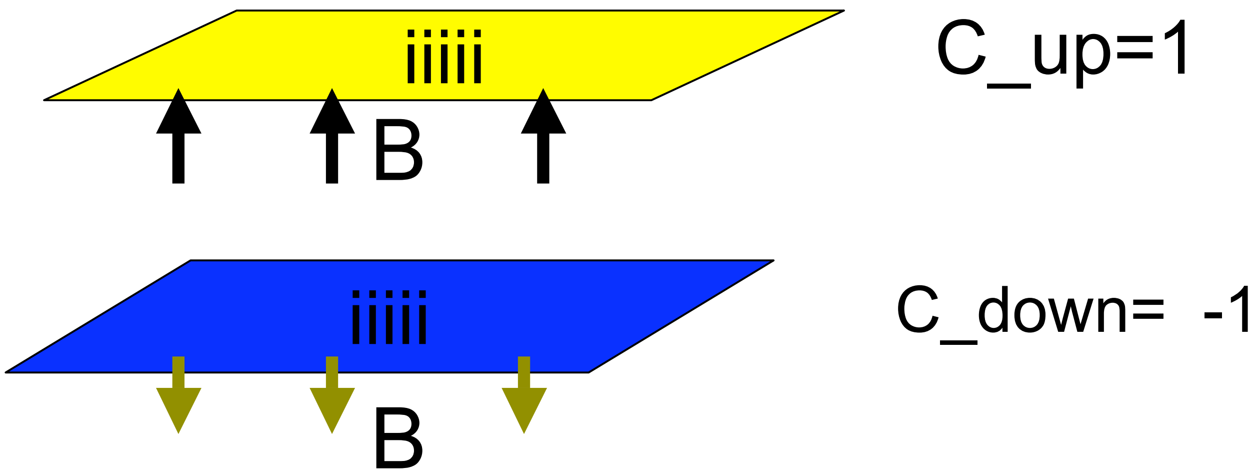
The $\nu=1$ IQHE is robust in both pseudo-spin FM state and pseudo-spin liquid like state, protected by a mobility gap (importance of localization in interacting system)

Bilayer (two single layer graphenes and possible counter flowing superfluid state is dicussed

Quantitative results of activation (mobility) gap at different disorder strengths can be compared with future experiments

Symmetry broken states (stripes and bubbles) are predicted to be stable states in higher (n=3 and n=2) Dirac LLs

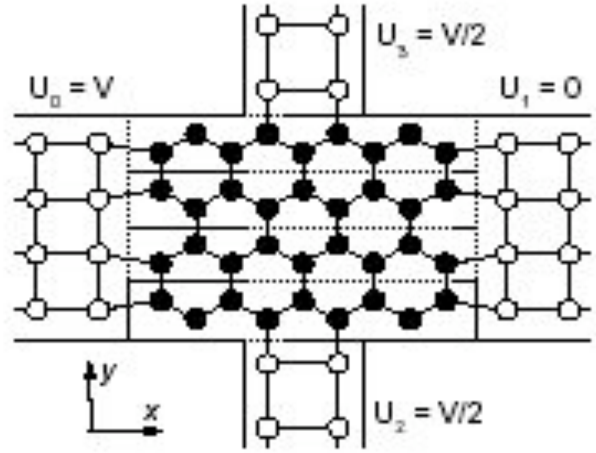
Starting state for QSHE models:

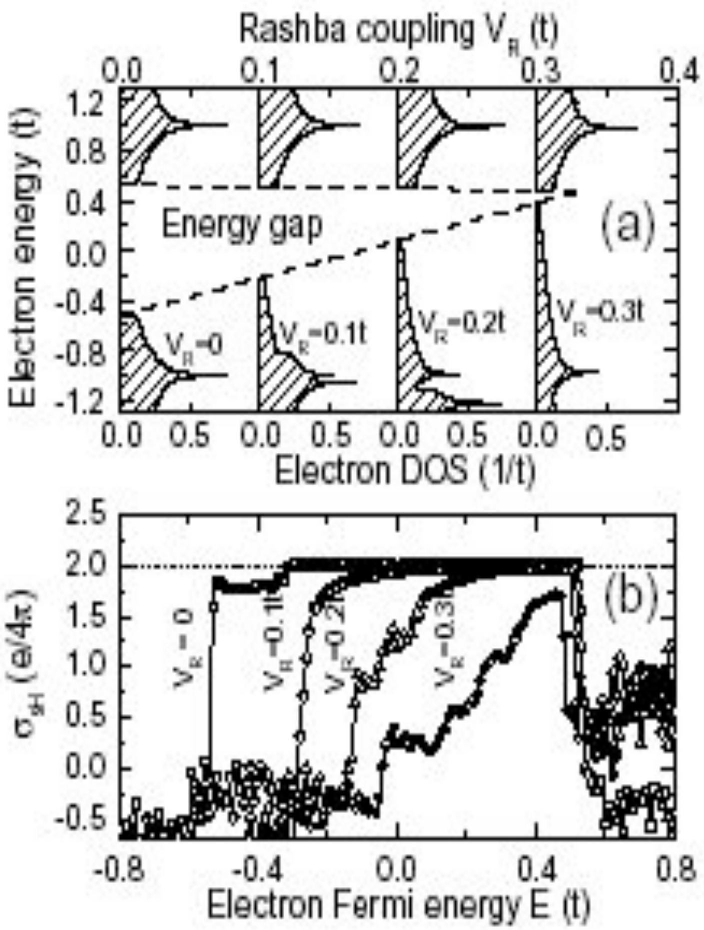


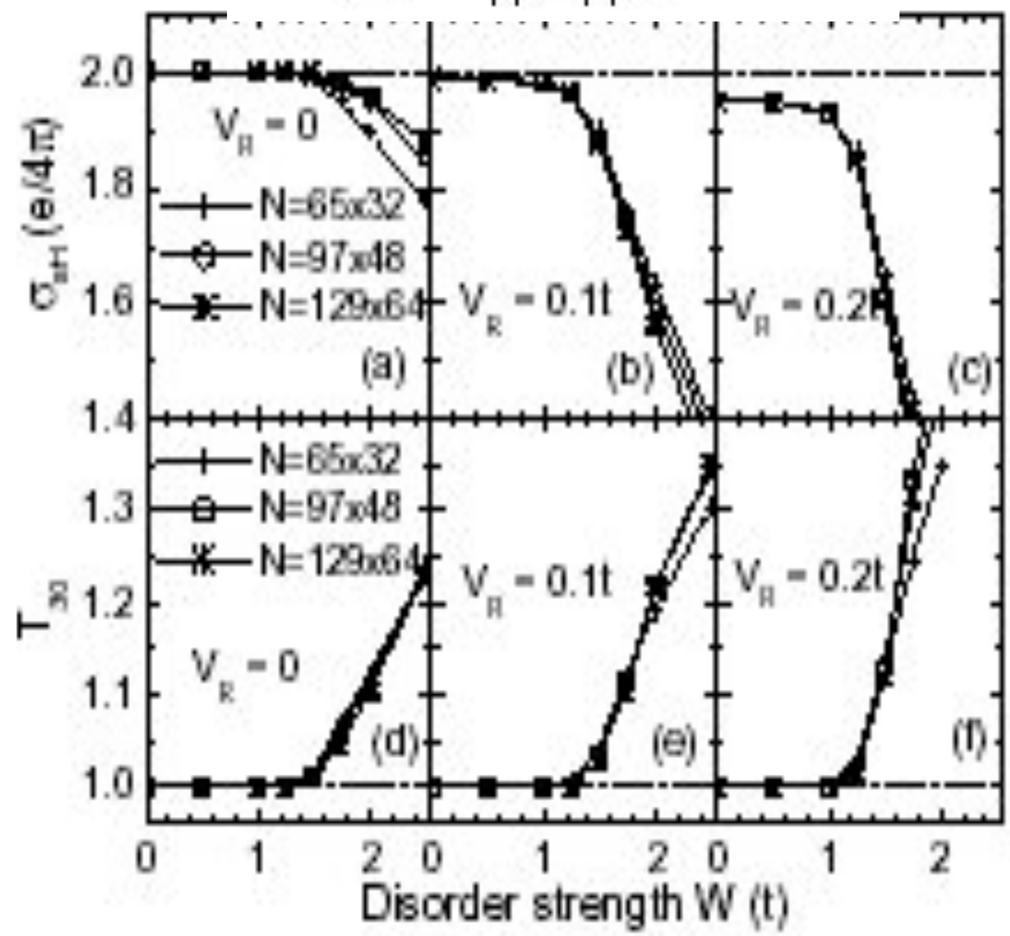
with Rashba SOC, coupled 2D system
is still topologically nontrivial

$$1+(-1)=?$$

LB form

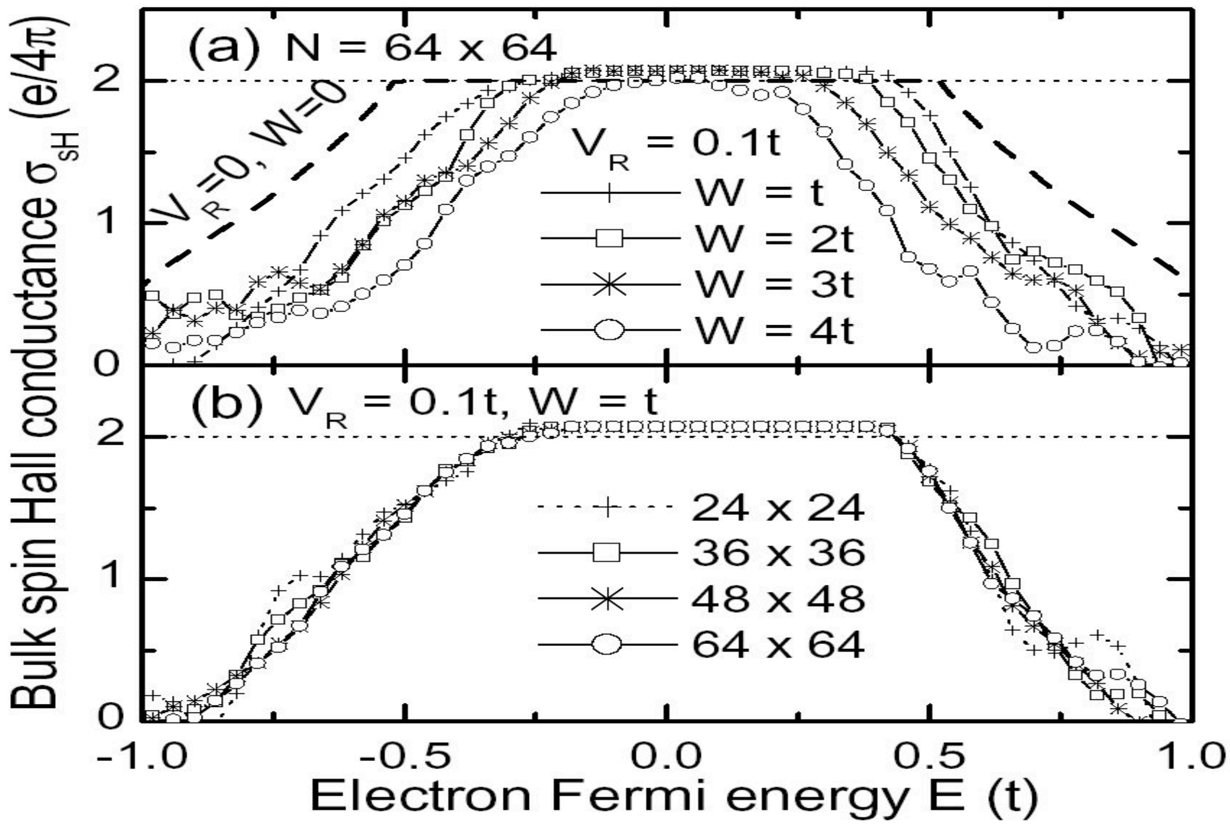

U_3 = V/2
U_0 = V
U_1 = 0
U_2 = V/2
y
x


Rashba coupling V_R (t)
0.0
0.1
0.2
0.3
0.4
Electron energy (t)
1.2
0.8
0.4
0.0
-0.4
-0.8
-1.2
Energy gap
(a)
V_R=0
V_R=0.1t
V_R=0.2t
V_R=0.3t
0.0 0.5 0.0 0.5 0.0 0.5 0.0 0.5
Electron DOS (1/t)
σ_sH (e/4π)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
(b)
V_R=0
V_R=0.1t
V_R=0.2t
V_R=0.3t
-0.8
-0.4
0.0
0.4
0.8
Electron Fermi energy E (t)


σ_sH (e/4π)
2.0
1.8
1.6
1.4
V_R = 0
N=65x32
N=97x48
N=129x64
V_R = 0.1t
V_R = 0.2t
(a)
(b)
(c)
T_30
1.3
1.2
1.1
1.0
N=65x32
N=97x48
N=129x64
V_R = 0
V_R = 0.1t
V_R = 0.2t
(d)
(e)
(f)
0 1 2 0 1 2 0 1 2
Disorder strength W (t)

Kubo formula for bulk SHC

$C^{SC} = C^{CS} = 2$



G. 4: Bulk SHC σ_{sH} calculated from the Kubo formula

robust and system size independent SHE
only appears as $C^{SC} = C_1 - C_2 = 2$ phase,
carried by two dissipationless edge states

Chern number

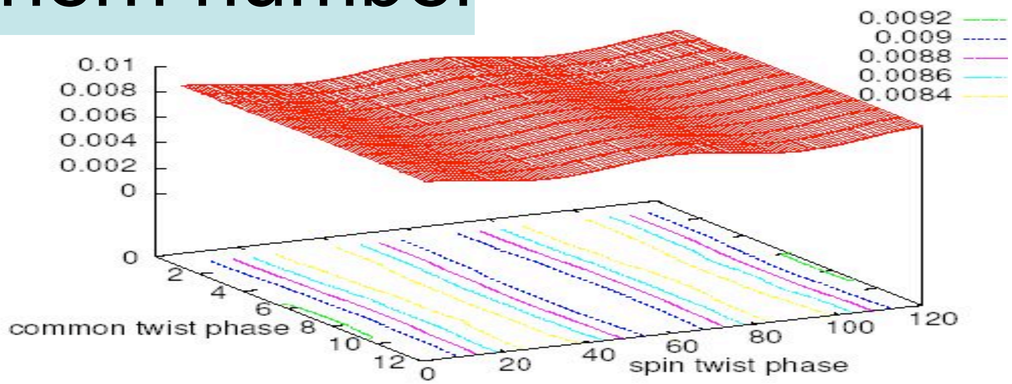


FIG. 1: Solid angle Ω_j as a function of two boundary phases (θ_x^s, θ_y^c), each θ unit cell is meshed into $N_{mesh} = 120 \times 12$ points for a pure system with $N_x = N_y = 60 \times 60$ at $V_{so} = 0.1$ and $V_R = 0.1$. Thus $\sum_{j=1}^{N_{mesh}} \Omega_j = 4\pi$, and $C_{sc} = 2$.

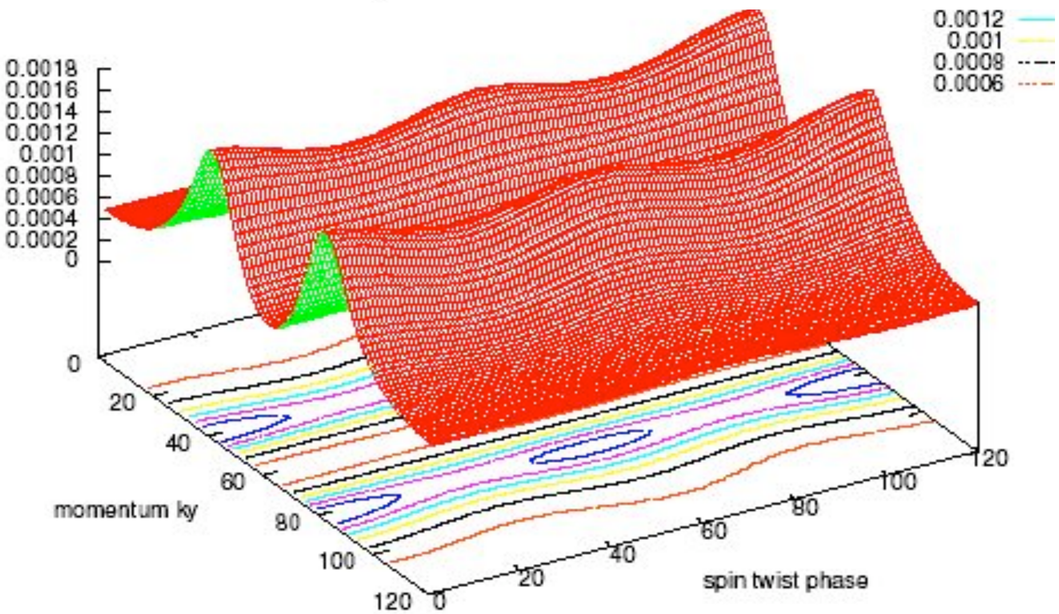
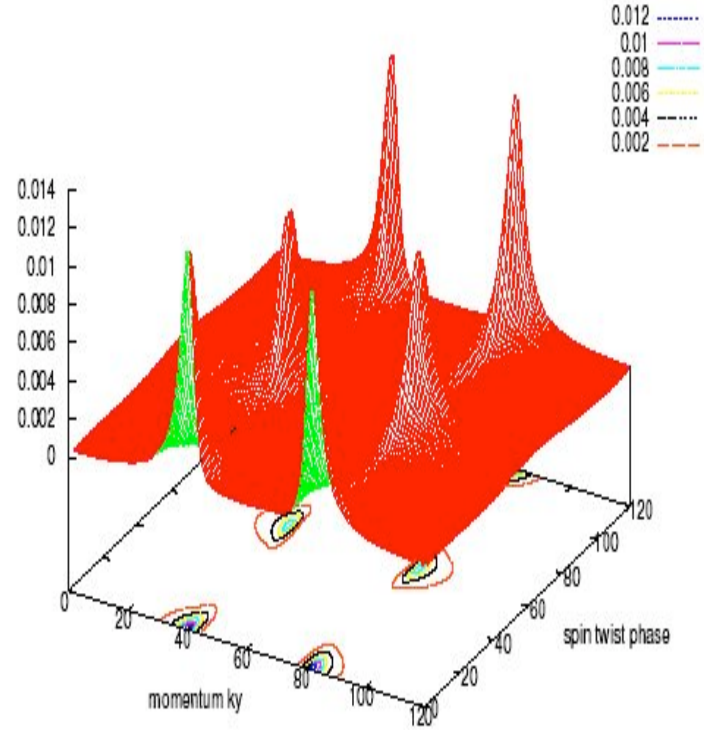
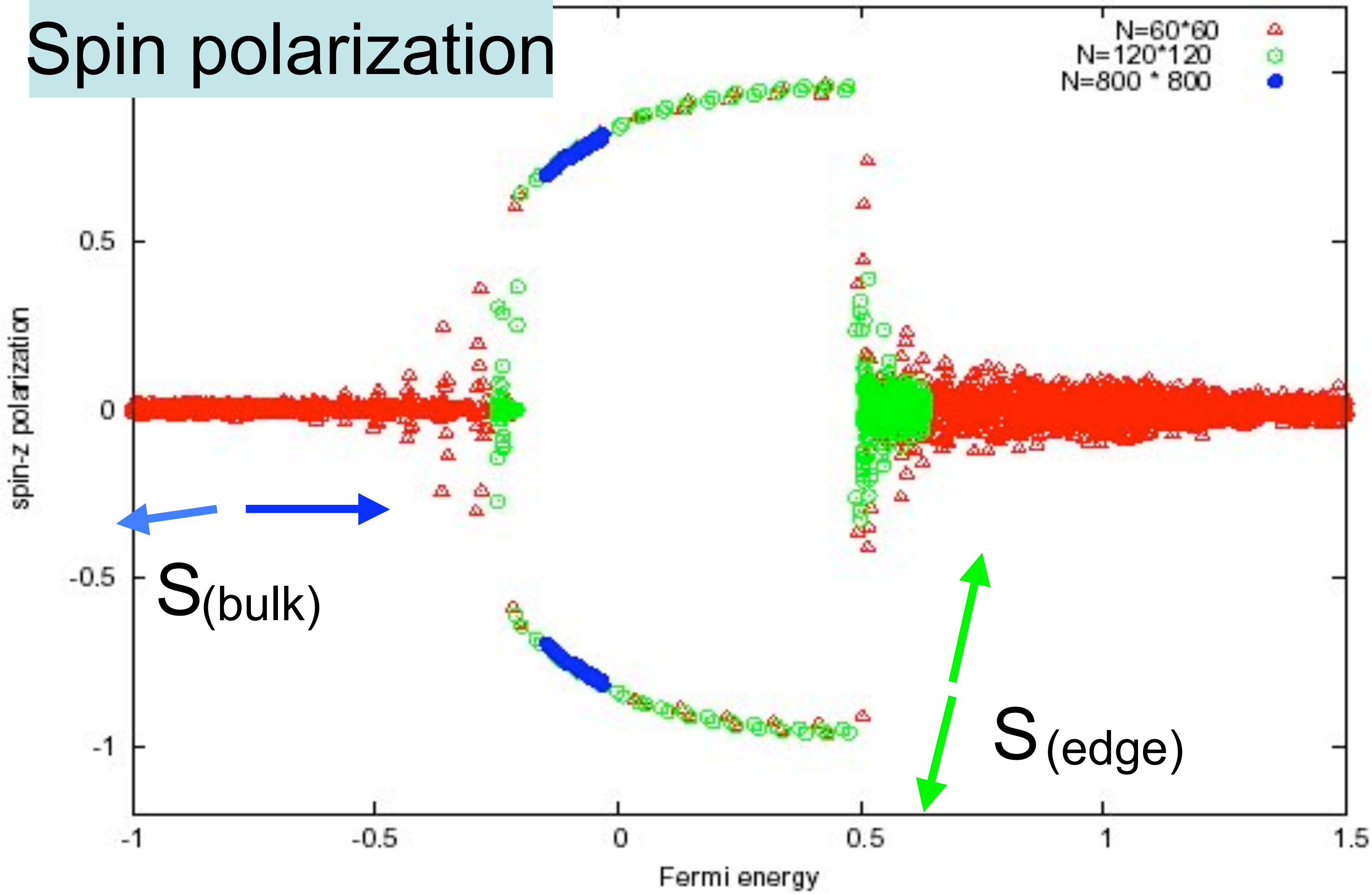


FIG. 2: Solid angle Ω_j as a function of spin twist and momentum k_y. Each $2\pi \times 2\pi$ unit cell is meshed into $N_{mesh} = 120 \times 120$ points for a pure system with $N_x = 60$ at $V_{so} = 0.1$ and $V_R = 0.1$. Thus $\sum_{j=1}^{N_{mesh}} \Omega_j = 4\pi$, and $C_{sc} = 2$.



Spin polarization



Spin 90% up state pumping to top, 90% down to bottom, SHC= 1.8 (e/4π)

Spin C in QSHE is charge-spin coupled (mixing) Chern number

Chern number matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ C^{SC}

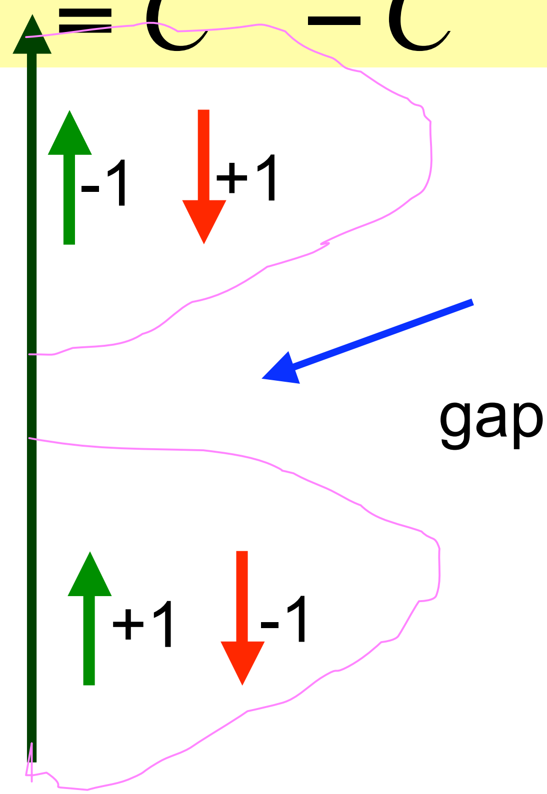
Charge twist $\theta_x^1 = \theta_x^2$

Spin twist $\theta_y^1 = -\theta_y^2$

$C^{CC}=0, \quad C^{SS}=0$

$$C^{SC} = C^{11} - C^{22} = 2$$

Density of states



QSHE to trivial state only happens

↑ -1 + ↑ +1 = 0

↓ -1 + ↓ +1 = 0